

Autonomous Equation

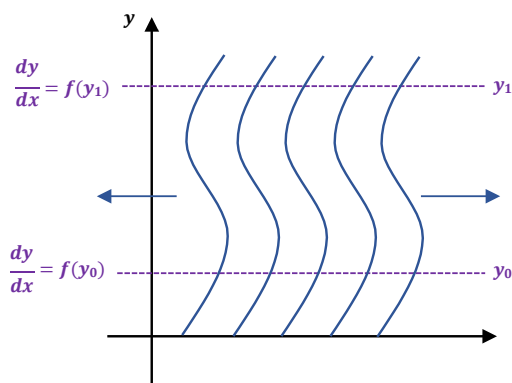
Autonomous Equation

$$\frac{dy}{dx} = f(y)$$

there is no independent variable x on the right-hand side

we can get information about the solution without actually solving it

Direction Field



the integral curve of autonomous equation are translationally equal

Critical point

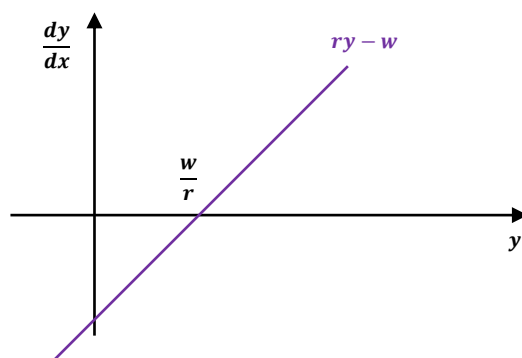
$$\frac{dy_0}{dx} = f(y_0) = 0$$

then $y = y_0$ is an absolute barrier

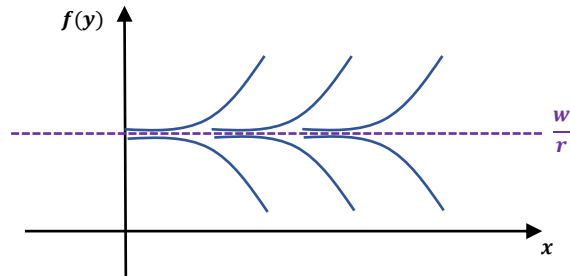
example: $dy/dx = ry - w$, r and w are constant

find critical point

$$\frac{dy}{dx} = ry - w = 0 \Rightarrow y_0 = \frac{w}{r}$$



when $y > w/r$, $dy/dx > 0$ and increases
 when $y < w/r$, $dy/dx < 0$ and decreases
 then we can graph $f(y)$



Logistic Equation

the model is mostly used to describe how population increases

$$\frac{dy}{dx} = ky = (a - by)y$$

y is the population, k is the growth rate

the model has feature that when y increases, the k decreases, and vice versa

$$(a - by)y = 0 \Rightarrow y_0 = 0 \text{ or } y_0 = a/b$$

when $0 < y < a/b$, $dy/dx > 0$, y first increases and then decreases and approach both barriers

