

DAA

$$K + L - M * N + (O^P) * W / U / V * T + Q$$

Input op.	Stack	Postfix exp.
K	-	K
+	+	K
L	+	KL
-	-	KL+
M	-	KL+M
*	-*	KL+M
N	-*	KL+MN
+	+	KL+MN*-
(, O	+()	KL+MN*-O
^, P	+(^)	KL+MN*-OP
)		KL+MN*-OP^
*	+*	KL+MN*-OP
W	+*	KL+MN*-OP^W
/	+ /	KL+MN*-OP^W*/
U	+ /	KL+MN*-OP^W*U
/	+ /	KL+MN*-OP^W*U/
V	+ /	KL+MN*-OP^W*U/V
*	+*	KL+MN*-OP^W*U/V/
T	+*	KL+MN*-OP^W*U/V/T
+	+ *	KL+MN*-OP^W*U/V/T*
Q	+	KL+MN*-OP^W*U/V/T+Q

$$T(n) = (C_1 \cdot n) + (C_2 \cdot (n-1)) + (C_3 \cdot (n-1)) + (C_4 (\leq t_i)) + \\ (C_5 (\leq t_{i-1})) + (C_6 (\leq t_{i-1})) + (C_7 \cdot (n-1))$$

Best Case $C_4 * \sum_{i=2}^n (t_i) \rightarrow \sum_{i=2}^n 1$

$$T_n = (C_1 \cdot (n)) + (C_2 \cdot (n-1)) + (C_3 \cdot (n-1)) + \\ (C_4 \cdot (n-1)) + (C_7 \cdot (n-1))$$

$$n(C_1 + C_2 + C_3 + C_4 + C_7) - (C_2 + C_3 + C_4 + C_7)$$

Worst Case

$$\sum_{i=0}^n t_i = 1 + 2 + 3 + \dots + (n-1)$$

$$\sum_{i=0}^n =$$

$$n = \frac{n(n+1)}{2}$$

For $[C_5 \cdot (t_{i-1})]$ = It will run 1 time for first multi
 $i-1=0$, that means 0

$$0 \dots (n-1)$$

we can apply $\frac{n(n+1)}{2}$

$$T(n) = C_1 \cdot n + C_2 \cdot (n-1) + C_3 \cdot (n-1) + C_4 \left[\frac{n(n+1)}{2} - 1 \right] +$$

$$C_5 \left[\frac{n(n-1)}{2} \right] + C_6 \left[\frac{n(n-1)}{2} \right] + C_7 \cdot (n-1)$$

USA

$p=0$
for ($i=1$; $p \leq n$; $i++$) {
 $p = p+i$ }

$O\sqrt{n}$

i	p
1	$0+1$
2	$0+1+2$
3	$0+1+2+3$
4	$0+1+2+3+4$
5	$0+1+2+3+4+5$

~~R~~ $\frac{R}{2}$ $\frac{k(k+1)}{2}$

Dense - More connections are there

Sparse - Less connection are there

Complexity classes

$g(n)$? $f(n)$?

Insertion Sort

Algorithm	Cost	Cost	Time
① Repeat through step 7 for $i=2, 3, \dots, n$	C_1	n	
② Set $t \leftarrow A[i]$	C_2	$(n-1)$	
③ set $p \leftarrow p-1$	C_3	$(n-1)$	
④ Repeat while ($p > 0$ and $A[p] > t$) through step 6	C_4	$\sum_{i=2}^n t_i$	
⑤ Set $A[p+1] \leftarrow A[p]$	C_5	$\sum_{i=2}^n (t_i - 1)$	
⑥ Set $p \leftarrow p-1$ $p \leftarrow p-1$	C_6	$\sum_{i=2}^n (t_i - 1)$	
⑦ Set $A[p+1] \leftarrow t$	C_7	$(n-1)$	

AAD

LL

2 3 1 * + 9 -

Symbol Scanned	Stack
2	2
3	2 3
1	2 3 1
*	2 [3*1] \Rightarrow 2*3 = 23
+	2+3 \Rightarrow 5
9	59
-	5-9 \Rightarrow 4

for ($i=0$; $i < N$; $i++$) \leftarrow it will run $(n+1)$ times
{} including false statement
statement; $\leftarrow (n)$ times
}

$$\text{Order of } n \mid O(n) \rightarrow f(n) = 2n + 1$$

if $\left(\frac{n}{2} + 1\right)$ then $\rightarrow O(n)$

for ($i=0$; $i < n$; $i++$) {
 for ($j=0$; $j < i$; $j++$) {
 statement;
 }
}

$$O(1+2+3+4+5+\dots+n) \\ \frac{n(n+1)}{2} = \frac{n^2+n}{2}$$

$O(n^2)$

f

1 / 1

$$\begin{aligned} \text{if } \log f(n) = O(\log g(n)) &\Rightarrow f(n) = O(g(n)) \\ \text{if } \log f(n) = \omega(\log g(n)) &\Rightarrow f(n) = \omega(g(n)) \end{aligned}$$

Q)

Constant are removed while finding asymptotic

$$\begin{aligned} f(n) &= 2^n \\ \log 2^n &= n \log 2 \end{aligned}$$

$$g(n) = n^5$$

$$\log n^5 = 5 \log n$$

$$\begin{aligned} \cancel{f(n)} &< g(n) \\ n > \log n & \\ f(n) &> g(n) \\ 2^n &= \omega(n^5) \end{aligned}$$

$$f(n) < g(n)$$

Log approach is applicable where power is there

Q)

$$\begin{array}{ll} 2^n & 2^{2n} \\ \log 2^n & \log 2^{2n} \\ n \log 2 & 2n \log 2 \\ m & 2m \end{array}$$

Constants
are removed

$$\begin{aligned} f(n) &< g(n) \\ f(n) &= O(g(n)) \\ f(n) &> g(n) \\ f(n) &= \omega(g(n)) \end{aligned}$$

$$m \approx n$$

Asymptotic mean there
is no space for constant

$$\lim_{n \rightarrow \infty} \frac{1}{2^n} = \frac{1}{2^\infty} = \frac{1}{\infty} = 0$$

DO NOT USE LOG WITH
BIG O OR Θ (theta)

$$[g(n) > f(n) / g(n) < f(n)]$$

→ That means rapid growth is
true

If (equal to) is used then it is not

$$\frac{km - p + 2n - 2np}{2} = \frac{-np - p + 2n}{2}$$

Q) Determine for a fixed n for the values of P for which we get max value of cost of avg & min val of cost avg.

Unsuccessful search = $P(\text{Probability}) = 0$

$$-\frac{n}{2}(0) - 0 + 2n = \frac{2n}{2}$$

$$P = 1, \frac{n}{2}$$

Q) How many comparisons, both successful and unsuccessful with which we made with brute algorithm in searching for each of the following method in binary text. 6000 zeros and patterns are

- (1) 00001
- (2) 10000
- (3) 01010

Naive algorithm $\Rightarrow (n - m + 1)$

- (1) $596 * 5$
- (2) $596 * 1$ (because loop will run only once, inner loop)
- (3) $596 * 2$

UNIT - 2 (Searching Algorithm)

This unit consist of Hybrid searching Algo.

Sentential form

a b a l a c l b a b a b c

We extend the array instead of making new array
We need single space now.

String matching problem

Normal sequential approach

[Naive - string - Matching]

a b c a c d e
a b c
a c a
c a c
c d e
d e e

$m \leftarrow \text{length}[T]$

$m \leftarrow \text{length}[P]$

for $s \leftarrow 0$ to $n-m$

do if $P[1..m] = T[s+1..s+m]$

then print ("Pattern occur")

$O(n-m)$

If $P \leftarrow$ Probability of successful search

$1-P \leftarrow$ Probability of unsuccessful search

Average case

$$\left(\frac{1 \times P}{m} \right) + \left(2 \times \frac{P}{m} \right) + \dots + \left(\frac{(n-1)P}{m} \right) + n(1-P)$$

[Successful] + [Unsuccessful]

$$\frac{P}{m} \frac{n(n-1)}{2} + n(1-P) \rightarrow \frac{P(n-1) + 2n + 2np}{2}$$

Consider the following function :-

$$f(n) = 2^n \quad g(n) = n! \quad h(n) = n \log n$$

- (a) $f(n) = O(g(n)) T$ $g(n) = O(h(n)) F$
- (b) $f(n) = \Omega(g(n)) F$ $g(n) = O(h(n)) F \quad m \log n < 2^n < n!$
- (c) $g(n) = O(f(n)) F$ $h(n) = O(f(n)) T$
- (d) $h(n) = O(f(n)) T$ $g(n) = \Omega(f(n)) T$

Arrange all time function in ascending & descending

$$\begin{array}{lll} f(n) < g(n) & f(n) \leq g(n) & g(n) \leq h(n) \\ f(n) > g(n) & 2^n \leq n! & n! \leq n \log n \end{array}$$

$$\begin{array}{ll} f(n) \geq g(n) & g(n) \leq h(n) \\ 2^n \geq n! & n! \leq n \log n \end{array}$$

What is the

- 5 2 * 3 3 2 + * +
- ? Full & Complete BT
 - ? Ority operation
 - ? Parse expression (Scanning)

Pre-Post use to parse without parentheses.
In ~~with~~ ~~parse~~ with parentheses.

10	11
5	6

28

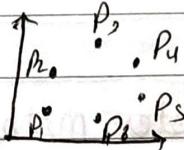
In division tree, making tree is used,

Brute force

A straight forward approach, based on direct approach

No Restriction

Closest Pair Problem



$O(n^2)$

$d_{min} \leftarrow \infty$

for $i \leftarrow 1$ to $n-1$ do

 for $j \leftarrow i+1$ to n do

$\text{desqrt}((x_i - x_j)^2 + (y_i - y_j)^2)$

Convex Hull Problem

Exhaustive Search

- Potential Solution
- Feasible / not
- Solution

Traveling Salesman Problem.

$\Theta((n-1)!)$

In brute force we will traverse every solution even after we find solution.

then "Pattern occur with shift" s

if $s < n-m$

then $t_{s+1} \leftarrow (d(t_s - T[s+1]h) + T[s+m+1] \bmod q)$

HashCode Q) 13240128
↓
401

$$\boxed{132} = 401$$

$$\boxed{324} = 401$$

Hashfunction

$$\frac{1^*10^2 + 3^*10^1 + 2^*10^0}{8^*10^2 + 2^*10 + 4^*10^0} \rightarrow 10 [3 \times 10^1 + 2 \times 10^0] + 4 \times 10^0$$
$$10 [t_0 - 1^*10^2] + 3^*10^1$$

$$(10) t_{s+1} = d [t_s - T(s+1)] * h$$

Rabin-Karp Algorithm

Q) When this algo. is applicable?

This algo is applicable where some pattern is repeated again and again.

Rabin-Karp uses mod value with prime number.

x = Length of pattern

$x \bmod (\text{prime})$

Spurious hit - When we get same hash value but value is different (tex) not match

Q) Why we take prime number?

If we take normal random number then more chances of spurious hits are there, but not with prime number.

- Prime number giving the largest value will be selected

$m \leftarrow \text{length}[T]$

$m \leftarrow \text{length}[P]$

$h \leftarrow d^{m-1} \bmod q \quad // \quad d = \text{Radix}, \quad q = \text{Prime}$

$p \leftarrow 0; \quad t_0 \leftarrow 0$

for $i \leftarrow 1$ to m

do $p \leftarrow (dp + P[i]) \bmod q$

$t_0 \leftarrow (dt_0 + T[i]) \bmod q$

for $s \leftarrow 0$ to $n-m$

do if $p = t_s$

then if $P[1..m] = T[s+1..s+m]$

Hash code we use is known
a Rolling Hash Function

\hookrightarrow Base value, what type values are being
decimal 10, hexadecimal = 16

Case 3: If $\log_b a < k$,

If $b \geq 0$, $T(n) = \Theta(n^k \log^p n)$

If $b < 0$, then $T(n) = \Theta(n^k)$

Eg $> T(n) = 2T\left(\frac{n}{2}\right) + n^2 \log^{-1} n$

$$\begin{cases} a = 2 \\ b = 2 \\ k = 2 \\ p = -1 \end{cases}$$

$$\begin{cases} n^2 \\ \log n \end{cases}$$

Case 3, 2nd part
will be used

Masters Method for Decreasing

Condition: ~~$a \leq 1$~~
 $b > 0$
 $k \geq 0$

Case 1: If $a \leq 1$, $T(n) = \Theta(n^k)$

Case 2: If $a = 1$, $T(n) = \Theta(n^{k-1})$

Case 3: If $a > 1$, $T(n) = \Theta(n^{m/b} * f(n))$

Merge Sort with Master theorem

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

$$\begin{array}{ll} a = 2 & b = 2 \\ c = 1 & d = 0 \end{array}$$

$$\begin{array}{l} \downarrow \\ n^k \not\propto \log^{p+1} n \end{array}$$

$$\downarrow \\ n \log n$$

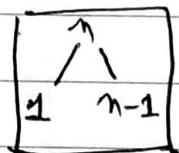
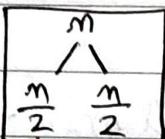
Knapsack Problem by Exhaustive search

The assignment Problem

Master Method for Dividing Function

$$T(n) = aT(n/b) + f(n)$$

$$\text{where } f(n) = \Theta(n^k \log^p n)$$



Dividing
method is
used.

Decreasing

Eg $\rightarrow T(n) = 2T(n/2) + n^2$

$$a = 2$$

$$b = 2$$

$$R = 2$$

To apply master theorem there are 3
condition to satisfy :-

$$R \geq 0$$

$$a \geq 1$$

$$b > 1$$

There are 3 cases :-

We will compare 2 value by ~~$\log_a n$~~ $\log_a n$

Case 1 : If $\log_b a > R$

$$T(n) = \Theta(n^{\log_b a})$$

Case 2 : If $\log_b a = R$

$$\rightarrow \text{If } p > -1, T(n) = \Theta(n^R \log^{(p-1)} n)$$

$$\rightarrow \text{If } p = -1, T(n) = \Theta(n^R \log \log n)$$

$$\rightarrow p < -1, \Theta(n^R)$$

Counting Sort

2	5	3	0	2	3	0	3
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- ① We will make new array $C[0 \dots k]$
- ② for $i = 0 \rightarrow k$
 $C[i] = 0;$
- ③ for $j = 1 \rightarrow n$
 $C[A[j]] = C[A[j]] + i$
- ④ for $i = 1 \rightarrow k$
 $C[i] = C[i] + C[i-1]$
- ⑤ for $i = n$

Radix Sort

Pick least significant first and arrange them on list.

- Q) Perform multiplication of given large integers
~~9577500~~ ... 957, 9873

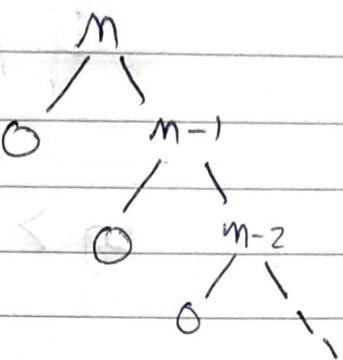


Recurrence eq. for Merge Sort

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n), \quad T(1) = 0$$

Quick Sort

Worst case \Rightarrow



$$\text{Then, } T(n) = T(n-1) + n$$

$$a = 1$$

$$b = 1$$

$$n = 1$$

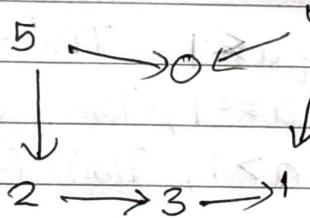
Bucket Sort

$$O(n+k)$$

Time complexity

Space complexity
 $O(\max \text{ element})$

Topological sort



Step 1

$0 \rightarrow$	
$1 \rightarrow$	
$2 \rightarrow 3$	
$3 \rightarrow 1$	
$4 \rightarrow 0, 1$	
$5 \rightarrow 2, 0$	

Stack \rightarrow

0	1	3	2	4	5
---	---	---	---	---	---

\hookrightarrow we add 3 first because it was dependent on 2 (DFS)

\bullet we will fill each element 1 by 1

In graph \Rightarrow Time complexity = $O(v+e)$

vertex + edge