Chapter 9 — Cosine and sine rules

Exercise 9.2 – Review of trigonometric ratios and the unit circle

1 a $\sin(\theta) = \dfrac{\text{opp}}{\text{hyp}}$

$\sin(68°) = \dfrac{x}{13}$

$x = 13\sin 68°$
$= 12.1\text{ cm}$

b $\tan(\theta) = \dfrac{\text{opp}}{\text{adj}}$

$\tan(49°) = \dfrac{y}{48}$

$y = 48\tan(49°)$
$= 55.2\text{ m}$

c $\cos(\theta) = \dfrac{\text{adj}}{\text{hyp}}$

$\cos(41°) = \dfrac{z}{12.5}$

$z = 12.5\cos(41°)$
$= 9.43\text{ km}$

2 a $\sin(50°) = \dfrac{h}{10}$

$\therefore h = 10\sin(50°)$
$\therefore h \simeq 7.66$

b Recognising the "3, 4, 5" Pythagorean triad gives

$\tan(a°) = \dfrac{5}{2}$

$\therefore a° = \tan^{-1}(2.5)$
$\therefore a° \simeq 68.20°$
Hence, $a \simeq 68.20°$.

3 a $5° = 5 \times \dfrac{\pi}{180} = \dfrac{\pi^c}{36}$

b $15° = 15 \times \dfrac{\pi}{180} = \dfrac{\pi^c}{12}$

c $120° = 120 \times \dfrac{\pi}{180} = \dfrac{2\pi^c}{3}$

d $130° = 130 \times \dfrac{\pi}{180} = \dfrac{13\pi^c}{18}$

e $63.9° = 63.9 \times \dfrac{\pi}{180} = 1.12^c$

f $78.82° = 78.82 \times \dfrac{\pi}{180} = 1.38^c$

g $235° = 235 \times \dfrac{\pi}{180} = 4.10^c$

h $260° = 260 \times \dfrac{\pi}{180} = 4.54^c$

i $310° = 310 \times \dfrac{\pi}{180} = 5.41^c$

j $350° = 350 \times \dfrac{\pi}{180} = 6.11^c$

4 a $3^c = 3 \times \dfrac{180}{\pi} = 171.89°$

b $5^c = 5 \times \dfrac{180}{\pi} = 286.48°$

c $4.8^c = 4.8 \times \dfrac{180}{\pi} = 275.02°$

d $2.56^c = 2.56 \times \dfrac{180}{\pi} = 146.68°$

e $\dfrac{7\pi^c}{20} = \dfrac{7\pi}{20} \times \dfrac{180}{\pi} = 63°$

f $\dfrac{3\pi^c}{10} = \dfrac{3\pi^c}{10} \times \dfrac{180}{\pi} = 54°$

g $\dfrac{5\pi^c}{6} = \dfrac{5\pi}{6} \times \dfrac{180}{\pi} = 150°$

h $\dfrac{5\pi^c}{4} = \dfrac{5\pi}{4} \times \dfrac{180}{\pi} = 225°$

5 a $\sin(0.4) = 0.389$
b $\sin(0.8) = 0.717$
c $\cos(1.4) = 0.170$
d $\cos(1.7) = -0.129$
e $\tan(2.9) = -0.246$
f $\tan(2.4) = -0.916$

6 a $\sin(75°) = 0.966$
b $\sin(68°) = 0.927$
c $\cos(160°) = -0.940$
d $\cos(185°) = -0.996$
e $\tan(265°) = 11.430$
f $\tan(240°) = 1.732$

7 a $\sin(0) = 0$
b $\sin(\pi) = 0$
c $\cos(2\pi) = 1$
d $\cos(\pi) = -1$
e $\tan\left(\dfrac{3\pi}{2}\right) = \text{undefined}$
f $\tan\left(\dfrac{\pi}{2}\right) = \text{undefined}$

8 a $\sin(90°) = 1$
b $\sin(360°) = 0$
c $\cos(180°) = -1$
d $\cos(0°) = 1$
e $\tan(270°) = \text{undefined}$
f $\tan(720°) = 0$

9 a $\sin^2(20) + \cos^2(20) = 1$
b $\cos^2(50) + \sin^2(50) = 1$
c $\sin^2(\pi) + \cos^2(\pi) = 1$
d $\sin^2(2.5) + \cos^2(2.5) = 1$
e $\sin^2\left(\dfrac{\pi}{2}\right) + \cos^2\left(\dfrac{\pi}{2}\right) = 1$
f $\sin^2\left(\dfrac{\theta}{2}\right) + \cos^2\left(\dfrac{\theta}{2}\right) = 1$

g $2\sin^2(\alpha) + 2\cos^2(\alpha)$
$= 2(\sin^2(\alpha) + \cos^2(\alpha))$
$= 2 \times 1$
$= 2$

h $5\sin^2(\beta) + 5\cos^2(\beta)$
$= 5(\sin^2(\beta) + \cos^2(\beta))$
$= 5 \times 1$
$= 5$

10 a $\sin(35°) = 0.574$
$\sin(70°) = 0.940$
$\sin(120°) = 0.866$

$\sin(150°) = 0.5$
$\sin(240°) = -0.866$
smallest to largest
$\sin(240°), \sin(150°), \sin(35°), \sin(120°), \sin(70°),$

b $\cos(0.2) = 0.980$
$\cos(1.5) = 0.071$
$\cos(3.34) = -0.980$
$\cos(5.3) = 0.554$
$\cos(6.3) = -0.999$
smallest to largest
$\cos(3.34), \cos(1.5), \cos(5.3), \cos(0.2), \cos(6.3)$

11 $\sin(\theta) = \dfrac{8}{17}$

$\cos(\theta) = \dfrac{15}{17}$

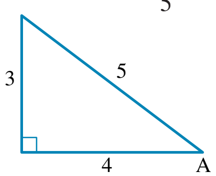
$\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$

$= \dfrac{\frac{8}{17}}{\frac{15}{17}}$

$\tan(\theta) = \dfrac{8}{15}$

12 $\sin(A) = 0.6 = \dfrac{3}{5}$

$\cos(A) = 0.8 = \dfrac{4}{5}$



$\tan(A) = \dfrac{3}{4}$ or 0.75

13 a $\cos(x) = -0.6591$
reference angle $= 0.8512$
2nd and 3rd quadrant
$x = \pi - 0.8512, \pi + 0.8512, 3\pi - 0.8512, 3\pi - 0.8512$
$x = 2.2904, 3.9928, 8.5736, 10.2760$

b $\sin(x) = 0.9104$
reference angle $= 1.1442$
1st and 2nd quadrant
$x = 1.1442, \pi - 1.1442, 2\pi + 1.1442, 3\pi - 1.1442$
$x = 1.1442, 1.9973, 7.4274, 8.2805$

c $\cos(x) = 0.48$
reference angle $= 1.0701$
1st and 4th quadrants
$x = 1.0701, 2\pi - 1.0701, 2\pi + 1.0701, 4\pi - 1.0701$
$x = 1.0701, 5.2130, 7.3533, 11.4962$

d $\sin(x) = -0.371$
reference angle $= 0.3801$
3rd and 4th quadrants
$x = \pi + 0.3801, 2\pi - 0.3801, 3\pi + 0.3801, 4\pi - 0.3801$
$x = 3.5217, 5.9031, 9.8049, 12.1863$

14 a $\cos(2x) = 1 \quad 0° \le x \le 360°$
$2x = 0, 360°, 720° \quad 0° \le 2x \le 720°$
$x = 0°, 180°, 360°$

b $2\sin(2x) = -1 \quad 0° \le x \le 360°$

$\sin(2x) = -\dfrac{1}{2} \quad 0° \le 2x \le 720°$

reference angle $= 30°$
3rd and 4th quadrants
$2x = 180 + 30, 360 - 30, 540 + 30, 720 - 30$
$= 210, 330, 570, 690$
$x = 105°, 165°, 285°, 345°$

c $2\cos(3x) = -\sqrt{2} \quad 0° \le x \le 360°$

$\cos(3x) = -\dfrac{\sqrt{2}}{2} \quad 0 \le 3x \le 1080°$

reference angle $= 45°$
2nd and 3rd quadrant
$3x = 180 - 45, 180 + 45, 540 - 45, 540$
$+ 45, 900 - 45, 900 + 45$
$3x = 135, 225, 495, 585, 855, 945$
$x = 45°, 75°, 165°, 195°, 285°, 315°$

d $2\sin(3x) = \sqrt{3} \quad 0° \le x \le 360°$

$\sin(3x) = \dfrac{\sqrt{3}}{2} \quad 0° \le 3x \le 1\,080°$

reference angle $= 60°$
1st and 2nd quadrant
$3x = 60, 180 - 60, 360 + 60, 540 - 60, 720 + 60, 900 - 60$
$3x = 60, 120, 420, 480, 780, 840$
$x = 20°, 40°, 140°, 160°, 260°, 280°$

e $\sin(3x) = -0.1254 \quad 0° \le x \le 360°$
reference angle $= 7.20° \quad 0° \le 3x \le 1\,080°$
3rd and 4th quadrants
$3x = 180 + 7.20, 360 - 7.20, 540 + 7.20, 720 - 7.20,$
$900 + 7.20, 1080 - 7.20$
$3x = 187.20, 352.80, 547.20, 712.8, 907.20, 1072.8$
$x = 62.40, 117.60, 182.40, 237.6, 302.40, 357.6$ degrees.

f $3\cos(2x) = 0.5787 \quad 0° \le x \le 360°$
$\cos(2x) = 0.1929 \quad 0° \le 2x \le 720°$
reference angle $= 78.88°$
1st and 4th quadrants
$2x = 78.88°, 360 - 78.88°, 360 + 78.88, 720 - 78.88$
$2x = 78.88°, 281.12°, 438.88°, 641.12$
$x = 39.44, 140.56, 219.44, 320.56$ degrees.

g $4\sin\left(\dfrac{x}{2}\right) = 0.913 \quad 0° \le x \le 360°$

$\sin\left(\dfrac{x}{2}\right) = 0.2283 \quad 0° \le \dfrac{x}{2} \le 180°$

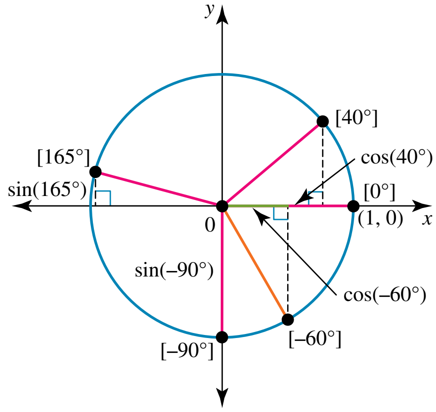
reference angle $= 13.19°$
1st and 2nd quadrants

$\dfrac{x}{2} = 13.19°, 180 - 13.19°$

$x = 26.39°, 333.61°$

h $\sqrt{2}\cos(x) = -0.2751 \quad 0° \le x \le 360°$
$\cos(x) = -0.1945$
reference angle $= 78.78°$
2nd and 3rd quadrants
$x = 180 - 78.78, 180 + 78.78$
$x = 101.22°, 258.78°$

15



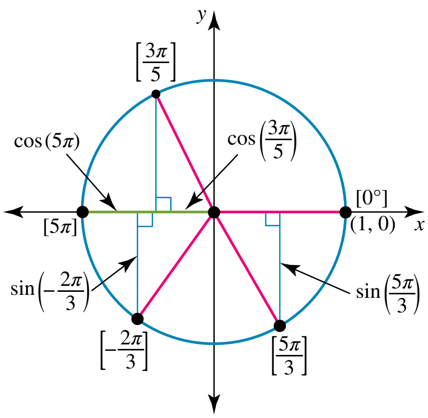
a cos(40°) is the x co-ordinate of the trigonometric point [40°] which lies in the first quadrant.

b sin(165°) is the y co-ordinate of the trigonometric point [165°] which lies in the second quadrant.

c cos(−60°) is the x co-ordinate of the trigonometric point [−60°] which lies in the fourth quadrant.

d sin(−90°) is the y co-ordinate of the trigonometric point [−90°] which lies on the boundary between the third and fourth quadrants.

16



a $\sin\left(\frac{5\pi}{3}\right)$ is the y co-ordinate of the trigonometric point $\left[\frac{5\pi}{3}\right]$ which lies in the fourth quadrant.

b $\cos\left(\frac{3\pi}{5}\right)$ is the x co-ordinate of the trigonometric point $\left[\frac{3\pi}{5}\right]$ which lies in the second quadrant.

c $\cos(5\pi)$ is the x co-ordinate of the trigonometric point $[5\pi]$ which lies on the boundary between the second and third quadrants.

d $\sin\left(-\frac{2\pi}{3}\right)$ is the y co-ordinate of the trigonometric point $\left[-\frac{2\pi}{3}\right]$ which lies in the third quadrant.

17 a $P\left[\frac{\pi}{4}\right]$

$x = \cos(\theta)$ and $y = \sin(\theta)$ where $\theta = \frac{\pi}{4}$.

$$\therefore x = \cos\left(\frac{\pi}{4}\right) \quad \text{and } y = \sin\left(\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{2}}{2} \qquad\qquad = \frac{\sqrt{2}}{2}$$

P has Cartesian co-ordinates $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

b The point P (0, −1) lies on the boundary between quadrants 3 and 4.

Therefore, P could be the trigonometric point $\left[\frac{3\pi}{2}\right]$ or the trigonometric point $\left[-\frac{\pi}{2}\right]$. In degrees, the answers are 270° and −90°.

Exercise 9.3 – The sine rule

1 a $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

b $\frac{x}{\sin(X)} = \frac{y}{\sin(Y)} = \frac{z}{\sin(Z)}$

c $\frac{p}{\sin(P)} = \frac{q}{\sin(Q)} = \frac{r}{\sin(R)}$

2 a
$$\frac{b}{\sin(B)} = \frac{c}{\sin(c)}$$
$$\frac{16}{\sin(50°)} = \frac{x}{\sin(45°)}$$
$$x = \frac{16\sin(45°)}{\sin(50°)}$$
$$= 14.8\text{ cm}$$

b
$$\frac{l}{\sin(L)} = \frac{n}{\sin(N)}$$
$$\frac{q}{\sin(63°)} = \frac{1.9}{\sin(59°)}$$
$$q = \frac{1.9\sin(63°)}{\sin(59°)}$$
$$= 1.98\text{ km}$$

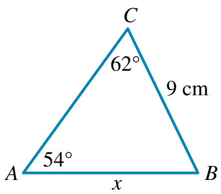
c
$$\frac{t}{\sin(T)} = \frac{r}{\sin(R)}$$
$$\frac{t}{\sin(84°)} = \frac{89}{\sin(52°)}$$
$$t = \frac{89\ \sin(84°)}{\sin(52°)}$$
$$= 112\text{ mm}$$

3 a
$$\angle\text{HIG} = 180° - (74° + 74°)$$
$$= 32°$$
$$\frac{x}{\sin(32°)} = \frac{18.2}{\sin(74°)}$$
$$x = \frac{18.2\sin(32°)}{\sin(74°)}$$
$$= 10.0\text{ mm}$$

b
$$\angle\text{NMP} = 180° - (80° + 62°)$$
$$= 38°$$
$$\frac{m}{\sin(38°)} = \frac{35.3}{\sin(80°)}$$
$$m = \frac{35.3\sin(38°)}{\sin(80°)}$$
$$= 22.1\text{ cm}$$

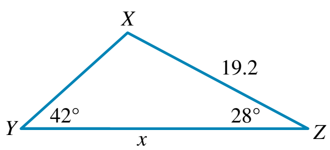
c
$$\angle\text{BAC} = 180° - (85° + 27°)$$
$$= 68°$$
$$\frac{y}{\sin(68°)} = \frac{19.4}{\sin(27°)}$$
$$y = \frac{19.4\sin(68°)}{\sin(27°)}$$
$$= 39.6\text{ km}$$

4



$$\frac{x}{\sin(62°)} = \frac{9}{\sin(54°)}$$
$$x = \frac{9\sin(62°)}{\sin(54°)}$$
$$= 9.8\,\text{cm}$$

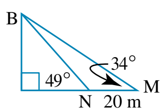
5



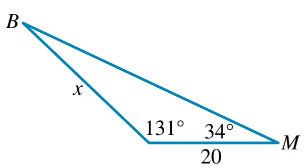
$$\angle YXZ = 180° - (42° + 28°)$$
$$= 110°$$
$$\frac{x}{\sin(110°)} = \frac{19.2}{\sin(42°)}$$
$$x = \frac{19.2\sin(110°)}{\sin(42°)}$$
$$= 27.0\,\text{m}$$

6 $\angle XZY = 180° - (59° + 72°)$
$$= 49°$$
$$\frac{y}{\sin(72°)} = \frac{30}{\sin(49°)}$$
$$y = \frac{30\sin(72°)}{\sin(49°)}$$
$$= 37.8\,\text{m}$$

7 a



b



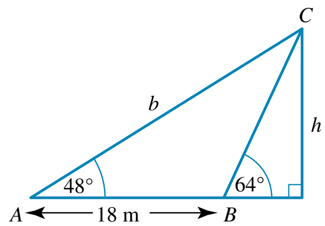
Angle at B is $180° - (131° + 34°)$
$= 15°$
$$\frac{x}{\sin(34°)} = \frac{20}{\sin(15°)}$$
$$x = \frac{20\sin(34°)}{\sin(15°)}$$
$$= 43.2$$

Distance NB is 43.2 m

c $\sin(49°) = \dfrac{\text{height}}{43.2}$
$$\text{height} = 43.2 \times \sin(49°)$$
$$= 33$$

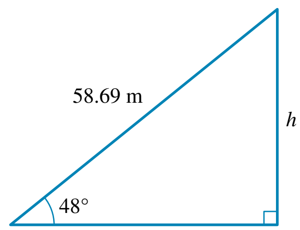
The building's height is 33 m.

8



$$B = 180° - 64°$$
$$= 116°$$
$$C = 180° - (48° + 116°)$$
$$= 16°$$
$$\frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$
$$\frac{b}{\sin(116°)} = \frac{18}{\sin(16°)}$$
$$b = \frac{18\sin(116°)}{\sin(16°)}$$
$$\approx 58.69$$

(Note: Rounding could be done later.)



$$\sin(\theta) = \frac{opp}{hyp}$$
$$\sin(48°) = \frac{h}{58.69}$$
$$h = 58.69\ \sin(48°)$$
$$= 43.62$$

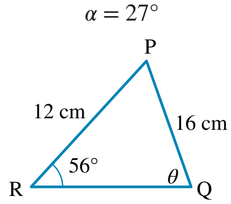
The height of the building is 43.62 metres.

9 a $\dfrac{\sin(A)}{a} = \dfrac{\sin(B)}{b}$
$$\frac{\sin(100°)}{46} = \frac{\sin(\theta)}{32}$$
$$\sin(\theta) = \frac{32\sin(100°)}{46}$$
$$\theta = 43°$$

b $\dfrac{\sin(\phi)}{18.9} = \dfrac{\sin(60°)}{29.5}$
$$\sin(\phi) = \frac{18.9\sin(60°)}{29.5}$$
$$\phi = 34°$$

c $\dfrac{\sin(\alpha)}{79} = \dfrac{\sin(117°)}{153}$
$$\sin(\alpha) = \frac{79\sin(117°)}{153°}$$
$$\alpha = 27°$$

10

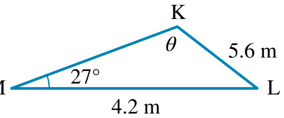


$$\frac{\sin(\theta)}{12} = \frac{\sin(56°)}{16}$$

$$\sin(\theta) = \frac{12\sin(56°)}{16}$$

$$\theta = 38°$$

11

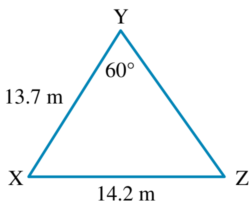


$$\frac{\sin(\theta)}{4.2} = \frac{\sin(27°)}{5.6}$$

$$\sin(\theta) = \frac{4.2\sin(27°)}{5.6}$$

$$\theta = 20°$$

12 a



Let $\angle XZY = \theta$

$$\frac{\sin(\theta)}{13.7} = \frac{\sin(60°)}{14.2}$$

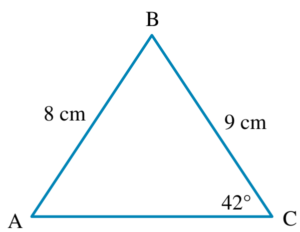
$$\sin(\theta) = \frac{13.7\sin(60°)}{14.2}$$

$$\theta = 57°$$

b $\angle YXZ = 180° - (60° + 57°)$

$= 63°$

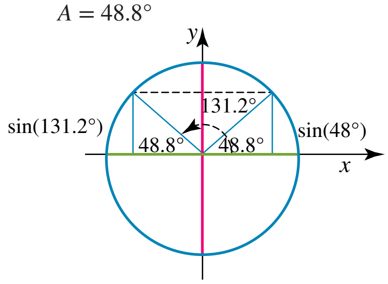
13



$$\frac{\sin(A)}{9} = \frac{\sin(42°)}{8}$$

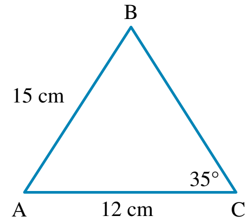
$$\sin(A) = \frac{9\ \sin(42°)}{8}$$

$$A = 48.8°$$



Referring to the unit circle, it can be seen that:
$\sin(48.8°) = \sin(180° - 48.8°) = \sin(131.2°)$
Therefore, either $A = 48.8°$ or $A = 131.2°$

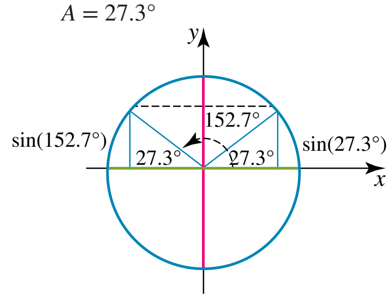
14



$$\frac{\sin(B)}{12} = \frac{\sin\,(35°)}{15}$$

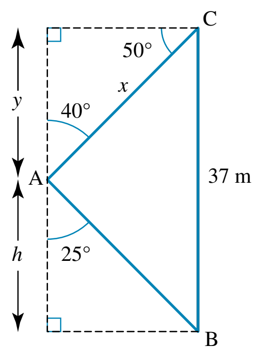
$$\sin(A) = \frac{12\ \sin(35°)}{15}$$

$$A = 27.3°$$



Referring to the unit circle, it can be seen that:
$\sin(27.3°) = \sin(180° - 27.3°) = \sin(152.7°)$
Therefore, either $A = 27.3°$ or $A = 152.7°$

15



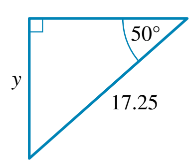
$$A = 180° - (40° + 25°)$$
$$= 115°$$
$$B = 25°$$
$$C = 40°$$

$$\frac{b}{\sin(B)} = \frac{a}{\sin(A)}$$

$$\frac{x}{\sin(25°)} = \frac{37}{\sin(115°)}$$

$$x = \frac{37\ \sin(25°)}{\sin(115°)}$$

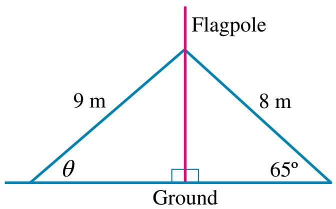
$$= 17.25$$



$$\sin(\theta) = \frac{opp}{hyp}$$
$$\sin(50°) = \frac{y}{17.25}$$
$$y = 17.25\ \sin(50°)$$
$$= 13.21$$
$$h = 37 - y$$
$$= 37 - 13.21$$
$$= 23.79$$

Total length of rope required
$= 2 + x + h$
$= 2 + 17.25 + 23.79$
$= 43.04$ metres
Therefore 45 m is enough rope since only 43 m is required.

16

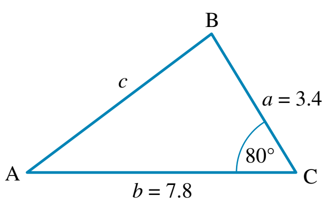


$$\frac{\sin(\theta)}{8} = \frac{\sin(65°)}{9}$$
$$\sin(\theta) = \frac{8\sin(65°)}{9}$$
$$\theta = 54°$$
(to the nearest degree.)

Exercise 9.4 – The cosine rule

1 a $a^2 = b^2 + c^2 - 2bc\cos(A)$
b $r^2 = p^2 + q^2 - 2pq\cos(R)$
c $n^2 = l^2 + m^2 - 2lm\cos(N)$

2



$$c^2 = a^2 + b^2 - 2ab\cos(C)$$
$$= 3.4^2 + 7.8^2 - 2 \times 3.4 \times 7.8 \times \cos(80°)$$
$$= 11.56 + 60.84 - 9.2103$$
$$= 63.1897$$
$$c = \sqrt{63.1897}$$
$$= 7.9492$$
$$c = 7.95$$

3 a $b^2 = a^2 + c^2 - 2ac\cos(B)$
$$x^2 = 14^2 + 12^2 - 2 \times 14 \times 12\cos(35°)$$
$$= 196 + 144 - 275.235$$
$$= 64.765$$
$$x = \sqrt{64.765}$$
$$= 8.05\text{ m}$$

b $r^2 = p^2 + q^2 - 2pq\cos(R)$
$$= 21^2 + 13^2 - 2 \times 21 \times 13 \times \cos(42°)$$
$$= 441 + 169 - 405.757$$
$$= 204.243$$
$$r = \sqrt{204.243}$$
$$= 14.3\text{ cm}$$

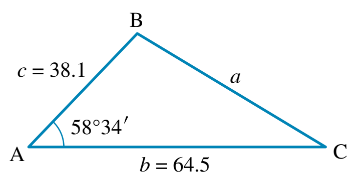
c $x^2 = y^2 + z^2 - 2yz\cos(X)$
$$= 12^2 + 12^2 - 2 \times 12 \times 12\cos(60°)$$
$$= 144 + 144 - 144$$
$$= 144$$
$$x = \sqrt{144}$$
$$= 12.0\text{ m}$$

4 a $x^2 = z^2 + y^2 - 2zy\cos(X)$
$$= 112^2 + 114^2 - 2 \times 112 \times 114\ \cos(110°)$$
$$= 34273.826$$
$$x = \sqrt{34273.826}$$
$$= 185.1\text{ cm}$$

b $b^2 = a^2 + c^2 - 2ac\cos(B)$
$$= 9.7^2 + 6.1^2 - 2 \times 9.7 \times 6.1\ \cos(130°)$$
$$= 207.3675$$
$$b = \sqrt{207.3675}$$
$$= 14.4\text{ m}$$

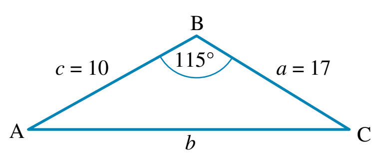
c $q^2 = p^2 + r^2 - 2pr\cos(Q)$
$$= 63^2 + 43^2 - 2 \times 63 \times 43\ \cos(160°)$$
$$= 10909.2546$$
$$q = \sqrt{10909.2546}$$
$$= 104.4\text{ mm}$$

5



$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$= 64.5^2 + 38.1^2 - 2 \times 64.5 \times 38.1 \times \cos(58°34')$$
$$= 4160.25 + 1451.61 - 4914.9 \times 0.5215$$
$$= 5611.86 - 2563.1504$$
$$= 3048.7096$$
$$a = \sqrt{3048.7096}$$
$$= 55.215$$
$$a = 55.22$$

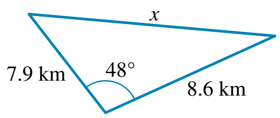
6



$$b^2 = c^2 + a^2 - 2ca\cos(B)$$
$$= 10^2 + 17^2 - 2 \times 10 \times 17 \times \cos(115°)$$
$$= 100 + 289 - 340 \times (-0.4226)$$
$$= 389 + 143.6902$$
$$b^2 = 532.6902$$
$$b = \sqrt{532.6902}$$
$$= 23.080$$
$$= 23.08$$

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{23.08^2 + 10^2 - 17^2}{2 \times 23.08 \times 10}$$
$$= \frac{532.6902 + 100 - 289}{461.6}$$
$$= \frac{343.6902}{461.6}$$
$$= 0.744\,563$$
$$A = \cos^{-1}(0.744\,563)$$
$$= 41.878$$
$$= 41°53'$$
$$C = 180° - (41°53' + 115°)$$
$$= 23°7'$$

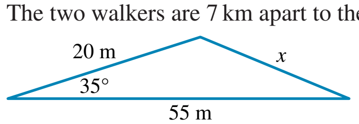
7



$$x^2 = 7.9^2 + 8.6^2 - 2 \times 7.9 \times 8.6 \cos(48°)$$
$$= 45.4485$$
$$x = \sqrt{45.4485}$$
$$= 6.7\text{ km}$$
$$= 7\text{ km}$$

The two walkers are 7 km apart to the nearest metre.

8



$$x^2 = 20^2 + 55^2 - 2 \times 20 \times 55 \cos(35°)$$
$$= 1622.8655$$
$$x = \sqrt{1622.8655}$$
$$= 40.3\text{ m}$$

The cricketer must run 40 metres to field the ball.

9 a
$$\cos(A) = \frac{b^2 + c^2 - a^2}{2\,bc}$$
$$= \frac{11^2 + 8^2 - 13^2}{2 \times 11 \times 8}$$
$$= \frac{16}{176}$$
$$A = \cos^{-1}\left(\frac{16}{176}\right)$$
$$= 85°$$

b
$$\cos(B) = \frac{a^2 + c^2 - b^2}{2\,ac}$$
$$= \frac{2.8^2 + 3.2^2 - 4.0^2}{2 \times 2.8 \times 3.2}$$
$$= \frac{2.08}{17.92}$$
$$B = \cos^{-1}\left(\frac{2.08}{17.92}\right)$$
$$= 83°$$

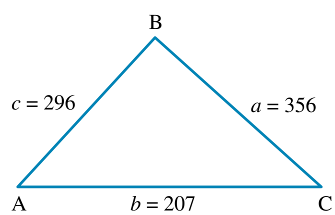
c
$$\cos(O) = \frac{n^2 + m^2 - o^2}{2 \times n \times m}$$
$$= \frac{5.4^2 + 6.2^2 - 4.5^2}{2 \times 5.4 \times 6.2}$$
$$= \frac{47.35}{66.96}$$
$$O = \cos^{-1}\left(\frac{47.35}{66.96}\right)$$
$$= 45°$$

10 a
$$\cos(\theta) = \frac{6^2 + 8^2 - 11^2}{2 \times 6 \times 8}$$
$$= \frac{-21}{96}$$
$$\theta = \cos^{-1}\left(\frac{-21}{96}\right)$$
$$= 103°$$

b
$$\cos(\alpha) = \frac{4.2^2 + 6.1^2 - 9.6^2}{2 \times 4.2 \times 6.1}$$
$$= \frac{-37.31}{51.24}$$
$$\alpha = \cos^{-1}\left(\frac{-37.31}{54.24}\right)$$
$$= 137°$$

c
$$\cos(\theta) = \frac{9.2^2 + 12.9^2 - 4.2^2}{2 \times 9.2 \times 12.9}$$
$$= \frac{233.41}{237.36}$$
$$\theta = \cos^{-1}\left(\frac{233.41}{237.36}\right)$$
$$= 10°$$

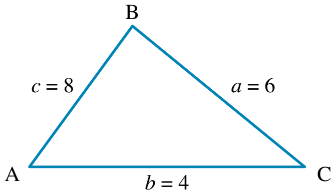
11



Largest angle is opposite the longest side.

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{207^2 + 296^2 - 356^2}{2 \times 207 \times 296}$$
$$= \frac{42\,849 + 87\,616 - 126\,736}{122\,544}$$
$$= \frac{3729}{122\,544}$$
$$= 0.030\,43$$
$$A = \cos^{-1}(0.030\,43)$$
$$= 88.256$$
$$= 88°15'$$

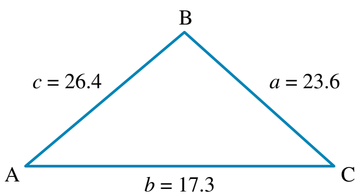
12



Smallest angle is opposite the shortest side.

$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$
$$= \frac{6^2 + 8^2 - 4^2}{2 \times 6 \times 8}$$
$$= \frac{36 + 64 - 16}{96}$$
$$= \frac{84}{96}$$
$$= 0.875$$
$$B = \cos^{-1}(0.875)$$
$$= 28.955$$
$$= 28°57'$$

13



$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{17.3^2 + 26.4^2 - 23.6^2}{2 \times 17.3 \times 26.4}$$
$$= \frac{439.29}{913.44}$$
$$= 0.480\,92$$
$$A = \cos^{-1}(0.480\,92)$$
$$= 61.2546$$
$$A = 61°15'$$
$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$
$$= \frac{23.6^2 + 26.4^2 - 17.3^2}{2 \times 23.6 \times 26.4}$$
$$= \frac{954.63}{1246.08}$$
$$= 0.7661$$
$$B = \cos^{-1}(0.7661)$$
$$= 39.994$$
$$= 39°60'$$
$$= 40°$$
$$C = 180° - (61°15' + 40°)$$
$$= 78°45'$$

14 Angle opposite the 12 – cm side

$$\cos(\theta) = \frac{14^2 + 17^2 - 12^2}{2 \times 14 \times 17}$$
$$= \frac{341}{476}$$
$$\theta = \cos^{-1}\left(\frac{341}{476}\right)$$
$$= 44°$$

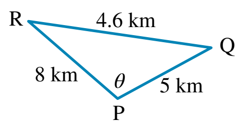
Angle opposite the 14 cm side

$$\cos(\alpha) = \frac{12^2 + 17^2 - 14^2}{2 \times 12 \times 17}$$
$$= \frac{237}{408}$$
$$\alpha = \cos^{-1}\left(\frac{237}{408}\right)$$
$$= 54°$$

Third angle $= 180° - (44° + 54°)$

$= 82°$

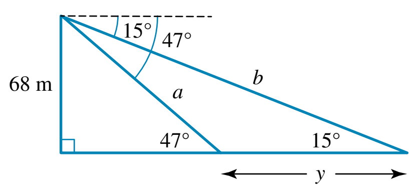
15



$$\cos(\theta) = \frac{8^2 + 5^2 - 4.6^2}{2 \times 8 \times 5}$$
$$= \frac{67.84}{80}$$
$$\theta = \cos^{-1}\left(\frac{67.84}{80}\right)$$
$$= 32°$$

The two roads diverge at 32°

16 Method 1:



$$\sin(\theta) = \frac{opp}{hyp}$$
$$\sin(47°) = \frac{68}{a}$$
$$a\sin(47°) = 68$$
$$a = \frac{68}{\sin(47°)}$$
$$a = 92.98$$
$$\sin(\theta) = \frac{opp}{hyp}$$
$$\sin(15°) = \frac{68}{b}$$
$$b\sin(15°) = 68$$
$$b = \frac{68}{\sin(15°)}$$
$$b = 262.73$$

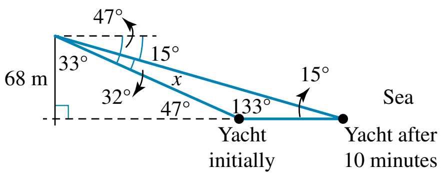
$47° - 15° = 32°$

$$y^2 = a^2 + b^2 - 2ab\cos(32°)$$
$$= 92.98^2 + 262.73^2 - 2 \times 92.98 \times 262.73 \times \cos(32°)$$
$$= = 77\,672.333 - 41\,433.315$$
$$= 36\,239.018$$
$$y = \sqrt{36\,239.018}$$
$$= 190.37$$

$$Speed = \frac{distance}{time}$$
$$= \frac{190.37 \div 1000}{\frac{10}{60}}$$
$$\approx 1.14$$

Speed of the yacht is 1.14 km/h.

Method 2:

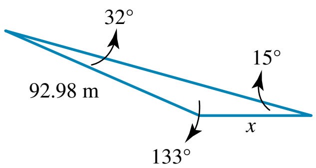


$$\sin(\theta) = \frac{opp}{hyp}$$

$$\sin(47°) = \frac{68}{x}$$

$$x = \frac{68}{\sin(47°)}$$

$$x = 92.98$$



$$\frac{x}{\sin(32°)} = \frac{92.98}{\sin(15°)}$$

$$x = \frac{92.98 \times \sin(32°)}{\sin(15°)}$$

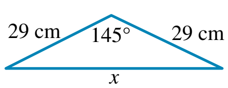
$$= 190.368\text{ m}$$

$$Speed = \frac{distance}{time}$$

$$= \frac{0.19037\text{ km}}{0.167\text{ hour}}$$

$$= 1.14\text{ km/h}$$

17



$$x^2 = 29^2 + 29^2 - 2 \times 29 \times 29\ \cos(145°)$$

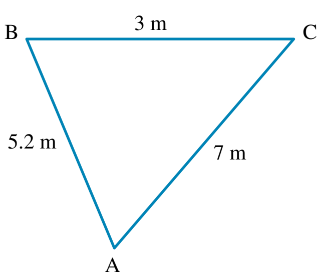
$$= 3059.8137$$

$$x = \sqrt{3059.5137}$$

$$= 55.316\text{ cm}$$

Length of backing is 55 cm.

18



$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{7^2 + 5.2^2 - 3^2}{2 \times 7 \times 5.2}$$

$$= \frac{67.04}{72.8}$$

$$= 0.920\,88$$

$$A = \cos^{-1}(0.920\,88)$$

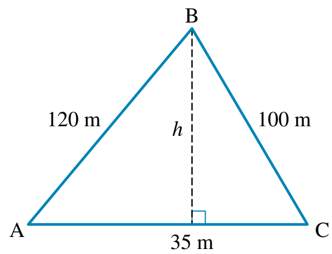
$$= 22.945$$

$$= 22°57'$$

$$\approx 23°$$

The shot must be made within 23°.

19



$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{35^2 + 120^2 - 100^2}{2 \times 35 \times 120}$$

$$= \frac{5625}{8400}$$

$$= 0.669\,64$$

$$A = \cos^{-1}(0.669\,64)$$

$$= 47.960$$

$$= 47°58'$$

$$\sin(\theta) = \frac{opp}{hyp}$$

$$\sin(47°58') = \frac{h}{120}$$

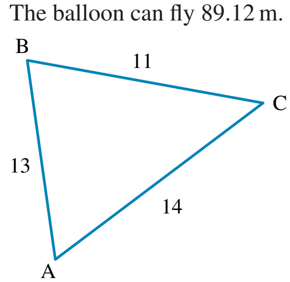
$$h = 120\ \sin(47°58')$$

$$h = 120\ \sin(47.96)$$

$$h = 89.12$$

The balloon can fly 89.12 m.

20



$a = 5 + 6 = 11$

$b = 6 + 8 = 14$

$c = 5 + 8 = 13$

Largest angle is opposite the longest side.

$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{11^2 + 13^2 - 14^2}{2 \times 11 \times 13}$$

$$= \frac{94}{286}$$

$$= 0.328\,67$$

$$B = \cos^{-1}(0.328\,67)$$

$$= 70.8118$$

$$B = 70°49'$$

The largest angle is 70°49′.

Exercise 9.5 – Area of a triangle

1 Area $= \frac{1}{2}ab\ \sin(C)$

$= \frac{1}{2}(11.9)(14.4)\sin(38°)$

$= 52.75\ \text{mm}^2$

2 Area $= \frac{1}{2}ab\ \sin(C)$

$= \frac{1}{2}(14.3)(6.5)\sin(32°)$

$= 24.63\ \text{mm}^2$

3 $a = 10,\ b = 6\sqrt{2},\ c = 2\sqrt{13}$ cm and $C = 45°$.

Area is $A_\Delta = \frac{1}{2}ab\sin(C)$

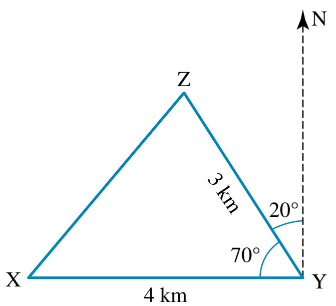
$\therefore A_\Delta = \frac{1}{2} \times 10 \times 6\sqrt{2} \times \sin(45°)$

$= 30\sqrt{2} \times \frac{\sqrt{2}}{2}$

$\therefore A_\Delta = 30$

The area is 30 sq cm.

4



In triangle XYZ , the angle of 70° is included between the sides XY and YZ of lengths 4 and 3 km respectively.

The area of triangle XYZ is $A_\Delta = \frac{1}{2} \times 3 \times 4 \times \sin(70°)$.

$\therefore A_\Delta = 6 \times \sin(70°)$

$\therefore A_\Delta \simeq 5.64$

Correct to two decimal places, the horses can graze over an area of 5.64 square km.

5 a Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 19.4 \times 11.7$

$= 113.49\ \text{cm}^2$

b Area $= \frac{1}{2}bc\sin(A)$

$= \frac{1}{2} \times 12.4 \times 9.1 \times \sin(57°)$

$= 47.3177\ldots$

$= 47.32\ \text{mm}^2$ (correct to 2 decimal places.)

c Area $= \frac{1}{2}bc\sin(A)$

$= \frac{1}{2} \times 31.2 \times 22.5\sin(38°)$

$= 216.097\ldots$

$= 216.10\ \text{cm}^2$ (correct to 2 decimal places.)

d Missing angle $= 180° - 65° - 41°$

$= 74°$

$\frac{x}{\sin(65°)} = \frac{19.9}{\sin(74°)}$

$x = \frac{19.9\sin(65°)}{\sin(74°)}$

$= 18.762\ldots$

Area $= \frac{1}{2}bc\sin(A)$

$= \frac{1}{2} \times 19.9 \times 18.76\ \ldots\ \sin(41°)$

$= 122.476\ldots$

$= 122.48\ \text{cm}^2$ (correct to 2 decimal places.)

6 a i In the isosceles triangle, the 20° angle is included between the two equal sides of 5 cm.

The area of the triangle is

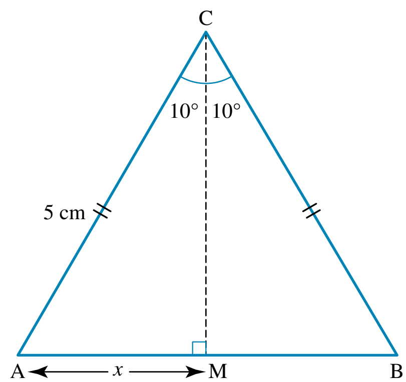
$A_\Delta = \frac{1}{2} \times 5 \times 5 \times \sin(20°)$

$= 12.5 \times \sin(20°)$

$\therefore A_\Delta = 4.275$

The area is 4.275 sq cm correct to three decimal places.

ii Divide the isosceles triangle into two right angled triangles by joining C to the midpoint M of the side AB.



CM bisects the angle ACB and the side AB.

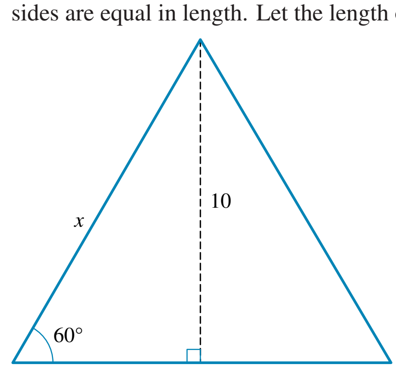
Let AM have length x metres so AB has length $2x$ metres.

$\sin(10°) = \frac{x}{5}$

$\therefore x = 5\sin(10°)$

The third side, AB has length $10\sin(10°) \simeq 1.736$ cm.

b The angles in an equilateral triangle are each 60° and the sides are equal in length. Let the length of a side be x cm.



$$\sin(60°) = \frac{10}{x}$$

$$\therefore \frac{\sqrt{3}}{2} = \frac{10}{x}$$

$$\therefore \sqrt{3}x = 20$$

$$\therefore x = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\therefore x = \frac{20\sqrt{3}}{3}$$

The perimeter is $3x = 20\sqrt{3}$ cm.

The base and height of the triangle are known so its area is:

$$A_\Delta = \frac{1}{2}bh$$

$$\therefore A_\Delta = \frac{1}{2} \times \frac{20\sqrt{3}}{3} \times 10$$

$$\therefore A_\Delta = \frac{100\sqrt{3}}{3}$$

Area is $\frac{100\sqrt{3}}{3}$ sq cm.

c In triangle ABC $a = 4\sqrt{2}$ cm, $b = 6$ cm and $C = 30°$.

$$A_\Delta = \frac{1}{2}ab\sin(C)$$

$$= \frac{1}{2} \times 4\sqrt{2} \times 6 \times \sin(30°)$$

$$= 12\sqrt{2} \times \frac{1}{2}$$

$$= 6\sqrt{2}$$

Area is $6\sqrt{2}$ sq cm.

7 If the 40.2° angle is between the two given sides:

$$\text{Area} = \frac{1}{2}bc\sin(A)$$

$$= \frac{1}{2} \times 9.5 \times 13.5\sin(40.2°)$$

$$= 41.389\ldots$$

$= 41.39\text{ cm}^2$ (correct to 2 decimal places.)

If the 40.2° angle is opposite the 9.5 cm side:

$$\frac{13.5}{\sin(x)} = \frac{9.5}{\sin(40.2°)}$$

$$\sin(x) = \frac{13.5\sin(40.2°)}{9.5}$$

$$x = \sin^{-1}\left(\frac{13.5\sin(40.2°)}{9.5}\right)$$

$$= 66.524\ldots°$$

$$180° - 40.2° - 66.524\ldots° = 73.275\ldots°$$

$$\text{Area} = \frac{1}{2}bc\sin(A)$$

$$= \frac{1}{2} \times 9.5 \times 13.5\sin(73.275\ldots°)$$

$$= 61.412\ldots$$

$= 61.41\text{ cm}^2$ (correct to 2 decimal places.)

If the 40.2° angle is opposite the 13.5 cm side:

$$\frac{9.5}{\sin(x)} = \frac{13.5}{\sin(40.2°)}$$

$$\sin(x) = \frac{9.5\sin(40.2°)}{13.5}$$

$$x = \sin^{-1}\left(\frac{9.5\sin(40.2°)}{13.5}\right)$$

$$= 27.014\ldots°$$

$$180° - 40.2° - 27.014\ldots° = 112.785\ldots°$$

$$\text{Area} = \frac{1}{2}bc\sin(A)$$

$$= \frac{1}{2} \times 9.5 \times 13.5\sin(112.785\ldots°)$$

$$= 59.120\ldots$$

$= 59.12\text{ cm}^2$ (correct to 2 decimal places.)

8 a $A = \frac{1}{2} \times (109 + 78) \times 62$

$$= 5797\text{ m}^2$$

b $A = \frac{1}{2} \times 40 \times 50 \times \sin(30°) + \frac{1}{2} \times 50 \times 35 \times \sin(40°)$

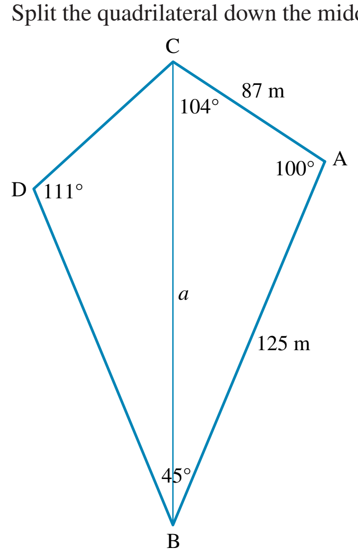
$$= 1062\text{ m}^2$$

c $A = \frac{1}{2} \times 121 \times 214 \times \sin(32°) + \frac{1}{2} \times 214 \times 190$

$$\times \sin(34°) + \frac{1}{2}190 \times 156 \times \sin(41°)$$

$$= 27952\text{ m}^2$$

9 Split the quadrilateral down the middle and label this length a.



$$a^2 = 87^2 + 125^2 - 2 \times 87 \times 125\cos(100°)$$

$$= 23\,194 - -3776.847\ldots$$

$$= 26\,970.847\ldots$$

$$a = \sqrt{26\,970.847\ldots}$$

$$= 164.228\ldots$$

$$\frac{87}{\sin(B)} = \frac{164.228\ldots}{\sin(100°)}$$

$$\sin(B) = \frac{87\sin(100°)}{164.228\ldots}$$

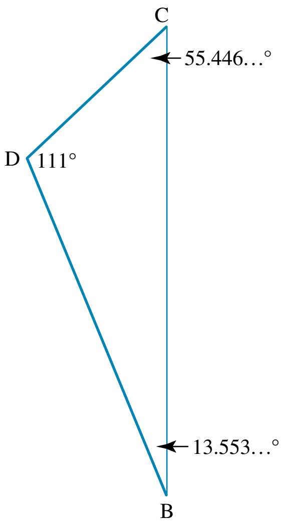
$$B = \sin^{-1}\left(\frac{87\sin(100°)}{164.228\ldots}\right)$$

$$= 31.446\ldots°$$

For ΔBCD:

$\angle DBC = 45° - 31.446\ldots°$
$= 13.553\ldots°$

$\angle DCB = 180° - 111° - 13.553\ldots°$
$= 55.446\ldots°$



$$\frac{x}{\sin(55.446\ldots°)} = \frac{164.228\ldots}{\sin(111°)}$$

$$x = \frac{164.228\ldots\sin(55.446\ldots°)}{\sin(111°)}$$

$= 144.880\ldots$

$$\text{Area of } \Delta BCD = \frac{1}{2} \times 144.880\ldots \times 164.228\ldots \times \sin(13.553\ldots°)$$

$= 2788.236\ldots$
$= 2788.24\text{ m}^2$ (correct to 2 decimal places.)

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 87 \times 125 \times \sin(100°)$$

$= 5354.892\ldots$
$= 5354.89\text{ m}^2$ (correct to 2 decimal places.)

Total area $= 2788.24 + 5354.89$
$= 8143.13\text{ m}^2$ (correct to 2 decimal places.)

Volume $= A \times H$
$= 8143.13 \times 0.001$
$= 8.14313$
$= 8.14\text{ m}^3$ (correct to 2 decimal places.)

10 a First, the lengths of FH, FA and AH are calculated:

$FH^2 = FG^2 + GH^2$
$= (6)^2 + (10)^2$
$= 136$

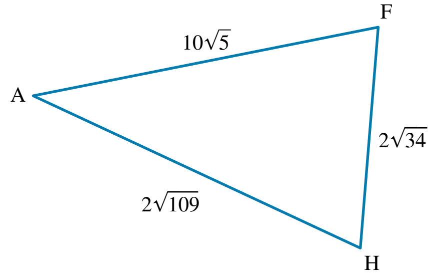
$FH = \sqrt{136} = 2\sqrt{34}$ cm

$FA^2 = FB^2 + FA^2$
$= (20)^2 + (10)^2$
$= 500$

$FA = \sqrt{500} = 10\sqrt{5}$ cm

$AH^2 = DH^2 + AD^2$
$= (20)^2 + (6)^2$
$= 436$

$AH = \sqrt{436} = 2\sqrt{109}$ cm



The angle A for Triangle AFH is calculated using the Cosine Rule:

$FH^2 = AH^2 + AF^2 - 2(AH)(AF)\cos(A)$

$136 = 436 + 500 - 2\left(2\sqrt{109}\right)\left(10\sqrt{5}\right)\cos(A)$

$136 = 936 - 40\sqrt{545}\cos(A)$

$$\cos(A) = \frac{936 - 136}{40\sqrt{545}}$$

$A = 31.1°$

Using the area equation:

$$\text{Area} = \frac{1}{2}(AH)(AF)\sin(A)$$

$$= \frac{1}{2}\left(2\sqrt{109}\right)\left(10\sqrt{5}\right)\sin(31.1°)$$

$= 120.6\text{ cm}^2$

b The total surface area of the block will be equal to the sum of the exposed faces:

Total Surface Area = Area (AFH) + Area (ABF)
+ Area (ADH) + Area (FGH)
+ Area $(ABCD)$ + Area $(BFGC)$
+ Area $(DCGH)$

$$\text{Area}(ABF) = \frac{1}{2}(10)(20) = 100\text{ cm}^2$$

$$\text{Area}(ADH) = \frac{1}{2}(6)(20) = 60\text{ cm}^2$$

$$\text{Area}(FGH) = \frac{1}{2}(6)(10) = 30\text{ cm}^2$$

$\text{Area}(ABCD) = (6)(10) = 60\text{ cm}^2$

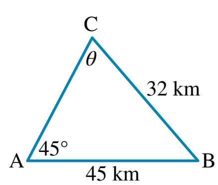
$\text{Area}(BFGC) = (6)(20) = 120\text{ cm}^2$

$\text{Area}(DCGH) = (10)(20) = 200\text{ cm}^2$

Total Surface Area $= 120.6\text{ cm}^2 + 100\text{ cm}^2 + 60\text{ cm}^2$
$+ 30\text{ cm}^2 + 60\text{ cm}^2 + 120\text{ cm}^2$
$+ 200\text{ cm}^2$
$= 690.6\text{ cm}^2$

Exercise 9.6 – Applications of the sine and cosine rules

1

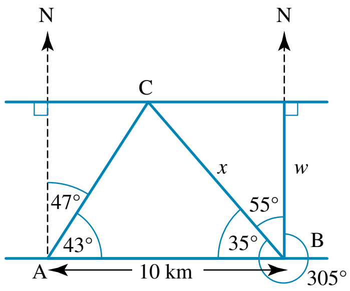


$$\frac{\sin(\theta)}{45} = \frac{\sin(45°)}{32}$$

$$\sin\theta = \frac{45\sin(45°)}{32}$$

$\theta = 84°$

2



a $C = 180° - (43° + 35°)$

$= 102°$

$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)}$$

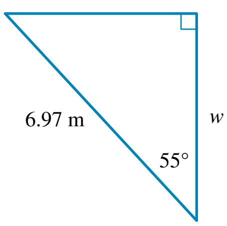
$$\frac{x}{\sin(43°)} = \frac{10}{\sin(102°)}$$

$$x = \frac{10\ \sin(43°)}{\sin(102°)}$$

$$= 6.97$$

The distance from the second bearing to the tree is 6.97 metres.

b



$$\cos(\theta) = \frac{adj}{hyp}$$

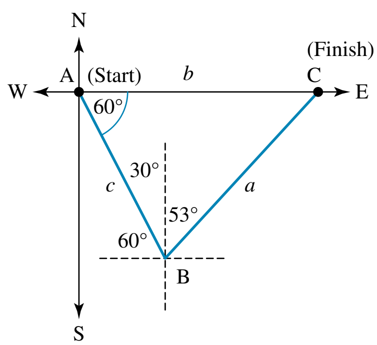
$$\cos(55°) = \frac{w}{6.97}$$

$$w = 6.97\ \cos(55°)$$

$$= 3.9978$$

The width of the river is 4 metres.

3



a $B = 30° + 53°$

$= 83°$

$C = 180° - (60° + 83°)$

$= 37°$

Distance = speed × time

$$c = 8 \times \frac{45}{60}$$

$$= 6\ \text{km}$$

$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)}$$

$$\frac{a}{\sin(60°)} = \frac{6}{\sin(37°)}$$

$$a = \frac{6\ \sin(60°)}{\sin(37°)}$$

$$= 8.634$$

The second leg is 8.63 km long.

b $$Speed = \frac{distance}{time}$$

$$= \frac{8.634}{\frac{80}{60}}$$

$$= \frac{8.634 \times 60}{80}$$

$$= 6.48$$

Her speed was 6.48 km/h.

c $$\frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

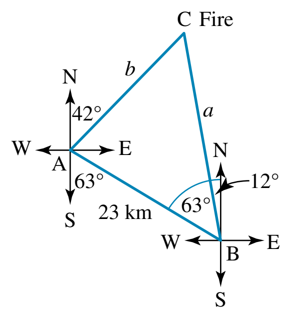
$$\frac{b}{\sin(83°)} = \frac{6}{\sin(37°)}$$

$$b = \frac{6\ \sin(83°)}{\sin(37°)}$$

$$= 9.896$$

She needs to run 9.90 km to get back to the start.

4



$A = 180° - (42° + 63°)$

$= 75°$

$B = 63° - 12°$

$= 51°$

$C = 180° - (75° + 51°)$

$= 54°$

$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)}$$

$$\frac{a}{\sin(75°)} = \frac{23}{\sin(54°)}$$

$$a = \frac{23\ \sin(75°)}{\sin(54°)}$$

$$= 27.46$$

$$\frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

$$\frac{b}{\sin(51°)} = \frac{23}{\sin(54°)}$$

$$b = \frac{23\ \sin(51°)}{\sin(54°)}$$

$$= 22.09$$

The fire is 22.09 km from A and 27.46 km from B.

5 a $\angle ABT = 180° - 35°$ (straight angle)

$= 145°$

so, angle ATB $= 180° - (20° + 145°)$

$= 15°$

b $\dfrac{a}{\sin(A)} = \dfrac{t}{\sin(T)}$

$\dfrac{BT}{\sin(20°)} = \dfrac{30}{\sin(15°)}$

$BT = \dfrac{30\sin(20°)}{\sin(15°)}$

c In Δ BTC,

$\sin(\theta) = \dfrac{opp}{hyp}$

$\sin(35°) = \dfrac{b}{\frac{30\sin(30°)}{\sin(15°)}}$

$h = \dfrac{30\sin(20°)\sin(35°)}{\sin(15°)}$

d $height = \dfrac{30\sin(20°)\sin(35°)}{\sin(15°)}$

$= 22.7\text{ m}$

6 a i $\text{CM} = \dfrac{120}{2}$

$= 60\text{ mm}$

ii $\text{BD} = \sqrt{200^2 + 200^2 - 2\times 200\times 200\times \cos 160}$

$= 394\text{ mm}$

iii $\angle MBC = 10°$

b i $\text{CM} = 125\text{ mm}$

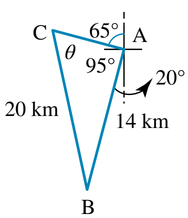
$\text{BD} = 2\times\sqrt{200^2 - 125^2}$

$= 312\text{ mm}$

ii $\text{BD} = 2\times BM = 2\times 200\cos(70) = 136.8 \approx 137\text{ mm}$

$\text{AC} = 2\times CM = 2\times 200\sin(70) = 375.88 \approx 376\text{ mm}$

7



a $\dfrac{\sin(\theta)}{14} = \dfrac{\sin(95°)}{20}$

$\sin(\theta) = \dfrac{14\sin(95°)}{20}$

$\theta = 44°13'$

Third angle is $180° - (95° + 44°13')$

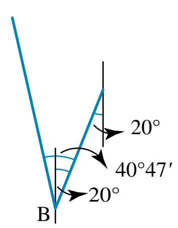
$= 40°47'$

$\dfrac{x}{\sin(40°47')} = \dfrac{20}{\sin(95°)}$

$x = \dfrac{20\sin(40°47')}{\sin(95°)}$

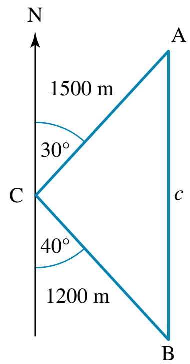
$= 13.11\text{ km}$

b



The bearing for the 20 km leg is N 20°47′W.

8



$C = 180° - (30° + 40°)$

$= 110°$

$c^2 = a^2 + b^2 - 2ab\cos(C)$

$= 1200^2 + 1500^2 - 2\times 1200\times 1500\cos(110°)$

$= 1\,440\,000 + 2\,250\,000 - 3\,600\,000\times(-0.342\,021\,43)$

$= 3\,690\,000 + 1\,231\,272.516$

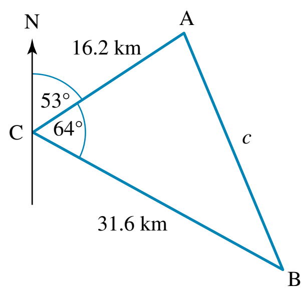
$= 4\,921\,272.516$

$C = \sqrt{492\,127\,2.516}$

$= 2218.394$

The two rowers are 2218 m apart.

9



$C = 117° - 53° = 64°$

$c^2 = a^2 + b^2 - 2ab\cos(C)$

$= 16.2^2 + 31.6^2 - 2\times 16.2\times 31.6\cos(64°)$

$= 1261 - 448.8219$

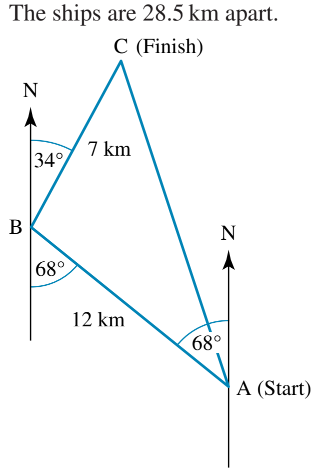
$= 812.1781$

$c = \sqrt{812.1781}$

$= 28.499$

The ships are 28.5 km apart.

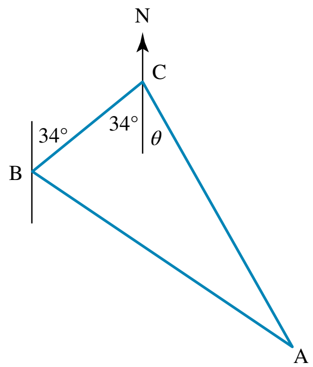
10



a $B = 180° - (34° + 68°)$
$= 78°$
$b^2 = a^2 + c^2 - 2ac\cos(B)$
$= 7^2 + 12^2 - 2 \times 7 \times 12\cos(78°)$
$= 49 + 144 - 168 \times 0.2079$
$= 193 - 34.9292$
$= 158.0708$
$b = \sqrt{158.0708}$
$= 12.573$

He is 12.57 km from his starting point.

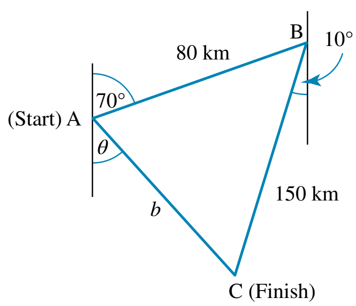
b $\cos(C) = \dfrac{a^2 + b^2 - c^2}{2ab}$
$= \dfrac{7^2 + 12.57^2 - 12^2}{2 \times 7 \times 12.57}$
$= \dfrac{63.0049}{175.98}$
$= 0.358\,023$
$C = \cos^{-1}(0.358\,023)$
$= 69.021$
$= 69°1'$



$\theta = 69°1' - 34°$
$\theta = 35°1'$

The bearing of the starting point from the finishing point is S 35°1′ E.

11



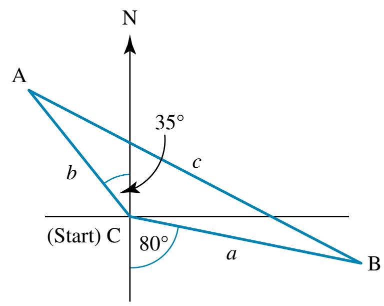
$B = 70° - 10°$
$= 60°$
$b^2 = a^2 + c^2 - 2ac\cos(B)$
$= 150^2 + 80^2 - 2 \times 150 \times 80 \times \cos(60°)$
$= 28\,900 - 12\,000$
$= 16\,900$
$b = \sqrt{16\,900}$
$= 130$

The plane is 130 km from its starting point.

b $\cos(A) = \dfrac{b^2 + c^2 - a^2}{2bc}$
$= \dfrac{130^2 + 80^2 - 150^2}{2 \times 130 \times 80}$
$= \dfrac{800}{20\,800}$
$= 0.038\,462$
$A = \cos^{-1}(0.038\,462)$
$= 87.796$
$= 87°48'$
$\theta = 180° - (87°48' + 70°)$
$= 22°12'$

The plane is on a bearing of S 22°12′E from its starting point.

12



$C = 35° + 90° + 10°$
$= 135°$

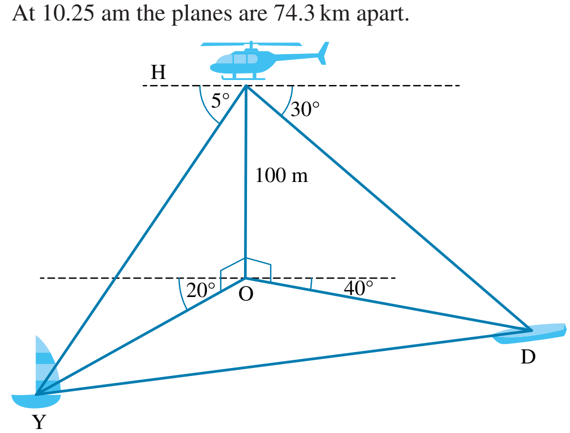
Distance travelled by plane A
$= 120 \times \dfrac{25}{60}$
$= 50\text{ km}$

Distance travelled by plane B
$= 90 \times \dfrac{20}{60}$
$= 30\text{ km}$

So $a = 50\text{ km}$, $b = 30\text{ km}$.
$c^2 = a^2 + b^2 - 2ab\cos(C)$
$= 50^2 + 30^2 - 2 \times 50 \times 30\cos(135°)$
$= 3400 - 3000 \times (-0.70711)$
$= 3400 + 2121.3203$
$= 5521.3203$
$c = \sqrt{5521.3203}$
$= 74.3$

At 10.25 am the planes are 74.3 km apart.

13



$$\tan(30°) = \frac{OH}{OD} = \frac{100}{OD}$$
$$OD = \frac{100}{\tan(30°)}$$
$$OD = 173.2\text{ m}$$
$$\tan(5°) = \frac{OH}{OY} = \frac{100}{OY}$$
$$OY = \frac{100}{\tan(5°)}$$
$$OY = 1\,143\text{ m}$$
$$\angle YOD = 180° - (20° + 40°)$$
$$\angle YOD = 120°$$

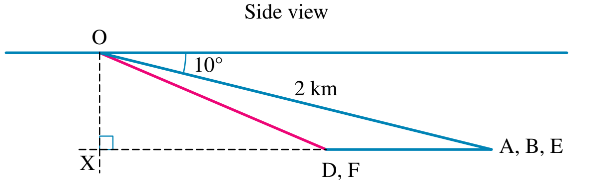
Using the cosine rule:

$$YD^2 = OD^2 + OY^2 - 2(OD)(OY)\cos(\angle YOD)$$
$$= (173.2)^2 + (1143)^2 - 2(173.2)(1143)\cos(120°)$$
$$YD = 1\,238.7\text{ m} \approx 1239\text{ m}$$

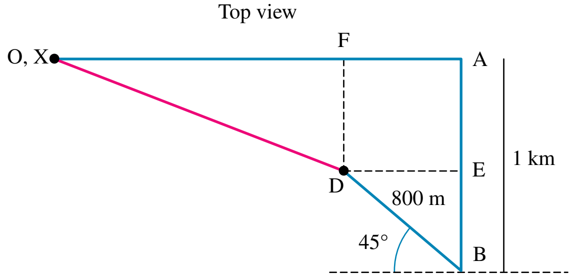
The dinghy and the yacht are 1 239 m apart.

14 Two separate diagrams are drawn showing the mine system as viewed from the side (the vertical plane) and from above (the horizontal plane)

Side view



Top view



a Considering the mine system viewed from the side, it can be seen that

$$XA = OA\cos(\angle OAX)$$
$$= 2000\cos(10°)$$
$$XA = 1969.6\text{ m}$$

Now, looking at the top view of the mine system, we see that

$$XD^2 = XF^2 + FD^2$$

We note the following:

$$XF = XA - AF$$
$$AF = ED$$
$$FD = AE = AB - BE$$
$$ED = DB\sin\angle DBE$$
$$= 800\sin(45°)$$
$$ED = 565.7\text{ m}$$
$$\therefore AF = 565.7\text{ m}$$
$$XF = XA - AF$$
$$= 1969.6 - 565.7$$
$$XF = 1403.9\text{ m}$$
$$BE = DB\cos(\angle DBE)$$
$$= 800\cos(45°)$$
$$BE = 565.7\text{ m}$$
$$FD = AB - BE$$
$$= 1000 - 565.7$$
$$FD = 434.3\text{ m}$$
$$XD^2 = XF^2 + FD^2$$
$$XD^2 = (1403.9)^2 + (434.3)^2$$
$$XD = 1469.5\text{ m}$$

Returning to the side view of the mine system:

$$OX = OA\sin(\angle OAX)$$
$$OX = 2\,000\sin(10°)$$
$$OX = 347.3\text{ m}$$
$$OD^2 = OX^2 + XD^2$$
$$= (347.3)^2 + (1469.5)^2$$
$$OD = 1\,510\text{ m}$$

Therefore, the new mine shaft will be 1510 metres long.

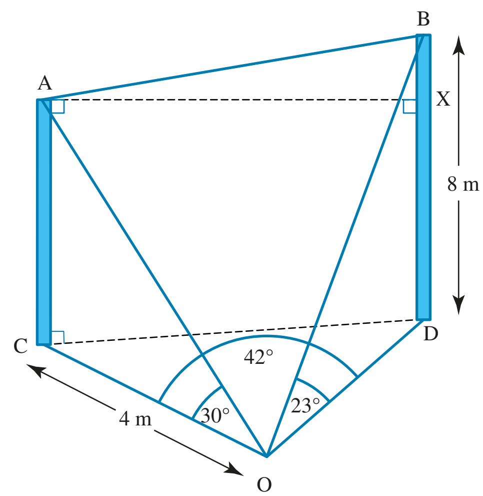
b $\tan(\angle XOD) = \dfrac{XD}{OX}$

$$\angle XOD = \tan^{-1}\left(\frac{XD}{OX}\right)$$
$$= \tan^{-1}\left(\frac{1469.5}{347.3}\right)$$
$$\angle XOD = 76.7°$$

angle with surface $= 90° - 76.7° = 13.3°$

Therefore, the new mine shaft will make an angle of 13.3° to the Earth's surface at the entrance.

15 Let the bases of poles A and B be marked C and D respectively as shown:



$$\frac{AC}{\sin(\angle AOC)} = \frac{CO}{\sin(\angle CAO)}$$
$$\frac{AC}{\sin(30°)} = \frac{4}{\sin(90° - 30°)}$$
$$AC = \frac{4\sin(30°)}{\sin(60°)}$$
$$AC = 2.3\text{ m}$$
$$AO^2 = AC^2 + CO^2$$
$$= (2.3)^2 + (4)^2$$
$$AO = 4.6\text{ m}$$

$$BO = \frac{BD}{\sin(\angle DOB)}$$
$$= \frac{8}{\sin(23°)}$$
$$BO = 20.5\text{ m}$$
$$\tan(\angle DOB) = \frac{BD}{DO}$$
$$DO = \frac{8}{\tan(23°)}$$
$$DO = 18.8\text{ m}$$

Using the Cosine rule:

$$CD^2 = CO^2 + DO^2 - 2(CO)(DO)\cos(\angle COD)$$
$$= (4)^2 + (18.8)^2 - 2(4)(18.8)\cos(42°)$$
$$CD = 16.1\text{ m}$$

As $AX \parallel CD$ and $AC \parallel BD$,

$$AX = CD = 16.1\text{ m}$$
$$BX = BD - AC$$
$$BX = 8 - 2.3 = 5.7\text{ m}$$
$$AB^2 = AX^2 + BX^2$$
$$= (16.1)^2 + (5.7)^2$$
$$AB = 17.1\text{ m}$$

We now use the cosine rule to determine the angle $\angle AOB$:

$$AB^2 = AO^2 + BO^2 - 2(AO)(BO)\cos(\angle AOB)$$
$$(17.1)^2 = (4.6)^2 + (20.5)^2 - 2(4.6)(20.5)\cos(\angle AOB)$$
$$\cos(\angle AOB) = \frac{(17.1)^2 - ((4.6)^2 + (20.5)^2)}{-2(4.6)(20.5)}$$
$$\angle AOB = \cos^{-1}(0.79)$$
$$\angle AOB = 37.8°$$

Area of the sail can now be determined.

$$\text{Area} = \frac{1}{2}(AO)(BO)\sin(\angle AOB)$$
$$= \frac{1}{2}(4.6)(20.5)\sin(37.8°)$$
$$\text{Area} = 28.9\text{ m}^2$$

Therefore, the area of sail cloth required is 28.9 m^2.

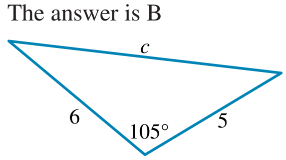
9.7 Review: exam practice

1 $100° \times \frac{\pi}{180} = \frac{5\pi^c}{9}$ OR 1.75^c

Answer is A.

2 There is not enough information to solve triangle B.
The answer is B

3



$$c^2 = a^2 + b^2 - 2ab\cos(C)$$
$$= 5^2 + 6^2 - 2 \times 5 \times 6\cos(105°)$$
$$= 76.5291$$
$$c = \sqrt{76.5291}$$
$$= 8.748$$

The answer is C.

4 $\theta = \pi - \frac{\pi}{5}$
$$= \frac{5\pi - \pi}{5}$$
$$= \frac{4\pi}{5}$$

The answer is B

5 For the unit circle, $\sin(\theta)$ will be the side opposite the angle PON. This is side NP.
The answer is D

6 $\frac{11\pi^c}{9} \times \frac{180}{\pi}$
$$= \frac{11 \times 180}{9}$$
$$= 220°$$

7 $3\sin(2\theta) = 1.56$

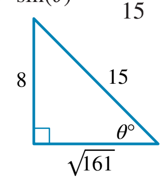
θ is in the 1st quadrant

$$\sin(2\theta) = 0.52$$

reference angle $= 0.5469$

$$2\theta = 0.5469, \pi - 0.5469$$
$$2\theta = 0.5469, 2.5947$$
$$\theta = 0.273, 1.297$$

8 $\sin(\theta) = -\frac{8}{15}$

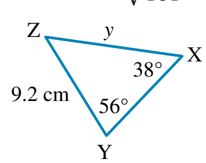


θ is in the 3rd quadrant

a $\cos(\theta) = -\frac{\sqrt{161}}{15}$

b $\tan(\theta) = \frac{8}{\sqrt{161}}$

9



$$\frac{y}{\sin(Y)} = \frac{x}{\sin(X)}$$
$$\frac{y}{\sin(56°)} = \frac{9.2}{\sin(38°)}$$
$$y = \frac{9.2\sin(56°)}{\sin(38°)}$$
$$= 12.4\text{ cm}$$

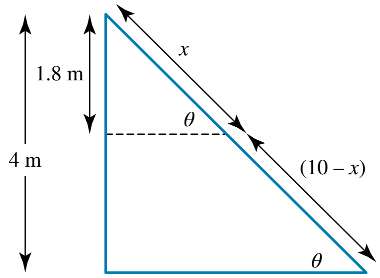
10 $\frac{\sin(\alpha)}{4.1} = \frac{\sin(123°)}{9.7}$
$$\sin(\alpha) = \frac{4.1\sin(123°)}{9.7}$$
$$\alpha = 21°$$

11 Third angle $= 180° - (31° + 28°)$
$$= 121°$$

$$\frac{d}{\sin(D)} = \frac{a}{\sin(A)}$$
$$\frac{d}{\sin(31°)} = \frac{136}{\sin(121°)}$$
$$d = \frac{136\ \sin(31°)}{\sin(121°)}$$
$$= 81.7\text{ mm}$$

12 $c^2 = 6.2^2 + 6.9^2 - 2 \times 6.2 \times 6.9 \times \cos(128°)$
$= 138.726$
$c = \sqrt{138.726}$
$= 11.8\text{ cm}$

13



$$\sin(\theta) = \frac{4}{10} = 0.4$$
$$\theta = \sin^{-1}(0.4) = 23.58°$$
$$x = \frac{1.8}{\sin(23.58°)}$$
$$x = 4.5\text{ m}$$
$10 - x = 10 - 4.5 = 5.5\text{ m}$

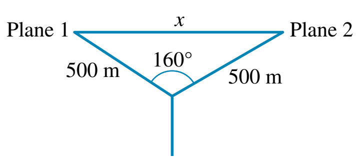
The person would need to climb 5.5 m up the ladder.

14 In an equilateral triangle, all angles are 60°.

$$A = \frac{1}{2}ab\ \sin(C)$$
$$= \frac{1}{2}\left(\sqrt{12}\right)\left(\sqrt{12}\right)\sin(60°)$$
$$= \frac{1}{2}\left(\sqrt{12}\right)\left(\sqrt{12}\right)\frac{\sqrt{3}}{2}$$
$$= 3\sqrt{3}$$

The area is $3\sqrt{3}\text{ m}^2$

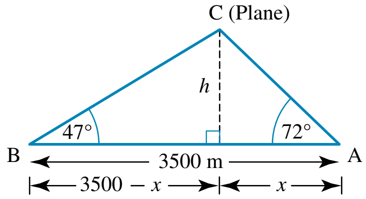
15



$x^2 = 500^2 + 500^2 - 2 \times 500 \times 500\ \cos(160°)$
$= 969846.3104$
$x = \sqrt{969846.3104}$
$= 948.8$

The planes are 949 m apart

16



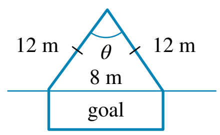
$$\tan(\theta) = \frac{opp}{adj}$$
$$\tan(47°) = \frac{h}{3500 - x}$$
$$1.072\,37 = \frac{h}{3500 - x}$$
$$h = (3500 - x) \times 1.072\,37$$
$$h = 3753.295 - 1.072\,37x \quad [1]$$
$$\tan(72°) = \frac{h}{x}$$
$$3.077\,68 = \frac{h}{x}$$
$$3.077\,68x = h$$
$$x = \frac{h}{3.077\,68} \quad [2]$$

Substitute x into equation [1]

$$h = 3753.295 - 1.072\,37 \times \left(\frac{h}{3.07768}\right)$$
$$= 3753.295 - 0.348\,43h$$
$$1.348\,43h = 3753.295$$
$$h = \frac{3753.295}{1.348\,43}$$
$$= 2783.46$$

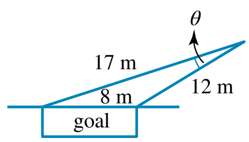
The height is 2783 m.

17 a



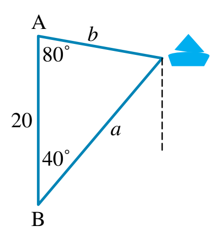
$$\cos(\theta) = \frac{12^2 + 12^2 - 8^2}{2 \times 12 \times 12}$$
$$= \frac{224}{288}$$
$$\theta = \cos^{-1}\left(\frac{224}{288}\right)$$
$$= 39°$$

b



$$\cos(\theta) = \frac{12^2 + 17^2 - 8^2}{2 \times 17 \times 12}$$
$$= \frac{369}{408}$$
$$\theta = \cos^{-1}\left(\frac{369}{408}\right)$$
$$= 25°$$

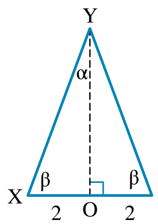
18



$$\frac{a}{\sin(80°)} = \frac{20}{\sin(60°)}$$
$$a = 20 \times \frac{\sin(80°)}{\sin(60°)}$$
$$= 22.7\text{ km}$$

$$\frac{b}{\sin(40°)} = \frac{20}{\sin(60°)}$$
$$b = 20 \times \frac{\sin(40°)}{\sin(60°)}$$
$$= 14.9\text{ km}$$

19



Consider one half of the isosceles triangle to be XOY as shown.

As $\cos(\beta) = \frac{\sqrt{5}}{3}$ (given), and

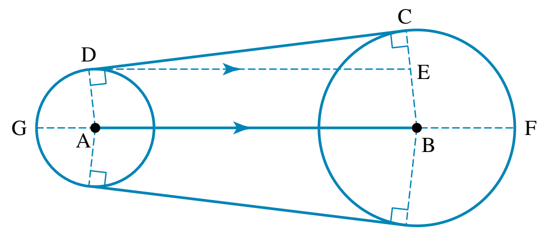
$\cos(\beta) = \frac{OX}{XY}$ (from diagram)

$$\frac{\sqrt{5}}{3} = \frac{OX}{XY}$$
$$\frac{2}{XY} = \frac{\sqrt{5}}{3}$$
$$XY = \frac{6}{\sqrt{5}}$$
$$XY^2 = OX^2 + OY^2$$
$$\Rightarrow OY^2 = XY^2 - OX^2$$
$$= \left(\frac{6}{\sqrt{5}}\right)^2 - (2)^2$$
$$= \frac{36}{5} - 4$$
$$= \frac{36-20}{5}$$
$$= \frac{16}{5}$$
$$OY = \frac{4}{\sqrt{5}}$$
$$\sin(\beta) = \frac{OY}{XY}$$
$$= \frac{\frac{4}{\sqrt{5}}}{\frac{6}{\sqrt{5}}}$$
$$\sin(\beta) = \frac{2}{3}$$
$$\text{Area}(XOY) = \frac{1}{2}(OX)(XY)\sin(\beta)$$
$$= \frac{1}{2}(2)\left(\frac{6}{\sqrt{5}}\right) \times \frac{2}{3}$$
$$\text{Area}(XOY) = \frac{4}{\sqrt{5}}$$

$$\text{Area of entire triangle} = 2 \times \text{Area}(XOY)$$
$$= 2 \times \frac{4}{\sqrt{5}}$$
$$= \frac{8}{\sqrt{5}} = \frac{8\sqrt{5}}{5}$$

The exact area of the block of land is $\frac{8\sqrt{5}}{5}$ km^2.

20 The following should be noted:



$AD = AG = 3$ cm
$BC = BF = 8$ cm
$AB = 13$ cm
As $\angle EDA = \angle DCB = 90°$, then $DA \parallel CB$.
This means that

$$CE = CB - DA$$
$$= 8 - 3$$
$$CE = 5\text{ cm},$$

and

$$DE = AB = 13\text{ cm},$$

and

$$\angle DAG = \angle CBA = \angle CED$$
$$\angle CED = \cos^{-1}\left(\frac{CE}{DE}\right)$$
$$= \cos^{-1}\left(\frac{5}{13}\right)$$
$$\angle CED = 67.38°$$
$$DC^2 = DE^2 - CE^2$$
$$= (13)^2 - (5)^2$$
$$DC = 12\text{ cm}$$

As $\angle DAG = \angle CED$,
$\angle DAG = 67.38°$

$$Arc\,(DG) = \frac{\angle DAG}{360°} \times 2\pi\,(AD)$$
$$= \frac{67.38°}{360°} \times 2\pi\,(3)$$
$$\text{Arc}\,(DG) = 3.528\text{ cm}$$
$$\angle CBF = 180° - \angle CBA$$
$$= 180° - 67.38°$$
$$\angle CBF = 112.62°$$
$$\text{Arc}\,(CF) = \frac{\angle CBF}{360°} \times 2\pi\,(BC)$$
$$= \frac{112.62°}{360°} \times 2\pi\,(8)$$
$$\text{Arc}\,(CF) = 15.72\text{ cm}$$

The length of the top half of the belt in the path GDCF can be calculated:

$L = \text{Arc}\,(DG) + DC + \text{Arc}\,(CF)$

$\quad = 3.528\text{ cm} + 12\text{ cm} + 15.72\text{ cm}$

$L = 31.248\text{ cm}$

As the system is symmetrical between the top and bottom halves of the diagram, it can be seen that:

$\textit{total belt length} = 2L = 2 \times 31.248\text{ cm}$

$\textit{total belt length} = 62.5\text{ cm (to 1 decimal place)}$

Therefore, the length of belt required is 62.5 cm