Direct Current Electricity

Electromotive Force

- the electromotive force(emf) ε of a battery is the maximum possible voltage that it can provide between its terminals
- the total power $P = I\varepsilon = I^2R + I^2r$
- the terminal voltage $\Delta V = \varepsilon - Ir$ where r is internal resistance

Kirchhoff's Rule

Junction Rule

$$\boxed{\sum_{junction} I = 0}$$

the sum of the currents entering any junction must equal 0

Loop Rule

$$\boxed{\sum_{closed\ loop} \Delta V = 0}$$

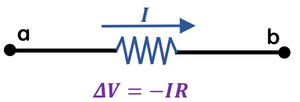
the sum of potential differences across all elements around a closed loop must equal 0

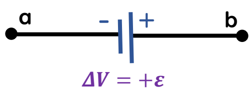
sign of ΔV in Loop Rule

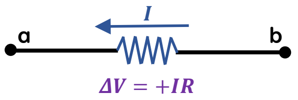
when we observe element from a to b

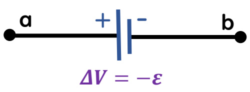
$\Delta V = V_b - V_a$

1. ΔV is negative when I is on the same direction with observation
2. The positive pole has higher electric potential

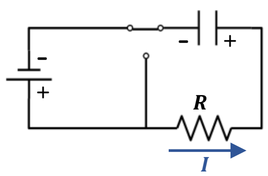








Charging a Capacitor in an RC Circuit



apply Loop Rule: $\varepsilon - \frac{q}{C} - IR = 0$

as $I = dq/dt$, so $\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} = -\frac{q - C\varepsilon}{RC}$

multiply the equation by dt and divided by $q - C\varepsilon$

$$\Rightarrow \frac{dq}{q - C\varepsilon} = -\frac{1}{RC}dt$$

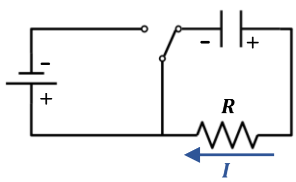
$$\Rightarrow \int_0^q \frac{dq}{q - C\varepsilon} = -\frac{1}{RC}\int_0^t dt \Rightarrow ln\left(1 - \frac{q}{C\varepsilon}\right) = -\frac{t}{RC}$$

$$\Rightarrow q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q(1 - e^{-t/RC})$$

the current can be found $I(t) = \frac{dq}{dt} = \frac{\varepsilon}{R}e^{-t/RC} = \frac{\varepsilon}{R}e^{-t/\tau}$

τ is the time constant $\tau = RC$

Discharging a Capacitor in an RC Circuit



apply Loop Rule: $-\frac{q}{C} - IR = 0$

as $I = \frac{dq}{dt}$, so $\frac{q}{C} = -R\frac{dq}{dt}$

$$\Rightarrow \frac{dq}{q} = -\frac{1}{RC}dt$$

$$\Rightarrow \int_0^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

use $q = Q$ at $t = 0$ $\Rightarrow ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$

$\Rightarrow q(t) = Qe^{-t/RC}$ and $I(t) = -\frac{Q}{RC}e^{-t/\tau}$