Electric Field

Coulomb's Law

electrostatic force to q_1 add by q_2

$$\boxed{\vec{F}_e = k_e \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{r}}$$

permittivity ε_0 in vacuum: $\varepsilon_0 = 8.854 \times 10^{-12} C^2 m^{-2} N^{-1}$

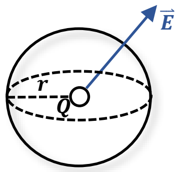
Electric Field

electric field on q_0 add by Q

$$\boxed{\vec{E} = \frac{\vec{F}_e}{q_0} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}}$$

here $\vec{E}$ is a vector field

Permittivity



define $1/\varepsilon_0$ electric field line generated per coulomb

at spherical surface of radius r, $\vec{E}$ at each point is given by

$$\vec{E} = \frac{\frac{1}{\varepsilon_0}Q}{4\pi r^2}\hat{r} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Superposition of Electric field

$$\vec{F} = \sum_{i=1}^{n} \vec{F}_i$$

$$\vec{E} = \sum_{i=1}^{n} \vec{E}_i = \sum_{i=1}^{n} \frac{q_i}{{r_i}^2}\hat{r}_i$$

Continuous Charge Distribution

- 2D plane

  assume charge distribution per unit area follows $\delta(x, y)$

  $$Q = \sum_i Q_i = \sum_i \delta(x, y)\Delta x \Delta y = \iint_S \delta(x, y)dxdy$$

- 3D volume

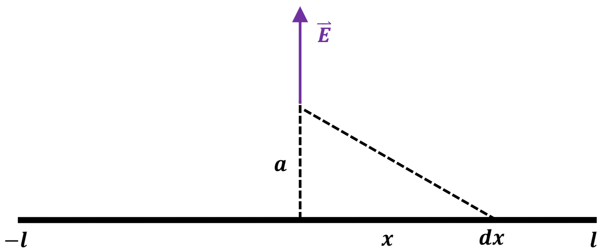
  assume charge distribution per unit volume follows $\delta(x, y, z)$

  $$Q = \sum_i Q_i = \sum_i \delta(x, y)\Delta x \Delta y\, \Delta z = \iiint_V \delta(x, y, z)dxdydz$$

E Field Calculation

- charged wire

  find $\vec{E}$ of distance a from symmetric position of $2l$ wire with charge density λ



$$\vec{E} = \int_{-l}^{l} d\vec{E}_y = \int_{-l}^{l} \frac{\lambda dx}{4\pi\varepsilon_0(x^2 + a^2)} \cdot \frac{a}{\sqrt{x^2 + a^2}} = \frac{2\lambda a}{4\pi\varepsilon_0} \int_{-l}^{l} \frac{1}{(x^2 + a^2)^{\frac{3}{2}}} dx$$

apply substitution $x = a\tan\theta$

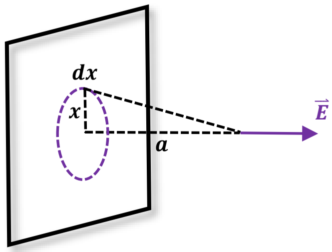
$$\Rightarrow \frac{2\lambda a}{4\pi\varepsilon_0} \int_{-l}^{l} \frac{asec^2\theta d\theta}{(a^2(tan^2\theta + 1))^{\frac{3}{2}}} = \frac{\lambda}{2\pi\varepsilon_0 a}\left(\frac{l}{\sqrt{l^2 + a^2}}\right)$$

when having an infinite long wire

$$l \to \infty \Rightarrow \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 a}$$

- charged plane

  find $\vec{E}$ of distance a from an infinite plane with charge density σ



$$E = \int_0^\infty dE = \int_0^\infty \frac{2\pi x dx \sigma}{4\pi\varepsilon_0(x^2 + a^2)} \cdot \frac{a}{\sqrt{x^2 + a^2}} = \frac{\sigma}{2\varepsilon_0}\int_0^{\frac{\pi}{2}} \frac{ax}{(x^2 + a^2)^{\frac{3}{2}}} dx$$

apply substitution $x = a\tan\theta$

$$\Rightarrow \frac{\sigma}{2\varepsilon_0}\int_0^{\frac{\pi}{2}} \frac{tan\theta}{(tan^2\theta + 1)^{\frac{3}{2}}} sec^2\theta d\theta = \frac{\sigma}{2\varepsilon_0}\int_0^{\frac{\pi}{2}} sin\theta d\theta = \frac{\sigma}{2\varepsilon_0}$$