## **Beats and Resonance**

## **Beats**

constructive and destructive interference of two or more frequencies of sound is the phenomenon of beats

consider interference of 2 waves: 
$$\begin{cases} y_1 = A\cos(k_1x - \omega_1t) \\ y_2 = A\cos(k_2x - \omega_2t) \end{cases}$$

resonance occurs when  $\omega_1 pprox \omega_2$ 

$$y = A\cos(k_1x - \omega_1t) + A\cos(k_2x - \omega_2t)$$

apply the trigonometric identities

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\Rightarrow 2A \cos \frac{(k_1 x - \omega_1 t) - (k_2 x - \omega_2 t)}{2} \cos \frac{(k_1 x - \omega_1 t) + (k_2 x - \omega_2 t)}{2}$$

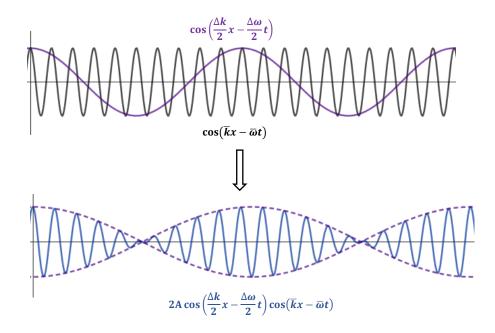
$$\Rightarrow 2A \cos \frac{(k_1 - k_2)x - (\omega_1 - \omega_2)t}{2} \cos \frac{(k_1 + k_2)x - (\omega_1 + \omega_2)t}{2}$$

$$\Rightarrow 2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\cos(\overline{k}x - \overline{\omega}t)$$

$$f_{beat} = \left| \frac{\Delta \omega}{2\pi} \right| = |f_1 - f_2|$$

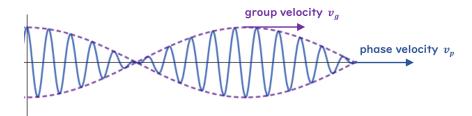
then we can sketch the graph

$$\overline{k} \gg \Delta k$$
,  $\overline{\omega} \gg \Delta \omega$ 



## **Group Velocity**

$$y = A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta \omega}{2}t\right)\cos(\overline{k}x - \overline{\omega}t)$$



ullet phase velocity  $v_p$ 

$$v_p = \frac{\overline{\omega}}{\overline{k}} \approx \frac{\omega_1}{k_1} \approx \frac{\omega_2}{k_2}$$

ullet group velocity  $v_g$  (envelope velocity)

$$v_g = \frac{\Delta \omega/2}{\Delta k/2} = \frac{\Delta \omega}{\Delta k}$$

$$\Rightarrow v_g = \frac{d\omega}{dk}$$

velocity of wave packet and energy transmission follow  $\,v_{g}\,$ 

for non-dispersive medium: v independent of k, like mechanical waves in string

$$v_g = \frac{d\omega}{dk} = \frac{d(vk)}{dk} = v$$

for dispersive medium: v dependent on k, like surface water wave

$$v_g = \frac{d\omega}{dk} = \frac{d(vk)}{dk} = v\frac{dk}{dk} + k\frac{dv}{dk} = v + k\frac{dv}{dk}$$

$$\frac{dv}{dk} < 0 \Rightarrow v_g < v$$