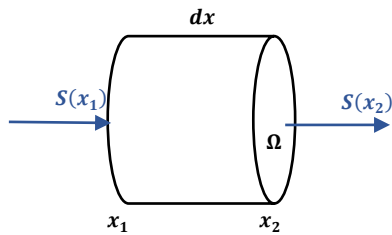


## Sound Wave

### Air Pressure

displacement of air:  $S(x, t) = A \cos(kx - \omega t)$

for a small segment of air



change of volume

$$dV = \Omega(S(x_2) - S(x_1)) = \Omega \cdot dS$$

change of pressure

$$dp = -B \frac{dV}{V}$$

here  $B$  is Bulk modulus (Young's modulus  $Y$  in solid)

$$\Rightarrow dp = -B \frac{\Omega \cdot dS}{\Omega \cdot dx} \Rightarrow \Delta p(x, t) = -B \frac{\partial S}{\partial x}$$

for  $S(x, t) = A \cos(kx - \omega t)$

$$\Delta p(x, t) = -B \frac{\partial S}{\partial x} = -B \frac{\partial}{\partial x} (A \cos(kx - \omega t))$$

$$\Rightarrow \Delta p(x, t) = BkA \sin(kx - \omega t) = p_0 \sin(kx - \omega t) \text{ where } p_0 = BkA$$

the total air pressure

$$p(x, t) = p_{atm} + p_0 \sin(kx - \omega t)$$

### Sound Wave

for a small segment  $dx$  of air

$$F_{net} = p(x_1)\Omega - p(x_2)\Omega = -dp \cdot \Omega = -\frac{\partial p}{\partial x} dx \cdot \Omega$$

$$\Rightarrow -\frac{\partial}{\partial x} \left( p_{atm} - B \frac{\partial S}{\partial x} \right) dx \cdot \Omega = \Omega B \frac{\partial^2 S}{\partial x^2} dx$$

$$F_{net} = ma$$

$$\Rightarrow \rho \Omega dx \frac{\partial^2 S}{\partial t^2} = \Omega B \frac{\partial^2 S}{\partial x^2} dx$$

$\Rightarrow$  wave equation of sound

$$\boxed{\frac{\partial^2 S}{\partial t^2} = v^2 \frac{\partial^2 S}{\partial x^2}}$$

where  $\boxed{v = \sqrt{B/\rho}}$ ,  $\rho$  is density of air

## Sound Intensity

Intensity of sound is power per area

$$I = \frac{P}{\Omega} = \frac{Fv}{\Omega} = pv = \Delta p(x, t) \cdot v(x, t)$$

$$\Rightarrow -B \frac{\partial S}{\partial x} \cdot \frac{\partial S}{\partial t} = BkA \sin(kx - \omega t) \cdot A\omega \sin(kx - \omega t) = Bk\omega A^2 \sin^2(kx - \omega t)$$

then we can get time-averaged intensity

$$I_{avg} = \frac{1}{T} \int_0^T Bk\omega A^2 \sin^2(kx - \omega t) dt$$

$$\Rightarrow I_{avg} = \frac{1}{2} \rho v \omega^2 A^2 = \frac{1}{2} Bk\omega A^2$$

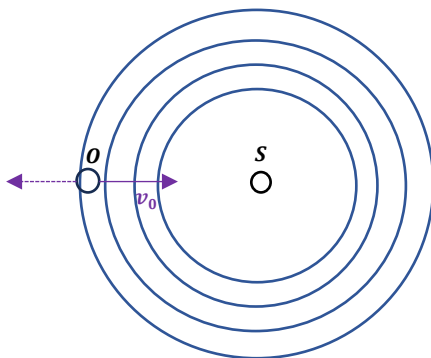
when sound wave spread spherically, at radius  $R$

$$I = \frac{P}{4\pi R^2}$$

## Doppler Effect

the movement will cause change of wavelength and frequency of sound wave

- when the sound source keep stationary



assume speed of sound in air  $v$

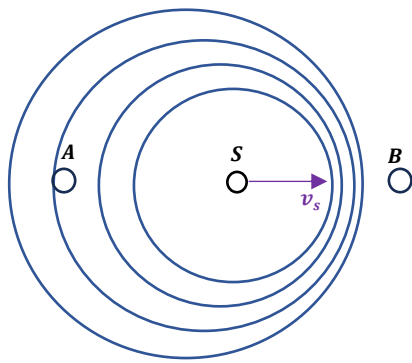
the frequency heard by observer is **higher when approaching the source**

$$f' = \frac{v'}{\lambda} = \frac{v + v_0}{\lambda} = \frac{v + v_0}{v} f$$

the frequency heard by observer is **lower when leaving the source**

$$f' = \frac{v'}{\lambda} = \frac{v - v_0}{\lambda} = \frac{v - v_0}{v} f$$

- when the sound source is moving



when the source **approaching the observer**, the apparent frequency is **higher**

$$f' = \frac{v}{\lambda'} = \frac{v}{\frac{v}{f} - \frac{v_s}{f}} = \frac{v}{v - v_s} f$$

when the source **leaving the observer**, the apparent frequency is **lower**

$$f' = \frac{v}{\lambda'} = \frac{v}{\frac{v}{f} + \frac{v_s}{f}} = \frac{v}{v + v_s} f$$

generally, when they are both moving

$$f_o = \left( \frac{v \pm v_o}{v \mp v_s} \right) f_s$$