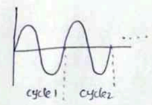
AC - CIRCUITS

→* Alternating Current or Voltage wave form :-

* Alternating current or voltage is one the circuit direction of which reverses at regularly recurring intervals.
* one complete set of positive and negative values of alternating quantity is known as cycle.

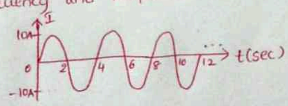
Eg:-



This wave form complete two cycles

* The time taken by an alternating quantity to complete one cycle is called its time period. (T). units of time is seconds.
* The no. of cycles/second is called Frequency of alternating quantity. units of frequency is Hertz. (or) $\boxed{\text{Frequency} = f = \frac{1}{\text{Time period}} = \frac{1}{T}}$
* Maximum value (either positive or negative) of an AC quantity (Alternating) is known as its Amplitude

Ⓢ. Calculate T, Frequency and Amplitude for the following wave form.



Sol:- Time period $= T = 4$ sec

Frequency $= f = \frac{1}{T} = \frac{1}{4}$ Hz $= 0.25$ Hz

Amplitude $= 10$ V

* phase :- The fraction of the time period of the Alternating current which has elapsed since the current last passed through zero position of reference.

→ There are several methods to express Amplitude value.
Eg: peak Amplitude value, peak-peak value, RMS value, Avg value .. etc.

* Mathematical Representation and Definitions of AC wave form :-

→ Alternating current wave form represented mathematically as follows.

$$i(t) = I_p \sin(\omega t + \phi)$$

where I_p = peak value of wave form. (Amplitude) (Amps)

$\omega = 2\pi f = \frac{2\pi}{T}$ = Angular Frequency (rad/sec)

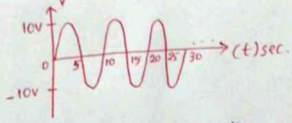
f = Frequency of the wave form. (Herz)

$T = \frac{1}{f}$ = Time period of the wave form (sec)

ϕ = phase difference

If $\phi = 0$ means, $i(t)$ is having phase of $0°$ and phase difference $0°$.

Ⓠ. Express the following sine wave form in Mathematical form.



Sol:- peak value or Amplitude $= V_p = 10$ V.

Time period $= T = 10$ sec

Frequency $= f = \frac{1}{T} = \frac{1}{10} = 0.1$ Hz

Angular frequency $= \omega = 2\pi(f) = 2\pi(0.1) = 0.2\pi$ rad/sec.

Phase $= \phi = 0$ (Because the wave form started at zero only)

$V(t) = V_p \sin(\omega t + \phi)$

$\boxed{V(t) = 10 \sin(0.2\pi t)}$

→ Root Mean Square Value Calculation for an AC signal:

(3)

* The RMS value of an AC current is given by the steady current (DC current) which when flowing through a given circuit for a given time produces the same heat as produced by alternating current when flowing through the same circuit for the same time.

* RMS value is also known as Effective value.

* Mathematically, RMS value for any periodic signal is given by

$$I_{rms} = \text{RMS value} = \sqrt{\frac{1}{T}\int_0^T I^2(t).dt}$$

* For an AC signal (sinusoidal signal) $I(t) = I_m \sin(\omega t)$, RMS value is given by

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T (I_m \sin(\omega t))^2 dt}$$

$$= \left[\frac{1}{T}\left[\int_0^T I_m^2 \sin^2(\omega t).dt\right]\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T I_m^2 \frac{(1-\cos 2\omega t)}{2}.dt\right]^{1/2}$$

$$= \left(\frac{1}{T}\int_0^T \frac{I_m^2}{2} dt\right]^{1/2} = \left(\frac{1}{\cancel{T}}.\frac{I_m^2(\cancel{T})}{2}\right)^{1/2} = \frac{I_m}{\sqrt{2}}$$

$$\therefore \boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

→ Average Value Calculation for a Sinusoidal Signal :-

* Average value I_a of an AC is expressed by the steady (DC) current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.

$$I_{avg} = \frac{1}{T}\int_0^T I(t).dt.$$

* For an AC Signal $I(t) = I \sin\omega t$, I_{avg} is given by

(4)

$$I_{avg} = \frac{1}{T}\int_0^T I_m \sin\omega t.dt$$

$$= \frac{1}{T}\left[-\frac{I_m \cos\omega t}{\omega}\right]_0^T = -\frac{I_m}{T}\left[\frac{\cos(\frac{2\pi}{T}.T) - \cos(\frac{2\pi}{T}.0)}{2\pi/T}\right]$$

$$= -\frac{I_m}{\cancel{T}}.\frac{\cancel{T}}{2\pi}\left[-1-1\right] = \frac{I_m}{\pi} = 0.637 I_m$$

$$\boxed{I_{avg} = \frac{I_m}{\pi}} \qquad \boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

For sinusoidal wave form :

* Form Factor $= K_f = \frac{\text{rms value}}{\text{Avg. Value}} = \frac{I_m/\sqrt{2}}{I_m/\pi} = 1.11$

* Crest or peak or Amplitude factor $= \frac{\text{Maximum value}}{\text{rms value}} = \frac{\text{Peak value}}{\text{rms value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2}$

→ Rectangle to polar form Conversion (will be discussed in class)

→ Addition and Substraction of AC wave forms (In polar form)

* In polar form, i) $V(t) = V_m \sin(\omega t + \phi)$ will be represented as $V_{rms}\angle\phi$

ii) $V(t) = V_m \cos(\omega t + \phi)$ will be represented as $V_{rms}\angle 90+\phi$.

(I have written formulas taking Sine wave as reference).

Q. ~~Add~~ $200 \sin\omega t + 100 \sin(\omega t + 30°) = A \sin(\omega t + \theta) = r\angle\theta$

Sol:- $200\angle 0° + 100\angle 30° = 290.93\angle 9.89 = 290.93 \sin(\omega t + 9.89)$

Q. $10 \sin(\omega t + 30°) + 20 \cos(\omega t + 45) + 10 \cos(\omega t - 10°) - 5 \sin(45° + \omega t)$

Sol:- $10\angle 30° + 20\angle(90° + 45°) + 10\angle(90° - 10°) - 5\angle -45° = 33.33\angle 102.61$

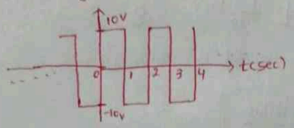
$= 33.33 \sin(\omega t + 102°.61)$

→ RMS value and Average value Calculation for Non-Sinusoidal waveform (5)

$$V_{rms} = \left[\frac{1}{T}\int_0^T v^2(t).dt\right]^{1/2}$$

$$V_{avg} = \frac{1}{T}\int_0^T v(t).dt$$

(Q). what is the average value and RMS value for the following wave form.



Sol:- $v(t) = 10 \quad (0 \le t \le 1)$ Time period = 2 sec.

$= -10 \quad (1 < t < 2)$

$$\therefore V_{avg} = \frac{1}{T}\int_0^T v(t).dt = \frac{1}{2}\int_0^2 v(t).dt$$

$$= \frac{1}{2}\left[\int_0^1 10.dt + \int_1^2 -10.dt\right]$$

$$= \frac{1}{2}\left[10[t]_0^1 + [-10(t)]_1^2\right] = \frac{1}{2}[10 - 10] = \underline{0}$$

$\Rightarrow \boxed{V_{avg} = 0}$

$$V_{rms} = \left[\frac{1}{T}\int_0^T v^2(t)\,dt\right]^{1/2} = \left[\frac{1}{2}\int_0^2 v^2(t)\,dt\right]^{1/2}$$

$$= \left[\frac{1}{2}\left[\int_0^1 10^2.dt + \int_1^2 (-10)^2 dt\right]\right]^{1/2} = \left(\frac{1}{2}[100 + 100]\right)^{1/2}$$

$\Rightarrow \boxed{V_{rms} = 10V}$ $\boxed{V_{avg} = 0V \quad V_{rms} = 10V}$

→ Apply Same Method for any other Non Sinusoidal Wave form.

→ while dealing with DC circuits, we have done problems with only Resistive element (R). But in AC, we have to consider Reactive elements (L and C) also. (6)

→ $\boxed{\text{Impedance} = \text{Resistance} + j(\text{Reactance})}$

→ Imaginary part of impedance is Reactance denoted with X Ω.

→ Real part of Re Impedance is Resistance. denoted with R Ω.

→ In DC circuits, we have only power. But in AC, we have three types of powers.

→ In DC Circuits, power will be dissipated across only R.

→ In AC Circuits, power will be available across R, L, C. also.

→ The power associated with Resistive elements in AC circuit is Real or True power. (denoted by P). units are watt.

→ The power associated with Reactive elements (L and C) combinely in AC circuit is Reactive power. (denoted by Q). Units are VAR (Volt Ampere Reactive)

→ The power associated with Impedance in AC circuit is Apparent power. denoted by S. and units are VA. (Volt Ampere).

→ In AC circuits, R ⟶ R Ω (Equivalent representation)

L ⟶ jωL Ω

C ⟶ $\frac{1}{j\omega C}$ Ω.

→ $\boxed{\text{Reactance Across Inductance} = X_L = \omega L\ \Omega}$ (or) $j\omega L$

→ $\boxed{\text{Reactance across Capacitance} = X_C = \frac{1}{\omega C}\ \Omega}$ (or) $\frac{1}{j\omega C}$

Note:- In solving Numericals, we will consider j also.

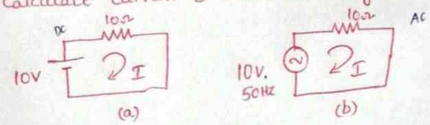
$\boxed{j \times j = -1, \quad -\frac{1}{j} = j}$

* $\boxed{\text{DC signal is having Frequency Zero.}}$ i.e $f = 0 \Rightarrow \omega = 2\pi(0) = 0$.

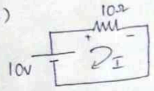
→ Concept to solve AC Numericals

i) Resistance (R) :-

Q. Calculate current I in the following circuits (a) and (b)



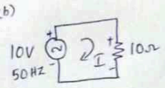
Sol:- (a)



$-10+10I=0$

$\Rightarrow I=\underline{\underline{1A}}$

(b)



$-10+10I=0$

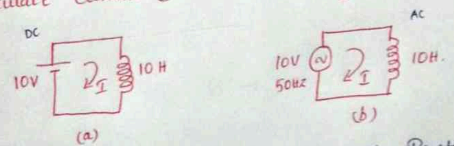
$\Rightarrow I=\underline{\underline{1A}}$

[For AC circuits don't take polarity signs]

only for convience I have taken.

ii) Inductance :-

Q) Calculate Current I in the following Circuits (a) and (b).



Sol:- For Inductance, convert Inductance into Reactance in both circuits.

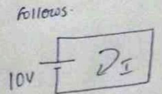
(a) For DC, $f=0.\Rightarrow \omega=0,\ L=10H$

$X_L=\text{Reactance}=\omega L\ \Omega$

$=0(10)$

$=0\ \Omega.$

∴ (a) will be converted as follows.



$I=\infty$ (Infinity)

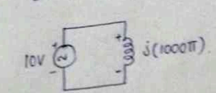
(b) $f=50Hz$ (given)

$L=10H$

$\omega=2\pi(f)=2\pi(50)=100\pi$

$X_L=\text{Reactance}=\omega L=1000\pi\ \Omega.$

(b) is converted as follows.

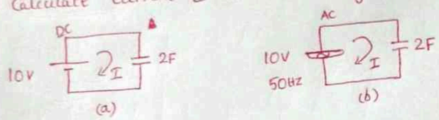


$I=\dfrac{10}{j(1000\pi)}=3.18\times10^{-3}\angle-90^\circ.$

$\boxed{\therefore\ I_{rms}=3.18\ mA}$

iii) Capacitance :-

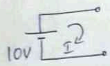
Q. Calculate current I in the following Circuits (a) and (b).



Sol:- (a) $X_C=\dfrac{1}{j(\omega C)}=\dfrac{1}{j(2\pi(0)(2))}$

$=\infty\ \Omega$

(a) will be drawn as follows.



$\underline{I=0A}$

(b) $X_C=\dfrac{1}{j(\omega C)}=\dfrac{1}{j(2\pi(50)2)}=1.59\times10^{-3}$

(b) will be drawn as follow

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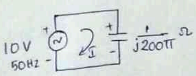


$I=\dfrac{10}{1/200\pi}=10(200\pi)$

$=\underline{\underline{6283.18\ A}}$

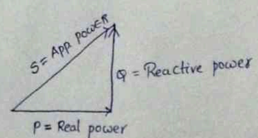
* Power Concept related formulas for AC circuits.

→ power factor :- $\dfrac{R}{Z}$ in AC circuit (or) $\cos(\phi)$ where ϕ is the phase difference b/w V and I in circuit (or) $\dfrac{\text{True power}}{\text{Apparent power}}$ in AC circuit.

→ True power :- $P=V_{rms}\,I_{rms}\cos\phi$ Watt

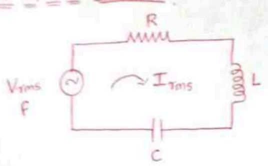
→ Reactive power :- $Q=V_{rms}\,I_{rms}\sin\phi$ VAR

→ Apparent power :- $S=V_{rms}\,I_{rms}$ VA $=\sqrt{P^2+Q^2}$ VA



Power triangle

→ Series R, L, C Circuit :-



$V_R =$ ___ $X =$ ___ $P_c =$ ___
$V_L =$ ___ $I_{rms} =$ ___ $P =$ ___
$V_c =$ ___ $Z =$ ___ $Q =$ ___
$X_L =$ ___ $P_R =$ ___ $S =$ ___
$X_c =$ ___ $P_L =$ ___ Power factor = ___

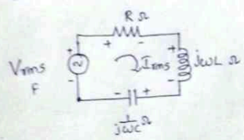
Sol:- Reactance across L $= X_L = \omega L = 2\pi f L\ \Omega$

Reactance across C $= X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}\ \Omega$

Resistance in the Circuit $= R\ \Omega$

Reactance in the Circuit $= X = X_L - X_c\ \Omega$ (or) $X_c - X_L\ \Omega$

The given Circuit can be drawn Like this.



Impedance $= Z = R + jX$
$= R + j(\omega L - \frac{1}{\omega C})$
$= \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}\ \Omega$

By KVL,

$$-V_{rms} + I_{rms} \cdot R + j\omega L \cdot I_{rms} + \frac{1}{j\omega C} \cdot I_{rms} = 0.$$

$$\Rightarrow I_{rms} = \left| \frac{V_{rms}}{R + j(\omega L - \frac{1}{\omega C})} \right| = \frac{V_{rms}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \left| \frac{V_{rms}}{Z} \right|.$$

→ power factor $= \frac{R}{Z} = \cos\phi \quad \Rightarrow \phi = \cos^{-1}(R/Z)$.

→ $V_R = I_{rms} \cdot R$ → $P_R = P =$ power across Resistor $= I_{rms}^2 \cdot R = V_{rms} I_{rms} \cos\phi$

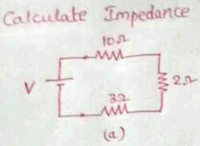
→ $V_L = I_{rms} \cdot \omega L$ → $P_L = V_L \cdot I_{rms} = I_{rms}^2 \cdot \omega L$

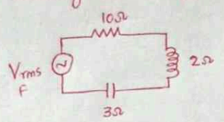
→ $V_c = I_{rms} \cdot \frac{1}{\omega C}$ → $P_c = V_c \cdot I_{rms} = I_{rms}^2 \cdot \frac{1}{\omega C}$

→ $P_L - P_c = Q =$ Reactive power $= V_{rms} I_{rms} \sin\phi$

→ $S = V_{rms} I_{rms} =$ Apparent power.

(9) Calculate Impedance for the following Circuits (a) and (b)





Sol:- (a) $Z = 10 + 2 + 3$
$= 15\ \Omega$

(b) $Z = R + jX_L - jX_c$

given data is $R = 10\Omega$ $X_L = 2\Omega$ $X_c = 3\Omega$.

$\therefore Z = 10 + j(2 - 3)$
$= 10 - j \Rightarrow |Z| = \sqrt{10^2 + 1}$
$= 10.04\ \Omega$.

→ Before this Question (above) i.e. 9th pg. we discussed about Series RLC Circuit. In that Circuit, If

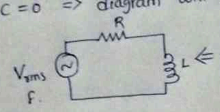
→ $L = 0$, $R = 0$ ⇒ C circuit ($C \neq 0$)
→ $C = 0$ $R = 0$ ⇒ L Circuit. ($L \neq 0$)
→ $L = 0$ $C = 0$ ⇒ R Circuit. ($R \neq 0$)

If $C = 0$ ⇒ RL Circuit (series) ($R \neq 0$, $L \neq 0$)
$L = 0$ ⇒ Series RC Circuit. ($R \neq 0$, $C \neq 0$)

If $R \neq 0$, $L \neq 0$, $C \neq 0$ then that Circuit is Series RLC. Circuit.

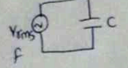
then All the formulas and diagrams will be Changed by placing (or) Substituting R, L, C values.

Eg: If $C = 0$ ⇒ diagram will be written as



⇐ series RL circuit

If $R = 0$, $L = 0$ ⇒



⇐ Capacitor Circuit.

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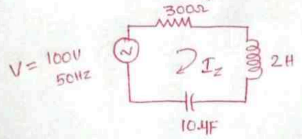
(9). What are the values of $X_L, X_c, X, Z, I_z, V_L, V_c, V_R, P_R, P_L, P_c$, powerfactor, P, Q, S for the following circuit.



$V = V_{rms} = V$
$I_z = I_{rms} = I$

Sol:- Given data is : $V = V_{rms} = 100V.$; $R = 300\Omega$; $L = 2H$, $C = 10\mu F$, $f = 50Hz$

$X_L = \omega L = 2\pi f L = 2\pi(50)(2) = 200\pi\ \Omega = 628.318\Omega$

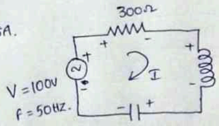
$X_c = \frac{1}{\omega c} = \frac{1}{2\pi f c} = \frac{1}{2\pi(50)\times 10\times 10^{-6}} = 318.3098\Omega$

Power factor $= \cos\phi = \frac{R}{Z} = \frac{300}{431.399} = 0.6954$

$X = X_L - X_c = 310.0087\Omega$

$Z = |R + j(X_L - X_c)| = \sqrt{R^2 + (X_L - X_c)^2} = 431.399\ \Omega.$

$I = I_z = \frac{V}{\sqrt{R^2 + (X_L - X_c)^2}} = \frac{100}{431.399} = 0.2318A.$



$$\boxed{\begin{aligned} V_R &= I.R = 0.2318(300) = 69.54V. \\ V_L &= IX_L = 0.2318(628.318) = 145.644V \\ V_c &= IX_c = 0.2318(318.3098) = 73.784V. \end{aligned}}$$

Verification By KVL, $V + V_R + jV_L - jV_c = 0.$
$\Rightarrow V = \sqrt{V_R^2 + (V_L - V_c)^2}$

$$\boxed{\begin{aligned} P = P_R &= V_R.I = 69.54(0.2318) = 16.1193W \\ P_L &= V_L.I = 33.760\ VAR \\ P_c &= V_c.I = 17.1031\ VAR \\ S &= V.I = \text{Apparent power} = 100(0.2318) = 23.18\ VA. = \sqrt{P^2+Q^2} \end{aligned}}$$

$\left.\begin{matrix} P_L \\ P_c \end{matrix}\right\}$ $Q = P_L - P_c = 16.6569\ VAR.$

You can calculate P, Q, S in another method by using direct formulas also. They are $P = VI\cos\phi$, $Q = VI\sin\phi$ $S = \sqrt{P^2+Q^2}$

→ **Admittance :-**

Admittance of a circuit is defined as the reciprocal of its Impedance. It is denoted with Y. Its unit is Siemens (S).

$Y = \frac{1}{Z} = \frac{1}{R+jX} = \left(\frac{R}{R^2+X^2}\right) + j\left(\frac{-X}{R^2+X^2}\right)$

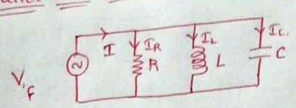
$= G + jB.$

where G = conductance and
B = Suceptance.

→ when $X=0$, Conductance(G) is the reciprocal of Resistance (R)
→ when $R=0$, Suceptance(B) is the reciprocal of Reactance (X)
→ Admittance (Y) is the Reciprocal of Impedance (Z)

→ In parallel branches Admittances are Added.

* **parallel RLC Circuit :-**



$V = V_{rms}$
$I = I_{rms}$.

→ $V_R = V_L = V_c = V$

→ $I_R = \frac{V}{R}$

→ $I_L = \frac{V}{X_L}$

→ $I_c = \frac{V}{X_c}$

→ $I = \sqrt{I_R^2 + (I_L - I_c)^2}$

$Y = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{jX_c}$

$= \frac{1}{R} + j\left(\frac{1}{X_c} - \frac{1}{X_L}\right)$

$= G + jB.$

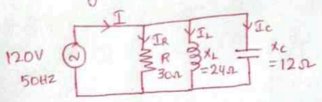
$Z = \frac{1}{G+jB}$

$P_R = P = VI\cos\phi = I_R^2 R. = V.I_R$

$\left.\begin{matrix} P_L = V_L.I_L \\ P_c = V_c.I_c \end{matrix}\right\}$ $Q = P_L - P_c = VI\sin\phi$

$S = VI = \sqrt{P^2+Q^2}$

9. What are the values of $I, I_R, I_L, I_c, Y, P_R, P_L, P_c, P, Q$, and S in the following circuit.



Sol:- Given data is $V = 120V$, $f = 50Hz$, $R = 30\Omega$, $X_L = 24\Omega$, $X_c = 12\Omega$.

$I_R = \frac{V}{R} = \frac{120}{30} = 4A$

$I_L = \frac{V}{X_L} = \frac{120}{24} = 5A$

$I_c = \frac{V}{X_c} = \frac{120}{12} = 10A$.

$I = \sqrt{I_R^2 + (I_L - I_c)^2}$

$= \sqrt{4^2 + (5-10)^2} = \sqrt{41}\ A$

$\Rightarrow I = \sqrt{41} = 6.4031\ A$

$\Rightarrow P = P_R = VI_R = 120(4) = 480\ W$

$P_L = VI_L = 120(5) = 600\ VAR$

$P_c = VI_c = 120(10) = 1200\ VAR$

$Q = P_L - P_c = -600\ VAR$

$= 600\ VAR$ (capacitive)

$S = \sqrt{P^2 + Q^2} = \sqrt{480^2 + 600^2}$

$\boxed{S = 768.37 VA}$

You can calculate and verify your P, Q, S using other formulas also.

i.e $P = VI\cos\phi$, $Q = VI\sin\phi$, $S = VI$.

* Resonant Frequency :-

In a series RLC circuit (or) parallel RLC circuit, the frequency at which $X_L = X_c$ is known as resonant frequency (or)

The frequency at which inductive reactance equal to capacitive reactance is known as Resonant Frequency.

$X_L = X_c$

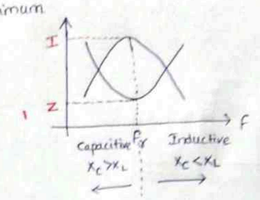
$\Rightarrow \omega L = \frac{1}{\omega C}$

$\Rightarrow \omega^2 = \frac{1}{LC}$

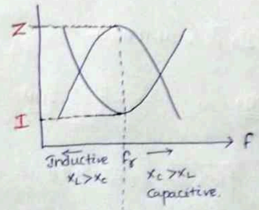
$\Rightarrow (2\pi f_r)^2 = \frac{1}{LC}$ $\Rightarrow$ $\boxed{f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hertz}}$

→ At Resonance, for series RLC circuit, Impedance is minimum so current is Maximum

power factor is 1 (unity) and circuit is purely Resistive.



→ At Resonance, for parallel RLC circuit, Admittance is Minimum. Means Impedance is Maximum and current is Minimum. powerfactor is unity (1) and circuit is purely resistive.



→ phasor concepts regarding series R, L, C, RL, RLC, and parallel will be discussed in class.

→ Rectangle to polar and polar to Rectangular conversion will also be explained in class.

———— x ————

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8. In a Series RL Circuit,

$i(t) = 5\sin(314t + 120^\circ)$

$V(t) = 15\sin(314t + 150^\circ)$.

a) Calculate RMS value of current and voltages.

b) Average power ?

c) Real power ?

d) Reactive power ?

e) Apparent power ?

Sol:- Given data is $i(t) = 5\sin(314t+120^\circ)$

$V(t) = 15\sin(314t+150^\circ)$.

$\boxed{I_{rms} = \frac{5}{\sqrt{2}}}$; $\boxed{V_{rms} = \frac{15}{\sqrt{2}}}$

$\phi = 150^\circ - 120^\circ = 30^\circ$. ; power factor $= \cos\phi = \cos 30^\circ = 0.866$ (lag).

$\sin\phi = \sin 30^\circ = 0.5$;

$\therefore$ Average power = Real power $= I_{rms}^2 R$ (or) $= V_{rms} I_{rms} \cos\phi$.

$= \frac{5}{\sqrt{2}} \cdot \frac{15}{\sqrt{2}} \cdot \cos 30^\circ = \boxed{32.475\,W}$

Reactive power $= V_{rms} \cdot I_{rms} \cdot \sin\phi$

$= \frac{5}{\sqrt{2}} \cdot \frac{15}{\sqrt{2}} \cdot \sin 30^\circ = \boxed{18.75\ VAR}$

Apparent power $= \sqrt{\text{Real power}^2 + \text{Reac power}^2}$ (or) $V_{rms} I_{rms}$

$= \sqrt{32.475^2 + 18.75^2}$ (or) $\frac{15}{\sqrt{2}} \frac{5}{\sqrt{2}}$

$= \boxed{37.499\ VA}$ (or) $37.5\ VA$

9. In a series RL Circuit, the current and Voltage expressed as

$i(t) = 5\sin(314t + \frac{2\pi}{3})$ and

$V(t) = 15\sin(314t + 5\pi/6)$.

a) what is the Magnitude of Impedance of the Circuit ?

b) Value of Resistance (R) ?

c) Value of Inductance (L) ?

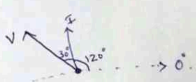
d) Average power drawn by the circuit ?

e) what is the power factor ? is it lagging or leading.

Sol:- Given data, $i(t) = 5\sin(314t + 120^\circ)$

$V(t) = 15\sin(314t + 150^\circ)$

i.e $\omega = 314$ rad/sec.



a) $\boxed{Z = \left|\frac{V(t)}{I(t)}\right| = \frac{15}{5} = 3\Omega}$

I lags V by 30°. $\phi = 30^\circ$.

e) power factor $= \cos\phi = \cos 30^\circ = \frac{\sqrt{3}}{2}$ (lagging)

$\Rightarrow \boxed{\cos\phi = 0.866(\text{lag})}$

b) power factor $= \cos\phi = \frac{R}{Z}$.

$\Rightarrow 0.866 = \frac{R}{3} \Rightarrow \boxed{R = 0.866 \times 3 = 2.59\,\Omega}$

c) $Z = \sqrt{R^2 + X_L^2} \Rightarrow 3^2 = \sqrt{(2.59)^2 + X_L^2}$

$\Rightarrow X_L = \sqrt{9 - 6.75} = 1.5\,\Omega$.

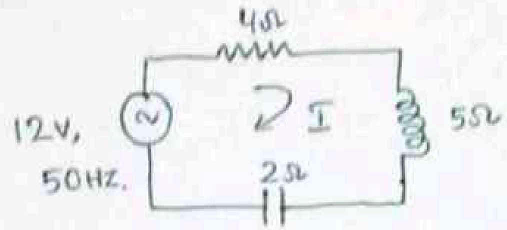
But $X_L = \omega L \Rightarrow 1.5 = 314(L) \Rightarrow \boxed{L = \frac{1.5}{314} = 4.7\,mH}$

d) Average power $= V_{rms} \cdot I_{rms} \cdot \cos\phi$ or $I_{rms}^2 R$

$= \frac{15}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} \cdot (0.866)$ $= \left(\frac{5}{\sqrt{2}}\right)^2 \cdot 2.59 = 32.5\,W$ (APROX)

$= 32.5\,W$ // (APROX)

(9). In following Series RLC Circuit, Calculate.



i) X_L ii) X_c iii) X iv) $|Z|$ v) $|Y|$ vi) V_L vii) V_c.

viii) V_R ix) I x) P_R xi) P_L xii) P_c. xiii) Real power,

xiv) Reactive power xv) Apparent power xvi) powerfactor

Sol:- $X_L = 5\Omega$; $X_c = 2\Omega$; $X = X_L - X_c = 3\Omega$ (Inductive)

iv) $|Z| = \left|\sqrt{R^2 + (X_L - X_c)^2}\right| = 5\Omega$ ix) $I = \frac{V}{Z} = \frac{12}{5} = 2.4A$.

v) $|Y| = \left|\frac{1}{Z}\right| = \frac{1}{5}$ simens.

vi) $V_L = IX_L = 2.4(5) = 12\ V$. ; vii) $V_c = 4.8V. = IX_c$.

viii) $V_R = IR = 2.4(4) = 9.6V$.

x) $P_R = I^2 R = 23.04\ W$.

xi) $P_L = I^2 X_L = 28.8\ VAR$

xii) $P_c = I^2 X_c = 11.52\ VAR$.

xvi) power factor $= \frac{R}{Z} = \frac{4}{5} = 0.8\ (lag). = \cos\phi$.

$X_L > X_c$ so I lag V.

$$\sin\phi = \sqrt{1 - \cos^2\phi} = \sqrt{1 - 0.8^2} = 0.6.$$

xiii) Real power $= P_R =$ Avg. power $= V_{rms} I_{rms} \cos\phi$.

$= VI\cos\phi = 23.04W$

xiv) Reactive power $= P_L - P_c = V_{rms} I_{rms} \sin\phi = 17.28\ VAR$.

xv) Apparent power $= \sqrt{P_R^2 + (P_L - P_c)^2} = V_{rms} I_{rms} = VI = 28.8\ VA$