BME2104 -《生物医学影像技术》Home Work #4

Due Date: June 5, 2024

Note: *Please prepare your answers to the problems in a single PDF, and upload your PDF to Blackboard.*

1. What two factors determine the precession frequency of resonance?

   The gyromagnetic ratio and the external magnetic field strength determine the precession frequency of resonance.

   Based on the Larmor equation:

$$\omega_0 = \gamma B_0$$

   where ω_0 is the precession frequency, γ is the gyromagnetic ratio, and B_0 is the external magnetic field strength. The strength of the external magnetic field is the primary determinant of the precession frequency while the gyromagnetic ratio is a constant that is specific to the type of nucleus or particle being studied.

2. How do we detect an NMR (or MRI) signal?

- First, the sample is placed in a strong, static magnetic field which is called B_0, causing the magnetic moments of the nuclei to align with the field. Then a radiofrequency (RF) pulse is applied at the Larmor frequency, exciting the nuclei to precess around the direction of the magnetic field.
- After the RF pulse is turned off, the nuclear precession induces a voltage in a receiver coil. The coil detects the induced voltage and the MR signal, known as the Free Induction Decay(FID), is an exponentially decaying oscillation at the Larmor frequency. The detected signal is typically weak so it should be amplified.
- The FID is in the spatial domain so it should be converted into a frequency domain using the Fourier transformation. FT decomposes the complex time signal into its constituent frequencies, providing a spectrum that shows the resonant frequencies of the nuclei in the sample.
- Finally, spatial information is obtained by applying magnetic field gradients during the signal acquisition. The gradients cause the Larmor frequency to vary with position, encoding spatial information into the MR signal. The raw data collected is reconstructed into images using reconstruction algorithms (FFT).

3. a. What is the Larmor equation?

The Larmor equation:

$$\omega_0 = \gamma B_0$$

where ω_0 is the precession frequency, γ is the gyromagnetic ratio (rad/s/T), and B_0 is the external magnetic field strength.

b. The gyromagnetic ratio of a proton, $\gamma/2\pi$, is approximately 43 MHz/T. What is its precession frequency at 3T?

Based on Larmor frequency, $f = \frac{\gamma}{2\pi} * B_0 = 43\ MHz/T * 3\mathrm{T} = 129\ \mathrm{MHz}$

c. What flip angle would be obtained with a B_1 at 23.3 μT applied for 0.5 ms?

The flip angle α is given by:

$$\alpha = \gamma B_1 t$$

where γ is the gyromagnetic ratio, B_1 is the amplitude of the RF magnetic field and t is the duration of the RF pulse.

So $\alpha = \gamma B_1 t = 2\pi * 43 * 10^6 * 23.3 * 10^{-6} * 0.5 * 10^{-3} \approx \pi$.

The flip angle is about 180 degrees.

4. Use the following T_1 and T_2 relaxation rates for gray matter and white matter for 1.5 T and 3 T.

Brain gray matter:

T_1 920 ms (1.5 T), 1600 ms (3.0 T)

T_2 100 ms 80 ms

Brain white matter:

T_1 790 ms (1.5 T), 1100 ms (3.0 T)

T_2 90 ms 60 ms

a. Write a program to plot (and label) the four T_1 curves.

b. Plot (and label) the four T_2 curves.

c. If faster T1 relaxation leads to brighter T1 images, which is brighter in a T1-weighted image - gray or white matter?

d. If slower T2 relaxation leads to brighter T2 images, which is brighter in a T2-weighted image - gray or white matter?

Based on the T1&T2 relaxation equation:

T1:

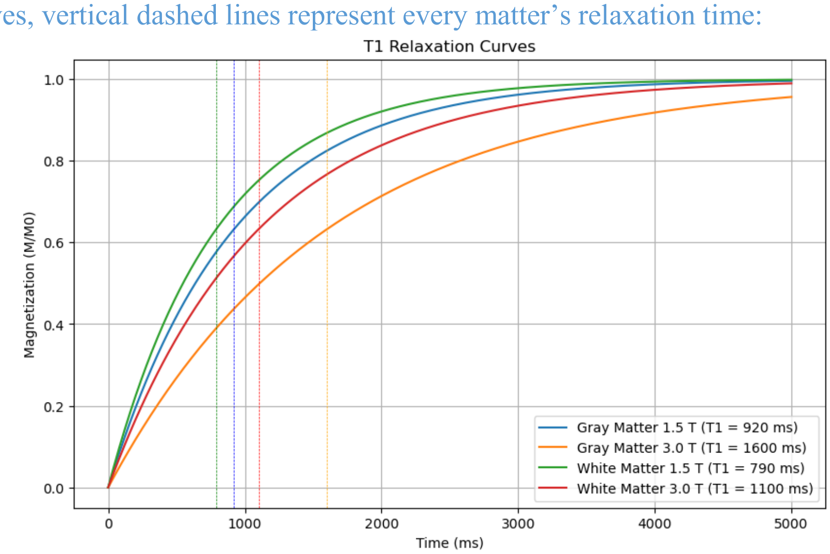
$$M_z(t) = M_0(1 - e^{-\frac{t}{T1}})$$

T2:

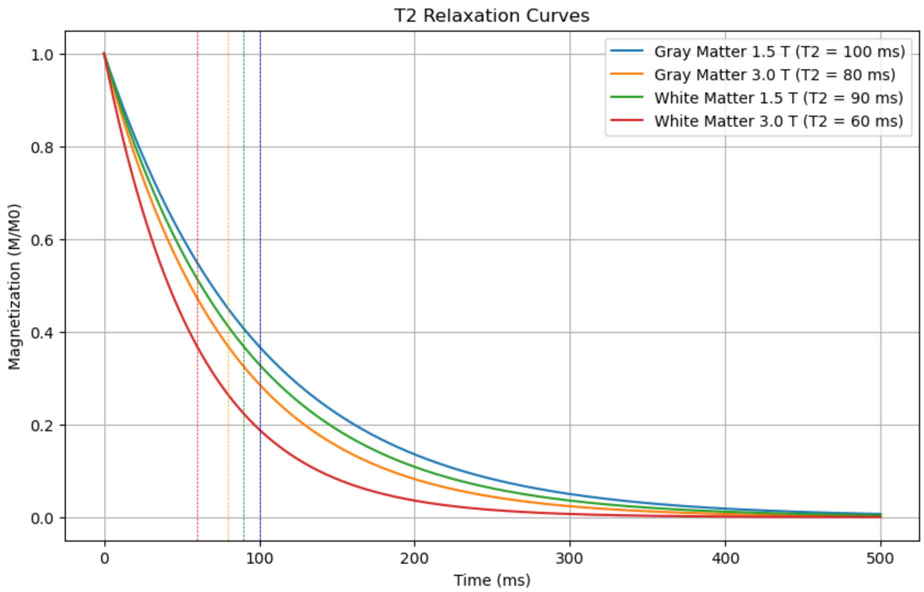
$$M_{xy}(t) = M_{xy}(0)e^{-\frac{t}{T2}}$$

In this case, the vertical coordinate was represented by the change of magnetic vector: $\frac{M}{M_0}$

The four T1 curves, vertical dashed lines represent every matter's relaxation time:



The four T2 curves：



In T1-weighted imaging, tissue with faster T1 relaxation times appears brighter. Comparing the T1 times provided, **the white matter** has faster T1 relaxation times at both 1.5 T and 3.0 T. So **white matter** will appear brighter than gray matter in T1-weighted images.

In T2-weighted imaging, tissue with slower T2 relaxation times appears brighter. Comparing the T2 times provided, **the gray matter** has slower T2 relaxation times at both 1.5 T and 3.0 T. So **gray matter** will appear brighter than white matter in T2-weighted images.

```
import numpy as np
import matplotlib.pyplot as plt


T1_gray_15T = 920
T1_gray_30T = 1600
T1_white_15T = 790
T1_white_30T = 1100
t = np.linspace(0, 5000, 500)

def T1_relaxation(t, T1):
    return 1 - np.exp(-t / T1)

T1_gray_15T_curve = T1_relaxation(t, T1_gray_15T)
T1_gray_30T_curve = T1_relaxation(t, T1_gray_30T)
T1_white_15T_curve = T1_relaxation(t, T1_white_15T)
T1_white_30T_curve = T1_relaxation(t, T1_white_30T)

plt.figure(figsize=(10, 6))
plt.plot(t, T1_gray_15T_curve, label=f'Gray Matter 1.5 T (T1 = {T1_gray_15T} ms)')
plt.plot(t, T1_gray_30T_curve, label=f'Gray Matter 3.0 T (T1 = {T1_gray_30T} ms)')
plt.plot(t, T1_white_15T_curve, label=f'White Matter 1.5 T (T1 = {T1_white_15T} ms)')
plt.plot(t, T1_white_30T_curve, label=f'White Matter 3.0 T (T1 = {T1_white_30T} ms)')
plt.axvline(T1_gray_15T, color='blue', linestyle='--', linewidth=0.5)
plt.axvline(T1_gray_30T, color='orange', linestyle='--', linewidth=0.5)
plt.axvline(T1_white_15T, color='green', linestyle='--', linewidth=0.5)
plt.axvline(T1_white_30T, color='red', linestyle='--', linewidth=0.5)
```

```python

plt.xlabel('Time (ms)')
plt.ylabel('Magnetization (M/M0)')
plt.title('T1 Relaxation Curves')
plt.legend()
plt.grid(True)
plt.show()

T2_gray_15T = 100  # ms
T2_gray_30T = 80   # ms
T2_white_15T = 90  # ms
T2_white_30T = 60  # ms
t = np.linspace(0, 500, 500)

def T2_relaxation(t, T2):
    return np.exp(-t / T2)

T2_gray_15T_curve = T2_relaxation(t, T2_gray_15T)
T2_gray_30T_curve = T2_relaxation(t, T2_gray_30T)
T2_white_15T_curve = T2_relaxation(t, T2_white_15T)
T2_white_30T_curve = T2_relaxation(t, T2_white_30T)

plt.figure(figsize=(10, 6))
plt.plot(t, T2_gray_15T_curve, label='Gray Matter 1.5 T (T2 = 100 ms)')
plt.plot(t, T2_gray_30T_curve, label='Gray Matter 3.0 T (T2 = 80 ms)')
plt.plot(t, T2_white_15T_curve, label='White Matter 1.5 T (T2 = 90 ms)')
plt.plot(t, T2_white_30T_curve, label='White Matter 3.0 T (T2 = 60 ms)')
plt.axvline(T2_gray_15T, color='blue', linestyle='--', linewidth=0.5)
plt.axvline(T2_gray_30T, color='orange', linestyle='--', linewidth=0.5)
plt.axvline(T2_white_15T, color='green', linestyle='--', linewidth=0.5)
plt.axvline(T2_white_30T, color='red', linestyle='--', linewidth=0.5)


plt.xlabel('Time (ms)')
plt.ylabel('Magnetization (M/M0)')
plt.title('T2 Relaxation Curves')
plt.legend()
plt.grid(True)
plt.show()
```