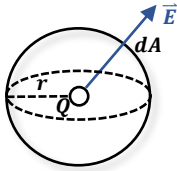


Gauss's Law

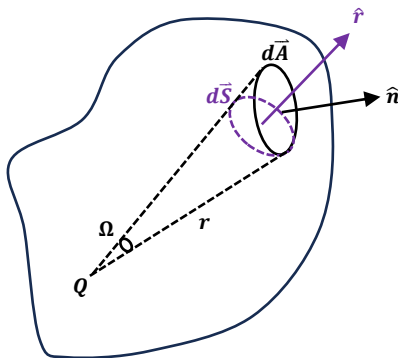
Gauss's Law for point charge



$$\oint_S \vec{E} \cdot d\vec{A} = \oint_S \frac{Q}{4\pi\epsilon_0 r^2} \hat{r} \cdot \hat{r} dA = \frac{Q}{4\pi\epsilon_0 r^2} \oint_S dA = \frac{Q}{4\pi\epsilon_0 r^2} \cdot 4\pi r^2$$

$$\Rightarrow \boxed{\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}}$$

for an irregular surface

solid angle Ω

$$\Omega = \frac{dS}{r^2}$$

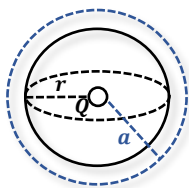
$$\oint_S \vec{E} \cdot d\vec{A} = \sum_i \vec{E} \cdot \hat{n} dA = \sum_i \frac{Q}{4\pi\epsilon_0 r^2} \hat{r} \cdot \hat{n} dA$$

make projection $\hat{r} \cdot \hat{n} dA = dS$

$$\Rightarrow \frac{Q}{4\pi\epsilon_0} \sum_i \frac{dS}{r^2} = \frac{Q}{4\pi\epsilon_0} \frac{4\pi r^2}{r^2} = \frac{Q}{\epsilon_0}$$

E Field Calculation by Gauss's Law

- uniformly charged sphere $a > r$

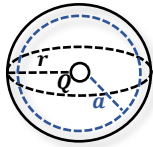


$$\oint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E \oint_S dA = E \cdot 4\pi a^2$$

$$\Rightarrow E = \frac{Q}{4\pi\epsilon_0 a^2}$$

- uniformly charged sphere $a < r$

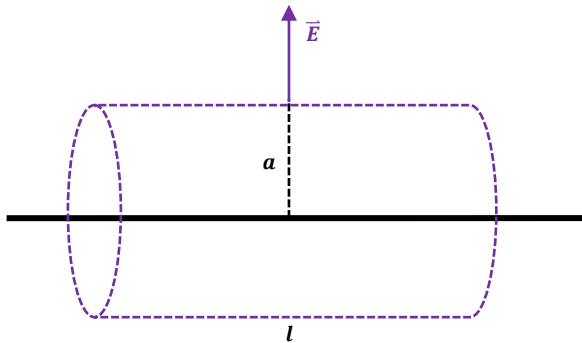


$$q_{in} = Q \frac{\frac{4}{3}\pi a^3}{\frac{4}{3}\pi r^3} = \frac{a^3}{r^3} Q$$

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} = E \oint_S dA = E \cdot 4\pi a^2 = \frac{a^3}{r^3 \epsilon_0} Q$$

$$\Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^3} a$$

- infinite charged wire, charge density λ

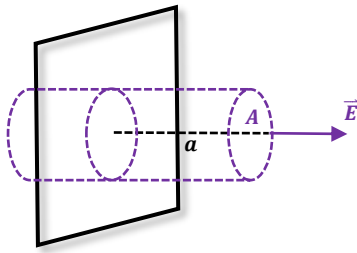


$$\oint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$\Rightarrow E \cdot 2\pi a l = \frac{\lambda l}{\epsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 a}$$

- infinite charged plane, charge density σ



$$\oint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} = 2E \cdot A = \frac{\sigma A}{\epsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{2\epsilon_0}$$

Electrostatic Equilibrium

when there is no net motion of charge within a conductor, it is said to be in electrostatic equilibrium

properties:

- $E_{in} = 0$
- if the conductor is **isolated** and charged, the charge **resides on its surface**
- the E field at the point on the surface is σ/ϵ_0 and is perpendicular to the surface
- on an irregular shaped conductor, the charge density is greater on the surface of smaller radius of curvature.