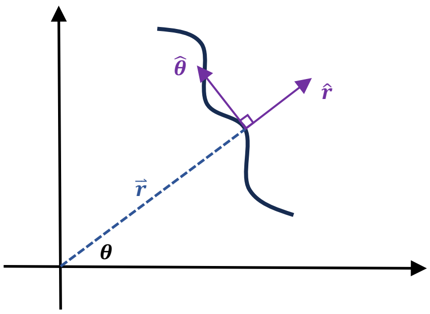
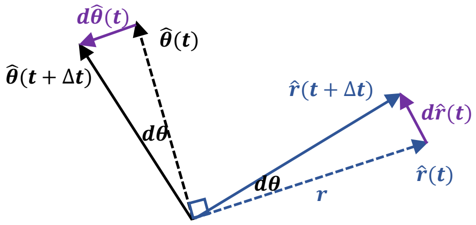
Kinematics Particles

2D Polar Coordinate System



$\hat{r}$ is unit vector parallel to $\vec{r}$ (radial)
$\hat{\theta}$ is unit vector vertical to $\vec{r}$ (tangential)

Velocity in 2D polar coordinates



$$\vec{r} = r\hat{r}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d(r\cdot\hat{r})}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt}$$

$$d\hat{r} \cong |\hat{r}|d\theta\hat{\theta} = d\theta\hat{\theta}$$

$$\frac{d\hat{r}}{dt} = \frac{d\theta}{dt}\hat{\theta}$$

$$\vec{v} = \underbrace{\frac{dr}{dt}}_{v_r}\hat{r} + \underbrace{r\frac{d\theta}{dt}}_{v_\theta}\hat{\theta} = v_r\hat{r} + v_\theta\hat{\theta}$$

$$\boxed{\vec{v} = v_r\hat{r} + v_\theta\hat{\theta}}$$

Acceleration in 2D polar coordinates

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\left(v_r\hat{r} + v_\theta\hat{\theta}\right)}{dt} = \frac{d^2r}{dt^2}\hat{r} + \frac{dr}{dt}\frac{d\hat{r}}{dt} + \frac{dr}{dt}\frac{d\theta}{dt}\hat{\theta} + r\frac{d\theta}{dt}\frac{d\hat{\theta}}{dt} + r\frac{d^2\theta}{dt^2}$$

$$d\hat{\theta} \cong \left|\hat{\theta}\right|d\theta(-\hat{r}) = -d\theta\hat{r}$$

$$\Rightarrow \boxed{\vec{a} = \left(\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right)\hat{r} + \left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)\hat{\theta}}$$