

Beats and Resonance

Beats

constructive and destructive interference of two or more frequencies of sound is the phenomenon of beats

consider interference of 2 waves: $\begin{cases} y_1 = A \cos(k_1 x - \omega_1 t) \\ y_2 = A \cos(k_2 x - \omega_2 t) \end{cases}$

resonance occurs when $\omega_1 \approx \omega_2$

$$y = A \cos(k_1 x - \omega_1 t) + A \cos(k_2 x - \omega_2 t)$$

apply the trigonometric identities

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

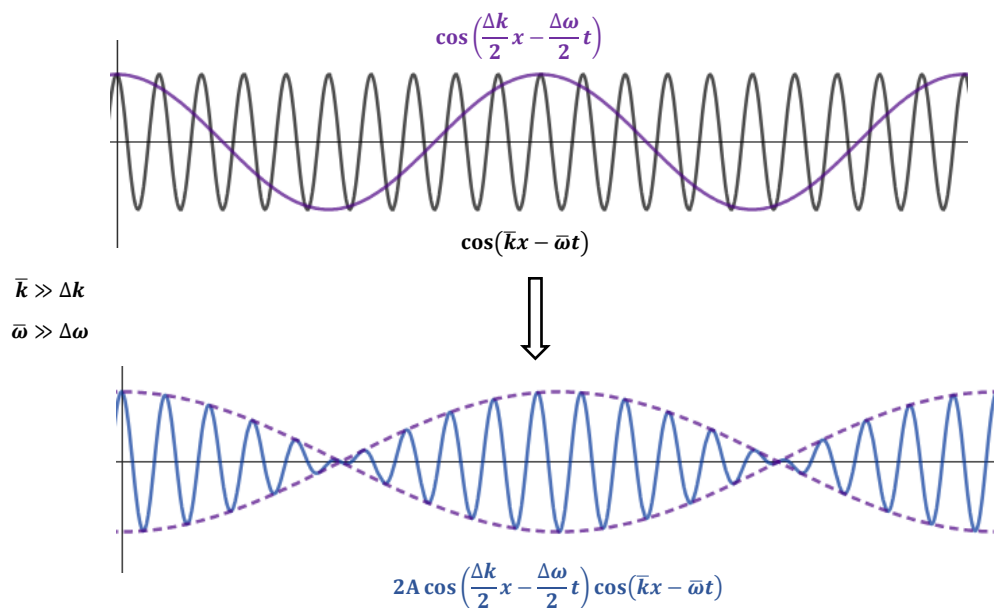
$$\Rightarrow 2A \cos \frac{(k_1 x - \omega_1 t) - (k_2 x - \omega_2 t)}{2} \cos \frac{(k_1 x - \omega_1 t) + (k_2 x - \omega_2 t)}{2}$$

$$\Rightarrow 2A \cos \frac{(k_1 - k_2)x - (\omega_1 - \omega_2)t}{2} \cos \frac{(k_1 + k_2)x - (\omega_1 + \omega_2)t}{2}$$

$$\Rightarrow 2A \cos \left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t \right) \cos(\bar{k}x - \bar{\omega}t)$$

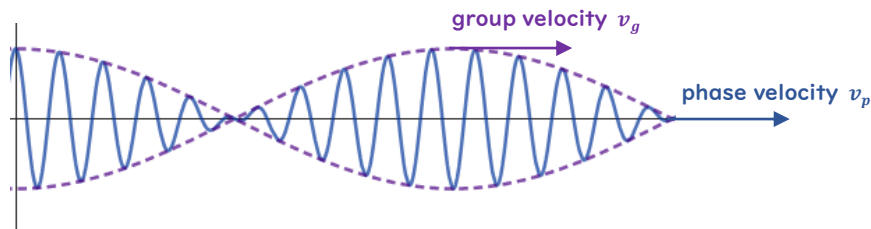
$$f_{\text{beat}} = \left| \frac{\Delta \omega}{2\pi} \right| = |f_1 - f_2|$$

then we can sketch the graph



Group Velocity

$$y = A \cos\left(\frac{\Delta k}{2}x - \frac{\Delta \omega}{2}t\right) \cos(\bar{k}x - \bar{\omega}t)$$



- phase velocity v_p

$$v_p = \frac{\bar{\omega}}{\bar{k}} \approx \frac{\omega_1}{k_1} \approx \frac{\omega_2}{k_2}$$

- group velocity v_g (envelope velocity)

$$v_g = \frac{\Delta \omega / 2}{\Delta k / 2} = \frac{\Delta \omega}{\Delta k}$$

$$\Rightarrow v_g = \frac{d\omega}{dk}$$

for non-dispersive medium: v independent of k , like mechanical waves in string

$$v_g = \frac{d\omega}{dk} = \frac{d(vk)}{dk} = v$$

for dispersive medium: v dependent on k , like surface water wave

$$v_g = \frac{d\omega}{dk} = \frac{d(vk)}{dk} = v \frac{dk}{dk} + k \frac{dv}{dk} = v + k \frac{dv}{dk}$$

$$\frac{dv}{dk} < 0 \Rightarrow v_g < v$$