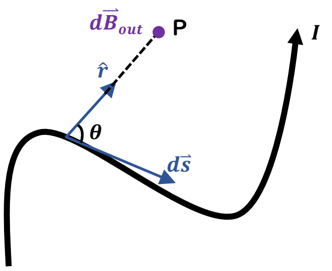
Sources of Magnetic Fields

Biot-Savart Law



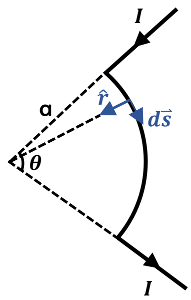
$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s} \times \hat{r}}{r^2}}$$

permeability of free space $\mu_0 = 4\pi \times 10^{-7} Tm/A$

for a single charge:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$

The Magnetic field for a curve wire segment



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s} \times \hat{r}}{r^2}$$

since $d\vec{s}$ and $\hat{r}$ are always perpendicular

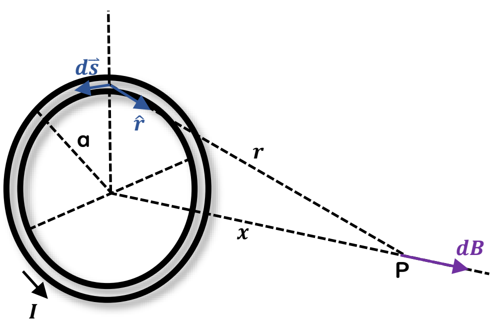
$$dB = \frac{\mu_0}{4\pi}\frac{Ids}{a^2}$$

$$B = \frac{\mu_0 I}{4\pi a^2}\int ds = \frac{\mu_0 I}{4\pi a^2}s = \frac{\mu_0 I}{4\pi a^2}(a\theta) = \frac{\mu_0 I}{4\pi a}\theta$$

for a circular loop of $\theta = 2\pi$

$$\boxed{B = \frac{\mu_0 I}{2a}}$$

The Magnetic field for a Circular Loop of Wire



$$B = \int dB = \frac{\mu_0 I}{4\pi}\oint \frac{ds}{a^2 + x^2}\frac{a}{\sqrt{a^2 + x^2}}$$

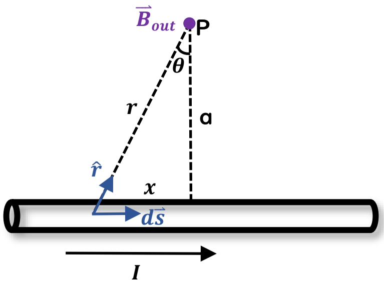
$$= \frac{\mu_0 I}{4\pi}\frac{a}{(a^2 + x^2)^{\frac{3}{2}}}(2\pi a) = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{\frac{3}{2}}}$$

two special cases:

$B = \frac{\mu_0 I}{2a}$ (at $x = 0$)

$B \approx \frac{\mu_0 I a^2}{2x^3}$ (at $x \gg a$)

The Magnetic field for a Long Conductor



$$dB = \frac{\mu_0 I}{4\pi}\frac{dx}{r^2}cos\theta$$

$$dx = d(atan\theta) = asec^2\theta d\theta$$

$$dB = \frac{\mu_0 I}{4\pi}(asec^2\theta d\theta)\left(\frac{cos^2\theta}{a^2}\right) = \frac{\mu_0 I}{4\pi a}cos\theta d\theta$$

$$B = \frac{\mu_0 I}{4\pi a}\int_{\theta_2}^{\theta_1} cos\theta d\theta = \frac{\mu_0 I}{4\pi a}(sin\theta_1 - sin\theta_2)$$

when conductor is infinitely long

$$B = \frac{\mu_0 I}{4\pi a}\left(sin\left(\frac{\pi}{2}\right) - sin\left(-\frac{\pi}{2}\right)\right)$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi a}}$$

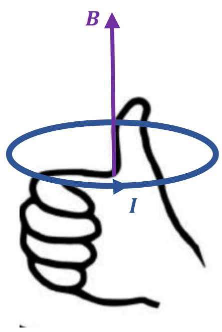
The Magnetic field Between Two Parallel Conductor

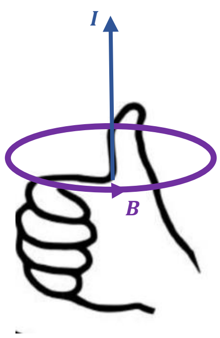
$$F_B = F_1 = F_2 = I_1 l B_2 = I_2 l B_1 = \frac{\mu_0 I_1 I_2}{2\pi a} l$$

$$\Rightarrow \boxed{\frac{F_B}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}}$$

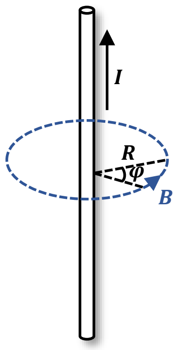
conductors carrying currents in the <u>same direction attract</u> each other
conductors carrying currents in the <u>opposite directions repel</u> each other

Direction of Magnetic Field





Ampere's Law



$$B = \frac{\mu_0 I}{2\pi R}\hat{e}_\varphi$$

$$\oint \vec{B}d\vec{s} = \oint \frac{\mu_0 I}{2\pi R}(Rd\vec{\varphi}) = \frac{\mu_0 I}{2\pi}\int_0^{2\pi} d\varphi = \mu_0 I$$

$$\Rightarrow \boxed{\oint \vec{B}d\vec{s} = \mu_0 I}$$

Field of a Long Straight Wire

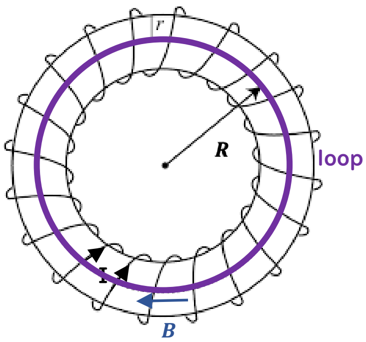
outside the wire, $r > a$

$$\oint \vec{B}d\vec{s} = B(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

inside the wire, $r < a$

$$\oint \vec{B}d\vec{s} = B(2\pi r) = \mu_0 I = \mu_0(\frac{r^2}{a^2})I \Rightarrow B = \frac{\mu_0 I}{2\pi a^2}r$$

Magnetic Field inside a Toroid



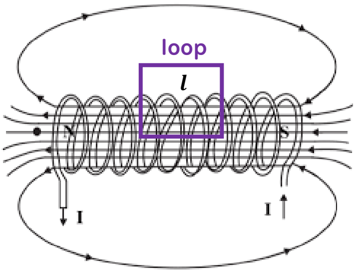
for the loop:

$$\oint \vec{B}d\vec{s} = \mu_0 I_{in} = B(2\pi R) = \mu_0(NI)$$

N is the total number of the wires

$$\Rightarrow B = \frac{\mu_0 NI}{2\pi R}$$

Magnetic Field of a Solenoid



assume the solenoid is infinitely long, there is no magnetic field outside

for the loop:

$$\oint \vec{B}d\vec{s} = \mu_0 I_{in} = Bl = \mu_0(NI)$$

N is the number of the wires inside the loop

$$\Rightarrow B = \mu_0 \frac{N}{l} I = \mu_0 nI$$

n is the number of the wires per unit length

Magnetic Flux

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BAcos\theta$$

Magnetic field lines are continuous, and form closed loop

Magnetic field lines end where they begin

$\Rightarrow \boxed{\oint \vec{B} \cdot d\vec{A} = 0}$