Kinetics Particles

Newton's Second Law

$$\boxed{\sum_i \vec{F}_i = m\vec{a}}$$

Cartesian coordinates

$$\sum F_x = ma_x = m\ddot{x}$$

$$\sum F_y = ma_y = m\ddot{y}$$

$$\sum F_z = ma_z = m\ddot{z}$$

Cylindrical coordinates

$$\sum F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$\sum F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$\sum F_z = ma_z = m\ddot{z}$$

Energy

Work Done

$$\boxed{W = \int \vec{F} \cdot d\vec{r}}$$

Power

$$P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$$\Rightarrow \boxed{P = \vec{F} \cdot \vec{v}}$$

Kinetic Energy

$$W = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} madx = \int_{x_i}^{x_f} m\frac{dv}{dt}dx = \int_{v_i}^{v_f} mvdv$$

$$= m\int_{v_i}^{v_f} vdv = m\frac{{v_f}^2 - {v_i}^2}{2} = E_f - E_i$$

$$\Rightarrow \boxed{E_K = \frac{1}{2}mv^2}$$

Gravitational Potential Energy

$$\vec{F}_g = -G\frac{M_E m}{r^2}\hat{r}$$

select infinity as zero potential point

$$E_g = \int_{\infty}^{r} -\left(-G\frac{M_E m}{r^2}\hat{r}\right) d\vec{r} = GM_E m \int_{\infty}^{r} \frac{1}{r^2}\hat{r}(\hat{r}dr + rd\hat{r}) = GM_E m \int_{\infty}^{r} \frac{1}{r^2} dr = -G\frac{M_E m}{r}$$

$$\Rightarrow \boxed{E_g = -G\frac{M_E m}{r}}$$

at Earth surface $r = R_E$

$$|E_g| = G\frac{M_E m}{{R_E}^2} = mg \Rightarrow GM_E = g{R_E}^2$$

First Cosmic Velocity: Orbital Velocity

least velocity to keep orbit around a celestial body

$$\frac{m{v_1}^2}{R_E} = G\frac{M_E m}{{R_E}^2} \Rightarrow v_1 = \sqrt{\frac{GM_E}{R_E}} = 7.91 \times 10^3 m/s$$

Second Cosmic Velocity: Escape Speed

least velocity to escape the celestial body

$$\frac{1}{2}m{v_2}^2 = \frac{GM_E m}{R_E} \Rightarrow v_2 = \sqrt{\frac{2GM_E}{R_E}} = 11.2 \times 10^3 m/s$$

Third Cosmic Velocity

least velocity to escape gravitational field of solar system

set v_r as the velocity of Earth relative to the sun

$$\frac{1}{2}m{v_3}^2 - \frac{1}{2}m{v_r}^2 = \frac{1}{2}m{v_2}^2 \Rightarrow v_3 = 16.7 \times 10^3 m/s$$

Linear Momentum

define linear momentum p: $\boxed{\vec{p} = m\vec{v}}$

Conservation of Linear Momentum

if there is no net external force, then linear momentum is conserved

$$\sum \vec{F}_i = 0 \Rightarrow \sum m_i \vec{a}_i = 0$$

$$\Rightarrow \sum m_i \frac{d\vec{v}_i}{dt} = \sum \frac{d(m_i\vec{v}_i)}{dt} = \sum \frac{d\vec{p}_i}{dt} = 0$$

$$\Rightarrow \boxed{\sum \vec{F} = \frac{d\vec{p}}{dt}}$$

Impulse

$$\boxed{I = \int \vec{p} = \int \sum \vec{F}\, dt}$$

Collision

for elastic collision, energy is conserved
for inelastic collision, energy is not conserved

Elastic Collision

for momentum

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\Rightarrow \frac{m_1}{m_2}(v_{1i} - v_{1f}) = (v_{2f} - v_{2i})$$

for kinetic energy

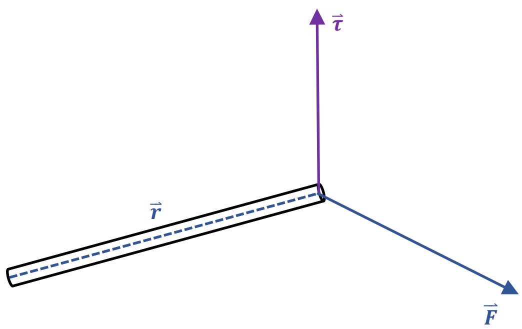
$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_1 v_{2f}^2$$

$$\Rightarrow \frac{m_1}{m_2}(v_{1i}^2 - v_{1f}^2) = (v_{2f}^2 - v_{2i}^2) = \frac{m_1}{m_2}(v_{1i} + v_{1f})(v_{1i} - v_{1f}) = (v_{2f} + v_{2i})(v_{2f} - v_{2i})$$

$$\Rightarrow v_{1f} - v_{2f} = -v_{1i} + v_{2i}$$

$$\Rightarrow \begin{cases} v_{1f} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\dfrac{2m_2}{m_1 + m_2}\right) v_{2i} \\ v_{2f} = \left(\dfrac{2m_1}{m_1 + m_2}\right) v_{1i} - \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right) v_{2i} \end{cases}$$

Torque



define torque $\vec{\tau}$: $\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$

Angular Momentum

define angular momentum L: $\boxed{\vec{L} = \vec{r} \times \vec{p}}$

Conservation of Angular Momentum

$$\frac{d\vec{L}}{dt} = \frac{d(\vec{r} \times m\vec{v})}{dt} = m\frac{d\vec{r}}{dt} \times \vec{v} + m\vec{r} \times \frac{d\vec{v}}{dt} = m\vec{v} \times \vec{v} + m\vec{r} \times \frac{d\vec{v}}{dt}$$
$$= m\vec{r} \times \frac{d\vec{v}}{dt} = \left(m\frac{d\vec{v}}{dt}\right) \times \vec{r} = \vec{\tau}$$
$$\Rightarrow \boxed{d\vec{L} = \int \sum \vec{\tau}\, dt}$$

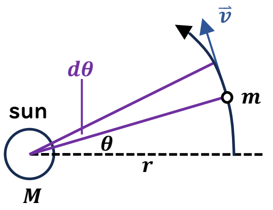
if there is no net external torque, then angular momentum is conserved

$$\vec{L} = \sum \vec{r} \times m\vec{v} = \sum \vec{r} \times m(\vec{r}\omega) = \left(\sum mr^2\right)\omega = I\omega$$

$$\Rightarrow \boxed{\vec{L} = I\omega}$$

Kepler's Law

- law 1:
  the orbit of a planet is an ellipse with the sun at one of the 2 focus
- law 2:
  a line segment joining a planet and the sun sweeps out equal areas during equal intervals of time



$$dA = \frac{1}{2}r(rd\theta)$$
$$\Rightarrow \frac{dA}{dt} = \frac{r^2}{2}\frac{d\theta}{dt} = \frac{r^2}{2}\omega$$

according to conservation of angular momentum

$\vec{L} = I\omega$, then ω is a constant

$$\Rightarrow \boxed{\frac{dA}{dt} = \text{constant}}$$

- law 3:

  $$\boxed{\frac{T^3}{a^3} = K}$$

  the square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit