11 - Normal-Form Games

知识点 & 题目

Definition L11 P5

Assumptions

- Agents are rational, self-interested and have perfect information
- Agents make simultaneous moves

Best Response & Nash Equilibria L11 P7

Examples

- Prisoner's Dilemma
- The Advertising Game P9
  - Firm 1 is likely to not advertise
  - Firm 2 can decide based on Firm's possible choice
- Split or Steal P10
  - Weak equilibria (0, 0) -> Multiple equilibria
- Matching Pennies P16
  - No pure equilibria
  - Mixed Strategy Equilibria -> Security Games

题目

Quiz

Q1: If all players have a weakly dominant strategy in a game, then there exists a unique Nash equilibria

- The answer is false.
- If all players have a *strictly* dominant strategy in a game, then there exists a unique Nash equilibria.
- However, if at least one player has only a dominant (or *weakly dominant)* strategy, then there may be multiple Nash equilibria.

Q2: Which of the following is correct?

- A pure strategy is also a mixed strategy ✔
  - A pure strategy is simply a mixed strategy where one of the underlying strategies has a probability of 1, and all others have a probability of 0.
- Pure strategy equilibria always exist
- Mixed strategy equilibria only exist for two-player games
- A mixed strategy is also a pure strategy

Question 3

2 / 2 pts

Consider the following two-player game, each with two strategies. Player 2 can play Left or Right, and Player 1 can play Up or Down. What is the Nash equilibrium for this game? (Hint, there is a unique Nash equilibrium)

	Player	2
Player 1	**Left**	**Right**
Up	10, 20	15, 8
Down	-10, 7	10, 10

- 10, 10
- 10, 20
- 15, 8
- -10, 7

Player 2 does not have a dominant strategy, however, Player 1 does:

A. If Player 2 plays Left, Player 1 should play Up
B. If Player 2 plays Right, Player 1 should play Up

Therefore, Up is a dominant strategy for Player 1, and Down is *dominated*

We can eliminate Down from the game. This now gives Player 2 a stragey: Player 1 will play Up so Player 2 should play Left.

Therefore, the Nash equilibrium is (10,20)

Question 4

2 / 2 pts

Consider an extension to the previous game, in which Player 2 has one additional strategy: Centre.

		Player 2	
Player 1	Left	Centre	Right
Up	10, 20	20, 10	10, 10
Down	10, 10	10, 10	20, 10

Are there any *weakly dominated* strategies in this game?

- Yes. For player 2, Right weakly dominates Left and Centre
- Yes. For player 2, Centre weakly dominates Left and Right
- Yes. Both players have one: Up for Player 1 and Right for Player 2
- Yes. For player 2, Centre weakly dominates Left but not Right
- Yes. For player 1, Up weakly dominates Down
- Yes. For player 2, Left weakly dominates Centre and Right
- Yes. For player 2, Left weakly dominates Centre and not Right
- No
- Yes. For player 1, Down weakly dominates Up

There is no dominated strategies for Player 1: Up is the best response to Centre while Down is the best response to Right.

For Player 2, Left weakly dominates for Centre and Right: the payoff for Center and Right is 10 no matter what Player 1 plays; but is 20 for Left is Player 1 plays Up, and 10 if Player 1 plays Down.

Question 5

4 / 4 pts

Consider the following game with with two players, each with two strategies.

	Player	2
Player 1	Left	Right
Up	2, -2	-1, 1
Down	-1, 1	1, -1

Calculate the probabilities of each of the following:

1. Probability of Player 1 playing Up = 2/5
2. Probability of Player 1 playing Down = 3/5
3. Probability of Player 2 playing Left = 2/5
4. Probability of Player 2 playing Right = 3/5

Write you answer as a fraction; e.g. 4/7

Answer 1:

2/5

Answer 2:

3/5

Answer 3:

2/5

Answer 4:

3/5

Assume that Player 2 plays Left with probability L and Right with probability (1-L).

If Player 1 plays Up, then Player 1's expected payoff is 2L - (1-L) = 3L - 1.

If Player 1 plays Down, the Player 1's expected payoff is -L + (1 - L) = 1 - 2L.

To make Player 2 *indifferent* to Player 1's moves, then 3L - 1 = 1 - 2L ==> 2 = 5L ==> L=2/5.

Therefore Player 2 plays Left with probability 2/5 and Right with probability 3/5.

The game is entirely symmetrical, therefore, Player 1 plays Up with probability 2/5 and Down with probability 3/5.

Question 6

3 / 3 pts

Consider the following two-player game.

		Player 2	
Player 1	Left	Centre	Right
Up	1, 4	10, 4	1, 3
Stay	3, 3	3, 4	5, 5
Down	4, 8	2, 2	2, 1

Select all of the pure Nash equilibria for this game.

- [ ] 1, 4
- [x] 5, 5
- [ ] 2, 2
- [ ] 1, 3
- [ ] 3, 3
- [ ] 2, 1
- [x] 10, 4
- [x] 4, 8
- [ ] 3, 4

For a game like this, we can look at all cells in our game matrix and determine whether either player could be better by switching their strategy. This answer analyses just the three pure equilibria, and one non-equilibria.

- (10, 4) -- This is an equilibrium because Player 1 cannot do better than Up (payoff 10): Stay has a payoff of 3 and Down has a payoff of 2; and Player 2 cannot do better than Centre (payoff 4): Left has a payoff of 4 and Right has a payoff of 3. Even though Player 2 could do *just as well* by playing Left (payoff 4) instead of Centre (payoff 4), the outcome (10, 4) is still an equilibrium because Player 2 is no better off switching.
- (5,5) -- This is an equilibrium because Player 1's other possible payoffs are 1 and 2, compared to 5 for Stay; and Player 2's other possible payoffs are 3 and 4, compared to 5 for Right,
- (4, 8) -- This is an equilibrium because Players 1's other possible payoffs

are 1 and 3, compared to 4 for Down; and Player 2's other possible payoff are 1 and 2, compared to 8 for Left.

- (1, 4) -- This is NOT an equilibrium because Player 1 can do better by playing Down (4). Player 2 can do no better by switching.
- (2, 1) -- This is NOT an equilibrium because Player 1 can do better by playing Stay (5); and also Player 2 can do better by playing left (8).

Question 7

0 / 1 pts

Recall the following security game from the lecture.

	Adversary	
Defender	**Terminal 1**	**Terminal 2**
Terminal 1	5, -3	-1, 1
Terminal 2	-5, 5	2, -1

Calculate the probability of the Adversary attacking Terminal 1 under a mixed strategy, expressed as a fraction; e.g. 4/7

7/13

3/13

We are calculating a strategy for the Adversary, so we need to consider the expected return of the Defender. If the Adversary attacks Terminal 1 with a probability of q, then the expected returns for our Defender are:

E_D(T1) = 5q +−1(1 −q) = 6q − 1
E_D(T2) = 5q + 2(1 −q) = 2 − 7q

To make the Defender *indifferent*, we need
E_D(T1) = E_D(T2)
6q − 1 = 2 − 7q
3 = 13q
q = 3/13