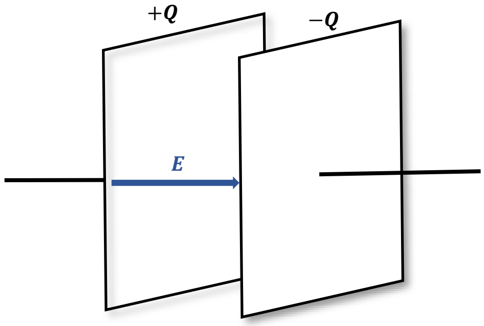
Capacitor

Definition of Capacitance



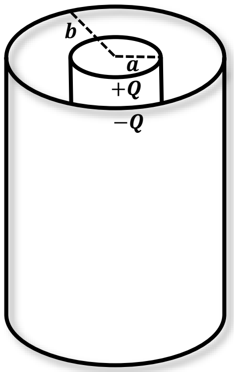
capacitance $\boxed{C = \frac{Q}{\Delta V}}$

for parallel plates, charge density $\sigma = \frac{Q}{A}$ and $E = \frac{\sigma}{\varepsilon_0}$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{\varepsilon_0 Q}{\sigma d} = \frac{\varepsilon_0 Q}{(Q/A)d} = \frac{\varepsilon_0 A}{d}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

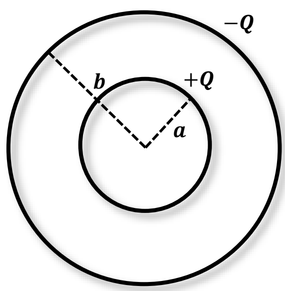
Cylindrical Capacitor



$$V_b - V_b = -\int_a^b \vec{E} \cdot d\vec{s} = -\int_a^b E_r dr = -2k_e\lambda \int_a^b \frac{dr}{r} = -2k_e\lambda \ln(\frac{b}{a})$$

$$C = \frac{Q}{|V_b - V_a|} = \frac{l\lambda}{2k_e\lambda \ln(\frac{b}{a})} = \frac{l}{2k_e \ln(\frac{b}{a})}$$

Spherical Capacitor



$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s} = -\int_a^b E_r dr = -k_e Q \int_a^b \frac{dr}{r^2} = k_e Q \left[\frac{1}{r}\right]_a^b = k_e Q(\frac{1}{b} - \frac{1}{a})$$

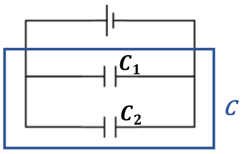
$$C = \frac{Q}{|V_b - V_a|} = \frac{Q}{k_e Q(\frac{1}{b} - \frac{1}{a})} = \frac{ab}{k_e(b-a)}$$

When a sphere is isolated,

$$C = \left.\frac{ab}{k_e(b-a)}\right|_{b\to\infty} = \frac{a}{k_e} = 4\pi\varepsilon_0 a$$

Capacitors in Circuit

Parallel

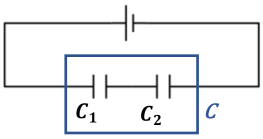


$$V = V_1 = V_2$$

$$Q = Q_1 + Q_2 = C_1V_1 + C_2V_2$$

$$C = \frac{Q}{V} = C_1 + C_2 \Rightarrow \boxed{C_{eq} = \sum C_i}$$

Series



$$Q = Q_1 = Q_2$$

$$V = V_1 + V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$C = \frac{Q}{V} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \boxed{C_{eq} = \sum \frac{1}{C_i}}$$

Energy Stored in a Capacitor

$$dW = \Delta V dq = \frac{q}{C} dq$$

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{1}{2} C(\Delta V)^2$$

$$\boxed{U = \frac{1}{2} C(\Delta V)^2}$$

The Work Done of Changing C

when the capacitor is charging $\Rightarrow$ the battery does work

$$W = \int_{q_0}^{Q} dqV = \int_{q_0}^{Q} dq \frac{q}{C} = \left[\frac{1}{2}\frac{q^2}{C}\right]_{q_0}^{Q} = U_1 - U_0$$

when the capacitor is discharging $\Rightarrow$ the capacitor does work

$$W = \int_{Q}^{q_f} dqV = \int_{Q}^{q_f} dq \left(\frac{Q-q}{C}\right) = \int_{Q}^{q_f} V_0 dq - \int_{Q}^{q_f} \frac{q}{C}$$

$$= [qV_0]_Q^{q_f} - \left[\frac{1}{2}\frac{q^2}{C}\right]_Q^{q_f} = U_0 - U_1 + V_0 \Delta Q$$

Capacitors with Dielectrics

$\boxed{C = \kappa C_0}$ κ is dielectric constant of the material

$\Rightarrow \boxed{C = \frac{\kappa \varepsilon_0 A}{d}}$

- if the capacitor remains connected to a battery, the voltage across the capacitor remains the same
- if the capacitor is disconnected from the battery, the charge remains the same