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★ Definitions
★ Examples

## Unit-3 → Counting Principles & Relations

★ **Sets** :- collection of well-defined objects.
↳ represented in capitals.

# **Relations** :- subset of cartesian product of 2 sets A & B

$$R \subseteq (A \times B)$$

$$\rightarrow \boxed{n(R) = 2^{n(A \times B)}}$$

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# **Set** :- A collection of elements which are well-defined

Example, students of KOCCF

# **Cartesian Product** :-

$$A = \{1, 2\}$$
$$B = \{3, 4, 5\}$$
$$A \times B = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)\}$$

# **Relations** :- Let A & B be the two sets, then the relation from the set A to the set B is a subset of $(A \times B)$.

$$R \subseteq A \times B$$

★ **Note** :- If set N is having n elements & the set M is having m elements then the number of elements in the set $N \times M$ is $mn$.

$$n(N \times M) = mn$$
$$n(R(N \rightarrow M)) = 2^{mn}$$

# **Types of Relations (Properties of Relations)** :-

1. **Reflexive** :- A relation R on the set A is called reflexive if ~~A,A~~ $(a, a) \in R \ \forall a \in A$ or

$$aRa \ \forall a \in A.$$

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Q. Is the divides relation on the set of positive integers reflexive?

S. $R = \{(a,b) ; a \text{ divides } b \ \forall\ a,b \in Z^+\}$

Yes, all the positive integers are divisible by itself.
Hence, Relation is reflexive.

★ matrix representation of reflexive relation $a_{ii} = 1$ always ($\Rightarrow$ diagnol elements are always 1).
Example,

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & & & \\ 2 & & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \\ 3 & & & \end{array}$$

2. <u>Symmetric & Anti-symmetric :-</u>

A relation R on a set A is called symmetric if $(a,b) \in R$ or $aRb$ then $(b,a) \in R$ or $bRa \ \forall\ a,b \in A$.
Example, $A = \{1,2,3,4\}$
$R_1 = \{(1,2),(2,1),(3,4),(4,3)\}$
$R_2 = \{(1,1),(1,2),(2,1)\}$

A relation R on a set A is called antisymmetric if $(a,b) \in R$ then $(b,a) \notin R$ 'or' $(a,b) \in R$ & $(b,a) \in R$ then $a=b$.
Example, $R_1 = \{(1,1),(1,2),(3,4)\}$

$R_2 = \{(1,1),(2,2),(3,3)\}$

★ If we have an ordered pair where the elements are related by itself then it is reflexive, symmetric & anti-symmetric. Eg, $A = \{1,2,3,4\}$
$R = \{(1,1),(2,2),(3,3),(4,4)\}$

Q1. Is the divide relation on the set of positive integers symmetric, anti symmetric or neither?
→ Anti-symmetric

2 divides 4 $(2,4) \in R$
4 divides 2 $(4,2) \notin R$
$(a,b) \in R$ then $(b,a) \notin R$
Hence, Anti-symmetric

3. Transitive :- A relation R on a set A is called transitive if $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$.
$A = \{1,2,3,4\}$
Example, $R = \{(1,2),(2,3),(1,3),(1,4)\}$

Q1. Is the divide relation on the set of $Z^+$ transitive?
→ Yes, it is transitive.

4. Composite :- Let R be a relation from a set A to the set B & S be the relation from set B to the set C. The composite of R & S is the relation consisting of the ordered pairs $(a,c)$, where $a \in A$ & $c \in C$ & for which there exists an element $b \in B$ such that $(a,b) \in R$ & $(b,c) \in S$.
Represented as $S \circ R$.
Example,
composite of relation R & S where R is defined from $R: \{\overbrace{1,2,3}^{A}\}$ to $\{1,\overbrace{2,3,4}^{B}\}$ &
$R = \{(1,1),(1,4),(2,3),(3,1),(3,4)\}$ &
$S: \{\overbrace{1,2,3,4}^{B}\}$ to $\{\overbrace{0,1,2}^{C}\}$ &
$S = \{(1,0),(2,0),(3,1),(3,2),(4,1)\}$

G $S \circ R = \{(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)\}$

Q1 suppose relation R on the set is represented as matrix

$$\begin{array}{c|c} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

. Is the relation R reflexive, symmetric or antisymmetric?

G

⇒ Reflexive ✓ → Diagnol elements are 1

$$R = \{(a,a), \underline{(a,b)}, \underline{(b,a)}, (b,b), \underline{(b,c)}, \underline{(c,b)}, (c,c)\}$$

⇒ Symmetric ✓

⇒ ~~No Anti-symmetric~~ X

⇒ ~~Transitive~~ ✗

5. **Equivalence** :- A relation R on a set A is called equivalence relation if it is reflexive, symmetric & transitive.

6. **Partial ordering** :- A relation R on a set S is called a partial ordering relation if it is reflexive, antisymmetric & transitive. A set S together with a partial ordering relation R is called partially ordered set (poset).

# Comparable & Incomparable :- The elements a & b of the Poset $(S, \leq)$ are called comparable if either $a \leq b$ or $b \leq a$.

When a & b are the elements of S such that neither $a \leq b$ nor $b \leq a$ then it is called as incomparable. Represented by $\boxed{a/b = \frac{a}{b}}$ & $\boxed{a|b = \frac{b}{a}}$.

Example, In the Poset $(Z^+, |)$ are the integers

(i) 3 & 9 comparable? → Yes

(ii) 5 & 7 comparable? → No

set → Relation

# Totally ordered :- If $(S, \leq)$ Poset & every 2 elements of S are comparable then S is called as totally ordered / linearly ordered set. A totally ordered set is also called a $\boxed{\text{chain}}$.

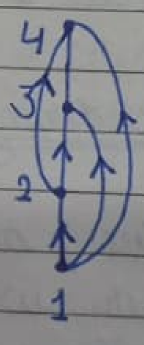
Most Impt. $\boxed{\text{for CA \& ETE}}$

★ # Hasse Diagram :- pictorial representation of Posets.

↳ always in upper direction (lowest to highest value)

1. $(\{1, 2, 3, 4\}, \leq)$


Hasse Diagram



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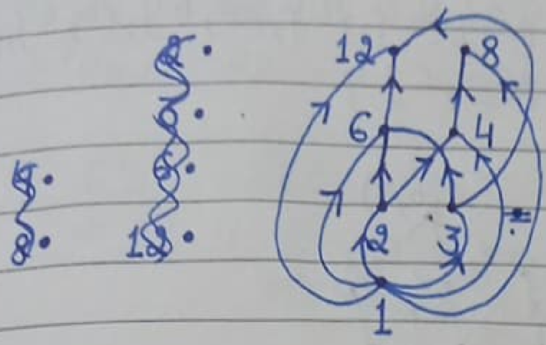
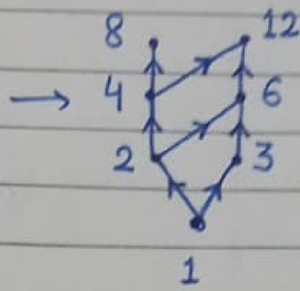
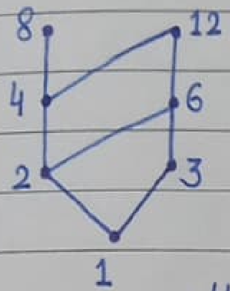
2. $(\{1, 2, 3, 4, 6, 8, 12\}, |)$

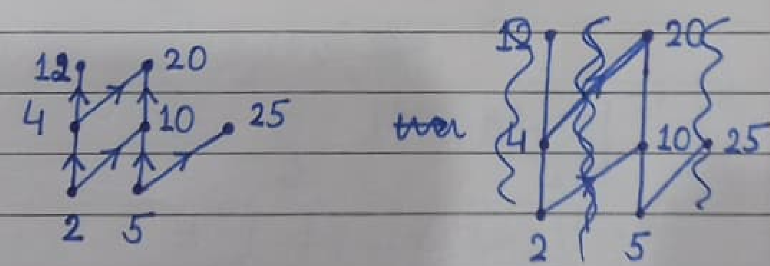


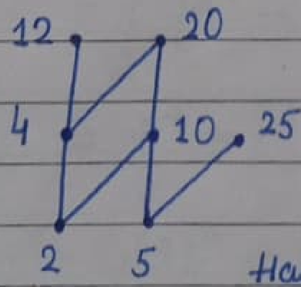



Hasse Diagram
(Final)

3. $(\{2, 4, 5, 10, 12, 20, 25\}, |)$






Hasse Diagram

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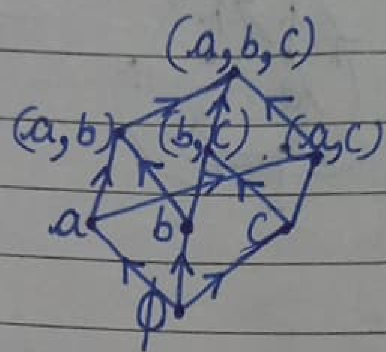
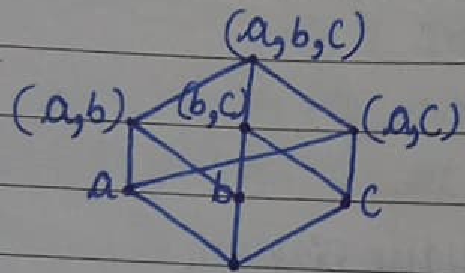
4. $S = \{a, b, c\}$

$R \to \subseteq$

$R = \{\{\phi\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\},$
$\{a, b, c\}\}$




$\phi$ <u>Hasse Diagram</u>