Standing Wave

Standing Waves

a standing wave just seem to be standing produced from superposition of several waves

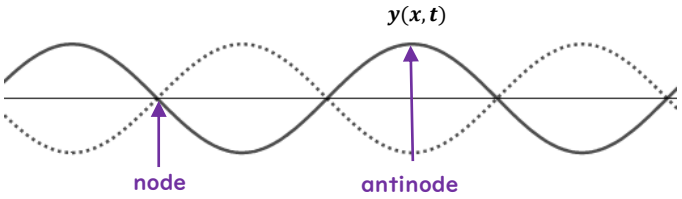
2 identical waves traveling in opposite direction will form a standing waves

$y(x,t) = y_1(x,t) + y_2(x,t) = A\sin(kx - \omega t) + A\sin(kx + \omega t)$

apply the trigonometric identities

$\Rightarrow y(x,t) = A[\sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t) + \sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t)]$

$\Rightarrow y(x,t) = 2A\sin(kx)\cos(\omega t)$



- there are positions always be 0, called node

  $\sin(kx) = 0 \Rightarrow kx = \frac{2\pi}{\lambda}x = n\pi$

  $\Rightarrow x = \frac{1}{2}n\lambda \quad n = 0, 1, 2\cdots$

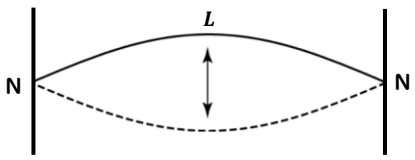
- there are positions that can reach $\pm 2A$ when oscillating, called antinode

  $\sin(kx) = \pm 1 \Rightarrow kx = \frac{2\pi}{\lambda}x = \left(n + \frac{1}{2}\right)\pi$

  $\Rightarrow x = \frac{1}{4}n\lambda \quad n = 1, 3, 5\cdots$

Standing Wave on a String

for first mode, $n = 1$, f_1 is called fundamental frequency



$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$

similarly, we calculate when n is bigger

$f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$

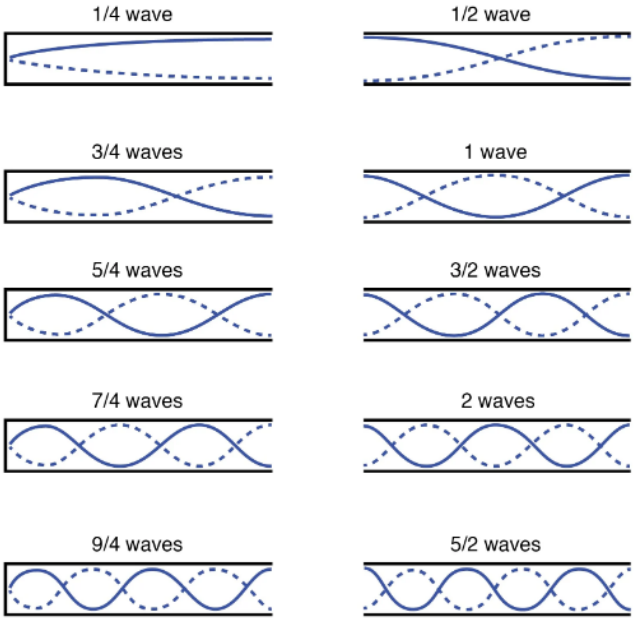
……

summarizing

$$\lambda_n = \frac{2}{n}L \quad n = 1, 2, 3 \cdots$$

$$f_n = \frac{v}{\lambda_n} = nf_1 \quad n = 1, 2, 3 \cdots$$

Standing Wave in Tube



- for an open tube: antinodes on both ends
  fundamental frequency $f_1 = v/2L$

  $\Rightarrow f_n = nf_1 = \frac{nv}{2L} \quad n = 1, 2, 3 \cdots$
- for tube closed on one end: node on the closed end
  fundamental frequency is $f_1 = v/4L$

  $\Rightarrow f_n = nf_1 = \frac{nv}{4L} \quad n = 1, 3, 5 \cdots$