Analysis of Perfect Frames (Analytical Method)

CHAPTER

13

Contents

13.1. INTRODUCTION

A frame may be defined as a structure, made up of several bars, riveted or welded together. these are made up of angle irons or channel sections, and are called members of the frame or framed structure. though these members are welded or riveted together, at their joints, yet for calculation purposes, the joints are assumed to be hinged or pin-jointed. the determination of force in a frame is an important problem in engineering- science, which can be solved by the application of the principles of either statics or graphics. in this chapter, we shall be using the principles of statics for determining the forces in frames.

13.2. TYPES OF FRAMES

Though there are many types of frames, yet from the analysis point of view, the frames may be classified into the following two groups:

1. Perfect frame. 2. Imperfect frame.

13.3. PERFECT FRAME

A perfect frame is that, which is made up of members just sufficient to keep it in equilibrium, when loaded, without any change in its shape.

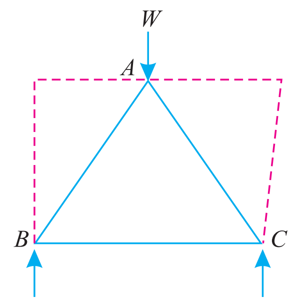


Fig. 13.1. Perfect Frame.

The simplest perfect frame is a triangle, which contains three members and three joints as shown in Fig. 13.1. It will be intersting to know that if such a structure is loaded, its shape will not be distorted. Thus, for three jointed frame, there should be three members to prevent any distortion. It will be further noticed that if we want to increase a joint, to a triangular frame, we require two members as shown by dotted lines in Fig. 13.1. Thus we see that for every additional joint, to a triangular frame, two members are required.

The no. of members, in a perfect frame, may also be expressed by the relation :

$$n = (2j - 3)$$
$$n = \text{No. of members, and}$$
$$j = \text{No. of joints.}$$

13.4. IMPERFECT FRAME

An imperfect frame is that which does not satisfy the equation :

$$n = (2j - 3)$$

Or in other words, it is a frame in which the no. of members are *more* or *less* than $(2j - 3)$. The imperfect frames may be further classified into the following two types :

1. Deficient frame. 2. Redundant frame.

13.5. DEFICIENT FRAME

A deficient frame is an imperfect frame, in which the no. of members are less than $(2j - 3)$.

13.6. REDUNDANT FRAME

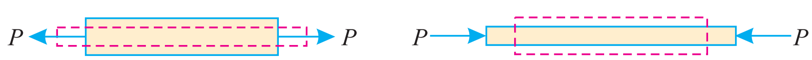
A redundant frame is an imperfect frame, in which the no. of members are more than $(2j - 3)$. In this chapter, we shall discuss only perfect frames.

13.7. STRESS

When a body is acted upon by a force, the internal force which is transmitted through the body is known as stress. Following two types of stress are important from the subject point of view :

1. Tensile stress. 2. Compressive stress.

13.8. TENSILE STRESS



(*a*) Tensile stress (*b*) Compressive stress

Fig. 13.2.

Sometimes, a body is pulled outwards by two equal and opposite forces and the body tends to extend, as shown in Fig 13.2. (*a*). The stress induced is called tensile stress and corresponding force is called tensile force.

13.9. COMPRESSIVE STRESS

Sometimes, a body is pushed inwards by two equal and opposite forces and the body tends to shorten its length as shown in Fig. 13.2 (*b*). The stress induced is called compressive stress and the corresponding force is called compressive force.

13.10. ASSUMPTIONS FOR FORCES IN THE MEMBERS OF A PERFECT FRAME

Following assumptions are made, while finding out the forces in the members of a perfect frame:

1. All the members are pin-jointed.
2. The frame is loaded only at the joints.
3. The frame is a perfect one.
4. The weight of the members, unless stated otherwise, is regarded as negligible in comparison with the other external forces or loads acting on the truss.

   The forces in the members of a perfect frame may be found out either by analytical method or graphical method. But in this chapter, we shall discuss the analytical method only.

13.11. ANALYTICAL METHODS FOR THE FORCES

The following two analytical methods for finding out the forces, in the members of a perfect frame, are important from the subject point of view :

1. Method of joints. 2. Method of sections.

13.12. METHOD OF JOINTS

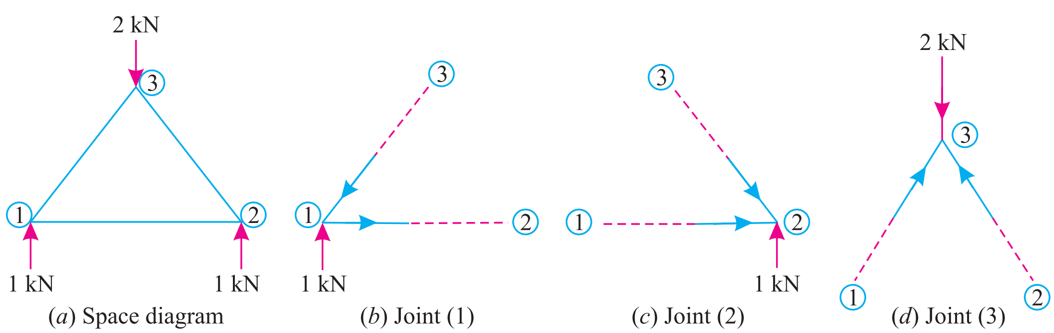


Fig. 13.3.

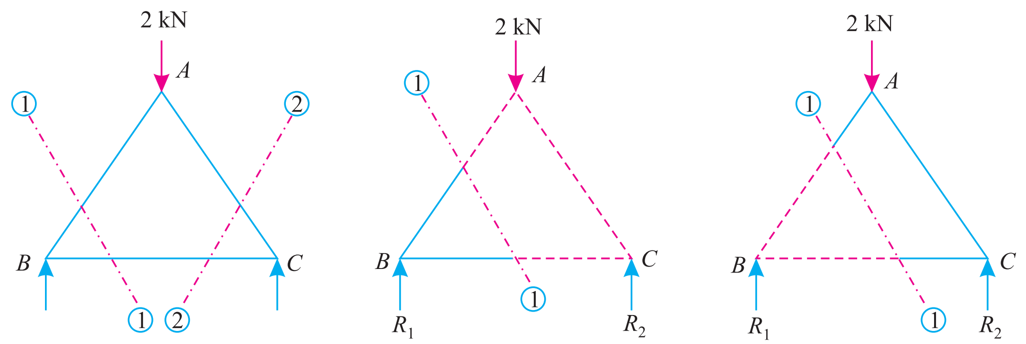
In this method, each and every joint is treated as a free body in equilibrium as shown in Fig. 13.3 (*a*), (*b*), (*c*) and (*d*). The unknown forces are then determined by equilibrium equations viz., $\Sigma V = 0$ and $\Sigma H = 0$. *i.e.,* Sum of all the vertical forces and horizontal forces is equated to zero.

Notes: 1. The members of the frame may be named either by Bow's methods or by the joints at their ends.

2. While selecting the joint, for calculation work, care should be taken that at any instant, the joint should not contain more than two members, in which the forces are unknown.

13.13. METHOD OF SECTIONS (OR METHOD OF MOMENTS)

This method is particularly convenient, when the forces in a few members of a frame are required to be found out. In this method, a section line is passed through the member or members, in which the forces are required to be found out as shown in Fig. 13.4 (*a*). A part of the structure, on any one side of the section line, is then treated as a free body in equilibrium under the action of external forces as shown in Fig. 13.4 (*b*) and (*c*).



(*a*) Space diagram (*b*) Left part (*c*) Right part

Fig. 13.4.

The unknown forces are then found out by the application of equilibrium or the principles of statics *i.e.*, $\Sigma M = 0$.

Notes:1. To start with, we have shown section line 1-1 cutting the members *AB* and *BC*. Now in order to find out the forces in the member *AC*, section line 2-2 may be drawn.

2. While drawing a section line, care should always be taken not to cut more than three members, in which the forces are unknown.

13.14. FORCE TABLE

Finally, the results are tabulated showing the members, magnitudes of forces and their nature. Sometimes, tensile force is represented with a + ve sign and compressive force with a – ve sign.

Note: The force table is generally prepared, when force in all the members of a truss are required to be found out.

Example 13.1. *The truss ABC shown in Fig. 13.5 has a span of 5 metres. It is carrying a load of 10 kN at its apex.*

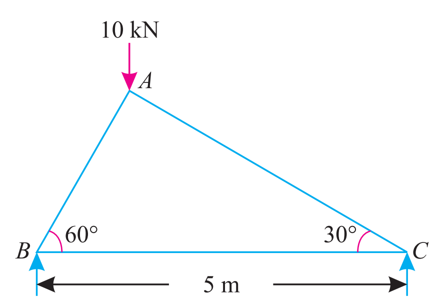


Fig. 13.5.

Find the forces in the members AB, AC and BC.

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Solution. From the geometry of the truss, we find that the load of 10 kN is acting at a distance 1.25 m from the left hand support *i.e.,* *B* and 3.75 m from *C*. Taking moments about *B* and equating the same,

$$R_C \times 5 = 10 \times 1.25 = 12.5$$

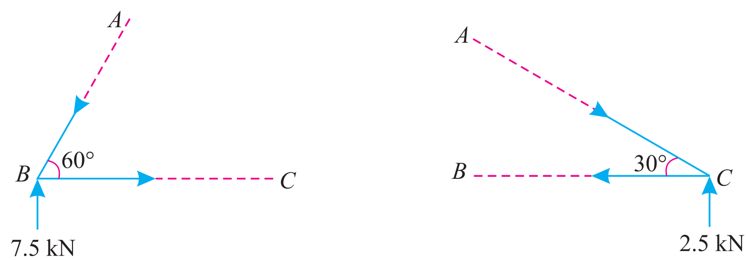
$$\therefore \qquad R_C = \frac{12.5}{5} = 2.5 \text{ kN}$$

and $$R_B = 10 - 2.5 = 7.5 \text{ kN}$$

The example may be solved by the method of joints or by the method of sections. But we shall solve it by both the methods.

Methods of Joints

First of all consider joint *B*. Let the *directions of the forces P_{AB} and P_{BC} (or P_{BA} and P_{CB}) be assumed as shown in Fig 13.6 (*a*).



(*a*) Joint *B* (*b*) Joint *C*

Fig. 13.6.

Resolving the forces vertically and equating the same,

$$P_{AB} \sin 60° = 7.5$$

or $$P_{AB} = \frac{7.5}{\sin 60°} = \frac{7.5}{0.866} = 8.66 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{BC} = P_{AB} \cos 60° = 8.66 \times 0.5 = 4.33 \text{ kN (Tension)}$$

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* The idea, of assuming the direction of the force P_{AB} to be downwards, is that the vertical component of the force P_{BC} is zero. Therefore in order to bring the joint *B* in equilibrium, the direction of the force P_{AB} must be downwards, or in other words, the direction of the force P_{AB} should be *opposite* to that of the reaction R_B. If, however the direction of the force P_{AB} is assumed to be upwards, then resolving the forces vertically and equating the same,

$$P_{AB} \sin 60° = -7.5 \text{ (Minus sign due to same direction of } R_B \text{ and } P_{AB}.)$$

$$\therefore \qquad P_{AB} = \frac{-7.5}{\sin 60°} = \frac{-7.5}{0.866} = -8.66 \text{ kN}$$

Minus sign means that the direction assumed is wrong. It should have been downwards instead of upwards. Similarly, the idea of assuming the direction of the force P_{BC} to be towards right is that the horizontal component of the reaction R_B is zero. Therefore in order to bring the joint *B* in equilibrium, the direction of the force P_{AB} must be towards right (because the direction of the horizontal component of the force P_{AB} is towards left).

Now consider the joint C. Let the *directions of the forces P_{AC} and P_{BC} (or P_{CA} and P_{CB}) be assumed as shown in Fig. 13.6 (*b*). Resolving the forces vertically and equating the same,

$$P_{AC} \sin 30° = 2.5$$

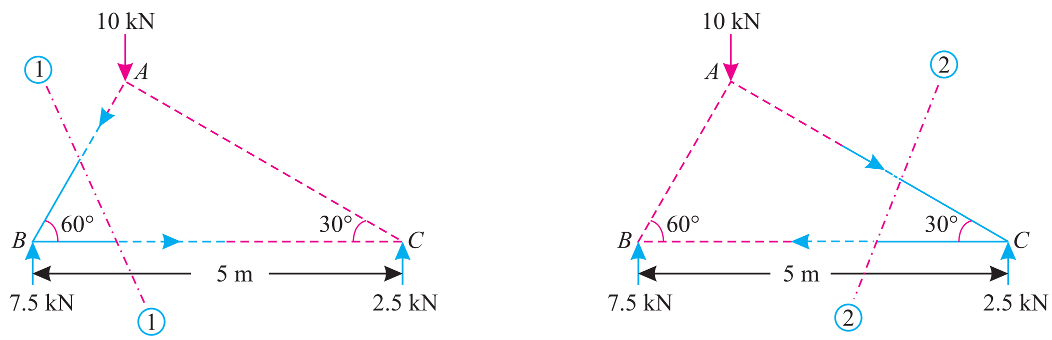
$$\therefore \qquad P_{AC} = \frac{2.5}{\sin 30°} = \frac{2.5}{0.5} = 5.0 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{BC} = P_{AC} \cos 30° = 5.0 \times 0.866 = 4.33 \text{ kN (Tension).}$$

...(As already obtained)

Method of Sections



(*a*) Section (1-1) (*b*) Section (2-2)

Fig. 13.7.

First of all, pass section (1-1) cutting the truss into two parts (one part shown by firm lines and the other by dotted lines) through the members AB and BC of the truss as shown in Fig 13.7 (*a*). Now consider equilibrium of the left part of the truss (because it is smaller than the right part). Let the directions of the forces P_{AB} and P_{AC} be assumed as shown in Fig 13.7 (*a*).

Taking** moments of the forces acting in the left part of the truss only about the joint C and equating the same,

$$P_{AB} \times 5 \sin 60° = 7.5 \times 5$$

$$\therefore \qquad P_{AB} = \frac{7.5 \times 5}{5 \sin 60°} = \frac{7.5}{0.866} = 8.66 \text{ kN (Compression)}$$

and now taking moments of the forces acting in the left part of the truss only about the joint A and equating the same,

$$P_{BC} \times 1.25 \tan 60° = 7.5 \times 1.25$$

$$\therefore \qquad P_{BC} = \frac{7.5 \times 1.25}{1.25 \tan 60°} = \frac{7.5}{1.732} = 4.33 \text{ kN (Tension)}$$

* For details, please refer to the foot note on last page.

** The moment of the force P_{AB} about the joint C may be obtained in any one of the following two ways :

1. The vertical distance between the member AB and the joint C (*i.e.*, AC in this case) is equal to 5 sin 60° m. Therefore moment about C is equal to $P_{AB} \times 5 \sin 60°$ kN-m.
2. Resolve the force P_{AB} vertically and horizontally at B. The moment of horizontal component about C will be zero. The moment of vertical component (which is equal to $P_{AB} \times \sin 60°$) is equal to $P_{AB} \times \sin 60° \times 5 = P_{AB} \times 5 \sin 60°$ kN-m.

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Now pass section (2-2) cutting the truss into two parts through the members AC and BC. Now consider the equilibrium of the right part of the truss (because it is smaller than the left part). Let the †direction of the forces P_{AC} and P_{BC} be assumed as shown in Fig 13.7 (*b*).

Taking moments of the force acting in the right part of the truss only about the joint B and equating the same,

$$P_{AC} \times 5 \sin 30° = 2.5 \times 5$$

$$\therefore \qquad P_{AC} = \frac{2.5}{\sin 30°} = \frac{2.5}{0.5} = 5 \text{ kN} \quad \text{(Compression)}$$

and now taking moments of the forces acting in the right part of the truss only about the joint A and equating the same,

$$P_{BC} \times 3.75 \tan 30° = 2.5 \times 3.75$$

$$\therefore \qquad P_{BC} = \frac{2.5 \times 3.75}{3.75 \tan 30°} = \frac{2.5}{0.577} = 4.33 \text{ kN} \quad \text{(Tension)}$$

...(As already obtained)

Now tabulate the results as given below :

S.No.	*Member*	*Magnitude of force in kN*	*Nature of force*
1	*AB*	8.66	Compression
2	*BC*	4.33	Tension
3	*AC*	5.0	Compression

Example 13.2. *Fig 13.8 shows a Warren girder consisting of seven members each of 3 m length freely supported at its end points.*

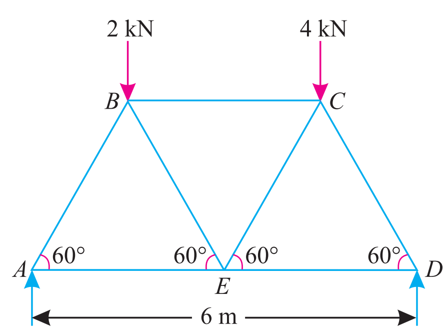


Fig. 13.8.

The girder is loaded at B and C as shown. Find the forces in all the members of the girder, indicating whether the force is compressive or tensile.

Solution. Taking moments about A and equating the same,

$$R_D \times 6 = (2 \times 1.5) + (4 \times 4.5) = 21$$

$$\therefore \qquad R_D = \frac{21}{6} = 3.5 \text{ kN}$$

and $\qquad R_A = (2 + 4) - 3.5 = 2.5 \text{ kN}$

† For details, please refer to the foot note on last page.

The example may be solved by the method of joints or method of sections. But we shall solve it by both the methods.

Method of Joints

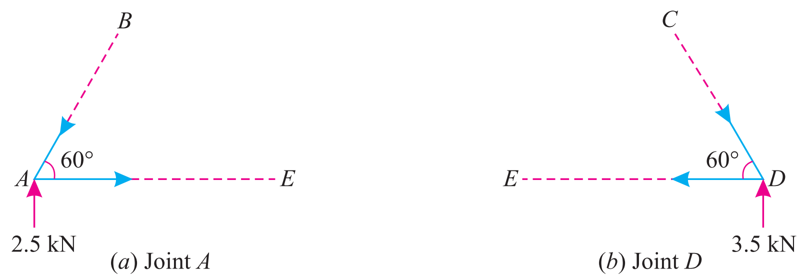


Fig. 13.9.

First of all, consider the joint A. Let the directions of P_{AB} and P_{AE} be assumed as shown in Fig. 13.9 (*a*) Resolving the forces vertically and equating the same,

$$P_{AB} \sin 60° = 2.5$$

$$\therefore \qquad P_{AB} = \frac{2.5}{\sin 60°} = \frac{2.5}{0.866} = 2.887 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{AE} = P_{AB} \cos 60° = 2.887 \times 0.5 = 1.444 \text{ kN (Tension)}$$

Now consider the joint D. Let the directions of the forces P_{CD} and P_{ED} be assumed as shown in Fig. 13.9 (*b*).

Resolving the forces vertically and equating the same,

$$P_{CD} \times \sin 60° = 3.5$$

$$\therefore \qquad P_{CD} = \frac{3.5}{\sin 60°} = \frac{3.5}{0.866} = 4.042 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{DE} = P_{CD} \cos 60° = 4.042 \times 0.5 = 2.021 \text{ kN (Tension)}$$

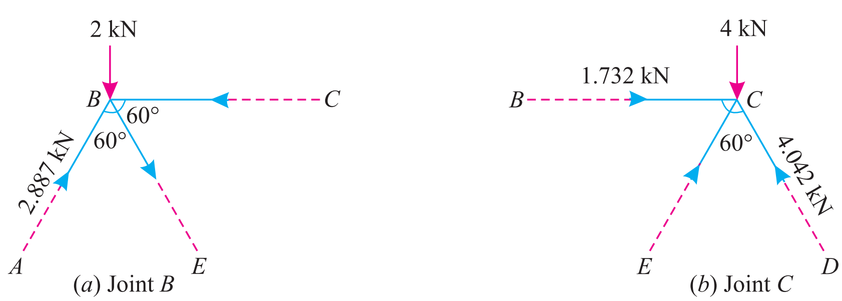


Fig. 13.10.

Now consider the joint B. We have already found that force in member AB *i.e.*, P_{AB} is 2.887 kN (Compression). Let the direction of the forces P_{BC} and P_{BE} be assumed as shown in Fig.13.10 (*a*).

Resolve the forces vertically and equating the same,

$$P_{BE} \sin 60° = P_{AB} \sin 60° - 2.0 = 2.887 \times 0.866 - 2.0 = 0.5 \text{ kN}$$

$$\therefore \quad P_{BE} = \frac{0.5}{\sin 60°} = \frac{0.5}{0.866} = 0.577 \text{ kN} \quad \text{(Tension)}$$

and now resolving the forces horizontally and equating the same,

$$P_{BC} = 2.887 \cos 60° + 0.577 \cos 60° \text{ kN}$$
$$= (2.887 \times 0.5) + (0.577 \times 0.5) \text{ kN} = 1.732 \text{ kN (Compression)}$$

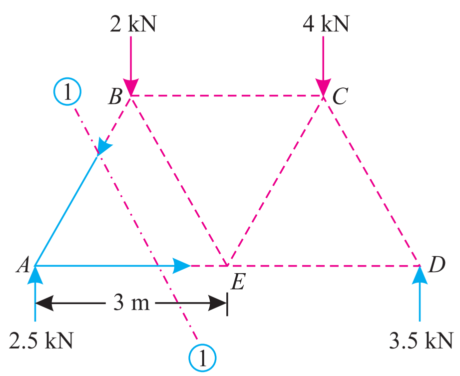
Now consider joint C. We have already found out that the forces in the members BC and CD (*i.e.* P_{BC} and P_{CD}) are 1.732 kN (Compression) and 4.042 kN (Compression) respectively. Let the directions of P_{CE} be assumed as shown in Fig. 13.10 (*b*). Resolving the forces vertically and equating the same,

$$P_{CE} \sin 60° = 4 - P_{CD} \sin 60° = 4 - (4.042 \times 0.866) = 0.5$$

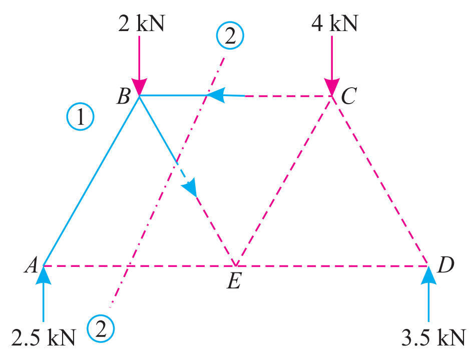
$$\therefore \quad P_{CE} = \frac{0.5}{\sin 60°} = \frac{0.5}{0.866} = 0.577 \text{ kN (Compression)}$$

Method of sections

First of all, pass section (1-1) cutting the truss through the members AB and AE. Now consider equilibrium of the left part of the truss. Let the directions of the forces P_{AB} and P_{AE} be assumed as shown in Fig. 13.11 (*a*).



(*a*) Section (1-1)



(*b*) Section (2-2)

Fig. 13.11.

Taking moments of the forces acting in the left part of the truss only, about the joint E and equating the same,

$$P_{AB} \times 3 \sin 60° = 2.5 \times 3$$

$$P_{AB} = \frac{2.5}{\sin 60°} = \frac{2.5}{0.866} = 2.887 \text{ kN (Compression)}$$

Now pass section (2-2) cutting the truss through the members BC, BE and AE. Now consider equilibrium of the left of the truss. Let the directions of the forces P_{BC} and P_{BE} be assumed as shown in Fig. 13.11 (*b*). Taking moments of the forces acting in left part of the truss only, about the joint E and equating the same,

$$P_{BC} \times 3 \sin 60° = (2.5 \times 3) - (2 \times 1.5) = 4.5$$

$$\therefore \quad P_{BC} = \frac{4.5}{3 \sin 60°} = \frac{4.5}{3 \times 0.866} = 1.732 \text{ kN (Compression)}$$

and now taking moments of the forces acting in the left part of the truss only about the joint A and equating the same,

$$P_{BE} \times 3 \sin 60° = (P_{BC} \times 3 \sin 60°) - (2 \times 1.5) = (1.732 \times 3 \times 0.866) - 3.0 = 1.5$$

$$P_{BE} = \frac{1.5}{3 \sin 60°} = \frac{1.5}{3 \times 0.866} = 0.577 \text{ kN (Tension)}$$

Now pass section (3-3) cutting the truss through the members BC, CE and ED. Now consider the equilibrium of the right part of the truss. Let the directions of the forces P_{CE} and P_{DE} be assumed as shown in Fig. 13.12 (*a*) Taking moments of the forces in the right part of the truss only, about the joint D and equating the same,

$$P_{CE} \times 3 \sin 60° = (4 \times 1.5) - (P_{BC} \times 3 \sin 60°)$$

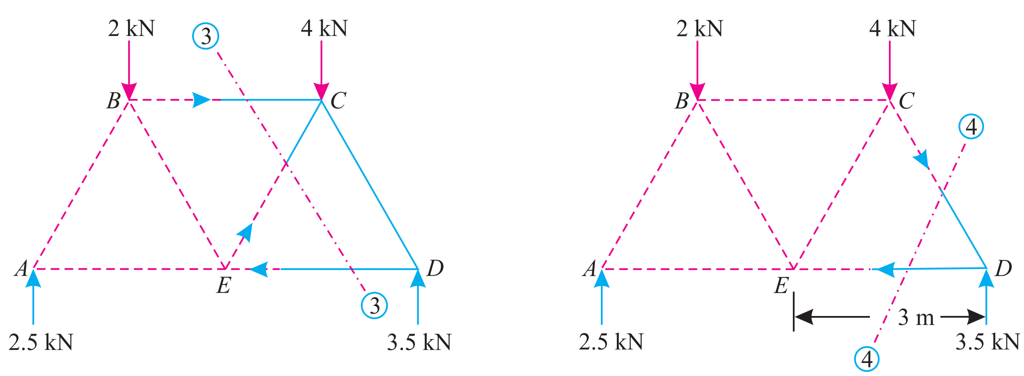
$$= 6.0 - (1.732 \times 3 \times 0.866) = 1.5$$

$$\therefore \quad P_{CE} = \frac{1.5}{3 \sin 60°} = \frac{1.5}{3 \times 0.866} = 0.577 \text{ kN (Compression)}$$

and now taking moments of the forces in the right part of the truss only about the joint C and equating the same,

$$P_{DE} \times 3 \sin 60° = 3.5 \times 1.5 = 5.25$$

$$\therefore \quad P_{DE} = \frac{5.25}{3 \sin 60°} = \frac{5.25}{3 \times 0.866} = 2.021 \text{ kN (Tension)}$$



(*a*) Section (3–3) (*b*) Section (4–4)

Fig. 13.12.

Now pass section (4-4) cutting the truss through the members CD and DE. Let the directions of the forces P_{CD} be assumed as shown in Fig 13.12 (*b*). Taking moments of the forces acting in the right part of the truss only about the joint E and equating the same,

$$P_{CD} \times 3 \sin 60° = 3.5 \times 3$$

$$P_{CD} = \frac{3.5}{\sin 60°} = \frac{3.5}{0.866} = 4.042 \text{ kN (Compression)}$$

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Now tabulate the results as given below :

S.No.	*Member*	*Magnitude of force in kN*	*Nature of force*
1	*AB*	2.887	Compression
2	*AE*	1.444	Tension
3	*CD*	4.042	Compression
4	*DE*	2.021	Tension
5	*BE*	0.577	Tension
6	*BC*	1.732	Compression
7	*CE*	0.577	Compression

Example 13.3. *A plane is loaded and supported as shown in Fig 13.13.*

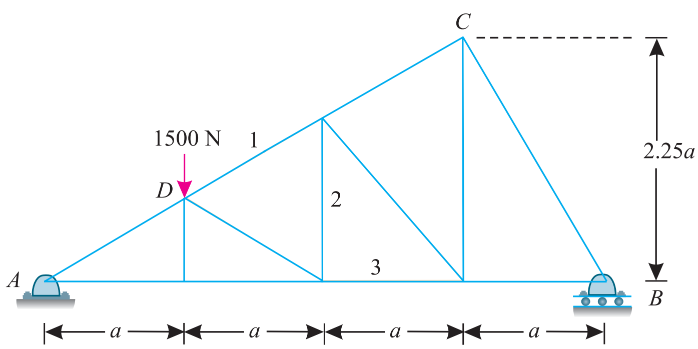

Fig. 13.13.

Determine the nature and magnitude of the forces in the members 1,2 and 3.

Solution. Taking moments about A and equating the same,

$$V_B \times 4\,a = 1500 \times a$$

$$\therefore \qquad V_B = \frac{1500}{4} = 375 \text{ N}$$

and $$V_A = 1500 - 375 = 1125 \text{ N}$$

From the geometry of the figure, we find that

$$\tan\,\theta = \frac{2.25\,a}{3\,a} = 0.75$$

and $$\sin\,\theta = \frac{3}{5} = 0.6 \text{ and } \cos\,\theta = \frac{4}{5} = 0.8$$

The example may be solved by any method. But we shall solve it by the method of sections, as one section line can cut the members 1, 2 and 3 in which the forces are required to be found out. Now let us pass section (1-1) cutting the truss into two parts as shown in Fig 13.14.

Now consider the equilibrium of the right part of the truss. Let the directions of P_1, P_2 and P_3 be assumed as shown in Fig. 13.14.

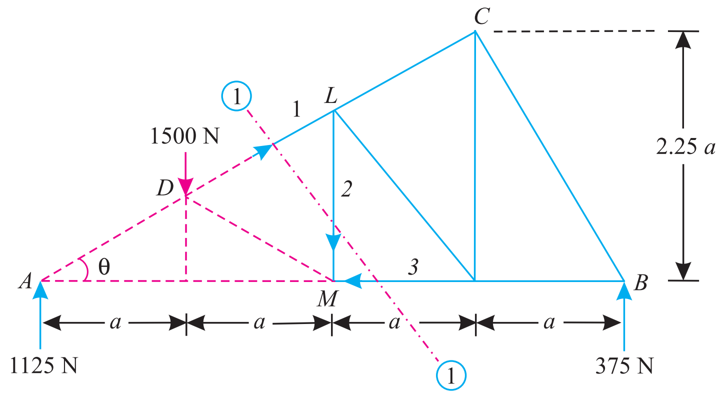

Fig. 13.14.

Taking moments about joint M and equating the same,

$$P_1 \times 2a \sin\theta = 375 \times 2a$$

$$\therefore \qquad P_1 = \frac{375}{\sin\theta} = \frac{375}{0.6} = 625 \text{ N (Compression)}$$

Similarly, taking moments about joint A and equating the same,

$$P_2 \times 2a = 375 \times 4a = 1500a$$

$$\therefore \qquad P_2 = \frac{1500a}{2a} = 750 \text{ N (Tension)}$$

and now taking moments about the joint L, and equating the same,

$$P_3 \times \frac{3a}{2} = 375 \times 2a = 750a$$

$$\therefore \qquad P_3 = \frac{750}{1.5} = 500 \text{ N (Tension)}$$

Example 13.4. *An inclined truss shown in Fig 13.15 is loaded as shown.*

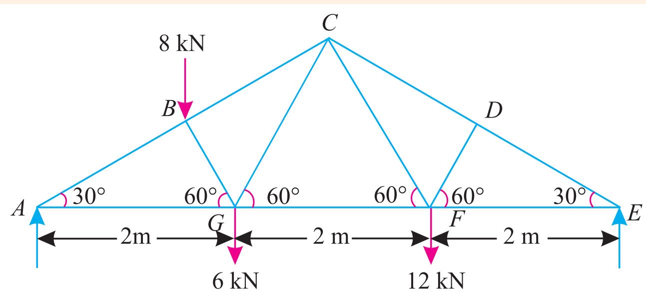

Fig. 13.15.

Determine the nature and magnitude of the forces in the members BC, GC and GF of the truss.

Solution. From the geometry of the figure, we find that the load 8 kN at B is acting at a distance of 1.5 m from the joint A. Taking moments about A and equating the same,

$$R_E \times 6 = (8 \times 1.5) + (6 \times 2) + (12 \times 4) = 72$$

$$\therefore \qquad R_E = \frac{72}{6} = 12 \text{ kN}$$

$$R_A = (8 + 6 + 12) - 12 = 14 \text{ kN}$$

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The example may be solved by any method. But we shall solve it by the method of sections, as one section line can cut the members *BC*, *GC*, and *GF* in which the forces are required to be found out. Now let us pass section (1-1) cutting the truss into two parts as shown in Fig. 13.16

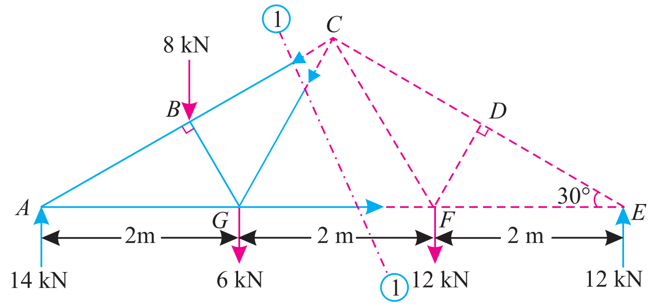

Fig. 13.16.

Now consider equilibrium of the left part of the truss. Let the directions of the force P_{BC}, P_{GC} and P_{GF} be assumed as shown in Fig 13.16. Taking moments about the joint *G* and equating the same,

$$P_{BC} \times 2 \sin 30° = (14 \times 2) - (8 \times 0.5) = 24$$

$$\therefore \qquad P_{BC} = \frac{24}{2 \sin 30°} = \frac{24}{2 \times 0.5} = 24 \text{ kN (Compression)}$$

Similarly, taking moments about the joint *B* and equating the same,

$$P_{GC} \times 1 \cos 30° = (14 \times 1.5) + (6 \times 0.5) = 24 \text{ kN}$$

$$P_{GC} = \frac{24}{\cos 30°} = \frac{24}{0.866} = 27.7 \text{ kN (Compression)}$$

and now taking moments about the joint *C* and equating the same,

$$P_{GF} \times 3 \tan 30° = (14 \times 3) - (6 \times 1) = 36$$

$$\therefore \qquad P_{GF} = \frac{36}{3 \tan 30°} = \frac{12}{0.5774} = 20.8 \text{ kN (Tension)}$$

Example 13.5. *A framed of 6 m span is carrying a central load of 10 kN as shown in Fig. 13.17.*

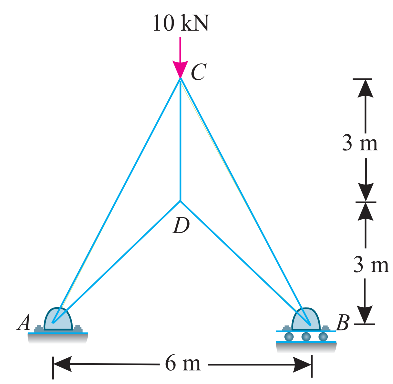

Fig. 13.17.

Find by any method, the magnitude and nature of forces in all members of the structure and tabulate the results.

Solution. Since the structure is symmetrical in geometry and loading, therefore reaction at A,

$$R_A = R_B = 5 \text{ kN}$$

From the geometry of the structure, shown in Fig. 13.18 (a). we find that

$$\tan\theta = \frac{3}{3} = 1.0 \quad \text{or} \quad \theta = 45°$$

$$\tan\alpha = \frac{6}{3} = 2.0 \quad \text{or} \quad \alpha = 63.4°$$

The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of joints only.

First of all, consider the joint A. Let the directions of the forces P_{AC} and P_{AD} be assumed as shown in Fig 13.18 (a). Resolving the forces horizontally and equating the same,

$$P_{AC}\cos 63.4° = P_{AD}\cos 45°$$

$$\therefore \quad P_{AC} = \frac{P_{AD}\cos 45°}{\cos 63.4°} = \frac{P_{AD}\times 0.707}{0.4477} = 1.58\,P_{AD}$$

and now resolving the forces vertically and equating the same,

$$P_{AC}\sin 63.4° = 5 + P_{AD}\sin 45°$$

$$1.58\,P_{AD}\times 0.8941 = 5 + P_{AD}\times 0.707 \qquad \text{...}(\because P_{AC} = 1.58\,P_{AD})$$

$$\therefore \quad 0.7056\,P_{AD} = 5$$

$$P_{AD} = \frac{5}{0.7056} = 7.08 \text{ kN (Tension)}$$

$$P_{AC} = 1.58\times P_{AD} = 1.58\times 7.08 = 11.19 \text{ kN (Compression)}$$

Now consider the joint D. Let the directions of the forces P_{CD} and P_{BD} be assumed as shown in Fig. 13.18 (b). Resolving the forces vertically and equating the same,

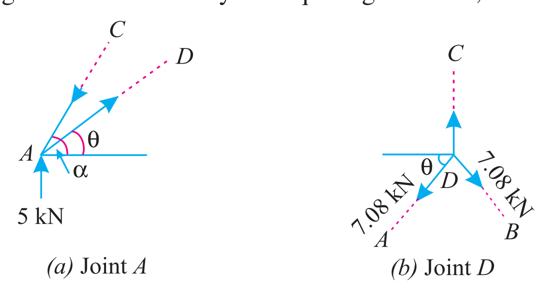

Fig. 13.18.

$$P_{CD} = P_{AD}\sin 45° + P_{BD}\sin 45° = 2\,P_{AD}\sin 45° \qquad \text{...}(\because P_{BD} = P_{AD})$$

$$= 2\times 7.08\times 0.707 = 10.0 \text{ kN (Tension)}$$

Now tabulate these results as given below :

S.No.	*Member*	*Magnitude of force in kN*	*Nature of force*
1	*AD, DB*	7.08	Tension
2	*AC, CB*	11.19	Compression
3	*CD*	10.0	Tension

EXERCISE 13.1

1. A truss of span 10 meters is loaded as shown in Fig. 13.19. Find the forces in all the members of the truss.

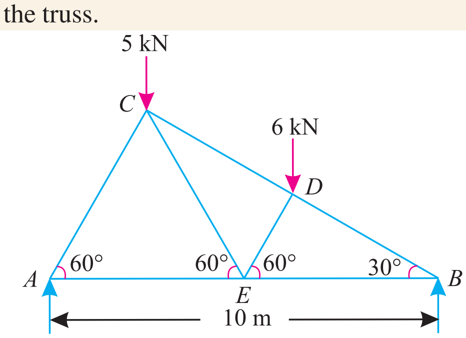

Fig. 13.19.

Ans. AC = 6.92 kN (Compression)
AE = 3.46 kN (Tension)
BD = 10.0 kN (Compression)
BE = 8.66 kN (Tension)
CD = 7.0 kN (Compression)
ED = 5.2 kN (Compression)
CE = 5.2 kN (Tension)

2. A king post truss of 8 m span is loaded as shown in Fig 13.20. Find the forces in each member of the truss and tabulate the results.

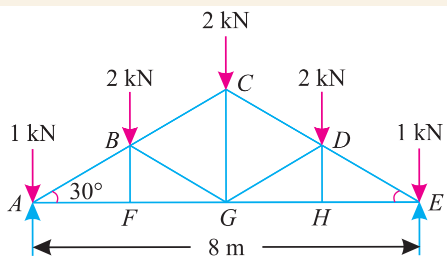

Fig. 13.20.

Ans. AC, DE =6.0 kN (Compression)
AF, EH = 5.2 kN (Tension)
FG, GH = 5.2 kN (Tension)
BF, DH = 0
BG, DG = 2.0 kN (Compression)
BC, CD = 4.0 kN (Compression)
CG = 2.0 kN (Tension)

3. A plane truss of 6 m span is subjected to a point load of 30 kN as shown in the figure 13.21. Find graphically, or otherwise, the forces in all the members of the truss and tabulate the results.

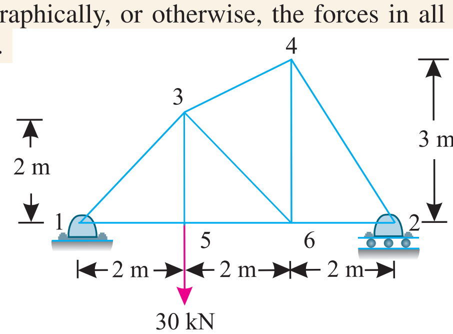

Fig. 13.21.

Ans. 1-3 = 28.3 kN (Compression)
1-5 = 20.0 kN (Tension)
2-4 = 12.0 kN (Compression)
2-6 = 6.7 kN (Tension)
1-5 = 20.0 kN (Tension)
3-5 = 30.0 kN (Tension)
3-6 = 18.8 kN (Compression)
4-6 = 13.3 kN (Tension)
3-4 = 7.5 kN (Compression)

4. A 9 m span truss is loaded as shown in Fig 13.22. Find the forces in the members BC, CH and HG of the truss.

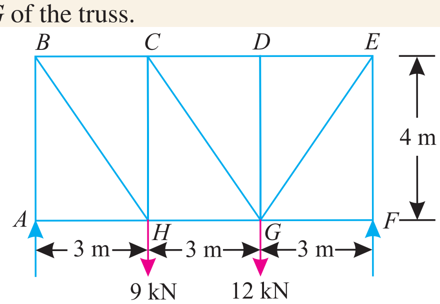

Fig. 13.22.

Ans. BC = 7.5 kN (Compression)
CH = 1.0 kN (Compression)
GH = 7.5 kN (Tension)

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5. The roof truss shown in Fig. 13.23 is supported at *A* and *B* and carries vertical loads at each of the upper chord points.

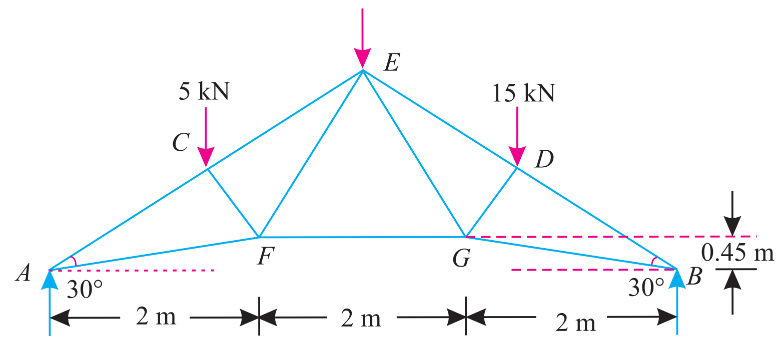


Fig. 13.23.

Using the method of sections, determine the forces in the members *CE* and *FG* of truss, stating whether they are in tension or compression.

[**Ans.** 38.5 kN (Compression); 24.2 kN (Tension)]

13.15. CANTILEVER TRUSSES

A truss, which is connected to a wall or a column at one end, and free at the other is known as a cantilever truss. In the previous examples, the determination of support reactions was absolutely essential to start the work. But in the case of cantilever trusses, determination of support reaction is not essential, as we can start the calculation work from the free end of the cantilever.

Example 13.6. *A cantilever truss of 3 m span is loaded as shown in Fig 13.24.*

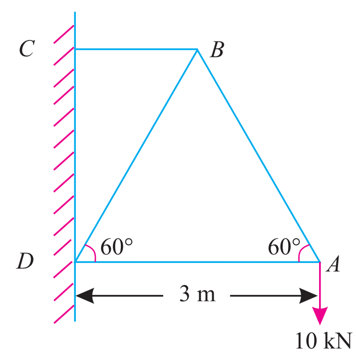


Fig. 13.24.

Find the forces in the various members of the framed truss, and tabulate the results.

Solution. The example may be solved either by the method of joints or method of sections. But we shall solve it by both the methods one by one.

Method of joints

First of all, consider the joint *A*, Let the directions of the forces P_{AB} and P_{AD} be assumed as shown Fig 13.25 (*a*).

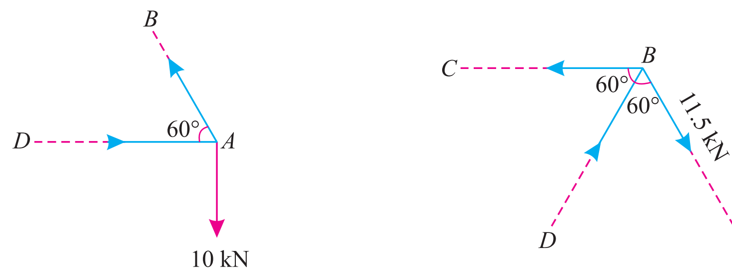
Resolving the forces vertically and equating the same,

$$P_{AB} \sin 60° = 10$$

$$\therefore \quad P_{AB} = \frac{10}{\sin 60°} = \frac{10}{0.866} = 11.5 \text{ kN (Tension)}$$

and now resolving the forces horizontally and equating the same,

$$P_{AD} = P_{AB} \cos 60° = 11.5 \times 0.5 = 5.75 \text{ kN (Compression)}$$



(*a*) Joint *A* (*b*) Joint *B*

Fig. 13.25.

Now consider the joint *B*. Let the directions of P_{BD} and P_{BC} be assumed as shown in Fig 13.25 (*b*). We have already found out that the force in member *AB* is 11.5 kN (Tension) as shown in the figure 13.25 (*b*). Resolving the forces vertically and equating the same,

$$P_{BD} \sin 60° = P_{AB} \sin 60° = 11.5 \sin 60°$$

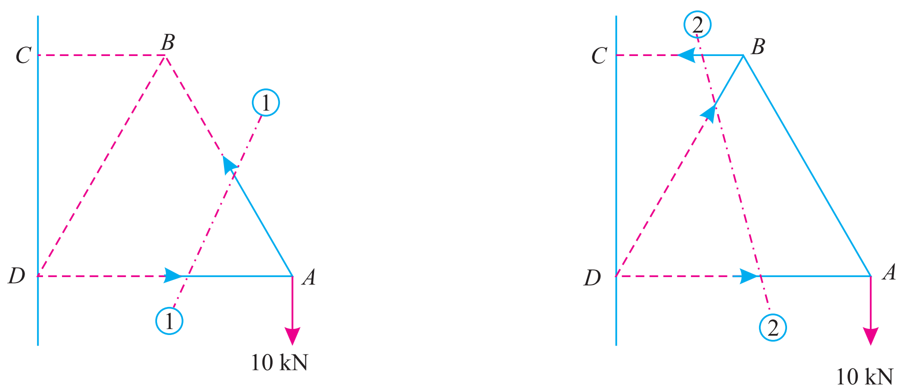
$$\therefore \quad P_{BD} = P_{AB} = 11.5 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{BC} = P_{AB} \cos 60° + P_{BD} \cos 60°$$
$$= (11.5 \times 0.5) + (11.5 \times 0.5) = 11.5 \text{ kN (Tension)}$$

Method of sections

First of all, pass section (1-1) cutting the truss through the members *AB* and *AD*. Now consider the equilibrium of the right part of the truss. Let the directions of the forces P_{AB} and P_{AD} be assumed as shown in Fig 13.26 (*a*).



(*a*) Section (1-1) (*b*) Section (2-2)

Fig. 13.26.

Taking moments of the forces acting on right part of the truss only, about the joint D and equating the same,

$$P_{AB} \times 3 \sin 60° = 10 \times 3$$

$$\therefore \qquad P_{AB} = \frac{10}{\sin 60°} = \frac{10}{0.866} = 11.5 \text{ kN (Tension)}$$

and now taking moments of the forces in the right part of the truss only about the joint B and equating the same,

$$P_{AD} \times 3 \sin 60° = 10 \times 1.5 = 15$$

$$\therefore \qquad P_{AD} = \frac{15}{3 \sin 60°} = \frac{15}{3 \times 0.866} = 5.75 \text{ kN (Compression)}$$

Now pass section (2-2) cutting the truss through the members BC, BD and AD. Now consider the equilibrium of the right part of the truss. Let the directions of the forces P_{BC} and P_{BD} be assumed as shown in Fig. 13.26 (*b*)

Taking moments of the forces acting on the right part of the truss only, about the joint D and equating the same,

$$P_{BC} \times 3 \sin 60° = 10 \times 3$$

$$\therefore \qquad P_{BC} = \frac{10}{\sin 60°} = \frac{10}{0.866} = 11.5 \text{ kN (Tension)}$$

and now taking moments of the forces in the right part of the truss only, about the joint C and equating the same,

$$P_{BD} \times 1.5 \sin 60° = (10 \times 3) - P_{AD} \times 3 \sin 60° = 30 - (5.75 \times 3 \times 0.866) = 15$$

$$P_{BD} = \frac{15}{1.5 \sin 60°} = \frac{15}{1.5 \times 0.866} = 11.5 \text{ kN (Compression)}$$

Now tabulate the results as given below :

S.No.	*Members*	*Magnitude of force in kN*	*Nature of force*
1	*AB*	11.5	Tension
2	*AD*	5.75	Compression
3	*BD*	11.5	Compression
4	*BC*	11.5	Tension

Example 13.7. *A cantilever truss is loaded as shown in Fig 13.27.*

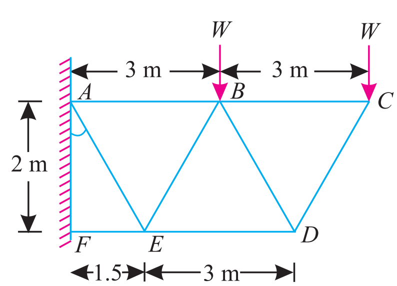


Fig. 13.27.

Find the value W, which would produce the force of magnitude 15 kN in the member AB.

Solution. The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of section only as we have to find out the force in member *AB* only.

First of all, let us find out the force in the member *AB* of the truss in terms of *W*. Now pass section (1-1) cutting the truss through the members *AB, BE* and *ED* as shown in Fig. 13.28.

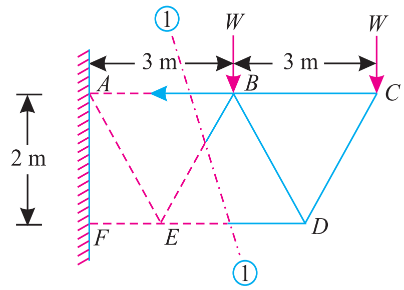


Fig. 13.28.

Now consider the equilibrium of the right part of the truss. Let the direction P_{AB} be assumed as shown in Fig 13.28. Taking moments of the forces in the right part of the truss only, about the joint *E* and equating the same,

$$P_{AB} \times 2 = (W \times 1.5) + (W \times 4.5) = 6\ W$$

$$P_{AB} = \frac{6W}{2} = 3W$$

Thus the value of *W*, which would produce the force of 15 kN in the member *AB*

$$= \frac{W}{3W} \times 15 = 5 \text{ kN} \qquad \textbf{Ans.}$$

Example 13.8. *Figure 13.29 shows a cantilever truss having a span of 4.5 meters. It is hinged at two joints to a wall and is loaded as shown.*

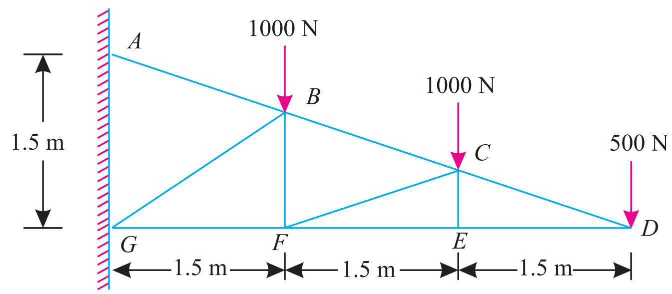
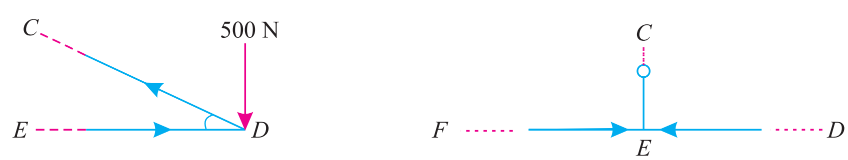


Fig. 13.29.

Find the forces in all the member of the truss.

Solution. The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of joints as we have to find out forces in all members of the truss.

Force in all the members of the truss



(*a*) Joint *D* (*b*) Joint *E*

Fig. 13.30.

First of all, consider the joint *D*. Let the directions of P_{CD} and P_{DE} be assumed as shown in Fig. 13.30 (*a*).

From the geometry of the figure, we find that

$$\tan \angle CDE = \frac{1.5}{4.5} = 0.3333 \qquad \text{or} \qquad \angle CDE = 18.4°$$

Resolving the forces vertically at D

$$P_{CD} \sin \angle CDE = 500 \qquad \text{or} \qquad P_{CD} \sin 18.4° = 500$$

$$\therefore \qquad P_{CD} = \frac{500}{\sin 18.4°} = \frac{500}{0.3156} = 1584 \text{ N (Tension)}$$

and now resolving the forces horizontally at D

$$P_{DE} = P_{CD} \cos \angle CDE = 1584 \cos 18.4°$$

$$\therefore \qquad P_{DE} = 1584 \times 0.9488 = 1503 \text{ N (Compression)}$$

Now consider the joint E. A little consideration will show that the value of the force P_{FE} will be equal to the force P_{ED} *i.e.,* 1503 N (Compression). Since the vertical components of the forces P_{FE} and P_{ED} are zero, therefore the value of the force P_{CE} will also be zero.

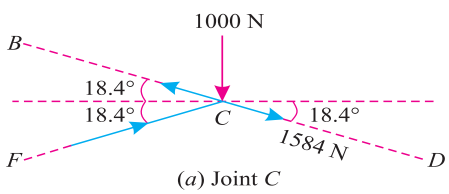


Fig. 13.31.

Now consider the joint C. Let the directions of P_{BC} and P_{FC} be assumed as shown in Fig. 13.31 (*a*). From the geometry of the figure, we find that the members CD, BC and FC make angle of 18.4° with the horizontal. Resolving the forces horizontally and equating the same,

$$P_{BC} \cos 18.4° = 1584 \cos 18.4° + P_{FC} \cos 18.4°$$

$$\text{or} \qquad P_{BC} = 1584 + P_{FC} \qquad \ldots(i)$$

and now resolving the forces vertically and equating the same,

$$1000 + 1584 \sin 18.4° = P_{FC} \sin 18.4° + P_{BC} \sin 18.4°$$

$$1000 + (1584 \times 0.3156) = (P_{FC} \times 0.3156) + (P_{BC} \times 0.3156)$$

$$1000 + (1581 \times 0.3156) = 0.3156\, P_{FC} + (1584 + P_{FC}) \times 0.3156$$

$$\ldots(\because P_{BC} = 1584 + P_{FC})$$

$$1000 + (1581 \times 0.3156) = 0.3156\, P_{FC} + (1584 \times 0.3156) + 0.3156\, P_{FC}$$

$$\therefore \qquad P_{FC} = \frac{1000}{0.6312} = 1584 \text{ N (Compression)}$$

Substituting the value of P_{FC} in equation (i)

$$P_{BC} = 1584 + 1584 = 3168 \text{ N (Tension)}$$

Now consider the joint F. Let the directions of the forces P_{GF} and P_{FB} be assumed as shown in Fig 13.31 (*b*). Resolving the forces horizontally,

$$P_{GF} = 1584 + 1584 \cos 18.4° = 1584 + (1584 \times 0.9488) \text{ N}$$

$$= 1584 + 1503 = 3087 \text{ N (Compression)}$$

and now resolving the forces vertically and equating the same,

$$P_{BF} = 1584 \sin 18.4° = 1584 \times 0.3156 = 500 \text{ N (Tension)}$$

Now consider the joint B. Let the direction of P_{BG} and P_{AB} be assumed as shown in Fig 13.32.

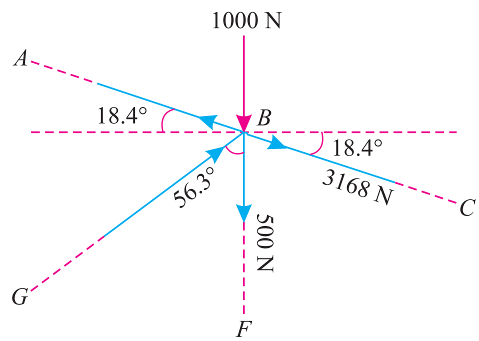


Fig. 13.32.

From the geometry of the figure, we find that

$$\tan \angle GBF = \frac{1.5}{1} = 1.5 \quad \text{or} \quad \angle GBF = 56.3°$$

Resolving the forces horizontally at B and equating the same,

$$P_{AB} \cos 18.4° = P_{BG} \sin 56.3° + 3168 \cos 18.4°$$

$$P_{AB} \times 0.9488 = P_{BG} \times 0.832 + 3168 \times 0.9488$$

$$\therefore \qquad 0.9488\, P_{AB} = 0.832\, P_{BG} + 3000 \qquad \text{....}(ii)$$

Dividing the above equation by 3,

$$0.3156\, P_{AB} = 0.2773\, P_{BG} + 1000 \qquad \text{....}(iii)$$

and now resolving the forces vertically at B and equating the same,

$$P_{AB} \sin 18.4° + P_{BG} \cos 56.3° = 1000 + 500 + 3168 \sin 18.4°$$

$$= 1500 + (3168 \times 0.3156)$$

$$P_{AB} \times 0.3156 + P_{BG} \times 0.5548 = 1500 + 1000$$

$$0.3156\, P_{AB} + 0.5548\, P_{BG} = 2500 \qquad \text{...}(iv)$$

Substracting equation (iii) from equation (iv),

$$0.8321\, P_{BG} = 1500$$

or

$$P_{BG} = \frac{1500}{0.8321} = 1801 \text{ N (Compression)}$$

Substituting the value of P_{BG} in equation (iii)

$$0.3156\, P_{AB} = (0.2773 \times 1801) + 1000$$

$$0.3156\, P_{AB} = 500 + 1000 = 500$$

$$P_{AB} = \frac{1500}{0.3156} = 4753 \text{ N (Tension)}$$

Now tabulate the results as given below :

S.No.	*Member*	*Magnitude of force in kN*	*Nature of force*
1	*AB*	4753	Tension
2	*BC*	3168	Tension
3	*CD*	1584	Tension
4	*DE*	1503	Compression
5	*CE*	0	—
6	*FE*	1503	Compression
7	*FC*	1584	Compression
8	*BF*	500	Tension
9	*GF*	3087	Compression
10	*BG*	1801	Compression

Example 13.9. *A truss shown in Fig 13.33 is carrying a point load of 5 kN at E.*

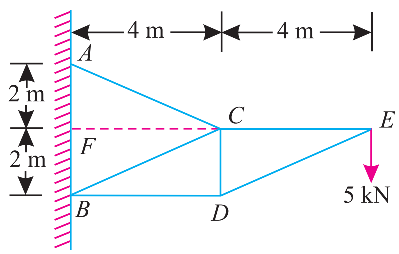


Fig. 13.33.

Find graphically, or otherwise, the force in the members CE, CD and BD of the truss.

Solution. The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of sections, as one section line can cut the members *CE*, *CD* and *BD* in which the forces are required to be found out. Now let us pass section (1-1) cutting truss into two parts as shown in Fig. 13.34.

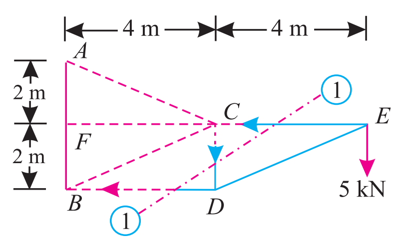


Fig. 13.34.

Now consider equilibrium of the right parts of the truss. Let the directions of the force P_{CE} P_{CD} and P_{BD} be assumed as shown in Fig. 13.34. Taking moments about the joint *D* and equating the same,

$$P_{CE} \times 2 = 5 \times 4 = 20$$

$$\therefore \qquad P_{CE} = \frac{20}{2} = 10 \text{ kN (Tension)}$$

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Similarly, taking moments about the joint B and equating the same,

$$P_{CD} \times 4 = (5 \times 8) - (P_{CE} \times 2) = 40 - (10 \times 2) = 20$$

$$\therefore \quad P_{CD} = \frac{20}{4} = 5 \text{ kN (Compession)}$$

and now taking moments about the joint C and equating the same,

$$P_{BD} \times 2 = 5 \times 4 = 20$$

$$\therefore \quad P_{BD} = \frac{20}{2} = 10 \text{ kN (Tension)}$$

Example 13.10. *A pin-joined cantilever frame is hinged to a vertical wall at A and E and is loaded as shown in Fig 13.35.*

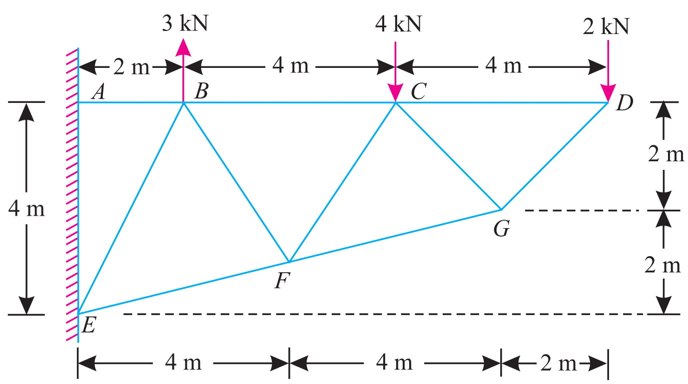

Fig. 13.35.

Determine the forces in the members CD, CG and FG.

Solution. First of all, extend the lines through the joints B, C and D as E, F and G meeting at O. Through G, draw GP perpendicular to CD. Similarly, through C, draw CQ perpendicular to FG.

Now extend the line of action of the member CG, and through O, draw a perpendicular to this line meeting at R as shown in Fig. 13.36.

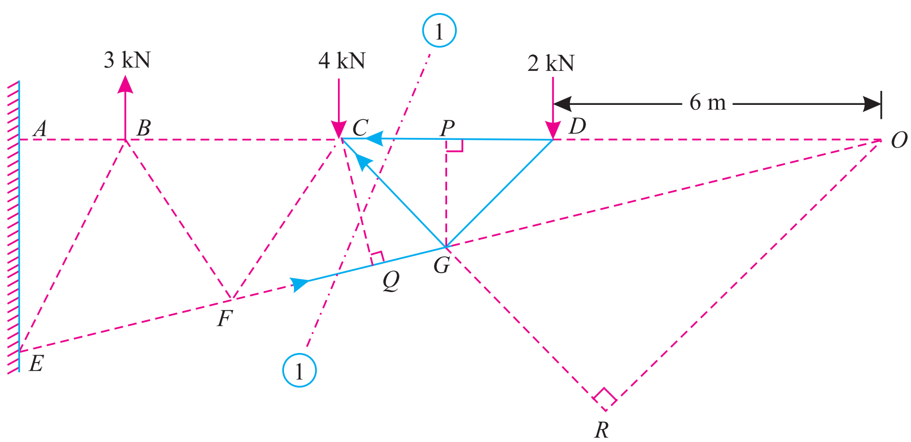

Fig. 13.36.

We know that in similar triangles *OPG* and *OAE*,

$$\frac{AO}{AE} = \frac{AP}{PG} \quad \text{or} \quad \frac{AO}{4} = \frac{8}{2} = 4$$

$\therefore$ $AO = 4 \times 4 = 16$ m

and $DO = 16 - 10 = 6$ m

Now in triangle *CGP*, we find that

$$\tan \angle GCP = \frac{2}{2} = 1 \quad \text{or} \quad \angle GCP = 45°$$

$\therefore$ $\angle COR = 90° - 45° = 45°$

and $OR = OC \cos 45° = 10 \times 0.707 \text{ m} = 7.07 \text{ m}$

From the geometry of the triangle *OPG*, we find that

$$\tan \angle GOP = \frac{2}{8} = 0.25 \quad \text{or} \quad \angle GOP = 14°$$

Similarly, in triangle *OCQ*, we find that

$$CQ = CO \sin 14° = 10 \times 0.2425 = 2.425 \text{ m}$$

Now pass section (1-1) cutting the frame through the members *CD*, *CG* and *FG*. Let the directions of the forces P_{CD}, P_{CG} and P_{FG} be assumed as shown in Fig. 13.36. Taking moments of the forces acting on right part of the frame only, about the joint *G* and equating the same,

$$P_{CD} \times 2 = 2 \times 2 \quad \text{or} \quad P_{CD} = 2 \text{ kN (Tension)} \qquad \textbf{Ans.}$$

Similarly, taking moments of the forces acting in the right part of the truss only about the imaginary joint *O* and equating the same,

$$P_{CG} \times 7.07 = 2 \times 6$$

or $$P_{CG} = \frac{12}{7.07} = 1.7 \text{ kN (Tension)} \qquad \textbf{Ans.}$$

and now taking moments of the forces acting in the right part of the truss only about the joint *C* and equating the same,

$$P_{FG} \times 2.425 = 2 \times 4 = 8$$

$\therefore$ $$P_{FG} = \frac{8}{2.425} = 3.3 \text{ kN (Compression)}$$

EXERCISE 13.2

1. Determine the forces in the various members of a pin-joined frame as shown in Fig. 13.37. Tabulate the result stating whether they are in tension or compression.

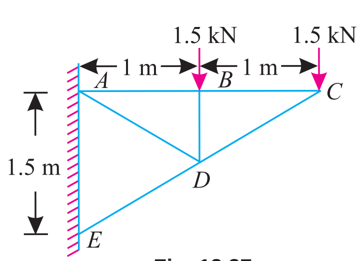

Fig. 13.37.

Ans. $CD = 2.5$ kN (Compression)
$BC = 2.0$ kN (Tension)
$AB = 2.0$ kN (Tension)
$BD = 1.5$ kN (Compression)
$AD = 1.25$ kN (Tension)
$ED = 3.75$ kN (Compression)

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2. A cantilever truss of 4 m span is carrying two point loads of 1.5 kN each as shown in Fig. 13.38 Find the stresses in the members BC and BD of the truss.

Ans. 2.52 kN (Tension) ; zero

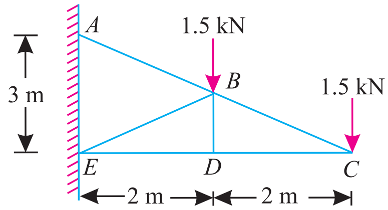


Fig. 13.38.

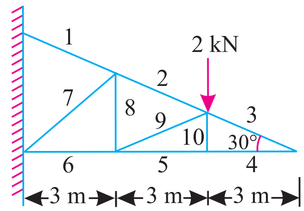


Fig. 13.39.

3. A cantilever truss carries two vertical load as shown in the Fig. 13.39. Find the magnitude and nature of strees in the members 2, 9, 5 and 10 of the truss.

Ans. $P_2 = 6.0$ kN (Tension)
$P_9 = 2.9$ kN (Compression)
$P_5 = 3.46$ kN (Compression)
$P_{10} = 0$

4. A cantilever truss is subjected to two point loads of 3 kN each at B and C as shown in Fig 13.40. Find by any method the forces in the members AB. BE and ED of the truss.

Ans. $AB = 8.6$ kN (Tension)
$BE = 2.0$ kN (Tension)
$ED = 2.0$ kN (Compression)

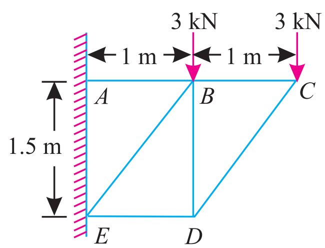


Fig. 13.40.

13.16. STRUCTURES WITH ONE END HINGED (OR PIN-JOINTED) AND THE OTHER FREELY SUPPORTED ON ROLLERS AND CARRYING HORIZONTAL LOADS

Sometimes, a structure is hinged or pin-jointed at one end, and freely supported on rollers at the other end. If such a truss carries vertical loads only, it does not present any special features. Such a structure may be solved just as a simply supported structure.

But, if such a structure carries horizontal loads (with or without vertical loads) the support reaction at the roller supported end will be normal to the support; where the support reaction at the hinged end will consist of :

1. Vertical reaction, which may be found out, by substracting the vertical support reaction at the roller supported end from the total vertical load.
2. Horizontal reaction, which may be found out, by algebraically adding all the horizontal loads.

After finding out the reactions, the forces in members of the frame may be found out as usual.

Example. 13.11. *Figure 13.41 shows a framed of 4 m span and 1.5 m height subjected to two point loads at B and D.*

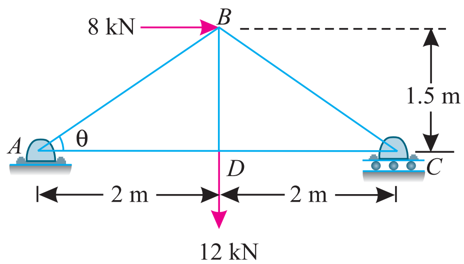


Fig. 13.41.

Find graphically, or otherwise, the forces in all the members of the structure.

Solution. Since the structure is supported on rollers at the right hand support (*C*), therefore the reaction at this support will be vertical (because of horizontal support). The reaction at the left hand support (*A*) will be the resultant of vertical and horizontal forces and inclined with the vertical.

Taking moments about *A* and equating the same,

$$V_C \times 4 = (8 \times 1.5) + (12 \times 2) = 36$$

$$V_C = \frac{36}{4} = 9 \text{ kN } (\uparrow)$$

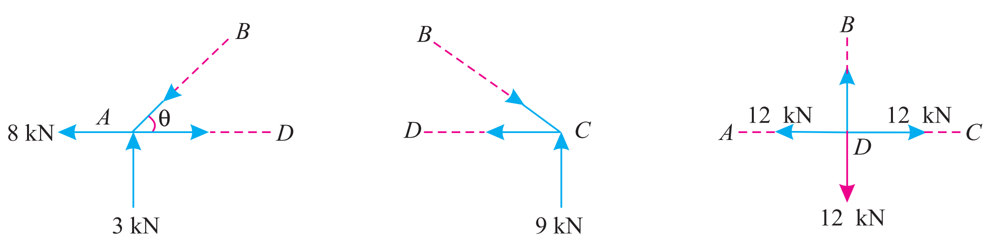
$$V_A = 12 - 9 = 3 \text{ kN } (\uparrow) \quad \text{and} \quad H_A = 8 \text{ kN } (\leftarrow)$$

From the geometry of the figure, we find that

$$\tan\theta = \frac{1.5}{2} = 0.75 \quad \text{or} \quad \theta = 36.9°$$

Similarly $\quad \sin\theta = \sin 36.9° = 0.6 \quad$ and $\quad \cos\theta = \cos 36.9° = 0.8$

The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of joints as we have to find forces in all the members of the structure.



(*a*) Joint *A* (*b*) Joint *C* (*c*) Joint *D*

Fig. 13.42.

First of all, consider joint *A*. Let directions of the forces P_{AB} and P_{AD} be assumed as shown in Fig. 13.42 (*a*). We have already found that a horizontal force of 8 kN is acting at *A* as shown in Fig. 13.42 (*a*).

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Resolving the forces vertically and equating the same,

$$P_{AB} \sin 36.9° = 3$$

$$\therefore \qquad P_{AB} = \frac{3}{\sin 36.9°} = \frac{3}{0.6} = 5.0 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{AD} = 8 + P_{AB} \cos 36.9° = 8 + (5 \times 0.8) = 12.0 \text{ kN (Tension)}$$

Now consider the joint C. Let the directions of the forces P_{BC} and P_{CD} be assumed as shown in Fig. 13.42 (*b*).

Resolving the forces vertically and equating the same,

$$P_{BC} \sin 36.9° = 9$$

$$P_{BC} = \frac{9}{\sin 36.9°} = \frac{9}{0.6} = 15 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{CD} = P_{BC} \cos 36.9° = 15 \times 0.8 = 12.0 \text{ kN (Tension)}$$

Now consider the joint D. A little consideration will show that the value of the force P_{BD} will be equal to the load 12 kN (Tension) as shown in Fig 13.42. (*c*). This will happen as the vertical components of the forces P_{AD} and P_{CD} will be zero.

Now tabulate the results as given below :

S.No.	*Member*	*Magnitude of force in kN*	*Nature of force*
1	*AB*	5.0	Compression
2	*AD*	12.0	Tension
3	*BC*	15.0	Compression
4	*CD*	12.0	Tension
5	*BD*	12.0	Tension

Example 13.12. *2 A truss of 8 metres span, is loaded as shown in Fig. 13.43.*

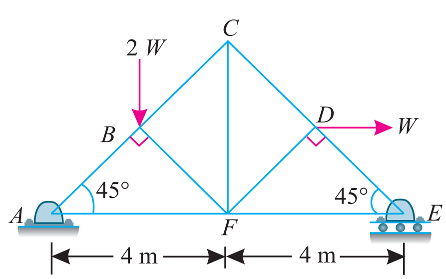

Fig. 13.43.

Find the forces in the members CD, FD and FE of the truss.

Solution. Since the truss is supported on rollers at the right hand support (*E*), therefore the reaction at this support will be vertical (because of horizontal support). The reaction at the left hand support (*A*) will be the resultant of vertical and horizontal forces and inclined with vertical.

Taking moments about *A* and equating same,

$$V_E \times 8 = (2\,W \times 2) + (W \times 2) = 6\,W$$

$$\therefore \qquad V_E = \frac{6W}{8} = 0.75W \;(\uparrow)$$

and $\quad {}^*V_A = 2\,W - 0.75\,W = 1.25\,W\,(\uparrow) \quad$ and $\quad H_A = W\,(\leftarrow)$

The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of sections, as one section line can cut the members *CD*, *FD* and *FE* in which the forces are required to be found out. Now let us pass section (1-1) cutting the truss into two parts as shown in Fig. 13.44.

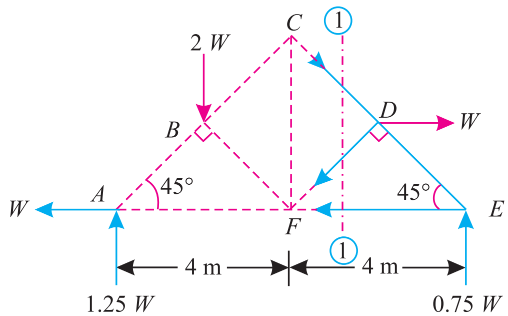

Fig. 13.44.

Now consider equilibrium of the right part of the truss. Let the directions of the forces P_{CD}, P_{FD} and P_{FE} be assumed as shown in Fig. 13.44. Taking moments about the joint *F* and equating the same,

$$P_{CD} \times 4 \sin 45° = (0.75\,W \times 4) - (W \times 2) = W$$

$$\therefore \qquad P_{CD} = \frac{W}{4 \sin 45°} = \frac{W}{4 \times 0.707} = 0.354\,W \text{ (Compression)}$$

Similarly, taking moments about the joint *E* and equating the same,

$$P_{FD} \times 4 \cos 45° = W \times 2 = 2\,W$$

$$\therefore \qquad P_{FD} = \frac{2W}{4 \cos 45°} = \frac{2W}{4 \times 0.707} = 0.707\,W \text{ (Tension)}$$

and now taking moments about the joint *D* and equating the same,

$$P_{FE} \times 2 = 0.75\,W \times 2 = 1.5\,W$$

$$\therefore \qquad P_{FE} = \frac{1.5W}{2} = 0.75\,W \text{ (Tension)}$$

* There is no need of finding out the vertical and horizontal reaction at A, as we are not considering this part of the truss.

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Example 13.13. *Figure 13.45 shows a pin-jointed frame carrying a vertical load at B and a horizontal load at D*

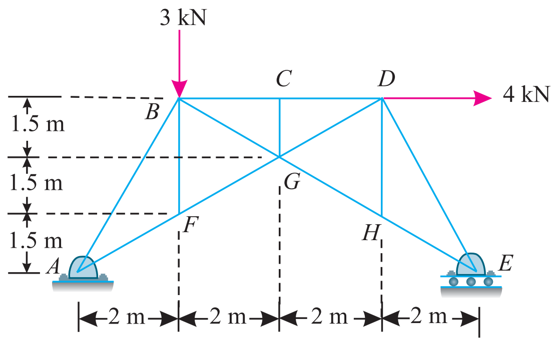


Fig. 13.45.

Find the forces in the members DF, HE and DH of the frame.

Solution. Since the frame is supported on rollers at the right hand support (E), therefore the reaction at this support will be vertical (because of horizontal support). The reaction at the left hand support (A) will be the resultant of vertical and horizontal forces and inclined with the vertical.

Taking moments about the joint* A and equating the same,

$$R_E \times 8 = (3 \times 2) + (4 \times 4.5) = 24$$

$$\therefore \qquad R_E = \frac{24}{8} = 3\,\text{kN}$$

From the geometry of the figure, we find that

$$\tan\theta = \frac{3}{4} = 0.75 \qquad \text{or} \qquad \theta = 36.9°$$

$$\tan\alpha = \frac{4.5}{2} = 2.25 \qquad \text{or} \qquad \alpha = 66°$$

The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of joints, as we can resolve the force in the members at joint E in which the force are required to be found out. Now consider the point E. Let the directions of the forces P_{DE} and P_{HE} be assumed as shown in Fig. 13.46.

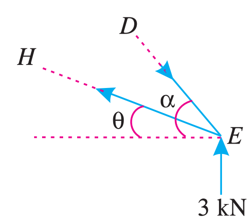


Fig. 13.46.

Resolving the forces horizontally and equating the same,

$$P_{DE} \cos 66° = P_{HE} \cos 36.9° = P_{HE} \times 0.8$$

$$\therefore \qquad P_{DE} = \frac{P_{HE} \times 0.8}{\cos 66°} = \frac{P_{HE} \times 0.8}{0.4062} = 1.97\ P_{HE}$$

and now resolving the forces vertically and equating the same,

$$P_{DE} \sin 66° = P_{HE} \sin 36.9° + 3$$

$$1.97\ P_{HE} \times 0.9137 = (P_{HE} \times 0.6) + 3$$

$$1.2\ P_{HE} = 3$$

$$\text{or} \qquad P_{HE} = \frac{3}{1.2} = 2.5 \text{ kN (Tension)}$$

$$\text{and} \qquad P_{DE} = 1.97\ P_{HE} = 1.97 \times 2.5 = 4.93 \text{ (Compression)}$$

* There are no need of finding out the vertical and horizontal reaction at A, as we are not considering this part of the truss.

Now consider the joint *H*. We have already found out that P_{HE} = 2.5 kN (Tension). It will be interesting to know that the force P_{DH} will be zero, as there is no other member at joint *H* to balance the component of this forces (if any) at right angle to the member *GHE*.

13.17. STRUCTURES WITH ONE END HINGED (OR PIN-JOINTED) AND THE OTHER FREELY SUPPORTED ON ROLLERS AND CARRYING INCLINED LOADS

We have already discussed in the last article that if a structure is hinged at one end, freely supported on rollers at the other, and carries horizontal loads (with or without vertical loads), the support reaction at the roller- supported end will be normal to the support. The same principle is used for structures carrying inclined loads also. In such a case, the support reaction at the hinged end will be the resultant of :

1. Vertical reaction, which may be found out by subtracting the vertical component of the support reaction at the roller supported end from the total vertical loads.
2. Horizontal reaction, which may be found out algebraically by adding all the horizontal loads.

Example 13.14. *Figure 13.47 represents a north-light roof truss with wind loads acting on it.*

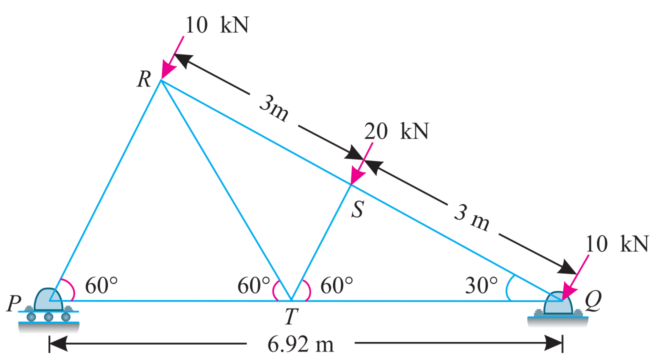


Fig. 13.47.

Find graphically, or otherwise, the forces in all the members of the truss Give your results in a tabulated form.

Solution. Since the truss is supported on rollers at *P*, therefore the reaction at this end will be vertical (because of horizontal support). Moreover, it is hinged at *Q*, therefore the reaction at this end will be the resultant of horizontal and vertical forces and inclined with the vertical.

Taking moments about *Q* and equating the same,

$$V_P \times 6.92 = (20 \times 3) + (10 \times 6) = 120$$

$$\therefore \qquad V_P = \frac{120}{6.92} = 17.3 \text{ kN}$$

We know that total wind loads on the truss

$$= 10 + 20 + 10 = 40 \text{ kN}$$

∴ Horizontal component of wind load,

$$H_Q = 40 \cos 60° = 40 \times 0.5 = 20 \text{ kN} (\rightarrow)$$

and vertical component of the wind load

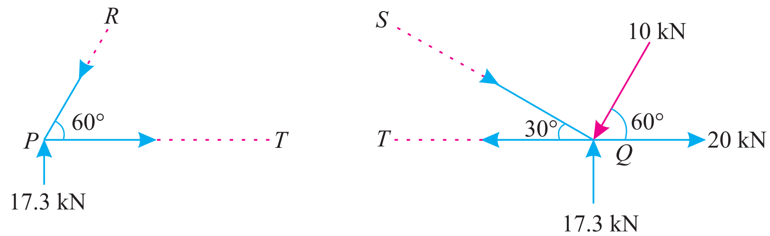
$$= 40 \sin 60° = 40 \times 0.866 = 34.6 \text{ kN} (\downarrow)$$

∴ Vertical reaction at *Q*,

$$V_Q = 34.6 - 17.3 = 17.3 \text{ kN} (\uparrow)$$

The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of joints, as we have to find out the forces in all the members of the truss.

First of all, consider the joint P. Let the directions of the forces P_{PR} and P_{PT} be assumed as shown in Fig 13.48(a). We know that a horizontal force of 20 kN is acting at Q as shown in Fig. 13.48 (b).



(a) Joint P (a) Joint Q

Fig. 13.48.

Resolving the forces vertically and equating the same,

$$P_{PR} \sin 60° = 17.3$$

$$\therefore \quad P_{PR} = \frac{17.3}{\sin 60°} = \frac{17.3}{0.866} = 20 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{PT} = P_{PR} \cos 60° = 20 \times 0.5 = 10 \text{ kN (Tension)}$$

Now consider the joint Q. Let the directions of the forces P_{SQ} and P_{QT} be assumed as shown in Fig. 13.48 (b). We know that a horizontal force of 20 kN is acting at Q as shown in Fig 13.48 (b).

Resolving the forces vertically and equating the same,

$$P_{SQ} \sin 30° = 17.3 - 10 \cos 30° = 17.3 - (10 \times 0.866) = 8.64$$

$$\therefore \quad P_{SQ} = \frac{8.64}{\sin 30°} = \frac{8.64}{0.5} = 17.3 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{QT} = P_{SQ} \cos 30° + 20 - 10 \sin 30°$$
$$= (17.3 \times 0.866) + 20 - (10 \times 0.5) = 30 \text{ kN (Tension)}$$

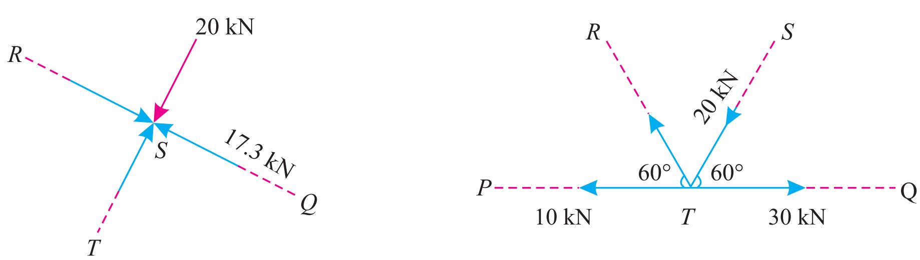


Fig. 13.49.

Now consider the joint S. We have already found out that P_{SQ} = 17.3 kN (Compression). A little consideration will show that the value of the force P_{TS} will be equal to the force 20 kN (Compression). Similarly, the value of the force P_{RS} will be equal to P_{SQ} *i.e.*, 17.3 kN (Compression) as shown in Fig. 13.49 (a).

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Now consider the joint T. Let the directions of the force P_{RT} be assumed as shown in Fig. 13.49 (*b*). We have already found out that P_{ST} = 20 kN (Compression).

Resolving the forces vertically and equating the same,

$$P_{RT} \sin 60° = P_{ST} \sin 60° = 20 \sin 60°$$

or
$$P_{RT} = 20 \text{ kN (Tension)}$$

Now tabulate the results as given below:

S.No.	*Member*	*Magnitude of force in kN*	*Nature of force*
1	*PR*	20.0	Compression
2	*PT*	10.0	Tension
3	*SQ*	17.3	Compression
4	*QT*	30.0	Tension
5	*ST*	20.0	Compression
6	*RS*	17.3	Compression
7	*RT*	20.0	Tension

Example 13.15. *A truss of 12 m span is loaded as shown in Fig 13.50.*

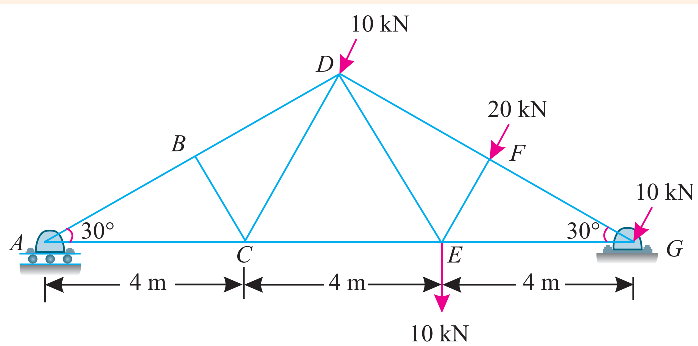


Fig. 13.50.

Determine the force in the members BD, CE and CD of the truss.

Solution. Since the truss is supported on rollers on the left end (A), therefore the reaction at this end will be vertical (because of horizontal support). Moreover, it is hinged at the right hand support (G), therefore the reaction at this end will be the resultant of horizontal and vertical forces and will be inclined with the vertical.

Taking * moments about G and equating the same,

$$V_A \times 12 = (10 \times 4) (20 \times 4 \cos 30°) + (10 \times 8 \cos 30°)$$
$$= 40 + (80 \times 0.866) + (80 \times 0.866) = 178.6$$

$$\therefore \quad V_A = \frac{178.6}{12} = 14.9 \text{ kN}$$

The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of sections, as one section line can cut the members *BD*, *CE* and *CD* in which forces are required to be found out.

* There is no need of finding out the vertical and horizontal reaction at G, as we are not considering this part of the truss.

Now let us pass section (1-1) cutting the truss into two parts as shown in Fig 13.51.

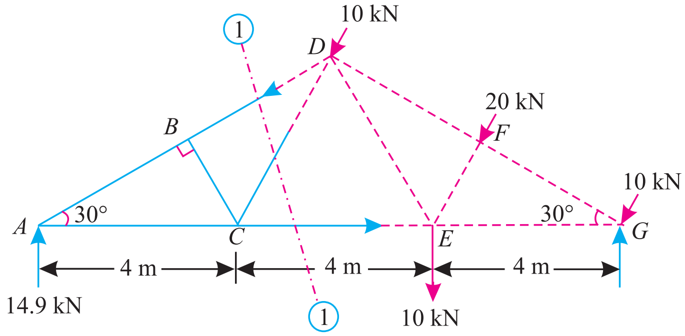


Fig. 13.51.

Now consider equilibrium of the left part of the truss. Let the directions of the forces P_{BD}, P_{CE} and P_{CD} be assumed as shown in Fig 13.51. Taking moments about the joint C and equating the same,

$$P_{BD} \times 2 = 14.9 \times 4 = 59.6$$

$$\therefore \qquad P_{BD} = \frac{59.6}{2} = 29.8 \text{ kN (Compression)}$$

Similarly taking moments about the joint D and equating the same,

$$P_{CE} \times 6 \tan 30° = 14.9 \times 6 = 89.4$$

$$\therefore \qquad P_{CE} = \frac{89.4}{6 \tan 30°} = \frac{89.4}{6 \times 0.5774} = 25.8 \text{ kN} \quad \text{(Tension)}$$

Now for finding out P_{CD}, we shall take moments about the A (where the other two members meet). Since there is no force in the lift of the truss (other than the reaction V_A, which will have zero moment about A), therefore the value of P_{CD} will be zero.

Note: The force P_{CD} may also be found out as discussed below :

At joint B, the force in member BC is zero, as there is no other member to balance the force (if any) in the member BC. Now at joint C, since the force in member BC is zero, therefore the force in member CD is also equal to zero.

Example 13.16. *A truss hinged at A and supported on rollers at D, is loaded as shown in Fig. 13.52.*

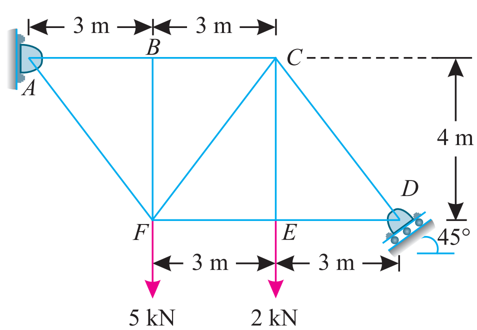


Fig. 13.52.

Find the forces in the members BC, FC, FE of the truss.

Solution. Since the truss is supported on rollers at the right end D, therefore the reaction at this support will be normal to the support *i.e.*, inclined at 45° with the horizontal. The reaction at A will be the resultant of horizontal and vertical forces. It will be interesting to know that as the reaction at D is inclined at 45° with the horizontal, therefore horizontal component (R_{DH}) and vertical component (R_{DV}) of this reaction will be equal. Mathematically $R_{DH} = R_{DV}$.

Taking moments about A and equating the same,

$$(R_{DV} \times 9) - (R_{DH} \times 4) = (5 \times 3) + (2 \times 6)$$

$$5\,R_{DH} = 27 \qquad [\because R_{DH} = R_{DV}]$$

$$R_{DH} = \frac{27}{5} = 5.4\text{ kN } (\leftarrow)$$

and $$R_{DV} = 5.4\text{ kN } (\uparrow)$$

The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of sections, as one section line can cut the members BC, FE and FC and in which forces are required to be found out.

Now let us pass section (1-1) cutting the truss into two parts as shown in Fig. 13.53.

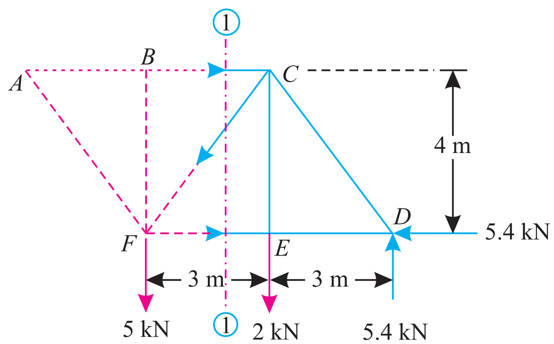


Fig. 13.53.

Now consider equilibrium of right part of the truss. Let the directions of the forces P_{BC} and P_{FE} be assumed as shown in Fig 13.53. Taking moments about the joint F and equating the same,

$$P_{BC} \times 4 = (5.4 \times 6) - (2 \times 3) = 26.4$$

$$\therefore \qquad P_{BC} = \frac{26.4}{4} = 6.6\text{ kN (Compression)}$$

Similarly, taking moments about the joint C and equating the same,

$$P_{FE} \times 4 = (5.4 \times 4) - (5.4 \times 3) = 5.4$$

$$\therefore \qquad P_{FE} = \frac{5.4}{4} = 1.35\text{ kN (Compression)}$$

and now taking moments about the joint B and equating the same,

$$P_{FC} \times 2.4 = (P_{FE} \times 4) - (2 \times 3) + (5.4 \times 6) - (5.4 \times 4)$$

$$= (1.35 \times 4) - 6 + 32.4 - 21.6 = 10.2$$

$$\therefore \qquad P_{FC} = \frac{10.2}{2.4} = 4.25\text{ kN (Tension)}$$

13.18. MISCELLANEOUS STRUCTURES

In the previous articles we have been analysing the regular frames subjected to vertical, horizontal and inclined loads. We have been solving such examples by the methods of joints and sections. But sometimes we come across irregular structures.

Such structures may be analysed in the same way as that for regular structures. The casual look at such a structure, gives us a feeling that it is complicated problem. But a patient and thoughtful procedure helps us in solving such problems. The following examples will illustrate this point.

Example 13.17. *Figure 13.54 shows a bridge truss of 130 m span subjected to two points loads.*

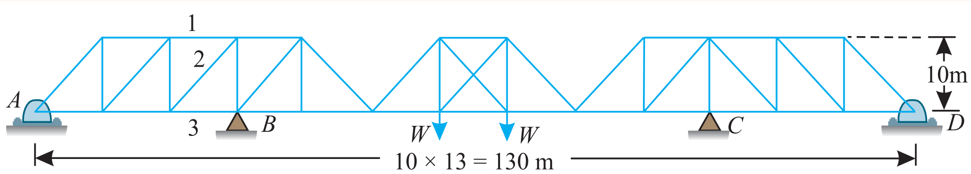


Fig. 13.54.

Determine the forces in the members 1, 2 and 3 of the bridge truss by any suitable method.

Solution. The whole structure may be considered to consist of two cantilever trusses supporting an intermediate truss. As a matter of fact, the two point loads acting at the intermediate truss are transferred to the ends of the cantilever trusses.

Since the two cantilever trusses are symmetrical and the point loads on the intermediate truss are also symmetrical, therefore each cantilever truss is subjected to a point load as shown in Fig. 13.55 (*a*).

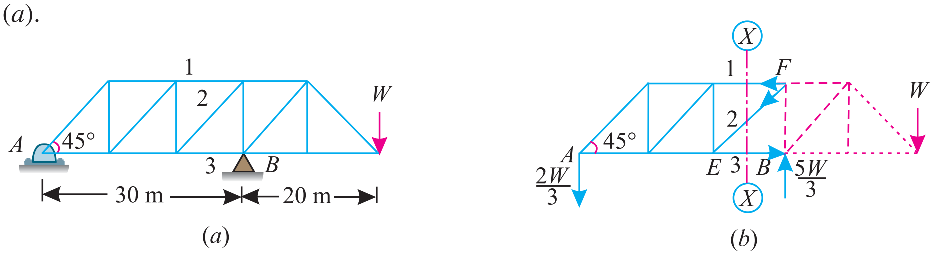


Fig. 13.55.

Let V_B = Vertical reaction at the support B.

Taking moments about the support A and equating the same,

$$V_B \times 30 = W \times 50 = 50\ W$$

$$V_B = \frac{50\,W}{30} = \frac{5\,W}{3}\ (\uparrow)$$

and
$$V_A = \frac{5\,W}{3} - W = \frac{2\,W}{3}\ (\downarrow)$$

First of all, pass section (*X-X*) cutting the truss into two parts and consider the equilibrium of the left part of the truss as shown in Fig. 13.55 (*b*). Now let the directions of the forces P_1, P_2, P_3 and P_4 be assumed as shown in Fig 13.55 (*b*). First of all, let us consider the joint *B*. A little consideration will show that the magnitude of the force P_4 will be equal and opposite to the reaction V_B *i.e.*, $5W/3$ (Compression). This will happen as the vertical components of the horizontal members at *B* will be zero.

Resolving the forces vertically and equating the same,

$$P_2 \times \cos 45° = \frac{2W}{3}$$

or
$$P_2 = \frac{2W}{3} \times \frac{1}{\cos 45°} = \frac{2W}{3 \times 0.707} = 0.943\ W \text{ (Compression)}$$

Taking moments of the forces acting on the left part of the truss only about the joint *E* and equating the same,

$$P_1 \times 10 = \frac{2W}{3} \times 20 = \frac{40W}{3}$$

$$\therefore \quad P_1 = \frac{40W}{3} \times \frac{1}{10} = \frac{4W}{3} \text{ (Compression)} \qquad \textbf{Ans.}$$

and now taking moments of the forces acting on the left part of the truss only about the joint *F* and equating the same,

$$P_3 \times 10 = \frac{2W}{3} \times 30 = 20W$$

$$\therefore \quad P_3 = \frac{20W}{10} = 2W \text{ (Tension)} \qquad \textbf{Ans.}$$

Example 13.18. *A pin-jointed frame shown in Fig 13.56 is hinged at A and loaded at D. A horizontal chain is attached to C and pulled so that AD is horizontal.*

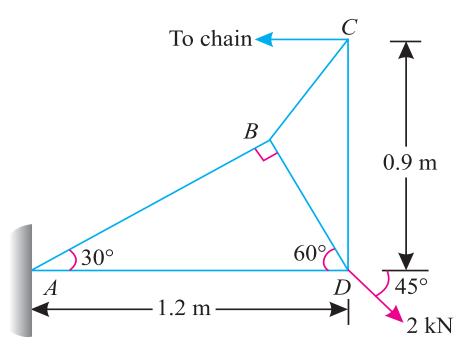


Fig. 13.56.

Determine the pull in the chain and also the force in each member. Tabulate the results.

Solution. The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of joints, as we have to find the force in each member.

Contents

Pull in the chain

Let P = Pull in the chain.

Taking moments about the joint A and equating the same,

$$P \times 0.9 = 2 \cos 45° \times 1.2 = 2 \times 0.707 \times 1.2 = 1.7$$

$$\therefore \quad P = \frac{1.7}{0.9} = 1.889 \text{ kN} \qquad \textbf{Ans.}$$

Force in each member

We know that horizontal reaction at A,

$$H_A = 1.889 - (2 \cos 45°) = 1.889 - (2 \times 0.707) = 0.475 \text{ kN} (\rightarrow)$$

and vertical reaction at A,

$$V_A = 2 \sin 45° = 2 \times 0.707 = 1.414 \text{ kN} (\uparrow)$$

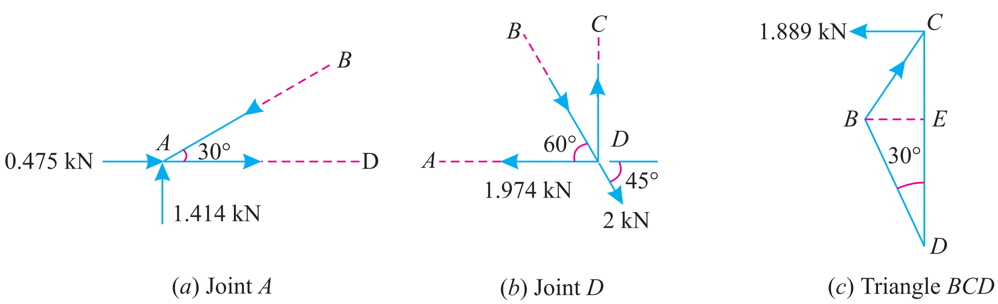


Fig. 13.57.

First of all, consider the joint A. Let the directions of the forces P_{AB} and P_{AD} be assumed as shown in Fig 13.57 (a). We have already found out that zthe horizontal and vertical reactions at A are 0.475 kN and 1.414 kN repectively as shown in the figure.

Resolving the forces vertically and equating the same,

$$P_{AB} \sin 30° = 1.414$$

$$P_{AB} = \frac{1.414}{\sin 30°} = \frac{1.414}{0.5} = 2.828 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{AD} = P_{AB} \cos 30° - 0.475 = (2.828 \times 0.866) - 0.475$$

$$= 1.974 \text{ kN (Tension)}$$

Now consider the joint D. Let the directions of the forces P_{BD} and P_{CD} be assumed as shown in Fig 13.57 (b). We have already found out that P_{AD} = 1.974 kN (Tension) as shown in the figure.

Resolving the forces horizontally and equating the same,

$$P_{BD} \cos 60° = 1.974 - 2 \cos 45° = 1.974 - (2 \times 0.707) = 0.56 \text{ kN}$$

$$\therefore \quad P_{BD} = \frac{0.56}{\cos 60°} = \frac{0.56}{0.5} = 1.12 \text{ kN} \quad \text{(Compression)}$$

and now resolving the forces vertically and equating the same,

$$P_{CD} = P_{BD} \sin 60° + 2 \sin 45°$$

$$= (1.12 \times 0.866) + (2 \times 0.707) = 2.384 \text{ kN (Tension)}$$

Now consider the triangle BCD. From B, draw BE perpendicular to CD. Let the direction of P_{BC} be assumed as shown in Fig 13.57 (c).

From the geometry of this triangle, we find that

$$BD = AD \sin 30° = 1.2 \times 0.5 = 0.6 \text{ m}$$

and $$BE = BD \sin 30° = 0.6 \times 0.5 = 0.3 \text{ m}$$

$\therefore$ $$DE = BD \cos 30° = 0.6 \times 0.866 = 0.52 \text{ m}$$

and $$CE = DC - DE = 0.9 - 0.52 = 0.38 \text{ m}$$

$\therefore$ $$\tan \angle BCE = \frac{BE}{CE} = \frac{0.3}{0.38} = 0.7895$$

or $$\angle BCE = 38.3°$$

Resolving the forces horizontally at C and equating the same,

$$P_{BC} \sin 38.3° = 1.889$$

$\therefore$ $$P_{BC} = \frac{1.889}{\sin 38.3°} = \frac{1.889}{0.6196} = 3.049 \text{ kN (Compression)}$$

Now tabulate the results as given below :

S.No.	*Member*	*Magnitude of force in kN*	*Nature of force*
1	*AB*	2.828	Compression
2	*AD*	1.974	Tension
3	*BD*	1.12	Compression
4	*CD*	2.384	Tension
5	*BC*	3.049	Compression

Example 13.19. *The truss shown in the Fig. 13.58 is made up of three equilateral triangles loaded at each of the lower panel pains.*

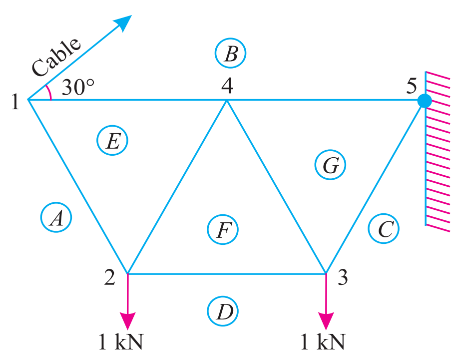


Fig. 13.58.

It is supported at the wall on the right hand side and by a cable on the left as shown. Determine (a) the tension in the cable (b) the reaction at the wall and (c) the nature and magnitude of the force in each bar.

Solution. The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of joints, as we have to find out the forces in all the members of the truss.

(*a*) *Tension in the cable*

Let T = Tension in the cable and

a = Length of each side of the equilateral triangle.

Taking moments about the joint 5 and equating the same,

$$(T \cos 60°) \times 2a = (1 \times 1.5\ a) + (1 \times 0.5\ a)$$

$$(T \times 0.5)\ 2a = 2a$$

$$\therefore \quad T = 2 \text{ kN} \qquad \textbf{Ans.}$$

(*b*) *Nature and magnitude of the force in each bar*

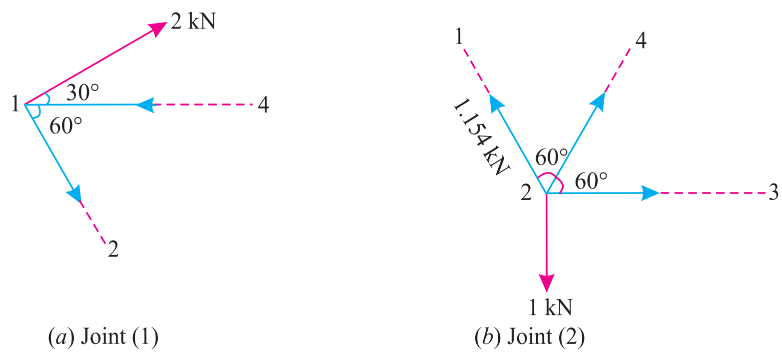


Fig. 13.59.

First of all consider the joint 1. We have already found out that tension in the cable is 2 kN as shown in the figure. Let the directions of P_{1-2} and P_{1-4} be assumed as shown in Fig. 13.59 (*a*). Resolving the forces vertically and equating the same,

$$P_{1-2} \sin 60° = 2 \sin 30°$$

$$\therefore \quad P_{1-2} = \frac{2 \sin 30°}{\sin 60°} = \frac{2 \times 0.5}{0.866} = 1.154 \text{ kN (Tension)}$$

and now resolving the forces horizontally and equating the same,

$$P_{1-4} = 2 \cos 30° + 1.154 \cos 60° \text{ kN}$$

$$= (2 \times 0.866) + (1.154 \times 0.5) = 2.309 \text{ kN (Compression)}$$

Now consider the joint 2. We have already found out that the force in member 1-2 (*i.e.* P_{1-2}) is 1.54 kN (Tension). Let the directions of the forces P_{2-4} and P_{2-3} be assumed as shown in Fig 13.59 (*b*). Resolving the forces vertically and equating the same,

$$P_{2-4} \sin 60° = 1 - 1.154 \sin 60° = 1 - (1.154 \times 0.866) = 0$$

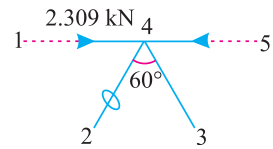
$$\therefore \quad P_{2-4} = 0$$

and now resolving the forces horizontally and equating the same,

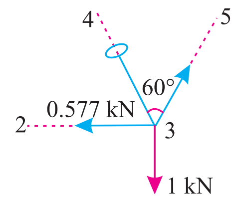
$$P_{2-3} = 1.154 \cos 60° = 1.154 \times 0.5 = 0.577 \text{ kN (Tension)}$$

Now consider the joint 4. A little consideration will show that the force P_{3-4} will be zero. This will happen as the force P_{2-4} is zero and the vertical components of the forces P_{1-4} and P_{4-5} are also zero. Moreover, the force P_{4-5} will be equal to the force P_{1-4} *i.e.*, 2.309 kN (Compression). This will happen as the forces P_{2-4} and P_{2-5} (being zero) will have their vertical components as zero.

Now consider the joint 3. Let the direction of the force P_{3-5} be assumed as shown in Fig. 13.60 (*b*). We have already found out that the force P_{2-3} is 0.577 kN (Tension) and force P_{3-4} is zero.



(*a*) Joint (4)



(*b*) Joint (3)

Fig. 13.60.

Resolving the forces vertically and equating the same,

$$P_{3-5} \cos 30° = 1$$

$$\therefore \qquad P_{3-5} = \frac{1}{\cos 30°} = \frac{1}{0.866} = 1.154 \text{ kN (Tension)}$$

Now tabulate the results as given below :

S.No.	*Member*	*Magnitude of force in kN*	*Nature of force*
1	1-2 (*AE*)	1.154	Tension
2	1-4 (*BE*)	2.309	Compression
3	2-4 (*EF*)	0	—
4	2-3 (*FD*)	0.577	Tension
5	3-4 (*FG*)	0	—
6	4-5 (*BG*)	2.309	Compression
7	3-5 (*GD*)	1.154	Tension

(*C*) *Reaction at the wall*

We know that the reaction at the wall will be the resultant of the forces P_{4-5} (*i.e.*, 2.309 kN Compression) and P_{3-5} (*i.e.*, 1.154 kN Tension). This can be easily found out by the parallelogram law of forces *i.e.*,

$$R = \sqrt{(1.154)^2 + (2.309)^2 + 2 \times 1.154 \times 2.309 \cos 120°}$$

$$= \sqrt{1.332 + 5.331 + 5.329(-0.5)} = 2 \text{ kN} \qquad \textbf{Ans.}$$

Example 13.20. *A frame ABCD is hinged at A and supported on rollers at D as shown in Fig. 13.61.*

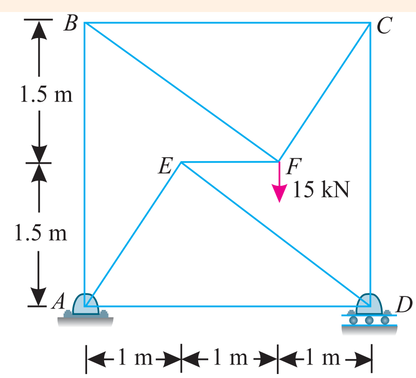


Fig. 13.61.

Determine the forces in the member AB, CD and EF,.

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Solution. The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of sections, as we have to determine forces in three members of the frame only.

First of all pass section (1-1) cutting the truss through the members *AB*, *EF* and *CD* as shown in Fig 13.62. Now consider equilibrium of the upper portion of the frame. Let the directions of the forces P_{AB} and P_{CD} be assumed as shown in Fig 13.62. Now consider the joint *F*. We know that horizontal component of 15 kN load is zero. Therefore force in member *EF* is also zero. **Ans.**

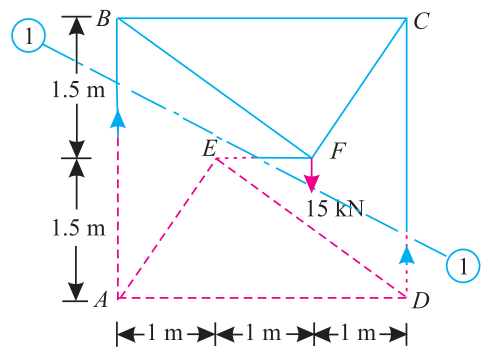

Fig. 13.62.

Now taking moments of the forces acting on the upper portion of the frame about the joint *A* and equating the same,

$$P_{CD} \times 3 = 15 \times 2 = 30$$

or $$P_{CD} = \frac{30}{3} = 10 \text{ kN} \qquad \textbf{Ans.}$$

and now taking moments of the forces about the joint *D* and equating the same,

$$P_{AB} \times 3 = 15 \times 1 = 15$$

or $$P_{AB} = \frac{15}{3} = 5 \text{ kN} \qquad \textbf{Ans.}$$

Example 13.21. *A framed structure of 6 m span is carrying point loads as shown in Fig 13.63.*

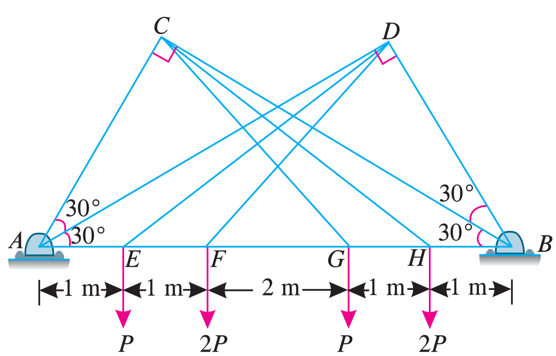

Fig. 13.63

Find by any method the forces in the members AC, BD and FG of the structure.

Solution. First of all, from *D* draw *DK* perpendicular to *AB* as shown in Fig 13.63. From the geometry of the figure, we find that

$$AD = AB \cos 30° = 6 \times 0.866 = 5.196 \text{ m}$$

and $$DK = AD \sin 30° = 5.196 \times 0.5 = 2.598 \text{ m}$$

Similarly $$AK = AD \cos 30° = 5.196 \times 0.866 = 4.5 \text{ m}$$

$$\therefore \qquad \tan \alpha = \frac{DK}{EK} = \frac{2.598}{3.5} = 0.7423 \quad \text{or} \quad \alpha = 36.6°$$

and $$\tan \beta = \frac{DK}{FK} = \frac{2.598}{2.5} = 1.0392 \quad \text{or} \quad \beta = 46.1°$$

Taking moments about B and equating the same,

$$R_A \times 6 = (P \times 5) + (2\,P \times 4) + (P \times 2) + (2\,P \times 1) = 17\,P$$

$$\therefore \qquad R_A = \frac{17\,P}{6} = 2.83\,P.$$

Let the directions of the various forces be assumed as shown in Fig 13.64. Now resolving the forces vertically at E and equating the same,

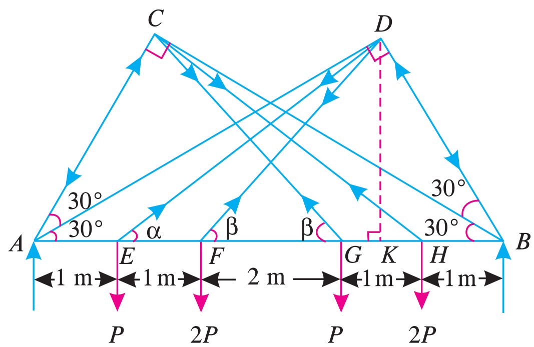

Fig. 13.64.

$$P_{ED} \sin 36.6° = P$$

$$\therefore \qquad P_{ED} = \frac{P}{\sin 36.6°} = \frac{P}{0.5960} = 1.68\ P \text{ (Tension)}$$

and now resolving the forces vertically at F and equating the same,

$$P_{FD} \sin 46.1° = 2\,P$$

$$\therefore \qquad P_{FD} = \frac{2\,P}{\sin 46.1°} = \frac{2\,P}{0.7206} = 2.78\ P \text{ (Tension)}$$

Similarly, resolving the forces vertically at G and equating the same,

$$P_{CG} \sin 46.1° = P$$

$$\therefore \qquad P_{CG} = \frac{P}{\sin 46.1°} = \frac{P}{0.7206} = 1.39\ P \text{ (Tension)}$$

and now resolving the forces vertically at H and equating the same,

$$P_{CH} \sin 36.6° = 2\,P$$

$$\therefore \qquad P_{CH} = \frac{2\,P}{\sin 36.6°} = \frac{2\,P}{0.5960} = 3.36\ P \text{ (Tension)}$$

From the geometry of the figure, we also find that

$$\angle EDB = \angle ACH = 180° - (36.6° + 60°) = 83.4°$$

$\frac{2.24}{12.74}$

and $\angle FDB = \angle ACG = 180° - (46.1° + 60°) = 73.9°$

Now at D, resolving the forces along BD and equating the same,

$$P_{BD} = P_{ED} \cos 83.4° + P_{FD} \cos 73.9°$$

....(The component of force P_{AD} about BD is zero)

$$= (1.68\,P \times 0.1146) + (2.78\,P \times 0.2773)$$

$$= 0.963\,P \text{ (Compression)} \qquad \textbf{Ans.}$$

and at C resolving the forces along AC and equating the same,

$$P_{AC} = P_{CH} \cos 83.4° + P_{CG} \cos 73.9°$$

....(The component of force P_{BC} about AC is zero)

$$= (3.36\,P \times 0.1146) + (1.39\,P \times 0.2773)$$

$$= 0.772\,P \text{ (Compression)} \qquad \textbf{Ans.}$$

Taking moments about B and equating the same,

$$R_A \times 6 = (P \times 5) + (2\,P \times 4) + (P \times 2) + (2\,P \times 1) = 17\,P$$

$$R_A = \frac{17\,P}{6} = 2.83\,P$$

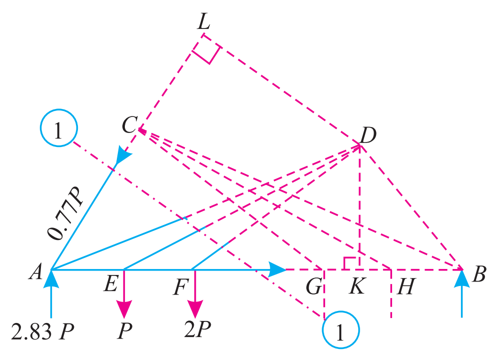

Fig. 13.65.

Now pass section (1-1) cutting the truss into two parts as shown in Fig 13.65. Let us extend the line AC and through D draw DL perpendicular to AC. From the geometry of the figure, we find that

$$DL = AD \sin 30° = 5.196 \times 0.5 = 2.598 \text{ m}$$

Taking moments of the forces in the left part of the truss about D and equating the same,

$$2.83\,P \times 4.5 = (0.772\,P \times 2.598) + (P \times 3.5) + (2\,P \times 2.5) + (P_{FG} \times 2.598)$$

$$12.74\,P = 10.5\,P + (P_{FG} \times 2.598)$$

$$\therefore \quad 2.598\,P_{IG} = 12.74\,P - 10.5\,P = 2.24\,P$$

or $$P_{FG} = \frac{2.24P}{2.598} = 0.862\,P \text{ (Tension)} \qquad \textbf{Ans.}$$

EXERCISE 13.3

1. A truss shown in Fig. 13.66 is subjected to two points loads at *B* and *F*. Find the forces in all the members of the truss and tabulate the results.

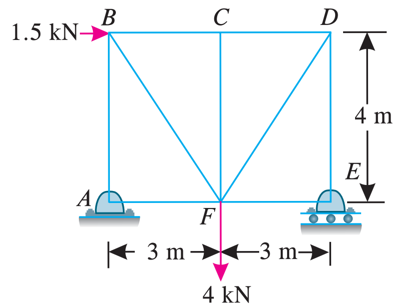

Fig. 13.66.

Ans. AB = 1.0 kN (Compression)
AF = 1.5 kN (Tension)
AE = 3.0 kN (Compression)
EF = 0
BF = 1.25 kN (Tension)
BC = 2.25 kN (Compression)
DF = 3.75 kN (Tension)
CD = 2.25 kN (Compression)
CF = 0

2. A cantilever braced truss supported on rollers at E and hinged at A is loaded as shown in Fig 13.67. Determine graphically or otherwise, the forces in the members of the truss, also determine the reactions at *A* and *E*.

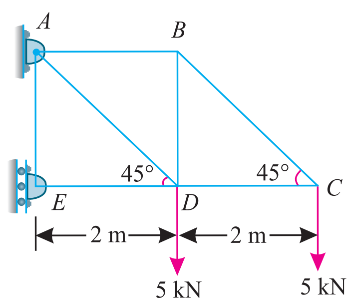

Fig. 13.67.

Ans. BC = 7.1 kN (Compression)
CD = 5.0 kN (Tension)
AB = 5.0 kN (Compression)
BD = 5.0 kN (Tension)
AD = 14.1 kN (Tension)
ED = 15.0 kN (Compression)
R_E = 15 kN
R_E = 18 kN

Note: Since the truss is freely supported on rollers at *E*, therefore the reaction at this support will be horizontal (because of vertical support).

3. A truss of 5 m span and 2.5 m height is subjected to wind load as shown in Fig. 13.68. Find by any method the magnitude of forces in all the members of the truss. Also state their nature.

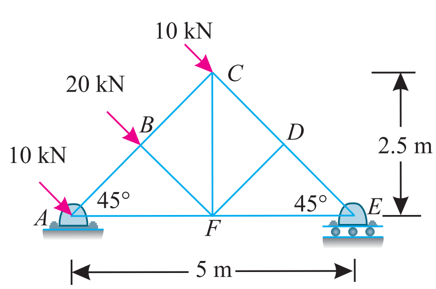

Fig. 13.68.

Ans. AB = 10.0 kN (Compression)
AF = 28.28 kN (Tension)
DE = 20.0 kN (Compression)
EF = 14.14 kN (Tension)
BF = 20.0 kN (Compression)
BC = 10.0 kN (Compression)
CF = 14.11 kN (Tension)
CD = 20.0 kN (Compression)
DF = 0

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4. A truss 15 m long is subjected to a point load of 10 kN as shown in Fig. 13.69. Find the forces in the members 1, 2 and 3 of the truss.

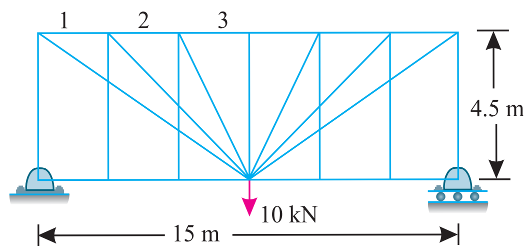

Fig. 13.69.

Ans. 1 = 40 kN (Compression)
2 = 10 kN (Compression)
3 = 10 kN (Compression)

Hint. Pass vertical sections cutting the members 1, 2 and 3 and take moments about the joint containing 100 kN load. Each time, all the members (except 1, 2 and 3) pass through the joint about which moments are taken.

QUESTIONS

1. What is a 'frame' ? Discuss its classification.

2. State clearly the difference between a perfect frame and an imperfect frame.

3. How would you distinguish between a deficient frame and a redundant frame ?

4. What are the assumptions made, while finding out the forces in the various members of a framed structure ?

5. Name the methods, which are employed, for finding out the forces in a frame.

6. What is the difference between a simply supported frame and a cantilever frame ? Discuss the method of finding out reactions in both the cases.

OBJECTIVE TYPE QUESTIONS

1. A framed structure is perfect, if the number of members are .....(2*j* – 3), where *j* is the number of joints.

(*a*) less than (*b*) equal to (*c*) greater than (*d*) either (*a*) or (*c*)

2. A framed structure is imperfect, if the number of members are .....(2*j* – 3), where *j* is the number of joints.

(*a*) less than (*b*) equal to (*c*) greater than (*d*) either (*a*) or (*c*)

3. A redundant frame is also called ......frame

(*a*) perfect (*b*) imperfect (*c*) deficient (*d*) none of these

4. A framed structure of a triangular shape is

(*a*) perfect (*b*) imperfect (*c*) deficient (*d*) redundant

5. In a cantilever truss, it is very essential to find out the reactions before analyzing it.

(*a*) agree (*b*) disagree

ANSWERS

1. (*b*) **2.** (*d*) **3.** (*b*) **4.** (*a*) **5.** (*b*)

Contents

$$P_{CD} = P_{AD} \sin 45° + P_{BD} \sin 45° = 2\,P_{AD} \sin 45° \qquad ...(\because P_{BD} = P_{AD})$$

$$= 2 \times 7.08 \times 0.707 = 10.0 \text{ kN (Tension)}$$

C *C* *C*

B *B* *B*

D *D* *D*

D *D* *D*

Top