(a). $90^\circ_{x'} - \tau - 180^\circ_{y'}$ The effect of the first $90^\circ_{x'}$:

$$\begin{cases} M_{1,x'} = 0 \\ M_{1,y'} = M^\circ_{1,z} \\ M_{1,z'} = 0 \end{cases} \quad \text{and} \quad \begin{cases} M_{2,x'} = 0 \\ M_{2,y'} = M^\circ_{2,z} \\ M_{2,z'} = 0 \end{cases}$$

During the time delay, $\vec{M}_1$ stays the same, but $\vec{M}_2$ precesses at an angular frequency of $\delta\omega_0$ in the rotating frame. As a result, $\vec{M}_2$ takes the following value immediately before the second pulse:

$$\begin{cases} M_{2,x'} = \sin -\delta\omega_0\tau \, M^\circ_{2,z} = \sin \delta\omega_0 \cdot \frac{\pi}{3} \, M^\circ_{2,z} = \frac{\sqrt{3}}{2} M^\circ_{2,x} \\ M_{2,y'} = \cos -\delta\omega_0\tau \, M^\circ_{2,z} = \cos \frac{\pi}{3} M^\circ_{2,z} = \frac{\sqrt{3}}{2} M^\circ_{2,z} \\ M_{2,z'} = 0. \end{cases}$$

The second pulse is applied along the $y'$-axis and, therefore, has no effect on the first isochromat because it has been lying along the $y'$-axis since the first pulse. The final value for $\vec{M}_1$ after two pulses is

$$\begin{bmatrix} M_{1,x'}(0+) \\ M_{1,y'}(0+) \\ M_{1,z'}(0+) \end{bmatrix} = \begin{bmatrix} 0 \\ M^\circ_{1,z} \\ 0 \end{bmatrix}. \qquad \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

For the second ischromat, the magnetization vector will be flipped 180° around the $y'$-axis the following postpulse value is:

$$\begin{bmatrix} M_{2,x'}(0+) \\ M_{2,y'}(0+) \\ M_{2,z'}(0+) \end{bmatrix} = \begin{bmatrix} \cos 180^\circ & 0 & -\sin 180^\circ \\ 0 & 1 & 0 \\ \sin 180^\circ & 0 & \cos 180^\circ \end{bmatrix} \begin{bmatrix} -\sin\frac{\pi}{3}\delta\omega_0 \\ \cos\frac{\pi}{3}\delta\omega_0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \sin\frac{\pi}{3}\delta\omega_0 M^\circ_{2,z} \\ \cos\frac{\pi}{3}\delta\omega_0 M^\circ_{2,z} \\ 0 \end{bmatrix}$$

(b). $45^\circ_{x'} - \frac{\tau}{2} - 90^\circ_{y'}$

The effect of the first $45^\circ_{x'}$ on both isochromat is the same, resulting in:

$$\begin{cases} M_{1,x'} = 0 \\ M_{1,y'} = \frac{\sqrt{2}}{2} M^0_{1,z} \\ M_{1,z'} = \frac{\sqrt{2}}{2} M^0_{1,z} \end{cases} \quad \text{and} \quad \begin{cases} M_{2,x'} = 0 \\ M_{2,y'} = \frac{\sqrt{2}}{2} M^0_{2,z} \\ M_{2,z'} = \frac{\sqrt{2}}{2} M^0_{2,z} \end{cases}$$

During the time delay, $\vec{M_1}$ stays the same (ignoring any relaxation effects), but $\vec{M_2}$ precesses at an angular frequency of $-\delta\omega_0$ in the rotating frame. As a result, $M_2$ takes the following value immediately before the second pulse:

$$\begin{cases} M_{2,x'} = \sin(-\delta\omega_0\tau) M^0_{2,z} = -\sin\frac{\pi}{\delta}\delta\omega_0 M^0_{2,z} \\ M_{2,y'} = \cos(-\delta\omega_0\tau) M^0_{2,z} = \cos\frac{\pi}{\delta}\delta\omega_0 M^0_{2,z} \\ M_{2,z'} = \frac{\sqrt{2}}{2} M^0_{2,z} \end{cases}$$

The second pulse is applied along the y-axis and therefore, has no effect on the first isochromat because it has been lying along the y-axis since the first pulse. The final value for $\vec{M_1}$ after the two pulses is:

$$\begin{bmatrix} M_{1,x'}(0_+) \\ M_{1,y'}(0_+) \\ M_{1,z'}(0_+) \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\sqrt{2}}{2} M^0_{1,z} \\ \frac{\sqrt{2}}{2} M^0_{1,z} \end{bmatrix}$$

For the second isochromat:

$$\begin{bmatrix} M_{2,x'}(0_+) \\ M_{2,y'}(0_+) \\ M_{2,z'}(0_+) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -\sin\frac{\pi}{\delta}\delta\omega_0 M^0_{2,z} \\ \cos\frac{\pi}{\delta}\delta\omega_0 M^0_{2,z} \\ \frac{\sqrt{2}}{2} M^0_{2,z} \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} M^0_{2,z} \\ \cos\frac{\pi}{\delta}\delta\omega_0 M^0_{2,z} \\ -\sin\frac{\pi}{\delta}\delta\omega_0 M^0_{2,z} \end{bmatrix}$$

3.28. We assume $\alpha$ for this equation.

So we have $R_\varphi(\alpha) = R_{z'}(\varphi) R_{x'}(\alpha) R_{z'}(-\varphi)$

For simplicity, consider only the transverse magnetization. we have

$$R_\varphi(\alpha) = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \cos\alpha\sin\varphi & \cos\alpha\cos\varphi \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\varphi + \sin^2\varphi\cos\alpha & -\sin\varphi\cos\varphi + \sin\varphi\cos\varphi\cos\alpha \\ -\sin\varphi\cos\varphi + \cos\varphi\sin\varphi\cos\alpha & \sin^2\varphi + \cos^2\varphi\cos\alpha \end{bmatrix}$$

This equation implies that

$M_{x'}(0_+) = M_{x'}(0_-)(\cos^2\varphi + \sin^2\varphi\cos\alpha) - M_{y'}(0_-)(-\sin\varphi\cos\varphi \sim$

$M_{y'}(0_+) = (-\sin\varphi\cos\varphi + \sin\varphi\cos\varphi\cos\alpha) M_{x'}(0_-) \sim$

~~There fore:~~ Because $M^*_{x'y'}(0_-)e^{-i2\varphi} = M_{x'y'}(0$

~~$M_{x'y'}(0_+) = M_{x'}(0_+) + i M_{y'}(0_+)$~~

~~$= M^*_{x'y'}(0_-)e^{-i2\varphi}$~~

So.
$$\begin{cases} \cos^2\varphi + \sin^2\varphi\cos\alpha = \cos 2\alpha \\ -\sin\varphi\cos\varphi + \sin\varphi\cos\varphi\cos\alpha = -\sin 2\alpha \end{cases}$$

~~we got~~ ~~$\alpha = \pi$~~

we can get
$$\begin{cases} \alpha = \pi \\ \varphi = 0 \end{cases}$$
pulse: $\pi_{x'}$

and. $M_{x'y'}(0_+) = -M^*_{x'y'}(0_-)e^{-i2\varphi}$

we can get
$$\begin{cases} \cos^2\varphi + \sin^2\varphi\cos\alpha = -\cos 2\alpha \\ -\sin\varphi\cos\varphi + \sin\varphi\cos\varphi\cos\alpha = \sin 2\alpha \end{cases}$$

So
$$\begin{cases} \alpha = \pi \\ \varphi = \frac{\pi}{2} \end{cases}$$
pulse: $\pi_{y'}$