# Inductance

## Self-inductance

the induced emf tends to oppose the direction of emf of the battery (Lenz's Law)

## Inductor

self-induced emf $\varepsilon_L$

$$\boxed{\varepsilon_L = -L\frac{dI}{dt}}$$

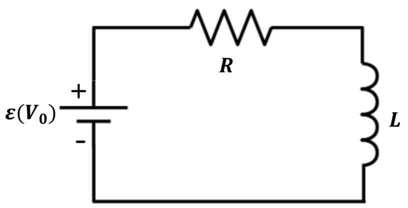
$L$ is inductance of the inductor coil

### Inductance of the Inductor

$$\varepsilon_L = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

$$\Rightarrow L = \frac{\mathrm{N}\Phi_B}{I}$$

## RL Circuit Charging




apply Kirchhoff's rule clockwise:

$$\varepsilon - IR - L\frac{dI}{dt} = 0 \Rightarrow IR + L\frac{dI}{dt} = V_0$$

when time $t \to \infty$, $I$ tends to be a stable value $I_\infty$, $dI \to 0$

$$I_\infty = \frac{V_0}{R}$$

assume $I(t) = I_\infty + \tilde{I}(t)$, where $\tilde{I}(t)$ is the difference

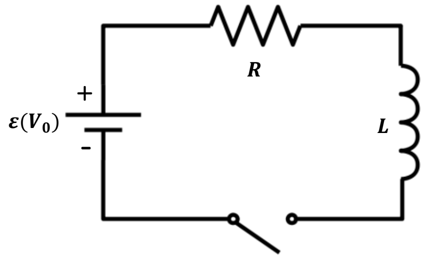
$$\Rightarrow L\frac{d\tilde{I}}{dt} + R\tilde{I} + RI_\infty = V_0 \Rightarrow \frac{d\tilde{I}}{dt} = -\frac{R}{L}\tilde{I}$$

$$\Rightarrow \tilde{I} = \tilde{I}_0 e^{-\frac{Rt}{L}}$$

$$\Rightarrow I(t) = \frac{V_0}{R} + \tilde{I}_0 e^{-\frac{Rt}{L}} = \frac{V_0}{R} + \left(-\frac{V_0}{R}\right)e^{-\frac{Rt}{L}}$$

$$\Rightarrow I(t) = \frac{V_0}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

## RL Circuit Discharging




after the current reaches the maximum, remove the switch

$$L\frac{dI}{dt} = IR$$

$$\Rightarrow I(t) = \frac{V_0}{R}e^{-\frac{Rt}{L}}$$

## Energy in a Magnetic Field

$$U = \int I^2 R = \int_0^\infty {I_0}^2 e^{-\frac{2Rt}{L}} R = {I_0}^2 R\frac{L}{2R} = \frac{1}{2}L{I_0}^2$$

$$\Rightarrow \boxed{U = \frac{1}{2}L{I_0}^2}$$