Chapter 1 — Arithmetic sequences

Exercise 1.2 — Arithmetic sequences

1 a 2, 7, 12, 17, 22. …

$t_2 - t_1 = 7 - 2$
$= 5$
$t_3 - t_2 = 12 - 7$
$= 5$
$t_4 - t_3 = 17 - 12$
$= 5$

There is a common difference of 5, therefore $d = 5$. This is an arithmetic sequence.

b 3, 7, 11, 15, 20. …

$t_2 - t_1 = 7 - 3$
$= 4$
$t_3 - t_2 = 11 - 7$
$= 4$
$t_4 - t_3 = 15 - 11$
$= 4$
$t_5 - t_4 = 20 - 15$
$= 5$

There is no common difference, therefore this is not an arithmetic sequence.

c 0, 100, 200, 300, 400…

$t_2 - t_1 = 100 - 0 = 100$
$t_3 - t_2 = 200 - 100$
$= 100$
$t_4 - t_3 = 300 - 200$
$= 100$

There is a common difference, therefore $d = 100$. This is an arithmetic sequence.

d −123, −23, 77, 177, 277...

$t_2 - t_1 = -23 + 123 = 100$
$t_3 - t_2 = 77 - -23 = 100$
$t_4 - t_3 = 177 - 77 = 100$

There is a common difference $d = 100$, so this is an arithmetic sequence.

e 1, 0, −1, −3, 5…

$t_2 - t_1 = 0 - 1 = -1$
$t_3 - t_2 = -1 - 0 = -1$
$t_4 - t_3 = -3 + 1 = -2$

This is not an arithmetic sequence, because there is no common difference.

f 6.2, 9.3, 12.4, 15.5, 16.6…

$t_2 - t_1 = 9.3 - 6.2 = 3.1$
$t_3 - t_2 = 12.4 - 9.3 = 3.1$
$t_4 - t_3 = 15.5 - 12.4 = 3.1$
$t_5 - t_4 = 16.6 - 15.5 = 1.1$

This is not an arithmetic sequence because there is no common difference.

g $\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}...$

$t_4 - t_3 = 3\frac{1}{2} - 2\frac{1}{2} = 1$

$t_5 - t_4 = 4\frac{1}{2} - 3\frac{1}{2} = 1$

$t_2 - t_1 = 1\frac{1}{2} - \frac{1}{2} = 1$

$t_3 - t_2 = 2\frac{1}{2} - 1\frac{1}{2} = 1$

Common difference of 1, therefore this is an arithmetic sequence.

h $\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}, 2\frac{1}{4}$

$t_2 - t_1 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$t_3 - t_2 = 1\frac{1}{4} - \frac{3}{4} = \frac{1}{2}$

$t_4 - t_3 = 1\frac{3}{4} - 1\frac{1}{4} = \frac{1}{2}$

$t_5 - t_4 = 2\frac{1}{4} - 1\frac{3}{4} = \frac{1}{2}$

Common difference $= \frac{1}{2}$

Therefore this is an arithmetic sequence.

2 a Yes, an arithmetic sequence where $a = 2$, $d = 2$

b No, not an arithmetic sequence

c No, not an arithmetic sequence 2, 4, 8… .

d Yes, an arithmetic sequence $a = ?$ $d = 2$ i.e. 37, 39, 41… etc

e Using example, Yes, it is an arithmetic sequence 8, 16, 24 $a = 8$, $d = 8$

3 $d = t_2 - t_1 = 2 - -7 = 9$

$t_1 = -7$
$t_6 = -7 + 9(6 - 1) = 38$
$t_7 = -7 + 9(7 - 1) = 47$
$t_8 = -7 + 9(8 - 1) = 56$
$t_9 = -7 + 9(9 - 1) = 65$
$t_{10} = -7 + 9(10 - 1) = 74$

4 $t_1 = 5 \times 1 + 7$
$= 12$
$t_2 = 5 \times 2 + 7$
$= 17$
$t_3 = 5 \times 3 + 7$
$= 22$
$t_4 = 5 \times 4 + 7$
$= 27$
$t_5 = 5 \times 5 + 7$
$= 32$

The first five terms of the sequence are 12, 17, 22, 27 and 32.

5 $t_1 = 3 \times 1 - 5$
$= -2$
$t_2 = 3 \times 2 - 5$
$= 1$
$t_3 = 3 \times 3 - 5$
$= 4$
$t_4 = 3 \times 4 - 5$
$= 7$
$t_5 = 3 \times 5 - 5$
$= 10$

The first five terms of the sequence are −2, 1, 4, 7 and 10.

6 a Given $t_n = 5 + 3(n - 1)$,

$t_1 = 5 + 3(1 - 1)$
$= 5$
$t_2 = 5 + 3(2 - 1)$
$= 8$
$t_3 = 5 + 3(3 - 1)$
$= 11$
$t_4 = 5 + 3(4 - 1)$
$= 14$
$t_5 = 5 + 3(5 - 1)$
$= 17$

The first five terms of the sequence are 5, 8, 11, 14 and 17.

b Given $t_n = -1 - 7(n - 1)$,

$t_1 = -1 - 7(1 - 1)$
$= -1$
$t_2 = -1 - 7(2 - 1)$
$= -8$
$t_3 = -1 - 7(3 - 1)$
$= -15$
$t_4 = -1 - 7(4 - 1)$
$= -22$
$t_5 = -1 - 7(5 - 1)$
$= -29$

The first five terms of the sequence are −1, −8, −15, −22 and − 29.

c Given $t_n = \frac{1}{3} + \frac{2}{3}(n - 1)$,

$t_1 = \frac{1}{3} + \frac{2}{3}(1 - 1)$

$= \frac{1}{3}$

$t_2 = \frac{1}{3} + \frac{2}{3}(2 - 1)$

$= 1$

$t_3 = \frac{1}{3} + \frac{2}{3}(3 - 1)$

$= \frac{5}{3}$

$t_4 = \frac{1}{3} + \frac{2}{3}(4-1)$
$= \frac{7}{3}$
$t_5 = \frac{1}{3} + \frac{2}{3}(5-1)$
$= 3$
The first five terms of the sequence are $\frac{1}{3}$, 1, $\frac{5}{3}$, $\frac{7}{3}$ and 3.

d Given $t_n = 3.3 - 0.7(n+1)$,
$t_1 = 3.3 - 0.7(1+1)$
$= 1.9$
$t_2 = 3.3 - 0.7(2+1)$
$= 1.2$
$t_3 = 3.3 - 0.7(3+1)$
$= 0.5$
$t_4 = 3.3 - 0.7(4+1)$
$= -0.2$
$t_5 = 3.3 - 0.7(5+1)$
$= -0.9$
The first five terms of the sequence are 1.9, 1.2, 0.5, −0.2, −0.9.

7 a $t_2 - t_1 = 68 - 23$
$= 45$
$t_3 - t_2 = 113 - 68$
$= 45$
$t_4 - t_3 = 158 - 113$
$= 45$
$t_5 - t_4 = 203 - 158$
$= 45$
The common differences are constant, so the sequence is arithmetic.
$t_1 = 23$ and $d = 45$.

b $t_2 - t_1 = 8 - 3$
$= 5$
$t_3 - t_2 = 23 - 8$
$= 15$
$t_4 - t_3 = 68 - 23$
$= 45$
$t_5 - t_4 = 203 - 68$
$= 135$
The common differences are not constant, so the sequence is not arithmetic.

c $t_2 - t_1 = \frac{3}{4} - \frac{1}{2}$
$= \frac{1}{4}$
$t_3 - t_2 = 1 - \frac{3}{4}$
$= \frac{1}{4}$
$t_4 - t_3 = \frac{5}{4} - 1$
$= \frac{1}{4}$
$t_5 - t_4 = \frac{3}{2} - \frac{5}{4}$
$= \frac{1}{4}$
$t_6 - t_5 = \frac{7}{4} - \frac{3}{2}$
$= \frac{1}{4}$
The common differences are constant, so the sequence is arithmetic.
$t_1 = \frac{1}{2}$ and $d = \frac{1}{4}$.

8 a This is an infinite sequence as evidenced by the ellipsis (…) at the end of the sequence which indicates an ongoing sequence.

b The sequence is increasing; for each value of n, $t_n > t_{n-1}$

c $t_1 = -3.6$
$d = t_2 - t_1 = -2.1 - -3.6 = 1.5$

9 a $d = t_2 - t_1$
$= 3 - (-1)$
$= 4$
$t_{n+1} = t_n + d$
$= t_n + 4$
The recursive function is $t_{n+1} = t_n + 4$.

b $d = t_2 - t_1$
$= -2 - 1.5$
$= -3.5$
$t_{n+1} = t_n + d$
$= t_n - 3.5$
The recursive function is $t_{n+1} = t_n - 3.5$.

c $d = t_2 - t_1$
$= \frac{11}{2} - \frac{7}{2}$
$= \frac{4}{2}$
$= 2$
$t_{n+1} = t_n + d$
$= t_n + 2$
The recursive function is $t_{n+1} = t_n + 2$.

d $d = t_2 - t_1$
$= 4.3 - 6.2$
$= 1.9$
$t_{n+1} = t_n + d$
$= t_n + 1.9$
The recursive function is $t_{n+1} = t_n + 1.9$

10 a $t_2 - t_1 = -12 - 13$
$= -25$
Therefore $d = -25$.
The missing value f is the fourth term in the sequence.
$t_4 - t_3 = -25$
$f - -37 = -25$
$f = -62$
The missing value is −62.

b $t_4 - t_3 = 12.1 - 8.9$
$= 3.2$
Therefore $d = 3.2$.
The missing value j is the second term in the sequence.
$t_2 - t_1 = 3.2$
$j - 2.5 = 3.2$
$j = 5.7$
The missing value k is the fifth term in the sequence.
$t_5 - t_4 = 3.2$
$k - 12.1 = 3.2$
$k = 15.3$
The missing values are 5.7 and 15.3.

c $t_5 - t_4 = \frac{25}{4} - \frac{9}{2}$
$= \frac{7}{4}$
Therefore $d = \frac{7}{4}$.
The missing value r is the third term in the sequence.
$t_4 - t_3 = \frac{7}{4}$
$\frac{9}{2} - r = \frac{7}{4}$
$\frac{9}{2} - \frac{7}{4} = r$
$r = \frac{11}{4}$
The missing value q is the second term in the sequence.
$t_3 - t_2 = \frac{7}{4}$
$\frac{11}{4} - q = \frac{7}{4}$
$\frac{11}{4} - \frac{7}{4} = q$
$q = 1$
The missing value p is the first term in the sequence.
$t_2 - t_1 = \frac{7}{4}$
$1 - p = \frac{7}{4}$
$1 - \frac{7}{4} = p$
$p = -\frac{3}{4}$
The missing values are $p = -\frac{3}{4}$, $q = 1$ and $r = \frac{11}{4}$.

d Since

$d = s - \frac{1}{2}$

$d = t - s$

$d = 2 - t$

To find the common difference we can rearrange the first and third equations and substitute them into the second.

$$d = (2 - d) - \left(d + \frac{1}{2}\right)$$

$$d = 2 - d - d - \frac{1}{2}$$

$$d = \frac{3}{2} - 2d$$

$$3d = \frac{3}{2}$$

$$d = \frac{3}{6} = \frac{1}{2}$$

Therefore $s = \frac{1}{2} + \frac{1}{2} = 1$ and $t = 2 - \frac{1}{2} = \frac{3}{2}$.

11 a $t_1 = 3$

$t_2 = t_1 - 5$
$= 3 - 5$
$= -2$

$t_3 = t_2 - 5$
$= -2 - 5$
$= -7$

$t_4 = t_3 - 5$
$= -7 - 5$
$= -12$

The first four terms of the arithmetic sequence are 3, −2, −7, −12.

b $t_1 = -0.6$

$t_2 = t_1 + 1.4$
$= -0.6 + 1.4$
$= 0.8$

$t_3 = t_2 + 1.4$
$= 0.8 + 1.4$
$= 2.2$

$t_4 = t_3 + 1.4$
$= 2.2 + 1.4$
$= 3.6$

The first four terms of the arithmetic sequence are −0.6, 0.8, 2.2, 3.6.

c $t_1 = -23$

$t_2 = t_1 + 32$
$= -23 + 32$
$= 9$

$t_3 = t_2 + 32$
$= 9 + 32$
$= 41$

$t_4 = t_3 + 32$
$= 41 + 32$
$= 73$

The first four terms of the arithmetic sequence are −23, 9, 41, 73.

d $t_1 = 10$

$t_2 = t_1 - 3$
$= 10 - 3$
$= 7$

$t_3 = t_2 - 3$
$= 7 - 3$
$= 4$

$t_4 = t_3 - 3$
$= 4 - 3$
$= 1$

The first four terms of the arithmetic sequence are 10, 7, 4, 1.

12 23, 33, 43

$a = 23,\ d = 10$

$t_n = 23 + (n - 1)10$
$= 23 + 10n - 10$

$t_n = 13 + 10n$

13 Row 1 2 3 n

40, 43, 46 … $a = 40,\quad d = 3$

$t_n = 40 + (n - 1)3$
$= 40 + 3n - 3$

$t_n = 37 + 3n$

14 12, 15.5, 19, ... $a = 12,\ d = 3.5$

a $t_n = 12 + (n - 1)3.5$
$= 12 + 3.5n - 3.5$

$t_n = 8.5 + 3.5n$

b $t_n = 8.5 + 3.5 \times 100$
$= 358.5$

The last post will be 358.5 metres from the road.

Exercise 1.3 — The general form of an arithmetic sequence

1 a $t_1 = 4$

$d = t_2 - t_1 = 13 - 4 = 9$

Therefore, the sequence is described by the equation $t_n = 4 + 9\,(n - 1)$

b $t_1 = 9$

$d = t_2 - t_1 = 4.5 - 9 = -4.5$

Therefore, the sequence is described by the equation $t_n = 9 - 4.5\,(n - 1)$

c $t_1 = -60$

$d = t_2 - t_1 = (-49) - (-60) = 11$

Therefore, the sequence is described by the equation $t_n = -60 + 11\,(n - 1)$

d $t_1 = 100$

$d = t_2 - t_1 = (87) - (100) = -13$

Therefore, the sequence is described by the equation $t_n = 100 - 13\,(n - 1)$

2 a As the sequence has a common difference of −13, it is an arithmetic sequence.

$a = 85, d = -13, n = 20$

$t_n = a + (n - 1)d$

$t_{20} = 85 + (20 - 1)(-13)$
$= 85 + (19)(-13)$
$= 85 - 247$
$= -162$

The 20th term of the sequence is −162.

b $d = -43$, $n = 70$, $t_{70} = 500$

$$\begin{aligned} a &= t_n - (n-1)d \\ &= 500 - (70-1)(-43) \\ &= 500 - (69)(-43) \\ &= 500 - -2967 \\ &= 500 + 2967 \\ &= 3467 \end{aligned}$$

The first term of the sequence is 3467.

3 a $a = -32$, $n = 8$, $t_8 = 304$

$$\begin{aligned} d &= \frac{t_n - a}{n-1} \\ &= \frac{304 - -32}{8-1} \\ &= \frac{336}{7} \\ &= 48 \end{aligned}$$

The common difference is 48.

b $a = 5$, $d = 40$, $t_n = 85$

$$\begin{aligned} n &= \frac{t_n - a}{d} + 1 \\ &= \frac{85-5}{40} + 1 \\ &= 3 \end{aligned}$$

The 3rd term in the sequence has a value of 85.

c $a = 40$, $d = 12$, $t_n = 196$

$$\begin{aligned} n &= \frac{t_n - a}{d} + 1 \\ &= \frac{196-40}{12} + 1 \\ &= 14 \end{aligned}$$

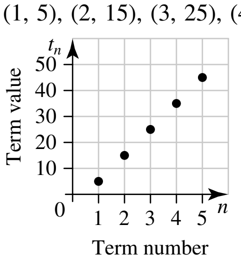
The 14th term in the sequence has a value of 196.

4 a Given $t_n = 5 + 10(n-1)$,

$$\begin{aligned} t_1 &= 5 + 10(1-1) \\ &= 5 \\ t_2 &= 5 + 10(2-1) \\ &= 15 \\ t_3 &= 5 + 10(3-1) \\ &= 25 \\ t_4 &= 5 + 10(4-1) \\ &= 35 \\ t_5 &= 5 + 10(5-1) \\ &= 45 \end{aligned}$$

Term number	1	2	3	4	5
Term value	5	15	25	35	45

b The points to be plotted are
$(1, 5)$, $(2, 15)$, $(3, 25)$, $(4, 35)$ and $(5, 45)$.



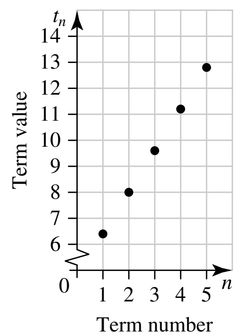
c Extend the straight line on the graph to cover future values of the sequence. Then read the value of when $n = 9$ from the graph. The 9th term of the sequence is 85.

5 a Given $t_n = 6.4 + 1.6(n-1)$,

$$\begin{aligned} t_1 &= 6.4 + 1.6(1-1) \\ &= 6.4 \\ t_2 &= 6.4 + 1.6(2-1) \\ &= 8 \\ t_3 &= 6.4 + 1.6(3-1) \\ &= 9.6 \\ t_4 &= 6.4 + 1.6(4-1) \\ &= 11.2 \\ t_5 &= 6.4 + 1.6(5-1) \\ &= 12.8 \end{aligned}$$

Term number	1	2	3	4	5
Term value	6.4	8	9.6	11.2	12.8

b The points to be plotted are
$(1, 6.4)$, $(2, 8)$, $(3, 9.6)$, $(4, 11.2)$ and $(5, 12.8)$.



c Extend the straight line on the graph to cover future values of the sequence. Then read the value of when $n = 13$ from the graph. The 13th term of the sequence is 25.6.

6 a $a = 6$

$$\begin{aligned} d &= t_2 - t_1 \\ &= 13 - 6 \\ &= 7 \end{aligned}$$

$$\begin{aligned} t_n &= a + (n-1)d \\ &= 6 + (n-1) \times 7 \\ &= 6 + 7(n-1) \end{aligned}$$

Given $n = 15$,

$$\begin{aligned} t_{15} &= 6 + 7(15-1) \\ &= 6 + 7 \times 14 \\ &= 6 + 98 \\ &= 104 \end{aligned}$$

b $a = 9$

$$\begin{aligned} d &= t_2 - t_1 \\ &= 23 - 9 \\ &= 14 \end{aligned}$$

$$\begin{aligned} t_n &= a + (n-1)d \\ &= 9 + (n-1) \times 14 \\ &= 9 + 14(n-1) \end{aligned}$$

Given $n = 20$,

$$\begin{aligned} t_{20} &= 9 + 14(20-1) \\ &= 9 + 14 \times 19 \\ &= 9 + 266 \\ &= 275 \end{aligned}$$

c $a = 56$

$d = t_2 - t_1$

$= 48 - 56$

$= -8$

$t_n = a + (n-1)d$

$= 56 + (n-1) \times (-8)$

$= 56 - 8(n-1)$

Given $n = 30$,

$t_{30} = 56 - 8(30-1)$

$= 56 - 8 \times 29$

$= 56 - 232$

$= -176$

d $a = \dfrac{72}{5}$

$d = t_2 - t_1$

$= \dfrac{551}{40} - \dfrac{72}{5}$

$= -\dfrac{5}{8}$

$t_n = a + (n-1)d$

$= \dfrac{72}{5} + (n-1) \times \left(-\dfrac{5}{8}\right)$

$= \dfrac{72}{5} - \dfrac{5}{8}(n-1)$

Given $n = 55$,

$t_{55} = \dfrac{72}{5} - \dfrac{5}{8}(55-1)$

$= \dfrac{72}{5} - \dfrac{5}{8} \times 54$

$= \dfrac{72}{5} - \dfrac{135}{4}$

$= -\dfrac{387}{20}$

7 a 2, 7, 12, 17, 22...

$t_1 = 2,\ d = 5$

$t_{25} = 2 + (25-1)5$

$t_{25} = 2 + 24 \times 5$

$t_{25} = 122$

The 25th term is 122

b 0, 100, 200, 300, 400...

$t_1 = 0,\ d = 100$

$t_{30} = 0 + (30-1)100$

$= 0 + 29 \times 100$

$= 2900$

The 30th term is 2900

c 5, −2, −9, −16, −23...

$t_1 = 5,\ d = -7$

$t_{33} = 5 + (33-1) \times -7$

$= 5 + 32 \times -7$

$= 5 - 224$

$= -219$

The 33rd term is −219

8 a $t_2 = t_1 + d = 13 \quad (1)$

$t_5 = t_1 + 4d = 31 \quad (2)$

(2) − (1)

$3d = 18$

$d = 6$

If $d = 6,\ t_1 = ?$

$t_1 + 6 = 13$

$t_1 = 7$

$t_{17} = 7 + 16 \times 6$

$= 103$

The 17th term is 103

b $t_2 = t_1 + d = -23 \quad (1)$

$t_5 = t_1 + 4d = 277 \quad (2)$

(2) − (1)

$3d = 300$

$d = 100$

If $d = 100,\ t_1 = ?$

$t_1 + 100 = -23$

$t_1 = -123$

$t_{20} = -123 + 19 \times 100$

$= 1777$

The 20th term is 1777

c $t_2 = t_1 + d = 0 \quad (1)$

$t_6 = t_1 + 5d = -8 \quad (2)$

(2) − (1)

$4d = -8$

$d = -2$

If $d = -2,\ t_1 = ?$

$t_1 - 2 = 0$

$t_1 = 2$

$t_{32} = t + 31 \times d$

$= 2 + 31 \times -2$

$= -60$

The 32nd term is −60

d $t_3 = a + 2d = 5 \quad (1)$

$t_7 = a + 6d = -19 \ (2)$

(2) − (1)

$4d = -24$

$d = -6$

If $d = -6,\ t_1 = ?$

$a_1 - 12 = 5$

$a_1 = 17$

$t_{40} = 17 + 39 \times -6$

$= -217$

The 40th term is −217

e $t_9 = t_1 + 3d = 2$ (1)

$t_9 = t_1 + 8d = -33$ (2)

(2) − (1)

$5d = -35$

$d = -7$

If $d = -7,\ t_1 = ?$

$t_1 - 21 = 2$

$t_1 = 23$

$t_{26} = 23 + 25 \times -7$

$= -152$

The 26th term is −152

9 a $d = 6, n = 31, t_{31} = 904$

$t_1 = t_n - (n-1)d$

$= 904 - (31 - 1) \times 6$

$= 904 - (30) \times 6$

$= 904 - 180$

$= 724$

The first term of the sequence is 724.

b $d = \frac{2}{5}, n = 40, t_{31} = -37.2$

$t_1 = t_n - (n-1)d$

$= -37.2 - (40 - 1) \times \frac{2}{5}$

$= -37.2 - (39) \times \frac{2}{5}$

$= -37.2 - \frac{78}{5}$

$= -37.2 - 15.6$

$= -52.8$

The first term of the sequence is −52.8.

c $t_1 = 564, n = 51, t_{51} = 54$

$d = \frac{t_n - t_1}{n - 1}$

$= \frac{54 - 564}{51 - 1}$

$= \frac{-510}{50}$

$= -10.2$

The common difference is −10.2.

d $t_1 = -87, n = 61, t_{61} = 43$

$d = \frac{t_n - t_1}{n - 1}$

$= \frac{43 - -87}{61 - 1}$

$= \frac{130}{60}$

$= \frac{13}{6}$

The common difference is $\frac{13}{6}$.

10 a $t_1 = 120, d = 16, t_n = 712$

$n = \frac{t_n - t_1}{d} + 1$

$= \frac{712 - 120}{16} + 1$

$= 38$

The 38th term in the sequence has a value of 712.

b $a_1 = 320, d = 4, t_n = 1160$

$n = \frac{t_n - t_1}{d} + 1$

$= \frac{1160 - 320}{4} + 1$

$= 211$

The 211th term in the sequence has a value of 1160.

11 Since the sequence is arithmetic, the difference between consecutive terms is constant.

$t_2 - t_1 = t_3 - t_2$

$x + 4 - (x - 5) = 2x - 7 - (x + 4)$

$x + 4 - x + 5 = 2x - 7 - x - 4$

$9 = x - 11$

$9 + 11 = x$

$x = 20$

12 a Using the points (2, 12) and (4, 9),

$d = \frac{9 - 12}{4 - 2}$

$= \frac{-3}{2}$

$= -1.5$

b Using the point (2, 12),

$d = -1.5, n = 2, t_2 = 12$

$t_1 = t_n - (n-1)d$

$= 12 - (2 - 1)(-1.5)$

$= 12 - (1)(-1.5)$

$= 12 - -1.5$

$= 12 + 1.5$

$= 13.5$

The first term of the sequence is 13.5.

c $t_1 = 13.5$ and $d = -1.5$

$t_n = t_1 + (n-1)d$

$= 13.5 + (n - 1)(-1.5)$

$= 13.5 - 1.5(n - 1)$

$t_{12} = 13.5 - 1.5(12 - 1)$

$= 13.5 - 1.5 \times 11$

$= 13.5 - 16.5$

$= -3$

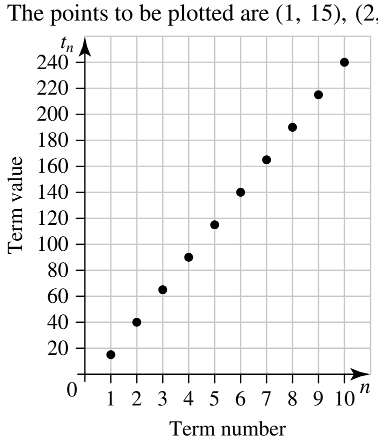
The 12th term in the sequence is −3.

13 Given $t_n = 15 + 25\,(n - 1)$,

$$
\begin{aligned}
t_1 &= 15 + 25(1 - 1) \\
&= 15 \\
t_2 &= 15 + 25(2 - 1) \\
&= 40 \\
t_3 &= 15 + 25(3 - 1) \\
&= 65 \\
t_4 &= 15 + 25(4 - 1) \\
&= 90 \\
t_5 &= 15 + 25(5 - 1) \\
&= 115 \\
t_6 &= 15 + 25(6 - 1) \\
&= 140 \\
t_7 &= 15 + 25(7 - 1) \\
&= 165 \\
t_8 &= 15 + 25(8 - 1) \\
&= 190 \\
t_9 &= 15 + 25(9 - 1) \\
&= 215 \\
t_{10} &= 15 + 25(10 - 1) \\
&= 240
\end{aligned}
$$

Term number	1	2	3	4	5	6	7	8	9	10
Term value	15	40	65	90	115	140	165	190	215	240

The points to be plotted are (1, 15), (2, 40), (3, 65), (4, 35), (5, 115), (6, 140), (7, 165), (8, 190), (9, 215), and (10, 240).



14 **a** $t_1 = 60\,000,\ d = 2500$

$$
\begin{aligned}
t_n &= t_1 + (n - 1)d \\
&= 60\,000 + (n - 1)2500 \\
&= 60\,000 + 2500(n - 1)
\end{aligned}
$$

For $n = 6$,

$$
\begin{aligned}
t_n &= 60\,000 + 2500(6 - 1) \\
&= 60\,000 + 2500 \times 5 \\
&= 60\,000 + 12\,500 \\
&= 72\,500
\end{aligned}
$$

In the 6th year her salary will be \$72 500.

b $t_1 = 60\,000,\ d = 2500,\ t_n = 85\,000$

$$
\begin{aligned}
n &= \frac{t_n - t_1}{d} + 1 \\
&= \frac{85\,000 - 60\,000}{2500} + 1 \\
&= 11
\end{aligned}
$$

It will take 11 years for her salary to reach \$85 000.

Exercise 1.4 — The sum of an arithmetic sequence

1 By regular addition: $S_n = 5 + 7 + 9 + 11 = 32$

By rule:

$t_1 = 5$

$t_4 = 11$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$= \frac{4(5 + 11)}{2}$$

$$= 32$$

2 $n = 6$

$t_1 = 2$

$$t_6 = 2 + 2(6 - 1)$$
$$= 2 + 2 \times 5$$
$$= 2 + 10$$
$$= 12$$

$$S_n = \frac{n(t_n + t_1)}{2}$$

$$S_6 = \frac{6(t_6 + t_1)}{2}$$

$$S_6 = \frac{6(12 + 2)}{2}$$

$$= \frac{6 \times 14}{2}$$

$$= \frac{84}{2}$$

$$= 42$$

The sum of the first six terms of the sequence represented by $t_n = 2 + 2(n - 1)$ is 42.

3 $n = 4$

$t_1 = -4$

$$t_4 = -4 - 5(4 - 1)$$
$$= -4 - 5 \times 3$$
$$= -4 - 15$$
$$= -19$$

$$S_n = \frac{n(t_n + t_1)}{2}$$

$$S_4 = \frac{4(-19 + (-4))}{}$$

$$= \frac{4(-19 - 4)}{2}$$

$$= \frac{4 \times (-23)}{2}$$

$$= \frac{-92}{2}$$

$$= -46$$

The sum of the first four terms of the sequence represented by $t_n = -4 - 5(n - 1)$ is -46.

4 $S_n = -30,\ t_1 = 5,\ t_n = -15$

$$S_n = \frac{n(t_n + t_1)}{2}$$

$$-30 = \frac{n(-15 + 5)}{2}$$

$$-60 = -10n$$

$$n = 6$$

5 $S_n = -14,\ t_1 = -14,\ t_n = 7$

$$S_n = \frac{n(t_n + t_1)}{2}$$

$$-14 = \frac{n(7 - 14)}{2}$$

$$-28 = -7n$$

$$n = 4$$

6 $n = 5, S_5 = 75, t_5 = 33$

$$S_n = \frac{n(t_n + t_1)}{2}$$

$$S_5 = \frac{5(t_5 + t_1)}{2}$$

$$75 = \frac{5(33 + t_1)}{2}$$

$$150 = 5(33 + t_1)$$

$$33 + t_1 = 30$$

$$t_1 = -3$$

7 $n = 7, S_7 = 0, t_1 = -27$

$$S_n = \frac{n(t_n + t_1)}{2}$$

$$S_7 = \frac{7(t_7 + t_1)}{2}$$

$$0 = \frac{7(t_7 - 27)}{2}$$

$$t_7 = 27$$

8 $n = 5, t_1 = 20, d = 4$

$$t_5 = t_1 + d(5 - 1)$$
$$= 20 + 4 \times 4$$
$$= 36$$

$$S_n = \frac{n(t_n + t_1)}{2}$$

$$S_5 = \frac{5(t_5 + t_1)}{2}$$

$$= \frac{5(36 + 20)}{2}$$

$$= \frac{5 \times 56}{2}$$

$$= \frac{280}{2}$$

$$= 140$$

9 $n = 5, S_5 = 55, t_1 = 3$

$$S_n = \frac{n(t_n + t_1)}{2}$$

$$S_5 = \frac{5(t_5 + t_1)}{2}$$

$$55 = \frac{5(t_5 + 3)}{2}$$

$$110 = 5(t_5 + 3)$$

$$22 = t_5 + 3$$

$$t_5 = 19$$

Since $t_1 = 3$, $t_5 = 19$, we can find t_4 by calculating the common difference.

$$t_5 = t_1 + 4d$$
$$19 = 3 + 4d$$
$$16 = 4d$$
$$d = 4$$

$$t_5 = t_4 + d$$
$$19 = t_4 + 4$$
$$t_4 = 15$$

10 $t_1 = -10,\ t_4 = 29$

To find the common difference d:

$$t_4 = t_1 + 3\text{d}$$
$$29 = -10 + 3d$$
$$39 = 3d$$
$$d = 13$$

Find the fifth term:

$$t_5 = t_4 + d$$
$$= 29 + 13$$
$$= 42$$

Now we can calculate the sum of the first five terms with $n = 5$, $t_5 = 42$, $t_1 = -10$.

$$S_n = \frac{n(t_n + t_1)}{2}$$

$$S_5 = \frac{5(t_5 + t_1)}{2}$$

$$= \frac{5(42 - 10)}{2}$$

$$= \frac{5 \times 32}{2}$$

$$= \frac{160}{2}$$

$$= 80$$

11 $n = 3, s_3 = 24, d = 7.$

$$S_n = \frac{n(t_n + t_1)}{2}$$

$$S_3 = \frac{3(t_3 + t_1)}{2}$$

$$24 = \frac{3(t_3 + t_1)}{2}$$

$$48 = 3(t_3 + t_1)$$

$$t_3 + t_1 = 16$$

We also know that

$$t_3 = t_1 + 2d$$
$$= t_1 + 2 \times 7$$
$$= t_1 + 14$$

Substituting $t_3 = t_1 + 14$ into $t_3 + t_1 = 16$ gives us

$$(t_1 + 14) + t_1 = 16$$
$$2t_1 = 2$$
$$t_1 = 1$$

And therefore we can calculate t_2 and t_3.

$$t_2 = t_1 + d$$
$$= 1 + 7$$
$$= 8$$

$$t_3 = t_2 + d$$
$$= 8 + 7$$
$$= 15$$

Therefore the first three terms are 1, 8 and 15.

12 $t_1 = 100$

$d = 75$

$S_{15} = ?$

$$S_{15} = \frac{15}{2}[2 \times 100 + 14 \times 75]$$

$= \$9375$

Sam's total profit is \$9375.

13 $t_1 = 36\,000,\ d = 1200,\ S_{10} = ?$

$$S_{10} = \frac{10}{2}[36\,000 \times 2 + 9 \times 1200]$$

$= \$414\,000$

George would have earned \$414 000.

14 $d = 0.8,\ t = 15,\ S_{17} = ?$

$$S_{17} = \frac{10}{2}[2 \times 15 + 16 \times 0.8]$$

$= 363.8\,\text{cm}$

The total height of the first 17 steps is 363.8 cm.

15 $t_1 = 250,\ d = 15,\ t_{60} = ?$

$t_{60} = 250 + 59 \times 15$

$= 1135$

After 5 years Paula will have 1135 stamps.

16

2000	2001	2002		2019
3000	3400	3800	…	

$t_1 = 3000,\ d = 400,\ t_{20} = ?$

$t_{20} = 3000 + 19 \times 400$

$= \$10\,600$

a The proceeds of the 2019 Fete should be \$10 600.

b The total amount would be \$136 000.

$$S_{20} = \frac{20}{2}(2 \times 3000 + 19 \times 400).$$

$= \$136\,000.$

Exercise 1.5 — Applications of arithmetic sequences

1 a Given that the rate is 4.8% per year,

$$r = \frac{4.8\%}{12\text{ months}} = 0.4\%\text{ per month}$$

$$I = \frac{PrT}{100}$$

$$= \frac{1500 \times 0.4 \times 1}{100}$$

$$= \frac{600}{100}$$

$= 6$

$t_1 = 1500 + 6$

$= 1506$

$t_1 = 1506,\ d = 6$

$t_n = 1506 + 6(n - 1)$

b $t_1 = 1506 + 6(1 - 1)$

$= 1506 + 6 \times 0$

$= 1506 + 0$

$= 1506$

$t_2 = 1506 + 6(2 - 1)$

$= 1506 + 6 \times 1$

$= 1506 + 6$

$= 1512$

$t_3 = 1506 + 6(3 - 1)$

$= 1506 + 6 \times 2$

$= 1506 + 12$

$= 1518$

$t_4 = 1506 + 6(4 - 1)$

$= 1506 + 6 \times 3$

$= 1506 + 18$

$= 1524$

$t_5 = 1506 + 6(5 - 1)$

$= 1506 + 6 \times 4$

$= 1506 + 24$

$= 1530$

$t_6 = 1506 + 6(6 - 1)$

$= 1506 + 6 \times 5$

$= 1506 + 30$

$= 1536$

The amounts in Grigor's account at the end of each of the first 6 months are \$1506, \$1512, \$1518, \$1524, \$1530 and \$1536.

c $t_{18} = 1506 + 6(18 - 1)$

$= 1506 + 6 \times 17$

$= 1506 + 102$

$= 1608$

$t_{18} = \$1608$

2 a Given $t_n = 8050 + 50\,(n - 1)$,

$t_1 = 8050$ and $d = 50$ by inspection.

amount invested $= a - d$

$= 8050 - 50$

$= 8000$

Justine invested \$8000 in the account.

b

$$I = \frac{PrT}{100}$$

$$50 = \frac{8000 \times r \times 1}{100}$$

$5000 = 8000r$

$$r = \frac{5000}{8000}$$

$= 0.625$

annual rate $= 0.625 \times 12$

$= 7.5\%$

3 a First, calculate the value of the car after 1 km of use.

$t_1 = 24\,000 - 0.25$

$= 23\,999.75$

$t_1 = 23\,999.75,\ d = -0.25$

$t_n = t_1 + (n - 1)d$

$= 23\,999.75 - 0.25(n - 1)$

b Given $n = 12\,000$,

$t_{12\,000} = 23\,999.75 - 0.25(12\,000 - 1)$

$= 23\,999.75 - 0.25 \times 11\,999$

$= 23\,999.75 - 2999.75$

$= 21\,000$

After 12 000 km the car will be worth \$21 000.

4 a Given $t_n = 5399.999 - 0.001\,(n - 1)$, $t_1 = 5399.999$ and $d = -0.001$ by inspection.

original cost $= t_1 - d$

$= 5399.999 - -0.001$

$= 5400$

The photocopier cost \$5400.

b Since $d = 0.001$,

rate of depriction $= \$0.001$

$= 0.1$ cents

5 a Given that the rate is 6% per year,

$$I = \frac{PrT}{100}$$

$$= \frac{90\,000 \times 6 \times 1}{100}$$

$$= \frac{540\,000}{100}$$

$= 5400.$

Nadia will receive \$5400 interest after one year.

b $t_1 = 90\,000 + 5400$

$= 95\,400$

$t_1 = 95\,400,\ d = 5400$

$t_n = 95\,400 + 5400(n - 1)$

c $t_1 = 95\,400,\ d = 5400,\ t_n = 154\,800$

$$n = \frac{t_n - t_1}{d} + 1$$

$$= \frac{154\,800 - 95\,400}{5400} + 1$$

$= 12$

Nadia should keep her money invested for 12 years.

6 a First, calculate the value of the car after 1 km of use.

$t_1 = 23\,000 - 210$

$= 22\,790$

$t_1 = 22\,790,\ d = -210$

$t_n = t_1 + (n - 1)d$

$= 22\,790 - 210(n - 1)$

For $n = 18$,

$t_{18} = 22\,790 - 210(18 - 1)$

$= 22\,790 - 210 \times 17$

$= 22\,790 - 3570$

$= 19\,220$

After 18 months the car will be worth \$19 220.

b $n = 3 \times 12$

$= 36$

$t_{36} = 22\,790 - 210(36 - 1)$

$= 22\,790 - 210 \times 35$

$= 22\,790 - 7350$

$= 15\,440$

$$\begin{aligned}\text{amount depricated} &= 23\,000 - 15\,440\\ &= 7560\end{aligned}$$

In 3 years the value of the car depreciates by \$7560.

c $t_1 = 22\,970$, $d = -210$, $t_n = 6200$

$$\begin{aligned}n &= \frac{t_n - t_1}{d} + 1\\ &= \frac{6200 - 22\,970}{-210} + 1\\ &= 80\end{aligned}$$

It will take 80 months.

7 a $t_3 = \$2875$

$t_3 = t_1 + d\,(3 - 1)$

$2875 = t_1 + 2\,d \qquad (1)$

$t_5 = \$3125$

$t_5 = t_1 + d\,(5 - 1)$

$3125 = t_1 + 4\,d \qquad (2)$

Rearranging (1) and (2) to make t_1 the subject and equating them, we find

$$\begin{aligned}2875 - 2\,d &= 3125 - 4d\\ 2d &= 250\\ d &= \$125\end{aligned}$$

Therefore, Sanchia receives \$125 a year in interest.

b Substituting d = 125 into equation (1):

$$\begin{aligned}2875 &= t_1 + 2\,d\\ 2875 &= t_1 + 2\,(125)\\ t_1 &= 2875 - 250 = 2625\end{aligned}$$

Therefore, Sanchia initially deposited \$2625 into her account.

c $$\begin{aligned}I &= \frac{PrT}{100}\\ \Rightarrow r &= \frac{100\,I}{PT}\\ &= \frac{100\ \times 125}{2625 \times 1}\\ &= 4.76\ \%\end{aligned}$$

8 a $t_1 = 50\,000$

$$\begin{aligned}d &= 50\,000 \times \frac{30}{100}\\ &= 15\,000\end{aligned}$$

$$\begin{aligned}t_n &= t_1 + (n-1)d\\ &= 50\,000 + (n-1)15\,000\\ &= 50\,000 + 15\,000(n-1)\end{aligned}$$

For $n = 20$,

$$\begin{aligned}t_{20} &= 50\,000 + 15\,000(n-1)\\ &= 50\,000 + 15\,000(20-1)\\ &= 50\,000 + 15\,000 \times 19\\ &= 50\,000 + 285\,000\\ &= 335\,000\end{aligned}$$

335 000 packets are manufactured in the 20th week.

b $t_1 = 50\,000$, $d = 15\,000$,

$t_n = 5\,540\,000$

$$\begin{aligned}n &= \frac{t_n - t_1}{d} + 1\\ &= \frac{5\,540\,000 - 50\,000}{15\,000} + 1\\ &= 367\end{aligned}$$

In the 367th week.

9 a $$\begin{aligned}\text{unit deprication} &= -\frac{250\,000}{500\,000\,000}\\ &= -0.0005\end{aligned}$$

The machine depreciates by \$0.0005 or 0.05 cents per can.

b Given the machine makes 40 200 000 cans per year,

$$\begin{aligned}d &= -0.0005 \times 40\,200\,000\\ &= -20\,100\end{aligned}$$

So the machine depreiates in value by \$20 100 each year.

$$\begin{aligned}t_1 &= \text{original price} + d\\ &= 250\,000 - 20\,100\\ &= 229\,900\end{aligned}$$

The write-off value is the point at which the machine has a value of \$0.

For $t_n = 0$

$$\begin{aligned}n &= \frac{t_n - t_1}{d} + 1\\ &= \frac{0 - 229\,900}{-20\,100} + 1\\ &= 12.44\end{aligned}$$

It will be written off in the 13th year.

c For $t_n = 89\,200$

$$\begin{aligned}n &= \frac{t_n - t_1}{d} + 1\\ &= \frac{89\,200 - 229\,900}{-20\,100} + 1\\ &= 8\end{aligned}$$

After 8 years.

10

1st yr		2nd yr		3rd yr
520		560		600
	+40		+40	

a $t_{15} = 520 + 14 \times 40$

1080 donations made in 15th year.

b $$\begin{aligned}S_{15} &= \frac{15}{2}[2 \times 520 + 14 \times 40]\\ &= 12\,000\end{aligned}$$

12 000 donations over 15 year.

11 $t_1 = 0.8$

$t_2 = 2.4$

$t_3 = 4$

$t_4 = 5.6$

$d = t_2 - t_1 = 2.4 - 0.8 = 1.6$

$t_n = t_1 + d(n-1)$

$t_9 = 0.8 + 1.6(9-1)$

$t_9 = 13.6$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$S_n = \frac{9(0.8 + 13.6)}{2}$$

$S_n = 94.8$

Thus, the rock will have fallen a total of 94.8 m at the end of the ninth second.

12 $t_1 = a$

$S_4 = 32$

$\Rightarrow 32 = \dfrac{4\,(a + t_4)}{2}$

$64 = 4\,(a + t_4)$

$16 = a + t_4$

$\Rightarrow t_4 = 16 - a \qquad (1)$

$S_7 = 77$

$\Rightarrow 77 = \dfrac{7\,(a + t_7)}{2}$

$154 = 7\,(a + t_7)$

$22 = a + t_7$

$\Rightarrow t_7 = 22 - a \qquad (2)$

average of t_4 and $t_7 = \dfrac{t_4 + t_7}{2} = 14$

Substituting (1) and (2) into this equation, we get

$$14 = \frac{(16 - a) + (22 - a)}{2}$$

$28 = 38 - 2a$

$2a = 10$

$\therefore a = 5$ (i.e $t_1 = 5$)

Substituting this value for a into (1):

$t_4 = 16 - 5$

$t_4 = 11$

As $t_n = t_1 + d\,(n-1)$,

$t_4 = t_1 + d(4-1)$

$t_4 = t_1 + 3d$

$\Rightarrow 11 = 5 + 3d$

$6 = 3d$

$\therefore d = 2$

We can now calculate how much Lucca earns in the ninth round:

$t_9 = 5 + 2(9-1)$

$t_9 = 21$

As we now have values for t_1, t_9 and n, we can calculate Lucca's total winnings at the end of the ninth round:

$$S_9 = \frac{9(5 + 21)}{2}$$

$S_9 = 117$

Therefore, Lucca's parents will pay him \$117 if he completes nine rounds.

13 a $t_1 = 5000,\ d = 1200$

$t_n = 5000 + 1200\,(n-1)$

For $n = 15$,

$$\begin{aligned} t_{15} &= 5000 + 1200\,(15-1) \\ &= 5000 + 1200 \times 14 \\ &= 5000 + 16\,800 \\ &= 21\,800 \end{aligned}$$

In 15 years' time they will have 21 800 members.

b $t_1 = 200,\ n = 6,\ t_8 = 320$

$$\begin{aligned} d &= \frac{t_n - a}{n-1} \\ &= \frac{320-200}{6-1} \\ &= \frac{120}{5} \\ &= 24 \end{aligned}$$

$t_n = 200 + 24(n-1)$

For $n = 15$,

$$\begin{aligned} t_{15} &= 200 + 24\,(15-1) \\ &= 200 + 24 \times 14 \\ &= 200 + 336 \\ &= 536 \end{aligned}$$

In 15 years' time the tickets would cost \$536 each.

c For the first year,

total membership income

$= \$200 \times 5000$

$= \$1\,000\,000$

For the 15th year,

total membership income

$= \$536 \times 21\,800$

$= \$11\,684\,800$

14 $8 \xrightarrow{+3.5} 11.5 \xrightarrow{+3.5} 15$

a In the 4th month

$15 + 3.5 = 18.5$ tonnes of rock will be crushed.

b
$$\begin{aligned} t_n &= 8 + (n-1)3.5 \\ &= 8 + 3.5n - 3.5 \\ t_n &= 4.5 + 3.5n \end{aligned}$$

c
$$\begin{aligned} t_{60} &= 4.5 + 3.5 \times 60 \\ &= 214.5 \text{ tonnes crushed in } 60^{\text{th}} \text{ month} \end{aligned}$$

d $100 = 4.5 + 3.5n$

$27.28 = n$

100 tonnes will be exceeded in the 28th month.

e
$$3050 = \frac{n}{2}\,[2 \times 8 + (n-1)\,3.5]$$
$$3050 = \frac{n}{2}\,[16 + 3.5n - 3.5]$$
$$\frac{3050 \times 2}{n} = 12.5 + 3.5n$$
$$\frac{6100}{n} = 12.5 + 3.5n$$
$$6100 = 12.5n + 3.5n^2$$
$$0 = 3.5n^2 + 12.5n - 6100$$

$a = 3.5,\ b = 12.5,\ c = -6100$

$$n = \frac{-12.5 \pm \sqrt{12.5^2 + 4 \times 3.5 \times 6100}}{2 \times 3.5}$$
$$n = \frac{-12.5 \pm \sqrt{85556.25}}{7}$$
$$n = \frac{-12.5 + 292.5}{7} \text{ or } \frac{-12.5 - 292.5}{2}$$

$n = 40$ Not feasible

After 40 months the environmental impact survey needs to be completed.

1.6 Review: exam practice

1 a $16 - 12 = 20 - 16 = 24 - 20 = 28 - 24 = 4$

As the common difference is the same for all terms (i.e.d = 4), this is an arithmetic sequence

b $4.8 - 12 = -7.2$ but $1.92 - 4.8 \neq -7.2$

As there is no common difference, this is not an arithmetic sequence

c $15 - 12 = 3$ but $21 - 15 \neq 3$

As there is no common difference, this is not an arithmetic sequence

d $6.2 - 12 = -5.8$ but $3.3 - 6.2 \neq -5.8$

As there is no common difference, this is not an arithmetic sequence

e $14.8 - 12 = 17.6 - 14.8 = 20.4 - 17.6 = 23.2 - 20.4 = 2.8$

As the common difference is the same for all terms (i.e.d = 2.8), this is an arithmetic sequence

The correct answers are **a** and **e**.

2 $t_1 =$ the first term $= -16$

$d = t_2 - t_1 = (-11.2) - (-16) = 4.8$

The correct answer is **C**

3 $54.5 - 58 = -3.5$, therefore $d = -3.5$

$65 - 3.5 = 61.5$

The correct answer is **D**.

4 $-4.3, -2.1, 0.1, 2.3, 4.5$

$t_{41} = t_1 = -4.3,\ d = 2.2$

$t_{41} = -4.3 + 40 \times 2.2$

$t_{41} = 83.7$

$\therefore$ the answer is **A**.

5 $t_1 = 5.2$

$t_2 = 6$

$d = t_2 - t_1 = 6 - 5.2 = 0.8$

$t_{22} = 5.2 + 0.8\ (22-1) = 22$

$$S_{22} = \frac{22\,(5.2 + 22)}{2} = 299.2$$

The correct answer is **B**.

6 a
$$\begin{aligned} t_n &= 2 + 5\,(n-1) \\ t_1 &= 2 + 5\,(1-1) = 2 \\ t_2 &= 2 + 5\,(2-1) = 7 \\ t_3 &= 2 + 5\,(3-1) = 12 \\ t_4 &= 2 + 5\,(4-1) = 17 \\ t_5 &= 2 + 5\,(5-1) = 22 \end{aligned}$$

The first 5 terms of the sequence are: 2, 7, 12, 17, 22

b $t_1 =$ first term $= 2$

$d = t_2 - t_1 = 7 - 2 = 5$

c
$$\begin{aligned} 72 &= 2 + 5(n-1) \\ 70 &= 5(n-1) \\ 16 &= (n-1) \\ n &= 17 \end{aligned}$$

The 17th term will have a value of 72.

7 As the first term of the sequence is 3, we can write $t_1 = 3$

Each term is separated from the succeeding term by –2.5, so the common difference

$d = -2.5$

$t_n = t_1 + d(n-1)$

so the equation describing this sequence is $t_n = 3 - 2.5\,(n-1)$

8 a $29.2 - 27 = 2.2$, therefore the common difference is 2.2.

b $t_1 = 27$

c $t_n = 27 + 2.2(n-1)$

d
$$\begin{aligned} t_9 &= 27 + 2.2(9-1) \\ &= 27 + 2.2 \times 8 \\ &= 27 + 17.6 \\ &= 44.6 \end{aligned}$$

9 a $t_1 = 4,\ d = 8 - 4 = 4$

$t_8 = 4 + 4(8-1) = 32$

$$S_8 = \frac{8(4 + 32)}{2} = 144$$

The sum is 144

b $t_1 = 2,\ d = -2.5 - 2 = -4.5$

$t_8 = 2 - 4.5(8-1) = -29.5$

$$S_8 = \frac{8(2 - 29.5)}{2} = -110$$

The sum is -110

c $t_1 = \frac{1}{2},\ d = 2 - \frac{1}{2} = 1.5$

$$t_8 = \frac{1}{2} + 1.5(8-1) = 11$$

$$S_8 = \frac{8\left(\frac{1}{2} + 11\right)}{2} = 46$$

The sum is 46

10 In this sequence, $t_1 = 3$ and $d = 2$

As $t_{n+1} - t_n = d$

$t_{n+1} - t_n = 2$

$\Rightarrow t_{n+1} = t_n + 2$ where $t_1 = 3$

11 Given $t_1 = 0$

$$\begin{aligned} t_2 &= t_1 - 2 = 0 - 2 = -2 \\ t_3 &= t_2 - 2 = -2 - 2 = -4 \\ t_4 &= t_3 - 2 = -4 - 2 = -6 \\ t_5 &= t_4 - 2 = -6 - 2 = -8 \end{aligned}$$

The first five terms in the sequence are 0, −2, −4, −6, −8

12 P = 30 000
$r = 16\%$
$T = 4$ years
$depreciation = \dfrac{PrT}{100} = \dfrac{30\,000 \times 16 \times 4}{100} = 19\,200$
depreciated value = original value − depreciation
= \$30 000 − \$19 200
= \$10 800
The depreciated value after four years is \$10 800.

13 **a** $P = 900,\ r = 8.2\%,\ T = \dfrac{1}{12}$
$I = \dfrac{\Pr T}{100}$
$= \dfrac{900 \times 8.2 \times \frac{1}{12}}{100}$
$= 6.15$
After 1 month: $t_1 = 900 + 6.15$
$= 906.15$
$d = 6.15$
$t_n = t_1 + d(n-1)$
$t_n = 906.15 + 6.15(n-1)$

b $t_1 = 906.15$(as given)
$t_2 = 906.15 + 6.15(2-1)$
$= 906.15 + 6.15$
$= 912.30$
$t_3 = 906.15 + 6.15(3-1)$
$= 906.15 + 6.15 \times 2$
$= 918.45$
$t_4 = 906.15 + 6.15(4-1)$
$= 906.15 + 6.15 \times 3$
$= 924.60$
$t_5 = 906.15 + 6.15(5-1)$
$= 906.15 \times 6.15 \times 4$
$= 930.75$
The amount in Chirs' account after each of the first 5 months is \$906.15, \$912.30, \$918.45, \$924.60 and \$930.75.

c $t_{20} = 906.15 + 6.15(20-1)$
$= 906.15 + 6.15 \times 19$
$= 1023$
After 20 months there will be \$1023 in the savings account.

d $1200 = 906.15 + 6.15(n-1)$
$1200 - 906.15 = 6.15(n-1)$
$1200 - 906.15 = 6.15n - 6.15$
$1200 - 906.15 + 6.15 = 6.15n$
$n = \dfrac{1200 - 906.15 + 6.15}{6.15}$
$n = 48.78...$
$n \approx 49$
It will take Chirs 49 months to save \$1200.

14 $t_1 = 1$
$d = t_2 - t_1 = 3 - 1 = 2$
The student with the best attendance record will be $n = 30$
$t_{30} = t_1 + d\,(30-1)$
$t_{30} = 1 + 2\,(30-1) = 59$
The student with the best attendance will get 59 lollies
$S_{30} = \dfrac{30\,(t_1 + t_{30})}{2}$
$S_{30} = \dfrac{30\,(1 + 59)}{2} = 900$
The teacher will give out a total of 900 lollies.

15 $t_1 = 28\,000$
$d = 2\,500$
n = 15
$t_{15} = 28\,000 + 2\,500\,(15-1) = 63\,000$
$S_{15} = \dfrac{15\,(28\,000 + 63\,000)}{2}$
$S_{15} = 682\,500$
Average wage $= \dfrac{685\,200}{15} = 45\,500$
Her average wage was \$45 500

16 $I = \dfrac{PrT}{100}$
Hales Bank:
$P = 2\,000;\ r = 8;\ T = 5$
$I = \dfrac{2000 \times 8 \times 5}{100}$
$I = 800$
Country Bank:
$P = 2\,000;\ r = 12;\ T = 3$
$I = \dfrac{2000 \times 12 \times 3}{100}$
$I = 720$
Hales Bank gives the highest return on interest.

17 The easiest approach is to investigate increasing values of n starting with the provided values for the sum of the first n terms of an arithmetic sequence. This will give a relationship between t_n and n.
$S_n = \dfrac{n(t_n + t_1)}{2}$
$60 = \dfrac{n(t_n + 3)}{2}$
$120 = n(t_n + 3)$
$\dfrac{120}{n} = t_n + 3$
$t_n = \dfrac{120}{n} - 3$
We can now test for values of n that divide into 120, to determine t_n and therefore d from the general equation.

n	***tn***	***d***
1	$t_1 = \frac{120}{1} - 3 = 117$ (Reject as $t_1 = 3$ which is a contradiction.)	N/A
2	$t_2 = \frac{120}{2} - 3 = 57$	$d = t_2 - t_1$ $= 57 - 3$ $= 54$
3	$t_3 = \frac{120}{3} - 3 = 37$	$t_n = t_1 + (n-1)d$ $t_3 = t_1 + (3-1)d$ $37 = 3 + 2d$ $2d = 34$ $d = 17$
4	$t_4 = \frac{120}{4} - 3 = 27$	$t_n = t_1 + (n-1)d$ $t_4 = t_1 + (4-1)d$ $27 = 3 + 3d$ $3d = 24$ $d = 8$
5	$t_5 = \frac{120}{5} - 3 = 21$	$t_n = t_1 + (n-1)d$ $t_5 = t_1 + (5-1)d$ $21 = 3 + 4d$ $4d = 18$ $d = 4.5$
6	$t_6 = \frac{120}{6} - 3 = 17$	$t_n = t_1 + (n-1)d$ $t_6 = t_1 + (6-1)d$ $17 = 3 + 5d$ $5d = 14$ $d = 2.8$
8	$t_8 = \frac{120}{8} - 3 = 12$	$t_n = t_1 + (n-1)d$ $t_8 = t_1 + (8-1)d$ $12 = 3 + 7d$ $7d = 9$ $d \approx 1.2857$
10	$t_{10} = \frac{120}{10} - 3 = 9$	$t_n = t_1 + (n-1)d$ $t_{10} = t_1 + (10-1)d$ $9 = 3 + 9d$ $9d = 6$ $d \approx 0.6667$
12	$t_{12} = \frac{120}{12} - 3 = 7$	$t_n = t_1 + (n-1)d$ $t_{12} = t_1 + (12-1)d$ $7 = 3 + 11d$ $11d = 4$ $d \approx 0.3636$

We can see that as n increases, d decreases. From $n = 10$ onwards the values of d fall below 1 which means they are no longer integers, and therefore there is no need to investigate any further.

18 $t_4 - t_2 = 4$

$(t_1 + d(4-1)) - (t_1 + d(2-1)) = 4$

$(t_1 + 3d) - (t_1 + d) = 4$

$2d = 4$

$d = 2$

$S_n = \frac{n(t_1 + t_n)}{2}$

As $t_n = t_1 + d(n-1)$

then, $S_n = \dfrac{n(t_1 + t_1 + d(n-1))}{2}$

$S_n = \dfrac{n(2t_1 + d(n-1))}{2}$

$S_6 = \dfrac{6(2t_1 + 2(6-1))}{2}$

As we are given that $S_6 = 36$

$36 = \dfrac{6(2t_1 + 2(6-1))}{2}$

$36 = \dfrac{6(2t_1 + 2(6-1))}{2}$

$12 = 2t_1 + 10$

$t_1 = 1$

To find when $S_n > 200$,

$200 < \dfrac{n(t_1 + t_n)}{2}$

$200 < \dfrac{n(t_1 + t_n)}{2}$

$200 < \dfrac{n(2t_1 + d(n-1))}{2}$

$200 < \dfrac{n(2(1) + (2)(n-1))}{2}$

$200 < \dfrac{n(2 + (2n-2))}{2}$

$200 < \dfrac{n(2n)}{2}$

$200 < n^2$

$\sqrt{200} < n$

$14.14 < n$

As n is an integer value, the culture mass will exceed 200 milligrams on day15.

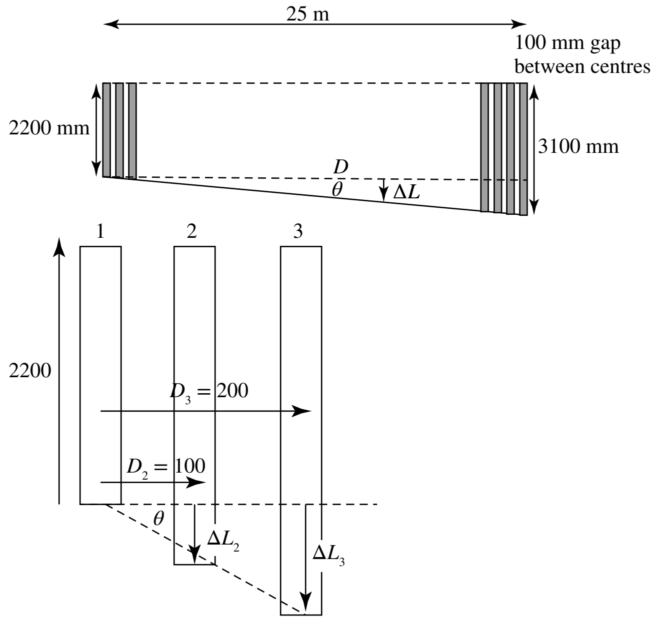
19 A table may be used to model the first few years of the Lunx population:

End of Year	Number of 1 year old Lunx	Number of 2 year old Lunx	Number of 3 year old Lunx	Number of 4 year old Lunx	Number of 5 year old Lunx	Total number of Lunx
1	A					1
2		A				1
3	B		A			2
4		B		A		2
5	C		B		A	3
6		C		B		2
7	D		C		B	3
8		D		C		2
9	E		D		C	3
10		E		D		2
11	F		E		D	3

It can be seen that, after the 5^{th} year, the population alternates between 2 at the end of even number years and 3 at the end of odd number years.

As year 24 is an even number year, this means that there will be a population of 2 Lunx at the end of the 24^{th} year.

20 a It can be seen that the length of each batten will be equal to 2200 mm + ΔL_n where L_n is the length of nth batten.



For each batten, $\tan\theta = \dfrac{\Delta L_n}{D_n}$ where D_n is the separation of the middle of the nth batten from the centre of the first batten.

This can be rearranged:

$\Delta L_n = D_n \tan\theta$

As a result, we can write

$L_n = 2200 + D_n \tan\theta$

As the battens are evenly spaced with a gap of 100 mm between each batten,

$D_n = 100\ (n-1)$

and so

$L_n = 2200 + 100\,(n-1)\tan\theta$

$L_n = 2200 + 100\ \tan\theta\,(n-1)$

The difference in length between any two successive battens L_n and L_{n+1} will be:

$L_{n+1} - L_n = (2200 + 100\ \tan\theta\,(n+1-1)) - (2200 + 100\ \tan\theta\,(n-1))$

$= 2200 + 100\ \tan\theta\, n - 2200 - 100\ \tan\theta\,(n-1)$

$= 100\ \tan\theta\, n - 100\ \tan\theta\, n + 100\ \tan\theta$

$= 100\ \tan\theta$

Thus, each batten is a constant $100\ \tan\theta$ longer than the batten preceding it.

b As shown previously, $L_n = 2200 + 100\ \tan\theta\,(n-1)$

$\tan\theta = \dfrac{3100 - 2200}{25\,000} = 0.036$

So $L_n = 2200 + 3.6\,(n-1)$

c $number\ of\ battens = \dfrac{25000\text{ mm}}{100\text{ mm}} = 250$

d $L_1 = 2200$; $L_{25} = 3100$; $n = 250$

$S_n = \dfrac{n\,(t_1 + t_n)}{2}$

$S_n = \dfrac{250\ (2200 + 3100)}{2} = 388\,600\text{ mm}$

The combined length of all battens is 388.6 metres