Chapter 7 — Indices

Exercise 7.2 — Index laws

1 a $x^2 \times x^5 \times x^3 = x^{2+5+3}$
$= x^{10}$

b $m^3 \times m^2 \times p \times p^4 = m^3 \times m^2 \times p^1 \times p^4$
$= m^5 p^5$

c $4y^3 \times 2y \times y^7 = 4 \times 2 \times y^3 \times y^1 \times y^7$
$= 8y^{11}$

2 a $5^2 \times 5^7 \times (5^3)^3 = 5^9 \times 5^9$
$= 5^{18}$

b $(xy)^3 \times x^4 y^5 = x^3 y^3 \times x^4 y^5$
$= x^3 \times x^4 \times y^3 \times y^5$
$= x^7 y^8$

c $(2x^4)^2 \times (4x^2)^5 = 4x^8 \times 4^5 x^{10}$
$= 4 \times 4^5 \times x^8 \times x^{10}$
$= 2^{12} x^{18}$

d $3m^2 p^5 \times \left(mp^2\right)^3 \times 2m^4 p^6$
$= 3m^2 p^5 \times m^3 p^6 \times 2m^4 p^6$
$= 3 \times 2 \times m^2 \times m^3 \times m^4 \times p^5 \times p^6$
$= 6m^9 p^{17}$

e $5x^2 y^3 \times \left(5xy^2\right)^4 \times \left(5x^2 y\right)^2$
$= 5x^2 y^3 \times 5^4 x^4 y^8 \times 5^2 x^4 y^2$
$= 5 \times 5^4 \times 5^2 \times x^2 \times x^4 \times x^4 \times y^3 \times y^8 \times y^2$
$= 5^7 x^{10} y^{13}$

3 a $a^7 b^8 \div a^2 b^5 = a^{7-2} b^{8-5}$
$= a^5 b^3$

b $2a^{12} b^9 \div (2a)^3 b^4 = 2^1 a^{12} b^9 \div 2^3 a^3 b^4$
$= 2^{1-3} a^{12-3} b^{9-4}$
$= 2^{-2} a^9 b^5$
$= \frac{a^9 b^5}{4}$

c $\left(3x^5\right) y^{11} \div 6x^2 y^2 = 3x^5 y^{11} \div 6x^2 y^2$
$= \frac{3x^5 y^{11}}{6x^2 y^2}$
$= \frac{x^3 y^9}{2}$

d $p^{13} q^{10} \div \left(pq^4\right)^2 = \frac{p^{13} q^{10}}{p^2 q^8}$
$= p^{11} q^2$

e $\left(4mn^4\right)^2 \div 14n^3 = \frac{16m^2 n^8}{14n^3}$
$= \frac{8m^2 n^5}{7}$

4 a $\frac{a^3 b^4}{ab^2} = a^{3-1} b^{4-2}$
$= a^2 b^2$

b $25r^{15} s^{10} t^4 \div r^5 \left(s^5\right)^2 (5t)^3$
$= \frac{5^2 r^{15} s^{10} t^4}{r^5 s^{10} 5^3 t^3}$
$= 5^{2-3} r^{15-5} s^{10-10} t^{4-3}$
$= 5^{-1} r^{10} s^0 t^1$
$= \frac{r^{10} t}{5}$

c $\frac{15a^6 b^7}{3a^3 b^4} = 5a^{6-3} b^{7-4}$
$= 5a^3 b^3$

d $\frac{24x^4 y^7}{20x^2 y^3} = \frac{6x^{4-2} y^{7-3}}{5}$
$= \frac{6x^2 y^4}{5}$

5 a $\frac{6p^8 m^4 \times 2p^7 m^6}{9p^5 m^2}$
$= \frac{6 \times 2 \times p^8 \times p^7 \times m^4 \times m^6}{9p^5 m^2}$
$= \frac{4p^{8+7-5} m^{4+6-2}}{3}$
$= \frac{4p^{10} m^8}{3}$

b $\frac{(3x)^2 y^2 \times 5x^6 y^3}{10x^7 y}$
$= \frac{3^2 x^2 y^2 \times 5x^6 y^3}{10x^7 y}$
$= \frac{9 \times 5 \times x^2 \times x^6 \times y^2 \times y^3}{10x^7 y^1}$
$= \frac{9x^{2+6-7} y^{2+3-1}}{2}$
$= \frac{9xy^4}{2}$

c $\frac{14u^{11} v^9 \times (3u^2)^3 v}{21u^6 v^5}$
$= \frac{14u^{11} v^9 \times 3^3 u^6 v}{21u^6 v^5}$
$= \frac{14 \times 27 \times u^{11} \times u^6 \times v^9 \times v^1}{21u^6 v^5}$
$= 18u^{11+6-6} v^{9+1-5}$
$= 18u^{11} v^5$

d $\dfrac{(5e^3)^2 f^4 \times 8e^4 f^3}{20e f^5}$

$= \dfrac{5^2 e^6 f^4 \times 8e^4 f^3}{20e f^5}$

$= \dfrac{25 \times 8 \times e^6 \times e^4 \times f^4 \times f^3}{20e^1 f^5}$

$= 10e^{6+4-1} f^{4+3-5}$

$= 10e^9 f^2$

e $\dfrac{6w^2t^7 \times 9w^4t^{12}}{(3w)^5 t^{13}}$

$= \dfrac{6w^2t^7 \times 9w^4t^{12}}{3^5 w^5 t^{13}}$

$= \dfrac{6 \times 9 \times w^2 \times w^4 \times t^7 \times t^{12}}{243 w^5 t^{13}}$

$= \dfrac{2w^{2+4-5} t^{7+12-13}}{9}$

$= \dfrac{2wt^6}{9}$

6 a $\dfrac{(2x)^4 y \times (3x^7 y)^2}{18x^5 (2y)^3}$

$= \dfrac{2^4 x^4 y \times 3^2 x^{14} y^2}{18x^5 2^3 y^3}$

$= \dfrac{16 \times 9x^4 \times x^{14} \times y^1 \times y^2}{8 \times 18x^5 y^3}$

$= x^{4+14-5} y^{1+2-3}$

$= x^{13} y^0$

$= x^{13}$

b $\dfrac{(-3x^3y^2)^3}{2x^3y^6} \times \dfrac{6x^7y^5}{(x^2y)^2}$

$= \dfrac{(-3)^3 \times x^{3\times3} \times y^{2\times3} \times 6x^7y^5}{2x^3y^6 \times x^{2\times2} \times y^{1\times2}}$

$= \dfrac{-27 \times 6 \times x^9 \times x^7 \times y^6 \times y^5}{2x^3 \times x^4 \times y^6 \times y^2}$

$= -81x^{9+7-3-4} y^{6+5-6-2}$

$= -81x^9 y^3$

c $\dfrac{(-3mp)^2 \times 4m^4 p}{12(mp)^2}$

$= \dfrac{(-3)^2 m^2 p^2 \times 4m^2 p}{12m^2 p^2}$

$= \dfrac{9 \times 4 \times m^2 \times m^4 \times p^2 \times p^1}{12m^2 p^2}$

$= 3 \times m^{2+4-2} \times p^{2+1-2}$

$= 3m^4 p$

d $\dfrac{m^3 p^4 \times (mp^3)^2}{(-mp^2)^4}$

$= \dfrac{m^3 p^4 \times m^2 p^6}{m^4 p^8}$

$= \dfrac{m^3 \times m^2 \times p^4 \times p^6}{m^4 p^8}$

$= m^{3+2-4} p^{4+6-8}$

$= mp^2$

e $\dfrac{4(u^7v^6)^3}{(-2u^3v^2)^2 \times u^4 (3v^5)^2}$

$= \dfrac{4u^{7\times3} v^{6\times3}}{(-2)^2 \times u^{3\times2} \times v^{2\times2} \times u^4 \times 3^2 \times v^{5\times2}}$

$= \dfrac{4u^{21} v^{18}}{4 \times 9 \times u^6 \times u^4 \times v^4 \times v^{10}}$

$= \dfrac{u^{21-6-4} v^{18-4-10}}{9}$

$= \dfrac{u^{11} v^4}{9}$

7 a $\dfrac{15a^8b^3}{9a^4b^5} \div \left(\dfrac{2a^3b}{3ab^2}\right)^2$

$= \dfrac{15a^8b^3}{9a^4b^5} \times \left(\dfrac{3ab^2}{2a^3b}\right)^2$

$= \dfrac{15a^8b^3}{9a^4b^5} \times \dfrac{3^2a^2b^4}{2^2a^6b^2}$

$= \dfrac{15 \times 9 \times a^{8+2-4-6} b^{3+4-5-2}}{9 \times 4}$

$= \dfrac{15a^0b^0}{4}$

$= \dfrac{15}{4}$

b $\dfrac{5k^{12}d}{(2k^3)^2} \div \dfrac{6kd^4}{25(k^2d^3)^3}$

$= \dfrac{5k^{12}d}{(2k^3)^2} \times \dfrac{25(k^2d^3)^3}{6kd^4}$

$= \dfrac{5k^{12}d^1}{2^2k^6} \times \dfrac{25k^6d^9}{6k^1d^4}$

$= \dfrac{5 \times 25k^{12+6-6-1} d^{1+9-4}}{24}$

$= \dfrac{125k^{11}d^6}{24}$

c $\dfrac{4g^4(2p^{11})^2}{g^3p^7} \div \dfrac{8g^4p}{(2gp)^3}$

$= \dfrac{4g^4(2p^{11})^2}{g^3p^7} \times \dfrac{(2gp)^3}{8g^4p}$

$= \dfrac{4g^4 2^2 p^{22}}{g^3p^7} \times \dfrac{2^3g^3p^3}{8g^4p^1}$

$= \dfrac{4 \times 4 \times 8g^{4+3-3-4} p^{22+3-7-1}}{8}$

$= 16g^0p^{17}$

$= 16p^{17}$

d $\left(\dfrac{3jn^2}{n^5}\right)^3 \div \dfrac{(4j^2n)^2}{n^{13}(2j)^4}$

$= \left(\dfrac{3jn^2}{n^5}\right)^3 \times \dfrac{n^{13}(2j)^4}{(4j^2n)^2}$

$= \dfrac{3^3j^3n^6}{n^{15}} \times \dfrac{n^{13}2^4j^4}{4^2j^4n^2}$

$= \dfrac{27 \times 16j^{3+4-4} n^{6+13-15-2}}{16}$

$= 27j^3n^2$

e $\dfrac{x^4y^7}{x^3y^2} \div \dfrac{x^3y^2}{x^5y}$

$= \dfrac{x^4y^7}{x^3y^2} \times \dfrac{x^5y^1}{x^3y^2}$

$= x^{4+5-3-3} y^{7+1-2-2}$

$= x^3y^4$

f $\dfrac{6x^3y^8}{(x^2y^3)^3} \div \dfrac{(2xy^3)^2}{8x^5y^7}$

$= \dfrac{6x^3y^8}{(x^2y^3)^3} \times \dfrac{8x^5y^7}{(2xy^3)^2}$

$= \dfrac{6x^3y^8}{x^6y^9} \times \dfrac{8x^5y^7}{2^2x^2y^6}$

$= \dfrac{6 \times 8x^{3+5-6-2} y^{8+7-9-6}}{4}$

$= 12x^0y^0$

$= 12$

8 a $\dfrac{3p^3m^4}{p^1m^2} = 3p^{3-1}m^{4-2}$

$= 3p^2m^2$

b $\dfrac{6x^6y^5}{x^5y^3} \times \dfrac{x^4}{(2y)^2}$

$= \dfrac{6x^6y^5}{x^5y^3} \times \dfrac{x^4}{2^2y^2}$

$= \dfrac{6x^{6+4-5} y^{5-3-2}}{4}$

$= \dfrac{3x^5y^0}{2}$

$= \dfrac{3x^5}{2}$

c $\dfrac{3ab^3}{-ab} \div \left(\dfrac{a^2b}{a^5}\right)^2$

$= \dfrac{3ab^3}{-ab} \times \left(\dfrac{a^5}{a^2b}\right)^2$

$= \dfrac{3a^1b^3}{-a^1b^1} \times \dfrac{a^{10}}{a^4b^2}$

$= \dfrac{3a^{1+10-1-4} b^{3-1-2}}{-1}$

$= -3a^6b^0$

$= -3a^6$

9 a $\dfrac{x^{n+1} \times y^5 \times z^{4-n}}{x^{n-2} \times y^{4-n} \times z^{3-n}}$

$= x^{n+1-(n-2)} y^{5-(4-n)} z^{4-n-(3-n)}$

$= x^{n+1-n+2} y^{5-4+n} z^{4-n-3+n}$

$= x^3 y^{1+n} z$

b $\dfrac{(x^n y^{m+3})}{x^{n+2} y^{3-m}} \times \dfrac{x^2y}{x^{n-5} \times y^{5-3m}}$

$= \dfrac{x^{2n} y^{2m+6}}{x^{n+2} y^{3-m}} \times \dfrac{x^2y}{x^{n-5} y^{5-3m}}$

$= x^{2n+2-(n+2)-(n-5)} y^{2m+6+1-(3-m)-(5-3m)}$

$= x^{2n+2-n-2-n+5} y^{2m+6+1-3+m-5+3m}$

$= x^5 y^{6m-1}$

10 a $2^4 \times 4^2 \times 8$
$= 2^4 \times (2 \times 2)^2 \times (2 \times 2 \times 2)$
$= 2^4 \times \left(2^2\right)^2 \times 2^3$
$= 2^4 \times 2^4 \times 2^3$
$= 2^{11}$

b $3^7 \times 9^2 \times 27^3 \times 81$
$= 3^7 \times (3 \times 3)^2 \times (3 \times 3 \times 3)^3$
$\times (3 \times 3 \times 3 \times 3)$
$= 3^7 \times \left(3^2\right)^2 \times \left(3^3\right)^3 \times 3^4$
$= 3^7 \times 3^4 \times 3^9 \times 3^4$
$= 3^{24}$

c $5^3 \times 15^2 \times 3^2 = 5^3 \times (5 \times 3)^2 \times 3^2$
$= 5^3 \times 5^2 \times 3^2 \times 3^2$
$= 5^5 \times 3^4$

11 a $20^5 \times 8^4 \times 125$
$= (2 \times 2 \times 5)^5 \times (2 \times 2 \times 5)^4$
$\times (5 \times 5 \times 5)$
$= \left(2^2 \times 5\right)^5 \times \left(2^3\right)^4 \times 5^3$
$= 2^{10} \times 5^5 \times 2^{12} \times 5^3$
$= 2^{22} \times 5^8$

b $\dfrac{3^4 \times 27^2}{6^4 \times 3^5} = \dfrac{3^4 \times (3 \times 3 \times 3)^2}{(2 \times 3)^4 \times 3^5}$
$= \dfrac{3^4 \times (3^3)^2}{2^4 \times 3^4 \times 3^5}$
$= \dfrac{3^4 \times 3^6}{2^4 \times 3^4 \times 3^5}$
$= \dfrac{3^{4+6-4-5}}{2^4}$
$= \dfrac{3}{2^4}$

c $\dfrac{8 \times 5^2}{2^3 \times 10} = \dfrac{(2 \times 2 \times 2) \times 5^2}{2^3 \times (5 \times 2)}$
$= \dfrac{2^3 \times 5^2}{2^3 \times 5^1 \times 2^1}$
$= 2^{3-3-1} \times 5^{2-1}$
$= 2^{-1} \times 5$
$= \dfrac{5}{2}$

12 a $\dfrac{(625)^4}{(5^3)^5} = \dfrac{(5 \times 5 \times 5 \times 5)^4}{(5^3)^5}$
$= \dfrac{(5^4)^4}{(5^3)^5}$
$= \dfrac{5^{16}}{5^{15}}$
$= 5$

b $\dfrac{(25)^4}{(125)^3} = \dfrac{(5 \times 5)^4}{(5 \times 5 \times 5)^3}$
$= \dfrac{(5^2)^4}{(5^3)^3}$
$= \dfrac{5^8}{5^9}$
$= 5^{-1}$
$= \dfrac{1}{5}$

c $\dfrac{4^{11} \div 8^2}{16^3} = \dfrac{(2 \times 2)^{11} \div (2 \times 2 \times 2)^2}{(2 \times 2 \times 2 \times 2)^3}$
$= \dfrac{(2^2)^{11} \div (2^3)^2}{(2^4)^3}$
$= \dfrac{2^{22} \div 2^6}{2^{12}}$
$= 2^{22-6-12}$
$= 2^4$
$= 16$

d $\dfrac{27^2 \times 81}{9^3 \times 3^5}$
$= \dfrac{(3 \times 3 \times 3)^2 \times (3 \times 3 \times 3 \times 3)}{(3 \times 3)^3 \times 3^5}$
$= \dfrac{(3^3)^2 \times 3^4}{(3^2)^3 \times 3^5}$
$= \dfrac{3^6 \times 3^4}{3^6 \times 3^5}$
$= 3^{6+4-6-5}$
$= 3^{-1}$
$= \dfrac{1}{3}$

13 a $\dfrac{2^n \times 9^{2n+1}}{6^{n-2}} = \dfrac{2^n \times (3 \times 3)^{2n+1}}{(2 \times 3)^{n-2}}$
$= \dfrac{2^n \times (3^2)^{2n+1}}{(2 \times 3)^{n-2}}$
$= \dfrac{2^n \times 3^{4n+2}}{2^{n-2} \times 3^{n-2}}$
$= 2^{n-(n-2)} \times 3^{4n+2-(n-2)}$
$= 2^{n-n+2} \times 3^{4n+2-n+2}$
$= 2^2 \times 3^{3n+4}$

b $\dfrac{25^{3n} \times 5^{n-3}}{5^{4n+3}}$
$= \dfrac{(5 \times 5)^{3n} \times 5^{n-3}}{5^{4n+3}}$
$= \dfrac{(5^2)^{3n} \times 5^{n-3}}{5^{4n+3}}$
$= \dfrac{5^{6n} \times 5^{n-3}}{5^{4n+3}}$
$= 5^{6n+n-3-(4n+3)}$
$= 5^{6n+n-3-4n-3}$
$= 5^{3n-6}$

c $\dfrac{12^{x-2} \times 4^x}{6^{x-2}}$
$= \dfrac{(2 \times 2 \times 3)^{x-2} \times (2 \times 2)^x}{(2 \times 3)^{x-2}}$
$= \dfrac{(2^2 \times 3)^{x-2} \times (2^2)^x}{(2 \times 3)^{x-2}}$
$= \dfrac{2^{2x-4} \times 3^{x-2} \times 2^{2x}}{2^{x-2} \times 3^{x-2}}$
$= 2^{2x-4+2x-(x-2)} \times 3^{x-2-(x-2)}$
$= 2^{2x-4+2x-x+2} \times 3^{x-2-x+2}$
$= 2^{3x-2} \times 3^0$
$= 2^{3x-2}$

d $\dfrac{12^{n-3} \times 27^{1-n}}{9^{2n} \times 8^{n-1} \times 16^n}$
$= \dfrac{(2^2 \times 3)^{n-3} \times (3^3)^{1-n}}{(3^2)^{2n} \times (2^3)^{n-1} \times (2^4)^n}$
$= \dfrac{2^{2n-6} \times 3^{n-3} \times 3^{3-3n}}{3^{4n} \times 2^{3n-3} \times 2^{4n}}$
$= 2^{2n-6-(3n-3)-4n} \times 3^{n-3+3-3n-4n}$
$= 2^{2n-6-3n+3-4n} \times 3^{n-3+3-3n-4n}$
$= 2^{-5n-3} \times 3^{-6n}$

e $\dfrac{4^n \times 7^{n-3} \times 49^{3n+1}}{14^{n+2}}$
$= \dfrac{(2 \times 2)^n \times 7^{n-3} \times (7 \times 7)^{3n+1}}{(2 \times 7)^{n+2}}$
$= \dfrac{(2^2)^n \times 7^{n-3} \times (7^2)^{3n+1}}{(2 \times 7)^{n+2}}$
$= \dfrac{2^{2n} \times 7^{n-3} \times 7^{6n+2}}{2^{n+2} \times 7^{n+2}}$
$= 2^{2n-(n+2)} \times 7^{n-3+6n+2-(n+2)}$
$= 2^{2n-n-2} \times 7^{n-3+6n+2-n-2}$
$= 2^{n-2} \times 7^{6n-3}$

14 a $\dfrac{35^2 \times 5^5 \times 7^6}{25^4 \times 49^3}$
$= \dfrac{(5 \times 7)^2 \times 5^5 \times 7^6}{(5 \times 5)^4 \times (7 \times 7)^3}$
$= \dfrac{5^2 \times 7^2 \times 5^5 \times 7^6}{(5^2)^4 \times (7^2)^3}$
$= \dfrac{5^2 \times 7^2 \times 5^5 \times 7^6}{5^8 \times 7^6}$
$= 5^{2+5-8} \times 7^{2+6-6}$
$= 5^{-1} \times 7^2$
$= \dfrac{7^2}{5}$
$= \dfrac{49}{5}$

b $\dfrac{3^{5n-4} \times 16^n \times 9^3}{4^{n+1} \times 18^{1-n} \times 6^{3-2n}}$
$= \dfrac{3^{5n-4} \times (2^4)^n \times (3^2)^3}{(2^2)^{n+1} \times (2 \times 3^2)^{1-n} \times (2 \times 3)^{3-2n}}$
$= \dfrac{3^{5n-4} \times 2^{4n} \times 3^6}{2^{2n+2} \times 2^{1-n} \times 3^{2-2n} \times 2^{3-2n} \times 3^{3-2n}}$
$= 3^{5n-4+6-(2-2n)-(3-2n)}$
$\times 2^{4n-(2n+2)-(1-n)-(3-2n)}$
$= 3^{5n-4+6-2+2n-3+2n} \times 2^{4n-2n-2-1+n-3+2n}$
$= 3^{9n-3} \times 2^{5n-6}$

c $\dfrac{3^n + 3^{n+1}}{3^n + 3^{n-1}} = \dfrac{3^n(1+3)}{3^n(1+3^{-1})}$
$= \dfrac{1+3}{1+3^{-1}}$
$= \dfrac{4}{1\frac{1}{3}}$
$= 3$

d $\dfrac{5^n - 5^{n+1}}{5^{n+1} + 5^n} = \dfrac{5^n(1+5)}{5^n(5+1)}$
$= \dfrac{-4}{6}$
$= -\dfrac{2}{3}$

15 $\dfrac{36^{2n} \times 6^{n+3}}{216^{n-2}} = \dfrac{(6^2)^{2n} \times 6^{n+3}}{(6^3)^{n-2}}$
$= \dfrac{6^{4n} \times 6^{n+3}}{6^{3n-6}}$
$= 6^{4n+n+3-(3n-6)}$
$= 6^{4n+n+3-3n+6}$
$= 6^{2n+9}$

16 a $\dfrac{4^5}{2^7} = \dfrac{(2 \times 2)^5}{2^7}$
$= \dfrac{(2^2)^5}{2^7}$
$= \dfrac{2^{10}}{2^7}$
$= 2^3$
$= 8$

b $9^4 \times 3^5 \times 27 = \dfrac{9^4 \times 3^5}{27}$
$= \dfrac{(3 \times 3)^4 \times 3^5}{(3 \times 3 \times 3)}$
$= \dfrac{(3^2)^4 \times 3^5}{3^3}$
$= \dfrac{3^8 \times 3^5}{3^3}$
$= 3^{10}$
$= 59\,049$

c $\dfrac{(16^2)^3}{(2^5)^4} = \dfrac{16^6}{2^{20}}$
$= \dfrac{(2 \times 2 \times 2 \times 2)^6}{2^{20}}$
$= \dfrac{(2^4)^6}{2^{20}}$
$= \dfrac{2^{24}}{2^{20}}$
$= 2^4$
$= 16$

d $\dfrac{27^2}{(3^2)^3} = \dfrac{(3 \times 3 \times 3)^2}{(3^2)^3}$
$= \dfrac{(3^3)^2}{(3^2)^3}$
$= \dfrac{3^6}{3^6}$
$= 3^0$
$= 1$

Exercise 7.3 — Negative and rational indices

1 a $6^{-2} = \dfrac{1}{6^2}$
$= \dfrac{1}{36}$

The answer is D

b $\left(\dfrac{27}{8}\right)^{-\frac{1}{3}} = \left(\dfrac{3^3}{2^3}\right)^{-\frac{1}{3}}$
$= \dfrac{3^{-1}}{2^{-1}}$
$= \dfrac{2}{3}$

The answer is C

c $\sqrt[3]{25} \times \sqrt{125} = 25^{\frac{1}{3}} \times 125^{\frac{1}{2}}$
$= (5^2)^{\frac{1}{3}} \times (5^3)^{\frac{1}{2}}$
$= 5^{\frac{2}{3}} \times 5^{\frac{3}{2}}$
$= 5^{\frac{2}{3}+\frac{3}{2}}$
$= 5^{\frac{13}{6}}$

The answer is D

2 a $6^{-3} = \dfrac{1}{6^3}$

b $5^{-4} = \dfrac{1}{5^4}$

c $\left(\dfrac{3}{5}\right)^{-2} = \dfrac{3^{-2}}{5^{-2}}$
$= \dfrac{5^2}{3^2}$

d $\left(\dfrac{7}{4}\right)^{-5} = \dfrac{7^{-5}}{4^{-5}}$
$= \dfrac{4^5}{7^5}$

3 a $\left(\dfrac{1}{9}\right)^{-2} = \dfrac{1^{-2}}{9^{-2}}$
$= \dfrac{9^2}{1^2}$
$= 9^2$

b $(64^{-2})^3 = 64^{-6}$
$= \dfrac{1}{64^6}$

c $(-3)^{-1} = \dfrac{1}{-3}$

d $\left(\dfrac{3^4}{2^3}\right)^{-4} = \dfrac{3^{-16}}{2^{-12}}$
$= \dfrac{2^{12}}{3^{16}}$

4 a $\dfrac{(-2)^3 \times 2^{-4}}{2^{-3}} = \dfrac{(-1 \times 2)^3 \times 2^{-4}}{2^{-3}}$
$= \dfrac{(-1)^3 \times 2^3 \times 2^{-4}}{2^{-3}}$
$= -1 \times 2^{3-4--3}$
$= -1 \times 2^{3-4+3}$
$= -1 \times 2^2$
$= -2^2$

b $\dfrac{(x^{-2})^3 \times (y^4)^{-2}}{x^{-5} \times (y^{-2})^3} = \dfrac{x^{-6} \times y^{-8}}{x^{-5} \times y^{-6}}$
$= x^{-6--5} \times y^{-8--6}$
$= x^{-6+5} \times y^{-8+6}$
$= x^{-1}y^{-2}$
$= \dfrac{1}{xy^2}$

c $\dfrac{(-m)^2 \times m^{-3}}{(p^{-2})^{-1} \times p^{-4}} = \dfrac{m^2 \times m^{-3}}{p^2 \times p^{-4}}$
$= \dfrac{m^{-1}}{p^{-2}}$
$= \dfrac{p^2}{m}$

5 a $\dfrac{x^5}{x^{-3}} \div \dfrac{(x^4)^{-2}}{(x^2)^{-3}} = \dfrac{x^5}{x^{-3}} \times \dfrac{(x^2)^{-3}}{(x^4)^{-2}}$
$= \dfrac{x^5}{x^{-3}} \times \dfrac{x^{-6}}{x^{-8}}$
$= \dfrac{x^{-1}}{x^{-11}}$
$= x^{-1--11}$
$= x^{10}$

b $\dfrac{(3^{-2})^2 \times (2^{-5})^{-1}}{(2^4)^{-2} \times (3^4)^{-3}} = \dfrac{3^{-4} \times 2^5}{2^{-8} \times 3^{-12}}$
$= 3^{-4--12} \times 2^{5--8}$
$= 3^8 \times 2^{13}$

c $\dfrac{x^3y^{-2} \times (xy^2)^{-3}}{(2x^3)^2 \times (y^{-3})^2} = \dfrac{x^3y^{-2} \times x^{-3}y^{-6}}{2^2x^6 \times y^{-6}}$
$= \dfrac{x^0y^{-8}}{4x^6y^{-6}}$
$= \dfrac{x^{0-6}y^{-8--6}}{4}$
$= \dfrac{x^{-6}y^{-2}}{4}$
$= \dfrac{1}{4x^6y^2}$

6 a $9^{\frac{1}{2}} = (3^2)^{\frac{1}{2}}$
$= 3$

b $27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}}$
$= 3$

c $625^{\frac{1}{4}} = (5^4)^{\frac{1}{4}}$
$= 5$

d $256^{\frac{1}{8}} = (2^8)^{\frac{1}{8}}$
$= 2$

e $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}}$
$= 2^2$
$= 4$

f $81^{\frac{3}{4}} = (3^4)^{\frac{3}{4}}$
$= 3^3$
$= 27$

g $125^{\frac{4}{3}} = (5^3)^{\frac{4}{3}}$
$= 5^4$
$= 625$

h $\left(\frac{8}{125}\right)^{\frac{1}{3}} = \left(\frac{2^3}{5^3}\right)^{\frac{1}{3}}$
$= \frac{2}{5}$

7 a $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \left(\frac{2^4}{3^4}\right)^{\frac{1}{4}}$
$= \frac{2}{3}$

b $\left(\frac{25}{16}\right)^{\frac{3}{2}} = \left(\frac{5^2}{4^2}\right)^{\frac{3}{2}}$
$= \frac{5^3}{4^3}$
$= \frac{125}{64}$

c $\left(\frac{27}{64}\right)^{\frac{2}{3}} = \left(\frac{3^3}{4^3}\right)^{\frac{2}{3}}$
$= \frac{3^2}{4^2}$
$= \frac{9}{16}$

d $32^{-\frac{2}{5}} = (2^5)^{-\frac{2}{5}}$
$= 2^{-2}$
$= \frac{1}{2^2}$
$= \frac{1}{4}$

e $81^{-\frac{3}{4}} = (3^4)^{-\frac{3}{4}}$
$= 3^{-3}$
$= \frac{1}{3^3}$
$= \frac{1}{27}$

f $\left(\frac{8}{27}\right)^{-\frac{2}{3}} = \left(\frac{2^3}{3^3}\right)^{-\frac{2}{3}}$
$= \frac{2^{-2}}{3^{-2}}$
$= \frac{3^2}{2^2}$
$= \frac{9}{4}$

g $\left(\frac{16}{121}\right)^{-\frac{1}{2}} = \left(\frac{4^2}{11^2}\right)^{-\frac{1}{2}}$
$= \frac{4^{-1}}{11^{-1}}$
$= \frac{11}{4}$

h $\left(\frac{125}{216}\right)^{-\frac{1}{3}} = \left(\frac{5^3}{6^3}\right)^{-\frac{1}{3}}$
$= \frac{5^{-1}}{6^{-1}}$
$= \frac{6}{5}$

8 a $\sqrt{9} \times \sqrt[3]{81} = 9^{\frac{1}{2}} \times 81^{\frac{1}{3}}$
$= (3^2)^{\frac{1}{2}} \times (3^4)$
$= 3^1 \times 3^{\frac{4}{3}}$
$= 3^{1+\frac{4}{3}}$
$= 3^{\frac{7}{3}}$

b $x^{\frac{2}{3}} \times x^{\frac{1}{6}} = x^{\frac{2}{3}+\frac{1}{6}}$
$= x^{\frac{5}{6}}$

c $x^{-\frac{3}{4}} \times x^{\frac{9}{8}} = x^{-\frac{3}{4}+\frac{9}{8}}$
$= x^{\frac{3}{8}}$

d $x^{\frac{5}{2}} \div (x^{\frac{1}{3}})^4 = x^{\frac{5}{2}} \div x^{\frac{4}{3}}$
$= x^{\frac{5}{2}-\frac{4}{3}}$
$= x^{\frac{7}{6}}$

e $\sqrt[3]{(xy^3)} \div \sqrt{(x^2y)} = (xy^3)^{\frac{1}{3}} \div (x^2y)^{\frac{1}{2}}$
$= x^{\frac{1}{3}}y^1 \div x^1y^{\frac{1}{2}}$
$= x^{\frac{1}{3}-1}y^{1-\frac{1}{2}}$
$= x^{-\frac{2}{3}}y^{\frac{1}{2}}$
$= \frac{y^{\frac{1}{2}}}{x^{\frac{2}{3}}}$

f $\sqrt[5]{32} \times \sqrt[4]{8} = 32^{\frac{1}{5}} \times 8^{\frac{1}{4}}$
$= (2^5)^{\frac{1}{5}} \times (2^3)^{\frac{1}{4}}$
$= 2^1 \times 2^{\frac{3}{4}}$
$= 2^{1+\frac{3}{4}}$
$= 2^{\frac{7}{4}}$

9 a $2^{\frac{5}{4}} \times 4^{-\frac{1}{2}} \times 8^{-\frac{2}{3}}$
$= 2^{\frac{5}{4}} \times \left(2^2\right)^{-\frac{1}{2}} \times \left(2^3\right)^{-\frac{2}{3}}$
$= 2^{\frac{5}{4}} \times 2^{-1} \times 2^{-2}$
$= 2^{\frac{5}{4}-1-2}$
$= 2^{-\frac{7}{4}}$
$= \frac{1}{2^{\frac{7}{4}}}$

b $27^{-\frac{1}{4}} \times 9^{\frac{2}{3}} \times 3^{-\frac{5}{4}}$
$= \left(3^3\right)^{-\frac{1}{4}} \times \left(3^2\right)^{\frac{2}{3}} \times 3^{-\frac{5}{4}}$
$= 3^{-\frac{3}{4}} \times 3^{\frac{4}{3}} \times 3^{-\frac{5}{4}}$
$= 3^{-\frac{3}{4}+\frac{4}{3}-\frac{5}{4}}$
$= 3^{-\frac{8}{12}}$
$= 3^{-\frac{2}{3}} = \frac{1}{3^{\frac{2}{3}}}$

c $\frac{18^{\frac{1}{2}}}{9^{\frac{4}{3}} \times 4^{\frac{3}{4}}} = \frac{(2 \times 3^2)^{\frac{1}{2}}}{(3^2)^{\frac{4}{3}} \times (2^2)^{\frac{3}{2}}}$
$= \frac{2^{\frac{1}{2}} \times 3^1}{3^{\frac{8}{3}} \times 2^{\frac{3}{2}}}$
$= 2^{\frac{1}{2}-\frac{3}{2}} \times 3^{1-\frac{8}{3}}$
$= 2^{-1} \times 3^{-\frac{5}{3}}$
$= \frac{1}{2 \times 3^{\frac{5}{3}}}$

d $(\sqrt[4]{x^3})^{\frac{2}{3}} \times (\sqrt[3]{x^4})^{\frac{3}{8}} = (x^{\frac{3}{4}})^{\frac{2}{3}} \times (x^{\frac{4}{3}})^{\frac{3}{8}}$
$= x^{\frac{1}{2}} \times x^{\frac{1}{2}}$
$= x^{\frac{1}{2}+\frac{1}{2}}$
$= x^1$
$= x$

e $\frac{(64m^6)^{\frac{4}{3}}}{4m^{-2}} = \frac{(4^3m^6)^{\frac{4}{3}}}{4m^{-2}}$
$= \frac{4^4m^8}{4^1m^{-2}}$
$= 4^{(4-1)}m^{8--2}$
$= 4^3m^{10}$
$= 64m^{10}$

f $\frac{\sqrt{x^3}}{\sqrt{x}} = \frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$
$= x^{\frac{3}{2}-\frac{1}{2}}$
$= x^1$
$= x$

10 a $\frac{1}{\sqrt{x^{-4}}} = \frac{1}{x^{-\frac{4}{2}}}$
$= \frac{1}{x^{-2}}$
$= x^2$

b $\frac{(x+1)^2}{\sqrt{x+1}} = \frac{(x+1)^2}{(x+1)^{\frac{1}{2}}}$
$= (x+1)^{2-\frac{1}{2}}$
$= (x+1)^{1\frac{1}{2}}$

c $\sqrt{x} - \frac{1}{\sqrt{x}} = x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{2}}}$
$= \frac{x-1}{x^{\frac{1}{2}}}$

d $\sqrt{x+2} + \frac{x}{\sqrt{x+2}}$
$= (x+2)^{\frac{1}{2}} + \frac{x}{(x+2)^{\frac{1}{2}}}$
$= \frac{x+2+x}{(x+2)^{\frac{1}{2}}}$
$= \frac{2x+2}{(x+2)^{\frac{1}{2}}}$

e $(y-4)\sqrt{y-4}$
$= (y-4)^1 \times (y-4)^{\frac{1}{2}}$
$= (y-4)^{1\frac{1}{2}}$

f $(p+3)(p+3)^{-\frac{2}{5}} = (p+3)^{1-\frac{2}{5}}$
$= (p+3)^{\frac{3}{5}}$

11 a i $\sqrt{a^3b^4} = (a^3b^4)^{\frac{1}{2}}$
$= a^{\frac{3}{2}}b^2$

ii $\sqrt{\frac{a^5}{b^{-4}}} \times \sqrt[3]{a^2b} = \left(\frac{a^5}{b^{-4}}\right)^{\frac{1}{2}} \times (a^2b)^{\frac{1}{3}}$
$= \frac{a^{\frac{5}{2}}}{b^{-2}} \times a^{\frac{2}{3}}b^{\frac{1}{3}}$
$= a^{\frac{5}{2}+\frac{2}{3}}b^{\frac{1}{3}+2}$
$= a^{\frac{19}{6}}b^{\frac{7}{3}}$

b i $a^{\frac{1}{2}} \div b^{\frac{3}{2}} = \sqrt{a} \div \sqrt{b^3}$
$= \sqrt{\frac{a}{b^3}}$

ii $2^{\frac{5}{2}} = \sqrt{2^5}$
$= \sqrt{32}$

iii $3^{-\frac{2}{5}} = \sqrt[5]{3^{-2}}$
$= \sqrt[5]{\frac{1}{3^2}}$
$= \sqrt[5]{\frac{1}{9}}$

12 a $4^{\frac{3}{2}} = \sqrt{4^3}$
$= \sqrt{64}$
$= 8$

b $3^{-1} + 5^0 - 2^2 \times 9^{-\frac{1}{2}} = \frac{1}{3} + 1 - 4 \times \frac{1}{\sqrt{9}}$
$= \frac{4}{3} - \frac{4}{3}$
$= 0$

c $2^3 \times \left(\frac{4}{9}\right)^{-\frac{1}{2}} \div \left(6 \times \left(3^{-2}\right)^2\right) = 8 \times \sqrt{\frac{9}{4}} \div \left(6 \times 3^{-4}\right)$
$= 8 \times \frac{3}{2} \div \frac{6}{3^4}$
$= 12 \times \frac{81}{6}$
$= 162$

d $\frac{15 \times 5^{\frac{3}{2}}}{125^{\frac{1}{2}} - 20^{\frac{1}{2}}} = \frac{15 \times \sqrt{5^3}}{\sqrt{125} - \sqrt{20}}$
$= \frac{15 \times 5\sqrt{5}}{5\sqrt{5} - 2\sqrt{5}}$
$= \frac{^{5}\cancel{15} \times 5\sqrt{5}}{\cancel{3}\sqrt{5}}$
$= 25$

13 a $\frac{3\left(x^2y^{-2}\right)^3}{\left(3x^4y^2\right)^{-1}} = \frac{3x^6y^{-6}}{3^{-1}x^{-4}y^{-2}}$
$= 3^2x^{10}y^{-4}$
$= \frac{9x^{10}}{y^4}$

b $\dfrac{2a^{\frac{2}{3}}b^{-3}}{3a^{\frac{1}{3}}b^{-1}} \times \dfrac{3^2 \times 2 \times (ab)^2}{\left(-8a^2\right)^2 b^2} = \dfrac{2a^{\frac{1}{3}}b^{-2}}{3} \times \dfrac{18a^2b^2}{64a^4b^2}$

$= \dfrac{2a^{\frac{1}{3}}}{3b^2} \times \dfrac{9}{32a^2}$

$= \dfrac{a^{\frac{1}{3}}}{b^2} \times \dfrac{3}{16a^2}$

$= \dfrac{3}{16a^{\frac{5}{3}}b^2}$

c $\dfrac{\left(2mn^{-2}\right)^{-2}}{m^{-1}n} \div \dfrac{10n^4m^{-1}}{3\left(m^2n\right)^{\frac{3}{2}}} = \dfrac{2^{-2}m^{-2}n^4}{m^{-1}n} \times \dfrac{3m^3n^{\frac{3}{2}}}{10n^4m^{-1}}$

$= \dfrac{n^3}{2^2m} \times \dfrac{3m^4}{10n^{\frac{5}{2}}}$

$= \dfrac{3n^3m^4}{40mn^{\frac{5}{2}}}$

$= \dfrac{3m^3n^{\frac{1}{2}}}{40}$

d $\dfrac{4m^2n^{-2} \times -2\left(m^2n^{\frac{3}{2}}\right)^2}{\left(-3m^3n^{-2}\right)^2} = \dfrac{4m^2n^{-2} \times -2m^4n^3}{9m^6n^{-4}}$

$= \dfrac{-8m^6n}{9m^6n^{-4}}$

$= \dfrac{-8n^5}{9}$

e $\dfrac{m^{-1} - n^{-1}}{m^2 - n^2} = \left(\dfrac{1}{m} - \dfrac{1}{n}\right) \div \left(m^2 - n^2\right)$

$= \left(\dfrac{n-m}{mn}\right) \times \dfrac{1}{m^2 - n^2}$

$= \dfrac{n-m}{mn} \times \dfrac{1}{(m-n)(m+n)}$

$= \dfrac{-(m-n)}{mn} \times \dfrac{1}{(m-n)(m+n)}$

$= \dfrac{-1}{mn(m+n)}$

f $\sqrt{4x-1} - 2x(4x-1)^{-\frac{1}{2}} = (4x-1)^{\frac{1}{2}} - \dfrac{2x}{(4x-1)^{\frac{1}{2}}}$

$= \dfrac{(4x-1)^{\frac{1}{2}}(4x-1)^{\frac{1}{2}} - 2x}{(4x-1)^{\frac{1}{2}}}$

$= \dfrac{(4x-1) - 2x}{(4x-1)^{\frac{1}{2}}}$

$= \dfrac{2x-1}{(4x-1)^{\frac{1}{2}}}$

14 $\dfrac{2^{1-n} \times 8^{1+2n}}{16^{1-n}} = \dfrac{2^{1-n} \times 2^{3(1+2n)}}{2^{4(1-n)}}$

$= \dfrac{2^{4+5n}}{2^{4-4n}}$

$= 2^{9n}$

Exercise 7.4 — Indicial equations and scientific notation

1 a $2^x = 32$

$2^x = 2^5$

$x = 5$

b $5^x = 625$

$5^x = 5^4$

$x = 4$

c $3^x = 243$

$3^x = 3^5$

$x = 5$

d $10^{-x} = \dfrac{1}{100}$

$10^{-x} = \dfrac{1}{10^2}$

$10^{-x} = 10^{-2}$

$-x = -2$

$x = 2$

e $4^{-x} = 16$

$4^{-x} = 4^2$

$-x = 2$

$x = -2$

f $6^x = \dfrac{1}{216}$

$6^x = \dfrac{1}{6^3}$

$6^x = 6^{-3}$

$x = -3$

g $3^{-x} = \dfrac{1}{81}$

$3^{-x} = \dfrac{1}{3^4}$

$3^{-x} = 3^{-4}$

$-x = -4$

$x = 4$

h $2^{-x} = 1$

$2^{-x} = 2^0$

$-x = 0$

$x = 0$

i $8^x = 2^6$

$\left(2^3\right)^x = 2^6$

$2^{3x} = 2^6$

$3x = 6$

$x = 2$

2 a $2^{3n+1} = 64$

$2^{3n+1} = 2^6$

$3n + 1 = 6$

$3n = 5$

$n = \dfrac{5}{3}$

b $5^{2n+3} = 25$
$5^{2n+3} = 5^2$
$2n + 3 = 2$
$2n = -1$
$n = -\frac{1}{2}$

c $3^{2-n} = 27$
$3^{2-n} = 3^3$
$2 - n = 3$
$-n = 1$
$n = -1$

d $16^{n+3} = 2^3$
$\left(2^4\right)^{n+3} = 2^3$
$2^{4n+12} = 2^3$
$4n + 12 = 3$
$4n = -9$
$n = -\frac{9}{4}$

e $49^{5-3n} = \frac{1}{7}$
$\left(7^2\right)^{5-3n} = 7^{-1}$
$7^{10-6n} = 7^{-1}$
$10 - 6n = -1$
$-6n = -11$
$n = \frac{11}{6}$

f $36^{4n-3} = 216$
$\left(6^2\right)^{4n-3} = 6^3$
$6^{8n-6} = 6^3$
$8n - 6 = 3$
$8n = 9$
$n = \frac{9}{8}$

3 a $4^{2x} = 8^{x-1}$
$\left(2^2\right)^{2x} = \left(2^3\right)^{x-1}$
$2^{4x} = 2^{3x-3}$
$4x = 3x - 3$
$x = -3$

b $27^{4-x} = 9^{2x+1}$
$\left(3^3\right)^{4-x} = \left(3^2\right)^{2x+1}$
$3^{12-3x} = 3^{4x+2}$
$12 - 3x = 4x + 2$
$-7x = -10$
$x = \frac{10}{7}$

c $16^{3x+1} = 128^{x-2}$
$\left(2^4\right)^{3x+1} = \left(2^7\right)^{x-2}$
$2^{12x+4} = 2^{7x-14}$
$12x + 4 = 7x - 14$
$5x = -18$
$x = -\frac{18}{5}$

d $25^{2x-3} = \frac{1}{125}$
$\left(5^2\right)^{2x-3} = 5^{-3}$
$5^{4x-6} = 5^{-3}$
$4x - 6 = -3$
$4x = 3$
$x = \frac{3}{4}$

e $32^{5-x} = 4^{3x+2}$
$\left(2^5\right)^{5-x} = \left(2^2\right)^{3x+2}$
$2^{25-5x} = 2^{6x+4}$
$25 - 5x = 6x + 4$
$-11x = -21$
$x = \frac{21}{11}$

f $64^{2-3x} = 16^{x+1}$
$\left(2^6\right)^{2-3x} = \left(2^4\right)^{x+1}$
$2^{12-18x} = 2^{4x+4}$
$12 - 18x = 4x + 4$
$-22x = -8$
$x = \frac{8}{22}$
$x = \frac{4}{11}$

g $9^{3x+5} = \frac{1}{243}$
$\left(3^2\right)^{3x+5} = \frac{1}{3^5}$
$3^{6x+10} = 3^{-5}$
$6x + 10 = -5$
$6x = -15$
$x = -\frac{15}{6}$
$= -\frac{5}{2}$

h $16^{4-3x} = \frac{1}{8^{x+3}}$
$16^{4-3x} = 8^{-x-3}$
$\left(2^4\right)^{4-3x} = \left(2^3\right)^{-x-3}$
$2^{16-12x} = 2^{-3x-9}$
$16 - 12x = -3x - 9$
$-9x = -25$
$x = \frac{25}{9}$

4 $\frac{2^{5x-3} \times 8^{9-2x}}{4^x} = 1$
$\frac{2^{5x-3} \times 2^{3(9-2x)}}{2^{2x}} = 1$
$\frac{2^{24-x}}{2^{2x}} = 1$
$2^{24-3x} = 1$
$2^{24-3x} = 2^0$
Equating indices:
$24 - 3x = 0$
$\therefore x = 8$

5 a $2 \times 5^x + 5^x < 75$
Adding:
$3 \times 5^x < 75$
$5^x < \frac{75}{3}$
$5^x < 25$
$5^x < 5^2$
Hence the indices give $x < 2$

b $\left(\frac{1}{9}\right)^{2x-3} > \left(\frac{1}{9}\right)^{7-x}$
$9^{-(2x-3)} > 9^{-(7-x)}$
$9^{-2x+3} > 9^{-7+x}$
$-2x + 3 > -7 + x$
$10 > 3x$
$x < \frac{10}{3}$

6 a $2^x \times 8^{3x-1} = 64$
$2^x \times \left(2^3\right)^{3x-1} = 2^6$
$2^x \times 2^{9x-3} = 2^6$
$2^{10x-3} = 2^6$
$10x - 3 = 6$
$10x = 9$
$x = \frac{9}{10}$

b $5^{2x} \times 125^{3-x} = 25$
$5^{2x} \times \left(5^3\right)^{3-x} = 5^2$
$5^{2x} \times 5^{9-3x} = 5^2$
$5^{-x+9} = 5^2$
$-x + 9 = 2$
$-x = -7$
$x = 7$

c $3^{4x} \times 27^{x+3} = 81$
$3^{4x} \times \left(3^3\right)^{x+3} = 81$
$3^{4x} \times 3^{3x+9} = 3^4$
$3^{7x+9} = 3^4$
$7x + 9 = 4$
$7x = -5$
$x = \frac{-5}{7}$

d $16^{x+4} \times 2^{3+2x} = 4^{5x}$
$\left(2^4\right)^{x+4} \times 2^{3+2x} = \left(2^2\right)^{5x}$
$2^{4x+16} \times 2^{3+2x} = 2^{10x}$
$2^{6x+19} = 2^{10x}$
$6x + 19 = 10x$
$4x = 19$
$x = \frac{19}{4}$

e $3125 \times 25^{2x+1} = 5^{3x+4}$

$5^5 \times \left(5^2\right)^{2x+1} = 5^{3x+4}$

$5^5 \times 5^{4x+2} = 5^{3x+4}$

$5^{4x+7} = 5^{3x+4}$

$4x + 7 = 3x + 4$

$x = -3$

f $\dfrac{81^{2-x}}{27^{x+3}} = 9^{2x}$

$\dfrac{(3^4)^{2-x}}{(3^3)^{x+3}} = \left(3^2\right)^{2x}$

$\dfrac{3^{8-4x}}{3^{3x+9}} = 3^{4x}$

$3^{-1-7x} = 3^{4x}$

$-1 - 7x = 4x$

$11x = -1$

$x = \dfrac{-1}{11}$

7 a $3^{2x} - 4\,(3^x) + 3 = 0$

$(3^x)^2 - 4\,(3^x) + 3 = 0$

and now let $y = 3x$

$y^2 - 4y + 3 = 0$

$(y - 3)(y - 1) = 0$

$y = 3$ or $y = 1$

$3^x = 3^1$ or $3^x = 1$ or 3^0

$x = 1$ or $x = 0$

b $2^{2x} - 6(2^x) + 8 = 0$

$(2^x)^2 - 6(2^x) + 8 = 0$

and let $y = 2^x$

$y^2 - 6y + 8 = 0$

$(y - 4)(y - 2) = 0$

$y = 4$ or $y = 2$

$2^x = 2^2$ or $2^x = 2^1$

$x = 2$ or $x = 1$

c $3(2^{2x}) - 36(2^x) + 96 = 0$

$2^{2x} - 12(2^x) + 32 = 0$

$(2^x)^2 - 12(2^x) + 32 = 0$

and let $y = 2^x$

$y^2 - 12y + 32 = 0$

$(y - 8)(y - 4) = 0$

$y = 8$ or $y = 4$

$2^x = 8$ or $2^x = 4$

$2^x = 2^3$ or $2^x = 2^2$

$x = 3$ or $x = 2$

d $2(5^{2x}) - 12(5^x) + 10 = 0$

$5^{2x} - 6(5^x) + 5 = 0$

$(5^x)^2 - 6(5^x) + 5 = 0$

and let $y = 5^x$

$y^2 - 6y + 5 = 0$

$(y - 5)(y - 1) = 0$

$y = 5$ or $y = 1$

$5^x = 5$ or $5^x = 1$

$5^x = 5^1$ or $5^x = 5^0$

$x = 1$ or $x = 0$

e $3(4^{2x}) = 15(4^x) - 12$

$3(4)^{2x} - 15(4^x) + 12 = 0$

$4^{2x} - 5(4^x) + 4 = 0$

$(4^x)^2 - 5(4^x) + 4 = 0$

and let $y = 4^x$

$y^2 - 5y + 4 = 0$

$(y - 4)(y - 1) = 0$

$y = 4$ or $y = 1$

$4^x = 4$ or $4^x = 1$

$4^x = 4^1$ or $4^x = 4^0$

$x = 1$ or $x = 0$

f $25^x - 30(5^x) + 125 = 0$

$5^{2x} - 30(5^x) + 125 = 0$

$(5^x)^2 - 30(5^x) + 125 = 0$

and let $y = 5^x$

$y^2 - 30y + 125 = 0$

$(y - 25)(y - 5) = 0$

$y = 25$ or $y = 5$

$5^x = 25$ or $5^x = 5$

$5^x = 5^2$ or $5^x = 5^1$

$x = 2$ or $x = 1$

8 a $\dfrac{32 \times 4^{3x}}{16^x} = \dfrac{2^5 \times \left(2^2\right)^{3x}}{\left(2^4\right)^x}$

$= \dfrac{2^5 \times 2^{6x}}{2^{4x}}$

$= \dfrac{2^{5+6x}}{2^{4x}}$

$= 2^{5+6x-4x}$

$= 2^{5+2x}$

b $\dfrac{3^{1+n} \times 81^{n-2}}{243^n} = \dfrac{3^{1+n} \times \left(3^4\right)^{n-2}}{\left(3^5\right)^n}$

$= \dfrac{3^{1+n} \times 3^{4n-8}}{3^{5n}}$

$= \dfrac{3^{5n-7}}{3^{5n}}$

$= 3^{-7}$

c $0.001 \times \sqrt[3]{10} \times 100^{\frac{5}{2}} \times (0.1)^{-\frac{2}{3}}$

$= \dfrac{1}{1000} \times 10^{\frac{1}{3}} \times \left(10^2\right)^{\frac{5}{2}} \times \left(\dfrac{1}{10}\right)^{-\frac{2}{3}}$

$= \dfrac{1}{10^3} \times 10^{\frac{1}{3}} \times 10^5 \times \left(10^{-1}\right)^{-\frac{2}{3}}$

$= 10^{-3} \times 10^{\frac{1}{3}} \times 10^5 \times 10^{\frac{2}{3}}$

$= 10^{-3+\frac{1}{3}+5+\frac{2}{3}}$

$= 10\dfrac{-45 + 5 + 75 + 10}{15}$

$= 10^{\frac{45}{15}}$

$= 10^3$

d $\dfrac{5^{n+1} - 5^n}{4} = \dfrac{5^n \times 5^1 - 5^n}{4}$

$= \dfrac{5^n(5 - 1)}{4}$

$= 5^n$

9 a $2^{2x} \times 8^{2-x} \times 16^{-\frac{3x}{2}} = \frac{2}{4^x}$

$\therefore 2^{2x} \times (2^3)^{2-x} \times \left(2^4\right)^{-\frac{3x}{2}} = \frac{2}{(2^2)^x}$

$\therefore 2^{2x} \times 2^{6-3x} \times 2^{-6x} = \frac{2^1}{2^{2x}}$

$\therefore 2^{-7x+6} = 2^{1-2x}$

Equating indices,

$-7x + 6 = 1 - 2x$

$\therefore 5 = 5x$

$\therefore x = 1$

b $25^{3x-3} \leq 125^{4+x}$

$\therefore \left(5^2\right)^{3x-3} \leq \left(5^3\right)^{4+x}$

$\therefore 5^{6x-6} \leq 5^{12+3x}$

As the base is greater than 1,

$6x - 6 \leq 12 + 3x$

$\therefore 3x \leq 18$

$\therefore x \leq 6$

c $9^x \div 27^{1-x} = \sqrt{3}$

$\therefore \left(3^2\right)^x \div \left(3^3\right)^{1-x} = 3^{\frac{1}{2}}$

$\therefore 3^{2x} \div 3^{3-3x} = 3^{\frac{1}{2}}$

$\therefore 3^{5x-3} = 3^{\frac{1}{2}}$

$\therefore 5x - 3 = \frac{1}{2}$

$\therefore 5x = \frac{7}{2}$

$\therefore x = \frac{7}{10}$

d $\left(\frac{2}{3}\right)^{3-2x} > \left(\frac{27}{8}\right)^{-\frac{1}{3}} \times \frac{1}{\sqrt{2\frac{1}{4}}}$

$\therefore \left(\frac{2}{3}\right)^{3-2x} > \left(\frac{8}{27}\right)^{\frac{1}{3}} \times \frac{1}{\sqrt{\frac{9}{4}}}$

$\therefore \left(\frac{2}{3}\right)^{3-2x} > \sqrt[3]{\frac{8}{27}} \times \frac{1}{\frac{3}{2}}$

$\therefore \left(\frac{2}{3}\right)^{3-2x} > \frac{2}{3} \times \frac{2}{3}$

$\therefore \left(\frac{2}{3}\right)^{3-2x} > \left(\frac{2}{3}\right)^2$

As the base is less than 1,

$3 - 2x < 2$

$\therefore -2x < -1$

$\therefore x > \frac{1}{2}$

10 a $3^{2x} - 10 \times 3^x + 9 = 0$

Let $a = 3^x$

$\therefore a^2 - 10a + 9 = 0$

$\therefore (a - 1)(a - 9) = 0$

$\therefore a = 1 \text{ or } a = 9$

$\therefore 3^x = 1 \text{ or } 3^x = 9$

$\therefore x = 0 \text{ or } x = 2$

b $24 \times 2^{2x} + 61 \times 2^x = 2^3$

Let $a = 2^x$

$\therefore 24a^2 + 61a = 8$

$\therefore 24a^2 + 61a - 8 = 0$

$\therefore (8a - 1)(3a + 8) = 0$

$\therefore a = \frac{1}{8} \text{ or } a = -\frac{8}{3}$

$\therefore 2^x = \frac{1}{8} \text{ or } 2^x = -\frac{8}{3}$

Reject $2^x = -\frac{8}{3}$ since $2^x > 0$

$\therefore 2^x = \frac{1}{8}$

$\therefore 2^x = 2^{-3}$

$\therefore x = -3$

c $25^x + 5^{2+x} - 150 = 0$

$\therefore 5^{2x} + 5^2 \times 5^x - 150 = 0$

Let $a = 5^x$

$\therefore a^2 + 25a - 150 = 0$

$\therefore (a - 5)(a + 30) = 0$

$\therefore a = 5 \text{ or } a = -30$

$\therefore 5^x = 5 \text{ or } 5^x = -30$

$\therefore 5^x = 5, (5^x > 0)$

$\therefore x = 1$

d $(2^x + 2^{-x})^2 = 4$

$(2^x + 2^{-x})^2 = 4$

$\therefore 2^x + 2^{-x} = \pm\sqrt{4}$

$\therefore 2^x + 2^{-x} = \pm 2$

However, $2^x + 2^{-x} > 0$

$\therefore 2^x + 2^{-x} = 2$

$\therefore 2^x + \frac{1}{2^x} = 2$

Let $a = 2^x$

$\therefore a + \frac{1}{a} = 2$

$\therefore a^2 + 1 = 2a$

$\therefore a^2 - 2a + 1 = 0$

$\therefore (a - 1)^2 = 0$

$\therefore a = 1$

$\therefore 2^x = 1$

$\therefore x = 0$

e $10^x - 10^{2-x} = 99$

$\therefore 10^x - \frac{10^2}{10^x} = 99$

Let $a = 10^x$

$\therefore a - \frac{100}{a} = 99$

$\therefore a^2 - 100 = 99a$

$\therefore a^2 - 99a - 100 = 0$

$\therefore (a - 100)(a + 1) = 0$

$\therefore a = 100 \text{ or } a = -1$

$\therefore 10^x = 100 \text{ or } 10^x = -1$

$\therefore 10^x = 100 \, (10^x > 0)$

$\therefore 10^x = 10^2$

$\therefore x = 2$

f $2^{3x} + 3 \times 2^{2x-1} - 2^x = 0$

$\therefore 2^{3x} + 3 \times \dfrac{2^{2x}}{2^1} - 2^x = 0$

Let $a = 2^x$

$\therefore a^3 + 3 \times \dfrac{a^2}{2} - a = 0$

$\therefore 2a^3 + 3a^2 - 2a = 0$

$\therefore a(2a^2 + 3a - 2) = 0$

$\therefore a(2a - 1)(a + 2) = 0$

$\therefore a = 0 \text{ or } a = \dfrac{1}{2} \text{ or } a = -2$

$\therefore 2^x = 0 \text{ or } 2^x = \dfrac{1}{2} \text{ or } 2^x = -2$

$\therefore 2^x = \dfrac{1}{2} \; (2^x > 0)$

$\therefore 2^x = 2^{-1}$

$\therefore x = -1$

11 a i $1\,409\,000 = 1.409 \times 10^6$ and it contains 4 significant figures.

ii $0.000\,130\,6 = 1.306 \times 10^{-4}$ and it contains 4 significant figures.

b i $3.04 \times 10^5 = 304\,000$

ii $5.803 \times 10^{-2} = 0.058\,03$

12 a i $-0.000\,000\,050\,6 = -5.06 \times 10^{-8}$

ii Diameter is $2 \times 6370 = 12\,740$km.
In scientific notation, the diameter is 1.274×10^4km.

iii $3.2 \times 10^4 \times 5 \times 10^{-2}$
$= (3.2 \times 5) \times \left(10^4 \times 10^{-2}\right)$
$= 16 \times 10^2$
$= 1.6 \times 10^3$

iv 16, 878.7km is equal to $1.687\,87 \times 10^4$km.

b i $6.3 \times 10^{-4} + 6.3 \times 10^4 = 0.00063 + 63\,000$
$= 63000.00063$

ii $(1.44 \times 10^6)^{\frac{1}{2}} = (1.44)^{\frac{1}{2}} \times (10^6)^{\frac{1}{2}}$
$= \sqrt{1.44} \times 10^3$
$= 1.2 \times 10^3$
$= 1200$

13 a 60 589
$= 6.0589 \times 10^4$
$\simeq 6.1 \times 10^4$
$\therefore 60\,589 \simeq 61\,000$
Correct to 2 significant figures, 61 000 people attended the match.

b $1.994 \times 10^{-2} \simeq 2.0 \times 10^{-2}$
The probability, correct to 2 significant figures, is 0.020.

c -0.00634
$= -6.34 \times 10^{-3}$
$\simeq -6.3 \times 10^{-3}$
Correct to 2 significant figures, $x = -0.0063$.

d $26,597,696$
$= 2.6597696 \times 10^7$
$\simeq 2.7 \times 10^7$
Correct to 2 significant figures, the distance flown is 27 000 000 km.

14 a $\dfrac{x^2 y^{-2}}{2x^{\frac{1}{3}}\sqrt{y^5}}$

$= \dfrac{x^2}{2x^{\frac{1}{3}} y^2 y^{\frac{5}{2}}}$

$= \dfrac{x^{\left(2-\frac{1}{3}\right)}}{2y^{\left(2+\frac{5}{2}\right)}}$

$= \dfrac{x^{\frac{5}{3}}}{2y^{\frac{9}{2}}}$

b i $5^x \times 25^{2x} = \dfrac{1}{5}$

$5^x \times \left(5^2\right)^{2x} = 5^{-1}$

$5^x \times 5^{4x} = 5^{-1}$

$5^{5x} = 5^{-1}$

$\Rightarrow 5x = -1$

$\therefore x = -\dfrac{1}{5}$

ii $5^x \times 25^{2x} = 0.25$

$5^x \times \left(5^2\right)^{2x} = 0.25$

$5^x \times 5^{4x} = 0.25$

$5^{5x} = 0.25$

From here, the solution needs to be derived with the aid of a calculator either by

a using trial and error substituting in different values of x to narrow down the solution, or

b with the calculator in Decimal mode, solve the equation using Equation/Inequality.
The answer, to 4 significant figures, is $x = -0.1723$

7.5 Review: exam practice

1 $\dfrac{(2xy^3)^2}{7x^3} \times \dfrac{3x^5y^2}{4y}$

$= \dfrac{4x^2y^6}{7x^3} \times \dfrac{3x^5y^2}{4y}$

$= \dfrac{4 \times 3 \times x^2 \times x^5 \times y^6 \times y^2}{7 \times 4x^3y}$

$= \dfrac{3x^7y^8}{7x^3y^1}$

$= \dfrac{3x^4y^7}{7}$

The answer is B

2 $\dfrac{5m^4p^2}{2m^3p} \div \dfrac{(5m^2p^6)^3}{3m^7p}$

$= \dfrac{5m^4p^2}{2m^3p} \times \dfrac{3m^7p}{(5m^2p^6)^3}$

$= \dfrac{5m^4p^2}{2m^3p} \times \dfrac{3m^7p}{125m^6p^{18}}$

$= \dfrac{5 \times 3 \times m^4 \times m^7 \times p^2 \times p^1}{2 \times 125 \times m^3 \times m^6 \times p^1 \times p^{18}}$

$= \dfrac{3m^{11}p^3}{50m^9p^{19}}$

$= \dfrac{3m^2}{50p^{16}}$

The answer is C

3 $5^{-2}\left(\dfrac{64}{125}\right)^{-\frac{1}{3}} = 5^{-2}\left(\dfrac{4^3}{5^3}\right)^{-\frac{1}{3}}$

$= 5^{-2}\left(\dfrac{4^{-1}}{5^{-1}}\right)$

$= 5^{-2}\left(\dfrac{5^1}{4^1}\right)$

$= \dfrac{1}{5 \times 4}$

$= \dfrac{1}{20}$

The answer is A

4 $25^{2-x} = 125$

$\left(5^2\right)^{2-x} = 5^3$

$5^{4-2x} = 5^3$

$4 - 2x = 3$

$2x = 1$

$x = \dfrac{1}{2}$

The answer is B

5 $\left(3.2 \times 10^{-2}\right) \times \left(5 \times 10^5\right)$

$= (3.2 \times 5) \times \left(10^{-2} \times 10^5\right)$

$= 16 \times 10^3$

$= 1.6 \times 10^4$

Answer is D.

6 $\left(9a^3b^{-4}\right)^{\frac{1}{2}} \times 2\left(a^{\frac{1}{2}}b^{-2}\right)^{-2} = 3a^{\frac{3}{2}}b^{-2} \times 2 \times a^{-1}b^4$

$= 6a^{\frac{1}{2}}b^2$

7 $27^{-\frac{2}{3}} + \left(\dfrac{49}{81}\right)^{\frac{1}{2}} = \dfrac{1}{27^{\frac{2}{3}}} + \left(\dfrac{49}{81}\right)^{\frac{1}{2}}$

$= \dfrac{1}{\left(\sqrt[3]{27}\right)^2} + \dfrac{\sqrt{49}}{\sqrt{81}}$

$= \dfrac{1}{(3)^2} + \dfrac{7}{9}$

$= \dfrac{1}{9} + \dfrac{7}{9}$

$= \dfrac{8}{9}$

8 $\left(16x^{-6}y^{10}\right)^{\frac{1}{2}} \div \sqrt[3]{\left(27x^3y^9\right)}$

$= \left(4^2x^6y^{10}\right)^{\frac{1}{2}} \div \left(27x^3y^9\right)^{\frac{1}{3}}$

$= \left(4x^{-3}y^5\right) \div \left(3^3x^3y^9\right)^{\frac{1}{3}}$

$= 4x^{-3}y^5 \div 3x^1y^3$

$= \dfrac{4x^{-3}y^5}{3x^1y^3}$

$= \dfrac{4y^2}{3x^4}$

9 a $2x^5 = 100$

$x^5 = 50$

$x = 50^{\frac{1}{5}}$

$x = 2.187$

Using graphics calculator.

b $8^{x+1} \times 2^{2x} = 4^{3x} - 1$

$\left(2^3\right)^{x+1} \times 2^{2x} = \left(2^2\right)^{3x-1}$

$2^{3x+3} \times 2^{2x} = 2^{6x-2}$

$2^{5x+3} = 2^{6x-2}$

$5x + 3 = 6x - 2$

$x = 5$

10 a $a^{\frac{1}{2}} \div b^{\frac{3}{2}} = \sqrt{a} \div \sqrt{b^3}$

$= \sqrt{\dfrac{a}{b^3}}$

b $2^{\frac{5}{2}} = \sqrt{2^5}$

$= \sqrt{32}$

c $3^{-\frac{2}{5}} = \sqrt[5]{3^{-2}}$

$= \sqrt[5]{\dfrac{1}{3^2}}$

$= \sqrt[5]{\dfrac{1}{9}}$

11 $\left(4 \times 10^6\right)^2 \times (5 \times 10^{-3}) = 16 \times 10^{12} \times 5 \times 10^{-3}$

$= 16 \times 5 \times 10^{12} \times 10^{-3}$

$= 80 \times 10^9$

$= 8.0 \times 10^1 \times 10^9$

$= 8 \times 10^{10}$

12 a $4^{\frac{3}{2}} = \sqrt{4^3}$

$= \sqrt{64}$

$= 8$

b $3^{-1} + 5^0 - 2^2 \times 9^{-\frac{1}{2}} = \dfrac{1}{3} + 1 - 4 \times \dfrac{1}{\sqrt{9}}$

$= \dfrac{4}{3} - \dfrac{4}{3}$

$= 0$

13 $\dfrac{20p^5}{m^3q^{-2}} \div \dfrac{5\left(p^2q^{-3}\right)^2}{-4m^{-1}} = \dfrac{20p^5}{m^3q^{-2}} \times \dfrac{-4m^{-1}}{5p^4q^{-6}}$

$= \dfrac{^{4}\cancel{20}p^5q^2}{m^3} \times \dfrac{-4q^6}{\cancel{5}p^4m}$

$= \dfrac{-16p^5q^8}{m^4p^4}$

$= \dfrac{-16pq^8}{m^4}$

14 $2^x - 48 \times 2^{-x} = 13$

$\therefore 2^x - \frac{48}{2^x} = 13$

Let $a = 2^x$.

$a - \frac{48}{a} = 13$

$a^2 - 48 = 13a$

$a^2 - 13a - 48 = 0$

$(a - 16)(a + 3) = 0$

$a = 16,\ a = -3$

$\therefore 2^x = 16,\ 2^x = -3$

Reject $2^x = -3$ since there are no real solutions.

$2^x = 16$

$2^x = 2^4$

$x = 4$

15 a $4^{5x} + 4^{5x} = \frac{8}{2^{4x-5}}$

$\therefore 2 \times 4^{5x} = \frac{2^3}{2^{4x-5}}$

$\therefore 2 \times \left(2^2\right)^{5x} = 2^{8-4x}$

$\therefore 2 \times 2^{10x} = 2^{8-4x}$

$\therefore 2^{1+10x} = 2^{8-4x}$

$\therefore 1 + 10x = 8 - 4x$

$\therefore 14x = 7$

$\therefore x = \frac{7}{14}$

$\therefore x = \frac{1}{2}$

b $5^{\frac{2x}{3}} \times 5^{\frac{3x}{2}} = 25^{x+4}$

• $\therefore 5^{\frac{2x}{3}+\frac{3x}{2}} = \left(5^2\right)^{x+4}$

$\therefore 5^{\frac{4x+9x}{6}} = 5^{2x+8}$

$\therefore \frac{13x}{6} = 2x + 8$

$\therefore 13x = 12x + 48$

$\therefore x = 48$

16 $\frac{a - a^{-1}}{a + 1}$

$= \left(a - \frac{1}{a}\right) \div (a + 1)$

$= \left(\frac{a^2 - 1}{a}\right) \times \frac{1}{a + 1}$

$= \frac{\cancel{(a+1)}(a-1)}{a} \times \frac{1}{\cancel{a+1}}$

$\frac{a - 1}{a}$

17 Given $x = 3^{\frac{1}{3}} + 3^{-\frac{1}{3}}$, show the that $x^3 - 3x = \frac{10}{3}$.

let $x = a + \frac{1}{a}$ where $a = 3^{\frac{1}{3}}$.

$x^3 - 3x = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$

$= a3 + 3a^2 \times \frac{1}{a} + 3a \times \frac{1}{a^2} + \frac{1}{a^3} - 3a - \frac{3}{a}$

$= a^3 + 3a + \frac{3}{a} + \frac{1}{a^3} - 3a - \frac{3}{a}$

$= a^3 + \frac{1}{a^3}$

Substitute back that $a = 3^{\frac{1}{3}}$.

$\therefore x^3 - 3x = \left(3^{\frac{1}{3}}\right)^3 + \frac{1}{\left(3^{\frac{1}{3}}\right)^3}$

$\therefore x^3 - 3x = 3 + \frac{1}{3}$

$\therefore x^3 - 3x = \frac{10}{3}$

as required.

18 a consider the system of equations:

$5^{2x-y} = \frac{1}{125}$.....(1)

$10^{2y-6x} = 0.01$....(2)

From equation(1),

$5^{2x-y} = 5^{-3}$

$\therefore 2x - y = -3$.....(3)

From equation (2),

$10^{2y-6x} = \frac{1}{100}$

$\therefore 10^{2y-6x} = 10^{-2}$

$\therefore 2y - 6x = -2$

$\therefore -3x + y = -1$.....(4)

Consider the simultaneous equations (3) and (4)

$2x - y = -3$....(3)

$-3x + y = -1$....(4)

Add equations (3) and (4)

$\therefore -x = -4$

$\therefore x = 4$

Substitute $x = 4$ in equation (4)

$\therefore -12 + y = -1$

$\therefore y = 11$

Answer. $x = 4,\ y = 11$

b Consider the system of equations

$a \times 2^{k-1} = 40$....(1)

$a \times 2^{2k-2} = 10$.....(2)

Divide equation (1) by equation (2)

$\therefore \frac{\cancel{a} \times 2^{k-1}}{\cancel{a} \times 2^{2k-2}} = \frac{40}{10}$

$\therefore 2^{k-1-2k+2} = 4$

$\therefore 2^{-k+1} = 2^2$

$\therefore -k + 1 = 2$

$\therefore k = -1$

Substitute $k = -1$ in equation (1)

$\therefore a \times 2^{-2} = 40$

$\therefore a \times \frac{1}{4} = 40$

$\therefore a = 160$

Answer: $a = 160,\ k = -1$

19 $\left(\frac{2x^2}{3a}\right)^{n-1} \div \left(\frac{3x}{a}\right)^{n+1} = \left(\frac{x}{4}\right)^3$

$\therefore \frac{2^{n-1}x^{2n-2}}{3^{n-1}a^{n-1}} \times \frac{a^{n+1}}{3^{n+1}a^{n+1}} = \frac{x^3}{64}$

$\therefore \frac{2^{n-1}x^{2n-2-n-1}a^{n+1-n+1}}{3^{n-1+n+1}} = \frac{x^3}{64}$

$\therefore \frac{2^{n-1}x^{n-3}a^2}{3^{2n}} = \frac{x^3}{64}$

$\therefore \frac{2^{n-1}a^2}{3^{2n}}x^{n-3} = \frac{1}{64}x^3$

For the equality to hold, the powers of x must be equal.

$\therefore n - 3 = 3$

$\therefore n = 6$

And, for the equality to hold, the coefficient of x^3 must be equal.

$\therefore \frac{2^{n-1}a^2}{3^{2n}} = \frac{1}{64}$

Substitute $n = 6$

$\therefore \frac{2^5a^2}{3^{12}} = \frac{1}{2^6}$

$\therefore a^2 = \frac{3^{12}}{2^5} \times \frac{1}{2^6}$

$\therefore a^2 = \frac{3^{12}}{2^{11}}$

$\therefore a^2 = \pm\frac{3^6}{2^{\frac{11}{2}}}$

$\therefore a = \pm\left(3^6 \times 2^{\frac{11}{2}}\right)$

Answer: $a = \pm\left(3^6 \times 2^{-\frac{11}{2}}\right)$, $n = 6$

20 $x^2 = 2^{32}$

$x = 2^{\frac{32}{2}}$

$x = 2^{16}$

Substituting x into $x^x = 2^y$

$\left(2^{16}\right)^{2^{16}} = 2^y$

$\left(2^{2^4}\right)^{2^{16}} = 2^y$

$\Rightarrow \left(2^4\right)\left(2^{16}\right) = 2^y$

$2^{20} = 2^y$

$\therefore y = 20$