Chapter 4 — Calculus of trigonometric functions

Exercise 4.2 – Review of the unit circle, symmetry and exact values

1 a $5^c = 5 \times \dfrac{180}{\pi} = 286.48°$

b $4.8^c = 4.8 \times \dfrac{180}{\pi} = 275.02°$

c $2.56^c = 2.56 \times \dfrac{180}{\pi} = 146.68°$

d $\dfrac{3\pi}{10} = \dfrac{3 \times 180}{10} = 54°$

e $\dfrac{5\pi}{6} = \dfrac{5 \times 180}{6} = 150°$

f $\dfrac{5\pi}{4} = \dfrac{5 \times 180}{4} = 225°$

2 a $15° = 15 \times \dfrac{\pi}{180} = \dfrac{\pi^c}{12}$

b $120° = 120 \times \dfrac{\pi}{180} = \dfrac{2\pi^c}{3}$

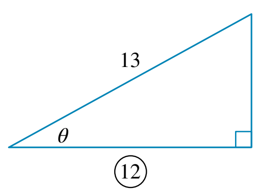
c $130° = 130 \times \dfrac{\pi}{180} = \dfrac{13\pi^c}{18}$

d $63.9° = 63.9 \times \dfrac{\pi}{180} == 1.12^c$

e $78.82° = 78.82 \times \dfrac{\pi}{180} = 1.38^c$

f $310° = 310 \times \dfrac{\pi}{180} = \dfrac{31\pi}{18}$

3



a $\sin(\pi - \alpha) = +\sin(\alpha)$

$= \dfrac{5}{13}$

b $\cos(\pi + \alpha) = -\cos(\alpha)$

$= \dfrac{-12}{13}$

c $\tan(2\pi - \alpha) = -\tan(\alpha)$

$= \dfrac{-5}{12}$

d $\sin(3\pi + \alpha) = -\sin(\alpha)$

$= \dfrac{-5}{13}$

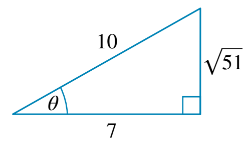
e $\cos(2\pi - \alpha) = \cos(\alpha)$

$= \dfrac{12}{13}$

f $\tan(-\alpha) = -\tan(\alpha)$

$= \dfrac{-5}{12}$

4



a $\cos(\pi - \theta) = -\cos(\theta)$

$= \dfrac{-7}{10}$

b $\sin(\pi - \theta) = +\sin(\theta)$

$= \dfrac{\sqrt{51}}{10}$

c $\tan(2\pi - \theta) = -\tan(\theta)$

$= \dfrac{-\sqrt{51}}{7}$

d $\cos(3\pi + \theta) = -\cos(\theta)$

$= \dfrac{-7}{10}$

e $\tan(\pi + \theta) = +\tan(\theta)$

$= \dfrac{\sqrt{51}}{7}$

f $\cos(-\theta) = +\cos(\theta)$

$= \dfrac{7}{10}$

5 a $\tan\left(\dfrac{3\pi}{4}\right) = \tan\left(\pi - \dfrac{\pi}{4}\right) = -\tan\left(\dfrac{\pi}{4}\right) = -1$

b $\cos\left(\dfrac{5\pi}{6}\right) = \cos\left(\pi - \dfrac{\pi}{6}\right) = -\cos\left(\dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

c $\sin\left(-\dfrac{\pi}{4}\right) = -\sin\left(\dfrac{\pi}{4}\right) = -\dfrac{1}{\sqrt{2}}$

d $\cos\left(\dfrac{7\pi}{3}\right) = \cos\left(2\pi + \dfrac{\pi}{3}\right) = \cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$

e $\tan\left(-\dfrac{\pi}{3}\right) = -\tan\left(\dfrac{\pi}{3}\right) = -\sqrt{3}$

f $\sin\left(\dfrac{11\pi}{6}\right) = \sin\left(2\pi - \dfrac{\pi}{6}\right) = -\sin\left(\dfrac{\pi}{6}\right) = -\dfrac{1}{2}$

6 a $\tan\left(\dfrac{5\pi}{6}\right) = \tan\left(\pi - \dfrac{\pi}{6}\right) = -\tan\left(\dfrac{\pi}{6}\right) = -\dfrac{1}{\sqrt{3}}$

b $\cos\left(\dfrac{14\pi}{3}\right) = \cos\left(5\pi - \dfrac{\pi}{3}\right) = -\cos\left(\dfrac{\pi}{3}\right) = -\dfrac{1}{2}$

c $\tan\left(-\dfrac{5\pi}{4}\right) = -\tan\left(\dfrac{5\pi}{4}\right) = -\tan\left(\pi + \dfrac{\pi}{4}\right)$

$= -\tan\left(\dfrac{\pi}{4}\right) = -1$

d $\cos\left(-\dfrac{3\pi}{4}\right) = \cos\left(\dfrac{3\pi}{4}\right) = \cos\left(\pi - \dfrac{\pi}{4}\right) = -\cos\left(\dfrac{\pi}{4}\right)$

$= -\dfrac{1}{\sqrt{2}}$

e $\sin\left(-\frac{2\pi}{3}\right) = -\sin\left(\frac{2\pi}{3}\right) = -\sin\left(\pi - \frac{\pi}{3}\right)$

$= -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

f $\sin\left(\frac{17\pi}{6}\right) = \sin\left(\frac{5\pi}{6}\right) = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

7 a $\sin(\pi - \theta) = \sin(\theta)$

b $\cos(6\pi - \theta) = \cos(\theta)$

c $\tan(\pi + \theta) = \tan(\theta)$

d $\cos(-\theta) = \cos(\theta)$

e $\sin(180° + \theta) = -\sin(\theta)$

f $\tan(720° - \theta) = -\tan(\theta)$

8 a $\cos\left(\frac{\pi}{2}\right) = 0$

b $\tan(270°) =$ undefined

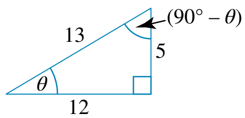
c $\sin(-4\pi) = 0$

d $\tan(\pi) = 0$

e $\cos(-6\pi) = 1$

f $\sin\left(\frac{3\pi}{2}\right) = -1$

9



a $\sin(\theta) = \frac{5}{13}$

b $\tan(\theta) = \frac{5}{12}$

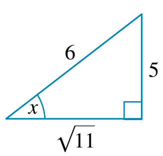
c $\cos(\theta) = \frac{12}{13}$

d $\sin(90° - \theta) = \frac{12}{13}$

e $\cos(90° - \theta) = \frac{5}{13}$

f $\tan(90° - \theta) = \frac{12}{5}$

10



a $\sin^2(x) + \cos^2(x) = \left(\frac{5}{6}\right)^2 + \left(\frac{\sqrt{11}}{6}\right)^2$

$= \frac{25}{36} + \frac{11}{36}$

$= \frac{36}{36}$

$= 1$ as required.

b LHS $= 1 + \tan^2(x)$ RHS $= \frac{1}{\cos^2(x)}$

LHS $= 1 + \left(\frac{5}{\sqrt{11}}\right)^2$ RHS $= \frac{1}{\left(\frac{\sqrt{11}}{6}\right)^2}$

LHS $= 1 + \frac{25}{11} = \frac{36}{11}$ RHS $= \frac{36}{11}$

LHS = RHS as required

c Since the ratios are squared, there is no need to consider the quadrant for the angle.

11 $3\sin(2x)$ if $x = \frac{\pi}{12}$

$= 3\sin\left(2 \times \frac{\pi}{12}\right)$

$= 3\sin\left(\frac{\pi}{6}\right)$

$= 3 \times \frac{1}{2}$

$= \frac{3}{2}$

12 a $\cos\left(\frac{7\pi}{6}\right) + \cos\left(\frac{2\pi}{3}\right) = \cos\left(\pi + \frac{\pi}{6}\right) + \cos\left(\pi - \frac{\pi}{3}\right)$

$= -\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{3}\right)$

$= -\frac{\sqrt{3}}{2} - \frac{1}{2} = -\frac{\left(\sqrt{3}+1\right)}{2}$

b $2\sin\left(\frac{7\pi}{4}\right) + 4\sin\left(\frac{5\pi}{6}\right) = 2\sin\left(2\pi - \frac{\pi}{4}\right) + 4\sin\left(\pi - \frac{\pi}{6}\right)$

$= -2\sin\left(\frac{\pi}{4}\right) + 4\sin\left(\frac{\pi}{6}\right)$

$= -2 \times \frac{\sqrt{2}}{2} + 4 \times \frac{1}{2} = -\sqrt{2} + 2$

c $\sqrt{3}\tan\left(\frac{5\pi}{4}\right) - \tan\left(\frac{5\pi}{3}\right) = \sqrt{3}\tan\left(\pi + \frac{\pi}{4}\right) - \tan\left(2\pi - \frac{\pi}{3}\right)$

$= \sqrt{3}\tan\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{3}\right)$

$= \sqrt{3} + \sqrt{3} = 2\sqrt{3}$

d $\sin^2\left(\frac{8\pi}{3}\right) + \sin\left(\frac{9\pi}{4}\right) = \sin^2\left(3\pi - \frac{\pi}{3}\right) + \sin\left(2\pi + \frac{\pi}{4}\right)$

$= \sin^2\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{4}\right)$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{\sqrt{2}}$

$= \frac{3 + 2\sqrt{2}}{4}$

e $2\cos^2\left(-\frac{5\pi}{4}\right) - 1 = 2\cos^2\left(\frac{5\pi}{4}\right) - 1$

$= 2\left(-\frac{1}{\sqrt{2}}\right)^2 - 1$

$= 1 - 1$

$= 0$

f $\dfrac{\tan\left(\frac{17\pi}{4}\right)\cos(-7\pi)}{\sin\left(-\frac{11\pi}{6}\right)} = \dfrac{\tan\left(4\pi + \frac{\pi}{4}\right)\cos(-\pi)}{-\left(-\sin\left(\frac{\pi}{6}\right)\right)}$

$= \dfrac{\tan\left(\frac{\pi}{4}\right) \times -1}{-\left(-\sin\left(\frac{\pi}{6}\right)\right)}$

$= (1 \times -1) \div +\frac{1}{2}$

$= -2$

13 a $v = 12 + 3\sin\left(\frac{\pi t}{3}\right)$

Initially $t = 0$

$v = 12 + 3\sin(0) = 12$ cm/s

b When $t = 5$

$v = 12 + 3\sin\left(\frac{5\pi}{3}\right)$

$v = 12 + 3\sin\left(2\pi - \frac{\pi}{3}\right)$

$v = 12 - 3\sin\left(\frac{\pi}{3}\right)$

$v = 12 - \frac{3\sqrt{3}}{2}$ cm/s

c When $t = 12$

$v = 12 + 3\sin\left(\frac{12\pi}{3}\right)$

$v = 12 + 3\sin(4\pi) = 12$ cm/s

14 $h(t) = 0.5\cos\left(\frac{\pi t}{12}\right) + 1.0$

a At 6 am $t = 0$

$h(0) = 0.5\cos(0) + 1.0 = 1.5$ m or $\frac{3}{2}$ m

b At 2 pm $t = 8$

$h(8) = 0.5\cos\left(\frac{8\pi}{12}\right) + 1.0$

$h(8) = 0.5\cos\left(\frac{4\pi}{3}\right) + 1.0$

$h(8) = 0.5\cos\left(\pi - \frac{\pi}{3}\right) + 1.0$

$h(8) = 0.5\cos\left(\frac{\pi}{3}\right) + 1.0$

$h(8) = \frac{1}{2} \times -\frac{1}{2} + 1 = 0.75$ m or $\frac{3}{4}$ m

c At 10 pm $t = 16$

$h(16) = 0.5\cos\left(\frac{16\pi}{12}\right) + 1.0$

$h(16) = 0.5\cos\left(\frac{4\pi}{3}\right) + 1.0$

$h(16) = 0.5\cos\left(\pi + \frac{\pi}{3}\right) + 1.0$

$h(16) = -0.5\cos\left(\frac{\pi}{3}\right) + 1.0$

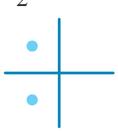
$h(16) = -\frac{1}{2} \times \frac{1}{2} + 1 = 0.75$ m or $\frac{3}{4}$ m

Exercise 4.3 – Review of solving trigonometric equations with and without the use of technology

1 a $2\cos(\theta) + \sqrt{3} = 0 \qquad 0 \le \theta \le 2\pi$

$\cos(\theta) = -\frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2}$ suggests $60°$. Since cos is negative



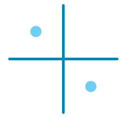
$\theta = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

b $\tan(x) + \sqrt{3} = 0 \qquad 0 \le x \le 720°$

$\tan(x) = -\sqrt{3}$

$\sqrt{3}$ suggests $60°$. Since tan is negative



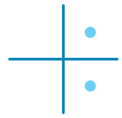
$x = 180° - 60°, 360° - 60°, 540° - 60°, 720° - 60°$

$x = 120°, 300°, 480°, 660°$

c $2\cos(\theta) = 1 \qquad -\pi \le \theta \le \pi$

$\cos(\theta) = \frac{1}{2}$

$\frac{1}{2}$ suggests $\frac{\pi}{3}$. Since cos is positive



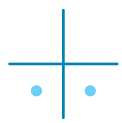
$\theta = -\frac{\pi}{3}, \frac{\pi}{3}$

2 a $2\sin\theta + 1 = 0$

$2\sin\theta = -1$

$\sin\theta = \frac{-1}{2}$

sin is negative in the 3rd & 4th quadrants



$\sin^{-1}(\theta) = 30°$

$\theta = 180° + 30° = 210°$

$\theta = 360° - 30° = 330°$

So, $\theta° = 210°, 330°$

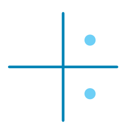
b $\sin(x) = 1$

$x = -\frac{3\pi}{2}, \frac{\pi}{2}$ for $-2\pi \le x \le 2\pi$.

3 $2\cos(3\theta) - \sqrt{2} = 0 \qquad 0 \le \theta \le 2\pi$

$\cos(3\theta) = \frac{\sqrt{2}}{2} \quad 0 \le 3\theta \le 6\pi$

$\frac{\sqrt{2}}{2}$ suggests $\frac{\pi}{4}$. Since cos is positive



$3\theta = \frac{\pi}{4}, 2\pi - \frac{\pi}{4}, 2\pi + \frac{\pi}{4}, 4\pi - \frac{\pi}{4}, 4\pi + \frac{\pi}{4}, 6\pi - \frac{\pi}{4}$

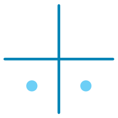
$3\theta = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}, \frac{17\pi}{4}, \frac{23\pi}{4}$

$\theta = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}$

b $2\sin(2x+\pi)+\sqrt{3}=0 \qquad -\pi \le x \le \pi$

$$\sin(2x+\pi) = -\frac{\sqrt{3}}{2} \qquad -\pi \le 2x+\pi \le 3\pi$$

$\frac{\sqrt{3}}{2}$ suggests $\frac{\pi}{3}$. Since sin is negative



$$2x+\pi = -\pi+\frac{\pi}{3}, -\frac{\pi}{3}, \pi+\frac{\pi}{3}, 2\pi-\frac{\pi}{3}$$

$$2x+\pi = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$2x = -\frac{2\pi}{3}-\pi, -\frac{\pi}{3}-\pi, \frac{4\pi}{3}-\pi, \frac{5\pi}{3}-\pi$$

$$2x = -\frac{5\pi}{3}, -\frac{4\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

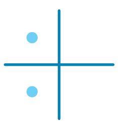
$$x = -\frac{5\pi}{6}, -\frac{2\pi}{3}, \frac{\pi}{6}, \frac{\pi}{3}$$

4 $2\cos\left(3\theta-\frac{\pi}{2}\right)+\sqrt{3}=0 \qquad 0 \le \theta \le 2\pi$

$$\cos\left(3\theta-\frac{\pi}{2}\right) = -\frac{\sqrt{3}}{2} \qquad 0 \le 3\theta \le 6\pi$$

$$\cos\left(3\theta-\frac{\pi}{2}\right) = -\frac{\sqrt{3}}{2} \qquad -\frac{\pi}{2} \le 3\theta-\frac{\pi}{2} \le 6\pi-\frac{\pi}{2}$$

$\frac{\sqrt{3}}{2}$ suggests $\frac{\pi}{6}$, Since cos is negative



$$3\theta-\frac{\pi}{2} = \pi-\frac{\pi}{6},\ \pi+\frac{\pi}{6},\ 3\pi-\frac{\pi}{6},\ 3\pi+\frac{\pi}{6},\ 5\pi-\frac{\pi}{6},\ 5\pi+\frac{\pi}{6}$$

$$3\theta-\frac{\pi}{2} = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{29\pi}{6}, \frac{31\pi}{6}$$

$$3\theta = \frac{5\pi}{6}+\frac{3\pi}{6}, \frac{7\pi}{6}+\frac{3\pi}{6}, \frac{17\pi}{6}+\frac{3\pi}{6}, \frac{19\pi}{6}+\frac{3\pi}{6}, \frac{29\pi}{6}+\frac{3\pi}{6}, \frac{31\pi}{6}+\frac{3\pi}{6}$$

$$3\theta = \frac{4\pi}{6}, \frac{5\pi}{6}, \frac{10\pi}{6}, \frac{11\pi}{6}, \frac{16\pi}{6}, \frac{17\pi}{6}$$

$$\theta = \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$$

5 $\cos^2(\theta)-\sin(\theta)\cos(\theta)=0 \quad 0 \le \theta \le 2\pi$

$\cos(\theta)(\cos(\theta)-\sin(\theta))=0$

$\cos(\theta)=0$ or $\cos(\theta)-\sin(\theta)=0$

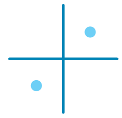
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos(\theta)=\sin(\theta)$

$\frac{\cos(\theta)}{\cos(\theta)} = \frac{\sin(\theta)}{\cos(\theta)}$

$1=\tan(\theta)$

1 suggests $\frac{\pi}{4}$. Since tan is positive



$$\theta = \frac{\pi}{4}, \pi+\frac{\pi}{4}$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

Therefore $\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$

6 $2\cos^2(\theta)+3\cos(\theta)=-1 \qquad 0 \le \theta \le 2\pi$

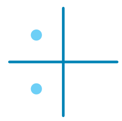
$2\cos^2(\theta)+3\cos(\theta)+1=0 \qquad 0 \le \theta \le 2\pi$

$(2\cos(\theta)+1)(\cos(\theta)+1)=0$

$2\cos(\theta)+1=0$ or $\cos(\theta)+1=0$

$\cos(\theta) = -\frac{1}{2}$ $\qquad \cos(\theta) = -1$ so $\theta=\pi$

$\frac{1}{2}$ suggests $\frac{\pi}{3}$. Since cos is negative



$$\theta = \pi-\frac{\pi}{3}, \pi+\frac{\pi}{3}$$

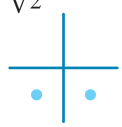
$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

Therefore $\theta = \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$

7 a $\sqrt{2}\sin(\theta)=-1 \qquad 0 \le \theta \le 2\pi$

$$\sin(\theta) = -\frac{1}{\sqrt{2}}$$

$\frac{1}{\sqrt{2}}$ suggests $\frac{\pi}{4}$. Since sin is negative



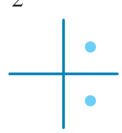
$$\theta = \pi+\frac{\pi}{4},\ 2\pi-\frac{\pi}{4}$$

$$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$$

b $2\cos(\theta)=1 \quad 0 \le \theta \le 2\pi$

$$\cos(\theta)=\frac{1}{2}$$

$\frac{1}{2}$ suggests $\frac{\pi}{3}$. Since cos is positive



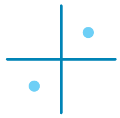
$$\theta = \frac{\pi}{3},\ 2\pi-\frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

c $\tan(3\theta) - \sqrt{3} = 0 \qquad 0 \le \theta \le 2\pi$

$\tan(3\theta) = \sqrt{3} \qquad 0 \le 3\theta \le 6\pi$

$\sqrt{3}$ suggests $\frac{\pi}{3}$. Since tan is positive



$3\theta = \frac{\pi}{3}, \pi + \frac{\pi}{3}, 2\pi + \frac{\pi}{3}, 3\pi + \frac{\pi}{3}, 4\pi + \frac{\pi}{3}, 5\pi + \frac{\pi}{3}$

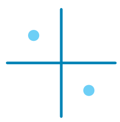
$3\theta = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3}, \frac{13\pi}{3}, \frac{16\pi}{3}$

$\theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{10\pi}{9}, \frac{13\pi}{9}, \frac{16\pi}{9}$

d $\tan\left(\theta - \frac{\pi}{2}\right) + 1 = 0 \qquad 0 \le \theta \le 2\pi$

$\tan\left(\theta - \frac{\pi}{2}\right) = -1 \qquad -\frac{\pi}{2} \le \theta - \frac{\pi}{2} \le 2\pi - \frac{\pi}{2}$

1 suggests $\frac{\pi}{4}$. Since tan is negative



$\theta - \frac{\pi}{2} = -\frac{\pi}{4}, \pi - \frac{\pi}{4}$

$\theta - \frac{\pi}{2} = -\frac{\pi}{4}, \frac{3\pi}{4}$

$\theta = -\frac{\pi}{4} + \frac{\pi}{2}, \frac{3\pi}{4} + \frac{\pi}{2}$

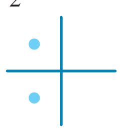
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

8 a $2\cos(x) + 1 = 0 \qquad 0° \le x \le 360°$

$2\cos(x) = -1$

$\cos(x) = -\frac{1}{2}$

$\frac{1}{2}$ suggests 60°. Since cos is negative



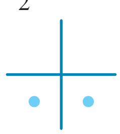
$x = 180° - 60°, 180° + 60°$

$x = 120°, 240°$

b $2\sin(2x) + \sqrt{2} = 0 \qquad 0° \le x \le 360°$

$\sin(2x) = -\frac{\sqrt{2}}{2} \qquad 0° \le 2x \le 720°$

$\frac{\sqrt{2}}{2}$ suggests 45°. Since sin is negative



$2x = 180° + 45°, 360° - 45°, 540° + 45°, 720° - 45°$

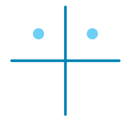
$2x = 225°, 315°, 585°, 675°$

$x = 112.5°, 157.5°, 292.5°, 337.5°$

9 a $3\sin(\theta) - 2 = 0 \qquad 0 \le \theta \le 2\pi$

$\sin(\theta) = \frac{2}{3}$

$\frac{2}{3}$ suggests 0.7297^c. Since sin is positive



$\theta = 0.7297, \pi - 0.7297$

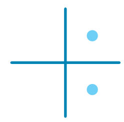
$\theta = 0.73, 2.41$

or solve on CAS

b $7\cos(x) - 2 = 0 \qquad 0° \le x \le 360°$

$\cos(x) = \frac{2}{7}$

$\frac{2}{7}$ suggests 73.3985°. Since cos is positive



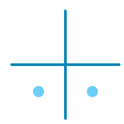
$x = 73.3985°, 360° - 73.3985°$

$x = 73.40°, 286.60°$

10 a $2\sin(2\theta) + \sqrt{3} = 0 \qquad -\pi \le \theta \le \pi$

$\sin(2\theta) = -\frac{\sqrt{3}}{2} \qquad -2\pi \le \theta \le 2\pi$

$\frac{\sqrt{3}}{2}$ suggests $\frac{\pi}{3}$. Since sin is negative



$2\theta = -\pi + \frac{\pi}{3}, -\frac{\pi}{3}, \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

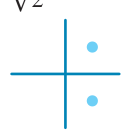
$2\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\theta = -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{6}$

b $\sqrt{2}\cos(3\theta) = 1 \qquad -\pi \le \theta \le \pi$

$\cos(3\theta) = \frac{1}{\sqrt{2}} \qquad -3\pi \le \theta \le 3\pi$

$\frac{1}{\sqrt{2}}$ suggests $\frac{\pi}{4}$. Since cos is positive



$3\theta = -2\pi - \frac{\pi}{4}, -2\pi + \frac{\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, 2\pi - \frac{\pi}{4}, 2\pi + \frac{\pi}{4}$

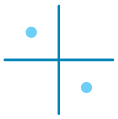
$3\theta = -\frac{9\pi}{4}, -\frac{7\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}$

$\theta = -\frac{3\pi}{4}, -\frac{7\pi}{12}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}$

c $\tan(2\theta) + 1 = 0 \qquad -\pi \le \theta \le \pi$

$\tan(2\theta) = -1 \qquad -2\pi \le \theta \le 2\pi$

1 suggests $\frac{\pi}{4}$. Since tan is negative



$$2\theta = -\pi - \frac{\pi}{4}, -\frac{\pi}{4}, \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$$

$$2\theta = -\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$$

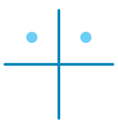
$$\theta = -\frac{5\pi}{8}, -\frac{\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}$$

11 a $2\sin\left(2x + \frac{\pi}{4}\right) = \sqrt{2}, \ -\pi \le x \le \pi$

$$\sin\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \ -2\pi \le 2x \le 2\pi$$

$$\sin\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \ -2\pi + \frac{\pi}{4} \le 2x + \frac{\pi}{4} \le 2\pi + \frac{\pi}{4}$$

$\frac{\sqrt{2}}{2}$ suggests $\frac{\pi}{4}$. Since sin is positive



$$2x + \frac{\pi}{4} = -2\pi + \frac{\pi}{4}, -\pi - \frac{\pi}{4}, \frac{\pi}{4}, \pi - \frac{\pi}{4}, 2\pi + \frac{\pi}{4}$$

$$2x + \frac{\pi}{4} = -\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}$$

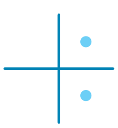
$$2x = -2\pi, -\frac{3\pi}{2}, 0, \frac{\pi}{2}, 2\pi$$

$$x = -\pi, -\frac{3\pi}{4}, 0, \frac{\pi}{4}, \pi$$

b $2\cos(x + \pi) = \sqrt{3} \qquad -\pi \le x \le \pi$

$$\cos(x + \pi) = \frac{\sqrt{3}}{2} \qquad 0 \le x + \pi \le 2\pi$$

$\frac{\sqrt{3}}{2}$ suggests $\frac{\pi}{6}$. Since cos is positive



$$x + \pi = \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$

$$x + \pi = \frac{\pi}{6}, \frac{11\pi}{6}$$

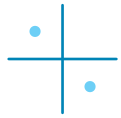
$$x = \frac{\pi}{6} - \pi, \frac{11\pi}{6} - \pi$$

$$x = -\frac{5\pi}{6}, \frac{5\pi}{6}$$

c $\tan(x - \pi) = -1, \ -\pi \le x \le \pi$

$\tan(x - \pi) = -1, \ -2\pi \le x - \pi \le 0$

1 suggests $\frac{\pi}{4}$. Since tan is negative



$$x - \pi = -\pi - \frac{\pi}{4}, -\frac{\pi}{4}$$

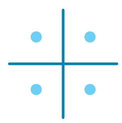
$$x - \pi = -\frac{5\pi}{4}, -\frac{\pi}{4}$$

$$x = -\frac{5\pi}{4} + \pi, -\frac{\pi}{4} + \pi$$

$$x = -\frac{\pi}{4}, \frac{3\pi}{4}$$

12 a $\tan(\theta) = 1$ or $\tan(\theta) = -1$

1 suggests $\frac{\pi}{4}$ and tan is positive & negative in all quadrants



$$\theta = \frac{\pi}{4}, \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$$

$$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

b $4\sin^2(\theta) - (2 + 2\sqrt{3})\sin(\theta) + \sqrt{3} = 0, \quad 0 \le \theta \le 2\pi$

Let $A = \sin(\theta)$

$$4A^2 - \left(2 + 2\sqrt{3}\right)A + \sqrt{3} = 0$$

$$(2A - 1)\left(2A - \sqrt{3}\right) = 0$$

$$2A = 1 \quad \text{or} \quad 2A = \sqrt{3}$$

$$A = \frac{1}{2} \quad \text{or} \quad A = \frac{\sqrt{3}}{2}$$

$$\therefore \sin(\theta) = \frac{1}{2} \qquad \sin(\theta) = \frac{\sqrt{3}}{2}$$

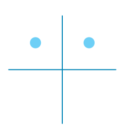
$$(2\sin(\theta) - \sqrt{3})(2\sin(\theta) - 1) = 0$$

$$2\sin(\theta) - \sqrt{3} = 0 \quad \text{or} \quad 2\sin(\theta) - 1 = 0$$

$$\sin(\theta) = \frac{\sqrt{3}}{2} \qquad 2\sin(\theta) = \frac{1}{2}$$

$\frac{\sqrt{3}}{2}$ suggests $\frac{\pi}{3}$ and $\frac{1}{2}$ suggests $\frac{\pi}{6}$

Since sin is positive



$$\theta = \frac{\pi}{3}, \pi - \frac{\pi}{3} \qquad \theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3} \qquad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$$

13 a

$$\sin(\alpha) - \cos^2(\alpha)\sin(\alpha) = 0, \quad -\pi \le \alpha \le \pi$$
$$\sin(\alpha)(1 - \cos^2(\alpha)) = 0$$
$$\sin(\alpha)(1 - \cos(\alpha))(1 + \cos(\alpha)) = 0$$

$1 - \cos(\alpha) = 0 \qquad 1 + \cos(\alpha) = 0$

$\sin(\alpha) = 0$ or $\cos(\alpha) = 1$ or $\cos(\alpha) = -1$

$\alpha = -\pi, 0, \pi \qquad \alpha = 0 \qquad \alpha = -\pi, \pi$

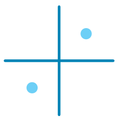
Thus $\alpha = -\pi, 0, \pi$.

b $\sin(2\alpha) = \sqrt{3}\cos(2\alpha), \quad -\pi \le \alpha \le \pi$

$$\frac{\sin(2\alpha)}{\cos(2\alpha)} = \sqrt{3}\frac{\cos(2\alpha)}{\cos(2\alpha)}, \quad -2\pi \le 2\alpha \le 2\pi$$

$\tan(2\alpha) = \sqrt{3}$

$\sqrt{3}$ suggests $\frac{\pi}{3}$. Since tan is positive

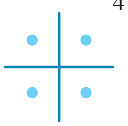


$$2\alpha = -2\pi + \frac{\pi}{3} - \pi + \frac{\pi}{3}, \frac{\pi}{3}, \pi + \frac{\pi}{3}$$
$$2\alpha = -\frac{5\pi}{3}, -\frac{2\pi}{3}, \frac{\pi}{3}, \frac{4\pi}{3}$$
$$\alpha = -\frac{5\pi}{6}, -\frac{\pi}{3}, \frac{\pi}{6}, \frac{2\pi}{3}$$

c $\sin^2(\alpha) = \cos^2(\alpha), \ -\pi \le \alpha \le \pi$

$$\frac{\sin^2(\alpha)}{\cos^2(\alpha)} = 1$$
$$\tan^2\alpha = 1$$
$$\tan\alpha = \pm 1$$

1 suggests $\frac{\pi}{4}$ in all quadrants



$$\alpha = \frac{-\pi}{4}, -\pi + \frac{\pi}{4}, \frac{\pi}{4}, \pi - \frac{\pi}{4}$$
$$\therefore \alpha = -\frac{\pi}{4}, \frac{-3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$$

d $4\cos^2(\alpha) - 1 = 0, \quad -\pi \le \alpha \le \pi$

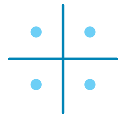
$(2\sin(\alpha))^2 - 1^2 = 0$ or $2\sin(\alpha) + 1 = 0$

$(2\sin(\alpha) - 1)(2\sin(\alpha) + 1) = 0 \qquad \sin(\alpha) = -\frac{1}{2}$

$2\sin(\alpha) - 1 = 0$

$\sin(\alpha) = \frac{1}{2}$

$\frac{1}{2}$ suggests $\frac{\pi}{3}$. Since sin is both positive and negative, all four quadrants.



$$\alpha = -\pi + \frac{\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \pi - \frac{\pi}{3}$$
$$\alpha = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

14 a $x = 3 + 4\sin(2t)$

When $t = 0$:

$x = 3 + 4\sin(0)$

$x = 3$ metres

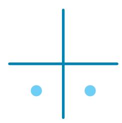
b when $x = 0$:

$3 + 4\sin(2t) = 0$

$4\sin(2t) = -3$

$\sin(2t) = -\frac{3}{4}$

$\frac{3}{4}$ suggests 0.8481. Since sin is negative



$2t = \pi + 0.8481, \ 2\pi - 0.8481$

For first time:

$2t = 3.98965$

$t = 1.9948$

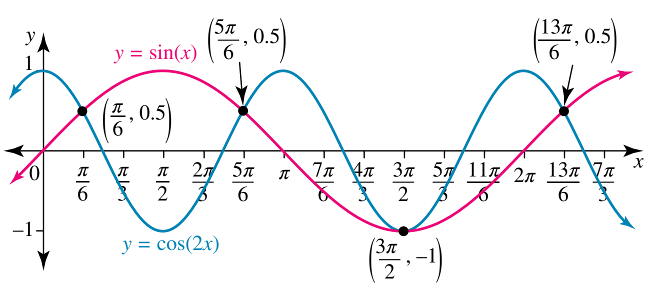
Time taken is 1.99 seconds (2 decimal places)

15 a See figure at foot of the page.*

b For $0 \le x \le 2\pi$: $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \backsimeq 0.52, 2.62, 4.71$

c The trigonometric ratios involved different angles, so could not be combined to solve.

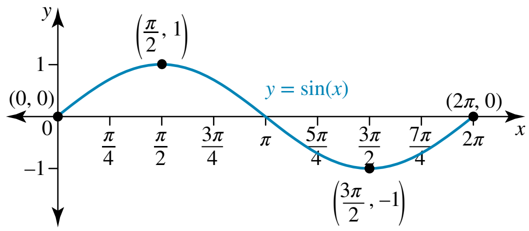
15a*



Exercise 4.4 – Review of graphs of trigonometric functions of the form $y = A\sin(B(x + C)) + D$ and $y = A\cos(B(x + C)) + D$

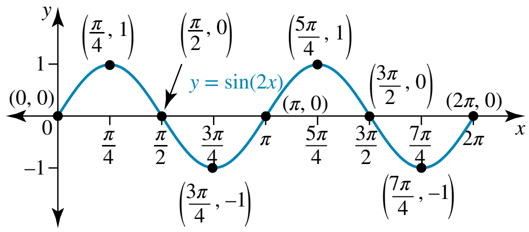
1 a period: 2π

amplitude: 1



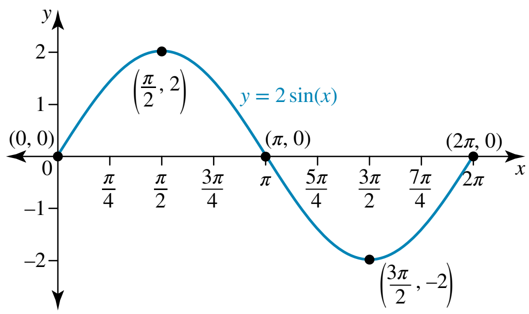
b period: $\frac{2\pi}{2} = \pi$

amplitude: 1



c period: 2π

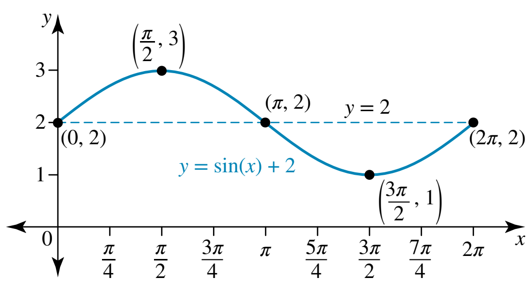
amplitude: 2



d period: 2π

amplitude: 1

vertical translation of +2, giving line of oscillation: $y = 2$

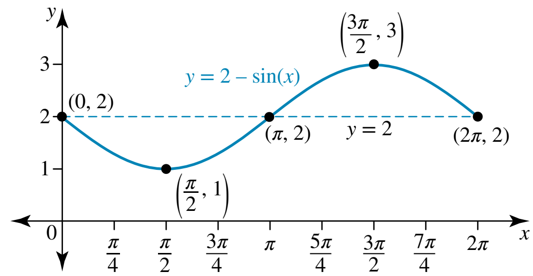


e period: 2π

amplitude: 1

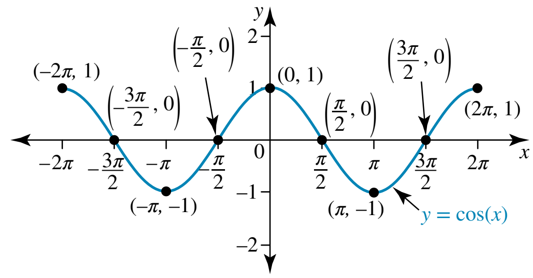
vertical translation of +2, giving line of oscillation: $y = 2$

reflection in the x-axis.



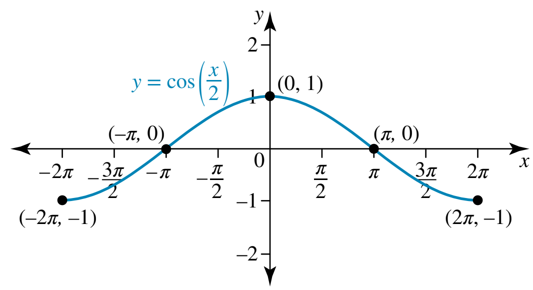
2 a period: 2π

amplitude: 1



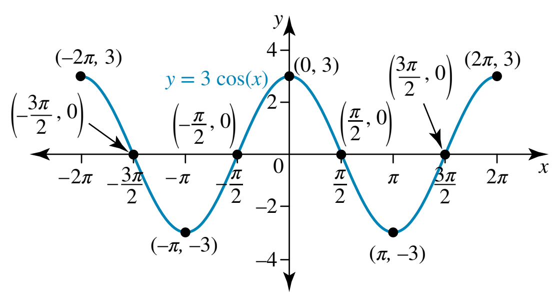
b period: $\frac{2\pi}{1/2} = 4\pi$

amplitude: 1



c period: 2π

amplitude: 3

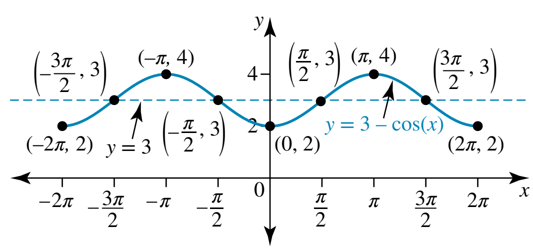


d period: 2π

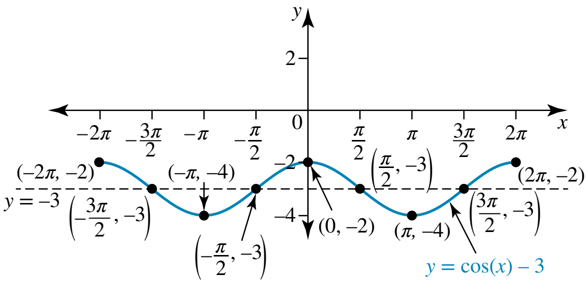
amplitude: 1

vertical translation of +3, giving line of oscillation: $y = 3$

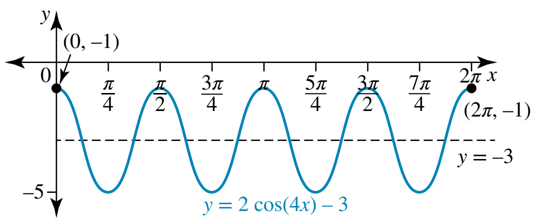
reflection in x-axis.



e period: 2π
amplitude: 1
vertical translation of -3, giving line of oscillation: $y = -3$



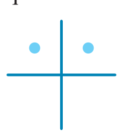
3 $y = 2\cos(4x) - 3, 0 \leq x \leq 2\pi$
Period $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$
Amplitude: 2
Mean position $y = -3$
Range $[-3-2,\ -3+2] = [-5, -1]$
No x-intercepts



4 $y = 2 - 4\sin(3x), 0 \leq x \leq 2\pi$
Period: $\dfrac{2\pi}{3}$
Amplitude: 4
Line of oscillation (mean position): $y = 2$
Reflection in the x-axis
Range: $[2-4,\ 2+4] = [-2, 6]$
x-intercepts: $y = 0$
$2 - 4\ \sin(3x) = 0$

$$\sin(3x) = \frac{1}{2}$$

$\dfrac{1}{2}$ suggests $\dfrac{\pi}{6}$ and since sin positive, in the first and second quadrants.



For $0 \leq x \leq 2\pi$
then $0 \leq 3x \leq 6\pi$

$$(3x) = \frac{\pi}{6}, \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}, 3\pi - \frac{\pi}{6}, 4\pi + \frac{\pi}{6}, 5\pi - \frac{\pi}{6}$$

$$(3x) = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$$

$$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$$

See figure at foot of the page.*

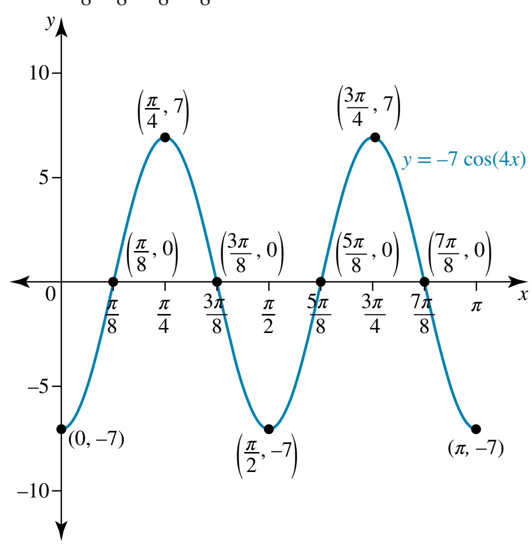
5 $y = -7\cos(4x)$ for $0 \leq x \leq \pi$
Period: $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$
Amplitude: 7
Reflection in the x-axis
Range: $[-7, 7]$
x-intercepts: $y = 0$
$-7\cos(4x) = 0$
$\cos(4x) = 0$

For $0 \leq x \leq \pi$
then $0 \leq 4x \leq 4\pi$

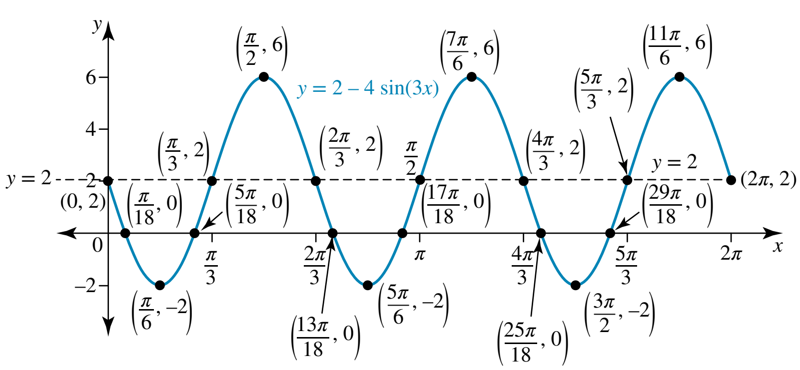
$$(4x) = \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi + \frac{\pi}{2}, 2\pi + \frac{3\pi}{2}$$

$$4x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$$



4*



6 $y = \frac{1}{2}\cos(2x) + 3$ for $-\pi \le x \le 2\pi$

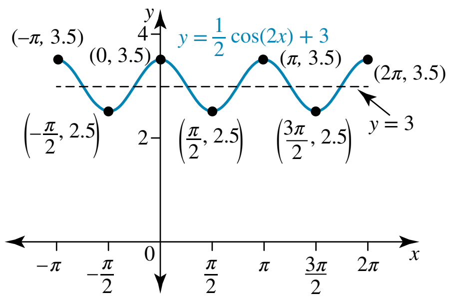
Period: $\frac{2\pi}{2} = \pi$

Amplitude: $\frac{1}{2}$

Line of oscillation (or mean position): $y = 3$

Range: $\left[3 - \frac{1}{2}, 3 + \frac{1}{2}\right] = \left[\frac{5}{2}, \frac{7}{2}\right]$

No x-intercepts



7 $f: [0, 2\pi] \to R, f(x) = 1 - 2\sin\left(\frac{3x}{2}\right)$

$y = f(x) = 1 - 2\sin\left(\frac{3x}{2}\right)$

Period: $2\pi \div \frac{3}{2} = \frac{4\pi}{3}$

Amplitude 2, reflection in the x-axis

Mean position $y = 1$

Range $[-1, 3]$

x-intercepts: Let $y = 0$

$0 = 1 - 2\sin\left(\frac{3x}{2}\right), 0 \le x \le 2\pi$

$\therefore \sin\left(\frac{3x}{2}\right) = \frac{1}{2}, 0 \le \frac{3x}{2} \le 3\pi$

$\therefore \frac{3x}{2} = \frac{\pi}{6}, \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}, 3\pi - \frac{\pi}{6}$

$\therefore \frac{3x}{2} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

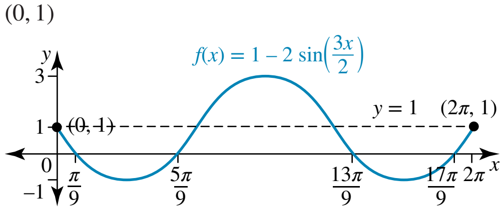
$\therefore 3x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}$

$\therefore x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$

y-intercepts: Let $x = 0$

$y = 1 - 2\sin(0) = 1$

$(0, 1)$



8 Let the equation be $y = a\sin(nx) + k$.

The period of the graph is 2.

$\therefore \frac{2\pi}{n} = 2$

$\therefore n = \pi$

The mean position is 5.

$y = 5 \Rightarrow k = 5$.

The range is $[-3, 13]$ which means the amplitude is 8.

As the graph has an inverted sine shape, $a = -8$

The equation is $y = -8\sin(\pi x) + 5$.

9 Let equation be $y = a\cos(nx) + k$.

range: $[-2, 4]$

Line of oscillation is mid point of range: $y = 1$

Amplitude from line of oscillation to largest y value:

amplitude $= 3$

One complete curve in π: period $= \pi$

$\therefore \frac{2\pi}{n} = \pi$

$n = 2$

Equation: $y = 1 + 3\cos(2x)$

10 a $f: \left[0, \frac{3\pi}{2}\right] \to R, f(x) = -6\sin\left(3x - \frac{3\pi}{4}\right)$

$y = f(x) = -6\sin\left(3x - \frac{3\pi}{4}\right)$

$\therefore y = -6\sin\left(3\left(x - \frac{\pi}{4}\right)\right)$

Horizontal translation $\frac{\pi}{4}$ units to the right.

Period $\frac{2\pi}{3}$

Amplitude 6, graph is reflected in the x-axis

Mean position $y = 0$ so range is $[-6, 6]$.

Endpoints:

$$\begin{aligned} f(0) &= -6\sin\left(-\frac{3\pi}{4}\right) \\ &= -6 \times \frac{-\sqrt{2}}{2} \\ &= 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} f\left(\frac{3\pi}{2}\right) &= -6\sin\left(\frac{15\pi}{4}\right) \\ &= -6 \times \frac{-\sqrt{2}}{2} \\ &= 3\sqrt{2} \end{aligned}$$

Endpoints are $\left(0, 3\sqrt{2}\right)$ and $\left(\frac{3\pi}{2}, 3\sqrt{2}\right)$.

x-intercepts: Either translate those of $y = -6\sin(3x)$ $\frac{\pi}{4}$ units to the right or solve

$-6\sin\left(3x - \frac{3\pi}{4}\right) = 0$

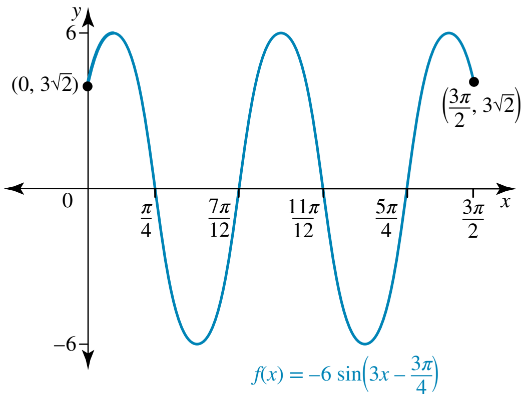
Solving the equation:

$\sin\left(3x - \frac{3\pi}{4}\right) = 0$

$\therefore 3x - \frac{3\pi}{4} = 0, \pi, 2\pi, 3\pi$

$\therefore 3x = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$

$\therefore x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{5\pi}{4}$



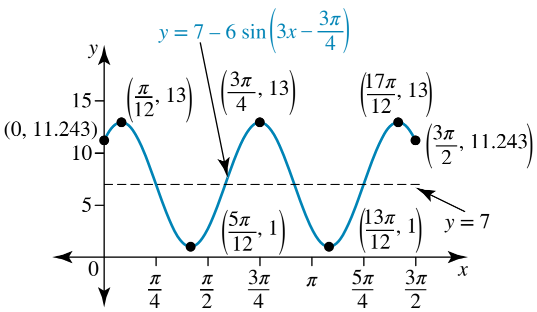
b $g: \left[0, \frac{3\pi}{2}\right] \to R, g(x) = 7 - 6\sin\left(3x - \frac{3\pi}{4}\right)$

$g(x)$ is $f(x)$ translated vertically up by 7, oscillating around $y = 7$ with same period and amplitude.

Range: $[7 - 6, 7 + 6 = 1, 13]$

No x-intercepts

Endpoints: $(0, 7 + 3\sqrt{2})$ and $\left(\frac{3\pi}{2}, 7 + 3\sqrt{2}\right)$



11 a $y = 2\sin\left(x + \frac{\pi}{4}\right), 0 \le x \le 2\pi$

Period: 2π

Amplitude: 2

Line of oscillation (or mean position): $y = 0$

Range: $[-2, 2]$

Horizontal translation of $\frac{\pi}{4}$ to the left, or in the negative x direction.

Endpoints:

at $x = 0$ at $x = 2\pi$

$y = 2\sin\left(\frac{\pi}{4}\right)$ $\quad y = 2\sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$

$y = 2 \times \frac{1}{\sqrt{2}}$ $\quad y = 2\sin\left(\frac{3\pi}{4}\right)$

$y = \sqrt{2}$ $\quad y = 2 \times \frac{1}{\sqrt{2}}$

$y = \sqrt{2}$

Endpoints are: $(0, \sqrt{2})$ and $(2\pi, \sqrt{2})$

For x-intercepts: $y = 0$

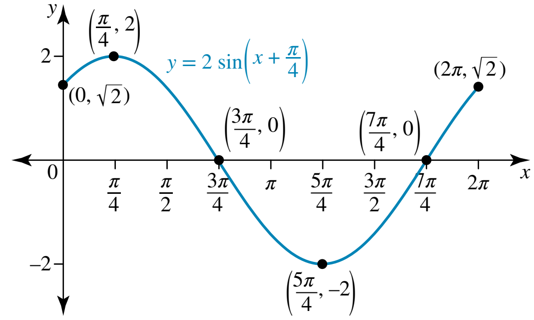
$2\sin\left(x + \frac{\pi}{4}\right) = 0$

$\sin\left(x + \frac{\pi}{4}\right) = 0$

$\left(x + \frac{\pi}{4}\right) = 0, \pi, 2\pi$

$x = \frac{-\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$

For $0 \le x \le 2\pi$: $x = \frac{3\pi}{4}, \frac{7\pi}{4}$



b $y = 2\sin\left(x + \frac{\pi}{4}\right) - 1, 0 \le x \le 2\pi$

This is the curve in part **a**. translated vertically down by 1 unit, oscillating around $y = -1$, with the same period and amplitude.

Range: $[-1 - 2, -1 + 2] = [-3, 1]$

Endpoints are: $(0, -1 + \sqrt{2})$ and $(2\pi, -1 + \sqrt{2})$

For x-intercepts: $y = 0$

$2\sin\left(x + \frac{\pi}{4}\right) - 1 = 0$

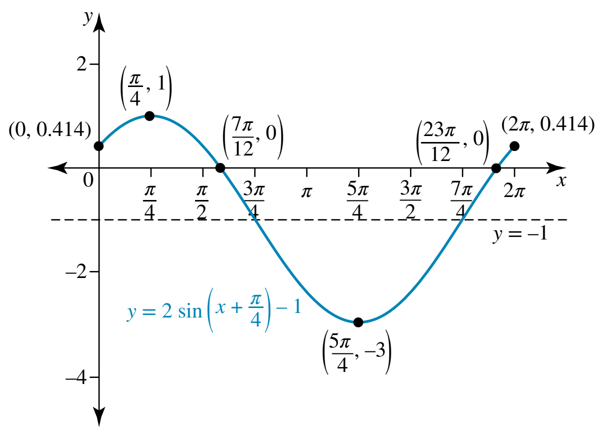
$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{2}$

$\frac{1}{2}$ suggests an angle of $\frac{\pi}{6}$ in the first and second quadrants

$\left(x + \frac{\pi}{4}\right) = \frac{\pi}{6}, \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$

$x = \frac{-\pi}{12}, \frac{7\pi}{12}, \frac{23\pi}{12}$

For $0 \le x \le 2\pi$: $x = \frac{7\pi}{12}, \frac{23\pi}{12}$



12 a $f: \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right] \to R, f(x) = 4\cos\left(3x - \frac{\pi}{2}\right)$

$f: \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right] \to R, f(x) = 4\cos\left(3\left(x - \frac{\pi}{6}\right)\right)$

Period: $\frac{2\pi}{3}$

Amplitude: 4

Line of oscillation (or mean position): $y = 0$

Range: $[-4, 4]$

Horizontal translation of $\frac{\pi}{6}$ to the right, or in the positive x direction.

Endpoints: $x = -\frac{\pi}{2}, x = \frac{3\pi}{2}$

$f\left(-\frac{\pi}{2}\right) = 4\cos\left(3\left(\frac{-\pi}{2}\right) - \frac{\pi}{2}\right)$ $\quad f\left(\frac{3\pi}{2}\right) = 4\cos\left(3 \times \frac{3\pi}{2} - \frac{\pi}{2}\right)$

$f\left(-\frac{\pi}{2}\right) = 4\cos(-2\pi)$ $\quad f\left(\frac{3\pi}{2}\right) = 4\cos(4\pi)$

$f\left(-\frac{\pi}{2}\right) = 4\cos(0)$ $\quad f\left(\frac{3\pi}{2}\right) = 4\cos(0)$

$f\left(-\frac{\pi}{2}\right) = 4$ $\quad f\left(\frac{3\pi}{2}\right) = 4$

Endpoints are: $\left(-\frac{\pi}{2}, 4\right)$ and $\left(\frac{3\pi}{2}, 4\right)$

Axis intercepts:

y-intercepts: $x = 0$

$f(0) = 0$

$(0, 0)$

x-intercepts: $y = 0$

$$0 = 4\cos\left(3x - \frac{\pi}{2}\right)$$

$$\cos\left(3x - \frac{\pi}{2}\right) = 0$$

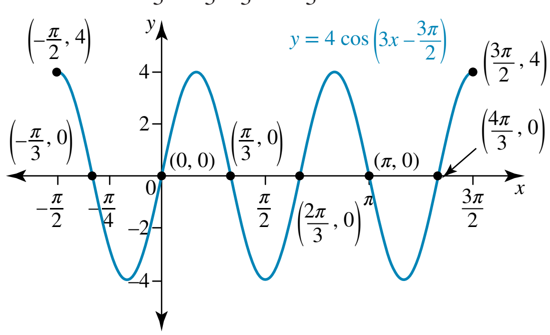
For $\frac{-\pi}{2} \le x \le \frac{3\pi}{2}$, then $\frac{-3\pi}{2} \le 3x \le \frac{9\pi}{2}$ and

$-2\pi \le \left(3x - \frac{\pi}{2}\right) \le 4\pi$

$\left(3x - \frac{\pi}{2}\right) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{-\pi}{2}, \frac{-3\pi}{2}$

$$3x = \pi, 2\pi, 3\pi, 4\pi, 0, -\pi$$

$$x = -\frac{\pi}{3}, 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$$



b $g: \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right] \to R,\ g(x) = 4 - 4\cos\left(3x - \frac{\pi}{2}\right)$

$g: \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right] \to R, g(x) = 4 - 4\cos\left(3\left(x - \frac{\pi}{6}\right)\right)$

This is the curve in part a. reflected in the x-axis and translated vertically up by 4 unit, oscillating around $y = 4$. with the same period and amplitude.

Range: $[4 - 4, 4 + 4] = [0, 8]$

Endpoints: $x = -\frac{\pi}{2}$ $x = \frac{3\pi}{2}$

$g\left(-\frac{\pi}{2}\right) = 4 - 4\cos\left(3\left(\frac{-\pi}{2}\right) - \frac{\pi}{2}\right)$

$g\left(\frac{3\pi}{2}\right) = 4 - 4\cos\left(3 \times \frac{3\pi}{2} - \frac{\pi}{2}\right)$

$g\left(-\frac{\pi}{2}\right) = 4 - 4\cos(-2\pi)$ $\quad g\left(\frac{3\pi}{2}\right) = 4 - 4\cos(4\pi)$

$g\left(-\frac{\pi}{2}\right) = 4 - 4\cos(0)$ $\quad g\left(\frac{3\pi}{2}\right) = 4 - 4\cos(0)$

$g\left(-\frac{\pi}{2}\right) = 0$ $\quad g\left(\frac{3\pi}{2}\right) = 0$

Endpoints are: $\left(-\frac{\pi}{2}, 0\right)$ and $\left(\frac{3\pi}{2}, 0\right)$

Axis intercepts:

y-intercepts: $x = 0$

$g(0) = 4 - 4\cos\left(-\frac{\pi}{2}\right)$

$g(0) = 4$

$(0, 4)$

x-intercepts: $y = 0$

$$0 = 4 - 4\cos\left(3x - \frac{\pi}{2}\right)$$

$$\cos\left(3x - \frac{\pi}{2}\right) = 1$$

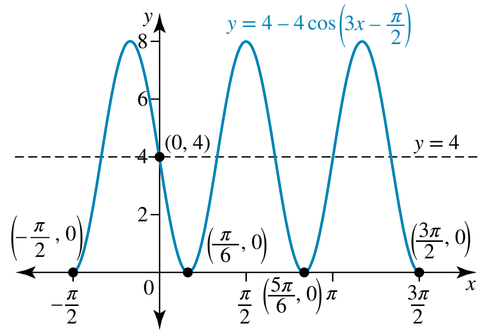
For $\frac{-\pi}{2} \le x \le \frac{3\pi}{2}$, then $\frac{-3\pi}{2} \le 3x \le \frac{9\pi}{2}$ and

$-2\pi \le \left(3x - \frac{\pi}{2}\right) \le 4\pi$

$\left(3x - \frac{\pi}{2}\right) = -2\pi, 0, 2\pi, 4\pi$

$$3x = \frac{-3\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{9\pi}{2}$$

$$x = \frac{-\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$



c The curve $g(x)$ is $f(x)$ reflected in the x-axis (or inverted) and translated vertically up by 4 unit, oscillating around $y = 4$. Neither the period nor the amplitude have been changed.

13 a $y = 2\cos(3x)$

Period: $\frac{2\pi}{3}$

Amplitude: 2

Line of oscillation (or mean position): $y = 0$

Range: $[-2, 2]$

For one complete cycle: $0 \le x \le \frac{2\pi}{3}$

Endpoints:

At $x = 0$ $\quad$ At $x = \frac{2\pi}{3}$

$y = 2\cos(0)$ $\quad y = 2\cos(2\pi)$

$y = 2$ $\quad y = 2$

Endpoints are: $(0, 2)$ and $\left(\frac{2\pi}{3}, 2\right)$

x-intercepts: $y = 0$

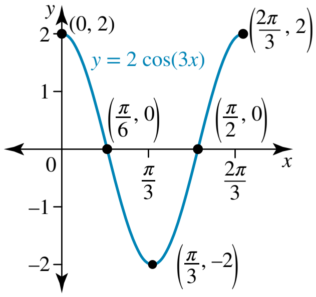
$$0 = 2\cos(3x)$$

$\cos(3x) = 0$

For $0 \le x \le \frac{2\pi}{3}$ then $0 \le (3x) \le 2\pi$

$(3x) = \frac{\pi}{2}, \frac{3\pi}{2}$

$$x = \frac{\pi}{6}, \frac{\pi}{2}$$



b Translate the points $\frac{\pi}{3}$ to the right and up by 3.

Looking at maximum and minimum points:

$$(0, 2) \rightarrow \left(\frac{\pi}{3}, 5\right)$$

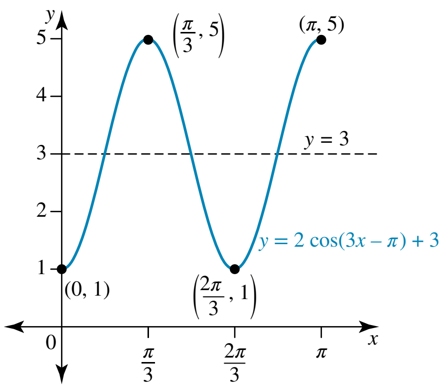
$$\left(\frac{\pi}{3}, -2\right) \rightarrow \left(\frac{2\pi}{3}, 1\right)$$

$$\left(\frac{2\pi}{3}, 2\right) \rightarrow (\pi, 5)$$

The range would now be: $[-2 + 3, 2 + 3] = [1, 5]$

Therefore no x-intercepts.

Note: no restricted domain was stated.



c To translate $\frac{\pi}{3}$ to the right, the curve becomes:

$y = 2\cos\left(3\left(x - \frac{\pi}{3}\right)\right)$

To translate vertically up by 3 units, the curve becomes:

$y = 2\cos\left(3\left(x - \frac{\pi}{3}\right)\right) + 3$

Translated curve: $y = 2\cos(3x - \pi) + 3$

14 $f(x) = 2 - 3\cos\left(x + \frac{\pi}{12}\right)$

Amplitude: 3

Line of oscillation (mean position): $y = 2$

Range: $[2 - 3, 2 + 3] = [-1, 5]$

Maximum occurs when $y = 5$

$$5 = 2 - 3\cos\left(x + \frac{\pi}{12}\right)$$

$$\cos\left(x + \frac{\pi}{12}\right) = -1$$

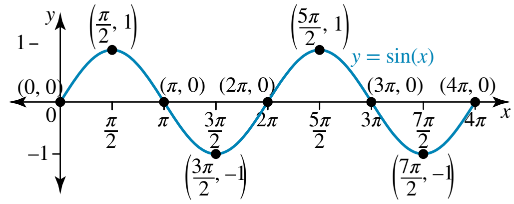
$$\left(x + \frac{\pi}{12}\right) = \pi \text{ for first positive value.}$$

$$x = \frac{11\pi}{12}$$

15 a $y = \sin(x), 0 \le x \le 4\pi$

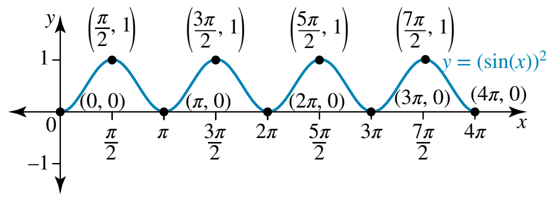
Period: 2π so two complete cycles of the sine curve.

No transformations.



b $y = \sin^2(x), 0 \le x \le 4\pi$

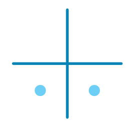
This is the same as $y = (\sin(x))^2$, so all the negative y-values with become positive, giving the following curve.



16 a i $2\sin(2x) + \sqrt{3} = 0$ for $x \in [0, 2\pi]$

$\therefore \sin(2x) = -\frac{\sqrt{3}}{2}, 2x \in [0, 4\pi]$

$\frac{\sqrt{3}}{2}$ suggests $\frac{\pi}{3}$ and sine is negative in 3rd & 4th quadrants



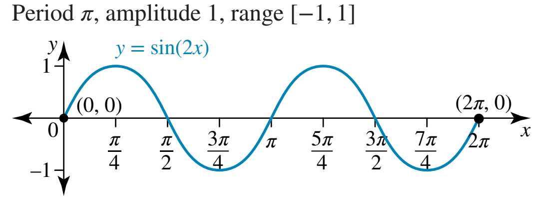
$\therefore 2x = \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, 3\pi + \frac{\pi}{3}, 4\pi - \frac{\pi}{3}$

$\therefore 2x = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$

$\therefore x = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

ii Graph of $y = \sin(2x)$ for $x \in [0, 2\pi]$

Period π, amplitude 1, range $[-1, 1]$



iii $x\colon \sin(2x) < -\frac{\sqrt{3}}{2}, 0 \le x \le 2\pi$

Draw the line $y = -\frac{\sqrt{3}}{2}$ on the graph of $y = \sin(2x)$.

At their intersections, $\sin(x) = -\frac{\sqrt{3}}{2}$ and therefore $2\sin(2x) + \sqrt{3} = 0$, the solutions to which were found in part **ai** as $x = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$.

The sine curve lies below the line for $\frac{2\pi}{3} < x < \frac{5\pi}{6}$ and $\frac{5\pi}{3} < x < \frac{11\pi}{6}$.

The solution set is

$$\left\{x\colon \frac{2\pi}{3} < x < \frac{5\pi}{6}\right\} \cup \left\{x\colon \frac{5\pi}{3} < x < \frac{11\pi}{6}.\right\}$$

Exercise 4.5 – Derivatives of the sine and cosine functions

1 a $y = \sin 8x$

$\frac{dy}{dx} = 8\cos x$

b $y = \sin(-6x)$

$\frac{dy}{dx} = -6\cos x(-6x)$

c $y = \sin x$

$\frac{dy}{dx} = \cos x$

d $y = \sin\frac{x}{3}$

$\frac{dy}{dx} = \frac{1}{3}\cos\frac{x}{3}$

e $y = \sin\left(-\frac{x}{2}\right)$

$\frac{dy}{dx} = -\frac{1}{2}\cos\left(-\frac{x}{2}\right)$

f $y = \sin\frac{2x}{3}$

$\frac{dy}{dx} = \frac{2}{3}\cos\frac{2x}{3}$

2 a $y = \cos 3x$

$\frac{dy}{dx} = -3\sin 3x$

b $y = \cos(-2x)$

$\frac{dy}{dx} = 2\sin(-2x)$

c $y = \cos\frac{x}{3}$

$\frac{dy}{dx} = -\frac{1}{3}\sin\frac{x}{3}$

d $y = \cos 21x$

$\frac{dy}{dx} = -21\sin 21x$

e $y = \cos(-7x)$

$\frac{dy}{dx} = 7\sin(-7x)$

f $y = \cos\frac{\pi x}{4}$

$\frac{dy}{dx} = -\frac{\pi}{4}\sin\frac{\pi x}{4}$

3 a $y = \sin(2x+3)$

$\frac{dy}{dx} = 2\cos(2x+3)$

b $y = \sin(6-7x)$

$\frac{dy}{dx} = -7\cos(6-7x)$

c $y = \sin(5x-4)$

$\frac{dy}{dx} = 5\cos(5x-4)$

d $y = \sin\left(\frac{3x+2}{4}\right)$

$\frac{dy}{dx} = \frac{3}{4}\cos\left(\frac{3x+2}{4}\right)$

e $y = \sin\left(\frac{8-7x}{3}\right)$

$\frac{dy}{dx} = -\frac{7}{3}\cos\left(\frac{8-7x}{3}\right)$

f $y = 5\pi\sin 2\pi x$

$\frac{dy}{dx} = 10\pi^2\cos 2\pi x$

4 a $y = \cos(8-x)$

$\frac{dy}{dx} = -1\times-\sin(8-x)$

$= \sin(8-x)$

b $y = \cos(6-5x)$

$\frac{dy}{dx} = -\sin(6-5x)\times(-5)$

$\frac{dy}{dx} = 5\sin(6-5x)$

c $y = \cos\left(\frac{2x+3}{3}\right)$

$\frac{dy}{dx} = -\frac{2}{3}\sin\left(\frac{2x+3}{3}\right)$

d $y = \cos\left(\frac{4x-1}{5}\right)$

$\frac{dy}{dx} = \frac{-4}{5}\sin\left(\frac{4x-1}{5}\right)$

e $y = 4\pi\cos 10\pi x$

$\frac{dy}{dx} = -40\pi^2\sin 10\pi x$

f $y = -6\cos(-2x)$

$\frac{dy}{dx} = -6\times-2\times-\sin(-2x)$

$= -12\sin(-2x)$

5 a $y = \cos(x^2-4x+3)$

$\frac{dy}{dx} = (2x-4)\times-\sin(x^2-4x+3)$

$= -2(x-2)\sin(x^2-4x+3)$

$= 2(2-x)\sin(x^2-4x+3)$

b $y = \sin(10-5x+x^2)$

$\frac{dy}{dx} = (-5+2x)\cos(10-5x+x^2)$

$= (2x-5)\cos(10-5x+x^2)$

c $y = \sin(e^x)$

Let $u = e^x$

$\frac{du}{dx} = e^x$

$y = \sin u$

$\frac{dy}{du} = \cos u$

$\frac{dy}{dx} = \cos u\times e^x$

$= e^x\cos(e^x)$

d $y = \cos(x^2 + 7x)$

$$\frac{dy}{dx} = (2x + 7) \times -\sin(x^2 + 7x)$$
$$= -(2x + 7)\sin(x^2 + 7x)$$

e $y = \cos(4x - x^2)$

$$\frac{dy}{dx} = (4 - 2x) \times -\sin(4x - x^2)$$
$$= -2(2 - x)\sin(4x - x^2)$$
$$= 2(x - 2)\sin(4x - x^2)$$

f $y = \sin(x^2 + 3x)$

$$\frac{dy}{dx} = (2x + 3)\cos(x^2 + 3x)$$

6 $\cos(10x°) = \cos\left(10 \times \frac{\pi}{180}x\right)$

$$= \cos\left(\frac{\pi}{18}x\right)$$

$$y = 9\cos\left(\frac{\pi}{18}x\right)$$

$$\frac{dy}{dx} = 9 \times -\sin\left(\frac{\pi}{18}x\right) \times \frac{\pi}{18}$$

$$\frac{dy}{dx} = \frac{-\pi}{2}\sin\left(\frac{\pi}{18}x\right)$$

7 a $y = 2\cos(3x)$

$$\frac{dy}{dx} = -6\sin(3x)$$

b $y = \cos(x°)$

$$y = \cos\left(\frac{\pi x}{180}\right)$$

$$\frac{dy}{dx} = -\frac{\pi}{180}\sin\left(\frac{\pi x}{180}\right)$$

c $y = 3\cos\left(\frac{\pi}{2} - x\right)$

$$\frac{dy}{dx} = 3\left(-\cos\left(\frac{\pi}{2} - x\right)\right) \times -1$$

$$\frac{dy}{dx} = 3\sin\left(\frac{\pi}{2} - x\right)$$

d $y = -4\sin\left(\frac{x}{3}\right)$

$$\frac{dy}{dx} = -\frac{4}{3}\cos\left(\frac{x}{3}\right)$$

e $y = \sin(12x°)$

$$y = \sin\left(\frac{\pi x}{15}\right)$$

$$\frac{dy}{dx} = \frac{\pi}{15}\cos\left(\frac{\pi x}{15}\right)$$

f $y = 2\sin\left(\frac{\pi}{2} + 3x\right)$

$$\frac{dy}{dx} = 6\cos\left(\frac{\pi}{2} + 3x\right)$$

8 $y = -\cos(x)$

$$m_T = \frac{dy}{dx} = \sin(x)$$

When $x = \frac{\pi}{2}$; $m_T = \sin\left(\frac{\pi}{2}\right) = 1$

When $x = \frac{\pi}{2}$; $y = -\cos\left(\frac{\pi}{2}\right) = 0$

Equation of tangent with $m_T = 1$ which passes through the point

$(x_1, y_1) = \left(\frac{\pi}{2}, 0\right)$ is given by

$$y - y_1 = m_T(x - x_1)$$
$$y - 0 = 1\left(x - \frac{\pi}{2}\right)$$
$$y = x - \frac{\pi}{2}$$

9 When $x = \frac{\pi}{6}, y = 3\cos\left(\frac{\pi}{6}\right) = 3 \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

$$m_T = \frac{dy}{dx} = -3\sin(x)$$

When $x = \frac{\pi}{6}, m_T = -3\sin\left(\frac{\pi}{6}\right) = -3 \times \frac{1}{2} = -\frac{3}{2}$

Equation of tangent with $m_T = -\frac{3}{2}$ which passes through the point

$(x_1, y_1) = \left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right)$ is given by

$$y - y_1 = m_T(x - x_1)$$
$$y - \frac{3\sqrt{3}}{2} = -\frac{3}{2}\left(x - \frac{\pi}{6}\right)$$
$$y - \frac{3\sqrt{3}}{2} = -\frac{3}{2}x + \frac{\pi}{4}$$
$$y = -\frac{3}{2}x + \frac{\pi}{4} + \frac{3\sqrt{3}}{2}$$

10 $y = -2\sin\left(\frac{x}{2}\right)$ for $x \in [0, 2\pi]$

$$\frac{dy}{dx} = -\cos\left(\frac{x}{2}\right)$$
$$\frac{1}{2} = -\cos\left(\frac{x}{2}\right)$$
$$-\frac{1}{2} = \cos\left(\frac{x}{2}\right) \text{ for } \frac{x}{2} \in [0, \pi]$$

$\frac{1}{2}$ suggests $\frac{\pi}{3}$. Since cos is negative in the second quadrant.

$$\frac{x}{2} = \pi - \frac{\pi}{3}$$
$$\frac{x}{2} = \frac{2\pi}{3}$$
$$x = \frac{4\pi}{3}$$

When $x = \frac{4\pi}{3}, y = -2\sin\left(\frac{4\pi}{3} \times \frac{1}{2}\right)$

$$= -2\sin\left(\frac{2\pi}{3}\right)$$
$$= -2\sin\left(\pi - \frac{\pi}{3}\right)$$
$$= -2\sin\left(\frac{\pi}{3}\right) = -\sqrt{3}$$

Point is $\left(\frac{4\pi}{3}, -\sqrt{3}\right)$.

11 a $f(x) = \sin(x) - \cos(x)$

$f(0) = \sin(0) - \cos(0) = -1$

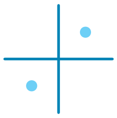
b $f(x) = 0$

$\sin(x) - \cos(x) = 0$

$\sin(x) = \cos(x)$

$\tan(x) = 1$

1 suggests $\frac{\pi}{4}$. Since tan is positive 1st and 3rd quadrants.



$x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

c $f'(x) = \cos(x) + \sin(x)$

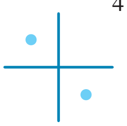
d $f'(x) = 0$

$\cos(x) + \sin(x) = 0$

$\sin(x) = -\cos(x)$

$\tan(x) = -1$

1 suggests $\frac{\pi}{4}$. Since tan is negative 2nd and 4th quadrants.



$x = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

12 a $f(x) = \sqrt{3}\cos(x) + \sin(x)$

$f(0) = \sqrt{3}\cos(0) + \sin(0) = \sqrt{3}$

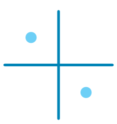
b $f(x) = 0$

$\sqrt{3}\cos(x) + \sin(x) = 0$

$\sin(x) = -\sqrt{3}\cos(x)$

$\tan(x) = -\sqrt{3}$

$\sqrt{3}$ suggests $\frac{\pi}{3}$. Since tan is negative 2nd and 4th quadrants.



$x = -\frac{\pi}{3}, \pi - \frac{\pi}{3}$

$x = -\frac{\pi}{3}, \frac{2\pi}{3}$

c $f'(x) = -\sqrt{3}\sin(x) + \cos(x)$

d $f'(x) = 0$

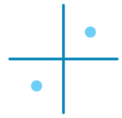
$-\sqrt{3}\sin(x) + \cos(x) = 0$

$\cos(x) = \sqrt{3}\sin(x)$

$1 = \sqrt{3}\tan(x)$

$\frac{1}{\sqrt{3}} = \tan(x)$

$\frac{1}{\sqrt{3}}$ suggests $\frac{\pi}{6}$. Since tan is positive 1st and 3rd quadrants.



$x = -\pi + \frac{\pi}{6}, \frac{\pi}{6}$

$x = -\frac{5\pi}{6}, \frac{\pi}{6}$

13 $f(x) = \sin(2x)$ so $f'(x) = 2\cos(2x)$

$f(x) = \cos(2x)$ so $f'(x) = -2\sin(2x)$

When the gradients are equal

$2\cos(2x) = -2\sin(2x)$ where $x \in [-\pi, \pi]$

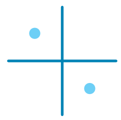
$\cos(2x) = -\sin(2x)$ where $2x \in [-2\pi, 2\pi]$

$\frac{\cos(2x)}{\cos(2x)} = \frac{-\sin(2x)}{\cos(2x)}$

$1 = -\tan(2x)$

$-1 = \tan(2x)$

1 suggests $\frac{\pi}{4}$. Since tan is negative 2nd and 4th quadrants.



$2x = -\pi - \frac{\pi}{4}, -\frac{\pi}{4}, \pi - \frac{\pi}{4}$ and $2\pi - \frac{\pi}{4}$

$2x = -\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}$ and $\frac{7\pi}{4}$

$x = -\frac{5\pi}{8}, -\frac{\pi}{8}, \frac{3\pi}{8}$ and $\frac{7\pi}{8}$

14 $f(x) = x - \sin(2x), -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

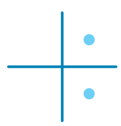
$f'(x) = 1 - 2\cos(2x)$

For gradient of zero: $f'(x) = 0$

$0 = 1 - 2\cos(2x)$

$\cos(2x) = \frac{1}{2}$

$\frac{1}{2}$ suggests $\frac{\pi}{3}$. Since cos is positive in 1st and 4th quadrants, and $-\pi \le 2x \le \pi$



$(2x) = -\frac{\pi}{3}, \frac{\pi}{3}$

$x = -\frac{\pi}{6}, \frac{\pi}{6}$

For coordinates of points:

$f\left(-\frac{\pi}{6}\right) = -\frac{\pi}{6} - \sin\left(-\frac{\pi}{3}\right)$ $\quad f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} - \sin\left(\frac{\pi}{3}\right)$

$f\left(-\frac{\pi}{6}\right) = -\frac{\pi}{6} + \sin\left(\frac{\pi}{3}\right)$ $\quad f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} - \frac{\sqrt{3}}{2}$

$f\left(-\frac{\pi}{6}\right) = -\frac{\pi}{6} + \frac{\sqrt{3}}{2}$

Points are: $\left(-\frac{\pi}{6}, -\frac{\pi}{6} + \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\pi}{6}, \frac{\pi}{6} - \frac{\sqrt{3}}{2}\right)$

To three decimal places: $(-0.524, 0.342)$ and $(0.524, -0.342)$

15 $f(x) = 2x + \cos(3x), 0 \le x \le \dfrac{\pi}{2}$

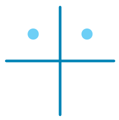
$f'(x) = 2 - 3\sin(3x)$

For gradient of zero: $f'(x) = 0$

$0 = 2 - 3\sin(3x)$

$\sin(3x) = \dfrac{2}{3}$

$\dfrac{2}{3}$ suggests 0.7297. Since sin is positive in 1st and 2nd quadrants, and $0 \le 3x \le \dfrac{3\pi}{2}$



$(3x) = 0.7297$ or $\pi - 0.7297$

$x = 0.24324, 0.803955$

For coordinates of points:

$f(0.24324) = 2(0.24324) + \cos(3(0.24324))$

$= 1.2318$

$f(0.803955) = 2(0.803955) + \cos(3(0.803955))$

$= 0.862554$

Points, to three decimal places: (0.243, 1.232) and (0.804, 0.863)

Exercise 4.6 – Applications of trigonometric functions

1 $d(t) = 6 + 2.5\sin\dfrac{\pi t}{6}$

a greatest depth is $6 + 2.5 = 8.5$ m

this occurs when $\sin\dfrac{\pi t}{6} = 1$

$\Rightarrow \dfrac{\pi t}{6} = \dfrac{\pi}{2}$

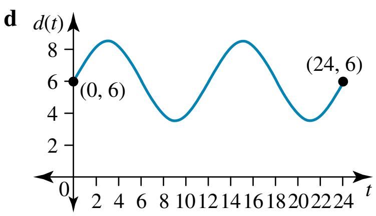
$t = 3$ pm

b period $= \dfrac{2\pi}{\frac{\pi}{6}} = 12$ hrs

max depth again after 12 hours

c least depth is $6 - 2.5 = 3.5$ m

d



e $6 + 2.5\sin\dfrac{\pi t}{6} = 7.25$

$2.5\sin\dfrac{\pi t}{6} = 1.25$

$\sin\dfrac{\pi t}{6} = \dfrac{1}{2}$

basic angle $= \dfrac{\pi}{6}$

1st and 2nd quadrants.

$\dfrac{\pi t}{6} = \dfrac{\pi}{6}, \pi - \dfrac{\pi}{6}, 2\pi + \dfrac{\pi}{6}, 3\pi - \dfrac{\pi}{6}$

$\dfrac{\pi t}{6} = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}$

$t = 1, 5, 13, 17$

Fred is able to enter and leave the inlet between 1 pm and 5 pm and again between 1 am and 5 am the next day.

2 $d = 7 + 3\sin\dfrac{\pi t}{6}$

a maximum depth of water is when

$\sin\dfrac{\pi t}{6} = 1$

$d = 7 + 3 = 10$ m

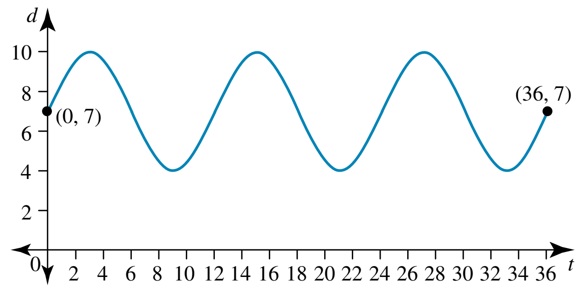
minimum depth of water is when

$\sin\dfrac{\pi t}{6} = -1$

$d = 7 - 3 = 4$ m

b From midnight Friday to midday on Sunday is 36 hours.

So $0 \le t \le 36$



c at $\sin\dfrac{\pi t}{6} = 1$

$\dfrac{\pi t}{6} = \dfrac{\pi}{2}$

$t = 3$ am

d $7 + 3\sin\dfrac{\pi t}{6} = 8.5$

$3\sin\dfrac{\pi t}{6} = 1.5$

$\sin\dfrac{\pi t}{6} = 0.5$

basic angle $= \dfrac{\pi}{6}$

1st and 2nd quadrants

$\dfrac{\pi t}{6} = \dfrac{\pi}{6}, \pi - \dfrac{\pi}{6}, 2\pi + \dfrac{\pi}{6}, 3\pi - \dfrac{\pi}{6}, 4\pi + \dfrac{\pi}{6}, 5\pi - \dfrac{\pi}{6}$

$= \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}, \dfrac{25\pi}{6}, \dfrac{29\pi}{6}$

$t = 1, 5, 13, 17, 25, 29$

Student should be on the pier between 1 am and 5 am on Saturday, 1 pm and 5 pm on Saturday, 1 am and 5 am on Sunday.

e She should fish between 3 am and 5 am Sunday morning.

3 $T(m) = 18 + 7\cos\dfrac{\pi}{6}m$

a max temp in March means $m = 3$

$T(3) = 18 + 7\cos\dfrac{3\pi}{6}$

$= 18°C$

max temp in August means $m = 8$

$T(8) = 18 + 7\cos\left(\dfrac{8\pi}{6}\right)$

$= 14.5°C$

b Highest temp is $18 + 7 = 25°$.

This occurs when $\cos\dfrac{\pi m}{6} = 1$

$$\frac{\pi m}{6} = 0, 2\pi$$

$$m = 0, 12$$

i.e. January and December

c In February, $m = 14$

$$T(2) = 18 + 7\cos\frac{14\pi}{6}$$

$$= 21.5°\text{C}$$

d $$18 + 7\cos\frac{\pi m}{6} = 21.5$$

$$7\cos\frac{\pi m}{6} = 3.5$$

$$\cos\frac{\pi m}{6} = \frac{1}{2}$$

$$\text{basic angle} = \frac{\pi}{3}$$

1st and 4th quadrant

$$\frac{\pi m}{6} = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$

$$= \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$$

$$m = 14, 22$$

next time it is 21.5° it is month 22, i.e. October, 8 months later.

4 $h = a\sin nt + c$

a max height of rope $= 1.8$ m

median is 0.9

amplitude is 0.9

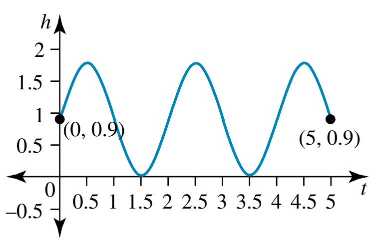
$$\text{period} = \frac{2\pi}{n} = 2$$

$$n = \pi$$

$a = 0.9, n = \pi, c = 0.9$

$h = 0.9\sin\pi t + 0.9$

b



c $0.9\ \sin\pi t + 0.9 = 0.25$

$0.9\ \sin\pi t = -0.65$

$\sin\pi t = -0.7\dot{2}$

basic angle is 0.807

3rd and 4th quadrants

$$\pi t = \pi + 0.807$$

$$= 3.9486$$

$$t = 1.2569$$

$$t = 1.3 \text{ seconds}$$

5 $P(t) = 100\sin\left(\dfrac{\pi}{2}t\right) + 500$

a at $t = 0$

$P(0) = 100\sin(0) + 500$

$P(0) = 500$

Initial population $= 500$ frogs

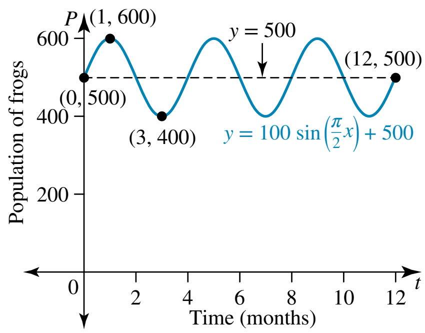
b $P(t) = 100\sin\left(\dfrac{\pi}{2}t\right) + 500$

period: $\dfrac{2\pi}{\frac{\pi}{2}} = 4$ months (3 complete cycles in 1 year)

amplitude: 100

line of oscillation (mean position): $y = 500$

range: $[500 - 100, 500 + 100] = [400, 600]$



c greatest population $= 600$ frogs.

Occurs when $\sin\left(\dfrac{\pi}{2}t\right) = 1$

$$\frac{\pi}{2}t = \frac{\pi}{2}$$

$t = 1$ or read off from graph

population greatest the first time after 1 month.

6 a $L(t) = 2\sin(\pi t) + 10$

When $t = 0$; $L(0) = 2\sin(0) + 10 = 10$ cm

b $\dfrac{dL}{dt} = 2\pi\cos(\pi t)$

c When $t = 1$ second then $\dfrac{dL}{dt} = 2\pi\cos(\pi) = -2\pi$ cm/s.

7 $T = 2\sin\left(\dfrac{\pi}{9}t\right) + 12,\ 0 \le t \le 24$ (Note time in hours after 8:00 am)

a at 12 noon, $t = 4$

$$T(4) = 2\sin\left(\frac{4\pi}{9}\right) + 12$$

$$T = 13.969616$$

Temperature is 14° (to nearest degree)

b $$\frac{dT}{dt} = 2\cos\left(\frac{\pi}{9}t\right) \times \frac{\pi}{9}$$

$$\frac{dT}{dt} = \frac{2\pi}{9}\cos\left(\frac{\pi}{9}t\right)$$

c at midnight, $t = 16$

$$\frac{dT}{dt} = \frac{2\pi}{9}\cos\left(\frac{\pi}{9} \times 16\right)$$

$$\frac{dT}{dt} = \frac{2\pi}{9}\cos\left(\frac{16\pi}{9}\right)$$

$$\frac{dT}{dt} = 0.53479991$$

Rate of change of temperature at midnight $= 0.535°$C/hr (correct to 3 d.p.)

8 $h = 4\cos\left(\dfrac{\pi}{25}d\right) + 5,\ 0 \le d \le 25$

a at $d = 0$:

$h = 4\cos(0) + 5$

$h = 4 + 5$

height of rollercoaster car at the beginning is 9 metres.

b $\dfrac{dh}{dd} = 4 \times \left(-\sin\left(\dfrac{\pi}{25}d\right)\right) \times \dfrac{\pi}{25}$

$\dfrac{dh}{dd} = -\dfrac{4\pi}{25}\sin\left(\dfrac{\pi}{25}d\right)$

c **i** at $d = 5$:

$\dfrac{dh}{dd} = -\dfrac{4\pi}{25}\sin\left(\dfrac{\pi}{25} \times 5\right)$

$\dfrac{dh}{dd} = -\dfrac{4\pi}{25}\sin\left(\dfrac{\pi}{5}\right)$

$\dfrac{dh}{dd} = -0.29545$

Gradient $= -0.295$ metres/metre

ii at $d = 15$:

$\dfrac{dh}{dd} = -\dfrac{4\pi}{25}\sin\left(\dfrac{\pi}{25} \times 15\right)$

$\dfrac{dh}{dd} = -\dfrac{4\pi}{25}\sin\left(\dfrac{3\pi}{5}\right)$

$\dfrac{dh}{dd} = -0.47805$

Gradient $= -0.478$ metres/metre

9 **a** Period of function is $2\pi \div \dfrac{\pi}{6} = 12$ hours

b Low tide occurs when $\sin\left(\dfrac{\pi t}{6}\right) = -1$ so

$H_{LOWTIDE} = 1.5 + 0.5(-1) = 1$ m.

$1.5 + 0.5\sin\left(\dfrac{\pi t}{6}\right) = 1$

$0.5\sin\left(\dfrac{\pi t}{6}\right) = -0.5$

$\sin\left(\dfrac{\pi t}{6}\right) = -1$

1 suggests $\dfrac{\pi}{2}$. Since sin is negative 3rd quadrant.

$\dfrac{\pi t}{6} = \pi + \dfrac{\pi}{2}$

$\dfrac{\pi t}{6} = \dfrac{3\pi}{2}$

$t = \dfrac{3\pi}{2} \times \dfrac{6}{\pi} = 9$ or 3 pm

Low tide $= 1$ metre at 3 pm

c $\dfrac{dH}{dt} = \dfrac{\pi}{6} \times \dfrac{1}{2}\cos\left(\dfrac{\pi t}{6}\right) = \dfrac{\pi}{12}\cos\left(\dfrac{\pi t}{6}\right)$

d When $t = 7.30$ am then 1.5 hours.

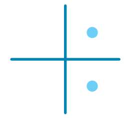
$\dfrac{dH}{dt} = \dfrac{\pi}{12}\cos\left(\dfrac{\pi}{6} \times \dfrac{3}{2}\right) = \dfrac{\pi}{12}\cos\left(\dfrac{\pi}{4}\right) = \dfrac{\pi}{12} \times \dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2}\pi}{24}$

e $\dfrac{\pi}{12}\cos\left(\dfrac{\pi t}{6}\right) = \dfrac{\sqrt{2}\pi}{24}$

$\cos\left(\dfrac{\pi t}{6}\right) = \dfrac{\sqrt{2}\pi}{24} \times \dfrac{12}{\pi} = \dfrac{\sqrt{2}}{2}$

$\dfrac{\sqrt{2}}{2}$ suggests $\dfrac{\pi}{4}$. Since cos is positive then 1st and 4th quadrants.



$\dfrac{\pi t}{6} = \dfrac{\pi}{4}, 2\pi - \dfrac{\pi}{4}$

$\dfrac{\pi t}{6} = \dfrac{\pi}{4}, \dfrac{7\pi}{4}$

$t = \dfrac{\pi}{4} \times \dfrac{6}{\pi}, \dfrac{7\pi}{4} \times \dfrac{6}{\pi}$

$t = \dfrac{3}{2}, \dfrac{21}{2}$

The second time when $t = 10.5$ hours or at 4.30 pm.

10 **a** $h = a\cos(nt) + c$

Amplitude $= 50$

Reflection in x-axis so $a = -50$

Period $= 1$ second so $1 = \dfrac{2\pi}{n}$ and $n = 2\pi$

Vertical translation is 50 so $c = 50$

Thus $h = -50\ \cos(2\pi t) + 50$

$h = 50 - 50\ \cos(2\pi t)$

b $\dfrac{dh}{dt} = 100\pi\sin(2\pi t)$

c When $t = 0.25$ seconds

$\dfrac{dh}{dt} = 100\pi\sin(2\pi \times 0.25) = 100\pi$ mm/ sec

11 **a** $h = 5 - 3.5\left(\dfrac{\pi}{30}\right)$

When $t = 0$; $h = 5 - 3.5\cos(0) = 5 - 3.5 = 1.5$ m

b $h_{\max} = 5 - 3.5(-1) = 8.5$ m

c period $= \dfrac{2\pi}{\frac{\pi}{30}} = 60$ s

Therefore 1 rotation takes 60 seconds

d For $5 - 3.5\ \cos\left(\dfrac{\pi t}{30}\right) > 7$ for $0 \le t \le 60$

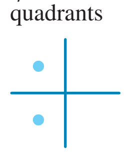
Solve:

$5 - 3.5\ \cos\left(\dfrac{\pi t}{30}\right) = 7$

$3.5\ \cos\left(\dfrac{\pi t}{30}\right) = -2$

$\cos\left(\dfrac{\pi t}{30}\right) = \dfrac{-4}{7}$

$\dfrac{4}{7}$ implies 0.962 551 and cos negative in 2nd & 3rd quadrants



$\therefore \left(\dfrac{\pi t}{30}\right) = \pi - 0.962\,551$ or $\pi + 0.962\,551$

$t = 20.8083$ or 39.1917

time spent above 7 metres $= 39.1917 - 20.8083$

$= 18.3834$

$= 18.4$ seconds (1 dp)

e $\dfrac{dh}{dt} = \dfrac{3.5\pi}{30}\sin\left(\dfrac{\pi t}{30}\right)$

$\dfrac{dh}{dt} = \dfrac{7\pi}{60}\sin\left(\dfrac{\pi t}{30}\right)$

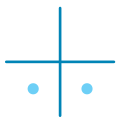
f $\frac{dh}{dt} = -0.2$ m/s

$-0.2 = \frac{7\pi}{60} \sin\left(\frac{\pi t}{30}\right)$

$\frac{-0.2 \times 60}{7\pi} = \sin\left(\frac{\pi t}{30}\right)$

$-0.5456 = \sin\left(\frac{\pi t}{30}\right)$

0.5456 suggests 0.5772. Since sin is negative 3rd and 4th quadrants.



$\frac{\pi t}{30} = \pi + 0.5772, 2\pi - 0.5772$

$\frac{\pi t}{30} = 3.7188, 5.7060$

$t = 3.7188 \times \frac{30}{\pi}, 5.7060 \times \frac{30}{\pi}$

$t = 35.51$ s, 54.49 seconds

12 a $y = \frac{7}{2} \cos\left(\frac{\pi x}{20}\right) + \frac{5}{2}\ 0 \le x \le 20$

$y_{\text{max}} = \frac{7}{2} \times 1 + \frac{5}{2} = 6$ m

b $\frac{dy}{dx} = -\frac{7\pi}{40} \sin\left(\frac{\pi x}{20}\right)$

c **i** When $x = 5$; $\frac{dy}{dx} = -\frac{7\pi}{40} \sin\left(\frac{\pi}{4}\right) = \frac{-7\sqrt{2}\pi}{80}$

ii When $x = 10$; $\frac{dy}{dx} = -\frac{7\pi}{40} \sin\left(\frac{\pi}{2}\right) = \frac{-7\pi}{40}$

d **i** When $y = 0$ then

$\frac{7}{2} \cos\left(\frac{\pi x}{20}\right) + \frac{5}{2} = 0$

$7 \cos\left(\frac{\pi x}{20}\right) + 5 = 0$

$7 \cos\left(\frac{\pi x}{20}\right) = -5$

$\cos\left(\frac{\pi x}{20}\right) = -\frac{5}{7}$

$\frac{\pi x}{20} = \cos^{-1}\left(-\frac{5}{7}\right)$

$\frac{\pi x}{20} = 2.3664$

$x = \frac{2.3664 \times 20}{\pi}$

$x = 15$ m

ii When $x = 15.0649$ then

$\frac{dy}{dx} = -\frac{7\pi}{40} \sin\left(\frac{\pi \times 15.0649}{20}\right)$

$\frac{dy}{dx} = -0.3847$

If θ is the required angle, then

$\tan(\theta) = -0.3847$

$\theta = \tan^{-1}(-0.3847)$

$\theta = 180° - 21.0452°$

$\theta = 158.96°$

13 $D(t) = 2.5 + 0.5 \sin\left(\frac{\pi t}{3}\right)$, $0 \le t \le 24$ time after 4 a.m.

a at 4 a.m.: $t = 0$

$D(0) = 2.5 + 0.5 \sin(0)$

$D(0) = 2.5$

Depth of water at 4 a.m. is 2.5 metres

b at midday: $t = 8$

$D(8) = 2.5 + 0.5 \sin\left(\frac{8\pi}{3}\right)$

$D(8) = 2.5 + 0.5 \sin\left(2\pi + \frac{2\pi}{3}\right)$

$D(8) = 2.5 + 0.5 \sin\left(\frac{2\pi}{3}\right)$

$D(8) = 2.5 + 0.5 \times \frac{\sqrt{3}}{2}$

$D(8) = 2.9330127$

Depth of water at midday is 2.93 metres (to 2 d.p.)

c maximum depth $= 2.5 + 0.5 = 3$ metres.

First occurred when $\sin\left(\frac{\pi}{3}t\right) = 1$

$\frac{\pi}{3}t = \frac{\pi}{2}$

$t = \frac{3}{2}$

Maximum depth first at $t = 1.5$ hours after 4 a.m.

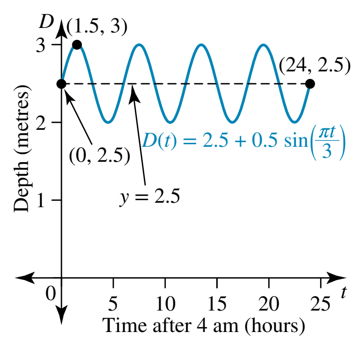
Maximum depth of 3 metres at 5:30 a.m.

d period: $\frac{2\pi}{\frac{\pi}{3}} = 6$ hours (4 complete cycles in a 24 hour period)

amplitude: 0.5 metres

line of oscillation (mean position): $y = 2.5$

range: $[2.5 - 0.5, 2.5 + 0.5] = [2, 3]$



e $\frac{dD}{dt} = 0.5 \cos\left(\frac{\pi}{3}t\right) \times \frac{\pi}{3}$

$\frac{dD}{dt} = \frac{\pi}{6} \cos\left(\frac{\pi}{3}t\right)$

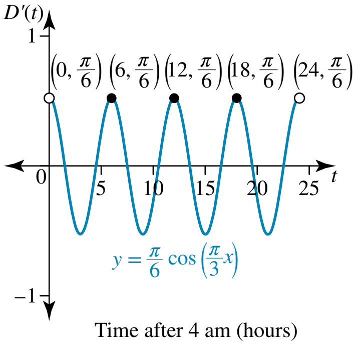
Rate of change at any time: $D'(t) = \frac{\pi}{6} \cos\left(\frac{\pi}{3}t\right)$

period: $\frac{2\pi}{\frac{\pi}{3}} = 6$ hours (4 complete cycles in a 24 hour period)

amplitude: $\frac{\pi}{6}$

line of oscillation (mean position): $y = 0$

range: $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$



f Greatest flow of water into the inlet at maximum turning points of the rate of change.
From the graph, the maximums occur at $t = 0, 6, 12, 18, 24$
Since within the 24 hour period, only include $t = 6, 12, 18$.
Greatest flow of water into the inlet during the 24 hour period occurs at 10 a.m., 4 p.m. and 10 p.m.

14 a $y = 2.5 - 2.5\cos\left(\frac{x}{4}\right), -4\pi \le x \le 4\pi$

period: $\frac{2\pi}{\frac{1}{4}} = 8\pi$

amplitude: 2.5 metres

reflected in the x-axis (or inverted)

translated vertically up by 2.5, line of oscillation (mean position): $y = 2.5$

range: $[2.5 - 2.5, 2.5 + 2.5] = [0, 5]$

y-intercepts: $x = 0$

$y = 2.5 - 2.5\cos(0)$

$y = 2.5 - 2.5$

$y = 0$

x-intercepts: $y = 0$

$$0 = 2.5 - 2.5\cos\left(\frac{x}{4}\right)$$

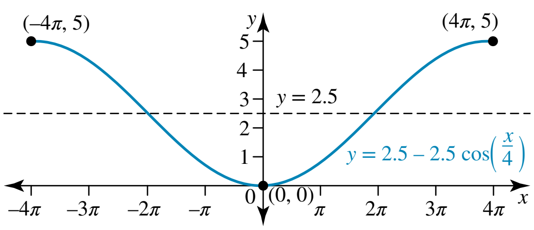
$$\cos\left(\frac{x}{4}\right) = 1$$

If $-4\pi \le x \le 4\pi$ then $-\pi \le \frac{x}{4} \le \pi$

$\frac{x}{4} = 0$

$x = 0$

Axis intercept at $(0, 0)$



b i $h(x) = 2.5 - 2.5\cos\left(\frac{x}{4}\right) \quad -5 \le x \le 5$

$$h(5) = 2.5 - 2.5\cos\left(\frac{5}{4}\right)$$

$h(5) = 1.7117$

Maximum depth is 1.7 metres.

ii $\frac{dh}{dx} = \frac{2.5}{4}\sin\left(\frac{x}{4}\right)$

$$\frac{dh}{dx} = 0.625\sin\left(\frac{x}{4}\right)$$

iii When $x = 3$ then

$$\frac{dh}{dx} = 0.625\sin\left(\frac{3}{4}\right)$$

$$\frac{dh}{dx} = 0.426 \text{ m/m}$$

iv When $\frac{dh}{dx} = 0.58$ then

$$0.58 = 0.625\sin\left(\frac{x}{4}\right)$$

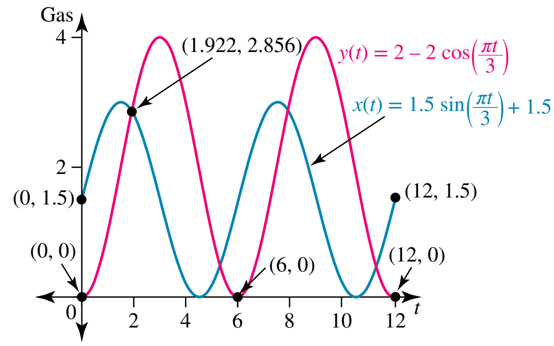
$$0.928 = \sin\left(\frac{x}{4}\right) \quad \frac{-5}{4} \le \frac{x}{4} \le \frac{5}{4}$$

0.928 suggests 1.1890. Since sin is positive 1st quadrant because of the domain.

$$\frac{x}{4} = 1.1890$$

$x = 4.756$ metres

15 a graphs drawn using technology



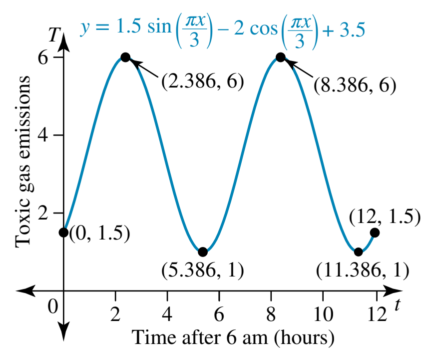
b first point of intersection of the curves is at $(1.922, 2.856)$
time = 1.922 hours after 6:00 a.m.
1.922 hours = 1 hour and 55 minutes (to nearest minute)
Time of day: 7:55 a.m.

c Amount of gas emitted is 2.856 units

d i $T(t) = 1.5\sin\left(\frac{\pi x}{3}\right) + 1.5 + 2 - 2\cos\left(\frac{\pi x}{3}\right)$

$$T(t) = 1.5\sin\left(\frac{\pi x}{3}\right) - 2\cos\left(\frac{\pi x}{3}\right) + 3.5$$

Graph using technology



ii Reading from the graph:
Maximum gas emissions = 6 unit;
minimum gas emissions = 1 unit
Maximums occur at: $t = 2.386, 8.386$
t = 2 hours and 23 minutes, or 8 hours and 23 minutes after 6 a.m.
time of day: 8:23 a.m. and 2:23 p.m.
minimums occur at: $t = 5.386, 11.386$

$t = 5$ hours and 23 minutes, or 11 hours and 23 minutes after 6 a.m.
time of day: 11:23 a.m. and 5:23 p.m.

iii Since the maximum gas emission is 6 units and the minimum is 1 unit, they lie within the range of 0 to 7 units, so the company works within the guidelines.

4.7 Review: exam practice

1 a $\dfrac{3\pi}{4} = \dfrac{3 \times 180}{4}$

$= 135°$

b $\dfrac{13\pi}{12} = \dfrac{13 \times 180}{12}$

$= 195°$

c $2.1 = 2.1 \times \dfrac{180}{\pi}$

$= 120.32114°$
$= 120°19'$

d $1.76 = 1.76 \times \dfrac{180}{\pi}$

$= 100.84057°$
$= 100°50'$

2 a $35° = 35 \times \dfrac{\pi}{180}$

$= \dfrac{7\pi}{36}$

b $280° = 280 \times \dfrac{\pi}{180}$

$= \dfrac{14\pi}{9}$

c $128.5° = 128.5 \times \dfrac{\pi}{180}$

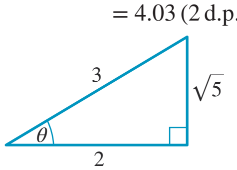
$= 2.24275$
$= 2.24$ (2 d.p.)

d $230°48' = 230.8°$

$230.8° = 230.8 \times \dfrac{\pi}{180}$

$= 4.02822$
$= 4.03$ (2 d.p.)

3



a $\cos(\pi - \theta)$
$= -\cos(\theta)$
$= -\dfrac{2}{3}$

b $\sin(\pi - \theta)$
$= \sin(\theta)$
$= \dfrac{\sqrt{5}}{3}$

c $\tan(\pi + \theta)$
$= \tan(\theta)$
$= \dfrac{\sqrt{5}}{2}$

d $\sin(3\pi + \theta)$
$= -\sin(\theta)$
$= -\dfrac{\sqrt{5}}{3}$

e $\tan(\pi - \theta)$
$= -\tan(\theta)$
$= -\dfrac{\sqrt{5}}{2}$

f $\cos(-\theta)$
$= \cos(\theta)$
$= \dfrac{2}{3}$

4 a $\sin 120° = \sin(180 - 60)°$
$= \sin 60°$
$= \dfrac{\sqrt{3}}{2}$

b $\cos 135° = \cos(180 - 45)°$
$= -\cos 45°$
$= -\dfrac{\sqrt{2}}{2}$ or $\dfrac{-1}{\sqrt{2}}$

c $\tan 330° = \tan(360 - 30)°$
$= -\tan 30°$
$= -\dfrac{\sqrt{3}}{3}$ or $-\dfrac{1}{\sqrt{3}}$

d $\cos 225° = \cos(180 + 45)°$
$= -\cos 45°$
$= -\dfrac{\sqrt{2}}{2}$ or $-\dfrac{1}{\sqrt{2}}$

e $\sin 210° = \sin(180 + 30)°$
$= -\sin 30°$
$= -\dfrac{1}{2}$

f $\tan 150° = \tan(180 - 30)°$
$= -\tan 30$
$= -\dfrac{\sqrt{3}}{3}$ or $-\dfrac{1}{\sqrt{3}}$

5 a $\sin\dfrac{3\pi}{4} = \sin\left(\pi - \dfrac{\pi}{4}\right)$
$= \sin\dfrac{\pi}{4}$
$= \dfrac{\sqrt{2}}{2}$

b $\cos\frac{5\pi}{6} = \cos\left(\pi - \frac{\pi}{6}\right)$
$-\cos = \frac{\pi}{6}$
$= -\frac{\sqrt{3}}{2}$

c $\tan\frac{2\pi}{3} = \tan\left(\pi - \frac{\pi}{3}\right)$
$= -\tan\frac{\pi}{3}$
$= -\sqrt{3}$

d $\cos\frac{4\pi}{3} = \cos\left(\pi + \frac{\pi}{3}\right)$
$= -\cos\frac{\pi}{3}$
$= -\frac{1}{2}$

e $\sin\frac{5\pi}{4} = \sin\left(\pi + \frac{\pi}{4}\right)$
$= -\sin\frac{\pi}{4}$
$= -\frac{\sqrt{2}}{2}$

f $\tan\frac{7\pi}{6} = \tan\left(\pi + \frac{\pi}{6}\right)$
$= \tan\frac{\pi}{6}$
$= \frac{\sqrt{3}}{3}$

g $\sin\frac{11\pi}{6} = \sin\left(2\pi - \frac{\pi}{6}\right)$
$= -\sin\frac{\pi}{6}$
$= -\frac{1}{2}$

h $\cos\frac{5\pi}{3} = \cos\left(2\pi - \frac{\pi}{3}\right)$
$= \cos\frac{\pi}{3}$
$= \frac{1}{2}$

i $\tan\frac{7\pi}{4} = \tan\left(2\pi - \frac{\pi}{4}\right)$
$= -\tan\frac{\pi}{4}$
$= -1$

j $\cos\frac{9\pi}{4} = \cos\left(2\pi + \frac{\pi}{4}\right)$
$= \cos\frac{\pi}{4}$
$= \frac{\sqrt{2}}{2}$

k $\sin\frac{13\pi}{6} = \sin\left(2\pi + \frac{\pi}{6}\right)$
$= \sin\frac{\pi}{6}$
$= \frac{1}{2}$

l $\tan\frac{7\pi}{6} = \tan\left(\pi + \frac{\pi}{6}\right)$
$= \tan\frac{\pi}{6}$
$= \frac{\sqrt{3}}{3}$

6 a $\tan\left(\frac{-\pi}{4}\right) = -\tan\frac{\pi}{4}$
$= -1$

b $\cos\left(\frac{-3\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$
$= \cos\left(\pi - \frac{\pi}{4}\right)$
$= -\cos\frac{\pi}{4}$
$= -\frac{\sqrt{2}}{2}$

c $\sin\left(\frac{-2\pi}{3}\right) = -\sin\frac{2\pi}{3}$
$= -\sin\left(\pi - \frac{\pi}{3}\right)$
$= -\sin\frac{\pi}{3}$
$= -\frac{\sqrt{3}}{2}$

d $\tan\left(\frac{-5\pi}{6}\right) = -\tan\frac{5\pi}{6}$
$= -\tan\left(\pi - \frac{\pi}{6}\right)$
$= -\left(-\tan\frac{\pi}{6}\right)$
$= \tan\frac{\pi}{6}$
$= \frac{\sqrt{3}}{3}$

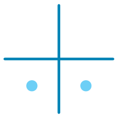
e $\sin\left(\frac{-7\pi}{6}\right) = -\sin\frac{7\pi}{6}$
$= -\sin\left(\pi + \frac{\pi}{6}\right)$
$= -\left(-\sin\frac{\pi}{6}\right)$
$= \frac{1}{2}$

f $\cos\left(\frac{-5\pi}{4}\right) = \cos\frac{5\pi}{4}$
$= \cos\left(\pi + \frac{\pi}{4}\right)$
$= -\cos\frac{\pi}{4}$
$= -\frac{\sqrt{2}}{2}$

7 a $\cos\theta = 0$
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

b $\sin\theta = \frac{-1}{\sqrt{2}}$
basic angle $= \frac{\pi}{4}$

3rd and 4th quadrants



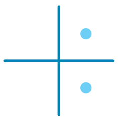
$\theta = \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

c $\cos\theta = \frac{1}{\sqrt{2}}$

basic angle $= \frac{\pi}{4}$

1st and 4th quadrants



$\theta = \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

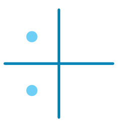
d $\sin\theta = -1$

$\theta = \frac{3\pi}{2}$

e $\cos\theta = \frac{-\sqrt{3}}{2}$

basic angle $= \frac{\pi}{6}$

2nd and 3rd quadrants



$\theta = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

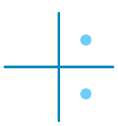
8 a $\sin\theta = 1$

$\theta = 90°$

b $\cos\theta = \frac{1}{2}$

basic angle $= 60°$

1st and 4th quadrants



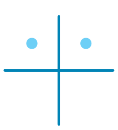
$\theta = 60°, 360 - 60$

$\theta = 60°, 300°$

c $\sin\theta = \frac{\sqrt{3}}{2}$

basic angle $= 60°$

1st and 2nd quadrants



$\theta = 60°, 180 - 60$

$\theta = 60°, 120°$

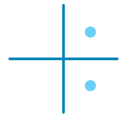
d $\cos\theta = -1$

$\theta = 180°$

e $\sin\theta = \frac{1}{\sqrt{2}}$

basic angle $= 45°$

1st and 2nd quadrants



$\theta = 45°, 180 - 45$

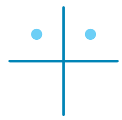
$\theta = 45°, 135°$

9 a $2\sin x = 1$

$\sin x = \frac{1}{2}$

basic angle $= \frac{\pi}{6}$

1st and 2nd quadrants



$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

b $3\cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

c $2\sin x = -\sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$

basic angle $= \frac{\pi}{3}$

3rd and 4th quadrants

$x = \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3},$

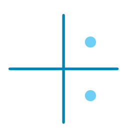
$x = \frac{4\pi}{3}, \frac{5\pi}{3}$

d $\sqrt{2}\cos x = 1$

$\cos x = \frac{1}{\sqrt{2}}$

basic angle $= \frac{\pi}{4}$

1st and 4th quadrants



$x = \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

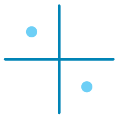
$x = \frac{\pi}{4}, \frac{7\pi}{4}$

e $\sqrt{3}\tan x + 1 = 0$

$\tan x = -\frac{1}{\sqrt{3}}$

basic angle $= \frac{\pi}{6}$

2nd and 4th quadrants



$x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

10 a $4\sin x + 2 = 6$ for $-\pi \le x \le \pi$

$\sin x = 1$

$x = \frac{\pi}{2}$

b $3\cos x - 3 = 0$ for $-\pi \le x \le \pi$

$\cos x = 1$

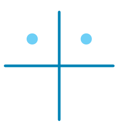
$x = 0$

c $2\sin(3x) - 5 = -4$ for $-\pi \le x \le \pi$

$\sin(3x) = \frac{1}{2}$ for $-3\pi \le (3x) \le 3\pi$

basic angle $\frac{\pi}{6}$

1st and 2nd quadrants in both positive and negative directions.



$(3x) = \frac{\pi}{6}, \frac{5\pi}{6}, 2\pi + \frac{\pi}{6}, 2\pi + \frac{5\pi}{6}, -\frac{7\pi}{6}, -\frac{11\pi}{6}$

$(3x) = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, -\frac{7\pi}{6}, -\frac{11\pi}{6}$

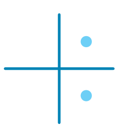
$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, -\frac{7\pi}{18}, -\frac{11\pi}{18}$

d $\sqrt{2}\cos(3x) + 2 = 3$ for $-\pi \le x \le \pi$

$\cos(3x) = \frac{1}{\sqrt{2}}$ for $-3\pi \le (3x) \le 3\pi$

basic angle $\frac{\pi}{4}$

1st and 4th quadrants in both positive and negative directions.



$(3x) = \frac{\pi}{4}, 2\pi - \frac{\pi}{4}, 2\pi + \frac{\pi}{4}, -\frac{\pi}{4}, -2\pi + \frac{\pi}{4}, -2\pi - \frac{\pi}{4}$

$(3x) = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{-\pi}{4}, \frac{-7\pi}{4}, \frac{-9\pi}{4}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12}, \frac{-\pi}{12}, \frac{-7\pi}{12}, \frac{-9\pi}{12}$

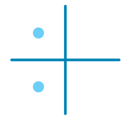
$x = \frac{-3\pi}{4}, \frac{-7\pi}{12}, \frac{-\pi}{12}, \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}$

e $2\cos(2x) + \sqrt{3} = 0$ for $-\pi \le x \le \pi$

$\cos(2x) = -\frac{\sqrt{3}}{2}$ for $-2\pi \le (2x) \le 2\pi$

basic angle $\frac{\pi}{6}$

2nd and 3rd quadrants in both positive and negative directions.



$(2x) = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, -\pi + \frac{\pi}{6}, -\pi - \frac{\pi}{6}$

$(2x) = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{-5\pi}{6}, \frac{-7\pi}{6}$

$x = \frac{-7\pi}{12}, \frac{-5\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}$

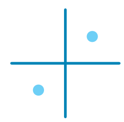
11 a $\sin x = \cos x \quad 0 \le x \le 2\pi$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

basic angle $= \frac{\pi}{4}$

1st and 3rd quadrants



$x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

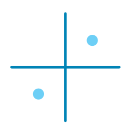
b $\sin\ 2x \ \cos\ 2x \quad 0 \le x \le 2\pi$

$\frac{\sin 2x}{\cos 2x} = 1 \qquad 0 \le 2x \le 4\pi$

$\tan\ 2x = 1$

basic angle $= \frac{\pi}{4}$

1st and 3rd quadrants



$2x = \frac{\pi}{4}, \pi + \frac{\pi}{4}, 2\pi + \frac{\pi}{4}, 3\pi + \frac{\pi}{4}$

$2x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$

$x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

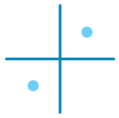
c $\sin\ 2x = \sqrt{3}\cos 2x \quad 0 \le x \le 2\pi$

$\frac{\sin 2x}{\cos 2x} = \sqrt{3} \qquad 0 \le 2x \le 4\pi$

$\tan 2x = \sqrt{3}$

basic angle $= \frac{\pi}{3}$

1st and 3rd quadrants



$2x = \frac{\pi}{3}, \pi + \frac{\pi}{3}, 2\pi + \frac{\pi}{3}, 3\pi + \frac{\pi}{3}$

$2x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3}$

$x = \frac{\pi}{6}, \frac{4\pi}{6}, \frac{7\pi}{6}, \frac{10\pi}{6}$

$= \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

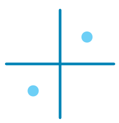
d $\sqrt{3}\sin 3x = \cos 3x \quad 0 \le x \le 2p$

$\frac{\sin 3x}{\cos 3x} = \frac{1}{\sqrt{3}} \quad 0 \le 3x \le 6p$

$\tan 3x = \frac{1}{\sqrt{3}}$

basic angle $= \frac{\pi}{6}$

1st and 3rd quadrants



$3x = \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi + \frac{\pi}{6}, 3\pi + \frac{\pi}{6}, 4\pi + \frac{\pi}{6}, 5\pi + \frac{\pi}{6}$

$3x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}, \frac{25\pi}{6}, \frac{31\pi}{6}$

$x = \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}$

e $\sin 3x + 2\cos 3x = 0 \quad 0 \le x \le 2\pi$

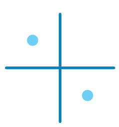
$\sin 3x = -2\cos 3x \quad 0 \le 3x \le 6\pi$

$\frac{\sin 3x}{\cos 3x} = -2$

$\tan 3x = -2$

basic angle $= 1.1071$

2nd and 4th quadrants



$3x = \pi - 1.1071, 2\pi - 1.1071, 3\pi - 1.1071, 4\pi - 1.1071,$
$5\pi - 1.1071, 6\pi - 1.1071$

$3x = 2.0345, 5.1761, 8.3177, 11.4593, 14.6009, 17.7425$

$x = 0.6782, 1.7254, 2.7726, 3.8198, 4.8670, 5.9142$

f $\sin x + 3\cos x = 0 \quad 0 \le x \le 2\pi$

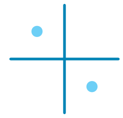
$\sin x = -3\cos x$

$\frac{\sin x}{\cos x} = -3$

$\tan x = -3$

basic angle $= 1.2490$

2nd and 4th quadrants



$x = \pi - 1.2490, 2\pi - 1.2490$

$x = 1.8926, 5.0342$

12 a $\sin^2(2\alpha) + \sin(2\alpha) - 2 = 0$ for $0 \le \alpha \le 2\pi$

Let $x = \sin(2\alpha)$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2$ or 1

$\sin(2\alpha) = -2$ or 1

$\sin(2\alpha) \ne -2 \therefore \sin(2\alpha) = 1$

$\sin(2\alpha) = 1$ for $0 \le 2\alpha \le 4\pi$

basic angle $= \frac{\pi}{2}$

1st and 2nd quadrants

$(2\alpha) = \frac{\pi}{2}, 2\pi + \frac{\pi}{2}$

$(2\alpha) = \frac{\pi}{2}, \frac{5\pi}{2}$

$\alpha = \frac{\pi}{4}, \frac{5\pi}{4}$

b $2\cos^2(3\alpha) + \cos(3\alpha) - 1 = 0$ for $0 \le \alpha \le 2\pi$

Let $x = \cos(3\alpha)$

$2x^2 + x - 1 = 0$

$(2x - 1)(x + 1) = 0$

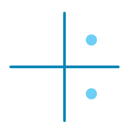
$x = \frac{1}{2}$ or -1

$\cos(3\alpha) = \frac{1}{2}$ or -1

$\cos(3\alpha) = \frac{1}{2}$ for $0 \le (3\alpha) \le 6\pi$

basic angle $\frac{\pi}{3}$

1st and 4th quadrants



$(3x) = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, 2\pi + \frac{\pi}{3}, 4\pi - \frac{\pi}{3}, 4\pi + \frac{\pi}{3}, 6\pi - \frac{\pi}{3}$

$(3x) = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}$

$x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$

$\cos(3\alpha) = -1$ for $0 \le (3\alpha) \le 6\pi$

basic angle π

$(3x) = \pi, 3\pi, 5\pi$

$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

Therefore: $x = \frac{\pi}{9}, \frac{\pi}{3}, \frac{5\pi}{9}, \frac{7\pi}{9}, \pi, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{5\pi}{3}, \frac{17\pi}{9}$

c $2\sin^2\left(\alpha - \frac{\pi}{2}\right) = \sin\left(\alpha - \frac{\pi}{2}\right)$ for $0 \le \alpha \le 2\pi$

Let $x = \sin\left(\alpha - \frac{\pi}{2}\right)$

$2x^2 = x$

$x(2x - 1) = 0$

$x = 0$ or $x = \frac{1}{2}$

$\sin\left(\alpha - \frac{\pi}{2}\right) = 0$ or $\sin\left(\alpha - \frac{\pi}{2}\right) = \frac{1}{2}$

$\sin\left(\alpha - \frac{\pi}{2}\right) = 0$ for $\frac{-\pi}{2} \le \left(\alpha - \frac{\pi}{2}\right) \le \frac{3\pi}{2}$

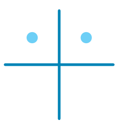
$\left(\alpha - \frac{\pi}{2}\right) = 0, \pi$

$\alpha = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin\left(\alpha - \frac{\pi}{2}\right) = \frac{1}{2}$ for $\frac{-\pi}{2} \le \left(\alpha - \frac{\pi}{2}\right) \le \frac{3\pi}{2}$

basic angle $\frac{\pi}{6}$

1st and 2nd quadrants



$\left(\alpha - \frac{\pi}{2}\right) = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$\alpha = \frac{2\pi}{3}, \frac{4\pi}{3}$

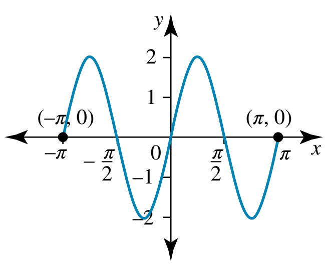
Therefore: $\alpha = \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}$

13 $y = 2\sin 2x, -\pi \le x \le \pi$

amplitude $= 2$

period $= \pi$

range is $-2 \le y \le 2$



14 a $y = 2\sin(2x + \pi)$ for $0 \le x \le 2\pi$

$y = 2\sin\left(2\left(x + \frac{\pi}{2}\right)\right)$

Period: $\frac{2\pi}{2} = \pi$

Amplitude: 2

Line of oscillation (or mean position): $y = 0$

Range: $[-2, 2]$

Horizontal translation of $\frac{\pi}{2}$ to the left, or in the negative x direction.

Endpoints:

at $x = 0$ at $x = 2\pi$

$y = 2\sin(\pi)$ $\quad$ $y = 2\sin(4\pi + \pi)$

$y = 0$ $\quad$ $y = 2\sin(\pi)$

$y = 0$

Endpoints are: $(0, 0)$ and $(2\pi, 0)$

For x-intercepts: $y = 0$

$2\sin(2x + \pi) = 0$ for $\pi \le (2x + \pi) \le 5\pi$

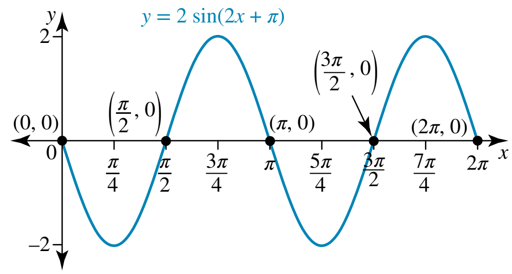
$\sin(2x + \pi) = 0$

$(2x + \pi) = \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$(2x) = 0, \pi, 2\pi, 3\pi, 4\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

For $0 \le x \le 2\pi$: $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$



b $y = 3\cos(3x + \pi)$ for $0 \le x \le 2\pi$

$y = 3\cos\left(3\left(x + \frac{\pi}{3}\right)\right)$

Period: $\frac{2\pi}{3}$

Amplitude: 3

Line of oscillation (or mean position): $y = 0$

Range: $[-3, 3]$

Horizontal translation of $\frac{\pi}{3}$ to the left, or in the negative x direction.

Endpoints:

at $x = 0$ at $x = 2\pi$

$y = 3\cos(\pi)$ $\quad$ $y = 3\cos(6\pi + \pi)$

$y = -3$ $\quad$ $y = 3\cos(\pi)$

$y = -3$

Endpoints are: $(0, -3)$ and $(2\pi, -3)$

For x-intercepts: $y = 0$

$3\cos(3x + \pi) = 0$ for $\pi \le (3x + \pi) \le 7\pi$

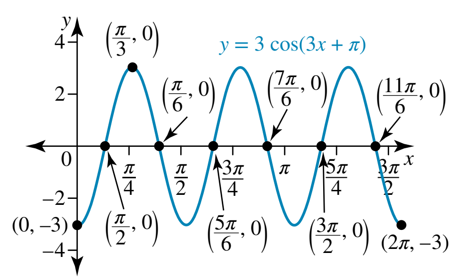
$\cos(3x + \pi) = 0$

$(3x + \pi) = \frac{3\pi}{2}, 2\pi + \frac{\pi}{2}, 2\pi + \frac{3\pi}{2}, 4\pi + \frac{\pi}{2}, 4\pi + \frac{3\pi}{2}, 6\pi + \frac{\pi}{2}$

$(3x) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

For $0 \le x \le 2\pi$: $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$



c $y = 2\sin\left(x - \frac{\pi}{4}\right) - 1$ for $0 \le x \le 2\pi$

Period: 2π

Amplitude: 2

Vertical translation of down by 1 unit; line of oscillation (or mean position): $y = -1$

Range: $[-1 - 2, -1 + 2] = [-3, 1]$

Horizontal translation of $\frac{\pi}{4}$ to the right, or in the positive x direction.

Endpoints:

at $x = 0$ at $x = 2\pi$

$y = 2\sin\left(-\frac{\pi}{4}\right) \quad y = 2\sin\left(2\pi - \frac{\pi}{4}\right)$

$y = -2\sin\left(\frac{\pi}{4}\right) \quad y = -2\sin\left(\frac{\pi}{4}\right)$

$y = -2 \times \frac{1}{\sqrt{2}} \quad y = -2 \times \frac{1}{\sqrt{2}}$

$y = -\sqrt{2} \quad y = -\sqrt{2}$

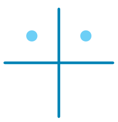
Endpoints are: $(0, -\sqrt{2})$ and $(2\pi, -\sqrt{2})$

For x-intercepts: $y = 0$

$0 = 2\sin\left(x - \frac{\pi}{4}\right) - 1$ for $-\frac{\pi}{4} \le \left(x - \frac{\pi}{4}\right) \le \frac{7\pi}{4}$

$\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{2}$

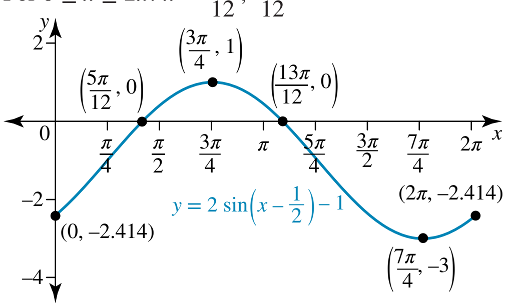
Basic angle $\frac{\pi}{6}$ in the 1st and 2nd quadrants



$\left(x - \frac{\pi}{4}\right) = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{5\pi}{12}, \frac{13\pi}{12}$

For $0 \le x \le 2\pi$: $x = \frac{5\pi}{12}, \frac{13\pi}{12}$



d $y = \cos\left(\frac{1}{2}(x - \pi)\right) + 1$ for $0 \le x \le 2\pi$

Period: $\frac{2\pi}{\frac{1}{2}} = 4\pi$

Amplitude: 1

Vertical translation up by 1; giving line of oscillation (or mean position): $y = 1$

Range: $[1 - 1, 1 + 1] = [0, 2]$

Horizontal translation of π to the right, or in the positive x direction.

Endpoints:

at $x = 0$ at $x = 2\pi$

$y = \cos\left(-\frac{\pi}{2}\right) + 1 \quad y = \cos\left(\frac{\pi}{2}\right) + 1$

$y = 1 \quad y = 1$

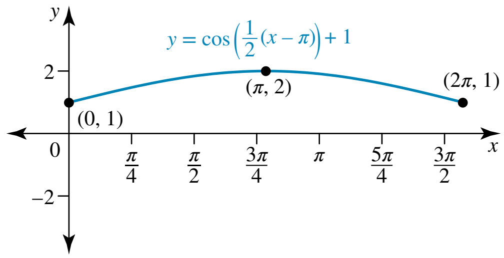
Endpoints are: $(0, 1)$ and $(2\pi, 1)$

For x-intercepts: $y = 0$

$\cos\left(\frac{1}{2}(x - \pi)\right) + 1 = 0$ for $-\frac{\pi}{2} \le \left(\frac{1}{2}(x - \pi)\right) \le \frac{\pi}{2}$

$\cos\left(\frac{1}{2}(x - \pi)\right) = -1$

No solutions for these restricted x values.



15 a $y = \cos(8x - 3)$

$\frac{dy}{dx} = -\sin(8x - 3) \times 8$

$\frac{dy}{dx} = -8\sin(8x - 3)$

b $y = 4 - 3\sin(2x + 1)$

$\frac{dy}{dx} = -3\cos(2x + 1) \times 2$

$\frac{dy}{dx} = -6\cos(2x + 1)$

c $y = 6\sin(2x) + 3\cos(2x)$

$\frac{dy}{dx} = 6\cos(2x) \times 2 + 3(-\sin(2x)) \times 2$

$\frac{dy}{dx} = 12\cos(2x) - 6\sin(2x)$

d $y = \cos(x^2 + 2x + 1)$

$\frac{dy}{dx} = -\sin(x^2 + 2x + 1) \times (2x + 2)$

$\frac{dy}{dx} = -(2x + 2)\sin(x^2 + 2x + 1)$

e $y = 2\sin(4 - 3x)$

$\frac{dy}{dx} = 2\cos(4 - 3x) \times -3$

$\frac{dy}{dx} = -6\cos(4 - 3x)$

f $y = \sin(-x) - \cos(2x)$

$\frac{dy}{dx} = \cos(-x) \times (-1) - (-\sin(2x)) \times 2$

$\frac{dy}{dx} = -\cos(-x) + 2\sin(2x)$

16 $y = 3\cos(x)$

$\frac{dy}{dx} = -3\sin(x)$

At $x = \pi$: $y = 3\cos(\pi)$

$\frac{dy}{dx} = -3\sin(\pi)$

$y = -3$

$\frac{dy}{dx} = 0$

Equation of tangent at $(\pi, -3)$, $m = 0$

$y = -3$

Equation of the line perpendicular to tangent at $(\pi, -3)$:

$x = \pi$

17 $D = 14 + 5\sin\frac{4\pi t}{13}$

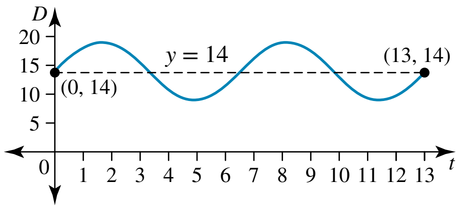
a maximum depth $= 14 + 5 = 19$ m

b minimum depth $= 14 - 5 = 9$ m

c period $= \frac{2\pi}{\frac{4\pi}{13}} = \frac{13}{2} = 6.5$ hrs

d amplitude = 5

e



18 a $T = 19 - 3\sin\left(\frac{\pi}{12}t\right)$

At midnight, $t = 0$

Therefore, at midnight, $T = 19 - 3\sin(0) \Rightarrow T = 19$.

The temperature was 19° at midnight.

b Temperature will be a maximum when $\sin\left(\frac{\pi}{12}t\right) = -1$

$\therefore T_{\max} = 19 - 3 \times (-1)$

$\therefore T_{\max} = 22$

maximum temperature is 22°.

Maximum occurs when $\sin\left(\frac{\pi}{12}t\right) = -1$

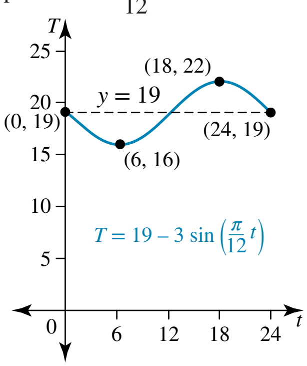
$\therefore \frac{\pi}{12}t = \frac{3\pi}{2}$

$\therefore t = 18$

temperature reaches its maximum of 22° at 6 pm.

c Since the amplitude is 3 and the equilibrium occurs at $T = 19$. The range of temperature is given by 19 ± 3 degrees. Therefore the temperature varied over the interval 16° to 22°.

d period $2\pi \div \frac{\pi}{12} = 24$ hours



e For the temperature to be below k for 3 hours, the interval must lie between $t = 6 - \frac{3}{2}$ and $t = 6 + \frac{3}{2}$, that is, $t = 4.5$ to $t = 7.5$.

When $t = 4.5$,

$T = 19 - 3\sin\left(\frac{\pi}{12} \times \frac{9}{2}\right)$

$= 19 - 3\sin\left(\frac{3\pi}{8}\right)$

$\simeq 16.2$

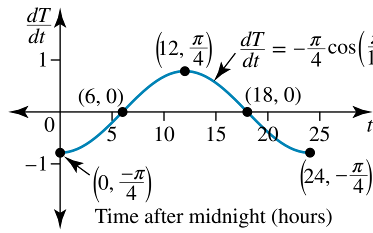
Therefore, $k = 16.2$.

f Rate of change $= \frac{dT}{dt}$

$$\frac{dT}{dt} = -3\cos\left(\frac{\pi}{12}t\right) \times \frac{\pi}{12}$$

$$\frac{dT}{dt} = -\frac{\pi}{4}\cos\left(\frac{\pi}{12}t\right)$$

$y = \frac{dT}{dt}$, using technology, gives the following curve.



From graph, the greatest rate of change of Temperature occurs at $t = 12$ which is 12 hours after midnight, or at 12 noon.

The greatest rate of change of Temperature is $\frac{\pi}{4}$ degrees/hour, or 0.785398 degrees/hour.

Therefore, the temperature rising the fastest at a rate of 0.785 degrees/hour at midday.

19 $h = 4\sin\left(\frac{\pi(t-2)}{6}\right)$

At 1 am, $t = 1$

a $\therefore h = 4\sin\left(\frac{\pi(-1)}{6}\right)$

$= -4\sin\left(\frac{\pi}{6}\right)$

$= -4 \times \frac{1}{2}$

$\therefore h = -2$

The tide is 2 metres below mean sea level at 1 am.

b Since the mean position is $h = 0$ and the amplitude is 4, the high tide level is 4 metres above mean sea level.

High tide occurs when $\sin\left(\frac{\pi(t-2)}{6}\right) = 1$

$\therefore \frac{\pi(t-2)}{6} = \frac{\pi}{2}$

$\therefore \frac{t-2}{6} = \frac{1}{2}$

$\therefore t - 2 = 3$

$\therefore t = 5$

High tide first occurs 5 hours after midnight, that is, at 5 am.

c There is half a period between high tide and the following low tide.

Period, in hours,

$= 2\pi \div \frac{\pi}{6}$

$= 2\pi \times \frac{6}{\pi}$

$= 12$

Therefore there is an interval of 6 hours between high tide and the following low tide.

d $h = 4\sin\left(\frac{\pi(t-2)}{6}\right)$

Period 12, amplitude 4, horizontal translation 2 to the right.

Domain [0, 12], range [−4, 4]

Endpoints: Let $t = 0$,

$\therefore h = 4\sin\left(\dfrac{\pi(-2)}{6}\right)$

$\therefore h = 4\sin\left(-\dfrac{\pi}{3}\right)$

$= -4\sin\left(\dfrac{\pi}{3}\right)$

$= -4 \times \dfrac{\sqrt{3}}{2}$

$\therefore h = -2\sqrt{3}$

$(0, -2\sqrt{3})$

Let $t = 12$,

$\therefore h = 4\sin\left(\dfrac{\pi(10)}{6}\right)$

$\therefore h = 4\sin\left(\dfrac{5\pi}{3}\right)$

$= -4\sin\left(\dfrac{\pi}{3}\right)$

$\therefore h = -2\sqrt{3}$

$\left(12, -2\sqrt{3}\right)$

t-intercepts: Let $h = 0$

$\therefore 4\sin\left(\dfrac{\pi(t-2)}{6}\right) = 0$

$\therefore \sin\left(\dfrac{\pi(t-2)}{6}\right) = 0$

$\therefore \dfrac{\pi(t-2)}{6} = 0, \pi, 2\pi$

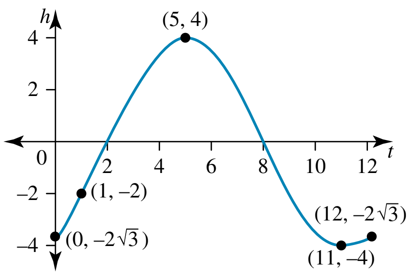
$\therefore \dfrac{t-2}{6} = 0, 1, 2$

$\therefore t - 2 = 0, 6, 12$

$\therefore t = 2, 8$ for $t \in [0, 12]$

As high tide is at $(5, 4)$, six hours later the minimum point is $(11, -4)$.

The point $(1, -2)$ is also known to lie on the graph.



e At 2 pm, $t = 14$.

$\therefore h = 4\sin\left(\dfrac{\pi(12)}{6}\right)$

$= 4\sin(2\pi)$

$= 0$

The tide is predicted to be at mean sea level.

f At 11:30 am, $t = 11.5$

$\therefore h = 4\sin\left(\dfrac{\pi(9.5)}{6}\right)$

$\simeq -3.86$

At low tide, $h = -4$.

Therefore the tide at 11:30 am is 0.14 metres higher than low tide.

20 $T = 19 + 6\sin\left(\dfrac{\pi t}{6}\right)$ with t the time in hours since 10 am.

a **i** As for any sine function, $-1 \le \sin\left(\dfrac{\pi t}{6}\right) \le 1$.

$\therefore T_{\max} = 19 + 6 \times 1$

$= 25$

The maximum temperature is 25°.

The maximum temperature occurs when $\sin\left(\dfrac{\pi t}{6}\right) = 1$

$\therefore \dfrac{\pi t}{6} = \dfrac{\pi}{2}$

$\therefore t = 3$ after 10 am

The maximum temperature occurs at 1 pm.

ii The minimum temperature occurs when $\sin\left(\dfrac{\pi t}{6}\right) = -1$.

$\therefore \dfrac{\pi t}{6} = \dfrac{3\pi}{2}$

$\therefore t = 9$ after 10 am

$T_{\min} = 19 + 6 \times (-1)$

$= 13°$

The minimum temperature of 13° occurs at 7 pm.

b **i** At 11:30 am, $t = 1.5$

$\therefore T = 19 + 6\sin\left(\dfrac{1.5\pi}{6}\right)$

$\therefore T = 19 + 6\sin\left(\dfrac{\pi}{4}\right)$

$= 19 + 6 \times \dfrac{\sqrt{2}}{2}$

$= 19 + 3\sqrt{2}$

$\therefore T \simeq 23.2$

The temperature at 11:30 am is 23.2°.

ii At 7:30 pm, $t = 9.5$

$\therefore T = 19 + 6\sin\left(\dfrac{9.5\pi}{6}\right)$

$\therefore T = 19 + 6\sin\left(\dfrac{19\pi}{12}\right)$

$\therefore T \simeq 13.2$

The temperature at 7:30 pm is 13.2°.

c $T = 19 + 6\sin\left(\dfrac{\pi t}{6}\right), t \in [0, 9.5]$.

Amplitude 6, equilibrium $T = 19$.

Period is $2\pi \div \dfrac{\pi}{6} = 12$, so for the domain specified the graph will not cover a full cycle.

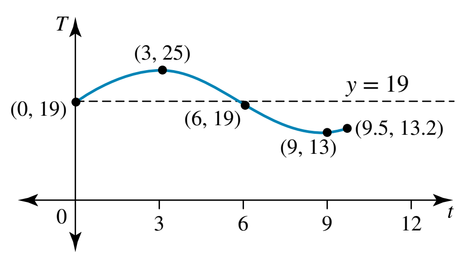
Right endpoint is $(9.5, 13.2)$, maximum point $(3, 25)$, minimum point $(9, 13)$.

Left endpoint: Let $t = 0$

$\therefore T = 19 + 6\sin(0)$

$\therefore T = 19$

$(0, 19)$



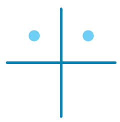
d Let $T = 24$

$$\therefore 24 = 19 + 6\sin\left(\frac{\pi t}{6}\right)$$

$$\therefore 5 = 6\sin\left(\frac{\pi t}{6}\right)$$

$$\therefore \sin\left(\frac{\pi t}{6}\right) = \frac{5}{6}$$

Quadrants 1 and 2, base $\sin^{-1}\left(\frac{5}{6}\right) \simeq 0.99$



$$\therefore \frac{\pi t}{6} = 0.99, \pi - 0.99$$

$$\therefore t = \frac{6}{\pi} \times 0.99, \frac{6}{\pi} \times (\pi - 0.99)$$

$$\therefore t = 1.88, 4.12$$

The air conditioner is switched on at $t = 1.88$ and switched off 2.24 hours later at $t = 4.12$.

e From the graph in part **c**, the coldest two hour period is between $t = 7.5$ and $t = 9.5$.

When $t = 7.5$,

$$\therefore T = 19 + 6\sin\left(\frac{7.5\pi}{6}\right)$$

$$\therefore T = 19 + 6\sin\left(\frac{15\pi}{12}\right)$$

$$\therefore T = 19 + 6\sin\left(\frac{5\pi}{4}\right)$$

$$\therefore T = 19 - 6\sin\left(\frac{\pi}{4}\right)$$

$$\therefore T = 19 - 6 \times \frac{\sqrt{2}}{2}$$

$$\therefore T = 19 - 3\sqrt{2}$$

The heating is switched on at 5:30 pm when the temperature is $(19 - 3\sqrt{2})°$ or approximately 14.8°.