Problem 1:

First, the Fourier coefficients will be calculated. The function is:

$$f(x) = \begin{cases} 1, & 0 \le x < 1/2 \\ 0, & 1/2 \le x < 1 \end{cases}$$

and extended to be periodic of period 1 (T=1). Obviously, the zeroth coefficient is $a_0 = 1$.
The general coefficients will be:

$$\widehat{f(n)} = \int_0^1 e^{-2\pi i n t} f(t)dt = \int_0^{\frac{1}{2}} e^{-2\pi i n t} * 1dt + \int_{\frac{1}{2}}^1 e^{-2\pi i n t} * 0dt = \int_0^{\frac{1}{2}} e^{-2\pi i n t} dt$$

$$= \frac{1}{2\pi i n}(1 - e^{-\pi i n})$$

The infinite Fourier series:

$$\sum_{n \neq 0} \frac{1}{2\pi i n}(1 - e^{-\pi i n})\, e^{2\pi i n t}$$

While $1 - e^{-\pi i n} = \begin{cases} 0\,, n\ even \\ 2\,, n\ \ odd \end{cases}$, so the series becomes

$$\sum_{n\ odd} \frac{1}{\pi i n} e^{2\pi i n t}$$

Based on the Eula equation, the result is

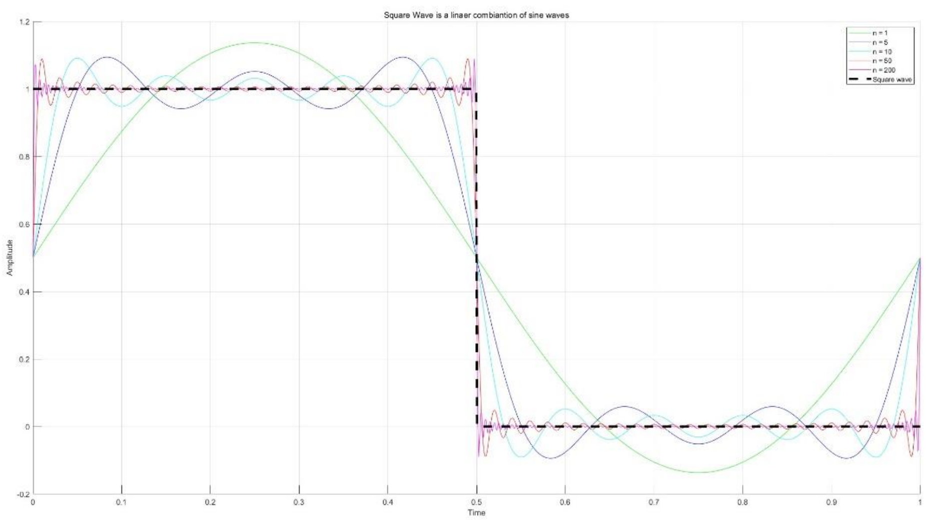
$$\sum_{n\ odd} \frac{2}{\pi n} sin2\pi n t$$

Writing n = 2k+1, the final answer is

$$f(t) = \sum_{k=0}^{\infty} \frac{2}{\pi(2k+1)} sin2\pi(2k+1)t + \frac{1}{2}$$

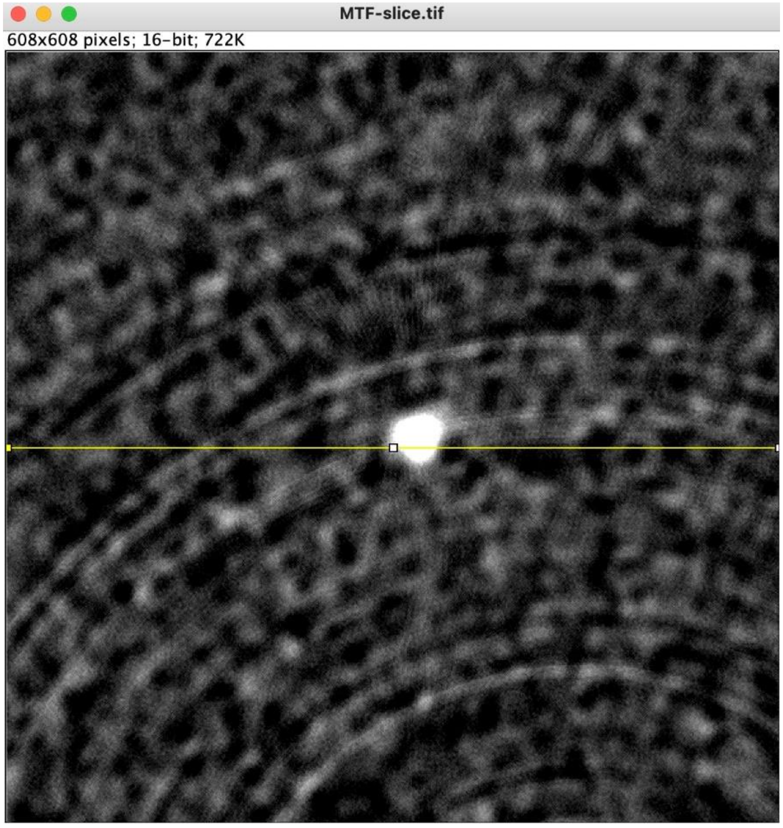
So this square wave can be represented as a linear combination of sine waves of different frequencies.

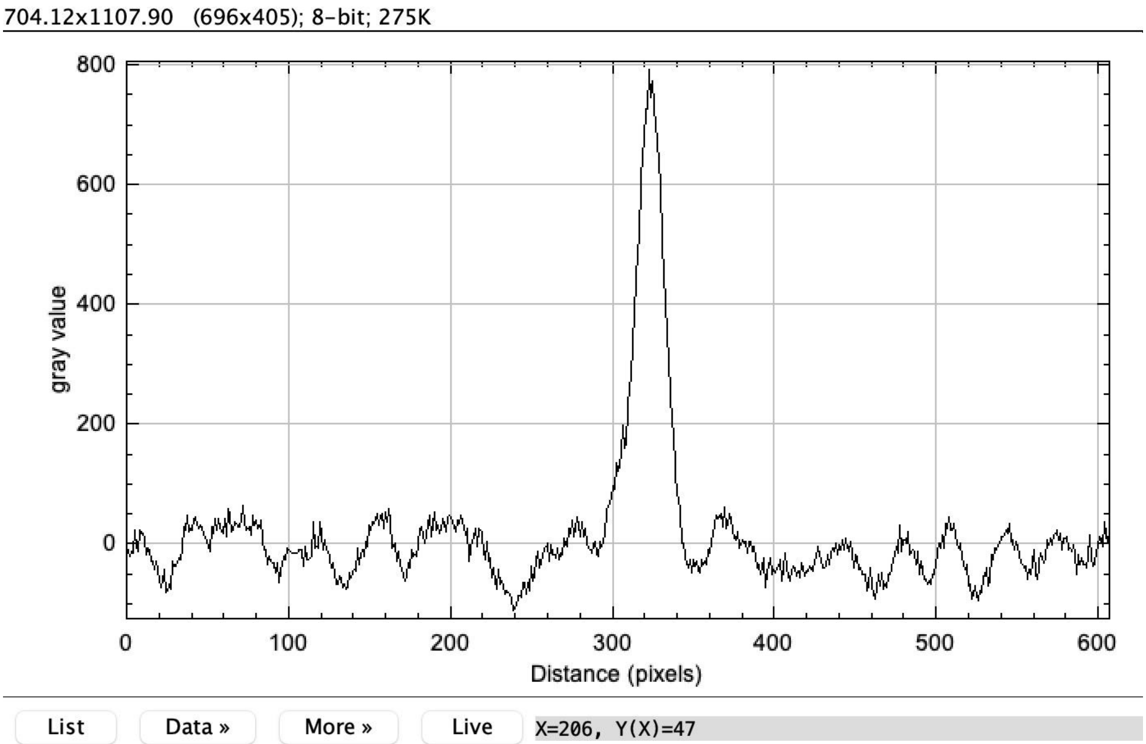
Now, the numerical model show the above square wave is indeed a linear combination of sine waves. The n is given by 1, 5, 10, 50, 200, by the increasing of n, the superposition of sine waves approximates square waves.



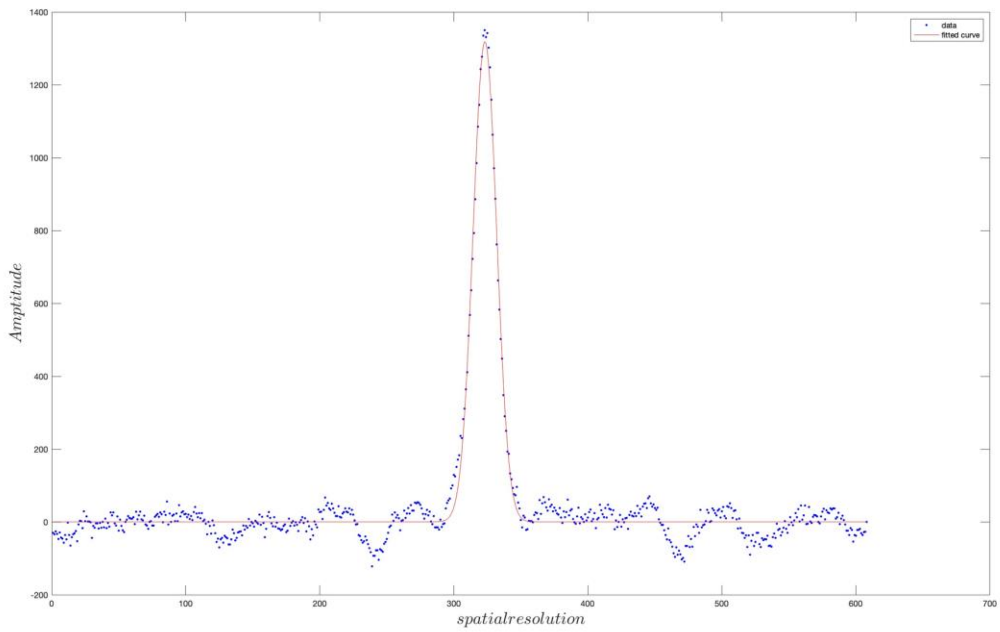
Problem2

First, plot the LIP of the wire in the center of the image and the result is shown below.



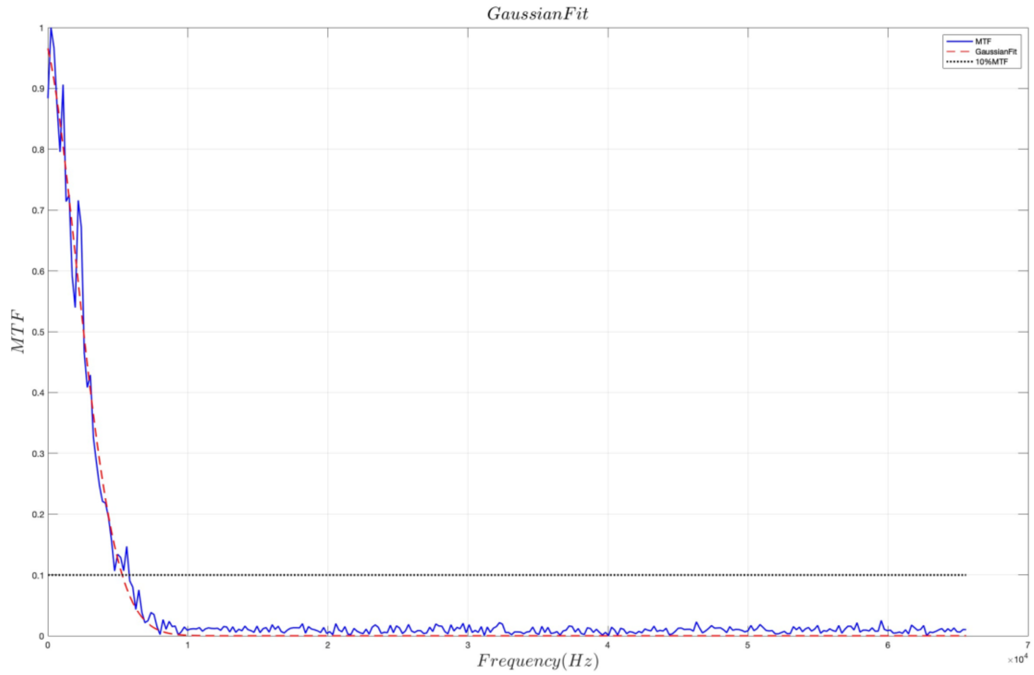


The function after fitting LIP with Gaussian is shown in the figure below



Assuming that LIP is the LSF of micro-CT, according to MTF=F{LSF}, the blue MTF curve in the figure below is obtained. The MTF curve is fitted with the same Gaussian as shown in the red dotted line in the figure below, and 10%MTF is marked at the same time.

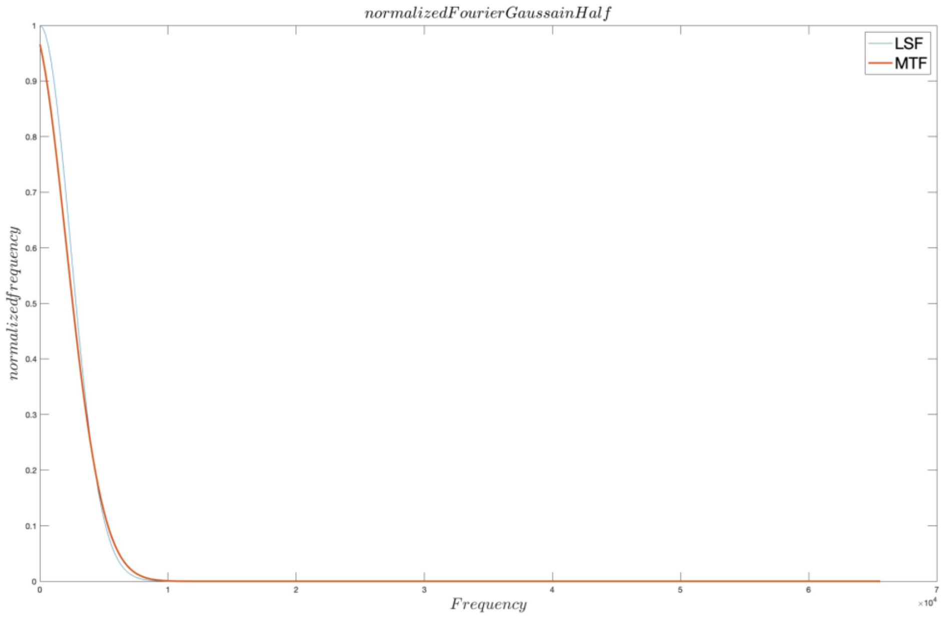
The spatial resolution is calculated as the number of points above the threshold (10%MTF) , half of the length of the 'MTF' vector, divided by the total length of the MTF curve times the pixel size in millimeters. The calculated spatial resolution is b=5.8431mm, which means that the distance between two details in the imaging system needs to be at least 5.8431mm for contrast distinction.

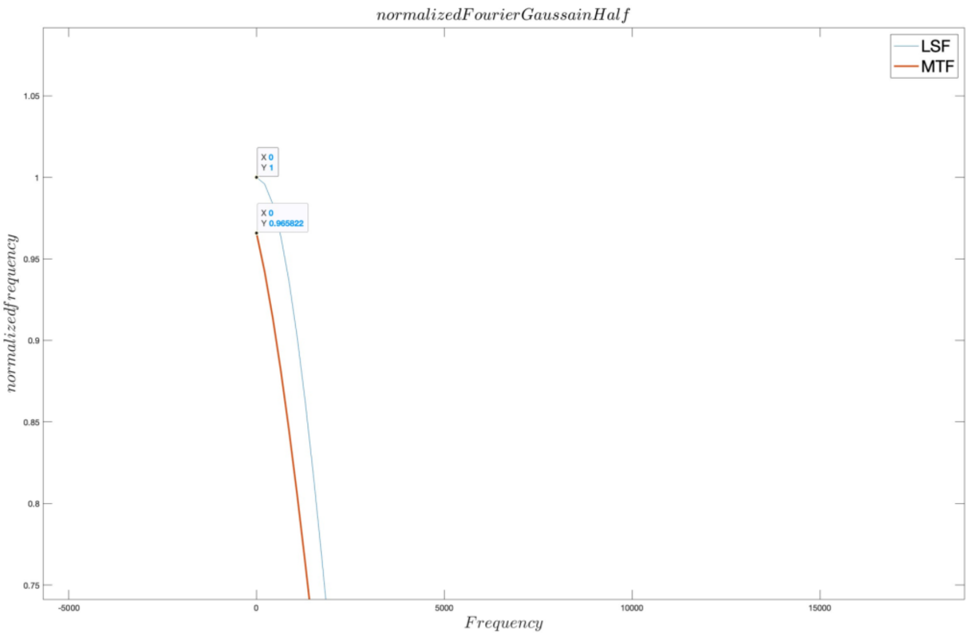


```
b =

    5.8431
```

Comparing the fitted MTF and the Fourier transformed LSF, it can be observed that the LSF still presents a Gaussian distribution after Fourier transformation (the mean and variance are slightly different). It is found that the fitted MTF curve is better than the fitted LSF. The peak value is slightly lower. It may be that the signal in the higher frequency part is limited and cannot be completely transmitted. There may also be errors in the fitting process.





Problem3

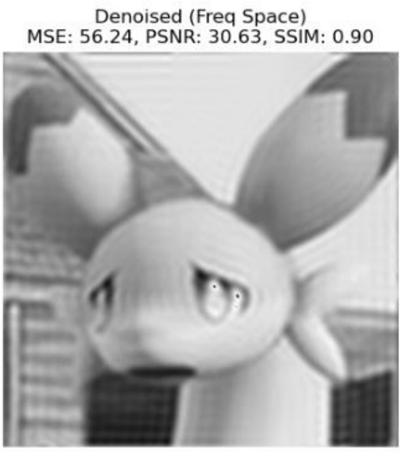
Filtering in frequency space and multiply the FT version with a low-pass filter of different filtration levels (kernel size =32,64,128):

Original Figure:

Different kernel size:

N=32

N=64



N=128

Denoised (Frequence Space)
MSE: 14.11, PSNR: 36.64, SSIM: 0.98

Filtering in image space and design a “denoising” filter kernel of 5, and convolute it with noisy grey-scale image. The denoised image is shown below.

Denoised (Image Space)
MSE: 0.08, PSNR: 59.26, SSIM: 1.00