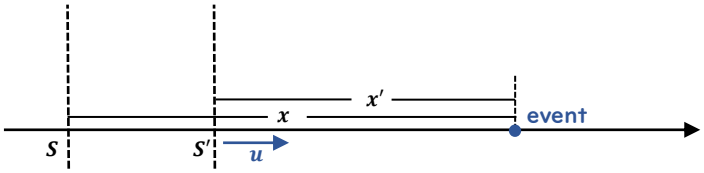
Introduction to Relativity

Newtonian Mechanics



when S' cross S, define

$$\begin{cases} S\ (x=0, t=0) \\ S'\ (x'=0, t=0) \end{cases}$$

observing the event

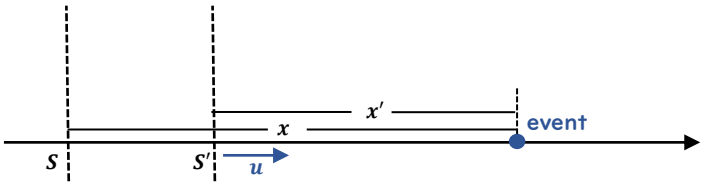
$$\begin{cases} S\ (x, t) \\ S'\ (x', t) \end{cases}$$

$$x' = x - ut$$

$$m\frac{d^2x}{dt^2} = F$$

$$m\frac{d^2x'}{dt^2} = F$$

Relativity



$$x' = \gamma(x - ut)$$

$$x = \gamma(x + ut')$$

observing the event

$$\begin{cases} S\ (x, t) \\ S'\ (x', t') \end{cases}$$

$$x' = ct'$$

$$x = ct$$

$$\Rightarrow 1 = \gamma^2\left[1 + u\frac{t'}{x'} - u\frac{t}{x} - u^2\left(\frac{t}{x}\right)\left(\frac{t'}{x'}\right)\right]$$

$$\Rightarrow 1 = \gamma^2\left(1 + \frac{u}{c} - \frac{u}{c} - u^2/c^2\right)$$

$$\Rightarrow \gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

Lorentz Transformation

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}} \qquad t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}$$