First Order Linear Equation

$a(x)y' + b(x)y = c(x)$

Standard Linear Form: $y' + \underbrace{p(x)}_{\frac{b(x)}{a(x)}} y = \underbrace{q(x)}_{\frac{c(x)}{a(x)}}$

Model Examples

Temperature-concentration model

$$\frac{dT}{dt} = k(T_e - T) \Rightarrow \frac{dT}{dt} + kT = kT_e$$

Salt-concentration model

$$\frac{dC}{dt} = k(C_e - C) \Rightarrow \frac{dC}{dt} + kC = kC_e$$

Integrity Factor

to solve $y' + p(x)y = q(x)$

set integrity factor $u(x)$, letting when multiply by $u(x)$ both size

$$u(x)y' + u(x)p(x)y = q(x)u(x) \Rightarrow (u(x)y)' = q(x)u(x)$$

$$\Rightarrow p(x)u(x) = u'(x)$$

$$\Rightarrow \frac{du}{dx} = p(x)u(x) \Rightarrow \frac{du}{u(x)} = p(x)dx$$

$$\Rightarrow \int \frac{du}{u(x)} = \int p(x)dx$$

$$\Rightarrow \ln u(x) = \int p(x)dx$$

$$\Rightarrow \boxed{u(x) = e^{\int p(x)dx}}$$

Constant Linear Equation

take Temp-concentration model for example

$$\frac{dT}{dt} + kT = kT_e$$

$$u(x) = e^{\int kdt} = e^{kt}$$

$$\Rightarrow e^{kt}\frac{dT}{dt} + e^{kT}kT = e^{kt}kT_e$$

$$\Rightarrow (e^{kt}T)' = e^{kt}kT_e$$

$$\Rightarrow e^{kt}T = \int ke^{kt}T_e(t)dt + C$$

$$\Rightarrow T = e^{-kt}\int ke^{kt}T_e(t)dt + Ce^{-kt}$$