Electric Field

Coulomb's Law

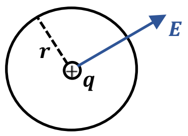
$$\boxed{F_e = k_e \frac{|q_1||q_2|}{r^2}}$$

Electric Field

$$\boxed{\vec{E} = \frac{F_e}{q_0} = k_e \frac{q}{r^2}\hat{r}}$$

$$\vec{E} = \sum_i E_i = \sum_i \frac{q_i}{r_i^2}\hat{r}$$

define $\frac{1}{\varepsilon_0}$ electric field line per coulomb

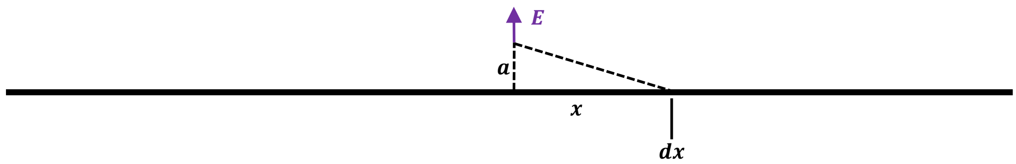


at each point on the surface

$$E = q_0 \frac{1}{\varepsilon_0}\frac{1}{4\pi r^2} \Rightarrow k_e = \frac{1}{4\pi\varepsilon_0}$$

Find the E Field

1. Find the E field of distance a from the infinite wire with charge density $\lambda(C/m)$

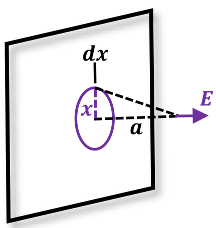


$$E = \int_{-\infty}^{\infty} dE_y = \int_{-\infty}^{\infty} \frac{\lambda dx}{4\pi\varepsilon_0(x^2+a^2)} \cdot \frac{a}{\sqrt{x^2+a^2}} = \frac{2\lambda a}{4\pi\varepsilon_0}\int_0^{\infty} \frac{dx}{\sqrt{x^2+a^2}}$$

Let $x = a\tan\theta$

$$\Rightarrow \frac{2\lambda a}{4\pi\varepsilon_0}\int_0^{\frac{\pi}{2}} \frac{asec^2\theta d\theta}{(a^2(tan^2\theta+1))^{\frac{3}{2}}} = \frac{\lambda}{2\pi\varepsilon_0 a}\int_0^{\frac{\pi}{2}} cos\theta d\theta = \frac{\lambda}{2\pi\varepsilon_0 a} = 2k_e\frac{\lambda}{a}$$

2. Find the E field of distance a from the infinite plane with charge density $\sigma(C/m^2)$

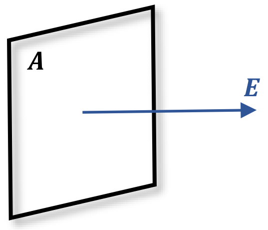


$$E = \int_0^{\infty} dE = \int_0^{\infty} \frac{2\pi x dx\sigma}{4\pi\varepsilon_0(x^2+a^2)} \cdot \frac{a}{\sqrt{x^2+a^2}} = \frac{\sigma}{2\varepsilon_0}\int_0^{\frac{\pi}{2}} \frac{ax}{(x^2+a^2)^{\frac{3}{2}}}dx$$

Let $x = a\tan\theta$

$$\Rightarrow \frac{\sigma}{2\varepsilon_0}\int_0^{\frac{\pi}{2}} \frac{tan\theta}{(tan^2\theta+1)^{\frac{3}{2}}} sec^2\theta d\theta = \frac{\sigma}{2\varepsilon_0}\int_0^{\frac{\pi}{2}} sin\theta d\theta = \boxed{\frac{\sigma}{2\varepsilon_0}}$$

Electric Flux



$\boxed{\Phi_E = \vec{A}\vec{E} = AEcos\theta}$

generally, $\Phi_E = \int E \cdot dA$