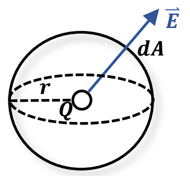
Gauss's Law

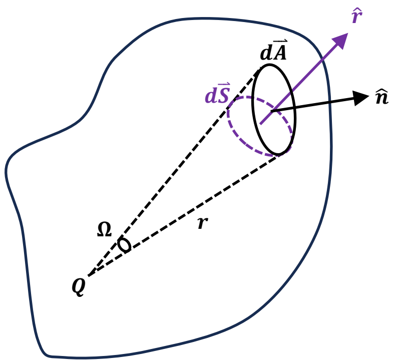
Gauss's Law for point charge



$$\oint_S \vec{E}\cdot d\vec{A} = \oint_S \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}\cdot\hat{r}dA = \frac{Q}{4\pi\varepsilon_0 r^2}\oint_S dA = \frac{Q}{4\pi\varepsilon_0 r^2}\cdot 4\pi r^2$$

$$\Rightarrow \boxed{\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}}$$

for an irregular surface



solid angle Ω

$$\Omega = \frac{dS}{r^2}$$

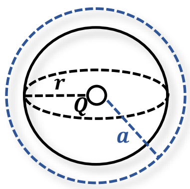
$$\oint_S \vec{E}\cdot d\vec{A} = \sum_i \vec{E}\cdot\hat{n}dA = \sum_i \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}\cdot\hat{n}dA$$

make projection $\hat{r}\cdot\hat{n}dA = dS$

$$\Rightarrow \frac{Q}{4\pi\varepsilon_0}\sum_i \frac{dS}{r^2} = \frac{Q}{4\pi\varepsilon_0}\frac{4\pi r^2}{r^2} = \frac{Q}{\varepsilon_0}$$

E Field Calculation by Gauss's Law

- uniformly charged sphere $a > r$

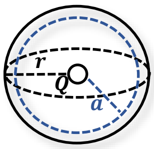


$$\oint_S \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E\oint_S dA = E\cdot 4\pi a^2$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 a^2}$$

- uniformly charged sphere $a < r$

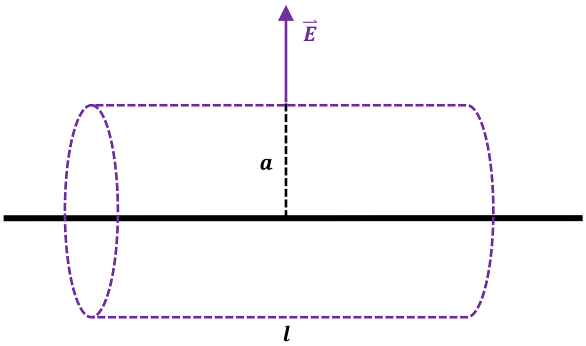


$$q_{in} = Q\frac{\frac{4}{3}\pi a^3}{\frac{4}{3}\pi r^3} = \frac{a^3}{r^3}Q$$

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = E\oint dA = E\cdot 4\pi a^2 = \frac{a^3}{r^3\varepsilon_0}Q$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^3}a$$

- infinite charged wire, charge density λ

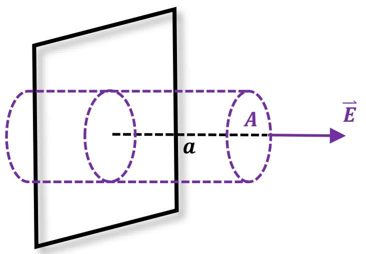


$$\oint_S \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow E\cdot 2\pi a l = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 a}$$

- infinite charged plane, charge density σ



$$\oint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = 2E \cdot A = \frac{\sigma A}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

Electrostatic Equailibrium

when there is no net motion of charge within a conductor, it is said to be in electrostatic equilibrium

properties:

- $E_{in} = 0$
- if the conductor is isolated and charged, the charge resides on its surface
- the E field at the point on the surface is σ/ε_0 and is perpendicular to the surface
- on an irregular shaped conductor, the charge density is greater on the surface of smaller radius of curvature.

Differential Form of Gauss's Law

$$\boxed{\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$$

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

write integral form of Gauss law

$$\oint_S \vec{E} \cdot dS = \int_V \frac{\rho}{\varepsilon_0} dV$$

apply the divergence theorem

$$\oint_S \vec{E} \cdot dS = \int_V \nabla \cdot \vec{E}\, dV$$

$$\Rightarrow \int_V \nabla \cdot \vec{E}\, dV = \int_V \frac{\rho}{\varepsilon_0} dV$$

$$\Rightarrow \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$