Lorentz Transformation

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$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}}$$

$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}$$

if $x' - y'$ axis equals $x - y$ axis rotated by angle θ:

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

for changing variable:

$$\Delta x' = \frac{\Delta x - u\Delta t}{\sqrt{1 - u^2/c^2}}$$

$$\Delta t' = \frac{\Delta t - u\Delta x/c^2}{\sqrt{1 - u^2/c^2}}$$

relative speed w

$$w = \frac{x'}{t'} = \frac{\Delta x - u\Delta t}{\Delta t - u\Delta x/c^2} = \frac{\frac{\Delta x}{\Delta t} - u}{1 - \frac{u}{c^2}\frac{\Delta x}{\Delta t}}$$

$$\Rightarrow \boxed{w = \frac{v - u}{1 - \frac{vu}{c^2}}}$$

Time Travel

assume 2 events happen time Δt apart

$$\Delta t' = \frac{\Delta t - u\Delta x/c^2}{\sqrt{1 - u^2/c^2}}$$

if $\Delta t' < 0$, the time order of 2 events will be reversed for the observer

$$\Rightarrow \frac{u}{c} > \frac{c\Delta t}{\Delta x}$$

$$\Rightarrow c\Delta t < \Delta x$$

meaning no enough time for light to travel from first event to the second
(observer need to travel faster than light)