CHAPTER 1 Arithmetic sequences

1.1 Overview

1.1.1 Introduction

We learn to recognise mathematical patterns and sequences from an early age — perhaps the earliest being the recognition of even and odd numbers — and continue to encounter them throughout life. Being able to recognise and identify sequences allows us to infer certain properties and manipulate them as needed.

One of the most famous numerical patterns is the Fibonacci sequence. This sequence takes the name of an Italian mathematician, Fibonacci (also known as Leonardo of Pisa), who lived around 1200 CE. In the Fibonacci sequence each term (after the first two) is the sum of the two previous terms:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots$$

Fibonacci numbers are significant not only due to their widespread application in mathematics, but perhaps more famously, because they are widely observed in the natural world. Fibonacci numbers describe patterns in nature such as the structure of seashells, the arrangement of petals in a flower and the clustering of seeds in a pine cone.

An interesting, if mysterious, aspect of the Fibonacci sequence is its significance in art and design. For centuries it has been recognised that some shapes are more appealing than others. The ratio of height to width (or width to height) of rectangles that appeal to the eye is $\frac{1+\sqrt{5}}{2} : 1$ (approximately $1.6 : 1$). This ratio is called the 'golden ratio'. As the number of terms in a Fibonacci sequence increases, we find that the ratio between two consecutive numbers approaches the golden ratio.

LEARNING SEQUENCE

Fully worked solutions for this chapter are available in the resources section of your eBookPLUS at www.jacplus.com.au.

1.2 Arithmetic sequences

1.2.1 Defining mathematical sequences

A **sequence** is a related set of objects or events that follow each other in a particular order. Sequences can be found in everyday life, with some examples being:

- the opening share price of a particular stock each day
- the daily minimum temperature readings in a particular city
- the lowest petrol prices each day
- the population of humans counted each year.

When data is collected in the order that the events occur, patterns often emerge. Some patterns can be complicated, whereas others are easy to define.

In mathematics, sequences are always ordered, and the links between different terms of sequences can be identified and expressed using mathematical equations.

You may already be familiar with some mathematical sequences, such as the multiples of whole numbers or the square numbers.

Multiples of 3: 3, 6, 9, 12, …

Multiples of 5: 5, 10, 15, 20, …

Square numbers: 1, 4, 9, 16, …

For each of these patterns there is a link between the numbers in the sequence (known as **terms**) and their position in the sequence (known as the **term number**).

In general, mathematical sequences can be displayed as:

$$t_1,\ t_2,\ t_3,\ t_4,\ t_5 \ldots t_n$$

where t_1 is the first term, t_2 is the second term, and so on. The nth term is referred to as t_n where n represents the ordered position of the term in the sequence, for example 1st, 2nd, 3rd, …

1.2.2 Sequences expressed as functions

If we consider the term numbers in a sequence as the inputs of a **function** then the term values of that sequence are the outputs of that function.

INPUT → OUTPUT

Term number → $\boxed{\text{Function}}$ → Term value

If we are able to define a sequence as a function, then we can input term numbers into that function to determine any term value in the sequence.

WORKED EXAMPLE 1

Determine the first five terms of the sequence $t_n = 2n + 3$.

THINK	WRITE
1. Substitute $n = 1$ into the function.	$t_1 = 2 \times 1 + 3$ $= 5$
2. Substitute $n = 2$ into the function.	$t_2 = 2 \times 2 + 3$ $= 7$

3. Substitute $n = 3$ into the function.

$$t_3 = 2 \times 3 + 3 \\ = 9$$

4. Substitute $n = 4$ into the function.

$$t_4 = 2 \times 4 + 3 \\ = 11$$

5. Substitute $n = 5$ into the function.

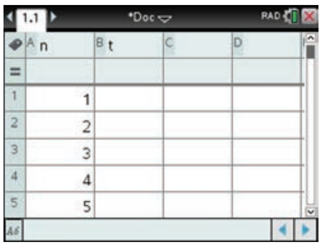
$$t_5 = 2 \times 5 + 3 \\ = 13$$

6. State the answer.

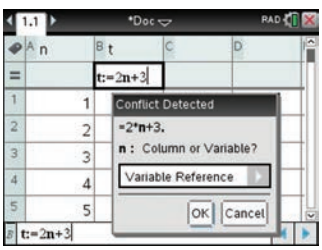
The first five terms of the sequence are 5, 7, 9, 11 and 13.

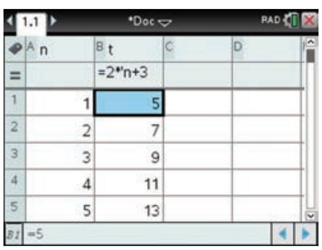
TI | THINK — **WRITE**

1. On a Lists & Spreadsheet page, label the first column as n and the second column as t. Enter the numbers 1 to 5 in the first column.



2. In the function cell below the label t, complete the entry line as
$t:=2n+3$
then press ENTER.
Select the variable reference for n when prompted.
Select OK.



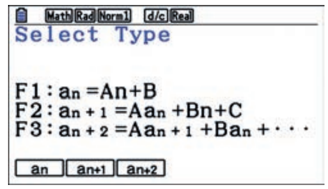


3. The first five terms can be read from the table.

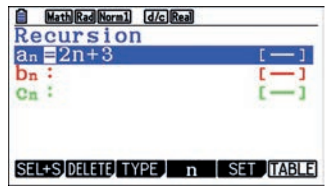
The first five terms of the sequence are 5, 7, 9, 11 and 13.

CASIO | THINK — **WRITE**

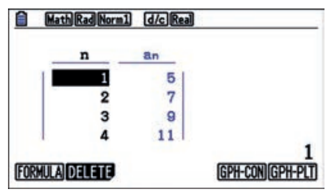
1. On a Recursion screen, select TYPE by pressing F3, then select the first option by pressing F1.



Complete the entry line for a_n as
$a_n = 2n + 3$
then press EXE.



2. Select TABLE by pressing F6.



3. The first five terms can be read from the table.

The first five terms of the sequence are 5, 7, 9, 11 and 13.

1.2.3 Arithmetic sequences

An **arithmetic sequence** is a sequence in which the difference between any two successive terms in the sequence is the same. The next term in an arithmetic sequence can be found by adding or subtracting a fixed value. This fixed value is known as the **common difference** (represented in sequence equations as d).

Consider the arithmetic sequence:

$$4,\ 7,\ 10,\ 13,\ 16,\ 19,\ 22.$$

The difference between each successive term is +3, or similarly, the next term is found by adding 3 to the previous term. We can see that a *positive common difference* gives a sequence that is *increasing*. We say that the common difference is +3, stated as $d = +3$.

$$4 \xrightarrow{+3} 7 \xrightarrow{+3} 10 \xrightarrow{+3} 13 \xrightarrow{+3} 16 \xrightarrow{+3} 19 \xrightarrow{+3} 22$$

The first term of the sequence is 4. We refer to the first term of a sequence as 't_1'. So in this example, $t_1 = 4$. In the arithmetic sequence above, the first term is 4, the second term is 7, the third term is 10, and so on.

Another way of writing this is:

$$t_1 = 4,\ t_2 = 7 \text{ and } t_3 = 10.$$

There are 7 terms in this sequence. Because there are a countable number of terms in the sequence, it is referred to as a *finite sequence*.

The arithmetic sequence below

$$37 \xrightarrow{-7} 30 \xrightarrow{-7} 23 \xrightarrow{-7} 16 \xrightarrow{-7} 9\ldots$$

is an infinite sequence since it continues endlessly as indicated by the dots. The first term, t_1, is 37 and the common difference, d, is -7. We can see that a *negative common difference* gives a sequence that is *decreasing*.

Now consider the sequence 1, 3, 6, 10, 15. This is not an arithmetic sequence, as each term does not increase by the same constant value.

Resources

Interactivity: Terms of an arithmetic sequence (int-6261)

WORKED EXAMPLE 2

Determine which of the following sequences are arithmetic sequences, and for those sequences which are arithmetic, state the values of t_1 and d.

a. **2, 5, 8, 11, 14, …**

b. **4, −1, −6, −11, −16, …**

c. **3, 5, 9, 17, 33, …**

THINK	WRITE
a. 1. Calculate the difference between consecutive terms of the sequence.	**a.** $t_2 - t_1 = 5 - 2 = 3$ $t_3 - t_2 = 8 - 5 = 3$ $t_4 - t_3 = 11 - 8 = 3$ $t_5 - t_4 = 14 - 11 = 3$
2. If the differences between consecutive terms are constant, then the sequence is arithmetic. The first term of the sequence is t_1 and the common difference is d.	The common differences are constant, so the sequence is arithmetic. $t_1 = 2$ and $d = 3$

b. 1. Calculate the difference between consecutive terms of the sequence.

b. $t_2 - t_1 = -1 - 4$
$= -5$
$t_3 - t_2 = -6 - -1$
$= -6 + 1$
$= -5$
$t_4 - t_3 = -11 - -6$
$= -11 + 6$
$= -5$
$t_5 - t_4 = -16 - -11$
$= -16 + 11$
$= -5$

2. If the differences between consecutive terms are constant, then the sequence is arithmetic. The first term of the sequence is t_1 and the common difference is d.

The common differences are constant, so the sequence is arithmetic.
$t_1 = 4$ and $d = -5$

c. 1. Calculate the difference between consecutive terms of the sequence.

c. $t_2 - t_1 = 5 - 3$
$= 2$
$t_3 - t_2 = 9 - 5$
$= 4$
$t_4 - t_3 = 17 - 9$
$= 8$
$t_5 - t_4 = 33 - 17$
$= 16$

2. If the differences between consecutive terms are constant, then the sequence is arithmetic.

The common differences are not constant, so the sequence is not arithmetic.

1.2.4 The recursive definition of arithmetic sequences

Each term of an arithmetic sequence is dependent on the previous terms. We can represent this relationship using a **recursive function** of the form:

$$t_{n+1} = t_n + d$$

where d is the common difference.

WORKED EXAMPLE 3

Determine the recursive function for the following arithmetic sequences.

a. **−5, −1, 3, 7, 11**

b. **81, 63, 45, 27, ...**

THINK	WRITE
a. 1. Find d by calculating the difference between any two terms of the sequence. (Here we use t_1 and t_2, however, any pair of consecutive terms can be used.)	$d = t_2 - t_1$ $= (-1) - (-5)$ $= -1 + 5$ $= 4$
2. Recall the recursive function formula, then substitute d and state the answer.	$t_{n+1} = t_n + d$ The recursive function is $t_{n+1} = t_n + 4$.
b. 1. Find d by calculating the difference between any two terms of the sequence. (Here we use t_1 and t_2, however, any pair of consecutive terms can be used.)	$d = t_2 - t_1$ $= 63 - 81$ $= -18$
2. Recall the recursive function formula, then substitute d into the recursive function, and state the answer.	$t_{n+1} = t_n + d$ The recursive function is $t_{n+1} = t_n - 18$.

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Units 1 & 2 > Area 1 > Sequence 1 > Concept 1

Arithmetic sequences Summary screen and practice questions

Exercise 1.2 Arithmetic sequences

Technology free

1. State which of the following are arithmetic sequences.
   - **a.** 2, 7, 12, 17, 22, ...
   - **b.** 3, 7, 11, 15, 20, ...
   - **c.** 0, 100, 200, 300, 400, ...
   - **d.** −123, −23, 77, 177, 277, ...
   - **e.** 1, 0, −1, −3, −5, ...
   - **f.** 6.2, 9.3, 12.4, 15.5, 16.6, ...
   - **g.** $\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}, \ldots$
   - **h.** $\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}, 2\frac{1}{4}, \ldots$
2. State which of the following situations are arithmetic sequences.

   - **a.** A teacher hands out 2 lollies to the first student, 4 lollies to the second student, 6 lollies to the third student and 8 lollies to the fourth student.
   - **b.** The sequence of numbers after rolling a die 8 times.
   - **c.** The number of layers of paper after each folding in half of a large sheet of paper.
   - **d.** The house numbers on the same side of a street on a newspaper delivery route.
   - **e.** The cumulative total of the number of seats in the first ten rows in a regular cinema (for example, with 8 seats in each row, so there are 8 seats after the first row, 16 seats after the first 2 rows, and so on).
3. Determine the next five terms in the sequence, $-7, 2, 11, 20, 29$.
4. **WE1** Determine the first five terms of the sequence $t_n = 5n + 7$.
5. Determine the first five terms of the sequence $t_n = 3n - 5$.

6. Determine the first five terms of the following arithmetic sequences.

a. $t_n = 5 + 3(n - 1)$
b. $t_n = -1 - 7(n - 1)$
c. $t_n = \frac{1}{3} + \frac{2}{3}(n + 1)$
d. $t_n = 3.3 - 0.7(n + 1)$

7. **WE2** Determine which of the following sequences are arithmetic sequences, and for those sequences that are arithmetic, state the values of t_1 and d.

a. 23, 68, 113, 158, 203, ...
b. 3, 8, 23, 68, 203, ...
c. $\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, \ldots$

8. Consider the following sequence:

$$-3.6, -2.1, -0.6, 0.9, 2.4, \ldots$$

a. Is this a finite or an infinite sequence?
b. Is the sequence increasing or decreasing?
c. State the values of t_1 and d for the sequence.

9. **WE3** Determine the recursive function for the following arithmetic sequences.

a. −1, 3, 7, 11, 15, ...
b. 1.5, −2, −5.5, −8, −11.5, ...
c. $\frac{7}{2}, \frac{11}{2}, \frac{15}{2}, \frac{19}{2}, \frac{23}{2}, \ldots$
d. 6.2, 4.3, 2.4, 0.5, ...

10. Find the missing values in the following arithmetic sequences.

a. 13, −12, −37, f, −87, ...
b. 2.5, j, 8.9, 12.1, k, ...
c. $p, q, r, \frac{9}{2}, \frac{25}{4}, \ldots$
d. $\frac{1}{2}, s, t, 2, \ldots$

11. For each value of t_1 and the corresponding recursive function, determine the first four terms of the following arithmetic sequences.

a. $t_1 = 3,\ t_{n+1} = t_n - 5$
b. $t_1 = -0.6,\ t_{n+1} = t_n + 1.4$
c. $t_1 = -23,\ t_{n+1} = t_n + 32$
d. $t_1 = 10,\ t_n = t_{n-1} - 3$

12. A batsman made 23 runs in his first innings, 33 in his second and 43 in his third. If he continued to add 10 runs each innings, write down a rule for the number of runs he would have made in his nth innings.

13. In a vineyard, rows of wire fences are built to support the vines. The length of the fence in row 1 is 40 m, the length of the fence in row 2 is 43 m, and the length of the fence in row 3 is 46 m. If the lengths of the fences continue in this pattern, write down a rule for the length of a fence in row number n.

14. The first fence post in a fence is 12 m from the road, the next is 15.5 m from the road and the next is 19 m from the road. The rest of the fence posts are spaced in this pattern.

a. Write down a rule for the distance of fence post n from the road.
b. If 100 posts are to be erected, how far will the last post be from the road?

1.3 The general form of an arithmetic sequence

1.3.1 The general term of an arithmetic sequence

Consider the recursive function for arithmetic sequences for values up to $n = 3$:

$$\begin{aligned} n = 1 &: t_2 = t_1 + d \\ n = 2 &: t_3 = t_2 + d \\ n = 3 &: t_4 = t_3 + d \end{aligned}$$

What happens if we substitute the equation for $n = 1$ into the equation for $n = 2$, and then the result of that into the equation for $n = 3$?

$$t_2 = t_1 + d$$

$$\begin{aligned} t_3 &= t_2 + d \quad \text{Substitute } t_2 = t_1 + d \\ &= (t_1 + d) + d \\ &= t_1 + 2d \end{aligned}$$

$$\begin{aligned} t_4 &= t_3 + d \quad \text{Substitute } t_3 = t_1 + 2d \\ &= (t_1 + 2d) + d \\ &= t_1 + 3d \end{aligned}$$

What pattern can we see when we write out t_2, t_3, t_4 in terms of t_1?

$$\begin{aligned} t_3 &= t_1 + d \\ t_3 &= t_1 + 2d \\ t_4 &= t_1 + 3d \end{aligned}$$

We can see that for each t_n on the left-hand side, the coefficient of d on the right-hand side is equal to $n - 1$. This allows us to generalise the following formula for the general term of an arithmetic sequence.

$$\boldsymbol{t_n = t_1 + (n - 1)\,d}$$

Given sufficient information, we can use this formula to determine any term in a sequence, the common difference, or the term number of a given value.

on Resources

Interactivity: Arithmetic sequence (int-6258)

WORKED EXAMPLE 4

Determine the equations that represent the following arithmetic sequences.

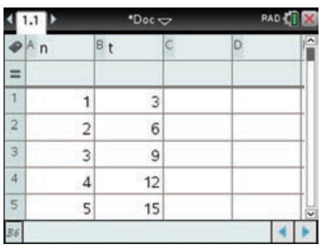
a. **3, 6, 9, 12, 15, …**

b. **40, 33, 26, 19, 12, …**

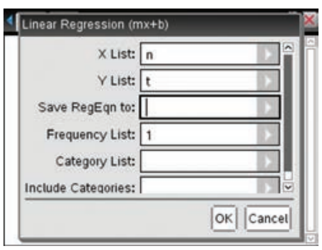
THINK	WRITE
a. 1. Determine the values of t_1 and d.	**a.** $t_1 = 3$ $d = t_2 - t_1$ $= 6 - 3$ $= 3$
2. Substitute the values for t_1 and d into the formula for arithmetic sequences.	$t_n = t_1 + (n - 1)d$ $= 3 + (n - 1) \times 3$ $= 3 + 3(n - 1)$ $= 3 + 3n - 3$ $= 3n$
b. 1. Determine the values of t_1 and d.	**b.** $t_1 = 40$ $d = t_2 - t_1$ $= 33 - 40$ $= -7$
2. Substitute the values for t_1 and d into the formula for arithmetic sequences.	$t_n = t_1 + (n - 1)d$ $= 40 + (n - 1) \times -7$ $= 40 - 7(n - 1)$ $= 40 - 7n + 7$ $= 47 - 7n$

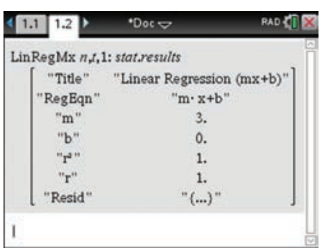
TI | THINK — **WRITE**

1. On a Lists & Spreadsheet page, label the first column as n and the second column as t. Enter the numbers 1 to 5 in the first column, and enter the given terms in the second column.



2. On a Calculator page, press MENU, then select:
6: Statistics
1: Stat Calculations
3: Linear Regression $(mx + b)$ ...
Complete the fields as:
X List: n
Y List: t
then select OK.





3. Interpret the output.

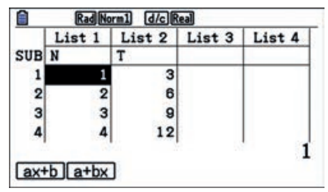
The equation is given in the form $y = mx + b$ where $y = t_n, m = 3, x = n$ and $b = 0$.

4. State the answer.

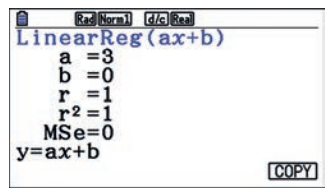
The equation is $t_n = 3n$.

CASIO | THINK — **WRITE**

1. On a Statistics screen, label List 1 as N and List 2 as T. Enter the numbers 1 to 5 in the first column, and enter the given terms in the second column.



2. Select CALC by pressing F2, then select REG by pressing F3. Select X by pressing F1, then select $ax + b$ by pressing F1.



3. Interpret the output.

The equation is given in the form $y = ax + b$ where $y = t_n,\ a = 3, x = n$ and $b = 0$.

4. State the answer.

The equation is $t_n = 3n$.

WORKED EXAMPLE 5

a. **Find the 15th term of the sequence 2, 8, 14, 20, 26, …**

b. **Find the first term of the arithmetic sequence in which $t_{22} = 1008$ and $d = -8$.**

c. **Find the common difference of the arithmetic sequence which has a first term of 12 and an 11th term of 102.**

d. **An arithmetic sequence has a first term of 40 and a common difference of 12. Which term number has a value of 196?**

THINK	WRITE
a. 1. As it has a common difference, this is an arithmetic sequence. State the known values.	**a.** $t_1 = 2, d = 6, n = 15$
2. Substitute the known values into the equation for an arithmetic sequence and solve.	$t_n = t_1 + (n-1)d$ $t_{15} = 2 + (15-1)6$ $= 2 + 14 \times 6$ $= 2 + 84$ $= 86$
3. State the answer.	The 15th term of the sequence is 86.
b. 1. State the known values of the arithmetic sequence.	**b.** $d = -8, n = 22, t_{22} = 1008$
2. Substitute the known values into the equation to determine the first term and solve.	$t_1 = t_n - (n-1)d$ $= 1008 - (22-1)(-8)$ $= 1008 - (21)(-8)$ $= 1008 - -168$ $= 1008 + 168$ $= 1176$
3. State the answer.	The first term of the sequence is 1176.
c. 1. State the known values of the arithmetic sequence.	**c.** $t_1 = 12, n = 11, t_{11} = 102$
2. Substitute the known values into the equation to determine the common difference and solve.	$d = \dfrac{t_n - t_1}{n-1}$ $= \dfrac{102-12}{11-1}$ $= \dfrac{90}{10}$ $= 9$
3. State the answer.	The common difference is 9.
d. 1. State the known values of the arithmetic sequence.	**d.** $t_1 = 40, d = 12, t_n = 196$
2. Substitute the known values into the equation to determine the term number and solve.	$n = \dfrac{t_n - t_1}{d} + 1$ $= \dfrac{196-40}{12} + 1$ $= 14$
3. State the answer.	The 14th term in the sequence has a value of 196.

1.3.2 Graphical display of sequences

We can plot graphs of mathematical sequences by treating the term number as our independent variable (x-values) and the term value as our dependent variable (y-values).

	t_1	t_2	t_3	t_4	t_5	...	t_n
Term number (x-values)	1	2	3	4	5	...	n
Term value (y-values)							

We will see in the following worked example that the graph will be a straight line, which is the case for all arithmetic sequences. The straight line indicates a linear relationship between the term number n and term value t_n. We can use this to find undetermined terms in the sequence.

WORKED EXAMPLE 6

An arithmetic sequence is given by the equation $t_n = 7 + 2(n - 1)$.

a. **Draw up a table of values showing the term number and term value for the first 5 terms of the sequence.**

b. **Plot the graph of the sequence.**

c. **Use your graph of the sequence to determine the 12th term of the sequence.**

THINK

a. 1. Set up a table with the term number in the top row and the term value in the bottom row.

WRITE/DRAW

a.

Term number	1	2	3	4	5
Term value					

2. Substitute the first 5 values of n into the equation to determine the missing values.

$$\begin{aligned} t_1 &= 7 + 2(1 - 1) \\ &= 7 + 2 \times 0 \\ &= 7 + 0 \\ &= 7 \end{aligned}$$

$$\begin{aligned} t_2 &= 7 + 2(2 - 1) \\ &= 7 + 2 \times 1 \\ &= 7 + 2 \\ &= 9 \end{aligned}$$

$$\begin{aligned} t_3 &= 7 + 2(3 - 1) \\ &= 7 + 2 \times 2 \\ &= 7 + 4 \\ &= 11 \end{aligned}$$

$$\begin{aligned} t_4 &= 7 + +2(4 - 1) \\ &= 7 + 2 \times 3 \\ &= 7 + 6 \\ &= 13 \end{aligned}$$

$$\begin{aligned} t_5 &= 7 + 2(5 - 1) \\ &= 7 + 2 \times 4 \\ &= 7 + 8 \\ &= 15 \end{aligned}$$

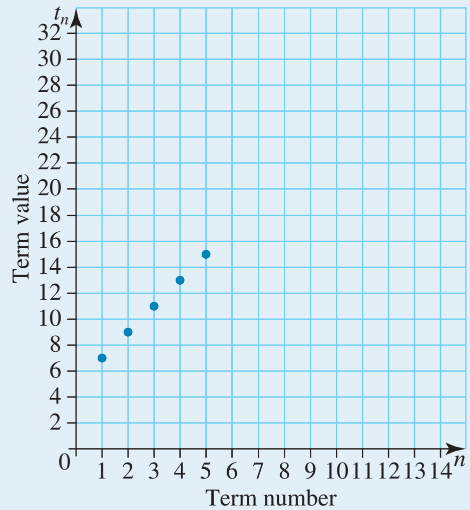
3. Complete the table with the calculated values.

Term number	1	2	3	4	5
Term value	7	9	11	13	15

b. 1. Use the table of values to identify the points to be plotted.

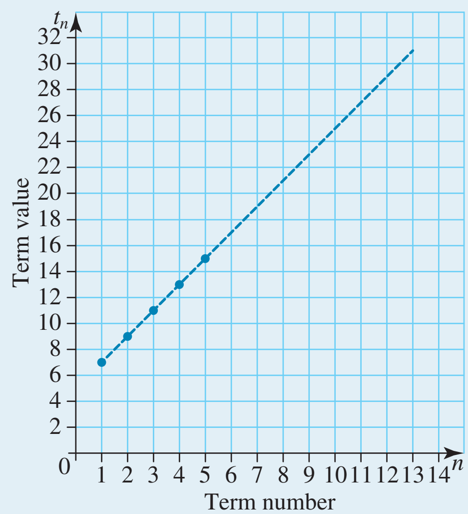
2. Plot the points on the graph.

b. The points to be plotted are (1, 7), (2, 9), (3, 11), (4, 13) and (5, 15).

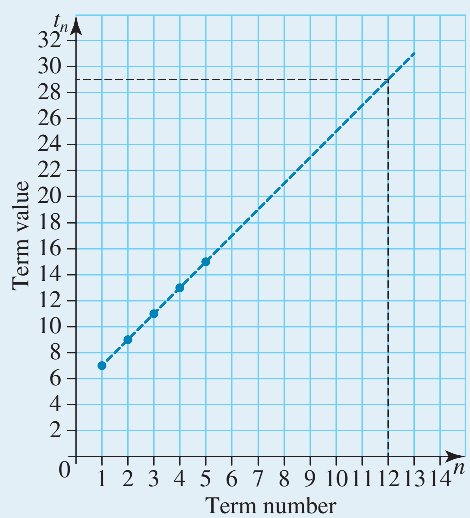


c. 1. Join the points with a dotted line (since the data values are discrete) and extend the line to cover future values of the sequence.

c.



2. Read the required value from the graph (when $n = 12$).



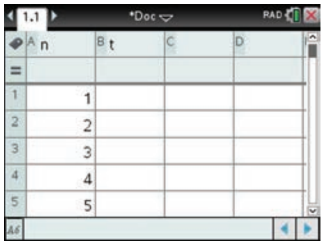
3. Write the answer.

The 12th term of the sequence is 29.

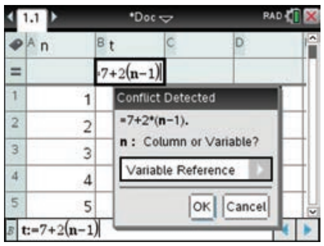
TI | THINK

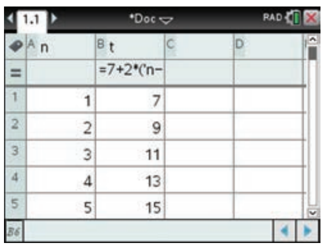
a.1. On a Lists & Spreadsheet page, label the first column as n and the second column as t. Enter the numbers 1 to 5 in the first column.

WRITE



2. In the function cell below the label t, complete the entry line as $t := 7 + 2\,(n - 1)$ then press ENTER. Select the variable reference for n when prompted. Select OK.

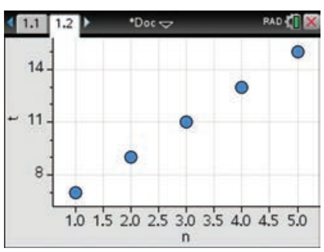




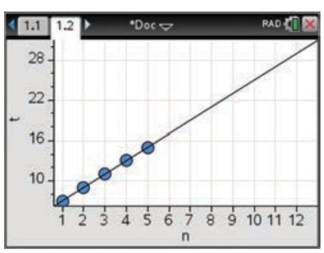
3. The table of values appears on the screen.

Term number	1	2	3	4	5
Term value	7	9	11	13	15

b.1. On a Data & Statistics page, click on the label of the horizontal axis and select n. Click on the label of the vertical axis and select t.



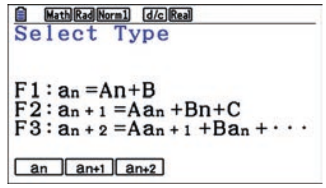
c.1. Press MENU, then select:
4: Analyze
6: Regression
1: Show Linear $(mx + b)$.
Note: the window settings can be changed by pressing MENU, then selecting:
4: Window/Zoom
1: Window Settings.



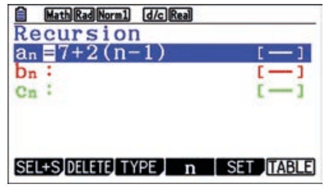
CASIO | THINK

a.1. On a Recursion screen, select TYPE by pressing F3, then select the first option by pressing F1.

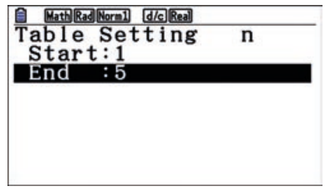
WRITE



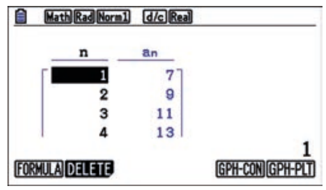
Complete the entry line for a_n as $a_n = 7 + 2\,(n - 1)$ then press EXE.



2. Select SET by pressing F5, then set the start value as 1 and the end value as 5. Press EXE.



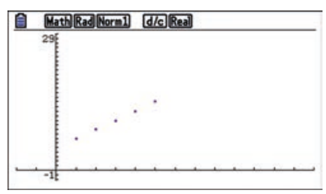
Select TABLE by pressing F6.



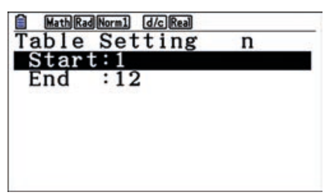
3. The table of values appears on the screen.

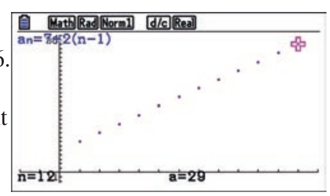
Term number	1	2	3	4	5
Term value	7	9	11	13	15

b.1. Select GPH-PLT by pressing F6.

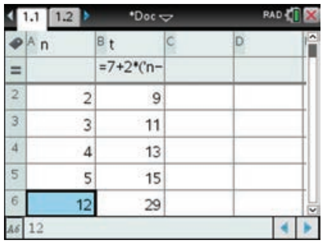


c.1. Select U by pressing F6, then select FORMULA by pressing F1. Select SET by pressing F5 and change the End value to 12. Press EXE. Select TABLE by pressing F6, then select GPH-PLT by pressing F6. Select Trace by pressing F1, then use the left/right arrows to move to the point where $n = 12$.





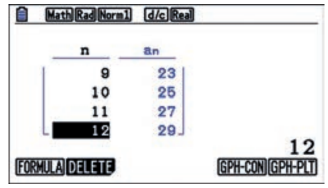
2. Alternatively, return to the Lists & Spreadsheet page, enter the value 12 into cell A6, then press ENTER.



3. The answer can be read from the screen.

The 12th term of the sequence is 29.

2. Alternatively, return to the table by pressing F6 then scroll down to find $n = 12$.



3. The answer can be read from the screen.

The 12th term of the sequence is 29.

studyon

Units 1 & 2 > Area 1 > Sequence 1 > Concept 2

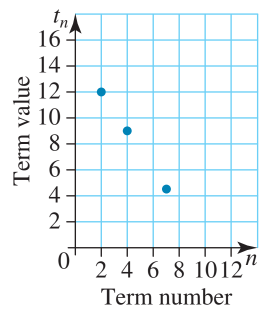
The recursive function and general term of an arithmetic sequence Summary screen and practice questions

Exercise 1.3 The general form of an arithmetic sequence

Technology free

1. **WE4** Determine the equations that represent the following arithmetic sequences.
   a. $4, 13, 22, 31, \ldots$ b. $9, 4.5, 0, \ldots$ c. $-60, -49, -38, \ldots$ d. $100, 87, 74, \ldots$
2. **WE5** a. Write the 20th term of the sequence 85, 72, 59, 46, 33, …
   b. Write the first term of the arithmetic sequence in which $t_{70} = 500$ and $d = -43$.
3. a. Determine the common difference of the arithmetic sequence that has a first term of -32 and an 8th term of 304.
   b. An arithmetic sequence has a first term of 5 and a common difference of 40. Which term number has a value of 85?
   c. An arithmetic sequence has a first term of 40 and a common difference of 12. Which term number has a value of 196?
4. **WE6** An arithmetic sequence is given by the equation $t_n = 5 + 10(n - 1)$.
   a. Draw up a table of values showing the term number and term value for the first 5 terms of the sequence.
   b. Plot the graph of the sequence.
   c. Use your graph of the sequence to determine the 9th term of the sequence.
5. An arithmetic sequence is defined by the equation $t_n = 6.4 + 1.6(n - 1)$.
   a. Draw up a table of values showing the term number and term value for the first 5 terms of the sequence.
   b. Plot the graph of the sequence.
   c. Use your graph of the sequence to determine the 13th term of the sequence.
6. a. Determine the 15th term of the arithmetic sequence 6, 13, 20, 27, 34, …
   b. Determine the 20th term of the arithmetic sequence 9, 23, 37, 51, 65, …
   c. Determine the 30th term of the arithmetic sequence 56, 48, 40, 32, 24, …
   d. Determine the 55th term of the arithmetic sequence $\frac{72}{5}, \frac{551}{40}, \frac{263}{20}, \frac{501}{40}, \frac{119}{10}, \ldots$
7. For each of the arithmetic sequences given, determine:
   a. the 25th term of the sequence 2, 7, 12, 17, 22, ...
   b. the 30th term of the sequence 0, 100, 200, 300, 400, ...
   c. the 33rd term of the sequence 5, -2, -9, -16, -23, ...

8. Evaluate the following.
   a. The 2nd term of an arithmetic sequence is 13 and the 5th term is 31. What is the 17th term of this sequence?
   b. The 2nd term of an arithmetic sequence is -23 and the 5th term is 277. What is the 20th term of this sequence?
   c. The 2nd term of an arithmetic sequence is 0 and the 6th term is -8. What is the 32nd term of this sequence?
   d. The 3rd term of an arithmetic sequence is 5 and the 7th term is -19. What is the 40th term of this sequence?
   e. The 4th term of an arithmetic sequence is 2 and the 9th term is -33. What is the 26th term of this sequence?
9. a. Determine the first value of the arithmetic sequence which has a common difference of 6 and a 31st term of 904.
   b. Determine the first value of the arithmetic sequence which has a common difference of $\frac{2}{5}$ and a 40th term of -37.2.
   c. Determine the common difference of an arithmetic sequence which has a first value of 564 and a 51st term of 54.
   d. Determine the common difference of an arithmetic sequence which has a first value of -87 and a 61st term of 43.
10. a. An arithmetic sequence has a first value of 120 and a common difference of 16. Which term has a value of 712?
    b. An arithmetic sequence has a first value of 320 and a common difference of 4. Which term has a value of 1160?



11. Three consecutive terms of an arithmetic sequence are $x - 5$, $x + 4$ and $2x - 7$. Calculate the value of x.
12. The graph at right shows some points of an arithmetic sequence.
    a. What is the common difference between consecutive terms?
    b. What is the value of the first term of the sequence?
    c. What is the value of the 12th term of the sequence?
13. Sketch the graph of $t_n = a + (n - 1)d$, where $a = 15$ and $d = 25$, for the first 10 terms.
14. An employee starts a new job with a \$60 000 salary in the first year and the promise of a pay rise of \$2500 a year.
    a. How much will her salary be in their 6th year?
    b. How long will it take for her salary to reach \$85 000?

1.4 The sum of an arithmetic sequence

1.4.1 Arithmetic sequences and series

When the terms of an arithmetic sequence are added together, an arithmetic series is formed.

So, 5, 9, 13, 17, 21, ... is an arithmetic sequence,

whereas $5 + 9 + 13 + 17 + 21 + ...$ is an arithmetic series.

The sum of n terms of an arithmetic sequence is given by S_n where

$$S_n = t_1 + t_2 + t_3 + ... + t_{n-1} + t_n$$

Consider the finite arithmetic sequence below.

$$5, 10, 15, 20, 25, 30, 35, 40, 45, 50$$

The sum of this arithmetic sequence is given by S_{10} since there are 10 terms in the sequence. So:

$$S_{10} = 5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 = 275.$$

Note that the sum of the first and last terms is 55. Also, the sum of the second and second-last terms is 55. Similarly, the sum of the third and third-last terms is 55. This pattern continues with the fourth and fourth-last terms as well as with the fifth and fifth-last terms. Thus, S_{10} is made up of five lots of 55.

We can formalise this pattern to obtain a rule that applies to all arithmetic sequences.

As $S_n = t_1 + t_2 + t_3 + ... + t_{n-1} + t_n$,

then $S_n = t_1 + (t_1 + d) + (t_1 + 2d) + ... + (t_n - 2d) + (t_n - d) + t_n$

where t_n is the last term of the sequence and where n represents the number of terms in the sequence.

By reversing the order of the series above, we obtain:

$$S_n = t_n + (t_n - d) + (t_n - 2d) + ... + (t_1 + 2d) + (t_1 + d) + t_1.$$

By adding these two equations, we obtain:

$$2S_n = (t_1 + t_n) + (t_1 + d + t_n - d) + (t_1 + 2d + t_n - 2d) + ... (t_1 + t_n) + (t_1 + t_n) + (t_1 + t_n)$$
$$2S_n = (t_1 + t_n) + (t_1 + t_n) + (t_1 + t_n) + ... (t_1 + t_n) + (t_1 + t_n) + (t_1 + t_n)$$
$$2S_n = n\,(t_1 + t_n)\,.$$

So, $S_n = \dfrac{n(t_n + t_1)}{2}$.

The sum of all arithmetic sequences is represented by:

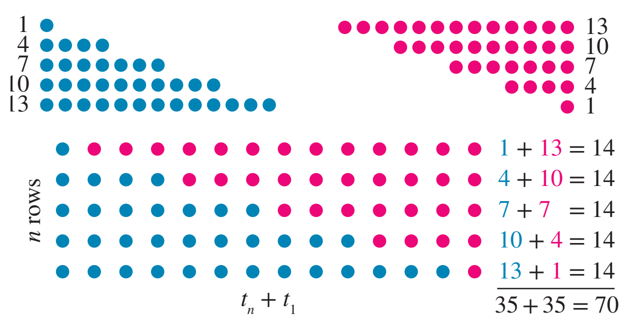
$$\boldsymbol{S_n = \frac{n\,(t_n + t_1)}{2}}$$

1.4.2 A visual explanation of S_n

Consider the arithmetic series below.

$$1 + 4 + 7 + 10 + 13 = 35$$

Let us represent two instances of this summation visually with coloured dots as given below, and join them together to make a rectangle.



Notice that the resulting rectangle has n rows and $t_n + t_1$ columns, and the total number of dots (representing the 'area' of the rectangle) can be found by multiplying them together:

Total number of dots $= n\,(t_n + t_1)$

As we know that the total number of dots is equal to twice the number of dots in the series:

$$2S_n = n\,(t_n + t_1)$$

$$\text{So, } S_n = \frac{n\,(t_n + t_1)}{2}$$

WORKED EXAMPLE 7

a. **Find the sum of the first five terms of the arithmetic sequence represented by $t_n = 1 + 3(n - 1)$.**

b. **If the first term of a sequence is 10, the nth term is 1 and the sum of the first n terms is 22, what is n?**

c. **If the sum of the first six terms of an arithmetic sequence is 132 and $t_6 = 37$, what is the first term t_1?**

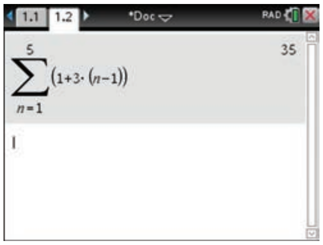
THINK	WRITE
a. 1. State the known values.	$n = 5$
2. Calculate t_1 and t_n.	$t_1 = 1 + 3 \times 0 = 1$ $t_5 = 1 + 3 \times 4 = 1 + 12 = 13$
3. Substitute these values into the rule for S_n and solve for $n = 5$.	$S_5 = \frac{n(t_5 + t_1)}{2} = \frac{5(13+1)}{2} = \frac{5 \times 14}{2} = \frac{70}{2} = 35$
4. State the answer.	The sum of the first five terms of the arithmetic sequence represented by $t_n = 1 + 3(n - 1)$ is 35.
b. 1. State the known values.	$t_1 = 10,\ t_n = 1,\ S_n = 22$
2. Substitute the values into the formula for S_n and solve for n.	$S_n = \frac{n\,(t_n + t_2)}{2}$ $22 = \frac{n\,(1 + 10)}{2}$ $22 \times 2 = n(11)$ $11n = 44$ $n = 4$
3. State the answer.	The number of terms summed together is 4.
c. 1. State the known values.	$t_6 = 37, S_n = 132, n = 6$
2. Substitute the values into the formula for S_n and solve for t_1.	$S_n = \frac{n(t_n + t_1)}{2}$ $132 = \frac{6(37 + t_1)}{2}$ $264 = 6(37 + t_1)$ $44 = t_1 + 37$ $t_1 = 7$

3. State the answer.	The first term of the arithmetic sequence is $t_1 = 7$.

TI | THINK

WRITE

a.1. On a Calculator page, press the t button and select $\sum\square$. Complete the entry line as $\sum_{n=1}^{5}(1+3\times(n-1))$ then press ENTER.



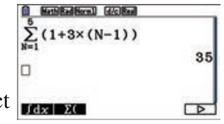
2. The answer appears on the screen.

The sum of the first 5 terms is 35.

CASIO | THINK

WRITE

a.1. On the Run-Matrix screen, select MATH by pressing F4, press F6 to scroll across to more menu options, then select $\sum($ by pressing F2. Complete the entry line as $\sum_{n=1}^{5}(1+3\times(n-1))$ then press EXE.



2. The answer appears on the screen.

The sum of the first 5 terms is 35.

studyon

Unit 1 & 2 > Area 1 > Sequence 1 > Concept 3

The sum of an arithmetic sequence Summary screen and practice questions

Exercise 1.4 The sum of an arithmetic sequence

Technology free

1. Evaluate the sum of the sequence 5, 7, 9, 11 first by using regular addition then by using the formula.
2. **WE7a** Evaluate the sum of the first six terms of the arithmetic sequence represented by $t_n = 2 + 2(n-1)$.
3. Evaluate the sum of the first four terms of the arithmetic sequence represented by $t_n = -4 - 5(n-1)$.
4. **WE7b** If the first term of a sequence is 5, the nth term is -15 and the sum of the first n terms is -30, what is n?
5. If the first term of a sequence is -14, the nth term is 7 and the sum of the first n terms is -14, what is n?
6. **WE7c** If the sum of the first five terms of an arithmetic sequence is 75 and $t_5 = 33$, what is the first term t_1?
7. If the sum of the first seven terms of an arithmetic sequence is 0 and $t_1 = -27$, what is t_7?
8. Evaluate the sum of the first five terms of the arithmetic sequence with first term $t_1 = 20$ and common difference $d = 4$.
9. If the sum of the first five terms of an arithmetic sequence is 55, and the first term is 3, what is t_4?
10. An arithmetic sequence is such that $t_1 = -10$ and $t_4 = 29$. What is the sum of the first five terms of this sequence?
11. If the sum of the first three terms of an arithmetic sequence is 24, and the common difference is $d = 7$, what are the first three terms?

Technology active

12. Sam makes \$100 profit in his first week of business. If his profit increases by \$75 each week, what would his profit be in total by the end of week 15?
13. George's salary is to start at \$36 000 a year and increase by \$1200 each year after that. How much will George have earned in total after 10 years?
14. A staircase is designed so that the height of each step increases by 0.8 cm for each step. If the height of the first step is 15 cm, what is the total height of the first 17 steps?

15. Paula collects stamps. She bought 250 in the first month to start her collection and added 15 stamps to the collection each month thereafter. How many stamps will she have collected after 5 years?

16. Proceeds from the school fete were \$3000 in 2000. In 2001 the proceeds were \$3400 and in 2002 they were \$3800. If they continued in this pattern:
    a. what were the proceeds from the year 2019 fete?
    b. how much in total would the proceeds from school fetes since 2000 have amounted to in the year 2019?

1.5 Applications of arithmetic sequences

1.5.1 Simple interest

If we have a practical situation involving linear growth or decay in discrete steps, this situation can be modelled by an arithmetic sequence.

Simple interest is calculated on the original amount of money invested. It is a fixed amount of interest paid at equal intervals, and as such it can be modelled by an arithmetic sequence.

Remember that simple interest is calculated by using the formula $I = \frac{PrT}{100}$, where I is the amount of simple interest, P is the principal, r is the percentage rate and T is the amount of periods.

WORKED EXAMPLE 8

Jelena puts \$1000 into an investment that earns simple interest at a rate of 0.5% per month.

a. **Set up an equation that represents Jelena's situation as an arithmetic sequence, where t_n is the amount in Jelena's account after n months.**

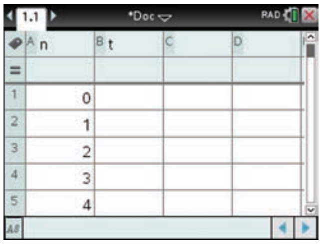
b. **Use your equation from part a to determine the amount in Jelena's account at the end of each of the first 6 months.**

c. **Calculate the amount in Jelena's account at the end of 18 months.**

THINK	WRITE
a. 1. Use the simple interest formula to determine the amount of simple interest Jelena earns in one month.	**a.** $\begin{aligned} I &= \frac{PrT}{100} \\ &= \frac{1000 \times 0.5 \times 1}{100} \\ &= \frac{500}{100} \\ &= 5 \end{aligned}$
2. Calculate the amount in the account after the first month.	$\begin{aligned} t_1 &= 1000 + 5 \\ &= 1005 \end{aligned}$
3. State the known values in the arithmetic sequence equation.	$t_1 = 1005,\ d = 5$
4. Substitute these values into the arithmetic sequence equation.	$t_n = 1005 + 5(n - 1)$
b. 1. Use the equation from part **a** to find the value of t_2, t_3, t_4, t_5 and t_6.	**b.** $\begin{aligned} t_2 &= 1005 + 5(2 - 1) \\ &= 1005 + 5 \times 1 \\ &= 1005 + 5 \\ &= 1010 \\ t_3 &= 1005 + 5(3 - 1) \\ &= 1005 + 2 \times 1 \\ &= 1005 + 10 \\ &= 1015 \\ t_4 &= 1005 + 5(4 - 1) \\ &= 1005 + 5 \times 3 \\ &= 1005 + 15 \\ &= 1020 \\ t_5 &= 1005 + 5(5 - 1) \\ &= 1005 + 5 \times 4 \\ &= 1005 + 20 \\ &= 1025 \\ t_6 &= 1005 + 5(6 - 1) \\ &= 1005 + 5 \times 5 \\ &= 1005 + 25 \\ &= 1030 \end{aligned}$
2. Write the answer.	The amounts in Jelena's account at the end of each of the first 6 months are \$1005, \$1010, \$1015, \$1020, \$1025 and \$1030.
c. 1. Use the equation from part **a** to find the value of t_{18}.	**c.** $\begin{aligned} t_{18} &= 1005 + 5(18 - 1) \\ &= 1005 + 5 \times 17 \\ &= 1005 + 85 \\ &= 1090 \end{aligned}$
2. Write the answer.	After 18 months Jelena has \$1090 in her account.

TI | THINK | **WRITE**

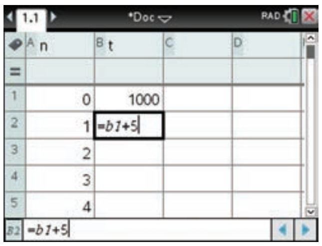
b.1. On a Lists & Spreadsheet page, label the first column as n and the second column as t. Enter the numbers 0 to 6 in the first column.

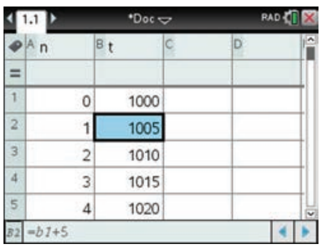


2. Calculate the amount of simple interest earned in 1 month.

$\$1000 \times 5\% = \$5$

3. Enter 1000 into cell B1. In cell B2, complete the entry line as
$= b1 + 5$
then press ENTER.
Highlight cell B2, press MENU, and select:
3: Data
3: Fill.
Click and drag to select cells B3 to B7.

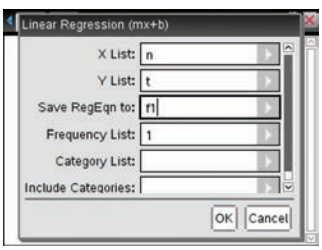


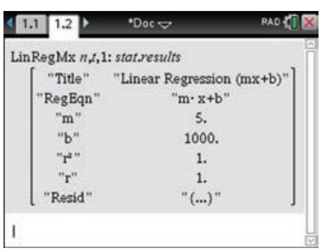


4. The answers can be read from the table.

The amounts in Jelena's account at the end of each of the first 6 months are \$1005, \$1010, \$1015, \$1020, \$1025 and \$1030.

a.1. On a Calculator page, press MENU, then select:
6: Statistics
1: Stat Calculations
3: Linear Regression($mx + b$).
Complete the fields as:
X List: n
Y List: t
Save RegEqn to: f1
then select OK.





2. Interpret the output.

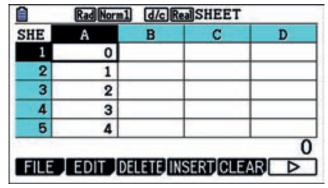
The equation is given in the form $y = mx + b$, where $y = t_n$, $m = 5$, $x = n$ and $b = 1000$.

3. State the answer.

The equation is $t_n = 5n + 1000$.

CASIO | THINK | **WRITE**

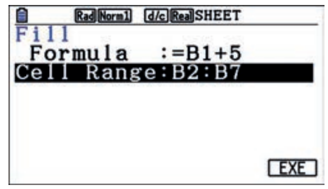
b.1. On a Spreadsheet page, enter the numbers 0 to 6 in the first column.

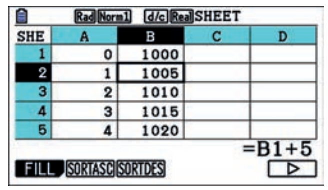


2. Calculate the amount of simple interest earned in 1 month.

$\$1000 \times 5\% = \$5$

3. Enter 1000 into cell B1. Select EDIT by pressing F2, press F6 to scroll across to more menu options, then select FILL by pressing F1.
Complete the fields as
Formula: $= B1 + 5$
Cell Range: $B2 : B7$
then press EXE.

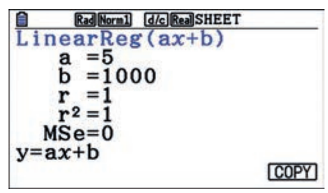




4. The answers can be read from the table.

The amounts in Jelena's account at the end of each of the first 6 months are \$1005, \$1010, \$1015, \$1020, \$1025 and \$1030.

a.1. Press EXIT to return to the original menu items, then press F6 to scroll across to more menu options. Select CALC by pressing F2, then select REG by pressing F3.
Select X by pressing F1, then select $ax + b$ by pressing F1.
Select COPY by pressing F6, then select Y1 by pressing EXE.



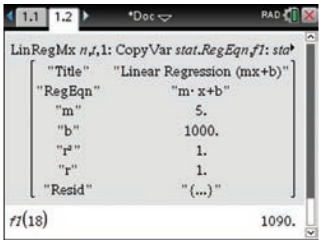
2. Interpret the output.

The equation is given in the form $y = mx + b$, where $y = t_n$, $a = 5$, $x = n$ and $b = 1000$.

3. State the answer.

The equation is $t_n = 5n + 1000$.

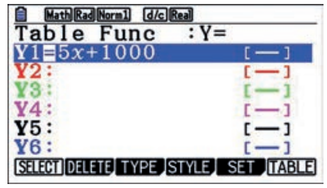
c.1. On the Calculator page, complete the entry line as $f1\ (18)$ then press ENTER.



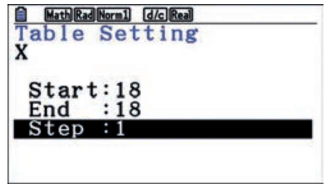
2. The answer appears on the screen.

After 18 months Jelena has \$1090 in her account.

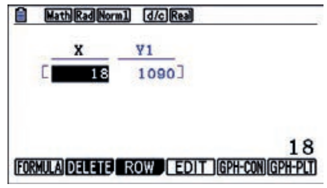
c.1. On a Table screen, highlight Y1 then select SELECT by pressing F1.



Select SET by pressing F5 and set both the Start and End values to18. Press EXE.



Select TABLE by pressing F6.



2. The answer appears on the screen.

After 18 months Jelena has \$1090 in her account.

1.5.2 Depreciating assets

Many items, such as automobiles or electronic equipment, decrease in value over time as a result of wear and tear. At tax time individuals and companies use depreciation of their assets to offset expenses and to reduce the amount of tax they have to pay.

Unit cost depreciation

Unit cost depreciation is a way of depreciating an asset according to its use. For example, you can depreciate the value of a car based on how many kilometres it has driven. The unit cost is the amount of depreciation per unit of use, which would be 1 kilometre of use in the example of the car.

Future value and write-off value

When depreciating the values of assets, companies will often need to know the **future value** of an item. This is the value of that item at that specific time.

The **write-off value** or scrap value of an asset is the point at which the asset is effectively worthless (i.e. has a value of \$0) due to depreciation.

WORKED EXAMPLE 9

Loni purchases a new car for \$25 000 and decides to depreciate it at a rate of \$0.20 per km.

a. Set up an equation to determine the value of the car after *n* km of use.

b. Use your equation from part a to determine the future value of the car after it has 7500 km on its clock.

THINK	WRITE
a. 1. Calculate the value of the car after 1 km of use.	**a.** $a = 25\,000 - 0.2$ $= 24\,999.8$

2. State the known values in the arithmetic sequence equation.

$a = 24\,999.8,\ d = -0.2$

3. Substitute these values into the arithmetic sequence equation.

$$\begin{aligned} t_n &= a + (n-1)d \\ &= 24\,999.8 + (n-1) \times -0.2 \\ &= 24\,999.8 - 0.2(n-1) \end{aligned}$$

b. 1. Substitute $n = 7500$ into the equation determined in part **a**.

b.
$$\begin{aligned} t_n &= 24\,999.8 - 0.2(n-1) \\ t_{7500} &= 24\,999.8 - 0.2(7500-1) \\ &= 24\,999.8 - 0.2 \times 7499 \\ &= 24\,999.8 - 1499.8 \\ &= 23\,500 \end{aligned}$$

2. Write the answer.

After 7500 km the car will be worth \$23 500.

studyon

Units 1 & 2 > Area 1 > Sequence 1 > Concept 4

Modelling with arithmetic sequences Summary screen and practice questions

Exercise 1.5 Applications of arithmetic sequences

Technology active

1. **WE8** Grigor puts \$1500 into an investment account that earns simple interest at a rate of 4.8% per year.
   a. Set up an equation that represents Grigor's situation as an arithmetic sequence, where t_n is the amount in Grigor's account after n months.
   b. Use your equation from part **a** to determine the amount in Grigor's account after each of the first 6 months.
   c. Calculate the amount in Grigor's account at the end of 18 months.
2. Justine sets up an equation to model the amount of her money in a simple interest investment account after n months. Her equation is $t_n = 8050 + 50(n-1)$, where t_n is the amount in Justine's account after n months.
   a. How much did Justine invest in the account?
   b. What is the annual interest rate of the investment?
3. **WE9** Phillipe purchases a new car for \$24 000 and decides to depreciate it at a rate of \$0.25 per km.
   a. Set up an equation to determine the value of the car after n km of use.
   b. Use your equation from part **a** to determine the future value of the car after it has 12 000 km on its clock.

4. Dougie is in charge of the equipment for his office. He decides to depreciate the value of a photocopier at the rate of x cents for every n copies made. Dougie's equation for the value of the photocopier after n copies is $t_n = 5399.999 - 0.001(n-1)$.
   a. How much did the photocopier cost?
   b. What is the rate of depreciation per copy made?

5. Nadia wants to invest her money and decided to place \$90 000 into a credit union account earning simple interest at a rate of 6% per year.

   a. How much interest will Nadia receive after one year?
   b. What is the total amount Nadia has in the credit union after n years?
   c. For how long should Nadia keep her money invested if she wants a total of \$154 800 returned?
6. Tom bought a car for \$23 000, knowing it would depreciate in value by \$210 per month.
   a. What is the value of the car after 18 months?
   b. By how much does the value of the car depreciate in 3 years?
   c. How many months will it take for the car to be valued at \$6200?
7. Sanchia deposits an amount of money in a bank account that earns simple interest every year. After three years she has \$2875 in the account and after five years she has \$3125.
   a. Calculate the yearly interest that Sanchia receives on her deposit.
   b. Determine the amount of money Sanchia originally deposited.
   c. Determine the yearly interest rate as a percentage.
8. A confectionary manufacturer introduces a new sweet and produces 50 000 packets of the sweets in the first week. The stores sell them quickly, and in the following week there is demand for 30% more. In each subsequent week the increase in production is 30% of the original production.

   a. How many packs are manufactured in the 20th week?
   b. In which week will the confectionary manufacturer produce 5 540 000 packs?
9. A canning machine was purchased for a total of \$250 000 and is expected to produce 500 000 000 cans before it is written off.
   a. By how much does the canning machine depreciate with each can made?
   b. If the canning machine were to make 40 200 000 cans each year, when will the machine theoretically be written off?
   c. When will the machine have a book value of \$89 200?
10. Blood donations at a suburban location increased by 40 each year. If there were 520 donations in the first year:
   a. how many donations were made in the 15th year?
   b. what was the total number of donations made over those 15 years?
11. A rock dropped from the top of a high ridge on the Moon's surface falls a distance of 0.8 metres during the first second of its descent. During the next second it falls 2.4 metres; during the third second it falls 4 meters; during the fourth second it falls 5.6 metres. If this arithmetic pattern continues, how far will the rock have fallen in total at the end of the ninth second?
12. Lucca's parents are very pleased with his progress in the Chess club. As an incentive to do well in the local competition, they will give him \$$a$ for getting through the first round, and will increase the amount he gets by \$$d$ for each successive round he gets through. Lucca's total earnings are \$32 after completing the fourth round and his total earnings are \$77 after getting through the seventh round. If the average of the individual amounts he earns in the fourth and seventh rounds equals \$14, what will be the total amount that Lucca's parents will have to pay him if he completes nine rounds?

13. The local rugby club wants to increase its membership. In the first year they had 5000 members, and so far they have managed to increase their membership by 1200 members per year.

a. If the increase in membership continues at the current rate, how many members will they have in 15 years' time?

Tickets for membership in the first year were \$200, and each year the price has risen by a constant amount, with memberships in the 6th year costing \$320.

b. How much will the tickets cost in 15 years' time?

c. What is the total membership income in both the first and 15th years?

14. A newly established quarry produces crushed rock for the building of roads and freeways. The amount of crushed rock, in tonnes, it produces increases by $3\frac{1}{2}$ tonnes each month and its production for the first 3 months of operation is shown in the table below.

Month	Crushed rock produced (tonnes)
1	8
2	11.5
3	15

a. Write down the amount of crushed rock produced in the 4th month.

b. Write down a rule for t_n, the amount of crushed rock produced in month n, expressed in terms of n, the nth month.

c. Write down the amount of crushed rock produced in the 60th month.

d. During which month will the amount of crushed rock coming from the quarry exceed 100 tonnes?

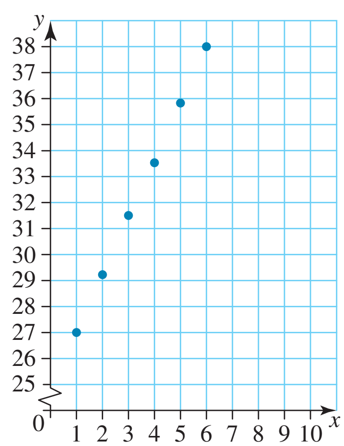
e. The local council has ordered that after a total of 3050 tonnes of crushed rock has been extracted from the quarry, an environmental impact survey must be completed. After how many months will that happen?

1.6 Review: exam practice

A summary of this chapter is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

Simple familiar

1. Which of the following sequences are arithmetic sequences?
   a. 12, 16, 20, 24, 28
   b. 12, 4.8, 1.92, 0.77, 0.31
   c. 12, 15, 21, 30, 42
   d. 12, 6.2, 3.3, 1.88, 0.42
   e. 12, 14.8, 17.6, 20.4, 23.2
2. **MC** For the sequence $-16, -11.2, -6.4, -1.6, 3.2$, the correct values for t_1 and d are:
   A. $t_1 = -16, d = -11.2$
   B. $t_1 = 4.8, d = 3.2$
   C. $t_1 = -16, d = 4.8$
   D. $t_1 = -19.2, d = 3.2$
3. **MC** The missing value in the arithmetic sequence of 65, x, 58, 54.5, 51 is:
   A. -3.5
   B. 68.5
   C. 60
   D. 61.5
4. **MC** The 41st term of the arithmetic sequence $-4.3, -2.1, 0.1, 2.3, 4.5, \ldots$ is:
   A. 83.7
   B. 85.9
   C. 92.3
   D. 172.4
5. **MC** The first term of an arithmetic sequence is 5.2 and the 2nd is 6. The sum of the first 22 terms of the sequence is:
   A. 598.4
   B. 299.2
   C. 242
   D. 137.2
6. Consider the sequence described by the equation $t_n = 2 + 5(n - 1)$.
   a. Determine the first five terms of the sequence.
   b. State the values of t_1 and d for the sequence.
   c. Which term of the sequence will have a value of 72?
7. Construct an equation to represent the following arithmetic sequence: $3, 0.5, -2, -4.5, -7, -9.5$
8. The following graph shows points of an arithmetic sequence.
   a. What is the common difference between consecutive terms?
   b. What is the value of the first term of the sequence?
   c. Determine the equation for this sequence.
   d. What is the value of the 9th term?



9. Determine the sum S_n of the first eight terms in each of the following sequences. You may choose to use technology to answer the question.
   a. $4, 8, 12, \ldots$
   b. $2, -2.5, -7, -11.5, \ldots$
   c. $\frac{1}{2}, 2, 3\frac{1}{2}, \ldots$
10. Write a recursive formula for t_{n+1} in terms of t_n for the sequence $3, 5, 7, \ldots$.
11. List the first five terms of the sequence defined by $t_{n+1} = t_n - 2$ given that $t_1 = 0$.
12. Calculate the depreciated value of a car four years after it was bought new for \$30 000 if it depreciates at a rate of 16% of its original value per year. You may choose a technology of your choice to answer this question.

Complex familiar

13. Chris is saving for his first car. He put \$900 into a simple interest savings account that earns 8.2% per year. You may choose to use technology to answer this question.

  a. Set up an equation that represents Chris's situation as an arithmetic sequence, where t_n is the amount in the account after n months.

  b. Use your equation from part **a** to determine the amount in Chris's account after each of the first 5 months.

  c. Calculate the amount in the savings account at the end of 20 months.

  d. At this interest rate, how many months will it take Chris to save \$1200?

14. A teacher decides to give his 30 students lollies at an end-of-year party. He wishes to give different numbers of lollies to each student based on their attendance record.

He decides to give 1 lolly to the student with the highest number of absences, 3 lollies to the student with the next highest number of absences, 5 to the third highest, then 7, and so on. How many lollies in total will the teacher need to give out to the class?

You may choose to use a technology of your choice to answer questions **15–20**.

15. An accountant has been working with the same company for 15 years. She commenced on a salary of \$28 000 and has received a \$2500 increase each year. Calculate her average wage over this time period.

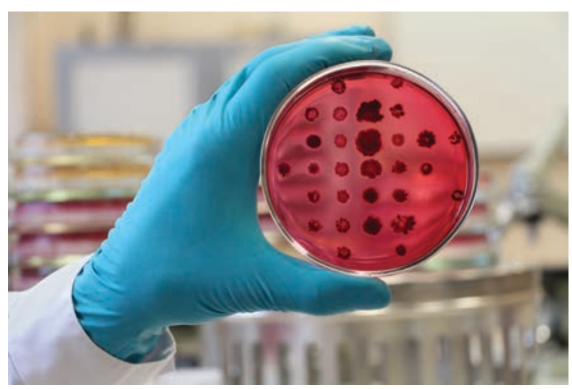
16. Two banks pay simple interest on short-term deposits. Hales Bank pays 8% p.a. over 5 years and Countrybank pays 12% p.a. for 3 years. Which bank will yield the highest return if \$2000 was invested in each account?

Complex unfamiliar

17. Let an arithmetic sequence be such that $t_1 = 3$ and $S_n = 60$. What are the possible values of n that result in integer values for both t_n and d?

18. A biologist is growing a tissue culture in a Petri dish. Between the end of the second day and the end of the fourth day, the culture's mass had increased by 4 milligrams. By the sixth day the culture had a mass of 36 milligrams.

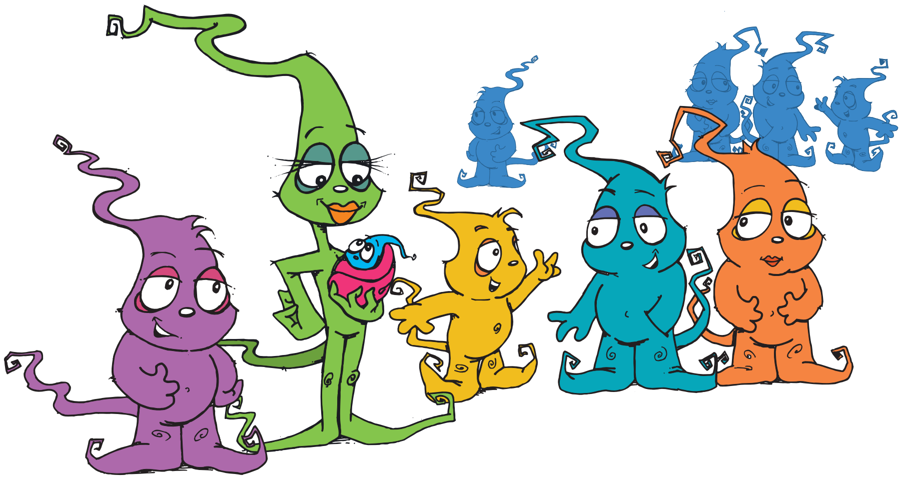
Assuming that the daily growth is arithmetic, on what day will the culture mass first exceed 200 milligrams?



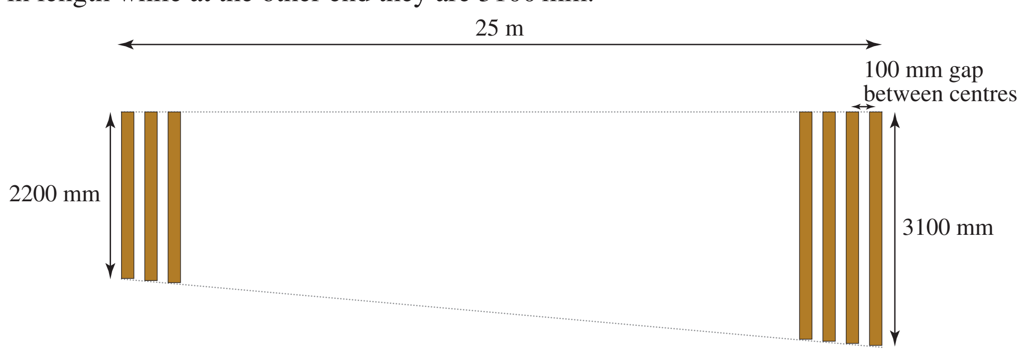
19. The Lunx is a creature that reproduces according to the following rules.

- All mature Lunx produce one baby each year.
- A Lunx becomes mature at 2 years and will produce an offspring at the beginning of its third year.
- Lunx die at the end of their 5th year.

At the end of year 1 the zoo has one Lunx aged 1 year. It will produce its first baby at the beginning of year 3. If the zoo population is left to grow uninhibited, what will the population be at the end of the 24th year?



20. Battens (thin strips of timber) are to be used to enclose the area underneath the house shown in the diagram. The centres of the strips are 100 mm apart. The block slopes so that at one end the battens are 220 mm in length while at the other end they are 3100 mm.



a. Prove that the length of successive battens differs by a constant amount.
b. If the first batten is 2200 mm long, give an expression for the length of the nth batten.
c. How many battens are needed?
d. Determine the combined length of all battens.

study on

Units 1 & 2 — Sit chapter test

Answers

Chapter 1 Arithmetic sequences

Exercise 1.2 Arithmetic sequences

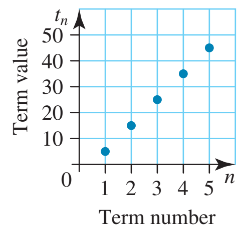
1. **a** is an arithmetic sequence where $d = 5$
   b is not an arithmetic sequence as there is no common difference
   c is an arithmetic sequence where $d = 100$
   d is an arithmetic sequence where $d = 100$
   e is not an arithmetic sequence as there is no common difference
   f is not an arithmetic sequence as there is no common difference
   g is an arithmetic sequence where $d = 1$
   h is an arithmetic sequence where $d = \frac{1}{2}$
2. **a** is an arithmetic sequence where $d = 2$
   b is not an arithmetic sequence as there is no common difference
   c is not an arithmetic sequence as there is no common difference
   d is an arithmetic sequence where $d = 2$
   e is an arithmetic sequence where $d = 8$
3. 38, 47, 56, 65, 74
4. 12, 17, 22, 27, 32
5. -2, 1, 4, 7, 10
6. **a.** 5, 8, 11, 14, 17 **b.** $-1, -8, -15, -22, -29$
   c. $\frac{1}{3}, 1, \frac{5}{3}, \frac{7}{3}, 3$ **d.** 1.9, 1.2, 0.5, -0.2, -0.9
7. **a.** Arithmetic sequence where $t_1 = 23$; $d = 45$
   b. is not an arithmetic sequence as there is no common difference
   c. Arithmetic sequence where $t_1 = \frac{1}{2}$; $d = \frac{1}{4}$
8. **a.** infinite
   b. increasing
   c. $t_1 = -3.6$; $d = 1.5$
9. **a.** $t_{n+1} = t_n + 4$ **b.** $t_{n+1} = t_n - 3.5$
   c. $t_{n+1} = t_n + 2$ **d.** $t_{n+1} = t_n + 1.9$
10. **a.** $f = -62$
    b. $j = 5.7$; $k = 15.3$
    c. $r = \frac{11}{4}$; $q = 1$; $p = -\frac{3}{4}$
    d. $s = 1$; $t = \frac{3}{2}$
11. **a.** 3, -2, -7, -12 **b.** -0.6, 0.8, 2.2, 3.6
    c. -23, 9, 41, 73 **d.** 10, 7, 4, 1
12. $t_n = 13 + 10n$
13. $t_n = 37 + 3n$
14. **a.** $t_n = 8.5 + 3.5n$
    b. 358.5 metres from the road

Exercise 1.3 The general form of an arithmetic sequence

1. **a.** $t_n = 4 + 9(n - 1)$ **b.** $t_n = 9 - 4.5(n - 1)$
   c. $t_n = -60 + 11(n - 1)$ **d.** $t_n = 100 - 13(n - 1)$
2. **a.** -162 **b.** 3467
3. **a.** $d = 48$ **b.** 3 **c.** 14
4. **a.**

Term number	1	2	3	4	5
Term value	5	15	25	35	45

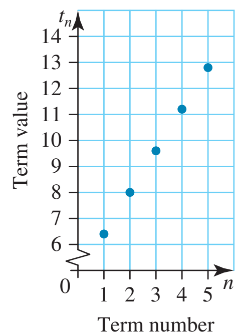
   b.



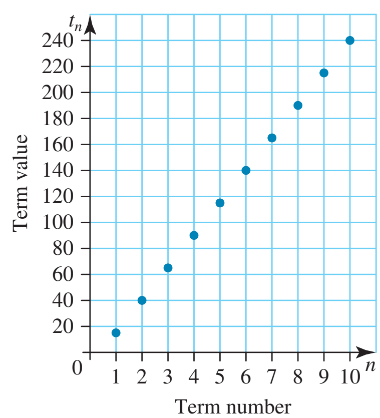
   c. 85
5. **a.**

Term number	1	2	3	4	5
Term value	6.4	8	9.6	11.2	12.8

   b.



   c. 25.6
6. **a.** 104 **b.** 275
   c. -176 **d.** $-\frac{387}{20}$
7. **a.** 122 **b.** 2900 **c.** -219
8. **a.** 103 **b.** 1777 **c.** -60 **d.** -217 **e.** -152
9. **a.** 724 **b.** -52.8
   c. -10.2 **d.** $\frac{13}{6}$
10. **a.** 712 **b.** 1160
11. $x = 20$
12. **a.** -1.5 **b.** 13.5 **c.** -3
13.



14. **a.** \$72 500 **b.** 11 years

Exercise 1.4 The sum of an arithmetic sequence

1. 32
2. 42
3. -46
4. $n = 6$
5. $n = 4$

6. $t_1 = -3$
7. $t_7 = 27$
8. 140
9. $t_4 = 15$
10. 80
11. 1, 8, 15
12. \$9375
13. \$414 000
14. 363.8 cm
15. 1135 stamps
16. a. \$10 600
    b. \$136 000

Exercise 1.5 Applications of arithmetic sequences

1. a. $t_n = 1506 + 6(n-1)$
   b. \$1506, \$1512, \$1518, \$1524, \$1530, \$1536
   c. \$1608
2. a. \$8000 b. 7.5 %
3. a. $t_n = 23\,999.75 - 0.25(n-1)$
   b. \$21 000
4. a. \$5400 b. 0.1 cents
5. a. \$5400
   b. $t_n = 95\,400 + 5400(n-1)$
   c. 12 years
6. a. \$19 220 b. \$7560 c. 80 months
7. a. \$125 b. \$2625 c. 4.76%
8. a. 335 000 packets b. 367th week
9. a. 0.05 cents per can
   b. 13th year
   c. after 8 years
10. a. 1080 b. 12 000
11. 98.4 m
12. \$117
13. a. 21 800 b. \$536 each c. \$11 684 800
14. a. 18.5 tonnes
    b. $t_n = 4.5 + 3.5n$
    c. 214.5 tonnes
    d. 28th month
    e. after 40 months

1.6 Review: exam practice

Simple familiar

1. a is an arithmetic sequence where $d = 4$
   b is not an arithmetic sequence as there is no common difference
   c is not an arithmetic sequence as there is no common difference
   d is not an arithmetic sequence as there is no common difference
   e is an arithmetic sequence where $d = 2.8$
   a and e
2. C
3. D
4. A
5. B
6. a. 2, 7, 12, 17, 22
   b. $t_1 = 2$; $d = 5$
   c. 72
7. $t_n = 3 - 2.5(n-1)$
8. a. 2.2 b. 27
   c. $t_n = 27 + 2.2(n-1)$ d. 44.6
9. a. 144 b. −110 c. 46
10. $t_{n+1} = t_n + 2$ where $t_1 = 3$
11. 0, −2, −4, −6, −8
12. \$10 800

Complex familiar

13. a. $t_n = 906.15 + 6.15(n-1)$
    b. \$906.15, \$912.30, \$918.45, \$924.60, \$930.75
    c. \$1023
    d. 49 months
14. 900 lollies
15. \$45 500
16. Hales Bank
17. $n = 2$, 3 or 4 only
18. day 15
19. 2 Lunx
20. a. Sample responses can be found in the worked solutions in the online resources.
    b. $L_n = 2200 + 3.6(n-1)$
    c. 250
    d. 388.6 m