Beats and Resonance

Beats

constructive and destructive interference of two or more frequencies of sound is the phenomenon of beats

consider interference of 2 waves: $\begin{cases} y_1 = A\cos(k_1x - \omega_1 t) \\ y_2 = A\cos(k_2x - \omega_2 t) \end{cases}$

resonance occurs when $\omega_1 \approx \omega_2$

$y = A\cos(k_1x - \omega_1 t) + A\cos(k_2x - \omega_2 t)$

apply the trigonometric identities

$$\boxed{\cos\alpha + \cos\beta = 2\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}}$$

$$\Rightarrow 2A\cos\frac{(k_1x - \omega_1 t) - (k_2x - \omega_2 t)}{2}\cos\frac{(k_1x - \omega_1 t) + (k_2x - \omega_2 t)}{2}$$

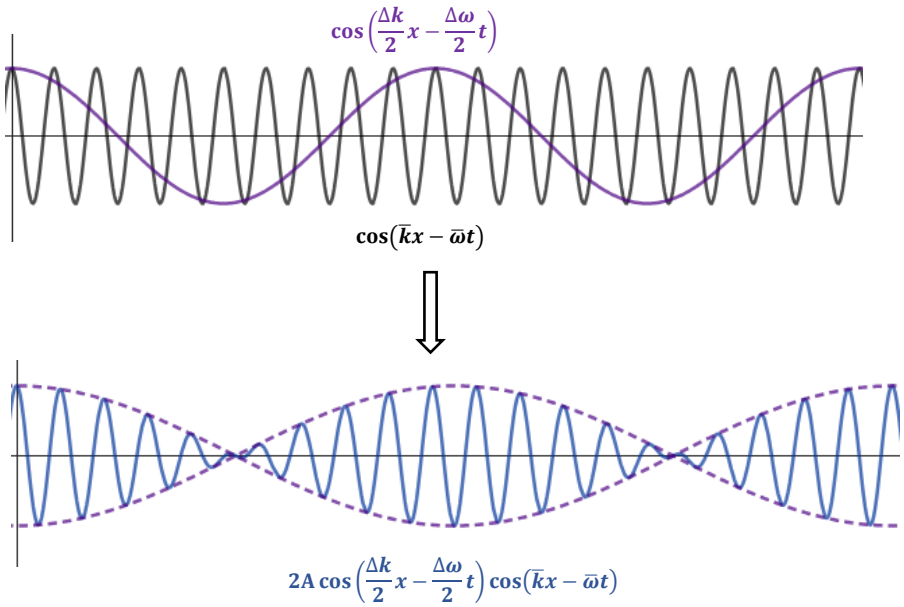
$$\Rightarrow 2A\cos\frac{(k_1 - k_2)x - (\omega_1 - \omega_2)t}{2}\cos\frac{(k_1 + k_2)x - (\omega_1 + \omega_2)t}{2}$$

$$\Rightarrow 2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\cos(\bar{k}x - \bar{\omega}t)$$

$$f_{beat} = \left|\frac{\Delta\omega}{2\pi}\right| = |f_1 - f_2|$$

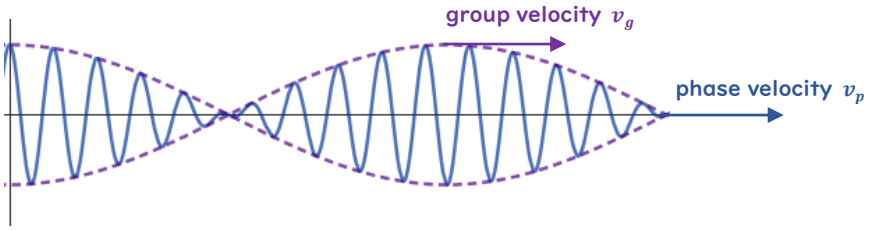
then we can sketch the graph

$\bar{k} \gg \Delta k,\ \bar{\omega} \gg \Delta\omega$



Group Velocity

$$y = \mathrm{A}\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\cos\left(\bar{k}x - \bar{\omega}t\right)$$



- phase velocity v_p

  $$v_p = \frac{\bar{\omega}}{\bar{k}} \approx \frac{\omega_1}{k_1} \approx \frac{\omega_2}{k_2}$$

- group velocity v_g (envelope velocity)

  $$v_g = \frac{\Delta\omega/2}{\Delta k/2} = \frac{\Delta\omega}{\Delta k}$$

  $$\Rightarrow v_g = \frac{d\omega}{dk}$$

  velocity of wave packet and energy transmission follow v_g

for non-dispersive medium: v independent of k, like mechanical waves in string

$$v_g = \frac{d\omega}{dk} = \frac{d(vk)}{dk} = v$$

for dispersive medium: v dependent on k, like surface water wave

$$v_g = \frac{d\omega}{dk} = \frac{d(vk)}{dk} = v\frac{dk}{dk} + k\frac{dv}{dk} = v + k\frac{dv}{dk}$$

$$\frac{dv}{dk} < 0 \Rightarrow v_g < v$$