# 12 - Extensive-Form Games

# 知识点 & 题目

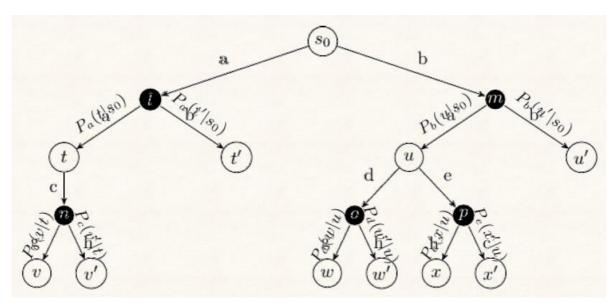
### **Definition L12 P3**

- Similar to normal-form games
  - Agents are rational, self-interested and have perfect information
- Agents do not have simultaneous moves

### **Backward Induction**

- Named as **Minimax** in zero-sum games
- Find sub-game perfect equilibria (SPE) and construct the equilibria path
- Buyer/Seller Game P4
- The Advertising Game P6
- Tic-Tac-Toe P7

## **Multi-Agent Reinforcement Learning**



## Algorithm - Multi-agent Q-Learning

**Input:** Stochastic game  $M=(S,s_0,A^1,\ldots A^n,r^1,\ldots,r^n,Agt,P,\gamma)$ 

**Output:** Q-function  $Q^j$  where j is the 'self' agent

Initialise  $Q^j$  arbitrarily; e.g.,  $Q^j(s,a)=0$  for all states s and joint actions a

Repeat (for each episode)

 $s \leftarrow$  the first state in episode e

Repeat (for each step in episode e)

Select action  $a^j$  to apply in s using Q-values in  $Q^j$  and a multi-armed bandit algorithm Execute action  $a^j$  in state s

Wait for other agents' actions (via simulation or via play)

Observe rewards  $r_t^j, \dots, r_{t+n}^j$  and new state  $s_{t+n}$ 

$$Q^j(s,a) \leftarrow Q^j(s,a) + \alpha \cdot [r_t^j + \ldots + r_{t+n}^j + \gamma \cdot \max_{a'} Q^j(s_{t+n},a') - Q^j(s,a)]$$
  
 $s \leftarrow s'$ 

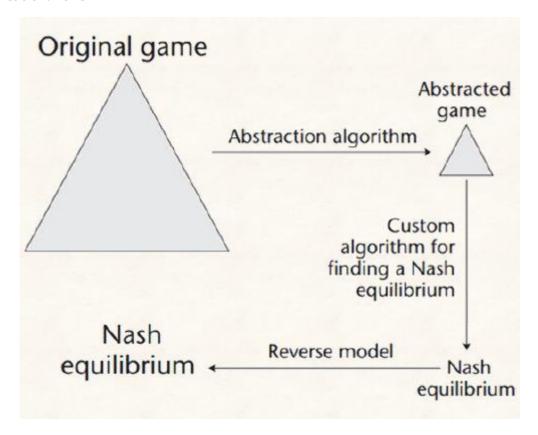
Until the end of episode e (a terminal state)

### **Simulating Opponent moves**

- Random selection
- Fixed policy
  - Hand coded: Not great but reasonably good
  - Learned policy
- Self play

#### **Multi-Agent MCTS P11**

**Application: Poker** 



## 题目

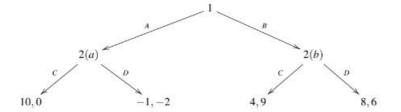
## Quiz

Q1: Extensive-form games differ to normal-form games in which of the following ways (only one answer is correct)?

- They do not have a Nash equilibrium
- Their Nash equilibria are never unique
- Their Nash equilibria are extensive
- They allow sequential moves 🗸

5 / 5 pts Question 2

Consider the following abstract extensive-form game of two players, 1 and 2, each with two available moves.



Select the right answers for the following:

[ Select ] What move will player 1 make: [Select]

What is the equilibrium of the sub-game starting at node 2(a): 10, 0

What is the equilibrium of the sub-game starting at node 2(b): 4, 9

What is the equilibrium of the game starting at node 1:



What move will player 2 make:

Answer 1:

Answer 2:

C

Answer 3:

10,0

Answer 4:

4,9

Answer 5:

10,0

Sub-game 2(a): Player 2 prefers a pay-off of 0 rather than -2, so will select C and the equilibrium is 10, 0

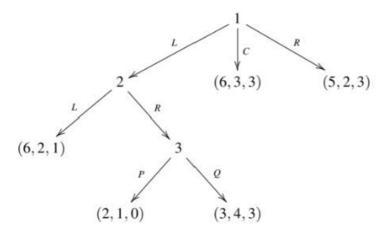
Sub-game 2(b): Player 2 prefers a pay-off of 9 rather than 6, so will select C and the equilibrium is 4, 9

Game 1: Player 1 prefers a pay-off of 10 over 4, so will select A and the equilibrium is 10, 0

Although a extensive-form game, Player 2 still has a dominant strategy of C: it is the best response in both sub-games.

Question 3 3 / 3 pts

Consider the following abstract game consisting of three players. The numbers in the nodes refer to the player number:



Select the right answers for the following:

What is the sub-game perfect equilibrium for sub-game 3:



What is the sub-game perfect equilibrium for sub-game 2:



What is the sub-game perfect equilibrium for the entire game:



Answer 1:

(3, 4, 3)

Answer 2:

(3, 4, 3)

Answer 3:

(6, 3, 3)

The only catch here is to remember the order of the players! Player 3 has the choice at sub-game 3, and their pay-off is in the third element of the tuple.

Player 3 prefers a pay-off of 3 over 0, so they prefer move Q and therefore the equilibrium is (3, 4, 3)

Now, player 2 has to choose between (6, 2, 1) and the sub-game perfect equilibrium of the sub-game starting at node 3, which is (3, 4, 3). Player 3

prefers a pay-off of 4 over 2, so prefers move R over L, and therefore the equilibrium is (3, 4, 3).

Finally, player 1 has to choose between the sub-game perfect equilibrium of the sub-game starting at node 2, which is (3, 4, 3), and the other two outcomes: (6, 3, 3) and (5, 2, 3). Player 1 prefers a pay-off of 6 over 5, so prefers move C, and the sub-game perfect equilibrium of the entire game is (6, 3, 3).

Q4: Model-free reinforcement learning cannot be applied in game theory because it is model-free and game-theory would require a model of other players?

False: Model-free reinforcement learning can be applied by exploring and exploiting actions and treating the other agents' actions as uncertain outcomes of our own agent's actions.