BME2104 -《生物医学影像技术》Home Work #5

Due Date: June 14th, 2024

Note: *Please prepare your answers to the problems in a single PDF, and upload your PDF to Blackboard.*

1. Calculate the intensity transmission coefficient T1 for the following interfaces, assuming that the ultrasound beam is exactly perpendicular to the interface: muscle/kidney, air/muscle, and bone/muscle. Discuss briefly the implications of these values of 11 for ultrasonic imaging.

Z_{muscle}	$1.7\times10^5 g\cdot cm^{-2}\cdot s^{-1}$
Z_{kidney}	$1.65\times10^5 g\cdot cm^{-2}\cdot s^{-1}$
Z_{air}	$0.0004\times10^5 g\cdot cm^{-2}\cdot s^{-1}$
Z_{bone}	$7.8\times10^5 g\cdot cm^{-2}\cdot s^{-1}$

The equation of calculating the intensity transmission coefficient T1 is:

$$T_1 = \frac{4Z_1Z_2}{(Z_1+Z_2)^2}$$

,where Z_1 and Z_2 are the acoustic impedances of two media.

For muscle/kidney:

$$T_1 = \frac{4*(1.7\times10^5)*(\,1.65\times10^5)}{(1.7\times10^5+1.65\times10^5)^2} \approx 0.9998$$

For air/muscle:

$$T_1 = \frac{4*(1.7\times10^5)*(\,0.0004\times10^5)}{(1.7\times10^5+0.0004\times10^5)^2} \approx 0.00094$$

For bone/muscle:

$$T_1 = \frac{4*(1.7\times10^5)*(\,7.8\times10^5)}{(1.7\times10^5+7.8\times10^5)^2} \approx 0.5877$$

Implications:

For muscle/kidney: The high transmission coefficient (0.9998) means that most of the ultrasound wave passes through the interface, resulting in clear imaging of internal structures. This is ideal for medical ultrasound as it minimizes reflection and signal loss.

For air/muscle: The low transmission coefficient (0.00094) means that almost all the ultrasound waves are reflected at the air/muscle interface. This is problematic for ultrasound imaging as it prevents effective penetration of ultrasound into the body. To mitigate this, coupling gels (which have impedances closer to that of soft tissues) are used to eliminate air gaps.

For bone/muscle: The moderate transmission coefficient (0.5877) suggests that a significant portion of the ultrasound wave is reflected at the bone/muscle interface. This can make imaging structures behind bones difficult and may lead to shadowing artifacts.

2. Calculate the distance at which the intensity of a 1-MHz and a 5-MHz ultrasound beam will be reduced by half traveling through (a) bone, (b) air, and (c) muscle.

μ_{bone}	45 dB·cm^{-1}·MHz^{-1}
μ_{air}	8.7 dB·cm^{-1}·MHz^{-1}
μ_{muscle}	1 dB·cm^{-1}·MHz^{-1}

The formula to calculate the distance d is:

$$\mathrm{d} = \frac{3\ dB}{\alpha}$$

where α is the attenuation coefficient of the medium in dB/cm, and each 3 dB corresponds to a reduction in intensity by a factor of 2.

The distances for each medium and frequency:

1 MHz:

$$d_{bone} = \frac{3\ dB}{\mu_{bone} \cdot 1MHz} \approx 0.067cm$$

$$d_{air} = \frac{3\ dB}{\mu_{air} \cdot 1\ MHz} \approx 0.345\ cm$$

$$d_{muscle} = \frac{3\ dB}{\mu_{muscle} \cdot 1\ MHz} \approx 3\ cm$$

5 MHz:

$$d_{bone} = \frac{3\ dB}{\mu_{bone} \cdot 5\ MHz} \approx 0.0134\ cm$$

$$d_{air} = \frac{3\ dB}{\mu_{air} \cdot 5\ MHz} \approx 0.0692\ cm$$

$$d_{muscle} = \frac{3\ dB}{\mu_{muscle} \cdot 5\ MHz} \approx 0.6\ cm$$

3. Consider a focused transducer with a radius of curvature of 10em and a diameter of 4 cm. This transducer operates at a frequency of 3.5 MHz and transmits a pulse of duration 0.857 us. What is the axial and the lateral resolution at the focal point of the transducer?

The spatial pulse length (SPL) can be determined by:

$$\mathrm{SPL} = Pd \cdot c$$

To calculate the axial resolution Δz:

$$\Delta z = 0.5 \cdot SPL$$

where Pd is the pulse duration and c is the speed of sound (c = 1540 m/s). The axial resolution is:

$$\Delta z = 0.5 \times 0.857\ \mu s \times 1540\ m/s\ = 0.00066\ m$$

To calculate the lateral resolution Δx:

$$\Delta x = \frac{\lambda F}{D}$$

where λ is the wavelength of ultrasound, F is the focal length, D is the diameter of the transducer.

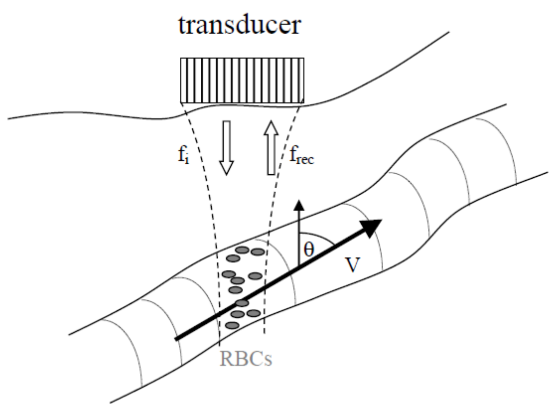
The wavelength can be calculated:

$$\lambda = \frac{c}{f} = \frac{1540\ m/s}{3.5\ MHz} = 0.00044\ m$$

The lateral resolution is:

$$\Delta x = \frac{0.00044\ m \cdot 10\ cm}{4\ cm} = 0.0011\ m$$

4. In Doppler ultrasound scan of a blood vessel (see below), show that the Doppler shift caused by the blood flow is $f_D = f_{rec} - f_0 = \frac{2f_0 v cos\theta}{c}$



1) Transducer to blood cells:

When the ultrasound wave is transmitted from the transducers and hit the blood cells, due to the Doppler effect, the frequency obtained by the moving blood cells (f_1) is different from f_0. f_1 can be calculated by:

$$f_1 = f_0 * (\frac{c + v * cos\theta}{c})$$

2) Blood cells to transducer:

The moving blood cells act as a source of sound with frequency f_1 and reflect the ultrasound wave back to the transducer. The frequency received by the transducer is:

$$f_{rec} = f_1 * (\frac{c + v * cos\theta}{c})$$

The Doppler shift caused by the moving blood flow is:

$$f_D = f_{rec} - f_0 = f_1 * \left(\frac{c + v * cos\theta}{c}\right) - f_0 = f_0 * (\frac{c + v * cos\theta}{c})^2 - f_0$$

$$= f_0 * (\frac{2v * cos\theta}{c} + \frac{v^2 * cos^2\theta}{c^2})$$

While values of v are always small, the term $\frac{v^2 * cos^2\theta}{c^2}$ can be neglected. Therefore

$$f_D = f_0 * \frac{2v * cos\theta}{c}$$