NETWORK THEOREMS

KISTM
17891 (1)

1. Superposition theorem
2. Thevenin's theorem
3. Norton's theorem
4. Maximum power transfer theorem.

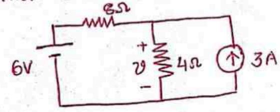
SUPERPOSITION THEOREM :-

→ This theorem states that the response in a linear circuit at any point due to multiple sources can be calculated by summing the effects of each source considered seperately, all other sources being made in operative.

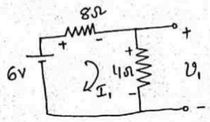
→ This theorem is applicable only to a linear network (containing independent or dependent sources).

Hint : Voltage source should be short circuited, Current source should be open circuited

Q) Use superposition theorem to find v in the circuit (a)



Sol :-

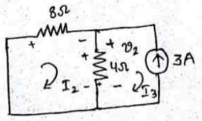


By Applying KVL

$-6+8I_1+4I_1=0.$

$\Rightarrow 12I_1=6 \quad \Rightarrow I_1=0.5A.$

$\therefore \boxed{v_1=4I_1=4(0.5)=2V.}$



By Applying KVL,

$8I_2+4(I_2-I_3)=0.$

$I_3=-3A.$

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(2)

$\Rightarrow 8I_2+4(I_2+3)=0.$

$\Rightarrow 12I_2=-12. \quad \Rightarrow I_2=-1A.$

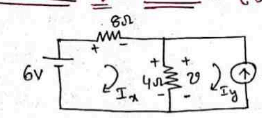
$\therefore v_2=4(I_2-I_3)$

$=4(-1+3)=8V$

$\therefore$ When both sources were present in the circuit,

$$\boxed{\begin{aligned} v &= v_1+v_2 \\ &= 2+8=10V. \end{aligned}}$$

Verification of Answer :- (By KVL)



By Applying KVL,

$-6+8I_x+4(I_x-I_y)=0.$

$I_y=-3.$

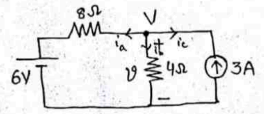
$\Rightarrow -6+8I_x+4I_x-4I_y=0.$

$\Rightarrow -6+12I_x+12=0.$

$\Rightarrow I_x=-6/12=-0.5.$

$\boxed{\therefore v=4(I_x-I_y)=4(-0.5+3)=10V.}$

Verification of Answer :- (By KCL)



By Applying KCL,

$i_a+i_b+i_c=0.$

$\frac{V-6}{8}+\frac{V}{4}-3=0.$

$\Rightarrow V-6+2V-24=0.$

$\Rightarrow 3V=30$

$\Rightarrow \boxed{v=10v}$

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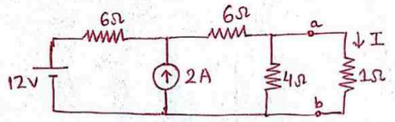
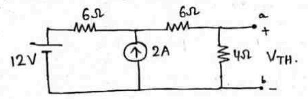
Thevenin's theorem

Thevenin's theorem states that it is possible to simplify any Linear Circuit containing independent and dependent Voltage and current Sources, no matter how Complex, to an equivalent Circuit with just a Single Voltage Source and a Series resistance between any two points of the Circuit.

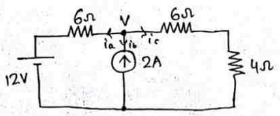
(Q) Using thevenin's theorem, find the equivalent Circuit to the left of the terminals in the circuit. Then find I.



Sol:- i) Remove 1Ω, then write or draw circuit again



ii) Calculate V_{TH} = Voltage across a,b terminals.



$i_a + i_b + i_c = 0$ (AS per KCL)

$\Rightarrow \frac{V-12}{6} + (-2) + \frac{V}{10} = 0.$

$\Rightarrow \frac{V}{6} + \frac{V}{10} = 4.$

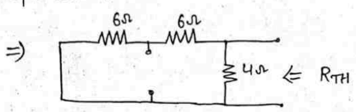
$\Rightarrow \frac{16V}{60} = 4 \Rightarrow V = \frac{60 \times 4}{16} = 15$

$i_c = \frac{V}{10} = \frac{15}{10} = 1.5A.$

$\Rightarrow \boxed{V_{4\Omega} = i_c(4) = 4(1.5) = 6V = V_{TH}}$

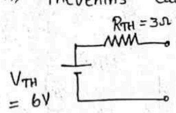
iii) Calculate R_{TH} by short circuit Voltage Source and open circuit current Source.

$\Rightarrow$



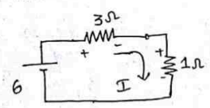
$\boxed{R_{TH} = \frac{12 \times 4}{12+4} = \frac{48}{16} = 3\Omega.}$

iv) Thevenin's Equivalent Circuit is drawn as follows.



v) Now connect load Resistor as 1Ω in equivalent ckt of thevenin then Calculate I.

i.e



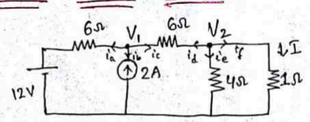
By Applying KVL,

$-6 + 3I + 1(I) = 0.$

$\Rightarrow I = 6/4 = 1.5A$

$\boxed{I = 1.5A}$

Cross check:-
Verification of I (using KCL) :-



By Applying KCL, at Node V_1,

$i_a + i_b + i_c = 0.$

$\Rightarrow \frac{V_1 - 12}{6} + (-2) + \frac{V_1 - V_2}{6} = 0 \rightarrow (1)$

By Applying KCL, at Node V_2,

$i_d + i_e + i_f = 0.$

$\frac{V_2 - V_1}{6} + \frac{V_2}{4} + \frac{V_2}{1} = 0. \quad -(2)$

Solving (1) and (2)

$V_2 = 1.5V$

$\Rightarrow \boxed{I = \frac{V_2}{1}}$

NORTON'S THEOREM :-

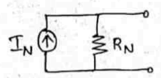
Norton's theorem states that a two-terminal Linear network Containing independent Voltage and Current Sources may be replaced by an Equivalent Current Source (I_N) in parallel with a Resistance (R_N)

→ The procedure for determining Norton's Equivalent is as follows.

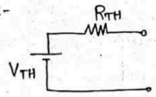
i) Short Circuit the two terminals of the Network and determine the Current through this Short circuit.

ii) Calculate R_N or R_{TH}.

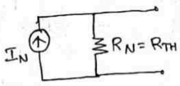
iii) Norton's Equivalent Circuit is as follows.



Note :-



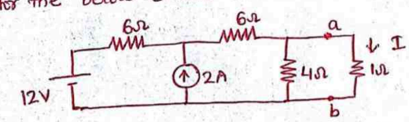
Thevenin's Equivalent



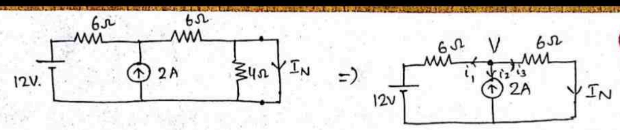
$I_N = \frac{V_{TH}}{R_{TH}}$.

Q. Using Norton's theorem, find the Norton's Equivalent circuit for the below Circuit and Calculate I using this theorem.

Sol



Sol:- i) Remove 1Ω, and short circuit a and b and Calculate Current in that path as I_N.



By Applying KCL, $i_1 + i_2 + i_3 = 0$.

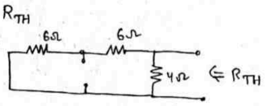
$$\Rightarrow \frac{V-12}{6} - 2 + \frac{V}{6} = 0.$$

$$\Rightarrow V - 12 - 12 + V = 0$$

$$\Rightarrow V = 12 \text{ Volt.}$$

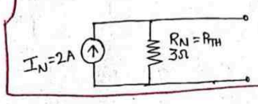
$$\therefore I_N = i_3 = \frac{V}{6} = \frac{12}{6} = 2A$$

ii) $R_N = R_{TH}$



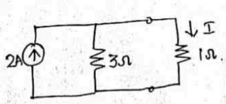
$\Leftarrow R_{TH} = \frac{12 \times 4}{12+4} = 3\Omega$.

iii) Norton Equivalent Circuit is



iv) Now Connect 1Ω Resistor and Calculate Current through it.

=)



By Current division rule,

$$I = \frac{2 \times 3}{3+1} = \frac{6}{4} = 1.5A$$

$$\boxed{I = 1.5A}$$

Verification of I using KCL

Already done in thevenin's theorem problem, please See there for Answer. (or) Contact your friends who understand KCL.

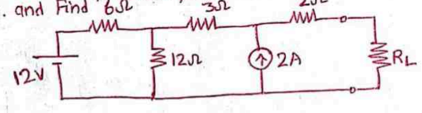
Refer Pg. 4 for Answer. _ 17891

MAXIMUM POWER TRANSFER THEOREM :-

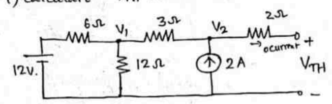
This theorem states that maximum power is absorbed from a network when the load resistance is equal to the output resistance of the network as seen from the terminals of the load.

$$\boxed{\text{Maximum power that can be extracted from a Circuit} = \frac{V_{TH}^2}{4R_{TH}}}$$

Q. Find the R_L for maximum power transfer in the following Circuit. and Find Maximum power.



Sol:- i) Calculate V_{TH} and R_{TH}.



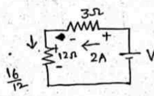
V_2 is nothing but V_{TH}.

By Apply KCL, at V_1 Node,

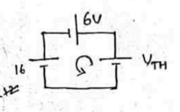
$$\frac{V_1-12}{6}+\frac{V_1}{12}-2=0.$$

$$\Rightarrow 2V_1-24+V_1-24=0.$$

$$\Rightarrow 3V_1=48 \Rightarrow \boxed{V_1=16V}$$



$\Rightarrow$

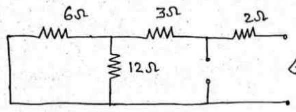


$$-V_{TH}+6+16=0$$

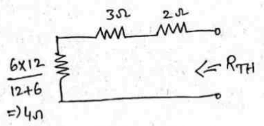
$$\Rightarrow V_{TH}=22V.$$

$$\boxed{V_{TH}=22V}$$

R_{TH} :

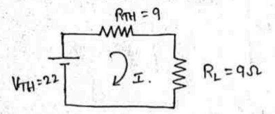


$\Rightarrow$



$$\Rightarrow R_{TH}=4+3+2$$
$$=9\Omega.$$

$\Rightarrow R_L$ must be R_{TH} So that Maximum power should be transfered.



Power across 9Ω is the Maximum power.

$$\therefore\ -22+9I+9I=0$$

$$\Rightarrow I=\frac{22}{9+9}$$

$$\text{Power across } R_L = I^2(R_L)$$

$$=\left(\frac{22}{9+9}\right)^2\times9=\frac{22^2}{(9+9)(9+9)}\times9.$$

$$\Rightarrow \frac{22^2\times\cancel{9}}{\cancel{9}(2)(18)}$$

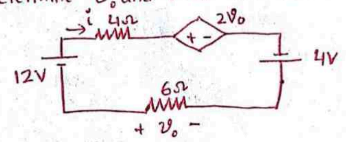
$$=\frac{22^2}{2\times2\times9}=\frac{22^2}{4\times9}.$$

$$\Rightarrow \frac{22^2}{4\times9}\ \text{means}\ \frac{V_{TH}^2}{4\times R_L}$$

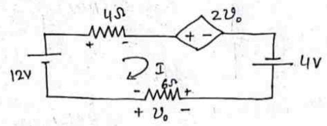
$$\boxed{\text{Maximum power} = 13.44\ \text{watt.}}$$

Numericals on dependent Sources :-

(Q) Determine v_o and i in the Circuit shown below.



Sol:-



By Applying KVL, $-12+4I+2v_o-4+6I=0.$ —①.

Two unknowns, single Equation not possible to get solution.
so search in ckt for Relation between I and v_o

$$v_o = -6I.$$

Put $v_o = -6I$ in ①

$$-12+4I+2(-6I)-4+6I=0.$$

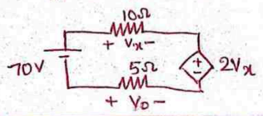
$$\Rightarrow -12+4I-12I-4+6I=0$$

$$\Rightarrow -2I = 16$$

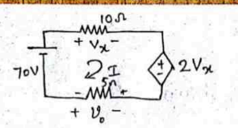
$$\Rightarrow I = -8A.$$

$$\boxed{\begin{array}{l} \therefore v_o = -6(-8) = 48V. \\ I = i = -8A \end{array}}$$

(Q) Find v_x and v_o in the following ckt



Sol:-



By Applying KVL,

$$-70+10I+2V_x+5I=0 \rightarrow ①$$

Two unknowns, single Eq. not possible.

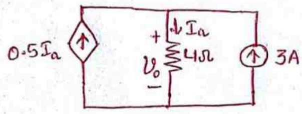
Relation between v_x and I $\Rightarrow$ $V_x = 10I$.

$$\therefore -70+10I+2(10I)+5I=0.$$

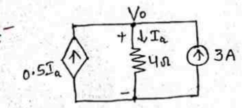
$$35I = 70 \quad \Rightarrow I = 2A$$

$$\boxed{\begin{array}{l} V_x = 10I = 10(2) = 20V. \\ v_o = -5I = -5(2) = -10V. \end{array}}$$

(Q) Find i_a and v_o in the following circuit



Sol:-



By Applying KCL at V_o,

$$-0.5I_a + \frac{V_o}{4} - 3 = 0. \rightarrow ①$$

Two unknowns, single Eq. not possible.

Relation between I_a and V_o is $v_o = 4I_a$

put in ① $\Rightarrow$ $-0.5I_a + \frac{\cancel{4}I_a}{\cancel{4}} - 3 = 0.$

$$\Rightarrow 0.5I_a = 3.$$

$$\Rightarrow I_a = 6A$$

and $v_o = 4I_a = 4\times 6 = 24V.$

$$\boxed{I_a = 6A, \quad V_o = 24V}$$

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