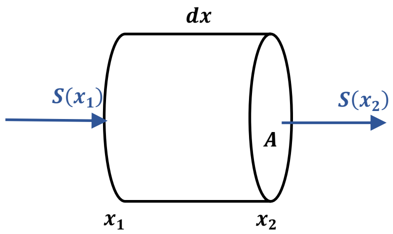
Sound Wave

Air Pressure

displacement of air: $S(x,t) = A\cos(kx - \omega t)$

for a small segment of air



change of volume

$$dV = A\big(S(x_2) - S(x_1)\big) = A \cdot dS$$

change of pressure

$$dp = -B\frac{dV}{V}$$

here B is Bulk modulus (Young's modulus Y in solid)

$$\Rightarrow dp = -B\frac{A \cdot dS}{A \cdot dx} \Rightarrow \Delta p(x,t) = -B\frac{\partial S}{\partial x}$$

for $S(x,t) = A\cos(kx - \omega t)$

$$\Delta p(x,t) = -B\frac{\partial S}{\partial x} == -B\frac{\partial}{\partial x}(A\cos(kx - \omega t))$$

$$\Rightarrow \Delta p(x,t) = BkA\sin(kx - \omega t) = p_0\sin(kx - \omega t)$$ where $p_0 = BkA$

the total air pressure

$$p(x,t) = p_{atm} + p_0\sin(kx - \omega t)$$

Sound Wave

for a small segment dx of air

$$F_{net} = p(x_1)A - p(x_2)A = -dp \cdot A = -\frac{\partial p}{\partial x}dx \cdot A$$

$$\Rightarrow -\frac{\partial}{\partial x}\left(p_{atm} - B\frac{\partial S}{\partial x}\right)dx \cdot A = AB\frac{\partial^2 S}{\partial x^2}dx$$

$$F_{net} = ma$$

$$\Rightarrow \rho A dx\frac{\partial^2 S}{\partial t^2} = AB\frac{\partial^2 S}{\partial x^2}dx$$

$\Rightarrow$ wave equation of sound

$$\boxed{\frac{\partial^2 S}{\partial t^2} = v^2\frac{\partial^2 S}{\partial x^2}}$$

where $\boxed{v = \sqrt{B/\rho}}$, ρ is density of air

Sound Intensity

Intensity of sound is power per area

$$I = \frac{P}{A} = \frac{Fv}{A} = pv = \Delta p(x,t) \cdot v(x,t)$$

$$\Rightarrow -B\frac{\partial S}{\partial x} \cdot \frac{\partial S}{\partial t} = BkA\sin(kx-\omega t) \cdot A\omega\sin(kx-\omega t) = Bk\omega A^2\sin^2(kx-\omega t)$$

then we can get time-averaged intensity

$$I_{avg} = \frac{1}{T}\int_0^T Bk\omega A^2\sin^2(kx-\omega t)\,dt$$

$$\Rightarrow \boxed{I_{avg} = \frac{1}{2}\rho v\omega^2A^2 = \frac{1}{2}Bk\omega A^2}$$

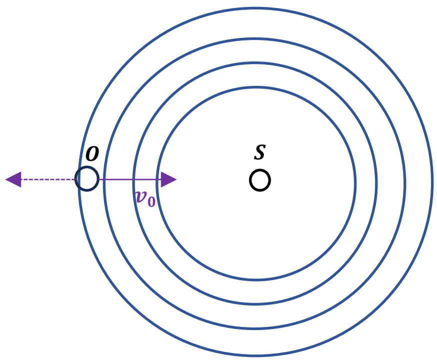
when sound wave spread spherically, at radius R

$$I = \frac{P}{4\pi R^2}$$

Doppler Effect

the movement will cause change of wavelength and frequency of sound wave

- when the sound source keep stationary



assume speed of sound in air v

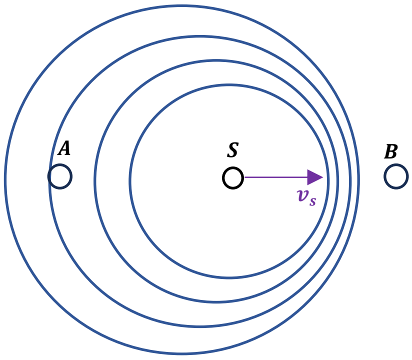
the frequency heard by observer is higher when approaching the source

$$f' = \frac{v'}{\lambda} = \frac{v+v_0}{\lambda} = \frac{v+v_0}{v}f$$

the frequency heard by observer is lower when leaving the source

$$f' = \frac{v'}{\lambda} = \frac{v-v_0}{\lambda} = \frac{v-v_0}{v}f$$

- when the sound source is moving



when the source approaching the observer, the apparent frequency is higher

$$f' = \frac{v}{\lambda'} = \frac{v}{\frac{v}{f} - \frac{v_s}{f}} = \frac{v}{v - v_s} f$$

when the source leaving the observer, the apparent frequency is lower

$$f' = \frac{v}{\lambda'} = \frac{v}{\frac{v}{f} + \frac{v_s}{f}} = \frac{v}{v + v_s} f$$

generally, when they are both moving

$$\boxed{f_o = \left(\frac{v \pm v_0}{v \mp v_0}\right) f_s}$$