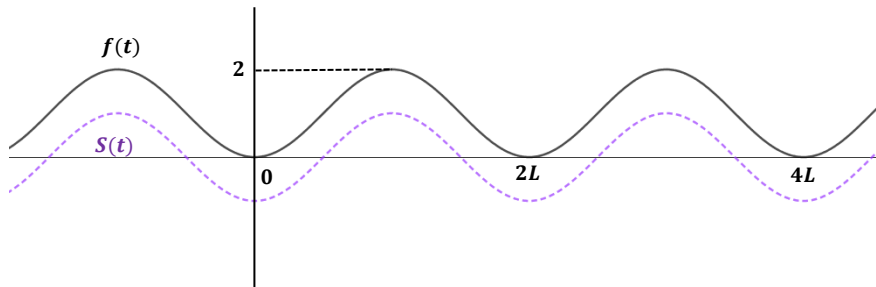# Special Solution by Fourier Series

Manage General Periodic Function

displace a function to get even or odd function

$S(t) = f(t) - 1$ and $S(t)$ is an even function

$$S(t) = \sum a_n \cos \frac{n\pi}{L} t \Rightarrow f(t) = 1 + \sum a_n \cos \frac{n\pi}{L} t$$

Special Solution of ODE by Fourier Series

for $y'' + \omega_0^2 y = f(t)$

when input is $\begin{cases} \sin \omega t \\ \cos \omega t \end{cases}$ or their linear combination, the solution is $\begin{cases} \sin \omega t / (\omega_0^2 - \omega^2) \\ \cos \omega t / (\omega_0^2 - \omega^2) \end{cases}$

more generally, when input can be present as Fourier series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \omega_n t + b_n \sin \omega_n t), \qquad \omega_n = \frac{\pi}{L} n$$

$$\Rightarrow x_p = \frac{a_0}{2\omega_0^2} + \sum_{n=1}^{\infty} \left(\frac{a_n \cos \omega_n t}{\omega_0^2 - \omega_n^2} + \frac{b_n \sin \omega_n t}{\omega_0^2 - \omega_n^2} \right)$$