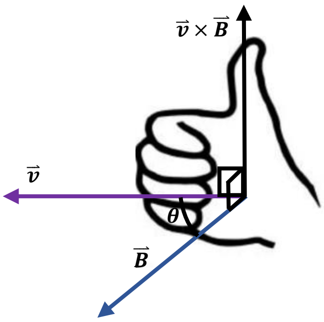
Magnetic Field

Magnetic Force on a Moving Charge

the magnetic force $\vec{F_B}$ will be exerted on a charged particle moving with a velocity $\vec{v}$

$\boxed{\vec{F}_B = q\vec{v} \times \vec{B}}$



the direction of $\vec{F_B}$ for negative charges are opposite

$|\vec{F_B}| = |q\vec{v} \times \vec{B}| = |qvBsin\theta|$

Charged particles in a magnetic field

Circular motion

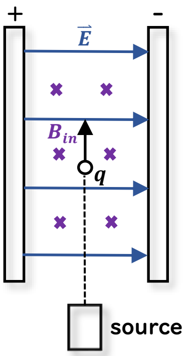
$$F_B = qvB = \frac{mv^2}{r} \Rightarrow \boxed{r = \frac{mv}{qB}}$$

$$T = \frac{2\pi r}{v} \Rightarrow \boxed{T = \frac{2\pi m}{qB}}$$

Lorentz Force

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

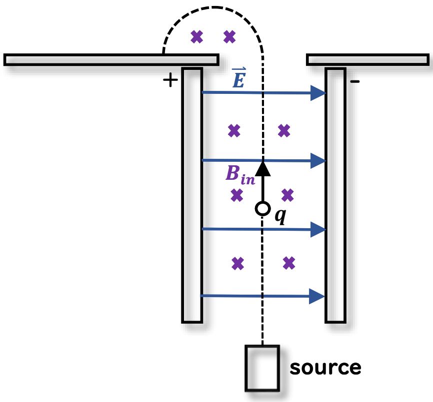
Velocity Selector



only the particles with the given speed can pass through two fields without deflection

$$F_E = F_B = qE = qvB \Rightarrow v = \frac{E}{B}$$

Mass Spectrometer

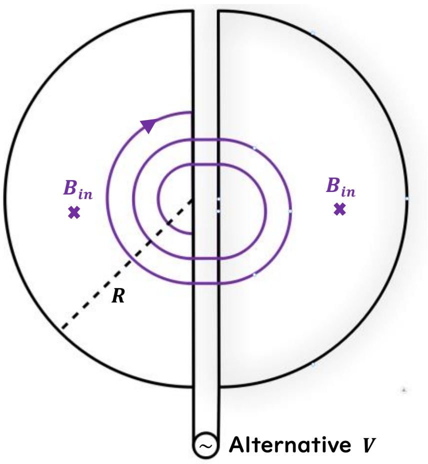


the mass to charge ratio can be determined by

measuring the radius: $r = \frac{mv}{qB}$

$$\Rightarrow \frac{m}{q} = \frac{rB}{v} = \frac{rB^2}{E}$$

The Cyclotron



the particles accelerate in the slits by the E field and perform uniform circular motion in the B field

$$K_E = \frac{1}{2}mv^2 = \frac{1}{2}m(\frac{qBR}{m})^2 = \frac{q^2B^2R^2}{2m}$$

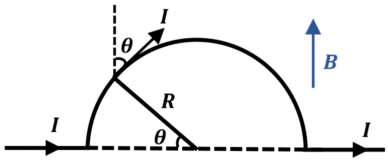
Force on a Wire

$\vec{F} = q\vec{v} \times \vec{B}$, total force $\vec{F} = N(q\vec{v} \times \vec{B})$

$\Rightarrow nAL(q\vec{v} \times \vec{B}) = (nqvA)(L\hat{v} \times \vec{B}) = I\vec{L} \times \vec{B}$

$\Rightarrow \boxed{\vec{F} = I\vec{L} \times \vec{B}}$

Force on a semicircular conductor

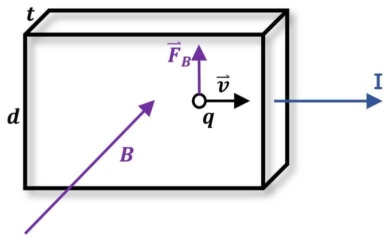


$d\vec{F} = IdLBsin\theta$

$dL = Rd\theta \Rightarrow F = IBR\int_0^{\pi} sin\theta d\theta = 2IRB$

Hall Effect

the charge carries deflect to one side of the conductor due to the magnetic force and form potential difference



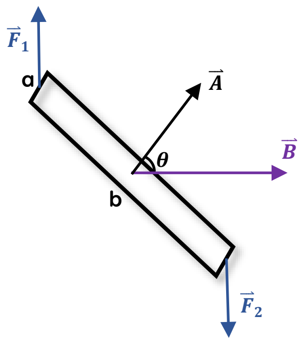
Hall voltage

$qvB = qE_H \Rightarrow E_H = vB$

$\Delta V_H = E_H d = vBd = \left(\frac{I}{nqA}\right)Bd = \left(\frac{I}{nqtd}\right)Bd = \frac{1}{nq} \cdot \frac{IB}{t} = R_H \frac{IB}{t}$

Hall coefficient $R_H = \frac{1}{nq}$

Torque on a Current Loop



$\tau = \vec{F}_1 \frac{b}{2} sin\theta + \vec{F}_2 \frac{b}{2} sin\theta = BIa\left(\frac{b}{2} sin\theta\right) + BIa\left(\frac{b}{2} sin\theta\right)$

$= IB(ab)sin\theta = IABsin\theta = I\vec{A} \times \vec{B}$

for any loop placed on a uniform B field:

$\boxed{\tau = I\vec{A} \times \vec{B}}$