CHAPTER 7
Indices

7.1 Overview

7.1.1 Introduction

Exponential (or indicial) notation provides a convenient means of writing multiples of the same variable in a succinct form. The French mathematician René Descartes — best known to us for his philosophy 'I think, therefore I am' — was the first to describe indices as we use them today. In his 1637 text *Discours de la méthode pour bien conduire sa raison et chercher* (roughly translated as 'Discourse on the methodology of logic and reasoning'), he described the notation that he used to write multiples of the same variable:

> **'… in order to multiply a by itself we write aa or a^2, and a^3 in order to multiply it once more by a, and thus to infinity'**

This notation is at its most useful when we are using numbers that involve very large or very small powers of ten. Hence, we write the speed of light as 3×10^8 metres per second rather than 3 00 000 000 metres per second and the mass of a carbon atom as 1.997×10^{-23} grams instead of 0.000 000 000 000 000 000 000 001 997 grams. As these extraordinary numbers are most commonly used in fields such as chemistry or physics, this form of notation is referred to as scientific notation (we shall examine this in more detail during the course of this chapter).

Without exponential notation it would take us a very long time to write numbers such as the googol (equal to 10^{100}), let alone the googolplex, which is $10^{10^{100}}$. In fact, astrophysicist Carl Sagan once noted that to write the decimal representation of the googolplex would take a piece of paper bigger than the boundaries of the known universe!

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7.2 Index laws

7.2.1 Introduction

Recall that a number, a, which is multiplied by itself n times can be represented in **index notation**.

$$\underbrace{a \times a \times a \times \ldots \times a}_{n \text{ lots of } a} = a^n$$

Index (or power or exponents)

Base

where a is the base number and n is the **index** (or power or exponent).

a^n is read as 'a to the power of n' or 'a to the n'.

When the number 8 is expressed as a power of 2, it is written as $8 = 2^3$. In this form, the base is 2 and the power (also known as the index or **exponent**) is 3. The form of 8 expressed as 2^3 is known both as its index form with index 3 and base 2, and its **exponential form** with exponent 3 and base 2. The words 'index' and 'exponent' are interchangeable.

For any positive number n where $n = a^x$, the statement $n = a^x$ is called an index or exponential statement. For our study:

- the base is a where $a \in R^+ \backslash \{1\}$
- the exponent, or index, is x where $x \in R$
- the number n is positive, so $a^x \in R^+$.

Index laws control the simplification of expressions which have the same base.

7.2.2 Review of the index laws

Multiplication

When multiplying two numbers in index form with the same *base, add* the indices.

$$a^m \times a^n = a^{m+n}$$

For example, $2^3 \times 2^4 = 2 \times 2 \times 2 \quad \times \quad 2 \times 2 \times 2 \times 2 = 2^7$

Division

When dividing two numbers in index form with the same *base, subtract* the indices.

$$a^m \div a^n = a^{m-n}$$

For example, $2^6 \div 2^2 = \dfrac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2} = 2^4$

Raising to a power

To raise an indicial expression to a power, *multiply* the indices.

$$(a^m)^n = a^{m \times n} = a^{mn}$$

For example, $\left(2^4\right)^3 = 2^4 \times 2^4 \times 2^4 = 2^{4+4+4} = 2^{12}$

Raising to the power of zero

Any number raised to the power of zero is equal to *one*.

$$a^0 = 1,\ a \neq 0$$

$$\begin{aligned}
&\text{For example} && 2^3 \div 2^3 = 2^3 - 3 = 2^0 && [1]\\
&\text{or} && 2^3 \div 2^3 = (2 \times 2 \times 2) \div (2 \times 2 \times 2) &&\\
& && \qquad\quad = 8 \div 8 &&\\
& && \qquad\quad = 1 &&\\
&\text{So} && 2^3 \div 2^3 = 1 && [2]
\end{aligned}$$

Using [1] and [2] we have $2^0 = 1$.

7.2.3 Products and quotients

Note the following examples of products or quotients.

$$(ab)^n = a^n b^n$$

Example:
$$\begin{aligned}(2 \times 3)^4 &= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)\\ &= 2 \times 3 \times 2 \times 3 \times 2 \times 3 \times 2 \times 3\\ &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3\\ &= 2^4 \times 3^4\end{aligned}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Example:
$$\begin{aligned}\left(\frac{2}{3}\right)^4 &= \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)\\ &= \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}\\ &= \frac{2^4}{3^4}\end{aligned}$$

WORKED EXAMPLE 1

Simplify each of the following.

a. $2x^3y^2 \times 4x^2y$ b. $(2x^2y^3)^2 \times xy^4$ c. $(3a)^5\, b^6 \div 9a^4b^3$ d. $\dfrac{8p^6m^2 \times (3p)^3m^5}{6p^4m}$

THINK	WRITE
a. 1. Collect 'plain' numbers (2 and 4) and terms with the same base.	a. $2x^3y^2 \times 4x^2y$ $= 2 \times 4 \times x^3 \times x^2 \times y^2 \times y$
2. Simplify by multiplying plain numbers and adding powers with the same base.	$= 8x^5y^3$
b. 1. Remove the bracket by multiplying the powers. (The power of the 2 inside the bracket is 1.)	b. $(2x^2y^3)^2 \times xy^4$ $= 2^2 \times x^4 \times y^6 \times xy^4$
2. Convert 2^2 to a plain number (4) first and collect terms with the same base.	$= 4 \times x^4 \times x \times y^6 \times y^4$
3. Simplify by adding powers with the same base.	$= 4x^5y^{10}$
c. 1. Write the quotient as a fraction.	c. $(3a)^5\, b^6 \div 9a^4b^3 = \dfrac{(3a)^5b^6}{9a^4b^3}$
2. Remove the bracket by multiplying the powers.	$= \dfrac{243^a5b^6}{9a^4b^3}$
3. Simplify by first cancelling plain numbers.	$= \dfrac{27a^5b^6}{a^4b^3}$

4. Complete simplification by subtracting powers with the same base. (*Note:* $a^1 = a$)

$= 27ab^3$

d. 1. Expand the brackets by raising each term to the power of 3.

d. $\dfrac{8p^6m^2 \times (3p)^3 m^5}{6p^4m} = \dfrac{8p^6m^2 \times 3^3p^3m^5}{6p^4m}$

2. Convert 3^3 to 27 and collect 'like' pronumerals.

$= \dfrac{8 \times 27 \times p^6 \times p^3 \times m^2 \times m^5}{6p^4m}$

3. Simplify by first reducing the plain numbers, then simplify the pronumerals by adding the indices for multiplication and subtracting the indices for division.

$= 36p^{6+3-4}m^{2+5-1}$

4. Simplify the indices of each base.

$= 36p^5m^6$

WORKED EXAMPLE 2

Simplify $\dfrac{6a^4b^3}{16a^7b^6} \div \left(\dfrac{3a^2b}{2a^3b^2}\right)^3$.

THINK

WRITE

1. Write the expression.

$\dfrac{6a^4b^3}{16a^7b^6} \div \left(\dfrac{3a^2b}{2a^3b^2}\right)^3$

2. Change the division sign to multiplication and replace the second term with its reciprocal (turn the second term upside down).

$= \dfrac{6a^4b^3}{16a^7b^6} \times \left(\dfrac{2a^3b^2}{3a^2b}\right)^3$

3. Remove the brackets by multiplying the powers.

$= \dfrac{6a^4b^3}{16a^7b^6} \times \dfrac{2^3a^9b^6}{3^3a^6b^3}$

4. Collect plain numbers and terms with the same base.

$= \dfrac{6 \times 8a^{4+9-7-6}b^{3+6-6-3}}{16 \times 27}$

5. Cancel plain numbers and apply index laws.

$= \dfrac{a^0b^0}{9}$

6. Simplify.

$= \dfrac{1}{9}$

Expressions involving just numbers and numerical indices can be simplified using index laws and then evaluated.

WORKED EXAMPLE 3

Write in simplest index notation and evaluate.

a. $2^3 \times 16^2$

b. $\dfrac{9^5 \times 3^4}{27^3}$

THINK

a. 1. Rewrite the bases in terms of their prime factors.
2. Simplify the brackets using index notation.
3. Remove the brackets by multiplying the powers.
4. Simplify by adding the powers.
5. Evaluate as a basic number.

b. 1. Rewrite the bases in terms of their prime factors.
2. Simplify the brackets using index notation.
3. Remove the brackets by multiplying the powers.
4. Write in simplest index form.
5. Evaluate as a basic number.

WRITE

a.
$$\begin{aligned}2^3 \times 16^2 &= 2^3 \times (2 \times 2 \times 2 \times 2)^2\\ &= 2^3 \times \left(2^4\right)^2\\ &= 2^3 \times 2^8\\ &= 2^{11}\\ &= 2048\end{aligned}$$

b.
$$\begin{aligned}\frac{9^5 \times 3^4}{27^3} &= \frac{(3 \times 3)^5 \times 3^4}{(3 \times 3 \times 3)^3}\\ &= \frac{(3^2)^5 \times 3^4}{(3^3)^3}\\ &= \frac{(3^{10}) \times 3^4}{3^9}\\ &= 3^5\\ &= 243\end{aligned}$$

Complex expressions involving terms with different bases have to be simplified by replacing each base with its prime factors.

WORKED EXAMPLE 4

Simplify $\dfrac{3^{4n} \times 18^{n+1}}{6^{3n-2}}$.

THINK

1. Rewrite the bases in terms of their prime factors.
2. Simplify the brackets using index notation.
3. Remove the brackets by multiplying powers.
4. Collect terms with the same base by adding the powers in the products and subtracting the powers in the quotients.
5. Simplify.

WRITE

$$\begin{aligned}\frac{3^{4n} \times 18^{n+1}}{6^{3n-2}} &= \frac{3^{4n} \times (3 \times 3 \times 2)^{n+1}}{(2 \times 3)^{3n-2}}\\ &= \frac{3^{4n} \times (3^2 \times 2^1)^{n+1}}{(2 \times 3)^{3n-2}}\\ &= \frac{3^{4n} \times 3^{2n+2} \times 2^{n+1}}{2^{3n-2} \times 3^{3n-2}}\\ &= 3^{4n+2n+2-(3n-2)} \times 2^{n+1-(3n-2)}\\ &= 3^{6n+2-3n+2} \times 2^{n+1-3n+2}\\ &= 3^{3n+4} \times 2^{-2n+3}\\ &= 3^{3n+4} \times 2^{3-2n}\end{aligned}$$

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Exercise 7.2 Index laws

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1. **WE1a** Simplify each of the following.
 a. $x^2 \times x^5 \times x^3$
 b. $m^3 \times m^2 p \times p^4$
 c. $4y^3 \times 2y \times y^7$
2. **WE1b** Simplify each of the following.
 a. $5^2 \times 5^7 \times \left(5^3\right)^3$
 b. $(xy)^3 \times x^4 y^5$
 c. $\left(2x^4\right)^2 \times \left(4x^2\right)^5$
 d. $3m^2p^5 \times \left(mp^2\right)^3 \times 2m^4p^6$
 e. $5x^2y^3 \times \left(5xy^2\right)^4 \times \left(5x^2y\right)^2$
3. **WE1c** Simplify each of the following.
 a. $a^7b^8 \div a^2b^5$
 b. $2a^{12}b^9 \div (2a)^3\, b^4$
 c. $\left(3x^5\right) y^{11} \div 6x^2y^2$
 d. $p^{13}q^{10} \div \left(pq^4\right)^2$
 e. $\left(4mn^4\right)^2 \div 14n^3$
4. Simplify each of the following.
 a. $\dfrac{a^3b^4}{ab^2}$
 b. $25r^{15}s^{10}t^4 \div r^5\left(s^5\right)^2 (5t)^3$
 c. $\dfrac{15a^6b^7}{3a^3b^4}$
 d. $\dfrac{24x^4y^7}{20x^2y^3}$
5. Simplify each of the following.
 a. $\dfrac{6p^8m^4 \times 2p^7m^6}{9p^5m^2}$
 b. $\dfrac{(3x)^2\, y^2 \times 5x^6y^3}{10x^7y}$
 c. $\dfrac{14u^{11}v^9 \times \left(3u^2\right)^3 v}{21u^6v^5}$
 d. $\dfrac{\left(5e^3\right)^2 f^4 \times 8e^4f^3}{20ef^5}$
 e. $\dfrac{6w^2t^7 \times 9w^4t^{12}}{(3w)^5\, t^{13}}$
6. Simplify each of the following.
 a. $\dfrac{(2x)^4\, y \times \left(3x^7y\right)^2}{18x^5\,(2y)^3}$
 b. $\dfrac{\left(-3x^3y^2\right)^3}{2x^3y^6} \times \dfrac{6x^7y^5}{\left(x^2y\right)^2}$
 c. $\dfrac{(-3mp)^2 \times 4m^4p}{12\,(mp)^2}$
 d. $\dfrac{m^3p^4 \times \left(mp^3\right)^2}{\left(-mp^2\right)^4}$
 e. $\dfrac{4\left(u^7v^6\right)^3}{\left(-2u^3v^2\right)^2 \times u^4\left(3v^5\right)^2}$
7. **WE2** Simplify each of the following.
 a. $\dfrac{15a^8b^3}{9a^4b^5} \div \left(\dfrac{2a^3b}{3ab^2}\right)^2$
 b. $\dfrac{5k^{12}d}{\left(2k^3\right)^2} \div \dfrac{6kd^4}{25\left(k^2d^3\right)^3}$
 c. $\dfrac{4g^4\left(2p^{11}\right)^2}{g^3p^7} \div \dfrac{8g^4p}{(2gp)^3}$
 d. $\left(\dfrac{3jn^2}{n^5}\right)^3 \div \dfrac{\left(4j^2n\right)^2}{n^{13}\,(2j)^4}$
 e. $\dfrac{x^4y^7}{x^3y^2} \div \dfrac{x^3y^2}{x^5y}$
 f. $\dfrac{6x^3y^8}{\left(x^2y^3\right)^3} \div \dfrac{\left(2xy^3\right)^2}{8x^5y^7}$
8. Simplify each of the following.
 a. $\dfrac{3p^3m^4}{p^1m^2}$
 b. $\dfrac{6x^6y^5}{x^5y^3} \times \dfrac{x^4}{(2y)^2}$
 c. $\dfrac{3ab^3}{-ab} \div \left(\dfrac{a^2b}{a^5}\right)^2$
9. Simplify each of the following.
 a. $\dfrac{x^{n+1} \times y^5 \times z^{4-n}}{x^{n-2} \times y^{4-n} \times z^{3-n}}$
 b. $\dfrac{\left(x^ny^{m+3}\right)^2}{x^{n+2}y^{3-m}} \times \dfrac{x^2y}{x^{n-5} \times y^{5-3m}}$

10. **WE3** Write in simplest index notation.
 a. $2^4 \times 4^2 \times 8$
 b. $3^7 \times 9^2 \times 27^3 \times 81$
 c. $5^3 \times 15^2 \times 3^2$

11. Write in simplest index notation.

a. $20^5 \times 8^4 \times 125$

b. $\dfrac{3^4 \times 27^2}{6^4 \times 3^5}$

c. $\dfrac{8 \times 5^2}{2^3 \times 10}$

12. Write in simplest index notation and evaluate.

a. $\dfrac{(625)^4}{\left(5^3\right)^5}$

b. $\dfrac{(25)^4}{(125)^3}$

c. $\dfrac{4^{11} \div 8^2}{16^3}$

d. $\dfrac{27^2 \times 81}{9^3 \times 3^5}$

13. WE4 Simplify each of the following.

a. $\dfrac{2^n \times 9^{2n+1}}{6^{n-2}}$

b. $\dfrac{25^{3n} \times 5^{n-3}}{5^{4n+3}}$

c. $\dfrac{12^{x-2} \times 4^x}{6^{x-2}}$

d. $\dfrac{12^{n-3} \times 27^{1-n}}{9^{2n} \times 8^{n-1} \times 16^n}$

e. $\dfrac{4^n \times 7^{n-3} \times 49^{3n+1}}{14^{n+2}}$

14. Simplify each of the following.

a. $\dfrac{35^2 \times 5^5 \times 7^6}{25^4 \times 49^3}$

b. $\dfrac{3^{5n-4} \times 16^n \times 9^3}{4^{n+1} \times 18^{1-n} \times 6^{3-2n}}$

c. $*\dfrac{3^n + 3^{n+1}}{3^n + 3^{n-1}}$

d. $*\dfrac{5^n - 5^{n+1}}{5^{n+1} + 5^n}$

**Hint*: Factorise the numerator and denominator first.

15. Simplify the following.

$\dfrac{36^{2n} \times 6^{n+3}}{216^{n-2}}$

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16. Write in simplest index notation and evaluate.

a. $\dfrac{4^5}{2^7}$

b. $9^4 \times 3^5 \div 27$

c. $\dfrac{\left(16^2\right)^3}{\left(2^5\right)^4}$

d. $\dfrac{27^2}{\left(3^2\right)^3}$

7.3 Negative and rational indices

7.3.1 Negative indices

Wherever possible, negative index numbers should be expressed with positive index numbers using the simple rule below.

When an index number is moved from the numerator to denominator or vice versa, the sign of the power changes.

$$\boldsymbol{a^{-n} = \frac{1}{a^n}, a \neq 0}$$

This is easily verified as follows.

$$\begin{aligned}\frac{1}{a^n} &= \frac{a^0}{a^n} \quad \text{since } a^0 = 1 \\ &= a^{0-n} \quad \text{using division rule for indices} \\ &= a^{-n} \quad \text{simplifying the index}\end{aligned}$$

In other words, $\dfrac{a^{-n}}{1} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = \dfrac{a^n}{1}$

Note: Change the level, change the sign.

WORKED EXAMPLE 5

Express each of the following with positive index numbers.

a. $\left(\frac{5}{8}\right)^{-4}$ b. $\dfrac{x^4y^{-2} \times (x^2y)^{-5}}{x^{-3}y^3}$

THINK	WRITE
a. 1. Remove the brackets by raising the denominator and numerator to the power of −4.	a. $\left(\frac{5}{8}\right)^{-4} = \frac{5^{-4}}{8^{-4}}$
2. Interchange the numerator and denominator, changing the signs of the powers.	$= \frac{8^4}{5^4}$
3. Simplify by expressing as a fraction to the power of 4.	$= \left(\frac{8}{5}\right)^4$
b. 1. Remove the brackets by multiplying powers.	b. $\dfrac{x^4y^{-2} \times (x^2y)^{-5}}{x^{-3}y^3} = \dfrac{x^4y^{-2} \times x^{-10}y^{-5}}{x^{-3}y^3}$
2. Collect terms with the same base by adding the powers on the numerator and subtracting the powers on the denominator.	$= \dfrac{x^{-6}y^{-7}}{x^{-3}y^3}$ $= x^{-6-(-3)}y^{-7-3}$ $= x^{-3}y^{-10}$
3. Rewrite the answer with positive powers.	$= \dfrac{1}{x^3y^{10}}$

7.3.2 Fractional indices

Up to now, we have only looked at cases in which the indices have all been integers. However, an index can be any number.

Since $\left(a^{\frac{1}{2}}\right)^2 = a$ and $\left(\sqrt{a}\right)^2 = a$, then $a^{\frac{1}{2}} = \sqrt{a}$.

Thus, surds such as $\sqrt{3}$ can be written in index form as $3^{\frac{1}{2}}$.

The symbols $\sqrt{\ }$, $\sqrt[3]{\ }$, $\ldots \sqrt[n]{\ }$ are radical signs. Any radical can be converted to and from a fractional index using the index law:

$$\boldsymbol{a^{\frac{1}{n}} = \sqrt[n]{a}}$$

A combination of index laws allows $a^{\frac{m}{n}}$ to be expressed as $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$. It can also be expressed as $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$.

$$\boldsymbol{a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m \text{ or } a^{\frac{m}{n}} = \sqrt[n]{a^m}}$$

WORKED EXAMPLE 6

Evaluate each of the following without technology.

a. $16^{\frac{3}{2}}$ **b.** $\left(\frac{9}{25}\right)^{-\frac{3}{2}}$

THINK	WRITE
a. 1. Rewrite the base number in terms of its prime factors.	**a.** $16^{\frac{3}{2}} = \left(2^4\right)^{\frac{3}{2}}$
2. Remove the brackets by multiplying the powers.	$= 2^6$
3. Evaluate as a basic number.	$= 64$
b. 1. Rewrite the base numbers of the fraction in terms of their prime factors.	**b.** $\left(\frac{9}{25}\right)^{-\frac{3}{2}} = \left(\frac{3^2}{5^2}\right)^{-\frac{3}{2}}$
2. Remove the brackets by multiplying the powers.	$= \frac{3^{-3}}{5^{-3}}$
3. Rewrite with positive powers by interchanging the numerator and denominator.	$= \frac{5^3}{3^3}$
4. Evaluate the numerator and denominator as basic numbers.	$= \frac{125}{27}$

WORKED EXAMPLE 7

Simplify the following, expressing your answer with positive indices.

a. $\sqrt[3]{64} \times \sqrt[4]{512}$ **b.** $\sqrt[3]{x^2y^6} \div \sqrt{x^3y^5}$

THINK	WRITE
a. 1. Write the expression.	**a.** $\sqrt[3]{64} \times \sqrt[4]{512}$
2. Write using fractional indices.	$= 64^{\frac{1}{3}} \times 512^{\frac{1}{4}}$
3. Write 64 and 512 in index form.	$= \left(2^6\right)^{\frac{1}{3}} \times \left(2^9\right)^{\frac{1}{4}}$
4. Multiply the powers.	$= 2^2 \times 2^{\frac{9}{4}}$
5. Simplify the powers.	$= 2^{\frac{17}{4}}$
b. 1. Write the expression.	**b.** $\sqrt[3]{x^2y^6} \div \sqrt{x^3y^5}$
2. Express the roots in index notation.	$= \left(x^2y^6\right)^{\frac{1}{3}} \div \left(x^3y^5\right)^{\frac{1}{2}}$
3. Remove the brackets by multiplying the powers.	$= x^{\frac{2}{3}}y^2 \div x^{\frac{3}{2}}y^{\frac{5}{2}}$

4. Collect terms with the same base by subtracting the powers. $= x^{\frac{2}{3}-\frac{3}{2}}y^{2-\frac{5}{2}}$

5. Simplify the powers. $= x^{-\frac{5}{6}}y^{-\frac{1}{2}}$

6. Rewrite with positive powers. $= \dfrac{1}{x^{\frac{5}{6}}y^{\frac{1}{2}}}$

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Exercise 7.3 Negative and rational indices

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1. **MC**

a. The exact value of 6^{-2} is:

A. -12 B. -36 C. $-\dfrac{1}{6}$ D. $\dfrac{1}{36}$

b. The exact value of $\left(\dfrac{27}{8}\right)^{-\frac{1}{3}}$ is:

A. $-\dfrac{2}{3}$ B. 6 C. $\dfrac{2}{3}$ D. $-\dfrac{3}{2}$

c. $\sqrt[3]{25} \times \sqrt{125}$ simplifies to:

A. $25^{\frac{5}{6}}$ B. $5^{\frac{7}{6}}$ C. $5^{\frac{3}{2}}$ D. $5^{\frac{13}{6}}$

2. **WE5a** Express each of the following with positive index numbers.

a. 6^{-3} b. 5^{-4} c. $\left(\dfrac{3}{5}\right)^{-2}$ d. $\left(\dfrac{7}{4}\right)^{-5}$

3. Express each of the following with positive index numbers.

a. $\left(\dfrac{1}{9}\right)^{-2}$ b. $\left(64^{-2}\right)^{3}$ c. $(-3)^{-1}$ d. $\left(\dfrac{3^4}{2^3}\right)^{-4}$

4. **WE5b** Simplify each of the following, expressing your answer with positive index numbers.

a. $\dfrac{(-2)^3 \times 2^{-4}}{2^{-3}}$ b. $\dfrac{\left(x^{-2}\right)^3 \times \left(y^4\right)^{-2}}{x^{-5} \times \left(y^{-2}\right)^3}$ c. $\dfrac{(-m)^2 \times m^{-3}}{\left(p^{-2}\right)^{-1} \times p^{-4}}$

5. Simplify each of the following, expressing your answer with positive index numbers.

a. $\dfrac{x^5}{x^{-3}} \div \dfrac{\left(x^4\right)^{-2}}{\left(x^2\right)^{-3}}$ b. $\dfrac{\left(3^{-2}\right)^2 \times \left(2^{-5}\right)^{-1}}{\left(2^4\right)^{-2} \times \left(3^4\right)^{-3}}$ c. $\dfrac{x^3y^{-2} \times \left(xy^2\right)^{-3}}{\left(2x^3\right)^2 \times \left(y^{-3}\right)^2}$

6. WE6 Evaluate the following without using technology.

a. $9^{\frac{1}{2}}$ b. $27^{\frac{1}{3}}$ c. $625^{\frac{1}{4}}$ d. $256^{\frac{1}{8}}$

e. $8^{\frac{2}{3}}$ f. $81^{\frac{3}{4}}$ g. $125^{\frac{4}{3}}$ h. $\left(\frac{8}{125}\right)^{\frac{1}{3}}$

7. Evaluate the following without using technology.

a. $\left(\frac{16}{81}\right)^{\frac{1}{4}}$ b. $\left(\frac{25}{16}\right)^{\frac{3}{2}}$

c. $\left(\frac{27}{64}\right)^{\frac{2}{3}}$ d. $32^{-\frac{2}{5}}$

e. $81^{-\frac{3}{4}}$ f. $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

g. $\left(\frac{16}{121}\right)^{-\frac{1}{2}}$ h. $\left(\frac{125}{216}\right)^{-\frac{1}{3}}$

8. WE7 Simplify each of the following, expressing your answer with positive indices.

a. $\sqrt{9} \times \sqrt[3]{81}$ b. $x^{\frac{2}{3}} \times x^{\frac{1}{6}}$

c. $x^{-\frac{3}{4}} \times x^{\frac{9}{8}}$ d. $x^{\frac{5}{2}} \div \left(x^{\frac{1}{3}}\right)^4$

e. $\sqrt[3]{(xy^3)} \div \sqrt{(x^2y)}$ f. $\sqrt[5]{32} \times \sqrt[4]{8}$

9. Simplify each of the following, expressing your answer with positive indices.

a. $2^{\frac{5}{4}} \times 4^{-\frac{1}{2}} \times 8^{-\frac{2}{3}}$ b. $27^{-\frac{1}{4}} \times 9^{\frac{2}{3}} \times 3^{-\frac{5}{4}}$ c. $\dfrac{18^{\frac{1}{2}}}{9^{\frac{4}{3}} \times 4^{\frac{3}{4}}}$

d. $\left(\sqrt[4]{x^3}\right)^{\frac{2}{3}} \times \left(\sqrt[3]{x^4}\right)^{\frac{3}{8}}$ e. $\dfrac{(64m^6)^{\frac{4}{3}}}{4m^{-2}}$ f. $\dfrac{\sqrt{x^3}}{\sqrt{x}}$

10. Simplify each of the following, expressing your answer with positive indices.

a. $\dfrac{1}{\sqrt{x^{-4}}}$ b. $\dfrac{(x+1)^2}{\sqrt{x+1}}$ c. $\sqrt{x} - \dfrac{1}{\sqrt{x}}$

d. $\sqrt{x+2} + \dfrac{x}{\sqrt{x+2}}$ e. $(y-4)\sqrt{y-4}$ f. $(p+3)(p+3)^{-\frac{2}{5}}$

11. a. Express the following in index (exponent) form.

i. $\sqrt{a^3b^4}$ ii. $\sqrt{\dfrac{a^5}{b^{-4}}} \times \sqrt[3]{a^2b}$

b. Express the following in surd form.

i. $a^{\frac{1}{2}} \div b^{\frac{3}{2}}$ ii. $2^{\frac{5}{2}}$ iii. $3^{-\frac{2}{5}}$

12. Evaluate the following without using technology.

a. $4^{\frac{3}{2}}$ b. $3^{-1} + 5^0 - 2^2 \times 9^{-\frac{1}{2}}$

c. $2^3 \times \left(\frac{4}{9}\right)^{-\frac{1}{2}} \div \left(6 \times (3^{-2})^2\right)$ d. $\dfrac{15 \times 5^{\frac{3}{2}}}{125^{\frac{1}{2}} - 20^{\frac{1}{2}}}$

13. Simplify and express the answer with positive indices.

a. $\dfrac{3\left(x^2y^{-2}\right)^3}{\left(3x^4y^2\right)^{-1}}$

b. $\dfrac{2a^{\frac{2}{3}}b^{-3}}{3a^{\frac{1}{3}}b^{-1}} \times \dfrac{3^2 \times 2 \times (ab)^2}{\left(-8a^2\right)^2 b^2}$

c. $\dfrac{\left(2mn^{-2}\right)^{-2}}{m^{-1}n} \div \dfrac{10n^4m^{-1}}{3\left(m^2n\right)^{\frac{3}{2}}}$

d. $\dfrac{4m^2n^{-2} \times -2\left(m^2n^{\frac{3}{2}}\right)^2}{\left(-3m^3n^{-2}\right)^2}$

e. $\dfrac{m^{-1} - n^{-1}}{m^2 - n^2}$

f. $\sqrt{4x-1} - 2x(4x-1)^{-\frac{1}{2}}$

g. Express $\dfrac{2^{1-n} \times 8^{1+2n}}{16^{1-n}}$ as a power of 2.

7.4 Indicial equations and scientific notation

7.4.1 Indicial equations

An **indicial equation** has the unknown variable as an exponent. In this section we shall consider indicial equations which have rational solutions.

7.4.2 Method of equating indices

If index laws can be used to express both sides of an equation as single powers of the same base, then this allows indices to be equated. For example, if an equation can be simplified to the form $2^{3x} = 2^4$, then for the equality to hold, $3x = 4$. Solving this linear equation gives the solution to the indicial equation as $x = \dfrac{4}{3}$.

In the case of an inequation, a similar method is used. If $2^{3x} < 2^4$, for example, then $3x < 4$. Solving this linear inequation gives the solution to the indicial inequation as $x < \dfrac{4}{3}$.

To solve for the exponent x in equations of the form $a^x = n$:
- **Express both sides as powers of the same base.**
- **Equate the indices and solve the equation formed to obtain the solution to the indicial equation.**

Inequations are solved in a similar manner. However, you need to ensure the base a is greater than 1 prior to expressing the indices with the corresponding order sign between them.

For example, you first need to write $\left(\dfrac{1}{2}\right)^x < \left(\dfrac{1}{2}\right)^3$ as $2^{-x} < 2^{-3}$ and then solve the linear inequation $-x < -3$ to obtain the solution $x > 3$.

WORKED EXAMPLE 8

Find the value of x in each of the following equations.

a. $\mathbf{3^x = 81}$ **b.** $\mathbf{4^{x-1} = 256}$

THINK	WRITE
a. 1. Write the equation.	a. $3^x = 81$
2. Express both sides to the same base.	$3^x = 3^4$
3. Equate the powers.	Therefore, $x = 4$,

b. 1. Write the equation.
2. Express both sides to the same base.
3. Equate the powers.
4. Solve the linear equation for x by adding one to both sides.

b.
$$4^{x-1} = 256$$
$$4^{x-1} = 4^4$$
Therefore, $x - 1 = 4$,
$$x = 5$$

WORKED EXAMPLE 9

Solve $5^{3x} \times 25^{4-2x} = \dfrac{1}{125}$ for x.

THINK

1. Use the index laws to express the left-hand side of the equation as a power of a single base.

WRITE

$$5^{3x} \times 25^{4-2x} = \frac{1}{125}$$
$$5^{3x} \times 5^{2(4-2x)} = \frac{1}{125}$$
$$5^{3x+8-4x} = \frac{1}{125}$$
$$5^{8-x} = \frac{1}{125}$$

2. Express the right-hand side as a power of the same base.

$$5^{8-x} = \frac{1}{5^3}$$
$$5^{8-x} = 5^{-3}$$

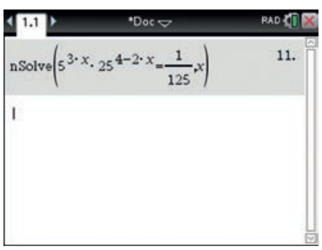
3. Equate indices and calculate the required value of x.

Equating indices,
$$8 - x = -3$$
$$x = 11$$

TI | THINK

1. On a Calculator page, press MENU then select
3: Algebra
1: Numerical Solve
Complete the entry line as
nSolve
$\left(5^{3x} \times 25^{4-2x} = \dfrac{1}{125}, x\right)$
then press ENTER.

WRITE



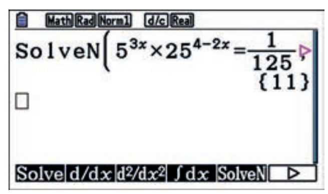
2. The answer appears on the screen.

$x = 11$

CASIO | THINK

1. On the Run-Matrix screen, press OPTN then select CALC by pressing F4. Select SolveN by pressing F5 then complete the entry line as
SolveN
$\left(5^{3x} \times 25^{4-2x} = \dfrac{1}{125}, x\right)$
and press EXE.

WRITE



2. The answer appears on the screen.

$x = 11$

7.4.3 Indicial equations which reduce to quadratic form

The technique of substitution to form a quadratic equation may be applicable to indicial equations.

To solve equations of the form $p \times a^{2x} + q \times a^x + r = 0$:

- Note that $a^{2x} = (a^x)^2$.
- Reduce the indicial equation to quadratic form by using a substitution for a^x.
- Solve the quadratic and then substitute back for a^x.
- Since a^x must always be positive, solutions for x can only be obtained for $a^x > 0$; reject any negative or zero values for a^x.

WORKED EXAMPLE 10

Solve $3^{2x} - 6 \times 3^x - 27 = 0$ for x.

THINK	WRITE
1. Use a substitution technique to reduce the indicial equation to quadratic form. *Note:* The subtraction signs prevent the use of index laws to express the left-hand side as a power of a single base.	$3^{2x} - 6 \times 3^x - 27 = 0$ Let $a = 3^x$ $\therefore a^2 - 6a - 27 = 0$
2. Solve the quadratic equation.	$(a-9)(a+3) = 0$ $a = 9, a = -3$
3. Substitute back and solve for x.	Replace a by 3^x. $\therefore 3^x = 9$ or $3^x = -3$ (reject negative value) $3^x = 9$ $\therefore 3^x = 3^2$ $\therefore x = 2$

7.4.4 Scientific notation (standard form)

Index notation provides a convenient way to express numbers which are either very large or very small. Writing a number as $a \times 10^b$ (the product of a number a where $1 \leq a < 10$ and a power of 10) is known as writing the number in **scientific notation** (or **standard form**). The age of the earth since the Big Bang is estimated to be 4.54×10^9 years, while the mass of a carbon atom is approximately 1.994×10^{-23} grams. These numbers are written in scientific notation.

To convert scientific notation back to a basic numeral:

- **Move the decimal point b places to the right if the power of 10 has a positive index, in order to obtain the large number $a \times 10^b$ represents**
  Or
- **Move the decimal point b places to the left if the power of 10 has a negative index, in order to obtain the small number $a \times 10^{-b}$ represents. This is because multiplying by 10^{-b} is equivalent to dividing by 10^b.**

7.4.5 Significant figures

When a number is expressed in scientific notation as either $a \times 10^b$ or $a \times 10^{-b}$, the number of digits in a determines the number of **significant figures** in the basic numeral. The age of the Earth is 4.54×10^9 years in scientific notation or 4 540 000 000 years to three significant figures. To one significant figure, the age would be 5 000 000 000 years.

WORKED EXAMPLE 11

a. **Express each of the following numerals in scientific notation and state the number of significant figures each numeral contains.**

i. **3 266 400** **ii.** **0.009 876 03**

b. **Express the following as basic numerals.**

i. $\mathbf{4.54 \times 10^9}$ **ii.** $\mathbf{1.037 \times 10^{-5}}$

THINK

a. i. 1. Write the given number as a value between 1 and 10 multiplied by a power of 10, discarding any trailing zeroes.
Note: The number is large so the power of 10 should be positive.

WRITE

a. i. In scientific notation,
$3\,266\,400 = 3.2\,664 \times 10^6$

THINK

2. Count the number of digits in the number a in the scientific notation form $a \times 10^b$ and state the number of significant figures.

WRITE

There are 5 significant figures in the number 3 266 400.

THINK

ii. 1. Write the given number as a value between 1 and 10 multiplied by a power of 10.
Note: The number is small so the power of 10 should be negative.

WRITE

ii. $0.00\,987\,603 = 9.87\,603 \times 10^{-3}$

THINK

2. Count the number of digits in the number a in the scientific notation form and state the number of significant figures.

WRITE

0.009 987 603 has 6 significant figures.

THINK

b. i. 1. Perform the multiplication.
Note: The power of 10 is positive, so a large number should be obtained.

WRITE

b. i. 4.54×10^9
Move the decimal point 9 places to the right.
$\therefore 4.54 \times 10^9 = 4\,540\,000\,000$

THINK

ii. 2. Perform the multiplication.
Note: The power of 10 is negative, so a small number should be obtained.

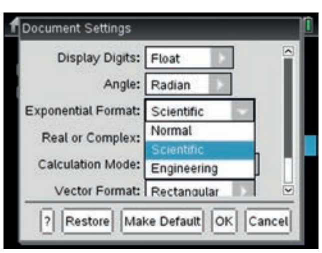
WRITE

ii. 1.037×10^{-5}
Move the decimal point 5 places to the left.
$\therefore 1.037 \times 10^{-5} = 0.00\,001\,037$

TI | THINK

a.i1. On the Home screen, select
5: Settings
2: Document settings
then change the Exponential Format to Scientific and select OK.

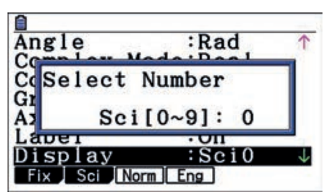
WRITE



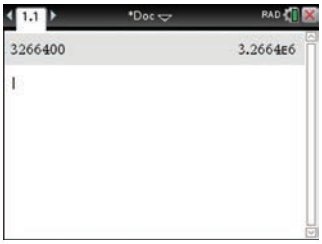
CASIO | THINK

a.i1. On a Run-Matrix screen, press SHIFT then MENU. Change the Display mode to Scientific, then type '0' and press ENTER.
Press EXIT.

WRITE



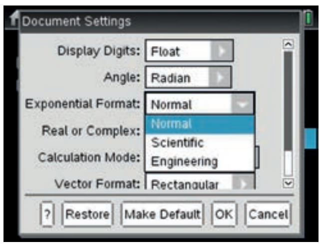
2. On a Calculator page, complete the entry line as 3 266 400E0, then press ENTER.
Note: The EE button is located above the π button.



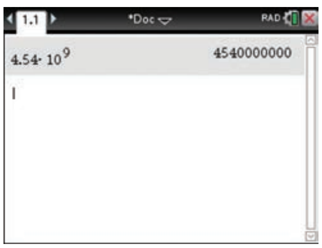
3. The answer appears on the screen.

$3\,266\,400 = 3.2664 \times 10^6$

b.i1. Change the Exponential Format back to Normal.



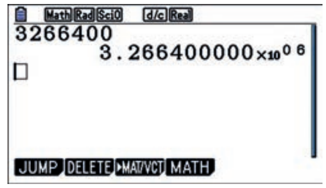
2. On a Calculator page, complete the entry line as
4.54×10^9
then press ENTER.



3. The answer appears on the screen.

$4.54 \times 10^9 = 4540000000$

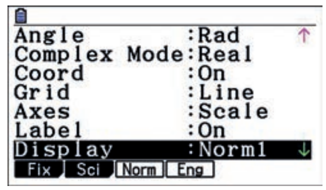
2. Complete the entry line as 3266400, then press EXE.



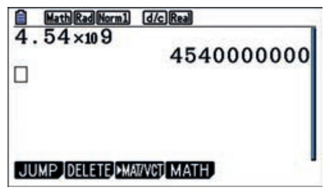
3. The answer appears on the screen.

$3\,266\,400 = 3.2664 \times 10^6$

b.i1. On the Run-Matrix screen, change the Display mode back to Normal.



2. Complete they entry line as 4.54×10^9, then press EXE.



3. The answer appears on the screen.

$4.54 \times 10^9 = 4540000000$

studyon

Units 1 & 2 › Area 4 › Sequence 1 › Concept 3

Scientific notation Summary screen and practice questions

Exercise 7.4 Indicial equations and scientific notation

Technology free

1. **WE8a** Solve for x in each of the following equations.

a. $2^x = 32$ **b.** $5^x = 625$ **c.** $3^x = 243$

d. $10^{-x} = \dfrac{1}{100}$ **e.** $4^{-x} = 16$ **f.** $6^x = \dfrac{1}{216}$

g. $3^{-x} = \dfrac{1}{81}$ **h.** $2^{-x} = 1$ **i.** $8^x = 2^6$

2. **WE8b** Solve for n in each of the following equations.

a. $2^{3n+1} = 64$ b. $5^{2n+3} = 25$ c. $3^{2-n} = 27$

d. $16^{n+3} = 2^3$ e. $49^{5-3n} = \frac{1}{7}$ f. $36^{4n-3} = 216$

3. **WE9** Solve for x in each of the following.

a. $4^{2x} = 8^{x-1}$ b. $27^{4-x} = 9^{2x+1}$ c. $16^{3x+1} = 128^{x-2}$ d. $25^{2x-3} = \frac{1}{125}$

e. $32^{5-x} = 4^{3x+2}$ f. $64^{2-3x} = 16^{x+1}$ g. $9^{3x+5} = \frac{1}{243}$ h. $16^{4-3x} = \frac{1}{8^{x+3}}$

4. Solve $\frac{2^{5x-3} \times 8^{9-2x}}{4^x} = 1$ for x.

5. Solve the following inequations.

a. $2 \times 5^x + 5^x < 75$ b. $\left(\frac{1}{9}\right)^{2x-3} > \left(\frac{1}{9}\right)^{7-x}$

6. Solve for x in each of the following equations.

a. $2^x \times 8^{3x-1} = 64$ b. $5^{2x} \times 125^{3-x} = 25$ c. $3^{4x} \times 27^{x+3} = 81$

d. $16^{x+4} \times 2^{3+2x} = 4^{5x}$ e. $3125 \times 25^{2x+1} = 5^{3x+4}$ f. $\frac{81^{2-x}}{27^{x+3}} = 9^{2x}$

7. **WE10** Solve for x in each of the following.

a. $3^{2x} - 4(3^x) + 3 = 0$ b. $2^{2x} - 6(2^x) + 8 = 0$ c. $3\left(2^{2x}\right) - 36(2^x) + 96 = 0$

d. $2\left(5^{2x}\right) - 12(5^x) + 10 = 0$ e. $3\left(4^{2x}\right) = 15(4^x) - 12$ f. $25^x - 30(5^x) + 125 = 0$

8. a. Express $\frac{32 \times 4^{3x}}{16^x}$ as a power of 2.

b. Express $\frac{3^{1+n} \times 81^{n-2}}{243^n}$ as a power of 3.

c. Express $0.001 \times \sqrt[3]{10} \times 100^{\frac{5}{2}} \times (0.1)^{-\frac{2}{3}}$ as a power of 10.

d. Express $\frac{5^{n+1} - 5^n}{4}$ as a power of 5.

9. Solve for x in each of the following.

a. $2^{2x} \times 8^{2-x} \times 16^{-\frac{3x}{2}} = \frac{2}{4^x}$ b. $25^{3x-3} \leq 125^{4+x}$

c. $9^x \div 27^{1-x} = \sqrt{3}$ d. $\left(\frac{2}{3}\right)^{3-2x} > \left(\frac{27}{8}\right)^{-\frac{1}{3}} \times \frac{1}{\sqrt{2\frac{1}{4}}}$

10. Use a suitable substitution to solve the following equations.

a. $3^{2x} - 10 \times 3^x + 9 = 0$ b. $24 \times 2^{2x} + 61 \times 2^x = 2^3$

c. $25^x + 5^{2+x} - 150 = 0$ d. $(2^x + 2^{-x})^2 = 4$

e. $10^x - 10^{2-x} = 99$ f. $2^{3x} + 3 \times 2^{2x-1} - 2^x = 0$

Technology active

11. **WE11** Express each of the following numbers in scientific notation, and state the number of significant figures each number contains.

a. i. 1 409 000 ii. 0.0 001 306

b. Express the following as basic numerals.

i. 3.04×10^5

ii. 5.803×10^{-2}

12. a. Express in scientific notation:
   i. $-0.0\,000\,000\,506$
   ii. the diameter of the Earth, given its radius is 6370 km
   iii. $3.2 \times 10^4 \times 5 \times 10^{-2}$
   iv. the distance between Roland Garros and Kooyong tennis stadiums of 16878.7 km.

b. Express the following as a basic numeral.
   i. $6.3 \times 10^{-4} + 6.3 \times 10^4$ ii. $\left(1.44 \times 10^6\right)^{\frac{1}{2}}$

13. Express the following to 2 significant figures.
   a. 60 589 people attended a football match.
   b. The probability of winning a competition is 1.994×10^{-2}.
   c. The solution to an equation is $x = -0.00634$.
   d. The distance flown per year by the Royal Flying Doctor Service is 26 597 696 km.

14. a. Simplify $\dfrac{x^2y^{-2}}{2x^{\frac{1}{3}}\sqrt{y^5}}$.
   b. Solve the following equations.
   i. $5^x \times 25^{2x} = \dfrac{1}{5}$ to obtain x exactly ii. $5^x \times 25^{2x} = 0.25$ to obtain x to 4 significant figures

7.5 Review: exam practice

A summary of this chapter is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

Simple familiar

1. MC When simplified, $\dfrac{\left(2xy^3\right)^2}{7x^3} \times \dfrac{3x^5y^2}{4y}$ is equal to:

   A. $\dfrac{x^4y^7}{7}$ B. $\dfrac{3x^4y^7}{7}$ C. $\dfrac{3y^7}{x^2}$ D. $\dfrac{3x^4}{y^6}$

2. MC $\dfrac{5m^4p^2}{2m^3p} \div \dfrac{\left(5m^2p^6\right)^3}{3m^7p}$ may be simplified to:

   A. $\dfrac{m^2}{47p^{16}}$ B. $\dfrac{3m^{10}}{2p^{32}}$ C. $\dfrac{3m^2}{50p^{16}}$ D. $\dfrac{m^{15}}{p^{29}}$

3. MC The value of $5^{-2}\left(\dfrac{64}{125}\right)^{-\frac{1}{3}}$ is:

   A. $\dfrac{1}{20}$ B. 5 C. $\dfrac{4}{5}$ D. $\dfrac{5}{4}$

4. MC If $25^{2-x} = 125$, then x is equal to:

   A. 1 B. $\dfrac{1}{2}$ C. -1 D. 2

5. MC In scientific notation, $\left(3.2 \times 10^{-2}\right) \times \left(5 \times 10^5\right)$ would equal:

   A. 2.56×10^8 B. 16×10^{-10} C. 1.6×10^{-9} D. 1.6×10^4

6. Simplify $\left(9a^3b^{-4}\right)^{\frac{1}{2}} \times 2\left(a^{\frac{1}{2}}b^{-2}\right)^{-2}$.

7. Evaluate $27^{-\frac{2}{3}} + \left(\dfrac{49}{81}\right)^{\frac{1}{2}}$.

8. Simplify the following expression with positive indices.

$$\left(16^{x-6}y^{10}\right)^{\frac{1}{2}} \div \sqrt[3]{\left(27x^3y^9\right)}$$

9. Solve the following equations.

a. $2x^5 = 100$ b. $8^{x+1} \times 2^{2x} = 4^{3x-1}$

10. Express the following in surd form.

a. $a^{\frac{1}{2}} \div b^{\frac{3}{2}}$ b. $2^{\frac{5}{2}}$ c. $3^{-\frac{2}{5}}$

11. Calculate $\left(4 \times 10^6\right)^2 \times \left(5 \times 10^{-3}\right)$ without using technology.

12. Evaluate the following without using technology.

a. $4^{\frac{3}{2}}$ b. $3^{-1}+5^0-2^2\times 9^{-\frac{1}{2}}$

Complex familiar

13. Simplify $\dfrac{20p^5}{m^3q^{-2}} \div \dfrac{5\left(p^2q^{-3}\right)^2}{-4m^{-1}}$.

14. Solve $2^x - 48 \times 2^{-x} = 13$ for x.

15. Solve for x.

a. $4^{5x} + 4^{5x} = \dfrac{8}{2^{4x-5}}$ b. $5^{\frac{2x}{3}} \times 5^{\frac{3x}{2}} = 25^{x+4}$

16. Simplify $\dfrac{a - a^{-1}}{a + 1}$, expressing your answer with positive indices.

Complex unfamiliar

17. If $x = 3^{\frac{1}{3}} + 3^{-\frac{1}{3}}$, show that $x^3 - 3x = \dfrac{10}{3}$.

18. a. Solve the pair of simultaneous equations for x and y.

$$5^{2x-y} = \frac{1}{125}$$
$$10^{2y-6x} = 0.01$$

b. Solve the pair of simultaneous equations for a and k.

$$a \times 2^{k-1} = 40$$
$$a \times 2^{2k-2} = 10$$

19. If $\left(\dfrac{2x^2}{3a}\right)^{n-1} \div \left(\dfrac{3x}{a}\right)^{n+1} = \left(\dfrac{x}{4}\right)^3$, determine the values of the constants a and n.

20. Given that $x^2 = 2^{32}$ and $x^2 = 2^y$, determine the value of y. Assume that $x > 0$.

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Units 1 & 2 Sit chapter test

Answers

Chapter 7 Indices

Exercise 7.2 Index laws

1. a. x^{10} b. m^5p^5 c. $8y^{11}$

2. a. 5^{18} b. x^7y^8 c. $2^{12}x^{18}$ d. $6m^9p^{17}$ e. $5^7x^{10}y^{13}$

3. a. a^5b^3 b. $\frac{a^9b^5}{4}$ c. $\frac{x^3y^9}{2}$ d. $p^{11}q^2$ e. $\frac{8m^2n^5}{7}$

4. a. a^2b^2 b. $\frac{r^{10}t}{5}$ c. $5a^3b^3$ d. $\frac{6x^2y^4}{5}$

5. a. $\frac{4p^{10}m^8}{3}$ b. $\frac{9xy^4}{2}$ c. $18u^{11}v^5$ d. $10e^9f^2$ e. $\frac{2wt^6}{9}$

6. a. x^{13} b. $-81x^9y^3$ c. $3m^4p$ d. mp^2 e. $\frac{u^{11}v^4}{9}$

7. a. $\frac{15}{4}$ b. $\frac{125k^{11}d^6}{24}$ c. $16p^{17}$ d. $27j^3n^2$ e. x^3y^4 f. 12

8. a. $3p^2m^2$ b. $\frac{3x^5}{2}$ c. $-3a^6$

9. a. $x^3y^{1+n}z$ b. x^5y^{6m-1}

10. a. 2^{11} b. 3^{24} c. $5^5 \times 3^4$

11. a. $2^{22} \times 5^8$ b. $\frac{3}{2^4}$ c. $\frac{5}{2}$

12. a. 5 b. $\frac{1}{5}$ c. 16 d. $\frac{1}{3}$

13. a. $2^2 \times 3^{3n+4}$ b. 5^{3n-6} c. 2^{3x-2} d. $2^{-5n-3} \times 3^{-6n}$ e. $2^{n-2} \times 7^{6n-3}$

14. a. $\frac{49}{5}$ b. $3^{9n-3} \times 2^{5n-6}$ c. 3 d. $-\frac{2}{3}$

15. 6^{2n+9}

16. a. 8 b. $59\,049$ c. 16 d. 1

Exercise 7.3 Negative and rational indices

1. a. D b. C c. D

2. a. $\frac{1}{6^3}$ b. $\frac{1}{5^4}$ c. $\frac{5^2}{3^2}$ d. $\frac{4^5}{7^5}$

3. a. 9^2 b. $\frac{1}{64^6}$ c. $\frac{1}{-3}$ d. $\frac{2^{12}}{3^{16}}$

4. a. -2^2 b. $\frac{1}{xy^2}$ c. $\frac{p^2}{m}$

5. a. x^{10} b. $3^8 \times 2^{13}$ c. $\frac{1}{4x^6y^2}$

6. a. 3 b. 3 c. 5 d. 2 e. 4 f. 27 g. 625 h. $\frac{2}{5}$

7. a. $\frac{2}{3}$ b. $\frac{125}{64}$ c. $\frac{9}{16}$ d. $\frac{1}{4}$ e. $\frac{1}{27}$ f. $\frac{9}{4}$ g. $\frac{11}{4}$ h. $\frac{6}{5}$

8. a. $3^{\frac{7}{3}}$ b. $x^{\frac{5}{6}}$ c. $x^{\frac{3}{8}}$ d. $x^{\frac{7}{6}}$ e. $\frac{y^{\frac{1}{2}}}{x^{\frac{2}{3}}}$ f. $2^{\frac{7}{4}}$

9. a. $\frac{1}{2^{\frac{7}{4}}}$ b. $\frac{1}{3^{\frac{2}{3}}}$ c. $\frac{1}{2 \times 3^{\frac{5}{3}}}$ d. x e. $64\,m^{10}$ f. x

10. a. x^2 b. $(x+1)^{1\frac{1}{2}}$ c. $\frac{x-1}{x^{\frac{1}{2}}}$ d. $\frac{2x+2}{(x+2)^{\frac{1}{2}}}$ e. $(y-4)^{1\frac{1}{2}}$ f. $(p+3)^{\frac{3}{5}}$

11. a. i. $a^{\frac{3}{2}}b^2$ ii. $a^{\frac{19}{6}}b^{\frac{7}{3}}$
b. i. $\sqrt{\frac{a}{b^3}}$ ii. $\sqrt{32}$ iii. $\sqrt[5]{\frac{1}{9}}$

12. a. 8 b. 0 c. 162 d. 25

13. a. $\frac{9x^{10}}{y^4}$ b. $\frac{3}{16a^{\frac{5}{3}}b^2}$ c. $\frac{3m^3n^{\frac{1}{2}}}{40}$ d. $\frac{-8n^5}{9}$ e. $\frac{-1}{mn(m+n)}$ f. $\frac{2x-1}{(4x-1)^{\frac{1}{2}}}$

14. 2^{9n}

Exercise 7.4 Indicial equations and scientific notation

1. a. 5 b. 4 c. 5 d. 2 e. -2 f. -3 g. 4 h. 0 i. 2

2. a. $\frac{5}{3}$ b. $-\frac{1}{2}$ c. -1 d. $-\frac{9}{4}$ e. $\frac{11}{6}$ f. $\frac{9}{8}$

3. a. -3 b. $\frac{10}{7}$ c. $-\frac{18}{5}$
d. $\frac{3}{4}$ e. $\frac{21}{11}$ f. $\frac{4}{11}$
g. $-\frac{5}{2}$ h. $\frac{25}{9}$

4. 8

5. a. $x < 2$ b. $x < \frac{10}{3}$

6. a. $\frac{9}{10}$ b. 7 c. $\frac{-5}{7}$
d. $\frac{19}{4}$ e. -3 f. $\frac{-1}{11}$

7. a. $x = 0$ or $x = 1$ b. $x = 2$ or $x = 1$
c. $x = 3$ or $x = 2$ d. $x = 0$ or $x = 1$
e. $x = 0$ or $x = 1$ f. $x = 2$ or $x = 1$

8. a. 2^{5+2x} b. 3^{-7}
c. 10^3 d. 5^n

9. a. $x = 1$ b. $x \leq 6$
c. $\frac{7}{10}$ d. $x > \frac{1}{2}$

10. a. $x = 0$ or $x = 2$ b. $x = -3$
c. $x = 1$ d. $x = 0$
e. $x = 2$ f. $x = -1$

11. a. i. 1.409×10^6; 4 significant figures
ii. 1.306×10^{-4}; 4 significant figures
b. i. 304 000 ii. 0.058 03

12. a. i. -5.06×10^{-8} ii. 1.274×10^4 km
iii. 1.6×10^3 iv. 1.68787×10^4 km
b. i. 63 000.000 63 ii. 1200

13. a. 61 000 b. 0.020
c. -0.0063 d. 27 000 000 km

14. a. $\frac{x^{\frac{5}{3}}}{2y^{\frac{9}{2}}}$
b. i. $-\frac{1}{5}$ ii. -0.1723

7.5 Review: exam practice

1. B
2. C
3. A
4. B
5. D
6. $6a^{\frac{1}{2}}b^2$
7. $\frac{8}{9}$
8. $\frac{4y^2}{3x^4}$
9. a. $50^{\frac{1}{5}}$ b. 5
10. a. $\sqrt{\frac{a}{b^3}}$ b. $\sqrt{32}$
c. $\sqrt[5]{\frac{1}{9}}$
11. 8×10^{10}
12. a. 8 b. 0
13. $\frac{-16pq^8}{m^4}$
14. $x = 4$
15. a. $\frac{1}{2}$ b. 48
16. $\frac{a-1}{a}$
17. $x^3 - 3x = \frac{10}{3}$
18. a. $x = 4, y = 11$ b. $a = 160, k = -1$
19. $a = \pm\left(3^6 \times 2^{-\frac{11}{2}}\right), n = 6$
20. $y = 20$