Solutions to the practice problems (Week 4, Module 18)

- **Find if a given functional dependency is implied from a set of Functional Dependencies:**
  - For: A → BC, CD → E, E → C, D → AEH, ABH → BD, DH → BC
    - Check: BCD → H

    Solution: (BCD)+=BCDAEH (Since D → AEH, CD → E)

    Hence BCD → H is true.
    - Check: AED→C

    Solution: (AED)+=AEDBCH (Since A → BC, E → C, D → AEH)\

    Hence AED→C is true.
  - For: AB → CD, AF → D, DE → F, C → G, F → E, G → A
    - Check: CF → DF

    Solution: (CF)+=CFGEAD (Since C → G, F → E, G → A, AF → D, DE → F)

    Hence, CF → DF is true.
    - Check: BG → E

    Solution: (BG)+=BGACD (Since G → A, AB → CD, C → G)

    Hence BG → E is false.
    - Check: AF → G

    Solution: (AF)+=AFED (Since F → E, AF → D, DE → F)

    Hence AF → G is false.
    - Check: AB → EF

    Solution: (AB)+=ABCDG (Since AB → CD, C → G, G → A)

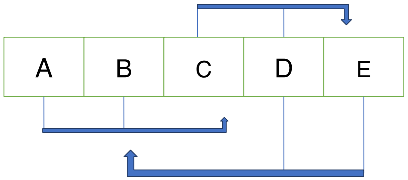
    Hence, AB → EF is false.
  - For: A → BC, B → E, CD → EF
    - Check: AD → F

    Solution: (AD)+=ADBCEF (Since, A → BC, B → E, CD → EF)

    Hence : AD → F is true.

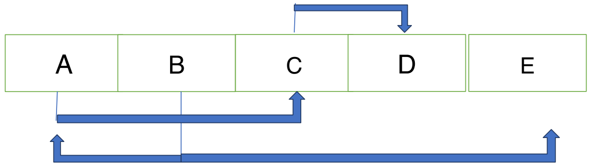
Find Candidate Key using Functional Dependencies:

Q1. Relational Schema R(ABCDE). Functional dependencies: AB → C, DE → B, CD → E



- First find out the closure step by step of the attribute A and D
- Closure of (A)+ ={A} //This is not a CK
- Closure of (D)+ ={D} //This is not a CK
- Closure of (AD)+ ={A,D} //This is not a CK
- Closure of (ABD)+ ={A,B,C,D,E} //USING AB → C, CD → E// This is a CK
- Closure of (ACD)+ ={A,C,D,E} ={A,B,C,D,E} //USING DE → B, CD → E// This is a CK
- Closure of (ADE)+ ={A,D,E,B} ={A,B,C,D,E} //USING DE → B, AB → C // This is a CK
- So candidate keys are ABD,ACD,ADE.

Q2.Find Candidate Key using Functional Dependencies: Relational Schema R(ABCDE).
Functional dependencies: AB → C, C → D, B → EA



- First find out the closure step by step of the attribute B
- Closure of (B)+ ={B,E,A} //using B → EA

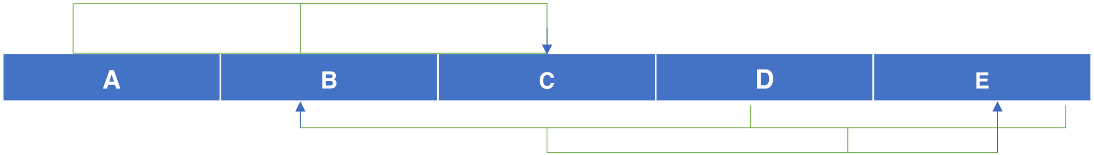
  ={A,B,C,E} //USING AB → C,

  ={A,B,C,D,E} //USING C → D

- So here we have only one Candidate key (B)
- Any other attribute cannot be a candidate key because to derived E we need only B.

- **Find Super Key using Functional Dependencies:**
  - Relational Schema R(ABCDE). Functional dependencies:

AB → C, DE → B, CD → E



According to the diagram, the candidate key must include AD.

(AD)+=AD [Hence not a candidate key]

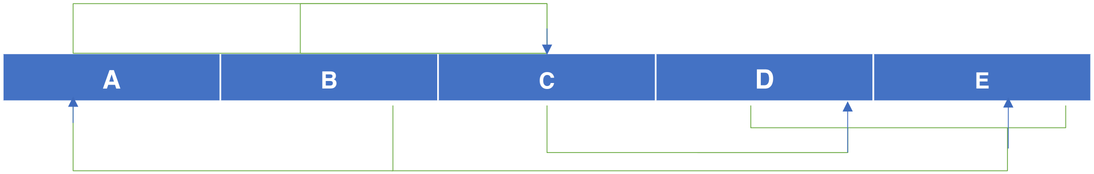
(ADB)+=ADBCE [Hence a candidate key]

(ADE)+=ADEBC [Hence a candidate key]

(ADC)+=ADCEB [Hence a candidate key]

Thus the super keys are ADB, ADE, ADC, ADBC, ADBE, ADEC, ABCDE.

- **Find Super Key using Functional Dependencies:**
    - Relational Schema R(ABCDE). Functional dependencies: AB → C, C → D, B → EA



According to the diagram, the candidate key must include B.

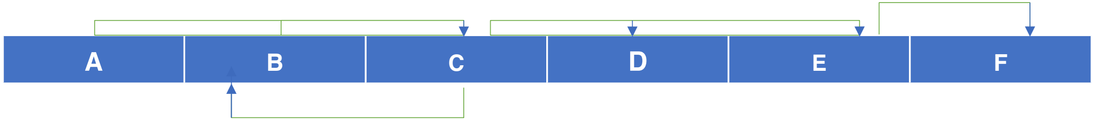
(B)+=BEACD [Hence a candidate key]

Thus the super keys are B, BA, BC, BD, BE, BAC, BAD, BAE, BCD, BDE, BCE, BACD, BADE, BCDE, BACE, ABCDE.

• Find Prime and Non Prime Attributes using Functional Dependencies:

- Note: for each relation, the candidate keys can be derived following the same process shown in the previous examples (also shown in details for the second practice problem in this slide).
  - R(ABCDEF) having FDs {AB→C, C→D, D→E, F→B, E→F}

  The candidate keys for R are AB,AC,AD,AE,AF. Thus the prime attributes are A,B,C,D,E,F.
  - R(ABCDEF) having FDs {AB → C, C → DE, E → F, C → B}



Thus, the candidate key must include A.

A+=A [Hence not a candidate key]

AB+=ABCDEF [Hence a candidate key]

AC+=ABDEBF [Hence a candidate key]

AD+=AD [Hence not a candidate key]

AE+=AEF [Hence not a candidate key]

AF+=AF [Hence not a candidate key]

Since AD, AE, AF are not candidate keys, these individual attributes are combined to check for the possible candidate keys:

ADE+=ADEF [Hence not a candidate key]

ADF+=ADF [Hence not a candidate key]

AEF+=AEF [Hence not a candidate key]

ADEF+=ADEF [Hence not a candidate key]

Thus, AB, AC are candidate keys and A,B, C are prime attributes. D, E, F are non prime.

- **Find Prime and Non Prime Attributes using Functional Dependencies:**
  - R(ABCDEFGHIJ) having FDs {AB → C, A → DE, B → F, F → GH, D → IJ}

  AB is the candidate key. Hence A, B are prime attributes. C,D,E,F,G,H,I,J are non prime.

  - R(ABDLPT) having FDs {B → PT, A → D, T → L}

  The candidate key for R is AB. Hence the prime attributes are A,B. Non-prime attributes are P, T, L, D.

  - R(ABCDEFGH) having FDs {E → G, AB → C, AC → B, AD → E,B → D, BC → A}

  The candidate keys are ABFH, ACFH, BCFH. Thus the prime attributes are A, B, C, F, H. While, D, E, G are non prime attributes,.

  - R(ABCDE) having FDs {A → BC, CD → E, B → D, E → A}

  The candidate keys are A, E, CD, BC. Thus A,B,C,D,E are prime attributes.

  - R(ABCDEH) having FDs {A → B, BC → D, E → C, D → A}

  The candidate keys are AEH, BHE, DHE. Thus the prime attributes are A, B, D, H, E. Non prime attribute is C.

Check the Equivalence of a Pair of Sets of Functional Dependencies:

1. Consider the two sets F and G with their FDs as below :

1. F : A → C, AC → D, E → AD, E → H
2. G: A → CD, E → AH

- Step 1 : Find out the closure of all lefthand attributes of F functional dependency(FD) using the FD of G and vice versa.

F : A → C, AC → D, E → AD, E → H	**G: A → CD, E → AH**
(A)+ ={A,C,D} //USING A → CD ,OF FD G	(A)+ ={A,C} //USING A → C, OF FD F ={A,C,D} //USING AC → D, OF FD F
(AC)+ ={A,C,D} //USING A → CD, OF FD G	(E)+={E,A,D,H} \\ USING E → AD, E → H ={E,A,C,D,H} \\ USING A → C
(E)+={A,E,H} // USING E → AH , OF FD G ={A,C,D,E,H} // USING A → CD	IF YOU WANT YOU CAN FIND FOR (AC)+ ={A,C,D} //USING AC → D OF F

- F⊆G and G ⊆F
- So F=G
- So two sets of functional dependency is equivalent.

Check the Equivalence of a Pair of Sets of Functional Dependencies:

Q2. Consider the two sets F and G with their FDs as below :

1. P : A → B, AB → C, D → ACE
2. Q : A → BC, D → AE

- Step 1 : Find out the clousure of all lefthand attributes of P functional dependency(FD) using the FD of Q and vice versa.

P : A → B, AB → C, D → ACE	**Q : A → BC, D → AE**
(A)+ ={A,B,C} //USING A → BC // Q FD	(A)+ ={A,B} //USING A → B //FD OF P ={A,B,C} //USING AB → C, //FD OF P
(AB)+ ={A,B,C} //USING A → BC //Q FD	(D)+={D,A,C,E} //USING D → ACE //FD OF P ={A,B,C,D,E} //USING A → B //FD OF P
(D)+={D,A,E} // USING D → AE OF Q FD ={A,B,C,D,E} // USING A → BC,	IF YOU WANT YOU CAN FIND FOR (AB)+ ={A,B,C} //USING A → B, AB → C, //FD OF P

- P⊆Q and Q⊆P
- So P=Q
- So two sets of functional dependency is equivalent.

Find the Minimal Cover or Irreducible Sets or Canonical Cover of a Set of Functional Dependencies:

1. $AB \rightarrow CD$, $BC \rightarrow D$

- Step 1:Use the union rule to replace any dependencies in *F*

$\alpha 1 \rightarrow \beta 1$ and $\alpha 1 \rightarrow \beta 2$ with $\alpha 1 \rightarrow \beta 1\ \beta 2$

So we can write AB → C, AB → D, BC → D

Step 2: find the redundancy step by step:

(AB)+ ={A,B,C,D} // Now check for AB → C

(AB)+ ={A,B,D} without considering this FD :AB → C

(A)+ ={A}

(B)+ ={B}

So AB → C is essential not redundancy.[left side multiple attribute is also essential]

(AB)+ ={A,B,C,D} // Now check for AB → D

(AB)+ ={A,B,C,D} without considering this FD :AB → D [we got the same closure]

So AB → C is redundancy.[SO delete this one]

(BC)+ ={B,C,D} // Now check for BC → D

(BC)+ ={B,C} without considering this FD :BC → D

SO this FD :BC → D is essential.

So canonical cover set of the above set of FD is AB → C, BC → D

Find the Minimal Cover or Irreducible Sets or Canonical Cover of a Set of Functional Dependencies;

2. $ABCD \rightarrow E$, $E \rightarrow D$, $AC \rightarrow D$, $A \rightarrow B$

- Step 1:Use the union rule to replace any dependencies in F

$\alpha 1 \rightarrow \beta 1$ and $\alpha 1 \rightarrow \beta 2$ with $\alpha 1 \rightarrow \beta 1\ \beta 2$

We have nothing like this.

Step 2: find the redundancy step by step:

Now check for $ABCD \rightarrow E$

$(ABCD)^+ = \{A,B,C,D,E\}$

$(ABCD)^+ = \{A,B,C,D\}$ without considering this FD : $ABCD \rightarrow E$

$(A)^+ = \{A,B\}$

$(B)^+ = \{B\}$

$(C)^+ = \{C\}$

$(D)^+ = \{D\}$

So $ABCD \rightarrow E$ is essential not redundancy.

Now check for $E \rightarrow D$

$(E)^+ = \{E,D\}$

$(E)^+ = \{E\}$ without considering this FD: $E \rightarrow D$,

So $E \rightarrow D$ is essential.

Find the Minimal Cover or Irreducible Sets or Canonical Cover of a Set of Functional Dependencies:

$ABCD \rightarrow E$, $E \rightarrow D$, $AC \rightarrow D$, $A \rightarrow B$

Now check for $AC \rightarrow D$,

(AC)+ ={A,C,D} //USING $AC \rightarrow D$

={A,B,C,D} //USING $A \rightarrow B$

={A,B,C,D,E} //USING $ABCD \rightarrow E$

(AC)+ ={A,B,C} without considering this FD : $AC \rightarrow D$

(A)+={A,B}

(C)+={C}

SO this FD : $AC \rightarrow D$ is essential.[LEFT HAND ATTRIBUTE IS ALSO ESSENTIAL]

Now check for $A \rightarrow B$

(A)+={A,B}

(A)+={A} without considering this FD: $A \rightarrow B$

SO $A \rightarrow B$ this FD is essential

So canonical cover set of the above set of FD is $ABCD \rightarrow E$, $E \rightarrow D$, $AC \rightarrow D$, $A \rightarrow B$