Gauss's Law

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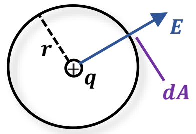
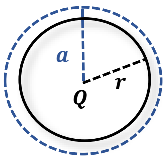


for a positive charge q in the sphere

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = E \oint d\vec{A} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot (4\pi r^2) = \frac{q}{\varepsilon_0}$$

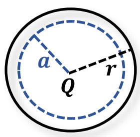
for any closed surface $\boxed{\Phi_E = \frac{q}{\varepsilon_0}}$

find the E field of a uniformly charged sphere



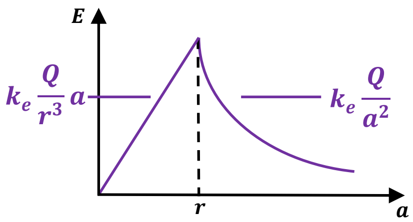
$a > r$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$
$$= E \oint d\vec{A} = E(4\pi a^2)$$
$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 a^2} = k_e \frac{Q}{a^2}$$



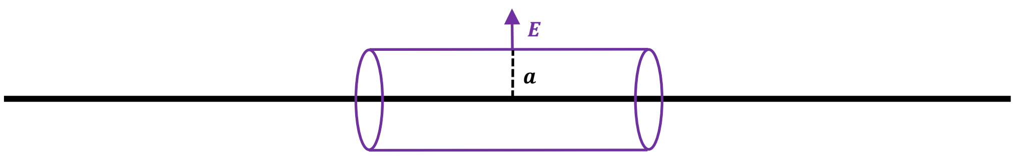
$a < r$

$$q_{in} = Q \frac{\frac{4}{3}\pi a^3}{\frac{4}{3}\pi r^3} = \frac{a^3}{r^3} Q$$
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = E \oint d\vec{A} = E(4\pi a^2) = \frac{a^3 Q}{r^3 \varepsilon_0}$$
$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^3} a = k_e \frac{Q}{r^3} a$$



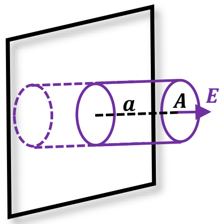
Find the E field by Gauss Law

1. Find the E field of distance a from the infinite wire with charge density $\lambda (C/m)$



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = E \oint d\vec{A} = E(2\pi a l) = \frac{\lambda l}{\varepsilon_0}$$
$$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 a} = 2k_e \frac{\lambda}{a}$$

2. Find the E field of distance a from the infinite plane with charge density $\sigma(C/m^2)$



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = E\oint d\vec{A} = 2EA = \frac{\sigma A}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

Electrostatic Equilibrium

- When there is no net motion of charge within a conductor, it is said to be in electrostatic equilibrium
- Properties:
  1. $E_{in} = 0$
  2. If the conductor is isolated and charged, the charge resides on its surface.
  3. The E field at the point on the surface is $\frac{\sigma}{\varepsilon_0}$ and is perpendicular to the surface.
  4. On an irregular shaped conductor, the charge density is greater on the surface of smaller radius of curvature.