Contents

CHAPTER

7

Moment of Inertia

Contents

7.1. INTRODUCTION

We have already discussed in Art. 3.2 that the moment of a force (P) about a point, is the product of the force and perpendicular distance (x) between the point and the line of action of the force (*i.e.* $P.x$). This moment is also called first moment of force. If this moment is again multiplied by the perpendicular distance (x) between the point and the line of action of the force *i.e.* $P.x\ (x) = Px^2$, then this quantity is called moment of the moment of a force or second moment of force or moment of inertia (briefly written as M.I.).

Sometimes, instead of force, area or mass of a figure or body is taken into consideration. Then the second moment is known as second moment of area or second moment of mass. But all such second moments are broadly termed as moment of inertia. In this chapter, we shall discuss the moment of inertia of plane areas only.

7.2. MOMENT OF INERTIA OF A PLANE AREA

Consider a plane area, whose moment of inertia is required to be found out. Split up the whole area into a number of small elements.

Let $a_1, a_2, a_3, \ldots =$ Areas of small elements, and

$r_1, r_2, r_3, \ldots =$ Corresponding distances of the elements from the line about which the moment of inertia is required to be found out.

Now the moment of inertia of the area,

$$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \ldots$$
$$= \Sigma\, a\, r^2$$

7.3. UNITS OF MOMENT OF INERTIA

As a matter of fact the units of moment of inertia of a plane area depend upon the units of the area and the length. *e.g.*,

1. If area is in m^2 and the length is also in m, the moment of inertia is expressed in m^4.
2. If area in mm^2 and the length is also in mm, then moment of inertia is expressed in mm^4.

7.4. METHODS FOR MOMENT OF INERTIA

The moment of inertia of a plane area (or a body) may be found out by any one of the following two methods :

1. By Routh's rule 2. By Integration.

Note : The Routh's Rule is used for finding the moment of inertia of a plane area or a body of uniform thickness.

7.5. MOMENT OF INERTIA BY ROUTH'S RULE

The Routh's Rule states, if a body is symmetrical about three mutually perpendicular axes*, then the moment of inertia, about any one axis passing through its centre of gravity is given by:

$$I = \frac{A\ (\text{or}\ M) \times S}{3} \qquad \text{... (For a Square or Rectangular Lamina)}$$

$$I = \frac{A\ (\text{or}\ M) \times S}{4} \qquad \text{... (For a Circular or Elliptical Lamina)}$$

$$I = \frac{A\ (\text{or}\ M) \times S}{5} \qquad \text{... (For a Spherical Body)}$$

where $A =$ Area of the plane area

$M =$ Mass of the body, and

$S =$ Sum of the squares of the two semi-axis, other than the axis, about which the moment of inertia is required to be found out.

Note : This method has only academic importance and is rarely used in the field of science and technology these days. The reason for the same is that it is equally convenient to use the method of integration for the moment of inertia of a body.

* *i.e.*, *X-X* axis, *Y-Y* axis and *Z-Z* axis.

Contents

7.6. MOMENT OF INERTIA BY INTEGRATION

The moment of inertia of an area may also be found out by the method of integration as discussed below:

Consider a plane figure, whose moment of inertia is required to be found out about X-X axis and Y-Y axis as shown in Fig 7.1. Let us divide the whole area into a no. of strips. Consider one of these strips.

Let dA = Area of the strip

x = Distance of the centre of gravity of the strip on X-X axis and

y = Distance of the centre of gravity of the strip on Y-Y axis.

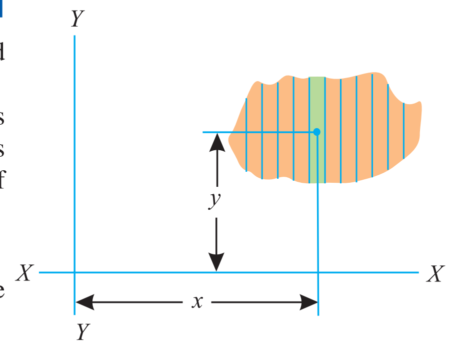

Fig. 7.1. Moment of inertia by integration.

We know that the moment of inertia of the strip about Y-Y axis

$$= dA \cdot x^2$$

Now the moment of inertia of the whole area may be found out by integrating above equation. *i.e.*,

$$I_{YY} = \Sigma\, dA \cdot x^2$$

Similarly $\quad I_{XX} = \Sigma\, dA \cdot y^2$

In the following pages, we shall discuss the applications of this method for finding out the moment of inertia of various cross-sections.

7.7. MOMENT OF INERTIA OF A RECTANGULAR SECTION

Consider a rectangular section $ABCD$ as shown in Fig. 7.2 whose moment of inertia is required to be found out.

Let b = Width of the section and

d = Depth of the section.

Now consider a strip PQ of thickness dy parallel to X-X axis and at a distance y from it as shown in the figure

∴ Area of the strip

$$= b.dy$$

We know that moment of inertia of the strip about X-X axis,

$$= \text{Area} \times y^2 = (b.\,dy)\,y^2 = b.\,y^2.\,dy$$

Now *moment of inertia of the whole section may be found out by integrating the above equation for the whole length of the lamina *i.e.* from $-\dfrac{d}{2}$ to $+\dfrac{d}{2}$,

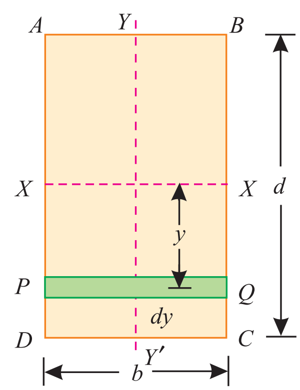

Fig. 7.2. Rectangular section.

* This may also be obtained by Routh's rule as discussed below :

$$I_{xx} = \frac{AS}{3} \qquad \text{...(for rectangular section)}$$

where area, $A = b \times d$ and sum of the square of semi axes Y-Y and Z-Z,

$$S = \left(\frac{d}{2}\right)^2 + 0 = \frac{d^2}{4}$$

$$\therefore \quad I_{xx} = \frac{AS}{3} = \frac{(b \times d) \times \dfrac{d^2}{4}}{3} = \frac{bd^3}{12}$$

$$I_{xx} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} b.y^2.dy = b\int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2\ .\,dy$$

$$= b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{+\frac{d}{2}} = b\left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right] = \frac{bd^3}{12}$$

Similarly, $$I_{YY} = \frac{db^3}{12}$$

Note. Cube is to be taken of the side, which is at right angles to the line of reference.

Example 7.1. *Find the moment of inertia of a rectangular section 30 mm wide and 40 mm deep about X-X axis and Y-Y axis.*

Solution. Given: Width of the section (b) = 30 mm and depth of the section (d) = 40 mm.

We know that moment of inertia of the section about an axis passing through its centre of gravity and parallel to X-X axis,

$$I_{XX} = \frac{bd^3}{12} = \frac{30 \times (40)^3}{12} = 160 \times 10^3\ \text{mm}^4 \qquad \textbf{Ans.}$$

Similarly $$I_{YY} = \frac{db^3}{12} = \frac{40 \times (30)^3}{12} = 90 \times 10^3\ \text{mm}^4 \qquad \textbf{Ans.}$$

7.8. MOMENT OF INERTIA OF A HOLLOW RECTANGULAR SECTION

Consider a hollow rectangular section, in which *ABCD* is the main section and *EFGH* is the cut out section as shown in Fig 7.3

Let b = Breadth of the outer rectangle,

d = Depth of the outer rectangle and

b_1, d_1 = Corresponding values for the cut out rectangle.

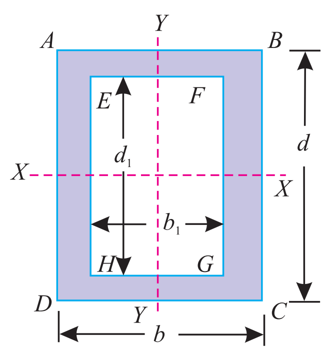

Fig. 7.3. Hollow rectangular section.

We know that the moment of inertia, of the outer rectangle *ABCD* about X-X axis

$$= \frac{bd^3}{12} \qquad ...(i)$$

and moment of inertia of the cut out rectangle *EFGH* about X-X axis

$$= \frac{b_1 d_1^3}{12} \qquad ...(ii)$$

∴ M.I. of the hollow rectangular section about X-X axis,

I_{XX} = M.I. of rectangle *ABCD* – M.I. of rectangle *EFGH*

$$= \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$$

Similarly, $$I_{yy} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12}$$

Note : This relation holds good only if the centre of gravity of the main section as well as that of the cut out section coincide with each other.

Example 7.2. *Find the moment of inertia of a hollow rectangular section about its centre of gravity if the external dimensions are breadth 60 mm, depth 80 mm and internal dimensions are breadth 30 mm and depth 40 mm respectively.*

Solution. Given: External breadth (b) = 60 mm; External depth (d) = 80 mm ; Internal breadth (b_1) = 30 mm and internal depth (d_1) = 40 mm.

We know that moment of inertia of hollow rectangular section about an axis passing through its centre of gravity and parallel to X-X axis,

$$I_{XX} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12} = \frac{60\,(80)^3}{12} - \frac{30\,(40)^3}{12} = 2400 \times 10^3 \text{ mm}^4 \qquad \textbf{Ans.}$$

Similarly,

$$I_{YY} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12} = \frac{80\,(60)^3}{12} - \frac{40\,(30)^3}{12} = 1350 \times 10^3 \text{ mm}^4 \qquad \textbf{Ans.}$$

7.9. THEOREM OF PERPENDICULAR AXIS

It states, *If I_{XX} and I_{YY} be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia I_{ZZ} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:*

$$I_{ZZ} = I_{XX} + I_{YY}$$

Proof :

Consider a small lamina (P) of area da having co-ordinates as x and y along OX and OY two mutually perpendicular axes on a plane section as shown in Fig. 7.4.

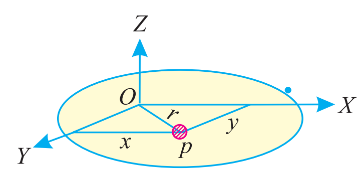

Fig. 7.4. Theorem of perpendicular axis.

Now consider a plane OZ perpendicular to OX and OY. Let (r) be the distance of the lamina (P) from Z-Z axis such that $OP = r$.

From the geometry of the figure, we find that

$$r^2 = x^2 + y^2$$

We know that the moment of inertia of the lamina P about X-X axis,

$$I_{XX} = da.\, y^2 \qquad \text{...[}\because I = \text{Area} \times (\text{Distance})^2\text{]}$$

Similarly,

$$I_{YY} = da.\, x^2$$

and

$$I_{ZZ} = da.\, r^2 = da\,(x^2 + y^2) \qquad \text{...(}\because r^2 = x^2 + y^2\text{)}$$

$$= da.\, x^2 + da.\, y^2 = I_{YY} + I_{XX}$$

7.10. MOMENT OF INERTIA OF A CIRCULAR SECTION

Consider a circle $ABCD$ of radius (r) with centre O and X-X' and Y-Y' be two axes of reference through O as shown in Fig. 7.5.

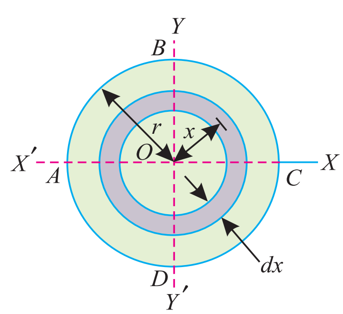

Fig. 7.5. Circular section.

Now consider an elementary ring of radius x and thickness dx. Therefore area of the ring,

$$da = 2\,\pi\, x.\, dx$$

and moment of inertia of ring, about X-X axis or Y-Y axis

$$= \text{Area} \times (\text{Distance})^2$$
$$= 2\,\pi\, x.\, dx \times x^2$$
$$= 2\,\pi\, x^3.\, dx$$

Now moment of inertia of the whole section, about the central axis, can be found out by integrating the above equation for the whole radius of the circle *i.e.*, from 0 to r.

$$\therefore \qquad I_{ZZ} = \int_0^r 2\pi x^3 \,.\, dx = 2\pi \int_0^r x^3 \,.\, dx$$

$$I_{ZZ} = 2\pi\left[\frac{x^4}{4}\right]_0^r = \frac{\pi}{2}(r)^4 = \frac{\pi}{32}(d)^4 \qquad \ldots\left(\text{substituting } r = \frac{d}{2}\right)$$

We know from the Theorem of Perpendicular Axis that

$$I_{XX} + I_{YY} = I_{ZZ}$$

$$\therefore \quad {}^{*}I_{XX} = I_{YY} = \frac{I_{ZZ}}{2} = \frac{1}{2} \times \frac{\pi}{32}(d)^4 = \frac{\pi}{64}(d)^4$$

Example 7.3. *Find the moment of inertia of a circular section of 50 mm diameter about an axis passing through its centre.*

Solution. Given: Diameter (d) = 50 mm

We know that moment of inertia of the circular section about an axis passing through its centre,

$$I_{XX} = \frac{\pi}{64}(d)^4 = \frac{\pi}{64} \times (50)^4 = 307 \times 10^3 \text{ mm}^4 \qquad \textbf{Ans.}$$

7.11. MOMENT OF INERTIA OF A HOLLOW CIRCULAR SECTION

Consider a hollow circular section as shown in Fig.7.6, whose moment of inertia is required to be found out.

Let D = Diameter of the main circle, and

d = Diameter of the cut out circle.

We know that the moment of inertia of the main circle about X-X axis

$$= \frac{\pi}{64}(D)^4$$

and moment of inertia of the cut-out circle about X-X axis

$$= \frac{\pi}{64}(d)^4$$

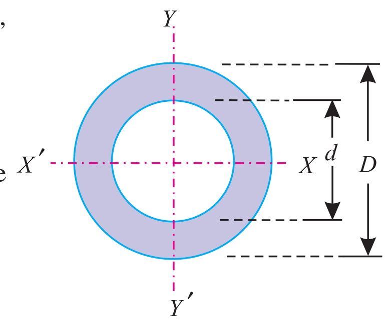

Fig. 7.6. Hollow circular section.

∴ Moment of inertia of the hollow circular section about X-X axis,

I_{XX} = Moment of inertia of main circle – Moment of inertia of cut out circle,

$$= \frac{\pi}{64}(D)^4 - \frac{\pi}{64}(d)^4 = \frac{\pi}{64}(D^4 - d^4)$$

Similarly, $$I_{YY} = \frac{\pi}{64}(D^4 - d^4)$$

Note : This relation holds good only if the centre of the main circular section as well as that of the cut out circular section coincide with each other.

* This may also be obtained by Routh's rule as discussed below

$$I_{XX} = \frac{AS}{4} \qquad \text{(for circular section)}$$

where area, $A = \frac{\pi}{4} \times d^2$ and sum of the square of semi axis Y-Y and Z-Z,

$$S = \left(\frac{d}{2}\right)^2 + 0 = \frac{d^2}{4}$$

$$\therefore \quad I_{XX} = \frac{AS}{4} = \frac{\left[\frac{\pi}{4} \times d^2\right] \times \frac{d^2}{4}}{4} = \frac{\pi}{64}(d)^4$$

Contents

Example 7.4. *A hollow circular section has an external diameter of 80 mm and internal diameter of 60 mm. Find its moment of inertia about the horizontal axis passing through its centre.*

Solution. Given : External diameter (D) = 80 mm and internal diameter (d) = 60 mm.

We know that moment of inertia of the hollow circular section about the horizontal axis passing through its centre,

$$I_{XX} = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}[(80)^4 - (60)^4] = 1374 \times 10^3 \text{ mm}^4 \qquad \textbf{Ans.}$$

7.12. THEOREM OF PARALLEL AXIS

It states, *If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by I_G, then moment of inertia of the area about any other axis AB, parallel to the first, and at a distance h from the centre of gravity is given by*:

$$I_{AB} = I_G + ah^2$$

where I_{AB} = Moment of inertia of the area about an axis AB,

I_G = Moment of Inertia of the area about its centre of gravity

a = Area of the section, and

h = Distance between centre of gravity of the section and axis AB.

Proof

Consider a strip of a circle, whose moment of inertia is required to be found out about a line AB as shown in Fig. 7.7.

Let δa = Area of the strip

y = Distance of the strip from the centre of gravity the section and

h = Distance between centre of gravity of the section and the axis AB.

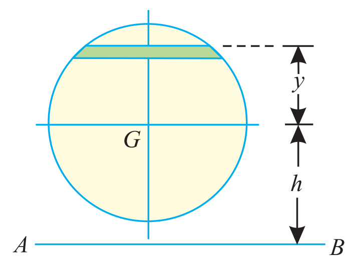

Fig. 7.7. Theorem of parallel axis.

We know that moment of inertia of the whole section about an axis passing through the centre of gravity of the section

$$= \delta a.\, y^2$$

and moment of inertia of the whole section about an axis passing through its centre of gravity,

$$I_G = \Sigma\, \delta a.\, y^2$$

∴ Moment of inertia of the section about the axis AB,

$$I_{AB} = \Sigma\, \delta a\, (h + y)^2 = \Sigma\, \delta a\, (h^2 + y^2 + 2\, h\, y)$$

$$= (\Sigma\, h^2.\, \delta a) + (\Sigma\, y^2.\, \delta a) + (\Sigma\, 2\, h\, y\, .\, \delta a)$$

$$= a\, h^2 + I_G + 0$$

It may be noted that $\Sigma\, h^2 .\, \delta a = a\, h^2$ and $\Sigma\, y^2 .\, \delta a = I_G$ [as per equation (i) above] and $\Sigma\, \delta a.y$ is the algebraic sum of moments of all the areas, about an axis through centre of gravity of the section and is equal to $a.\bar{y}$, where $\bar{y}$ is the distance between the section and the axis passing through the centre of gravity, which obviously is zero.

Contents

7.13. MOMENT OF INERTIA OF A TRIANGULAR SECTION

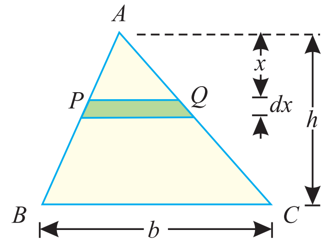

Fig. 7.8. Triangular section.

Consider a triangular section *ABC* whose moment of inertia is required to be found out.

Let b = Base of the triangular section and

h = Height of the triangular section.

Now consider a small strip *PQ* of thickness dx at a distance of x from the vertex *A* as shown in Fig. 7.8. From the geometry of the figure, we find that the two triangles *APQ* and *ABC* are similar. Therefore

$$\frac{PQ}{BC} = \frac{x}{h} \quad \text{or} \quad PQ = \frac{BC \cdot x}{h} = \frac{bx}{h} \qquad (\because BC = \text{base} = b)$$

We know that area of the strip *PQ*

$$= \frac{bx}{h} \cdot dx$$

and moment of inertia of the strip about the base *BC*

$$= \text{Area} \times (\text{Distance})^2 = \frac{bx}{h}\, dx\, (h-x)^2 = \frac{bx}{h}(h-x)^2\, dx$$

Now moment of inertia of the whole triangular section may be found out by integrating the above equation for the whole height of the triangle *i.e.*, from 0 to h.

$$I_{BC} = \int_0^h \frac{b\,x}{h}(h-x)^2\, dx$$

$$= \frac{b}{h}\int_0^h x\,(h^2 + x^2 - 2\,h\,x)\, dx$$

$$= \frac{b}{h}\int_0^h (x\,h^2 + x^3 - 2\,hx^2)\, dx$$

$$= \frac{b}{h}\left[\frac{x^2\,h^2}{2} + \frac{x^4}{4} - \frac{2\,hx^3}{3}\right]_0^h = \frac{b\,h^3}{12}$$

We know that distance between centre of gravity of the triangular section and base *BC*,

$$d = \frac{h}{3}$$

∴ Moment of inertia of the triangular section about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_G = I_{BC} - ad^2 \qquad ...(\because I_{XX} = I_G + a\,h^2)$$

$$= \frac{bh^3}{12} - \left(\frac{bh}{2}\right)\left(\frac{h}{3}\right)^2 = \frac{bh^3}{36}$$

Notes : 1. The moment of inertia of section about an axis through its vertex and parallel to the base

$$= I_G + a\,d^2 = \frac{b\,h^3}{36} + \left(\frac{b\,h}{2}\right)\left(\frac{2\,h}{3}\right)^2 = \frac{9\,b\,h^3}{36} = \frac{b\,h^3}{4}$$

2. This relation holds good for any type of triangle.

Example. 7.5. *An isosceles triangular section ABC has base width 80 mm and height 60 mm. Determine the moment of inertia of the section about the centre of gravity of the section and the base BC.*

Solution. Given : Base width (b) = 80 mm and height (h) = 60 mm.

Moment of inertia about the centre of gravity of the section

We know that moment of inertia of triangular section about its centre of gravity,

$$I_G = \frac{bh^3}{36} = \frac{80 \times (60)^3}{36} = 480 \times 10^3 \text{ mm}^4$$

Moment of inertia about the base BC

We also know that moment of inertia of triangular section about the base BC,

$$I_{BC} = \frac{bh^3}{12} = \frac{80 \times (60)^3}{12} = 1440 \times 10^3 \text{ mm}^4$$

Exmple 7.6. *A hollow triangular section shown in Fig. 7.9 is symmetrical about its vertical axis.*

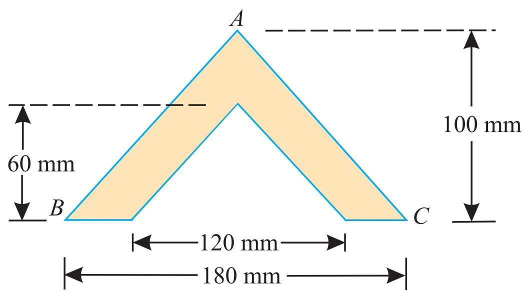

Fig. 7.9.

Find the moment of inertia of the section about the base BC.

Solution. Given : Base width of main triangle (B) = 180 mm; Base width of cut out triangle (b) = 120 mm; Height of main triangle (H) = 100 mm and height of cut out triangle (h) = 60 mm.

We know that moment of inertia of the triangular, section about the base BC,

$$I_{BC} = \frac{BH^3}{12} - \frac{bh^3}{12} = \frac{180 \times (100)^3}{12} - \frac{120 \times (60)^3}{12} \text{ mm}^4$$

$$= (15 \times 10^6) - (2.16 \times 10^6) = 12.84 \times 10^6 \text{ mm}^4 \qquad \textbf{Ans.}$$

7.14. MOMENT OF INERTIA OF A SEMICIRCULAR SECTION

Consider a semicircular section ABC whose moment of inertia is required to be found out as shown in Fig. 7.10.

Let $\quad r$ = Radius of the semicircle.

We know that moment of inertia of the semicircular section about the base AC is equal to half the moment of inertia of the circular section about AC. Therefore moment of inertia of the semicircular section ABC about the base AC,

$$I_{AC} = \frac{1}{2} \times \frac{\pi}{64} \times (d)^4 = 0.393\, r^4$$

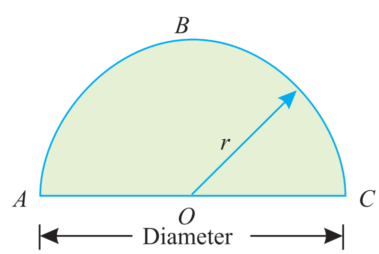

Fig. 7.10. Semicircular section *ABC*.

We also know that area of semicircular section,

$$a = \frac{1}{2} \times \pi r^2 \; \frac{\pi r^2}{2}$$

Contents

and distance between centre of gravity of the section and the base AC,

$$h = \frac{4r}{3\pi}$$

∴ Moment of inertia of the section through its centre of gravity and parallel to x-x axis,

$$I_G = I_{AC} - ah^2 = \left[\frac{\pi}{8} \times (r)^4\right] - \left[\frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right)^2\right]$$

$$= \left[\frac{\pi}{8} \times (r)^4\right] - \left[\frac{8}{9\pi} \times (r)^4\right] = 0.11\, r^4$$

Note. The moment of inertia about y-y axis will be the same as that about the base AC *i.e.*, $0.393\ r^4$.

Example 7.7. *Determine the moment of inertia of a semicircular section of 100 mm diameter about its centre of gravity and parallel to X-X and Y-Y axes.*

Solution. Given: Diameter of the section (d) = 100 mm or radius (r) = 50 mm

Moment of inertia of the section about its centre of gravity and parallel to X-X axis

We know that moment of inertia of the semicircular section about its centre of gravity and parallel to X-X axis,

$$I_{XX} = 0.11\ r^4 = 0.11 \times (50)^4 = 687.5 \times 10^3 \text{ mm}^4 \qquad \textbf{Ans.}$$

Moment of inertia of the section about its centre of gravity and parallel to Y-Y axis.

We also know that moment of inertia of the semicircular section about its centre of gravity and parallel to Y-Y axis.

$$I_{YY} = 0.393\ r^4 = 0.393 \times (50)^4 = 2456 \times 10^3 \text{ mm}^4 \qquad \textbf{Ans.}$$

Example 7.8. *A hollow semicircular section has its outer and inner diameter of 200 mm and 120 mm respectively as shown in Fig. 7.11.*

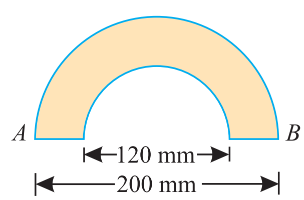

Fig. 7.11.

What is its moment of inertia about the base AB ?

Solution. Given: Outer diameter (D) = 200 mm or Outer Radius (R) = 100 mm and inner diameter (d) = 120 mm or inner radius (r) = 60 mm.

We know that moment of inertia of the hollow semicircular section about the base AB,

$$I_{AB} = 0.393\ (R^4 - r^4) = 0.393\ [(100)^4 - (60)^4] = 34.21 \times 10^6 \text{ mm}^4 \qquad \textbf{Ans.}$$

EXERCISE 7.1

1. Find the moment of inertia of a rectangular section 60 mm wide and 40 mm deep about its centre of gravity. [**Ans.** $I_{XX} = 320 \times 10^3$ mm^4 ; $I_{YY} = 720 \times 10^3$ mm^4]
2. Find the moment of inertia of a hollow rectangular section about its centre of gravity, if the external dimensions are 40 mm deep and 30 mm wide and internal dimensions are 25 mm deep and 15 mm wide. [**Ans.** $I_{XX} = 140\,470$ mm^4 : $I_{YY} = 82\,970$ mm^4]

3. Find the moment of inertia of a circular section of 20 mm diameter through its centre of gravity. [**Ans.** 7854 mm^4]
4. Calculate the moment of inertia of a hollow circular section of external and internal diameters 100 mm and 80 mm respectively about an axis passing through its centroid. [**Ans.** 2.898×10^6 mm^4]
5. Find the moment of inertia of a triangular section having 50 mm base and 60 mm height about an axis through its centre of gravity and base. [**Ans.** 300×10^3 mm^4: 900×10^3 mm^4]
6. Find the moment of inertia of a semicircular section of 30 mm radius about its centre of gravity and parallel to *X-X* and *Y-Y* axes. [**Ans.** 89 100 mm^4 : 381 330 mm^4]

7.15. MOMENT OF INERTIA OF A COMPOSITE SECTION

The moment of inertia of a composite section may be found out by the following steps :

1. First of all, split up the given section into plane areas (*i.e.*, rectangular, triangular, circular etc., and find the centre of gravity of the section).
2. Find the moments of inertia of these areas about their respective centres of gravity.
3. Now transfer these moment of inertia about the required axis (*AB*) by the Theorem of Parallel Axis, *i.e.*,

$$I_{AB} = I_G + ah^2$$

where I_G = Moment of inertia of a section about its centre of gravity and parallel to the axis.
a = Area of the section,
h = Distance between the required axis and centre of gravity of the section.

4. The moments of inertia of the given section may now be obtained by the algebraic sum of the moment of inertia about the required axis.

Example 7.9. *Figure 7.12 shows an area ABCDEF.*

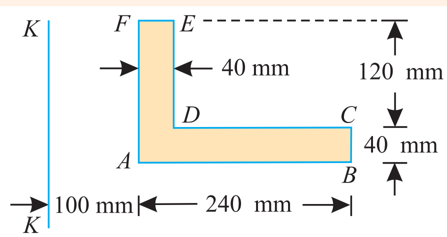

Fig. 7.12.

Compute the moment of inertia of the above area about axis K-K.

Solution. As the moment of inertia is required to be found out about the axis *K-K*, therefore there is no need of finding out the centre of gravity of the area.

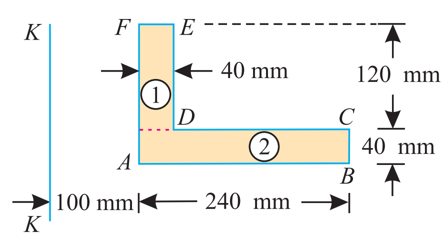

Fig. 7.13.

Let us split up the area into two rectangles 1 and 2 as shown in Fig. 7.13.

We know that moment of inertia of section (1) about its centre of gravity and parallel to axis *K-K*,

$$I_{G1} = \frac{120 \times (40)^3}{12} = 640 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of section (1) and axis *K-K*,

$$h_1 = 100 + \frac{40}{2} = 120 \text{ mm}$$

∴ Moment of inertia of section (1) about axis *K-K*

$$= I_{G1} + a_1 h_1^2 = (640 \times 10^3) + [(120 \times 40) \times (120)^2] = 69.76 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of section (2) about its centre of gravity and parallel to axis *K-K*,

$$I_{G2} = \frac{40 \times (240)^3}{12} = 46.08 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of section (2) and axis *K-K*,

$$h_2 = 100 + \frac{240}{2} = 220 \text{ mm}$$

∴ Moment of inertia of section (2) about the axis *K-K*,

$$= I_{G2} + a_2 h_2^2 = (46.08 \times 10^6) + [(240 \times 40) \times (220)^2] = 510.72 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole area about axis *K-K*,

$$I_{KK} = (69.76 \times 10^6) + (510.72 \times 10^6) = 580.48 \times 10^6 \text{ mm}^4 \qquad \textbf{Ans.}$$

Example 7.10. *Find the moment of inertia of a T-section with flange as 150 mm × 50 mm and web as 150 mm × 50 mm about X-X and Y-Y axes through the centre of gravity of the section.*

Solution. The given *T*-section is shown in Fig. 7.14.

First of all, let us find out centre of gravity of the section. As the section is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis. Split up the whole section into two rectangles *viz*., 1 and 2 as shown in figure. Let bottom of the web be the axis of reference.

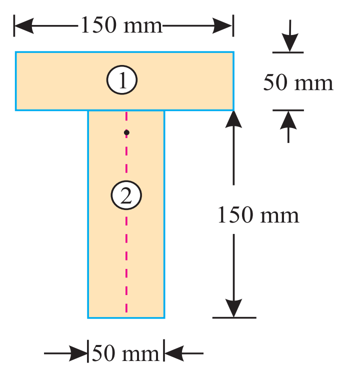

Fig. 7.14.

(*i*) *Rectangle* (1)

$$a_1 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_1 = 150 + \frac{50}{2} = 175 \text{ mm}$$

(*ii*) *Rectangle* (2)

$$a_2 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_2 = \frac{150}{2} = 75 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the web,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500} = 125 \text{ mm}$$

Moment of inertia about X-X axis

We also know that M.I. of rectangle (1) about an axis through its centre of gravity and parallel to *X-X* axis.

$$I_{G1} = \frac{150\,(50)^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of rectangle (1) and *X-X* axis,

$$h_1 = 175 - 125 = 50 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about X-X axis

$$I_{G1} + a_1 h_1^2 = (1.5625 \times 10^6) + [7500 \times (50)^2] = 20.3125 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G2} = \frac{50\,(150)^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of rectangle (2) and X-X axis,

$$h_2 = 125 - 75 = 50 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about X-X axis

$$= I_{G2} + a_2 h_2^2 = (14.0625 \times 10^6) + [7500 \times (50)^2] = 32.8125 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about X-X axis,

$$I_{XX} = (20.3125 \times 10^6) + (32.8125 \times 10^6) = 53.125 \times 10^6 \text{ mm}^4$$ **Ans.**

Moment of inertia about Y-Y axis

We know that M.I. of rectangle (1) about Y-Y axis

$$= \frac{50\,(150)^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

and moment of inertia of rectangle (2) about Y-Y axis,

$$= \frac{150\,(50)^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about Y-Y axis,

$$I_{YY} = (14.0625 \times 10^6) + (1.5625 \times 10^6) = 15.625 \times 10^6 \text{ mm}^4$$ **Ans.**

Example 7.11. *An I-section is made up of three rectangles as shown in Fig. 7.15. Find the moment of inertia of the section about the horizontal axis passing through the centre of gravity of the section.*

Solution. First of all, let us find out centre of gravity of the section. As the section is symmetrical about Y-Y axis, therefore its centre of gravity will lie on this axis.

Split up the whole section into three rectangles 1, 2 and 3 as shown in Fig. 7.15, Let bottom face of the bottom flange be the axis of reference.

(*i*) *Rectangle* 1

$$a_1 = 60 \times 20 = 1200 \text{ mm}$$

and $$y_1 = 20 + 100 + \frac{20}{2} = 130 \text{ mm}$$

(*ii*) *Rectangle* 2

$$a_2 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_2 = 20 + \frac{100}{2} = 70 \text{ mm}$$

(*iii*) *Rectangle* 3

$$a_3 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_3 = \frac{20}{2} = 10 \text{ mm}$$

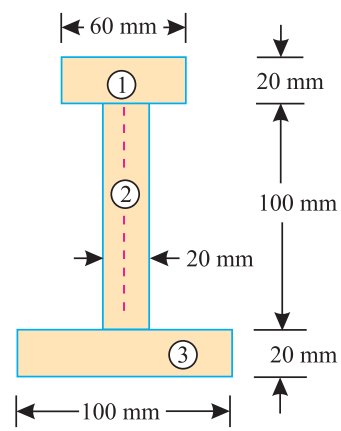

Fig. 7.15.

We know that the distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(1200 \times 130) + (2000 \times 70) + (2000 \times 10)}{1200 + 2000 + 2000} \text{ mm}$$

$$= 60.8 \text{ mm}$$

We know that moment of inertia of rectangle (1) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G1} = \frac{60 \times (20)^3}{12} = 40 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (1) and *X-X* axis,

$$h_1 = 130 - 60.8 = 69.2 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about *X-X* axis,

$$= I_{G1} + a_1 h_1^2 = (40 \times 10^3) + [1200 \times (69.2)^2] = 5786 \times 10^3 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G2} = \frac{20 \times (100)^3}{12} = 1666.7 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (2) and *X-X* axis,

$$h_2 = 70 - 60.8 = 9.2 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about *X-X* axis,

$$= I_{G2} + a_2 h_2^2 = (1666.7 \times 10^3) + [2000 \times (9.2)^2] = 1836 \times 10^3 \text{ mm}^4$$

Now moment of inertia of rectangle (3) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G3} = \frac{100 \times (20)^3}{12} = 66.7 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (3) and *X-X* axis,

$$h_3 = 60.8 - 10 = 50.8 \text{ mm}$$

∴ Moment of inertia of rectangle (3) about *X-X* axis,

$$= I_{G3} + a_3 h_3^2 = (66.7 \times 10^3) + [2000 \times (50.8)^2] = 5228 \times 10^3 \text{ mm}^4$$

Now moment of inertia of the whole section about *X-X* axis,

$$I_{XX} = (5786 \times 10^3) + (1836 \times 10^3) + (5228 \times 10^3) = 12\,850 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Example 7.12. *Find the moment of inertia about the centroidal X-X and Y-Y axes of the angle section shown in Fig. 7.16.*

Solution. First of all, let us find the centre of gravity of the section. As the section is not symmetrical about any section, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the angle section. Split up the section into two rectangles (1) and (2) as shown in Fig. 7.16.

Moment of inertia about centroidal X-X axis

Let bottom face of the angle section be the axis of reference.

Rectangle (1)

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

Rectangle (2)

$$a_2 = (80 - 20) \times 20 = 1200 \text{ mm}^2$$

and $$y_2 = \frac{20}{2} = 10 \text{ mm}$$

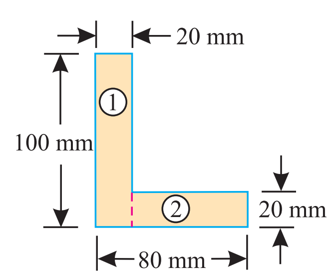

Fig. 7.16.

We know that distance between the centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35 \text{ mm}$$

We know that moment of inertia of rectangle (1) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G1} = \frac{20 \times (100)^3}{12} = 1.667 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangle (1) from *X-X* axis,

$$h_1 = 50 - 35 = 15 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about *X-X* axis

$$= I_{G1} + a h_1^2 = (1.667 \times 10^6) + [2000 \times (15)^2] = 2.117 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G2} = \frac{60 \times (20)^3}{12} \; 0.04 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangle (2) from *X-X* axis,

$$h_2 = 35 - 10 = 25 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about *X-X* axis

$$= I_{G2} + a h_2^2 = (0.04 \times 10^6) + [1200 \times (25)^2] = 0.79 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about *X-X* axis,

$$I_{XX} = (2.117 \times 10^6) + (0.79 \times 10^6) = 2.907 \times 10^6 \text{ mm}^4 \qquad \textbf{Ans.}$$

Moment of inertia about centroidal Y-Y axis

Let left face of the angle section be the axis of reference.

Rectangle (1)

$a_1 = 2000 \text{ mm}^2$ ...(As before)

and $x_1 = \dfrac{20}{2} = 10 \text{ mm}$

Rectangle (2)

$a_2 = 1200 \text{ mm}^2$ ...(As before)

and $x_2 = 20 + \dfrac{60}{2} = 50 \text{ mm}$

We know that distance between the centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \text{ mm}$$

We know that moment of inertia of rectangle (1) about an axis through its centre of gravity and parallel to *Y-Y* axis,

$$I_{G1} = \frac{100 \times (20)^3}{12} = 0.067 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangle (1) from *Y-Y* axis,

$$h_1 = 25 - 10 = 15 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about *Y-Y* axis

$$= I_{G1} + a_1 h_1^2 = (0.067 \times 10^6) + [2000 \times (15)^2] = 0.517 \times 10^6 \text{ mm}^4$$

Contents

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to *Y-Y* axis,

$$I_{G2} = \frac{20 \times (60)^3}{12} = 0.36 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangle (2) from *Y-Y* axis,

$$h_2 = 50 - 25 = 25 \text{ mm},$$

∴ Moment of inertia of rectangle (2) about *Y-Y* axis

$$= I_{G2} + a_2 h_2^2 = 0.36 \times 10^6 + [1200 \times (25)^2] = 1.11 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about *Y-Y* axis,

$$I_{YY} = (0.517 \times 10^6) + (1.11 \times 10^6) = 1.627 \times 10^6 \text{ mm}^4 \qquad \textbf{Ans.}$$

Example 7.13. *Figure 7.17 shows the cross-section of a cast iron beam.*

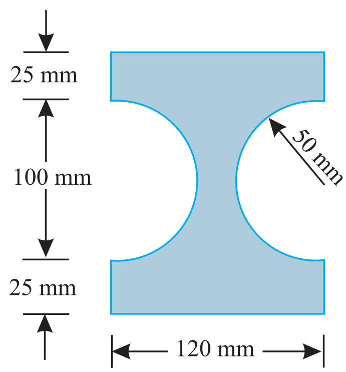

Fig. 7.17.

Determine the moments of inertia of the section about horizontal and vertical axes passing through the centroid of the section.

Solution. As the section is symmetrical about its horizontal and vertical axes, therefore centre of gravity of the section will lie at the centre of the rectangle. A little consideration will show that when the two semicircles are placed together, it will form a circular hole with 50 mm radius or 100 mm diameter.

Moment of inertia of the section about horizontal axis passing through the centroid of the section.

We know that moment of inertia of the rectangular section about its horizontal axis passing through its centre of gravity,

$$= \frac{bd^3}{12} = \frac{120 \times (150)^3}{12} = 33.75 \times 10^6 \text{ mm}^4$$

and moment of inertia of the circular section about a horizontal axis passing through its centre of gravity,

$$= \frac{\pi}{4}(r)^4 = \frac{\pi}{4}(50)^4 = 4.91 \times 10^6 \text{ mm}^4$$

∴ Moment of inertia of the whole section about horizontal axis passing through the centroid of the section,

$$I_{XX} = (33.75 \times 10^6) - (4.91 \times 10^6) = 28.84 \times 10^6 \text{ mm}^4 \qquad \textbf{Ans.}$$

Contents

Moment of inertia of the section about vertical axis passing through the centroid of the section

We know that moment of inertia of the rectangular section about the vertical axis passing through its centre of gravity,

$$I_{G1} = \frac{db^3}{12} = \frac{150 \times (120)^3}{12} = 21.6 \times 10^6 \text{ mm}^4 \quad ...(i)$$

and area of one semicircular section with 50 mm radius,

$$a = \frac{\pi r^2}{2} = \frac{\pi (50)^2}{2} = 3927 \text{ mm}^2$$

We also know that moment of inertia of a semicircular section about a vertical axis passing through its centre of gravity,

$$I_{G2} = 0.11\ r^4 = 0.11 \times (50)^4 = 687.5 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of the semicircular section and its base

$$= \frac{4r}{3\pi} = \frac{4 \times 50}{3\pi} = 21.2 \text{ mm}$$

∴ Distance between centre of gravity of the semicircular section and centre of gravity of the whole section,

$$h_2 = 60 - 21.2 = 38.8 \text{ mm}$$

and moment of inertia of one semicircular section about centre of gravity of the whole section,

$$= I_{G2} + a_2 h_2^2 = (687.5 \times 10^3) + [3927 \times (38.8)^2] = 6.6 \times 10^6 \text{ mm}^4$$

∴ Moment of inertia of both the semicircular sections about centre of gravity of the whole section,

$$= 2 \times (6.6 \times 10^6) = 13.2 \times 10^6 \text{ mm}^4 \quad ...(ii)$$

and moment of inertia of the whole section about a vertical axis passing through the centroid of the section,

$$= (21.6 \times 10^6) - (13.2 \times 10^6) = 8.4 \times 10^6 \text{ mm}^4$$ **Ans.**

Example 7.14. *Find the moment of inertia of a hollow section shown in Fig. 7.18. about an axis passing through its centre of gravity or parallel X-X axis.*

Solution. As the section is symmentrical about *Y-Y* axis, therefore centre of a gravity of the section will lie on this axis. Let $\bar{y}$ be the distance between centre of gravity of the section from the bottom face.

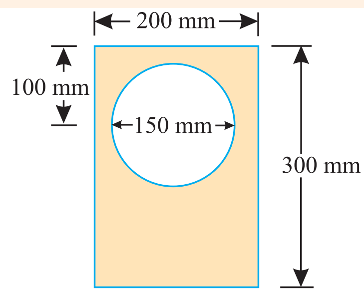

Fig. 7.18.

(*i*) *Rectangle*

$$a_1 = 300 \times 200 = 60\ 000 \text{ mm}^2$$

and $$y_1 = \frac{300}{2} = 150 \text{ mm}$$

(*ii*) *Circular hole*

$$a_2 = \frac{\pi}{4} \times (150)^2 = 17\ 670 \text{ mm}^2$$

and $$y_2 = 300 - 100 = 200 \text{ mm}$$

We know that distance between the centre of gravity of the section and its bottom face,

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{(60000 \times 150) - (17670 \times 200)}{60000 - 17670} = 129.1 \text{ mm}$$

∴ Moment of inertia of rectangular section about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G1} = \frac{200 \times (300)^3}{12} = 450 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangular section and *X-X* axis,

$$h_1 = 150 - 129.1 = 20.9 \text{ mm}$$

∴ Moment of inertia of rectangle about *X-X* axis

$$= I_{G1} + ah^2 = (450 \times 10^6) + [(300 \times 200) \times (20.9)]^2 = 476.21 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of circular section about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G2} = \frac{\pi}{64} \times (150)^4 = 24.85 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of the circular section and *X-X* axis,

$$h_2 = 200 - 129.1 = 70.9 \text{ mm}$$

∴ Moment of inertia of the circular section about *X-X* axis,

$$= I_{G2} + ah^2 = (24.85 \times 10^6) + [(17\ 670) \times (70.9)^2] = 113.67 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about *X-X* axis

$$= (476.21 \times 10^6) - (113.67 \times 10^6) = 362.54 \times 10^6 \text{ mm}^4 \quad \textbf{Ans.}$$

Example 7.15. *A rectangular hole is made in a triangular section as shown in Fig. 7.19.*

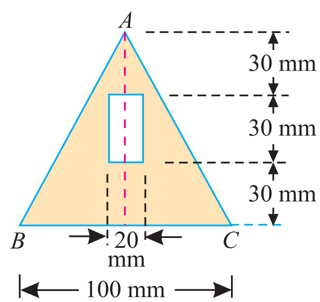

Fig. 7.19.

Determine the moment of inertia of the section about X-X axis passing through its centre of gravity and the base BC.

Solution. As the section is symmetrical about *Y-Y* axis, therefore centre of gravity of the section will lie on this axis. Let $\bar{y}$ be the distance between the centre of gravity of the section and the base *BC*.

(*i*) *Triangular section*

$$a_1 = \frac{100 \times 90}{2} = 4500 \text{ mm}^2$$

and $$y_1 = \frac{90}{3} = 30 \text{ mm}$$

(*ii*) *Rectangular hole*

$$a_2 = 30 \times 20 = 600 \text{ mm}^2$$

and $$y_2 = 30 + \frac{30}{2} = 45 \text{ mm}$$

We know that distance between the centre of gravity of the section and base *BC* of the triangle,

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{(4500 \times 30) - (600 \times 45)}{4500 - 600} = 27.7 \text{ mm}$$

Moment of inertia of the section about X-X axis.

We also know that moment of inertia of the triangular section through its centre of gravity and parallel to X-X axis,

$$I_{G1} = \frac{b\,d^3}{36} = \frac{100 \times (90)^3}{36} = 2025 \times 10^3 \text{ mm}^4$$

and distance between the centre of gravity of the section and X-X axis,

$$h_1 = 30 - 27.7 = 2.3 \text{ mm}$$

∴ Moment of inertia of the triangular section about X-X axis

$$= I_{G1} + a_2\, h_1^2 = 2025 \times 10^3 + [4500 \times (2.3)^2] = 2048.8 \times 10^3 \text{ mm}^4$$

Similarly moment of inertia of the rectangular hole through its centre of gravity and parallel to the X-X axis

$$I_{G2} = \frac{b\,d^3}{12} = \frac{20 \times (30)^3}{12} = 45 \times 10^3 \text{ mm}^4$$

and distance between the centre of gravity of the section and X-X axis

$$h_2 = 45 - 27.7 = 17.3 \text{ mm}$$

∴ Moment of inertia of rectangular section about X-X axis

$$= I_{G2} + a_2\, h_2^2 = (45 \times 10^3) + [600 \times (17.3)^2] = 224.6 \times 10^3 \text{ mm}^4$$

Now moment of inertia of the whole section about X-X axis.

$$I_{xx} = (2048.8 \times 10^3) - (224.6 \times 10^3) = 1824.2 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Moment of inertia of the section about the base BC

We know that moment of inertia of the triangular section about the base BC

$$I_{G1} = \frac{b\,d^3}{12} = \frac{100 \times (90)^3}{12} = 6075 \times 10^3 \text{ mm}^4$$

Similarly moment of inertia of the rectangular hole through its centre of gravity and parallel to X-X axis,

$$I_{G2} = \frac{b\,d^3}{12} = \frac{20 \times (30)^3}{12} = 45 \times 10^3 \text{ mm}^4$$

and distance between the centre of gravity of the section about the base BC,

$$h_2 = 30 + \frac{30}{2} = 45 \text{ mm}$$

∴ Moment of inertia of rectangular section about the base BC,

$$= I_{G2} + a_2\, h_2^2 = (45 \times 10^3) + [600 \times (45)^2] = 1260 \times 10^3 \text{ mm}^4$$

Now moment of inertia of the whole section about the base BC,

$$I_{BC} = (6075 \times 10^3) - (1260 \times 10^3) = 4815 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

7.16. MOMENT OF INERTIA OF A BUILT-UP SECTION

A built-up section consists of a number of sections such as rectangular sections, channel sections, I-sections etc., A built-up section is generally made by symmetrically placing and then fixing these section by welding or riveting. It will be interesting to know that a built-up section behaves as one unit. The moment of inertia of such a section is found out by the following steps.

1. Find out the moment of inertia of the various sections about their respective centres of gravity as usual.
2. Now transfer these moments of inertia about the required axis (say *X-X* axis or *Y-Y* axis) by the Theorem of Parallel Axis.

Note. In most of the standard sections, their moments of inertia of about their respective centres of gravity is generally given. However, if it is not given then we have to calculate it before transferring it to the required axis.

Example 7.16. *A compound beam is made by welding two steel plates 160 mm × 12 mm one on each flange of an ISLB 300 section as shown in Fig 7.20.*

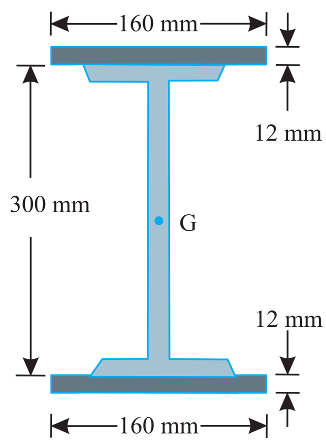

Fig. 7.20.

Find the moment of inertia the beam section about an axis passing through its centre of gravity and parallel to X-X axis. Take moment of inertia of the ISLB 300 section about X-X axis as 73.329×10^6 mm^4.

Solution. Given: Size of two steel plates = 160 mm × 12 mm and moment of inertia of ISLB 300 section about *X-X* axis = 73.329

From the geometry of the compound section, we find that it is symmetrical about both the *X-X* and *Y-Y* axes. Therefore centre of gravity of the section will lie at *G i.e.* centre of gravity of the beam section.

We know that moment of inertia of one steel plate section about an axis passing through its centre of gravity and parallel to *X-X* axis.

$$I_G = \frac{160 \times (12)^3}{12} = 0.023 \times 10^6 \text{ mm}^4$$

and distance between the centre of gravity of the plate section and *X-X* axis,

$$h = 150 + \frac{12}{2} = 156 \text{ mm}$$

∴ Moment of inertia of one plate section about *X-X* axis,

$$= I_G + a\,h^2 = (0.023 \times 10^6) + [(160 \times 12) \times (156)^2] = 46.748 \times 10^6 \text{ mm}^4$$

and moment of inertia of the compound beam section about *X-X* axis,

I_{XX} = Moment of inertia of ISLB section

+ Moment of inertia of two plate sections.

$= (73.329 \times 10^6) + 2\,(46.748 \times 10^6) = 166.825 \times 10^6 \text{ mm}^4$ **Ans.**

Example 7.17. *A compound section is built-up by welding two plates 200 mm × 15 mm on two steel beams ISJB 200 placed symmetrically side by side as shown in Fig. 7.21.*

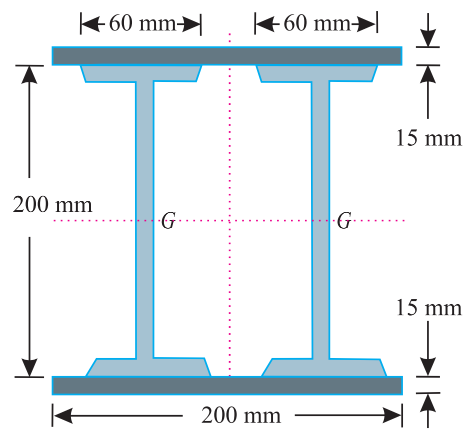

Fig. 7.21.

What is the moment of inertia of the compound section about an axis passing through its centre of gravity and parallel to X-X axis ? Take I_{XX} for the ISJB section as 7.807×10^6 mm^4.

Solution. Given: Size of two plates = 200 mm × 15 mm and moment of inertia of ISJB 200 section about *X-X* axis = 7.807×10^6 mm^4.

From the geometry of the compound section, we find that it is symmetrical about both the *X-X* and *Y-Y* axis. Therefore centre of gravity of the section will lie at *G i.e.*, centre of gravity of the beam sections.

We know that moment of inertia of one plate section about an axis passing through its centre of gravity and parallel to *X-X* axis,

$$I_G = \frac{200 \times (15)^3}{12} = 0.056 \times 10^6 \text{ mm}^4$$

and distance between the centre of gravity of the plate section and *X-X* axis,

$$h = 100 + \frac{15}{2} = 107.5 \text{ mm}$$

∴ Moment of inertia of the plate section about *x-x* axis

$= I_G + a\,h^2 = (0.056 \times 10^6) + (200 \times 15) \times (107.5)^2 = 34.725 \times 10^6 \text{ mm}^4$

and moment of inertia of the compound section about *x-x* axis,

I_{XX} = Moment of inertia of two ISJB sections

+ Moment of inertia of two plate sections

$= [2 \times (7.807 \times 10^6) + 2 \times (34.725 \times 10^6)] = 85.064 \times 10^6 \text{ mm}^4$ **Ans.**

Contents

Example 7.18. *A built up section is made by needing too stable and two channel sections as shown in Fig. 7.22.*

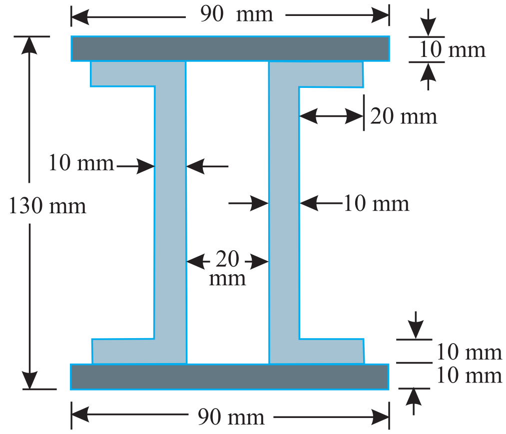

Fig. 7.22.

Determine moment of inertia of a built up section about X-X axis passing through centre of gravity of the section.

Solution. As the section is symmetrical about *X-X* axis and *Y-Y* axis therefore centre of gravity of the section will coincide with the geometrical centre of section.

We know that the moment of inertia of one top or bottom plate about an axis through its centre os gravity and parallel to *X-X* axis,

$$I_{G1} = \frac{90 \times (10)^3}{12} = 7500 \text{ mm}^4$$

and distance between centre of gravity of the plates from *X-X* axis,

$$h_1 = 65 - 5 = 60 \text{ mm}$$

∴ Moment of inertia of top and bottom plates about *X-X* axis,

$$= I_{G1} + a\,h^2 = 2\,[7500 + (90 \times 10) \times (60)^2] \text{ mm}^4$$

(because of two plates)

$$= 6.5 \times 10^6 \text{ mm}^4$$

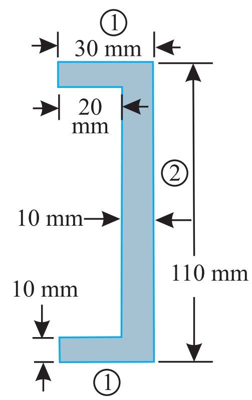

Fig. 7.23.

Now moment of inertia of part (1) of one channel section about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G2} = \frac{30 \times (10)^3}{12} = 2500 \text{ mm}^4$$

and distance of centre of gravity of this part from *X-X* axis,

$$h_2 = 55 - 5 = 50 \text{ mm}$$

∴ Moment of inertia of part (1) about *X-X* axis,

$$= I_{G2} + a\,h^2 = 4\,[2500 + (30 \times 10) \times (50)^2 \text{ mm}^4$$ ...(because of four plates)

$$= 3.0 \times 10^6 \text{ mm}^4$$

Similarly moment of inertia of part (2) of the channel about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{G3} = 2\left[\frac{10 \times (90)^3}{12}\right] = 0.6 \times 10^6 \text{ mm}^4$$ ...(because of two plates)

Contents

Now moment of inertia of the whole built-up section about an axis through its centre of gravity and parallel to *X-X* axis,

$$I_{XX} = (6.5 \times 10^6) + (3.0 \times 10^6) + (0.6 \times 10^6) = 10.1 \times 10^6 \text{ mm}^4 \qquad \textbf{Ans.}$$

EXERCISE 7.2

1. Find the moment of inertia of a *T*-section having flange and web both 120 mm × 30 mm about *X-X* axis passing through the centre of gravity of the section.

   [**Ans.** $14\,715 \times 10^3$ mm^4]

2. Calculate the moment of inertia of an I-section having equal flanges 30 mm × 10 mm and web also 30 mm × 10 mm about an axis passing through its centre of gravity and parallel to *X-X* and *Y-Y* axes. [**Ans.** 267.5×10^3 mm^4; 47×10^3 mm^4]

3. Find the moment of inertia of the lamina with a circular hole of 30 mm diameter about the axis *AB* as shown in Fig. 7.24. [**Ans.** 638.3×10^3 mm^4]

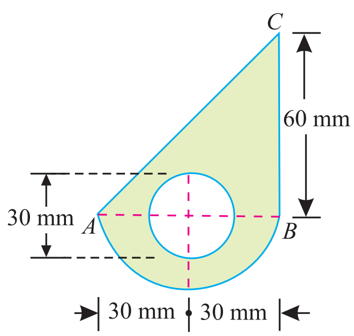

Fig. 7.24.

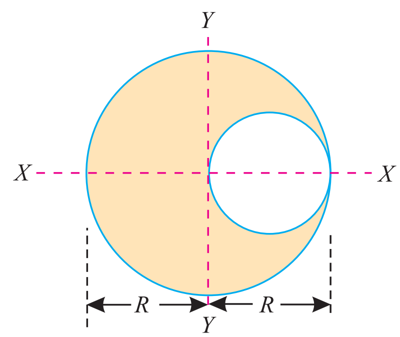

Fig. 7.25.

4. A circular hole of diameter *R* is punched out from a circular plate of radius *R* shown in Fig. 7.25. Find the moment of inertia about both the centroidal axes.

   $$\left[\textbf{Ans. } I_{XX} = \frac{15\pi R^4}{64};\ I_{YY} = \frac{29\pi R^4}{192}\right]$$

5. The cross-section of a beam is shown in Fig. 7.26. Find the moment of inertia of the section about the horizontal centroidal axis. [**Ans.** 1.354×10^6 mm^4]

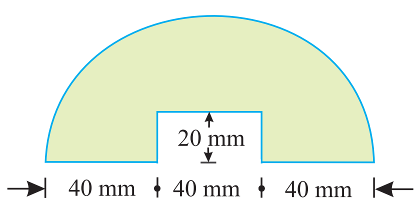

Fig. 7.26.

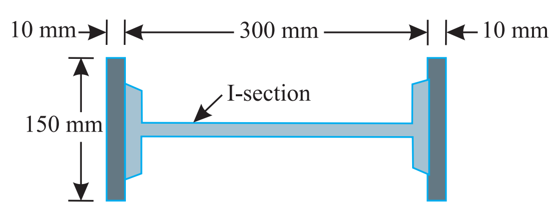

Fig. 7.27.

6. A built-up section consists of an I-section and two plates as shown in Fig 7.27. Find values of I_{XX} and I_{YY} of the section. Take values of I_{XX} as 3.762×10^6 mm^4 and I_{YY} as 73.329×10^6 mm^6 respectively for the I-section.

   [**Ans.** $I_{XX} = 17.095 \times 10^6$ mm^4 ; $I_{YY} = 169.46 \times 10^6$ mm^4]

Contents

QUESTIONS

1. How would you find out the moment of inertia of a plane area ?
2. What is Routh's rule for finding out the moment of inertia of an area ? Explain where it is used and why ?
3. Derive an equation for moment of inertia of the following sections about centroidal axis:

   (*a*) a rectangular section,

   (*b*) a hollow rectangular section,

   (*c*) a circular section, and

   (*d*) a hollow circular section.
4. State and prove the theorem of perpendicular axis applied to moment of inertia.
5. Prove the parallel axis theorem in the determination of moment of inertia of areas with the help of a neat sketch.
6. Describe the method of finding out the moment of inertia of a composite section.

OBJECTIVE TYPE QUESTIONS

1. If the area of a section is in mm^2 and the distance of the centre of area from a lines is in mm, then units of the moment of inertia of the section about the line is expressed in

   (*a*) mm^2 (*b*) mm^3 (*c*) mm^4 (*d*) mm^5
2. Theorem of perpendicular axis is used in obtaining the moment of inertia of a

   (*a*) triangular lamina (*b*) square lamina

   (*c*) circular lamina (*d*) semicircular lamina
3. The moment of inertia of a circular section of diameter (*d*) is given by the relation

   (*a*) $\frac{\pi}{16}(d)^4$ (*b*) $\frac{\pi}{32}(d)^4$ (*c*) $\frac{\pi}{64}(d)^4$ (*d*) $\frac{\pi}{96}(d)^4$
4. The moment of inertia of a triangular section of base (*b*) and height (*h*) about an axis through its *c.g.* and parallel to the base is given by the relation.

   (*a*) $\frac{bh^3}{12}$ (*b*) $\frac{bh^3}{24}$ (*c*) $\frac{bh^3}{36}$ (*d*) $\frac{bh^3}{48}$
5. The moment of inertia of a triangular section of base (*b*) and height (*h*) about an axis passing through its vertex and parallel to the base is ... as that passing through its C.G. and parallel to the base.

   (*a*) twelve times (*b*) nine times

   (*c*) six times (*d*) four times

ANSWERS

1. (*c*) **2.** (*b*) **3.** (*c*) **4.** (*c*) **5.** (*b*)

Top