Chapter 6 — Antidifferentiation

Exercise 6.2 – Antidifferentiation of rational functions

1 a $\int x^7 dx$

$= \frac{x^8}{8} + c$

b $\int (8x^3 + 4x) dx$

$= 8 \times \frac{x^4}{4} + 4 \times \frac{x^2}{2} + c$

$= 2x^4 + 2x^2 + c$

c $\int (3x^2 + 5x - 8) dx$

$= 3 \times \frac{x^3}{3} + 5 \times \frac{x^2}{2} - 8x + c$

$= x^3 + \frac{5}{2}x^2 - 8x + c$

d $\int (2x^3 + 3x^2 - 6x - 9) dx$

$= 2 \times \frac{x^4}{4} + 3 \times \frac{x^3}{3} - 6 \times \frac{x^2}{2} - 9x + c$

$= \frac{x^4}{2} + x^3 - 3x^2 - 9x + c$

2 a $\int (2x + 5) \mathrm{d}x = \frac{2x^2}{2} + 5x + c$

$= x^2 + 5x + c$

b $\int (3x^2 + 4x - 10) \mathrm{d}x = \frac{3x^3}{3} + \frac{4x^2}{2} - 10x + c$

$= x^3 + 2x^2 - 10x + c$

c $\int (10x^4 + 6x^3 + 2) \mathrm{d}x = \frac{10x^5}{5} + \frac{6x^4}{4} + 2x + c$

$= 2x^5 + \frac{3x^4}{2} + 2x + c$

d $\int (-4x^5 + x^3 - 6x^2 + 2x) \mathrm{d}x$

$= \frac{-4x^6}{6} + \frac{x^4}{4} - \frac{6x^3}{3} + \frac{2x^2}{2} + c$

$= \frac{-2}{3}x^6 + \frac{1}{4}x^4 - 2x^3 + x^2 + c$

e $\int (x^3 + 12 - x^2) \mathrm{d}x = \frac{x^4}{4} + 12x - \frac{x^3}{3} + c$

$= \frac{1}{4}x^4 + 12x - \frac{1}{3}x^3 + c$

3 a $\frac{dy}{dx} = 4\sqrt{x} - \frac{1}{x^2}$

$y = \int \left(4x^{\frac{1}{2}} - x^{-2}\right) dx$

$= 4 \times \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{-1}}{-1} + c$

$= \frac{8}{3}x\sqrt{x} + \frac{1}{x} + c$

b $\frac{dy}{dx} = 6\sqrt{x} + \frac{3}{\sqrt{x}} + 8$

$y = \int \left(6x^{\frac{1}{2}} + 3x^{\frac{-1}{2}} + 8\right) dx$

$= 6 \times \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 3 \times \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 8x + c$

$= 4x\sqrt{x} + 6\sqrt{x} + 8x + c$

4 a $\int \frac{x^4}{5} \mathrm{d}x = \frac{1}{25}x^5 + c$

b $\int \frac{x^3}{2} \mathrm{d}x = \frac{1}{8}x^4 + c$

c $\int \frac{x^{-4}}{3} \mathrm{d}x = -\frac{1}{9}x^{-3} + c$

$= -\frac{1}{9x^3} + c$

d $\int \sqrt{x} \mathrm{d}x = \int x^{\frac{1}{2}} \mathrm{d}x$

$= \frac{2}{3}x^{\frac{3}{2}} + c$

e $\int x^{\frac{2}{3}} \mathrm{d}x = \frac{3}{5}x^{\frac{5}{3}} + c$

f $\int 4x^{\frac{3}{4}} \mathrm{d}x = \frac{16}{7}x^{\frac{7}{4}} + c$

5 a $\int x^{\frac{-3}{7}} \mathrm{d}x = \frac{7}{4}x^{\frac{4}{7}} + c$

b $\int \frac{5}{x^3} \mathrm{d}x = \int 5x^{-3} \mathrm{d}x$

$= -\frac{5}{2}x^{-2} + c$

$= -\frac{5}{2x^2} + c$

c $\int \frac{9}{x^2} \mathrm{d}x = \int 9x^{-2} \mathrm{d}x$

$= -9x^{-1} + c$

$= -\frac{9}{x} + c$

d $\int \frac{-10}{x^6} \mathrm{d}x = \int -10x^{-6} \mathrm{d}x$

$= 2x^{-5} + c$

$= \frac{2}{x^5} + c$

e $\int \frac{8}{\sqrt{x}} \mathrm{d}x = \int 8x^{\frac{-1}{2}} \mathrm{d}x$

$= 16x^{\frac{1}{2}} + c$

$= 16\sqrt{x} + c$

f $\int \frac{-6}{x\sqrt{x}} \mathrm{d}x = \int -6x^{\frac{-3}{2}} \mathrm{d}x$

$= 12x^{\frac{-1}{2}} + c$

$= \frac{12}{\sqrt{x}} + c$

6 a $\int (x+3)(x-7)\mathrm{d}x = \int (x^2-4x-21)\,\mathrm{d}x$

$= \frac{x^3}{3} - \frac{4x^2}{2} - 21x + c$

$= \frac{1}{3}x^3 - 2x^2 - 21x + c$

b $\int 5(x^2+2x-1)\mathrm{d}x = 5\int (x^2+2x-1)\,\mathrm{d}x$

$= 5\left(\frac{x^3}{3} + \frac{2x^2}{2} - x + c\right)$

$= \frac{5}{3}x^3 + 5x^2 - 5x + c$

c $\int (x^2+4)(x-7)\mathrm{d}x = \int (x^3-7x^2+4x-28)\,\mathrm{d}x$

$= \frac{x^4}{4} - \frac{7x^3}{3} + \frac{4x^2}{2} - 28x + c$

$= \frac{1}{4}x^4 - \frac{7}{3}x^3 + 2x^2 - 28x + c$

d $\int x(x-1)(x+4)\mathrm{d}x = \int (x^3+3x^2-4x)\,\mathrm{d}x$

$= \frac{x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2} + c$

$= \frac{1}{4}x^4 + x^3 - 2x^2 + c$

7 a $\int \frac{x^2+x^4}{x}\,\mathrm{d}x = \int (x+x^3)\,\mathrm{d}x$

$= \frac{1}{2}x^2 + \frac{1}{4}x^4 + c$

b $\int \frac{x^2+2x-1}{\sqrt{x}}\,\mathrm{d}x = \int \left(\frac{x^2}{x^{\frac{1}{2}}} + \frac{2x}{x^{\frac{1}{2}}} - \frac{1}{x^{\frac{1}{2}}}\right)\mathrm{d}x$

$= \int \left(x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - x^{\frac{-1}{2}}\right)\mathrm{d}x$

$= \frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c$

c $\int \frac{10-x+2x^4}{x^3}\,\mathrm{d}x = \int \left(\frac{10}{x^3} - \frac{x}{x^3} + \frac{2x^4}{x^3}\right)\mathrm{d}x$

$= \int 10x^{-3} - x^{-2} + 2x)\,\mathrm{d}x$

$= -5x^{-2} + x^{-1} + x^2 + c$

8 $f'(x) = x^2 - \frac{1}{x^2}$

$f'(x) = x^2 - x^{-2}$

$f(x) = \int (x^2 - x^{-2})dx$

$f(x) = \frac{1}{3}x^3 + x^{-1} + c$

$f(x) = \frac{1}{3}x^3 + \frac{1}{x} + c$

9 a $\int x^3 dx = \frac{1}{4}x^4 + c$

b $\int \left(7x^2 - \frac{2}{5x^3}\right)dx = \int \left(7x^2 - \frac{2}{5}x^{-3}\right)dx$

$= \frac{7}{3}x^3 + \frac{1}{5}x^{-2} + c$

c $\int (4x^3 - 7x^2 + 2x - 1)dx = x^4 - \frac{7}{3}x^3 + x^2 - x + c$

d $\int \left(2\sqrt{x}\right)^3 dx$

$= \int 8x^{\frac{3}{2}} dx$

$= 8 \times \frac{2}{5}x^{\frac{5}{2}} + c$

$= \frac{16}{5}\sqrt{x^5} + c$

$= \frac{16}{5}x^2\sqrt{x} + c$

10 a $f'(x) = \frac{3}{2}x - 4x^2 + 2x^3$

$f(x) = \int \left(\frac{3}{2}x - 4x^2 + 2x^3\right)dx$

$f(x) = \frac{3}{4}x^2 - \frac{4}{3}x^3 + \frac{1}{2}x^4 + c$

b $\int \left(\frac{3}{\sqrt{x}} - 4x^3 + \frac{2}{5x^3}\right)dx$

$= \int \left(3x^{-\frac{1}{2}} - 4x^3 + \frac{2}{5}x^{-3}\right)dx$

$= 6x^{\frac{1}{2}} - x^4 - \frac{1}{5}x^{-2} + c$

$= 6\sqrt{x} - x^4 - \frac{1}{5x^2} + c$

c $\int x(x-3)(2x+5)dx$

$= \int \left(2x^3 - x^2 - 15x\right)dx$

$= \frac{1}{2}x^4 - \frac{1}{3}x^3 - \frac{15}{2}x^2 + c$

d $\int \frac{3x^3 - x}{2\sqrt{x}}dx$

$= \int \left(\frac{3}{2}x^{\frac{5}{2}} - \frac{1}{2}x^{\frac{1}{2}}\right)dx$

$= \frac{3}{7}x^{\frac{7}{2}} - \frac{1}{3}x^{\frac{3}{2}} + c$

$= \frac{3}{7}x^3\sqrt{x} - \frac{1}{3}x\sqrt{x} + c$

11 a $\int \left(\frac{2}{\sqrt{x}} + \frac{3}{x^2} - \frac{1}{2x^3}\right)dx$

$= \int \left(2x^{-\frac{1}{2}} + 3x^{-2} - \frac{1}{2}x^{-3}\right)dx$

$= 4x^{\frac{1}{2}} - 3x^{-1} + \frac{1}{4}x^{-2} + c$

$= 4\sqrt{x} - \frac{3}{x} + \frac{1}{4x^2} + c$

b $\int (x+1)(2x^2 - 3x + 4)dx$

$= \int \left(2x^3 - x^2 + x + 4\right)dx$

$= \frac{1}{2}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2 + 4x + c$

12 a $\int (2x+3)(3x-2)dx$

$= \int \left(6x^2 + 5x - 6\right)dx$

$= 2x^3 + \frac{5}{2}x^2 - 6x + c$

b $\int \frac{x^3+x^2+1}{x^2}\,dx$

$= \int \left(x+1+x^{-2}\right)dx$

$= \frac{1}{2}x^2 + x - x^{-1} + c$

$= \frac{1}{2}x^2 + x - \frac{1}{x} + c$

c $\int \left(2\sqrt{x} - \frac{4}{\sqrt{x}}\right)dx$

$= \int \left(2x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}\right)dx$

$= 2 \times \frac{2}{3}x^{\frac{3}{2}} - 4 \times 2x^{\frac{1}{2}} + c$

$= \frac{4}{3}x^{\frac{3}{2}} - 8x^{\frac{1}{2}} + c$

$= \frac{4}{3}x\sqrt{x} - 8\sqrt{x} + c$

d $\int \left(x^3 - \frac{2}{x^3}\right)^2 dx$

$= \int \left((x^3)^2 - 2(x^3)\left(\frac{2}{x^3}\right) + \left(\frac{2}{x^3}\right)^2\right)dx$

$= \int \left(x^6 - 4 + 4x^{-6}\right)dx$

$= \frac{1}{7}x^7 - 4x - \frac{4}{5}x^{-5} + c$

$= \frac{1}{7}x^7 - 4x - \frac{4}{5x^5} + c$

13 $\frac{dy}{dx} = x^3 - 3\sqrt{x} = x^3 - 3x^{\frac{1}{2}}$

$y = \frac{1}{4}x^4 - 3 \times \frac{2}{3}x^{\frac{3}{2}} + c$

$y = \frac{1}{4}x^4 - 2x\sqrt{x} + c$

14 $\frac{dy}{dx} = \frac{x^3+3x^2-3}{x^2} = x + 3 - 3x^{-2}$

$y = \frac{1}{2}x^2 + 3x + 3x^{-1} + c$

$y = \frac{1}{2}x^2 + 3x + \frac{3}{x} + c$

15 $\frac{dy}{dx} = \sqrt{x} + \frac{1}{\sqrt{x}} = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

$y = \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

$y = \frac{2}{3}x\sqrt{x} + 2\sqrt{x} + c$

Exercise 6.3 – Antidifferentiation of exponential functions

1 a $\int e^{2x}\,\mathrm{d}x = \frac{1}{2}e^{2x} + c$

b $\int e^{4x}\,\mathrm{d}x = \frac{1}{4}e^{4x} + c$

c $\int e^{-x}\,\mathrm{d}x = -e^{-x} + c$

d $\int e^{-3x}\,\mathrm{d}x = -\frac{1}{3}e^{-3x} + c$

e $\int 5e^{5x}\,\mathrm{d}x = e^{5x} + c$

f $\int 7e^{4x}\,\mathrm{d}x = \frac{7}{4}e^{4x} + c$

2 a $\int e^{\frac{x}{3}}\,\mathrm{d}x = 3e^{\frac{x}{3}} + c$

b $\int 0.1e^{\frac{x}{4}}\,\mathrm{d}x = 0.4e^{\frac{x}{4}} + c$

c $\int 3e^{\frac{x}{2}}\,\mathrm{d}x = 6e^{\frac{x}{2}} + c$

d $\int 3e^{\frac{-x}{3}}\,\mathrm{d}x = -9e^{\frac{-x}{3}} + c$

e $\int e^x + e^{-x}\,\mathrm{d}x = e^x - e^{-x} + c$

f $\int \frac{e^x - e^{-x}}{2}\,\mathrm{d}x = \frac{1}{2}e^x + \frac{1}{2}e^{-x} + c$

3 $\frac{dy}{dx} = (e^x - e^{-x})^2$

$\frac{dy}{dx} = e^{2x} - 2 + e^{-2x}$

$y = \int \left(e^{2x} - 2 + e^{-2x}\right)dx$

$y = \frac{1}{2}e^{2x} - 2x + \frac{1}{-2}e^{-2x} + c$

$y = \frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + c$

4 $\frac{dy}{dx} = \left(1 + e^{2x}\right)^2$

$\frac{dy}{dx} = 1 + 2e^{2x} + e^{4x}$

$y = \int \left(1 + 2e^{2x} + e^{4x}\right)dx$

$y = x + 2 \times \frac{1}{2}e^{2x} + \frac{1}{4}e^{4x} + c$

$y = x + e^{2x} + \frac{1}{4}e^{4x} + c$

5 $\frac{dy}{dx} = \left(e^{3x} + 6\right)^2$

$\frac{dy}{dx} = e^{6x} + 12e^{3x} + 36$

$y = \int \left(e^{6x} + 12e^{3x} + 36\right)dx$

$y = \frac{1}{6}e^{6x} + 12 \times \frac{1}{3}e^{3x} + 36x + c$

$y = \frac{1}{6}e^{6x} + 4e^{3x} + 36x + c$

6 a $\int \left(x^4 - e^{-4x}\right)dx = \frac{1}{5}x^5 + \frac{1}{4}e^{-4x} + c$

b $\int \left(\frac{1}{2}e^{2x} - \frac{2}{3}e^{-\frac{x}{2}}\right)dx = \frac{1}{4}e^{2x} + \frac{4}{3}e^{-\frac{x}{2}} + c$

7 a $\int \frac{e^{2x} + 3e^{-5x}}{2e^x}\,dx = \int \left(\frac{1}{2}e^x + \frac{3}{2}e^{-6x}\right)dx$

$= \frac{1}{2}e^x - \frac{1}{4}e^{-6x} + c$

b $\int \left(e^x - e^{2x}\right)^2 dx$

$= \int \left((e^x)^2 - 2(e^x)(e^{2x}) + (e^{2x})^2\right)dx$

$= \int \left(e^{2x} - 2e^{3x} + e^{4x}\right)\,dx$

$= \frac{1}{2}e^{2x} - \frac{2}{3}e^{3x} + \frac{1}{4}e^{4x} + c$

8 $\int\left(e^{\frac{x}{2}}-\frac{1}{e^x}\right)^2 dx$

$=\int\left(\left(e^{\frac{x}{2}}\right)^2-2\left(e^{\frac{x}{2}}\right)\left(\frac{1}{e^x}\right)+\left(\frac{1}{e^x}\right)^2\right)dx$

$=\int\left(e^x-2e^{-\frac{x}{2}}+e^{-2x}\right)dx$

$=e^x+4e^{-\frac{x}{2}}-\frac{1}{2}e^{-2x}+c$

$=e^x+\frac{4}{e^{\frac{x}{2}}}-\frac{1}{2e^{2x}}+c$

9 $f'(x)=4e^{2x}+8$

$f(x)=\int\left(4e^{2x}+8\right)$

$f(x)=4\times\frac{1}{2}e^{2x}+8x+c$

$f(x)=2e^{2x}+8x+c$

10 $\frac{dy}{dx}=e^{2x}\left(e^{2x}-e^{-2x}\right)$

$\frac{dy}{dx}=e^{4x}-1$

$y=\int\left(e^{4x}-1\right)dx$

$y=\frac{1}{4}e^{4x}-x+c$

11 $\frac{dy}{dx}=6e^{3x}+9x^2-2\sqrt{e^x}$

$\frac{dy}{dx}=6e^{3x}+9x^2-2e^{\frac{x}{2}}$

$y=\int\left(6e^{3x}+9x^2-2e^{\frac{x}{2}}\right)dx$

$y=6\times\frac{1}{3}e^{3x}+9\times\frac{x^3}{3}-2\times\frac{1}{\frac{1}{2}}e^{\frac{x}{2}}+c$

$y=2e^{3x}+3x^3-4e^{\frac{x}{2}}+c$

12 $\int\left(e^{2x}-e^{-3x}\right)^3 dx$

$=\int\left(\left(e^{2x}\right)^3-3\left(e^{2x}\right)^2\left(e^{-3x}\right)+3\left(e^{2x}\right)\left(e^{-3x}\right)^2-\left(e^{-3x}\right)^3\right)dx$

$=\int\left(e^{6x}-3e^{x}+3e^{-4x}-e^{-9x}\right)dx$

$=\frac{1}{6}e^{6x}-3e^x-\frac{3}{4}e^{-4x}+\frac{1}{9}e^{-9x}+c$

13 $f'(x)=4e^{-2x}+k$

a stationary point at $x=0$, therefore $f'(0)=0$

$f'(0)=4e^0+k$

$4+k=0$

$k=-4$

b $f'(x)=4e^{-2x}-4$

$f(x)=4\times\frac{1}{-2}e^{-2x}-4x+c$

$f(x)=-2e^{-2x}-4x+c$

14 $\int ae^{bx}dx=-3e^{3x}+c$

$\frac{a}{b}e^{bx}+c=-3e^{3x}+c$

$b=3$ and $\frac{a}{3}=-3$

$a=-9$

15 $\int\left(me^{nx}+px+q\right)dx$

$=m\times\frac{1}{n}e^{nx}+p\times\frac{x^2}{2}+qx+c$

$=\frac{m}{n}e^{nx}+\frac{p}{2}x^2+qx+c$

Given that $\int(me^{nx}+px+q)\,dx=5e^{2x}+2x^2-3x+c$:

Then $\frac{m}{n}e^{nx}+\frac{p}{2}x^2+qx+c=5e^{2x}+2x^2-3x+c$.

Equating: $\frac{m}{n}=5, n=2, \frac{p}{2}=2, q=-3$

$m=10, n=2, p=4, q=-3$

Exercise 6.4 – Antidifferentiation of logarithmic functions

1 a $\int\frac{3}{x}\,\mathrm{d}x=3\int\frac{1}{x}\mathrm{d}x$

$=3\ln(x)+c$

b $\int\frac{8}{x}\,\mathrm{d}x=8\int\frac{1}{x}\mathrm{d}x$

$=8\ln(x)+c$

c $\int\frac{6}{5x}\,\mathrm{d}x=\frac{6}{5}\int\frac{1}{x}\mathrm{d}x$

$=\frac{6}{5}\ln(x)+c$

d $\int\frac{7}{3x}\,\mathrm{d}x=\frac{7}{3}\int\frac{1}{x}\mathrm{d}x$

$=\frac{7}{3}\ln(x)+c$

e $\int\frac{4}{7x}\,\mathrm{d}x=\frac{4}{7}\int\frac{1}{x}\mathrm{d}x$

$=\frac{4}{7}\ln(x)+c$

2 a $\int\frac{1}{x+3}\mathrm{d}x=\ln(x+3)+c$

b $\int\frac{3}{x+3}\,\mathrm{d}x=3\int\frac{1}{x+3}\mathrm{d}x$

$=3\ln(x+3)+c$

c $\int\frac{-2}{x+4}\,\mathrm{d}x=-2\int\frac{1}{x+4}\mathrm{d}x$

$=-2\ln(x+4)+c$

d $\int\frac{-6}{x+5}\,\mathrm{d}x=-6\int\frac{1}{x+5}\mathrm{d}x$

$=-6\ln(x+5)\,\mathrm{d}x$

e $\int\frac{4}{3x+2}\,\mathrm{d}x=4\int\frac{1}{3x+2}\mathrm{d}x$

$=\frac{4}{3}\ln(3x+2)+c$

3 a $\int\frac{8}{5x+6}\,\mathrm{d}x=8\int\frac{1}{5x+6}\mathrm{d}x$

$=\frac{8}{5}\ln(5x+6)+c$

b $\int\frac{3}{2x-5}\,\mathrm{d}x=3\int\frac{1}{2x-5}\mathrm{d}x$

$=\frac{3}{2}\ln(2x-5)+c$

c $\int \frac{-5}{3+2x}\,dx = -5\int \frac{1}{3+2x}dx$
$= \frac{-5}{2}\ln(3+2x) + c$

d $\int \frac{-2}{6+7x}\,dx = -2\int \frac{1}{6+7x}dx$
$= \frac{-2}{7}\ln(6+7x) + c$

4 a $\int \frac{1}{5-x}dx = -\ln(5-x) + c$

b $\int \frac{3}{6-11x}\,dx = 3\int \frac{1}{6-11x}dx$
$= \frac{3}{-11}\ln(6-11x) + c$

c $\int \frac{-2}{4-3x}\,dx = -2\int \frac{1}{4-3x}dx$
$= \frac{-2}{-3}\ln(4-3x) + c$
$= \frac{2}{3}\ln(4-3x) + c$

d $\int \frac{-8}{5-2x}\,dx = -8\int \frac{1}{5-2x}dx$
$= 4\ln(5-2x) + c$

5 $\int \frac{(2x+5)^2}{x}dx$
$= \int \frac{4x^2+20x+25}{x}dx$
$= \int \left(\frac{4x^2}{x} + \frac{20x}{x} + \frac{25}{x}\right)dx$
$= \int \left(4x + 20 + \frac{25}{x}\right)dx$
$= 4 \times \frac{x^2}{2} + 20x + 25\ln(x) + c$
$= 2x^2 + 20x + 25\ln(x) + c$

6 $\int \frac{(3x+2)^2}{x^2}dx$
$= \int \frac{9x^2+12x+4}{x^2}dx$
$= \int \left(\frac{9x^2}{x^2} + \frac{12x}{x^2} + \frac{4}{x^2}\right)dx$
$= \int \left(9 + \frac{12}{x} + 4x^{-2}\right)dx$
$= 9x + 12\ln(x) + 4 \times \frac{x^{-1}}{-1} + c$
$= 9x + 12\ln(x) - \frac{4}{x} + c$

7 a $\int \frac{3-4x}{x}dx$
$= \int \left(\frac{3}{x} - \frac{4x}{x}\right)dx$
$= \int \left(\frac{3}{x} - 4\right)dx$
$= 3\ln(x) - 4x + c$

b $\int \frac{2x^2-3x+4}{x^2}dx$
$= \int \left(\frac{2x^2}{x^2} - \frac{3x}{x^2} + \frac{4}{x^2}\right)dx$
$= \int \left(2 - \frac{3}{x} + 4x^{-2}\right)dx$
$= 2x - 3\ln(x) + 4 \times \frac{x^{-1}}{-1} + c$
$= 2x - 3\ln(x) - \frac{4}{x} + c$

c $\int \frac{(4-3x)^2}{2x}dx$
$= \int \frac{16-24x+9x^2}{2x}dx$
$= \int \left(\frac{16}{2x} - \frac{24x}{2x} + \frac{9x^2}{2x}\right)dx$
$= \int \left(\frac{8}{x} - 12 + \frac{9}{2}x\right)dx$
$= 8\ln(x) - 12x + \frac{9}{2} \times \frac{x^2}{2} + c$
$= 8\ln(x) - 12x + \frac{9}{4}x^2 + c$

d $\int \frac{9+\sqrt{x}}{x}dx$
$= \int \left(\frac{9}{x} + \frac{\sqrt{x}}{x}\right)dx$
$= \int \left(\frac{9}{x} + x^{\frac{-1}{2}}\right)dx$
$= 9\ln(x) + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$
$= 9\ln(x) + 2\sqrt{x} + c$

8 $f'(x) = x - \frac{4}{x}$
$f(x) = \int \left(x - \frac{4}{x}\right)dx$
$f(x) = \frac{1}{2}x^2 - 4\ln(x) + c$

9 $\frac{dy}{dx} = 2x + 3 - \frac{4}{5-x}$
$y = \int \left(2x + 3 - \frac{4}{5-x}\right)dx$
$y = \int (2x+3)\,dx - 4\int \frac{1}{5-x}dx$
$y = 2 \times \frac{x^2}{2} + 3x - 4 \times \frac{1}{-1}\ln(5-x) + c$
$y = x^2 + 3x + 4\ln(5-x) + c$

10 $\frac{dy}{dx} = x\left(1 - \frac{1}{x}\right)^2$
$y = \int x\left(1 - \frac{1}{x}\right)^2 dx$
$y = \int x\left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx$
$y = \int \left(x - 2 + \frac{1}{x}\right)dx$
$y = \frac{1}{2}x^2 - 2x + \ln(x) + c$

11 a $1 - \frac{4}{x+1}$
$= \frac{x+1-4}{x+1}$
$= \frac{x-3}{x+1}$
$\therefore \frac{x-3}{x+1} = 1 - \frac{4}{x+1}$

b $\int \frac{x-3}{x+1}dx$

$= \int \left(1 - \frac{4}{x+1}\right) dx$

$= x - 4\ln(x+1) + c$

12 a $2 + \frac{1}{x-3}$

$= \frac{2(x-3)+1}{x-3}$

$= \frac{2x-6+1}{x-3}$

$= \frac{2x-5}{x-3}$

$\therefore \frac{2x-5}{x-3} = 2 + \frac{1}{x-3}$

b $\int \frac{2x-5}{x-3}dx$

$= \int \left(2 + \frac{1}{x-3}\right) dx$

$= 2x + \ln(x-3) + c$

13 a $x + 6 + \frac{16}{x-2}$

$= \frac{x(x-2)+6(x-2)+16}{x-2}$

$= \frac{x^2-2x+6x-12+16}{x-2}$

$= \frac{x^2+4x+4}{x-2}$

$= \frac{(x+2)^2}{x-2}$

$\therefore \frac{(x+2)^2}{x-2} = x + 6 + \frac{16}{x-2}$

b $\int \frac{(x+2)^2}{x-2}dx$

$= \int \left(x + 6 + \frac{16}{x-2}\right) dx$

$= \frac{1}{2}x^2 + 6x + 16\ln(x-2) + c$

14 $\int \left(\frac{a}{bx+3}\right) dx$

$= a\int \frac{1}{bx+3}dx$

$= a \times \frac{1}{b}\ln(bx+3) + c$

$= \frac{a}{b}\ln(bx+3) + c$

If $\int \frac{a}{bx+3}dx = 6\ln(2x+3) + c$

then $\frac{a}{b}\ln(bx+3) + c = 6\ln(2x+3) + c$

equate: $\frac{a}{b} = 6, b = 2$

$\therefore a = 12, b = 2$

15 a $f'(x) = \frac{k}{2x+3}$

when $x = 1, f'(1) = 2$

$f'(1) = \frac{k}{2+3}$

$2 = \frac{k}{5}$

$k = 10$

b $f'(x) = \frac{10}{2x+3}$

$f(x) = \int \frac{10}{2x+3}dx$

$f(x) = 10\int \frac{1}{2x+3}dx$

$f(x) = 10 \times \frac{1}{2}\ln(2x+3) + c$

$f(x) = 5\ln(2x+3) + c$

Exercise 6.5 – Antidifferentiation of sine and cosine functions

1 a $\int \sin(3x)\,dx = -\frac{1}{3}\cos(3x) + c$

b $\int \sin(4x)\,dx = -\frac{1}{4}\cos(4x) + c$

c $\int \cos(7x)\,dx = \frac{1}{7}\sin(7x) + c$

d $\int \frac{\cos(2x)}{3}dx = \frac{1}{6}\sin(2x) + c$

e $\int \sin(-2x)\,dx = \frac{1}{2}\cos(-2x) + c$

f $\int \cos(-3x)\,dx = -\frac{1}{3}\sin(-3x) + c$

2 a $\int \frac{4\sin(6x)}{3}\,dx = -\frac{2}{9}\cos(6x) + c$

b $\int 8\sin(4x)\,dx = -2\cos(4x) + c$

c $\int -6\sin(3x)\,dx = 2\cos(3x) + c$

d $\int -2\cos(-x)\,dx = 2\sin(-x) + c$

e $\int \sin\left(\frac{x}{3}\right)dx = -3\cos\left(\frac{x}{3}\right) + c$

f $\int \cos\left(\frac{x}{2}\right)dx = 2\sin\left(\frac{x}{2}\right) + c$

3 a $\int 3\sin\left(\frac{-x}{4}\right)dx = 12\cos\left(\frac{-x}{4}\right) + c$

b $\int -2\sin\left(\frac{x}{5}\right)dx = 10\cos\left(\frac{x}{5}\right) + c$

c $\int 4\cos\left(\frac{x}{4}\right)dx = 16\sin\left(\frac{x}{4}\right) + c$

d $\int -6\cos\left(\frac{-x}{2}\right)dx = 12\sin\left(\frac{-x}{2}\right) + c$

e $\int 4\sin\left(\frac{2x}{3}\right)dx = -6\cos\left(\frac{2x}{3}\right) + c$

f $\int 6\cos\left(\frac{3x}{4}\right)dx = 8\sin\left(\frac{3x}{4}\right) + c$

4 a $\int e^{4x} + \sin(2x) + x^3\,dx = \frac{1}{4}e^{4x} - \frac{1}{2}\cos(2x) + \frac{1}{4}x^4 + c$

b $\int 3x^2 - 2\cos(2x) + 6e^{3x}\,dx = x^3 - \sin(2x) + 2e^{3x} + c$

5 a $\int \sin(x) + \cos(x)\,dx = -\cos(x) + \sin(x) + c$

$= \sin(x) - \cos(x) + c$

b $\int \sin(2x) - \cos(x)\,dx = \frac{-1}{2}\cos(2x) - \sin(x) + c$

c $\int \cos(4x) + \sin(2x)\,dx = \frac{1}{4}\sin(4x) - \frac{1}{2}\cos(2x) + c$

d $\int \sin\left(\frac{x}{2}\right) - \cos(2x)\,dx = -2\cos\left(\frac{x}{2}\right) - \frac{1}{2}\sin(2x) + c$

6 a $\int 4\cos(4x) - \frac{1}{3}\sin(2x)\,dx = \sin(4x) + \frac{1}{6}\cos(2x) + c$

b $\int 5x + 2\sin(x)\,dx = \frac{5x^2}{2} - 2\cos(2x) + c$

c $\int 3\sin\left(\frac{\pi x}{2}\right) + 2\cos\left(\frac{\pi x}{3}\right)dx$

$= \frac{-6}{\pi}\cos\left(\frac{\pi x}{2}\right) + \frac{6}{\pi}\sin\left(\frac{\pi x}{3}\right) + c$

$= \frac{6}{\pi}\left(\sin\left(\frac{\pi x}{3}\right) - \cos\left(\frac{\pi x}{2}\right)\right) + c$

d $\int 3e^{6x} - 4\sin(8x) + 7\,dx = \frac{1}{2}e^{6x} + \frac{1}{2}\cos(8x) + 7x + c$

7 a $\int\left(\frac{1}{2}\cos(3x+4) - 4\sin\left(\frac{x}{2}\right)\right)dx = \frac{1}{6}\sin(3x+4)$

$+ 8\cos\left(\frac{x}{2}\right) + c$

b $\int\left(\cos\left(\frac{2x}{3}\right) - \frac{1}{4}\sin(5-2x)\right)dx$

$= \frac{3}{2}\sin\left(\frac{2x}{3}\right) - \frac{1}{8}\cos(5-2x) + c$

8 a $\int\left(\sin\left(\frac{x}{2}\right) - 3\cos\left(\frac{x}{2}\right)\right)dx = -2\cos\left(\frac{x}{2}\right)$

$- 6\sin\left(\frac{x}{2}\right) + c$

b $f'(x) = 7\cos(2x) - \sin(3x)$

$f(x) = \frac{7}{2}\sin(2x) + \frac{1}{3}\cos(3x) + c$

9 a $\int\left(e^{\frac{x}{3}} + \sin\left(\frac{x}{3}\right) + \frac{x}{3}\right)dx = 3e^{\frac{x}{3}} - 3\cos\left(\frac{x}{3}\right) + \frac{1}{6}x^2 + c$

b $\int\left(\cos(4x) + 3e^{-3x}\right)dx = \frac{1}{4}\sin(4x) - e^{-3x} + c$

10 $\int\left(\frac{1}{4x^2} + \sin\left(\frac{3\pi x}{2}\right)\right)dx$

$= \int\left(\frac{1}{4}x^{-2} + \sin\left(\frac{3\pi x}{2}\right)\right)dx$

$= -\frac{1}{4}x^{-1} - \frac{2}{3\pi}\cos\left(\frac{3\pi x}{2}\right) + c$

$= -\frac{1}{4x} - \frac{2}{3\pi}\cos\left(\frac{3\pi x}{2}\right) + c$

11 $\frac{dy}{dx} = \cos(2x) - e^{-3x}$

$y = \frac{1}{2}\sin(2x) + \frac{1}{3}e^{-3x} + c$

12 a $f'(x) = k\sin(3x)$

since function has a gradient of 2 when $x = \frac{\pi}{2}$:

$f'\left(\frac{\pi}{2}\right) = 2$

$f'\left(\frac{\pi}{2}\right) = k\sin\left(\frac{3\pi}{2}\right)$

$2 = k \times (-1)$

$k = -2$

b $f'(x) = -2\sin(3x)$

$f(x) = \int(-2\sin(3x))\,dx$

$f(x) = -2 \times \frac{-1}{3}\cos(3x) + c$

$f(x) = \frac{2}{3}\cos(3x) + c$

13 a $f'(x) = 4\cos(2x) + k$

since function has a gradient of -3 when $x = \frac{5\pi}{6}$

$f'\left(\frac{5\pi}{6}\right) = -3$

$f'\left(\frac{5\pi}{6}\right) = 4\cos\left(2 \times \frac{5\pi}{6}\right) + k$

$-3 = 4\cos\left(\frac{5\pi}{3}\right) + k$

$-3 = 4 \times \frac{1}{2} + k$

$-3 = 2 + k$

$k = -5$

b $f'(x) = 4\cos(2x) - 5$

$f(x) = \int(4\cos(2x) - 5)\,dx$

$f(x) = 4 \times \frac{1}{2}\sin(2x) - 5x + c$

$f(x) = 2\sin(2x) - 5x + c$

14 a $\frac{dy}{dx} = k\cos\left(2x + \frac{\pi}{3}\right)$

$\frac{dy}{dx} = 5$ when $x = \frac{\pi}{2}$

$\therefore 5 = k\cos\left(2 \times \frac{\pi}{2} + \frac{\pi}{3}\right)$

$5 = k\cos\left(\frac{4\pi}{3}\right)$

$5 = k \times \frac{-1}{2}$

$k = -10$

b $\frac{dy}{dx} = -10\cos\left(2x + \frac{\pi}{3}\right)$

$y = \int -10\cos\left(2x + \frac{\pi}{3}\right)dx$

$y = -10 \times \frac{1}{2}\sin\left(2x + \frac{\pi}{3}\right) + c$

$y = -5\sin\left(2x + \frac{\pi}{3}\right) + c$

15 $\int(3\sin(2x) + 8\cos(2x))\,dx = 3 \times \frac{-1}{2}\cos(2x) + 8$

$\times \frac{1}{2}\sin(2x) + c$

$= \frac{-3}{2}\cos(2x) + 4\sin(2x) + c$

$= p\sin(2x) + q\cos(2x)$

$\therefore p = 4, q = -\frac{3}{2}$

Exercise 6.6 – Further integration

1 a $\int(x+3)^2\,dx = \frac{1}{3}(x+3)^3 + c$

b $\int(x-5)^3\,dx = \frac{1}{4}(x-5)^4 + c$

c $\int 2(2x+1)^4\,dx = 2\int(2x+1)^4\,dx$

$= 2 \times \frac{1}{10}(2x+1)^5 + c$

$= \frac{1}{5}(2x+1)^5 + c$

d $\int -2(3x-4)^5\,dx = -2\int(3x-4)^5\,dx$

$- 2 \times \frac{1}{3 \times 6}(3x-4)^6 + c$

$- \frac{1}{9}(3x-4)^6 + c$

e $\int (6x+5)^4\,dx = \frac{1}{6\times 5}(6x+5)^5 + c$

$= \frac{1}{30}(6x+5)^5 + c$

f $\int 3(4x-1)^2\,dx = 3\int (4x-1)^2\,dx$

$= 3\times \frac{1}{4\times 3}(4x-1)^3 + c$

$= \frac{1}{4}(4x-1)^3 + c$

2 a $\int (4-x)^3\,dx = \frac{1}{-1\times 4}(4-x)^4 + c$

$= -\frac{1}{4}(4-x)^4 + c$

b $\int (7-x)^4\,dx = \frac{1}{-1\times 5}(7-x)^5 + c$

$= -\frac{1}{5}(7-x)^5 + c$

c $\int 4(8-3x)^4\,dx = 4\int (8-3x)^4\,dx$

$= 4\times \frac{1}{-3\times 5}(8-3x)^5 + c$

$= -\frac{4}{15}(8-3x)^5 + c$

d $\int -3(8-9x)^{10}\,dx = -3\int (8-9x)^{10}\,dx$

$= -3\times \frac{1}{-9\times 11}(8-9x)^{11} + c$

$= \frac{1}{33}(8-9x)^{11} + c$

e $\int (2x+3)^{-2}\,dx = \frac{1}{2\times -1}(2x+3)^{-1} + c$

$= -\frac{1}{2}(2x+3)^{-1} + c$

f $\int (6x+5)^{-3}\,dx = \frac{1}{6\times -2}(6x+5)^{-2} + c$

$= -\frac{1}{12}(6x+5)^{-2} + c$

3 a $\int (3x-5)^5\,dx = \frac{(3x-5)^6}{3\times 6} = \frac{1}{18}(3x-5)^6 + c$

b $\int \frac{1}{(2x-3)^{\frac{5}{2}}}\,dx = \int (2x-3)^{-\frac{5}{2}}\,dx$

$= \frac{(2x-3)^{-\frac{3}{2}}}{2\times -\frac{3}{2}}$

$= -\frac{1}{3(2x-3)^{\frac{3}{2}}} + c$

4 a $\int (2x+3)^4 dx = \frac{(2x+3)^5}{2\times 5}$

$= \frac{1}{10}(2x+3)^5 + c$

b $\int (1-2x)^{-5} dx = \frac{(1-2x)^{-4}}{-2\times -4}$

$= \frac{1}{8}(1-2x)^{-4}$

$= \frac{1}{8(1-2x)^4} + c$

5 a $\int \left(e^{2x+1}-4\right)^2 dx$

$= \int \left(e^{4x+2} - 8e^{2x+1} + 16\right) dx$

$= \frac{1}{4}e^{4x+2} - 8\times \frac{1}{2}e^{2x+1} + 16x + c$

$= \frac{1}{4}e^{4x+2} - 4e^{2x+1} + 16x + c$

b $\int \left(2e^{3-x} + 3e^{2-x}\right)^2 dx$

$= \int \left(4e^{6-2x} + 12e^{5-2x} + 9e^{4-2x}\right) dx$

$= 4\times \frac{1}{-2}e^{6-2x} + 12\times \frac{1}{-2}e^{5-2x} + 9\times \frac{1}{-2}e^{4-2x} + c$

$= -2e^{6-2x} - 6e^{5-2x} - \frac{9}{2}e^{4-2x} + c$

6 a $f'(x) = 4x + 1$

$f(x) = 2x^2 + x + c$

at $(0, 2)$

$2 = 2(0)^2 + (0) + c$

$2 = c$

$f(x) = 2x^2 + x + 2$

b $f'(x) = 5 - 2x$

$f(x) = 5x - x^2 + c$

at $(1, -1)$

$-1 = 5 - 1 + c$

$-1 = 4 + c$

$-5 = c$

$f(x) = 5x - x^2 - 5$

c $f'(x) = x^{-2} + 3$

$f(x) = -x^{-1} + 3x + c$

at $(1, 4)$

$4 = -1 + 3 + c$

$4 = 2 + c$

$2 = c$

$f(x) = \frac{-1}{x} + 3x + 2$

$= 3x + 2 - \frac{1}{x}$

d $f'(x) = x + \sqrt{x}$

$= x + x^{\frac{1}{2}}$

$f(x) = \frac{x^2}{2} + \frac{2}{3}x^{\frac{3}{2}} + c$

at $(4, 10)$

$10 = \frac{4^2}{2} + \frac{2}{3}(4)^{\frac{3}{2}} + c$

$10 = 8 + \frac{16}{3} + c$

$2 - \frac{16}{3} = c$

$\frac{-10}{3} = c$

$f(x) = \frac{1}{2}x^2 + \frac{2}{3}x^{\frac{3}{2}} - \frac{10}{3}$

e $f'(x) = x^{\frac{1}{3}} - 3x^2 + 50$

$f(x) = \frac{3}{4}x^{\frac{4}{3}} - x^3 + 50x + c$

at $(8, -100)$

$-100 = \frac{3}{4}(8)^{\frac{4}{3}} - 8^3 + 50 \times 8 + c$

$-100 = 12 - 512 + 400 + c$

$0 = c$

$f(x) = \frac{3}{4}x^{\frac{4}{3}} - x^3 + 50x$

f $f'(x) = \frac{1}{\sqrt{x}} - 2x$

$= x^{\frac{-1}{2}} - 2x$

$f(x) = 2x^{\frac{1}{2}} - x^2 + c$

at $(1, -5)$

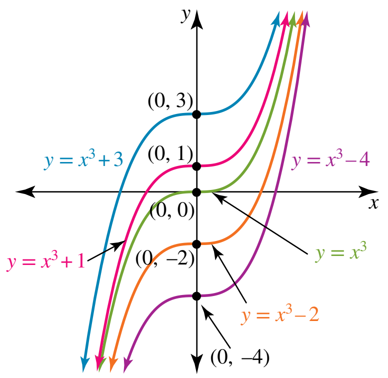
$-5 = 2 - 1 + c$

$-5 = 1 + c$

$-6 = c$

$f(x) = 2\sqrt{x} - x^2 - 6$

7 a $f'(x) = 3x^2$ so $f(x) = x^3 + c$



b $f(x) = x^3 + c$

$f(2) = 16$

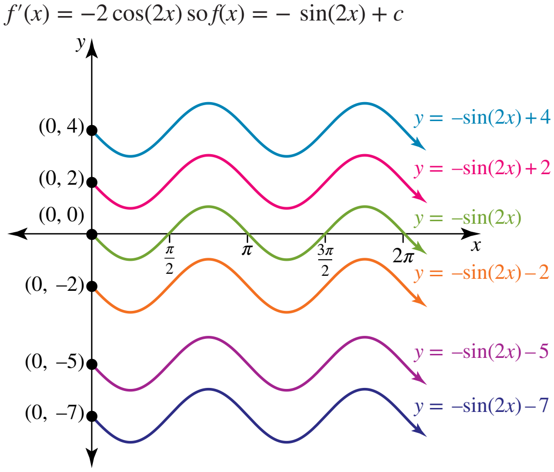
$2^3 + c = 16$

$8 + c = 16$

$c = 8$

$f(x) = x^3 + 8$

8 a $f'(x) = -2\cos(2x)$ so $f(x) = -\sin(2x) + c$



b $f(x) = -\sin(2x) + c$

$f\left(\frac{\pi}{2}\right) = 4$

$4 = -\sin(\pi) + c$

$4 = 0 + c$

$c = 4$

$f(x) = 4 - \sin(2x)$

9 $\frac{dy}{dx} = 2e^{2x} + e^{-x}$

$y = e^{2x} - e^{-x} + c$

When $x = 0,\ y = 3$

$3 = e^0 - e^0 + c$

$3 = 1 - 1 + c$

$c = 3$

$y = e^{2x} - e^{-x} + 3$

10 $f'(x) = \cos(2x) - \sin(2x)$

$f(x) = \frac{1}{2}\sin(2x) + \frac{1}{2}\cos(2x) + c$

$f(\pi) = 2$

$2 = \frac{1}{2}\sin(2\pi) + \frac{1}{2}\cos(2\pi) + c$

$2 = \frac{1}{2}(0) + \frac{1}{2}(1) + c$

$2 = \frac{1}{2} + c$

$c = \frac{3}{2}$

$f(x) = \frac{1}{2}\sin(2x) + \frac{1}{2}\cos(2x) + \frac{3}{2}$

11 a $\frac{dP}{dt} = 20e^{0.4t}$

$P = \int \left(20e^{0.4t}\right) dt$

$P = 20 \times \frac{1}{0.4}e^{0.4t} + c$

$P = 50e^{0.4t} + c$

When $t = 0, P = 35$

$35 = 50e^0 + c$

$c = -15$

$P = 50e^{0.4t} - 15$

b When $t = 6$:

$P = 50e^{0.4\times 6} - 15$

$P = 50e^{2.4} - 15$

$P = 536.159$

There are 536 bugs present after 6 days.

12 a $\frac{dP}{dt} = 30e^{0.3t}$

$P = \frac{30}{0.3}e^{0.3t} + c$

$P = 100e^{0.3t} + c$

When $t = 0,\ P = 50$

$50 = 100e^0 + c$

$50 = 100 + c$

$c = -50$

$P = 100e^{0.3t} - 50$

b When $t = 10,\ P = 100e^3 - 50 = 1959$

There are 1959 seals after 10 years.

13 $\dfrac{dh}{dt} = \dfrac{\pi}{2}\cos\left(\dfrac{\pi t}{4}\right)$

a $h = \dfrac{4}{\pi} \times \dfrac{\pi}{2}\sin\left(\dfrac{\pi t}{4}\right) + c = 2\sin\left(\dfrac{\pi t}{4}\right) + c$

When $t = 0$, $h = 3$

$3 = 2\sin(0) + c$

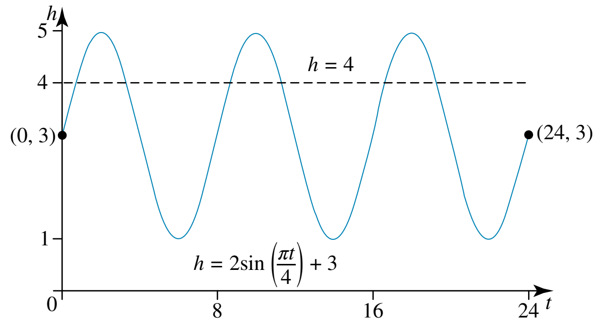
$c = 3$

$h = 2\sin\left(\dfrac{\pi t}{4}\right) + 3$

b Maximum depth $= 2(1) + 3 = 5$ m

Minimum depth $= 2(-1) + 3 = 1$ m

c



$4 = 2\sin\left(\dfrac{\pi t}{4}\right) + 3$

$1 = 2\sin\left(\dfrac{\pi t}{4}\right)$

$\dfrac{1}{2} = \sin\left(\dfrac{\pi t}{4}\right)$

$\dfrac{1}{2}$ indicates $\dfrac{\pi}{6}$. Since sin is positive then 1st and 2nd quadrants.

$\dfrac{\pi t}{4} = \dfrac{\pi}{6}, \pi - \dfrac{\pi}{6}, 2\pi + \dfrac{\pi}{6}, 3\pi - \dfrac{\pi}{6}, 4\pi + \dfrac{\pi}{6}, 5\pi - \dfrac{\pi}{6}$

$\dfrac{\pi t}{4} = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}, \dfrac{25\pi}{6}, \dfrac{29\pi}{6}$

$t = \dfrac{\pi}{6} \times \dfrac{4}{\pi}, \dfrac{5\pi}{6} \times \dfrac{4}{\pi}, \dfrac{13\pi}{6} \times \dfrac{4}{\pi}, \dfrac{17\pi}{6} \times \dfrac{4}{\pi}, \dfrac{25\pi}{6} \times \dfrac{4}{\pi}, \dfrac{29\pi}{6} \times \dfrac{4}{\pi}$

$t = \dfrac{2}{3}, \dfrac{10}{3}, \dfrac{26}{3}, \dfrac{34}{3}, \dfrac{50}{3}, \dfrac{58}{3}$

$h \geq 4$ when $\left\{h : \dfrac{2}{3} \leq t \leq \dfrac{10}{3}\right\} \cup \left\{h : \dfrac{26}{3} \leq t \leq \dfrac{34}{3}\right\} \cup$
$\left\{h : \dfrac{50}{3} \leq t \leq \dfrac{58}{3}\right\}$

This is $\dfrac{8}{3} + \dfrac{8}{3} + \dfrac{8}{3} = \dfrac{24}{3} = 8$ hours/day.

14 $y = \sqrt{x^2 + 1} = (x^2 + 1)^{\frac{1}{2}}$

$\dfrac{dy}{dx} = \dfrac{1}{2}(2x)\left(x^2 + 1\right)^{-\frac{1}{2}}$

$\dfrac{dy}{dx} = \dfrac{x}{\sqrt{x^2 + 1}}$

$\displaystyle\int \dfrac{5x}{\sqrt{x^2 + 1}}\,dx = 5\int \dfrac{x}{\sqrt{x^2 + 1}}\,dx$

$= 5\sqrt{x^2 + 1} + c$

15 $y = \left(5x^2 + 2x - 1\right)^4$

Let $y = u^4$, $u = 5x^2 + 2x - 1$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\dfrac{dy}{dx} = 4(10x + 2)\left(5x^2 + 2x - 1\right)^3$

$\dfrac{dy}{dx} = 8(5x + 1)\left(5x^2 + 2x - 1\right)^3$

$\displaystyle\int 16(5x + 1)(5x^2 + 2x - 1)^3 dx = 2\int 8(5x + 1)$
$(5x^2 + 2x - 1)^3 dx = 2\left(5x^2 + 2x - 1\right)^4 + c$

16 $y = \ln\left(3x^2 + 4\right)$

Let $y = \ln(u)$, $u = 3x^2 + 4$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\dfrac{dy}{dx} = \dfrac{1}{\left(3x^2 + 4\right)} \times 6x$

$\dfrac{dy}{dx} = \dfrac{6x}{\left(3x^2 + 4\right)}$

$\displaystyle\int \dfrac{6x}{\left(3x^2 + 4\right)}dx = \ln\left(3x^2 + 4\right)$

$6\displaystyle\int \dfrac{x}{\left(3x^2 + 4\right)}dx = \ln\left(3x^2 + 4\right)$

$\displaystyle\int \dfrac{x}{\left(3x^2 + 4\right)}dx = \dfrac{1}{6}\ln\left(3x^2 + 4\right) + c$

17 a $y = \ln(\cos(x))$

Let $y = \ln(u)$, $u = \cos(x)$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\dfrac{dy}{dx} = \dfrac{1}{\cos(x)} \times (-\sin(x))$

$\dfrac{dy}{dx} = -\tan(x)$

b $\displaystyle\int (-\tan(x))\,dx = \ln(\cos(x))$

$-\displaystyle\int (\tan(x))\,dx = \ln(\cos(x))$

$\displaystyle\int (\tan(x))\,dx = -\ln(\cos(x)) + c$

18 $y = x\sin(x)$

Use the product rule to differentiate:

$\dfrac{dy}{dx} = \sin(x) \times 1 + x \times \cos(x)$

$\dfrac{dy}{dx} = \sin(x) + x\cos(x)$

Therefore:

$\displaystyle\int (\sin(x) + x\cos(x))\,dx = x\sin(x)$

$\displaystyle\int (\sin(x))\,dx + \int (x\cos(x))\,dx = x\sin(x)$

$\displaystyle\int (x\cos(x))\,dx = x\sin(x) - \int (\sin(x))\,dx$

$\displaystyle\int (x\cos(x))\,dx = x\sin(x) - (-\cos(x))$

$\displaystyle\int (x\cos(x))\,dx = x\sin(x) + \cos(x) + c$

19 $y = x\ln(x)$

Use the product rule to differentiate:

$\dfrac{dy}{dx} = \ln(x) \times 1 + x \times \dfrac{1}{x}$

$\dfrac{dy}{dx} = \ln(x) + 1$

Therefore:

$\displaystyle\int (\ln(x) + 1)\,dx = x\ln(x) + c$

$\int \ln(x)\,dx + \int 1dx = x\ln(x) + c$

$\int \ln(x)\,dx + x = x\ln(x) + c$

$\int \ln(x)\,dx = x\ln(x) - x + c$

20 $y = 2xe^{3x}$

Use the product rule to differentiate

$\frac{dy}{dx} = 2e^{3x} + 6xe^{3x}$

$\int \left(2e^{3x} + 6xe^{3x}\right)\,dx = 2xe^{3x} + c$

$\int 2e^{3x}dx + 6\int xe^{3x}dx = 2xe^{3x} + c$

$6\int xe^{3x}dx = 2xe^{3x} - \int 2e^{3x}dx + c$

$6\int xe^{3x}dx = 2xe^{3x} - \frac{2}{3}e^{3x} + c$

$3\int xe^{3x}dx = xe^{3x} - \frac{1}{3}e^{3x} + c$

$\int xe^{3x}dx = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c$

21 a $v = \frac{dx}{dt} = (3t+1)^{\frac{1}{2}}$

$x = \int (3t+1)^{\frac{1}{2}}dt$

$= \frac{1}{3\left(\frac{3}{2}\right)}(3t+1)^{\frac{3}{2}} + c$

$= \frac{2}{9}(3t+1)^{\frac{3}{2}} + c$

When $t = 0,\ x = 0$

$0 = \frac{2}{9}(3(0)+1)^{\frac{3}{2}} + c$

$0 = \frac{2}{9} + c$

$c = -\frac{2}{9}$

$x = \frac{2}{9}(3t+1)^{\frac{3}{2}} - \frac{2}{9}$

$x = \frac{2}{9}\sqrt{(3t+1)^3} - \frac{2}{9}$

b $v = \frac{dx}{dt} = \frac{1}{(t+2)^2} = (t+2)^{-2}$

$x = \int (t+2)^{-2}dt$

$= -\frac{1}{1}(t+2)^{-1} + c$

$= -\frac{1}{(t+2)} + c$

When $t = 0,\ x = 0$

$0 = -\frac{1}{(0+2)} + c$

$0 = -\frac{1}{2} + c$

$c = \frac{1}{2}$

$x = \frac{1}{2} - \frac{1}{(t+2)}$

c $v = \frac{dx}{dt} = (2t+1)^3$

$x = \int (2t+1)^3 dt$

$= \frac{1}{2(4)}(2t+1)^4 + c$

$= \frac{1}{8}(2t+1)^4 + c$

When $x = 0,\ t = 0;$

$0 = \frac{1}{8}(2(0)+14) + c$

$0 = \frac{1}{8} + c$

$c = -\frac{1}{8}$

$x = \frac{1}{8}(2t+1)^4 - \frac{1}{8}$

22 a $v = 3t^2 + 6t$

$x = \int \left(3t^2 + 6t\right)dt$

$x = 3 \times \frac{t^3}{3} + 6 \times \frac{t^2}{2} + c$

$x = t^3 + 3t^2 + c$

When: $t = 0,\ x = -2$:

$-2 = c$

$x = t^3 + 3t^2 - 2$

b When $t = 5$:

$x = 5^3 + 3 \times 5^2 - 2$

$x = 198$

The particle is 198 metres from the origin after 5 seconds.

23 a $v = \frac{dx}{dt} = e^{(3t-1)}$

$x = \int e^{(3t-1)}dt$

$= \frac{1}{3}e^{(3t-1)} + c$

When $t = 0,\ x = 0$

$0 = \frac{1}{3}e^{(3(0)-1)} + c$

$0 = \frac{1}{3}e^{-1} + c$

$c = -\frac{1}{3e}$

$x = \frac{1}{3}e^{(3t-1)} - \frac{1}{3e}$

b $v = \frac{dx}{dt} = -\sin(2t+3)$

$x = \int -\sin(2t+3)dt$

$= \frac{1}{2}\cos(2t+3) + c$

When $t = 0,\ x = 0$

$0 = \frac{1}{2}\cos(2(0)+3) + c$

$0 = \frac{1}{2}\cos(3) + c$

$c = -\frac{1}{2}\cos(3)$

$x = \frac{1}{2}\cos(2t+3) - \frac{1}{2}\cos(3)$

24 $v = \frac{dx}{dt} = \sin(2t) + \cos(2t)$

a When $t = 0$,

$v = \sin(0) + \cos(0)$

$v = 1$ cm/s

b $x = \int (\sin(2t) + \cos(2t))\, dt$

$= -\frac{1}{2}\cos(2t) + \frac{1}{2}\sin(2t) + c$

When $t = 0,\ x = 0$

$0 = -\frac{1}{2}\cos(0) + \frac{1}{2}\sin(0) + c$

$0 = -\frac{1}{2} + c$

$c = \frac{1}{2}$

$x = \frac{1}{2} - \frac{1}{2}\cos(2t) + \frac{1}{2}\sin(2t)$

25 a $v = \frac{dx}{dt} = \frac{12}{(t-1)^2} + 6$

$v = \frac{dx}{dt} = 12(t-1)^{-2} + 6$

$x = -12(t-1)^{-1} + 6t + c$

$x = -\frac{12}{(t-1)} + 6t + c$

When $t = 0,\ x = 0;$

$x = -\frac{12}{(0-1)} + 6(0) + c$

$c = -12$

$x = 6t - \frac{12}{(t-1)} - 12$

b When $t = 3$,

$x = 6(3) - \frac{12}{(3-1)} - 12$

$= 18 - 6 - 12$

$= 0$

After 3 seconds the particle is at the origin again.

6.7 Review: exam practice

1 a $\int 3x^5\,dx = \frac{3}{6}x^6 + c = \frac{1}{2}x^6 + c$

b $\int 5x^{-2}\,dx = \frac{5}{-1}x^{-1} + c = -\frac{5}{x} + c$

c $\int -2x^4\,dx = \frac{-2}{5}x^5 + c$

d $\int 2\sqrt{x}\,dx = \int 2x^{\frac{1}{2}}dx$

$= \frac{2}{\frac{3}{2}}x^{\frac{3}{2}} + c$

$= \frac{4}{3}x^{\frac{3}{2}} + c$

e $\int \frac{x^4}{5}dx = \frac{1}{25}x^5 + c$

f $\int (3x-8)^{-6}\,dx = \frac{1}{3\times -5}(3x-8)^{-5} + c$

$= -\frac{1}{15}(3x-8)^{-5} + c$

$= \frac{-1}{15\,(3x-8)^5} + c$

g $\int (6-5x)^{-3}\,dx = \frac{1}{-5\times -2}(6-5x)^{-2} + c$

$= \frac{1}{10}(6-5x)^{-2} + c$

$= \frac{1}{10\,(6-5x)^2} + c$

h $\int (-10(7-5x)^{-4})\,dx = -10\int (7-5x)^{-4}\,dx$

$= -10 \times \frac{1}{-5\times -3}(7-5x)^{-3} + c$

$= \frac{-2}{3}(7-5x)^{-3} + c$

$= \frac{-2}{3\,(7-5x)^3} + c$

2 a $\int \left(x^4 + 2x + \frac{1}{x}\right) dx = \frac{x^5}{5} + \frac{2x^2}{5} + \ln(x) + c$

$= \frac{1}{5}x^5 + x^2 + \ln(x) + c$

b $\int (3x+1)^5\,dx = \frac{1}{6\times 3}(3x+1)^6 + c$

$= \frac{1}{18}(3x+1)^6 + c$

c $\int \frac{3x^2+2x-1}{x^2}\,dx = \int \left(\frac{3x^2}{x^2} + \frac{2x}{x^2} - \frac{1}{x^2}\right) dx$

$= \int \left(3 + \frac{2}{x} - x^{-2}\right) dx$

$= 3x + 2\ln(x) + \frac{1}{x} + c$

d $\int \frac{3}{2x+1}\,dx = 3\int \frac{1}{2x+1}\,dx$

$= \frac{3}{2}\ln(2x+1) + c$

e $\int \frac{-5}{6-10x}\,dx = -5\int \frac{1}{6-10x}\,dx$

$= \frac{-5}{-10}\log_e(6-10x) + c$

$= \frac{1}{2}\ln(6-10x) + c$

f $\int 3(4x+1)^{-3}\,dx = 3\int (4x+1)^{-3}\,dx$

$= \frac{3}{4\times -2}(4x+1)^{-2} + c$

$= -\frac{3}{8}(4x+1)^{-2} + c$

g $\int \frac{(x+4)^2}{2x}\,dx = \int \frac{x^2+8x+16}{2x}\,dx$

$= \int \frac{x^2}{2x} + \frac{8x}{2x} + \frac{16}{2x}\,dx$

$= \int \frac{x}{2} + 4 + \frac{8}{x}\,dx$

$= \frac{1}{4}x^2 + 4x + 8\ln(x) + c$

h $\int \left(\sqrt{x} + \frac{2}{3-x}\right) dx = \int x^{\frac{1}{2}} + \frac{2}{3-x}\,dx$

$= \frac{2}{3}x^{\frac{3}{2}} + -2\log_e(3-x) + c$

$= \frac{2}{3}x^{\frac{3}{2}} - 2\ln(3-x) + c$

3 a $f'(x) = (x+4)^3$

$f(x) = \frac{1}{4}(x+4)^4 + c$

$(-2, 5)$

$5 = \frac{1}{4}(2)^4 + c$

$5 = 4 + c$

$1 = c$

$f(x) = \frac{1}{4}(x+4)^4 + 1$

b $f'(x) = 8(1-2x)^{-5}$

$f(x) = \frac{8}{-2\times -4}(1-2x)^{-4} + c$

$= (1-2x)^{-4} + c$

at $(1, 3)$

$3 = (1-2)^{-4} + c$

$3 = 1 + c$

$2 = c$

$f(x) = (1-2x)^{-4} + 2$

c $f'(x) = (x+5)^{-1} = \frac{1}{x+5}$

$f(x) = \ln(x+5) + c$

at $(-4, 2)$

$2 = \ln(1) + c$

$2 = c$

$f(x) = \ln(x+5) + 2$

d $f'(x) = \frac{8}{7-2x}$

$f(x) = -4\ \ln(7-2x) + c$

at $(3, 7)$

$7 = -4\ln(1) + c$

$7 = c$

$f(x) = -4\ln(7-2x) + 7$

4 a gradient $= 8x + k$ at $(1, 5)$

$8x + k = 0$ at $x = 1$

$8 + k = 0$

$k = -8$

b if $\frac{dy}{dx} = 8x - 8$

then $y = 4x^2 - 8x + c$

at $(1, 5)$, $5 = 4 - 8 + c$

$5 = -4 + c$

$9 = c$

$y = 4x^2 - 8x + 9$

at $x = -2$

$y = 4(-2)^2 - 8(-2) + 9$

$y = 16 + 16 + 9$

$y = 41$

5 a $\int \left(e^x - 3\right)^2 dx$

$= \int \left(e^{2x} - 6e^x + 9\right) dx$

$= \frac{1}{2}e^{2x} - 6e^x + 9x + c$

b $\int \left(1 + e^{-x}\right)^3 dx$

$= \int \left(1 + 3e^{-x} + 3e^{-2x} + e^{-3x}\right) dx$

$= x + \frac{3}{-1}e^{-x} + \frac{3}{-2}e^{-2x} + \frac{1}{-3}e^{-3x} + c$

$= x - 3e^{-x} - \frac{3}{2}e^{-2x} - \frac{1}{3}e^{-3x} + c$

6 a $\int -2\sin\left(\frac{5x}{2}\right) dx = \frac{4}{5}\cos\left(\frac{5x}{2}\right) + c$

b $\int -3\cos\left(\frac{7x}{4}\right) dx = \frac{-12}{7}\sin\left(\frac{7x}{4}\right) + c$

c $\int 5\sin(\pi x)\, dx = \frac{-5}{\pi}\cos(\pi x) + c$

d $\int 3\cos\left(\frac{\pi x}{2}\right) dx = \frac{6}{\pi}\sin\left(\frac{\pi x}{2}\right) + c$

e $\int -2\cos\left(\frac{\pi x}{3}\right) dx = \frac{-6}{\pi}\sin\left(\frac{\pi x}{3}\right) + c$

f $\int -\sin\left(\frac{-4x}{\pi}\right) dx = \frac{-\pi}{4}\cos\left(\frac{-4x}{\pi}\right) + c$

7 a $\int \left(x^3 - \frac{1}{2x+3} + e^{2x}\right) dx = \frac{1}{4}x^4 - \frac{1}{2}\log_e(2x+3) + \frac{1}{2}e^{2x} + c$

b $\int (x^2 + 4\cos(2x) - e^{-x})\, dx = \frac{1}{3}x^3 + 2\sin(2x) + e^{-x} + c$

c $\int \left(\sin\left(\frac{x}{3}\right) + e^{\frac{x}{2}} - (3x-1)^4\right) dx = -3\cos\left(\frac{x}{3}\right) + 2e^{\frac{x}{2}} - \frac{1}{15}(3x-1)^5 + c$

d $\int \left(\frac{1}{3x-2} + e^{4x} + \cos\left(\frac{x}{5}\right)\right) dx = \frac{1}{3}\log_e(3x-2) + \frac{1}{4}e^{4x} + 5\sin\left(\frac{x}{5}\right) + c$

e $\int \left(3\sin\left(\frac{x}{2}\right) - 2\cos\left(\frac{x}{3}\right) - e^{\frac{-x}{5}}\right) dx = -6\cos\left(\frac{x}{2}\right) - 6\sin\left(\frac{x}{3}\right) + 5e^{\frac{-x}{5}} + c$

f $\int \left(\sqrt{x} + 2x - 2\sin\left(\frac{\pi x}{3}\right) + 5\right) dx = \frac{2}{3}x^{\frac{3}{2}} + x^2 + \frac{6}{\pi}\cos\left(\frac{\pi x}{3}\right) + 5x + c$

8 a $f'(x) = \cos(x)$

$f(x) = \sin(x) + c$

$5 = \sin\left(\frac{\pi}{2}\right) + c$

$5 = 1 + c$

$4 = c$

$f(x) = \sin(x) + 4$

b $f'(x) = 4\sin(2x)$

$f(x) = -2\cos(2x) + c$

$-1 = -2\cos(0) + c$

$-1 = -2 + c$

$1 = c$

$f(x) = -2\cos(2x) + 1$

$= 1 - 2\cos(2x)$

c $f'(x) = 3\cos\left(\frac{x}{4}\right)$

$f(x) = 12\sin\left(\frac{x}{4}\right) + c$

$9\sqrt{2} = 12\sin\left(\frac{\pi}{4}\right) + c$

$9\sqrt{2} = 12 \times \frac{\sqrt{2}}{2} + c$

$9\sqrt{2} = 6\sqrt{2} + c$

$3\sqrt{2} = c$

$f(x) = 12\sin\left(\frac{x}{4}\right) + 3\sqrt{2}$

d $f'(x) = \cos\left(\frac{x}{4}\right) - \sin\left(\frac{x}{2}\right)$

$f(x) = 4\sin\left(\frac{x}{4}\right) + 2\cos\left(\frac{x}{2}\right) + c$

$-2 = 4\sin\left(\frac{2\pi}{4}\right) + 2\cos\left(\frac{2\pi}{2}\right) + c$

$-2 = 4\sin\left(\frac{\pi}{2}\right) + 2\cos(\pi) + c$

$-2 = 4 - 2 + c$

$-2 = 2 + c$

$-4 = c$

$f(x) = 4\sin\left(\frac{x}{4}\right) + 2\cos\left(\frac{x}{2}\right) - 4$

9 a $f'(x) = 4\cos(2x) + ke^x = 0$ at $x = 0$

$4\cos(0) + k = 0$

$4 + k = 0$

$k = -4$

b $f'(x) = 4\cos(2x) - 4e^x$

$f(x) = 2\sin(2x) - 4e^x + c$

$-1 = 2\sin(0) - 4 + c$

$-1 = -4 + c$

$3 = c$

$f(x) = 2\sin(2x) - 4e^x + 3$

c $f\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{3}\right) - 4e^{\frac{\pi}{6}} + 3$

$= -2.02$

10 $\frac{d}{dx}\left(\ln\left(x^2+3\right)\right) = \frac{1}{(x^2+3)} \times 2x$

$= \frac{2x}{(x^2+3)}$

$\therefore \int \frac{2x}{(x^2+3)}dx = \ln(x^2+3)$

$6 \times \int \frac{2x}{(x^2+3)}dx = 6 \times \ln(x^2+3)$

$\int \frac{12x}{(x^2+3)}dx = 6\ln\left(x^2+3\right) + c$

11 $\frac{d}{dx}\left(\frac{\cos(x)}{\sin(x)}\right) = \frac{\sin(x) \times -\sin(x) - \cos(x) \times \cos(x)}{(\sin(x))^2}$

$= \frac{-(\sin(x))^2 - (\cos(x))^2}{(\sin(x))^2}$

$= \frac{-\left(\sin^2(x) + \cos^2(x)\right)}{\sin^2(x)}$

$= \frac{-1}{\sin^2(x)}$

$\therefore \int \frac{-1}{\sin^2(x)}dx = \frac{\cos(x)}{\sin(x)}$

$-1\int \frac{1}{\sin^2(x)}dx = \frac{\cos(x)}{\sin(x)}$

$\int \frac{1}{\sin^2(x)}dx = -\frac{\cos(x)}{\sin(x)} + c$

12 a $-3 + \frac{4}{(3-2x)}$

$= \frac{-3(3-2x)+4}{(3-2x)}$

$= \frac{-9+6x+4}{(3-2x)}$

$= \frac{6x-5}{3-2x}$

b $\int\left(-3 + \frac{4}{3-2x}\right)dx = \int -3dx + \int \frac{4}{3-2x}dx$

$= -3x - 2\ln(3-2x) + c$

13 $f'(x) = a\sin(mx) - be^{nx}$

$f(x) = \cos(2x) - 2e^{-2x} + 3$

$f'(x) = -2\sin(2x) + 4e^{-2x}$

Therefore $a = -2$, $b = -4$, $m = 2$, and $n = -2$

14 a $v = \frac{t}{400}(50 - t)$

$v = \frac{1}{8}t - \frac{1}{400}t^2$

Greatest velocity occurs when $\frac{dv}{dt} = 0$

$\frac{dv}{dt} = \frac{1}{8} - \frac{1}{200}t$

$0 = \frac{1}{8} - \frac{1}{200}t$

$t = 25$

When $t = 25$, $v = \frac{25}{400}(50 - 25)$

$v = 1.5625$ m/s

b $x = \int \frac{t}{400}(50 - t)\,dt$

$x = \int\left(\frac{1}{8}t - \frac{1}{400}t^2\right)dt$

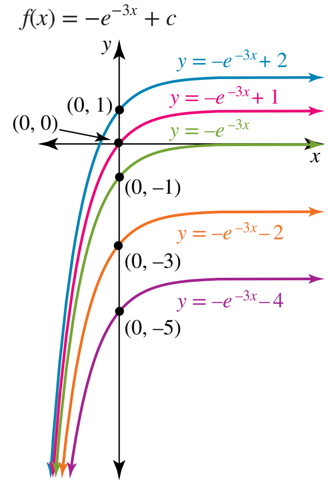
$x = \frac{1}{16}t^2 - \frac{1}{1200}t^3 + c$

When $t = 0$, $x = 0 \therefore c = 0$

$x = \frac{1}{16}t^2 - \frac{1}{1200}t^3$

15 a $f'(x) = 3e^{-3x}$

$f(x) = -e^{-3x} + c$



b $f(x) = -e^{-3x} + c$

When $x = 0,\ y = 1$

$1 = -e^{0} + c$

$1 = -1 + c$

$c = 2$

$f(x) = 2 - e^{-3x}$

16 a $f'(x) = 5 - 2x$

$f(x) = 5x - x^2 + c$

When $f(1) = 4$

$4 = 5(1) - (1)^2 + c$

$4 = 4 + c$

$c = 0$

$f(x) = 5x - x^2$

b $f'(x) = \sin\left(\frac{x}{2}\right)$

$f(x) = -2\cos\left(\frac{x}{2}\right) + c$

When $f(\pi) = 3$

$3 = -2\cos\left(\frac{\pi}{2}\right) + c$

$3 = 0 + c$

$c = 3$

$f(x) = 3 - 2\cos\left(\frac{x}{2}\right)$

c $f'(x) = \frac{1}{(1-x)^2} = (1-x)^{-2}$

$f(x) = \frac{1}{(-1)(-1)}(1-x)^{-1} + c$

$f(x) = \frac{1}{(1-x)} + c$

When $f(0) = 4$

$4 = \frac{1}{(1-0)} + c$

$4 = 1 + c$

$c = 3$

$f(x) = \frac{1}{(1-x)} + 3$

17 a $v = \frac{dx}{dt} = 3\pi\sin\left(\frac{\pi t}{8}\right)$

$x = -\frac{8}{\pi} \times 3\cos\left(\frac{\pi t}{8}\right) + c$

$x = -24\cos\left(\frac{\pi t}{8}\right) + c$

When $t = 0,\ x = 0$;

$0 = -24\cos(0) + c$

$c = 24$

$x = 24 - 24\cos\left(\frac{\pi t}{8}\right)$

b $x_{MAX} = 24 - 24(-1) = 24 + 24 = 48$

$x_{MIN} = 24 - 24(1) = 24 - 24 = 0$

Maximum displacement is 48 metres.

c When $t = 4,\ x = 24 - 24\cos\left(\frac{\pi}{2}\right) = 24$

After 4 seconds the particle is 24 metres above the stationary position.

18 a $\frac{dN}{dt} = 400 + 1000\sqrt{t}$

$\frac{dN}{dt} = 400 + 1000t^{\frac{1}{2}}$

$N = 400t + \frac{2000}{3}t^{\frac{3}{2}} + c$

$N = 400t + \frac{2000}{3}\sqrt{t^3} + c$

When $t = 0,\ N = 40$

$40 = 400(0) + \frac{2000}{3}\sqrt{0^3} + c$

$c = 40$

$N = 400t + \frac{2000}{3}t\sqrt{t} + 40$

b When $t = 5$,

$N = 400(5) + \frac{2000}{3}\sqrt{(5)^3} + 40$

$N = 2000 + \frac{2000}{3}\sqrt{125} + 40$

$N = 9494$ families

19 $\frac{dV}{dt} = 20t^2 - t^3$

$V = \frac{20}{3}t^3 - \frac{1}{4}t^4 + c$

When $t = 0,\ V = 0$ so $c = 0$

$V = \frac{20}{3}t^3 - \frac{1}{4}t^4$

When $t = 20$,

$V = \frac{20}{3}(20)^3 - \frac{1}{4}(20)^4$

$V = \frac{160\,000}{3} - 40\,000$

$V = 13\,333\frac{1}{3}\ \text{cm}^3$

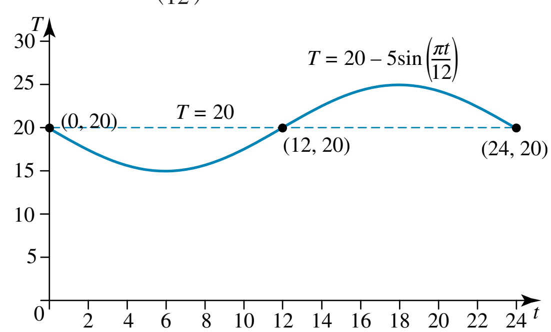
20 $\frac{dT}{dt} = \frac{-5\pi}{12}\cos\left(\frac{\pi t}{12}\right)$

a $T = -5\sin\left(\frac{\pi t}{12}\right) + c$

initially $T = 20$ at $t = 0$

$\Rightarrow c = 20$

$T = 20 - 5\sin\left(\frac{\pi t}{12}\right)$



b $20 - 5\sin\left(\frac{\pi t}{12}\right) = 13$

$-5\sin\left(\frac{\pi t}{12}\right) = -7$

No since $-1 \le \sin x \le 1$

c Max Temp is when $\sin\frac{\pi t}{12} = -1$

Max Temp = 25°C

$\frac{\pi t}{12} = \frac{3\pi}{2}$

$t = \frac{3\pi}{2} \times \frac{12}{\pi}$

$t = 18$ hrs $= 6$ pm.

d Min Temp is 15°C when $\sin\frac{\pi t}{12} = 1$

$\frac{\pi t}{12} = \frac{\pi}{2}$

$t = 6$ am.

e **i** at 2 am, $t = 2$ and $T = 20 - 5\sin\frac{\pi}{6}$

$T = 17.5$°C

ii at 3 pm, $t = 15$ and $T = 20 - 5\sin\frac{5\pi}{4}$

$T = 23.5$°C

f $20 - 5\sin\frac{\pi t}{12} = 22.5$

$-5\sin\frac{\pi t}{12} = 2.5$

$\sin\frac{\pi t}{12} = -\frac{1}{2}$

basic angle $= \frac{\pi}{6}$

3^{rd} & 4^{th} quadrants

$\frac{\pi t}{12} = \pi + \frac{\pi}{6}$

$\frac{\pi t}{12} = \frac{7\pi}{6}$

$t = \frac{7\pi}{6} \times \frac{12}{\pi}$

$t = 14$ hours after midnight

t reaches 22.5°C at 2 pm