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Pharmaceutical supply chain specifics and inventory solutions for a hospital case

Peter Kelle [a,1], John Woosley [b,*], Helmut Schneider [a,2]

[a] Louisiana State University, ISDS Department, E. J. Ourso College of Business, Baton Rouge, LA 70803, United States
[b] Southeastern Louisiana University, Department of Marketing and Supply Chain Management, College of Business, Hammond, LA 70402, United States

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ABSTRACT

Pharmaceuticals represent a large portion of the costs in the healthcare industry due to the significant costs of these products and their storage and control requirements. In this work we discuss the pharmacy supply chain and current managerial practices in a case hospital, examine the often conflicting goals in decision making amongst the various stakeholders, and explore the managerial tradeoffs present at the operational, tactical, and strategic levels of decision making. We focus on the inventory management at a local storage unit within an individual Care Unit (CU). For the operational inventory decision we provide the reorder point and order up to level (called*min and max par levels*) that control the automated ordering system. These parameters are based on a near-optimal allocation policy of cycle stock and safety stock under storage space constraint. Tactical decision support focuses on the relevant tradeoffs amongst three key performance indicators: the expected number of daily refills, the service level, and the storage space utilization. We analyze the tradeoffs amongst the refill workload, the emergency workload, and the variety of drugs offered (called *formulary*). The resulting decision support tool facilitates improvements to the current management practices.

1. Introduction

A significant part of healthcare costs is the pharmaceutical component, which represented approximately 10% of annual healthcare expenditures in the United States and about $600 billion globally in 2009 [1]. Despite the size and importance of this industry around the world, especially in developed countries, the area of healthcare Supply Chain Management (SCM) and inventory management has been given relatively little attention. Several researchers have estimated that inventory investments in healthcare range between 10% and 18% of total revenues [2,3]. Any measures taken to control expenditures in this area can have substantial impacts on the overall efficiency of the organization.

Burns [4] examined the healthcare value chain. In addition, Pitta and Laric [5] provide a model of the healthcare value and supply chains. This supply chain is not linear or sequential in nature but closely follows the flow of information through the system. Almarsdóttir and Traulsen [6] also identify a number of reasons why pharmaceuticals deserve extraordinary consideration in controlling inventory, including the fundamental differences between medicines and other consumer products: medicines are developed, manufactured, and distributed according to strict regulatory requirements; medicines are most often selected by a physician for a specific patient and reimbursed in whole or in part by a third-party insurer or the state. These specific characteristics make the pharmaceutical industry a very powerful force in its own right, accounting for 15.4% of total health expenditure [6], and the increase of pharmaceutical costs is much higher than GDP growth.

This paper is targeting the decision support for the pharmacy supply chain of a local hospital. We focus on the inventory management at a local storage unit within an individual Care Unit (CU). There are a number of stakeholders in the healthcare supply network. The three key stakeholders in our hospital pharmacy case are the physicians, pharmacists, and the Group Purchasing Organization (GPO). One can identify several relevant conflicts amongst the stakeholders with respect to prescribed drugs. Physicians and pharmacists/pharmacy directors clash over medications offered by the hospital [7]. The basic conflict here revolves around the issue of product variety (called *formulary*) versus economies of scale. Physicians value their individual freedom of choice in selecting the medications that they feel best meets the specific needs of the patients under their care so they want a larger variety of drugs. Physicians are also influenced by drug manufacturers and sales representatives to add new drugs to the formulary as they appear on the market [8–10]. In contrast, the pharmacy directors are motivated to decrease the formulary to get better prices due to economy of scale and to decrease the

* Corresponding author. Tel.: +1 985 549 3973; fax: +1 985 549 5010.
E-mail addresses: qmkell@lsu.edu (P. Kelle), john.woosley@selu.edu (J. Woosley), hschnei@lsu.edu (H. Schneider).
[1] Tel.: +1 225 578 2509; fax: +1 225 578 2511.
[2] Tel.: +1 225 578 2516; fax: +1 225 578 2511.

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cost associated with holding, refilling, and administering a larger variety of drugs. A different divergence appears between hospital administration and the GPO on issues of product variety. Hospitals focus on negotiating the best prices for a wider selection of drugs. The GPO, on the other hand, strives to minimize the drug variety to decrease the purchasing cost from manufacturers and ensure an economic administration and supply cost of drugs.

The above conflicts are so complex that a single model on one decision level cannot grasp the different aspects appropriately. The decisions cannot be characterized simply as a hierarchical top-down or bottom-up decision sequence. In addition, the hospital management is in need of a simplified, more practical solution to aid in inventory management decisions. In our approach we start from the current conditions set by the top level decisions and seek the optimal decisions on the lower levels under the conditions provided by the upper level. Thereafter, the question is how the lower level decisions change if the upper level conditions change. The goal of this procedure is to provide an iterative top-down and bottom-up decision support that improves the overall system performance measured by the key performance indicators, the service level (emergency refill workload), the available space (depending on the variety of drugs—*formulary*), and the number of orders (*refill workload*) per day while keeping in mind the hospital's need for a simplified approach.

In our approach we provide optimization of the operational level decisions constrained by the tactical decision using a simplified model. The tactical decision support is considering the strategic framework. This way, besides the above strategic conflicts, we consider different tradeoffs on the operational and tactical decision level in the hospital pharmacy management. Some of these conflicting goals are common in inventory management, but there are some specifics for hospital pharmacies.

There is a stream of publications handling hospital supply problems with some similarities to our case. Michelon et al. [11] published a tabu search method to optimize the distribution of supplies in a hospital, but they are not dealing with inventory management. Dellaert and van de Poel [12] derived a simple inventory rule for joint ordering in a university hospital in The Netherlands, but the capacity constraints are disregarded. Banerjea-Brodeur et al. [13] looked at an application of a routing model to match the different care units to be visited by a laundry department in a hospital. Vendor managed inventory (VMI) has been applied in healthcare since the 1990s. An analysis and the hospital saving potentials of VMI are demonstrated in Kim [14]. Our case hospital is considering a possible VMI agreement with the GPO, but it is far from implementation. Meijboom and Obel [15] investigated supply chain coordination facing a pharmaceutical company with a multi-location and multi-stage operations structure. They concentrated on the organizational issues while our major goal is the quantitative support. Lapierre and Ruiz [16] developed modeling approaches for improving healthcare inventory management by examining the impact of scheduling decisions on the coordination of supply activities while recognizing inventory capacities. While this research includes capacity considerations, the focus is scheduling not inventory management.

We start our discussion with the case hospital characteristics and problem description in Section 2, then we develop OR models for the practical decision problems in Section 3. In Section 4 we discuss the applicability and managerial implications of our models in the operational, tactical, and strategic decisions for our hospital pharmacy case. We close the paper with conclusions and extensions.

2. The pharmacy supply and the major decisions of the case hospital pharmacy

The case hospital houses 763 licensed beds, has an average daily census of 500–550 patients, and employs over 950 medical staff members. The hospital applies *advanced technology* in controlling medication throughout the hospital. These *Pyxis MedStations* ® (called *local depot* in the rest of the paper) house the drugs needed for patient care in that Care Unit (CU), as well as, track every inventory transaction, prompt replenishment orders, generate necessary documentation, and facilitate the billing processes related to pharmaceutical treatments. While the technology of *Pyxis MedStations* ® allows for the automation of a number of tasks associated with this supply network, it is important to recognize that pharmaceutical inventory management is still a very labor-intensive process due to the number of local depots in the hospital (∼85) that need to be replenished, the large volume of drugs in each depot (250–500), and the pharmacy staff workload required during the restocking process. The staff must be supervised by highly educated pharmacists, a valuable and sometimes scarce resource for the hospital.

The demand for each drug is uncertain and frequent seasonal changes occur. In the drug supply a high service level is essential. In case of a shortage at a local depot an emergency delivery is necessary and this *emergency refill* is very costly and can be dangerous for the patient's healing process, so for these drugs a very high service level is crucial.

One major goal of the hospital pharmacy is to minimize the *number of refills* (how many different drugs need to be refilled for all the locations) per day. If the number of refills per day is very large, it cannot be done in two shifts. To have overtime or an extra shift is difficult and costly. The refills from the central depot (*Tallyst* ®) must be delivered within a day. Shortage in the central depot is rare and in that case the wholesaler (*GPO*) refills within a day.

The physical volume (the total space) of the *Pyxis* ® system is limited. It consists of drawers. Each drawer is subdivided by spacers that can be relocated so different-sized cubicles are constructed and assigned to each drug. Different drugs cannot be stored in the same cubicle. The total room for the cubicles is fixed.

The *Pyxis MedStations* ® allow the automation of the orders in each local depot for each drug. As the inventory level of a drug decreases to the reorder point (called min par level) an automatic order is triggered by the local depot. The time of the order is determined by the reorder point and the order quantity is determined by the order up to level (called max par level). The amount of the order is the difference between the order up to level and the inventory available at the time of order. The par levels can be selected and fixed for each drug and each depot separately. Thus the local depots use an ordering policy called (s, S) policy in the literature with reorder point (min par level $= s_i$) and order up to level (max par level $= S_i$) as control parameters for each drug, i.

The current control values (min and max par levels) suggested by the pharmacy supplier (GPO) are fixed day supply levels (denoted by $T_{\min}$ and $T_{\max}$, where T-day supply is the quantity that is used on the average during T days), and do not consider demand variability or storage space requirements. The typical min par level of 3-days and max par level of 10-days supply is suggested for a large proportion of the drugs. These values are corrected occasionally by the local pharmacists using so-called "experiences"; however, no modeling or optimization is involved in setting these control values.

This simplistic policy results in frequent shortages and emergency refills for some drugs and also a large number of regular refills putting an overload on the pharmacy staff. Based on experiences the pharmacists are frequently modifying the fixed day supply policy, but there is a need for appropriate decision support in how to select the reorder point (s_i) and order up to (S_i) control parameters for each drug, *i*. The primary, sometimes conflicting, operational goals of the hospital are to (a) reduce workloads (emergency and daily refills) and refilling costs at the local storage depots, (b) reduce holding costs, and (c) help in a pharmacy decision support system to analyze the effect of upper-level decisions on the operational costs.

Next we describe the different tradeoffs and conflicts in the decisions of the case hospital pharmacy on operational, tactical, and strategic levels. Each level has different objectives and tradeoffs, but they are strongly linked.

The *operational decision* occurs on the item level: how to set the min and max par levels for each (250–300) drug in each one of the 86 local depots? This decision determines the safety stock and cycle stock for each item. The safety stock is the average buffer inventory held to protect against stockout in case if a larger than average demand occurs during the lead time; it is the difference between the reorder point, and the expected lead time demand. The cycle stock is the average inventory above the safety (buffer) inventory and it determines the frequency of the orders for that item. The safety stock determines the service level and the frequency of emergency refills when shortage occurs.

– In setting the reorder point (min par level, s_i) there is a major tradeoff:

- a high availability per item is required and a high *emergency refill cost* occurs which requires the provision of a *high service* level;
- needs additional *buffer inventory* that takes space from cycle stock and higher *refilling* cost arise.

– In setting the order up to level (max par level, S_i) the major tradeoff:

- *less frequent orders* decrease the *refilling* cost;
- *limited space* for inventory in a local depot.

The operational decision is constrained by the total available volume that is decided on a tactical level. There is also a connection between safety and cycle stock (s_i and S_i) for each drug, *i*, so it necessitates the joint consideration of s_i and S_i.

Additionally, there is a tradeoff among the different drugs in a local depot because the total space is constrained by the tactical decision. Thus, the *tactical decision* problem occurs at the multi-item level: how to allocate the space (by flexible drawer dividers) among the 250–300 drugs in each local depot? What is the best allocation strategy for safety and cycle stock? How to set the s_i and S_i control parameters according to the allocation strategy? These questions indicate various tradeoffs to consider among the different drugs in a local depot. Further requirements include:

- the limited space is subdivided into separate areas (cubicles) for each drug,
- the cubicles cannot be shared among drugs, and
- the max inventory level (S_i) must fit into the assigned cubicle.

In the *strategic level*, there are tradeoffs among the three key performance indicators:

- the service level (that influences the emergency refill workload),
- the available space (that depends on the variety of drugs—*formulary*), and
- the number of orders (that influences the *refill workload*) per day.

The major strategic decision problems include: what kind of tradeoffs are among refill workload, emergency workload, and variety of drugs offered (*formulary*)? What are their connections to the three key performance indicators: number of orders, the service level, and the available space? How can the effects of formulary change be estimated and used in the pharmaceutical directors' negotiations with physicians and GPOs?

In the next section, we develop OR models for the above described practical decision problems and in Section 4 we discuss the applicability and managerial implications of our models for the operational, tactical, and strategic decisions in the hospital pharmacy. We use the example of a particular *Pyxis MedStation* ® of the case hospital for illustration. All transactions (demand, refill, inventory position) and control parameters (s_i, S_i) for two years are available in Excel files. For this study we selected 70 drugs with the highest usage rate out of the 214 drugs. The slow moving items pose the most challenge. However, from a practical point of view, the contribution of the slow movers to the total value and cost (holding, refilling, and emergency supply cost) is almost negligible. After consultations with the pharmacy representatives of the hospital and GPO, we agreed in selecting 70 drugs for the pilot application of our methodology. These items make up 71% of the total usage and around 70% of the total volume of the *Pyxis* ®. This sample was used to illustrate the relevant managerial tradeoffs. In addition to the managerial insight, a number of quantitative models are utilized to determine the optimal inventory control parameters given the goals of the individual models and the various constraints.

3. OR modeling and iterative solutions for the hospital inventory management

In this section first we formulate the exact model for the operational level decision and check the relevant literature for this OR modeling area. It turns out that the analytical solution of this model is not feasible because of the complexity and dimension of the problem, so we formulate two other models as approximations that capture the essence of the practical decision problems and that can be efficiently solved using iterative procedures.

3.1. The general multi-product (s, S) model with space constraint and its approximation

The main goal of the hospital pharmacy on the operational level is to find the optimal values of the *decision variables*

s_i: the reorder point (the min par level) and

S_i: the reorder level (max par level or order up to level) for each drug i ($i = 1$ to n) to minimize the total expected *refilling* (ordering), inventory holding and shortage cost under volume constraint of the local depots. The total space is subdivided into separated and dedicated storage areas for each drug, and they must be large enough to hold the max par level, S_i, for each drug. The cost factors for item i are

K_i: cost of a refill (order),

$h_i = rc_i$: holding cost for a drug that is proportional with the value of the drug, c_i, and the holding cost rate, denoted by r,

p_i: shortage cost of an emergency refill that is independent of the size of the shortage.

The optimization problem of selecting the s_i, S_i decision variables can formally expressed in

Model 1:

$$\text{Min} \sum [K_i N_i(s_i, S_i) + h_i H_i(s_i, S_i) + p_i P_i(s_i, S_i)] \tag{1}$$

$$\text{s.t.} \quad \sum (v_i S_i) \le M' \tag{2}$$

with the notation for each item

$N_i(s_i, S_i)$ = the expected number of orders per period,
$H_i(s_i, S_i)$ = the expected average inventory per period,
$P_i(s_i, S_i)$ = the probability of shortage per period,
v_i = volume requirement for a unit,
M' = total volume of the space available for the n items.

There is a stream of publications handling similar multi-item constrained inventory problems with no reference to hospital management. The publications closest to our situation are listed next.

For an (s, S) inventory policy, Scarf [17] expressed the above expected values, $N_i(s_i, S_i)$ and $H_i(s_i, S_i)$, using renewal functions. The probability of a shortage in a period has been derived by Schneider et al. [18] using the steady state distribution of inventory on hand plus on order published in Iglehart [19]. Since these expressions use the renewal equation, we can get the exact solution only for the single-item case with specific demand distributions. A further problem is, as it was shown by Wagner et al. [20], that the cost function, even for a single item, is not convex. Thus, following the traditional constrained optimization solution for Model 1 using the derivatives of the Lagrangian with respect to the parameters s_i and S_i, setting them equal to zero and solving the equations will not necessarily provide the optimum. However, the Roberts [21] approximation based on the first derivatives of the Lagrangian produces a nearly optimal solution for a single item cost function. Schneider and Rinks [22] suggested an approximate solution for a constrained multi-item problem, similar to Model 1, based on the Roberts [21] approximation, and employed a search algorithm to verify the approximate optimality of the derived policy for examples up to 10 items. This procedure can also be applied as a benchmark in our case but besides the multi-tier iterative search it requires simulation to estimate the approximation error. So it is too cumbersome for our practical application having many items (70 and up) and frequent recalculations needed for changing data.

Ordering policies for multi-item inventory systems subject to multiple resource constraints were published in Güder and Zydiak [23] considering deterministic demand. A stochastic multi-item constrained model is discussed by Beyer et al. [24] but only for the case of base stock policy. Ohno and Ishigaki [25] examined a continuous review inventory system for multiple items with compound Poisson demands. Here the joint ordering was targeted but no space or budget constraints were considered. Minner and Silver [26] analyzed the stochastic demand, continuous review lot-size coordination problem without safety stock consideration. In a recent paper, Minner and Silver [27] analyze a replenishment decision problem where each replenishment has an associated setup cost and inventories are subject to holding costs. A second aspect of their analysis is that demands are random, which they modeled by a compound-Poisson demand process. This further complicates the analysis because safety stocks and cycle stocks share the limited warehouse space. In summary, the published papers handle several aspects of our problem; however, they fail to consider jointly the challenge of multi-item and joint constraint with demand uncertainty. An exception is the extension in Minner and Silver's [27] paper, but this model is too complex and time-consuming for our practical application.

The managerial problem is that the shortage cost factors are very difficult to provide. The technical problem is that the large number of items (n = 250–300 per local depot), the nonlinearity and stochastic demand make the solution challenging and time consuming. The changing composition of the drugs stored (*formulary*) and the dynamic demand characteristics (like flu season, changes in drug popularity) call for a fast solution. Two simplified approximate models and their simple iterative solutions are summarized in the next section. The solutions enable using Excel spreadsheets that are familiar and easy to interpret by the pharmacists controlling the system. As we show in Section 4, the applied simple numerical solutions are also advantageous to provide an efficient decision support in managerial tradeoffs and in allocation strategies for safety and cycle stock.

The notations and basic models are presented here; however, more details on the solution procedures are available in Woosley [28] and are summarized in Appendix B. Also, a short description of the results is published in the Proceedings of the AMCIS conference [29].

3.2. Optimal allocation based on ordering and holding costs

Due to the difficulty of quantifying the shortage cost factor we follow the common practice and consider a service level constraint instead of shortage cost in the next two models. Since the shortage means an *emergency refill* in our case with a fixed cost (depending only on the number of shortage occasions per day and not on the amount of the shortage) it is appropriate to consider the so-called α service level which is the chance that there is no shortage in an arbitrary period (day). Using the α service level constraint, Model 1 can be modified and simplified into the following form.

Minimize the total refills (ordering) cost plus the inventory holding cost for a local depot, subject to the constraints:

- the service level (chance of no shortage) for drug i is at least α_i, and
- the total space needed for the maximum possible inventory level for all drugs is not more than the available total space of the *local depot* (M').

Using the notation from the previous section and the same decision variables, s_i, S_i, we can formulate

Model 2:

$$\text{Min} \sum [K_i N_i(s_i, S_i) + h_i H_i(s_i, S_i)] \tag{3}$$

s.t.

$$\text{Prob. } (\textit{shortage for drug } i) \leq 1 - \alpha_i, \quad (i = 1 \text{ to } n) \tag{4}$$

$$\sum (v_i S_i) \leq M'. \tag{5}$$

The solution of Model 2 is still quite complex because of the large number of variables, the nonlinearity and the stochastic constraints. We consider the following notation and approximations

L = lead time (in our practical case it is one period),
$D_i = S_i - s_i$,
Q_i = expected order quantity,
d_i = expected demand per period,
σ_i = standard deviation of the demand per period,
u_i = *undershoot quantity*, the expected inventory position below s_i when an order is placed. It can be approximated by the first two moments of the demand per period in the form (see Schneider [30])

$$u_i \approx u_i(d_i, \sigma_i) = (d_i^2 + \sigma_i^2)/2d_i. \tag{6}$$

Using this approximation, we can express the expected order quantity as a function of d_i and σ_i:

$$Q_i \approx Q_i(d_i, \sigma_i) = D_i + (d_i^2 + \sigma_i^2)/2d_i. \tag{7}$$

Thus, we have the following simple approximate expressions as functions of d_i and σ_i:

$$N_i(s_i, S_i) \approx d_i / Q_i(d_i, \sigma_i)$$
$$H_i(s_i, S_i) \approx s_i - u_i(d_i, \sigma_i) - Ld_i + Q_i(d_i, \sigma_i)/2.$$

To simplify the solution, we consider the two types of constraints, service level (4) and space (5) constraints, separately and also the two sets of decision variables (s_i and S_i) separately and solve the optimization Model 2 iteratively. This method is very straightforward, intuitive and sheds light on the structure of the optimal allocation of the safety stock and cycle stock separately.

In the first iteration step we find the $s_i^{(1)}$ values that provide the required service level, α_i, for each item. Since the exact solution is very cumbersome (see Schneider [31]), we use the simple Power Approximation formula derived in Schneider [31] that provides a good accuracy according to our simulation experiments:

$$s_i = d_i(L_i+1) + p(y_i)\sigma_{i,L_i+1} - \delta(\sigma_i{}^2/d_i - 1) \\ \times(-1.95269 + 6.39059y_i)/(1 + 21.17036y_i) \tag{8}$$

with $\delta(x) = \max(x, 0)$ and $p(y_i)$ being a rational function of y_i (see Appendix B), where

$$y_i = (1-\alpha)Q_i/\surd(\sigma^2_{i,L_i+1}). \tag{9}$$

Thus, s_i depends on the service level α and also on Q_i.

$Q_i^{(0)}$ is initially selected using the economic order quantity formula. However, the resulting $S_i = S_i^{(0)}$ values may violate the space constraint (5). In this case we need to solve the sub-problem of minimizing the ordering and holding cost (3) under the space constraint (5), where for fixed $s_i = s_i^{(1)}$ values (5) is equivalent to

$$\sum(v_iQ_i) \le M' - \sum[v_i(s_i^{(1)} - u_i)] = M_1. \tag{10}$$

This deterministic constrained optimization is solved for Q_i using the convergent iterative solution procedure suggested by Ziegler [32]. This embedded iteration procedure provides the near-optimal value of the order quantities, $Q_i = Q_i^{(1)}$. For the new $Q_i^{(1)}$ values $s_i = s_i^{(2)}$ must be recalculated applying (8) and (9). Thus the solution procedure employs two embedded iterative procedures for the optimization of Model 2. The iterations are continued until convergence. According to formula (9), there is a minor effect of Q_i on s_i, thus the convergence is very rapid; in our experience two or three iterations are sufficient to get within 0.1% range.

The control parameters for item i ($i = 1$ to n) provided by the above procedure are: min par level, s_i, of the last iteration; max par level, S_i, is resulting from $S_i = s_i + D_i$. A summary of the procedure and its convergence is in Appendix B. For more details on the procedure and the convergence of the iterations, see Woosley [28].

3.3. Optimal allocation based on ordering cost

To resolve the high priority problem of the pharmacy director in managing the work force requirement we formulated a simplified optimization model, a specific case of Model 2. In this specific case we concentrate on the key goal of minimizing the total number of expected refills (orders) per day for a local depot, subject to the same service level and space constraints as in Model 2. The explicit decision variables are Q_i ($i = 1, \ldots, n$), but they are implicitly set by s_i and S_i:

Model 3:

$$\text{Min} \sum(d_i/Q_i) \tag{11}$$

s.t.

$$\text{Prob. }(\textit{shortage for drug } i) \le 1 - \alpha_i, \quad (i = 1 \text{ to } n) \tag{12}$$

$$\sum(v_iS_i) \le M'. \tag{13}$$

We apply an iterative solution similar to Model 2, but finding the optimal Q_i values, instead of the Ziegler iterative solution, we formulate the sub-problem of minimizing the number of orders per day under the remaining total free storage space for cycle stock with fixed reorder points, $s_i^{(1)}$ and the resulting $M = M_1$ expressed in (10).

The importance of the simplification of Model 2 in the form of Model 3 is that it provides an explicit analytic result for the optimal space allocation:

Proposition 1. *The optimal space allocation for Q_i to minimize the expected number of orders is proportional with the square root of the demand over volume rate, and the optimal solution of (11) s.t. (13) is*

$$Q_i = M/W^*\surd(d_i/v_i) \tag{14}$$

with notation

$$W = \sum\surd(v_id_i). \tag{15}$$

Proof follows from the Kuhn–Tucker conditions for the Lagrange function (see the details of the proof in Appendix A).

4. Analysis and managerial interpretation

This section identifies the manner in which the simplified approach of Models 2 and 3 supports decision making at multiple levels. The resulting decision support tool serves as a model for managers allowing them to examine changes in the formulary or item usage, to evaluate options for modifying the control parameters, and to choose the par values that best fit hospital and management criteria. First, the operational decisions are discussed. Second, the tactical considerations are presented along with the utility of this decision support tool in analyzing managerial tradeoffs. Third, a breakdown of the strategic implications is provided.

For this project we developed an application and simulation system using MS Excel. The most important reason why this software was chosen is that the pharmacy administrators are familiar with this application, and it is also used extensively by the staff to manipulate demand and usage data in Excel. They want to maintain control over the application by user interaction and modifications. The IT-based solution currently employed by the hospital to monitor and control pharmaceutical inventory works quite well with this software.

4.1. Operational decision support

At this level of decision making, the focus is on the management of individual items. The high service level requirement dictates a high reorder point, s_i, be used to maintain healthcare standards and to avoid expensive emergency refills. Additionally, the order up to level, S_i, for each item must be reduced to accommodate the other products in the Pyxis ® given the space constraints of the storage unit. The reduced space for cycle stock generates a need for additional daily refills, which results in higher refilling costs and workloads for the pharmacy. Any changes at the item level can impact the operations and workloads associated with the local depot. Managers needed help in evaluating their current practices and in improving these procedures.

Cost comparison of allocation strategies

Here comparisons are made between the current hospital inventory policy (HM) and the alternative approaches of Models 2 and 3. Model 2 is designed to find the optimal allocation of space across all items being considered by minimizing the sum of holding and ordering costs. Model 3 was established to satisfy the desire of the hospital pharmacy to minimize the total number of expected refills (orders) per day for a *Pyxis* ®, subject to the service level and storage capacity constraints. Fig. 1 illustrates the costs as calculated using actual product prices and demand data in the simulations.

As shown in Fig. 1, the holding cost, refill cost, and total cost of refill and holding are substantially different for the three policies. Holding costs are nearly identical for HM and Model 2 with the Model 3 policy resulting in holding costs around 40% higher than the other strategies. Refill costs are substantially higher for the HM policy than for either Model 2 or Model 3 with the refilling cost being 71.1% and 83.9% higher respectively. These higher refill costs result in a much higher total cost for the HM policy and is evidenced by a 52.8% higher cost than Model 2 and a 36.4% higher cost than Model 3. These differences are summarized in Table 1.

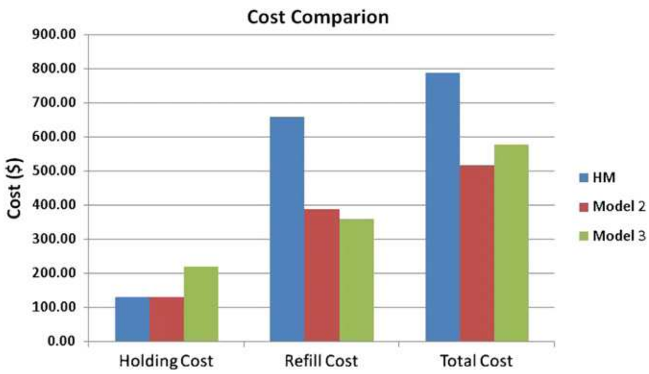

Fig. 1. Cost comparison for allocation methods.

Table 1
Percent differences in model costs to hospital policy costs.

	Percent difference		
	Holding cost (%)	Refill cost (%)	Total cost (%)
Model 2	1.3	−71.1	−52.8
Model 3	41.4	−83.9	−36.4

Table 2
Policy comparison with constant space utilization (M).

Inventory control policy	Avg. service level (%)	Service level range (%)	Avg. daily refills
GPO suggested (GPO)	99	7.15	5.56
Hospital modified (HM)	99	5.14	5.74
Our model 3 (OM)	99	0.44	4.70

4.2. Tactical decision support

In this section discussion shifts to the tactical implications. Specifically, we compare our Model 3 (OM), which was designed to address the explicit needs of the pharmacy, to the GPO Suggested (GPO) and Hospital Modified (HM) policies. As we demonstrate, the developed application allows managers to quickly evaluate the performance of numerous policies on our key performance indicators of service level, space utilization, and average daily refills.

Table 2 is an example of the potential comparisons available to management. Here we hold one performance measure constant in order to examine the effects on other measures by setting the space availability (M') equal to the value that is currently used by the HM policy. As shown above, all three policies perform very well on the average service level criteria with the OM policy outperforming the others in the consistency of the service level, measured by the service level range (max–min service level) achieved across all items in the *Pyxis* ® unit. With respect to the number of refills required per day, the OM method outperforms both of the other approaches. This is but a single example of the possible comparisons.

A primary result from a practical point of view is that the optimal allocation strategy of the space for the order quantities (cycle stock) of the items is proportional with the *square root of the demand over space rate*. The decision support tool allows for quick comparisons with different allocation schemes in reducing the total number of expected orders (refills) per day. We have compared five different allocation strategies for illustration. Table 3 summarizes the average percent increases in the total number of orders needed by applying an allocation rule that is different from the optimal one (that allocates the total space proportionally with the *square root of the demand over space rate*). The four other rules we considered are allocating the total space proportionally to the item's demand rate, unit space requirement, total space requirement, and demand over space rate.

Table 3
Comparison of the different space allocation strategies.

The expected refill size Q_i is proportional to	Percent increase in total # of orders per day (%)
SQRT(d_i/v_i)—the optimal	0
Demand rate, d_i	10
Unit space requirement, v_i	400
$d_i^* v_i$	177
d_i/v_i	38

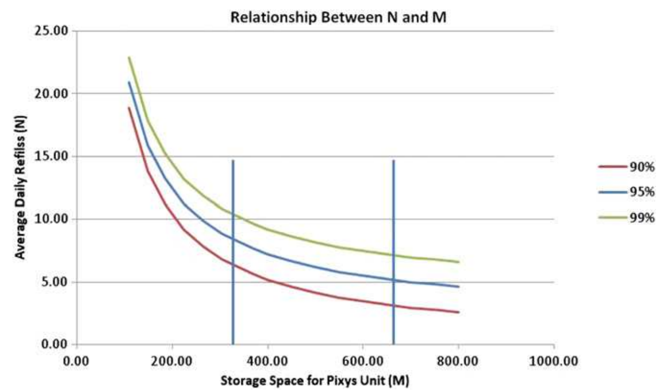

Fig. 2. Performance indicator relationships and tradeoffs.

4.3. Strategic decision support

Thus far, this research focused on decision support in pharmaceutical inventory management within the hospital; however, there are a number of additional stakeholders interested in the product formulary. According to pharmacy managers at the case hospital, efforts are made to accommodate doctors and to offer greater variety, but product and operational costs are also a concern. Unfortunately, the cost impacts of changes in the product formulary are not documented and remain essentially unknown to these parties. We estimate the influence of these costs using our models to provide valuable information for the hospital when negotiating with doctors or the GPO on issues related to the formulary and pharmaceutical inventory management.

A significant benefit of using this decision support tool is the allowance of quick evaluations of the managerial tradeoffs. When suggestions are made that impact the number of workers or work shifts available in the pharmacy or when changes to the formulary are discussed, the pharmacy director can quickly analyze the issues based on our results and identify the impacts on service levels, workloads, and operating costs.

Model 3 provides a simple and efficient strategic decision support tool to analyze the tradeoffs among the three key performance indicators:

- the service level (emergency refill workload),
- the available space (depending on the variety of drugs—*formulary*), and
- the number of orders (*refill workload*) per day.

Here we examine the impacts of changing formulary, available space, and worker capacity on our primary performance measures. For a fixed service level the tradeoff curve has a hyperbola shape that is shifted to the right (higher space requirement) with increasing service level requirement. This property allows a simple visual tradeoff analysis, and a simple example is illustrated next.

Fig. 2 illustrates the relationship between available space for cycle stock, M, and average daily refills, N, at a given service level. As space availability increases, there is a decrease in the average number of daily refills. If we change the service level at the same

Table 4
Impact of adding or subtracting items to local formulary.

Changes in formulary	Expected number of daily refills (N)	% Difference
Baseline (70 items)	3.58	–
20% Additional high usage items	5.31	48.32%
20% Additional low usage items	4.08	13.97%
20% Fewer items	2.70	−24.58%

space utilization, we can see the expected increase in the refill requirements.

Formulary changes

The available space is directly connected to the *formulary*. Even if the total demand for the drugs doesn't increase, a bigger number of items requires a larger amount of safety stocks. As such, we investigate the effects of adding or removing items from a *Pyxis* ®. The inclusion of extra items means that less space is available at the item level for cycle stock in the *Pyxis* ®. Reducing the number of items stored in the *Pyxis* ® will have the opposite effect as more space becomes available for cycle stock for individual items in the machine.

Table 4 provides a simple demonstration of the formulary changes and the resulting influence on pharmacy workload. Again, we start with the sample of 70 drugs found in a particular *Pyxis MedStation* ® and modify the product mix to include or omit 20% of items. To establish a reasonable idea of the impact such changes to the formulary will have on the number of expected refills (N), we consider both high usage and low demanded items for the situation where items are added. In contrast, we only consider the low usage items for circumstances where items are removed from the *Pyxis* ® given the small likelihood of high usage items being eliminated from the formulary. This allows pharmacy directors to gain valuable insight into the cost implications of such formulary changes.

As shown above in Table 4, increasing or decreasing the number of items in the *Pyxis* ® will influence the number of expected daily refills needed for the machine. Regardless of the daily demand, adding items will raise the number of refills. First, consider the situation where low usage items are added to the local depot. In this case one can observe approximately a 14% increase in the expected daily refills at the *Pyxis* ®. Considering Organization *XYZ* has over 85 *PyxisMedStations* ®, this increased workload poses significant problems from both a cost and worker capacity perspective. Second, we examine the effect of adding high usage items with daily demand values similar to the top 20% of items currently housed in the *Pyxis* ® unit. This results in an increase in expected daily refills of more than 48%. On the other hand, we noticed almost a 25% drop in refills when the bottom 20% of drugs were removed. This demonstrates the importance of both reasonably restricting the product formulary and evaluating the items in the *Pyxis* ® on a frequent basis for possible reductions. Again, these results provide decision support which is needed to support management.

Worker capacity

In this context the number of refills that can be accomplished by pharmacists and technicians during an 8-h shift is relatively fixed. As a result, any increases in the number of refills required at the CUs can only be satisfied through incremental increases in the number of work shifts. As the service level increases, one expects that smaller numbers of items require refills on a daily basis to prevent expensive *stockout* occasions. On the other hand, higher service level, and the resulting higher safety stock, takes away space from cycle stock increasing the expected number of regular refills per day. Table 5 provides a simple illustration of these relationships expressing the number of refills that can be achieved at different target service levels using a single shift per day or two shifts.

Table 5
Incremental increases to worker capacity.

Shifts	Refills	
	95% SL	99% SL
1	10	15
2	20	30

Knowing this relationship and the expected number of refills required at any given *Pyxis* ® allows managers to better schedule human resources and ensure that the workload does not exceed worker capacity. By optimizing the allocation of space within the *Pyxis* ® such that the number of expected refills is minimized, the hospital pharmacy has the ability to keep workloads at a controllable level and track changes over time. Control charts can be used to detect trends indicating workloads are approaching the pharmacy capacity limits and indicate a need to reevaluate pharmaceutical inventory control values and possibly adjust the number of technicians or work shifts refilling the machines.

5. Conclusion and extensions

The primary goal of our project was to improve the current pharmacy *inventory management* policy. This is the first step toward improving the supply chain by revealing the tradeoffs and providing quantitative tools for negotiations among the stakeholders.

For the *operational inventory decision* we provide the*min and max par levels* (reorder point and order up to level) that control the automated ordering system. These parameters are based on a near-optimal allocation policy of cycle stock and safety stock under storage space constraint. As we proved, the suggested selection of the control parameters s_i and S_i

- provides consistent service level,
- allocates the safety stock to decrease the workload of emergency refilling, and
- allocates the cycle stock space decreasing the workload of daily refilling.

Our research has illustrated that the implementation and use of Models 2 and 3 as a healthcare DSS for pharmaceutical inventory control purposes can reduce inventory related pharmaceutical expenditures up to 70%–80%.

For the *tactical and strategic decisions*, we demonstrated that our models can be applied as a simple visual decision support tool to analyze the tradeoffs among the refill workload, the emergency workload, and the variety of drugs offered (called *formulary*). We reveal the relationship of these tradeoffs to the three key performance indicators at a local care unit: the expected number of daily refills, the service level, and the storage space utilization.

The quantitative models and Excel spreadsheets prepared are providing further *managerial support* for pharmacists to analyze tradeoffs between service level and cycle stock, test implications on cost savings and effects of parameters, and test a variety of parameter settings. Furthermore, we are providing tools for managing *worker capacity*. Based on refill capacity requirements, pharmacy directors can manage the addition of pharmacists or overtime and the addition of another shift of pharmacy technicians responsible for filling the *Pyxis MedStations* ® around the hospital.

There are plenty of extension possibilities. Some of them are technical, quantitative in nature, while others are managerial extensions. The main technical improvements include examining the effect of special demand types and the consideration of multiple objectives in which Multiple Criteria Decision Making

(MCDM) tools may be applied. Specifically, more research is necessary to examine the effects of low volume demand, "lumpy" demand, and auto-correlated demand when subsequent days have a high demand applying a treatment followed by a longer period without demand. The correlation between items can also be an important factor to investigate in the future.

Some of the most important managerial extensions include examining the effect of demand uncertainties on workload. Specifically, it is important to determine the uncertainty of the workload by analytical estimation or by simulation and to determine if there is enough worker capacity with certain reliability. If not, the question becomes how many workers (or additional shifts) to add to satisfy that excess workload?

Another significant extension of this research is the opportunity to provide more specified quantitative support for negotiations between pharmacy administrators and medical doctors by analyzing the effect of extending the formulary and the cost vs. product variety tradeoff. The current research effort and these recommended extensions will provide further insight into stakeholder preferences and acceptable tradeoffs.

Appendix A

Proof of Proposition 1. Considering the Kuhn–Tucker conditions for the Lagrange function of Model 2

$$L(\underline{\mathbf{x}}, \lambda) = \sum (d_i/x_i) + \lambda \left[\sum (v_i x_i) - M\right] \tag{A.1}$$

for the optimal x_i^* and λ^*

$$x_i^* \partial L(x^*, \lambda^*)/\partial x_i = -d_i/x_i + \lambda^* v_i x_i = 0$$

$$\text{for } i = 1, \ldots, n \tag{A.2}$$

so if λ^* is known from (A.2) we get

$$x_i^* = \surd(d_i/\lambda^* v_i). \tag{A.3}$$

On the other hand, if any of x_i^* is known

$$\lambda^* = d_i/(v_i {x^*_i}^2). \tag{A.4}$$

Since for the optimal solution the capacity constraint is active

$$\sum (v_i x^*_i) = M. \tag{A.5}$$

Substituting (A.3) into Eq. (A.5) and using the notation

$$W = \sum \surd(v_i d_i) \tag{A.6}$$

the optimal value of λ^* can be expressed

$$\lambda^* = (W/M)^2 \tag{A.7}$$

and substituting (A.7) into (A.3) provides for $i = 1, \ldots, n$ the optimal

$$\begin{aligned} x^*_i &= \surd(d_i/\lambda^* v_i) = [M\surd d_i]/[W\surd v_i] \\ &= M/W\surd(d_i/v_i). \quad \Box \end{aligned} \tag{A.8}$$

Appendix B

Solution algorithm for *Model* 2:

To simplify the solution, we consider the two types of constraints, service level (4) and space (5) constraints, defined in Section 3.2, separately and also the two sets of decision variables (s_i and S_i) separately and solve the optimization Model 2 based on two embedded iteration procedures. The specific Power Approximation formula derived in Schneider [31] provides the reorder points, s_i, expressed in (8) and (9) providing the required service level α. The rational function $p(y_i)$ is defined as

$$p\,(y_i) = \frac{a_0 + a_1 w + a_2 w^2 + a_3 w^3}{b_0 + b_1 w + b_2 w^2 + b_3 w^3 + b_4 w^4} \tag{B.1}$$

with notation

$$w = \sqrt{\ln\left(25/y_i^2\right)} \tag{B.2}$$

and with given constants:

$a_0 = -5.3925569$; $a_1 = 5.6211054$;
$a_2 = -3.8836830$; $a_3 = 1.0897299$;
$b_0 = 1.0000$; $b_1 = -0.72496485$;
$b_2 = 0.507326622$; $b_3 = 0.0669136868$;
$b_4 = -0.00329129114$.

First, we check if the value of the expected order quantity Q_i using the Economic Order Quantity (EOQ) formula, which is noted as $Q_i{}'$:

$$Q_i{}' = \sqrt{\frac{2d_i K_i}{h_i}}. \tag{B.3}$$

To check the feasibility, using the above $Q_i{}'$ values and the fixed service level α we calculate the appropriate $s_i = s_i^{(0)}$ values from Power Approximation (8). The resulting decision variables provide the approximate optimal solution of Model 2 if the space constraint (10) is fulfilled which means the solution is feasible. Using the notation of Section 3 we get

$$S_i = s_i + D_i = s_i + Q_i - u_i \tag{B.4}$$

for the optimal parameters. In this case, the approximate optimal par levels, s_i, S_i, $(i = 1, \ldots, n)$ are provided by $s_i = s_i^{(0)}$, and $S_i = S_i^{(0)}$ expressed in (B.4) using the approximation (6) for the expected undershoot quantities, u_i.

If the space constraint (5) is not fulfilled, the above solution is not feasible. That means the equation

$$\sum (v_i Q_i) \le M' - \sum [v_i(s_i - u_i)] \tag{B.5}$$

is not fulfilled, and there is a need to decrease the Q_i values so that the required storage space for order quantities $\sum(v_i Q_i)$ is less than the remaining total free space for order quantities

$$M = M(s_i) = M' - \sum [v_i(s_i - u_i)]. \tag{B.6}$$

To find the optimal Q_i values that fit into the remaining space, M, defined by (B.6) we formulate the sub-problem 2A of minimizing the sum of ordering cost and cycle stock inventory holding cost under the remaining total free storage space. In this sub-problem, we fix the reorder points, $s_i = s_i^{(0)}$ and the resulting $M = M_0$ expressed in (B.6) using $s_i = s_i^{(0)}$.

Sub-problem 2A:

$$\text{Min} \sum \left[K_i \frac{d_i}{Q_i} + h_i \frac{Q_i}{2}\right] \tag{B.7}$$

$$\text{s.t.} \quad \sum v_i Q_i \le M. \tag{B.8}$$

This sub-problem 2A cannot be solved directly, but the iterative solution procedure suggested by Ziegler [32] can be applied.

To initialize the iteration of finding the optimal Q_i values of sub-problem 2A, we use the Economic Order Quantities (EOQ), noted before by $Q_i{}'$, which also provide the upper bound, $Q^{(u)}$, on the order quantities for the iteration.

Table B.1
Order quantities and reorder points for sample items.

	Iteration 1		Iteration 2		Change		
	$Q_i^{(1)}$	$s_i^{(1)}$	$Q_i^{(2)}$	$s_i^{(2)}$	Q (%)	s(%)	
Drug 1	122.89	59.95	122.89	59.90	0.00	0.08	
Drug 2	122.62	34.23	122.62	34.21	0.00	0.07	
Drug 3	120.03	26.00	120.03	25.98	0.00	0.10	
Drug 4	51.11	25.99	51.11	25.97	0.00	0.07	
Drug 5	71.48	17.86	71.48	17.85	0.00	0.08	
Drug 6	66.34	15.82	66.34	15.81	0.00	0.11	
Drug 65	24.95	2.96	24.95	2.96	0.00	0.09	
Drug 66	28.75	2.19	28.75	2.18	0.00	0.17	
Drug 67	35.88	6.87	35.88	6.85	0.00	0.27	
Drug 68	18.92	5.40	18.92	5.40	0.00	0.10	
Drug 69	18.77	4.48	18.77	4.47	0.00	0.10	
Drug 70	30.68	1.72	30.68	1.72	0.00	0.13	
					0.00	0.11	Average

The overall volume of using these Q values is

$$V' = \sum v_i Q_i'. \tag{B.9}$$

Here we have the decision rule as described before; if $V' = M$, we stop because we have the optimal solution as described before. Otherwise, we calculate the lower bound, $Q^{(l)}$, in the form Q_i'', as

$$Q_i'' = \frac{M}{V'} Q_i' \tag{B.10}$$

and calculate values for λ_i, for $i = 1, \ldots, n$

$$\lambda_i = \frac{d_i K_i}{v_i \left(Q_i''\right)^2} - \frac{h_i}{2v_i}. \tag{B.11}$$

Once the values of λ_i have been calculated, set $\lambda' = \min \lambda_i$ and $\lambda'' = \max \lambda_i$. In the iteration we systematically change the λ' and λ'' values until the required accuracy is achieved.

In the *first step*, we average the min and max values as determined before to set the overall λ:

$$\lambda = \frac{\lambda' + \lambda''}{2}. \tag{B.12}$$

In the *second step*, we calculate a new Q_i using the new λ provided in step one, which is noted as $Q_i(\lambda)$ for $i = 1, \ldots, n$:

$$Q_i(\lambda) = \sqrt{\frac{d_i K}{\frac{h_i}{2} + \lambda v_i}} \tag{B.13}$$

and recalculate V using expression (B.9) and substituting the new $Q_i(\lambda)$ for the original Q_i. Next, employ the following decision rules:

If $V < M$, then set $\lambda'' = \lambda$ and $Q^{(U)} = Q_i(\lambda)$
If $V > M$, then set $\lambda' = \lambda$ and $Q^{(L)} = Q_i(\lambda)$.

In the *third step*, calculate the functions for the upper and lower bounds as

$$F\left(Q^{(U)}\right) = \sum \left[K_i \frac{d_i}{Q_i^{(U)}} + h_i \frac{Q_i^{(U)}}{2}\right] \tag{B.14}$$

$$F\left(Q^{(L)}\right) = \sum \left[K_i \frac{d_i}{Q_i^{(L)}} + h_i \frac{Q_i^{(L)}}{2}\right] \tag{B.15}$$

and use the stopping rule for the iteration

$$\frac{F\left(Q^{(U)}\right) - F\left(Q^{(L)}\right)}{F\left(Q^{(L)}\right)} \le E. \tag{B.16}$$

If the relative difference is determined to be less than or equal to the preset acceptable error, E, we stop. Otherwise, continue the iteration beginning at step one and repeat this process until convergence in order to have the solution of sub-problem 2A. The proof of the convergence of the above iterative procedure is in Ziegler [32]. By our numerical experiences, 5–11 iterations were required to get within the 0.1% accuracy range.

In the above iteration process on Q_i, the optimal Q_i values are obtained for fixed $s_i = s_i^{(0)}$. The $s_i^{(0)}$ values provide the α service level for the old Q_i values defined by (B.3). However, for the new $Q_i = Q_i^{(1)}$ values we need to recalculate the $s_i = s_i^{(1)}$ values since they depend on the values of Q_i.

We then move to the next stage of the iterative procedure and with the new $s_i = s_i^{(1)}$, we recalculate $M = M_1$ and solve sub-problem 2A again with the iteration process above. We continue recalculating s_i and Q_i values until the difference in subsequent s_i and Q_i is smaller than the preset accuracy limit ($E = 0.1\%$). For our case example we needed 2–3 rounds of iterations with each round requiring less than 12 iterations to reach convergence. Upon completion of Round 2, the resulting Q_i values are used once again in the Power Approximation formula to determine the corresponding s_i values for the sample items. We don't have a formal proof of convergence but as Table B.1 shows the calculated values of Q_i and s_i for a sample and it supports the decision to stop the iterations after 2–3 rounds.

References

[1] The Plunkett Research Group, 2010, Health Care Trends. http://www.plunkettresearch.com/Industries/HealthCare/HealthCareTrends/tabid/294/Default.aspx [accessed 10.05.2010].

[2] H. Holmgren, J. Wentz, Material Management and Purchasing for the Health Care Facility, AUPHA Press, Washington, DC, 1982.

[3] P. Jarrett, Logistics in the health care industry, International Journal of Physical Distribution and Logistics Management 28 (9–10) (1998) 741–742.

[4] L. Burns, Wharton school colleagues, in: The Health Care Value Chain Producers, Purchasers, and Providers, Jossey-Bass, San Francisco, California, 2002.

[5] D.A. Pitta, M.V. Laric, Value chains in health care, Journal of Consumer Marketing 21 (7) (2004) 451–464.

[6] A.B. Almarsdóttir, J.M. Traulsen, Cost-containment as part of pharmaceutical policy, Pharmacy World and Science 27 (3) (2005) 144–148.

[7] H. Prosser, T. Walley, A qualitative study of GPs' and PCO stakeholders' views on the importance and influence of cost on prescribing, Social Science & Medicine 60 (6) (2005) 1335–1346.

[8] A.L. Greer, The state of the art versus the state of the science: the diffusion of new medical technologies into practice, International Journal of Technology Assessment in Health Care 4 (1) (1988) 5–26.

[9] H. Prosser, S. Almond, T. Walley, Influences on GPs' decision to prescribe new drugs—the importance of 'who says what', Family Practice 20 (1) (2003) 61–68.

[10] H. Prosser, T. Walley, New drug prescribing by hospital doctors: the nature and meaning of knowledge, Social Science & Medicine 62 (7) (2006) 1565–1578.

[11] P. Michelon, M. DibCruz, V. Gascon, Using the tabu search method for the distribution of supplies in a hospital, Annals of Operations Research 50 (1) (1994) 427–435.

[12] N. Dellaert, E. van de Poel, Global inventory control in an academic hospital, International Journal of Production Economics 46–47 (1996) 277–284.

[13] M. Banerjea-Brodeur, J-F. Cordeau, G. Laporte, A. Lasry, Scheduling linen deliveries in a large hospital, Journal of the Operational Research Society 49 (8) (1998) 777–780.
[14] D. Kim, An integrated supply chain management system: a case study in healthcare sector, Lecture Notes in Computer Science 3590 (2005) 218–227.
[15] B. Meijboom, B. Obel, Tactical coordination in a multi-location and multi-stage operations structure: a model and a pharmaceutical company case, Omega 35 (3) (2007) 258–273.
[16] S. Lapierre, A. Ruiz, Scheduling logistic activities to improve hospital supply systems, Computers and Operations Research 34 (3) (2007) 624–641.
[17] H.E. Scarf, The optimality of (s, S) policies in the dynamic inventory problem, in: Mathematical Methods in the Social Sciences, Stanford University Press, Stanford, California, 1960.
[18] H. Schneider, D.B. Rinks, P. Kelle, Power approximations for a two-echelon inventory system using service levels, Production and Operations Management 4 (4) (1995) 381–400.
[19] D. Iglehart, Dynamic programming and stationary analysis of inventory problems, in: H.E. Scarf, D.M. Gilford, M.W. Shelly (Eds.), Multistage Inventory Models and Techniques, Stanford University Press, Stanford, California, 1963.
[20] H. Wagner, M. O'Hagan, B. Lundh, An empirical study of exactly and approximately optimal inventory policies, Management Science 11 (7) (1965) 690–723.
[21] D. Roberts, Approximations to optimal policies in a dynamic inventory model studies, in: K. Arrow, S. Kardin, H.E. Scarf (Eds.), Applied Probability and Management Science, Stanford University Press, Stanford, California, 1962, pp. 207–229.
[22] H. Schneider, D.B. Rinks, Optimal policy surfaces for a multi-item inventory problem, European Journal of Operational Research 39 (2) (1989) 180–191.
[23] F. Güder, J.L. Zydiak, Ordering policies for multi-item inventory systems subject to multiple resource constraints, Computers and Operations Research 26 (6) (1999) 583–597.
[24] D. Beyer, S.P. Sethi, R. Sridhar, Stochastic multiproduct inventory models with limited storage, Journal of Optimization Theory and Applications 111 (3) (2001) 553–588.
[25] K. Ohno, T. Ishigaki, A multi-item continuous review inventory system with compound Poisson demands, Mathematical Methods of Operations Research 58 (1) (2001) 147–165.
[26] S. Minner, E.A. Silver, Multi-product batch replenishment strategies under stochastic demand and a joint capacity constraint, IIE Transactions on Scheduling and Logistics 37 (5) (2005) 469–479.
[27] S. Minner, E.A. Silver, Replenishment policies for multiple products with compound-Poisson demand that share a common warehouse, International Journal of Production Economics 108 (1–2) (2007) 388–398.
[28] J. Woosley, Improving Healthcare Supply Chains and Decision Making in the Management of Pharmaceuticals, Unpublished doctoral dissertation, Louisiana State University, Baton Rouge, LA, 2009.
[29] J. Woosley, S. Wiley-Patton, P. Kelle, H. Schneider, Decision support in healthcare supply chain management and pharmaceutical inventory control, in: Proceedings of the Fifteenth Americas Conference on Information Systems, AMCIS, August 6–9, San Francisco, CA, 2009.
[30] H. Schneider, Effect of service-levels on order-points or order levels in inventory models, International Journal of Production Research 19 (6) (1981) 615–631.
[31] H. Schneider, Methods for determining the reorder point of an (s, S) ordering policy when a service-level is specified, Journal of the Operational Research Society 29 (12) (1978) 1181–1193.
[32] H. Ziegler, Solving certain singly constrained convex optimization problems in production planning, Operations Research Letters 1 (1982) 246–252.