First Order Autonomous

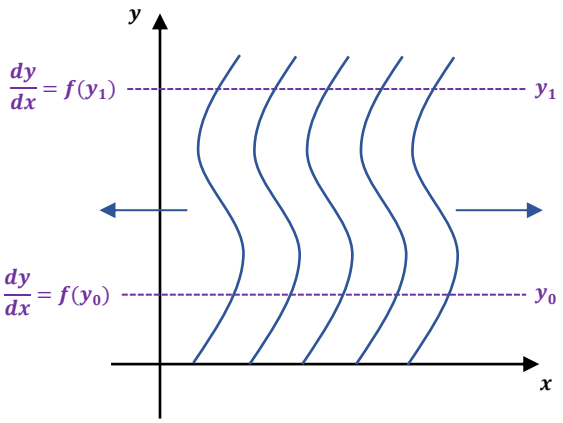
Autonomous

$$\frac{dy}{dx} = f(y)$$

where there is no independent variable on right hand side

we can get information about the solution without actually solving it

Direction Field



the integral curves of autonomous equation are translationally equal

Critical Point

$$\frac{dy_0}{dx} = f(y_0) = 0$$

then $y = y_0$ is an absolute barrier

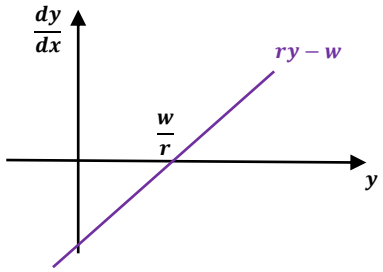
method:

1. find the critical points'
2. graph $f(y)$ and compare it with 0

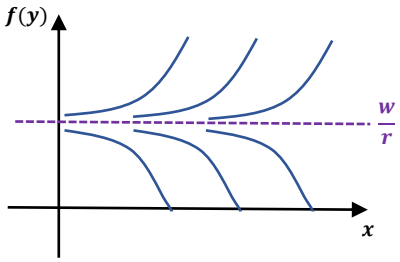
example: $dy/dx = r \cdot y - w$, where r and w are constant

find critical point:

$$\frac{dy}{dx} = r \cdot y - w = 0 \Rightarrow y = \frac{w}{r}$$



when $y > w/r,\ dy/dx > 0$ and increases
when $y < w/r,\ dy/dx < 0$ and decreases
then we can graph $f(y)$



Logistic Equation

the model is mostly used to describe how population increases

$$\frac{dy}{dx} = f(y) = ky = (a - by)y$$

where y is the population, k is the growth rate
the model has the feature that when y increases the k decreases, and vice versa

$(a - by)y = 0 \Rightarrow y = 0$ or $y = a/b$
when $0 < y < a/b,\ dy/dx > 0$ and it first increases and then decreases