

## Electric potential

### Electric Potential Energy

$$W = \vec{F} \cdot d\vec{s} = q_0 \vec{E} \cdot d\vec{s}$$

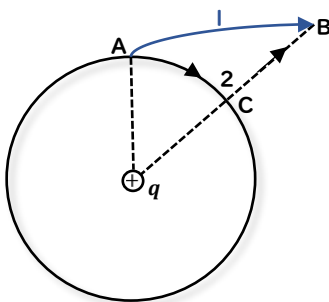
$$\Rightarrow \Delta U = U_B - U_A = - \int_A^B \vec{E} \cdot d\vec{s}$$

$$\text{energy is conservative} \Rightarrow q_0 \oint \vec{E} \cdot d\vec{s} = 0$$

### Electric Potential

$$\Delta V = \frac{\Delta U}{q_0} = - \int_A^B \vec{E} \cdot d\vec{s}$$

### V of a point charge



$$\begin{aligned} V_B - V_A &= \Delta V_1 = \Delta V_2 \\ &= - \int_A^B \vec{E} \cdot d\vec{s} = - \left( \int_A^C \vec{E} \cdot d\vec{s} + \int_C^B \vec{E} \cdot d\vec{s} \right) = - \left( 0 + \int_{r_A}^{r_B} k_e \frac{q}{r^2} dr \right) \\ &= k_e q \left( \frac{1}{r_B} - \frac{1}{r_A} \right) \\ &\text{choose } V = 0 \text{ at } r_A = \infty \\ &\Rightarrow V = k_e \frac{q}{r} \end{aligned}$$

$$\text{for } V = k_e \frac{q}{r} \text{ and } E_r = k_e \frac{q}{r^2}, \text{ we can find } E_r = - \frac{dV}{dr}$$

### Potential Energy of Multiple Charges

$$U_{AB} = q_A V_B = q_A \left( k_e \frac{q_B}{r_{AB}} \right) = k_e \frac{q_A q_B}{r_{AB}} \Rightarrow U = \sum_{i,j} k_e \frac{q_i q_j}{r_{ij}}$$