How to Prepare *for*

Quantitative Aptitude *for the* CAT

COMMON ADMISSION TEST

FOURTH EDITION

ARUN SHARMA

TATA McGRAW-HILL'S WE SERIES

How to Prepare for

Quantitative Aptitude for the CAT

FOURTH EDITION

Arun Sharma
Educational Consultant

Tata McGraw Hill Education Private Limited
NEW DELHI

McGraw-Hill Offices
New Delhi New York St Louis San Francisco Auckland Bogotá Caracas
Kuala Lumpur Lisbon London Madrid Mexico City Milan Montreal
San Juan Santiago Singapore Sydney Tokyo Toronto

PREFACE TO THE FOURTH EDITION

With the CAT going online, we felt the need to create a comprehensive and updated book that caters to CAT aspirants. Some salient features are given below:

1. Questions from this book: Over the past 5-7 years, it has been noticed that a minimum of 10–20% questions in CAT and other major management entrance examinations have been directly taken from the questions provided in this book. Furthermore, it was seen that between 2005 (when the book was first released) to 2008, 90–90% of the questions in CAT and other top management entrance tests were covered in this book.

The CAT having gone online saw no change in this trend. Many questions in each of the 22 test papers that the CAT administered in its online avatar in 2009 have been covered in this book.

In fact, 2009 can be said to be a watershed year for CAT preparation because this is the first year when there have been multiple CAT papers to study and base our writing on. So while in previous years we would have to second guess what the CAT would do based on just one test paper (the which we had access that year), we are now richer by 22 test papers when it comes to understanding what we need to generate our readers for their preparation. It is on the basis of this rather rich insight that we have based the changes in this edition. (Note: Similar changes have been incorporated in my other books *How to Prepare for Data Interpretation and Logical Reasoning for CAT* and *How to Prepare for Verbal Ability and Reading Comprehension for the CAT* published by Tata McGraw Hill.)

2. Not too many changes in the pattern: Looking rationally into the paper patterns of the 22 CAT papers in 2009, there have not been too many changes with regard to the pattern of the examination as compared to previous years. While the difficulty level of the QA section varied from one paper to another (which was to be expected really because it is close to impossible to create two test papers with the same level of difficulty), what you would do well to keep in mind while preparing for the CAT is that the QA section continues to remain a thorough test of your academic intelligence. Thus, although on the first few days of the online CAT in 2009, some of the papers had a lot of easy questions (that you would expect to see in less tougher exams), on the later days the quality of the questions and the difficulty level of the papers had become pretty much "CAT Standard". As we head into the future with CAT 2010 and beyond, it is naturally expected that the IIMs would ensure that they maintain the quality standards associated with this prestigious examination. Hence, the LOD 1, LOD 2 and LOD 3 scheme of questions followed in this book is even more relevant now than ever before.

In fact, by 2012, the CAT is expected to shift to an adaptive format like the GMAT where every test-taker would get a different set of questions. In the adaptive format in the future, question banks of varying difficulty levels would be loaded into the computers and questions would appear for the student one by one. The difficulty level of the next question would increase if the previous one has been answered correctly and vice versa.

In order to do this, the examiners would need to build a database of questions which would be parallel to the LOD scheme followed in this book. These factors make this book and its content within all the more relevant for CAT aspirants in 2011 and beyond.

3. The need for greater variety in your preparation: Prior to the CAT going online, preparing for QA used to be a battle for Blocks 1, 4, 5 and 6. Even out of these, if someone did Blocks 1 and 5 well, he stood a strong chance in QA section.

However, as explained in detail in the introductory note to the online CAT, the new avatar of this exam requires the aspirant to be more well-balanced in the content of portion coverage.

With these things in mind and also keeping in mind the fact that past users of this book have added the greater and more detailed solutions, following are the major highlights in this book.

Major Highlights:

1. A comprehensive solution to most questions in all LODs of all chapters.
2. Based on all experiences of any type we have missed in any chapter, we have introduced extra questions for the sake of completeness in the form of additional block-wise practice sets. The questions in these sets have been carefully selected to ensure that we do not miss out on any possible question type.
3. An attempt to set up on the QA section of the online CAT to give you a holistic view of how to approach the online CAT from the perspective of the examiner.

The book is totally in sync with the new trend and pattern of the examination.

Ultimately, the endeavour is to provide a one-stop solution for CAT and MBA exam aspirants to tackle the QA section of all major management entrance exams—an endeavour we feel we have managed to do pretty well.

Through this book, we are confident of giving you—the reader—an invaluable resource for enhancing your QA section significantly. Contained in this book is the very best advisory for each and every question type. Your job is simple—to ensure that you follow the process contained in this advisory.

KEY POINTS FOR YOUR PREPARATION

Outline and Strategy

The first issue I would like to deal with here is to focus on helping you with the formulation of your strategy with respect to the syllabus to be covered for the Quantitative Aptitude section of the various management entrance exams including the CAT, XAT, FMS, CET, IPT and other examinations.

Let us start by trying to understand some of the key areas in Quantitative Aptitude (QA).

Tackling Each Portion

My experience shows that very often students look at the vast number of chapters and concepts to be studied for QA and get exhausted. This is especially true for students who do not have a strong traditional background in Mathematics, India. If you want to work out with a chapter-wise approach, you can easily outline the course to be studied by dividing it into 20 chapters—preparation for which is such a long-drawn effort that it ends up draining the student's energy, enthusiasm and motivation.

It is in this context and for this precise reason that I have divided this book into 7 manageable blocks—the approach being rationalising the chapters and grouping them according to the amount of shared concepts these chapters have amongst each other.

The order as defined in the index in this book would divide your work into 7 major areas to prepare for. For your convenience and strengthening we have put down the relative importance of each of these seven blocks into perspective.

Block 1: Number Systems and Progressions

Importance: **Very High** for CAT, XAT, IPT, FMS & **High** for MAT, CET, SNAP, IRMA, etc.

Block 2: Averages and Alligations

Importance: **Low** for QAs in CAT, XAT, IPT, FMS but **High** for Data Interpretation as a lot of questions in DI are based on the concepts of averages and alligations. Also **High** for MAT, CET, IRMA, NMIMS, etc.

Block 3: Percentages, Ratio Proportions, Time and Work, Time Speed and Distance. (Subsidiary but utmost importance chapters in this block : Interests, Partnerships and Profit and Loss).

Importance: **Moderate to High** for QA in CAT, XAT, IIFT, FMS and **Very High** for Data Interpretation (DI) as DI is almost entirely based on the concept of Percentages and Ratio and Proportions. **Very High** for MAT, CET, IRMA, NMIMS, etc.

Note: *The chapter of "Time, Speed and Distance" is extremely important for these exams (especially for the CAT as this chapter has been a constant presence in the CAT for almost a decade.)*

Block 4: Geometry and Mensuration.

Importance: **Very High** for CAT, XAT. **Average** for MAT, CET, SNAP, IRMA, IIFT, FMS, etc.

Block 5: Functions, Inequalities, Logs and Quadratic Equations

Importance: **Very High** for CAT, XAT, **Low** for MAT, CET, IRMA, NMIMS, etc.

Block 6: Permutation and Combination and Probability

Importance: **Very High** for CAT, XAT, FMS, IIFT etc. **Average** for MAT, CET, IRMA etc.

Block 7: Set Theory, Coordinate Geometry

Importance: **Low to Moderate** for all exams. Set Theory is used for DI in CAT while coordinate geometry shows itself up in the IIFT exam.

Based on the experience of the online CAT, the strategic preparation imperative for you should be to do at least 4 blocks and if possible up to 6 blocks "really well".

What does it mean to prepare a block "really well"? This is something we feel needs emphasis here.

Well what we mean to say is that do not just focus on studying the theory in each of these areas but develop an intuitive knowledge of all problem scenarios which emerge out of each block.

Only then would you be able to reach a situation in the exam – that when the question presents itself to you in the exam – you would have had the logic for the same worked out before hand. This is something that can make a huge difference to your chances in the CAT.

Analysing Your Knowledge Level

The first thing you need to focus on is an analysis of your knowledge level in each of these seven parts. In each of the above areas, first analyse your level of knowledgeability. In order to do so the typical question you should ask yourself is: For the next 100 questions I face in each of these areas, how many would I be able to handle comfortably?

Think of a number as an answer to this question for each of the seven blocks.

Based on your answer, the following analysis would provide you a thumb rule which would tell you how much of a knowledge base you have:

1. 90+: You know pretty much every question type and variant in the area. You should focus your energies on other aspects rather than knowledge improvement in the area.
2. 80+: Maybe you need to increase your exposure to questions a little bit; around 200–300 more questions in this area would be sufficient.
3. 60–80+: You have a significant knowledge base in the area. You might need to go back to the basics, but it is less likely to be a theory issue but more of an exposure to questions issue.
4. <60: You need to work on both theory and exposure to questions.

Needless to say, the target and objective for preparations has to be to reach the 90+ range as explained above in any block you intend to do "really well".

Looking beyond Ability (Quick Reflexes)

A common frustrating experience for test takers while taking the test is to not being able to solve a known question/logic and subsequently, not being able to score marks in questions which they know.

In order to handle this problem, you would need to work on your reactions and reflexes when faced with QA questions. Once you have solved your knowledge/ability issue in a particular block, your next step is to improve your reactions and reflexes while solving a question. Needless to say, you would need to do this block wise.

So obviously the main issue is how to improve reflexes and reactions.

(a) For every block, once you have solved the LODs and the Pre-Assessment/Review tests, the most crucial exercise is the content would be a comprehensive revision and review of each and every question you have solved in that block. Take every question of every LOD and Pre-Assessment/Review test again and review the logic/process of problem solving used. This need to be done to the point where you almost "recollect" the logic of the question and are able to recognise the same if it is used again in a different context/problem.

(b) A thorough revision on the theory of the block.

1. Improve your ability to select what you know and leave what you do not

In the context of an examination where the required scores for 99 percentile would be 60–70% attempts with 100% accuracy, it is easy to see that perfect knowledge is perhaps not needed in order to crack the CAT. Hence, even if you have around 60–70% knowledge of the questions in an average test, you are perhaps good enough to crack the exam. A good way to test whether you have sufficient knowledge would be to pick up 10–20 test papers and divide your QA section into blocks of 5 questions each. Then test your knowledge by looking at the average number of questions you know. If on an average for every 5 QA test questions that you pick up, if you know more than 3, then the proposed would be that you have adequate knowledge for cracking the CAT. Thus, while you may want to score towards knowing 5 out of 5 in this context, there are other things that you should focus on—developing your ability to decide on whether you are going to be able to solve a question while reading it for the first time. This would help you stop fishing during the test. (Fishing can be described as the activity of trying to solve a question without knowing whether you would actually complete the question.)

Your mind should give you a clear indication of whether you would be able to do the last step in a question, before you start doing it. In the sense you should be able to clearly define three types of outcomes when you finish reading a question for the first time:

(a) **I see a clear flowchart** and the steps are manageable—Obviously you need to go on and solve these questions.

(b) **I see a clear flowchart** but the steps are too lengthy—In this case you need to see where you stand in your test-time and strength wise.

(c) **I do not see a clear flowchart but I can try as I see a starting point**—This is potentially the most dangerous situation for you in the duration of the test, as once you get sucked into a question, there is a strong tendency to lose track of the time you are losing up while trying the question. Our advice is that while taking the test you should not over stay during such questions.

(d) **I see no flowchart and no starting point to the question**—Obviously you should leave such questions and in fact if there are limited to around 20–30% of the paper there is no problem and you need not worry about them.

2. Focus on thorough knowledge of 'problem scenarios' rather than theoretical learning

In Endeavor this, we would like to start with a few examples.

Consider the following string of 3 questions. Before I come to my main point here, I would like you to start by solving these questions before looking at the explanation provided.

1. A boy starts adding consecutive natural numbers starting from 1. After some time he reaches a total of 1000 when he realises that he has double counted a number. Find the number double counted.

2. A boy starts adding consecutive natural numbers starting from 1. He reaches a total of 575 when he realises that he has missed a number. What can be said about the number missed?
3. Find the 288th term of the series AB8CCCDDDDEEEEFFFFFG...

We can now start to look at each of these 3 questions:

1. Consider the fact that when you add numbers as stated above (1+2+3+4+.....) the result is known as a triangular number. Hence, numbers like 1, 1+2=3, 1+2+3=6 and so on are triangular numbers. This question tries us to consider the possibility of making the mistake of double counting a number. So instead of 1+2+3+4 if you were to do by error 1+2+3+3+4 you would realise that the number you would get would be 13 which would be more than 10 (which should have been your correct addition) and less than 15 (the sum of 1 to 5) which is the next triangular number. And the double counted value could be achieved by spotting 10 as the immediately lower value—and the difference between 10 and 13 would give you the required double counted number.

   To carry forward this logic, into the given questions, we should realise that we are just bothered about finding the last triangular number below 1000—and in trying to work this out is where we really apply our intelligence.

   Before one writes about that though, one fully realises that a lot of readers (especially aspirants with an engineering background at this point are thinking about n $(n+1)/2$. Knowing that process, one chooses to write about the alternate way to think about in this question.

$$1 + 2 + 3 + 4 \ldots + 10 = 55.$$

   Hence, we can easily see that 11+12+13+14+15+...+20 would equal 155 and the sum of 21 to 30 would equal 255 and so on.

   Thus, in trying to find the last triangular number below 1000 we can just do: 55+155+255+355 = 820 (which is the sum of the first 40 natural numbers) and since we have still not reached close to 1000 we start by adding more numbers as: 820 + 41 + 42 + 43 + 44 = 990 and the difference between 990 and 1000 is 10 which is the required answer.

2. For this question we would just need to carry the learning from the previous question forward and realise that when we miss a number, we actually get a total which is lower than the correct total. Hence, if we want to find the number missed all we need to do is to find the first triangular number greater than 575. This can be got simply by 55+155+255+311+32+33+34 = 595, so the number missed has to be 20.
3. In this question, all you would need to notice is that in the series ABBCCCDDDDEEEEFF...

   A ends after the first term; B ends after the third (1+2) term; C ends after the sixth (1+2+3) term and so on. So we can infer that what we are looking at is how many numbers need to be added before we get to a number just below 288. So 55 + (56+57+58+59) gives us 276 which pretty much means that the 28th alphabet (i.e. z) would be running in this series when we reach the 288th term.

   So looking at the three questions above and the solutions, one wants the reader to only answer one specific question:

   How much does knowing the first question and developing your thought ability and your intelligence help you in solving the second and the third case? I hope you see the connection. For your information, the three questions presented above were asked in CAT 2000, CAT 2002 and CAT 2004!!!

CONCLUDING NOTE

You sit in front of your CAT question paper and the first question comes in front of you. If you have identified the logic of the question or seen the question itself earlier, your entire QA preparation is fructified. In fact, every question/logic that you write here is your task which you have seen earlier represents a triumph of your preparation process. It is for this very reason that you prepare for an aptitude exam like the CAT. Any other preparation is quite worthless.

Your battle for CAT would be won if you get a "YES I KNOW THIS PATTERN/LOGIC" reaction to 50-60% of the questions in your test.

Contained in this book is the finest collection questions which you would hope to find anywhere. Remember, each question solved needs to be a learning experience—one that is to be kept in your mind for future problem solving. Adopt this approach with the problems contained in this book and we are quite confident that you would KNOW over 50% of your actual CAT in a few pages since you have already solved something like that before!!

All The BEST !!!!!

ARUN SHARMA

PREFACE TO THE FIRST EDITION

Over the last few years, as a trainer of CAT and other aptitude tests, I have felt the need for a comprehensive book on the subject. Students appearing for the CAT and other aptitude tests usually struggle for appropriate study material to prepare for this vital section of the examination.

This book comes as a humble attempt to fulfil this gap.

Structure of the book

The book is divided into 19 chapters and five test papers. Each chapter is divided into three broad parts:

(a) Theory
(b) Solved examples
(c) Chapter end exercises (LODs I, II & III), with answer key

The questions in the chapter end exercises have been categorised into three levels of difficulty, viz. Level of Difficulty I, Level of Difficulty II and Level of Difficulty III.

Level of Difficulty I (LOD I): These are the basic types of questions pertaining to the chapter. A majority of the MBA entrance tests would test the students with LOD I questions. Tests which ask LOD I questions include MAT, DMT, IRMA, IIFT, NIFT, CET Maharashtra, Bank PO examinations, BBA, BCA, Law, and so on. Besides, there are about 10 questions of LOD I type in the CAT nowadays.

Level of Difficulty II (LOD II): These are questions, which are more advanced than the LOD I questions. These questions test all basic as well as applied concepts in the chapter. *LOD II questions are closest to the difficulty levels of the CAT.* Hence, the objective of LOD II questions should be to:

(a) Clearly understand the concept which underlies the question.
(b) Create a judgment of time required for different mental processes.
(c) Identify the time puzzles.
(d) Reinforce application of a method to mental processes through the question.
(e) Learn to flowchart complex questions.

Level of Difficulty III (LOD III): LOD III questions build on the previous questions and are a step beyond the LOD II questions. Although they are also normally more difficult than the average CAT question, approximately 5–10 LOD III questions could be asked in the CAT every year. Hence, the learning objectives at LOD III are to:

(a) Learn applications of the basic concepts at the highest level.
(b) Sharpen the flowcharting skills learnt at LOD II.
(c) Use each question as a learning opportunity.

One should not be disheartened if he/she is unable to solve LOD III questions. These questions are extremely tough and uncommon in the CAT and other aptitude tests. Questions in actual tests will appear very simple and elementary if one can solve LOD III questions.

Approach Taken in Writing This Book

In my experience, the math skill of students appearing for CAT can be classified into three levels:

Level 1: Students who are weak at Mathematics
Level 2: Students who are average at Mathematics
Level 3: Students who are strong at Mathematics

This book has been written keeping in mind all the three levels of students.

From my experience I have given below my perspective of what one should aim for (based on the category that he/she belongs to). It is important to clearly understand the starting level and accordingly define strategy for the QA section.

Level 1: Students who are weak at mathematics: Typically, these are students who were weak at mathematics in school and/or have left mathematics after their 10th or 12th class. They face a mental block in mathematics and have problems in solving equations. They also have severe problems in understanding mathematical language and are unable to convert the mathematical language into mathematical equations. They make mistakes even in interpretation of the most basic statements in mathematics (leave alone the complex statements). Besides, these students also have problems in solving equations. They suffer from the insecurity of knowing that they are unable to solve most problems which they face.

Level 2: Students who are average at mathematics: These students lie between the Level 1 and Level 3 students.

Level 3: Students who are strong at mathematics: These are the students who have got strong, structured and logical thinking ability. They not only understand the basic repetitive statements in mathematics but also complex statements. They are able to create their own flowcharts to arrive at solutions of these complex mathematical situations.

There are two alternative approaches that a student can take in solving this book.

Approach 1: "Start with basic concepts, solved examples then move on to LOD I, then LOD II in the chapter. Go ahead into LOD III in the chapter in the first go. Complete all 19 chapters and then re-start with Chapter 1 – review the basic concepts, resolve LOD I and LOD II, then move on to LOD III. This approach is advocated for students who are weak to average in mathematics (i.e. students of Level 1 and Level 2).

After completing the theory and practice exercises of the book for a second time, go to the practice sets 1-5 provided at the end of book. Set a time limit of 40 minutes for each set and take the tests. The questions contained in the sets are questions which have appeared in the CAT over the last 5 years (based on memory)."

Approach 2: "Start with the basic concepts, solved examples and then go through the exercises of LOD I, LOD II and LOD III. This is recommended for students who have strong concepts in mathematics (Level 3 students)."

Then go to the 5 practice sets given at the end of the book and take them one by one (time limit of 40 minutes for each test)."

An Important Point

Each of the questions contained in the LOD I, LOD II and LOD III exercises in the chapters have immense learning value. Hence, the approach that one takes while solving the questions should be one of learning. The reader should try to clearly understand the interpretation of each sentence used in the construction of the questions.

In other words the learning in every chapter should not be restricted to the solved examples or the theory contained in the chapter but should continue through each of the questions contained in the exercises.

In conclusion, this is a book which is unique in approach and coverage. Any CAT aspirant who goes through the questions contained in this book in the manner advised in this book would get a distinct advantage when he/she faces the CAT.

Akin Sinha

ONLINE CAT: FROM THE QA PERSPECTIVE

Welcome to the world of online CAT!

The advent of the online version of the Common Admission Test (CAT) in 2009 brought with it a whole lot of opinions and views about what has changed in the examination and what should be the ideal preparation pattern. Therefore, one objective in this revised edition of this widely read book is to look at the issues that an aspirant needs to consider while preparing for the online CAT. I would like to discuss this issue in the following parts:

1. *What has changed?* A comprehensive analysis of what are the critical dimensions of the changes that have taken place in the CAT in its online avatar and what it means for the aspirant, both in respect of positive and negative factors, taking into account the following:
   (a) Changes in the Test-Taking Experience
   (b) Changes in the Exam pattern
   (c) Changes in the Marking process
2. *What does all this mean for the Preparation Process?* How it should change in the context of an online examination and how has it remained constant- whether online or paper-and-pen?

While doing so I have taken the help of a varied experimental sample of test-takers from across India and also my own personal experience of taking (and may I add dominating) the CAT. Given below are some of the implications of the online version of the CAT in the context of the Quantitative Aptitude section (which this book is all about).

1. WHAT HAS CHANGED?

The 'Experience' of Taking the Test

1. Cleaner & More Efficient: Compared to the paper-and-pen based CAT, the online version is much cleaner as the clarity of questions, their visibility, as well as the overall feel of the question solving experience is much better. Consequently, the efficiency (of the thought processes) is much higher leading to a much superior test-solving experience.

2. Space Management on the Table: In the paper-and-pen version, the aspirant had to typically manage the test paper, admit card, watch, pencils (at least 2), eraser, sharpener as well as the answer sheet on the table. To add to their woes, the paper-and-pen versions of the exam were mostly conducted in schools. Very often the aspirants had to contend with the additional challenge of managing all this paraphernalia on a school boy's small table. In addition, if luck did not run your way and you were made to sit in a classroom meant for juniors, (between classes 5 to 6) you really had a challenge.

Most of these problems have disappeared in the new version. The fact that computer terminals at most colleges and universities are of standard shape and size eliminates the imbalance created due to non-uniformity. Besides, while writing the online version of the CAT, all you need to manage on the table are the mouse, the key board, a pencil and a sheet of paper for rough work, no watches, erasers, sharpeners and more importantly, no test paper and no answer sheet.

3. Moving Questions in the Test: Unlike the paper-and-pen version, where test-takers could scan the whole question paper in one look, in the online CAT, aspirants had to move one question at a time. This had both its advantages and disadvantages in terms of the overall test experience. The obvious disadvantage that most aspirants faced was the fact that when you could not really see the whole paper in one look, you could not make a judgment about the balance, the difficulty level or the pattern wise question distribution in the paper. (Although we are referring to the quantitative aptitude section here, this was also true for all the sections in the exam)

Ironically, the biggest advantage for the examinee in terms of our online CAT was exactly the same as, since you could see the paper entirely at one go, the only option while taking the test was to look at the questions one by one.

This turned out to be a huge advantage because of two main reasons mentioned below:

Higher Focus while Solving an Individual Question: Not knowing the exact pattern of questions from various areas and being able to estimate the difficulty level of the paper, left individuals with no choice but to focus on the one question that was visible to them on the screen. The result was that achieving the all important 'tunnel vision' while solving a question was much easier. The immediate result of this was that the focus on the 'problem at hand' was infinitely more in the online test than in the conventional paper-and-pen format. Thus, ironically, not knowing the pattern of the paper resulted in students their best chance to solve a question.

The main reason for this was that while solving the question in front of the computer screen the experience of the previous question was totally blocked out. In the paper-and-pen version, students who had a negative experience while solving a question or two carried that negativity to the next question.

Thus the specific advantage of the online version was that 'forgetting' a bad experience was relatively easier. The moment you navigated away from the question in front of you, it went away from your mind as well. So much so, that navigating back to a question that was just two questions back was close to impossible. Naturally the 'carry over' emotions from a previous question to experience were significantly reduced.

The Imperative for Faster Navigation (less time wasted on unsolvable questions): Since the examinees had not seen the entire paper right at the beginning, the imperative to move to the next question was extremely strong. This resulted in students moving a higher percentage of the questions in the online test than in the paper-and-pen version.

Author's Note: One of the problems I had noticed in the paper and pen version was that most examinees were totally unable to 'see' the entire paper i.e. the fraction of the quantitative aptitude section that they were able to process was a function of the entire test paper. As a result they tend to move too fast on a large number of sitters! On an average, out of a 5-page question paper in quantitative aptitude, students were able to process at most upto 2-3 pages. So they would remain stuck on all easy sums on the pages they did not process. A lot of time would get wasted in questions that they thought they were unable to solve or even if they solved, they were unable to get them correct.

But of the time mismanagement also occurred due to the fact that they did not have the clock ticking on the screen in front of them. Therefore, they naturally lost track of how much time they had spent in attempting to solve a question. A good percentage of the time the aspirants used to spend in the QA section was spent in trying to solve a question which they were eventually unable to solve.

All this changed for the better in the online version. There was a greater imperative to move to the next question due to the extra facts that you had not seen the entire paper as you were moving from one question to the other, and the ticking clock was omnipresent in front of your eyes on the screen. As a result, you were aware of the exact amount of time you had spent on a particular question. The net result was that after trying a question for maybe 60 to 90 seconds, in case you did not have a clue about what to do in the same, you moved to the next question. This time management technique eventually for the examinee.

I believe this is one of the main reasons why a lot of students who were trying to compare the two versions of our CAT said that the online version was easier. Since the amount of time spent in questions which they were eventually unable to solve, reduced drastically, they got a feeling that they were moving questions all the time as opposed to the paper-and-pen version where aspirants used to have an overall negative experience of the test (as they would end up spending a lot of time in attempting "unsolvable" questions.

4. Mark/Unmark Button & the Review Button: A very important feature in the online version was the mark/unmark button of the REVIEW button. In the paper and pen version, it was extremely difficult to track the number of your attempts

especially so in the context of questions that you were unsure about and/or questions which you wanted to come back to. There was usually no way in which you could keep a track of those and as a result there was effectively 'no second chance' at a question.

This too changed in the online CAT. For every question, apart from the facility to answer it, you also had a MARK button, which would give you easy access to the questions at the end of the paper. When you have completed the paper (reached the last question in the paper), you also get access to a review screen that (in our view) showed you all the questions you had solved as well as all the questions you had marked with the MARK button. So going back to a specific question in the paper was just the click of the mouse away.

To sum up, the net effect of the online CAT was a superior test-taking experience — something that gives you a chance to be more in control of your test — and thus aim for a higher score assuming that the same set of questions would have been asked in the paper and pen version.

What has Changed in Terms of the Exam Pattern?

Having seen the specific changes that have occurred in terms of the test-taking experience, let us now examine another crucial aspect.

Changes in Exam Pattern: Obviously for the purpose of this book, the analysis will pertain to the QA portion only. In order to read a similar analysis with respect to the other sections namely, *Verbal Ability & Data Interpretation* you can refer to my book on those subjects, also published by Tata McGraw-Hill. The major changes in the pattern of the Quantitative Aptitude paper can be summarized through the following points:

1. More balanced portion coverage
2. Reduction in number of questions
3. Lack of uniformity
4. Higher percentage cutoffs

1. More Balanced Portion Coverage: As per the scheme followed in this book, the QA portion can be divided into 7 major parts (or blocks as I call them in this book).

The underlying constant that used to exist in the paper and pen version (through the entire decade prior to the first online CAT) was the prominence of Block 1 and Block 5. (Block 1 comprising Number Systems and Progressions and Block 5 comprising the chapters of Functions, Inequalities, Quadratic Equations and Logarithms.)

In each of the years from 1999 to 2004, the QA section required you to get a net score of approximately 30 – 40% of the total marks in order to score a high 90 percentile in this section.

In the light of this fact, the importance of Block 1 and Block 5 can be gauged from the table below:

Block	Weightage (as a % of total marks)
Block 1	30 – 35%
Block 5	15 – 20%
Combined weightage of Blocks 1 & 5	40 – 45%

Add to this, the chapter of Time, Speed and Distance with a minimum weightage of 5 – 10% and you pretty much had the QA section well covered. In a nutshell, QA for CAT preparation had become "do 10 chapters well"...

However this scenario has changed in the context of the online version of the exam.

The balance of weightage of questions asked and each of the 7 blocks have become important. The segment of CAT online review saw a weightage distribution of the kind illustrated below.

Block	Total No. of 30 Questions
Block 1	5 – 6 questions
Blocks 2 & 3	5 – 5 questions
Block 4	3 – 6 questions
Block 5	4 – 7 questions
Block 6	1 – 3 questions
Block 7	0 – 2 questions

Reduction in Number of Questions – The second major change in the Q4 [illegible] Across 50 questions in the last session to 38 (at least 2000 to 2001) to 30 & then 25 in the last years of the paper and the number of questions has finally come down to 20 in the online version. Naturally this reduced the number of choices the aspirant had for having into questions.

For instance in CAT 2000 out of 50 questions, one needed to score 15 to get the level of [illegible] in CAT 2001 [illegible] could afford to score 20 questions. This scenario has now changed drastically as it stands from the table below.

Year	No. of Questions in Q4	Number of Slots	Cut off or highest number of marks	No. of Questions one could leave or Attempt accuracy	No. of Questions one could leave in Non-attempts
CAT 00	50	20	15-18	37%	[illegible]
CAT 2001-H	50	30	12-14	35%	[illegible]
CAT 2001	30	30	11-13	33%	[illegible]
2000-01	25	100	7-13	11%	[illegible]
Online CAT	20	40	11-00	9%	[illegible]

- [illegible] there is very little flexibility available in the online version now to leave any questions and yet [illegible] a good percentile score.
- The conclusion is the future is the random taking the CAT would have to really use the mathematical intelligence and attempt as many questions as possible in order to get a top percentile in the test.

Lack of Uniformity – The third major factor in terms of paper pattern was the lack of uniformity in the test paper. Different papers gave rise with differing difficulty levels. The papers on the first few days of CAT 2001 were quite simple, but after [illegible] that the most papers had a pretty good difficulty level.

[illegible] being discussed widely in the Pan Indian Scenario. A lot of reasons were assigned for the CAT committee and the selection of the more commercial, for fairness of the testing process.

The Conclusion – In the context of multiple papers with varying difficulty levels, how would the IIMs judge fairly between students who tried a high percentage of the questions in an easier test versus students who were able to solve a lower number of questions in a more difficult paper? The answer is this is simple really. Since the population size of each paper was probably large, the IIMs could easily define percentiles in each test independently (among) in all.

The key point to be noted here is that there are adequate statistical ways through which Institutions like IIMs can be made to understand the relative CAT aspirants. In reality what you need to worry about is preparing diligently and facing the test with a positive attitude.

Higher Percentages Globally – In the recent months, a great loss and is questions which may thought are [illegible] and [illegible] they could solve. The result – more students may plan to score high scores in the [illegible] percentile.

Consequently scores that used to be in the range of 50% of the cut marks now to around 40 – 45% of the marks.

Changes in the Marking Process

The key change that an analysis of CAT 2000 results showed us was that there was an increased emphasis on accuracy. Mistakes were heavily penalised. This was evident from the fact that two students solving the same test paper (December 3 evening slot) scored:

(a) 14 attempts 1 incorrect – score 98.23 percentile

(b) 19 attempts 5 incorrect – score 92.6 percentile

There were innumerable such examples where students solving more questions with higher errors scored significantly lower than students who attempted much less but got more correct.

Hence, the key learning for you while preparing is to focus on improving your accuracy as well as the belief in your process of solving. This is especially true while preparing for the QA section. While solving a QA question, you should be able to know that if your process is correct then so would your answer would also be correct. The need to check the answer to a QA question is something that is only required for minds used to Quantitative Aptitude. This is where an under-prepared aspirant loses out to the best—in the knowledge of whether what they are solving is correct or not.

Unfortunately, most students I see are more engrossed in seeing the answer to the questions as soon as they solve the same. This is a habit I would strongly discourage you from. The ideal preparation process for you should be:

(a) solve the question,

(b) review your process and tell yourself, "If your process is correct, so is your answer", and

(c) only check your answer after you have reviewed your process.

This is important, because when you are solving a QA question inside the CAT, you would not have the option to 'look' at the answer. The only thing you have is the question and the process you use in solving the same. Your mind should be able to tell you whether the answer you have got is correct or not. This is a key difference in solving questions in practice and solving them under exam pressure.

Hence, developing more confidence in your QA problem-solving processes becomes a key ingredient and objective of your preparation process for this section.

II. WHAT DOES ALL THIS MEAN FOR THE PREPARATION PROCESS? HOW HAS IT CHANGED AND HOW HAS IT REMAINED CONSTANT?

Let us look at this aspect in two broad parts:

1. What are the changes that need to happen in the preparation processes for the online CAT vis-a-vis the preparation process for the traditional paper-and-pen version?
2. What are the things and issues that remain constant in the preparation process?

1. For the first question, the specific things that come to my mind are:

   (a) **More balanced portion coverage needed:** As explained above, in the paper-and-pen version the best approach for Quantitative Aptitude preparation was "do 10 chapters well — really well". In fact, even 8 chapters done well were mostly sufficient to crack this section. However, in the new online version, since the weightage of distribution of questions is much more even, this approach is no longer going to work. Hence, the need to cover all aspects of the portion well and not ignore any particular portion is perhaps the first and the biggest change that needs to be done in the preparation process.

   (b) **Need to cover the basics well, namely, speedily solving LOD 1 questions and the ability to think through LOD 2 and LOD 3 questions:** In the early years (1990s and even the late 1990s), the CAT used to be essentially a speed test (including the QA section). There were times when the paper used to consist of upto 225+ questions to be solved

in 120 minutes. Questions used to be one-liners and could be solved in 1–2 steps. The key differentiation used to be the speed in which the aspirants could solve questions. However, from late 1990s onwards the QA section of the CAT had become a real test of quantitative intelligence. Questions ceased to be one-liners unless you had a very high degree of mathematical understanding and intelligence. The online CAT in its first year tended to be a mix of both these extremes. Papers consisted of between 4–6 one-liners topped up by LOD 2 and LOD 3 questions. While most aspirants found 4–6 very easy questions in each paper, they also had to really use their mathematical strengths to cross 10–12 attempts. In the future, as the IIMs improve the quality of the database of quantitative questions, one can expect the quality of the questions to improve drastically and hence the LOD 2 and LOD 3 questions contained in this book would be an extremely important resource to solve for maximising your score in this section in the exam. (In fact after the first 3–4 days of the exam in 2009, examinees taking the test on the subsequent days found the paper to be of really good quality.)

For the future CAT aspirant and the readers of my books, my advice is short and simple. Cover both the flanks—the short cuts and quicker methods to solve the easier LOD 1 questions and improving your mathematical and logical intelligence to cover the higher end questions of LOD 2 and LOD 3 level.

(c) **The need to take computer based tests in order to be able to think on the computer:** Thinking and solving questions from the computer screen is a slightly different experience than solving from a physical book. Thus students and aspirants are advised to experience this change by going for online solving experience. It is in this context that we have tied up with www.mindworkz.in to give our readers a feel of the online problem solving experience. However, in spite of these seemingly big external changes, my personal opinion is that the changes are mostly external in nature.

2. The essence of preparation of the Quantitative Aptitude section remains the same in a lot many fundamental ways. Some of these that come readily to mind are:

(a) **The need to develop mental structures for the CAT:** QA preparation has always been associated with the development of the mathematical thinking processes and thought structures for specific mathematical situations. The smart CAT aspirant is able to create mathematical thoughts in his mind to situations that he would encounter in the exam.

The whole battle for QA preparation in the CAT can be essentially summarised in terms of the quality of pre formed logic that you have to the specific questions that you are going to face in the exam. In other words, your battle in QA section is won if you have during your preparation process, encountered the logic to the question which is in front of you. Hence, the focus of your QA preparation process has to be on creating the logics for as many questions and mathematical situations as possible. You would have well and truly won your battle in the CAT if case you encounter 15 'known' mathematical situations out of the 20 questions in your set. Hence, **the imperative to form "thought algorithms" for standard and non-standard mathematical situations related to various chapters and concepts in the portion remains as strong as ever.** In fact, if anything this imperative is expanded to the entire portion base due to the wider and more balanced portion coverage in the exam.

(b) **The need for thoroughness in your preparation:** This is again something that does not change.

The key point you need to remember is that the CAT still remains a test of your intelligence and an aspirant should focus on this aspect. This book provides plenty of mathematical thinking situations and alternatives related to each and every part of this section that help you hone your skills in the QA section of the examination.

CONTENTS

[illegible] 5

[illegible] 7

BLOCK 1

[illegible] 1
[illegible] 2
[illegible] 3
[illegible] 4
[illegible] 6

1. [illegible] SYSTEMS 11

[illegible] 13
[illegible] 13
[illegible] 15
[illegible] 16
To Check Whether a Number is Prime or Not 18
A Brief [illegible] Turing Machine? 19
The Concept of U.D.S. Euclidean Computer (Euclidean machine) [illegible] 19
[illegible] 19
The Concept of UDS Linear Computer (ULC) 19
Rule for Finding the UDS [illegible] 19
Rule for Finding the ULC and UDS of Functions 19
HCF and LCM 20
Practice Problem 19
Complexity 21
[illegible] 22
[illegible] 24
[illegible] 24
[illegible] 27
[illegible] 28
[illegible] 28
[illegible] 29
[illegible] 29
[illegible] 30
[illegible] 30
[illegible] 30

Units Digit [illegible]
Miscellaneous Problems [illegible]
Counting [illegible]
Level of Difficulty (LOD) I [illegible]
Level of Difficulty (LOD) II [illegible]
Level of Difficulty (LOD) III [illegible]
Answer Key [illegible]

2 PROGRESSIONS

Arithmetic Progression [illegible]
To Find the Sum of the Given Number of Terms in an Arithmetic Progression [illegible]
To Find the Arithmetic Mean between two Two Given Quantities [illegible]
To Insert a Given Number of Arithmetic Means between Two Given Quantities [illegible]
Geometric Progression [illegible]
To Find the Geometric Mean between Two Given Quantities [illegible]
To Insert a Given Number of Geometric Means between Two Given Quantities [illegible]
To Find the Sum of a Number of Terms in a Geometric Progression [illegible]
Harmonic Progression [illegible]
To Find the Harmonic Mean between Two Given Quantities [illegible]
Theorems Related with Progressions [illegible]
Useful Results [illegible]
Miscellaneous Problems [illegible]
Level of Difficulty (LOD) I [illegible]
Level of Difficulty (LOD) II [illegible]
Level of Difficulty (LOD) III [illegible]
Answer Key [illegible]
Block Review Tests [illegible]

BLOCK 2

Matrices Formula [illegible]
Phase Assessment Test [illegible]
Block Mathematics Appendices [illegible]

1 AVERAGES

Introduction [illegible]
Theory [illegible]
Miscellaneous Problems [illegible]
Level of Difficulty (LOD) I [illegible]
Level of Difficulty (LOD) II [illegible]
Level of Difficulty (LOD) III [illegible]
Answer Key [illegible]

4. EQUATIONS 119

Introduction 120
Theory 120
Investigation Situation 120
Graphical Representation of Linear Equations 120
Use Straight Line Regression 122
Some Typical Situations Where Algebraic Can be Used 123
Some Ways to Solve A, B, and Mixed Differentiated Problems, and \ 124
A Typical Problem 125
Level of Difficulty (LOD) 126
Answer Key 126
Block Review Tests 127

BLOCK 3

141

Back to School 141
Preassessment Test 143
Sample Instructional Algorithm 147

5. PERCENTAGES 151

Introduction 151
Basic Definition and Uses of Percentages 151
Traditional Efficient Calculator for Percentage Calculations 152
Concept of Percentage Change 152
Difference between a Percentage Point Change and the Percentage Change 157
Percentage Rule for Calculating Percentage Values Through Addition 158
What is the Percent Change in the Increase in a Person's Value 161
Percentage Change Graphs and its Applications 163
Implications for Data Interpretation 166
Practice in Percentage Conversion Table 167
Some Limitations of the Table 167
Calculation of Multiplication by Percentage Like 1-25, 2-50, and so on 168
Review and Problems 168
Level of Difficulty (LOD) 168
Level of Difficulty (LOD) 169
Level of Difficulty (LOD) 169
Answer Key 169

6. TIME AND DATE 180

Introduction 180
Theory 180
Profit and Loss cases in Individual Transactions 180
Profit and Loss cases in Business and Commercial Transactions 187

Profit Calculation on the Basis of Costing the Amount Spent and the Amount Earned 198
Concept in Short cut 199
Worked out Problems 199
Level of Difficulty (LOD) I 201
Level of Difficulty (LOD) II 201
Level of Difficulty (LOD) III 202
Answer Key 202

7. INTEREST 203

Introduction 205
Concept of Time Value of Money 213
Terminology Pertaining to Interest 215
Simple Interest 217
Additional Simple Interest, Time, Rate Percent (i) Interest Performance and Total Interest 217
Compound Interest 217
Formula 217
Depreciated Value 218
Probability 218
Accumulated Interest in C.I. 219
Worked out Problems 220
Level of Difficulty (LOD) I 224
Level of Difficulty (LOD) II 227
Answer Key 232

8. RATIO, PROPORTION AND VARIATION 233

Introduction 234
Ratio 235
Some Important Properties of Ratios 236
Mathematical Uses of Ratios 237
Proportion 238
Variation 239
Worked out Problems 240
Level of Difficulty (LOD) I 242
Level of Difficulty (LOD) II 246
Level of Difficulty (LOD) III 248
Answer Key 250

9. TIME AND WORK 253

Introduction 255
Theory 255
Worked out Problems 257
The Concept of Reserve Time 260

Application of Position Coordinates Theorem to Time and Work [illegible]
Work Equivalent Method (To Solve Time and Work Problems) [illegible]
One Step for Correct Reasoning is Half Chosen as Problem of Work [illegible]
The Concept in Efficiency [illegible]
Element (s) Fitness [illegible]
Level of Difficulty (LOD) I [illegible]
Level of Difficulty (LOD) II [illegible]
Level of Difficulty (LOD) III [illegible]
Answer Key [illegible]

11. TIME, SPEED AND DISTANCE AND APPLICATIONS OF TIME, SPEED AND DISTANCE [illegible]

Introduction [illegible]
Theorem of TSD [illegible]
Concept of Relative and Relative Speed and Formulas of Distance [illegible]
Application of Relative in Time Speed and Distance [illegible]
Application of Time, Speed and Distance [illegible]
Level of Difficulty (LOD) I [illegible]
Level of Difficulty (LOD) II [illegible]
Level of Difficulty (LOD) III [illegible]
Answer Key [illegible]
Block Review Tests [illegible]

BLOCK 4 [illegible]

Back to Formal [illegible]
Assessment Test [illegible]

12. GEOMETRY AND MENSURATION [illegible]

Introduction [illegible]
Theory [illegible]
Straight Lines [illegible]
Polygons [illegible]
Triangles [illegible]
Equilateral Triangle [illegible]
Isosceles Triangle [illegible]
Right Angled Triangle [illegible]
Pythagoras Theorem and Related Results [illegible]
Quadrilaterals [illegible]
Types of Quadrilaterals [illegible]
Regular Polygon [illegible]
Circles [illegible]
Tangents [illegible]

Chance 388
Die 392
Mechanical Problems 395
Hints to Work in Geometry and Mensuration 397
Special Note About Old Questions 399
Geometric Level of Difficulty (LOD) I 399
Mensuration Level of Difficulty (LOD) I 399
Geometry Level of Difficulty (LOD) II 171
Mensuration Level of Difficulty (LOD) II 173
Extra Practice Exercise 399
Answer Key 400
Short Review Tables 408

BLOCK 5

Back to Basics 413
Assessment Test 418

11. FUNCTIONS 421

Some Methods of Representing Functions 421
Analytical Representation 421
Tabular Representation of Functions 421
Graphical Representation of Functions 422
Even and Odd Functions 422
Even Functions 422
Odd Functions 422
Inverse of a Function 424
Shifting of Graphs 424
Combining Movements 424
Inequalities 425
Translation Graphics Process for Exponential Functions 426
Logarithms 428
Graphical Plan of the Logarithmic Function 428
Wicked-up Problems 430
Level of Difficulty (LOD) I 438
Level of Difficulty (LOD) II 447
Level of Difficulty (LOD) III 448
Answer Key 452

12. INEQUALITIES 457

Properties of Inequalities 457
Properties of Inequalities 458
Sample Properties of Inequalities 460

Some Important Results 455
To Find the Greatest Value of a Product, the Sum of Whose Factors is Constant 457
Definition of Direction-angle Associated 457
Variation of Ranges 458
Solving Linear Trigonometric Equations 458
Solving Quadratic Trigonometric 459
Solution of Trigonometric Inequalities 459
Inequalities Containing Modulus 460
Inversion of Positions 461
Level of Difficulty (LOD) I 461
Level of Difficulty (LOD) II 465
Answer Key 470

14. QUADRATIC EQUATIONS 475

Introduction 477
Theory 475
Properties of Quadratic Equations and Their Roots 475
The Sign of a Quadratic Expression 475
Rational Algebraic Problems 476
Level of Difficulty (LOD) I 478
Level of Difficulty (LOD) II 479
Answer Key 480

15. LOGARITHMS 483

Introduction 485
Theory 485
The Properties of Logarithms 486
Laws of Logarithms 486
Exponentiation 486
The Characteristic and Mantissa of a Logarithm 487
Base Change Rule 487
Results on Logarithms Inequalities 487
Numerical Problems 487
Level of Difficulty (LOD) I 488
Level of Difficulty (LOD) II 489
Answer Key 490
Extra Practice Tests 491

BLOCK 6 499

Mock Tests 501
Practice Tests 504

10. PERMUTATIONS AND COMBINATIONS 211

Introduction 211
Fundamental Principle 211
Factorial Notation 214
Permutations 217
Combinations 221
Miscellaneous Problems 227
Level of Difficulty (Ex) 242
Level of Difficulty (LEX) 249
Level of Difficulty (LEX II) 249
Answer Keys 250

11. PROBABILITY 261

Concepts and Terminologies of Probability 261
Assigning Forces to Real Life Problems of Probability 267
The Case of Mutually Exclusive (MT) Events (Full No. 1) 270
The Case of Non Independent (NI) Events (Full No. 1) 281
Combination of MEE and DE 294
Bayes Theorem and Probability 297
Some Important Concepts in Other Existing Events 303
The Concepts of Terms in Conditional Equations 306
Distinction Between Ideas of Chance in the Probability Problems 309
Miscellaneous Problems 318
Level of Difficulty (LEX) 330
Level of Difficulty (LEX I) 332
Level of Difficulty (LEX II) 337
Answer Key 339
Short Format Tests 341

BLOCK 7

12. SET THEORY 379

Set Theory 381
Different Kinds of Representations of Sets 387
Types of Sets 393
Operations on Sets 402
Applications of Set Theory 407
Some Important Results 411
Some Important Results on Numbers in Elements in Sets 417
Miscellaneous Problems 420
Level of Difficulty (LEX) 424
Answer Key 430

9. COORDINATE GEOMETRY **[illegible]**

Cartesian Coordinate System [illegible]
Rectangular Coordinates [illegible]
Straight Line [illegible]
Level of Difficulty (LOD) [illegible]
Level of Difficulty (LOD) [illegible]
Level of Difficulty (LOD) [illegible]
Answer Key [illegible]

MCQs TEST PAPERS **[illegible]–[illegible]**

MCQs TEST PAPER **[illegible]–[illegible]**

BLOCK 1

CHAPTERS

- Number Systems
- Progressions

INTRODUCTION

[illegible]

[illegible]

[illegible]

[illegible] [illegible]

...BACK TO SCHOOL

- [illegible]
- [illegible]

[illegible]

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]

ASSESSMENT TEST

This consists of 20 questions based on the chapters of **a(OCE,OIE)** (Scalar Systems and Programming). Do your best in trying to solve each question.

The time limit to be dedicated to this test is 30 minutes. However, after the 30 minutes is over, continue solving till you have attempted each question and add a different colour to each question. After you have finished, review each and every question of the test, check your answers. Now go through the SCORE DETERMINATION ALGORITHM given at the end of the test to understand the way in which you need to approach the chapters inside this book.

1. The number of integers x satisfying $-x < 2 < x$ is and just 4 is
   (a) 0 (b) 1
   (c) 2 (d) 3
2. The sum of two integers is 10 and the sum of their cubes is 910. Find the larger of these integers.
   (a) 7 (b) 8
   (c) 9 (d) 6
3. If x is a positive integer such that $2x + 1$ is perfectly divisible by x, then the number of possible values of x is
   (a) 3 (b) 4
   (c) 6 (d) 1
4. Let K be a positive integer such that $x + 4$ is divisible by 7. Then the smallest positive integer x greater than 2 such that $x + 26$ is divisible by 7 equals
   (a) 9 (b) 8
   (c) 5 (d) 7
5. $2^{3} \cdot 2^{5} \cdot 2^{7}$ is the same as
   (a) 2^{15} (b) 2^{16}
   (c) 2^{17} (d) 2^{18}
6. How many distinct three-digit numbers can be formed using 1, 2, 3, 4, 5 and 6 without repetition?
   (a) 35 (b) 5
   (c) 11 (d) 7
7. x, y and z are three positive integers such that $x > y > z$. Which of the following is correct for the product xyz?
   (a) $xyz > 1$ (b) $xyz < 0$
   (c) $xyz = 1$ (d) $xyz = 0$
8. A positive integer is called a perfect square if it is an integer divisible by 36 and a positive integer other than itself and 1, and is a perfect square greater than 5. Then $2^{2} + 3^{2} = 13$
   (a) never divisible by 5.
   (b) always divisible by 6, and may or may not be divisible by 12.
   (c) always divisible by 12, and may or may not be divisible by 24.
   (d) always divisible by 24.
9. Mital shuffle some cards in Mathematics and Science. From a full pack of playing cards, she had the cards labelled and said to Monika: "If you give me a certain number of your cards, I will have five times as many cards as you will have. If I give you the same number of cards, I will have thrice as many cards as you will have." Of the given choices, which could represent the number of cards with Mital?
   (a) 9 (b) 24
   (c) 12 (d) 18
10. In Sirhan, each boy's quota of tickets to be sold is 80 times boys is not more than 200 per session. If he reduces the number of tickets purchased by 25, he can fill 3 more boys with the total number of tickets assigned to him. Which of the following is the possible number of tickets assigned to each boy?
    (a) 400 (b) 160
    (c) 125 (d) 75
11. Akash got an order from a garment manufacturer for 480 Denim Shirts. He bought 12 sewing machines and operated them with eight labour in 6 days. However, many labourers were lazy. As a result, each of them also did half as much. If one days than expected planned for Akash, with equal distribution of work, how many labourers from remained lazy and how many had not reported for work?
    (a) 12, 4 (b) 16, 3
    (c) 16, 4 (d) None of these
12. How many 3-digit even numbers can you form such that sum of the digits is 9, the following digit form be 9?
    (a) 9 (b) 405
    (c) 99 (d) 45
13. To find the number of ones of a given integer divisible by 9, we can define a process by which an integer is reduced as follows: $(x_1x_2x_3x_4x_5x_6x_7x_8x_9)$ are the digits of the number, starting from the most significant digit.
    $(x_1 \cdot 10^{8} + x_2 \cdot 10^{7} + x_3 \cdot 10^{6} + x_4 \cdot 10^{5} + x_5 \cdot 10^{4} + x_6 \cdot 10^{3} + x_7 \cdot 10^{2} + x_8 \cdot 10^{1} + x_9 \cdot 10^{0})$
    e.g. $234 = 2 \cdot 10^{2} + 3 \cdot 10^{1} + 4 \cdot 10^{0} = 2 \cdot 100 + 3 \cdot 10 + 4 = 234$
    (Formally, the resulting number will be which when divided by 9 gives a remainder equal to the sum of digits

After how many such steps, does the number 201 reduce to 0?

(a) 2 (b) 3
(c) 4 (d) 1

16. A three-digit number gives a remainder 2 when divided by 7. Interchange the digits of the number and add it to the original number. The sum is divisible by 7. Find the product of the digits of the number. Only one of the following pairs of numbers satisfy the description of the number. Which one is that?

(a) 11, 32 (b) 11, 67
(c) 10, 33 (d) 12, 30

17. If $x = 1.1 \times 2.1 \times 3.1 \times \ldots \times 10.1$ then $[x] + [x^2]$

(a) 10 (b) 9
(c) 8 (d) 7

18. Find the common integral value of x such that the distance that it lies is minimum.

(a) 100 (b) 101
(c) 8 (d) 47

19. [illegible] $2x - 5y = 5$ and $3x + 4y = 10$.

(a) 0 (b) 1
(c) −1 (d) 2

20. A positive integer is said to be a prime if it is not divisible by any positive integer other than 1 and itself. Let p be a prime number such that gives rise 3. Then, when $p^2 + 17$ is divided by 12, the remainder is

(a) 6 (b) 1
(c) 0 (d) 4

21. Consider the [illegible] Each [illegible] half of the [illegible] can be bought from [illegible] and [illegible] half the number of [illegible] the other for plus half a [illegible]. A second [illegible] and buys half the remaining number of [illegible] plus half a [illegible]. After this, the [illegible] the other [illegible] the [illegible] number?

(a) 2 (b) 1
(c) 4 (d) 14

22. [illegible] playing a game. Every one plays 10 [illegible] and each player [illegible] one point for a [illegible] and loses [illegible]. The person picking up the last coin loses. X starts. How many possible pick up is the [illegible] when [illegible] Y replies?

(a) 8 (b) 5
(c) 2 (d) 7

23. If $x \in R$, which of the [illegible] is always true?

(a) $x + |x| + 2x \geq 0$
(b) $x + 2|x| + 3x$
(c) $x + x^2 - x^3 \geq 0$
(d) $x + x^2 + 1$

24. The money order commission is calculated as follows. Every Rs. 4 or part for money orders [illegible] and [illegible] by 10. Get the product and add 5 [illegible] the result is X. The money order commission Rs. [illegible]. If a person sends two money orders for Rs. [illegible] and Rs. [illegible] for Rs. 11 and Rs. [illegible] respectively, the total commission is [illegible]

(a) Rs. 1.00 (b) Rs. 0.50
(c) Rs. 0.00 (d) Rs. 7.37

25. The area [illegible] the following [illegible] the metre reading. The meter reading is rounded up to the next higher multiple of 5. For example, if the meter reading is 47 [illegible], it is rounded up to 50 paise. The resultant is multiplied by 1.5. The final result is rounded off to nearest multiple of 10 paise. If 51 paise is the meter reading when A fill in the wheel [illegible]

(a) Rs. 6.75 (b) Rs. 6.90
(c) Rs. 6.25 (d) Rs. 7.50

26. [illegible] were playing simple [illegible] problem. Both [illegible] digit number and [illegible] Bhaskar [illegible] 11. Ashok [illegible] that the number [illegible] the product of [illegible] digits. What was the number [illegible]?

(a) 38 (b) 24
(c) 32 (d) 48

27. [illegible] (the maximum power of 2) [illegible] where $0 < x < 1$. $(x + 2x^2 + 3x^3 + \ldots)$. What will be the exponent of 2?

(a) 8 (b) 9
(c) 11 (d) [illegible]

Answers (Block 1 Permutations Sets)

1. (b)	2. (c)	3. (a)	4. (a)	5. (c)
6. (a)	7. (b)	8. (d)	9. (b)	10. (c)
11. (c)	12. (c)	13. (a)	14. (b)	15. (c)
16. (a)	17. (c)	18. (a)	19. (a)	20. (b)
21. (a)	22. (b)	23. (a)	24. (b)	25. (d)

SOLUTIONS (PREASSESSMENT TEST)

1. The only value that satisfies is [illegible].
2. [illegible] will give [illegible].
3. The possible values are [illegible], [illegible] and [illegible] (as the factors of 12).
4. Let the common of the ratio be [illegible]. Hence, the number the ratio of [illegible] will become [illegible]. The number will be divisible by [illegible]. The numbers [illegible] and [illegible] do not provide a [illegible] ratio.
5. [illegible] Hence option [illegible] is correct.
6. Solve through options.
7. The given value will be subtracted [illegible] the necessary change will be [illegible] which is largest number is added by one.
8. This is a property of prime numbers [illegible].
9. [illegible] Making [illegible] and [illegible] costs. On putting [illegible] and [illegible] Making the price would become [illegible] which is [illegible] and [illegible] and [illegible]. Hence, the price would become [illegible].
10. Looking at the options you realize that the [illegible] should be a multiple of 25 and 50 both. The option that satisfies the condition of increasing the number of times by [illegible] is [illegible]. This is found through trial and error.
11. [illegible] gives you [illegible] as the correct answer.
12. Hence, the number must be [illegible] and [illegible] be even as the same is [illegible]. The only numbers possible are [illegible], [illegible], [illegible] and [illegible]. Also, if [illegible] is even [illegible] the number one will get [illegible] numbers.
13. [illegible] Hence, clearly two [illegible] are required.
14. Two [illegible] can give [illegible] (because [illegible] is even) [illegible] [illegible] [illegible]
15. The solutions are defined as the [illegible] of the [illegible]. Hence, [illegible] is correct.
16. There are no common factors between [illegible] and [illegible]. Hence, the [illegible] should be [illegible].
17. [illegible]
18. [illegible]
19. [illegible]
20. [illegible]
21. [illegible]
22. Option [illegible] is correct since the average of any two numbers lies between the numbers.
23. [illegible]
24. The [illegible] will be [illegible]. Hence, [illegible].
25. The [illegible] will be given by [illegible].

(No [illegible] is required in the number system.) Chapter

SCORE INTERPRETATION ALGORITHM

Block 1 Preassessment Test

[illegible]

If You Scored: [illegible] (In Minutes Time)

Step One: [illegible] Back to School Series carefully. Grasp each of the concepts explained in that part carefully. In fact, [illegible] CBSE Class 8, [illegible] you feel necessary.

Step Two: [illegible] Block. [illegible] Number System. When you feel so, [illegible] and [illegible] of the concepts explained in the chapter.

Then [illegible] LOD 1 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1 of [illegible]. While doing so, do not [illegible] over the [illegible]. Once you feel [illegible] LOD 1, [illegible] the [illegible] and [illegible] questions.

Step Three: After finishing LOD 1 of [illegible] Number System of this Block, [illegible] the [illegible]. Thereafter, [illegible] followed by solving LOD 2 questions.

Step Four: [illegible] and [illegible] in the end of the Block and [illegible]. While doing so, [illegible] the score you get with the time [illegible]. This [illegible] is [illegible] the test [illegible] and [illegible] to evaluate the improvement in your [illegible].

If you [illegible] in your score [illegible], go back to the chapter, revise [illegible] and [illegible] of the LOD 2 questions in both the chapters.

Step Five: Move to LOD 2 and repeat the process that you followed in LOD 1—first in the chapter on Number Systems, then with the Chapter on Progressions. Concentrate on understanding each and every question and on understanding concepts.

Step Six: Go to the Hints (HINTS) section given at the end of the book and solve it. Again, while doing so pretend that you were sitting the provided exam hall. First and foremost try to solve the and further without using hints and try to evaluate the improvement that you have had in your score.

Step Seven: Move to LOD 3 and after you have attempted and understood each of the questions in LOD 1 and LOD 2, repeat the process that you followed in LOD 1—first in the chapter of Number Systems, then with the Chapter on Progressions.

If You Scored: 7–15 (In Unlimited Time)

Although you are better than the above following the instructions above, you really have a lot of scope for the improvement of your score. You will have to work hard on your concepts as well as good, heavily emphasis more on the practice and improve aspect of your preparation, then more you emphasis only speed development. The following process is recommended for you.

Step One: Go through the Hints and Hack to Solved Section carefully. Revise each of the concepts explained in that part. Going through your 5th, 6th and 10th attempt hours will be an optimal exercise for you. It will be recommended to take you second to eight digits, while if your score is in two digits. Choose the choice is yours.

Step Two: Move into the first chapter of the book, the Number Systems. Where you do not concentrate on clearly understanding each of the concepts explained in the chapter during.

Then move to the LOD 1 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1 of Number Systems. Once you finished solving LOD 1, revisit the questions and check and count your score.

Step Three: After finishing LOD 1 of the Number Systems, move into Chapter 1 of the Book, Progressions and repeat the process, viz. Revisit each concept carefully following by solving LOD 1 questions.

Step Four: Go to the Hints and solved section given at the end of the book and solve it. Again while doing so pretend that you are sitting the provided exam hall. First and foremost try to solve the and further without using hints and try to evaluate the improvement that you have had in your score.

Please be patient in your score. A Not confident in mind is the enemy of book (the chapter) and to solve LOD 1 and LOD 2 of both the chapters. With doing so concentrate more on the LOD 2 questions.

Step Seven: Move to LOD 2 and repeat the process that you followed in LOD 1—first in the chapter on Number Systems, then with the Chapter on Progressions.

If You Scored 15+ (In Unlimited Time)

Obviously you are much better than the previous category of students. Hence, either your focus should be on detailing your speed by picking up the shorter process explained in this book. Besides, you might also need to pick up concepts that might be hazy in your mind. The following process of development is recommended for you.

Step One: Go through the concepts given in the book and Hack to Solved Section. Only go deeper into a concept in case you find it hard. Going back to school level books is not required for you.

Step Two: Move into the first chapter of the book, Number Systems. Go through the theory explained more carefully. Concentrate specifically on clearly understanding the concepts which are new to you. Work out the short cuts and try hard to keep your thinking to bring in most of alternative and organized ways of questioning with respect to the concept you are studying.

[illegible]

Step Three: [illegible]

Step Four: [illegible]

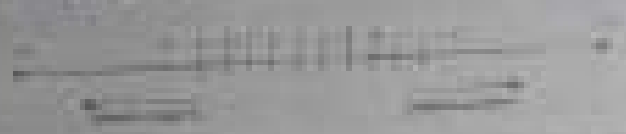
Step Six: [illegible]

Step Seven: [illegible]

CORE CONCEPTS

1. The concept of the number line is one of the most crucial concepts in [illegible] Arithmetic.

The number line is a line that starts from zero and goes towards positive infinity. Also it extends to the right and towards negative infinity when a zero is drawn at



The difference between the values of any two points is the number line that gives the distance between the points.

Thus, for example if we look at the distance between the points + 5 and − 2 it will be given by their difference 5 + (+ 2) = 5 + 2 = 7.

2. Types of numbers :

We will be looking at the types of numbers in much more detail when we go into the matters of number theory. Let us just look at the various types of numbers. Which may help us to understand the various types of numbers. Which may help us to understand the various types of numbers. Also we will also introduce the concept of one's complement (the other complement).

Natural Numbers and Whole Numbers:– Natural numbers, also called counting numbers, are the first numbers we have. They are the number set 1, 2, 3, 4 ..... ∞

On the other hand, whole numbers is the set of natural numbers plus the number zero.

Thus, 0, 1, 2, 3, ... ∞ is the set of whole numbers.

Integers and Decimals: All numbers that do not have a decimal or fractional remainder. Thus, −5, −17, 0, +4, 13, +1473 they are all integers.

Obviously, decimal numbers are numbers which have a decimal value attached to them. Thus, 1.3, 14.16, −12.23 etc are all decimal numbers, everything have certain values after the decimal point.

Dealing in more detail, let us prove a 'fact' which is taught in elementary decimals. As we shall see, the concept of decimals is closely related to the concept of factors and multiples. Suppose there happened to be four which were to divide equally between two people. It is easy for us to see that I can give one whole pizza to each of them.

However, if we also the children in such a way that I now have 2 pieces of pizza to distribute equally amongst 2 people, what do I do?

A pizza can not actually be split of them. The 2 pieces has to be divided equally between the two. Even so, I can do so, without in some way breaking the 2 pieces. Large do not, without in some way breaking the 2 pieces. Thus is the elementary concept that gives rise to the need for decimals to materialize.

Going back to my children (there, my only option is to give each the 2 pieces into two equal parts (which is exactly what is called).

This concept has huge implications for practical mathematics, [illegible] that a half (i.e. 0.5) is the equivalent (one pie, or pizza) of one half (i.e. 0.5) [illegible] decimal (or) [illegible] that can divide evenly into two parts.

Thus, it has all numbers decimals except of 10 [illegible] that divide evenly.

Hence, for example, the decimal 2.5 [illegible]

5.

Similarly the decimal 3 gives the 3 [illegible] 3 [illegible] and so on.

Prime Numbers and Composite Numbers: [illegible] natural numbers, there are two broad divisions.

Unity: It is representative of the number 1.

Prime Numbers: These are numbers which have no divisors (factors apart from 1 and itself).

Composite Numbers: On the other hand, are numbers which have at least one more factor (apart from 1 and itself).

Note: a brief word about factors and divisors – A number is said to divide Y or is said to be a divisor or factor of Y. Why the divisors of Y/X leaves no remainder.

All composite numbers have the property that they can be written as a product of their prime factors.

Thus, for instance the number 40 can be represented as $40 = 2 \times 2 \times 2 \times 5$ or $40 = 2^3 \times 5$

This form of writing is called as the standard form of the composite number.

The difference between Rational and Irrational numbers: The difference between one of the critical but unknowingly one of the less well understood differences in elementary Mathematics.

The definition of Rational numbers: Numbers which can be expressed in the form p/q where q ≠ 0 are called rational numbers.

Obviously, numbers which cannot be represented in the form p/q are called as irrational numbers.

However, one of the very well understood issues is [illegible] regard to what does this mean?

The difference between rational and irrational numbers are essential to grasp.

[illegible]

Infinite nonterminating decimal values. [illegible]

[illegible]

An exercise. Try:

[illegible]

[illegible] approximate value of the square root of 2. Is it a rational number? Why or why not? [illegible]

Cantor's argument. The area of any rectangle can be [illegible] the formula (base) × (height). Suppose that the base of the rectangle is 1 and the height is $\sqrt{2}$. Is this a rational number? [illegible] Hence, the area of an arbitrary rectangle with area equal to 1 is a pair of [illegible]

Before we move ahead we need to tackle some one [illegible]

[illegible]

[illegible] of the [illegible] of the decimal form. The question that arises is: Is there any process we can use to [illegible]

Yes, there is. In fact, it is easy to understand how. For example, you may need to understand that there are five kinds of terminating decimals. The process for converting an infinite nonterminating decimal into a rational number varies for each of these types. Let's look at them one by one.

Type 1: Pure terminating decimals. These are terminating decimals where the recurring digits immediately after the decimal point.

For example:

$$0.0555\ldots = 0.\overline{5}$$
$$0.142857\ldots = 0.\overline{142857}$$
$$0.06240\ldots = 0.\overline{0624}$$

If, instead, the recurring digits do not start immediately after the decimal point, we have:

$$0.1555\ldots = 0.1\overline{5}$$
$$0.242424\ldots = 0.\overline{24}$$
$$0.06240\ldots = 0.0\overline{624}$$

A little bit of computation will show you that what we have done is nothing but to get rid of the recurring part of the decimal by [illegible] the recurring part of the decimal by a power of 10 (e.g., 100 or 1000) equal to the number of digits in the recurring part of the decimal.

Thus, in the second case, the fraction is obtained by dividing by 10 (to get rid of the recurring part of the decimal and) leaving 2 digits because the number of digits is 2 (e.g., 24).

$$\text{Number} = \frac{\text{Decimal}}{10^{\text{Number of digits}}}$$

Type 2: Impure terminating decimals. Unlike pure terminating decimals, in these decimals the recurring digits start after a certain number of digits in the decimal. The procedure is similar to that for pure terminating decimals but we multiply [illegible] by the number.

Consider the following:

(i) $\frac{1}{3} = 0.3$

(ii) $\frac{1}{7} = 0.142857$

(iii) $\frac{1}{11} = 0.090909\ldots$

What is the difference between the decimal values of the three numbers above?

Well, simply, the difference between the values is, in every case, a finite decimal value. Each number can be expressed in a finite number of digits. Since $\frac{1}{3}$ can be represented as $\frac{10}{30}$ it is also equal to 0.33

Similarly, another fraction $\frac{1}{7}$ can be represented as 0.7777 (0.7777 = 7/9). Thus, numbers having a finite terminating decimal value are defined.

Now, let us consider the decimal value 0.7913

Interestingly, values with a finite value can have several. Hence, there are values **infinite decimals** or non-terminating decimals.

So, we can easily see that the number 0.7913... can be represented as 7/9. Hence, the number is discovered. By [illegible] numbers with finite decimal values, we have set terminating lines which can be represented in an easy way.

For example, the value of the number 1.1415926535 ... is PI.

What I mean to say is that whenever you have two terminating decimal numbers, the value of 'x' might not be finite, but it will always exist.

Thus, we can conclude that all numbers whose decimal values are infinite non-terminating are called irrational numbers or infinite non-terminating decimal values. Now, let us look at the non-terminating patterns, and do they exist at end—the point infinity. We have seen that it is not possible to find the value of a decimal, x^2 which can be known as rational or irrational. Hence, such numbers are called irrational numbers.

Now, let us look at the mathematical representation of the numbers (1, 2, 3, 4, 5, 6, 7, 8, 9, 10), which are rational numbers since they do not have a finite representation.

Now, let us look at the representation of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and $\sqrt{7}$ which are irrational as $\sqrt{x}$.

An Important Tip

Rational and irrational numbers do not exist. This means that [illegible] or irrational number are [illegible] represent which is infinite. It will become difficult for us to calculate it (or) we can be measured from the reference [illegible] completely divided in the same rational number.

Consider an example: The area of an equilateral triangle is given by the formula $(\sqrt{3}/4) \times a^2$ (where a is the side of the equilateral triangle). Now, $\sqrt{3}$ is an irrational number. A triangle is the answer in the real. Hence, the area of an equilateral triangle will consequently be a irrational number.

Before we move ahead, we need to understand one final thing about recurring decimals.

As I have already recommend, recurring decimals have the property of being able to be represented in the easy form. The question that arises is: Is there any process to convert a recurring decimal into a proper fraction?

Yes, there is. In fact, in order to understand how the process, you first need to understand that there are two kinds of recurring decimals. The process for converting an [illegible] recurring decimal into a fraction basically consists of two of these types. Let's look at these one by one.

Type 1-Pure recurring decimals: These are recurring decimals where the recurrence starts immediately after the decimal point.

For example:

$$0.777\ldots = x \Rightarrow 3$$

$$3.0016\ldots = x \Rightarrow 3.\overline{01}$$

$$3.9282\ldots = x \Rightarrow 3.\overline{927}$$

The method for converting these decimals to fractions can be illustrated as:

$$0.777\ldots = x \Rightarrow 7$$

$$3.0016\ldots = x \Rightarrow 3 + (3.\overline{01})$$

$$3.9282\ldots = x \Rightarrow 3 + (3.\overline{927})$$

A fraction of recurring decimal [illegible] that we have done is multiply the [illegible] for recurring part of the decimal so it is just dividing by a group of 9's. Also the number of 9's in the group equals the number of digits in the recurring part of the decimal.

Thus, in the second case, the fraction is derived by dividing 99 by 99 (Having the recurring part of the decimal and 99 having 2 digits because the number of digits is 2).

$$\text{Example, } 0.\overline{0101}\ldots = \frac{0101}{9999}$$

Type 2-Impure recurring decimals: Unlike pure recurring decimals, in these decimals, the recurrence occurs after a certain number of digits in the decimal. The process to convert these into a fraction is also very straightforward.

[illegible]

[illegible]

[illegible]

Table 1.1

Number	Square	Number	Square	Number	Square
1	1	21	441	41	1681
2	4	22	484	42	1764
3	9	23	529	43	1849
4	16	24	576	44	1936
5	25	25	625	45	2025
6	36	26	676	46	2116
7	49	27	729	47	2209
8	64	28	784	48	2304
9	81	29	841	49	2401
10	100	30	900	50	2500
11	121	31	961	51	2601
12	144	32	1024	52	2704
13	169	33	1089	53	2809
14	196	34	1156	54	2916
15	225	35	1225	55	3025
16	256	36	1296	56	3136
17	289	37	1369	57	3249
18	324	38	1444	58	3364
19	361	39	1521	59	3481
20	400	40	1600	60	3600

Kid, here does not get those numbers every time it wants! Why? Just one question: these values of n, to have 1 end pick [illegible]

Very natural! There is a very well-ordered sequence among 1 [illegible] the number of two n's. Let's [illegible] Third of all, you are [illegible] (Upper numbers [illegible] the Four Horsemen, [illegible] The problem [illegible] the [illegible] every [illegible] The [illegible] [illegible] Table 1.1. Trouble [illegible]

Now, follow the following processes and you'll know all numbers [illegible] 1.1.

Task 1: Find squares from 21 to 60. (**Note.** The [illegible] depends on your memory of the first thirty squares.)

The process is very simple and straightforward.

Suppose, you have to find out the square for the value of n^2. Look at $(n-1)^2$ or $(n-1)$ [illegible] The [illegible] right corner will be [illegible] as follows:

The last two digits will be the same as the last two digits of the square of the number 11. (The value $(n-1)$ derived by taking n out of brackets or (n) [illegible] 1.)

Now, $11^2 = 121$; so we are [illegible] the last two digits of 11^2 will be 21. (i.e. the last 2 digits of 121). Also, you will find the carry over the '1' in the hundreds place of 121 is the first part of the number.

The first two digits of the answer will be get by adding 1 [illegible] carry over. [illegible] carry over (1) is [illegible] the number 11. (Tricked number, [illegible] all cases.) Hence, the first two digits of the answer will be given by $(2n + 1) \times 2 = 44$.

Hence, the answer is 44^2 [illegible]

Similarly, suppose you have to find 59^2.

Step 1: $59 + 16 = 75$.

Step 2: 59^2 is of 59. Hence, the last 2 digits of the answer will be 59 and we will have a carry over 5.

Step 3: The first two digits of the answer will be $75 \times 59 = 4$ + 5.

Hence, the answer is 3481.

This technique will take care of squares from 11 to 60. If you remember the squares from 1 to 30, you are advised to use this process and see the answers for yourself.

Squares for Numbers from 31 to 60. Such another can be [illegible] the form $(30 + x)$ and the same process [illegible] to get the values of squares from 31 to 60. Again, [illegible] we will [illegible] the process you have to find the squares of x.

Step 1: Look at 30 as $(10 - 3)$.

Again, [illegible] in which we did [illegible] the [illegible] the square — the last two and the left two digits.

Step 2: The last two digits can be [illegible] $(10 - 3)^2$ [illegible] 49. Hence, 49 will represent the last two digits of x^2.

[illegible]

[illegible]

Note: In our [illegible] we have [illegible] digits to find the answer.

For example, in finding the value of 79^2 we look at $79 = (70 + 9)$.

Note: $(-10)^2 = 100$. Hence, the last 2 digits of the answer will be 00. The number "2" in the tens place will have no bearing on the last 2 digits of the answer.

Thus, the last two digits of $(70 + 9)^2$ will be 21.

Hence, $79^2 = 6241$.

Note: the process we use to find the square of numbers from 51 to 99.

Finding squares of numbers between 101 to 109:

Suppose we want to find the value of 102^2. The following process will give us the answer:

Step 1: Look at 102 as (100 + 10). The answer will have 4 digits whose values will be got by knowing or getting the value of the last two digits and one of the first two digits.

Step 2: The value of the last two digits will be equal to the last two digits of $(10)^2$.

Hence, $(10)^2 = 100$, the last two digits of 10^2 will be 00. The "1" in the candidate place of $(10)^2$ will be carried over to the other part of the answer i.e. the first two digits.

Step 3: The first two digits will be got by: $102 \times (102 - 10) + 1$

Where 102 is the original number, $(102 - 10)$ is the number obtained by looking at 102 as $(100 + x)$ and 1 is the carry over from $(10)^2$.

Similarly, 107^2 will give us the following 4-digit process:

$107 \times (107 - 7) + 1 = 107 \times 100 + 1 = 10701$. So, the last two digits of the answer is "01". Carry over 1. Consequently, $(7 \times 7) + 1 = 50$ will be the first 2 digits of the answer.

Hence, $107^2 = 11449$.

Note: Using this process you will be able to find the square of any number upto 109.

Properties of squares:

1. When a perfect square is written as a product of its prime factors, each prime factor will appear an even number of times.
2. The difference between the squares of two consecutive natural numbers is always equal to the sum of the natural numbers. Thus, $(n^2 - (n-1)^2) = (n + (n-1))$.

This property is very useful when want to find the square of a number. Given that we already know the square of the previous number, we just add consecutive integers to get the square of the next number. For example, we know that the square of 10 is 100. So, we can find the square of 11 as $100 + 10 + 11 = 121$.

3. The square of a number ending in 2, 3, 7 or 8 always ends in 4, 9, 9 or 4 respectively.
4. The square of any number ending in 5. The last two digits will always be 25. The digit before the last digit will always be odd. Ignoring the digit immediately to the right, we take the number by 1 more than itself.

Illustration

$$15^2 = __25$$

The unknown digit in the above answer will be gotten by $1 \times (1 + 1) = 2$. Hence, the square of 15 equals 225.

Similarly, $105^2 = __25$. The unknown digits are $5 \times 6 = 30$. Hence, $105^2 = 11025$.

5. The value of a perfect square has an odd (even) number of zeros if the number of zeros is even (odd).
6. If the units digit of the square of a number is 1, the units digit of the number is 1 or 9.
7. If the units digit of the square of a number is 4, the units digit of the number is 2 or 8.
8. If the units digit of the square of a number is 9, the units digit of the number is 3 or 7.
9. If the units of the square of a number is 6, the units digit of the number is 4 or 6.
10. The sum of the squares of the first 'n' natural numbers is given by:

$$\frac{n(n+1)(2n+1)}{6}$$

11. The square of a number is always non-negative.
12. Normally, by squaring any number we increase the value of the number. The only exception to this is when we use 0 and 1. (In these cases squaring the number has no effect on the value of the number).

Further, for values between 0 to 1, squaring a number reduces the value of the original number. For example, $0.5^2 = 0.25$.

Finding the Square Root of a Given Number

Step Five

Step 1: Write down the number whose square root is to be found.

Prime Factors: [illegible]

Step 3: The remaining factors [illegible] is dependent on knowing the values of the powers.

Hence: [illegible]

CUBES AND CUBE ROOTS:

Given a number [illegible] and [illegible], we get the value of the number.

Thus, [illegible]

Method to find out the value of 3-digit numbers: The product [illegible] of 3 parts, each of which has to be calculated separately.

The first part of the cube will be given by the value of the cube's digit.

Suppose we have to find the cube of 38.

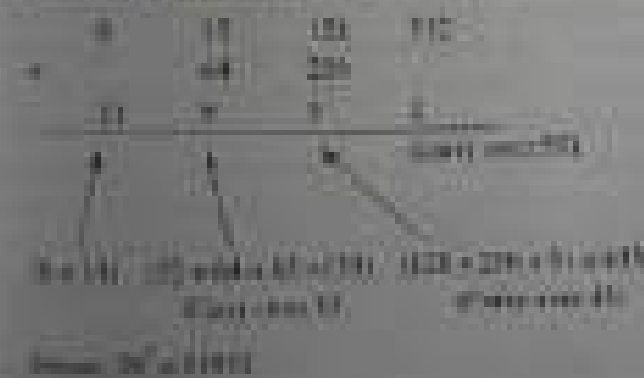
The first step is to find the cube of 3 and we've it known.

[illegible]

The second three parts of the number will be derived as follows (refer the values 12, 126, and 512).

By comparing 12, 126 and 512 with the [illegible] the cube of 8, and by [illegible] the value of the cube's digit, if the number will be one digit. (In this case, the cube is 512 = 8.)

Now, write the 3 parts as a cascade as shown. Also, write the middle two parts and delete the values.



Hence: [illegible]

Properties of Cubes

1. When a perfect cube is cubed, it is in standard form. The values of the powers in each prime factor will be a multiple of 3.
2. In order to find the cube root of a number, then value is in the standard form and then divide all powers by 3 to be standard form and then divide all powers by 3.

   Thus, the cube root of $2^6 \times 3^9 \times 5^{12} \times 7^{15}$ is given by $2^2 \times 3^3 \times 5^4 \times 7^5$.
3. The values of all cubes, integers and decimals greater than 1 are greater than the number itself.
4. [illegible] (Note: [illegible] when the value of the number is equal to the number itself.)
5. The value of the cube of a number between 0 and 1 is lesser than the number itself. Thus, $0.5^3 = 0.125 < 0.5$.
6. The value of a number between [illegible] is greater than the number itself. [illegible]
7. The cube of any number less than [illegible] is always lesser than the number. Thus, $(-1.5)^3 = (-3.375)$.

THE BODMAS RULE: [illegible] the ordering of mathematical operations in a mathematical statement.

In any mathematical statement, the first thing to be considered is Brackets (followed by Division, Multiplication, Addition and Subtraction in that order).

Thus, [illegible]

Also, [illegible]

Also, [illegible]

Therefore, we find out Even numbers:

ODDS:

Odd + odd	= Odd
Odd × odd	= Even
Odd − odd	= Even
Odd ÷ odd	= odd

EVENS:

Even + Even	= Even
Even × Even	= Even
Even − Even	= Even
Even ÷ Even	= Even (if exact)

ODDS & EVENS:

Odd + Even	= Even
Odd × Even	= Odd
Odd − Even	= Odd
Even ÷ odd	= Even

Odd + Even = Odd, Odd × Even = Odd

SERIES OF NUMBERS:

In many instances in Mathematics, we are presented with a series of numbers formed simply when a group of numbers is written together. The following are examples of series.

1. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 ...
2. 2, 4, 6, 8, 10, 12, 14 ... (Such series where the next term is derived by adding a certain fixed value to the previous [illegible])

4. 8, 32, 20, 40, ... (Each term before the next term is derived by multiplying the previous term by a fixed value and subtracting (Arithmetic Progression)
   (Note: Just as each such AP and HP is chosen to be instead of geometric which is chapter 2 of this book)
5. 1, 3, 9, 27, x^2, ...
6. 1, 8, 17, 19, 19, 19, ...
7. $2.5^2, 10^2, 19^2, 31^2, 47^2$
8. $1, 10^2, 11^2, 15^2, ...$

Remember the following points in this regard:

1. *AP* and *GP* are not specific instances of series. They are studied in details only because they have many applications and have defined rules.
2. Based on the behaviour of their terms, series can be classified as:

(Important: There are series whose sum to 'n' terms keeps increasing and smaller (which) for infinite terms.

Convergent: (series which sum to finite limit) Some be proven that series can tend to approach an upper boundary limit and sum values in the series. They have the property that exists series (infinite) terms of the series included only will only reach that value and no more.

For example consider the series:

$$(1)^2 + (1/2)^2 + (1/3)^2 + (1/4)^2 + ...$$

It is evident that subsequent terms of this series getting exactly finite. Sum value becomes negligible for some of the terms are taken into account.

If there is infinite series, the sum of the series reaches a value which is still finite. Such series are convergent series. Such series converges to a finite value even if it is infinite terms.

Note: Questions on finding infinite terms (finite values) are very commonly asked in exams specially including CAT and XAT.

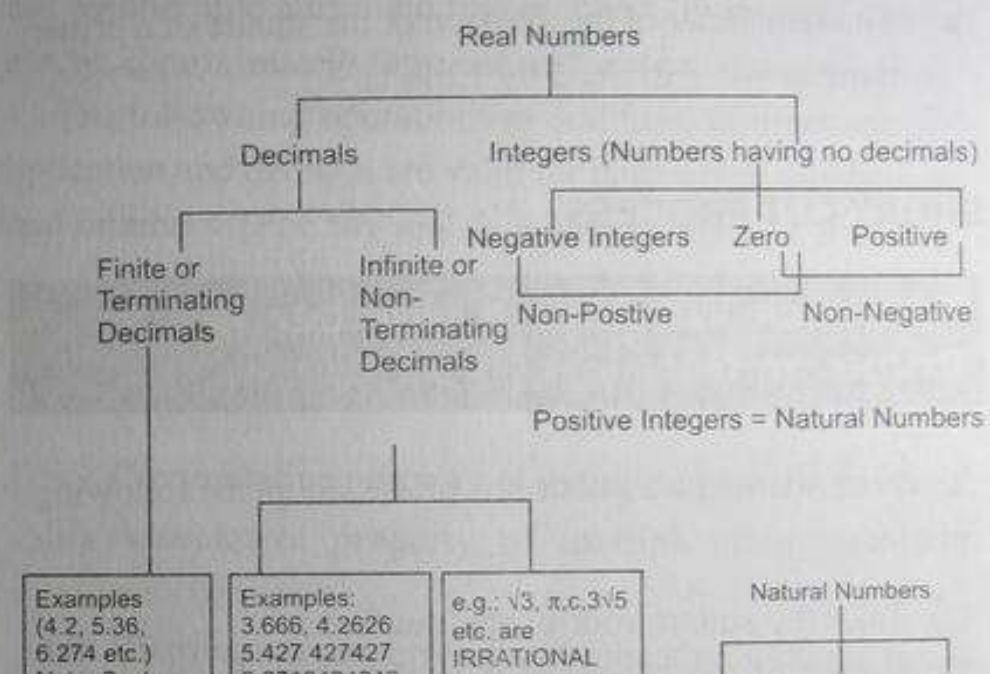
NOTE FOR THE READER: HUN, THIS YOU ARE THROUGH WITH THE MAIN TO THINK. BUT THAT YOU ARE READY TO FIGHTER AND THE CHAPTER OF THE DUCK HAPPY NEW YEAR!

1

NUMBER SYSTEMS

INTRODUCTION

The chapter of number systems is amongst the most important chapters in the whole of mathematics syllabus for the CAT examination. Students are advised to go through this chapter with utmost care understanding each and every question type on this topic. The CAT has consistently set between 10–15 marks based on the concepts of this chapter. Hence, going through this chapter and its concepts properly is very imperative for you. Seen in the context of the qualifying score in the CAT being in the range of 12–14 marks (for Q A), this leaves us with a fair idea of the critical importance



- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]

SHORT CUT PROCESS

To Check Whether a Number is Prime or Not

[illegible]

(a) Take the square root of the number.

(b) Round of the square root to the immediately lower integer. (Call this number [illegible]. For example if you have to check for 131, the square root is 11.4[illegible]. Round the value of [illegible] down to 11.)

(c) Check for divisibility of the number by all prime numbers below [illegible]. If there is a prime number below the value of [illegible] that is then the number is not the prime.

Example:

The square root of 131 (as found in (a)) is 11.4[illegible]. Round the value of [illegible] to 11.

Prime numbers less than 11 are 2, 3, 5, 7, 11. Since 131 is not divisible by any of these, [illegible] is a prime number.

A Brief Look into Why this Works?

[illegible]

The same task can be performed for a number, say, 143 as follows:

2	=	86
3	=	23
4	=	17
5	=	3

[illegible]

[illegible]

[illegible]

The first thing you will realise, when you first look at the list above is that all even numbers will get eliminated automatically (since no even number can divide an odd number and of course you will check a number for being prime only if it is odd).

This will leave you with the numbers 3, 5, 7, 9, 11 and 13 to check (9).

Why do we use to check with composite numbers below the square root? This will again be understood best if explained in the context of the example above. The only composite number in the list above is 9. You do not need to check with 9, because when you checked N for divisibility with 3 you would get either of two cases:

Case I: If N is divisible by 3: In such a case, N will automatically become non-prime and we can stop our checking. Hence, you will not need to check for the divisibility of the number by 9.

Case II: N is not divisible by 3: If N is not divisible by 3, it is obvious that it will not be divisible by 9. Hence, you will not need to check for the divisibility of the number by 9.

Thus, in either case, checking for divisibility by a composite number (9 in this case) will become useless. This will be true for all composite numbers.

Hence, when we have to check whether a number N is prime or not, we need to only check for its divisibility by prime factors below the square root of N.

Integers A set which consists of natural numbers, negative integers $(-1, -2, -3, \ldots, -n, \ldots)$ and zero is known as the set of integers. The numbers belonging to this set are known as integers.

Composite Numbers It is a natural number that has at least one divisor different from unity and itself.

Every composite number n can be factored into its prime factors. (This is sometimes called the canonical form of a number.)

In mathematical terms $n = p_1^{a} \cdot p_2^{b} \cdots p_k$, where $p_1, p_2, \ldots p_k$ are prime numbers called factors and $a, b, \ldots k$ are natural numbers.

Thus, $24 = 2^3 \cdot 3$, $44 = 2 \cdot 3 \cdot 2^2$ etc.

This representation of a composite number is known as the standard form of a composite number. It is an extremely useful form of writing a composite number as we shall see.

Whole Numbers The set of number that includes all natural numbers and the number zero are called whole numbers. Whole numbers are also called as fleeting integers.

The Concept of the Number Line The number line is a straight line between negative infinity on the left to infinity to the right.

The distance between any two points on the number line is got by subtracting the lower value from the higher value. Alternately, we can also start with the lower number and find the required addition to reach the higher number.

For example: The distance between the points 7 and -4 will be $7 - (-4) = 11$.

Real Numbers All numbers that can be represented on the number line are called real numbers. Every real number can be approximately replaced with a terminating decimal.

The following operations of addition, subtraction, multiplication and division are valid for both whole numbers and real numbers. [For any real or whole numbers a, b and c].

(a) Commutative property of addition: $a + b = b + a$

(b) Associative property of addition: $(a + b) + c = a + (b + c)$

(c) Commutative property of multiplication: $a \cdot b = b \cdot a$

(d) Associative property of multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

(e) Distributive property of multiplication with respect to addition: $(a + b)\, c = ac + bc$.

(f) Subtraction and division are defined as the inverse operations to addition and multiplication respectively.

Thus if $a + b = c$, then $c - b = a$ and if $a \cdot b = c$ then $c \div a = b$ (where $b \neq 0$).

Division by zero is not possible since there is no number q for which $b \cdot q$ equals a (non zero number) a.

Rational Numbers A rational number is defined as number of the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

The set of rational numbers includes the set of integers and fractions. The rules given above for addition, subtraction, multiplication and division also apply on rational numbers.

Rational numbers that are not integral will have decimal values. These values can be of two types:

(a) **Terminating (or finite) decimal fractions:** For example, $17/4 = 4.25$, $21/5 = 4.2$ and so forth.

(v) **Non-terminating recurring rational fractions**: Rational numbers [illegible]

(a) Non-terminating periodic decimals: These are those [illegible]

[illegible] For example, $\frac{1}{3} = 0.333333\ldots$ [illegible] These are all the form $\frac{p}{q}$, $p, q \in \mathbb{Z}$, $q \neq 0$. For example, 3.27(mod 1)(1119/96).

All the above categories, terminating rational and non-terminating periodic rational fractions belong to the set of rational numbers.

IRRATIONAL NUMBERS: Irrational numbers are those numbers which are not rational numbers.

Some examples of irrational numbers are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, etc. In other words, all square root and cube roots of natural numbers which are not squares and cubes of natural numbers are irrational. Other irrational numbers include π, e and $\ln 2$.

Every point on the real number line is either a rational or irrational number, covering all of $\mathbb{R}$.

All operations of addition, subtraction, multiplication and division applicable to rational numbers are also applicable to irrational numbers.

As briefly stated in (iii) Rule in Algebraic section, whenever we multiplied numbers, a rational and an irrational number together, the two cannot be cancelled together till the end. In other words, an irrational number was to appear in the solution of a problem till towards a special, till the end of the question. This concept is quite much related to Concepts that follow. For example, if you are asked to find the ratio of the area of a circle to that of an inscribed triangle, you can expect to see π in the answer. This is because the area of a circle will always have a component πr^2, while that of an inscribed triangle will always have r^2.

You should know that rational and irrational number operations in the solution to a question, it can only happen if π is multiplied or divided by the same irrational number.

THE CONCEPT OF GCD (GREATEST COMMON DIVISOR OR HIGHEST COMMON FACTOR)

Consider two natural numbers x_1 and x_2.

If the numbers x_1 and x_2 are divisible by the same number x_3, then x_3 is a common divisor of x_1 and x_2.

The highest of all the common divisors of x_1 and x_2 is called the GCD of x_1 and x_2. This is denoted as GCD(x_1, x_2).

Rules for Finding the GCD of Two Numbers x_1 and x_2

(i) Find the standard form of the numbers x_1 and x_2.

(ii) Write out all prime factors that are common in the standard forms of the numbers x_1 and x_2.

(iii) Raise each of the common prime factors found in the step of the process, to which it appears, the smallest power of the numbers x_1 and x_2.

(iv) The product of the results of the previous step will give the GCD of x_1 and x_2.

Illustration: Find the GCD of 108, 216, 270.

Step 1: Working down the standard form of number

$$108 = 2 \times 2 \times 3 \times 3 \times 3$$
$$216 = 2 \times 2 \times 2 \times 3 \times 3$$
$$270 = 2 \times 3 \times 3 \times 3 \times 5$$

Step 2: [illegible]

Step 3: [illegible]

Step 4: [illegible]

For practice: Find the GCD of the following

(a) 75, 75, 105

(b) 240, 144, 360

(c) 196, 252, 420

THE CONCEPT OF LCM (LEAST COMMON MULTIPLE)

Let x_1 and x_2 be two natural numbers whose [illegible] other. The smallest common number is the multiple of x_1 and x_2 is called the least common multiple of x_1 and x_2 and is denoted as LCM(x_1, x_2).

Rule for Finding the LCM of 2 [illegible] x_1 and x_2

(a) Find the standard form of the numbers x_1 and x_2.

(iv) Write down all the prime factors, taking the common ones only the smallest power of either of the numbers.

(v) Raise each of the prime factors found above to the highest power in which it appears in any one of the given forms of the numbers n_1 and n_2.

(vi) The product of powers of the prime factors thus obtained is the LCM of n_1 and n_2.

Illustration: Find the HCF and LCM of 150, 210, 375.

Step 1: Writing down the standard form of numbers

$$150 = 2 \times 3 \times 5 \times 5 = 2 \times 3 \times 5^2$$
$$210 = 2 \times 3 \times 5 \times 7 = 2 \times 3 \times 5 \times 7$$
$$375 = 3 \times 5 \times 5 \times 5 = 3 \times 5^3$$

Step 2: Write down all the prime factors that appear in any one of the numbers 2, 3, 5, 7.

Step 3: Raise each of the prime factors to their highest possible power (considering each of the numbers).

The LCM $= 2^1 \times 3^1 \times 5^3 \times 7^1 = 5250$

Important Rule:

$\text{GCD}(n_1, n_2) \times \text{LCM}(n_1, n_2) = n_1 \times n_2$

(i) The product of the HCF and the LCM equals the product of the numbers.

RULE FOR FINDING HCF AND LCM OF FRACTIONS

(i) HCF of two or more fractions is given by:

$$\frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

(ii) LCM of two or more fractions is given by:

$$\frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

Rules for HCF: (i) If HCF of x and $y = 1$, then the HCF of

(a) $x, x+1$ is also 1

(b) $x, x+2$ is also 2

(c) $x+y, x^2 - xy + y^2$ is also 1

HCF and LCM

Positive Exercise

(Figure in brackets indicate the marks)

1. Find the common factors for the numbers

(a) 16 and 64 (b) 42, 84 and 168

(c) 96, 120 and 156

2. Find the HCF of

(a) 42 and 168 (b) 96 and 16

(c) 36, 52, 108 (d) 42, 168 and 168

3. Find the LCM of

(a) 12, 22 and 60 (b) 20, 16, 14 and 32

(c) 22, 55, 66 and 22 (d) 15, 15, 15 and 15

4. Find the value of (where the factors are)

(a) 96 and 36 (b) 12, 40 and 16

(Note: Find the LCM and then divide it by the product of the given numbers. Note the difference in the LCM and the product [Illustration: 200 = 8 × 25])

5. The LCM of two numbers is 600. If the HCF is 6 and one of the numbers is 72, find other is [SET]

(a) 42 (b) 12

(c) 32 (d) None of these

Answer: The product of HCF and LCM is product of numbers.

6. Find the smallest number which when divided by 24 and 36 leaves 5 and 8 as remainders respectively. [Hint: Find the LCM of 24 and 36 and add the remainder to the LCM]

(a) 112 (b) 77

(c) 113 (d) None of these

Answer: (d) The LCM is 72 and 36 leaving 5 and 8 respectively. Adding 5 and 8 to the LCM, we get 85.

7. If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are in the ratio 3 : 4 : 5. The HCF of the 3 numbers is 1/12. The HCF of the 3 numbers is 1/60. The LCM of the 3 numbers is 1/60. The LCM of the 3 numbers is 1/12. Then the HCF of a, c, e and the LCM of b, d, f respectively are [SET, MAT]

(a) 3 (b) 4

(c) 5 (d) 6

Answer (c): The LCM of 3, 4, 5 and 12, 60, 60. Hence, the answer will be 5 and 12. The product integer value of the LCM of 3, 4, 5 and 12, 60, 60 is 6.

8. The bells ring three consecutively at 30PM, 45PM and 60PM. The bells ring at 9 AM. The bells ring at 9 AM. The bells ring at 9 A

(a) $2^n + 2^n = 4^n$ (b) None of these

Answer: (a)

10. Two numbers when divided by 3 and 5 leave remainders 2 and 4 respectively.

(a) The greatest length of rope that can measure both of them exactly is

Answer: (a) GCD of 3 and 5 is 1.

(b) How many such right triangles can be constructed?

Answer: $\frac{(n+1)(n+2)}{2}$

11. Two numbers are in the ratio 15:11. If their HCF is 13, what are the numbers?

Answer: (b) (15 and 11) × 13 i.e. 195 and 143 respectively

12. A farmer wants to plant 96 apple trees, 84 lemon trees and 132 mango trees in equal rows in terms of number of trees. Also, he wants to make the rows as long as possible. The number of trees in each row is

(a) 2 (b) 3

(c) 10 (d) 12

Answer: (c) HCF(96, 84, 132) = 12. Since 12 is the HCF

13. Three cousins starting around a circular track, completing one revolution in 12, 8 and 6 minutes respectively. When will they all meet at the starting point?

(a) 24 (b) 48

(c) 12 (d) 24

The answer is the LCM of 12, 8 and 6. They will meet after 24 minutes.

14. The HCF and LCM of two numbers are 23 and 184 respectively. When the first number is divided by 2, the quotient is 23. The other number is

(a) 46 (b) 92

(c) 184 (d) 46

Answer: (a) The other number is 2 times 23 = 46

15. The greatest number which divides 4455, 5005 and 6006 is

(a) 10 (b) 11

(c) 9 (d) None of these

The answer will be the HCF of the three numbers. (1) is the last choice.

16. Which of the cards in the numbers the largest 4-digit number which can be added to 7349 in order to make the resulting number divisible by each of 12, 14, 18, 75 and 84?

(a) 4229 (b) 4449

(c) 4229 (d) None of these

Solution: The LCM of the numbers 12, 14, 18, 75 and 84 is 4410. Hence, in order for the resulting number to be divisible by each of the numbers, the number must be of the form

$7349 + x = 4410 \times n$ (n = 2)

$x = 4429$

Hence,

17. Find the greatest number of 5 digits that will give a remainder of 9, when divided by 8, 12, 15 and 18.

(a) 99999 (b) 99999

(c) 99999 (d) None of these

Answer: The LCM of 8 and 9 is 72. The largest 5-digit number of 72 is 99936. Hence, the required number is 99945.

18. The least perfect square number which is divisible by 6, 9, 15 and 18 is

Solution: The number should have at least two 2's, one 3 and one 5 in its factor. Also, it should be divisible by 2, 18 and 5.

Hence, each of the prime factors should occur an even power. Thus, the number should be $2 \times 2 \times 3 \times 3 \times 5 \times 5 = 900$

19. Find the number of factors of form $2^p \times 3^q \times 5^r$ (where p, q and r are whole numbers) which are divisible by 10, 15 and 20 and also divisible by 30 (where p, q and r are whole numbers respectively)

(a) 8007 (b) 9007

(c) 8005 (d) None of these

Answer: First, find the greatest common factor of LCM of 10, 15, 20 and 30. This is found to be 60. So, subject to 3 factors to form the second

20. Find the HCF of $(3^{2^n} - 1)$ and $(3^{2^n} + 1)$

Answer: The solution of this problem is based on very basic

The HCF of $(3^{2^n} + 1)$ and $(3^{2^n} - 1)$ is equal to $(3^{2^n} + 1)$

Thus, in this case, the answer is $(3^{2^n} + 1)$ and hence HCF of 22 and 12.

21. What is the least positive number which when divided by 12, 16, 18 and 21 leaves remainder 8 in each case?

Hint: Find the least number which is divisible by 12, 16, 18 and 21 and add 8 to it. Provided the number is to be divided every prime of the same order of

(a) 7 (b) 8

(c) 12 (d) None of these

22. Find the greatest number which will divide 23, 37 and 73 and 43 leaves the same remainder in each case

(a) 16 (b) 32

(c) 10 (d) 11

19. Find the LCM of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
    (a) 2520 (b) 5
    (c) 56 (d) 150

20. The LCM of two numbers is 1008, and the HCF is 7. If one of the numbers is 56, find the other number.
    (a) 63 (b) 9
    (c) 122 (d) 129

21. The LCM of two numbers is 1008 and their HCF is 56. If one of them is 252, find the other number.
    (a) 224 (b) 128
    (c) 196 (d) 168

22. What is the smallest number which when increased by 4 is divisible by 16, 24 and 108?
    (a) 104 (b) 112
    (c) 204 (d) 76

23. The smallest square number that is exactly divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11 is
    (a) 900 (b) 9001
    (c) 3600 (d) None of these

24. The HCF of two numbers is 12, and their LCM is 420. If one of the numbers is 48, find the other?
    (a) 35 (b) 87
    (c) 37 (d) None of these

25. What is the greatest possible number which when divided with 37 leaves a remainder 12 and when divided with 43 leaves a remainder of 16?
    (a) 1 hour (b) 19 min
    (c) 27 min (d) None of these

26. The HCF and LCM of two numbers are 12 and 144 respectively. If one of the numbers is 36, the other number is:
    (a) 4 (b) 48
    (c) 72 (d) 432

Answers

17. (c)	18. (a)	19. (b)	20. (c)	21. (a)
22. (c)	23. (a)	24. (c)	25. (b)	26. (c)

DIVISIBILITY

A number a is said to be divisible by another number b if the remainder obtained by $a \div b$ is zero or the number a is exactly divisible by b.

It means it can be said that any integer P which can be divided by a certain number Q leaves a integer part of numbers X and R which are called the quotient and the remainder respectively.

$$\text{Thus, } P = QX + R$$

For any integer P and any natural number Q there is a unique set of numbers X and R such that:

$$P = QX + R$$

Where X is an integer and R is a natural number or zero and $0 \le R < Q$. R is remainder known as a whole number less than Q.

If the remainder is zero we say that the number P is divisible by Q.

When $R \neq 0$, we say that the number P is divisible by Q with a remainder.

Thus, 229 can be written as $20 \times 11 + 9$ $(11 \times 20 + 9)$, where 11 is the quotient and 9 is the remainder.

Again, $-25 = 7 \times (-4) + 3$ $(-4 \times 7 + 3)$, where the quotient is -4 and 3 is the remainder.

Note: An integer N $(N \neq 0)$ is said to be divisible by another integer D if there exists another integer Q such that $N = D \times Q$.

$$N = DQ$$

If an integer N is not divisible by another integer D, we can write $N = DQ + R$, where Q and R are integers and R is a positive number $(R \ge 0)$ called the remainder.

Suppose, we divide -32 by 7. Common sense says we should get the remainder to be -3 and -4. This is because the remainder is always very negative.

Thus, $-32 = 7 \times (-5) + 3$ $(-5 \times 7 + 3)$ is the correct remainder $= 3$.

The relationship between the remainder and the divisor:

1. Suppose we divide 42 by 5. The result has a quotient of 8 and remainder of 2.

   Here $42 \div 5 = 8.4$. And we can see that the result has an integer part and a fractional part. The integer part being 8 (quotient) the fractional part is 0.4 which is given by $2/5$.

   Now, we have concluded that we can express the set of numbers where the inequality $0 \le R < N$ not should mean that we divide N with maximum N possible remainders. For example, if we divide by 5, we can have 5 possible remainders $(0, 1, \ldots, 4)$. Hence, what 5 is the divisor we can have only 5 possible distinct values $(0, 1, 2, 3, 4)$. Now, if we divide by 6, we can have 6 possible remainders $(0, 1, \ldots, 5)$. Hence, what 6 is the divisor we can have only 6 possible distinct values $(0, 1, 2, 3, 4, 5)$. Now, if we divide by 7, we can have 7 possible remainders $(0, 1, \ldots, 6)$. Hence, what 7 is the divisor we can have only 7 possible distinct values $(0, 1, 2, 3, 4, 5, 6)$. Now, if we divide by 8, we can have 8 possible remainders $(0, 1, \ldots, 7)$. Hence, what 8 is the divisor we can have only 8 possible distinct values $(0, 1, 2, 3, 4, 5, 6, 7)$. Now, if we divide by 9, we can have 9 possible remainders $(0, 1, \ldots, 8)$. Hence, what 9 is the divisor we can have only 9 possible distinct values $(0, 1, 2, 3, 4, 5, 6, 7, 8)$. Now, if we divide by 10, we can have 10 possible remainders $(0, 1, \ldots, 9)$. Hence, what 10 is the divisor we can have only 10 possible distinct values $(0, 1, 2, 3, 4, 5, 6, 7, 8, 9)$. Now, if we divide by 11, we can have 11 possible remainders $(0, 1, \ldots, 10)$. Hence, what 11 is the divisor we can have only 11 possible distinct values $(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$.

2. If the sum of -42 is not divisible by 5, the value is -8.4. In this case the computation should be done "the integer case is -8 so look to divide the numbers of the fractional part. In this case the fractional part is 0.4 (corresponding to 2/5). Hence, that means the remainder should be negative, the number is negative.

[illegible] of [illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

(e) [illegible]

(f) The [illegible] of a prime number p, which divides n, is usually denoted by

$$[illegible]$$

where $v_p(n)$ denotes the greatest integer $[illegible]$ such that $p^{v_p(n)} \mid n$.

Any composite number can be written (prove it if you don't [illegible]) as a product of any prime factors (p_i) as follows (including 1 or 0):

Thus, for example, the number $[illegible]$ can be written as $[illegible]$.

The standard form of any number has a large amount of information stored in it. The best way to understand the information stored in the standard form of a number is to look at a concrete example. As a worked example (you do understand each of the processes defined below and are able to solve similar questions yourself), let's look at the number 240 (known and beyond).

5. **Using the standard form, [illegible] to find the sum and the number of factors of the number.**

   (a) **Sum of factors of a number**

   Suppose, for basis, we want to find the sum of factors and the number of factors of 240.

   $$240 = 2^4 \times 3 \times 5$$

   The sum of factors will be given by

   $$(2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1)(5^0 + 5^1)$$
   $$= 31 \times 4 \times 6 = 744$$

   Note: This is a useful process, allowing you to calculate the sum number of factors by number of distinct prime factors the number contains and then each. Hence, it is [illegible] to use it if [illegible] prime factors [illegible] to the highest power of that prime number contained in the standard form.

   (b) **Number of factors of a number**

   Suppose, for basis, we want to find the number of factors of 240.

   $$240 = 2^4 \times 3 \times 5$$

   The number of factors will be given by

   $$(2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1)(5^0 + 5^1)$$
   $$= 5 \times 2 \times 2 = 20$$

   Note: This is a useful process, allowing you to calculate the number of factors by number of distinct prime factors the number contains and then each. Hence, it is [illegible] to use it if [illegible] prime factors [illegible] to the highest power of that prime number contained in the standard form.

   (c) **Number of factors of the number**

   Let us explore the sum of factors of 40. It is a useful [illegible] to know:

   $$(2^0 + 2^1 + 2^2 + 2^3)(2^0 + 2^1)$$
   $$= (2^0 + 2^1 + 2^2 + 2^3)(2^0 + 2^1) + (2^0 + 2^1 + 2^2 + 2^3)$$
   $$= 2^0 + 2^1 + 2^2 + 2^3 + 2^1 + 2^2 + 2^3 + 2^4$$
   $$= 1 + 2 + 4 + 8 + 2 + 4 + 8 + 16 = 45$$

   A quick look at the numbers above will make you realize that it is nothing but the sum of factors of 20.

   Hence, any number that has number of factors is equivalent of $(2^0 + 2^1 + 2^2 + \ldots)(2^0 + 2^1 + \ldots)$ plus the number of factors of 40. Hence, if n has N factors:

   Note: This is a useful process that allows you to find the number of factors of the number of factors of a number. For example, $N = 2^3 \times 3^2 \times 5^1$ is equivalent to $(3+1)(2+1)(1+1) = 24$.

   (d) **Number of factors of the number**

   Let us explore the sum of factors of 40. It is a useful [illegible] to know:

   $$(2^0 + 2^1 + 2^2 + 2^3)(2^0 + 2^1)$$
   $$= (2^0 + 2^1 + 2^2 + 2^3)(2^0 + 2^1) + (2^0 + 2^1 + 2^2 + 2^3)$$
   $$= 2^0 + 2^1 + 2^2 + 2^3 + 2^1 + 2^2 + 2^3 + 2^4$$
   $$= 1 + 2 + 4 + 8 + 2 + 4 + 8 + 16 = 45$$

   A quick look at the numbers above will make you realize that it is nothing but the sum of factors of 20.

   Hence, any number that has number of factors is equivalent of $(2^0 + 2^1 + 2^2 + \ldots)(2^0 + 2^1 + \ldots)$ plus the number of factors of 40. Hence, if n has N factors:

   Note: This is a useful process that allows you to find the number of factors of the number of factors of a number. For example, $N = 2^3 \times 3^2 \times 5^1$ is equivalent to $(3+1)(2+1)(1+1) = 24$.

6. **Sum and Number of even and odd factors of a number**

   Suppose, you are trying to find out the number of factors of a number (represented as a product of prime factors) $N = 2^a \times 3^b \times 5^c$.

   As you are already aware, the process will be similar to $(2^0 + 2^1 + \ldots)(3^0 + 3^1 + \ldots)(5^0 + 5^1 + \ldots)$. Now, a basic [illegible] that the number of factors to the exponent $(2^0 + 2^1 + 2^2 + \ldots)$ [illegible] $(2^0 + 2^1 + 2^2 + \ldots)$ will be even. Let

(c) Since 2^n [illegible] is divisible by 3 [illegible]

[illegible]

[illegible]

(d) Given all [illegible] the sum of the numerator of the above expression will be the sum of the odd terms of the series. Consequently, the number of odd terms in the number of the series by the number of odd terms in the expansion of the above expression.

Thus, the number of odd factors in the number 2^n is $\frac{n+1}{2}$ if n is odd and $\frac{n}{2}$ if n is even.

4. Important number of factors satisfying given conditions for a composite number

These are explained through examples.

(i) Find the sum and the number of factors of 1200 such that the factors are divisible by 15.

Solution: $1200 = 2^4 \times 3^1 \times 5^2$

Every factor to be divisible by 15 is a multiple of 15. All factors 2^a and 3^b in a prime factor expression of 1200 are multiplied by 15 $(2^a \times 3^b \times 5^1)$ and consequently the number of factors will be given by $(a+1)(b+1)$.

(Note that there is a contradiction to every valid factor of the expression, since it is divisible by 5 or 25. This is due to an exactly powers of 5 and 15 should not be taken.)

Sol for the method (Thumb rule): the numerator in the expression above and by to convert the input into a binary algorithm.

MULTIPLICATIONS—the need for Google Hardware

I have often observed that the key difference between [illegible] is the presence or absence of a common [illegible] algorithm which further the [illegible] [illegible]. The Google [illegible] [illegible] [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

22. Find the sum the sum of divisors of 144 and 108.
(a) 360 (b) 364
(c) 374 (d) None of these

23. Find the sum of the sum of proper divisors of 36 and the sum of odd divisors of 360.
(a) 69 (b) 72
(c) 65 (d) 39

ANSWERS

1. (c)	6. (c)	11. (d)	16. (b)	21. (c)
22. (d)	23. (a)	24. (c)	25. (d)	26. (c)

NUMBER OF ZEROES IN AN EXPRESSION

Suppose you have to find the number of zeroes in a number $16 \times 25 \times 17 \times 20 \times 19 \times 2^3 \times 5^2 \times 12^5 \times 17^2 \times 22 \times 10$.

As you can assume, this product will have six zeroes.

Hence, it makes an expression like $5 \times 17 \times 27 \times 17 \times 22 \times 12$.

The above expression can be written in the detailed form as:

$$2^5 \times 5^2 \times 5^2 \times 23 \times 5 \times 5^2 \times 2^2 \times 12^5$$

Zeroes are formed by a combination of 2 x 5. Hence, the number of zeroes will depend on the number of pairs of 2's and 5's (we can be factors).

In the above product, there are five twos and five fives. Hence, we shall be able to form only one pair of 2 x 5. Hence, there will be 1 zero in the product.

[illegible]

Finding the Number of Zeroes in a Factorial Value

Suppose you had to find the number of zeroes in 8!

$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = (5 \times 2) \times 7 \times 3 \times 2 \times 4 \times 3 \times 2 \times 1$

The above expression will have only one pair of 2 x 5, meaning there is only one 5 and no greater than 5 (2).

It is clear that in any factorial value the number of 5's will always be lesser than the number of 2's. Hence, all we need to do is to count the number of 5's. The process for this is explained in Solved Examples 1 to 5.

■ EXERCISES FOR SELF-PRACTICE ■

Find the number of zeroes in the following:

1. 47!
2. 98!
3. $(15 \times 11 \times 13 \times 17 \times 66 \times 19 \times 11)$
4. $(32 \times 11 \times 9 \times 34 \times 12 \times 17)$
5. 101!
6. $(91 \times 6 \times 19 \times 17 \times 11 \times 44 \times 13)$
7. 300!
8. 1007
9. 1135
10. $(1427 \times 500 \times 1)$

A special trailing zero: Suppose you were to find the number of zeroes in the value of the following factorial values:

$$4!, 5!, 47!, 48!, 49!$$

What do you notice? The number of zeroes in factorial values will be equal in 48! Why does this happen? It is difficult to understand that the number of 5's in any factorial value is equal to 48. The number of zeroes will change at 50! It will become 12.

Extra Zeroes: The sum of factorial values [illegible] consecutive products of 5.

Thus, 25! = 5! × 5! × 5! and 50! = 5! times 12 times and they will have 12 zeroes.

Similarly, 75!, 99!, 77!, 95! and 98! will not have 12 zeroes.

After each 5 (5!), they are never another 5!. The whole deal are 10 zeroes in 49! Every are divisible 12 times 55. This means that there is an value of a factorial which gives 11 zeroes. This means there is no 10! or number of zeroes of 49! by 50. When you do so, the result is [illegible] 9 in the product. Hence, the number of zeroes [illegible] 50! value are never just any number of zeroes.

Notice at 10! you will get 24 × 6 = 12 zeroes.

At 15! you will get 10 × 3 × 1 × 1 = 1 zeroes, a prime number.

■ EXERCISES FOR SELF-ASSESSMENT ■

1. [illegible]
2. [illegible]
3. Find the number of zeroes in the product $1! \times 2! \times 3! \times 4! \times \ldots \times 49!$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Answers

1. This can never happen.
2. [illegible]
3. The factor 5 in the first the term (5!) is equal to zero, we need to count how many 5's there.

   Thus $4^5 \times 10^{10} \times 15^{10} \times 20^{10} \times 25^{10} \times 30^{10} \times 35^{10} \times 40^{10} \times 45^{10}$

   [illegible] 70 × 80 × 90 × 100 = [illegible] zeros. Thus, the product has 256 zeros.
4. Again, the key there is to find the number of times 5's are given by:

   $100^1 + 100^2 + 100^3 + 100^4 + 100^5 + 100^6 + \ldots + 100^{10}$

   $= 20 + 4 + 0 + 0 + 0 + 20 + 4 + 0 + 0 + 0 + \ldots + 20 + 4 + 0 + 0 + 0$

   $= 20 + 4 + 20 + 4 + 20 + 4 + 20 + 4 + 20 + 4 = 120 + 24 = 144$ (using form of A.P. as stated in the text/notes)

   $= 100 + 44 = 144$
5. The number of 2's in the number 10 5's factorial will be [illegible]
6. The number of 2's in 5's factorial, 5's will be [illegible]

   The number of 2's in 5's is given by the power of 5 in the product:

   $$5^1 + 5^2 + 5^3 + 5^4 + 5^5 + 5^6 + 5^7$$
   $$= 5 + 25 + 125 + 625 + 3125 + 15625 + 78125$$
7. A formula of sorts (structure will be less, more [illegible]

   [illegible]

   [illegible]

(a) 9 (b) 8
(c) 10 (d) 11

2. Find the maximum value of x such that 17^x is perfectly divisible by 17^3

(a) 3^4 (b) 36
(c) 79 (d) 36

3. Find the maximum value of x such that $(3!)^x$ is perfectly divisible by 3^6

(a) 3^6 (b) 18
(c) 36 (d) 45

4. Find the maximum value of x such that 30^x is perfectly divisible by 2520^2

(a) 5 (b) 6
(c) 7 (d) None of these

5. Find the maximum value of x such that $(n!)^x$ is perfectly divisible by $(n+1)^{n+1}$

(a) 7 (b) 6
(c) 8 (d) None of these

6. Find the maximum value of x such that $(7!)^x$ is perfectly divisible by 7^7

(a) 7 (b) 7
(c) 7 (d) 56

7. Find the maximum value of x such that

$42 \times 77 \times 52 \times 91 \times 11 \times 42 \times 55 \times 64 \times 13 \times 66 \times 67 \times$

$(x+1)$ is divisible by 11^7

(a) 6 (b) 7
(c) 3 (d) 6

8. Find the maximum value of x such that

$(100 \times 99 \times 98 \times 97) \times 300 \times 11 \times 7280 \times 81$

is perfectly divisible by 9^x

(a) 11 (b) 7
(c) 81 (d) 9

9. Find the maximum value of x such that

$11 \times 44 \times 55 \times 77 \times 52 \times 98 \times 13 \times 3600 \times 2800 \times 243$

$\times 167 \times (x+1)$ is perfectly divisible by 17^x

(a) 5 (b) 11
(c) 80 (d) 7

Find the maximum positive integer value of the exponent x for the following numbers.

10. 70

(a) 10^x (b) 9
(c) 8 (d) 40

11. $(n+1)!$

(a) 40^x (b) 9
(c) 20^x (d) 40^x

12. $100!$

(a) 75 (b) 50
(c) 31 (d) 32

13. $17^x \times 26 \times 34 \times 102 \times 51 \times 408 \times 425 \times 850 \times 51$

(a) 9 (b) 51
(c) 13 (d) 19

14. $(1 \times 2 \times 3) \times 4^x \times (2!) \times (\underbrace{\quad\quad}_{\quad\quad}) \times 20^x$

(a) 200 (b) 12
(c) 30 (d) 40

15. $(x^2 \times x^3 \times x^5 \times x^7 \times x^9 \times x^4 \times x^6 \times x^8 \times x^{10})$

(a) 25 (b) 17
(c) 23 (d) 30

16. $100! \times 200!$

(a) 26 (b) 72
(c) 19 (d) 7

Answers:

1.	(b)	2.	(a)	3.	(a)	4.	(c)	5.	(b)
6.	(c)	7.	(b)	8.	(b)	9.	(c)	10.	(a)
11.	(a)	12.	(b)	13.	(a)	14.	(c)	15.	(b)
16.	(b)								

Co-Prime or Relatively Prime Numbers: Two positive integers that do not have a common factor are said to be co-prime or relatively prime. In other words, two numbers have a highest common factor of unity.

If two numbers are not relatively prime and the sum of the numbers is divisible by 8, then the product of the numbers is also divisible by 8.

Key Concept 1: The sum of two numbers and their product has a very important implication in the study of two numbers using the factorization of the sum.

The concept is quite well understood through an example. Suppose, you are given an equation of the form:

Result: $AB + AC$ where A is the common factor.

Now, we can choose the two numbers such that A is not divisible by 8 and B is not divisible by 8. A is not divisible by 8 and C is not divisible by 8. A is not divisible by 8 and $B + C$ is divisible by 8. A is not divisible by 8 and $B - C$ is divisible by 8. A is not divisible by 8 and $B \times C$ is divisible by 8. A is not divisible by 8 and $B + C$ is divisible by 8. A is not divisible by 8 and $B - C$ is divisible by 8. A is not divisible by 8 and $B \times C$ is divisible by 8.

The reason for this is that A and B are co-prime, so the possibility of A is prime, B is prime, A and B are co-prime, and A and B are co-prime.

Now, this will not be the case if A is not co-prime.

$$AB + AC$$

In this case, A is not co-prime, B is not co-prime, A and B are co-prime, there is a possibility of the values of A and B being odd. Any odd number is not divisible by 8.

For example: $(1 \times 3) + (1 \times 5) = 8$

[illegible]

Note: [illegible]

[illegible]

Key Concept-3: [illegible]

Example: Find all five-digit numbers of the form [illegible] that are divisible by 36.

Solution: [illegible]

Hence the number [illegible] is divisible by 36.

Also for [illegible] the number [illegible] will be divisible by 36 [illegible]

■ EXERCISES FOR SELF-PRACTICE ■

Find all numbers of the form [illegible] that are divisible by 36.

Find all numbers of the form [illegible] that are divisible by 42.

Find all numbers of the form [illegible] that are divisible by 45.

Find all numbers of the form [illegible] that are divisible by 60.

Divisibility Rules

Divisibility by 2 (or 5): A number is divisible by 2 (or 5) if its last digit is divisible by 2 (or 5).

Divisibility by 3 (or 9): All such numbers in which sum of digits are divisible by 3 (or 9) are divisible by 3 (or 9).

Divisibility by 4: A number is divisible by 4 if its last 2 digits are divisible by 4.

Divisibility by 8: A number is divisible by 8 if its last 3 digits are divisible by 8.

Divisibility by 11: A number is divisible by 11 if the difference of the sum of the digits in its odd places and the sum of the digits in the even places is zero or is divisible by 11.

Divisibility by 12: All numbers divisible by 3 and 4 are divisible by 12.

Divisibility by 7, 11 or 13: The integer is divisible by 7, 11 or 13 if and only if the difference of the number of its thousands and the remainder of its division by 1001 is divisible by 7, 11 or 13.

Example: 4291 is divisible by 7 since the difference between 4291 − 1001 = 3290 is divisible by 7.

Even Numbers: All integers that are divisible by 2 are even numbers. They are of the form $2r$.

Example: $\ldots, -4, -2, 0, 2, 4, \ldots$

2 is the lowest positive even number.

Odd Numbers: All integers that are not divisible by 2 are odd numbers. Odd numbers are expressible of 2 or more, divisible by 2. They are denoted by $2r+1$ or $2r-1$.

Lowest positive odd number = 1

Example: $\ldots, -5, -3, -1, 1, 3, 5, \ldots$

Complex Numbers: The arithmetic combinations of real numbers and imaginary numbers are called complex numbers.

Alternate: All numbers of the form $a+ib$, where $i = \sqrt{-1}$ are called complex numbers.

Twin Primes: A pair of prime numbers are said to be twin prime numbers if they differ by 2.

Example: 3 and 5 are Twin Primes, so they are 11 and 13.

Perfect Numbers: A number n is called the perfect number if the sum of all the factors of n (including 1, excluding n) equal to n.

Example: $6 = 1 + 2 + 3$ sum of the divisors $6 = 1 + 2 + 3 = 6$

$$28 = 1 + 2 + 4 + 7 + 14, \quad 28 = 1 + 2 + \ldots + 27$$

[illegible]

Rational Numbers: A number that has both an integer and a fraction or a decimal or a natural number.

Irrational Numbers: A number which can be represented as the ratio of two natural numbers, meaning only it can be called a rational number.

e.g. $\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{10}$

Certain Rules

1. [illegible]
2. [illegible]
3. [illegible]
4. Any number n can be represented in the decimal system as follows as:

$$[illegible]$$

Example: [illegible]

5. [illegible]
6. [illegible]
7. The difference between 2 numbers [illegible] will be divisible by 9.
8. The sum of an odd number [illegible] always leaves a remainder of 3.
9. The product of 3 consecutive natural numbers is divisible by 6.
10. The product of 3 consecutive numbers [illegible] is always divisible by 24.
11. Powers:

$$[illegible]$$

12. All numbers are divisible by 3 have the property that their square will leave a remainder of 1 when divided by 3.
13. [illegible]
14. The product of any 5 consecutive integers is divisible by 7.
15. [illegible]
16. Difference between any number and the number obtained by writing the digits in reverse order is divisible by 9.
17. Any number [illegible] is divisible by 9 and 3.
18. If a numerical expression consists of [illegible] first the operations of the first [illegible] exponentiation or powers are performed, [illegible] operations of the second step [illegible] division) and, finally, the operations of the third step (addition and subtraction) are performed. [illegible] If an expression contains parentheses, then the [illegible] indicated in the parentheses are to be [illegible] first and foremost. By removing [illegible] operations of the brackets to [illegible] including [illegible] parentheses are performed in order indicated above.

   If in the [illegible] expression [illegible] operations indicated in the brackets are [illegible] of the functions are performed and the [illegible] divided by the second.
19. [illegible] Divisor is remainder of

$$[illegible]$$

20. [illegible] will always give a remainder of 1.
21. For any natural number n, n^2 has the remainder [illegible] or 1.
22. For any natural number $n^2 - n$ is divisible by [illegible]

THE REMAINDER THEOREM

Consider the following sequence:

$$[illegible]$$

Suppose you have to find the remainder if the [illegible] divided by 13.

We can write this as:

$$[illegible]$$

Which when expanded gives us:

$$[illegible]$$

You will notice that when this expression divided by 13, it will only depend on the last term.

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

This is the remainder when [illegible]

Common Pitfall: Students sometimes complete an $(-)+(-)$ sum divided by 12, and must be sure to look at the individual remainders (−4 and −3 when divided by 12). The remainder numbers (−4 and −3) will give you the remainder of the original numbers when divided by 12.

Mathematically, this can be written as:

The remainder of the expression $(a+b)+c+d+e$ divided by 12 is the same as the remainder of the expression $(a_r+b_r+c_r+d_r+e_r)$ divided by 12.

where a_r is the remainder when a is divided by 12,
b_r is the remainder when b is divided by 12,
c_r is the remainder when c is divided by 12,
d_r is the remainder when d is divided by 12 and
e_r is the remainder when e is divided by 12.

Use the remainder theorem to find the remainder when the sum is divided by 12 and compare it to the sum $-3+4$.

Note the remainder of
$(23+18)+(12)+(-12)$ when divided by 12 can be given as:

$$\frac{(23+18)+(12)+(-12)}{12} = \frac{5+7+0}{12} = \frac{12+0}{12}$$

$$= \frac{1+0}{12}$$

$= 1$ so gives a remainder of 1.

Adding or subtracting an even number of multiples of 12 (because 12 is a multiple of 12) does not change the remainder. Remember, remainders are always positive. Thus, $-3+4$ gives a remainder of 1.

Perform the following operations (divisible by 12) and express the remainder in relation to 12 (which is the divisor) using positive and negative expressions.

Try to solve the following questions on Remainder theorem:

Find the remainder in each of the following cases:

1. $(7\times 12)+(256\times 8)$ divided by 8.
2. $(-360)+(-240)+(-24)+(-24)+(-12)$ divided by 12.
3. $\frac{(-3)\times(-24)}{3}+\frac{(-24)\times(-12)^2}{12}$
4. $\frac{(-48)\times(-24)\times(-12)}{12}$
5. $\frac{(-4)\times(-12)}{3}+\frac{(-12)\times(-12)}{3}$

USING RELATIVE REMAINDERS

Consider the following questions:

Find the remainder when $(-8\times 11)\times 23$ divided by 9.

Also consider approach to the same would be:

$$\frac{(-8\times 11)\times 23}{9} = \frac{(-8)\times 2}{9} = \frac{-16}{9} = -8\times 2\ (\text{divisible})$$

However, there is another approach to the same question.

When (−8) is divided by 9, the remainder is −8 or equivalently −1 or +1. However, there might be times when using the negative value of the remainder might give us more convenient results. Which is why you should use the remainder process.

Concept Note: Remember by definition we always use positive remainders. Hence, even though we can say that −17 is −2 we say that the remainder is 5 and not −2. However, looking at the negative value in the remainder can be useful sometimes in Mathematics as it makes working calculations easier.

Thus, when a number is divisible by 8, the remainder is 7. The negative remainder is −1.

Point: It is definitely not the remainder theorem that 23, 28 is divisible by 5 or 36 − 7 is divisible by 7.

Thus $\frac{(-8)\times 23}{9}$ gives us $\frac{(-2)\times(-1)}{9}$ 2×1

Consider the advantage that process will give, with a divisibility process:

$$\frac{81\times 97}{31} = \frac{(-2)\times(-3)}{31} = 6 \times 3$$

(Remember we always use positive remainders whenever possible. Why? Because it makes the math easier.)

What If the Answer Comes Out Negative

$$\text{Let Answer} = \frac{(-2)\times(-3)\times(-6)}{30} = \frac{-4\times(-3)\times(-6)}{30} = \frac{-24}{30}$$

But we know that 1 remainder of −24 equals +6 remainder of −12 when divided by the divisor, the remainder is −12.

(If whose working steps are done using positive and negative remainders in the most like a circle in which the same product.)

$$\text{Thus } \frac{(-2)\times(-3)}{5} = \frac{(-2)\times(-3)}{5} = 1\times 1 = 1$$

Dealing with Large Numbers: Now, we take some more complex and complicated numbers and use the remainder theorem.

[illegible]

[illegible] DIVISIBILITY POINT

[illegible] to find the remainder of the number when divided by 5. It is clearly evident that the answer should be 3.

But remember the following question:

$1439 \times 1[illegible] \ldots$ (The answer has changed!)

What has happened?

We have multiplied the number 1439 by rounding the numerator and the denominator by 2. The result is that the original remainder 3 is now divisible by 2. (Answer: 1 is the REMAINDER.) In order to solve this problem, we need to reverse the effect of the factor on the numerator by 2. This is done by multiplying the denominator by 2 to get the correct answer.

Note: In our calculations we remember the rule, we should try to cancel the factor of the numerator and denominator as much as possible, since it greatly reduces the calculation required.

10. APPLICATION OF FERMAT'S THEOREM

Finding the unit digit of an expression:

Suppose we want to find the last 2 digits of the expression:

$$15 \times 17 \times 41 \times 19 \times 38 \times 44$$

The expression has about 6 numbers and give, when it is evaluated, 108 as the answer to the above expression.

Now, to answer the question, there, find the remainder of the expression when it is divided by 100.

Answer: $\frac{15 \times 17 \times 41 \times 19 \times 38 \times 44}{100}$

$$= \frac{15 \times 17 \times 41 \times 19 \times 38 \times 44}{25 \times 4}$

[illegible]

Answer:

1. (b)	2. (c)	3. (a)	4. (c)	5. (d)
6. (a)	7. (d)	8. (c)	9. (c)	10. (a)
11. (a)	12. (b)	13. (c)		

Unit digit

(1) By the logic of what we were just over above, the unit's digit of an expression will be given by getting the remainder when the expression is divided by 10.

This digit would be of use to us if we have to find the unit digit of the expression:

$$[illegible]$$

We can rewrite the expression as:

$$\frac{[illegible]}{10}$$

$$\frac{[illegible]}{10}$$

$$\frac{[illegible]}{10} = \frac{[illegible]}{10} = \frac{[illegible]}{10} = \frac{[illegible]}{10} = [illegible]$$

Hence, the required answer is 6.

The unit digit can also be got by multiplying the unit digit of each of the numbers, but the value digit.

(2) Unit's digit when 'N' is raised to a power

Number ending with	Value of power 1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	2	4	8	6	2	4	8	6	2
3	3	9	7	1	3	9	7	1	3
4	4	6	4	6	4	6	4	6	4
5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6
7	7	9	3	1	7	9	3	1	7
8	8	4	2	6	8	4	2	6	8
9	9	1	9	1	9	1	9	1	9
0	0	0	0	0	0	0	0	0	0

In the table above, if you look at the column corresponding to the power 5 or 9 you will notice that the unit's digit of each number is repeated. [illegible]

This means that whenever we have any number whose unit's digit is '2' and is to be multiplied a power of the form $4n+1$, the value of the unit's digit of the answer will be the same as the original unit's digit.

Alternative:

$(123)^{[illegible]}$ will give a unit digit of 3. $(1007)^{[illegible]}$ will give the unit digit of 7 and so forth.

Now, the above table can be modified and the form –

Value of Power

Number ending with	N = 1	N = 2	N = 3	N = 4
1	1	1	1	1
2	2	4	8	6
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1
0	0	0	0	0

The number of the power that we had said in the table is called cyclicity of Power 1. But it is not a number but the value digit.

[illegible]

Solution

[illegible]

Solution

[illegible]

EXERCISE FOR SELF PRACTICE

[illegible]

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]
6. [illegible]
   (a) 0 (b) 9
   (c) 5 (d) 1
7. [illegible]
   (a) 1 (b) 9
   (c) 5 (d) 0
8. [illegible]
   (a) 1 (b) 4
   (c) 0 (d) 1
9. [illegible]
   (a) 9 (b) 7
   (c) 5 (d) 3
10. [illegible]
    (a) 2 (b) 6
    (c) 8 (d) 0
11. [illegible]
    (a) 7 (b) 5
    (c) 3 (d) 9
12. [illegible]
    (a) 2 (b) 8
    (c) 6 (d) 4
13. [illegible]
    (a) 0 (b) 9
    (c) 5 (d) 1
14. [illegible]
    (a) 0 (b) 9
    (c) 5 (d) 1
15. [illegible]
    (a) 2 (b) 6
    (c) 4 (d) 0
16. [illegible]
    (a) 2 (b) 6
    (c) 4 (d) 0
17. [illegible]
    (a) 2 (b) 6
    (c) 4 (d) 0
18. [illegible]
    (a) 2 (b) 6
    (c) 4 (d) 0

Answers

6. (d)	7. (c)	8. (c)	9. (b)	10. (a)
11. (b)	12. (c)	13. (b)	14. (b)	15. (c)

WORKED-OUT PROBLEMS

PROBLEM 1 [illegible]

Solution [illegible]

PROBLEM 2 [illegible]

Solution [illegible]

PROBLEM 3 [illegible]

Solution [illegible]

EXERCISE FOR SELF PRACTICE

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible]

PROBLEM 1.8 [illegible]

Solution [illegible]

PROBLEM 1.9 [illegible]

Solution [illegible]

PROBLEM 1.10 [illegible]

Solution [illegible]

■ [illegible] FOR SELF-PRACTICE ■

[illegible]

PROBLEM 1.7 [illegible]

Solution [illegible]

PROBLEM 1.8 [illegible]

Solution [illegible]

Counting

PROBLEM 1.9 Find the number of numbers between [illegible] to [illegible]

(a) Both [illegible] and [illegible] are numbers.
(b) Only one of [illegible] and [illegible] is required.
(c) Neither [illegible] nor [illegible] is required.

Solution

(a) [illegible]
(b) [illegible]
(c) [illegible]

PROBLEM 1.10 Find the number of [illegible] between [illegible] and [illegible]

(a) Both ends are included.
(b) Only one end is included.
(c) Neither one is included.

Solution

(a) [illegible]
(b) [illegible]
(c) [illegible]

EXERCISES FOR SELF-PRACTICE

(i) Find the number of numbers between [illegible] which are divisible by 7.

(ii) Find the number of numbers between [illegible] which are divisible by 9.

PROBLEM 13 Find the number of numbers between 300 to [illegible] which are divisible by 2, 3, 4 and 5.

Solution: Total numbers: 99

Step 1: Not divisible by 2 = All even numbers = [illegible] 51

Number left: 48

Step 2: Of which divisible by 3 = Even number 300, last number 396. But even numbers have already been removed, hence count the odd numbers. Between 300 and 396 divisible by 3. This gives us five.

Then number 302, last number 398, common difference is 6.

So, remove: [(398 − 302)/6] + 1 = 17.

(81 − 17) = 55 numbers left.

[illegible] numbers which are divisible by 4 and 5 [illegible] which have already been removed.

Step 3: Numbers from 51 numbers left all odd numbers but not divisible by 3 and not divisible by 5.

Between 300 to 399, 20 [illegible] numbers 4 × 5 × 7 = 140 and the last = 399 [illegible] numbers left. We have 19 such numbers so [(399 − 301)/3] + 1 = 33.

However, some of these 33 numbers have already been removed to get to 51 numbers.

(Procedure left) Between 1 to 100, 33, 37, 39, [illegible] all numbers that are also divisible by 3. Quick process shows that the numbers that will 315 and have common difference 21.

Hence 11 are correct. – Type number (Difference = 21) = (399 − 315)/21 + 1) = 5.

Hence 5 numbers were already removed (remaining 19 good 100. Hence, for numbers divisible by 3, we need to remove only those numbers that are not divisible by 5 but not by 3. Hence we 7 such numbers between 300 and 399.

So numbers left are 51 − 7 = 45.

EXERCISES FOR SELF-PRACTICE

Find the number of numbers between 100 to 400 which are divisible by 6 but not 3, 5, 9 and 7.

PROBLEM 15 Find the number of numbers in the following series: 1 × 9 × 11 × 33 × 99 × 101 × 303 × [illegible]

Solution: The number of numbers depends on the number of 3's and the number of 9's. Here, there are [illegible] 3's, the number of 9's is 3's squared. Hence, we can [illegible] the solution as

[illegible]

Number of 3's = 13, Number of 9's = 8

Hence, 4 zeros.

PROBLEM 16 Find the remainder in [illegible]

Solution: The remainder is the exponent [illegible] [illegible] the value of the remainder is 2^[illegible] = [illegible]

Thus, [illegible] is divisible by 3.

PROBLEM 17 Find the remainder in [illegible]

Solution: [illegible]

[illegible] divisible by 11.

PROBLEM 18 Find the remainder when [illegible]

Solution: [illegible]

We are interested in finding the last two digits of [illegible]

Hence, by modular purpose [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

PROBLEM 19 [illegible] Q2, 3rd and 4th

Solution: The cyclical form of the numbers are

[illegible]

[illegible]

[illegible]

■ REINFORCEMENT SELF-PRACTICE ■

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(e) [illegible] (f) [illegible]

Problem 2.13 The sum of the individual elements (i.e. the sum of the squares of median numbers of each when increased by 500 gives the square of median, is 1.23. Find the median is

(a) 5 (b) 9

(c) 7 (d) None of these

Solution [illegible]

Problem 2.14 [illegible] x, y and z are such that the difference between the highest and the second highest was half the difference between the highest and the lowest. If the median was [illegible] and the mean [illegible] x [illegible] y [illegible] z [illegible] x [illegible] y [illegible] z [illegible] x [illegible] y [illegible] z

$$\text{Number of 5's} = \left[\frac{1000}{5}\right] + \left[\frac{1000}{25}\right] + \left[\frac{1000}{125}\right] + \left[\frac{1000}{625}\right] + \left[\frac{1000}{3125}\right] + \left[\frac{1000}{15625}\right] + \left[\frac{1000}{78125}\right] + \left[\frac{1000}{390625}\right]$$

$$= 200 + 40 + 8 + 1 + 0 + 0 + 0 + 0 = 249$$

Number of 2's = 500+

$\therefore$ Number of 2^3 = 250.

Similarly we find the number of 7's as

$$\left[\frac{1000}{7}\right] + \left[\frac{1000}{49}\right] + \left[\frac{1000}{343}\right] + \left[\frac{1000}{2401}\right]$$

$$= 142 + 20 + 2 + 0 = 164$$

Thus we have, 1000 zeroes, 250 twos and 249 sevens.

The required value of n will be given by the lowest of these three. The answer is expected to explain why this happens.

Hence, answer = 1000.

Short Cut We will look only for the number of 7's in this case. Because, $7 > 5 \times 2$, the number of 5's and 2's is more than the number of 2^3.

And $7 > 2 \times 5$, so the number of 7's must be less than the number of 5^3.

Sometimes the author are too asked about the finding of powers when the number only had prime factors. There for had now that we needed to check only for the highest power of the particular prime in the case.

In case of the divisors having composite factors, we have to be slightly careful in estimating the better one will reflect in wrong answer. In the above example, we saw a case where even though 7 was the lowest factor we related to 8 and 9, the calculation was not placed by 7 rather than by 9 but would be corrected based on the previous process of taking the highest number).

Problem 3.14 Find the units digit of the expression $3^{(4n+1)} + 9n^{(2n+1)} + 5^{(4n+1)}$.

Solution We can get the units digit in the expression by looking at the units digits of the 3, 9n and 5 when they are raised to odd powers.

Hence for the last digit we just need to consider the last digit of each part of the product.

A number like 3 having 4 as the units digit are the units digit as:

$3^1 \to 3$	$7^1 \to 7$	$5^{n+1} \to 5$
$3^2 \to 9$	$7^2 \to 9$	$5^{n+1} \to 5$
$3^3 \to 7$	$7^3 \to 3$	Hence $5^{(n+1)}$ will end with 5
$3^4 \to 1$	$7^4 \to 1$	

Similarly $9^{(2n+1)}$ will end with 9 for odd n.

Similarly

$9^1 \to 9$	$\to 9^{(2n+1)}$ will end with 9
$9^2 \to 1$	for odd n.
$9^3 \to 9$	

Similarly

$3^1 \to 3$	$3^{(4n+1)} \to 3$
$3^2 \to 9$	$3^{(4n+1)} \to 3$
$3^3 \to 7$	Hence $3^{(4n+1)}$ will end with 3
$3^4 \to 1$	

Hence, the required units digit is given by 4 with 1 less (answer).

Problem 3.15 Find the GCD and the LCM of the number P and Q where $P = 2^3 \times 3^2 \times 5^3$ and $Q = 5^2 \times 7^2$.

Solution GCD = HCF is given by the lowest power of the common factors.

Thus, $\text{GCD} = 5^2$.

LCM is simply the highest power of all factors.

Thus, $\text{LCM} = 2^3 \times 3^2 \times 5^3 \times 7^2$.

Problem 3.16 A school has 1176 girl students and 784 boy students. The school is divided into equal sections of girls and boys. All sections in the school have an equal number of students. Find the minimum value of the number of sections in the school.

Solution The answer will be given by the HCF of 1176 and 784.

$$1176 = 2 \times 3^2 \times 7^2$$
$$784 = 2^4 \times 7^2$$

Hence, HCF of the two is 2×7^2.

Hence, the number of sections is given by $\frac{1176}{14} + \frac{784}{14}$

= 84 + 56 = 140 sections.

Level of Difficulty (LOD) I

1. [illegible] of the [illegible] obtained by dividing [illegible] the numbers 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99 [illegible]

(a) 0 (b) 9 (c) 7 (d) 2

(e) 5

2. The last two digits of a five-digit number is 16, which is [illegible] digits are reversed. The difference between the old and the changed number

(a) 9 (b) 8 (c) 19 (d) 46

(e) 0

3. When we multiply a certain two-digit number by the sum of its digits, 405 is obtained. If we reverse the order of the digits, we get 486. Find the number.

(a) 45 (b) 45
(c) 36 (d) 54

(e) None of these

4. The sum of two numbers is 15 and their geometric mean is 6.25. Find the harmonic mean.

(a) 11.9 (b) 12.5
(c) 13.5 (d) 16.5

(e) 6.5

5. The difference between two numbers is 48 and the difference between the arithmetic mean and the greater of the two is less more than half of 15 of 96. Find the numbers.

(a) 46, 2 (b) 12, 60
(c) 62, 2 (d) 56, 64

(e) None of these

6. If 6(8) is divisible by 11, find the value of the smallest natural number x?

(a) 2 (b) 6
(c) 4 (d) 9

(e) None of these

7. If 6(x) is divisible by 9, find the value of smallest natural number x?

(a) 6 (b) 7
(c) 1 (d) 9
(e) 8

8. What will be the remainder obtained when $(9^{9} + 1)$ is divided by 8?

(a) 9 (b) 2 (c) 7 (d) 3

(e) 1

9. Find the ratio between the LCM and HCF of 3, 15 and 60?

(a) 1:1 (b) 3:1 (c) 15:2 (d) 12:1

(e) 1:12

10. Find the LCM of 132, 144, 168, 216?

(a) 1008 (b) 360
(c) 420 (d) 220

(e) None of these

11. If [illegible] A [illegible] what will be the [illegible]?

(a) 18 will always be divisible by 8
(b) 18 will always be divisible by 11
(c) $(a^{3} + 3)$ will be divisible by 7
(d) All of these
(e) None of these

12. A two-digit number is taken. Subtract the two-digit number (including the number in the units digit) equal to sum of all the digits. Which of the following will not divide the number necessarily?

(a) 10 (b) 2 (c) 3 (d) 5

(e) None of these

13. A number 15B is divisible by 6. Which of these will be true about the positive integer B?

(a) B will be even
(b) B will be odd
(c) B will be divisible by 6
(d) Both (a) and (c)
(e) None of these

14. Two numbers $p = 2^{3} \times 3^{4} \times 5$ and $q = 2^{2} \times 3^{2} \times 5^{3}$ are given. Find the GCD of P and Q.

(a) $2 \times 3 \times 5$ (b) 2×3^{2}
(c) $2^{2} \times 3^{2}$ (d) 2×3
(e) $2^{2} \times 3^{2} \times 2^{3} \times 3^{2}$

15. Find the unit digit of the expression $(29^{99} \times 31^{99} \times 77^{77})$?

(a) 4 (b) 0 (c) 8 (d) 1

(e) 3

16. What is the unit digit of the expression $(2^{99} \times 3^{99} \times 11^{99})$?

(a) 4 (b) 0 (c) 8 (d) 6

(e) 8

[illegible]

(a) 7 (b) 8 (c) 9 (d) 9

(e) 8

(e) [illegible]

(a) 0 (b) 1

(c) 7 (d) 8

(e) None of these

(e) Find the number of zeroes at the end of 1000!

(a) 256 (b) 200

(c) 249 (d) 253

(e) None of these

(e) If [illegible] divisible by 9, then find the minimum value of x.

(a) 36 (b) 32

(c) 34 (d) 33

(e) None of these

(e) Find the number of divisors of 1050

(a) 14 (b) 12

(c) 13 (d) 32

(e) None of these

(e) Find the HCF and LCM of the polynomials $(x^2 - 2x + 1)$ and $(x^2 - 3x + 1)$

(a) $(x-1)(x-2)(x-3)$, $(x-1)(x-3)$

(b) $(x-1)$, $(x-2)(x-3)$

(c) $(x-3)$, $(x-2)(x-1)(x-3)$, $x-3$

(d) $(x-2)(x-1)$, $(x-1)(x-3)$

(e) None of these

Directions for Questions 13-25: [illegible] given number # set. II find the number of divisors of the following

(a) [illegible] (b) 4 (c) 8 (d) 6

(e) 4

(e) [illegible]

(a) 2 (b) 8 (c) 6 (d) 6

(e) 8

(e) [illegible]

(a) 0 (b) 4 (c) 6 (d) 11

(e) 4

(e) The sum of contiguous [illegible] (a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible] (e) [illegible]

[illegible] Find the minimum length of the [illegible] so that the [illegible] may be measured [illegible]

(a) 7 (b) 28 (c) 31 (d) 8

(e) 31

(e) The [illegible] 176 kg and 400 gm [illegible] divided [illegible] equal [illegible] of [illegible] equal [illegible] Find the total number of [illegible]

(a) 24 (b) 32

(c) 40 (d) 20

(e) None of these

(e) [illegible] Find the [illegible] (a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible] (e) [illegible]

(a) 34 (b) 36 (c) 26 (d) 46

(e) 45

(e) What is the [illegible] (a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible] (e) [illegible]

(a) [illegible]

(b) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(e) [illegible]

(e) Find the least number that [illegible] 28 [illegible] 7.

(a) 364 (b) 1554

(c) 2364 (d) 1254

(e) 2364

(e) [illegible] (a) 72 (b) 36 (c) 48 (d) [illegible]

(e) 144

(e) Put question (e) find how many zeroes will [illegible] (a) 3 (b) 8

(c) 15 (d) 17

(e) None of these

(e) The units digit of the expression $(2^{2^{2^{2^{2}}}} + 1)^{2^{2^{2^{2^{2}}}}}$ is

(a) 4 (b) 2

(c) 8 (d) 5

(e) None of these

5. Which of the following is not a perfect square?
   (a) [illegible] (b) 5256121
   (c) 84[illegible]79 (d) 84 of these
   (e) None of these

6. Which of the following [illegible] is not ending as a perfect square?
   (a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
   (e) [illegible]

7. [illegible] is not divisible by [illegible]
   (a) 3 (b) 9
   (c) 6 (d) 8
   (e) None of these

8. Find the number of divisors of [illegible] including 1 and [illegible]
   (a) 28 (b) 35 (c) 34 (d) 36
   (e) 32

9. The LCM of [illegible] and [illegible]
   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]
   (e) None of these

10. HCF of [illegible] and [illegible]
    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]
    (e) None of these

11. The number [illegible] is divisible by 3. When [illegible] is divided by 9 it leaves remainder 2. When [illegible] is divided by 27 it leaves remainder 5. When [illegible] is divided by 27 it leaves remainder 8. When [illegible] is divided by 27 it leaves remainder 11. When [illegible] is divided by 27 it leaves remainder 14. When [illegible] is divided by 27 it leaves remainder 17. When [illegible] is divided by 27 it leaves remainder 20. When [illegible] is divided by 27 it leaves remainder 23. When [illegible] is divided by 27 it leaves remainder 26. When [illegible] is divided by 27 it leaves remainder 1. When [illegible] is divided by 27 it leaves remainder 4. When [illegible] is divided by 27 it leaves remainder 7. When [illegible] is divided by 27 it leaves remainder 10. When [illegible] is divided by 27 it leaves remainder 13. When [illegible] is divided by 27 it leaves remainder 16. When [illegible] is divided by 27 it leaves remainder 19. When [illegible] is divided by 27 it leaves remainder 22. When [illegible] is divided by 27 it leaves remainder 25. When [illegible] is divided by 27

[illegible]

[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) None of these

[illegible]

(a) 6
(b) 7
(c) 8
(d) 9
(e) None of these

[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]
(e) 28

[illegible]

(a) [illegible] and [illegible]
(b) [illegible] and [illegible]
(c) [illegible] and [illegible]
(d) [illegible] and [illegible]
(e) None of these

[illegible]

(a) [illegible] and [illegible]
(b) [illegible] and [illegible]
(c) [illegible] and [illegible]
(d) [illegible] and [illegible]
(e) None of these

[illegible]

(a) [illegible] and [illegible]
(b) [illegible] and [illegible]
(c) [illegible] and [illegible]
(d) [illegible] and [illegible]
(e) None of these

[illegible]

(a) [illegible] and [illegible]
(b) [illegible] and [illegible]
(c) [illegible] and [illegible]
(d) None of these

[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) None of these

[illegible]

(a) [illegible] and [illegible]
(b) [illegible] and [illegible]
(c) [illegible] and [illegible]
(d) None of these

[illegible]

(a) [illegible]
(b) [illegible]
(c)

(a) 5 (b) 5
(c) 6 (d) 7

[illegible]

(a) 0 (b) 2
(c) 3 (d) None of these

[illegible]

(a) 0 (b) 1
(c) 2 (d) 4

[illegible]

(a) 11 (b) 19
(c) 24 (d) Cannot be determined

[illegible]

(a) 10 (b) 15
(c) 14 (d) [illegible]

[illegible]

(a) 120 (b) [illegible]
(c) 220 (d) Cannot be determined

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[illegible]

(a) 1 (b) 2
(c) 3 (d) None of these

[illegible]

(a) 120 (b) 191
(c) 191 (d) Cannot be determined

[illegible]

(a) 10 (b) 15
(c) 15 (d) Cannot be determined

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[illegible]

(a) 2 (b) 3
(c) 4 (d) None of these

[illegible]

(a) 1 (b) 2
(c) 3 (d) None of these

[illegible]

(a) 100 (b) 106
(c) 108 (d) 112

[illegible]

(a) 18 (b) 30
(c) 36 (d) 7

[illegible]

(a) 15 (b) 30
(c) 13 (d) None of these

[illegible]

(a) 3 (b) 4
(c) 5 (d) 10

[illegible]

(a) 40 (b) 30
(c) 44 (d) 42

[illegible]

(a) 900 (b) 360
(c) 480 (d) 840

[illegible]

(a) 11 (b) 7
(c) 9 (d) 2

[illegible]

(a) 8000 (b) 9000
(c) 9900 (d) 9800

[illegible]

(a) 21 (b) −4
(c) 46 (d) None of these

[illegible]

(a) 10 (b) 30
(c) 40 (d) None of these

[illegible]

(a) 7 (b) 5
(c) 4 (d) Cannot be determined

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

Level of Difficulty (LOD)

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
(e) [illegible]

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
(e) [illegible]

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
(e) [illegible]

[illegible]

(a) $2^{2^n} + 2^{2^n}$ (b) $2^{2^n} + 2^{2^n}$
(c) $2^{2^n} + 2^{2^n}$ (d) $2^{2^n} + 2^{2^n}$
(e) $2^{2^n} + 2^{2^n}$

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
(e) [illegible]

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
(e) [illegible]

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) Any value above 6

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

(v) If we add the square of the digit to the tens place of a special two-digit number to the product of its digits of the number, we shall get 72, and if we add the square of the digit in the units place to the same product of the digits, we shall get 117. Find the two-digit number:

(a) 69 (b) 18 (c) 19 (d) 28

(e) 36

(b) Find two numbers such that the sum, difference and the difference of their squares are equal.

(a) $\left(\frac{1+\sqrt{3}}{2}\right)$ and $\left(\frac{1-\sqrt{3}}{2}\right)$ or $\left(\frac{1-\sqrt{3}}{2}\right)$ and $\left(\frac{1+\sqrt{3}}{2}\right)$

(b) $\left(\frac{1+\sqrt{3}}{2}\right)$ and $\left(\frac{1-\sqrt{3}}{2}\right)$ or $\left(\frac{1+\sqrt{3}}{2}\right)$ and $\left(\frac{1-\sqrt{3}}{2}\right)$

(c) $\left(\frac{1-\sqrt{3}}{2}\right)$ and $\left(\frac{1-\sqrt{3}}{2}\right)$ or $\left(\frac{1-\sqrt{3}}{2}\right)$ and $\left(\frac{1+\sqrt{3}}{2}\right)$

(d) All of these

(e) None of these

(c) The sum of the digits of a three-digit number is 13, and the sum of the squares of its digits is 108. If we subtract our from this number we shall get a number consisting of the same digits written in the reverse order. Find the number:

(a) 715 (b) 463

(c) 841 (d) 391

(e) 436

(d) Find the number of zeros in the product $1! \times 2! \times 3! \times \ldots 49! \times 50!$ $\times 1! \times 2! \times 3! \times \ldots 100!$

(a) 1200 (b) 1505

(c) 1000 (d) 1250

(e) None of these

(e) Find the pair of a natural number whose quotient when divided by 3 and the first common multiples is 351.

(a) 3 and 351 (b) 13 and 1

(c) 9 and 351 (d) 13 and 1

(e) 3 and 1 (f) 9 and 13

(g) 3 and 39 (h) 13 and 13

(i) None of these

(f) The denominator of an irreducible fraction is 1 less than the numerator. If we reduce the numerator by 2, if we reduce the numerator by 2, if we reduce the denominator by 1 and subtract the given fraction from the resulting one, we get $\frac{1}{6}$. Find the fraction.

(a) $\frac{11}{12}$ (b) $\frac{5}{6}$ (c) $\frac{7}{8}$ (d) $\frac{3}{4}$

(e) $\frac{5}{8}$

(g) A two-digit number exceeds by 27 the sum of the squares of its digits and by 14 the double product of its digits. Find the number.

(a) 72 (b) 81 (c) 32 (d) 15

(e) 18

(h) The sum of the squares of the digits constituting a three-digit positive number is 325 (from the sum of the squares of digits and a digit by which the sum of the squares of the digits. Find the number.

(a) 11 and 3 (b) 12 and 21

(c) 32 and 7 (d) 34 and 41

(e) None of these

(i) The units digit of a three-digit number is greater than twice that by 2, and the product of the number by the sum of its digits is 144. Find the number.

(a) 14 (b) 24 (c) 44 (d) 9

(e) 36

(j) Find the number of zeros in the product: $5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 45 \times 50$

(a) 8 (b) 9 (c) 11 (d) 5

(e) 10

(k) The power of 17 in the 81 factorial is $(17)^{x}$ (x)

(a) 20 (b) 30 (c) 31 (d) 32

(e) 20

(l) There numbers are such that the second is 4 times more than the third, the first is twice the second. If the product of the two smaller numbers and the product of the larger numbers is 144 find the middle number.

(a) 6 (b) 8 (c) 12 (d) 10

(e) 16

(m) Find the smallest natural number n such that $n = 2^{x} \times 3^{y}$ is divisible by 648.

(a) 5 (b) 7

(c) 11 (d) 13

(e) None of these

46. If $x^2 + y^2 = 9$ and $x + y = 3$, then only which

(a) $x = 0$ and $y = 3$ (b) $x = 3$ and $y = 0$

(c) $x = 0$ and $y = 0$ (d) All of these

(e) None of these

[illegible]

[illegible] the questions based on this.

In an examination [illegible] where we have the following type of questions for calculating the prize winners. Answer to the subject.

However, to avoid any risk, there are exceptions even in the qualifying limit [illegible] are known to be the exceptions in qualifying limit [illegible] to avoid any risk [illegible] mathematical operations [illegible]. If these questions fall within the limits, they would be called as LOH 1 questions. But when the option based [illegible] is concerned, [illegible] I have placed them in the LOH 2 category.

46. Find the two-digit number that satisfy the following [illegible]. If the number in the units place exceeds the number in the tens by 2 and the product of the required number with the sum of its digits is equal to 144.

47. The product of the digits of a two-digit number is one-sixth of the sum of its digits. If we subtract 27 from the original number, we get a number consisting of the same digits written in the reverse order. Find the number?

48. The product of the digits of a two-digit number is one-third that number. If we add 18 to the required number, we get a number consisting of the same digits written in the reverse order. Find the number?

49. The sum of the squares of the digits of a two-digit number is 13. If we subtract 9 from the number, we get a number consisting of the same digits written in the reverse order. Find the number?

50. A two-digit number is seven times larger than the sum of its digits and the square of the units digit is the tenth multiple of the tens digit. Find the number?

51. Find a two-digit number which exceeds by 12 the product of the squares of its digits and by 16 the arithmetic product of its digits.

52. The sum of the squares of the digits of a two-digit number is 37 and the product of the required two-digit number by the number consisting of the same digits written in the reverse order is 481. Find the 2 numbers satisfying these conditions?

53. If we divide a two-digit number by the sum of its digits we get 6 as quotient and 5 as remainder. Now, if we divide that two-digit number by the product of its digits we get 3 as quotient and 5 as remainder. Find the two-digit number.

54. There is a natural number that becomes equal to the square of a natural number when 100 is added to it and to the square of another natural number when 164 is added to it. Find the number?

55. Find two natural numbers whose sum is 85 and whose least common multiple is 102.

56. Find two three-digit numbers whose sum is [illegible] of the sum of the digits is a multiple of 5.

57. The difference between the digits in a two-digit number is equal to 7, and the sum of the squares of the same digits is 53. Find all the possible numbers?

58. If we divide a given two-digit number by the product of its digits, we get 3 as quotient and 5 as remainder. Find the two-digit number.

59. There is a natural number that becomes equal to the square of a natural number when 100 is added to it and to the square of another natural number when 164 is added to it. Find the number?

60. Find two natural numbers whose sum is 85 and whose least common multiple is 102.

61. Find two three-digit numbers whose sum is [illegible] of the sum of the digits is a multiple of 5.

62. The difference between the digits in a two-digit number is equal to 7, and the sum of the squares of the same digits is 53. Find all the possible numbers?

63. If we divide a given two-digit number by the product of its digits, we get 3 as quotient and 5 as remainder. If we subtract the product of the digits from the number, from the square of the sum of its digits, we get a prime number. Find the number.

64. Find the three-digit number if it is known that the sum of its digits is 17 and the sum of the squares of its digits is 109. If we subtract 297 from this number, we obtain another number consisting of the same digits written in reverse order.

65. The sum of the cubes of the digits constituting a three-digit number is 243 and the product of the sum of its digits by the product of its digits is 162. Find the two-digit number?

66. The difference between two numbers is 18. When each of them is divided by 3 that gives the quotients whose two numbers.

67. Arrange the following in descending order:

$(10)^{1}, (10)^{10}, (10)^{100}, (10)^{1000}, (10)^{10000}, (10)^{100000}, (10)^{1000000}$

68. If $(x + y) = 5$ and $(x - y) = 7$, Find the positive value of x and the least value of (x).

69. Which of these is greater

(a) 100^{99} or 99^{99} or 98^{99}

(b) 5^{40} and 2^{90}

(c) 10^{10} and 40^{5}

70. The sum of the two numbers is equal to 11 and their arithmetic mean is 25 per cent greater than the geometric mean. Find the numbers.

71. Define a number K such that K is the sum of the numbers of the form of natural numbers $a + b + 2^{2} + 2^{3} + \ldots + 2^{n}$

[illegible]

(a) 8 (b) 1

(c) 12 (d) None of these

[illegible]

[illegible]

(a) 36 (b) 48

(c) 60 (d) None of these

[illegible]

(a) 72 (b) 80

(c) 120 (d) None of these

[illegible]

(a) 3 (b) 4

(c) 6 (d) 7

[illegible]

(a) 1000 (b) 1101

(c) 1001 (d) 1000

[illegible]

(a) 1991 (b) 1997

(c) 1993 (d) None of these

[illegible]

(a) 6 (b) 8

(c) 7 (d) None of these

[illegible]

(a) 9 (b) 1

(c) 2 (d) None of these

[illegible]

(a) 7 (b) 5

(c) 9 (d) None of these

[illegible]

(a) 20 (b) 21

(c) 22 (d) 25

[illegible]

(a) 2^{2^n} (b) $2^{2^n}+1$

(c) 2^{2^n} (d) $2^{2^n}-1$

72. What is the remainder when $8+8^2+8^3+\ldots+8^{2000}$ is divided by 6?

(a) 1 (b) 2

(c) 3 (d) 4

73. The remainder when the number [illegible] is divided by 16 is

(a) 1 (b) 4

(c) 8 (d) 0

74. If $N=(1+2+3+\ldots+10)$ [illegible] then the highest power of 3 in $(N!)$ is

(a) 27 (b) 29

(c) 18 (d) None of these

75. Find the remainder when the number represented [illegible] is divided by 16.

(a) 7 (b) 1

(c) 2 (d) 4

(e) 0 (f) None of these

76. What is the total number of factors of the number [illegible]

(a) 4608 (b) 9216

(c) 2304 (d) None of these

77. [illegible] the sum of factors of the number [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

78. What is the remainder when [illegible] divided by 7?

(a) 28 (b) 252

(c) 32 (d) 360

79. [illegible] divisible by [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

[illegible] numbers in each row that satisfy the following conditions:

(i) No prime number of the set S is present.

(ii) When the numbers of the set S are arranged [illegible] a number that divides less than [illegible] prime number.

What is the number of elements in the set S?

(a) 2 (b) 12

(c) 0 (d) 1

[illegible] the first non-zero digit of the following numbers:

40. [illegible] 142 × 100 × 107 × 106 × 128

(a) 36 (b) 34

(c) 26 (d) 44

41. [illegible] 29 × 27 × 69 × 71 × 87

(a) 49 (b) 45

(c) 19 (d) 35

42. 45 × 29 × 27 × 63 × 71 × 87 × 61

(a) 29 (b) 35

(c) 19 (d) 47

43. [illegible] 29 × 27 × 63 × 71 × 87 × 62

(a) 36 (b) 31

(c) 29 (d) 93

44. [illegible] 29 × 27 × 63 × 71 × 87 × 62

(a) 36 (b) 31

(c) 18 (d) 49

45. [illegible] 2365 × 2309 × 204 × 246 × 257 × 108 × 245)

(a) 36 (b) 16

(c) 9 (d) 10

46. Find the remainder when $4^{[illegible]}$ is divided by 24?

(a) 1 (b) 321

(c) 19 (d) 7

47. Find the remainder when $(8^{[illegible]} \times 9^{[illegible]})^{[illegible]}$ is divided by 7?

(a) 2[illegible] (b) 1[illegible]

(c) 2[illegible] (d) 1

48. [illegible] Kiran and Deepika have a total of [illegible] coins with them. Kiran argues the amount of coins with the others by giving them some coins [illegible] Deepika argues the same amount by [illegible] Kiran argues the same amount [illegible] Deepika. Find the number of coins in hand [illegible] beginning?

(a) 2[illegible] (b) 3[illegible]

(c) 3[illegible] (d) 2[illegible]

49. The upper compass in Bora Station B is between [illegible] a angle of a number N and a E where E is a factor of an angle of a number N and a E where E is a factor of the number N. In a particular case, N is equal to 8 [illegible] angle E is equal to 15 where E is a factor of N and [illegible] angle of a number N is divisible by N? If [illegible] number N is divisible by N? [illegible]

(a) 6 (b) 5

(c) 2 (d) 9

50. [illegible] the Month i.e. 2016, Hours outside the [illegible] working Days Right Jul 2016, to avoid the [illegible] the previous day. [illegible] the [illegible] of the set of which [illegible] will be a [illegible] [illegible]

(a) 27th March 2016 (b) 10th April 2016

(c) 26th March 2016 (d) None of these

51. What is the rightmost digit preceding the zeros in the value of $20!$?

(a) 2 (b) 8

(c) 1 (d) 9

52. What is the remainder when $2(91) - 2(90)$ is divided by 7 [illegible] $(2^{[illegible]})$?

(a) 1 (b) 3

(c) 6 (d) 5

53. How many integer values of x satisfy any form [illegible] $4x + 7x = 2$, [illegible] $(x - 5)$?

(a) 104 (b) 141

(c) 30 (d) 34[illegible]

54. If $n = 1$ [illegible] the product of two consecutive prime numbers, then which of the following [illegible] true?

(A) n is odd (B) n is odd [illegible]

(C) n is a perfect square

(a) A Only (b) A and B only

(c) A and C only (d) None of these

55. How many two-digit numbers are there that are equal to the product of the factorials of their digits?

(a) 0 (b) 9

(c) 2 (d) None of these

56. A candidate takes a test and attempts all the 150 questions in it. While any correct answer fetches 1 mark, every incorrect one penalizes him by [illegible] marks of the questions carry 1 negative mark each, [illegible] of the questions carry 2 negative mark each. [illegible] attempted questions. What is the [illegible] he can forward the maximum and the minimum marks [illegible] in any case?

(a) 100 (b) 120

(c) 110 (d) None of these

Directions for Questions 57 to 60: [illegible] and [illegible] in a [illegible] Number System. The two small numbers of [illegible]

[illegible] in three levels of difficulty—[illegible], [illegible] and [illegible].

(a) Following table gives the details of the questions and [illegible] in each question type.

[illegible]	[illegible]	[illegible]
[illegible] 1	8	[illegible]
[illegible] 2	7	[illegible]
[illegible] 4	6	[illegible]

The section [illegible] consists of 1 [illegible] and [illegible] on [illegible] and [illegible].

(i) If a student has solved [illegible] questions correctly and scored [illegible] marks, the maximum number of incorrect questions that he/she might have attempted is:

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

(ii) If the [illegible] number of questions he/she got attempted of [illegible] and it is known that he/she attempted at least one of each type, then the number of questions he/she attempted is:

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

(iii) In the above question, what is the least number of questions he/she might have got incorrect?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

[illegible] Suppose [illegible] of [illegible], such that the [illegible] average is [illegible] and the [illegible] average is [illegible]. [illegible] average is [illegible] and [illegible] average is [illegible]. [illegible] average is [illegible] and [illegible] average is [illegible]. [illegible] average is [illegible] and

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

(e) None of these

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]

(e) [illegible]

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]

(e) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(e) None of these

(c) 100 (d) 120

(e) None of these

[illegible] digit of integers only in each case $x^2 + y^2$ [illegible] the remainder when divided by 8 digits to the two (2) [illegible] Find the number (x) is known that the difference between consecutive numbers [illegible] number [illegible] by the same digits by the reverse order' is 495.

(a) 63 (b) 549

(c) 693 (d) 792

(e) None of these

[illegible] the number 1.25 as a product of three positive factors so that the product of first factor by the square of the second is equal to 9. If we have to find the [illegible] of the third factor.

(a) $x_1 = 2.25, x_2 = 3, x_3 = 0.5$

(b) $x_1 = 1.25, x_2 = 6, x_3 = 4.5$

(c) $x_1 = 1.25, x_2 = 3, x_3 = 0.5$

(d) $x_1 = 1.25, x_2 = 6, x_3 = 2$

(e) None of these

[illegible] Find a number such that the sum of that number and its square is the least.

(a) -0.5 (b) 0.5 (c) -1.5 (d) 1.5

(e) 1

[illegible] When $2222^{3333} + 1111^{7777}$ is divided by 5, the remainder is

(a) 0 (b) 2 (c) 4 (d) 3

[illegible] If the number of two-digits which when divided by 3, 4, 5, 6 and 7 leaves remainders 2, 3, 4, 5, 6 respectively 3, 5, 12 and 13 is [illegible] Find [illegible] such that it is the smallest [illegible]

(a) 1007 (b) 1007 (c) 1080 (d) 1017

(e) None of these

[illegible] $2^{[illegible]}$ is divisible by $2^{[illegible]}$ for n is

(a) 1 (b) 2

(c) 3 (d) 4

(e) None of these

[illegible] $\sqrt{17}$ is divisible by $5^{[illegible]}$ for which [illegible] number value of x?

(a) 1 (b) 2 (c) 3 (d) 4

(e) None of these

[illegible] $\frac{[illegible]}{8}$ will leave a remainder

(a) 0 (b) 1 (c) 6 (d) 7

[illegible] Find the remainder when [illegible] is divided by 7.

(a) 0 (b) 1 (c) 2 (d) 3

(e) 4

[illegible] Find the remainder of $[illegible]^{[illegible]}$ when divided by 7

(a) 1 (b) 6 (c) 3 (d) 2

(e) None of these

[illegible] Find the remainder when the number $[illegible]^{[illegible]} + 1$ is divided by 7

(a) 0 (b) 5 (c) 6 (d) 1

(e) 2

[illegible] Find the last digit of the number $1 + 2^{[illegible]} + [illegible]$

(a) 1 (b) 3 (c) 7 (d) 5

(e) 9

[illegible] Find $(1)^{[illegible]} + (2)^{[illegible]} + [illegible]$

(a) $2^{[illegible]} - 1$ (b) $2^{[illegible]} + 1$

(c) $2^{[illegible]} - 1$ (d) $2^{[illegible]} + 1$

(e) $2^{[illegible]} - 1$

[illegible] Find the gcd $(111\ldots1)$ [illegible] 111 [illegible]

(a) 111... [illegible] (b) 111... [illegible]

(c) 111... [illegible] (d) 111... [illegible]

(e) None of these

[illegible] Find the last digit of the number $[illegible]$

(a) 1 (b) 3 (c) 2 (d) 7

(e) 9

[illegible] Find the GCD of the numbers [illegible]

(a) 1 (b) 2 (c) 4 (d) 7

(e) 5

[illegible] $\frac{[illegible]}{2}$

(a) 4 (b) 8 (c) 1 (d) 3

(e) 5

[illegible] The remainder when $[illegible]$ is divided by 7 is

(a) 0 (b) 1 (c) 3 (d) 5

(e) 4

[illegible] A positive number [illegible] Find the [illegible]

(a) 20 (b) 32

(c) 30 (d) None of these

[illegible] Suppose the [illegible]

[illegible] the number of the following numbers in the ascending order of magnitude?

(a) The number of letters can be 10
(b) The number of letters can be 5
(c) The number of letters can be 12
(d) The number of letters can be 16

[illegible]

(a) 2 (b) 3
(c) 4 (d) 5

[illegible]

(a) 32 (b) 40
(c) 48 (d) Cannot be determined

[illegible] Many the sets of problems on basic computer-science to the ER students of 'CAMPUS' were [illegible] in the ARD women team. The 14 students of such CAMPUS included 50 students of each month. 19 students were female in the 'ad' [illegible] the 13 students of each of the other 13 CAMPUS. What could be the minimum possible number of students participating the 14 students of the 13 CAMPUS are involved?

(a) 30 (b) 35
(c) 40 (d) 100

[illegible] In the above question, what could be the minimum possible number of students participated?

(a) 80 (b) 85
(c) 90 (d) None of these

Answer for Questions [illegible] Suppose in a particular way in the [illegible] information there were seven brackets, 1 the [illegible] [illegible] 10 students that [illegible] the [illegible] [illegible] that if one [illegible] the number of students taken by one of the 7 brackets of the [illegible] [illegible] the number of students taken by [illegible] [illegible] the [illegible] [illegible] [illegible]

[illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

(e) [illegible]

(f) [illegible]

(g) [illegible]

(h) [illegible]

(i) [illegible]

(j) [illegible]

(k) [illegible]

(l) [illegible]

(m) [illegible]

(n) [illegible]

(o) [illegible]

(p) [illegible]

(q) [illegible]

(r) [illegible]

(s) [illegible]

(t) [illegible]

(u) [illegible]

(v) [illegible]

(w) [illegible]

(x) [illegible]

(y) [illegible]

(z) [

[illegible]

[illegible]

[illegible]

[illegible]

(ii) [illegible]

Hence the average will be: $\frac{2^N - 1}{N} = 2^N - 1/N$

which lies between 2^N and 2^{N-1}

Hence the answer will be (c)

(iii) The determinant [illegible] that the determinant is given for [illegible] of the same [illegible]. The question is presented in the [illegible] of [illegible]. Options [illegible]

(iv) The value [illegible] has a lot [illegible]. Hence option [illegible] is the correct choice.

(v) For [illegible] given [illegible] (should be [illegible])

Further [illegible] [illegible]

Both these conditions are satisfied by [illegible]

Hence option (d) is correct

(vi) [illegible] by 11

(vii) For [illegible] the only [illegible] for the value of [illegible] should be either [illegible]

(viii) The decimal of [illegible] is a sum of all possible values of [illegible]

(ix) [illegible] condition

(a) When [illegible] [illegible] 4

Putting [illegible]

(b) When [illegible] should be [illegible] also be 2. This does not happen

(c) When [illegible] should be [illegible] given value

(x) [illegible] is odd power of [illegible] [illegible] [illegible]

[illegible]

[illegible]

(i) [illegible]

It can be easily seen that

[illegible]

[illegible] Hence the sum of the digits of the number [illegible]

(ii) [illegible] Hence the sum of the digits of the number [illegible]

(iii) [illegible]

(iv) [illegible]

(v) [illegible]

(vi) [illegible]

(vii) [illegible]

(viii) [illegible]

(ix) [illegible]

(x) [illegible]

Results in a convenient form for comparing.

6. Which is larger: the quotient obtaining (i) and (ii) [illegible]

(i) $\frac{2^{10}}{2^{5}}$. Hence it becomes $2^{10}/2^{5}$, [illegible] $2^{5} \times 2^{5}$ has 10 digits. Thus the number of digits is increased. Since the number is increased by 300, which is not what the question is asking for the number. Hence (i) is not the correct answer.

Checking for option (ii) [illegible]

[illegible]

Second Condition: If we the value of the number is increased by 2[illegible] would become [illegible] and the number of digits would be [illegible] which is the number of digits.

7. In order to solve this question, most of the numbers grouped in groups of 9 are

[illegible]

From such we can solve Conditions [illegible]

[illegible]

Now from this, we calculate [illegible]

[illegible]

[illegible]

8. Instead of any one complete number would have [illegible] and (ii) the difference of squares of two consecutive natural numbers. Thus we need to find the number of consecutive numbers that are odd.

[illegible]

9. The remainder when dividing [illegible] by 9 is [illegible] the remainder of the last 4 digits. Divided by 9, [illegible] 10000 is a multiple of 9. Thus, [illegible] very simply a logic would be look at the 2 digits [illegible] to find the last 2 digits [illegible] 6. Thus, the required answer would be the remainder of [illegible] divided by 9.

[illegible]

Essentially every number starting from 4[illegible] would be divisible perfectly by 1[illegible] since such number after that would have at least 1 zero and 2 threes.

Thus, the required remainder is got by the first three terms.

[illegible] gives us the 2[illegible] as the required remainder.

10. [illegible] number in terms of the number of [illegible]

[illegible]

Since of Rahul, [illegible] and [illegible] and [illegible] left with the number that the other 5 have to [illegible]. After three cases are helped. This means the before the dividing by Rahul, the whole thing [illegible] [illegible] Rahul [illegible] Rahul [illegible] [illegible] $R \Rightarrow 5k + 30$.

11. [illegible]

[illegible]

Hence the whole number should be [illegible] for the sake of simplicity sum of the numbers is the given sum is forty, out the 3 [illegible] [illegible] and the denominator is [illegible]

12. Product of factorials = Sum of factorials would mean [illegible]

The required number [illegible] 30, 31, 32, 33, 34, 35. Besides in the number 32, the product of factorials of the digits would be equal to the sum of factorials of the digits. Thus a total of 31 numbers.

13. The least number divisible [illegible] 9, 8, 7, 6 and [illegible] The second least number [illegible]

Hints and Solutions

1–18. [illegible] options.

4. [illegible]

7. [illegible]

8. [illegible]

16. [illegible]

17. [illegible]

19–26. Use options to check the given conditions.

19. [illegible]

20. [illegible]

21. [illegible]

22–28. Use options as usual.

26. [illegible]

27. [illegible]

28. [illegible]

29. [illegible]

30. [illegible]

31. [illegible]

32. [illegible]

33. [illegible]

34. [illegible]

35. [illegible]

36. [illegible]

37. [illegible]

38. [illegible]

ANSWER KEY

Section (Total 100)

1. a
2. a
3. a
4. a
5. a
6. b
7. b
8. a
9. c
10. a
11. a
12. d
13. a
14. c
15. b
16. a
17. a
18. c
19. a
20. a
21. a
22. a
23. a
24. a
25. a
26. a
27. a
28. a
29. b
30. a
31. a
32. a
33. a
34. a

35. b
36. b
37. a
38. c
39. c
40. a
41. 24
42. 32
43. 39
44. 56
45. 15, 6
46. 22
47. 1296
48. 32, 36
49. 144, 864
50. 40, 16
51. 30
52. 405
53. 4, 6, 4
54. $\cos\frac{\pi}{3}$ (or) $\frac{1}{2}$
According zero
Ka number.
55. $(11)^2$ x 100,101
112,113 x 116
100, 00,101
56. greater : x 75
more 105
57. a $(200)^{75}$
b) 1^{100}
c) 3.10^{10}
58. 12, 3
59. c
60. a
61. a
62. a
63. a

64. a
65. a
66. a
67. a
68. a
69. a
70. a
71. c
72. c
73. c
74. a
75. a
76. a
77. a
78. a
79. a
80. a
81. b
82. c
83. a
84. a
85. a
86. a
87. a
88. a
89. a
90. a
91. a
92. a
93. a
94. a
95. a
96. a
97. a
98. a
99. a
100. a

2

PROGRESSIONS

[illegible]

[illegible]

[illegible]

[illegible]

ARITHMETIC PROGRESSIONS

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

To Find the Sum of the Given Number of Terms in an Arithmetic Progression

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

To Find the Arithmetic Mean between the Two Given Quantities

[illegible]

$$b - A = A - a$$

Each being equal to the common difference.

This gives us $$A = \frac{(a+b)}{2}$$

Between any given quantities it is always possible to insert any number of terms such that the whole series thus formed shall be in AP. The series thus inserted are called the **arithmetic means**.

To Insert a Given Number of Arithmetic Means between Two Given Quantities

Let a and b be the given quantities and n be the number of means.

Including the extremes, the number of terms will then be $n + 2$ so that we have to find a series of $n + 2$ terms in AP, of which a is the first, and b is the last term.

Let d be the common difference

then $$b = \text{the } (n+2)\text{th term}$$
$$= a + (n+1)d$$

Hence, $$d = \frac{(b-a)}{(n+1)}$$

and the required means are

$$a + \frac{(b-a)}{n+1},\ a + \frac{2(b-a)}{n+1}, \ldots, a + \frac{n(b-a)}{n+1}$$

Till now we have studied APs in their mathematical context. This was important for you to understand the basic mathematical construct of A.Ps. However, you need to understand that questions on A.P. are seldom solved on a mathematical basis. (Especially under the time pressure that you are likely to face under the CAT and other aptitude exams). In such situations, the mathematical processes for solving progressions based questions are likely to fail. Hence, understanding the following logical aspects about Arithmetic Progressions is likely to help you solve questions based on APs in the context of an aptitude exam.

Let us look at these issues one by one.

1. Process for finding the nth term of an A.P.

Suppose you have to find the 17th term of the

A.P. 3, 7, 11, ...

The conventional mathematical process for this question would involve using the formula

$$T_n = a + (n-1)d$$

Thus, for the 17th term we would do

$$T_{17} = 3 + (17-1) \times 4 = 3 + 16 \times 4 = 67$$

Most students would mechanically insert the values for a, n and d and get this answer.

However, if you replace the above process with a thought algorithm, you will get the answer much faster.

The algorithm goes like this.

In order to find the 17th term of the above sequence add the common difference to the first term, sixteen times. (**Note:** Sixteen, since it is one less than 17).

Similarly in order to find the 37th term of the A.P. 3, 11, ..., All you need to do is add the common difference (4 in this case) 36 times.

Thus, the answer is $289 + 3 = 292$.

(**Note:** You ultimately end up doing the same thing, but you are at an advantage since the entire solution process is mechanical.)

2. Average of an A.P. and Corresponding terms of the A.P.

Consider the A.P., 2, 6, 10, 14, 18, 22. If you try to find the average of these six numbers, you will get. Average $= \frac{2 + 6 + 10 + 14 + 18 + 22}{6} = 12$.

Notice this 12 is also the average of the first and the last terms of the A.P. In fact, it is also the average of 6 and 18 (which correspond to the second and 5th terms of the A.P.). Further, 12 is also the average of the 3rd and 4th terms of the A.P.

(**Note:** In this A.P. of six terms, the average was the same as the average of the 1st and 6th terms. It was also given by the average of the 2nd and the 5th terms, as well as that of the 3rd and 4th terms.)

We can call each of these pairs as "CORRESPONDING TERMS" in an A.P.

What you need to understand is that every A.P. has an average.

And for any A.P., the average of any pair of corresponding terms will also be the average of the A.P.

If you try to notice the sum of the term numbers of the pair of corresponding terms given above:

1st and 6th (in that 1 + 6 = 7)
2nd and 5th(hence, 2 + 5 = 7)
3rd and 4th (hence, 3 + 4 = 7)

Note that: In each of these cases, the sum of the term numbers for the terms in a corresponding pair is one greater than the number of terms of the A.P.

This rule will hold true for all A.Ps.

For example, if an A.P. has 13 terms (this for instance), you can predict that the 7th term will have the 13th term as its corresponding term, or for that matter the 8th term will

[illegible]

2. Finding the common difference of an A.P., given 7 terms of the AP

[illegible]

3. Finding the common difference of an A.P., given 3 terms of the AP

[illegible]

Figure 1.11 Increasing AP

[illegible]

Hence $d = 0$

Note: Explore the difference between [illegible]

[illegible]

2. Types of APs: Increasing and Decreasing A.Ps

[illegible]

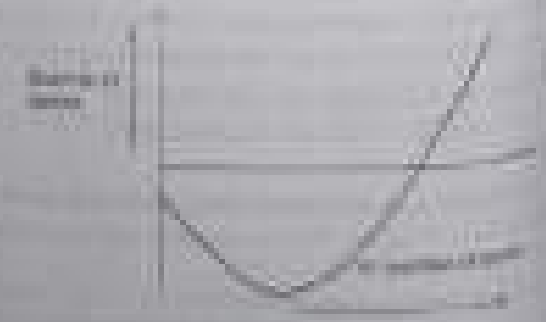
(A) Increasing A.Ps:

[illegible]

Consider the case of the [illegible] AP.

Case 1: Now, by knowing of the increasing A.P. [illegible] we know that the A.P. will [illegible] each increasing angle [illegible] like the [illegible] the figure below:

Case 2: When the terms of the increasing A.P. [illegible] the [illegible] AP. [illegible] of [illegible] the Decreasing Graph"



The specific case of the sum to n_1 terms being equal to the sum to n_2 terms.

In such a series, there is a possibility of the sum to 'n' terms being same for 2 values of 'n'. However, this will not necessarily occur.

This issue will get clear through the following example.

Consider the following series:

Series 1: $-15, -8, -1, 6, 13, 12$

So it is evident the sum to 2 terms and the sum to 5 terms is the same in the series. Similarly, the sum to 3 terms is the same as the sum to 4 terms. This can be written as

$$S_2 = S_5 \text{ and } S_3 = S_4$$

In other words the sum to n_1 terms is the same as the sum to n_2 terms.

Such situations arise for increasing A.P.'s where the first term is negative. But as we have already stated that this does not happen for all such cases.

Consider the following A.Ps.

Series 2 : $-6, -3, +2, +7, +12 \ldots$

Series 3 : $-13, -2, -1, +8, +11 \ldots$

Series 4 : $-12, -6, 0, 6, 12 \ldots$

Series 5 : $-15, -8, -3, +3, 9, 15 \ldots$

Series 6 : $-20, -12, -4, 4, 12 \ldots$

If we check the series listed above, you will realize that this phenomenon happens in the case of Series 1, Series 4, Series 5 and Series 6 while in the case of Series 2 and Series 3 the same value is not repeated for the sum of the Series.

A close look at the two series will reveal that this phenomenon occurs in series which have what can be called a balance about the number zero.

Another issue to notice is that in Series 4,

$$S_2 = S_4 \text{ and } S_3 = S_5$$

While in series 5

$$S_2 = S_4 \text{ and } S_3 = S_5$$

In the first case (where '0' is part of the series) the case is equal for two terms such that one of them is odd and the other is even.

In the second case on the other hand (when '0' is not part of the series) the sum is equal for two terms such that both are odd or both are even.

The notion that the sum of the term numbers which exhibit equal sums is constant for a given A.P.

Consider the following question which appeared in CAT 2008 and is based on this logic:

The sum to 12 terms of an A.P. is equal to the sum to 19 terms. What will be the sum to 30 terms for this series?

Solution: If $S_{12} = S_{19}$, $S_{11} = S_{18}$ and $S_2 = S_{29}$

But sum to zero terms for any series will always be 0.

Hence $S_{30} = 0$.

(Note: The solution to this problem does not take more than 40 seconds if you know this logic)

(B) Decreasing A.Ps.

Similar to the cases of the increasing A.Ps. we can have two cases for decreasing APs –

Case 1– Decreasing A.P. with first term negative.

Case 2– Decreasing A.P. with first term positive.

I leave it to the reader to understand these cases and deduce that whatever was true for increasing A.Ps. with first term negative will also be true for decreasing APs with first term positive.

GEOMETRIC PROGRESSION

Quantities are said to be in Geometric Progression when they increase or decrease by a constant factor.

The constant factor is also called the *common ratio* and it is found by dividing any term by the term immediately preceding it.

If we examine the series $a, ar, ar^2, ar^3, ar^4 \ldots$ *we notice that in any term the index of r is always less by one than the number of the term in the series.*

If n be the number of terms and l denote the last, or nth term, we have

$$l = ar^{n-1}$$

When three quantities are in geometrical progression, the middle one is called the geometric mean between the other two. While arbitrarily choosing these numbers in GP we take a/r, a and ar. This makes it easier since we confine down to two variables for the three terms.

To Find the Geometric Mean between Two Given Quantities

Let a and b be the two quantities, G the geometric mean. Then since a, G, b are in GP,

$$b/G = G/a$$

Each being equal to the common ratio

$$G^2 = ab$$

Hence $G = \sqrt{ab}$

To Insert a given Number of Geometric means between Two Given Quantities

Let a and b be the given quantities and n the required number of means to be inserted. In all there will be $n + 2$ terms so that we have to find a series of $n + 2$ terms in GP of which a is the first and b the last.

Let r be the common ratio.

Then b = the $(n + 2)$th term = ar^{n+1}

$$r^{n+1} = \frac{b}{a}$$

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} \quad (1)$$

Hence the required number of means are $ar, ar^2, \ldots ar^n$, where r has the value found in (1).

To Find the Sum of a Number of Terms in a Geometric Progression

Let a be the first term, r the common ratio, n the number of terms, and S_n be the sum to n terms.

If $r > 1$, then

$$S_n = \frac{a(r^n - 1)}{(r - 1)} \quad (1)$$

If $r < 1$, then

$$S_n = \frac{a(1 - r^n)}{(1 - r)} \quad (2)$$

Note: It will be convenient to remember both forms given above for it. Number (2) will be used in all cases except when r is positive and greater than **one**.

Sum of an infinite geometric progression when $r < 1$

$$S_\infty = \frac{a}{(1 - r)}$$

Obviously, this formula is used only when the common ratio of the GP is less than one.

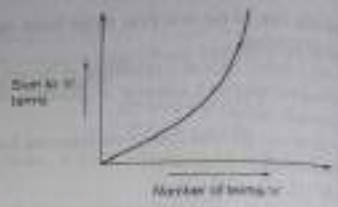
Similar to APs, GPs can also be logically viewed. Based on the value of the common ratio and its first term a G.P might have one of the following structures:

(1) Increasing GPs type 1:

A G.P. with first term positive and common ratio greater than 1. This is the most common type of G.P.

e.g. 1, 4, 12, 36 ..... (A G.P. with first term 1 and common ratio 3)

The plot of the sum of the series with respect to the number of terms in such a case will appear as follows:



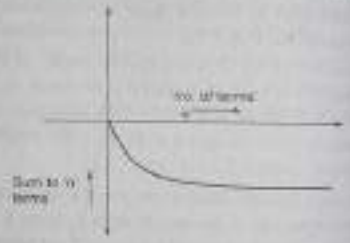
(2) Increasing GPs type 2:

A G.P. with first term negative and common ratio less than 1.

e.g. −8, −4, −2, −1, ...

As you can see in this GP all terms are greater than the previous terms.

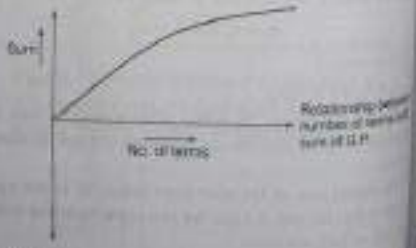
(The following figure will illustrate the relationship between the number of terms and the sum to 'n' terms in this case)



(3) Decreasing G.Ps type 1:

These GPs have their first term positive and common ratio less than 1.

e.g. 12, 6, 3, 1.5, 0.75 .....

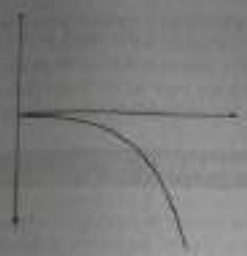


(4) Increasing GPs type 2:

First term negative and common ratio greater than 1.

e.g. −2, −6, −18 .....

In this case the relationship looks like



HARMONIC PROGRESSION

Three quantities a, b, c are said to be in Harmonic Progression when $a/c = \frac{(a-b)}{(b-c)}$.

In general, if a, b, c, d are in AP then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ and $\frac{1}{d}$ are all in HP.

Any number of quantities are said to be in harmonic progression when every three consecutive terms are in harmonic progression.

The reciprocals of quantities in harmonic progression are in arithmetic progression. This can be proved as:

By definition, if a, b, c are in harmonic progression,

$$\frac{a}{c} = \frac{(a-b)}{(b-c)}$$

$$\Rightarrow \quad ab - ac = cb - cb,$$

dividing every term by abc, we get

$$\left[\frac{1}{c} - \frac{1}{b} = \frac{1}{b} - \frac{1}{a}\right]$$

which proves the proposition.

There is no general formula for the sum of any number of quantities in harmonic progression. Questions in HP are generally solved by inverting the terms and making use of the properties of the corresponding AP.

To Find the Harmonic Mean between Two Given Quantities

Let a, b be the two quantities. If their harmonic mean then $\frac{1}{a}, \frac{1}{H}$ and $\frac{1}{b}$ are in A.P.

$$\Rightarrow \quad \frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H}$$

$$\frac{2}{H} = \frac{1}{a} + \frac{1}{b}$$

$$\text{i.e.} \quad H = \frac{2ab}{(a+b)}$$

THEOREMS RELATED WITH PROGRESSIONS

If A, G, H are the arithmetic, geometric, and harmonic means between a and b, we have

$$A = \left(\frac{a+b}{2}\right) \quad (1)$$

$$G = \sqrt{ab} \quad (2)$$

$$H = \frac{2ab}{(a+b)} \quad (3)$$

Therefore, $A \times H = \frac{(a+b)}{2} \times \frac{2ab}{(a+b)} = ab = G^2$

that is, G is the geometric mean between A and H.

From these results we see that

$$A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{(a+b-2\sqrt{ab})}{2}$$

$$= \left[\frac{(\sqrt{a} - \sqrt{b})}{\sqrt{2}}\right]^2$$

which is positive if a and b are positive. Therefore, the arithmetic mean of any two positive quantities is greater than their geometric mean.

Also from the equation $G^2 = AH$, we see that G is intermediate in value between A and H, and it has been proved that $A > G$, therefore $G > H$ and $A > G > H$.

The arithmetic, geometric, and harmonic means between any two positive quantities are in descending order of magnitude.

As we have already seen in the Hints to solved sections of this book there are some number series which have a continuously decreasing value from one term to the next and such series have the property that they have what can be defined as the sum of infinite terms. Questions on such series are very common in most aptitude exams. Even though they cannot be strictly said to be under the domain of progressions, we choose to deal with them here.

[illegible]

[illegible]

[illegible]

Let the sum be denoted by S, then

$$S = 1^2 + 2^2 + 3^2 + \ldots + n^2$$

This is given by $= S = \left[\frac{n(n+1)(2n+1)}{6}\right]$

8. To find the sum of the cubes of the first n natural numbers:

Let the sum be denoted by S, then

$$S = 1^3 + 2^3 + 3^3 + \ldots + n^3$$

$$S = \left[\frac{n(n+1)}{2}\right]^2$$

Thus, the sum of the cubes of the first n natural numbers is equal to the square of the sum of these numbers.

10. To find the sum of the first n odd natural numbers:

$$S = 1 + 3 + 5 + \ldots + (2n-1) \rightarrow n^2$$

11. To find the sum of the first n even natural numbers:

$$S = 2 + 4 + 6 + \ldots + 2n \rightarrow n(n+1)$$
$$= n^2 + n$$

12. To find the sum of odd numbers $\leq n$ where n is a natural number:

Case A: If n is odd $\rightarrow [(n+1)/2]^2$
Case B: If n is even $\rightarrow [n/2]^2$

13. To find the sum of even numbers $\leq n$ where n is a natural number:

Case A: If n is odd $\rightarrow [(n/2)][(n/2)+1]$
Case B: If n is even $\rightarrow [(n-1)/2][(n+1)/2]$

14. Number of terms in a count:

- If we are counting in steps of 1 from n_1 to n_2 including both the end points, we get $(n_2 - n_1) + 1$ numbers.
- If we are counting in steps of 1 from n_1 to n_2 including only one end, we get $(n_2 - n_1)$ numbers.
- If we are counting in steps of 1 from n_1 to n_2 excluding both ends, we get $(n_2 - n_1) - 1$ numbers.

Example: Between 16 and 25 both included there are $9 + 1 = 10$ numbers.

Between 100 and 200 both excluded there are $100 - 1 = 99$ numbers.

- If we are counting in steps of 2 from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/2] + 1$ numbers.
- If we are counting in steps of 2 from n_1 to n_2 including only one end, we get $[(n_2 - n_1)/2]$ numbers.
- If we are counting in steps of 2 from n_1 to n_2 excluding both ends, we get $[(n_2 - n_1)/2] - 1$ numbers.

Example: Number of even numbers between 20 and 100 (both included) is $100 - 20 \rightarrow (80/2) + 1 = 41$

- If we are counting in steps of 3 from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/3] + 1$ numbers.
- If we are counting in steps of 3 from n_1 to n_2 including only one end, we get $[(n_2 - n_1)/3]$ numbers.
- If we are counting in steps of 3 from n_1 to n_2 excluding both ends, we get $[(n_2 - n_1)/3] - 1$ numbers.

Example: Number of numbers between 100 and 200 divisible by three.

Solution. The first number is 102 and the last number is 198. Hence, answer $= (96/3) + 1 = 33$ (since both 102 and 198 are included).

Alternately, highest number below 100 that is divisible by 3 is 99, and the lowest number above 200 which is divisible by 3 is 201.

Hence, $201 - 99 = 102 \rightarrow 102/3 = 34 \rightarrow$ Answer $= 34 - 1 = 33$ (Since both ends are not included.)

In General

- If we are counting in steps of x from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/x] + 1$ numbers.
- If we are counting in steps of "x" from n_1 to n_2 including only one end, we get $(n_2 - n_1)/x$ numbers.
- If we are counting in steps of "x" from n_1 to n_2 excluding both ends, we get $[(n_2 - n_1)/x] - 1$ numbers.

For instance, if we have to find how many terms are there in the series 107, 114, 121, 128 ... 254, then we have

$(254 - 107)/7 + 1 = 147/7 + 1 = 21 + 1 = 22$ terms in the series

Of course, an appropriate adjustment will have to be made when n_2 does not fall into the series. This will be done as follows.

For instance, if we have to find how many terms of the series 107, 114, 121, 128 ... are below 256, then we have by the formula:

$\therefore$ Value $= 100(\sqrt{2} + 1 + 1/\sqrt{2} + 1) = 21.37 + 1 = 22.37$.

This will be adjusted by taking the lower integral value $= 22$. $\therefore$ The number of terms in the series is 200.

The student is advised to try and experiment on these principles to get a clear picture.

WORKED-OUT PROBLEMS

Problem 2.1 Two persons Ramu Dhobi and Kalu Mochi have joined Donkey-work Associates. Ramu Dhobi and Kalu Mochi started with an initial salary of Rs. 500 and Rs. 360 respectively with annual increments of Rs. 25 and Rs. 20 each respectively. In which year will Ramu Dhobi start earning more salary than Kalu Mochi?

Solution The current difference between the salaries of the two is Rs. 140. The annual rate of reduction of this difference is Rs. 5 per year. At this rate, it will take Ramu Dhobi 28 years to equalise his salary with Kalu Dhobi's salary.

Thus, in the 29th year he will earn more.

This problem should be solved while reading and the straight process should be $140/5 = 28$. Hence, answer is 29th year.

Problem 2.2 Find the value of the expression

$1 - 6 + 2 - 7 + 3 - 8 + _____$ to 100 terms

(a) -250 (b) -300 (c) -450 (d) -300

Solution The series $(1 - 6 + 2 - 7 + 3 - 8 + \ldots$ to 100 terms) can be rewritten as:

$\Rightarrow (1 + 2 + 3 + \ldots$ to 50 terms$) - (6 + 7 + 8 + \ldots$ to 50 terms$)$

Both these are AP's with values of a and d as $\rightarrow$ $a = 1$, $n = 50$ and $d = 1$ and $a = 6$, $n = 50$ and $d = 1$ respectively.

Using the formula for sum of an AP we get

$$25(2 + 49) - 25(12 + 49)$$
$$\Rightarrow 25(51 - 61) = -250$$

Alternatively, we can do this faster by considering $(1 - 6)$, $(2 - 7)$, and so on as one unit or one term.

$1 - 6 = 2 - 7 = \ldots = -5$. Thus the above series is equivalent to a series of fifty -5's added to each other.

So, $(1 - 6) + (2 - 7) + (3 - 8) + \ldots$ 50 terms $= -5 \times 50 = -250$

Problem 2.3 Find the sum of all numbers divisible by 6 in between 100 to 400.

Solution Here 1st term $= a = 102$ (which is the 1st number greater than 100 that is divisible by 6).

The last term less than 400, which is divisible by 6 is 396.

The number of terms in the AP: 102, 108, 114, ..., 396 is given by $[(396 - 102)/6] + 1 = 50$ numbers.

Common difference $= d = 6$

So, $S = 25(204 + 294) = 12450$

Problem 2.4 If x, y, z are in GP, then $1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ will be in

(a) AP (b) GP
(c) HP (d) Cannot be said

Solution Go through the options.

Checking option (a), the three will be in AP if the 2nd expression is the average of the 1st and 3rd expression. This can be mathematically written as

$$2/(1 + \log_{10}y) = [1/(1 + \log_{10}x)] + [1/(1 + \log_{10}z)]$$
$$= \frac{[1 + (1 + \log_{10}z)] + [1 + (1 + \log_{10}x)]}{[(1 + \log_{10}x)(1 + \log_{10}z)]}$$
$$= \frac{[2 + \log_{10}xz]}{[(1 + \log_{10}x)(1 + \log_{10}z)]}$$

Applying our judgement, there seems to be no indication that we are going to get a solution.

Checking option (b)

$$[1/(1 + \log_{10}y)]^2 = [1/(1 + \log_{10}x)][1/(1 + \log_{10}z)]$$
$$= [1/(1 + \log_{10}(xz))]$$

Again we are trapped and any solution is not a right one.

Checking option (c).

$1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ are in HP then $1 + \log_{10}x$, $1 + \log_{10}y$ and $1 + \log_{10}z$ will be in AP.

So, $\log_{10}x$, $\log_{10}y$ and $\log_{10}z$ will also be in AP.

Hence, $2\log_{10}y = \log_{10}x + \log_{10}z$

or $y^2 = xz$ which is given.

So, (c) is the correct option.

Alternatively, you could have solved through the straight forward process.

x, y and z are given in logarithmic functions.

Assume $x = 1$, $y = 10$ and $z = 100$ so x, y, z are in GP.

So, $1 + \log_{10}x = 1$, $1 + \log_{10}y = 2$ and $1 + \log_{10}z = 3$

□ Thus we find that since 1, 2 and 3 are in AP, we can assume that

$1 + \log_{10}x$, $1 + \log_{10}y$ and $1 + \log_{10}z$ are in AP

□ Hence, by definition of an HP we have that $1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ are in HP. Hence, option (c) is the required answer.

Author's Note: In my experience I have always found that the toughest questions and Factorisations get solved very easily when their site options, by assuming values in place of the variables in the equation. The values of the variables should be taken in such a manner that the basic constraints put on the variables should be respected. For example, if an expression in three variables a, b and c is given and it is mentioned that $a + b + c = 0$ then the values that you assume for a, b and c should satisfy this restriction. Hence, you should look at values like 1, 2 and -3 or 2, -1, -1 etc.

This process is especially useful in the case where the question as well as the options both contain expressions. Factorisation and advanced techniques of maths are then not required. This process will be very beneficial for students who are weak in Mathematics.]

Problem 2.5 Find t_{10} and S_{10} for the following series:

1, 8, 15,....

Solution This is an AP with first term 1 and common difference 7.

$t_{10} = a + (n - 1)d = 1 + 9 \times 7 = 64$

$$S_{10} = \frac{n[2a + (n-1)d]}{2}$$

$$= \frac{10[2 \times 1 + (10-1)7]}{2} = 325$$

Alternatively, if the number of terms is small, you can sum it directly.

Problem 2.6 Find t_{10} and S_{10} for the following series:

2, 8, 32,....

Solution This is a GP with first term 2 and common ratio 4.

$$t_{10} = ar^{n-1} = 2 \times 4^9$$

$$S_{10} = \frac{a(r^n - 1)}{r - 1} = \frac{2(4^{10} - 1)}{(4 - 1)}$$

Problem 2.7 Is the series 1, 4,.... in a series as AP, or a GP, or an HP, or a series which cannot be determined?

Solution To determine any progression, we should have at least three terms.

If the series is an AP then the next term of this series will be 7.

Again, if the next term is 16, then this will be a GP series (1, 4, 16,....)

Yes, we cannot determine the nature of the progression of this series.

Problem 2.8 Find the sum to 200 terms of the series:

$1 + 8 + 6 + 5 + 11 + 6 +$

(a) 30,200 (b) 29,800

(c) 36,200 (d) None of these

Solution Spot that the above series is a combination of two APs.

The 1st AP is $(1 + 5 + 11 + \ldots)$ and the 2nd AP is $(8 + 5 + 6 + \ldots)$

Since the terms of the two series alternate, $S = (1 + 5 + 11 + \ldots$ to 100 terms$) + (4 + 5 + 6 + \ldots$ to 100 terms$)$

$$= \frac{100[2 \times 1 + 99 \times 5]}{2} + \frac{100[2 \times 4 + 99 \times 1]}{2} \rightarrow \text{(Using the formula for the sum of an AP)}$$

$= 50[497 + 107] = 50[604] = 30200$

Alternatively, we can treat every two consecutive terms as one.

So we will have a total of 100 terms of the nature:

$$(1 + 4) + (8 + 5) + (11 + 6) \ldots \rightarrow 5, 13, 17, \ldots$$

Now, $a = 5$, $d = 6$ and $n = 100$

Hence the sum of the given series is

$$S = \frac{100}{2} \times [2 \times 5 + 99 \times 6]$$

$= 50[604] = 30200$

Problem 2.9 How many terms of the series $-12, -8, -4, \ldots$ must be taken that the sum may be 54?

Solution Here $S = 54$, $a = -12$, $d = 5$, n is unknown and has to be calculated. To do so we use the formula for the sum of an AP and get

$$54 = \frac{[2(-12) + (n-1)5]n}{2}$$

$$\Rightarrow n^2 - 2n - 3n + 3n^2 \text{ or } 3n^2 - 5n - 108 = 0$$

$$\Rightarrow n^2 - 9n - 36 = 0, \text{ or } n^2 - 12n + 3n - 36 = 0$$

$$\Rightarrow n(n - 12) + 3(n - 12) = 0, \text{ or } (n + 3)(n - 12) = 0$$

The value of n (the number of terms) cannot be negative. Hence, −3 is rejected.

So, we have n = 12.

Alternatively, we can directly add up individual terms and keep adding mentally till we get a sum of 24. We will observe that 24 will occur after adding 12 terms. In this case, as also in all cases where the number of terms is actually manageable, mentally adding the terms till we get the required sum will turn out to be much faster than the equation based process.]

Problem 1.32 Find the sum of n terms of the series 1.2.4 + 2.3.5 + 3.4.6 + ...

(a) $n(n + 1)(n + 2)$

(b) $n(n + 1)(2n^2 + 8n + 26)$

(c) $(n + 1)(n + 2)(n + 3)/4$

(d) $n^2(n + 1)(n + 2)(n + 3)/8$

Solution In order to solve such problems in the examination, the option based approach is the best. Even if you can find out the required expression mathematically, it is advisable to solve through the options as this will end up saving a lot of time for you. Use the options as follows:

If we put n = 1, we should get the sum as 1.2.4 = 8. By substituting n = 1 in each of the four options we will get the following values for the sum to 1 term:

Option (a) gives a value of: 6

Option (b) gives a value of: 8

Option (c) gives a value of: 6

Option (d) gives a value of: 8

From this check we can reject the options (a) and (c).

Now put n = 2. Now we see that up to 2 terms, the expression is 1.2.4 + 2.3.5 = 38.

The correct option should also give 38 if we put n = 2 in the expression. Since (a) and (c) have already been rejected, we only need to check the options (b) and (d).

Option (b) gives a value of 38

Option (d) gives a value of 30

Hence, we eliminate option (d) and get (b) as the answer.

Note: The above process is very effective for solving problems having options. The student should try to keep an eye open for the possibility of solving questions through options. In my opinion, approximately 50–75% of the questions asked in CAT in the QA section can be solved with options (at least partially).

Level of Difficulty (LOD) I

1. How many terms are there in the AP 20, 25, 30, ... 130?
   (a) 22 (b) 23 (c) 21 (d) 24
   (e) 25
2. Bobby was appointed in AMS Careers in the pay scale of Rs. 5000–500–12,500. Find how many years he will take to reach the max limits of the scale.
   (a) 11 years (b) 10 years
   (c) 9 years (d) 8 years
   (e) None of these
3. Find the 1st term of an AP whose 8th and 12th terms are respectively 39 and 59.
   (a) 5 (b) 6 (c) 4 (d) 3
   (e) 7
4. A number of squares are described whose perimeters are in GP. Then their sides will be in
   (a) AP (b) GP
   (c) HP (d) Nothing can be said
   (e) None of these
5. There is an AP 1, 3, 5, ... Which term of this AP is 81?
   (a) 27th (b) 26th (c) 25th (d) 28th
   (e) 29th
6. How many terms are identical in the two APs 1, 3, 5, ... up to 120 terms and 3, 6, 9, ... up to 80 terms?
   (a) 38 (b) 39 (c) 40 (d) 41
   (e) 42
7. Find the lowest number in an AP such that the sum of all the terms is 105 and greatest term is 6 times the least.
   (a) 5 (b) 10 (c) 15 (d) 20
   (e) (a) & (c)
8. Find the 15th term of the sequence 20, 15, 10, ...
   (a) −45 (b) −55 (c) −50 (d) 0
   (e) −65
9. A sum of money kept in a bank increases to Rs. 1,200 in 4 years and Rs. 1600 in 10 years at simple interest. Find the sum.

(a) Rs. 800 (b) Rs. 900
(c) Rs. 1150 (d) Rs. 1000
(e) Rs. 1200

10. A number 15 is divided into three parts which are in *AP* and the sum of their squares is 83. Find the smallest number.
(a) 5 (b) 3 (c) 6 (d) 8
(e) 7

11. The sum of the first 16 terms of an *AP* whose first term and third term are 5 and 15 respectively is
(a) 600 (b) 765 (c) 640 (d) 680
(e) 690

12. The number of terms of the series 54 + 51 + 48 +... such that the sum is 513 is
(a) 18 (b) 19
(c) Both *a* and *b* (d) 15
(e) None of these

13. The least value of *n* for which the sum of the series 5 + 8 + 11... *n* terms is not less than 670 is
(a) 20 (b) 19 (c) 22 (d) 21
(e) 18

14. A man receives Rs. 60 for the first week and Rs. 3 more each week than the preceding week. How much does he earn by the 20th week?
(a) Rs. 1770 (b) Rs. 1620
(c) Rs. 1890 (d) Rs. 1790
(e) None of these

15. How many terms are there in the GP 5, 20, 80, 320,... 20480?
(a) 6 (b) 5 (c) 7 (d) 8
(e) 9

16. A boy agrees to work at the rate of one rupee on the first day, two rupees on the second day, four rupees on the third day and so on. How much will the boy get if he starts working on the 1st of February and finishes on the 20th of February?
(a) 2^{20} (b) $2^{20} - 1$
(c) $2^{19} - 1$ (d) 2^{19}
(e) None of these

17. If the fifth term of a GP is 81 and first term is 16, what will be the 4th term of the GP?
(a) 36 (b) 18 (c) 54 (d) 24
(e) 27

18. The seventh term of a GP is 8 times the fourth term. What will be the first term when its fifth term is 48?
(a) 4 (b) 3 (c) 5 (d) 2
(e) 6

19. The sum of three numbers in a GP is 14 and the sum of their squares is 84. Find the largest number.
(a) 8 (b) 6
(c) 4 (d) 12
(e) None of these

20. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this AP?
(a) 4551 (b) 10091
(c) 7881 (d) 13531
(e) 13539

21. How many natural numbers between 300 to 500 are multiples of 7?
(a) 29 (b) 28
(c) 27 (d) 30
(e) None of these

22. The sum of the first and the third term of a geometric progression is 20 and the sum of its first three terms is 26. Find the progression.
(a) 2, 6, 18,... (b) 18, 6, 2,...
(c) Both of these (d) None of these
(e) Cannot be determined

23. If a man saves Rs. 4 more each year than he did the year before and if he saves Rs. 20 in the first year, after how many years will his savings be more than Rs. 1000 altogether?
(a) 19 years (b) 20 years
(c) 21years (d) 18 years
(e) 17 years

24. A man's salary is Rs. 800 per month in the first year. He has joined in the scale of 800–40–1600. After how many years will his savings be Rs. 64,800?
(a) 8 years (b) 7 years
(c) 6 years (d) None of these
(e) Cannot be determined

25. The 4th and 10th term of an GP are 1/3 and 243 respectively. Find the 2nd term.
(a) 3 (b) 1 (c) 1/27 (d) 1/9
(e) 9

26. The 7th and 21st terms of an AP are 6 and −22 respectively. Find the 26th term.
(a) −34 (b) −32 (c) −12 (d) −10
(e) −16

27. The sum of 3 numbers in AP is 30 and the sum of their squares is 220. Which of the following is the third term?
(a) 5 (b) 6 (c) 8 (d) 9
(e) 10

28. Find the sum of all numbers in between 10–50 excluding all those numbers which are divisible by 8. (include 10 and 50 for counting.)
(a) 1030 (b) 1220
(c) 1230 (d) 1380
(e) 1000

29. The sum of the first four terms of an AP is 24 and sum of the first eight terms of the same AP is 68. Find the sum of the first 16 terms of the AP?
(a) 240 (b) 340
(c) 308 (d) 208
(e) None of these

30. Find the general term of the GP with the third term 1 and the seventh term 8.
(a) $12^{3/2}r^{n-1}$ (b) $12^{4/2}r^{n-1}$
(c) $12^{3/2}r^{n-2}$ (d) $12^{3/2}r^{n}$
(e) None of these

31. Find the number of terms of the series 1/81, −1/27, 1/9, ... −729.
(a) 11 (b) 12 (c) 10 (d) 13
(e) 14

32. Four geometric means are inserted between 1/8 and 128. Find the third geometric mean.
(a) 4 (b) 16
(c) 32 (d) 8
(e) None of these

33. A and B are two numbers whose AM is 25 and GM is 2. Which of the following may be a value of A?
(a) 10 (b) 30 (c) 40 (d) 25
(e) 26

34. Two numbers A and B are such that their GM is 20% lower than their AM. Find the ratio between the numbers.
(a) 3 : 2 (b) 4 : 1 (c) 2 : 1 (d) 5 : 1
(e) 3 : 1

35. A man saves Rs. 100 in January 2002 and increases the saving by Rs. 10 every month over the previous month. What is the annual saving for the year in the year 2002?
(a) Rs. 2200 (b) Rs. 4000
(c) 4000 (d) 4100
(e) None of these

Level of Difficulty (LOD) I

1. If a times the nth term of an AP is equal to b times its mth term, find the (a + b)th term.
(a) 0 (b) $a^2 - b^2$
(c) a − b (d) 1
(e) $a^2 + b^2$

2. A number 20 is divided into four parts that are in AP such that the product of the first and fourth is to the product of the second and third is 2 : 3. Find the largest part.
(a) 12 (b) 4 (c) 8 (d) 5
(e) 15

3. Find the value of the expression 1 − 4 + 5 − 8 ... 61 terms.
(a) −120 (b) −75 (c) −50 (d) 75
(e) 150

4. If a clock strikes once at one o'clock, twice at two o'clock and twelve times at 12 o'clock and again once at one o'clock and so on, how many times will the clock be struck in the course of 2 days?
(a) 156 (b) 312 (c) 78 (d) 26
(e) 144

5. What will be the minimum sum of 44, 42, 40, ...?
(a) 502 (b) 504
(c) 506 (d) 300
(e) None of these

6. Find the sum of the integers between 1 and 100 that are multiples of 7.
(a) 2542 (b) 2642
(c) 2646 (d) 2546
(e) 2546

7. If the nth term of an AP is 1/n and mth term is 1/m, find the sum to mn terms.
(a) (mn − 1)/4 (b) (mn + 1)/4
(c) (mn + 1)/2 (d) (mn − 1)/2
(e) None of these

8. Find the sum of all odd numbers lying between 100 and 200.
(a) 7500 (b) 2400

(c) 2000 (d) 3000
(e) None of these

8. Find the sum of all integers of 3 digits that are divisible by 7.
(a) 68,156 (b) 71,356
(c) 70,156 (d) 72,356
(e) None of these

10. The first and the last terms of an AP are 107 and 253. If there are five terms in this sequence, find the sum of sequence.
(a) 330 (b) 720
(c) 400 (d) 620
(e) 1000

11. Find the value of $1 - 2 - 3 + 2 - 3 - 4 + \ldots$ upto 100 terms.
(a) −694 (b) −626
(c) −629 (d) −549
(e) −678

12. What will be the sum to n terms of the series $8 + 88 + 888 + \ldots$?
(a) $\frac{8(10^n - 9n)}{81}$ (b) $\frac{8(10^{n+1} - 10 - 9n)}{81}$
(c) $8(10^{n+1} - 10)$ (d) $8(10^{n+1} - 10)$
(e) None of these

13. If a, b, c are in GP, then $\log a, \log b, \log c$ are in
(a) AP (b) GP
(c) HP (d) None of these
(e) Cannot be said

14. After striking the floor, a rubber ball rebounds to 4/5th of the height from which it has fallen. Find the total distance that it travels before coming to rest if it has been gently dropped from a height of 120 meters.
(a) 840 meters (b) 960 meters
(c) 1080 meters (d) 1020 meters
(e) 1120 meters

15. If a be the first term, y be the nth term and p be the product of n terms of a GP, then the value of p^2 will be
(a) $(xy)^{n-1}$ (b) $(xy)^n$
(c) $(xy)^{n+1}$ (d) $(xy)^{n/2}$
(e) None of these

16. The sum of an infinite GP whose common ratio is numerically less than 1 is 32 and the sum of the first two terms is 24. What will be the third term?
(a) 2 (b) 16 (c) 8 (d) 12
(e) 4

17. What will be the value of $x^{1/2} \cdot x^{1/4} \cdot x^{1/8} \ldots$ to infinity?
(a) $x^{\frac{1}{2}}$ (b) 1
(c) $x^{\frac{1}{32}}$ (d) x^2
(e) None of these

18. Find the sum to n terms of the series
$1.2.3 + 2.3.4 + 3.4.5 + \ldots$
(a) $\frac{n}{3}(n + 1)(n + 7)(n + 5)/3$
(b) $n(n + 1)(2n + 2)(n + 2)/4$
(c) $n(n + 1)(n + 2)$
(d) $n(n + 1)(n + 2)(n + 3)/4$
(e) None of these

19. Determine the first term of the geometric progression, the sum of whose first term and third term is 40 and the sum of the second term and fourth term is 80.
(a) 12 (b) 36 (c) 8 (d) 4
(e) 6

20. Find the second term of an AP if the sum of its first five even terms is equal to 15 and the sum of the first three terms is equal to −3.
(a) −3 (b) −2 (c) −1 (d) 0
(e) 1

21. The sum of the second and the fifth term of an AP is 8 and that of the third and the seventh term is 14. Find the eleventh term.
(a) 19 (b) 17 (c) 15 (d) 16
(e) 18

22. How many terms of an AP must be taken for their sum to be equal to 120 if its third term is 9 and the difference between the seventh and the second term is 20?
(a) 6 (b) 9 (c) 7 (d) 8
(e) 5

23. Four numbers are inserted between the numbers 4 and 39 such that an AP results. Find the biggest of these four numbers.
(a) 31.5 (b) 31 (c) 32 (d) 30
(e) 32

24. Find the sum of all three digit natural numbers, which on being divided by 6, leave a remainder equal to 4.
(a) 82,250 (b) 96,750
(c) 89,200 (d) 98,250
(e) 90,250

25. The sum of the first three terms of the arithmetic progression is 30 and the sum of the squares of the first term and the second term of the same progression is

[illegible] Find the sixth term of the progression if its fifth term is known to be exactly divisible by 24.

(a) 36 (b) 40 (c) 48 (d) 32

(e) 30.

26. A and B set out to meet each other from two places 100 km apart. A travels 15 km the first day, 14 km the second day, 13 km the third day and so on. B travels 10 km the first day, 12 km the second day, 14 km the third day and so on. After how many days will they meet?

(a) 8 days (b) 5 days

(c) 6 days (d) 7 days

(e) 9 days

27. If a man saves Rs. 1000 each year and invests at the end of the year at 5% compound interest, how much will the amount be at the end of 15 years?

(a) Rs. 21,478 (b) Rs. 21,578

(c) Rs. 22,578 (d) Rs. 22,478

(e) Rs. 22,178

28. If sum to n terms of a series is given by $(n + 8)$, then its second term will be given by

(a) 10 (b) 9

(c) 8 (d) 7

(e) None of these

29. If A is the sum of the n terms of the series $1 + 1/4 + 1/16 + \ldots$ and B is the sum of $2n$ terms of the series $1 + 1/3 + 1/9 + \ldots$, then find the value of A/B.

(a) 1/3 (b) 1/2 (c) 2/3 (d) 3/4

(e) 4/5

30. A man receives a pension starting with Rs. 100 for the first year. Each year he receives 90% of what he received the previous year. Find the maximum total amount he can receive even if he lives forever.

(a) Rs. 1100 (b) Rs. 1000

(c) Rs. 1200 (d) Rs. 900

(e) Rs. 1250

31. The sum of the series represented as $1/(1 \times 2) + 1/(2 \times 3) + 1/(3 \times 4) + \ldots + 1/(21 \times 22)$ is

(a) 20/21 (b) 16/21

(c) 19/22 (d) None of these

32. The sum of the series $1 \times 1/2 + 1 \times 2 + 1/2 \times 3 + 1 \times 4 + \ldots + 1/(11 \times 12) + 1 \times 12$ is

(a) 10 (b) 11

(c) 12 (d) None of these

33. Find the infinite sum of the series $1/1 + 1/3 + 1/6 + 1/10 + 1/15 + \ldots$

(a) 2 (b) 2.25

(c) 3 (d) 4

34. The sum of the series $5 \times 8 + 8 \times 11 + 11 \times 14 + \ldots$ will be

(a) $(n + 1)[3(n + 1)^2 + 6(n + 1) + 1] - 30$

(b) $(n + 1)[3(n + 1)^2 + 6(n + 1) + 1] + 30$

(c) $(n + 1)[3(n + 1) + 6(n + 1)^2 + 1] - 30$

(d) $(n + 1)[3(n + 1) + 6(n + 1)^2 + 1] + 30$

35. The sum of the series $1/2 + 1/6 + 1/12 + 1/20 + \ldots + 1/182$ is

(a) 12/13 (b) 13/14

(c) 14/13 (d) None of these

36. For the above question 35, what is the sum of the series if taken to infinite terms?

(a) 1.1 (b) 1

(c) 14/13 (d) None of these

Directions for Questions 37–39: Answer the questions based on the following information.

There are 250 integers $a_1, a_2, \ldots, a_{250}$, not all of them necessarily different. Let the greatest integer of these integers be referred to as Max, and the smallest integer referred to as Min. The integers a_1 through a_{50} form sequence A, and the rest form sequence B. Each number in A is less than or equal to each number in B.

37. All values in A are changed in sign, while the values in sequence B remain unchanged. Which of the following statements is true?

(a) Every number of A is greater than or equal to every number of B.

(b) Max is in A.

(c) If all numbers originally in A and B had the same sign, then after the change of sign, the smallest number of A and B is in A.

(d) None of these

38. Elements of A are in ascending order, and those of B are in descending order. a_{125} and a_{126} are consecutive. Then which of the following statements is true?

(a) A continues to be in ascending order.

(b) B continues to be in descending order.

(c) A continues to be in ascending order and B in descending order.

(d) None of the above

39. Every element of A is made greater than or equal to every element of B by adding to each element of A an integer x. Then, x is given by [illegible]

(a) 27
(b) the smallest value of B
(c) the largest value of B
(d) (Also Men)

40. Rohit has a rectangular grid of 320 cells, arranged in 23 rows and 23 columns, and filled each cell with a number. The numbers with which he filled each cell were such that the numbers of each row taken from left to right formed an arithmetic series and the numbers of each column taken from top to bottom also formed an arithmetic series. The seventh and the seventeenth numbers of the fifth row were 47 and 63 respectively, while the seventh and the seventeenth numbers of the fifteenth row were 53 and 77 respectively. What is the sum of all the numbers in the grid?
(a) 32768 (b) 65536
(c) 52800 (d) None of these

41. How many three digit numbers have the property that their digits taken from left to right form an Arithmetic or a Geometric Progression?
(a) 15 (b) 36
(c) 20 (d) 42

Directions for Questions 42 and 43: These questions are based on the following data.

At Burger King – a famous fast food centre on Main Street in Pune, burgers are made only on an automatic burger making machine. The machine continuously makes different sorts of burgers by adding different sorts of fillings on a common bread. The machine makes the burgers at the rate of 1 burger per half a minute. The various fillings are added to the burgers in the following manner. The 1st, 5th, 9th, ... burgers are filled with a chicken patty; the 2nd, 9th, 14th, ... burgers with vegetable patty; the 1st, 5th, 9th, ... burgers with mushroom patty; and the rest with plain cheese and tomato filings.

The machine makes exactly 600 burgers per day.

42. How many burgers per day are made with cheese and tomato as fillings?
(a) 426 (b) 238
(c) 237 (d) None of these

43. How many burgers are made with all three fillings Chicken, vegetable and mushroom?
(a) 21 (b) 19
(c) 20 (d) 36

44. An arithmetic progression P consists of n terms. From the progression three different progressions P_1, P_2 and P_3 are created such that P_1 is obtained by the 1st, 4th, 7th,..... terms of P; P_2 has the 2nd, 5th, 8th, ... terms of P and P_3 has the 3rd, 6th, 9th, ... terms of P. It is found that of P_1, P_2 and P_3 two progressions have the property that their average is itself a term of the original Progression P. Which of the following can be a possible value of n?
(a) 20 (b) 36
(c) 36 (d) Both 1 and 2

45. For the above question, if the Common Difference between the terms of P_1 is 6, what is the common difference of P?
(a) 2 (b) 3
(c) 9 (d) Cannot be determined

Level of Difficulty (LOD) III

1. If in any decreasing arithmetic progression, sum of all its terms, except for the first term, is equal to -36, the sum of all its terms, except for the last term, is zero, and the difference of the tenth and the sixth term is equal to -16, then what will be first term of this series?
(a) 16 (b) 20 (b) -16 (d) -20(in) -24

2. The sum of all terms of the arithmetic progression having ten terms except for the first term, is 99, and except for the sixth term, 88. Find the third term of the progression if the sum of the first and the fifth term is equal to 10.
(a) 15 (b) 5
(c) 8 (d) 10
(e) None of these

3. Product of the fourth term and the fifth term of an arithmetic progression is 456. Division of the tenth term by the fourth term of the progression gives quotient as 11 and the remainder as 80. Find the first term of the progression.
(a) 52 (b) -42 (c) -56 (d) -66
(e) 80

4. A number of saplings are lying at a place by the side of a straight road. These are to be planted in a straight line at a distance interval of 50 meters between two consecutive saplings. Mithilesh, the country's greatest farmer, can carry only one sapling at a time and has to move back to the original point to get the next sapling

in his journey he covers a total distance of 1.32 km. How many replicas does he place in the process if he ends at the starting point?

(a) 11 (b) 14 (c) 15 (d) 12

(e) 10

5. A geometric progression consists of 500 terms. Sum of the terms occupying the odd places is P_1 and the sum of the terms occupying the even places is P_2. Find the common ratio.

(a) P_1/P_2 (b) P_2/P_1

(c) $P_1 + P_2/P_1$ (d) $P_1 + P_2/P_2$

(e) None of these

6. The sum of the first ten terms of the geometric progression is S_1 and the sum of the next ten terms (11th through 20th) is S_2. Find the common ratio.

(a) $(S_2/S_1)^{1/10}$ (b) $-(S_1/S_2)^{1/10}$

(c) $\sqrt[10]{S_2/S_1}$ (d) $(S_1/S_2)^{1/5}$

(e) None of these

7. The first and the third terms of an arithmetic progression are equal, respectively, to the first and the third term of a geometric progression, and the second term of the arithmetic progression exceeds the second term of the geometric progression by 0.25. Calculate the sum of the first five terms of the arithmetic progression if its first term is equal to 2.

(a) 2.25 or 25 (b) 2.5 or 27.5

(c) 2.5 (d) 3.25

(e) None of these

8. If $(2 + 4 + 6 + \ldots 50 \text{ terms})(1 + 2 + 3 + \ldots n \text{ terms}) = 51/2$, then find the value of n.

(a) 12 (b) 15 (c) 9 (d) 10

(e) 11

9. $(666\ldots n \text{ digits})^3 + (888\ldots n \text{ digits})$ is equal to

(a) $(10^n - 1) \times \frac{4}{9}$ (b) $(10^{2n} - 1) \times \frac{4}{9}$

(c) $\frac{4(10^n - 10^{n-1} - 1)}{9}$ (d) $\frac{4(10^n + 1)}{9}$

(e) None of these

10. The smallest angles of a polygon are in AP. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon.

(a) 7 (b) 8 (c) 9 (d) 10

(e) 11

11. Find the sum to n terms of the series $11 + 101 + 1001 + \ldots$

(a) $\frac{10(10^n - 1)}{9} + 1$ (b) $\frac{10(10^n - 1)}{9} + n$

(c) $\frac{10(10^n - 1)}{9} + n^2$ (d) $\frac{10(10^n + 1)}{11} + n^2$

(e) None of these

12. The sum of the first term and the fifth term of an AP is 26 and the product of the second term by the third term is 160. Find the sum of the first seven terms of the AP.

(a) 110 (b) 114 (c) 112 (d) 116

(e) 120

13. The sum of the third and the ninth term of an AP is 66. Find a possible sum of the first 11 terms of this AP.

(a) 55 (b) 44

(c) 66 (d) 48

(e) 67

14. The sum of the squares of the fifth and the sixth terms of an AP is 1 and the product of the second by the fourteenth term is equal to P. Find the product of the first and the fifteenth term of the AP.

(a) $156P - 39/45$ (b) $156P + 39/72$

(c) $116P - 39/90$ (d) $156P + 39/90$

(e) None of these

15. If the ratio of harmonic mean of two numbers to their geometric mean is 12 : 13, find the ratio of the numbers.

(a) 4/9 or 9/4 (b) 2/3 or 3/2

(c) 2/5 or 5/2 (d) 3/4 or 4/3

(e) 1/4 or 4/1

16. Find the sum of the series $1.2 + 2.2^2 + 3.2^3 + \ldots 100.2^{100}$.

(a) $100.2^{101} + 2$ (b) $99.2^{100} + 2$

(c) $99.2^{101} + 2$ (d) $100.2^{100} + 2$

(e) None of these

17. The sequence $\{x_n\}$ is a GP with $x_2/x_1 = (4x)^{1/3}$ and $x_3 = 16x^{2/3}$. What will be the value of x_5?

(a) 42 (b) 48 (c) 44 (d) 36

(e) 52

18. If x, y, z are in GP and a^x, b^y and c^z are equal, then b, a are in

(a) AP (b) GP

(c) HP (d) None of these

(e) Cannot be determined

19. Find the sum of all possible whole number divisors of 720.

(c) 2012 (d) 2024 (e) 2210 (f) 2018
(g) 2220

21. Sum to n terms of the series $\log_e x + \log_e x(x+1) + \log_e x(x+1)^2 + \log_e x(x+1)^3 + \ldots$ is

(a) $\log\left(\frac{x^{n+1}}{(x+1)^{n-1}}\right)^{\frac{n}{2}}$ (b) $\log\left(\frac{x^{n+1}}{(x+1)^{n+1}}\right)^{\frac{n}{2}}$

(c) $\log\left(\frac{x^{n+1}}{(x+1)^{n}}\right)^{\frac{n}{2}}$ (d) $\log\left(\frac{x^{n-1}}{(x+1)^{n-1}}\right)^{\frac{n}{2}}$

(e) $\log\left(\frac{x^{n+1}}{(x+1)^{n+1}}\right)$

22. The sum of first 20 and first 50 terms of an AP is 420 and 2500. Find the eleventh term of a GP whose first term is the same as the AP and the common ratio of the GP is equal to the common difference of the AP.

(a) 500 (b) 512 (c) 1024 (d) 1120
(e) 1032

23. If three positive real numbers x, y, z are in AP such that $xyz = 4$, then what will be the minimum value of y?

(a) $2^{1/3}$ (b) $2^{2/3}$ (c) $2^{1/4}$ (d) $2^{3/4}$
(e) $2^{4/3}$

24. If a_n be the nth term of an AP and if $a_2 = 15$, then the value of the common difference that would make $a_1a_2a_3$ greatest is

(a) 1 (b) 3/2
(c) 7 (d) 0
(e) None of these

25. If $a_1, a_2, a_3 \ldots a_n$ are in AP, where $a_i > 0$, then what will be the value of the expression

$(1/\sqrt{a_1} + \sqrt{a_2}) + (1/\sqrt{a_2} + \sqrt{a_3}) + (1/\sqrt{a_3} + \sqrt{a_4})$
$+ \ldots$ to n terms?

(a) $(1 - n)(\sqrt{a_1} + \sqrt{a_n})$

(b) $(n - 1)(\sqrt{a_1} + \sqrt{a_n})$

(c) $(n - 1)(\sqrt{a_1} - \sqrt{a_n})$

(d) $(1 - n)(\sqrt{a_1} + \sqrt{a_n})$

(e) All of these

26. If the first two terms of a HP are 2/7 and 1/13 respectively, which of the following terms is the largest term?

(a) 4th term (b) 5th term
(c) 6th term (d) 7th term
(e) 8th term

27. One side of a staircase is to be closed by rectangular planks from the floor to each step. The width of each plank is 9 inches and their height are successively 6 inches, 12 inches, 18 inches and so on. There are 24 planks required in total. Find the area in square feet.

(a) 112.5 (b) 107
(c) 118.5 (d) 106
(e) None of these

27. The middle points of the sides of a triangle are joined forming a second triangle. Again a third triangle is formed by joining the middle points of this second triangle and this process is repeated infinitely. If the perimeter and area of the outer triangle are P and A respectively, what will be the sum of perimeters of triangles thus formed?

(a) 2P (b) P^2 (c) 3P (d) $p^2/2$
(e) $2P^2/A$

28. In Problem 27, find the sum of areas of all the triangles.

(a) $\frac{4}{3}$A (b) $\frac{4}{5}$A (c) $\frac{3}{4}$A (d) $\frac{5}{4}$A
(e) 2A

29. A square has a side of 40 cm. Another square is formed by joining the mid-points of the sides of the given square and this process is repeated infinitely. Find the perimeter of all the squares thus formed.

(a) $160(1 + \sqrt{2})$ (b) $160(2 + \sqrt{2})$
(c) $160(2 - \sqrt{2})$ (d) $160(1 - \sqrt{2})$
(e) $160(3 - \sqrt{2})$

30. In problem 29, find the area of all the squares thus formed.

(a) 1600 (b) 3400
(c) 2800 (d) 1200
(e) 3000

31. The sum of the first n terms of the arithmetic progression is equal to half the sum of the next n terms of the same progression. Find the ratio of the sum of the first 3n terms of the progression to the sum of its first n terms.

(a) 5 (b) 8 (c) 7 (d) 6
(e) 9

9. In a certain colony of parameciums cells, each cell breaks into two new cells every hour. If there is a single parameciums cell at the start and this process continues for 8 hours, how many cells will the colony have at the end of 8 hours? It is known that the life of an individual cell is 24 hours.

(a) $2^7 - 1$ (b) 2^{24}

(c) 2^8 (d) $2^{24} - 1$

(e) $2^{24} - 8$

10. Find the sum of all three-digit whole numbers less than 700 that leave a remainder of 2 when they are divided by 3.

(a) 84657 (b) 30867

(c) 29657 (d) 30870

(e) 30778

11. If a is the arithmetic mean and b, c be the two geometric means between any two positive numbers, then $(b^3 + c^3)/(ab + ac)$ equals

(a) $(a+c)^2/a$ (b) 1

(c) $a^2/(b+c)$ (d) $2a$

(e) None of these

12. If p, q, r are three consecutive distinct natural numbers then the expression $(q + r - p)(q + r - q)(p + q - r)$ is

(a) Positive (b) Negative

(c) Non-positive (d) Non-negative

(e) Either (c) or (d)

Hints and Solutions

1. $a + (a + d) + (a + 2d) + (a + 3d) = 20$
and $(a + 3d) + (a + d) + (a + 2d)$
4. Calculate the sum of an AP with first term 1, common difference 1 and last term 12. Multiply this sum by 4 for 2 days.
5. The maximum sum will occur when the last term is either 2 or 0.
6. Consider the AP $= 7, 14, \ldots 196$.
8. The AP is 100, 113, ..., 994.
9. The common difference $= \frac{149}{5} = 29.2$.
10. Sum the sums of the series in 33 blocks of 3 each. This will give the AP $-4, -3, -6, \ldots 93$. Further, the hundreds sum will be 54.
12. Solve through options.
14. The first drop is 120 metres. After this the ball rises by 96 metres and fall by 96 metres. This process will continue in the form of an infinite GP with common ratio 0.8 and first term 96.
The required answer will be got by
$$120 + 96 * 1.25 * 2$$
15. Take any GP and solve by using values.
18. Solve by using values to check options.
22. The difference between the seventh and third term is given by
$$(a + 6d) - (a + d)$$
23. $\frac{(39 - 4)}{5} = 7$.
27. The required answer will be by adding 20 terms of the GP starting with the first term as 1000 and common ratio as 1.05.
30. Visualise it as an infinite GP with common ratio

Hints and Solutions

1. Difference between the tenth and the sixth term $= -16$
or $(a + 9d) - (a + 5d) = -16$
$\Rightarrow d = -4$
2. Sum of the first term and the fifth term $= 10$
or
$$a + a + 4d = 10$$
or
$$a + 2d = 5$$
and, the sum of all terms of the AP except the first term $= 99$
or
$$9a + 45d = 99$$
$$a + 5d = 11$$
Solve (1) and (2) to get the answer.
3. The second statement gives the equation as $a + 3(a + 3d) = 6$
or
$$a = -2d = 5$$
Now, use the options to find the value of d and these values to check the equation obtained from first statement.
i.e.
$$(a + 2d)(a + 5d) = 400$$
4. To plant the 1st sapling, Mitulkem will cover 20 m. To plant the 2nd sapling he will cover 40 m and so on. But for the last sapling, he will cover

absence from the starting point to the place where the sapling has to be planted.

5. Assume a series having a few number of terms e.g. 1, 2, 4, 8, 16, 32 ...
Now sum of all the terms at the even places = 42 (P_1) and sum of all the terms at the odd places = 21 (P_2)
common ratio of this series = $\frac{42}{21} = 2 = P_1/P_2$

6. Do the same process as illustrated above.

7. Check the options by putting $n = 1, 2, \ldots$ and then repeat it with the original equation given in question.

10. Sum of the AP for n sides = Sum of interior angles of a polygon of n sides.

$$\frac{n}{2} \times (2a + (n-1)d) = 12n - 4 \times 90$$

where $a = 120°$ and $d = 5°$.

11. Solve using options to check for the correct answer.

12. $a + (a + 4d) = 26$ and $(a + d)(a + 3d) = 160$
Alternatively, you can try to look at the factors of 160 and create an AP such that it meets the criteria.
Thus, 160 can be written as

$$2 \times 80$$
$$4 \times 40$$
$$8 \times 20$$
$$10 \times 16$$

and so on.
If we consider 8×20, then one possibility is that $d = 8$ and the first and fifth terms are 2 and 26. But $2 + 2b = 26$. Hence, this cannot be the correct factors.
Try 10×16. This will give you,
$a = 2$, $d = 3$ and 5th term = 19
and $7 + 19 = 26$ satisfies the condition.

13. A possible AP satisfying this condition is
0, 1, 2, 3, 4, 5, 6, 7, 8, ...

14. Assume the fifth term as $(a + 4d)$, the eleventh term as $(a + 10d)$, the second term as $(a + d)$, the fourteenth term as $(a + 13d)$, the first term as (a) and the fifteenth term as $(a + 14d)$.
Then First term × Fifteenth term
$= a \times (a + 14d) = a^2 + 14ad$
Also $(a + d)(a + 13d) = P$
and $(a + 4d)^2 = (a + 10d)^2 = 3$

15. $a = 2$ and $a + ar^2 = 108$.

20. Solve by checking options and using principles of Transition.

21. The sum of the first 20 terms will be
$a + (a + d) + (a + 2d) \ldots + (a + 19d)$
i.e. $20a + 190d = 400$
Similarly, use the sum to fifty terms.

23. For a product $a \times b \times c$ to be maximum, given $a + b + c$ = constant, the condition is $a = b = c$.

27. The length of sides of successive triangles form a GP with common ratio 1/3.

28. The area of successive triangles form an infinite GP with a common ratio 1/4.

29. Common ratio = $1/\sqrt{2}$.

31. Ratio of sum of the first 2n terms to the first n terms is equal to 3.
Thus,

$$\frac{2n\left(\frac{a_1 + a_{2n}}{2}\right)}{n\left(\frac{a_1 + a_n}{2}\right)} = 3$$

Solve to get, $3a = (n + 1)d$.
Put values for a and n to get a value for d and check for the conditions given in the question.

33. Visualize the AP and solve using standard formulae.

34. Take any values for the numbers.
Say, the two positive numbers are 1 and 27.
Then, $a = 14$, $b = 3$ and $c = 9$.

Thought Processes, Solutions and Short Cuts to Select Questions

LOD 1

5. The number of terms in a series are found by

$$\frac{\text{Difference between first and last terms}}{\text{Common Difference}} + 1$$

6. The first common term is 5, the next will be 9 (Notice that the second common term is exactly 6 away from the first common term. 6 is also the LCM of 2 and 3 which are the respective common differences of the two series.)
Thus, the common terms will be given by the AP 5, 9, 13 ..... last term. To find the answer you need to find the last term that will be common in the two series.
The first series is 5, 7, 9 239

Hence, the last common term is 237.

Thus our answer becomes $\frac{237 - 8}{11} + 1 = 20$

2. Prompt Option (c).

We get three terms 9 and largest term 39 (since the largest term is 6 times the least term).

The average of the A.P becomes $(9 + 39)/2 = 17.5$

Thus, $17.5 \times n = 105$ gives us:

to get a total of 105 we need $n = 6$ i.e. 6 terms in the A.P. Thus means the A.P. should look like:

$9, __, __, __, __, 39$

It can be easily seen that the common difference should be 5. The A.P. 9, 14, 19, 24, 29, 34 fits the situation.

The same process used for option (b) gives us the A.P. 10, 35, 60 $(10 + 35 + 60 = 105)$ and in the third option 15, 90 $(15 + 90 = 105)$.

Hence, all the three options are correct.

3. The difference between the amounts at the end of 4 years and 10 years will be the simple interest on the initial capital for 6 years.

Hence, $3600/6 = 600$ = simple interest.

Also, the simple interest for 4 years when added to the sum gives 1240 as the amount.

Hence, the original sum must be 1000.

4. Sum of a G.P. with first term 1 and common ratio 2 and no. of terms 20

$$\frac{1 \times (2^{20} - 1)}{(2 - 1)} = 2^{20} - 1$$

5. In the case of a G.P. the 7th term is derived by multiplying the fourth term thrice by the common ratio. (Note: this is very similar to what we had seen in the case of an A.P.)

Hence, the seventh term is derived by multiplying the fourth term by 8, the relationship

$r^3 = 8$ must be true

Hence, $r = 2$

If the fifth term is 48, the series in reverse from the fifth to the first term will look like:

48, 24, 12, 6, 3. Hence, option (b) is correct.

6. The series will be 910, 898, ..., 487

Hence, Answer $= \frac{(910 - 487)}{12} + 1 = 36$

7. Substitute the value for each term in equation (4).

8th term is $P = 1000$ terms

The 4th term here is 3^{-1} and the sixth term is 3.

Hence, $3^{-1} \times r^2 = 3^1$

Gives us $r = 3$.

Hence, the second term will be given by (the second term r^{-1})

$$3^{-1} \times 3^{-1} = 3^{-2}$$

(Note: To go forward in a G.P. you multiply by the common ratio, to go backward in a G.P. you divide by the common ratio.)

8. The average will be given by:

$(16 + 18 + 1.5 + ____ + 50)/(16 + 24 + ____ + 40)$

$= 4 \times 30 / 32 = 3$

$= (2 \times 30 - 16)/40 = 10/10$

9. Think like this:

The average of the first 4 terms is 7, while the average of the first 8 terms must be 11.

So we visualize this

1st 2nd 3rd 4th 5th 6th 7th 8th

average = 7 average = 11

Hence, $x = 4/2 = 2$ (Note: understand this as a property of an A.P.)

Hence, the average of the 6th and 7th terms = 9.

the average of the 8th and 9th terms = 14.

But this (19) also represents the average of the 8th and 9th terms of the A.P.

Hence, required answer $= (6 \times 19 = 114)$.

10. Go through options. The correct option should give value as 1, when $n = 0$ and as 8 when $n = 8$.

Only option (a) satisfies both conditions.

LOD 2

1. Identify an A.P. which satisfies the given conditions. Suppose we are talking about the second and third terms of the A.P.

Then an A.P. with second term 3 and third term 6 satisfies the condition.

a times the 4th term is 6 times the 6th term.

In this case the value of $a = 2$ and $b = 3$.

Hence, for the $(a + b)^{th}$ term, we have to find the 5th term.

It is clear that the fifth term of this A.P. must be 15.

Check the other three options to see whether any other option gives 15 when $a = 2$ and $b = 3$.

Since none of the options (b, c, or d) gives value for the particular value, the option a is correct.

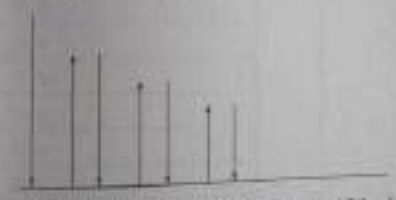
1. Here $1 - 4 + 9 - 8 + 9 - 12 \ldots$ 50 terms as
$(1 - 4) + (5 - 8) + (9 - 12) \ldots$ 25 terms
Hence, $-3 + -3 + -3 \ldots$ 25 terms
$= 25 \times -3 = -25$

5. Since this is a decreasing A.P. with first term positive, the minimum sum will occur upto the point where the progression remains non – negative.
40, 42, 44 ... 0
Hence, 22 terms = 22 × 510.

7. A little number juggling would give you 2nd term is 11 3 and 3rd term is 7/3 is a possible situation that satisfies the condition.
The A.P. will become:
1/6, 1/3, 1/2, 2/3, 5/6, 1
or in decimal terms, 0.166, 0.333, 0.5, 0.666, 0.833, 1
Sum to 6 terms = 3.5
Check the option with $n = 2$ and $n = 3$. Only option (c) gives 3.5. Hence, must be the answer.

11. The first 100 terms of this series can be viewed as:
$(1 - 2 + 3) + (2 - 3 + 4) + \ldots \quad + (33 - 34 + 35) + 34$
The first 33 terms of the above series (indicated inside the brackets) will give an A.P. $-4, -5, -6, \ldots -36$
Sum of this A.P. $= 33 \times -20 = -660$
Answer $= 660 + 34 = -626$

14. The path of the rubber ball is



In the figure above, every bounce is 4/5th of the previous drop.
In the above movement, there are two infinite G.Ps (The GP representing the falling distances and the GP representing the Rising distances.)
The required answer (Using $a/(1-r)$ formula)

$$\frac{120}{1/5} + \frac{96}{1/5} = 1080$$

18. For $n = 1$, the sum should be 8. Option (b), (c) and (d) all give 8 as the answer.
For $n = 2$, the sum should be 30.
Only option of given this value. Hence must be the answer.

24. Find sum of the series:
104, 109, 114 ... 909
Average $= a = 501.5 \times 180 = 90270$

28. Since, sum to n terms is given by $(n + 8)$.
Sum to 1 terms $= 9$
Sum to 2 terms $= 10$
Thus, the 2nd term must be 1.

31. Questions such as these have to be solved on the basis of a reading of the pattern of the question. The sum upto the first term is: 1/5. Upto the second term it is 2/9 and upto the third term it is 3/13. It can be easily seen that for the first term, second term and third term the numerators are 1, 2 and 3 respectively. Also, for 1/5 – the 5 is the second value in the denominator of 1/5 = 5 (the first term); for 2/9 also the same pattern is followed — as 9 comes out of the denominator of the second term of series and for 3/13 the 13 comes out of the denominator of the third term of the series and so on. The given series has 56 terms and hence the correct answer would be 56/225.

35. For this question too you would need to read the pattern of the values being followed. The given sum has 13 terms.
It can be seen that the sum in 1 term = ½
Sum to 2 terms = 2/3
Sum to 3 terms = ¾
Hence, the sum to 13 terms would be 13/14.

37. All members of A are smaller than all members of B. In order to visualize the effect of the change in sign in A, assume that $A = \{1, 2, 3 \ldots 124\}$ and $B = \{126, 127 \ldots 250\}$. It can be seen that for this assumption of values neither options a, b or c is correct.

38. If elements of A are in ascending order a_{124} would be the largest value in A. Also a_{125} would be the largest value in B. On interchanging a_{124} and a_{125} it continues to be in ascending order, but B would lose its descending order arrangement since a_{125} would be the lesser value in B. Hence, option d is correct.

39. Since the minimum is in A and the maximum is in B, the value of x cannot be less than Max-Min.

40. It is evident that the whole question is built around Arithmetic progressions. The 5th row has an average of 33, while the 15th row has an average of 63. Hence

over cohorts, when each cohorts is arranged in an AP we can conclude the following:

1st sum – average 51 – total = 23 × 51

2nd sum – average 52 – total 23 × 52 ....

23rd sum – average 71 – total 23 × 73

The overall total can be got by using averages as:

23 × 23 × 62 = 32798

42. Total burgers made = 660

Burgers with chicken and mushroom patty = 165 (Number of terms in the series 1, 5, 9 ... 657)

Burgers with vegetable patty = 95 (Number of terms in the series 2, 9, 16, ... 660)

Burgers with chicken, mushroom and vegetable patty = 24 (Number of terms in the series 9, 37, 65, ... 653)

Required answer = 660 – 165 – 95 + 24 = 424

43. From the above question, we have 24 such burgers.

LOD 3

4. For 1 term, the value should be

$$6^1 + 8 = 44$$

Only option (b) gives 44 for n = 1

16. The solution (from the options) has got something to do with either 2^{100} or 2^{101} for 100 terms. Since the terms increase, the options and crosscheck with the actual sum.

For 5 terms. Sum = 2 + 6 + 34 = 34

(a) $100 \times 2^{102} + 2$ for 100 terms becomes 1×2^{2} for 2 terms.

$= 48 + 2 = 50 \neq 34$. Hence is not correct

(b) $99 \times 2^{100} + 2$ for 100 terms becomes $1 \times 2^{3} + 2$ for 5 terms.

But this does not give 34. Hence is not correct

(c) $99 \times 2^{101} + 2 \rightarrow 1 \times 2^{4} + 2 = 34$

(d) $100 \times 2^{100} + 2 \rightarrow 1 \times 2^{3} + 2 \neq 34$

Hence, option (c) is correct.

ANSWER KEY

LOIII

1. b
2. a
3. a
4. b
5. a
6. a
7. a
8. a
9. d
10. b
11. d
12. a
13. a
14. a
15. c
16. b
17. a
18. b
19. a
20. c
21. a
22. c
23. a
24. a
25. c
26. b
27. b
28. a
29. c
30. a
31. a
32. d
33. a
34. b
35. b

LEVEL II

1. a
2. a
3. b
4. b
5. c
6. b
7. c
8. a
9. a
10. c
11. b
12. b
13. a
14. c
15. b
16. a
17. b
18. d
19. c
20. c
21. a
22. d
23. c
24. d
25. b
26. c
27. b
28. d
29. c
30. b
31. c
32. a
33. a
34. a
35. b
36. b
37. a
38. a
39. d
40. a
41. d
42. a
43. b
44. d
45. a

LOD III

1. a
2. b
3. d
4. d
5. a
6. c
7. b
8. d
9. b
10. c
11. c
12. c
13. d
14. d
15. c
16. a
17. b
18. b
19. d
20. a
21. a
22. b
23. d
24. b
25. d
26. a
27. a
28. b
29. b
30. d
31. b
32. d
33. b
34. c
35. d

BLOCK REVIEW TESTS

REVIEW TEST ONE

1. [illegible] the same number of sisters as she has brothers, but for brother Shyam he has twice as many sisters as he has brothers. How many children are there in the family?
   (a) 7 (b) 8 (c) 5 (d) 3
2. How many times does the digit 6 appear when you count from 11 to 100?
   (a) 9 (b) 10 (c) 19 (d) 20
3. If $m > n$, then
   (a) $m - n > n - m$
   (b) $m + n > n$
   (c) $m - n > n + m$, m
   (d) $m - m, m < n + m$, m
4. A square is drawn by joining the midpoints of the side of a given square. A third square is drawn in side the second square in the same way and this process is continued indefinitely. If a side of the first square is 4 cm, the sum of the areas of all the squares (in sq. cm) is
   (a) 128 (b) 120
   (c) 96 (d) None of these
5. Find the least number which when divided by 6, 15, 17 leaves a remainder 2, but when divided by 7 leaves no remainder
   (a) 211 (b) 212 (c) 1027 (d) 86
6. The number of positive integers not greater than 100, which are not divisible by 2, 3 or 5 is
   (a) 26 (b) 16 (c) 36
   (d) None of these
7. The smallest number which when divided by 4, 6 or 7 leaves a remainder of 2, is
   (a) 44 (b) 82 (c) 86 (d) 86
8. An intelligence agency decides on a code of 2 digits selected from 0, 1, 2, ... 9. But the clip on which the code is made within shows confusion between zero and one, because these are indistinguishable. Thus, for example, the code 01 could be confused with 16. How many codes are there such that there is no possibility of any confusion?
   (a) 21 (b) 36
   (c) 81 (d) None of these
9. Suppose one wishes to find distinct positive integers x, y such that $(x + y)/(x)$ is also a positive integer. Identify the correct alternative.
   (a) This is never possible
   (b) This is possible and the pair (x, y) satisfying the stated condition is unique
   (c) This is possible and there exist more than one but a finite number of ways of choosing the pairs (x, y)
   (d) This is possible and the pair (x, y) can be chosen in infinite ways.
10. A young girl counted in the following way on the fingers of her left hand. She started calling the thumb 1, the index finger 2, middle finger 3, ring finger 4, little finger 5, then reversed direction, calling the ring finger 6, middle finger 7, index finger 8, thumb 9, then back to the index finger for 10, middle finger for 11, and so on. She counted up to 1994. She ended on her
    (a) thumb (b) index finger
    (c) middle finger (d) ring finger
11. 139 persons have signed up for an elimination tournament. All players are to be paired up for the first round, but because 139 is an odd number one player gets a bye which promotes him to the second round, without actually playing in the first round. The pairing continues on the next round, with a bye to any player left over. The schedule is planned so that a minimum number of matches is required to determine the champion. The number of matches which must be played is
    (a) 136 (b) 137 (c) 138 (d) 139
12. The product of all integers from 1 to 100 will have the following numbers of zeros at the end
    (a) 20 (b) 24 (c) 19 (d) 22
13. There are ten 50 paise coins placed on a table so that three show tails four show heads. A coin chosen at random and flipped over twice. This operation is performed seven times. One of the coins is then picked at end. Of the remaining nine coins five show tails and four show heads. The covered coin shows
    (a) a head (b) a tail
    (c) more likely a head (d) more likely a tail
14. A five digit number is formed using digits 1, 2, 3, 4, 5, 6, 7, 8, 9 without repeating any one of them. What is the sum of all such possible numbers?
    (a) 888888 (b) 666666
    (c) 444444 (d) None

13. From each of two given numbers, half the smaller number is subtracted. Of the resulting numbers the larger one is three times as large as the smaller. What is the value of the two numbers?
(a) 2.1 (b) 3.1 (c) 3.2 (d) None

14. If the harmonic mean between two positive numbers is to the inverse of their geometric mean as 12:13, then the numbers could be in the ratio
(a) 12:13 (b) 10:12:6:13
(c) 4:9 (d) 2:3

15. Fourth term of an arithmetic progression is 8. What is sum of the first 7 terms of the arithmetic progression?
(a) 7 (b) 64
(c) 56 (d) Cannot be determined

16. It takes the pendulum of a clock 7 seconds to strike 4 o'clock. How much time will it take to strike 11 o'clock?
(a) 18 seconds (b) 20 seconds
(c) 19.25 seconds (d) 23.33 seconds

19. Along a road, lie an odd number of stones placed at intervals of 10 m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried out the job starting with the stone in the middle, carrying stones in succession, thereby covering a distance of 4.8 km. Then the number of stones is
(a) 35 (b) 15 (c) 29 (d) 31

20. What is the smallest number which when increased by 5 is completely divisible by 8, 11 and 24?
(a) 264 (b) 251
(c) 269 (d) None of these

21. Which is the least number that must be subtracted from 1856, so that the remainder when divided by 7, 12 and 16 will leave the same remainder 4
(a) 127 (b) 1364 (c) 140 (d) 172

22. Two positive integers differ by 4 and sum of their reciprocals is 10/21. Then one of the numbers is
(a) 3 (b) 1 (c) 5 (d) 21

23. $5^{2n} - 1$ is divisible by
(a) 19 (b) 31
(c) 5 (d) None of these

24. For the product $n(n+1)(2n+1)$, $n \in N$, which one of the following is necessarily false?
(a) It is always even
(b) Divisible by 3
(c) Always divisible by the sum of the square of first n natural numbers
(d) Never divisible by 25?

25. The remainder obtained when a prime number greater than 6 is divided by 6 is
(a) 1 or 3 (b) 1 or 2
(c) 2 or 5 (d) 4 or 5

REVIEW TEST TWO

Directions for Questions 1–4

Four sisters Savanna, Tara, Uma and Vibha playing a game such that the loser doubles the money of each of the other player. They played four games and each sister lost one game in alphabetical order. At the end of fourth game each sister had Rs. 32.

1. Who started with the lowest amount?
(a) Savanna (b) Tara
(c) Uma (d) Vibha

2. Who started with the highest amount?
(a) Savanna (b) Tara
(c) Uma (d) Vibha

3. What was the amount with Uma at the end of the second round?
(a) 36 (b) 72
(c) 76 (d) None of these

4. How many rupees did Savanna start with?
(a) 40 (b) 34 (c) 66 (d) 28

5. If n is an integer, how many values of n will give an integral value of $(16n^2+3n+6)/n$?
(a) 2 (b) 3
(c) 4 (d) None of these

6. A student instead of finding the value of $7/8^{th}$ of a number found the value of $7/10^{th}$ of the number. It his answer differed from the actual one by 770. Find the number.
(a) 2860 (b) 2530 (c) 1728 (d) 1896

7. P and Q are two integers such that $PQ = 8^3$. Which of the following cannot be the value of $P+Q$?
(a) 20 (b) 40 (c) 16 (d) 38

8. If m and n are integers divisible by 5, which of the following is not necessarily true?
(a) $m - n$ is divisible by 5
(b) $m^2 - n^2$ is divisible by 25
(c) $m + n$ is divisible by 10
(d) None of the above

9. Which of the following is true?
(a) $x^{0.5} > (x^2)^{0.5}$ (b) $x^{0.5} = (x^2)^{0.5}$
(c) $x^{0.5} < (x^2)^{0.5}$ (d) $x^{0.5} = (x^2)^{-0.5}$
(e) None of these

10. P, Q and R are three consecutive odd numbers in ascending order. If the value of three times P is three less than two times R, find the value of R.

(a) 7 (b) 9 (c) 11 (d) 13

11. ABC is a three-digit number in which $A > 0$. The value of ABC is equal to the sum of the factorials of its three digits. What is the value of B?

(a) 0 (b) 3 (c) 4 (d) 7

12. A, B and C are defined as follows:

$A = (2.000004) + (2.000004)^2 + (2.000004)^3$

$B = (3.000019) + (3.000019)^2 + (3.000019)^3$

$C = (4.000002) + (4.000002)^2 + (4.000002)^3$

Which of the following is true about the value of the above three expressions?

(a) All of them lie between 0.10 and 0.20

(b) A is twice C

(c) C is the smallest

(d) B is the smallest

13. Let $x \neq 0.5$, $0 < y < 1$, $z > 1$. Given a set of numbers, the middle number, when they arranged in ascending order is called the median. So the median of the numbers x, y and z would be

(a) less than one (b) between 0 and 1

(c) greater than one (d) cannot say

14. Let a, b, c, d, and e be integers such that $a = 6b = 12c$, and $2b = 3d = 12e$. Then which of the following pairs contains a number that is not an integer?

(a) $\left(\frac{a}{d}, \frac{b}{e}\right)$ (b) $\left(\frac{a}{36}, \frac{c}{e}\right)$

(c) $\left(\frac{a}{3}, \frac{2d}{9}\right)$ (d) $\left(\frac{a}{7}, \frac{c}{d}\right)$

15. If a, $a+2$, and $a+4$ are prime numbers, then the number of possible solutions for a is

(a) One (b) Two

(c) Three (d) More than three

16. Let x and y be positive integers such that x is prime and y is composite. Then,

(a) $2 - x$ cannot be an even integer

(b) xy cannot be an even integer

(c) $(x + y)$ cannot be an even integer

(d) None of the above statement is true

17. Let n (> 1) be a composite natural number such that the square root of n is not an integer. Consider the following statements:

A: n has a factor which is greater than 1 and less than square root n

B: n has a factor which is greater than square root n but less than n

Then

(a) Both A and B are false

(b) Both A and B are true

(c) A is false but B is true

(d) Both A and B are true

18. What is the remainder when 4^{96} is divided by 6?

(a) 0 (b) 2 (c) 3 (d) 4

19. What is the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7?

(a) 646 (b) 676 (c) 683 (d) 707

20. The infinite sum $1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4} + \cdots$

(a) $\frac{27}{14}$ (b) $\frac{29}{13}$

(c) $\frac{49}{27}$ (d) $\frac{256}{147}$

21. If the product of n positive real numbers is unity then their sum is necessarily

(a) a multiple of n (b) equal to $n + \frac{1}{n}$

(c) never less than n (d) None of these

22. How many three digit positive integer, with digits x and z in the hundred's, ten's and unit's place respectively, exist such that $x < y$, $z < y$ and $x \neq 0$?

(a) 245 (b) 285 (c) 240 (d) 320

23. How many even integers n, where $100 \le n \le 200$, are divisible neither by seven nor by nine?

(a) 40 (b) 37 (c) 39 (d) 38

24. A positive whole number M less than 100 is expressed in base 2 notation, base 3 notation, and base 5 notation. It is found that in all three cases the last digit is 1, while in exactly two out of the three cases the leading digit is 1. Then M equals:

(a) 31 (b) 63 (c) 75 (d) 81

25. In a certain examinations paper, there are n questions. For $j = 1, 2, \ldots n$, there are 2^{n-j} students who answered j or more questions wrongly. If the total number of wrong answers is 4095, then the value of n is

(a) 12 (b) 11 (c) 10 (d) 9

REVIEW TEST THREE

1. In 1970, I was as old as the number formed by the last two digits of my year of birth. Find my date of birth of my father who is 25 years older to me.
(a) 1900 (b) 1903
(c) 1902 (d) 1901
(e) Can't be determined

2. Find the total number of integral solutions of the equation $(a+b)^2 - (ab)(a+b) = 2500$, where 'abab' is a three digit number.
(a) 0 (b) 1 (c) 2 (d) 3
(e) Can't be determined

3. Find the digit at the ten's place of the number $N = 3^{200} + 3^{204}$.
(a) 0 (b) 1 (c) 6 (d) 5
(e) None of these

4. Raju went to a shop to buy a certain number of pens and pencils. Raju calculated the amount payable to the shopkeeper and offered that amount to him. Raju was surprised when the shopkeeper returned him Rs. 24 as balance. When he came back home, he realized that the shopkeeper had actually transposed the number of pens with the number of pencils. Which of the following is certainly an invalid statement?
(1) The number of pencils that Raju wanted to buy was 8 more than the number of pens.
(2) The number of pens that Raju wanted to buy was 6 less than the number of pencils.
(3) A pen cost Rs.4 more than a pencil.
(4) A pencil cost Rs.12 less than a pen.
(5) None of the above

5. HCF of 18a and a'b' is 18ab. What is the correct relation between a and b?
(a) $a = 2b$
(b) $a + b = 8$
(c) $a - b = 3$
(d) $a + b = 5$
(e) Can't be determined

6. In ancient India, 0 to 25 years of age was called Brahmacharya and 25 to 50 was called Grahastha. I am in Grahastha and my younger brother is also in Grahastha such as the difference in our ages is 4 years and both of our ages are prime numbers. Also twice my brother's age is 31 more than my age. Find the sum of our ages.
(a) 80 (b) 68
(c) 70 (d) 71
(e) None of these

7. Volume of a cube with integral sides is the same as the area of a square with integral sides. Which of these can be the volume of the cube formed by using the square and its replicas as the 6 faces?
(a) 1000 (b) 512
(c) 256 (d) Both (a) and (b)
(e) Both (b) and (c)

8. Let A be a two-digit number and B be another two-digit number formed by reversing the digits of A. If A + B + (Product of digits of the number A) = 145, then what is the sum of the digits of A?
(a) 9 (b) 10 (c) 11 (d) 12
(e) 13

9. When a two-digit number N is divided by the sum of its digits, the result is Q. Find the minimum possible value of Q.
(a) 10 (b) 2 (c) 5.5 (d) 1.9
(e) 7

10. A one-digit number, which is the ten's digit of a two-digit number X, is subtracted from X to give Y which is the quotient of the division of 990 by the cube of a number. Find the sum of the digits of X.
(a) 5 (b) 7 (c) 6 (d) 8
(e) 9

11. After Yuvraj hit 6 sixes in an over, Geoffery Boycott commented that Yuvraj just made 210 runs in the over. Harsha Bhogle was shocked and he asked Geoffery which base system was he using? What must have been Geoffery's answer?
(a) 9 (b) 2 (c) 5 (d) 4
(e) 7

12. Find the ten's digit of the number 7^{2010}.
(a) 0 (b) 1 (c) 2 (d) 3
(e) 4

13. Find the HCF of 14! and the number 'aaa' where 'a' is number between 1 and 9 (both included).
(a) 72 (b) 1 (c) 27 (d) 32
(e) Can't be determined

14. The number of positive integer valued pairs (x, y) satisfying $4x - 12y = 1$ and $x \le 1000$ is
(a) 90 (b) 85 (c) 80 (d) 36
(e) None of these

15. Let a, b, c be distinct digits. Consider a two digit number 'ab' and a three digit number 'acb'. Both the hundred number

[illegible] Test Papers for Aptitude Aptitude for the CAT

[illegible] number system. If $(ab)^2 = c$ [illegible]

[illegible] the value of b is

(a) 1 (b) 0 (c) 2 (d) 9

(e) None of these

[illegible] The parameter $1^{[illegible]}$ is divided by 343 is

(a) 0 (b) 1 (c) 49 (d) 343

(e) None of these

13. Let x, y and z be distinct integers. x and z are odd and positive, and y is even and positive. Which one of the following statements can be true?

(a) $(x+y)^2$ is even (b) $(x-z)^2$ is odd

(c) $(x-y)$ is odd (d) $(x-y)^2 z$ is even

(e) None of these

14. A boy starts adding consecutive natural numbers starting with 1. After some time by mistake a total of 1000 when he realizes that he has made the mistake of double counting a number. Find the number double counted.

(a) 44 (b) 48

(c) 10 (d) 12

(e) None of these

15. Six number system the product of 44 and 11 is 1004. The number 2113 of this system, when converted to decimal number system, becomes:

(a) 436 (b) 1006

(c) 251 (d) 691

(e) None of these

20. Ashish is given Rs 158 in one rupee denominations. He has been asked to allocate them into a number of bags such that any amount required between Re 1 and Rs 158 can be given by handing out a certain number of bags without opening them. What is the minimum number of bags required?

(a) 10 (b) 12

(c) 13 (d) 14

(e) None of these

REVIEW TEST FOUR

1. Find the number of 6-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 once such that the 6-digit number is divisible by its unit digit.

(a) 360 (b) 720

(c) 720 (d) 120

(e) None of these

2. Which is the highest 3-digit number that divides the number 111111... 1(27 times) perfectly without leaving any remainder?

(a) 111 (b) 333 (c) 666 (d) 999

(e) None of these

3. W1, W2, ..., W7 are 7 positive integral values such that by attaching the coefficients of +1, 0 and -1 to each value available and adding the resultant values, any number from 1 to 1093 (both included) could be formed. If W1, W2, ..., W7 are in ascending order, then what is the value of W3?

(a) 10 (b) 9 (c) 0 (d) 1

(e) None of these

4. What is the unit digit of the number $63^{27} + 25^{[illegible]}$?

(a) 3 (b) 5 (c) 8 (d) 2

(e) None of these

5. Find the remainder when $(2222^{[illegible]} + 5555^{[illegible]})$ is divided by 7.

(a) 1 (b) 0 (c) 2 (d) 5

(e) None of these

6. What is the number of nines used in numbering a 45 page book?

(a) 86 (b) 87 (c) 84 (d) 88

(e) 85

7. How many four digit numbers are divisible by 5 but not by 25?

(a) 2000 (b) 8000 (c) 1440 (d) 999

(e) None of these

8. The sum of two integers is 10 and the sum of their reciprocals is 5/12. What is the value of larger of the integers?

(a) 7 (b) 5 (c) 6 (d) 4

(e) None of these

9. Sneesh was born in 1989. His elder brother Sudhish was also born in the 1980's such that the last two digits of his date of birth form a prime number P. Find the remainder when $(P^2 + 11)$ is divided by 5.

(a) 0 (b) 1 (c) 2 (d) 3

(e) Can't be determined

10. The HCF of x and y is H. Find the HCF of $(x+y)^{[illegible]}$ $(x^3 + y^3)(x^2 - xy + y^2)$

(a) H − 1 (b) H^2 (c) H (d) H + 1

(e) None of these

11. 4 bells toll together at 6:00 A.M. They toll after 5, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 4 hours?
(a) 3 (b) 5 (c) 6 (d) 9
(e) None of these

12. What power of 210 will exactly divide 142!
(a) 22 (b) 11 (c) 34 (d) 38
(e) None of these

13. Find the total number between 122 and 462 that are divisible by 3 but not by 9.
(a) 50 (b) 70 (c) 72 (d) 75
(e) None of these

14. If 340! is divisible by 6^n, then find the maximum value of n.
(a) 74 (b) 78 (c) 76 (d) 75
(e) 70

15. If we add the square of the digit in the ten place of a positive two digit number to the product of the digits of that number, we shall get 52, and if we add the square of the digit in the units place to the same product of the digits, we shall get 117. Find the two-digit number.
(a) 38 (b) 36 (c) 40 (d) 28
(e) 30

16. Find the smallest natural number n such that $(n+1)(n-11)$ is divisible by 900.
(a) 2 (b) 4 (c) 10 (d) 11
(e) None of these

17. If x, y and z are odd, even and odd respectively, then $(x^2 + y^2 + z^2)$ and $(x^3 + y^3 + z^3)$ are respectively.
(a) Odd & Odd (b) Even & Odd
(c) Odd & Even (d) Odd & Odd
(e) Can't be determined

18. A two digit number N has its digits reversed to form another two digit number M. What is the unit digit of M if product of M and N is 574?
(a) 1 (b) 3 (c) 6 (d) 9
(e) None of these

19. For what relation between b and c is the number should divisible by 7, if $b = c^3$
(a) $b + c = 7$
(b) $b = c + 7$
(c) $bc = 7$
(d) $c = 7b$
(e) None of these

20. What is the remainder when n^n is divided by $(n - 1)^2$
(a) $n + 1$ (b) n (c) 0 (d) 1
(e) None of these

REVIEW TEST FIVE

1. What is the last digit of $62^*43^*54^*65^*76^*87$
(a) 2 (b) 4 (c) 6 (d) 8
(e) None of these

2. $N = 84^3 \cdot 36^2 \cdot 63^3$, how many factors does N have?
(a) 38 (b) 96 (c) 128 (d) 192
(e) None of these

3. Find the highest power of 2 in $11 + 21 + 31 + \ldots + 81$ ____600!
(a) 1 (b) 494 (c) 3, 0 (d) 256
(e) None of these

4. 100! is divisible by 180^n...what is the max. integral value of n^2
(a) 39 (b) 34 (c) 26 (d) 28
(e) 20

5. What is the sum of the digits of the decimal form of the product $2^{999} \times 5^{1001}$?
(a) 2 (b) 4 (c) 5 (d) 7
(e) 10

6. What is the remainder when 1*1 + 11*11 + 111*111 + 1111*1111 + ___+ (2001 times 1)*(2001 times 1) is divided by 100 ?
(a) 99 (b) 22 (c) 01 (d) 21
(e) 89

7. What is the remainder when 789456123 is divided by 999?
(a) 123 (b) 369 (c) 564 (d) 189
(e) 948

8. What is the total number of the factors of 36!
(a) 2016 (b) 1024 (c) 2080 (d) 4032
(e) None of these

9. Find the sum of the first 125 terms of the sequence 1, 2, 1, 3, 2, 1, 4, 3, 2, 1, 5, 4, 3, 2,
(a) 816 (b) 480 (c) 750 (d) 720
(e) 980

10. Unnati purchased a Tata Nano recently, but the faulty car odometer of Tata Nano proceeds from digit 4 to digit 6, always skipping the digit 5, regardless of position. If the odometer now reads 000008 (starting with 000000), how many km has Nano actually travelled?
(a) 2000 (b) 1000 (c) 2046 (d) 2199
(e) None of these

11. What is the remainder when 1234123412341234 ____ 4000 digits is divided by 9001

(a) 625 (b) 600 (c) 685 (d) 675
(e) None of these

12. Mr. Kambil lived his entire life during the 1800s. In the last year of his life, Rambil stated: Once I was x years old in the year x^2. He was born in the year
(a) 1822 (b) 1851 (c) 1853 (d) 1806
(e) 1800

13. Find the unit's digit of LCM of $13^{301} - 1$ and $13^{301} + 1$
(a) 2 (b) 4 (c) 5 (d) 8
(e) 9

14. If you were to add all odd numbers between 1 and 2007 (both inclusive), the result would be
(a) A perfect square (b) Divisible by 2008
(c) Multiple of 251 (d) All of the above
(e) None of these

15. Find the remainder when $97(30^{99} + 61^{100}) + (1148)^{24}$ is divided by 31
(a) 25 (b) 0 (c) 11 (d) 21
(e) 10

16. What is the remainder when 2^{100} is divided by 10?
(a) 1 (b) 100 (c) 0 (d) 96
(e) None of these

17. Find the last two digits of 2^{134}
(a) 04 (b) 84 (c) 34 (d) 64
(e) None of these

18. Find the remainder when $(10^3 + 9^3)^{1000}$ by 12^2
(a) 01 (b) 41 (c) 1001 (d) 727
(e) None of these

19. The number of factors of the number 3000 are
(a) 16 (b) 32 (c) 24 (d) 28
(e) 40

20. If N! has 73 zeroes at the end then find the value of N
(a) 295 (b) 300 (c) 290 (d) 25
(e) Not possible

ANSWER KEY

Review Test One

1. a
2. c
3. d
4. a
5. b
6. a
7. d
8. c
9. a
10. b
11. c
12. b
13. a
14. a
15. a
16. c
17. c
18. d
19. d
20. b
21. d
22. a
23. b
24. d
25. b

Review Test Two

1. a
2. a
3. b
4. e
5. c
6. a
7. d
8. c
9. b
10. e
11. e
12. d
13. b
14. d
15. a
16. d
17. b
18. d
19. b
20. e
21. c
22. e
23. c
24. d
25. a

Review Test Three

1. a
2. a
3. c
4. c
5. d
6. a
7. d
8. c
9. d
10. a
11. d
12. c
13. a
14. c
15. a
16. b
17. a
18. c
19. a
20. c

Review Test Paper

1. a
2. d
3. b
4. c
5. b
6. c
7. d
8. c
9. a
10. c
11. b
12. a
13. b
14. c
15. c
16. d
17. c
18. a
19. b
20. d

Review Test Five

1. a
2. b
3. a
4. a
5. d
6. c
7. b
8. d
9. c
10. c
11.
12. d
13. b
14. d
15. b
16. a
17. b
18. a
19. b
20. a

BLOCK 2

CHAPTERS

- Averages
- Alligations

...BACK TO SCHOOL

- **The Relevance of Averages**

Average is one of the most important mathematical concepts that we see in our day to day life. In fact, even the most non mathematical individuals regularly utilize the concept of averages on a day to day basis.

So, we use averages in all the following and many more instances:

- How a class of students fared in an exam is assessed by looking at the average score.
- What is the average price of items purchased by an individual.
- A person might be interested in knowing his average telephone expenditure, electricity expenditure, petrol expenditure, etc.
- A manager might be interested in finding out the average sales per territory or even the average growth rate (sales in month).
- Clearly there can be numerous application of averages that you might be able to visualize on your own.

- **The Meaning of an Average**

The average is basically seen as a representative value which can be used to represent the value of the general item in a group of values.

For instance, suppose that a cricket team had 10 partnerships as follows:

1st wicket 28	5th wicket 42
2nd wicket 157	6th wicket 52
3rd wicket 0	7th wicket 23
4th wicket 44	8th wicket 18
9th wicket 9	10th wicket 17

On adding the ten values above, we get a total of 340 which gives an average of 34 runs per wicket, i.e. the average partnership of the team was 34 runs.

Contd.

In other words, if we were to replace the value of all the six partnerships by 34 runs, we would get the same total runs. Hence, 34 represents the average partnership value for the team.

Suppose, in a cricket series of 5 matches between 2 countries, you are given that Team A had an average partnership of 38 Runs per wicket while Team B had an average partnership of 34 runs per wicket. What conclusion can you draw about the performance of the two teams, given that both the teams played 5 complete test matches?

Obviously, Team B would have performed much worse than Team A. For that matter, if I tell you that the average daytime high temperature of Lucknow was 19° C for a particular month, you can easily draw some kind of conclusion in your mind about the month we could possibly be talking about.

Thus, you should realize that the beauty of averages lies in the fact that it is one single number that tells you a lot about the group of numbers – hence, it is one number that represents an entire group of numbers.

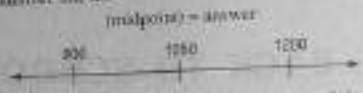
But one of the key concepts that you need to understand before you move into the chapters of this book is the Concept of WEIGHTED AVERAGES.

As always, the concept is best explained through a concrete example.

Suppose I had to buy a shirt and a trouser and let us say that the average cost of a shirt was Rs. 1200 while that of a trouser was Rs. 900.

In such a case, the average cost of a shirt and a trouser would be given by (1200 + 900)/2 = 1050.

This can be visualized on the number line as:



As you can easily see in the figure, the average occurs at the midpoint of the two numbers.

Now, let us try to modify the situation.

Suppose, I were to buy 2 trousers and 1 shirt. In such a case I would end up spending (900 + 900 + 1200) = Rs. 3000 in buying a total of 3 items. What would be my average in this case?

Obviously, 3000/3 = Rs. 1000!!

Clearly, the average has shifted!!

On the number line we could visualize this as follows:

It is clearly visible that the average has shifted towards 900 (which was the cost price of the trousers — the larger purchased item.)

In a way, this shift is similar to the way a two pan weighing balance shifts on weights being put on it. The balance shifts towards the pan containing the larger weight.

Similarly, in this case, the correct average (1000) is closer to 900 than it is to 1200. This has happened because the number of elements in the group of average 900 is greater than the number of elements in the group having average 1200. Since, this is very similar to the system of weights, we call this as a weighted average situation.

At this stage, you should realize that weighted averages are not solely restricted to two groups. We can also come up with a weighted average situation for three groups (although in such a case the representation of the weighted average on the number line might not be so easily possible.) In fact, it is the number line representation of a weighted average situation that is defined as alligation (where 2 groups are involved).

PROGRESSMENT TEST

1. X's age is $\frac{1}{3}$rd of Y's present age. Y's age will be twice of X's age after 10 years. If Z's eighth birthday was celebrated two years ago, then the present age of X and Y?
   (a) 5 years (b) 10 years
   (c) 15 years (d) 20 years
2. David was twice as old as Rahul 10 years back. How old is Rahul today if David will be 45 years old 15 years hence?
   (a) 20 years (b) 30 years
   (c) 25 years (d) None of these
3. A demographic survey of 100 families in which two parents were present revealed that the average age A of the oldest child is 15 years less than ½ the sum of the ages of the two parents. If X represents the age of one parent and Y the age of the other parent, then which of the following is equivalent to A?
   (a) $\frac{X+Y}{2} - 15$ (b) $\frac{X+Y}{2} + 15$
   (c) $\frac{X+Y}{2} - 15$ (d) X + Y – 7.5
4. 8 10 years are subtracted from the present age of Randy and the remainder divided by 12, then you would get the present age of his grandson Sandy. If Sandy is 19 years younger to Sandy whose age is 24, then what is the present age of Randy?
   (a) 80 years (b) 70 years
   (c) 60 years (d) None of these
5. Two groups of students, whose average ages are 15 years and 21 years, combine to form a third group whose average age is 23 years. What is the ratio of the number of students in the first group to the number of students in the second group?
   (a) 4 : 2 (b) 2 : 4
   (c) 4 : 5 (d) None of these
6. A year ago Mohit was four times his son's age. In six years, his age will be 9 more than twice his son's age. What is the present age of the son?
   (a) 10 years (b) 9 years
   (c) 20 years (d) None of these
7. In 1991, I was as old as the number formed by the last two digits of my birth year. When I mentioned this interesting coincidence to my grandfather, he surprised me by saying that the same applied to him also. The difference in our ages is:
   (a) 40 years (b) 50 years
   (c) 60 years (d) None of these
8. The average age of three boys is 18 years. If their ages are in the ratio 4:5:9, then the age of the youngest boy is
   (a) 8 years (b) 9 years
   (c) 12 years (d) 16 years
9. "I am eight times as old as you were when I was as old as you are", said a man to his son. Find out their present ages if the sum of their ages is 75 years.
   (a) 40 years and 35 years
   (b) 56 years and 19 years
   (c) 48 years and 27 years
   (d) None of these
10. My brother was 3 years of age when my sister was born, while my mother was 26 years of age when I was born. If my sister was 4 years of age when I was born, then what was the age of my father and mother respectively when my brother was born?
    (a) 35 years, 13 years (b) 35 years, 29 years
    (c) 32 years, 23 years (d) None of these
11. Namrata's father is now four times her age. In five years, he will be three times her age. In how many years, will he be twice her age?
    (a) 5 (b) 20
    (c) 25 (d) 15
12. A father is twice as old as his daughter. 20 years back, he was seven times as old as the daughter. What are their present ages?
    (a) 24, 12 (b) 44, 22
    (c) 48, 24 (d) none of these
13. The present ages of three persons are in the proportion of 5:6:7. Eight years ago, the sum of their ages was 76. Find the present age of the youngest person.
    (a) 20 (b) 25
    (c) 30 (d) None of these
14. The average age of a class is 14.8 years. The average age of the boys in the class is 15.4 years and that of girls is 14.4 years. What is the ratio of boys to girls in the class?
    (a) 1 : 2 (b) 3 : 2
    (c) 2 : 3 (d) None of these
15. In an organization, the daily average wages of 20 different employees is decreased from Rs 25 to Rs 10, thus the average salary of all the female employees and different employees is decreased by Rs 10 per day.

The number of educated employees working in the organization are:

(a) 25 (b) 20
(c) 30 (d) 35

16. Mr Abhishek Bajpai while going from Lucknow to Jaunpur covered half the distance by train at the speed of 96 km/hr, half the rest of the distance by his scooter at the speed of 60 km/hr and the remaining distance at the speed of 40 km/hr by car. The average speed at which he completed his journey is:

(a) 64 km/hr (b) 56 km/hr
(c) 60 km/hr (d) 56 km/hr

17. There are four types of candidates in AMS Learning System preparing for the CAT. The number of students of Engineering, Science, Commerce and Humanities is 400, 600, 500 and 300 respectively and the respective percentage of students who qualified the CAT is 80%, 75%, 60% and 50% respectively the overall percentage of successful candidates in our institute is:

(a) 67.7% (b) 66.66%
(c) 68.5% (d) None of these

18. Mr. Jagmohan calculated the average of 10 "Three digit numbers". But due to mistake he reversed the digits of a number and thus his average increased by 29.7. The difference between the unit digit and hundreds digit of that number is

(a) 4 (b) 3
(c) 2 (d) can't be determined

Directions for questions 19–20: Answer the questions based on the following information.

Production pattern for number of units in cubic feet per day.

Days	1	2	3	4	5	6	7
Number of units	150	180	120	250	100	120	150

Per a unit that can carry 2,000 cubic feet, hiring cost per day is Rs. 1,000. Storing cost per cubic feet is Rs. 5 per day. Any residual material left at the end of the seventh day has to be transferred.

19. If all the units should be sent to the market, then on which days should the trucks be hired to minimize the cost:

(a) 2nd, 4th, 6th, 7th (b) 7th
(c) 2nd, 4th, 5th, 7th (d) None of these

20. If the storage cost is reduced to Rs. 0.9 per cubic foot per day, then on which days/days, should the trucks be hired?

(a) 4th (b) 7th
(c) 4th and 7th (d) None of these

Answers (Block 2 Preassessment Test)

1. (a)	2. (a)	3. (a)	4. (b)	5. (b)
6. (b)	7. (b)	8. (c)	9. (c)	10. (a)
11. (b)	12. (c)	13. (b)	14. (c)	15. (c)
16. (d)	17. (a)	18. (b)	19. (a)	20. (a)

SCORE INTERPRETATION ALGORITHM

Block 2 Preassessment Test

If You Scored: <12: (In Unlimited Time)

Step One Go through the first chapter of the block, viz. Averages. When you do so, concentrate on clear understanding each of the concepts explained in the chapter theory.

Then move onto the LOD 1 exercises. These exercises provide you with the first level of challenge. Try to solve each and every question provided under LOD 1. While doing so do not think about the time requirement. Once you have solving LOD 1, revise the questions and their solution processes.

Also at this stage study the concept of averages from your school text books (Class 8, 9 & 10) and solve all the questions which are available to you in those books.

Step Two After finishing LOD 1 of Averages, move to LOD 2 and then LOD 3 of this chapter.

Step Three Go to the chapter of alligations and study the shortcuts provided carefully. Understand the use of the alligation process as clearly as possible.

Then move to the LOD 1 exercise of the same chapter. The chapter of alligations does not have an LOD2 & LOD3 exercise.

Step Four Go to the practice test given at the end of the block and solve it. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve

do not feel within a few hour and try to evaluate the [illegible] in your unlimited time score.

If you feel that in your score is not significant, go back to the start of each chapter and review each of the [illegible] you have solved for both the chapters.

If You Scored:> 12 (In Unlimited Time)

Follow the same process as above. The only difference is that the school book work is optional – do it only if you feel you need to. However, your concentration during the solving of the two chapters has to be on developing your speed of solving questions on this chapter.

3

AVERAGES

INTRODUCTION

The concept of averages is important since questions based on this topic regularly appear in all aptitude exams. While our best-fit MBA exams as well as Bank PO, NDA, NIFT and other exams of averages through direct questions, and the XLRI examination have used the concept in an intuitive problems that will not be found in most textbooks. Such problems have been given in this chapter in LRI T and LRO III questions.

While studying this chapter, make sure that you understand the technique of some illustrated computations in the text. These techniques are essentially based on certain concepts of relationship that will, in turn, help you get a better grip on these concepts.

THEORY

The average of a number is a measure of the central tendency of several numbers. In other words, it is an estimate of where the mean point of a set of numbers lies.

The basic formula for the average of n numbers $x_1, x_2, x_3, \ldots, x_n$ is

$$A_n = (x_1 + x_2 + x_3 + \ldots + x_n)/n = (\text{Total of set of } n \text{ numbers})/n$$

The term mean A_n is a total of the set of numbers.

The average is always calculated for a set of numbers.

Concept of Weighted Average: When we have 2 or more groups whose individual averages are known, then to find the combined average of all the elements of all the groups we use weighted average. Thus, if we have 3 groups with averages $A_1, A_2, \ldots A_k$ and having $n_1, n_2, \ldots n_k$ elements then the weighted average is given by the formula:

$$A_w = \frac{n_1 A_1 + n_2 A_2 + n_3 A_3 + \ldots + n_k A_k}{n_1 + n_2 + n_3 + \ldots + n_k}$$

Another Meaning of Average The average (also known as arithmetic mean (AM)) of a set of numbers can also be defined as the number by which we can replace each and every number of the set without changing the total of the set of numbers.

Properties of Average (AM) The properties of averages (arithmetic mean) can be elucidated by the following examples.

Example 1: The average of 4 numbers 12, 13, 17 and 18 is:

Solution: Required average = (12 + 13 + 17 + 18)/4 = 60/4 = 15

This means that if each of the 4 numbers of the set were replaced by 15 each, there would be no change in the total.

This is an important way to look at averages. In fact, whenever you come across any situation where the average of a group of 'n' numbers is given, you should visualise that there are 'n' numbers, each of whose value is the average of the group. This view is a very important way to visualise averages.

This can be visualised as

$$\begin{array}{l} 12 \to +3 \to 15 \\ 13 \to +2 \to 15 \\ 17 \to -2 \to 15 \\ 18 \to -3 \to 15 \\ \hline 60 \to +0 \to 60 \end{array}$$

Example 2: In Example 1, imagine addition of a fifth number, which increases the average by 1.

$$14 + 1 = 15$$
$$15 + 1 = 16$$
$$16 + 1 = 17$$
$$17 + 1 = 18$$

The 4 appearing 4 times is due to the fifth number, which is able to maintain the average of 16 first and then "give one" to each of the first 4.

Hence, the fifth number in this case is 20.

Example 3: The average always lies above the lowest number of the set and below the highest number of the set.

Example 4: The set deficit due to the numbers below the average always equals the set surplus due to the numbers above the average.

Example 5: *Ages and averages:* If the average age of a group of persons is x years today then after n years their average age will be $(x + n)$.

Also, n years ago their average age would have been $(x - n)$. This happens due to the fact that for a group of people, 1 year is added to each person's age every year.

Example 6: A man travels at 60 kmph on the journey from A to B and returns at 100 kmph. Find his average speed for the journey.

Solution:

Average speed = (total distance)/(total time)

If we assume distance between 2 points to be d.

Then

Average speed = $2d/[(d/60) + (d/100)] = (2 \times 60 \times 100)/(60 + 100) = (2 \times 60 \times 100)/160 = 75$

Average speed = $(2S_1 \times S_2)/(S_1 + S_2)$

(S_1 and S_2 are speeds of going and coming back respectively.

Short Cut The average speed will always come out by the following process:

The ratio of speeds is 60 : 100 = 3 : 5 (say $r_1 : r_2$)

Then divide the difference of speeds (40) in this case by $r_1 + r_2$ (3 + 5 = 8, in this case) to get one part. (40/8 = 5, in this case)

The required answer will be three parts away (i.e. r_1 parts away) from the lower speed.

Check out how this works with the following speeds:

$$S_1 = 20 \quad \text{and} \quad S_2 = 40$$

Step 1: Ratio of speeds = 20 : 40 = 1 : 2

Step 2: Divide difference of 20 into 3 parts ($r_1 + r_2$) = 20/3 = 6.66

Required average speed = 20 + 1 × 6.66

Note: This process is essentially based on alligations and we shall see it again in the next chapter.

EXERCISES FOR SELF-PRACTICE

Find the average speed for the above problem if

(1) $S_1 = 20$	$S_2 = 200$
(2) $S_1 = 60$	$S_2 = 120$
(3) $S_1 = 100$	$S_2 = 50$
(4) $S_1 = 60$	$S_2 = 180$

WORKED-OUT PROBLEMS

Problem 2.1 The average of a batsman after 25 innings was 56 runs per innings. If after the 26th inning his average increased by 2 runs, then what was his score in the 26th inning?

Solution *Normal process:*

Runs in 26th inning = Runs total after 26 innings − Runs total after 25 innings

$$= 26 \times 58 - 25 \times 56$$

For mental calculation use

$$(56 + 2) \times 26 - 56 \times 25$$
$$= 2 \times 26 + (56 \times 26 - 56 \times 25)$$
$$= 52 + 56 = 108$$

Short Cut Since the average increases by 2 runs per innings it is equivalent to 2 runs being added to each innings in the first 25 innings. Now, since these runs can only be added by the runs scored in the 26th inning, the score in the 26th inning must be 25 × 2 = 50 runs higher than the average after 26 innings (i.e. new average = 58).

Hence, runs scored in 26th inning = New Average + (No. of innings × Change in average)

$$= 58 + 25 \times 2 = 108$$

Situation this is:

Average in first 29 innings	Average after 29 innings
x	x
x	x
x	x
x	x

29 times 29 times

Difference is and is seen 29 times and 58 times, that is, 58 × 0.5 = 2.

Problem 2.2 The average age of a class of 30 students and a teacher is increased by 0.5 years if we exclude the teacher. If the total average is 14 years, find the age of the class teacher.

Solution *Normal process:*

Age of teacher = Total age of (students + teacher)
− Total age of students
= 31 × 14 − 30 × 13.5
= 434 − 405
= 29 years

Short Cut The teacher after fulfilling the average of 14 for the group (which he belonged) is also able to give 0.5 years to the age of each of the 30 students. Hence, he has 30 × 0.5 = 15 years to give over and above maintaining his own average age of 14 years.

Age of teacher = 14 + 30 × 0.5 = 29 years

(Note: This problem should be viewed as change of average from 13.5 to 14 when teacher is included.)

Problem 2.3 The average marks of a group of 20 students is 4 and is reduced by 4 when the topper who scored 90 marks is replaced by a new student. How many marks did the new student get?

Solution *Normal process:*

Let x be average for a.
Then the total sum is 20x.
Then average will be (x − 4) and the new total will be 20 × (x − 4) = 20x − 80.
The difference of 80 is created by the replacement.
Hence, the new student has 80 marks less than the student he replaced. Hence, he must have scored 10 marks.

Short Cut The replacement has the effect of reducing the average marks for each of the 20 students by 4. Hence, the replacement must be 20 × 4 = 80 marks below the original.

Hence, answer = 10 marks.

Problem 2.4 The average age of 3 students A, B and C is 48 marks. Another student D joins the group and the new average becomes 44 marks. If another student E, who has 3 marks more than D, joins the group, the average of the 4 students B, C, D and E becomes 43 marks. Find how many marks A got in the exam.

Solution Solve while reading. The first sentence gives you a total of 144 for A, B and C's marks. Second sentence: When D joins the group, the total becomes 44 × 4 = 176. Hence, D must get 32 marks.

Alternatively, you can reach this point by considering the first 2 statements together as

D's joining the group reduces the average from 48 to 44 marks (i.e. 4 marks).

This means that to maintain the average of 44 marks, D has to take 4 marks from A, 4 from B and 4 from C → A total of 12 marks. Hence, he must have got 32 marks.

From here:

The first part of the third sentence gives us information about E getting 3 marks more than 32 → Hence, E gets 35 marks.

Now, it is further stated that when A is replaced by E, the average marks of the students reduces by 1 to 43.

Mathematically this can be shown as

$$A + B + C + D = 44 \times 4 = 176 \text{ while, } B + C + D + E = 43 \times 4 = 172$$

Subtracting the two equations, we get A − E = 4 marks.

Hence, A would have got 39 marks.

Alternatively, you can think of this as:

The replacement of A with E results in the reduction of 1 mark from each of the 4 people who belong to the group. Hence, the difference is 4 marks. Hence, A would get 4 marks more than E i.e. A gets 39 marks.

Problem 2.5 The mean temperature of Monday to Wednesday was 37 °C and of Tuesday to Thursday was 34 °C. If the temperature on Thursday was 2/3rd of the temperature on Monday, what was the temperature on Thursday?

Solution From the first statement, we get that the total temp Monday to Wednesday was 81 while from Tuesday to Thursday was 72. The difference is arising out of the replacement of Monday by Thursday.

This can be mathematically written as

$$\text{Mon} + \text{Tue} + \text{Wed} = 81 \quad (1)$$

$$\text{Tue} + \text{Wed} + \text{Thu} = 72 \quad (2)$$

Hence:

$$\text{Mon} - \text{Thu} = 9$$

We have two unknown variables in the above equation. To solve for 2 unknowns, we need a new equation. Looking back at the problem we get the equation:

$$\text{Thu} = (2/3) \times \text{Mon}$$

Solving the two equations we get: Thursday = 18 °C.

However, in the exam, you should avoid using equation solving as much as possible. You should, ideally, be able to reach half way through the solution during the first reading of the question, and then meet the gap through the use of options.

The answer to this problem should be got by the time you finish reading the question for the first time.

Then suppose we have the equations

$M - T = 9$ and $T = 2M/3$ or $T/M = 2/3$ and have the options for T as

(a) 12 (b) 15 (c) 18 (d) 21

To check which of these options is the appropriate value, we need to check one by one.

Option (a) gives $T = 12$, then we have $M = 21$. But $12/21 \neq 2/3$. Hence, this is not the correct option.

Option (b) gives $T = 15$, then $M = 24$. But again $15/24 \neq 2/3$. Hence, this is not the correct option.

Option (c) gives $T = 18$, then $M = 27$. Now $18/27 = 2/3$. Hence, this is the correct option.

So we no longer need to check for option (d).

However, if we had checked for option (d), then $T = 21$, so $M = 30$. But again $21/30 \neq 2/3$. Hence, this is not the correct option.

In the above, we used 'solving while reading' and 'option based' approaches.

These two approaches are very important and by combining the two, you can reach amazing speeds in solving the exercise.

You are advised to practice both these approaches while solving questions, which will surely improve your efficiency and speed. You will see that, with practice, you will be able to arrive at the solution in most of the LOD questions (given later in this chapter) even as you finish reading the questions. And since it is the LOD type problems that appear in most examinations like CAT, Maharashtra Bank PO, MAT, India MAT, NMIMS, XAT, NLS and most other aptitude exams, you will gain a significant advantage in solving these problems.

On LOD II, LOD III and CAT type problems, you will find that using solving-while-reading and option-based approaches together would take you through anywhere between 10–70% of the question by the time you finish reading the question for the first time.

This will give you a tremendous time advantage over the other students appearing in the examination.

Problem 2.6 A person covers half his journey by train at 60 kmph, the remainder half by bus at 30 kmph and finally by cycle at 10 kmph. Find his average speed during the entire journey.

Solution Recognize that the journey by bus and that by cycle are of equal distance. Hence, we can use the shortcut illustrated earlier to solve this part of the problem.

Using the process explained above, we get average speed of the second half of the journey as

$$10 + 1 \times 5 = 15 \text{ kmph}$$

Then we employ the same technique for the first part and get

$$15 + 1 \times 9 = 24 \text{ kmph (Answer)}$$

Problem 2.7 A school has only 3 classes that contain 10, 20 and 30 students respectively. The pass percentage of these classes are 20%, 30% and 40% respectively. Find the pass percentage of the entire school.

Solution

Using weighted average $\dfrac{10 \times 0.2 + 20 \times 0.3 + 30 \times 0.4}{10 + 20 + 30}$

$$= \frac{20}{60} = 33.3\%$$

Alternatively, we can also use solving while reading as

Recognize that the pass percentage would be given by

$$\frac{\text{Passed students}}{\text{Total students}}$$

[illegible] is 2nd and was the second time on the quarterly [illegible] the first examination and get the total number of [illegible] = 2 + x + 15 and you are through with the [illegible]

Level of Difficulty (LOD) I

1. The average age of 24 students and the principal is 15 years. When the principal's age is excluded, the average age decreases by 1 year. What is the age of the principal?
   (a) 38 (b) 40
   (c) 39 (d) 32
   (e) Data inadequate

2. The average weight of 3 men A, B and C is 84 kg. Another man D joins the group and the average now becomes 80 kg. If another man E whose weight is 5 kg more than that of D replaces A then the average weight of B, C, D and E becomes 79 kg. The weight of A is
   (a) 70 kg (b) 72 kg
   (c) 74 kg (d) 78 kg
   (e) 80 kg

3. The mean temperature of Monday to Wednesday was 37 °C and of Tuesday to Thursday was 34 °C. If the temperature on Thursday was 4/5 that of Monday, the temperature on Thursday was
   (a) 28 °C (b) 36 °C
   (c) 40 °C (d) 30 °C
   (e) None of these

4. Three years ago the average age of A, B and C was 27 years and that of B and C 5 years ago was 20 years. A's present age is
   (a) 36 years (b) 35 years
   (c) 30 years (d) 40 years
   (e) None of these

5. A cricketer has a certain average for 9 innings. In the tenth inning he scores 100 runs thereby increasing his average by 5 runs. His new average is
   (a) 50 (b) 55 (c) 54 (d) 55
   (e) 56

6. The average of the first five multiples of 3 is
   (a) 10 (b) 20 (c) 30 (d) 40
   (e) 9

7. There are three fractions A, B and C. If $A = \frac{7}{4}$ and B is 1/6 and the average of A, B and C is 1/12. What is the value of C?
   (a) −1/2 (b) −1/6
   (c) −1/3 (d) −1/4
   (e) None of these

8. The marks obtained by Hare Rama in Mathematics, English and Biology are respectively 93 out of 100, 78 out of 150 and 177 out of 200. Find his average score in percent.
   (a) 87.83 (b) 86.83 (c) 76.33 (d) 75.33
   (e) 77.33

9. The average monthly expenditure of a family was Rs 2750 for the first 3 months, Rs 3150 for the next three months and Rs 6750 for the next three months. Find the average income of the family for the 9 months, if they save Rs 850 per month.
   (a) 4866.66 (b) 5123.33
   (c) 4666.66 (d) 4216.66
   (e) None of these

10. The average age of a family of 6 members is 22 years. If the age of the youngest member be 7 years, what was the average age of the family at the birth of the youngest member?
    (a) 15 (b) 18 (c) 20 (d) 12
    (e) None of these

11. The average age of 8 persons in a committee is increased by 2 years when two men aged 35 years and 45 years are substituted by two women. Find the average age of the two women.
    (a) 44 (b) 45 (c) 51 (d) 42
    (e) 48

12. The average temperature for Wednesday, Thursday and Friday was 40 °C. The average for Thursday, Friday and Saturday was 44 °C. If the temperature on Saturday was 42 °C, what was the temperature on Wednesday?
    (a) 36 °C (b) 44 °C (c) 38 °C (d) 41 °C
    (e) None of these

13. The speed of the train in going from Meerut to Allahabad is 100 km/hr while when coming back from Allahabad to Meerut, the speed is 150 km/hr. Find the average speed during the whole journey.
    (a) 123 (b) 75 (c) 133 (d) 140
    (e) 125

14. The average weight of a class of 20 students is 40 kg. If the weight of the teacher be included, the average rises by unit one. What is the weight of the teacher?
(a) 45.5 kg (b) 50.5 kg
(c) 45 kg (d) 55 kg
(e) 52 kg

15. The average of 3 numbers is 17 and that of the first two is 16. Find the third number.
(a) 15 (b) 18 (c) 19 (d) 20
(e) 18

16. The average weight of 10 men in a ship is increased by 3.7 kg when one of the men, who weighs 79 kg, is replaced by a new man. Find the weight of the new man upto 2 decimal places.
(a) 105.75 (b) 107.55
(c) 145.50 (d) 140.50
(e) None of these

17. The age of Shaurya and Kaustubh is in the ratio 2 : 6. After 5 years, the ratio of their ages will become 6 : 8. Find the average of their ages after 10 years.
(a) 32 (b) 15 (c) 17 (d) 24
(e) None of these

18. Find the average of the first 97 natural numbers.
(a) 47 (b) 57 (c) 48 (d) 49
(e) 49.5

19. Find the average of all prime numbers between 30 and 50.
(a) 39.8 (b) 36.8 (c) 37.8 (d) 42.8
(e) 40.8

20. If we take four numbers, the average of the first three is 24 and that of the last three is 15. If the last number is 18, the first number is
(a) 30 (b) 31 (c) 23 (d) 25
(e) 24

21. The average of 5 consecutive numbers is x. If the next two numbers are also included, the average will
(a) increase by 1 (b) remain the same
(c) increase by 1.4 (d) increase by 2
(e) None of these

22. The average of 50 numbers is 38. If two numbers, namely, 45 and 55 are discarded, the average of the remaining numbers is
(a) 36.5 (b) 37 (c) 37.6 (d) 38
(e) 37.5

23. The average of ten numbers is 7. If each number is multiplied by 12, then the average of the new set of numbers is
(a) 7 (b) 19 (c) 87 (d) 84
(e) 84

24. In a family of 8 males and a few ladies, the average monthly consumption of grain per head is 10.5 kg. If the average monthly consumption per head be 15 kg in the case of males and 6 kg in the case of females, find the number of females in the family.
(a) 8 (b) 7 (c) 9 (d) 15
(e) 48

25. Average marks obtained by a student in 3 papers is 51 and in the fourth paper he obtains 60 marks. Find his new average.
(a) 54 (b) 52 (c) 55 (d) 53.5
(e) 54.5

26. The average earning of Shambhu Nath Pandey during initial three months of the calendar year 2002 is Rs. 1200. If his average earning for the second and third month is Rs. 1300 find his earning in the first month.
(a) 900 (b) 1100 (c) 1000 (d) 1200
(e) 1300

27. In a hotel where rooms are numbered from 101 to 150, each room gives an earning of Rs. 3000 for the first fifteen days of a month and for the latter half, Rs. 2000 per room. Find the average earning per room per day over the month. (Assume 30 day month)
(a) 2250 (b) 2500 (c) 2750 (d) 2666.66
(e) 2483.33

28. The average weight of 5 men is decreased by 3 kg when one of them weighing 150 kg is replaced by another person. Find the weight of the new person?
(a) 165 kg (b) 135 kg
(c) 138 kg (d) 162 kg
(e) 165 kg

29. The average age of a group of men is increased by 7 years when a person aged 18 years is replaced by a new person of aged 38 years. How many men are there in the group?
(a) 3 (b) 4 (c) 5 (d) 6
(e) 7

30. The average score of a cricketer in three matches is 27 runs and in two other matches, it is 17 runs. Find the average in all the five matches.

(a) 20 (b) 10.6 (c) 21 (d) 19.5

(e) 20.5

32. The average of 11 papers is 40. The average of the first 5 papers is 42 and of the last seven papers is 35. Find the marks obtained in the 5th paper?

(a) 23 (b) 38

(c) 19 (d) 34

(e) None of these

33. The average age of the Indian cricket team playing the Asian test is 20. The average age of 5 of the players is 17 and that of another set of 5 players totally different from the first five, is 29. If it is the captain who was not included in either of these two groups, then find the age of the captain.

(a) 20 (b) 35

(c) 29 (d) 38

(e) Cannot be determined

34. Gokhale has earned an average of 4200 dollars for the first seven months of the year. If he satisfies his staying on in the US on the basis of his ability to earn at least 6000 dollars per month for the entire year, how much should he earn (in dollars) in the last month so as to achieve his required average for the whole year?

(a) 14800 (b) 3600 (c) 12800 (d) 13000

(e) 11600

35. A bus goes from Ranchi to Patna at the rate of 66 km per hour. Another bus leaves Ranchi for Patna at the same time as the first bus at the rate of 70 km per hour. Find the average speed for the journeys of the two buses, mentioned, if it is known that the distance from Ranchi to Patna is 420 kilometers.

(a) 64.615 km/h (b) 64.5 km/h

(c) 64.625 km/h (d) 64.83 km/h

(e) None of these

36. A man travels 8 km in the first quarter of an hour, 6 km in the second quarter and 40 km in the third quarter. Find the average speed of the man per hour over the entire journey.

(a) 32 km/h (b) 28 km/h

(c) 35.24 km/h (d) 36.5 km/h

(e) 30 km/h

37. The average weight of 6 men is 68 kg. If it is known that four men weigh 65 kg each, find the average weight of the others.

(a) 72.25 kg (b) 75 kg

(c) 78 kg (d) 80 kg

(e) None of these

37. The average score of a class of 40 students is 52. What will be the average score of the rest of the students if the average score of 10 of the students is 61

(a) 50 (b) 47 (c) 48 (d) 51

(e) 49

38. The average age of 80 students of IIM, Bangalore of the 2005 batch is 22 years. What will be the new average if we include the 20 faculty members whose average age is 37 years?

(a) 32 years (b) 24 years

(c) 28 years (d) 26 years

(e) None of these

39. Out of three numbers, the first is twice the second and three times the third. The average of the three numbers is 44. The smallest number is

(a) 72 (b) 36 (c) 42 (d) 48

(e) 50

40. The sum of three numbers is 98. If the ratio between the first and second is 2 : 3 and that between the second and the third is 5 : 8, then the second number is

(a) 30 (b) 20 (c) 58 (d) 48

(e) 32

41. The average height of 30 girls out of a class of 40 is 160 cm and that of the remaining girls is 156 cm. The average height of the whole class is

(a) 158 cm (b) 158.5 cm

(c) 159 cm (d) 157 cm

(e) 160 cm

42. The average weight of 8 persons is increased by 2.5 kg when one of them whose weight is 50 kg is replaced by a new man. The weight of the new man is

(a) 65 kg (b) 75 kg (c) 55 kg (d) 60 kg

(e) 66 kg

43. The average age of three boys is 15 years. If their ages are in the ratio 5 : 3 : 7, the age of the youngest boy is

(a) 21 years (b) 18 years

(c) 15 years (d) 9 years

(e) 12 years

44. The average age of A, B, C and D five years ago was 47 years. By including E, the present average age of all the five is 49 years. The present age of E is

(c) 44 years (d) 48 years
(e) 45 years
(f) None of these

45. The average salary of 20 workers in an office is Rs.1900 per month. If the manager's salary is added, the average salary increases Rs. 2000 per month. What is the manager's monthly salary?
(a) Rs.24,000 (b) Rs.25,200
(c) Rs.47,000 (d) Rs.44,000
(e) None of these

46. If a, b, c, d and e are five consecutive odd numbers, then their average is
(a) $2(a+b)$ (b) $(a+b+c+d+e)/5$
(c) $5(a+b+c+d+e)$ (d) $\frac{a+b}{5}$
(e) None of these

47. The average of first five multiples of 3 is
(a) 3 (b) 9 (c) 12 (d) 15
(e) None of these

48. The average weight of a class of 40 students is 40 kg. If the weight of the teacher be included, the average weight increases by 500 gm. The weight of the teacher is
(a) 40.5 kg (b) 60 kg
(c) 62 kg (d) 60.5 kg
(e) 64 kg

49. In a management entrance test, a student scores 2 marks for every correct answer and loses 0.5 marks for every wrong answer. A student attempts all the 100 questions and scores 126 marks. The number of questions he answered correctly was
(a) 30 (b) 45 (c) 60 (d) 68
(e) None of these

50. The average age of four children is 8 years, which is increased by 4 years when the age of the father is included. Find the age of the father
(a) 32 (b) 28 (c) 36 (d) 34
(e) 30

51. The average of the first ten natural numbers is
(a) 5 (b) 5.5 (c) 6.5 (d) 4

52. The average of the first ten whole numbers is
(a) 4.5 (b) 5 (c) 5.5 (d) 4

53. The average of the first ten even numbers is
(a) 10 (b) 22 (c) 9 (d) 11

54. The average of the first ten odd numbers is
(a) 11 (b) 10 (c) 12 (d) 9

55. The average of the first ten prime numbers is
(a) 15.5 (b) 12.5 (c) 10 (d) 12.9

56. The average of the first ten composite numbers is
(a) 12.9 (b) 11 (c) 11.2 (d) 9

57. The average of the first ten prime numbers, which are odd, is
(a) 12.0 (b) 13.8 (c) 17 (d) 15.8

58. The average weight of a class of 30 students is 40 kg. If, however, the weight of the teacher is included, the average becomes 41 kg. The weight of the teacher is
(a) 31 kg (b) 62 kg (c) 71 kg (d) 70 kg

59. Ram bought 2 toys for Rs. 5.50 each, 3 toys for Rs. 3.66 each and 6 toys for Rs. 1.833 each. The average price per toy is
(a) Rs. 3 (b) Rs. 10 (c) Rs. 5 (d) Rs. 8

60. 50 oranges and 75 apples were purchased for Rs. 18. If the price per apple was Rs. 2, then the average price of oranges was
(a) Rs. 12 (b) Rs. 14 (c) Rs. 10 (d) Rs. 11

61. The average income of Sambhu and Ganesh is Rs. 3,000 and that of Arun and Vinay is Rs. 500. What is the average income of Sambhu, Ganesh, Arun and Vinay?
(a) Rs. 1750 (b) Rs. 1250
(c) Rs. 1000 (d) Rs. 2500

62. A batsman made an average of 40 runs in 4 innings. In the fifth inning, he was out on zero. What is his average after fifth inning?
(a) 32 (b) 22 (c) 36 (d) 48

63. The average weight of a school of 40 teachers is 30 kg. If, however, the weight of the principal be included, the average decreases by 1 kg. What is the weight of the principal?
(a) 100 kg (b) 29 kg
(c) 40 kg (d) None of these

64. The average temperature of 1st, 2nd and 3rd December was 24.4 °C. The average temperature of the first two days was 24 °C. The temperature on the 3rd of December was
(a) 20 °C (b) 25 °C
(c) 28.2 °C (d) None of these

65. The average age of Ram and Shyam is 20 years. The average age 5 years hence will be

(a) 28 years (b) 22 years
(c) 24 years (d) 20 years

66. The average of 20 results is 30 and that of 30 more results is 20. For all the results taken together, the average is
(a) 25 (b) 50 (c) 15 (d) 54

67. The average of 5 consecutive numbers is 18. The highest of these numbers will be
(a) 20 (b) 19 (c) 20 (d) 22

68. The average of 6 students is 11 years. If 2 more students of age 14 and 15 years join, their average will become
(a) 12 years (b) 13 years
(c) 21 years (d) 19 years

69. The average of 8 numbers is 12. If each number is increased by 2, the new average will be
(a) 12 (b) 14 (c) 15 (d) 18

70. Three years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average age of the family is the same today. What is the age of the baby?
(a) 1 year (b) 2 years
(c) 9 months (d) 9 months

71. Sunitha's average daily expenditure is Rs. 10 during May, Rs. 14 during June and Rs. 15 during July. Her approximate daily expenditure for the 3 months is
(a) Rs. 13 approximately (b) Rs. 12
(c) Rs. 12 approximately (d) Rs. 10

72. A ship sailed out to a reach at the rate of 15 km per hour and sailed back at the rate of 20 km/h. What is its average rate of sailing?
(a) 16.80 km (b) 17.14 km
(c) 17.80 km (d) 18 km

73. The average temperature on Monday, Tuesday and Wednesday was 41 °C and on Tuesday, Wednesday and Thursday it was 40 °C. If on Thursday it was exactly 30 °C, then on Monday, the temperature was
(a) 42 °C (b) 46 °C (c) 25 °C (d) 26 °C

74. The average of 20 results is 30 out of which the first 10 results are having an average of 30. The average of the last 10 results is
(a) 30 (b) 40 (c) 20 (d) 35

75. A man had seven children. When their average age was 11 years a child aged 6 years died. The average age of the surviving children is
(a) 8 years (b) 13 years
(c) 15 years (d) 15 years

76. The average income of Ram and Shyam is Rs. 200. The average income of Rohit and Rohit is Rs. 250. The average income of Ram, Shyam, Rohit and Rohit is
(a) Rs. 275 (b) Rs. 225
(c) Rs. 450 (d) Rs. 250

77. The average weight of 35 students is 35 kg. If the teacher is also included, the average weight increases to 36 kg. The weight of the teacher is
(a) 36 kg (b) 71 kg (c) 70 kg (d) 45 kg

78. The average of x, y and z is 45. x is as much more than the average as y is less than the average. Find the value of z.
(a) 45 (b) 25 (c) 35 (d) 15

79. Find the average of four numbers $2\frac{3}{4}, 4\frac{1}{3}, 4\frac{1}{6}, 8\frac{1}{2}$
(a) $5\frac{3}{16}$ (b) $3\frac{1}{16}$ (c) $16\frac{5}{3}$ (d) $3\frac{16}{5}$

80. The average salary per head of all the workers in a company is Rs. 95. The average salary of 15 officers is Rs. 525 and the average salary per head of the rest is Rs. 85. Find the total number of workers in the workshop.
(a) 600 (b) 500 (c) 650 (d) 400

81. The average age of 8 men is increased by 2 years when one of them whose age is 24 years is replaced by a woman. What is the age of the woman?
(a) 35 years (b) 28 years
(c) 32 years (d) 40 years

82. The average monthly expenditure of Ravi was Rs. 1100 during the first 3 months, Rs. 2200 during the next 4 months and Rs. 4620 during the subsequent five months of the year. If the total saving during the year was Rs. 2100, find Ravi's average monthly income.
(a) Rs. 1850 (b) Rs. 1636.33
(c) Rs. 1100 (d) None of these

83. Sujan bought 2 articles for Rs. 5.50 each, and 2 articles for Rs. 3.50 each and 3 articles for Rs. 8.50 each and 5 articles for Rs. 1.50 each. The average price for one article is
(a) Rs. 4 (b) Rs. 3.10
(c) Rs. 8.50 (d) Rs. 2

84. In a bag, there are 150 coins of Rs. 1, 50 p and 25 p denominations. If the total value of coins is Rs. 150, then find how many times can be constituted by 50 p coins.
(a) 30 (b) 20
(c) 20 (d) None of these

Level of Difficulty (LOD) II

1. With an average speed of 40 km/h, a train reaches its destination in time. If it goes with an average speed of 30 km/h, it is late by 15 minutes. The length of the total journey is:
   (a) 40 km (b) 70 km (c) 80 km (d) 90 km
   (e) 100 km

2. In the month of July of a certain year, the average daily expenditure of an organisation was Rs. 68. For the first 15 days of the month, the average daily expenditure was Rs. 85 and for the last 17 days, Rs. 51. Find the amount spent by the organisation on the 15th of the month.
   (a) Rs. 42 (b) Rs. 36
   (c) Rs. 34 (d) Rs. 52
   (e) Rs. 40

3. In 1919, W. Rhodes, the Yorkshire cricketer, scored 891 runs for his county in an average of 34.27, in 1920, he scored 849 runs at an average of 28.25, in 1921, 1329 runs at an average of 42.87 and in 1922, 1101 runs at an average of 36.70. What was his county batting average for the four years?
   (a) 36.23 (b) 37.01 (c) 35.98 (d) 28.72
   (e) None of these

4. A train travels with a speed of 20 km/h in the first 10 minutes, goes 8.5 km in the next 10 minutes, 13 km in the next 10, 8.5 km in the next 10 and 6 km in the next 10 minutes. What is the average speed of the train in kilometre per hour for the journey described?
   (a) 42 km/h (b) 35.8 km/h
   (c) 55.2 km/h (d) 46 km/h
   (e) 52 km/h

5. One fourth of a certain journey is covered at the rate of 25 km/h, one third at the rate of 30 km/h and the rest at 50 km/h. Find the average speed for the whole journey.
   (a) 40.05 km/h (b) 1300/53 km/h
   (c) 1800/53 km/h (d) 1600/53
   (e) None of these

6. There is one type a chart in 6 minutes, type b in 9 minutes and type c in 8 minutes. The average number of charts typed per hour per typist for all three types is
   (a) 20/3 (b) 30/43
   (c) 65/69 (d) 530/53
   (e) None of these

7. Find the average increase rate of increase in the price below in the first year is 30% and that in the second year is 40%.
   (a) 41 (b) 36 (c) 40 (d) 38
   (e) 39

8. The average income of a person for the first 6 days is Rs. 29, for the next 6 days it is Rs. 24, for the next 12 days it is Rs. 32 and for the remaining days of the month it is Rs. 36. Find the average income per day.
   (a) Rs. 31.64 (b) Rs. 30.65
   (c) Rs. 29.28 (d) Rs. 31.22
   (e) Cannot be determined

9. In hotel Jayaramia, the rooms are numbered from 101 to 130 on the first floor, 221 to 280 on the second floor and 300 to 345 on the third floor. In the month of June 2002, the room occupancy was 60% on the first floor, 85% on the second floor and 95% on the third floor. If it is also known that the room charges are Rs. 200, Rs. 300 and Rs. 250 on each of the floors, then find the average income per room for the month of June 2002.
   (a) Rs. 151.5 (b) Rs. 88.18
   (c) Rs. 78.3 (d) Rs. 65.7
   (e) None of these

10. A salesman gets a bonus according to the following structure: If he sells articles worth Rs. x then he gets bonus of Rs. $(x/100 - 1)$. In the month of January the sales value was Rs. 300, in February it was Rs. 200 and March to November it was Rs. 500 for every month and in December it was Rs. 1200. Apart from this, he also receives a basic salary of Rs. 30 per month from his employer. Find his average income per month during the year.
   (a) Rs. 31.25 (b) Rs. 30.34
   (c) Rs. 32.5 (d) Rs. 34.5
   (e) Rs. 33.5

11. A man covers half of his journey by train at 60 km/h, half of the remainder by bus at 30 km/h and the rest by car at 30 km/h. Find his average speed during the whole journey.
   (a) 36 km/h (b) 30 km/h
   (c) 34 km/h (d) 18 km/h
   (e) 42 km/h

12. The average weight of 3 men is decreased by 2 kg when one of them weighing 150 kg is replaced by another person. This new person is again replaced by another person whose weight is 30 kg lower than the person he replaced. What is the overall change in the average due to this shift change?

(a) 4 kg (b) 8 kg (c) 12 kg (d) 15 kg

(e) 18 kg

13. Find the average weight of four containers, if it is known that the weight of the first container is 100 kg and the rest of the second, third and fourth containers' weight is defined by $f(x) = x^2 - 2x(x^2)$ where $x = 100$

(a) 850 kg (b) 900 kg

(c) 750 kg (d) 450 kg

(e) 600 kg

14. There are five boxes in a cargo hold. The weight of the first box is 200 kg and the weight of the second box is 20% higher than the weight of the third box, whose weight is 25% higher than the first box's weight. The fourth box is 200 kg is 30% lighter than the fifth box. Find the difference in the average weight of the four heaviest boxes and the four lightest boxes.

(a) 52.5 kg (b) 75 kg

(c) 37.5 kg (d) 112.5 kg

(e) None of these

15. Per question 14, find the difference in the average weight of the heaviest three and the lightest three.

(a) 131.25 kg (b) 125 kg

(c) 150 kg (d) 112.5 kg

(e) None of these

16. A batsman makes a score of 270 runs in the 17th innings and thus increases his average by a certain number of runs that is a whole number. Find the possible values of the new average.

(a) 18 (b) 19

(c) 21 (d) All of these

17. 10 persons went to a hotel for a combined dinner party. 10 plates upon Rs. 70 each on their dinner and the rest gave Rs. 4 more than the average expenditure of all the 10. What was the total money spent by them?

(a) 1234 (b) 1344

(c) 1402 (d) 1406

(e) None of these

18. There were 42 students in a hostel. Due to the admission of 11 new students, the expenses of the mess increase by Rs. 11 per day while the average expenditure per head diminished by Rs. 3. What was the original expenditure of the mess?

(a) Rs. 631.23 (b) Rs. 583.3

(c) Rs. 623.3 (d) Rs. 633

(e) None of these

19. The average price of 3 precious diamond studded platinum thrones is Rs. 976104983.3 if their prices are in the ratio 4:7:9. The price of the cheapest is:

(a) 5, 85, 06, 298.972 (b) 5, 85, 06, 29, 897.2

(c) 58, 56, 62, 889.72 (d) 58576298 17.2

(e) None of these

20. The average weight of 47 balls is 4 gm. If the weight of the bag (in which the balls are kept) be included, the calculated average weight per ball increases by 0.3 gm. What is the weight of the bag?

(a) 14.8 gm (b) 15.6 gm

(c) 18.6 gm (d) 18 gm

(e) None of these

21. The average of 71 results is 48. If the average of the first 59 results is 46 and that of the last 11 is 52. Find the 60th result.

(a) 132 (b) 122 (c) 134 (d) 128

(e) 136

22. A man covers 1/3rd of his journey by cycle at 50 km/h, the next 1/3 by car at 30 km/h, and the rest by walking at 7 km/h. Find his average speed during the whole journey.

(a) 14.2 km/h (b) 15.3 km/h

(c) 18.2 km/h (d) 12.8 km/h

(e) 16.2 km/h

23. The average age of a group of 14 persons is 27 years and 9 months. Two persons, each 42 years old, left the group. What will be the average age of the remaining persons in the group?

(a) 26.875 years (b) 26.25 years

(c) 25.375 years (d) 25 years

(e) 27 years

24. In an exam, the average was found to be x marks. After deducting compensatory error, the average marks of 84 candidates got reduced from 44 to 44. The average thus came down by 10.8 marks. The number of candidates who took the exam was:

(a) 180 (b) 160 (c) 190 (d) 105
(e) 130

25. The average salary of the entire staff is an office is Rs. 3000 per month. The average salary of officers is Rs. (x+6) and that of non officers is Rs. 3000. If the number of officers is 5, then find the number of non-officers in the office?
(a) 8 (b) 12 (c) 15 (d) 7
(e) 10

26. A person travels three equal distances at a speed of x km/h, y km/h and z km/h respectively. What will be the average speed during the whole journey?
(a) $xyz/(xy + yz + zx)$ (b) $(xy + yz + zx)/3xyz$
(c) $3xyz/(xy + yz + zx)$ (d) None of these

Directions for Questions 27–30: Read the following passage and answer the questions that follows.

In a family of five persons A, B, C, D and E, each and everyone loves one another very much. Their birthdays are in different months and on different dates. A remembers that his birthday is between 25th and 30th, of B it is between 20th and 25th, of C it is between 10th and 20th, of D it is between 5th and 10th and of E it is between 1st to 5th of the month. The sum of the date of birth is defined as the addition of the date and the month. For example 12th January will be written as 1.12 and will add to a sum of the date of 13. (Between 25th and 30th includes both 25 and 30).

27. What may be the minimum average of their sum of the dates of birth?
(a) 24.6 (b) 15.2
(c) 18 (d) 32

28. What may be the maximum average of their sum of the dates of birth?
(a) 24.6 (b) 15.2
(c) 28 (d) 32

29. If it is known that the dates of birth of three of them are even numbers then find minimum average of their sum of the dates of birth.
(a) 12.6 (b) 15.2
(c) 21.6 (d) 28

30. If the dates of birth of four of them are prime numbers, then find the maximum average of the sum of their dates of birth.
(a) 27.2 (b) 24.6
(c) 28 (d) None of these

31. The average age of a group of persons going for a picnic is 16.75 years. 20 new persons with an average age of 13.25 years join the group on the spot due to which the average of the group becomes 15 years. Find the number of persons initially going for the picnic?
(a) 24 (b) 20 (c) 15 (d) 18

32. A school has only four classes that contain 10, 20, 30 and 40 students respectively. The pass percentage of these classes are 20%, 30%, 60% and 100% respectively. Find the pass % of the entire school.
(a) 56% (b) 70% (c) 34% (d) 66%

33. Find the average of $f(x)$, $g(x)$, $h(x)$, $d(x)$ (or $x = 10$, $f(x)$ is equal to $x^2 + 2$, $g(x) = 5x^2 - 3$, $h(x) = \log x^2$ and $d(x) = (4/5)x^2$
(a) 170 (b) 170.25 (c) 70.25 (d) 70

34. Find the average of $f(x) - g(x)$, $g(x) - h(x)$, $h(x) - d(x)$, $d(x) - f(x)$
(a) 0 (b) −2.25 (c) 4.5 (d) 2.25

35. $\sum_{i=1}^{n} (i+1)i$ where $i = n$.
(a) $\frac{(n-1)(n)(n+1)}{2}$ (b) $\frac{n(n+1)^2}{2}$
(c) $\frac{n(n-1)^2}{2}$ (d) $\frac{n^2}{2}$

36. The average of 'n' numbers is 'z'. If the number 'x' is replaced by the number x^3, then the average become z^3. Find the relation between n, z, z^3, x and x^3
(a) $\left[\frac{z^3-2}{x^3-x}=\frac{1}{n}\right]$ (b) $\left[\frac{x^3-x}{z^3}=\frac{1}{n}\right]$
(c) $\left[\frac{z-z^3}{x-x^3}=\frac{1}{n}\right]$ (d) $\left[\frac{x-x^3}{z-z^3}=\frac{1}{n}\right]$

37. The average salary of workers in AMS centre is Rs. 2,000, the average salary of faculty being Rs. 4,000 and the management trainees being Rs. 1,250. The total number of workers could be
(a) 420 (b) 300 (c) 110 (d) 500

Directions for Questions 38–41: Read the following and answer the questions that follows.

During a cricket match, India playing against NZ scored in the following manner:

Partnership	*Runs scored*
1st wicket	112

[illegible]	78
[illegible]	72
[illegible]	92
[illegible]	46
[illegible]	20

8. Find the average runs scored by the first four batsmen.
   (a) 50.5 (b) 40.5
   (c) 60 (d) Cannot be determined

9. The minimum average runs scored by the last five batsmen could be
   (a) 30.8 (b) 66.8
   (c) 76 (d) Cannot be determined

10. The maximum average runs scored by the last five batsmen is not exceed be
    (a) 114 (b) 44.4 (c) 66.8 (d) 0

11. If the fifth down batsman gets out for a duck, then find the average runs scored by the first six batsmen.
    (a) 37.1 (b) 63.3
    (c) 46.5 (d) Cannot be determined

12. The weight of a body as calculated by the average of 3 different experiments is 53.733 gm. The average of the first two experiments is 54.005 gm, of the fourth is 0.008 gm greater than the fifth, which the average of the sixth and seventh experiment was 0.010 gm less than the average of six first three. Find the weight of the body obtained by the fourth experiment.
    (a) 49.351 gm (b) 51.712 gm
    (c) 50.02 gm (d) 54.512 gm

13. A man's average expenditure for the first 4 months of the year was Rs. 295.25. For the next 5 months the average monthly expenditure was Rs. 26.27 more than what it was during the first 4 months. If the person spent Rs. 200 in all during the remaining 3 months of the year, find what percentage of his annual income of Rs. 3000 he saved in the year.
    (a) 14% (b) –2.08%
    (c) 11.5% (d) None of these

14. A certain number of trucks were required to transport the sum of coal were from the TISCO factory to Jamshedpur. However, it was found that given each truck could take 0.5 tons of cargo less, another 4 trucks were needed. How many trucks were actually planned to be used?
    (a) 8 (b) 10 (c) 20 (d) 2

15. The collective farm got an average harvest of 21 tons of wheat and another collective farm that had 12 acres of land less given to wheat, got 25 tons from a hectare. At a month, the second farm harvested 300 tons of wheat more than the first. How many tons of wheat did each farm harvest?
    (a) 3150, 3450 (b) 3250, 3550
    (c) 2150, 2450 (d) None of these

Level of Difficulty (LOD) III

Directions for (Questions 1–28). Read the following:

There are 3 classes having 20, 25 and 30 students respectively having average marks in an examination as 20, 25 and 30 respectively. If the three classes are represented by A, B and C and you have the following information about the three classes, answer the questions that follow:

$A \rightarrow$ Highest score 27, Lowest score 18
$B \rightarrow$ Highest score 31, Lowest score 23
$C \rightarrow$ Highest score 33, Lowest score 26

If five students are transferred from A to B.

1. What will happen to the average score of B?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say
2. What will happen to the average score of A?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say

In a number of 5 students from A to C

3. What will happen to the average score of C?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say
4. What will happen to the average score of A?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say

In a number of 5 students from B to C (Questions 5–6)

5. What will happen to the average score of C?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say
6. Which of these can be used about the average score of B?
   (a) Increases if C decreases

(b) Increases if C increases
(c) Increases if C decreases
(d) Decreases if C decreases
(e) Cannot say

5. In a transfer of 5 students from A to B, the maximum possible average achievable for group B is

(a) 25 (b) 24.5
(c) 26.5 (d) 24
(e) Cannot be determined

6. For the above case, the maximum possible average achieved for group A will be

(a) 20.66 (b) 21.5
(c) 20.75 (d) 20.5
(e) Cannot be determined

7. What will be the minimum possible average of Group A if 5 students are transferred from A to B?

(a) 19.55 (b) 21.5
(c) 19.22 (d) 20.5
(e) Cannot be determined

8. If 5 students are transferred from B to A, what will be the minimum possible average of A?

(a) 20.66 (b) 21
(c) 20.75 (d) 20.6
(e) 20.2

11. For question 10, what will be the maximum average of A?

(a) 20.2 (b) 22.2
(c) 18.75 (d) 19
(e) Cannot be determined

Directions for Questions 12–17: Read the following and answer the questions that follow.

If 5 people are transferred from A to B and another independent set of 5 people are transferred back from B to A, then after this operation (Assume that the set transferred from B to A contains some from the set of students that came to B from A)

12. What will happen to B's average?

(a) Increase if A's average decreases
(b) Increases always
(c) Cannot be said
(d) Increase if A's average decreases
(e) Increase if A's average increases

13. What can be said about A's average?

(a) Will decrease
(b) Will always increase if B's average changes
(c) May increase or decrease
(d) Will increase only if B's average decreases
(e) Will decrease only if B's average increases

14. At the end of the 2 steps mentioned above (in the direction) what could be the maximum value of the average of class B?

(a) 25.4 (b) 25 (c) 24.8 (d) 24.6
(e) 25.2

15. For question 14, what could be the minimum value of the average of class B?

(a) 22.4 (b) 24.2 (c) 25 (d) 27
(e) 23

16. What could be the maximum possible average achieved by class A at the end of the operation?

(a) 25.2 (b) 26 (c) 23.25 (d) 23.75
(e) None of these

17. What could be the minimum possible average of class A at the end of the operation?

(a) 21.4 (b) 19.2 (c) 28.5 (d) 20.25
(e) 20.4

Directions for Questions 18–23: Read the following and answer the questions that follow.

If 5 people are transferred from C to B, further 5 more people are transferred from B to A, then 5 are transferred from A to B and finally, 5 more are transferred from B to C.

18. What is the maximum possible average achieved by class C?

(a) 30.833 (b) 30
(c) 29.66 (d) 30.66
(e) Cannot say

19. What is the maximum possible average of class B?

(a) 26 (b) 27
(c) 25 (d) 28
(e) Cannot say

20. What is the maximum possible average value achieved by class A?

(a) 22.75 (b) 23.75
(c) 23.5 (d) 24
(e) Cannot say

21. The minimum possible value of the average of group A is

(a) 26.1 (b) 27.5

(c) 25.5 (d) 26

(e) Cannot say

27. The minimum possible average of group B after the set of operations is

(a) 23.5 (b) 21.4

(c) 20.5 (d) 20.2

(e) Cannot say

28. The minimum possible average of group A after the set of 3 operations is

(a) 20 (b) 20.2

(c) 20.4 (d) 19.8

(e) Cannot say

29. Which of these will definitely not constitute an operation for getting the minimum possible average value for group A

(a) Transfer of five 3's from B to A

(b) Transfer of five 2's from C to B

(c) Transfer of five 12's from A to B

(d) Transfer of five 10's from C to B

(e) None of these

30. For getting the lowest possible value of C's average, the sequence of operations could be

(a) Transfer five 12's from C to B, five 2's from B to A, five 10's from A to B, five 10's from B to C

(b) Transfer five 12's from C to B, 3's from B to A

(c) Both a and b

(d) None of the above

(e) Cannot say

31. If we set the highest possible average of class C as the primary objective and want to achieve the highest possible value for class B as the secondary objective, what is the maximum value of class B's average that is attainable?

(a) 27 (b) 26 (c) 25 (d) 24

(e) 23

32. For question 31, if the secondary objective is changed to achieving the minimum possible average value of class B's average, the lowest value of class B's average that could be attained is

(a) 20.5 (b) 23 (c) 22.5 (d) 22

(e) 21

33. For question 31, what can be said about class A's average?

(a) Will be determined automatically at 22.25

(b) Will have a minimum possible value of 22.25

(c) Will have a minimum possible value of 22.25

(d) Will be determined automatically at 22.5

(e) None of these

34. A team of miners planned to mine 1400 tons of ore during a certain number of days. Due to technical difficulties in one-third of the planned number of days, the team was able to achieve an output of 20 tons of ore less than the planned output. To make up for this, the team overachieved for the rest of the days by 20 tons. The end result was that the team completed the task one day ahead of time. How many tons of ore did the team initially plan to ore per day?

(a) 50 tons (b) 120 tons

(c) 150 tons (d) 200 tons

(e) 250 tons

35. According to a plan, a team of agriculturists decided to harvest 216 km^2 of wheat in several days. In the first three days, the team fulfilled the daily assignments, and then it harvested 8 km^2 of wheat more and above the plan everyday. Therefore, a day before the planned date, they had already harvested 232 km^2 of wheat. How many extra tonnes of wheat a day did the team have to cut according to the plan?

(a) 12 (b) 10 (c) 24 (d) 25

(e) 26

36. On an average, two liters of milk and one liter of water are needed to be mixed to make 1 kg of sodha shorthand of type A, and 1 liters of milk and 2 liters of water are needed to be mixed to make 1 kg of sodha shorthand of type B. How many kilograms of each type of shorthand was manufactured if it is known that 80 liters of milk and 80 liters of water were used?

(a) 25 of type A and 30 of type B

(b) 30 of type A and 20 of type B

(c) 35 of type A and 30 of type B

(d) 30 of type A and 35 of type B

(e) None of these

37. There are 500 seats in Meenra Cinema, Mumbai, placed in similar rows. After the moviegoers enter, of the hall, the total number of seats became 10% less. The number of rows was reduced by 5 but each row contained 5 seats more than before. How many rows and how many seats in a row were there initially in the hall?

(a) 20 rows and 25 seats

(b) 20 rows and 30 seats

(c) 10 men and 30 women
(d) 50 men and 10 women
(e) 40 men and 20 women

33. One fashion house has to make 810 dresses and another one 600 dresses during the same period of time. In the first house, the order was ready 3 days ahead of time and in the second house, 6 days ahead of time. How many dresses did each fashion house make a day if the second house made 21 dresses more a day than the first?

(a) 54 and 75 (b) 24 and 48
(c) 44 and 68 (d) 04 and 25
(e) None of these

34. A shop sold 64 kettles of two different capacities. The smaller kettle cost a rupee less than the larger one. The shop made 300 rupees from the sale of large kettles and 36 rupees from the sale of small ones. How many kettles of either capacity did the shop sell and what was the price of each kettle?

(a) 20 kettles for 2.5 rupees each and 14 kettles for 1.5 rupees each
(b) 40 kettles for 4.5 rupees each and 24 kettles for 2.5 rupees each
(c) 40 kettles for 2.5 rupees each and 24 kettles for 1.5 rupees each
(d) either a or b
(e) None of these

35. An enterprise got a bonus and decided to share it in equal parts between the exemplary workers. It turned out, however, that there were 3 more exemplary workers than it had been assumed. In that case, each of them would have got 4 rupees less. The administration had found the possibility to increase the total sum of the bonus by 90 rupees and so a month each exemplary worker got 20 rupees. How many people got the bonus?

(a) 8 (b) 28 (c) 4 (d) 16
(e) 20

Directions for Questions 36–38: Read the following and answer the questions that follows.

In the island of Hoola Boola Moola, the inhabitants have a strange process of calculating their average incomes and expenditures. According to the old legend, prevalent on that island, the average monthly income had to be calculated on the basis of 14 months in a calendar year while the average monthly expenditure was to be calculated on the basis of 9 months per year. This would lead to people having an underestimation of their savings since there would be an underestimation of the income and an overestimation of the expenditure per month.

36. If the minister for economic affairs decides to reverse the process of calculation of average income and average expenditure, what will happen to the estimated savings of a person living on Hoola Boola Moola island?

(a) It will increase
(b) It will decrease
(c) It will remain constant
(d) Will depend on the value

37. If it is known that Mr. Magoo Hoola Boola estimates his savings at 10 Moolahs and if it is further known that his actual expenditure is 288 Moolahs in an year (Moolahs, for those who are not aware, is the official currency of Hoola Boola Moola), then what will happen to his estimated savings if he suddenly calculates on the basis of a 12 month calendar year?

(a) Will increase by 5
(b) Will increase by 15
(c) Will increase by 10
(d) Will triple

38. Mr. Boogie Woogie comes back from the USA to Hoola Boola Moola and convinces his community comprising 546 families to start calculating the average income and average expenditure on the basis of 11 months per calendar year. Now if it is known that the average estimated income on the island is (according to the old system) 57 Moolahs per month, then what will be the change in the average estimated savings for the island of Hoola Boola Moola. (Assume that there is no other change).

(a) 251.40 Moolahs (b) 565.5 Moolahs
(c) 625.5 Moolahs (d) Cannot be determined

39. Mr Boogie Woogie comes back from the USSR and convinces his community comprising 273 families to start calculating the average income on the basis of 11 months per calendar year. Now if it is known that the average estimated income in his community (according to the old system) 87 Moolahs per month, then what will be the change in the average estimated savings for the island of Hoola Boola Moola. (Assume that there is no other

(c) 291.00 Mandate (d) 282.75 Mandate
(e) 312.75 Mandate (e) Cannot be determined

Directions for Questions 40–44: Read the following and answer the questions that follows.

The Indian cricket team has to score 360 runs on the last day of a test match in 90 overs, to win the test match. This is the target set by the opposing captain Brian Lara after he declared his innings closed at the overnight score of 411 for 7.

The Indian team coach has the following information about the batting rates (in terms of runs per over) of the different batsmen:

Assume that the run rate of a partnership is the weighted average of the individual batting rates of the batsmen involved in the partnership (on the basis of the ratio of the strike each batsman gets, i.e. the run rate of a partnership is defined as the weighted average of the run rates of the two batsmen involved weighted by the ratio of the number of balls faced by each batsman).

Since decimal fractions of runs are not possible for any batsman, assume that the estimated runs scored by a batsman in an innings (on the basis of his run rate and the number of overs faced by him) is rounded off to the next higher integer immediately above the estimated value of the runs scored during the innings.

For example, if a batsman scores at an average of 3 runs per over for 2.1666 overs, then he will be estimated to have scored 2.1666 × 3 = 6.5 runs in his innings, but since this is not possible, the actual number of runs scored by the batsman will be taken as 7 (the next higher integer above 6.5).

Also, this rounding off can take place only once for one innings of a batsman.

Runs scored per over in different batting styles

Name of Batsman	Defensive	Normal	Aggressive
Das	3	4	5
Dasgupta	3	4	4
Dravid	2	3	4
Tendulkar	4	6	8
Laxman	4	5	6
Sehwag	4	5	6
Ganguly	3	4	5
Kaif	2	3	4
Harbhajan	3	4	5
Irfan	3	4	5
Yuvraj	3	3	4

Assume no extras (unless otherwise stated).

Assume that the strike is equally shared unless otherwise stated.

40. If the first wicket pair of Das and Dasgupta bats for 22 overs and during this partnership Das has started hesitating normally and turned aggressive after 15 overs while Dasgupta started off defensively but shifted gears to bat normally after batting for 20 overs, find the expected score after 22 overs.
(a) 65 (b) 71 (c) 82 (d) 58

41. Of the first wicket partnership between Das and Dasgupta as per the previous question, the ratio of the number of runs scored by Das to those scored by Dasgupta is
(a) 46 : 25 (b) 96: 46
(c) 41 : 32 (d) Cannot be determined

42. The latest time by which Tendulkar can come to bat and still win the game, assuming that the run rate at the time of his walking the wicket is into 2.5 runs per over, (assuming he shares strike equally with his partner and that he gets the maximum possible support at the other end from his batting partner and both play till the last ball).
(a) After 50 overs (b) After 55 overs
(c) After 60 overs (d) Cannot be determined

43. For question 42, where Tendulkar batted aggressively and assuming that it is the Tendulkar-Laxman pair that wins the game for India (after Tendulkar walks into bat with the current run rate at 2.5 per over, and at the latest possible time for him to win the game with maximum possible support from the opposite end), what will be Tendulkar's score for the innings (assume equal str. e)?
(a) 105 (b) 120
(c) 135 (d) None of these

44. For questions 42 and 43, if it was Laxman who batted with Tendulkar for his entire innings, then how many runs would Laxman score in the innings?
(a) 105 (b) 75
(c) 90 (d) Cannot be determined

Directions for Questions 45–49: Read the following and answer the questions that follow.

If Sachin Tendulkar walks into bat after the fall of the fifth wicket and has to share partnerships with Ganguly, Kumble, Harbhajan, Irfan and Yuvraj, who have batted normally, defensively, aggressively, defensively and aggressively respectively while Tendulkar has batted normally.

aggressively, aggressively, aggressively and aggressively respectively in each of the five partnerships that lasted for 12, 10, 8, 5 and 3 years respectively, sharing strike equally with Ganguly and holding two-thirds of the strike in his other four partnerships, then answer the following questions:

45. How many runs did Sachin score during his innings?
(a) 119 (b) 232
(c) 126 (d) None of these

46. The highest partnership that Tendulkar shared in was worth:
(a) 66 (b) 93 (c) 42 (d) 54

47. The three partnerships Kaif shared with:
(a) Ganguly (b) Yuvraj
(c) Raina (d) All three
(e) Only b and c

48. If India proceeded to win the match based on the runs scored by these last five partnerships (assuming the last wicket pair not scored anything), what could be the maximum score at which Tendulkar (only) have come to bat:
(a) 103 for 5 (b) 97 for 5
(c) 100 for 5 (d) 104 for 5

49. For question 48, what could be the minimum score at which Tendulkar could have come to bat:
(a) 103 for 5 (b) 92 for 5
(c) 104 for 5 (d) 98 for 5

Hints and Solutions II

1. The train needs to travel 15 minutes extra @ 35 kmph. Hence, it is behind by 8.75 kms. The rate of losing distance is 7 kmph. Hence, the train must have travelled for 8.75/7 = 1 hour 45 minutes. @ 40 kmph → 70 kms.
Alternatively, you can also see that 22.5% drop in speed results in 14.28% increase in time. Hence, total time required is 105 minutes @ 40 kmph → 70 kilometers.
Alternatively, solve through options.

2. Standard question requiring speed calculation speed. Obviously, the 15th day is being double counted. Calculations can be reduced by thinking as:
Surplus in first 15 days = Deficit in last 15 days = 293 − 264 = the deficit of 29. This means that the average is reducing by 34 due to the double counting of the 15th day. This can only mean that the 15th day expenditure is Rs. 68 − 34 = 34.
(Lengthy calculations would have yielded the following calculations:
35*15 + 51*15 − 68*31 = 34)

3. Find out the number of innings in each year. Then the answer will be given by:
$$\frac{\text{Total runs in 4 years}}{\text{Total innings in 4 years}} = (4270/119 = 35.88)$$

4. Find the total distance covered in each segment of 30 minutes. You will get total distance = 46 kilometers in 50 mins.

5. Assume that the distance is 120 km. Hence, 30 km is covered @ 25 kmph, 40 @ 30 kmph and so on. Then average speed is 120/total time.

6. In three hours the total number of sheets typed will be 6009 + 6007 + 6019 = 10 + 8.33 + 6.66.
Hence the number of sheets/hour is 25.2/3 = 8.4 is equivalent to 5300/3.

7. 106 → 130 → 182. Hence, 82/2 = 41.

8. You do not know the number of days in the month. Hence, the question cannot be answered.

9. The number of rooms is 18 + 16 + 30 in the three lots respectively.
Total revenues are: 18*2 + 16*1 + 30*1.5 = 184 + 1.515625 → 151.5625.
Note here that if you could visualize here that since the number of rooms is 64 the decimal value could be .5 or .7 which effectively means that options 3 and 4 are rejected.

10. Replace x with the sales value to calculate the profit in a month.

11. Use the same process as Q. No. 5 above.

12. The weight of the second man is 135 and that of the third is 105. Hence, net result is a drop of 45 for 5 people. Hence, 9 kg is the drop.

13. Put x = 100 to get the weight of the containers. Use these weights to find average weight is 2600/4 = 650.

14. The weight of the boxes are 1st box → 200, 3rd box → 250 kg, 2nd box → 300 kg, 4th box → 350 and 5th box → 500 kg. Hence difference between the heaviest and the lighter it is 300. Hence, difference in the average is 75.

15. Difference between heaviest three and lightest three loads is: (350 + 500) − (100 + 200) = 350.
Difference in average weights is 350/3 = 116.66

30. Half of the runs scored in the 47th innings will go towards increasing the average of the first 46 innings to the new average and the remaining half of the runs will go towards increasing the new average for the 47th innings. The only constraint in this problem is that there is an increase in the average by a whole number of runs. This is possible for all three options.
31. Answer: (c) is the average expenditure of 19 people. Thus, $19x = 13*29 + 6(x + 4)$.
32. The average weight per ball is asked. Hence, the bag does not have to be counted in the 48th item.
33. $71*48 + 79*46 = x + 11*52 \rightarrow x = 72$.
Alternately, this can be solved by using the concepts of surpluses and deficits as:
2*59 (deficit) – 4 * 11(surplus) + 48 (average to be maintained by the 60th member) = 118-44 + 48 = 122.
34. Solve through the same process as the Q. No. 5 of this chapter.
35. $(16*333 – 2*504)/12$.
36. $\frac{(34-64)*98}{16.4}$
37. Use allligation to solve. 20———32———66. Thus, 5 corresponds to 12, hence for 36 the answer will be 15.
38. Let the equal distances be 'x' each. Then $3x/3x + x/y + x/z = 3x/x/x + y + z$.
39-40. You have to take between 25th and 30th to mean that both these dates are also included.
37. The maximum average will occur when the minimum possible values are used. Thus:
A should have been born on 30th, B on 25th, C on 20th, D on 10th and E on 5th. Further, the months of birth in random order will have to be between August to December to maximize the average.
Hence the total will be 30 + 25 + 20 + 10 + 5 + 12 + 11 + 10 + 9 + 8 = 140. Hence average is 28.
38. The minimum average will be when we have 1 + 5 + 10 + 20 + 25 + 1 + 2 + 3 + 4 + 5 = 76. Hence, average is 15.2.
39. This does not change anything. Hence the answer is the same as Q. 37.
40. The prime dates must be 29th, 23rd, 19th and 5th. Hence, the maximum possible average will reduce by 45 = 0.6. Hence, answer will be 27.2.
41. Solve using allligation. Since 15 is the mid-point of 12.25 and 19.75, the ratio is 1:1 and hence there are 20 people who were going for the picnic initially.
42. The number of pass candidates are 2 + 6 + 18 + 40 = 66 out of a total of 100. Hence, 66%.

Q.43-44. For x = 10 in the given equations and find the average of the resultant values.

43. Solve through options.
44. $x^2 + x^3 + x^4 \rightarrow$ Simplify to get Option (c) correct.
45. By allligation the ratio is 3:8. Hence, only 110 is possible.

Q.46-47.

46. You don't know who got out when. Hence, cannot be determined.
47. Since possibilities are asked about, you will have to consider all possibilities. Assume, the sixth and seventh batsmen have scored zero. Only then will the possibility of the first 5 batsmen scoring the highest possible average arise. In this case the maximum possible average for the first 5 batsmen could be 403/5 = 80.6.
48. Again it is possible that only the first batsman has scored runs.
41. We cannot find out the number of runs scored by the 5th batsman. Hence answer is (d).
42. You can take 53 as the base to reduce your calculations. Otherwise the question will become highly calculation intensive.
41. 251.25*4 + 272.52 * 5 + 360 =
44. Solve using options. 20 is the only possible value.
45. Check through options to solve.

Hints and Solutions III

1. Definitely decrease, since the highest marks in Class A is less than the lowest marks in Class B.
2. Cannot say since there is no indication of the values of the numbers which are transferred.
3. It will definitely decrease since the highest possible number is lower than the lowest value in C.
4. The effect on A will depend on the profile of the people who are transferred. Hence, anything can happen.
5. Cannot say since there is a possibility that the numbers transferred are such that the average can either increase, decrease or remain constant.
6. If C increases, then the average of C goes up from 30. For this to happen it is sufficient that the average of B should drop.

7. The maximum possible average for *B* will occur if all the 8 transfers from *A* have 33 marks.
8. The average of Group *A* after the transfer in Q. 7 above is:
   $(400 - 10*5)/15 = 310/15 = 20.66$
9. $(400 - 10*5)/15 = 19.33$
10. $400 + 20*5 = 500$. Average = $500/25 = 20$ A
11. $400 + 31*5 = 555$. Average = $555/25 = 22.2$
12. Will always decrease since the net value transferred from *B* to *A* will be higher than the net value transferred from *A* to *B*.
13. Since the lowest score in Class *B* is 23 which is more than the highest score of any student in Class *A*. Hence, *A*'s average will always increase.
14. The maximum possible value for *B* will happen when the *A* to *B* transfer has the minimum possible value and the reverse transfer has the maximum possible value.
15. For the minimum possible value of *B* we will need the *A* to *B* transfer to be the lowest possible value while the *B* to *A* transfer must have the highest possible value. Thus, *A* to *B* transfer → 14*5 while *B* to *A* transfer will be 24*5. Hence answer is 22 A.
16. The maximum value for *A* will happen in the case of Q. 13. Then the increment for group *A* is:
   $31*5 - 10*5 = 5*(31 - 18) = 65$.
   Thus maximum possible value is $465/20 = 23.25$.
17. Minimum possible average will happen for the transfer we saw in Q. 14. Thus the answer will be $405/20 = 20.25$.
18. The maximum possible value for *C* will be achieved when the transfer from *C* is of five 26's and the transfer back from *B* is of five 31's. Hence, difference in totals will be 425. Hence, max. average = $(900 + 25)/30 = 30.33$.
   (Note here that 900 has come by 30* 30)
19. For the minimum possible value of Class *B* the following set of operations will have to hold:
   First 10's are transferred from *C* to *B*, whatever goes from *B* to *A* comes back from *A* to *B*, then five 23's are transferred from *B* to *C*. This leaves us with:
   Increase of 10 marks → average increases by 2 to 21.
20. *A* will show maximum value if five 31's come to *A* from *C* through *B* and five 18's leave *A*. In such a case the net result is going to be a change of +16. Thus the average *A*(f) ar to be $(315/15 + 1.2) = 23.2$.
21–23. Will be solved by the same pattern as the above three items.
24. Only option *A* will give us the required situation where the transfer of five 31's increases the value of the average of group *A*.
25–28. Will be solved by the same pattern as above questions.
29–35. These are standard questions using the concept of averages. Hence, analyse each and every sentence by itself and link the interpretations. If you are getting stuck, the only reason is that you have not used the information in the questions fully.
36. Monthly estimate of income is reduced as the *A*-nominator is increased from 12 to 14 at the same time the monthly estimate of expenditure is increased as the denominator is reduced from 12 to 9. Hence, the savings will be undoubtedly and
37–39. Use the averages formulae and common sense to answer.
40–49. The questions are commonsensical with a lot of calculations and assumptions involved. You have to solve these using all the information provided.
   45. Dan's score = $15*2 + 7*2.5 = 47.5 \rightarrow 48$.
   Dangupta's score = $20*1 + 2*1.5 = 23$
   41. From the above the answer is $48.23 = 96.46$.
42–44. By maximum possible support from the other end, we have to assume that he has Laxman or Sehwag batting aggressively for the entire innings or the entire. Strike has to be shared equally.
   42. Through options, After 40 overs, score would be 150. Then Tendulkar can score @ 4 runs per over (shared the strike and having aggressively) and get maximum support @ 3 runs per over. Thus in 30 overs left the target will be achieved.
   43. Tendulkar's score for the innings will be $30*4 = 120$.
   44. We do not know when Laxman would have come to bat. Hence this cannot be determined.
45–49. Build in each of the conditions in the problem to the table like:

Partnership Partner Overs faced Tendulkar's score
Partner's score

6th wicket	Ganguly	12	6 overs × 6 = 6 runs × 1
7th wicket	and so on		
8th wicket			
9th wicket			
10th wicket			

ANSWER KEY

Answer Table

1. c
2. c
3. a
4. c
5. a
6. b
7. c
8. a
9. a
10. b
11. a
12. a
13. c
14. c
15. d
16. c
17. c
18. d
19. a
20. a
21. a
22. c
23. c
24. b
25. a
26. a
27. b
28. b
29. b
30. a
31. c
32. c
33. c
34. a
35. a
36. a
37. d
38. c
39. d
40. c
41. c
42. a
43. d
44. c
45. c
46. c
47. b
48. d
49. d
50. b
51. b
52. a
53. d
54. b
55. d
56. c
57. d
58. c
59. a
60. a
61. a
62. a
63. c
64. c
65. a
66. d
67. c
68. a
69. b
70. b
71. a
72. b
73. a
74. a
75. b
76. b
77. b
78. a
79. a
80. a
81. d
82. b
83. c
84. d

Answer Key II

1. b
2. c
3. c
4. d
5. c
6. b
7. a
8. c
9. b
10. c
11. c
12. a
13. a
14. b
15. a
16. d
17. c
18. a
19. c
20. c
21. b
22. b
23. c
24. a
25. a
26. c
27. c
28. b
29. d
30. a
31. b
32. d
33. b
34. a
35. b
36. c
37. c
38. d
39. a
40. d
41. d
42. c
43. b
44. c
45. a

Antwoorden LOO III

1. a
2. a
3. b
4. d
5. d
6. b
7. a
8. a
9. c
10. d
11. b
12. b
13. a
14. c
15. a
16. c
17. d
18. a
19. b
20. b
21. b
22. b
23. a
24. c
25. a
26. a
27. c
28. c
29. b
30. c
31. b
32. a
33. a
34. c
35. b
36. a
37. b
38. d
39. b
40. b
41. b
42. c
43. b
44. d
45. b
46. b
47. b
48. d
49. b

4

ALLIGATIONS

INTRODUCTION

The chapter of alligation is nothing but a faster technique of solving problems based on the weighted average situation as applied to the case of two groups being mixed together. I have often seen students having a lot of difficulty in solving questions on alligation. Please remember that all problems on alligation can be solved through the weighted average method. Hence, the student is advised to revert to the weighted average formula in case of any confusion.

The use of the techniques of this chapter for solving weighted average problems will help you in saving valuable time whenever a direct question based on the mixing of two groups is asked. Besides, in the case of questions that use the concept of the weighted average as a part of the problem, you will gain a significant edge if you are able to use the techniques illustrated here.

The relevance of this chapter for the CAT is the use of the alligation technique for solving problems where the calculation is used to solve a part of the whole question. Besides questions of this chapter are directly relevant for the exams like CAT (Mahindra), SNAP, NMIMS, NIFT, Symbiosis, MAT and all other B level management entrance exams as well as for the bank PO exams and the MCA exams, which are based on the pattern of an aptitude test.

THEORY

In the chapter on Averages, we had seen the use of the weighted average formula. To therefore, the weighted average is used when a number of similar groups are mixed together to form one larger group.

If the average of the measured quantity was

A_1 for group	1	containing	n_1	elements
A_2 for group	2	containing	n_2	elements
A_3 for group	3	containing	n_3	elements
A_k for group	k	containing	n_k	elements

We say that the weighted average, Av is given by

$$Av = (n_1A_1 + n_2A_2 + n_3A_3 + \ldots\ldots + n_kA_k)/(n_1 + n_2 + n_3 \ldots + n_k)$$

That is, the weighted average

$$= \frac{\text{Sum total of all groups}}{\text{Total number of elements in all groups together}}$$

In the case of the situation where just two groups are being mixed, we can write this as:

$$Av = (n_1A_1 + n_2A_2)/(n_1 + n_2)$$

Rewriting this equation we get $(n_1 + n_2)Av = n_1A_1 + n_2A_2$

$$n_1(Av - A_1) = n_2(A_2 - Av)$$

or $n_1/n_2 = (A_2 - Av)/(Av - A_1)$ → The alligation equation

The Alligation Situation

Two groups of elements are mixed together to form a third group containing the elements of both the groups.

If the average of the first group is A_1 and the number of elements is n_1 and the average of the second group is A_2 and the number of elements is n_2, then to find the average of the new group formed, we can use either the weighted average equation or the alligation equation.

As a convenient convention, we take $A_1 > A_2$. Then, by the principle of averages, we get $A_1 > Av > A_2$.

Illustration 1

Two varieties of rice at Rs. 10 per kg and Rs. 12 per kg are mixed together in the ratio 1 : 2. Find the average price of the resulting mixture.

Solution: $1/2 = (12 - Av)/(Av - 10) \Rightarrow Av - 10 = 24 - 2Av$

$\Rightarrow 3Av = 34 \quad \Rightarrow Av = 11.33$ Rs./kg

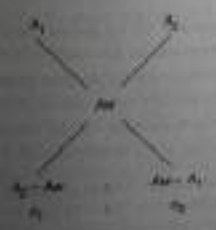
Illustration 2

On combining two groups of students having 30 and 40 marks respectively in an exam, the resultant group has an average score of 34. Find the ratio of the number of students in the first group to the number of students in the second group.

Solution: $n_1/n_2 = (40 - 34)/(34 - 30) = 6/4 = 3/2$

Graphical Representation of Alligation

The formula illustrated above can be represented by the following cross diagram:



Here, the cross method yields nothing but the alligation equation. Hence, the cross method is nothing but a graphical representation of the alligation equation.

In my view, there are five variables embedded in this the alligation equation. These being:

the three averages $\rightarrow A_1, A_2$ and Av

and the two weights $\rightarrow n_1$ and n_2

Based on the problem situation, one of the following cases may occur with respect to the knowns and the unknowns in the problem.

Case	Knowns	Unknowns
I	(a) A_1, A_2, Av	(a) $n_1 : n_2$
	(b) A_1, A_2, Av, n_1	(b) n_2 and $n_1 : n_2$
		Av
II	A_1, A_2, n_1, n_2	A_2
III	A_1, Av, n_1, n_2	

Now, let us try to evaluate the effectiveness of the cross method for each of the three cases illustrated above.

Case I: A_1, A_2, Av are known; may be one of n_1 or n_2 is known.

To find: $n_1 : n_2$ and n_2 if n_1 is known OR n_1 if n_2 is known.

Let us illustrate through an example.

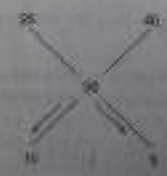
Illustration 3

On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find:

(a) The ratio of students in the classes

(b) The number of students in the first class if the second class had 30 students.

Solution:



(a) Hence, solution is 2 : 1.

(b) If the ratio is 2 : 1 and the second class has 30 students, then the first class has 60 students.

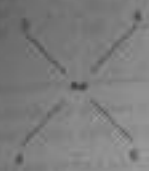
Note: The cross method becomes prompt effective in the situation where all the three averages are known and the ratio is to be found out.

Case 2: A_1, A_2, n_1 and n_2 are known; Av is unknown.

Illustration 4

4 kg of rice at Rs. 7 per kg is mixed with 6 kg of rice at Rs. 9 per kg. Find the average price of the mixture.

Solution:



$$x(6 - Av) = (Av - 7)$$

$$\therefore 6 \times 6(Av - 7) = 4S \quad -42 - 2 \cdot Av = Av - 7$$

$$5 \cdot Av = 17$$

$$Av = 3.40 \text{ Rs/kg. (Answer)}$$

Task for student: Solve through the alligation formula approach and through the weighted average approach to get the solution. Notice the amount of time required in doing the same.

Note: The cross method becomes quite cumbersome in this case, as the method results in the formula being written. Hence, there seems to be no logic in using the cross method in this case.

Case 3: A_1, Av, n_1 and n_2 are known; A_2 is unknown.

Illustration 5

5 kg of rice at Rs. 6 per kg is mixed with 4 kg of rice to get a mixture costing Rs. 7 per kg. Find the price of the costlier rice.

Solution: Using the cross method

$$x(6 - 7) = 5$$

$$\therefore -7x = 5x \Rightarrow 4x = 20 \Rightarrow x$$

$$x = \text{Rs. } 5.25$$

Task for student: Solve through the alligation formula approach and through the weighted average approach to get the solution. Notice the amount of time required in doing the same.

Note: The cross method becomes quite cumbersome in this case, since this method results in the formula being written. Hence, there seems to be no logic in using the cross method in this case.

The above problems are quite effectively dealt by using the straight line approach, which is explained below.

The Straight Line Approach

As we have seen, the cross method becomes quite cumbersome in Case 2 and Case 3. We will now proceed to modify the cross method so that the question can be solved graphically in all the three cases.

Consider the following diagram, which results from closing the cross like a pair of scissors. Then the positions of A_1, A_2, Av, n_1 and n_2 are as shown.

Visualise this as a fragment of the number line with points A_1, Av and A_2 in that order from left to right. Then,

(a) n_1 is responsible for the distance between A_1 and Av or n_1 corresponds to $Av - A_1$.

(b) n_2 is responsible for the distance between Av and A_2 or n_2 corresponds to $A_2 - Av$.

(c) $(n_1 + n_2)$ is responsible for the distance between A_1 and A_2, or $(n_1 + n_2)$ corresponds to $A_2 - A_1$.

The processes for the 3 cases illustrated above can now be illustrated below.

Illustration 6

On mixing two classes of students having average marks 25 and 40 respectively, the overall average obtained is 30 marks. Find

(a) the ratio in which the classes were mixed.

(b) the number of students in the first class if the second class had 30 students.

Solution:

Hence, ratio is 2 : 1, and the second class has 60 students.

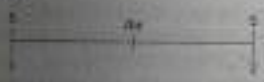
Case 2: A_1, A_2, n_1 and n_2 are known. A_0 is unknown.

Illustration 7

4 kg of rice at Rs. 5 per kg is mixed with 8 kg of rice at Rs. 6 per kg. Find the average price of the mixture.

Solution:

or the same as



Then, by solitary method

$n_1 : n_2$ corresponds to $A_2 - A_1$

$\rightarrow 1 : 2$ corresponds to $6 - 5$

That is, 5 corresponds to 1

$\therefore$ n_2 will correspond to $\dfrac{(A_2 - A_1) \times n_1}{(A_1 - n_1)}$

In the case $(1/1) \times 2 = 0.66$.

Hence, the required answer is 5.66.

Note: In this case, the problem associated with the cross method is erroneous and the solution becomes graphical.

Case 3: A_1, A_0, n_1 and n_2 are known. A_2 is unknown.

Illustration 8

3 kg of rice at Rs. 8 per kg is mixed with 4 kg of rice at Rs. x because costing Rs. 7 per kg. Find the price of the cheaper rice.

Using straight line method:

4 corresponds to $7 - 8$ and 3 corresponds to $x - 7$.

The thought process should go like:

$4 \rightarrow 1$

$\therefore$ $3 \rightarrow 1.25$

Hence, $x - 7 = 1.25$

and $x = 8.25$

SOME TYPICAL SITUATIONS WHERE ALLIGATIONS CAN BE USED

Given below are typical alligation situations, which students should be able to recognize. This will help them improve upon the time required in solving questions. Although in this chapter we have illustrated problems based on alligation at most 1 ratio, alligation is used in more complex problems where the weighted average is an intermediate step in the solution process.

The following situations should help the student identify alligation problems better as well as spot the way A_1, A_2, n_1 and n_2 and A_0 are mentioned in a problem.

In each of the following problems the following magnitudes represent these variables:

$A_1 = 20$, $A_2 = 30$, $n_1 = 40$, $n_2 = 60$

Each of these problems will yield an answer of 26 as the value of A_0.

1. A man buys 40 kg of rice at Rs. 20/kg and 60 kg of rice at Rs. 30/kg. Find his average price. (26/kg)
2. Prabhuji mixes two mixtures of milk and water. He mixes 40 liters of the first containing 20% water and 60 liters of the second containing 30% water. Find the percentage of water in the final mixture. (26%)
3. Two classes are combined to form a larger class. The first class having 40 students scored an average of 20 marks on a test while the second having 60 students scored an average of 30 marks on the same test. What was the average score of the combined class on the test. (26 marks)
4. A trader mixes a grade at 20% on 40% of his grade milk while he mixes a grade of 30% on 60% of his

goods sold. Find the percentage profit on the whole. (26%)

3. A car travels at 20 kmph for 40 minutes and at 30 kmph for 60 minutes. Find the average speed of the car for the journey. (28 kmph)
4. 40% of the revenues of a school come from the junior classes while 60% of the revenues of the school come from the senior classes. If the school raises its fees by 20% for the junior classes and by 10% for the senior classes, find the percentage increase in the revenues of the school. (26%)

Some Keys to spot A_1, A_2 and Aw and differentiate these from n_1 and n_2

1. Normally, there are 3 averages mentioned in the problem, while there are only 2 quantities. This isn't *Surprised thought*, since at times the question might confuse the student by giving 3 values for quantities representing n_1, n_2 and $n_1 + n_2$ respectively.
2. A_1, A_2 and Aw are always rate units, while n_1 and n_2 are quantity units.
3. The denominator of the average unit corresponds to the quantity unit (i.e. unit for n_1 and n_2).
4. All percentage values represent the average values.

A Typical Problem

A typical problem related to the topic of alligation goes as follows:

> 2 litres of wine are drawn from a cask containing 40 litres of wine. It is replaced by water. The process is repeated 3 times.
>
> (a) What is the final quantity of wine left in the cask.
> (b) What is the ratio of wine to water finally.

If we try to chart out the process, we get: Out of 40 litres of wine, 4 are drawn out.

This leaves 36 litres wine and 4 litres water. (Ratio of 9 : 1)

Now, when 4 litres are drawn out of this mixture, we will get 3.6 litres of wine and 0.4 litres of water (as the ratio is 9 : 1). Thus at the end of the second step we get 32.4 litres of wine and 7.6 litres of water. Further, the process is repeated, drawing out 2.24 litres wine and 1.76 litres water leaving 30.16 litres of wine and 5.84 litres of water.

This gives the final values and the ratio required.

A closer look at the process will yield that we can find the amount of wine left by:

$$40 \times 36/40 \times 36/40 \times 36 \times \text{Quantity}$$
$$\Rightarrow 40 \times (1 - 4/40)^3$$

This yields the formula:

Wine left: Capacity × (1 − fraction of Ale withdrawn)^n for n operations.

Thus, you could have multiplied

$$40 \times (0.9)^3 \text{ to get the answer}$$

That is, reduce 40 by 10% successively three times to get the required answer.

Thus, the thought process would be:

40 − 10% → 36 − 10% → 32.4 − 10% → 29.16

Level of Difficulty (LOD) I

(Please note that there are no LOD II and LOD III in this chapter since the questions are based on only certain basic situations that cannot be extended.)

1. 15 kg of salt costing Rs. 5/kg and 5 kg of salt costing Rs. 4/kg are mixed. Find the average cost of the mixture per kilogram.
   (a) Rs. 4.5 (b) Rs. 4.625
   (c) Rs. 4.75 (d) Rs. 4.125
   (e) Rs. 4.25
2. Two types of oils having the rates of Rs. 4/kg and Rs. 5/kg respectively are mixed in order to produce a mixture having the rate of Rs. 4.50/kg. What should be the amount of the second type of oil if the amount of the first type of oil in the mixture is 40 kg?
   (a) 75 kg (b) 50 kg
   (c) 60 kg (d) 40 kg
   (e) None of these
3. How many kilograms of sugar worth Rs. 5/kg should be mixed with 8 kg of sugar worth Rs. 4.50/kg such that by selling the mixture at Rs. 5.40/kg there may be a gain of 10%?
   (a) 6 kg (b) 3 kg (c) 2 kg (d) 5 kg
   (e) 4 kg

2. A mixture of 125 gallons of wine and water contains 20% water. How much water must be added to the mixture in order to increase the percentage of water to 25% of the new mixture?

(a) 10 gals (b) 8.5 gals
(c) 9 gals (d) 8.66 gals
(e) 8.33 gals

3. Ravi took Rs. 3600 on simple interest to Harsh for a period of 3 years. He lent a part of the amount at 4% interest and the rest at 6% and receives Rs. 960 as the amount of interest. How much money did he lend on 4% interest rate?

(a) Rs. 2800 (b) Rs. 2100
(c) Rs. 2400 (d) Rs. 1200
(e) Rs. 1600

4. 400 students took the AMS Mock Cat in Lucknow, 40% of the boys and 30% of the girls cleared the cut off in the examination. If the total percentage of students qualifying is 65%, how many girls appeared in the examination?

(a) 100 (b) 120 (c) 150 (d) 300
(e) 170

5. A man purchased a cow and a calf for Rs. 1300. He sold the calf at a profit of 20% and the cow at a profit of 25%. In this way, his total profit was $23\frac{1}{2}$%. Find the cost price of the cow.

(a) Rs. 1100 (b) Rs. 600
(c) Rs. 500 (d) Rs. 400
(e) Rs. 800

6. The average salary per head of all employees of a company is Rs. 600. The average salary of 120 officers is Rs. 4000. If the average salary per head of the rest of the employees is Rs. 560, find the total number of workers in the company?

(a) 10000 (b) 10120
(c) 10200 (d) 10880
(e) 10600

7. A dishonest milkman purchased milk at Rs. 10 per litre and mixed 5 litres of water in it. By selling the mixture at the rate of Rs. 10 per litre he earns a profit of 25%. The quantity of the amount of the mixture that he had was?

(a) 15 litres (b) 20 litres
(c) 25 litres (d) 30 litres
(e) None of these

10. A cistern contains 50 litres of water. 5 litres of water is taken out of it and replaced by wine. The process is repeated again. Find the proportion of wine and water in the resulting mixture.

(a) 1 : 4 (b) 41 : 50
(c) 19 : 61 (d) 61 : 19
(e) 50 : 41

11. A container has a capacity of 30 gallons and is full of spirit. 4 gallons of spirit is drawn out and the container is again filled with water. This process is repeated 5 times. Find out how much spirit is left in the resulting mixture finally?

(a) $6\frac{257}{525}$ gallons (b) $6\frac{346}{625}$ gallons
(c) 6.5 gallons (d) 6.25 gallons
(e) None of these

12. A vessel is full of refined oil. 1/4 of the refined oil is taken out and the vessel is filled with mustard oil. If the process is repeated 4 times and 10 litres of refined oil is finally left in the vessel, what is the capacity of the vessel?

(a) 33 litres (b) $\frac{2400}{81}$ litres
(c) $\frac{2560}{81}$ litres (d) 80 litres
(e) None of these

13. In what ratio should two qualities of coffee powder having the rates of Rs. 47 per kg and Rs. 32 per kg be mixed in order to get a mixture that would have a rate of Rs. 37 per kg?

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1
(e) 4 : 1

14. A thief steals four gallons of liquid soap kept in a trim compartment's bathroom from a container that is full of liquid soap. He then fills it with water to avoid detection. Unable to resist the temptation he steals 4 gallons of the mixture again, and fills it with water. When the liquid soap is checked at a station it is found that the ratio of the liquid soap now left in the container to that of the water in it is 36 : 13. What was the initial amount of the liquid soap in the container if it is known that the liquid soap is neither used nor augmented by anybody else during the entire period?

(a) 7 litres (b) 14 litres

(c) 31 litres (d) 28 litres
(e) 40 litres

17. In what ratio should water be mixed with milk costing Rs. 12 per litre so as to make a profit of 25% by selling the mixed liquid at Rs. 13.75 per litre?
(a) 10 : 1 (b) 11 : 2
(c) 1 : 11 (d) 12 : 1
(e) 1 : 12

18. A note of Rs. 36.40 is made up of 90 coins that are either 20 paise coins or 50 paise coins. Find out how many 20 paise coins are there in the total amount?
(a) 47 (b) 43 (c) 27 (d) 63
(e) 70

19. A unknown grocer professes to sell pure butter at cost price, but he mixes it with adulterated fat and thereby gains 25%. Find the percentage of adulterated fat in the mixture assuming that adulterated fat is freely available.
(a) 20% (b) 25% (c) 33.33% (d) 40%
(e) 35%

20. A mixture of 30 litres of alcohol and water contains 10% of water. How much water must be added to the above mixture to make the water 12.5% of the resulting mixture?
(a) 1 litre (b) 1.5 litre
(c) 2 litres (d) 2.5 litres
(e) 3 litres

21. A mixture of 20 litres of brandy and water contains 10% water. How much water should be added to it to increase the percentage of water to 25%?
(a) 2 litres (b) 3 litres
(c) 3.5 litres (d) 4 litres
(e) 5 litres

22. A merchant purchased two qualities of pulses at the rate of Rs. 200 per quintal and Rs. 300 per quintal. In 50 quintals of the second quality, how much pulse of the first quality should be mixed so that by selling the resulting mixture at Rs. 300 per quintal, he gains a profit of 25%?
(a) 300 quintals (b) 100 quintals
(c) 20 quintals (d) None of these

23. A man buys milk at Rs. 8.5 per litre and dilutes it with water. He sells the mixture at the same rate and thus gains 11.11%. Find the quantity of water mixed by him in every litre of milk.
(a) 0.111 litres (b) 0.909 litres
(c) 0.1 litre (d) 0.125 litres

22. There are two mixtures of honey and water, the quantity of honey in them being 25% and 75% of the mixture. If 2 gallons of the first are mixed with three gallons of the second, what will be the ratio of honey to water in the new mixture?
(a) 11 : 2 (b) 11 : 9 (c) 9 : 11 (d) 2 : 11

23. There are two kinds of alloys of tin and copper. The first alloy contains tin and copper such that 93.33% of it is tin. In the second alloy there is 86.66% tin. What weight of the first alloy should be mixed with some weight of the second alloy so as to make a 50 kg mixture containing 90% of tin?
(a) 15 kg (b) 30 kg (c) 20 kg (d) 25 kg

24. Two containers of equal capacity are full of a mixture of oil and water. In the first, the ratio of oil to water is 4 : 7 and in the second it is 7 : 11. Now both the mixtures are mixed in a bigger container. What is the resulting ratio of oil to water.
(a) 149 : 247 (b) 247 : 149
(c) 143 : 241 (d) 241 : 143

25. Two vessels contain spirit and water mixed respectively in the ratio of 1 : 3 and 3 : 5. Find the ratio in which these are to be mixed to get a new mixture in which the ratio of spirit to water is 1 : 2.
(a) 2 : 1 (b) 3 : 1 (c) 1 : 2 (d) 1 : 3

26. The price of a pen and a pencil is Rs. 35. The pen was sold at a 20% profit and the pencil at a 10% loss. If in the transaction a man gains Rs. 4, how much is cost price of the pen?
(a) Rs. 10 (b) Rs. 25
(c) Rs. 20 (d) None of these

27. A person purchased a cupboard and a cot for Rs. 18,000. He sold the cupboard at a profit of 20% and the cot at a profit of 30%. If his total profit was 25.833%, find the cost price of the cupboard.
(a) Rs. 10,500 (b) Rs. 12,000
(c) Rs. 7,500 (d) Rs. 10,000

28. A vessel is full of a mixture of kerosene and petrol in which there is 18% kerosene. Eight litres are drawn off and then the vessel is filled with petrol. If the kerosene is now 15%, how much does the vessel hold?
(a) 40 litres (b) 32 litres
(c) 36 litres (d) 48 litres

30. Two solutions of 90% and 97% purity are mixed in ratio of 2:1 times of solution of 94% purity. How much is the quantity of the first solution in the resulting mixture?

(a) 13 times (b) 12 times
(c) 9 times (d) 8 times

31. In the Singapore zoo, there are deers and birds. If the heads are counted, there are 140, while the legs are 448. What will be the number of deers in the zoo?

(a) 28 (b) 48 (c) 24 (d) 22

32. A bonus of Rs. 3,85,000 was divided among 500 workers of a factory. Each male worker gets 5000 rupees and each female worker gets 2500 rupees. Find the number of male workers in the factory.

(a) 253 (b) 47 (c) 94 (d) 300

33. What will be the ratio of petrol and kerosene in the final solution formed by mixing petrol and kerosene kept in three vessels in the ratio 4 : 1, 5 : 2 and 6 : 3 respectively?

(a) 108 : 22 (b) 83 : 22
(c) 43 : 44 (d) None of these

34. A mixture worth Rs. 3.25 a kg is formed by mixing two types of flour, one costing Rs. 3.10 per kg while the other Rs. 3.60 per kg. In what proportion must they have been mixed?

(a) 3 : 7 (b) 7 : 10
(c) 10 : 3 (d) 7 : 3

35. A gas percent of 20 is made by selling the mixture of two types of ghee at Rs. 480 per kg. If the type costing 450 per kg was mixed with 120 kg of the other, how many kilograms of the former was mixed?

(a) 120 kg (b) 34.5 kg
(c) 90 kg (d) Cannot be determined

36. In what proportion must water be mixed with milk so as to gain 20% by selling the mixture at the cost price of the milk? (Assume that water is freely available)

(a) 1 : 4 (b) 1 : 5
(c) 1 : 8 (d) 1 : 12

36. A bartender stole champagne from a bottle that contained 50% of spirit and he replaced what he had stolen with champagne having 20% spirit. The bottle then contained only 25% spirit. How much of the bottle did he steal?

(a) 40% (b) 83.33%
(c) 45.5% (d) 88.88%

37. A bag contains a total of 105 coins of Rs. 1, 50 p and 25 p denominations. Find the total number of coins of Re-1 if there are a total of 50.5 rupees in the bag and it is known that the number of 25 paise coins are 133.33% more than the number of 1 rupee coins.

(a) 56 (b) 25
(c) 24 (d) None of these

38. A man possessing Rs. 6800, lent a part of it at 10% simple interest and the remaining at 7.5% simple interest. His total income after $3\frac{1}{2}$ years was Rs. 1904. Find the sum lent at 10% rates

(a) Rs. 1200 (b) Rs. 1700
(c) Rs. 1300 (d) None of these

39. If a man decides to travel 80 kilometres in 8 hours partly by foot and partly on a bicycle, his speed on foot being 8 km/h and that on bicycle being 16 km/h, what distance would he travel on foot?

(a) 20 km (b) 30 km
(c) 48 km (d) 60 km

40. Two vessels contain a mixture of spirit and water. In the first vessel the ratio of spirit to water is 8 : 3 and in the second vessel the ratio is 5 : 1. A 35 litre cask is filled from these vessels so as to contain a mixture of spirit and water in the ratio of 4 : 1. How many litres are taken from the first vessel?

(a) 11 litres (b) 22 litres
(c) 16.5 litres (d) 17.5 litres

ANSWER KEY

Alligation LOD I

1. a
2. c
3. c
4. a
5. d
6. a
7. e
8. b
9. c
10. c
11. b
12. c
13. a
14. d
15. a
16. c
17. a
18. c
19. d
20. c
21. a
22. b
23. d
24. a
25. c
26. b
27. c
28. d
29. c
30. c
31. a
32. c

33. d
34. d
35. b
36. b
37. c
38. c
39. c
40. a

BLOCK REVIEW TESTS

REVIEW TEST ONE

1. Rakesh bought 10 erasers for Rs. 10. He paid 20 paise more for each white eraser than for each brown eraser. What could be the price of a white eraser and how many white erasers could he have bought?
   (a) 60 paise, 8 (b) 60 paise, 12
   (c) 50 paise, 8 (d) 50 paise, 10
2. After paying all your bills, you find that you have Rs. 5.30 in your pocket. You have equal number of 50 paise and 10 paise coins; but no other coins nor any other currency notes. How many coins do you have?
   (a) 8 (b) 24 (c) 27 (d) 30
3. Ramesh Kumar went to the market with Rs. 100. If he buys three pens and six pencils he uses up all his money. On the other hand if he buys three pencils and six pens he would fall short by 20%. If he wants to buy equal number of pens & pencils, how many pencils can he buy?
   (a) 4 (b) 5 (c) 6 (d) 7
4. For the above question, what is the amount of money he would save if he were to buy 3 pens and 3 pencils?
   (a) Rs. 50 (b) Rs.25
   (c) Rs.75 (d) Rs.40
5. Abdul goes to the market to buy bananas. If he can bargain and reduce the price per dozen by Rs. 2, he can buy 3 dozen bananas instead of 2 dozen with the money he has. How much money does he have?
   (a) Rs. 6 (b) Rs. 12 (c) Rs. 18 (d) Rs. 24
6. Two oranges, three bananas and four apples cost Rs.15. Three oranges, two bananas and one apple cost Rs.10. I bought 5 oranges, 3 bananas and 5 apples. How much did I pay?
   (a) Rs.10 (b) Rs.8
   (c) Rs.15 (d) cannot be determined
7. John bought five mangoes and ten oranges together for forty rupees. Subsequently, he returned one orange and got two oranges in exchange. The price of an orange would be
   (a) Rs. 1 (b) Rs. 2 (c) Rs. 3 (d) Rs. 4
8. Two towns A and B are 100 km apart. A school is to be built for 100 students of Town B and 30 students of Town A. The Expenditure on transport is Rs.1.20 per km per person. If the total expenditure on transport by all 130 students is to be as small as possible, then the school should be built at
   (a) 55 km from Town A
   (b) 55 km from Town B
   (c) Town A
   (d) Town B
9. A person who has a certain amount with him goes to the market. He can buy 50 oranges or 40 mangoes. He retains 10% of the amount for bus fare and buys 20 mangoes and of the balance he purchases oranges. Number of oranges he can purchase is
   (a) 36 (b) 40 (c) 15 (d) 20
10. 72 hens costs Rs._96.7_. Then what does each hen cost, where numbers at "_" are not visible or are written in illegible hand?
    (a) Rs.3.43 (b) Rs.5.31
    (c) Rs.3.51 (d) Rs.6.72

Directions for Questions 10–12:
There are 90 students in a class. Those students are divided into three groups A, B and C of 15, 30 and 25 students each. The groups A and C are combined to form group D.

11. What is the average weight of the students in group D?
    (a) more than the average weight of A.
    (b) more than the average weight of C.
    (c) less than the average weight of C.
    (d) Cannot be determined.
12. If one student from Group A is shifted to group B, which of the following will be true?
    (a) The average weight of both groups increases.
    (b) The average weight of both groups decreases.
    (c) The average weight of the class remains the same.
    (d) Cannot be determined.
13. If all the students of the class have the same weight then which of the following is false?
    (a) The average weight of all the four groups is the same.
    (b) The total weight of A and C is twice the total weight of B.
    (c) The average weight of D is greater than the average weight of A.
    (d) The average weight of all the groups remains the same even if a number of students are shifted from one group to another.

14. The average marks of a student in six papers are 80. If the highest and the lowest score are not considered the average is 81. If his highest score is 92 find the lowest.
(a) 48 (b) 40
(c) 42 (d) cannot be determined

15. A shipping clerk has five boxes of different but unknown weights each weighing less than 100 kg. The clerk weighs the boxes in pairs. The weights obtained are 110,112,113, 114, 115, 116, 117,118, 120 and 121 kg. What is the weight of the heaviest box?
(a) 60 kg (b) 62 kg
(c) 64 kg (d) Cannot be determined

16. The total expenses of a boarding house are partly fixed and partly varying linearly with the number of boarders. The average expense per boarder is Rs. 700 when there are 25 boarders and Rs. 600 when there are 50 boarders. What is the average expense per boarder when there are 100 boarders?
(a) 590 (b) 580
(c) 540 (d) 550

17. A yearly payment to a servant is Rs. 90 plus one turban. The servant leaves the job after 8 months and receives Rs.65 and a turban. Then find the price of the turban.
(a) Rs.80 (b) Rs.25
(c) Rs.7.50 (d) Cannot be determined

18. A leather factory produces two kinds of bags, Standard and deluxe. The profit margin is Rs.20 on a standard bag and Rs.30 on a deluxe bag. Every bag must be processed on machine A and on Machine B. The processing times per bag on the two machines are as follows:

Time required (Hours/bag)

	Machine A	Machine B
Standard Bag	4	6
Deluxe Bag	5	10

The total time available on machine A is 700 hours and on machine B is 1250 hours. Among the following production plans, which one meets the machine availability constraints and maximizes the profit?
(a) Standard 75 bags, Deluxe 80 bags
(b) Standard 100 bags, Deluxe 60 bags
(c) Standard 50 bags, Deluxe 100 bags
(d) Standard 60 bags, Deluxe 90 bags

19. Three math classes X, Y, and Z, take an algebra test. The average score of class X is 83.
The average score of class Y is 76.
The average score of class Z is 85.
What is the average score of classes X, Y, Z ?
(a) 81.5 (b) 80.5
(c) 83 (d) Cannot be determined

20. Prabhat ordered 4 Arrow shirts and some additional Park Avenue shirts. The price of one Arrow shirt was twice that of one Park Avenue shirt. When the order was executed it was found that the number of the two brands had been interchanged. This increased the bill by 40%. The ratio of the number of Arrow shirts to the number of Park Avenue shirts in the original order was:
(a) 1:3 (b) 1:4
(c) 1:2 (d) 1:5

21. Three groups of companies Tata, Birla and Reliance announce the average of the annual profit for all years since their establishment.
The average profit of Tata is Rs. 75,000 lakh
The average profit of Birla is Rs. 64000 lakh
The average profit of Reliance is Rs. 73000 lakh
The average profit of all results of Tata and Birla together is Rs. 70000 lakh.
The average profit of all results of Birla and Reliance together is Rs. 69000 lakh.
Approximately what is the average profit for all the three group of companies?
(a) Rs. 70800 lakh
(b) Rs. 71000 lakh
(c) Rs. 70666 lakh
(d) Can't be determined

ANSWER KEY

Review Test Chart

1. b
2. b
3. a
4. b
5. b
6. c
7. b
8. d
9. d
10. c
11. d
12. c
13. c
14. b
15. b
16. a
17. c
18. a
19. d
20. c
21. b

BLOCK 3

CHAPTERS

- Percentages
- Profit and Loss
- Interest
- Ratio, Proportion and Variation
- Time and Work
- Time, Speed and Distance

...BACK TO SCHOOL

As you are already aware, this block consists of the following chapters

Percentages,
Profit & Loss,
Ratio & Proportion,
Interest,
Time and Work,
Time, Speed and Distance

To put it very simply, the reason for these seemingly diverse chapters to be under one block of chapters are

Linear Equations

Yes, the solving of linear equations is the common thread that binds all the chapters in this block.

But before we start going through what a linear equation is, let us first understand the concept of a variable and it's need in the context of solving mathematical expressions.

Let us start off with a small exercise first:

Think of a number.

Contd

Add 2 to it.
Double the number to get a new number.
Add half of this new number to itself.
Divide the sum by 3.
Take away the original number from it.
The number you now have is..... ?"
How do I know this result?
The answer is pretty simple. Take a look. I am assuming that you had taken the initial number as 5 to show you what has happened in this entire process.

Instruction	Use	Me
Think of a number.	5	X
Add 2 to it.	5 + 2 = 7	X + 2
Double the number to get a new number.	7 × 2 = 14	2X + 4
Add half of this new number to itself.	14 + 7 = 21	3X + 6
Divide the result by 3.	21/3 = 7	X + 2
Take away the original number from it.	7 − 5 = 2	X + 2 − X = 2
The number you now have is..... ?!	The number you now have is..... 2!	The number you now have is..... ?!

The above is a perfect illustration of what a variable is and how it operates.

In this entire process, it did not matter to me as to what number you have assumed. All I set up is a kind of a parallel world wherein the number in your mind is represented by the variable X in my mind.

By ensuring that the final value does not have an X in it, I have ensured that the answer is independent of the value you would have assumed. Thus, even if someone had assumed 7 as the original value, his values would go as: 7, 9, 18, 27, 9, 2.

What you need to understand is that in Mathematics, whenever we have to solve for the value of an unknown we represent that unknown by using some letter (like *x*, *y*, etc.). These letters are then called as the variable representations of the unknown quantity.

Thus, for instance, if you come across a situation where a question says: The temperature of a city increases by 1°C on Tuesday from its value on Monday, you assume that if Monday's temperature was *t*, then Tuesday's temperature will be *t* + 1.

The opposite of a variable is a constant. Thus if it is said in the same problem that the temperature on Wednesday is 34° C, then 34 becomes a constant value in the context of the problem.

Thus, although you do not have the actual value in your mind, you can still move ahead in the question by assuming a variable to represent the value of the unknowns.

All problems in Mathematics ultimately take you to a point which will give the value of the unknowns—which then becomes the answer to the question.

Hence, in case you are stuck in a problem in this block of chapters it could be due to only one of three reasons:

Reason 1: You are stuck because you have either not used all the information given in the problem or have used them in the incorrect order.

In such a case, go back into the problem and try to identify each statement and see whether you have utilized it or not. If you have already used all the information, you might be interested in knowing whether you have used the information given in the problem in the correct order. If both of these have been done, you might want to explore the next reason for getting stuck.

Reason 2: You are stuck because even though you might have used all the information given in the problem, you have not utilized some of the information completely.

In such a case, you need to review each of the parts of the information given in the question and look at whether any additional details can be derived out of the same information. Very often, in Quants, you have situations wherein one sentence might have more than one connotation. If you have used this sentence only in one perspective, then doing it in the other perspective will solve the question.

Reason 3: You are stuck because the problem does not have a solution.

In such a case, check the question once and if it is correct go back to Reasons 1 and 2. Your solution has to be there.

My experience in training students tells me that the 1st case is the most common reason for not being able to solve questions correctly, more than 90% of the times. Hence, if you consider yourself to be weak in Maths, concentrate on the following process in this block of chapters.

THE LOGIC OF THE STANDARD STATEMENT

What I have been trying to tell you is that most of the times, you will get stuck in a problem only when you are not able to interpret a statement in the problem. Hence, my advice to students especially those who are weak in these chapters:

Concentrate on developing your ability to decode the mathematical meaning of a sentence in a problem.

To do this, even in problems that you are able to solve easily or with difficulty go back into the language of the question and work out the mathematical reaction that you should have with each statement.

It might not be a bad idea to make a list of standard statements along with their mathematical reactions for each chapter in this block of chapters. You will realize that in almost no time, you will come to a situation where you will only write statements in language.

Coming back to the issue of **linear equations**.

Linear equations are expressions about variables that might help us get the value of the variable if we can solve the equation.

Depending upon the number of variables in a problem, a linear equation might have one variable, two variables or even three or more variables. The only thing you should know is that in order to get the value of a variable, the number of equations needed is always equal to the number of variables. In other words, if you have more variables in a system of equations than the number of equations, you cannot solve for the individual values of the variables.

The basic mathematical principle goes like this:

For a system of equations to be solvable, the number of equations should be equal to the number of variables in the equations.

Thus for instance, if you have two variables, you need two equations to get the values of the two variables, while if you have three variables you will need three equations.

This situation is best exemplified by the situation where you might have the following equation: $x + y = 5$. If it is known that both x and y are natural numbers, it yields a set of possibilities for the values of x & y as follows: (1, 4), (2, 3), (3, 2), (4, 1). One of these possibilities has to be the answer.

In fact, it might be a good idea to think of all linear equation situations in this fashion. Hence, before you attempt to read about the linear equations, you should set up the set of possibilities based on the first equation.

Consider the following variation where a question yields a set of possibilities:

Four cousins A, B, C and D gather together for a picnic at a park with their wives. A's wife consumes 5 times as many glasses of juice as A. B's wife consumes 4 times as many glasses of juice as B. C's wife consumes 3 times as many glasses of juice as C and D's wife consumes 2 times as many glasses of juice as D. In total, the wives of the four cousins consume a total of 44 glasses of juice. If A consumes at least 5 glasses of juice while each of the other men have at least one glass, find the least number of drinks that could have been consumed by the 4 cousins together.

(1) 9 (2) 12 (3) 11 (4) 10

In the question above, we have 8 variables – A, B, C & D and a, b, c, d – the number of glasses consumed by the four men and the number of glasses consumed by the four wives.

Also, the question gives us five information which can be summarized into 5 equations as follows.

$a = 5A$

$b = 4B$

$c = 3C$

$d = 2D$

and $a + b + c + d = 44$

Also, $A \geq 5$

Under this condition, you do not have enough information to get all values and hence you will get a set of possibilities.

Since the minimum value of A is 5, a can take the values 25, 30, 35 and 40 when A takes the values 5, 6, 7 and 8 respectively. Based on these, and on the realization that b has to be a multiple of 4, c a multiple of 3 and d a multiple of 2, the following possibilities emerge:

At $A = 5$

a	(multiple of 5)	25	25	25	25	25
b	(multiple of 4)	12	8	8	4	4
c	(multiple of 3)	3	9	3	3	9
d	(multiple of 2)	4	2	8	12	6
	sum=44	44	44	44	44	44

a		A=6, a=30	A=7, a=35	A=8, a=40
b	(multiple of 4)	4	4	No solution
c	(multiple of 3)	6	3	
d	(multiple of 2)	4	2	
	sum=44	44	44	

In this case the answer will be 10, since in the case of $a=35$, $b=4$, $c=3$ & $d=2$, the values for A, B, C and D will be respectively 7, 1, 1 and 1. This solution is the least number of drinks consumed by the 4 cousins together as in all the other possibilities the number of glasses is greater than 10.

Such utilisations of linear equations are very common in CAT and top level aptitude examinations.

The relationship between the decimal value and the percentage value of a ratio:

Every ratio has a percentage value and a decimal value and the difference between the two is just in the positioning of the decimal point.

Thus, 2/5 can be represented as 0.4 in terms of its decimal value and can be represented as 40% in terms of its percentage value.

1. In a race, A, B and C run a race, with runner A finishing 10 metres ahead of runner B and 30 metres ahead of runner C, while runner B finishes 10 metres ahead of runner C. Each runner travels the entire distance at a constant speed. What was the length of the race?
   (a) 75 metres (b) 90 metres
   (c) 120 metres (d) 144 metres

2. A seller buys dry fruits at Rs.100, Rs. 80 and Rs. 60 per kilogram, the latter three in the ratio 4:3:6 by weight and sells at a profit of 50%. At what price per kilogram does he sell the dry fruits?
   (a) Rs.114 (b) Rs.106
   (c) Rs.102 (d) None of these

3. There are two containers, the first contains 500 ml of alcohol, while the second contains 300 ml of water. Five ml of alcohol from the first container is taken out and is added well in the second container. Then, five ml of this mixture is taken out from the second container and put back into the first container. Let X and Y denote the proportion of alcohol in the first and the proportion of water in the second container. Then what is the relationship between X & Y? (Assume the size of the cups to be identical)
   (a) X>Y (b) X<Y
   (c) X=Y (d) Cannot be determined

4. Akhilesh took five papers in an examination, where each paper was of 200 marks. His marks in these papers were in the proportions of 7 : 8 : 9 : 10 : 11. In all papers together, the candidate obtained 60% of the total marks. Then, the number of papers in which he got more than 50% marks is:
   (a) 1 (b) 3
   (c) 4 (d) 5

5. A and B walk up an escalator (moving stairway). The escalator moves at a constant speed. A takes six steps for every four of B's steps. A gets to the top of the escalator after having taken 50 steps, while B reaches the top of the escalator a little later. A takes only 40 steps to reach the top. If the escalator were turned off, how many steps would they need to take to walk up?
   (a) 80 (b) 100
   (c) 120 (d) 160

6. Fifty per cent of the employees of a certain company are men, and 60% of the men earn more than Rs. 2.5 lacs per year. If 60% of the company's employees earn more than Rs. 2.5 lacs per year, then what fraction of the women employed by the company earn more than Rs. 2.5 lacs per year?
   (a) 2/5 (b) 1/4
   (c) 1/3 (d) 3/4

7. A piece of string is 80 centimetres long. It is cut into three pieces. The longest piece is 4 times as long as the middle-sized and the shortest piece is 16 centimetres shorter than the longest piece. Find the length of the shortest piece (in cm).
   (a) 14 (b) 10
   (c) 8 (d) 18

8. Three members of a family A, B, and C, work together to get all household chores done. The time it takes them to do the work together is six hours less than A would have taken working alone, one hour less than B would have taken alone, and half the time C would have taken working alone. How long did it take them to do these chores working together?
   (a) 20 minutes (b) 30 minutes
   (c) 40 minutes (d) 50 minutes

9. Fresh grapes contain 90% water by weight while dried grapes contain 20% water by weight. What is the weight of dry grapes available from 20 kg of fresh grapes?
   (a) 2kg (b) 2.4kg
   (c) 2.2kg (d) None of these

10. At the end of the year 2008, a shepherd bought twelve dozen goats. Henceforth, every year he added p% of the goats at the beginning of the year and sold q% of the goats at the end of the year where p > 0 and q > 0. If the shepherd had twelve dozen goats at the end of the year 2012, (after making the sales for that year), which of the following is true?
   (a) p = q (b) p>q
   (c) p<q (d) p = 4/3

Directions for Questions 11–12: Answer the questions based on the following information.

An Indian company purchases components X and Y from UK and Germany respectively. X and Y form 40% and 30% of the total production cost. Current gain is 25%. Due to change in the international exchange rate scenario, the cost of the German mark increased by 50% and that of

UK pound increased by 20%. Due to tough competitive market conditions, the selling price cannot be increased beyond 10%.

11. What is the maximum current gain possible?
    (a) 10% (b) 12.5%
    (c) 8% (d) 7.5%

12. If the UK pound becomes cheap by 15% over its original cost and the cost of German mark increased by 20%, what will be the gain if the selling price is not altered?
    (a) 10% (b) 20%
    (c) 25% (d) 7.5%

13. A college has raised 80% of the amount it needs for a new building by receiving an average donation of Rs. 800 from the people already solicited. The people already solicited represent 50% of the people the college will ask for donations. If the college is to raise exactly the amount needed for the new building, what should be the average donation from the remaining people to be solicited?
    (a) 300 (b) 200
    (c) 400 (d) 500

14. A student gets an aggregate of 60% marks in five subjects in the ratio 10 : 9 : 8 : 7 : 6. If the passing marks are 41% of the maximum marks and each subject has the same maximum marks, in how many subjects did he pass the examination?
    (a) 2 (b) 3
    (c) 4 (d) 5

15. After allowing a discount of 12.5 % a trader still makes a gain of 40%. At what per cent above the cost price does he mark on his goods?
    (a) 45% (b) 60%
    (c) 75% (d) None of these

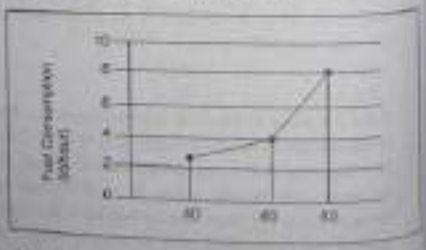
16. The owner of an art shop conducts his business in the following manner. Every year in a while he raises his prices by X%, then a while later he reduces all his new prices by X%. After one such up-down cycle, the price of a painting decreased by Rs. 441. After a second up-down cycle, the painting was sold for Rs. 1296.81. What was the original price of the painting (in Rs.)?
    (a) 2916.25 (b) 1296.25
    (c) 1960 (d) 2000

17. Mohan, Mitra, Shanti and Kapil bought a newspaper for Rs. 100. Mohan paid 50% of the amount paid by the other boys. Mitra paid one third of the amount paid by the other boys. Shanti paid one fourth of the sum of the amounts paid by the other boys. Kapil paid twice as much as the other boys. How much did Kapil have to pay?
    (a) $19000 (b) $13000
    (c) $17000 (d) None of these

18. A train X departs from station A at 11.00 a.m. for station B, which is 140 km away. Another train Y departs from station B at 11.30 a.m. for station A. Train X travels at an average speed of 70 km/hr and does not stop anywhere until it arrives at station B. Train Y travels at an average speed of 50 km/hr but has to stop for 10 minutes at station C, which is 35 km away from station B enroute to station A. Ignoring the lengths of the trains, what is the distance, to the nearest km, from station A to the point where the trains cross each other?
    (a) 110 (b) 112
    (c) 115 (d) None of these

19. In a survey of political preferences, 85% of those asked were in favour of at least one of the three budgetary proposals A, B and C. 50% of those asked favoured proposal A, 30% favoured proposal B and 20% favoured proposal C. If 5% of those asked favoured all the three proposals, what percentage of those asked favoured more than one of the four proposals?
    (a) 10% (b) 12%
    (c) 9% (d) 14%

Directions for Questions 20–21: The petrol consumption rate of a new model car 'Ruby' depends on its speed and may be described by the graph below.



20. Mumtaz makes the 240 km trip from Mumbai to Pune at a steady speed of 60 km per hour. What is the average of petrol consumed for the journey?
    (a) 12.5 litres (b) 16 litres
    (c) 13 litres (d) 19.25 litres

22. Marvin would like to maximize the first messages he can see by doing so the appropriate speed. How should he change the speed?
    (a) Increase the speed
    (b) Decrease the speed
    (c) Maintain the speed at 60km/hour
    (d) Cannot be determined

Directions for Questions 22–23: Answer the questions based on the following information.

There are five machines – A, B, C, D, and E – situated on a straight line in distances of 10m, 20 m, 30 m, 40 m and 50m respectively from the origin of the line. A robot is stationed at the origin of the line. The robot serves the machines with raw material whenever a machine becomes idle. All the raw materials are located at the origin. The robot is in an idle state at the origin at the beginning of a day. As soon as one or more machines become idle, they send messages to the robot-station and the robot starts and visits all the machines from which it received messages. If a message is received at the station while the robot is away from it, the robot takes notice of the message only when it returns to the station. While moving, it serves the machines in the sequence in which they are encountered, and then returns to the origin. If any messages are pending at the station when it returns, it repeats the process again. Otherwise, it remains idle at the origin till the next message(s) is (are) received.

22. Suppose on a certain day, machines A and D have sent the first two messages to the origin at the beginning of the first second. C has sent a message at the beginning of the 7th second, B at the beginning of the 8th second and E at the beginning of the 10th second. How much distance has the robot travelled since the beginning of the day, when it receives the message of E? Assume that the speed of movement of the robot is 10m/s.
    (a) 140 m (b) 80 m
    (c) 340 m (d) 360 m

23. Suppose there is a second station with raw material by the robot at the other extreme of the line which is 50 m from the origin, i.e., 10m from E. After it is at rest at the origin, i.e., 10m from E. After finishing the services in a trip, the robot returns to the nearest station. If both stations are equidistant, it chooses the origin as the station to return to. Assuming that both stations receive the messages sent by the machines and that all the other data remains the same, what would be the answer to the previous question?
    (a) 120 (b) 160
    (c) 140 (d) 170

24. One bacteria splits into eight bacteria of the next generation. But due to environmental factors only 50% of a generation survive. If the eighth generation number is 8192 million, what is the number in the first generation?
    (a) 1 million (b) 2 million
    (c) 4 million (d) 8 million

25. I bought 10 pens, 14 pencils and 4 erasers. Ram bought 12 pens, 8 erasers and 28 pencils for an amount which was half more when I had paid. What percent of the total amount paid by me was paid for the pens?
    (a) 77.5% (b) 62.5%
    (c) 50% (d) None of the a

Answers (Block 3 Preassessment Test)

1. (b)	2. (a)	3. (c)	4. (c)	5. (b)
6. (a)	7. (c)	8. (c)	9. (c)	10. (c)
11. (a)	12. (c)	13. (b)	14. (d)	15. (b)
16. (a)	17. (b)	18. (a)	19. (d)	20. (b)
21. (b)	22. (c)	23. (a)	24. (b)	25. (b)

SCORE INTERPRETATION ALGORITHM

Block 3 Preassessment Test

If You Scored: <7: (In Unlimited Time)

Step One: Go through the block three **Back to School** Section carefully. Grasp each of the concepts explained in that part carefully. I would recommend that you go back to your Mathematics school books (CBSE/ CBSE) Class 8,9 & 10 if you feel you need it.

Step Two: Move into each of the chapters of the block three by one.

When you do so, concentrate on clearly understanding each of the concepts explained in the chapter theory.

Step Three: Then move onto the LOD 1 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1. While doing so do not think about the time requirement. Once you finish solving LOD 1, review the questions and their solution processes.

Step Four: Go to the first review test given at the end of the block and solve it. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve the test further without a time limit and try to evaluate the improvement in your unlimited time score.

In case the growth in your score is not significant, go back to the theory of each chapter and review each of the LOD 1 questions for all the chapters.

Step Five: Move to LOD 2 and repeat the process that you followed in LOD 1 in each of the chapters. Concentrate on understanding each and every question and it's underlying concepts.

Step Six: Go to the second review test given at the end of the block and solve it. Again, while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

Step Seven: Move to LOD 3 only after you have solved and understood each of the questions in LOD 1 & LOD 2. Repeat the process that you followed in LOD 1 – going into each chapter one by one.

Step Eight: Go to the remaining review tests given at the end of the block and solve them. Again, while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

In case the growth in your score is not significant, go back to the theory of each chapter and review each of the LOD 1 & 2 questions for all the chapters.

If You Scored:7–15 (In Unlimited Time)

Although you are better than the persons following the instructions above, obviously there is a lot of scope for the development of your score. You will need to work both on your concepts as well as speed. Initially emphasize more on the concept development aspect of your preparations. Then slowly shift your emphasis onto speed development. The following process is recommended for you.

Step One: Go through the **Block three Back to Basics Section** carefully. Revise each of the concepts explained in that part. Going through your 8th, 9th and 10th standard books will be an optional exercise for you. If with the recommended is case you scored in single digits, and if your score is in two digits, I leave the choice to you.

Step Two: Move into each of the chapters of the Block three by one.

When you do so, concentrate on clearly understanding each of the concepts explained in the chapter theory.

Step Three: Then move onto the LOD 1 & LOD 2 exercises. These exercises will provide you with the level of challenge. Try to solve each and every question provided under LOD 1 & 2. While doing so do not think about the time requirement. Once you finish solving LOD 1, review the questions and their solution processes. Repeat the same process for LOD 2.

Step Four: Go to the first review test given at the end of the block and solve it. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve the test further without a time limit and try to evaluate the improvement in your unlimited time score.

Step Five: Go to the second review test given at the end of the block and solve it. Again, while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

In case the growth in your score is not significant, go back to the theory of each chapter and review each of the LOD 1& LOD 2 questions for all the chapters.

Step Six: Move to LOD 3 only after you have solved and understood each of the questions in LOD 1 & LOD 2. Repeat the process that you followed in LOD 1 – going into each chapter one by one.

Step Seven: Go to the remaining review tests given at the end of the block and solve them. Again, while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

In case the growth in your score is not significant, go back to the theory of both the chapters and to solve all the LOD 1 & LOD 3 questions.

If You Scored 15+ (In Unlimited Time)

Congrats you are much better than the first two category of students. Hence unlike them, your focus should be on developing your speed by picking up the shortcut processes explained in this book. Besides, you might also need to pick up concepts that might be hazy in your mind. The following process of development is recommended for you:

Step One: Quickly review the concepts given in the block time back to School Sections. Only go deeper into a concept in case you find it new. Going back to school level books is not required for you.

Step Two: Move into each of the chapters of the block one by one.

When you do so, concentrate on clearly understanding each of the concepts explained in the chapter theory.

Then move onto the LOD 1 & LOD 3 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1 & 3. While doing so try to work on faster processes for solving the same questions. Concentrate on how you could have solved the same question faster. Also try to think of how each time you took over the calculations.

Step Three: Go to the first review test given at the end of the block and solve it. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve the test further without a time limit and try to evaluate the improvement in your unlimited time score.

Step Four: Go to the second review test given at the end of the block and solve it. Again, while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

Step Five: In case the growth in your score is not significant (esp. under time limit), review each of the LOD 1 & LOD 2 questions for all the chapters.

Step Six: Move to LOD 3 only after you have solved and understood each of the questions in LOD 1 & LOD 2. Repeat the process that you followed in LOD 1 – going into each chapter one by one.

Step Seven: Go to the remaining review tests given at the end of the block and solve them. Again, while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

5

PERCENTAGES

INTRODUCTION

In our opinion, the chapter on Percentages forms the most important chapter (apart from Number Systems) in the syllabus of the CAT and the XLRI examination. The importance of percentages is accentuated by the fact that there are a lot of questions related to the use of percentage in all chapters of commercial arithmetic (especially Profit and Loss, Ratio and Proportion, Time and Work, Time, Speed and Distance).

Besides, the calculation skills that you can develop while going through the chapter on percentages will help in a Reading Data Interpretation (DI) calculations. A close look at the topic will yield that at least 80% of the DI questions in any DI paper is constituted of calculations in ratios and percentage.

BASIC DEFINITION AND UTILITY OF PERCENTAGE

Percent literally means 'for every 100' and is derived from the French word 'cent', which is French for 100.

The basic utility of Percentage arises from the fact that it is one of the most powerful tools for comparison of numerical data and information. It is also one of the simplest tools for comparison of data.

In the context of business and economic performance, it is basically used for comparing data such as profits, growth rate, performance, magnitudes and so on.

Mathematical Definition of Percentage : The concept of percentage literally implies to ratios, and the percentage value of a ratio is arrived at by multiplying by 100 the decimal value of the ratio.

For example, a student scores 20 marks out of a maximum possible 30 marks. His marks can then be denoted as 20 out of 30 = (20/30) or (20/30) × 100% = 66.66%.

The process for getting this is perfectly illustrated through the unitary method:

	Marks get		*Out of*
then,	20	out of →	30
	x	out of →	100

Then the value of $x \times 30 = 20 \times 100$

$x = (20/30) \times 100 \rightarrow$ the percentage equivalent of a ratio.

Now, let us consider a classic example of the application of percentage:

Example: Student *A* scores 20 marks in an examination out of 30 while another student *B* scores 40 marks out of 70. Who has performed better?

Solution: Just by considering the marks as 20 and 40, we do not a get clear picture of the actual performance of the two students. In order to get a clearer picture, we consider the percentage of marks.

Thus, A gets (20/30) × 100 = 66.66%

While B gets (40/70) × 100 = 57.14%

Now, it is clear that the performance of A is better. Consider another example.

Example: Company *A* increases its sales by 1 crore rupees while company *B* increases its sales by 10 crore rupees. Which company has grown more?

Solution: Apparently the answer to the question seems to be company *B*. The question cannot be answered since we

[illegible] the previous year's sales figure. Although on the face of it, Company B seems to have grown more.

If we [illegible] information saying that company A had a sales turnover of Rs. 1 crore in the previous year and complete B had a sales turnover of Rs. 100 crore in the previous year, we can compute growth rates and say that A is growing at a rate of 100% and B is growing at 10%. Hence, company A has a higher growth rate, even though in terms of absolute value terms of sales, company B has grown much more.

IMPORTANCE OF BASE/ DENOMINATOR FOR PERCENTAGE CALCULATIONS

Mathematically, the percentage value can only be calculated for ratio that by definition, must have a denominator. Hence, one of the most critical aspects of the percentage is the denominator, which in other words is also called the base value of the percentage. No percentage calculation is possible without knowing the base to which the percentage is to be calculated.

Hence, whenever faced with the question 'What is the percentage ...?' always try first to find out the answer to the question 'Percentage to what base?'

CONCEPT OF PERCENTAGE CHANGE

Whenever the value of a measured quantity changes, the change can be captured through

(a) Absolute value change or
(b) Percentage change.

Both these measurements have their own advantages and disadvantages.

Absolute value change: It is the actual change in the measured quantity. For instance, if sales in year 1 is Rs. 2500 crore and the sales in year 2 is Rs. 2600 crore, then the absolute value of the change is Rs. 100 crore.

Percentage change: It is the percentage change got by the formula

$$\text{Percentage change} = \frac{\text{Absolute value change}}{\text{Original quantity}} \times 100$$

$$= \frac{100}{2500} \times 100 = 4\%$$

As seen earlier, this often gives us a better picture of the effect of the change.

Note: The base used for the sake of percentage change calculations is always the original quantity unless otherwise stated.

Example: The population of a city grows from 20 lakh to 22 lakh. Find the

(a) percentage change
(b) percentage change based on the final value of population

Solution: (a) percentage change = (2/20) × 100 = 10%
(b) percentage change on the final value = (2/22) × 100 = 9.09%

Difference between the Percentage Point Change and the Percentage Change

The difference between the percentage point change and the percentage change is best illustrated through an example. Consider this:

The savings rate as a percentage of the GDP was 25% in the first year and 30% in the second year. Assume that there is no change in the GDP between the two years. Then:

Percentage point change in savings rate = 30% − 25% = 5 percentage points.

$$\text{Percentage change in savings rate} = \frac{30-25}{25} \times 100$$

= 25%.

PERCENTAGE RULE FOR CALCULATING PERCENTAGE VALUES THROUGH ADDITIONS

Illustrated below is a powerful method of calculating percentages. In my opinion, the ability to calculate percentages through this method depends on your ability to handle 2-digit additions. Unless you develop the skill to add 2-digit additions in your mind, you are always likely to face problems in calculating percentage through the method illustrated below. In fact, trying this method without being strong at 2-digit additions/subtractions (including 2-digit after decimal point) would prove to be a disadvantage in your attempt at calculating percentages fast.

This process, essentially being a commonsense process, is best illustrated through a few examples.

Example: What is the percentage value of the ratio 55/81?

Answer: The process involves removing of the 100%, 50%, 25%, 12.5% and so forth of the denominator from the numerator.

Thus, 55/81 can be rewritten as: (40.5 + 12.5x81) + 40.5/81 + (12.5x81) = 50% + (12.5%)

= 50% + (81 ÷ 4 x 12.5) = 50% + 10% + (4.4%)

= 60% + (4.4%)

At this stage you know that the answer to the question lies between 60 – 70%. (Since 4.4 is less than 10% of 81)

At this stage, you know that the answer to the calculation will be in the form: 6x.3x.dc

All you need to do is find out the value of the missing digit.

In order to do this, calculate the percentage value of 4.4/81 through the normal process of multiplying the numerator by 100.

Thus the % value of $\frac{4.4}{81} = \frac{4.4 \times 100}{81} = \frac{440}{81}$

(Note: Use the multiplication by 100 since you have the 10% stage. This step reduces the decimal calculation.)

Thus $\frac{440}{81} = 5.0$, with a remainder of 35

Our answer is now refined to 65.5x.dc (1% Range)

Next, in order to find the next digit (first one after the decimal) add a zero to the remainder:

Hence, the value of 'x' will be the quotient of

$4 \rightarrow 350/81 = 4$ Remainder 26

Answer: 65.4x0b (0.1% Range)

$x \rightarrow 260/81 = 3$ Remainder 17

Answer: 65.43 (0.01% Range)

and so forth.

The advantages of this process are two fold:

(1) You only calculate as long as you need to in order to eliminate the options. Thus, in case there was only a single option between 60 – 70% in the above question, you could have stopped your calculations right here.

(2) This process allows you to go through with the calculation as long as you need to.

Finally, remember what I had advised you right at the start. Fraction skills are a primary requirement for using this method properly.

To illustrate another example:

What is the percentage value of the ratio $\frac{223}{72}$?

223/72 → 300 – 310% Remainder 7

300/72 → 4. Hence, 300 + 310%. Remainder 52

520/72 → 7. Hence, 309.7. Remainder 16

160/72 → 2. Hence, 309.72 Remainder 16

Hence, 309.7222 (2 recurs since we enter an infinite loop of 160/72 calculations).

In any view, percentage rule (at J-call 10) is one of the best ways to calculate percentages since it gives you the flexibility to calculate the percentage value up to as many digits after decimals as you are required to and at the same time allows you to stop the moment you attain the required accuracy range.

Effect of a Percent Change in the Numerator on a Ratio's Value

The numerator has a direct relationship with the ratio, that is, if the numerator increases the ratio increases. The percentage increase in the ratio is the same as the percentage increase in the numerator, if the denominator is constant.

Thus, $\frac{22}{40}$ is exactly 10% more than $\frac{20}{40}$ (in terms of percentage change).

Percentage Change Graphic and its Applications

In mathematics there are many situations where one is required to work with percentage changes. In such situations the following thought structure (Something I call Percentage Change Graphic) is a very useful tool.

What I call Percentage Change Graphic (PCG) is best illustrated through an example:

Suppose you have to increase the number 20 by 20%. Visualize this as follows:

$$20 \xrightarrow[\times 1.2]{20\% \uparrow} 24$$

The PCG has 6 major applications listed and explained below. PCG applied to:

1. Successive changes
2. Positive change applications

3. Product variables application
4. $x - R \times x$ application
5. Transitionally similar to Ratio Change application
6. Use of PCG to calculate Ratio Changes

Application 1: PCG Applied to Successive Changes

This is a very common situation in most questions.

Suppose you have to solve a question in which a number 30 has two successive percentage increases (20% and 10% respectively).

The solution is handled in the following way using PCG:

$$30 \xrightarrow[\times 1.2]{20\% \text{ increase}} 36 \xrightarrow[\times 1.1]{10\% \text{ increase}} 39.6$$

Illustration

A's salary increases by 20% and then decreases by 20%. What is the net percentage change in A's salary?

Solution:

$$100 \xrightarrow[\times 1.2]{20\% \text{ inc.}} 120 \xrightarrow[\times .8]{20\% \text{ decrease}} 96$$

Thus, A's salary has gone down by 4%.

Illustration

A trader gives successive discounts of 10%, 20% and 10% respectively. The percentage of the original cost price he is selling at is:

Solution:

$$100 \xrightarrow[\times .9]{10\% \text{ discount}} 90 \xrightarrow[\times .8]{20\% \text{ discount}} 72 \xrightarrow[\times .9]{10\% \text{ discount}} 64.8$$

Hence the overall discount is 35.2% and the answer is 64.8%.

Illustration

A trader marks up the price of his goods by 20%, but to a particularly haggling customer he ends up giving a discount of 10% on the marked price. What is the percentage profit he makes?

Solution:

$$100 \xrightarrow[\times 1.2]{20\% \text{ increase}} 120 \xrightarrow[\times .9]{10\% \text{ discount}} 108$$

Hence, the percentage profit is 8%.

Application 2: PCG applied to Product Change

Suppose you have a product of two variables say 10×10. If the first variable changes to 11 and the second variable changes to 12, what will be the percentage change in the product? (Note there is a 10% increase in one part of the product and a 20% increase in the other part.)

The formula given for this situation goes as: $10 + 20 + \frac{10 \times 20}{100}$

Hence, Required % change $= 10 + 20 + \frac{10 \times 20}{100}$

(Where 10 and 20 are the respective percentage changes in the two parts of the product) (This is being taught as a shortcut at most institutes across the country currently.)

However, a much easier solution for this case can be visualised as:

$100 \xrightarrow[\times 1.2]{20\% \uparrow} 120 \xrightarrow[\times 1.1]{10\% \uparrow} 132$. Hence, the final product shows a 32% increase.

Similarly suppose $10 \times 10 \times 10$ becomes $11 \times 12 \times 13$. In such a case the following PCG will be used:

$$100 \xrightarrow[\times 1.2]{20\% \uparrow} 120 \xrightarrow[\times 1.2]{20\% \uparrow} 156 \xrightarrow[\times 1.1]{10\% \uparrow} 171.6$$

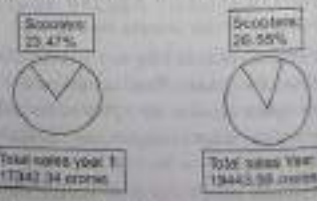
Hence, the final product sees a 71.6 percent increase (Since, the product changes from 100 to 171.6)

Note: You will get the same result irrespective of the order in which you use the respective percentage changes.

Also note that this process is very similar to the one used for calculating successive percentage changes.

Application for DI:

Suppose you have two pie charts as follows:



If you are asked to calculate the percentage change in the sales revenue of scooters for the company from year one to year two, what would you do?

The formula for percentage change would give as:

$$\frac{(0.2655 \times 19443.56) - (0.2347 \times 17342.34) \times 100}{(0.2347 \times 17342.34)}$$

$$\text{i.e.} \quad \frac{\text{New Sales Revenue} - \text{Original Sales Revenue}}{\text{Original Sales Revenue}} \times 100$$

Obviously this calculation is easier and that does not involve the Product-change application of PCG where we do involve this calculation with a lot of case computations. Consider the following situation.

Product for year one is: 0.2347 × 17942.34

Product for year two is: 0.2655 × 19463.39

There can be approximations into 234 × 175 and 265 × 194 respectively. (Note that by moving two decimal digits we do not end up losing any accuracy. We have elaborated this point in the chapter on Ratio and Proportion.)

The overall percentage change depends on two individual percentage changes:

234 increases to 265. A % change of 31.2794 = 31.2 % approx. This calculation has to be done using the percentage rule for calculating the percentage value of the ratio.

175 increases to 194. A percentage change of approximately 12%.

Then PCG will give the answer as follows:

$$100 \xrightarrow[+31]{\text{31.2\% ↑}} 131.2 \xrightarrow[+12\%]{\text{12\% ↑}} 135.7\%$$

Hence, 36 % % increase in the product's value. (Note that the value on the calculator for the full calculation was any approximation is 26.82 %, and given the fact that we have come extremely close to the answer—the method is proven enough to solve the question with a reasonable degree of accuracy.)

Application 3 of PCG: Product Constancy Application (Inverse proportionality)

Suppose you have a situation wherein the price of a commodity has gone up by 25%. In case you are required to keep the total expenditure on the commodity constant, you would obviously need to cut down on the consumption. By what percentage? Well, PCG gives you the answer as follows:

$$100 \xrightarrow[+25]{\text{25\% ↑}} 125 \xrightarrow[-25]{\text{25\% ↓}} 100$$

Hence, the percentage drop in consumption to offset the price increase is 20%.

Now it is the mistake to discover the percentage drop required in the second part of the product if we put increase by 50 percent.

Note: Product constancy is just another name for inverse proportionality.

Table 5.1 gives you some standard values for this kind of a situation.

Application 4 of PCG: A→B→A

Very often we are faced with a situation where we compare two entities say A and B. In such cases, if we are given a relationship from A to B, then the reverse relationship can be determined by using PCG in much the same way as the product constancy was shown above.

Illustration

B's salary is 25% more than A's salary. By what percent is A's salary less than B's salary?

$$100(A) \xrightarrow[+25]{\text{25\% ↑}} 125(B) \xrightarrow[-25]{\text{25\% ↓}} 100(A)$$

A drop of 25 on 125 gives a 20% drop.

Hence A's salary is 20% less than B's.

Note: The values which applied for Product Constancy also apply here. Hence Table 5.1 is useful for this situation also.

Application 5 of PCG → Effect of change in Denominator on the Value of the Ratio

The denominator has an inverse relationship with the value of a ratio.

Hence the process used for product constancy (and explained above) can be used for calculating percentage change in the denominator.

For instance, suppose you have to evaluate the difference between two ratios:

Ratio 1 10/20

Ratio 2 10/25

As is evident the denominator is increasing from 20 to 25 by 25%.

If we calculate the value of the two ratios we will get:

Ratio 1 = 0.5, Ratio 2 = 0.4.

$$\text{\% change between the two ratios} = \frac{0.1}{0.5} \times 100 = 20\% \text{ Drop}$$

This value can be get through PCG as:

$$100 \longrightarrow 125 \longrightarrow 100 \text{ Hence, 20\% drop}$$

Note: This is exactly the same as Product constancy and works here because the numerator is constant.

Hence, $R_1 = N/D_1$ and $R_2 = N/D_2$

i.e. $R_1 = D_2 = N$ and $R_2 = D_1 = N$, which is the product constancy situation.

Table 5.1 **Product Consistency Table, Inverse Proportionality Table, A → B → A table, Ratio Change to Denominator table**

Product FF is Constant F increases (%)		F Decreases (%)
A → B	A → B % increase B → A% decrease	
F is directly proportional to F	F increases (%)	F decreases (%)
Ratio change effect of Denominator change	Denominator increases (%)	(Ratio decreases%)
Denominator change effect of Ratio change	Ratio increases (%)	As Denominator decreases (%)
Standard Value 1	8.33	8.33
Standard Value 2	10	9.09
Standard Value 3	11.11	10
Standard Value 4	12.5	11.11
Standard Value 5	14.28	12.5
Standard Value 6	16.66	14.28
Standard Value 7	20	16.66
Standard Value 8	25	20
Standard Value 9	33.33	25
Standard Value 10	50	33.33
Standard Value 11	100	33.3
Standard Value 12	66.66	40
Standard Value 13	75	42.85
Standard Value 14	100	50

Application to Use of PCG to Calculate Ratio Changes:

Under inverse situations, you will be faced with ratios where both numerator and denominator change. The process to handle and calculate such changes is also quite convenient if you go through PCG.

Illustration

Calculate the percentage change between the Ratios.
Ratio 1 = 10/20 Ratio 2 = 15/25

The answer in this case is 0.5 → 0.6 (20% increase). However, in most cases calculating the values of the ratios will not be easy. The following PCG process can be used to get the answer.

When 10/20 changes to 15/25, the change occurs primarily due to two reasons:

(A) Change in the numerator (Numerator effect)

(B) Change in the denominator (Denominator effect)

By segregating the two effects and calculating the effect due to each separately, we can get the answer easily as follows:

Numerator Effect The numerator effect on the value of the ratio is the same as the change in the numerator. Hence to calculate the numerator effect, just calculate the percentage change in the numerator.

In this case the numerator 1 clearly changing from 10 to 15 (i.e. a 50% increase.) This signifies that the numerator effect is also 50%.

Denominator Effect As we have just seen above, the effect of a percentage change in the denominator on the value of the ratio is seen by calculating the denominator's percentage change in the reverse order.

In this case, the denominator is changing from 20 to 25. Hence the denominator effect will be seen by going from 25 to 20 i.e. 20% drop.

With these two values, the overall percentage change in the Ratio is seen by:

$$100 \xrightarrow[\text{Numerator Effect}]{50\%\uparrow} 150 \xrightarrow[\text{Denominator Effect}]{20\%\downarrow} 120$$

This means that the ratio has increased by 20%.
I leave it to the student to practice such calculations with more complicated values for the ratios.

Implications for Data Interpretation

Percentage is perhaps one of the most critical links between QA and Data Interpretation.

In the chapter theory mentioned above, the Percentage Rule for Percentage Calculations and the PCG applied to product change and ratio change are the most critical.

As already shown, the use of PCG to calculate the percentage change in a product (as exhibited through the product example above) as well as the use of PCG to calculate ratio changes are two extremely useful applications of the concepts of percentages into DI.

Step 3: Percentages for the special case of comparing two ratios to find the larger one.

Suppose you have two ratios to compare. Say $R_1 = N_1/D_1$ and $R_2 = N_2/D_2$.

The first step is to find the two percent ranges for each of these ratios. In case they belong to different ranges of 10% (say R_1 lies between 30-40% while R_2 lies between 50-60%), it is relatively simple to say which one will be larger.

In case both of these values for percentage of the ratios belong to the same ten percent range, then we can use the following process.

Step 1: Calculate the percentage change in the numerator.
Step 2: Calculate percentage change in the denominator.

Then look for four cases in this situation, when we have two Ratio, in Ratio,

Case 1: Numerator is increasing while denominator is decreasing → obviously the net effect of the two changes will be an increase in the ratio. Hence, R_2 will be greater.
Case 2: Numerator is decreasing while denominator is increasing → obviously the net effect of the two changes will be a decrease in the ratio. Hence, R_1 will be greater.

Now only in the following cases that we need to look at the respective changes in the Numerator and denominator.

Case 3: Numerator and denominator are both increasing
Calculate the percentage value of the respective increases. If the numerator is increasing more than the denominator the ratio will go up. On the other hand, if the denominator is increasing more than the numerator, Ratio, will be smaller than Ratio, (Note: Increase in percentage values).
Case 4: Numerator and denominator are both decreasing → Calculate the percentage value of the respective decreases. If the numerator is decreasing more than the denominator the ratio will go down. On the other hand, if the denominator is decreasing more than the numerator, Ratio, will be greater than Ratio,

FRACTION TO PERCENTAGE CONVERSION TABLE

The following percentage values appear repeatedly over the entire area where questions can be framed on the topic of percentage. Further, it would be of great help to you if you are able to recognize these values separately from values that do not appear in the Table 5.2.

Some Utilisations of the Table

- The values that appear in the table are all percentage values. These can be converted into decimals by just

Table 5.2 Percentage Conversion Table

	1	2	3	4	5	6	7	8	9	10	11	12
1	100											
2	50	100										
3	33.33	66.66	100									
4	25	50	75	100								
5	20	40	60	80	100							
6	16.66	33.33	50	66.66	83.33	100						
7	14.28	28.57	42.85	57.14	71.42	85.71	100					
8	12.5	25	37.5	50	62.5	75	87.5	100				
9	11.11	22.22	33.33	44.44	55.55	66.66	77.77	88.88	100			
10	10	20	30	40	50	60	70	80	90	100		
11	9.09	18.18	27.27	36.36	45.45	54.54	63.63	72.72	81.81	90.90	100	
12	8.33	16.66	25	33.33	41.66	50	58.33	66.66	75	83.33	91.66	100
13	7.69	15.38	23	30.76	38.46	46.15						
14	7.14	14.28	21.42	28.57	35.71	42.85						
15	6.66	13.33	20	26.66	33.33	40						
16	6.25	12.5	18.75	25								
17	5.88	11.76	17.64	23.52	29.41	35.29						
18	5.55	11.11	16.66	22.22	27.77	33.33	38.88	44.44	50			
19	5.26	10.52	15.78	21.05	26.31	31.57	36.84	42.10	47.36			
20	5	10	15	20	25	30	35	40	45	50		
21	4.76	9.52	14.28	19.04	23.80	28.57	33.33	38.09	42.85	47.61	52.38	
22	4.54	9.09	13.63	18.18	22.72	27.27	31.81	36.36	40.90	45.45	50	54.54
23	4.34	8.69	13.04	17.39	21.74	26.08	30.43	34.78	39.13	43.47	47.82	52.17
24	4.16	8.33	12.5	16.66	20.83	25	29.16	33.33	37.5	41.66	45.83	50
25	4	8	12	16	20	24	28	32	36	40		
26	3.84	7.69	11.53	15.38	19.23	23.07	26.92	30.76	34.61	38.46	42.30	46.15
27	3.70	7.40	11.11	14.81	18.51	22.22	25.92	29.63	33.33	37.03	40.74	44.44
28	3.57	7.14	10.71	14.28	17.85	21.42	25	28.57	32.14	35.71	39.28	42.85
29	3.44	6.89	10.34	13.79	17.24	20.69	24.13	27.58	31.03	34.48	37.93	41.38
30	3.33	6.66	10	13.33	16.66	20	23.33	26.66	30	33.33	36.66	40
31	3.22	6.45	9.67	12.9	16.12	19.35	22.58	25.81	29.03	32.25	35.48	38.71
32	3.12	6.25	9.37	12.5	15.62	18.75	21.87	25	28.12	31.25	34.37	37.5
33	3.03	6.06	9.09	12.12	15.15	18.18	21.21	24.24	27.27	30.30	33.33	36.36
34	2.94	5.88	8.82	11.76	14.70	17.64	20.58	23.52	26.46	29.41	32.35	35.29
35	2.85	5.71	8.57	11.42	14.28	17.14	20	22.85	25.71	28.57	31.42	34.28
36	2.77	5.55	8.33	11.11	13.88	16.66	19.44	22.22	25	27.77	30.55	33.33
37	2.70	5.40	8.10	10.81	13.51	16.21	18.91	21.62	24.32	27.02	29.72	32.42
38	2.63	5.26	7.89	10.52	13.15	15.78	18.42	21.05	23.68	26.31	28.94	31.57
39	2.56	5.12	7.69	10.25	12.82	15.38	17.94	20.51	23.07	25.64	28.20	30.76
40	2.50	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
41	2.44	4.88	7.32	9.76	12.19	14.63	17.07	19.51	21.95	24.39	26.82	29.26
42	2.38	4.76	7.14	9.52	11.90	14.28	16.66	19.04	21.42	23.80	26.18	28.57
43	2.32	4.65	6.97	9.30	11.62	13.95	16.27	18.60	20.93	23.25	25.58	27.90
44	2.27	4.54	6.81	9.09	11.36	13.63	15.90	18.18	20.45	22.72	25	27.27
45	2.22	4.44	6.66	8.88	11.11	13.33	15.55	17.77	20	22.22	24.44	26.66
46	2.17	4.34	6.52	8.69	10.86	13.04	15.21	17.39	19.56	21.73	23.91	26.08
47	2.12	4.25	6.38	8.51	10.63	12.76	14.89	17.02	19.14	21.27	23.40	25.53
48	2.08	4.16	6.25	8.33	10.41	12.5	14.58	16.66	18.75	20.83	22.91	25
49	2.04	4.08	6.12	8.16	10.20	12.24	14.28	16.32	18.36	20.40	22.44	24.48
50	2	4	6	8	10	12	14	16	18	20	22	24

Denotes the row and a column value is the/their value

shifting the decimal point by two places to the left. Thus 30.769 = 0.0769 in decimal value.

- A second meaning from this table is in the process of division by any of the numbers such as 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 15, 16, 24 and so on, students normally face problems in calculating the decimal values of these divisions. However, if one gets used to the decimal values that appear in the Table 2.2, calculation of decimals in divisions will become very simple. For instance, when an integer is divided by 3, the decimal values can only be .04, .26, .42, .57, .71, .85 or .00. (These are approximate values)
- This also means that the difference between two values like $\frac{5}{6} - \frac{5}{7}$ can be integral if and only if x is divisible by both 6 and 7.

This principle is very useful as an advanced short cut for option based solution of some questions. I leave it to the student to discover applications of this principle.

Calculation of Multiplication by Numbers like 1.21, 0.83 and so on

In my opinion, the calculation of multiplication of any number by a number of the form 0.xy or of the form 1.ab should be viewed as a subtraction/addition situation, and not as a multiplication situation. This can be explained as follows.

Example: Calculate 1.23 × 473.

Solution: If we try to calculate this by multiplying, we will end up going through a very time taking process, which will yield the final value at the end but nothing before that (i.e. you will have no clue about the answer's range till you reach the end of the calculation).

Instead, one should view this multiplication as an addition of 23% to the original number. This means, the answer can be got by adding 23% of the number to itself.

Thus 473 × 1.23 = 473 + 23% of 473 = 473 + 94.6 + 5% of 473 = 567.6 + 14.18 = 581.78

(The percentage rule can be used to calculate the addition and get the answer.)

The similar process can be utilised for the calculation of decimals such as 0.87

(Answer can be got by subtracting 13% of the number from itself and this calculation can again be done by percentage rule.)

Hence, the student is advised to become thoroughly with the percentage rules. Percentage calculations & additions of 2 & 3 digit numbers.

WORKED-OUT PROBLEMS

Problem 5.1 A sells his goods 30% cheaper than B and 30% dearer than C. By what percentage is the cost of C's goods cheaper than B's goods.

Solution There are two alternative processes for solving this question.

1. Assume the price of C's goods as p. Then A's goods are at 1.3 p and B's goods are such that A's goods are 30% cheaper than B's goods. i.e. A's goods are priced at 70% of B's goods.

Hence, 1.3 p → 70

B's price → 100

B's price = 130 p/70 = 1.8571 p

Then, the percentage by which C's price is cheaper than B's price =

(1.8571 p − p) × 100/(1.8571 p) = 600/13 = **46.15%**

Learning task for student Could you answer the question: Why did we assume C's price as a variable p and then work out the problem on its basis. What would happen if we assumed B's price as p or if we assumed A's price as p?.

2. Instead of assuming the price of one of the three we assume the price as 100.

Let B = 100. Then A = 70, which is 30% more than B. Hence C = 23.07% less than A (from Table 4.1) = approx 53.84. Hence answer is 46.15% approximately.

(This calculation can be done mentally if you are able to work through the calculations by the use of percentage rules. The students are advised to try to assume the value of one for each of the variables A, B and C and see what happens to the calculations involved in the problem. Since the value of 100 is assumed for a variable to minimize the number of calculations to solve the problems, we should ensure that the variable assumed as 100 should have the minimum calculations associated with it.)

Note: In fact this problem and the ones that follow come under one of the most basic exercises in the chapter of percentages. The questions at the first level of difficulty appear in examinations like CET, Mahanirmitha, Bank PO, MAT, Bank MAT, NMIMS, NIFT, NLSI and many other similar exams. Hence, if you are able to do the questions illustrated here mentally, you would be able to solve LOD 1 questions easily and gain a significant time advantage over your competitors.

However, for the actual CAT exercise, the hours used in LOD 1 problems would normally be used as a part of the warm-up. But would be wise to see this in the context of the second and the third level of difficulties in the exercises later in the chapter. Hence, developing the process for solving questions of the LOD 1 level would help you gain an improved speed for the LOD and questions.

The remainder for these percentages are the basis for a lot of conceptual explanations as well as for calculations in the Data Interpretation section, developing skills in calculation and problem solving illustrated here would prove very useful in helping you clear aptitude exams.

Problem 5.2 The length and the breadth of a rectangle are found by +20% and by −10% respectively. What is the percentage change in the area of the rectangle.

Solution The area of a rectangle is given by: length × breadth. If an equation, then by:

Area = $L \times B = LB$ → then we will get the changed area as

$$\text{Area}_{\text{changed}} = 1.2\ L \times 0.9\ B = 1.08\ LB$$

Hence the change in area is 8% increase.

Note: You can solve (and in fact, think the problem) using your mind reading by using percentage change rules as follows:

So $\xrightarrow{+20\%} 120 \xrightarrow{-10\%} 108$, Hence, the percentage change is 8%.

Problem 5.3 Due to a 25% price hike in the price of rice, Jenny is able to purchase 20 kg less of rice for Rs. 400. Find the initial price.

Solution Since price is rising by 25%, consumption has to decrease by 20%. But there is an actual reduction in the consumption by 20 kg. Thus, 20% decrease in consumption is equal to a 20 kg drop in consumption.

Hence, original consumption is 100 kg of rice.

Money spent being Rs. 400, the original price of rice is Rs. 5 per kg.

(Thus, you use the benefit of internalising the product consumer table. It is left to the student to analyse why and how the product consumer table applies here.)

Problem 5.4 A's salary is 20% lower than B's salary, which is 15% higher than C's salary. By how much percent is C's salary more than A's salary?

Solution The equation approach here would be

$$A = 0.8\ B$$
$$B = 0.85\ C$$
$$A = 0.8 \times 0.85\ C$$

Then

$A = 0.68\ C$ (Use percentage change graphic to calculate the value of 0.68)

Thus, A's salary is 68 % of C's salary.

If A's salary is 68, B's salary is 100.

Using percentage change graphic

$$68 \xrightarrow[+32]{(3200/68)\%} 100$$

Students are advised to refrain from using equations to solve questions of this nature. In fact, you can adopt the following process, which can be used while you are reading the problem, to get the result faster.

Assume one of the values as 100. (Remember, selection of the right variable that has to take the value of 100 may make a major difference to your solving time and effort required. The thumb rule for selecting the variable whose value is to be taken as 100 is based on three principal considerations:

Select as 100, the variable

1. With the maximum number of percentage calculations associated with it.
2. Select as 100 the variable with the most difficult calculations associated with it.
3. Select as 100 the variable at the start of the problem solving chain.

The student will have to develop his own judgement in applying these principles in specific cases.

Here I would take C as 100, setting B as 85 and A as 68. Hence, the answer is (32 × 100/68)%.

[illegible] [illegible]

Solution: [illegible]

[illegible]

[illegible] [illegible]

Solution: [illegible]

Return process for this problem: [illegible]

[illegible]

Through options: [illegible]

[illegible]

Level of Difficulty (LOD)

1. [illegible]

2. [illegible]

3. [illegible]

4. [illegible]

[illegible] Banerjee is worried about the balance of his monthly budget. The price of petrol has increased by 40%. By what percent should he reduce the consumption of petrol so that he is able to balance his budget?

(a) 33.33 (b) 28.56
(c) 25 (d) 14.28
(e) None of these

[illegible] 5, if Mr. Banerjee wanted to limit the increase in his expenditure to 5% on his basic expenditure on petrol then what should be the corresponding decrease in consumption so that expenditure exceeds only by 5%?

(a) 33.33 (b) 28.56
(c) 25 (d) 20
(e) None of these

7. Ram sells his goods 25% cheaper than Shyam and 25% costlier than Rham. How much percentage is Rham's goods cheaper than Shyam's?

(a) 33.33% (b) 50%
(c) 66.66% (d) 40%
(e) None of these

8. In an election between 2 candidates, Bhola gets 65% of the total valid votes. If the total votes were 6000, what is the number of valid votes that the other candidate Bhawn gets if 25% of the total votes were declared invalid?

(a) 1625 (b) 1975 (c) 1675 (d) 1525
(e) None of these

9. In a medical certificate, by mistake a candidate gave his height as 25% more than normal. In the interview panel, he clarified that his height was 5 feet 5 inches. Find the percentage correction made by the candidate from his stated height to his actual height.

(a) 20 (b) 28.56
(c) 25 (d) 16.66
(e) None of these

10. Ajit Bhawn generally wears his father's coat. Unfortunately, his cousin Shaurya poked him one day that he was wearing a coat of length more than his height by 15%. If the length of Ajit's father's coat is 126 cm, find the actual length of his coat.

(a) 95 (b) 108 (c) 104.34 (d) 101.52
(e) 100.14

11. A number is mistakenly divided by 5 instead of being multiplied by 5. Find the percentage change in the result due to this mistake.

(a) 96% (b) 95%
(c) 140% (d) 200%
(e) None of these

12. Harsh wanted to subtract 5 from a number. Unfortunately, he added 5 instead of subtracting. Find the percentage change in the result.

(a) 300% (b) 66.66%
(c) 50% (d) 33.33%
(e) Cannot be determined

13. If 63% of x = 13% of y, then find the value of x if y = 2000.

(a) 200 (b) 300 (c) 400 (d) 500
(e) None of these

14. In a mixture of 80 litres of milk and water, 25% of the mixture is milk. How much water should be added to the mixture so that milk becomes 20% of the mixture?

(a) 20 litres (b) 15 litres
(c) 25 litres (d) 24 litres
(e) None of these

15. 30% of a% of b is 75% of b% of c. Which of the following is c?

(a) 1.5a (b) 0.667a (c) 0.5a (d) 1.25a
(e) 1.00a

16. A landowner increased the length and the breadth of a rectangular plot by 10% and 20% respectively. Find the percentage change in the cost of the plot assuming land prices are uniform throughout his plot.

(a) 53% (b) 35%
(c) 22.22% (d) 28.56%
(e) None of these

17. The height of a triangle is increased by 40%. What can be the maximum percentage increase in length of the base so that the increase in area is restricted to a maximum of 40%?

(a) 50% (b) 20% (c) 14.28% (d) 25%
(e) 33.33%

18. The length, breadth and height of a room in the shape of a cuboid are increased by 10%, 20% and 50% respectively. Find the percentage change in the volume of the cuboid.

(a) 77% (b) 75% (c) 88% (d) 90%
(e) 99%

19. The salary of Amit is 30% more than that of Varun. Find by what percentage is the salary of Varun less than that of Amit?

(a) 28.12% (b) 23.08%

(c) 21.25% (d) 27.27%
(e) None of these

20. The price of sugar is reduced by 20% but inspite of the decrease Anjesh ends up increasing his expenditure on sugar by 20%. What is the percentage change in his monthly consumption of sugar ?
(a) 44% (b) 40% (c) +33.33% (d) 50%
(e) None of these

21. The price of rice falls by 20%. How much rice can be bought now with the money that was sufficient to buy 20 kg of rice previously ?
(a) 5 kg (b) 15 kg (c) 25 kg (d) 30 kg
(e) 20 kg

22. 20% of a number when subtracted from 91, gives the number itself. Find the number.
(a) 80 (b) 65
(c) 70 (d) 75
(e) None of these

23. When 60% of a number A is added to another number B, B becomes 175% of its previous value. Then which of the following is true regarding the values of A and B?
(a) A > B
(b) B > A
(c) B ≥ A
(d) Either (a) or (b) can be true depending upon the values of A and B
(e) Nothing can be said

24. In an election, the candidate who got 55% of the votes won by 144 votes. Find the total number of voters on the voting list if 80% people cast their vote and there were no invalid votes.
(a) 360 (b) 720 (c) 1600 (d) 1500
(e) 1000

25. The population of a village is 1,00,000. The rate of decrease is 10% per annum. Find the population at the end of the third year?
(a) 1,35,100 (b) 1,21,000
(c) 1,18,800 (d) 1,30,000
(e) None of these

26. The population of the village of Gaon is 10,000 at the beginning. It increases by 20% in the first year. However, in the second year, due to emigration, the population drops by 5%. Find the population at the end of the third year if in the first year the population increases by 30%.
(a) 11,140 (b) 13,340
(c) 1,27,540 (d) 12,540
(e) 11,240

27. A man invests Rs. 10,000 in some shares in the ratio 2 : 3 : 5 which pay dividends of 10%, 25% and 20% (on his investment) for that year respectively. Find the dividend income.
(a) 1000 (b) 2000
(c) 2050 (d) 1950
(e) 1850

28. In an examination, Mohit obtained 20% more than Sushant but 10% less than Rajesh. If the marks obtained by Sushant is 1080, find the percentage marks obtained by Rajesh if the full marks is 2000.
(a) 86.66% (b) 72%
(c) 38.33% (d) 77.77%
(e) None of these

29. In a class, 25% of the students were absent for an exam. 30% failed by 20 marks and 10% just passed because of grace marks of 5. Find the average score of the class if the remaining students scored an average of 60 marks and the pass marks are 35 (counting the final scores of the candidates).
(a) 37.266 (b) 37.6
(c) 37.8 (d) 36.93
(e) 37.5

30. Ram spends 20% of his monthly income on his household expenditure, 15% of the rest on books, 30% of the rest on clothes and saves the rest. On counting it comes to know that he has finally saved Rs. 875. Find his monthly income.
(a) 10000 (b) 15000
(c) 20000 (d) 12000
(e) None of these

31. Hans and Bhaskar have salaries that jointly amount to Rs. 10,000 per month. They spend the same amount monthly and then it is found that the ratio of their savings is 6 : 1. Which of the following can be Hans' salary?
(a) Rs. 8000 (b) Rs. 5000
(c) Rs. 4000 (d) Rs. 3000

32. The population of a village is 5500. If the number of males increases by 11% and the number of females increases by 20%, then the population becomes 6150. Find the population of females in the town.
(a) 2500 (b) 3000
(c) 2000 (d) 3500

32. Vicky's salary is 75% more than Asha's. Vicky got a raise of 20% on his salary while Asha got a raise of 25% on his salary. By what percent is Vicky's salary more than Asha's?

(a) 46% (b) 51.1%
(c) 50% (d) 52.1%

33. In a shelf, the first row contains 25% more books than the second row and the third row contains 25% less books than the second row. If the total number of books contained in all the rows is 800, then find the number of books in the first row.

(a) 250 (b) 225
(c) 300 (d) None of these

34. An ore contains 25% of an alloy that has 60% iron. Other than this, in the remaining 75% of the ore, there is no iron. How many kilograms of the ore are needed to obtain 40 kg of pure iron?

(a) 250 kg (b) 275 kg
(c) 300 kg (d) 266.66 kg

35. Last year the Indian cricket team played 40 one-day cricket matches out of which they managed to win only 60%. This year so far it has played some matches, which has made it mandatory for it to win 60% of the remaining matches to maintain its existing winning percentage. Find the number of matches played by India so far this year?

(a) 30 (b) 25
(c) 28 (d) Insufficient Information

36. The population of a village is 1,00,000. Increase rate per annum is 10%. Find the population at the starting of the fourth year.

(a) 133,100 (b) 121,000
(c) 131,000 (d) None of these

37. In the recent climate conference in New York, out of 700 men, 500 women, 800 children present inside the meeting premises, 20% of the men, 40% of the women and 10% of the children were Indians. Find the percentage of people who were not Indian?

(a) 75% (b) 77% (c) 79% (d) 83%

38. A cow and a calf cost Rs. 2000 and Rs. 1400 respectively. If the price of the cow and that of the calf increased by 20% and 30% respectively then the price of 3 dozen cows and 2 dozens calves is

(a) 72,000 (b) 71,360
(c) 74,240 (d) None of these

40. Rice sells for grade 20% cheaper than Bobby and 20% dearer than Chandiya. How much percentage is Chandiya's goods cheaper/dearer than Bobby's

(a) 33.33% (b) 50%
(c) 42.85% (d) None of these

41. During winter, an elder can run 'x' metres on one bottle of Glucose. But in the summer, he can only run 0.5 x metres on one bottle of Glucose. How many bottles of Glucose are required to run 400 metres during summer?

(a) 800/x (b) 800/x
(c) 4x (d) 4x/0.5

42. Out of the total production of iron from hematite, an ore of iron, 20% of the ore gets wasted, and out of the remaining iron, only 25% is pure iron. If the pure iron obtained in a year from a mine of hematite was 80,000 kg, then the quantity of hematite mined from that mine in the year is

(a) 5,00,000 kg (b) 4,00,000 kg
(c) 4,50,000 kg (d) None of these

43. A man buys a truck for Rs. 2,50,000. The annual repair cost comes to 2.0% of the price of purchase. Besides, he has to pay an annual tax of Rs. 2000. At what monthly rent must he run out the truck to get a return of 15% on his net investment of the first year?

(a) Rs. 3330 (b) Rs. 2500
(c) Rs. 4000 (d) Rs. 3212.50

44. Recently, while shopping in Patna Market in Bihar, I came across two new shirts selling at a discount. I decided to buy one of them for my little boy Sharry. The shopkeeper offered me the first shirt for Rs. 42 and said that it usually sold for 8/7 of that price. He then offered me the other shirt for Rs. 36 and said that it usually sold for 7/6th of that price. Of the two shirts which one do you think is a better bargain and what is the percentage discount on it?

(a) first shirt, 12.5% (b) second shirt, 14.28%
(c) Both are same (d) None of these

45. 40% of the voters in Bellary promised to vote for Sonia Gandhi and the rest promised to vote for Sushma Swaraj. Of these voters, 30% of the voters who had promised to vote for Sonia Gandhi, did not vote on the election day, while 20% of the voters who had promised to vote for Sushma Swaraj did not vote on the election day. What is the total no. of votes polled if Sonia Gandhi got 216 votes?

(a) 280 (b) 300 (c) 260 (d) 190

46. In an examination, 40% students passed in Physics, 35% in Chemistry while 12% failed in both the subjects. If 528 students passed in both the subjects. Find the total number of students who appeared in the examination.
(a) 800 (b) 400 (c) 300 (d) 600

47. Rohan spends 30% of his salary on home rent, 30% of the rest he spends on his children's education and 24% of the total salary he spends on clothes. After his expenditure, he is left with Rs. 2500. What is Rohan's salary?
(a) Rs. 11,904.25 (b) Rs. 20,000
(c) Rs. 10,000 (d) Rs. 12,000

48. The ticket value at the Minerva theatre in Mumbai is worth Rs. 250. When the price of the ticket was lowered, the sale of tickets increased by 50% while the collection recorded a decrease of 17.5%. Find the deduction in the ticket price.
(a) Rs. 150 (b) Rs. 112.5
(c) Rs. 165 (d) Rs. 120

49. Ravi's monthly salary is A rupees. Of this he spends B rupees. The next month he has an increase of C% in his salary and D% in his expenditure. The new amount saved is
(a) $A(1 + C/100) - B(1 + D/100)$
(b) $(A/100)(C - D) + (B/100)$
(c) $A(C - D)/100$
(d) $B(C + D)/100$

50. In the year 2000, the luxury car industry had two car manufacturers—Maruti and Honda with market shares of 25% and 75% respectively. In 2001, the overall market for the product increased by 50% and a new player BMW also entered the market and captured 12% of the market share. If we know that the market share of Maruti increased to 50% in the second year, the share of Honda in that year is an
(a) 50% (b) 40% (c) 40% (d) 35%

Level of Difficulty (LOD) II

1. Bill Anderson, a very clever businessman, started off a business with very little capital. In the first year, he earned a profit of 20% and invested half of the total capital before capital + profit in a charitable institution. The same course was followed in the 2nd and 3rd years also. If at the end of these years, he is left with Rs. 16,875, then find the amount donated by him at the end of the 2nd year.
(a) Rs. 45,000 (b) Rs. 12,500
(c) Rs. 22,500 (d) Rs. 20,000
(e) None of these

2. In an examination, 40% students failed in Hindi and 12% students in History. 20% students failed in both the subjects. If the number of students who passed the examination was 880, how many students appeared in the examination of the examination conducted only in these two subjects?
(a) 2000 (b) 2200
(c) 2500 (d) 1400
(e) None of these

3. At IIM, Bangalore, 60% of the students are boys and the rest are girls. Further 15% of the boys and 15% of the girls are getting a fee waiver. If the number of those getting a fee waiver is 90. Find the total number of students getting 50% concession, if it is given that 50% of those not getting a fee waiver are eligible to get half fee concession?
(a) 360 (b) 280 (c) 320 (d) 330
(e) 350

4. A machine depreciates in value each year at the rate 10% of its previous value. However, every second year there is some maintenance work so that in that particular year, depreciation is only 5% of its previous value. If at the end of the fourth year, the value of the machine stands at Rs. 1,46,205, then find the value of machine at the start of the first year.
(a) Rs. 1,90,000 (b) Rs. 2,00,000
(c) Rs. 1,95,000 (d) Rs. 2,10,000
(e) Rs. 2,20,000

5. Anshi's project report consists of 25 pages each of 40 lines with 75 characters on each line. In case the number of lines is reduced to 55 but the number of characters is increased to 90 per lines, what is the percentage change in the number of pages. (Assume the number of pages to be a whole number.)
(a) +10% (b) +5% (c) −8% (d) −10%
(e) −12%

6. The price of soap is collectively decided by four factors: research, raw materials, labour, advertisement

[illegible] Assume that the [illegible] remains [illegible]

Price of [illegible] = [illegible] + Raw material cost + Labour costs + Advertising cost + Transportation cost

If there are respective changes of 10%, 20%, −20%, [illegible] and 30% in the five factors, then find the change in the price of soap.

(a) 47% (b) 37%
(c) 46% (d) 36%
(e) Cannot be determined

[illegible] successive raises, Hanif's salary became equal to 1.331 times of his initial salary. By the third percent was the salary raised the first time, if the second raise was twice as high (in percent) as the first?

(a) 1% (b) 20% (c) 25% (d) 30%
(e) 10.1%

The ratio of Ani's salary for October to his salary for November was 1.5 : 1.333 and the ratio of the salary for November to that for December was 2 : 3.666. The worker got 40 rupees more for December than for October and received a bonus constituting 40 percent of the salary for three months. Find the bonus, assuming that the number of workdays is the same in every month.

(a) 44,000 rupees (b) 132.5555 rupees
(c) 22222 rupees (d) 265.6 rupees
(e) None of these

After three successive equal percentage rise in the value the sum of 100 rupees turned into 140 rupees and 10 paise. Find the percentage rise in the salary.

(a) 12% (b) 23% (c) 66% (d) 42%
(e) 7%

Prem goes to a shop to buy a sofa set costing Rs. 1000. The rate of sales tax is 6%. She tells the shopkeeper to reduce the price of the sofa set to such an extent that she has to pay Rs.1000 inclusive of sales tax. Find the percentage reduction needed in the price of the sofa set to just satisfy her requirement.

(a) 6.12% (b) 6.20%
(c) 5% (d) 8.3%
(e) None

The price of a certain article was raised by 10% in 1996. The consumption of the same article was increased from 200 units to 225 units. By how much percent will the expenditure on the article rise in the Indian economy?

(a) 34.25% (b) 22.5%
(c) 20.35% (d) 26.75%
(e) None of these

12. In the university examination last year, Rajesh scored 81% in English and 92% in History. What is the minimum percent he should score in Sociology, which is out of 50 marks (if English and History were for 100 marks each), if he aims at getting 78% overall?

(a) 84% (b) 92% (c) 98% (d) 96%
(e) 86%

13. King Dushasha, at his eleventh hour, called his three queens and distributed his gold in the following way: the gave 50% of his wealth to his first wife, 50% of the rest to his second wife and again 50% of the rest to his third wife. If their combined share is worth 1,93,000 kilograms of gold, find the quantity of gold King Dushasha was having initially?

(a) 1,50,000 kg (b) 1,49,000 kg
(c) 1,51,600 kg (d) 1,52,000 kg
(e) None of these

14. The population of New Foundland increases with a uniform rate of 8% per annum, but due to immigration, there is a further increase of population by 1%. However, this 1% increase in population is to be calculated on the population after the 8% increase and not on the previous years population. Find what will be the percentage increase in population after 2 years?

(a) 16.984 (b) 16.981 (c) 18.24 (d) 17.91
(e) 18.62

15. 10% of Mexico's population migrated to South Asia, 10% of the remaining migrated to America and 10% of the rest migrated to Australia. If the female population, which was left in Mexico, remained only 7,84,900, find the population of Mexico City before the migration and its effects, if it is given that before the migration the female population was half the male population and this ratio did not change after the migration?

(a) 10,00,000
(c) 15,30,000 (b) 12,00,000
(e) 12,50,000 (d) 16,00,000

16. According to a recent library report issued by the Commerce Ministry, Government of India, 30% of the total FDI goes to Chennai and 20% of this goes to

[illegible] If the FDI in Gujarat, which goes to urban areas, is $\frac{1}{3}$ th, then find the size of FDI in rural Andhra Pradesh, which accounts [illegible] of the FDI that comes to Andhra Pradesh, which accounts for 20% of the total FDI?

(a) $\frac{[illegible]}{[illegible]}$ (b) $\frac{[illegible]}{[illegible]}$
(c) $\frac{[illegible]}{[illegible]}$ (d) $\frac{[illegible]}{[illegible]}$
(e) None of these

17. If in question 16, the growth in the size of FDI for the next year with respect to the previous year is 20%, then find the share of urban Maharashtra next year if 12% of the total FDI going to Maharashtra, went to urban areas (provided Maharashtra attracted only 10% of the total share for both years).

(a) $\$30$ m (b) $\$4.32$ m
(c) $\$3$ m (d) $\$5$ m
(e) None of these

18. The cost of food accounted for 25% of the income of a particular family. If the income gets raised by 20%, then what should be the percentage point decrease in the food expenditure as a percentage of the total income to keep the food expenditure unchanged between the two years?

(a) 3.5 (b) 8.33 (c) 4.16 (d) 5
(e) 6.25

19. If the length, breadth and height of a cube are decreased, decreased and increased by 5%, 5% and 20% respectively, then what will be the impact on the surface area of the cube (in percentage terms)?

(a) 7.25% (b) 1% (c) 8.33% (d) 6.9833%
(e) 8.5%

20. A's salary is first increased by 25% and then decreased by 20%. The result is the same as B's salary increased by 20% and then reduced by 25%. Find the ratio of B's salary to that of A's?

(a) 4 : 3 (b) 11 : 10
(c) 10 : 9 (d) 12 : 11
(e) None of these

21. The minimum quantity of milk in three tin whole numbers that should be mixed in a mixture of 65 litres in which the initial ratio of milk to water is 1 : 4 so that the resulting mixture has 15% milk is

(a) 3 (b) 4
(c) 5 (d) 6
(e) This is not possible

22. A person saves 6% of his income. Two years later his income shoots up by 15% but his savings remain the same. Find the hike in his expenditure.

(a) 15.95% (b) 15%
(c) 14.8% (d) 15.5%
(e) None of these

23. A is 50% more than B, C is 2/3 of A and D is 60% more than C. Now, each of A, B, C and D is increased by 10%. Find what per cent of B is D (rounded to the nearest integer).

(a) 150% (b) 160%
(c) 175% (d) 176%
(e) None of these

24. A and B have, between them, Rs. 1200. A spends 75% of his money while B spends 20% of his money. They are then left with a sum that constitutes 30% of the whole sum. Find what amount is left with A.

(a) Rs. 780 (b) Rs. 800
(c) Rs. 700 (d) Rs. 660
(e) Rs. 400

25. Maya has Rs. M with her and her friend Chanda has Rs. C with her. Maya spends 12% of her money and Chanda also spends the same amount as Maya does. What percentage of her money did Chanda spend?

(a) $\frac{18M}{C}$ (b) $\frac{18C}{M}$
(c) $\frac{12M}{C}$ (d) $\frac{12C}{M}$
(e) None of these

26. In a village consisting of p persons, x% can read and write. Of the males alone, y%, and of the females alone, z% can read and write. Find the number of males in the village in terms of p, x, y and z if $y > x$.

(a) $\frac{p(y-z)}{(y+x-z)}$ (b) $\frac{p(x-z)}{(y+x-2z)}$
(c) $\frac{p(y-x)}{(x-z)}$ (d) $\frac{p(x-z)}{(y-z)}$

27. In order to maximise his gain, a theatre owner decides to reduce the price of tickets by 20% and as a result of this, the sales of tickets increase by 40%. As a result of these changes, he is able to increase his weekly collection by Rs. 1,68,000. Find by what amount did the gross collection increase per day?

(a) 14,000 (b) 18,000
(c) 24,000 (d) 20,000

[illegible] consisting of three families A, B and C, the population of the town in which A, B and C are in the ratio [illegible]. In family A, 80% of the people are literate, in family B, 30% of the people are literate. If 500 people in family C are literate, find the percentage literacy in that town.

(a) 41.5% (b) 78%

(c) 75% (d) None of these

29. A number is such that if the double of the numerator and its cube of the denominator is changed by +10% and 30% respectively then we get 11% of 1621. Find the fraction.

(a) $\frac{4}{15}$ (b) $\frac{7}{25}$

(c) $\frac{3}{28}$ (d) None of these

30. In past an examinee, 40% marks are essential. A score 10% marks less than the pass marks and B score 11.11% marks less than A. What percent less was the sum of A's and B's marks should C obtain to pass the exam?

(a) 45% (b) 41.67%

(c) 25% (d) Any of these

31. The hourly wages of a female labour are increased by 12.5% whereas the weekly working hours are reduced by 8%. Find the percentage change in the weekly wages if she was getting Rs. 1200 per week for 50 hours previously.

(a) +12% (b) 4%

(c) 4.5% (d) None of these

32. Two numbers X and Y are 20% and 28% less than a third number Z. Find by what percentage is the number Y less than the number X.

(a) 8% (b) 12% (c) 10% (d) 9%

33. Price of a commodity is first increased by x% and then decreased by x%. If the new price is R1000, find the original price.

(a) $(x + 100)/100R$ (b) $(x^2 - 100^2)/100R$

(c) $(100 - x)/100R$ (d) $100R/(100^2 - x^2)$

34. The salary of a person is increased by Rs. 4800 and the rate of tax is decreased by 2% from 12% to 10%. The effect is such that he is now paying the same as before. If in both the cases, the standard tax deduction is fixed at 20% of the total income, find the increased salary?

(a) Rs. 22,800 (b) Rs. 30,000

(c) Rs. 28,800 (d) None of these

35. Rahul goes to a shop to buy a radio costing Rs. 2568. The rate of sales tax is 7%, and the final value is rounded off to the next higher integer. Shri tells the shopkeeper to reduce the price of the radio so that she has to pay Rs. 2568 inclusive of sales tax. Find the reduction needed in the price of the radio.

(a) Rs. 180 (b) Rs. 210

(c) Rs. 140 (d) None of these

Directions for Questions 36–38: Read the following passage and answer the questions.

In a recent youth fair organised by AMS Careers, the many tickets were sold out according to the following scheme:

Tickets bought in one lot	6	12	18
Percentage discount	10%	20%	25%

Original price per ticket, Rs. 40

This offer could have been availed only when tickets were bought in a fixed lot according to the scheme and any additional ticket was available at its original price.

36. If a person has to buy 25 tickets, then what will be the minimum price per ticket?

(a) Equal to Rs. 32 (b) 32.32

(c) 31.84 (d) Cannot be determined.

37. In the above question, what will be the approximate possible maximum price per ticket (if discounts have been availed for 24 tickets)?

(a) Rs. 30 (b) Rs. 32

(c) Rs. 36 (d) Rs. 36.16

38. On the last day of the fair, with the objective of maximising participation, the number of tickets sold in a lot was halved with the same discount offer. Mr. X is in a fix regarding the number of tickets he can buy with Rs. 552. The maximum number of tickets he can purchase with this money is

(a) 14 (b) 18 (c) 16 (d) 17

39. 800 people were supposed to vote on a resolution, but 15% of the people who had decided to vote for the motion were abducted. However, the opponents of the motion, through some means managed to increase their strength by 100%. The motion was then rejected by a majority, which was 50% of that by which it would have been passed if none of these changes would have occurred. How many people finally voted for the motion and against the motion.

(a) 200 (b) 400 (c) [illegible]

(c) 100 (boy) and 200 (captain)
(d) 150 (boy), 300 (captain)
(e) 200 (boy) and 300 (captain)

40. Of the adult population in Nagpur, 45% are men and 25% of women are married. What percentage of the total population of adults is married (assume that no man marries more than one woman and vice versa)?

(a) 33.33% (b) 32.14%
(c) 33.1% (d) None of these

41. The weight of an iron bucket increases by 33.33% when filled with water to 80% of its capacity. Which of these may be 50% of the weight of the bucket when it is filled with water (assume the weight of bucket and its capacity in kg to be integers)?

(a) 7 kg (b) 6 kg (c) 5 kg (d) 8 kg

42. Australia scored a total of x runs in 50 overs. India tied the scores in 20% less overs. If India's average run rate had been 33.33% higher the scores would have been tied 10 overs earlier. Find how many runs were scored by Australia?

(a) 250 (b) 240
(c) 200 (d) Cannot be determined

43. Due to a 25% hike in the price of rice per kilogram, a person is able to purchase 20 kg less for Rs. 400. Find the increased price of rice per kilogram?

(a) Rs. 5 (b) Rs. 6 (c) Rs. 10 (d) Rs. 4

44. A salesman is appointed on the basic salary of Rs. 1200 per month and the condition that for every sales of Rs. 10,000 above Rs. 10,000, he will get 50% of basic salary and 10% of the sales as a reward. This incentive scheme does not operate for the first Rs. 10000 of sales. What should be the value of sales if he wants to earn Rs. 7600 in a particular month?

(a) Rs. 40,000 (b) Rs. 50,000
(c) Rs. 40,800 (d) None of these

45. In question 44, which of the following income cannot be achieved in a month?

(a) Rs. 6000
(b) Rs. 9000
(c) Both a and b
(d) Any income can be achieved

46. In question 44 despite a 5 percentage point increment in the commission from 20% the total remuneration remained unchanged. Find the change in the volume of the salesman?

(a) −10% (b) −16%
(c) −25% (d) −20%

47. In an assembly election at Surat, the total turnout was 85% out of which 16% of the total votes on the voting list were declared invalid. Find which of the following can be the percentage votes got by the winner of the election if the candidate who came second got 20% of the total votes on the voting list. (There were only three contestants, only one winner and the total number of voters on the voters' list was 20000.)

(a) 44.8% (b) 46.6%
(c) 48% (d) None of these

48. A watch gains by 2% per hour when the temperature is in the range of 40°C–50°C and it loses at the same rate when the temperature is in the range of 20°C–30°C. However, the watch owner is fortunate since it runs on time in all other temperature ranges. On a sunny day, the temperature started soaring up from 8 a.m. in the morning at the uniform rate of 2°C per hour and sometime during the afternoon it started coming down at the same rate. Find what time will it be by the watch at 7 p.m. If at 8 a.m. the temperature was 32°C and at 4 p.m. it was 40°C.

(a) 6 : 55 p.m. (b) 6 : 35 : 12 p.m.
(c) 6 : 55 : 24 p.m. (d) None of these

Questions 49–50: Study the following table and answer the questions that follow.

Beverages	% of Vitamin	% of Minerals	% of Micronutrients	Cost per 25 gram (Rs.)
Pepsi	12	30	30	8
Coke	15	20	10	10
Sprite	20	10	40	7

49. Which of the following beverages contains the maximum amount of vitamins?

(a) Pepsi worth Rs. 16
(b) Coke worth Rs. 15
(c) Sprite worth Rs. 8
(d) All the three worth Rs. 12.5 (125 grams of each)

50. Which of these is the cheapest?

(a) 300 grams of Pepsi + 200 grams of Coke
(b) 300 grams of Coke +100 grams of Pepsi
(c) 100 grams of Coke + 100 grams of Pepsi + 100 grams of Sprite
(d) 900 grams of Coke +100 grams of Sprite

Level of Difficulty (LOD) II

1. The price of raw materials has gone up by 15%. Labour cost has also increased from 25% of the cost of raw material to 30% of the cost of raw material. By how much percentage should there be a reduction in the usage of raw materials so as to keep the cost same?
   (a) 13% (b) 24%
   (c) 28% (d) 25%
   (e) Cannot be determined

2. Mr. A is a computer programmer. He is assigned three jobs (in which time allotted is in the ratio of 4 : 4 : 2 jobs are needed to be done individually). But due to some technical snag, 10% of the time allotted to each job gets wasted. Therefore, owing to the lack of master, he invests only 40%, 30%, 20% of the hours of what was actually allotted to do the three jobs individually. Find how much percentage of the total time allotted is the time invested by A?
   (a) 36.33% (b) 39.4545%
   (c) 32.72% (d) 36.66%
   (e) Cannot be determined

3. In the MOCK CAT paper or AMS, questions were asked in five sections. Out of the total students, 5% candidates cleared the cut-off in all the sections and 3% cleared none. Of the rest, 25% cleared only one section and 20% cleared four sections. If 24.5% of the rest candidates cleared two sections and 30% candidates cleared three sections, find out how many candidates appeared in the MOCK CAT at AMS?
   (a) 1000 (b) 1200 (c) 1500 (d) 2000
   (e) None

4. There are three galleries in a coal mine. On the first day two galleries are operative and after some time, the third gallery is made operative. With this, the output of the mine became half as large again. What is the capacity of the second gallery as a percentage of the first, if it is given that a four-month output of the first and the third galleries was the same as the annual output of the second gallery?
   (a) 75% (b) 84%
   (c) 45% (d) 65%
   (e) None of these

5. 10% of salty sea water contained in a flask was poured out into a beaker. After this, a part of the water contained in the beaker was evaporated by heating and due to this, the percentage of salt in the beaker increased M times. It is known that after the content of the beaker was poured into the flask, the percentage of salt in the flask increased by x%. Find the original quantity of sea water in the flask.
   (a) $\frac{9M+1}{M-1}\%$ (b) $\frac{19M+11}{M-1}\%$
   (c) $\frac{9M-1}{M+1}\%$ (d) $\frac{9M+1}{M+1}\%$
   (e) None of these

6. In an election of 3 candidates A, B and C, A gets 50% votes. If more votes than B, A also beats C by 1,80,00 votes. If it is known that B gets 5 percentage point more votes than C, find the number of voters on the voting list (given 90% of the voters on the voting list voted and no votes were illegal).
   (a) 72,000 (b) 81,000
   (c) 90,000 (d) 1,00,000
   (e) 1,10,000

7. A clock is set right at 12 noon on Monday. It loses 1/2% on the correct time in the first week but gains 1/4% on the true time during the second week. The time shown on Monday after two weeks will be
   (a) 12 : 28 : 12 (b) 11 : 34 : 48
   (c) 12 : 50 : 24 (d) 12 : 24 : 16
   (e) None of these

8. The petrol prices shot up by 7% as a result of the hike in the price of crudes. The price of petrol before the hike was Rs. 28 per litre. Vawal travels 2400 kilometres every month and his car gives a mileage of 18 kilometres to a litre. Find the increase in the expenditure that Vawal has to incur due to the increase in the price of petrol (to the nearest rupee)?
   (a) Rs. 270 (b) Rs. 282
   (c) Rs. 276 (d) Rs. 272
   (e) Rs. 280

9. For question 8, by how many kilometers should Vawal reduce his travel if he wants to maintain his expenditure at the previous level (prior to the price increase)?
   (a) 137 km (b) 127 km
   (c) 168 km (d) 180 km
   (e) None of these

10. In question 9, if Vimal wants to limit the increase in expenditure to Rs. 200, what strategy should he adopt with regard to his travel?
(a) Reduce travel to 2500 kilometres
(b) Reduce travel to 2340 kilometres
(c) Reduce travel to 2360 kilometres
(d) Reduce travel to 2370 kilometres
(e) None of these

11. A shopkeeper announces a discount scheme as follows: for every purchase of Rs. 5000 to Rs. 6000, the customer gets a 15% discount on a ticket that entitles him to get a 7% discount on a further purchase of goods costing more than Rs. 6000. The customer, however, would have the option of trading his right to the shopkeeper at 4% of his initial purchase value (as per the right-value to the 7% discount ticket). In an enthusiastic response to the scheme, 10 people purchase goods worth Rs. 4000 each. Find the maximum possible revenue for the shopkeeper.
(a) Rs. 38,400 (b) Rs. 38,000
(c) Rs. 39,400 (d) Rs. 39,000
(e) Rs. 40,000

12. For question 11, find the maximum possible discount that the shopkeeper would have to offer to the customer.
(a) Rs. 1600 (b) Rs. 3000
(c) Rs. 4000 (d) Rs. 4800
(e) None of these

Directions for Questions 13–16: Read the following and answer the questions that follow.

Two friends Shayan and Kailash own two versions of a car. Shayan owns the diesel version of the car, while Kailash owns the petrol version.

Kailash's car gives an average that is 20% higher than Shayan's (in terms of litres per kilometre). It is known that petrol costs 60% of its price higher than diesel.

13. The ratio of the cost per kilometre of Kailash's car to Shayan's car is
(a) 3 : 1 (b) 1 : 3
(c) 1.92 : 1 (d) 2 : 1
(e) Cannot be determined

14. If Shayan's car gives an average of 20 km per litre, then the difference in the cost of travel per kilometre between the two cars is
(a) Rs. 0.5 (b) Rs. 1.5
(c) Rs. 2.5 (d) Rs. 3
(e) Cannot be determined

15. For question 14, the ratio of the cost per kilometre of Shayan's travel to Kailash's travel is
(a) 3 : 1 (b) 1 : 3
(c) 1 : 1.92 (d) 2 : 1
(e) Cannot be determined

16. If diesel costs Rs. 12.5 per litre, then the difference in the cost of travel per kilometre between Kailash's and Shayan's is (assume an average of 20 km per litre for Shayan's car and also assume that petrol is 50% of its own price higher than diesel)
(a) Rs. 1.75 (b) Rs. 0.875
(c) Rs. 1.25 (d) Rs. 1.125
(e) None of these

Directions for Questions 17–23: Read the following and answer the questions that follow.

In the island of Hoola Boola Moola, the inhabitants have a strange process of calculating their average income and expenditures. According to an old legend provided on the island, the average monthly income had to be calculated on the basis of 14 months in a calendar year, while the average monthly expenditure was to be calculated on the basis of 11 months per year. This would lead to people having an underestimation of their savings since there would be an underestimation of the income and an overestimation of the expenditure per month.

17. Mr. Boogie Woogie comes back from the USSR and convinces his community comprising 273 families to start calculating the average income and the average expenditure on the basis of 12 months per calendar year. Now if it is known that the average monthly income in his community is (according to the old system) 67 moolahs per month, then what will be the percentage change in the savings of the community if Mr. Boogie Woogie (assume that there is no other change)?
(a) 12.31% (b) 22.22%
(c) 31.31% (d) 33.33%
(e) Cannot be determined

18. For question 17, if it is known that the average estimated monthly expenditure is 19 moolahs per month for the island of Hoola Boola Moola, then what will be the percentage change in the estimated savings of the community?
(a) 12.42% (b) 38.05%
(c) 28.23% (d) 26.66%
(e) Cannot be determined

19. For question 18, if it is known that the average estimated monthly expenditure was 22 thousands per month for the community of Bongli Wongli having 275 families, then what will be the percentage change in the estimated savings of the community?

(a) 30.57% (b) 28.14%
(c) 25.23% (d) 25.71%
(e) Cannot be determined

20. For question 19, what will be the percentage change in the estimated average income of the community recalculated on the basis of the new estimated savings?

(a) 16.28% increase (b) 14.26% decrease
(c) 16.66% increase (d) 16.66% decrease
(e) None of these

21. If the finance minister of the island Mr. Bhola Ram declares that henceforth the average monthly income has to be estimated on the basis of 12 months per year while the average monthly expenditure is to be estimated on the basis of 11 months in the year, what will happen to the savings in the economy of Bhola Bonla Manla?

(a) Increase (b) Decrease
(c) Remain constant (d) Either (b) or (c)
(e) Cannot be determined

22. For question 21, what will be the percentage change in savings?

(a) 3.1% (b) 1.32%
(c) 2.1% (d) 3.2%
(e) Cannot be determined

23. For question 22, what will be the percentage change in the estimated monthly expenditure?

(a) 22.32% decrease (b) 22.22% increase
(c) 18.18% decrease (d) 18.18% increase
(e) None of these

24. Abhisarini Banerjee has 72% vision in his left eye and 68% vision in his right eye. On consulting through a mass wearing contact lenses, which improve his vision by 15% in the left eye and 11% in the right eye. Find out the percentage of normal vision that he gets after consulting therapy. (Assume that a mass after consulting therapy (Assume that a person's eyesight is a multiplicative summer of the eyesight of his left and right eyes)

(a) 32.5% (b) 42.5% (c) 72.5% (d) 68.5%
(e) None of these

25. A shopkeeper gives 3 consecutive discounts of 10%, 15% and 15%, after which he sells his goods at a percentage profit of 40.05% on the C.P. Find the value of the percentage profit that the shopkeeper would have earned if he had given discounts of 10% and 15% only.

(a) 53% (b) 62.5% (c) 72.5% (d) 68.6%
(e) 69.2%

26. If the third discount in question 25 was Rs. 2,29,50, then find the original marked price of the item.

(a) Rs. 1,00,000 (b) Rs. 1,25,000
(c) Rs. 2,00,000 (d) Rs. 2,50,000
(e) Rs. 2,25,000

27. Kishan Bari, a motorist uses 24% of his fuel in covering the first 20% of his total journey (in city driving conditions). If he knows that he has to cover another 25% of his total journey in city driving conditions, what should be the minimum percentage increase in the fuel efficiency for non-city driving over the city driving fuel efficiency, so that he is just able to cover his entire journey without having to refuel? (Approximately)

(a) 39.2% (b) 43.5%
(c) 45.6% (d) 41.2%

Directions for Questions 28–30: Read the following and answer the questions that follow. The BSNL announced a cut in the STD rates on 27 December 2001. The new rates and slabs are given in the table below and are to be implemented from the 14 January 2002.

Slab Details

Distance	Rates (Rs./min)			
	Peak Rates		Off Peak	
	Old	New	Old	New
50–200	4.8	2.4	1.2	1.2
201–500	11.6	4.8	3	2.4
501–1000	17.56	6.00	4.8	4.5
1000+	17.56	6.00	6	4.5

28. The maximum percentage reduction in costs will be experienced for calls over which of the following distances?

(a) 50–200 (b) 501–1000
(c) 1000+ (d) 201–500

29. The percentage difference in the cost of a set of telephone calls made on the 13th and 14th January having durations of 4 minutes over a distance of 300 km, 5 minutes for a distance of 700 km and 3 minutes for a

(b) 31.7%
(c) 33.2%
(d) 33.3%
(e) Cannot be determined

30. If one of the three calls in question 29 were made in an off peak time on both days, then the percentage reduction in the total cost of the calls between 12th and 18th January will

(a) Definitely reduce
(b) Definitely increase
(c) Will depend on which particular call was made in an off peak time
(d) Cannot be determined

Directions for Questions 31–33: Read the following carefully and answer the questions that follow.

The circulation of the *Deccan Herald* newspaper is 1,71,000 copies, while its closest competitors are *The Times of Hindustan* and *India's Times*, which sell 2,47,000 and 20% more than that respectively (rounded off to the higher thousand). All the newspapers cost Rs. 2 each. The hawkers' commissions obtained by the three papers are 20%, 25% and 30% respectively (these commissions are calculated on the sale price of the newspaper). Also, it is known that newspapers earn primarily through sales and advertising.

31. Taking the base as the net revenue of *Deccan Herald*, the percentage difference of the net revenue (newspaper − commission obtained to have earn) between *Deccan Herald* and *India's Times* is

(a) 24.82%
(b) 30.32%
(c) 26.26%
(d) None of these

32. The ratio of the percentage difference in the total net revenue between Deccan Herald and India's Times to the percentage difference in the total revenue between Deccan Herald and India's Times is

(a) 1.000
(b) 0.3727
(c) 0.6726
(d) Cannot be determined

33. If the cost of printing the newspaper is Rs. 8, 1.5 and 1 (approximately) per day for Deccan Herald, Times of Hindustan and India's Times respectively and on any day the available advertising space in the Deccan Herald newspaper is 900 sq. inches (approximate) and the advertising rate for Deccan Herald is Rs. 3000 per sq. inch, then the percentage of the advertising space that must be utilized to recover the full recovery of the day's cost for Deccan Herald is

(a) 95.87%
(b) 99.46%
(c) 97.28%
(d) Cannot be determined

34. Based on the data in the previous questions and the additional information that the space available in India's Times is 1000 sq. inch and that in the Times of Hindustan is 1100 sq. inch, find the percentage point difference in the percentage of advertising space to be utilized in India's Times and that which must be utilized in Times of Hindustan so that both newspapers just break even.

(a) 4.5
(b) 5.2
(c) 10
(d) Cannot be determined

35. For the data in questions 33 and 34 if it is known that the advertising rate in Times of Hindustan is Rs. 3000 per sq. inch and that in the India's Times is Rs. 2500 per sq. inch, then what is the percentage point difference in the percentage of advertising space to be utilized by Times of Hindustan and India's Times so that both of them are just able to break even?

(a) 4.18
(b) 5.0
(c) 4.00
(d) Cannot be determined

36. On a train journey, there are 3 kinds of tickets AC I, AC II, AC III, 3-tier, and general. The relationship between the rates of the tickets for the Rand is: AC II is 50% higher than AC III and AC I is 50% of AC III's rates higher than the AC II ticket's rate. The 3-tier ticket is 25% of the AC I's ticket and the general ticket is 1/3 the price of the AC II ticket. The AC II ticket costs 700 euros between London and Paris. The difference in the rates of 3-tier and general ticket is

(a) 41.25 euros
(b) 55.8 euros
(c) 48.75 euros
(d) 52.75 euros

37. For the above question, the total cost of one ticket in each class will be

(a) 3233.75
(b) 3533.75
(c) 4233.75
(d) 3733.75

Directions for Question 38–40: Read the following and answer the questions that follow.

A Rajadhipriya train has 2 AC I boggies having 24 berths each, 3 AC II boggies having 43 berths each, 5 AC III boggies having 64 berths each and 12 3-tier boggies having 96 berths each. There are no general boggies in the train. ₹ 200 euros is the cost of an AC 3-tier berth from London to Glasgow, answer the following questions.

40. The value of the investment, in rupees, transfer from the Indian Rupee Account to London and back is

(a) 2,77,000 (b) 2,44,000
(c) 2,48,400 (d) 2,96,500

41. For a Bombay-London journey from London to Glasgow, one of the three non-stop tickets booked across classes, what percentage of the total revenue came out of the sale of 1st class tickets

(a) 44.23% (b) 52.14%
(c) 36.23% (d) 38.9%

42. If bookings for the above operation was 40% in AC I, 35% in AC II, 10% in AC III and 35% in 3 tier, then what will happen to the percentage contribution of Train to the total revenues, on the main journey

(a) Increase (b) Decrease
(c) Remain constant (d) Cannot be determined

43. A gas gas cylinder lasts for 304 hours when the meter valve on the gas stove is fully opened while it lasts for 80 hours when the larger burner on the gas stove is fully opened. Which of these values are the closest to the percentage difference in the usage of gas gas between the smaller and the larger burner.

(a) 26.25% (b) 30%
(c) 0.25% (d) 29.07%

44. In question 43, assume that the rate of gas dispersal is directly proportional to the degree of opening of the valve of the gas. If we are given that the smaller burner is open to 60% of its maximum and the larger burner is open to 50% of its maximum, the percentage increase in the percentage difference between the smaller burner and the larger burner (in terms of hours of use) is

(a) 72.25% (b) 73.33%
(c) 9.44% (d) None of these

45. Sunil Sharma has a salary of Rs 15,000 per month. In the first month of the year, he spends 40% of his income on food, 20% on clothing and saves 11,110 of his income for the month. In the next two months, he saves 10% of what he has spent expending 34.15% of his income on food. In the fourth month, he gets an increment of 11.11%, on his salary and spends every single Rupee on celebrating his raise. But from the fifth month onwards, good sense prevails on him and he spends 15%, 15%, 15%, 20%, 20%, 4.5%, 12.5%, 15% and 5% on his new income per month. The value of the rate of the savings for the two months having the highest savings in the rate of the savings for the two months having the lowest savings is

(a) 2 times (b) 5.5125
(c) 8 (d) None of these

46. In an economy, the rate of savings has a relation to the investments in industry for that year and the following three years. The relation is such that a percentage point change in investment in industry for that year has a relation to the total production output in the next 4 years. A 2 percentage point increase in the savings rate in a year increases the investment in the industry of the economy by 19. Further, the rate of investment also goes up by 0.5% in the next year, by 0.25% in the second year and again by 0.25% in the third year. Also assume that the investment in an economy is only dependent on the pattern of savings in the previous 5 years in the economy. Also, the percentage change in the investment in a particular year is got by adding the effect of the previous three years savings pattern.

In fiscal 2006-08, the rate of savings in the Indian economy is 25% while that in the Pakistani economy is 20%. This has remained constant since 1999. In 2009-06 the savings rate in the Indian economy suddenly rose by 5 percentage points to 30% while that in the Pakistani economy rose by 2 percentage points to 22%. It is further known that the value of the investment in the industry in the 2 countries was 2 million dollars and 1.8 million dollars respectively (for the previous year). The percentage difference between the investment in the Pakistani economy to the investment in the Indian economy in 2006-07 will be (if it is known that there is no change in the savings rate in 2006-07)

(a) 13.6% (b) 15.12%
(c) 11.19% (d) 17.2%

Directions for Questions 45–48: In an economy the rate of savings has a relation to the investment in industry for that year and for the following three years and the investment in industry for that year has a relation to the total production output in the next 4 years.

47. For question 44, if there is an additional change in the savings rate with 2007-08, then the percentage difference in the value of the investment in India to the investment in Pakistan in 2007-08 (as a percentage of the investment in India) is

(a) 11.26% (b) 14.16%

(c) 14.42% (d) None of these

46. If the change in production is directly related to the change in investment in the previous year, and if the rate of the average rate change for the previous 2 decades are to be assumed rate, then for which year the difference between the production in the Indian economy and the production in the Pakistani economy show the maximum percentage change?

(a) 2006-07 (b) 2007-08
(c) 2008-09 (d) Cannot be determined

47. For question 46, it is known that the percentage change is to consider in a year leads to a corresponding way equal percentage increase in the manufacturing production in the next year. Further, if the growth rate of manufacturing production is 27% of the GDP growth rate of the country, then what is the GDP growth rate of India in 2006-07?

(a) 4.52% (b) 2.26%
(c) 9.29% (d) None of these

48. The Euro was adopted as on the 1st January 2002 and the old currencies of the European countries were exchanged into Euro. In France, 4 Francs were exchanged for 1 Euro while in Germany 5 Deutsche Marks were exchanged for 1 Euro and in Italy 3 Lirae were exchanged for 1 Euro. The exchange rate for Moolah, the official currency of Hoola Boola Moola, was set at 50 Moolahs per Euro. Dr. Krishna Iyer, an NRI desire travel in Europe, had a practice across each of these three countries and he sends back money orders to his native island of Hoola Boola Moola. The existing exchange rate of Moolahs with the above mentioned currencies was 51 moolahs per Franc, 96 Moolahs per Deutsche Mark and 70 moolahs per Lira. If Dr. Iyer has the information, then what should he do with his currency holdings in these three currencies as on the 31st December 2001 so that he maximizes his moolah value on the 1st of January 2002. (Assume no arbitrage possibilities between the three currencies)

(a) Change to Francs
(b) Change to Deutsche Marks
(c) Change to Lirae
(d) Remain indifferent

49. For the above questions, the exchange rates for the three currencies with respect to a dollar were 25 per Franc, 1.25 per Euro and 1.4 dollar per Deutsche Mark. If Dr. Iyer has 100 lirae, 100 Deutsche Marks and 50 Francs on 31st December 2001, the maximum percentage change he can achieve in his net holdings in terms of dollars due to the arbitrage created by the three currencies could be

(a) 17.23% (b) 7.23%
(c) 11.2% (d) Cannot be determined

50. For question 48, which one of the following will allow the calculation of all possibilities of percentage change in terms of moolah value of Dr. Iyer's portfolio. That is possible through currency conversions

(a) Dr. Iyer's money holding in all three currencies
(b) Dr. Iyer's monthly earnings in all three currencies
(c) The inter-currency conversion rates between Lirae, Deutsche Mark and Francs
(d) Both a and c

Hints and Solutions

1. Assume the initial value to be 100 and solve.
3. Total number of students = Full fee waiver + 50% concession + No concession.
6. Assume initial value of price = 100.
Since, the price is a multiplicative function, we have
$100 \times 1.1 \times 1.2 \times 0.8 \times 1.25 \times 1.5$
Solve using percentage change graphic.
8. Salary ratio is 2.25 : 2.26666.
Hence 0.41666 = Rs. 40
Then, 6.9166 = Rs. 664
9. Solve using options
13. Assume initial amount of gold to be 100.
Then he gives away: 50 + 25 + 12.5 = 87.5
But 87.5 = 1209900 kg
Hence, 100 = 148600
18. If initial income = 100, initial food expenditure = 25
New income = 120
Since, food expenditure is constant at 25, the percentage of the new income = 20.833.
Percentage point change = 25 − 20.833 = 4.166
24. Solve using options.
35. Use standard formulae of percentage.

Alternatively, this problem can also be solved by assuming values for A, B, C and Z. Then compare the options to see which one fits.

10. Solve using options.

11. $100 \times 1.125 \times 0.92 = 103.5$

14. Solve using options.

15. Kilometers required $= \frac{7}{10^{11}} = 256$

16–18. To maximize the discount, tickets need to be bought in two groups of 18 and 6 respectively. The minimum possible price per ticket will occur when tickets are bought in sets of 6.

19. Solve through options.

Checking for options (a) → Final voting is 200 for and 400 against.

Hence, the motion is rejected by 200 votes. The extra five if none of the things had occurred then the motion would have been passed by 400 votes, i.e. 600 (for) and 200 (against). 1/3 of 600 were distracted and the opponents doubled their votes from 200 to 400. Thus all the values fit the answer is (a).

20. Take 45% runs = 29% wickets.

22. Since, run scored = overs × run rate. If overs reduce by 25%, run rate will go up by 33.33%. Hence, Australia could have scored any number of runs.

23. Solid votes = 64%.

The second placed candidate gets 20% votes.

Thus the winner can get between 20.01% to 44% votes.

Hints and Solutions

1. Assume initial raw material price to be 100. This means that the initial labour cost is 25. Hence the sale price is 125. Now, since there is a 15% increment in raw material cost and the labour cost has gone up to 30% of the raw material cost, it is clear that the new total expenditure is $115 + 1.5 = 116.5$. Reduce the sale price to 125 by reducing the usage of raw materials and.

2. Assume that 50, 40 and 20 hours are available. Then it is clear that 100 units of time is two operators.

3. Fall in large agriculture sector 1.5 times for an addition of 10%.

5. Assume values for M and x and solve through options.

6. $A = 1.5B$, $A + C = 150000$ and $B = 1.05 \cdot C$. Solve to get A, B and C. Also, $A + B + C =$ 90% of total votes on voting. Use this will give you the answer. Finally solve this question through options.

7. Clock loses 0.5% of 168 hours in the first week and gains 0.25% of 168 hours in the second week. Hence, net loss is 0.25% of 168 hours.

8. Would save 133.33 litres of petrol every month, while the price of petrol has gone up by Rs. 1.96. Hence, the increase in expenditure = 133.33 × 1.96 = Rs. 261 approximately.

11. Maximum revenue for the shopkeeper will occur when the minimum discount offer is used by the customer. This level is 4%.

12. This is the case of maximum discomfort.

Hints for questions 13–16

	Diesel Shyam	Petrol Kailash
Average (in litre per km)	x	$1.2x$
Cost of fuel (in Rs./litre)	$0.4\,p$	p

13. Average in litre per kilometre multiplied by the Cost of fuel in Rs./litre will give the required cost per kilometre.

14. Shyam's car gives 20 km/litre means 0.05 litres per kilometre than Kailash's car gives 0.06 litre/km. However, since we do not know the price of petrol or diesel we cannot find out the difference in the cost of travel.

15. This question is the opposite of question 13.

16. Cost of petrol is Rs. 25 per litre. Cost per kilometre for Shyam = 12.5 × 0.05.

Also, cost per kilometre for Kailash = 25 × 0.06.

Hints for Questions 17–23

Estimated Average savings

$$= \frac{\text{Annual Income}}{14} - \frac{\text{Annual Expenditure}}{9}$$

17. The value will depend on the values of annual expenditure which is not available.

18. Average estimated monthly expenditure is given for the island of Hindu Basin Mumbai and not for Mr. Bunglie Bunglie's community.

19. Original estimated savings = 87 − 22 = 65 Mundaka.

New estimated savings = 121.5/12 − 100/12 = 85.

24. $0.72 = 1.15 \times 0.84 = 1.14$.

25. Solve through options. A 15% reduction on the new price will give a profit of 36.36%.

26. For the last decrease being 22.00%, it means that the value price in this 18th decade must have been 1,31,000 (making with increase

$$\frac{22.00}{100} = \frac{100-x}{x} \Rightarrow \frac{100-x}{x} = \frac{22}{100} \Rightarrow x = 1,31,000$$

Hence option (c) is correct.

27. For 42% of the journey in city driving conditions, 30% of the fuel is consumed.

Hence, for the remaining 58% journey, 48% fuel is left.

Required increase in fuel efficiency

$$= \frac{\frac{52}{30} - \frac{48}{54}}{\frac{48}{54}} \times 100$$

28. The maximum percentage reduction in peak rates is for the 200 – 300 category.

29. $\frac{(4\times1.16+3\times17.38+5\times17.36)-(4\times4.8+1\times9+5\times9)}{4\times1.16+3\times17.36+1\times17.36}$

30. Loss to be made up over the $= 37,80,000 \times (1.60) = 6.4 \times 37,800$

So 'x' is required to be only $= \frac{37,800 \times 6.4}{3000}$

34. Interesting rates have not been mentioned. Hence, we cannot solve the question.

36-40. The rates are as

AC III → 100 Economy, AC – II → 120

AC I → 140, 2 Tier → 47.5, General → 40

Also, AC – II = 180 Rows for a Lencher – Paris journey

36. $147.5 \times 40 \times 6.5 = 38,39$

37. $100 \times 120 \times 300 \times 47.5 \times 40 \times 6.5$

38. Maximum revenue on a return journey means 100% bookings both ways.

39. $\frac{\text{Revenues from 3-Tier}}{\text{Total Revenues}} \times 100$

41. $\frac{\frac{114}{100} - \frac{114}{90}}{\frac{114}{100}} = \frac{\frac{114}{100} - \frac{114}{90}}{\frac{114}{100}} = 36\%$

42. Original percentage difference = 50%

In new system reducing the smaller 60% will lay 100 × 120 = 120 hours.

Similarly the larger one will be $\frac{80}{0.4} = 200$ hours.

Thus, the smaller one lasts $\frac{(133.33 - 100)}{133.33} \times 100 = 25\%$ 8.33% more than the larger one.

Thus, required answer $= \frac{30 - 8.33}{30} \times 100 = 72.23\%$

44. The 8% point increase in savings rate will increase for a 2.5% increase in investment in 2015-06 and further 1.25% increase in investment in 2006-07.

Thus, Indian investment in 2006-07 = 2 million × 1.025 × 1.0125 similarly, calculate for Pakistan.

45. Use the same process as for the previous question.

46. Cannot be determined since we do not know the initial values of the production output.

47. Since there is a 2.5% increase in investment in 2005-06, there will be a 2.5% increase in manufacturing production in 2006-07.

Thus, GDP growth rate $= \frac{2.5}{0.27} = 9.26\%$

Detailed Solutions & Short Cuts

LOD 1

3. 25% of 50% of 75% of 70% = 20/100 × 50/100 × 75/100 × 70 = 0.2 × 0.5 × 0.75 × 70 = 5.25.

A quicker way to think here would be: 20% of 35 = 14 → 50% of 14 = 7 → 75% of 7 = 5.25

5. The following PCG will give the answer

$$100 \xrightarrow[\text{Price hikes}]{40\%\uparrow} 140 \xrightarrow[\text{Consumption after}]{} 105$$

Hence, the percentage reduction required is 25% (= 140)

7. Assume Ram as 100, Shyam will be 133.33 and Bram will be 80

Thus, Bram's goods are 40% cheaper than Shyam's (53.33/133.33)

8. If the candidate has inadvertently increased his height by 25% the correction he would need to make to go back to his original height would be to reduce the stated height by 20%.

12. The percentage difference would be given by thinking of the percentage change between two numbers $(x - 5)$ to $(x + 5)$ ['What he wanted to get as what he was ...

The value of the percentage difference in this case depends on the value of x. Hence, this cannot be answered. Option (e) is correct.

14. From the first statement we get that out of 80 litres of the mixture, 20 litres must be milk. Since, we are adding water to this and keeping the milk constant, it is quite evident that 20 litres of milk should correspond to 20% of the total mixture. Thus, the amount in the total mixture must be 100, which means we need to add 20 litres of water to make 100 litres of the mixture.

16. $100 \xrightarrow{10\% \uparrow} 110 \xrightarrow{20\% \uparrow} 132$

Hence, the required answer is 32%

17. The area of a triangle depends on the product base × height.

Since, the height increases by 40% and the area has to increase by 60% overall, the following PCG will give the answer.

$$100 \xrightarrow[\text{base}]{40\% \uparrow} 140 \xrightarrow[\text{height}]{?} 160$$

The required answer will be 20/140 = 14.28%

18. The volume goes up by:

$$100 \xrightarrow[\text{Height}]{50\% \uparrow} 150 \xrightarrow[\text{breadth}]{20\% \uparrow} 180 \xrightarrow[\text{length}]{10\% \uparrow} 198$$

Hence, 98%

19. $100 \longrightarrow 130 \longrightarrow 100$ (A→B→A use of PCG)

∴ Answer = 30/130 = 23.07%

20. $100 \xrightarrow{25\% \downarrow} 75 \xrightarrow{?} 120$

We have assumed initial expenditure to be 100, in the above figure. Then the final expenditure is 120. The percentage change in consumption can be seen to be 45/75 × 100 = 60%

21. If the price of rice has fallen by 20% the quantity would be increased by 25% (if we keep the expenditure constant.)

This means that 20 kgs would increase by 25% to 25 kgs.

23. B + 60% of A = 175% of B →

60% of A = 75% of B.

i.e. $0.6A = 0.75B$

$A/B = 5/4$

Apparently it seems that A is bigger, but if you consider A and B to be negative the opposite would be true.

Hence, option (D) is correct.

25. $100000 \xrightarrow{10\% \uparrow} 110000 \xrightarrow{10\uparrow} 121000 \xrightarrow{10\% \uparrow} 133100.$

29. 30% students got a final score of 13. 10% students got a final score of 33 (inclusive of grace marks.) 35% students got a final score of 60

Hence, average score of the class

$$= \frac{0.35 \times 60 + 0.1 \times 33 + 0.3 \times 13}{0.75}$$

$$= 37.6$$

31. Since the ratio of their savings shows that Hans saves more after spending the same amount, we can conclude that he earns more.

Hence, Hans' income should be over 5000. Only option (a) satisfies this.

32. By using options, you can easily see that option (a) satisfies.

2500 females means 3000 males.

Increase = 2500 × 0.2 + 3000 × 0.11 = 830

35. Since the only iron contained in the ore is 90% of 25%, the net iron percentage would be 22.5%. Thus, 60 kg should be 22.5% of the ore → 60/0.225 = 266.66

36. The data is insufficient since the number of matches to be played by India this year is not given. (You cannot assume that they will play 40 matches.)

38. Total people present = 700 + 500 + 800 = 2000.

Indians = 0.2 × 700 + 0.4 × 500 + 0.1 × 800 = 420 = 21% of the population. Thus, 79% of the people were not Indians.

41. 1 Bottle → $0.5x$ metres

? Bottles → 400 meters

Using unitary method, we get no. of bottles = 400/$0.5x$ = 800/x Bottles.

43. The total cost for a year = 2,50,000 + 2% of 2,50,000 + 2000

= 2,55,000 + 2000 = 2,57,000

To get a return of 15% he must earn: 2,57,000 × 0.15 = 38,550 in twelve matches.

Hence, the monthly rent should be 38550/12 = 3212.5.

44. The sales price of the first shirt is 8/7 × 42 = Rs. 48. Hence, I am being offered a discount of Rs. 6 on a price of Rs. 48 – a 12.5% discount.

The sales price of the second shirt is 7/6 × 36 = Rs. 42.

Hence, 7 are being offered a discount of Rs. 4 or Rs. 47 = 14.29% discount. Hence, the second shirt is a better bargain.

xi. 75% were have voted for Shiv Gandhi and 16% for Rajesh Swami. Hence, 68 × 3 = 204.

xii. $100 \xrightarrow[\text{Sale price}]{\text{Sale}} 100 \xrightarrow[\text{Price rise}]{} 825$ (final sales figure)

Hence, the required price drop is 47.6/850 = 45% drop.

xiii. In 2001, BMW = 15%, Maruti = 50%, and hence Honda = 35%.

LOD II

1. $100 \longrightarrow 120 \longrightarrow \underset{\text{(cost 1)}}{25} \longrightarrow 112.5 \longrightarrow \underset{\text{(cost 2)}}{56.25}$

$\longrightarrow 64.375 \longrightarrow 42.1875$

Now, 42.1875 = Rs. 16.875

Hence 1 ⟶ 400

Also, your 2 situation is 56.25 × 400 = 22500

3. The thought process would go like:

If we assume 100 students

Total : 60 boys and 40 girls

For waiver : 6 boys and 3 girls.

This means that a total of 12 people are getting a fee waiver. (but this figure is given as 90.)

Hence, 1 corresponds to 7.5.

Now, number of students not getting a fee waiver = 51 boys and 37 girls

50% concession → 25.5 boys and 18.5 girls (i.e. a total of 44.)

Hence, the required answer = 44 × 7.5 = 330

4. Solve using options. Checking for option (b), gives us:

$$200000 \rightarrow 180000 \rightarrow 171000 \rightarrow 153900 \rightarrow 146205$$

(by successively decreasing 200000 by 10% and 5% successively)

5. Total characters in her report = 25 × 60 = 75.

Let the new no. of pages be x.

Then

$$x \times 30 + 25 \times 60 = 75$$
$$x = 22.75$$

This means that her report would require 23 pages.

A loss of 8% in terms of the pages.

7. The total value of salary is 87.5% (That is also 7/8 means here).

Using the options and PCD, you get option (c) as the correct answer.

8. October : November : December = 9 : 8 : 11 (as since, he gets Rs 40 more in December than October, we can conclude that 1.600 × 40) → 9 + 24.

Thus, total Bonus for the three months is:

$$0.4 \times 27.600 \times 24 = 265.6$$

10. 9% increase is offset by 8.26% decrease. Hence option (b) is correct.

11. The expenditure increase can be calculated using PCD as:

$$100 \rightarrow 112.5 \rightarrow 123.75$$

A 23.75% increase.

14. Population at the start = 100.

Population after 2 years = 100 × 1.08 × 1.01 = 108 × 1.01 = 108.984

Thus, the required percentage increase = 10.984%

16. 24% of the total goes to urban Gujarat 572 ⇒

∴ 1% = 5.3 mn.

The required value for Rural AP

= 50% of 20% = 10%

Hence, required answer = 5.30 mn

17. In the previous quarter, the total FDI was $ 80m. A growth of 20% this year means a total FDI of $360 mn.

The required answer is 17% of 10% of 360 mn

= 1.2% of 360 = 34.32 mn

18. The income goes to 120. Food expenditure has not maintained at 25. (i.e. 20.833%)

Hence, percentage point drop from 25 to 20.83% = 4.16%

25. Chanda would have spent 12% of Maya.

Thus, her percentage of expenditure would be 4.2

M = 100C = 12 M/C

27. The weekly change is equal to Rs. 1,68,000.

Hence, the daily collection will go up by 1,68,000/7 = 24,000.

29. Solve using options. 2/25 fits the requirement.

32. If Z = 100, X = 80 and Y = 72.

Thus, Y is less than X by 10%

34. The correct answer should satisfy the following condition. If 'x' is the increased salary

$$x \times 0.8 \times 0.1 = (x + 4800) \times 0.8 \times 0.12$$

None of the first 3 options satisfies this.

In fact, solving for x we get x = 23000.
Option (d) is correct.

46. The minimum price tickets are:

$$10 \times 20 + 8 \times 26 + x \times 30$$

Hence, 398/25 = 31.84

47. $36 \times 24 \times 40 \times 1 = 864$

Required income = 90025 − 56.16

48. If the ticket fees are halved, the maximum discount will be available for 9 tickets (25%). A maximum number of 36 tickets can be bought in Rs. 532 as 9 tickets for Rs. 30 each, 8 tickets for Rs. 32 each and 1 ticket for Rs. 40

49. Now using options:

Checking for option (A) will go as: According to this option 40% people have voted against the motion, hence, originally 200 people must have favoured the motion (hence there is a 100% increase in the opposition)

This means that 200 people who were for the motion initially went against it.

This leaves us with 400 people who were for the motion initially (after the abduction)

(10% of the original having been abducted, they could amount to half what is left

This means that 600 (for) and 200 (against) were the original distribution of 800.

This option fits perfectly (given all the constraints) and hence is the correct answer.

50. 1 man is married to 1 woman

Hence, 40% of men = 25% of women.

i.e. $0.45\ M = 0.25\ W$

Hence $\frac{0.45}{0.25} = \frac{M}{W}$

Women to men ratio of 9:5

Using alligation, the required answer is 32.14

51. The required weight of the bucket to the water when full is 3:2.

If both the weights (bucket and water) are integers, then the total weight must be a multiple of 5.

Only option (c) shows this characteristic.

52. We do not have sufficient information to solve the question.

53. Income of the salesman = 1200 + 1600 × O

Where, x is the number of Rs. 10000 sales he achieves over the initial Rs. 10000

So, $1200 + 1600 \times x = 7600$

So get $x = 4$

This means that the sales value must be Rs. 50000

54. A sales value of Rs. 90000 cannot be achieved.

55. Out of a total of 100% votes, 80% voted, 16% were invalid and 20% went to the second placed candidate. This means that the maximum the winner can get is 44%. Options a, b and c are greater than 44% and hence cannot be correct. Hence, none of these.

56. At 12 noon, the watch would show the correct time (since till then the temperature range was below 40°C). The watch would gain 2% every hour between 12 and 4. An hour having 3600 seconds, it would gain 72 seconds in each of these hours. Thus, at 1 pm it would be 72 × 4 = 288 seconds ahead. The time exhibited would be 7: 04: 48.

57. Pepsi worth Rs. 16 would be containing 60 grams of sugar.

58. Option (a) would cost: 8 + 7.5 = 15.5
Option (b) would cost: 12 + 3.2 = 15.2
Option (c) would cost: 8 + 3.2 + 2.8 = 14
Option (d) would cost: 12 + 2.8 = 14.8
Option (c) is the cheapest.

LOD III

1. Let the initial price of raw materials be 100. The new cost of the same raw material would be 115
The initial cost of labour would be 25 and the new cost would be 98% of 115 = 34.5
The total cost initially would be Rs.125
The total cost for the same usage of raw material would now be: 115+34.5= 149.5
This cost has to be reduced to 125. The percentage reduction will be given by 24.5/149.5 = 17 % approx.

2. Let the initial times allotted be: 30,40 and 20 hours. Then, the time used in each activity is 20,12 and 4 hours. Thus, 36 hours out of 110 are used in all.
Hence, the answer is 36/110 = 32.72 %

3. The following situation would follow:
Passed all: 5%
Passed 4: 20% of 90% =18%
Passed 3: 35% of 90% = 32.5%
Passed 2: 24.5%
Passed None: 5%
Passed 1: Rest (100% − 5 − 32.5 − 24.5 − 5 = 33%)
But it is given that 300 people passed 1. Hence, 33% = 300

Hence, 1750 students must have appeared in the test.

4. The first policy yielding the expected 'Sell at large' profit is given that, sales = profit = 17 (second).

Problem is given that, sales = profit = 17 (second). In order to get of the second case we try to fit in the options into the question.

Observe that the question is asking you to find the capacity of the second policy as a percentage of the first.

If we reverse option (a) as correct – 70% the following scenario follows:

If second is 70, then first is 100 and first + second is 170. Then third will be 68 (70% of first + second).

Then the equation:

$4 \times (100 + 70)$ should be equal to 12×70

But this is not true.

Through trial and error, we can see that the third option fits correctly.

$4 \times (100 + 80) = 12 \times 80$

Hence, it is the correct answer.

5. Let the initial percentage of salt be 30% in 100 liters of sea water in the flask.

30% of this is poured out i.e. 10 liters are poured out and the water boiled so as to increase the percentage of salt in the beaker 3 times (we have assumed M = 3 here.)

This means that there will be 30% salt in the beaker. Since, the salt concentration is increased by only evaporating water, the amount of salt remains the same.

Initially, the salt was 30% of 10 liters i.e. worth 3 liters. Hence, the water must have been worth 9 liters.

Now, since this amount of salt becomes worth 50% of the total solution, the amount of water left after evaporation would have been 3 liters and the total would be 2 liters.

When the 2 liters are mixed back again. The new concentration of salt in sea water would go up. In this specific case by allegation we would get the following allegation situation:

We mix 8 liters of 100% salted sea water with 2 liters of 50% salted sea water.

The weak group allegation will be: (10 × 40/46) % concentration of salted sea water. The value of the unknown percentage will be 40/46, (this will be the value of x).

Now, try to use the given options in order to account the fact that originally the flask contained 100 liters of sea water.

Use : M = 5, x = 40/46.

Only option (b) matches the situation.

$$\frac{(9 \times 5 + 1) \cdot 40/46}{(5 - 1)} = 100$$

6. The only values that fit this situation are c 20% and 30%, and d 40%. These are the percentage of votes polled. Now, these values can be got either through trial and error or through solving. $(x + x + 5 = 1)$ $(x + 5) = 100\%$

Then, 20% = 18000 (the difference between 4 & 1). Hence, 90000 people must have voted and 100000 people must have been on the voter's list.

7. The net time lost over two weeks would be 0.29% of a week's time (since in the first week the clock loses 0.5% and in the second week the clock gains lost of the true time.)

A week contains 168 hours. Hence, the clock loses 0.42 hours i.e. 25.2 minutes or 25 minutes 12 seconds. Hence, the correct time would be 12:25:12.

8. Traveling for 2400 kms at 18 kmph. Vawal will use 133.33 liters of petrol every month. The increase in expenditure for Vawal will be 133.33 × 0.7 = 2[illegible] Rs. 262 (approx).

9. The required answer will be given by: (7/107)×[illegible] = 157 km

10. The original expenditure is 28× 133.33 = Rs. 3733.333

The new expenditure will be given by 28 × 1.07 × 18 where x = the no. of kilometers to travel.

Since the new expenditure should increase by Rs. 200, its value has to be equal to Rs. 3933.33

This gives us x = 2363.15

Hence, the answer is c

11. The shopkeeper would get the maximum revenue when everybody opts for a 4% resale of the right. In such a case, the revenue for the shopkeeper from each customer would be: 96% of 4000 = 4000 − [illegible] = 3840. Hence, total revenue is 38400.

12. Similarly, the highest discount would be if everybody opts for the 15% discount. In such a case the total discount would be: 600 × 10 = 6000.

13-16. Detailed solutions for 13-16 is given in the [illegible] solutions for 1-12[illegible]

15-19. The average income (estimated) would be: Annual income (estimated) (savings)

The average monthly expenditure would be: Annual expenditure (Demonetised expenditure)

19. It can explained in the form and solution section of 1150 III.

20. $x/y = 47$. Hence, annual income = 1218

New income = 1218/12 = 101.5

Change in estimated income due to the change in prices of postage calculation = 14.5/87 → 16.66% increase

21. Estimated monthly Income would go up, while the estimated monthly expenditure would go down. Hence, Savings (estimated) would increase.

22. Cannot be determined since the percentage change would depend on the actual values which are not available in this question.

23. The estimated monthly expenditure would change from 49 to 11. Hence, percentage drop in the ratio will be 291 → 38.18%.

24-25 are explained in the form and solutions in 1150 III.

26-28 The following table will give a clearer picture of the situation.

Newspaper	Circulation (in 000)	Revenue	Commission	Net Revenue
Dainik Jagrati	573	546	10%	589.8
Times of Midstate	247	494	25%	365.1
India's Times	240	384	30%	419.8

29. Reduction of $\frac{181 * 100}{598.8} = 30.32\%$

30. The percentage difference between the revenues in Dai. Jag. = 589/546 = 30.37

Hence the required value is 30.32/30.37 = 1.000

31. The day's cost of printing 17,9000 copies of Dainik Jagrati is: 17,9000 × 8 = 2984000

Out of this, the paper recovers revenue. The revenue to be recovered would be: 2387200.

At Rs. 3000 per cc, 795.733 cc will have to be sold in any given day in order to obtain the cost. This represents 99.46% of the total value.

32. Time of Handouts

Total cost = 2,87,000 ÷ 7.5 = 18,33,500

Net revenue from newspaper sales is 3,95,200

Cost to be covered through advertising = 18,33,500 − 3,95,200 = 14,52,300.

At an ad rate of Rs 1800 per cc, they would have to sell 800.61 cc i.e. 33.6%

Similar calculations for India's Times will give 38.2%.

Hence, the percentage point difference = 5.6

33. If Ac. 1st costs 100, Ac. 2nd would cost 130 and AC 1st would cost 190. 3 Tier ticket would cost : 47.5 and general ticket would cost 40.

AC 2nd → 780 − 120

Then the difference between 3 Tier and general ticket would be: 7.5 × 780 = 46.75

34. Total cost → 100 + 120 + 190 + 47.5 + 40 = 497.5

The price (497.5/520) × 780 = 745.75

35. Harsh Kumar's savings

Month	Salary	Savings
1	10800	1080
2	10800	900
3	10800	900
4	10800	0
5	12000	1500
6	12000	1800
7	12000	3400
8	12000	1200
9	12000	1000
10	12000	1500
11	12000	1800
12	12000	2400

Required Ratio = 18000/900 = 5.333

36. Assume he has 1200 francs, 1200 DM and 1200 Lira. If he converts everything to francs, the result will be:

1200 DM will convert to 240 francs which will convert to 840 francs, the 51 Mirdits = 3 francs. Thus the value of 1200 DM in terms of Mirdits goes up from 1200 × 36 = 43200 to 840 × 51 = 40800. This increase in value has occurred only because of the change of currency. Hence, he should convert all his DM into France. However, before concluding on this you also will want to consider the effect of Lira. It is evident that 1200 DM will yield 240 francs which would yield 720 Lira (since 1 franc is 3 lira) which in turn would yield 720 × 36 = 5040 Mirdits.

Thus, it is evident that by converting DM into Liras the increase in value is higher than that achieved by converting DM into Francs.

Similarly, converting Francs to Liras also increases the value of the Francs.

1200 × 51 becomes equivalent to 900 × 70.

Note: The thought process goes like this: 1200 Francs = 300 Euros (since 1 euro = 4 francs). Further 300 Euros equals 900 liras which equal 900 × 3, Meeles.

49. Cannot be determined since the conversion from dollar to Euro is not given, neither is the inter-currency exchange rate between Lira, Francs and DM.

50. Obviously, both a and c are required in order to answer this question.

ANSWER KEY

Percentages LOD 1

1. c
2. d
3. a
4. b
5. b
6. c
7. d
8. b
9. a
10. c
11. a
12. c
13. c
14. a
15. b
16. c
17. c
18. d
19. b
20. a
21. a
22. c
23. d
24. d
25. b
26. b
27. d
28. b
29. b
30. c
31. c
32. a
33. a
34. a
35. d
36. d
37. a
38. c
39. a
40. a
41. a
42. b
43. d
44. b
45. c
46. a
47. a
48. b
49. a
50. d

Percentages LOD II

1. c
2. b
3. d
4. b
5. c
6. c
7. c
8. d
9. a
10. b
11. c
12. d
13. b
14. a
15. c
16. a
17. b
18. c
19. d
20. c
21. a
22. a
23. b
24. d
25. c
26. d
27. c
28. d
29. b
30. d
31. a
32. a
33. d
34. d
35. a
36. c
37. d
38. c
39. a
40. b
41. c
42. d
43. a
44. b
45. b
46. d
47. d
48. d
49. a
50. c

1. a
2. c
3. b
4. c
5. b
6. d
7. a
8. b
9. a
10. b
11. b
12. c
13. a
14. c
15. c
16. b
17. a
18. c
19. a
20. c
21. a
22. b
23. a
24. b
25. a
26. c
27. b
28. d
29. b
30. a
31. b
32. a
33. b
34. c
35. b
36. c
37. a
38. c
39. a
40. a
41. b
42. a
43. b
44. a
45. c
46. a
47. c
48. b
49. d
50. d

6

PROFIT AND LOSS

INTRODUCTION

Traditionally, Profit & Loss has always been an important chapter for CAT. Besides, all other Management entrance exams like MAT, CET, MAT, ATMA as well as Bank PO exams extensively use questions from this chapter. From the point of view of CAT, the relevance of this chapter has been gradually reducing and infact there have been no questions from this chapter for the past 3 years. However, CAT being a highly unpredictable exam, my advice to students and readers would be to go through this chapter and solve it at least up to LOD 2, so that they are ready for any changes in pattern.

Further, the Level of Difficulties at which questions are set in the various exams can be set as under:

LOD I: CAT, XLRI, IRMA, IIFT, CET, Bank PO exams, MAT, NIFT, NMIMS, and FMS, Symbiosis and all other management exams.

LOD II: CAT, XLRI, FMS, IRMA (partially), etc.

LOD III: CAT, XLRI and FMS (produces arising for 60% also is likely in CAT).

THEORY

Profit and Loss are part and parcel of every commercial transaction. In fact, the entire economy and the concept of capitalism is based on the so called "Profit Motive".

Profit and Loss in Case of Individual Transactions

We will here investigate the concept of Profit and Loss in the case of individual transactions. Certain concepts are common to each transaction. They are:

The price at which a person buys a product is the cost price of the product for that person. In other words, the amount paid or expended in either purchasing or producing an object is known as its Cost Price (also written as CP).

The price at which a person sells a product is the selling price of the product for that person. In other words, the amount got when an object is sold is called as the *Selling Price (SP)* of the object from the seller's point of view.

When a person is able to sell a product at a price higher than its cost price, we say that he has earned a profit. That is,

If SP > CP, the difference, SP − CP is known as the profit or gain.

Similarly, if a person sells an item for a price lower than its cost price, we say that a loss has been incurred.

The basic concept of profit and loss is as simple as that.

If, however, SP < CP, then the difference, CP − SP is called the loss.

It must be noted here that the Selling Price of the seller is the Cost Price of the buyer.

Thus we can say that in the case of profit the following formulae hold true:

1. Profit = SP − CP
2. SP = Profit + CP
3. CP = SP − Profit
4. Percentage Profit = $\frac{\text{Profit} \times 100}{\text{CP}}$

Percentage Profit is always calculated on CP unless otherwise stated.

Hence that SP = CP + Gain

$\Rightarrow$ CP + (Gain on Re 1) × CP

$\Rightarrow$ CP + (Gain %/100) × CP

Example: A man purchases an item for Rs. 120. If he sells it at a gain of 20 per cent profit find his selling price.

Solution: The selling price is given by 120 + 120 × 0.2 = 144

$$= CP + (Gain\%/100) \times CP = CP\left[1 + \frac{\%Gain}{100}\right]$$

For the above problem, the selling price is given by the method as Selling Price = 1.2 × 120 = 144

Hence, we also have the following

1. $SP = \left[1 + \frac{\%Gain}{100}\right] \times CP = \frac{(100 + Gain\%) \times CP}{100}$
2. $CP = \frac{100 \times SP}{(100 + Gain\%)}$

In case of loss

1. Loss = CP − SP
2. SP = CP − Loss
3. CP = SP + Loss
4. Loss% = Loss on Rs. 100 = $\frac{Loss \times 100}{CP}$

Percentage Loss is always calculated on CP unless otherwise stated.

The above situation (although it is the basic building block of Profit and Loss) is not the normal situation where we face Profit and Loss problems. In fact, there is a wide spectrum of profit and loss in day-to-day business and economic transactions. It is in these situations that we normally face or work out profit and loss problems.

Having investigated the basic concept of profit and loss in an individual transaction of selling and buying one unit of product, let us now look at the concept of profit and loss in a day-to-day business and commercial transactions.

Profit and Loss as Applied to Business and Commercial Transactions

Profit and Loss when Multiple Units of a Product are Being Bought and Sold The basic concept of profit and loss remains unchanged for this situation. However, a common mistake in this type of problem can be avoided if the following basic principle is adopted.

Profit or Loss in terms of money can only be calculated when the number of items bought and sold are equal.

That is, Profit or Loss in money terms cannot be calculated unless we equate the number of products bought and sold.

This is normally achieved by equating the number of items bought and sold to 1 or 100 or some other convenient figure as per the problem asked.

Overlooking of this basic fact is one of the most common mistakes that students are prone to making in the solving of profit and loss problems.

Types of Costs In any business dealing, there is a situation of selling and buying of products and services. From the sellers point of view, his principle interest, apart from maximising the sales price of a product/service, is to minimise the costs associated with the selling of that product/service. The costs that a businessman/trader faces in the process of day-to-day business transaction can be subdivided into three basic categories.

1. **Direct Costs or Variable Costs** This is the cost associated with direct selling of product/service. In other words, this is the cost that varies with every unit of the product sold. Hence, if the variable cost in selling a pen for Rs. 20 is Rs. 5, then the variable cost for selling 10 units of the same pen is 10 × 5 = Rs. 50.

   As is clear from the above example, that part of the cost that varies directly for every additional unit of the product sold is called as direct or variable cost.

   Typical examples of direct costs are: Raw material used in producing one unit of the product, wages to labour in producing one unit of the product when the wages are given on a piece rate basis, and so on. In the case of traders, the cost price per unit bought is also a direct cost. (i.e. every such expense that can be tied down to every additional unit of the product sold is a direct cost).

2. **Indirect Costs (Overhead Costs) or Fixed Costs** There are some types of costs that have to be incurred irrespective of the number of items sold and are called as fixed or indirect costs. For example, irrespective of the number of units of a product sold, the rent of the corporate office is fixed. Thus, whether the company sells 10 units or 100 units, this rent is fixed and is hence a fixed cost.

... other overhead or indirect or fixed costs. Salary to salesmen and managers, cost for offices, office telephone charges, office electricity charges.

Apportionment of Indirect or Fixed Costs: Fixed Costs are apportioned equally among each unit of the product sold. Thus, if a unit of a product is sold, then the fixed cost is to be apportioned to each unit sold is given by

$$\frac{\text{Fixed costs}}{n}$$

2. **Semi-Variable Costs**: Some costs are such that they behave as fixed costs under normal circumstances but have to be increased when a certain level of sales figure is reached. For instance, if the sales increase to such an extent that the company needs to take up additional office space to accommodate the increase in work due to the increase in sales then the rent for the office space becomes a part of the semi-variable cost.

The Concept of Margin or Contribution Per Unit

The difference between the value of the selling price and the variable cost for a product is known as the margin or the contribution of the product. This margin goes towards the recovery of the fixed costs incurred in selling the product/ service.

The Concept of the Break-even Point

The break-even point is defined as the volume of sale at which there is no profit or no loss. In other words, the sales value in terms of the number of units sold at which the company breaks even is called the break-even point. This point is also called the break-even sales.

Thus for every unit of the product the contribution goes towards recovering the fixed costs, as soon as a company sells more than the break-even sales, the company starts earning a profit. Conversely, when the sales value in terms of the number of units is below the break-even sales, the company makes losses.

The above concepts are best described through the following examples.

Let us suppose that a poor shop has to pay a rent of Rs. 1000 per month and salaries of Rs. 4000 to the assistants.

Now suppose that this poor shop sells only one variety of pens for Rs. 5 each. Further, the direct cost (variable cost) for making one pen is Rs. 2 in per pen, then the margin = Rs. (5 – 2) = Rs. 3 per pen.

Now, break-even sales will be given by

Break-even sales = Fixed costs/Margin per unit = Rs. 2,7 = 2000 pens.

Hence, the pen shop breaks even on a monthly basis by selling 2000 pens.

Selling every additional pen after the 2000th pen does towards increasing the profit of the shop. Also, in the case of the shop incurring a loss, the number of pens that are left to be sold to break-even will determine the quantum of the loss.

Now, the following formulae:

Profit = (Actual sales – Break-even sales) × Contribution per unit

Also in the case of a loss

Loss = (Break-even sales – Actual sales) × Contribution per unit

Also, if the break-even sales equals the actual sales, then we reach the point of no profit no loss, which is also the technical definition of the break-even point.

Note that the break-even point can be calculated on the basis of any time period that is normally done annually or monthly.

Profit Calculation on the Basis of Equating the Amount Spent and the Amount Earned

We have already seen that profit can only be calculated in the case of the number of items being bought and sold being equal. In such a case, we take the difference of the money got and the money given to get the calculation of the profit or the loss in the transaction.

There is another possibility, however, of calculating the profit. This is done by equating the money got and the money spent. In such a case, the profit can be represented by the amount of goods left. This is so because in terms of money the person going through the transaction has got back all the money that he has spent, but has ended up with some amount of goods left over after the transaction. These left over items can then be viewed as the profit or gain to the individual in consideration.

Hence, profit when money is equated is given by Goods left. Also, cost in this case is represented by Goods sold and

$$\text{hence percentage profit} = \frac{\text{Goods left}}{\text{Goods sold}} \times 100$$

Example A man makes discounts the cost of 25 mangoes by selling 20 mangoes. Find his percentage profit.

Solution Since the money spent is equal to the market value, the percentage profit is given by

$$\text{Profit} = \frac{(\text{Goods left})}{(\text{Goods sold})} = 100 \times 5 \times 100/20 = 25\%$$

Concept of Mark Up

Tradesmen/businessmen, while selling goods, add a certain percentage on the cost price. This addition is called percentage mark up (if it is in money terms) and the price thus obtained is called as the marked price (also is also the price printed on the product in the shops).

The relative relationship is

$$\text{CP} + \text{Mark up} = \text{Marked price}$$

or $\text{CP} + \%\ \text{Mark up on CP} = \text{Marked Price}$

The product is normally sold at the marked price in which case the marked price = the selling price.

If the tradesmen/shopkeeper gives a discount, he does so on the marked price and after the discount the product is sold at a discounted price.

Hence the following relationship operates:

$$\text{CP} + \%\ \text{Mark up (Calculated on CP)} = \text{Marked Price}$$
$$\text{Marked price} - \%\ \text{Discount} = \text{Selling price}$$

Use of PCG in Profit and Loss

- The relationship between CP and SP is typically defined through a percentage relationship. As we have seen earlier, this percentage value is called as the percentage mark up. (And is also equal to the percentage profit if there is no discount).

Under the following situation —

Assume the SP is 25% greater than the CP. This relationship can be seen in the following diagram

$$\text{CP} \xrightarrow{\text{add } 25\%} \text{SP}$$

In such a case the reverse relationship will be got by the PCG application of PCG and will be seen as follows:

If the profit is 25%

$$\text{Example: CP} \xrightarrow{\text{add } 25\%} \text{SP} \xrightarrow{\text{sub } 1} \text{CP}$$

Suppose you know that by selling at loss of 25%, the sale price of a bottle of wine is Rs. 1000.

Both the information, you can easily calculate the cost price by reducing the sales price by 20%. Thus, the CP is

$$1000 \times \frac{100 \times 5}{100} = 1250$$

WORKED-OUT PROBLEMS

Before we go into problems based on profit and loss, the reader should realize that there are essentially four phases of a profit and loss problem. These are connected together to get higher degrees of difficulty.

These are: (a) Cost calculations (b) Marked price calculations (c) Selling price calculations (d) Overhead/fixed cost calculations.

It is left to the reader to understand the interrelationships between a, b, c and d above. (These have already been stated in the earlier part of this chapter.)

Problem 5.1 A shopkeeper sold goods for Rs. 2000 at a profit of 50%. Find the cost price for the shopkeeper.

Solution The shopkeeper sells his items at a profit of 50%. This means that the selling price is 150% of cost price (Since CP + % Profit = SP).

For short you should view this as SP = 1.5 CP.

The problem with this calculation is that we know what 150% of the cost price is but we do not know what the cost price itself is. Hence we face difficulty in directly working out this problem. The calculation will become easier if we know the percentage calculation to be done on the basis of the selling price of the goods.

Hence look at the equation from the angle — (CP = SP/1.5, SP/1.5). This means that the denominator is increasing by 50%. But, from the table of denominator change in ratio change of the chapter of percentages, we saw that when the denominator increases by 50% the ratio decreases by 33.3%.

Interpret this as the CP is to be got from the SP by reducing the SP by 33.3%. Hence the answer is 2000 − (33.3% × 2000) = Rs. 1333.33

Also, this question can also be solved through operations by going from CP (assumed from the value of the expected) to the SP by increasing the assumed CP by 50% and then by checking whether the SP comes out to 2000. If it does, assume the

[illegible] CP was two while the SP equal that it means that the desired CP is [illegible]. Hence, you should move to the [illegible] option. On trying to understand whether you go for the higher options or the lower options based on your estimate of the [illegible] option.

Note: The above question will never appear as a full question in the examination but might appear as a part of a more complex question. If you are able to interpret the statement through the dimensions change to value change ratio, the time requirement will reduce significantly and you will gain a significant time advantage over the statement.

Problem 6.4 A man buys a shirt and a trousers for Rs. 371. If the trousers costs 120% more than the shirt, find the cost of the shirt.

Solution Here, we can write the equation:
$x + 1.12x = 371 \rightarrow x = 371/2.12$ (however, this calculation is not very easily done)

An alternate approach will be to go through options. Suppose the options are

(a) Rs. 125 (b) Rs. 150 (c) Rs. 175 (d) Rs. 200

Checking the opt. Rs. 150, the thought process should go like

Let x = cost of a shirt

If x = 150, 1.12x will be got by increasing x by 120% i.e. 120% of 150 = 18. Hence the value of 1.12x = 150 + 18 = 168 and $x + 1.12x$ = 318 is not equal to 371. Hence check the next higher option.

If x = 175, 1.12x = x + 120% of x = 175 + 21 = 196, i.e. $x + x$ = 371.

Hence, the option is correct.

Problem 6.5 A shopkeeper sells two items at the same price. He sells one of them at a profit of 10% and the other at a loss of 10%. Find the percentage profit/loss.

Generic question: A shopkeeper sells two items at the same price. He sells one of them at a profit of x% and the other at a loss of x%. Find the percentage profit/loss.

Solution The result will always be a loss of $[x/10]^2$%. Hence, the answer here is $(10/10)^2$% = 1% loss.

Problem 6.6 For the Problem 6.5, find the value of the loss incurred by the shopkeeper if the price of selling each item is Rs. 180.

Solution Where there is a loss of 10% → 180 = 90% of CP, ∴ CP₁ = 177.77

Where there is a profit of 10% → 160 = 110% of CP₂

∴ CP₂ = 145.45

Hence total cost price = 177.77 + 145.45 = 323.22 while the net realisation is Rs. 320.

Hence loss is Rs. 3.22.

Short cut for calculation: Since by selling the two items for Rs. 320 the shopkeeper gets a loss of 1% (from the previous problem), we can say that Rs. 320 is 99% of the value of the cost price of the two items. Hence the total cost is given by 32000/99 (solution of this calculation can be approximately done on the percentage change graphic).

Problem 6.5 If by selling 2 items for Rs. 180 each the shopkeeper gains 20% on one and loses 20% on the other, find the value of the loss.

Solution The percentage loss in this case will always be $(20/10)^2$ = 4% loss.

We can see this directly as 360 → 96% of the CP → CP = 360/0.96. Hence, by percentage change graphic 360 has to be increased by 4.166 per cent = 360 + 4.166% of 360 = 360 + 14.4 + 0.6 = Rs. 375.

Hence, the loss is Rs. 15.

Problem 6.6 By selling 15 mangoes, a fruit vendor recovers the cost price of 20 mangoes. Find the profit percentage.

Solution Here since the expenditure and the revenue are equal, we can use percentage profit = goods left × 100/goods sold = 5 × 100/15 = 33.33%.

Problem 6.7 A dishonest shopkeeper uses a 900 gm weight instead of 1 kilogram weight. Find his profit percent if he sells per kilogram at the same price as he sells a kilogram.

Solution Here again the money spent and the money got are equal. Hence, the percentage profit is got by goods left × 100/goods sold.

This gives us 11.11%.

Problem 6.8 A manufacturer makes a profit of 15% selling a colour TV for Rs. 6900. If the cost of manufacturing increases by 30% and the price paid by the [illegible]

is increased by 50%, find the profit percent made by the manufacturer.

Solution For this problem, the first line gives us that the cost price of the TV to the manufacturer is Rs. 6000.

The question now wants you to be able to solve this part of the problem in the first reading and reach at the figure of 6000 as cost, before you look further. This can be achieved subsequently if your percentage rule calculations are every Bluey work on it. The better you can get at it, the more it will benefit you. In fact, one of the people in my coaching classes I got through the CAT every year is the change in percentage calculation. Besides, percentage calculations will also go a long way in improving your speed in data interpretation.)

Further, if you have got to the 6000 figure by the end of the first line, reading further you can increase this advantage by calculating while reading as follows:

Marked selling price increase by 80% → New marked selling price 7200 and new selling price is 6000 + 20% of 6000 = 6000 × 1.20 = 7200.

Hence profit = 7200 − 7000 = 400 and profit percent = 400 × 100/7000 = 5.71%.

Problem 5.12 Find a single discount in equal three successive discounts of 10%, 12% and 5%.

Solution Using percentage change graphic starting from 100 → 90 → 81.6 → 75.24 (Note we can keep percentages in any order).

Hence the single discount is 24.76%.

Problem 5.13 A reduction in the price of actual by 10% enables a student to buy 5 gallons more for $288. Find the actual price of petrol.

Solution 10% reduction in price → 11.11% increase in consumption.

So 11.11% increase in consumption is equal to 5 gallons. Hence original consumption is equal to 45 gallons for $288. Hence, original price = 6$ per gallon.

Problem 5.14 Ashish bought an article and spent Rs. 110 on repairs. He then sold it to Bhushan at a profit of 20%. Bhushan sold it to Chetan at a loss of 10%. Chetan finally sold it for Rs. 198 at a profit of 10%. How much did Ashish pay for the article.

Solution Solve through options using percentage rule and keep checking options as you read. Try to finish the first option check before you finish reading the question for the first time. Also as a thumb rule always start with the middle most convenient option. This way you are likely to be required lesser number of options, on an average.

Also note that LOD II and LOD III questions will always essentially use the same statements as used in LOD I question. The only requirement that you need to have to handle LOD II and III questions is the ability to string together a set of statements and interconnect them.

Problem 5.15 A dishonest businessman professes to sell his articles at cost price but he uses false weights with which he cheats by 10% while buying and by 10% while selling. Find his percentage profit.

Solution Assume that the businessman buys and sells 1 kg of items. While buying he cheats by 10%, which means that when he buys 1 kg he actually takes 1100 grams. Similarly, he cheats by 10% while selling, that is, he gives only 900 grams when he sells a kilogram. Also, it must be understood that since he purportedly buys and sells the same amount of goods and he is trading at the same price while buying and selling, money is already squared in this case. Hence, we can directly use: % Profit = (Goods left × 100/Goods sold) − 100 × 100/900 = 22.22%. (Note that you should not need to do this calculation since this value comes from the fraction to percentage conversion table).

If you are looking at 70% plus net score in quantitative ability, you should be able to come to the solution in about 25 seconds inclusive of problem reading time. And the calculation should go like this:

Money is squared → % profit = 2/9 = 22.22%.

The longer process of calculation in this case would be involving the use of equating the amount of goods bought and sold and the money value of the profit. However, if you try to do this you will easily see that it requires a much higher degree of calculations and the process will need to get messy.

The options for doing this problem by equating goods would more to comparing the price per gram bought or sold. Alternatively, we could use the price per kilogram bought and sold (which would be preferable to equating on a per gram basis for this problem).

Here the thought process would be:

Assume price per kilogram = Rs. 1000. Therefore, he buys 1000 grams while purchasing and sells 900 grams while selling.

To repeat the case, use the following procedure:

	Market Price	Amount of goods
Buying	Rs. 1000	1100 grams (Reduce this by 10%)
After reduction	Rs. 900	900 grams
Selling	Rs. 1000	900 grams (Increase this by 10%)
After increase	Rs. 1100	990

Problem 5.13 RFO Tripathi bought some oranges for Rs. 12. He has to sell it off in Yeoman. He is able to sell off all the oranges in Yeoman and on reflection finds that he has made a profit equal to the cost price of 40 oranges. How many oranges did RFO Tripathi buy?

Solution Suppose we take the number of oranges bought as x. Then, the cost price per orange would be Rs. 12/x, and his profit would be 40 × 12/x = 120/x.

To solve for x, we need to equate this value with some value on the other side of the equation. But, we have no information provided here to find out the value of the variable x. Hence, we cannot solve this equation.

Problem 5.14 By selling 5 articles for Rs. 15, a man makes a profit of 20%. Find his gain or loss percentage if he sells 8 articles for Rs. 16 n?

Questions of this type normally appear as part of a more complex problem in an exam like the CAT.

Remember, such a question should be solved by you as soon as you finish reading the question by solving while-reading process, as follows:

By selling 5 articles for Rs. 15, a man makes a profit of 20% → SP = 3. Hence, CP = 2.5, if he sells 8 articles for Rs. 16 n → SP = 2.5. Hence percentage loss = 8%. For solving this question through this method will spend you much working out skill and ability to calculate percentage changes through the percentage change graphic. For this problem, you should not be required to use a pencil and a paper.

Problem 5.15 Oranges are bought at 12 for a rupee and sold at 10 for a rupee. Find the percentage profit or loss.

Solution Since money spent and gain are equal, use the formula for profit calculation in terms of goods bought goods sold.

This will give you percentage profit = 2/10 = 20%.

Alternatively, you can also equate the goods and calculate the percentage profit on the basis of money as

CP of 1 orange = 8.33 paise

SP of 1 orange = 10 paise

8.33 paise → 10 paise (corresponds to a percentage increase of 20% on CP)

Problem 5.16 In order to maximize its profits, AMS Corporate defines a function. Its unit sales price is Rs. 700 and the function representing the cost of production = 300 + 2p², where p is the total units produced or sold. Find the most profitable production level. Assume that everything produced is necessarily sold.

Solution The function for profit is a combination of revenue and costs. It is given by Profit = Revenue − Cost = 700 p − (300 + 2p²) = −2p² + 700p − 300.

In order to find the maxima or minima of any quadratic function, we differentiate it and equate the differentiated equation to zero.

Thus, the differentiated profit function is −4p + 700 = 0 → p = 175. This value of production will yield maximum profits in this case.

Note: Whether a quadratic function is maximum or minimum is decided by redifferentiating the differentiated equation. We then look at the sign of the coefficient term to determine whether the value got by equating the differentiated equation to zero corresponds to the maximum or the minimum. In the case of the current solution being negative, we say that the function is a maximum function and hence the solution point get would be a maximum point. In the event that the final coefficient term is positive, it is a minimum function.

Short cut Just look at the coefficient of x^2 in the function. If it is positive, equating the first differentiated term would yield the minimum point, and if the coefficient of x^2 is negative, the function is a maximum function.

Problem 5.17 For problem 5.16, what is the value of maximum profits for AMS Corporate?

Solution: In this reasoning from the previous question's solution, we just put the value of $p = 175$ in the equation for profit. Then, substitute $p = 175$ in the equation: Profit = $-p^2 + 10p - 300$ and get the answer.

Problem 5.22 A shopkeeper allows a rebate of 25% to an honest customer who only smuggled goods and at a loss, to pay 20% of the cost of the article. If his cost price is Rs. 1600, then find what should be the marked price of the article to make a profit of 9.06%.

Solution: The solving-when-reading are follows: Cost price is 1600 + Duty (= 10% of cost of article = 200) = Total cost to the shopkeeper (1600 + 200 = 2000).

To make a profit of 9.06 percent on this value → Using percentage change table we get 2200 + 9.06% of 2200 = 2200 + 200 = Rs. 2400.

He gets Rs. 1600 or gets after a rebate of 25%. Since we do not know the value of the marked price on which 25% rebate is to be calculated, it would be a good idea to work backwards through the percentage change graphic.

Going from the marked price to Rs. 2000 implies a 25% rebate. Hence the reverse process will be got by increasing Rs. 2000 by 33.33% and getting Rs. 4000.

(Note: the use of percentage change graphic is gracious and the product consistency table in particular in the solving of this question).

Problem 5.23 A man sells three articles, one at a loss of 25%, another at a profit of 20% and the third one at a loss of 0%. If the selling price of all the three is the same, find by how much percent is their average CP lower than or higher than SP.

Solution:

Note: It is always convenient to solve questions involving percentages by using the number 100. The one reason for this is that it reduces the amount of effort required in calculating the solution. Hence, it goes without saying that the variable to be fixed at 100 should be the one with the highest number of calculations associated with it. Another thumb rule for this is that the variable to be fixed at 100 should be the one with which the most difficult calculation are associated.

We have to calculate: average CP − average SP/average SP.

Here, the selling price is equal in all three cases. Since the maximum number of calculations are associated with the SP, we assume it to be 100. This gives us an average SP of 100 for the three articles. Then, the first article will be sold at 111.11, the second at 83.33 and the third at 133.33. (The student is advised to be fluent at these calculations). Further, the CP of the three articles is 111.11 + 83.33 + 133.33 = 327.77.

The average CP of the three articles is 327.77/3 = 109.2566.

Hence, (average CP − average SP)/average SP = 9.2566% higher.

Any other process adopted for this problem is likely to require much more effort and time.

Note: This process will be feasible if you have worked well with the percentage calculation techniques of the previous chapter.

Level of Difficulty (LOD) I

1. By selling a watch for Rs. 495, a shopkeeper incurs a loss of 10%. Find the cost price of the watch for the shopkeeper.
   (a) Rs. 545 (b) Rs. 550
   (c) Rs. 555 (d) Rs. 565
   (e) None of these
2. By selling a cap for Rs. 34.40, a man gains 7.5%. What will be the CP of the cap?
   (a) Rs. 32.60 (b) Rs. 32
   (c) Rs. 32.40 (d) Rs. 28.80
   (e) Rs. 31.40
3. A cellular phone when sold for Rs. 4600 fetches a profit of 25%. Find the cost price of the cellular phone.
   (a) Rs. 4300 (b) Rs. 4150
   (c) Rs. 4400 (d) Rs. 4800
4. A machine cost Rs. 135. If it is sold at a loss of 20%, what will be its cost price as a percentage of its selling price?
   (a) 80% (b) 120%
   (c) 110% (d) 125%
   (e) None of these

5. A shopkeeper sold goods for Rs. 2400 and made a profit of 25% on the purchase. Find his profit per cent if he had sold his goods for Rs. 2640.
(a) 6.25% (b) 7%
(c) 6.25% (d) 8.5%
(e) 6.75%

6. A digital clock is sold for Rs. 315 at a profit of 10%. What would have been the actual profit or loss on Rs. 6 if it had been sold for Rs. 800?
(a) Rs. 45 (b) Rs. 40 (c) Rs. 48 (d) Rs. 50
(e) Rs. 99

7. A music system when sold for Rs. 3500 gives a loss of 16.66% to the merchant who sells it. Calculate his loss or gain per cent, if he sells it for Rs. 3703.75.
(a) Loss of 5.625% (b) Profit of 8.33%
(c) Loss of 7% (d) Profit of 5.625%
(e) Profit of 7%

8. By selling bouquets for Rs. 63, a florist gains 5%. At what price should he sell the bouquets to gain 10% on the cost price?
(a) Rs. 66 (b) Rs. 69
(c) Rs. 72 (d) Rs. 72.50
(e) Rs. 75

9. A shopkeeper bought 240 chocolates at Rs. 8 per dozen. If he sold all of them at Rs. 1 each, what was his profit per cent?
(a) 66.66% (b) 33.33%
(c) 24% (d) 27%
(e) None of these

10. A trading bottle is sold for Rs. 120. Sales tax accounts for one fifth of this and profit one third of the remainder. Find the cost price of the trading bottle.
(a) Rs. 66 (b) Rs. 72 (c) Rs. 68 (d) Rs. 76
(e) Rs. 78

11. A fruit merchant makes a profit of 20% by selling oranges at Rs. 25 per quintal. If he sells the fruit at Rs. 22.50 per quintal, what is his profit per cent on the whole transaction?
(a) 8% (b) 6.66% (c) 7.5% (d) 8%
(e) 9%

12. The cost price of a chair and a pair of trousers is Rs. 375. If the chair costs 12% more than the trousers, find the cost price of the trousers.
(a) Rs. 125 (b) Rs. 150
(c) Rs. 175 (d) Rs. 200
(e) Rs. 225

13. A pet shop owner sells two puppies at the same price. On one he makes a profit of 20% and on the other he suffers a loss of 20%. Find his loss or profit per cent on the whole transaction.
(a) Gain of 4% (b) No profit no loss
(c) Loss of 10% (d) Loss of 8%
(e) None of these

14. The marked price of a table is Rs. 1200, which is 20% above the cost price. It is sold at a discount of 10% on the marked price. Find the profit per cent.
(a) 10% (b) 8% (c) 7.5% (d) 6%
(e) 8.33%

15. 135 toffees cost Rs. 75. Find the cost of one toffee if he sells if there is a discount of 40% on the selling price for this quantity.
(a) Rs. 3.00.000 (b) Rs. 1.20.000
(c) Rs. 3.60.000 (d) Rs. 4.00.000
(e) Rs. 4.50.000

16. A shopkeeper marks the price of an article at Rs. 80. Find the cost price of after allowing a discount of 20% he still gains 20% on the cost price.
(a) Rs. 53.33 (b) Rs. 70
(c) Rs. 75 (d) Rs. 60
(e) Rs. 66

17. In the question 16, what will be the selling price of the article if he allows two successive discounts of 5% each.
(a) Rs. 72 (b) Rs. 72.20
(c) Rs. 75 (d) Rs. 71.66
(e) Rs. 71.2

18. A dozen pairs of gloves quoted at Rs. 80 are available at a discount of 10%. Find how many pairs of gloves can be bought for Rs. 24.
(a) 4 (b) 5 (c) 6 (d) 8
(e) 7

19. Find a single discount equivalent to the discount series of 20%, 10%, 5%.
(a) 30% (b) 31.6% (c) 68.4% (d) 35%
(e) 32.6%

20. The printed price of a calculator is Rs. 180. A customer pays Rs. 137.7 for it by getting successive discounts of 10% and another rate which is illegible. What is the second discount rate?
(a) 12% (b) 12.5% (c) 15% (d) 20%
(e) 16.66%

21. How much percent more than the cost price should a shopkeeper mark his goods, so that after allowing a discount of 12.5% he should have a gain of 8% on his sales?
(a) 0.375 (b) 16.66% (c) 20% (d) 25%
(e) 18.17%

22. In order to maintain the price line, a trader allows a discount of 10% on the marked price of goods in his shop. However, he still makes a gross profit of 17% on his cost price. Find the profit per cent he would have made on the selling price had he sold at the marked price.
(a) 23.05% (b) 40% (c) 21.21% (d) 25%
(e) 40%

23. A whole seller allows a discount of 20% on the list price to a retailer. The retailer sells at 5% discount on the list price. If the customer paid Rs. 38 for an article, what profit is made by the retailer?
(a) Rs. 30 (b) Rs. 8
(c) Rs. 6 (d) Rs. 12
(e) None of these

24. In question 23, find also the retailer's percentage profit to his cost giving your answer correct to two decimal places.
(a) 12.5% (b) 10.66% (c) 19.77% (d) 20%
(e) 25%

25. The cost of production of a cordless phone set in 2006 is Rs. 900, divided between material, labour and overheads in the ratio 3 : 4 : 2. If the cordless phone set is marked at a price that gives a 20% profit on the component of price accounted for by labour, what is the marked price of the set?
(a) Rs. 980 (b) Rs. 1080
(c) Rs. 960 (d) Rs. 1020
(e) None of these

26. Subsequently in 2007, the cost of material, labour and overheads increased by 20%, 30% and 10% respectively, calculate the cost of manufacturing in 2007.
(a) Rs. 1120 (b) Rs. 1050
(c) Rs. 1080 (d) Rs. 1100

27. What should be the new marked price of the cordless phone set in question 25 above?
(a) Rs. 1120 (b) Rs. 1204
(c) Rs. 1144 (d) None of these

28. By selling a computer for Rs. 960, a man incurs a loss of 4%. At what price should he sell the computer to gain 10%?
(a) Rs. 1100 (b) Rs. 1080
(c) Rs. 1120 (d) None of these

29. A man sells 5 articles for Rs. 15 and makes a profit of 20%. Find his gain or loss percent if he sells 8 such articles for Rs. 18.40.
(a) 2.22% profit (b) 2.22% loss
(c) 8% loss (d) 8% profit

30. The cost price of 50 sarees is equal to the selling price of 40 sarees. Find the percentage profit?
(a) 20% (b) 25%
(c) 30% (d) None of these

31. A owns a house worth Rs. 10,000. He sells it to B at a profit of 15%. After some time, B sells it back to A at 10% loss. Find A's loss in gain percent.
(a) 2.25% gain (b) 6.25% gain
(c) 17.00% gain (d) 17.25% gain

32. A shopkeeper bought locks at the rate of 8 locks for Rs. 54 and sold them at the rate of 12 locks for Rs. 57. Calculate his gain percent.
(a) 9.33% (b) 12.5%
(c) 10.11% (d) 11.76%

33. Anil bought an article at Rs. 200 and sold it at a profit of 10%. What would have been the increase in the profit percent if it was sold for Rs. 230?
(a) 5% (b) 10%
(c) 15% (d) None of these

34. A makes an article for Rs. 120 and sells it to B at a profit of 25%. B sells it to C who sells it for Rs. 198, making a profit of 10%. What profit percent did B make?
(a) 25% (b) 20% (c) 16.66% (d) 15%

35. A reduction of 10% in the price of sugar enables a housewife to buy 0.2 kg more for Rs. 270. Find the reduced price per kilogram.
(a) Rs. 5 (b) Rs. 4.5
(c) Rs. 4.05 (d) None of these

36. A man buys 50 kg of oil at Rs. 10 per kilogram and another 40 kg of oil at Rs. 12 kilogram and mixes them. He sells the mixture at the rate of Rs. 11 per kilogram. What will be his gain percent if he in order to sell the whole lot?
(a) $\frac{250}{9}$% (b) [illegible]
(c) 30 [illegible] (d) [illegible]
(e) None of these

[illegible] If the cost price of 30 articles is equal to the selling price of 30 articles, find the profit percent.

(a) 11.1% (b) 40%
(c) 50% (d) 60%

[illegible] A shopkeeper sells sugar in such a way that the selling price of 950 gm is the same as the cost price of one kilogram. Find his gain percent.

(a) 100/17% (b) 150/17%
(c) 5/950% (d) 10%

[illegible] A dealer buys eggs at Rs. 30 per gross. He sells the eggs at a profit of 10/22% on the cost price. What is the selling price per egg (approximately)?

(a) 28 paise (b) 30 paise
(c) 26 paise (d) 24 paise

[illegible] A sold a table to B at a profit of 20%. B sold the same table to C for Rs. 15 thereby making a profit of 25%. Find the price at which A bought the table from X if it is known that X gained 25% in the transaction.

(a) Rs. 30 (b) Rs. 40 (c) Rs. 50 (d) Rs. 60

[illegible] A sold a table to B at a profit of 15%. Later on, B sold it back to A at a profit of 20%, thereby gaining Rs. 60. How much did A pay for the table originally?

(a) Rs. 300 (b) Rs. 320
(c) Rs. 340 (d) Rs. 360

[illegible] A dealer sold two TV sets for Rs. 2400 each, gaining 20% on one and losing 20% on the other set. Find his net gain or net loss.

(a) Rs. 300 loss (b) Rs. 200 loss
(c) Rs. 200 gain (d) Rs. 300 gain

[illegible] The selling rate is Rs. 40 per kg a loss of 10% is incurred. Calculate the amount of tea (in kg) sold if the total loss incurred is Rs. 80.

(a) 12 kg (b) 15 kg (c) 18 kg (d) 20 kg

[illegible] A colour TV and a VCP were sold for Rs. 12,000 each. The TV was sold at a loss of 20% whereas the VCP was sold at a gain of 20%. Find gain or loss in the whole transaction.

(a) Rs. 1200 loss (b) Rs. 1000 loss
(c) Rs. 800 loss (d) Rs. 1040 loss

Hint: In this case there will always be a loss.

[illegible] A man sells a TV set for Rs. 5400 and makes a profit of 12%. He also sells another TV at a loss of 12%. If on the whole, he neither gains nor loses, find the selling price of the second TV set.

(a) Rs. 4800 (b) Rs. 4900
(c) Rs. 4950 (d) Rs. 4980

46. A man sells an article at 5% above its cost price. If he had bought it at 5% less than what he paid for it and sold it for Rs. 2 less, he would have gained 10%. Find the cost price of the article.

(a) Rs. 500 (b) Rs. 300
(c) Rs. 425 (d) Rs. 400

47. A necklace was sold at a profit of 10%. If its cost price was 5% less and it was sold for Rs. 7 more, the gain would have been 20%. Find the cost price of the necklace.

(a) Rs. 175 (b) Rs. 200
(c) Rs. 225 (d) Rs. 300

48. A man sells a plot of land at 6% profit. If he had sold it at 10% profit, he would have received Rs. 200 more. What is the selling price of the land?

(a) Rs. 5000 (b) Rs. 5300
(c) Rs. 4800 (d) Rs. 5600

49. Ashok bought an article and spent Rs. 110 on its repairs. He then sold it to Bhushan at a profit of 20%. Bhushan sold it to Charan at a loss of 10%. Charu finally sold it for Rs. 1188 at a profit of 10%. How much did Ashok pay for the article?

(a) Rs. 890 (b) Rs. 1000
(c) Rs. 780 (d) Rs. 940

50. A man buys two cycles for a total cost of Rs. 800. He selling one for 4/5 of its cost and other for 5/4 of its cost, he makes a profit of Rs. 90 on the whole transaction. Find the cost price of lower priced cycle.

(a) Rs. 300 (b) Rs. 250
(c) Rs. 500 (d) Rs. 425

51. A merchant bought two transistors, which together cost him Rs. 480. He sold one of them at a loss of 5% and other at a gain of 10%. If the selling price of both the transistors are equal, find the cost of the lower priced transistor.

(a) Rs. 300 (b) Rs. 180
(c) Rs. 200 (d) Rs. 280

52. A manufacturer makes a profit of 15% by selling a colour TV for Rs. 5750. If the cost of manufacture increases by 30% and the price paid by the retailer increased by 20%, find the profit percent made by the manufacturer.

(a) 62/13% (b) 8 (8/13)%
(c) 63/13% (d) 7 (4/13)%

53. The cost of manufacturing an article is made up of materials, labour and overheads in the ratio 4 : 2 : 3

the cost of labour is Rs. 45, find the profit percent if the article is sold for Rs. 180.

(a) 20% (b) 25.5%
(c) 24% (d) 25%

74. Two dealers X and Y selling the same model of refrigerators sold them under the same selling price. X gives successive discounts of 25% and 5% and Y gives successive discounts of 15% and 12%. Then whom is it more profitable to purchase the refrigerator?

(a) from X
(b) from Y
(c) Indifferent between the two
(d) Cannot be determined

75. A car is set priced at Rs. 36,000. He gives a discount of 6% on the first Rs. 20,000 and 5% on the remaining Rs. 16,000. His competitor B sells a car of the same make priced at Rs. 36,000. If he wants to be competitive what percent discount should B offer on the marked price.

(a) 5% (b) 5.5%
(c) 6.6% (d) 5.75%

76. An article costs Rs. 750 to a manufacturer who has set price of Rs. 800. He sells it to a dealer at a discount of 5%. The dealer gets a further discount of 5% on his purchase for paying in cash. Calculate the amount that the dealer pays to the manufacturer.

(a) Rs. 722 (b) Rs. 720
(c) Rs. 725 (d) None of these

77. In picture 36, find the profit percent that the manufacturer makes on the sale.

(a) 150.0% (b) 22.7%
(c) 300.0% (d) None of these

78. A furnishing to furniture shows 4% discount on the marked price of each item. What price must be marked on a dining table that cost Rs. 480 to manufacture, so as to make a profit of 20%.

(a) Rs. 475 (b) Rs. 480
(c) Rs. 500 (d) Rs. 520

79. A shopkeeper allows a discount of 11.5% on the marked price of a certain article and makes a profit of 6%. If the article cost the shopkeeper Rs. 210, what price must be marked on the article.

(a) Rs. 240 (b) Rs. 280
(c) Rs. 310 (d) None of these

80. A Cotton shop allows a discount of 10% on the marked price of a camera. What price must be marked on the camera that costs him Rs. 600, marked he makes a profit of 30%?

(a) Rs. 900 (b) Rs. 750
(c) Rs. 780 (d) Rs. 850

81. A watch dealer pays 10% custom duty on a watch that costs Rs. 250 abroad. For how much should he mark it if he desires to make a profit of 30% after giving a discount of 25% to the buyer?

(a) Rs. 400 (b) Rs. 440
(c) Rs. 375 (d) Rs. 350

82. A shopkeeper buys an article for Rs. 300 and marks it for sale at a price that gives him 30% profit on his cost. He however, gives a 15% discount on the marked price to his customer. Calculate the actual percentage profit made by the shopkeeper.

(a) 12% (b) 14% (c) 15% (d) 34%

83. In the land of the famous milkman Marghesh Dumas, a milkman sells his buffalo for Rs. 720 at some profit. Had he sold his buffalo at Rs. 510, the quantum of the loss incurred would have been double that of the profit earned. What is the cost price?

(a) Rs. 600 (b) Rs. 625
(c) Rs. 637 (d) None of these

84. A trader purchases apples at Rs. 90 per hundred. He spends 15% on the transportation. What should be the selling price per 100 to earn a profit of 20%?

(a) Rs. 77 (b) Rs. 81.8
(c) Rs. 82.8 (d) Rs. 83.8

85. A dishonest dealer professes to sell at cost price but uses a 900 gram weight instead of a 1 kilogram weight. Find the percent profit to the dealer.

(a) 10% (b) 11.11%
(c) 12.5% (d) None of these

Level of Difficulty (LOD) III

1. Mukesh makes 720 articles at a cost of Rs 40 each. The article fetches the selling price such that if only 1000 articles are sold, he would have made a profit of 30% on the selling. However, 120 articles got spoilt and he was able to sell 480 articles at this price. Find the actual profit percent in the transaction. Given that assuming that the whole articles are sold.

(a) 42% (b) 24% (c) 41% (d) [illegible]

2. A manufacturer estimates that on inspection 15% of the articles he produces will be rejected. He accepts an order to supply 22,000 articles at Rs. 5.50 each. He estimates the profit on his outlay including the manufacturing of rejected articles, to be 20%. Find the cost of manufacturing each article.

(a) Rs. 6 (b) Rs. 5.50
(c) Rs. 5 (d) Rs. 4.50
(e) Rs. 4.5

3. The cost of setting up the type of a magazine is Rs. 3000. The cost of running the printing machine is Rs. 120 per 500 copies. The cost of paper, ink and so on is 60 paise per copy. The magazines are sold at Rs. 2.75 each. 900 copies are printed, but only 744 copies are sold. What is the sum to be obtained from advertisements to give a profit of 10% on the total?

(a) Rs. 730 (b) Rs. 735
(c) Rs. 736 (d) Rs. 736
(e) Rs. 790

4. A tradesman fixed his selling price of goods at 30% above the cost price. He sells half the stock at this price, one-quarter of his stock at a discount of 15% on the original selling price and rest at a discount of 30% on the original selling price. Find the gain percent altogether.

(a) 14.875% (b) 15.375%
(c) 15.575% (d) 16.375%
(e) 16.5%

5. A tradesman marks an article at Rs. 205 more than the cost price. He allows a discount of 10% on the marked price. Find the profit percent if the cost price is Rs. x.

(a) $\left[\frac{\left(\frac{x}{(18450)}-50\right)}{x}\right]$ (b) $\frac{\left[(18450)\right]-10x}{x}$

(c) $\left[\frac{\left(\frac{x}{(18450)}-100\right)}{x}\right]$ (d) $\left[\frac{\left(\frac{18450}{x}-100\right)}{x}\right]$

(e) None of these

6. Daily goes to a shop to purchase a dial priced at Rs. 400. He is offered 4 discount options by the shopkeeper. Which of these options should she opt for to gain maximum advantage of the discounts offered.

(a) Single discount of 30%
(b) 3 successive discounts of 15% each
(c) 3 successive discounts of 30% and 10%
(d) 3 successive discounts of 20% and 12%
(e) Either (b) or (c)

7. A dishonest dealer marks up the price of his goods by 20% and gives a discount of 10% to the customer. He also uses a 900 gram weight instead of a kilogram weight. Find his percentage profit from these transactions.

(a) 8% (b) 12%
(c) 20% (d) 16%
(e) None of these

8. A dishonest dealer marks up the price of his goods by 20% and gives a discount of 10% to the customer. Besides, he also cheats both his supplier and his buyer by 100 grams while buying or selling 1 kilogram. Find the percentage profit earned by the shopkeeper.

(a) 28% (b) 25%
(c) 32% (d) 37.5%
(e) None of these

9. For question 8, if it is known that the shopkeeper has a discount of 10% from his supplier and he changes this discount while marking up (i.e. he marks up at the undiscounted price), find the percentage profit from shopkeeper if there is no other change from the previous problem.

(a) 32% (b) 36.66%
(c) 40.33% (d) 46.66%
(e) 50%

10. Cheap and Best, a kirana shop bought some apples 4 per rupee and an equal number at 5 per rupee. He then sold the entire quantity at 9 for 2 rupees. What is his percentage profit or loss?

(a) 1.23% loss (b) 6.66%
(c) 8.88% (d) No profit no loss
(e) None of these

11. A watch dealer sells watches at Rs. 600 per watch. However, he is forced to give two successive discounts of 10% and 5% respectively. However, he must also collect tax on the net sale price from the customer at 5% of the net price. What price does a customer have to pay him to buy the watch.

(a) Rs. 539.75 (b) Rs. 539.65
(c) Rs. 534.75 (d) Rs. 538.65
(e) Rs. 540.55

12. Deb bought 300 kg of rice for Rs. 1100 and sold it at a loss of as much money as he received for 20 kg rice. At what price did he sell the rice?

(a) Rs. 9 per kg (b) Rs. 9.1 per kg
(c) Rs. 8.5 per kg (d) Rs. 10.33 per kg
(e) None of these

17. A salesman wants to sell 40 chairs. If he sells them at Rs. 150 per chair, he would be able to sell all the 40 with one additional beautiful chair. At what selling price would he be able to maximise his profit (assuming all chairs remain with him)?

(a) 195 (b) 180
(c) 200 (d) 210
(e) Cannot be determined

Directions for Questions 18–21: Read the following and answer the questions that follow.

Parents have advised Rama, a chocolate freak, not to eat more than 20 chocolates in one day. When she went to the market to buy her daily quota, she found that if she buys chocolates from the market complex she would have to pay Rs. 1 more for the same number of chocolates than she would have spent had she bought them from her uncle Scrooge's shop, getting two pennies less per rupee. She finally decided to get them from Uncle Scrooge's shop paying only in one rupee coins.

18. How many chocolates did she buy?

(a) 12 (b) 9
(c) 18 (d) 15
(e) More data required

19. How much would she have spent at the market complex?

(a) Rs. 6 (b) Rs. 12 (c) Rs. 9 (d) Rs. 5
(e) Rs. 11

20. A shopkeeper makes a profit of 5% by selling an object for Rs. 24. Had the cost price and selling price been interchanged, it would have led to a loss of 12.5%. With the latter cost price, what should be the new selling price to get a profit of 5%?

(a) Rs. 34.40 (b) Rs. 32.50
(c) Rs. 29.60 (d) Rs. 36.4
(e) None of these

21. Find the change in the percentage profit for a four wheeler sale, after finding 20% of the trade mark, increased his selling price by 10% over and above 1% that he was already charging?

(a) –11 (b) +11.2
(c) –11.8 (d) –10.5
(e) Cannot be determined

Directions for Questions 22–24: Read the following and answer the questions that follow.

Rama and Shyamla decided to sell their cars each in Rs. 30,000. While Rama decided to give a discount of 6% on the first Rs. 8000, 5% on next Rs. 12,000 and 3% on the rest to buyer Sudhi. Shyamla decided to give a discount of 4% on the first 12,000, 6% on the next 8,000 and 2% on the rest to buyer Rajesh. These discounts were, however, subject to the buyers making the payment on time failing which the discount percentage reduced by 1% for every delay of a week. In each case, the selling price of 36,000 was arrived at by increasing the cost price by 25%.

22. If each of them got the payments on time what is the approximate percentage profit of the person getting the higher profit?

(a) 19% (b) 21% (c) 25% (d) 17%
(e) 20%

23. If both defaults by 1 and 2 weeks in the second and third payments respectively, what would be the profit or loss in the sale of the car?

(a) Rs. 5920 (b) Rs. 6240
(c) Rs. 5840 (d) Rs. 5080
(e) Rs. 6040

24. What would be the difference in the profits if both the buyers default in each payment by a week?

(a) Rs. 200 (b) Rs. 900
(c) Rs. 400 (d) Rs. 500
(e) None of these

21. Find the selling price of goods if two salesmen claim to make 25% profit each, one calculating it on cost price while another on the selling price, the difference in the profits earned being Rs. 100 and selling price being the same in both the cases?

(a) Rs. 2000 (b) Rs. 1600
(c) Rs. 2400 (d) Rs. 2800
(e) Rs. 3000

22. A shopkeeper calculates percentage profit on the buying price and another on the selling price. What will be their difference in profits if both claim a profit of 20% on goods sold for Rs. 3000?

(a) Rs. 200 (b) Rs. 100
(c) Rs. 400 (d) Rs. 150
(e) None of these

23. A pharmaceutical company made some strips of tablets at a cost of Rs. 4400. The company gave every 100th strip of tablets as dozens on free samples. A discount of 25% was allowed on the printed price. Find the ratio of profit if the price is raised from Rs. 4.25 to Rs. 4.25 per strip and if at the same price, samples in dozens were done away with. (Treat profit=0 profit)

(a) 16.6 (b) 43.5 (c) 74 (d) 66.25
(e) 80

24. A merchant makes a profit of 20% by selling an article. What would be the percentage change in the profit percent had he paid 10% less for it and the customer paid 10% more for it?
(a) 120% (b) 125%
(c) 133.33% (d) 180%
(e) 150%

25. An article costing Rs. 20 was marked 25% above the cost price. After two successive discounts of the same percentage, the customer now pays Rs. 20.25. What would be the percentage change in profit had the price been increased by the same percentage twice successively instead of reducing it?
(a) 360% (b) 520%
(c) 280% (d) 400%
(e) 380%

26. Dora goes to buy fruits and after a lot of bargaining is able to get the price of a dozen apples reduced by Re. 1 from the initial price, thereby enabling her to get 1 apple extra for every rupee saved. (Going by the rates on the extra apple). What is the initial price of a dozen apples?
(a) Rs. 30 (b) Rs. 13
(c) Rs. 12 (d) Rs. 15

27. The accounts of a company show sales of Rs. 72,000. The primary cost is 35% of sales and trading cost accounts for 25% of the gross profit. Gross profit is arrived at by excluding the primary cost plus the cost of advertising expenses of Rs. 1400, director's salary of Rs. 600 per annum plus 2% of annual sales as miscellaneous costs. Find the percentage profit (approx) on a capital investment of Rs. 14,000?
(a) 15% (b) 31%
(c) 28% (d) Cannot be determined

28. Jenny has two cycles and one rickshaw. The rickshaw is worth Rs. 500. If she sells the rickshaw along with the first cycle, she has an amount equal to that of the value of the second cycle. But if she decides to sell the rickshaw along with the second cycle, the amount received would be less than the value of this cycle by Rs. 300. What is the value of this cycle?
(a) Rs. 800 (b) Rs. 600
(c) Rs. 400 (d) None of these

29. David sells his Laptop to Goliath at a loss of 20% and subsequently sells it to Hercules at a profit of 25%. Hercules, after finding some defect in the laptop returns it to Goliath but could recover only Rs. 4.50 for every Rs. 5 he had paid. Find the amount of Hercules' loss if David had paid Rs. 1,75 lakh for the laptop.
(a) Rs. 1500 (b) Rs. 2500
(c) Rs. 17,500 (d) None of these

30. A dishonest shopkeeper, at the time of selling and purchasing, weights 10% less and 20% more per kilogram respectively. Find the percentage profit earned by treachery. (Assuming he sells at Cost Price)
(a) 30% (b) 20% (c) 25% (d) 33.3%

31. A dealer marks articles at a price that gives him a profit of 30%. 6% of the consignment of goods was lost in a fire in his premises. 24% was sold and he is to be sold at half the cost price. If the remainder was sold at the marked price, what percentage profit or loss did the dealer make on that consignment?
(a) 2% (b) 2.5% (c) 3% (d) 4.2%

32. A book was sold for a certain cost and there was a loss of 20%. Had it been sold for Rs. 12 more, there would have been a gain of 30%. What would be the profit if the book were sold for Rs. 4.8 more than what it was sold for?
(a) No profit, no loss (b) 20%
(c) 10% (d) 25%

For Questions 33–36 use the following data:

33. Two thousand people invest in Business Village I which 55% were male and the rest were female. The male population earned a profit of 5% and the female population earned 8% on an investment of Rs. 50 each. Find the change in the percentage profit of the village if the ratio of male to female gets reversed the next year, population remaining the same.
(a) Drop of 0.3 (b) Increase of 0.3
(c) Increase of 0.45 (d) Drop of 0.45

34. In the question 33, find the change in the percentage profit of the village, if the population increases by 10%. (Assume the ratio remains the same)
(a) Increase of 10% (b) Increase of 11.11%
(c) No change (d) Cannot be determined

35. For question 34, find the percentage change in the profit.

(c) Increase of 10% (d) Increase of 11.11%
(e) No change (f) Cannot be determined

39. For question 38, what would be the change in the percentage profit, if along with the increase of the ratio of prices to features, the profit also increases by 1% for both items and formula?
(a) Drop of 1.1 (b) Increase of 1.1
(c) Increase of 0.9 (d) None of these

40. A wholesaler dealer buys 50 multibrand for Rs.4725. Of them, 8 are four-seaters and the rest are two seaters. At what price must he sell the four-seaters so that if he sells the two seaters at twice of this price, he makes a profit of 60% on his outlay.
(a) Rs. 180 (b) Rs. 270
(c) Rs. 360 (d) Rs. 450

41. A flat and a piece of land were bought by two friends Raghav and Sita respectively at prices of Rs. 2 lakh and Rs. 2.2 lakh. The price of the flat rises by 20 percent every year and that of land by 10% every year. After two years, they decide to exchange their properties. What is percentage gain of the gainer?
(a) 7.59% (b) 6.36%
(c) 4.59% (d) None of these

42. A, B and C form a company. A invests half of C expecting a return of 10%. B invests three-fourths of C expecting a return of 15% on it. C invests Rs. 1000 and the profit of the firm is 25%. How much would B's share of profit be more than that of A's share if B gets an additional 8% for managing the business? Assume that their respectives with respect to returns invested are met before profit is divided in the ratio of capitals invested.
(a) 20% (b) 16%
(c) 15% (d) Cannot be determined

43. A driver of a motorcycle makes a profit of 20% on every trip when he carries 3 passengers and the price of petrol is Rs. 30 a litre. Find the percentage profit for the same journey if he goes for four passengers per trip and the price of petrol reduces to Rs. 24 litre? (Assume that expenses per passenger in the case is half the cases.)
(a) 33.33% (b) 43.00
(c) 100% (d) Data inadequate

44. Raghav bought 23 washing machines and microwave ovens for Rs. 2,05,000. He sold half of the washing ovens for Rs. 80,000 and microwave ovens for a profit of Rs. 80,000. Each washing machine was marked up by 20% over cost and each microwave oven was sold at a profit of Rs. 2,000. The remaining washing machines and 3 microwave ovens could not be sold. What is Raghav's overall profit/loss?
(a) Rs. 1000 profit (b) Rs. 1000 loss
(c) Rs. 1000 zero (d) Cannot be determined

45. After selling a watch, Shyam found that he had made a loss of 10%. He also found that had he sold it for Rs. 27 more, he would have made a profit of 5%. The actual actual loss and actual percentage of the profit earned, had he sold the watch for a 5% profit?
(a) 22% (b) 15% (c) 20% (d) 18.75%

46. Sandhu buys rice at Rs. 16/kg and puts a price tag on it so as to earn a profit of 20%. However, his faulty balance shows 1000 gm when it is actually 800 gm. What is his actual gain percentage?
(a) 50% (b) 40% (c) 16% (d) 10%

47. The profit earned when an article is sold for Rs. 800 is 20 times the loss incurred when it is sold for Rs. 275. At what price should the article be sold if it is desired to make a profit of 25%.
(a) Rs. 200 (b) Rs. 350
(c) Rs. 375 (d) Rs. 400

48. A sells to B goods at five-thirds the rate of profit at which B has decided to sell it to C. C, on other hand, sells it to D at one-third the rate of profit at which B sold it to C. If D gives Rs. 2145 to C at 10% profit, how much did A buy it for?
(a) Rs. 1000 (b) Rs. 2000
(c) Rs. 1500 (d) Rs. 1800

49. In the town of Anthar Nagar Chappa Ways, shopkeepers have to buy and sell goods in the range of Rs. 500 to Rs. 999. A shopkeeper in such a town decides not to buy or sell goods for amounts that contain the digit 0 or for amounts that add up to 13 or are a multiple of 13. What is the maximum possible profit he can earn?
(a) Rs. 200 (b) Rs. 389
(c) Rs. 400 (d) None of these

47. Manish bought a continental total of 25 monitors and printers. He marked up the monitors by 20% on the cost price, while each printer was marked up by Rs. 300. He was able to sell 72% of the monitors and 7 printers and made a profit of Rs. 41,000. The remaining monitors and 7 printers could not be sold by

loss. Find the overall profit or loss if he gives an extra on unsold items and is to ensure that a profit costs 50% of a revenue.

(a) Loss of Rs. 40,000 (b) Loss of 20,000
(c) Loss of Rs. 41,000 (d) Inadequate data

48. For question 47, Mamta's approximate percentage profit or loss is

(a) 14.30% loss (b) 10.5% loss
(c) 12.10% loss (d) Insufficient information

49. An orange vendor makes a profit of 20% by selling oranges at a certain price. If he charges Rs. 1.2 higher per orange he would gain 40%. Find the original price at which he sold an orange.

(a) Rs. 5 (b) Rs. 4.8
(c) Rs. 6 (d) None of these

50. The AMS magazine prints 5000 copies for Rs. 5,00,000 every month. In the July issue of the magazine, AMS distributed 500 copies free. Besides, it was able to sell 20% of the remaining magazines at 20% discount. Besides, the remaining magazines were sold at the printed price of the magazine (which was Rs. 200). Find the percentage profit of AMS in the magazine venture in the month of July (assume a uniform 30% of the sale price as the vendor's discount and also assume that AMS earns no income from advertising for the issue).

(a) 36% (b) 24.8% (c) 26.5% (d) 22.6%

Level of Difficulty (LOD) III

The charges of a taxi journey are decided on the basis of the distance covered and the amount of the waiting time during a journey. Distance wise, for the first 2 kilometres (or any part thereof) of a journey, the meter reading is fixed at Rs. 10 (if there is no waiting). Also, if a taxi is hounded and it does not move, then the meter reading is again fixed at Rs. 10 for the first ten minutes of waiting. For every additional kilometre the meter reading changes by Rs. 5 (with changes in the meter reading being in multiples of Rs. 1 for every 200 meters travelled). For every additional minute of waiting, the meter reading changes by Rs. 1, (the account is taken of a fraction of a minute waited for or of a distance less than 200 meters travelled). The net meter reading is a function of the minutes of time waited for and the distance travelled.

The cost of running a taxi depends on the fuel efficiency (in terms of mileage/litre), depreciation (straight line over 10 years and the driver's salary (not taken into account if the taxi is self owned).

Depreciation is Rs. 100 per day everyday of the first 10 years. This depreciation has to be added equally to the cost for every customer while calculating the profit for a particular trip. Similarly, the driver's daily salary is also apportioned equally across the customers of the particular day. Assume, for simplicity, that there are 50 customers every day (unless otherwise mentioned). The cost of fuel is Rs. 35 per litre (unless otherwise stated).

The customer has to pay 20% over the meter reading while settling his bill. Also assume that there is no fuel cost for waiting time (unless otherwise stated).

Based on the above facts, answer the following.

1. If Sardar Preetpal Singh's taxi is 14 years old and has a fuel efficiency of 12 km/litre of fuel, find his profit in a run from Howrah Station to Park Street (a distance of 7 km) if the stoppage time is 8 minutes. (Assume he owns the taxi)

(a) Rs. 32.25 (b) Rs. 40.85
(c) Rs. 34.25 (d) Rs. 42.85
(e) Rs. 44.85

2. For question 2, Sardar Preetpal Singh's percentage profit is

(a) 391.42% (b) 390%
(c) 489.71% (d) 438.23%
(e) 444.25%

3. For the same journey as in question 1 (on another day, with heavier traffic, the waiting time increases to 13 minutes, find the percentage change in the profit.

(a) 12% (b) 14%
(c) 13% (d) 16%
(e) 17%

4. For question 3, if Sardar Preetpal Singh idled his taxi for 7 minutes and if the fuel consumption during idling is 20 ml per minute, find the percentage decrease in the profits.

(a) 10.74% (b) 11.21%
(c) 10.87% (d) 9.94%
(e) 10.44%

Directions for Questions 5–10: Answer questions based on this additional information.

Mr. Vikas Verma owns a fleet of 3 taxis, where he pays his driver Rs. 3000 per month. He also insists on keeping an attendant for Rs. 1500 per month in each of his taxis

[illegible] requires 90 ml of fuel for every minute of idling. For a normal taxi, the fuel consumption is given by 12 kmpl. On a particular day, he received the following expense [illegible] the three taxis.

Taxi	Fuel Expenses	Running cost with idling	Running cost without idling
A	280	190 min	34 min
B	200	170 min	80 min
C	175	180 min	50 min

5. The maximum revenue has been generated by which taxi?
   (a) A (b) B
   (c) C (d) A & B
   (e) Cannot be determined

   It is to be assumed that every customer travelled at least 2 kilometers.

6. Which of the three taxis generated the maximum revenue?
   (a) A (b) B
   (c) C (d) Both A & B
   (e) Cannot be determined

7. What percentage of the total revenue was generated by taxi B?
   (a) 32.39 (b) 33.36
   (c) 34.42 (d) 34.36
   (e) None of these

8. The highest profit was yielded by which taxi?
   (a) A (b) B
   (c) C (d) Both A & B
   (e) Cannot be determined

9. The taxi which had the highest percentage profit for the day was
   (a) A (b) B
   (c) C (d) B & C
   (e) Cannot be determined

10. The profit as a percentage of costs for the day was
    (a) 179.84% (b) 150.79%
    (c) 169.29% (d) 177.46%
    (e) Cannot be determined

Directions for Questions 11–15: Read the following and answer the questions that follow.

The Coca-Cola Company is setting up a plant for manufacture and sale of the soft drink.

The investment for the plant is Rs. 10 crore (to be invested in plant, machinery, advertising, infrastructure etc.).

The following information is known about the different bottle sizes planned:

Bottle size	Bottle cost	Cost of liquid	Transportation cost	Bulk price	Dealer margin
300 ml	Rs. 2	Rs. 0.6	10 paise per bottle	Rs. 10	Rs. 2
500 ml	Rs. 3	Rs. 1	15 paise per bottle	Rs. 18	Rs. 6
1.5 litre	Rs. 10	Rs. 3	30 paise per bottle	Rs. 40	Rs. 12

Based on the information answer the questions below:

11. For which bottle should Coca-Cola try to maximise sales to maximise its profits. (Assume that the total number of liters of Coca-Cola sold is constant irrespective of the break up of the sales in terms of the bottle sizes)
    (a) 300 ml
    (b) 500 ml
    (c) 1.5 liters
    (d) Indifferent between the three sizes
    (e) Cannot be determined

12. If the company sells only 300 ml bottles in the first year, how many bottles should it sell to recover the investment made in the first year only?
    (a) 33,255,814 (b) 282,558,140
    (c) 32,755,814 (d) 322,558,140
    (e) None of these

13. If sales of 300 ml bottles to 500 ml bottles is 4 : 1, and there is no sale of 1500 ml bottles how many 300 ml bottles will be required to recover the investment?
    (a) 1,75,53,580 (b) 2,93,25,512
    (c) 16,25,848 (d) 10,25,774
    (e) Cannot be determined

14. For question 13, the total number of both the types to be sold in India in order to recover the whole investment is
    (a) 3665896 (b) 5055310
    (c) 21091975 (d) 23123195
    (e) Cannot be determined

15. If we add administrative costs of Rs. 1 per litre, which bottle size will have the maximum profitability?
    (a) 300 ml (b) 500 ml (c) 1.5 litres
    (d) Indifferent between the three sizes
    (e) Cannot be determined

[illegible]

(a) 35,000 (b) 28,000
(c) 12,000 Loss (d) 17,500 Loss
(e) None of these

17. For the above situation, keeping the A/C occupancy constant at 50%, what should be the minimum selling price for one A/C room for making no loss for the month?
(a) 23,000 (b) 10,000
(c) 12,500 (d) 16,000
(e) 8000

18. For the questions 15 and 16, Rs. 25,000 worth of advertising a sales promotion of 20% off on the bill doubles the occupancy rate. If this is done, what is the change in the profit or loss?
(a) Reduction of loss by Rs. 5,000
(b) Reduction of loss by Rs. 3,000
(c) Reduction of loss by Rs. 20,000
(d) Both b and c
(e) None of these

19. Advertising worth Rs. 30,000 is done for the sales promotion of A/C rooms providing a 20% reduction on the bill for A/C rooms. This leads to a doubling of the occupancy rate of A/C rooms. Besides, it also has an effect of increasing non A/C room occupancy by 20%. Is this advisable?
(i) Yes
(ii) No
(iii) Cannot be determined
(iv) None of these

A restaurant has a pricing policy that allows for the following mark ups:

Items		
Beverages	Mark up of	40%
Entrees	Mark up of	50%
Breads	Mark up of	25%
Desserts	Mark up of	75%
	Mark up of	75%

20. Mr. Amarnath and his family of 4 went to a restaurant and got a bill for: Soups (Rs. 120), Starters (Rs. [illegible]), Meals (Rs. 600), Breads (Rs. 240) and Desserts (Rs. 240). Find the profit for the restaurant.
(a) Rs. 340 (b) Rs. 351
(c) Rs. 361 (d) Rs. 371
(e) Rs. 381

21. The approximate percentage profit for the restaurant on the bill is
(a) 40% (b) 45% (c) 50% (d) 55%
(e) 60%

22. Which of these are true.
(i) Profit increases if a part of the money spent on starters was spent on breads and another part of the money was spent on snacks.
(ii) Profit increases if a part of the money spent on meal items was spent on starters and another part spent on soups was spent on breads.
(iii) Profit decreases if a certain amount (say x) of the spending on soups was spent on starters and the same amount (Rs. x) of the spending on soups is spent on meal items.
(a) (ii) only (b) (iii) only
(c) (ii) and (iii) (d) All the three
(e) None of these

Directions for Questions 23–28: Read the following and answer the questions that follow.

Prabhat Ranjan manipulates his internet cafe on the 1st of January 2003. He intends to 10 computers @ Rs. 3000 per computer. Besides, he also invests in the other infrastructure of the center, a sum of Rs. 1 lakh only. He charges his customers on the time spent on the internet at the rate of Rs. 50 per hour. His initial investment on computers has to be written off equally in 3 years (1 lakh per year) and the infrastructure has to be written off in 5 years (@ Rs. 20,000 per year).

He has to pay a fixed rental of Rs. 8000 per month for the space and also hires an assistant at Rs. 2000 per month.

For every hour that he is connected to the internet, he has to bear a telephonic charge of Rs. 20 irrespective of the number of machines operational on the internet at that time. On top of this, he also has to pay an electricity charge of Rs. 5 per computer per hour. Assume that there are no other costs involved unless otherwise mentioned. The internet cafe is open 12 hours a day and is open on all 7 days of the

(iii) Assume that if a machine is not occupied, it is put off and hence expenses are eliminated.

(iv) Assuming a uniform 60% occupancy rate for the month of June 2003, find the profit or loss for the month.

(a) Rs. 1,62,400 (b) Rs. 1,22,400
(c) Rs. 1,15,600 (d) Rs. 1,20,725.33
(e) None of these

(v) If the occupancy rate drops to 40% in the month of June, what is the value of the profit for the month?

(a) Rs. 30,000 (b) Rs. 20,000
(c) Rs. 1,23,000 (d) Rs. 90,611.33
(e) None of these

27. If Prabhat estimates a fixed occupancy rate of 80% during the peak hours of 2 to 8 pm and 40% in the off-peak hours of 8 am to 2 pm find the expected profit for him in the month of July 2003.

(a) Rs. 73,000 (b) Rs. 95,100
(c) Rs. 96,000 (d) Rs. 1,27,920
(e) None of these

28. The percentage margin is defined as the margin as a percentage of the variable cost for an hour of operation. Find the percentage margin of the cyber cafe Prabhat runs.

(a) 600% (b) 533.33%
(c) 33% (d) None of these
(e) Cannot be determined

29. For question 25 above, how many 30 day months will be required for Prabhat to recover back the investment?

(a) 1.98 months (b) 3.72 months
(c) 3.71 months (d) 3.81 months
(e) Cannot be determined

30. If the tariff rates per hour have to be dropped downwards to Rs. 20 per hour in the fourth year of operation, what is Prabhat's expected profit for the calendar year 2003 assuming an average of 60% occupancy rate for the year?

(a) Rs. 2,66,000 (b) Rs. 1,66,800
(c) Rs. 95,200 (d) Rs. 40,500
(e) Cannot be determined

Directions for Questions 29–33: Read the following and answer the questions that follow.

A train journey from Patna to Delhi for the Magadh Express has 4 classes.

The fares of the 4 classes are as follows.

1 tier Rs. 200	No. of berths per bogey: 72	No. of bogeys: 8
AC 3 tier Rs. 800	No. of berths per bogey: 64	No. of bogeys: 2
AC 2 tier Rs. 1200	No. of berths per bogey: 48	No. of bogeys: 2
AC first Rs. 1600	No. of berths per bogey: 20	No. of bogeys: 1

Patna to Delhi distance: 1100 kilometres. Assume the train does not stop at any station unless inform for indicated. Running cost per kilometre: AC bogey → Rs. 25, non AC bogey → Rs. 10.

29. Assuming full occupancy, a bogey of which class exhibits the highest profit margin?

(a) AC 3 tier (b) AC 2 tier
(c) AC first class (d) 3 tier

30. Assuming full occupancy in all the classes, for a journey between Patna to Delhi, the profit margin (as a percentage of the running cost) of the class showing the lowest profit is approximately:

(a) 194% (b) 177%
(c) 109% (d) None of these

31. What is the approximate price for the railways to repay if the Magadh Express runs at full occupancy on a particular day?

(a) Rs. 250,000 (b) Rs. 275,000
(c) Rs. 300,000 (d) Cannot be determined

32. For question 31, the percentage of the total profit that comes out of AC bogeys is (approximately):

(a) 50% (b) 60% (c) 70% (d) none

33. The highest revenue for a journey from Patna to Delhi will always be generated by

(a) 3 tier (b) AC 3 tier
(c) AC 2 tier (d) Cannot be determined

34. A newspaper vendor sells three kinds of periodicals: dailies, weeklies and monthlies.

The weeklies sell for Rs. 12 at a profit of 20%, the monthlies sell for Rs. 30 at a profit of 25%, while the dailies sell at Rs. 3 at a profit of 50%. If there is a government restriction on the total number of periodicals that one particular news vendor can sell, and Rs. 6 a day cannot vendor has sufficient capital for all the three types of periodicals, what should be the maximum profit?

(a) Sell maximum weeklies

(b) Sell maximum mandarins
(c) Sell maximum apples
(d) Cannot be determined

35. Without the restriction mentioned in the problem above, what should the newspaper vendor do to maximize his profits if his capital is limited?
(a) Sell maximum watches
(b) Sell maximum mandarins
(c) Sell maximum dailies
(d) Cannot be determined

36. A fruit vendor buys fruits from the fruit market at wholesale prices and sells them at his shop at retail prices. He operates his shop 30 days a month, as a rule. He buys in multiples of 100 fruits and sells them in multiples of a dozen fruits. He purchases mangoes for Rs. 420 per hundred and sells at Rs. 55 per dozen, he buys apples at Rs. 150 per hundred and sells at Rs. 30 per dozen, he buys watermelons (always of equal size) at Rs. 1800 per hundred and sells at Rs. 360 per dozen. Which of the three fruits yields him the maximum percentage profit?
(a) Mangoes (b) Apples
(c) Watermelons (d) Both b and c

37. For question 36, if he sells oranges, which he buys at Rs. 180 per hundred and sells at Rs. 33 per dozen, what can be its maximum profit on a particular day if he invests Rs. 1800 in purchasing fruits everyday and he sells everything that he buys?
(a) Rs. 1200 (b) Rs. 1180
(c) Rs. 1260 (d) Rs. 1320

38. For questions 36 and 37, if the fruit vendor hires you as a consultant and gives you 20% of his profit in the month of July 2006 as a service charge, what can be the maximum fees that you will get for your consultancy charges?
(a) Rs. 7200 (b) Rs. 14,400
(c) Rs. 7440 (d) Cannot be determined

39. A newspaper costs Rs. 11 to print on a daily basis. Its sale price (printed) is Rs. 3. The newspaper gives a sales incentive of 40% on the printed price to the newspaper vendors. The newspaper makes up for the loss through advertisements, which are charged on the basis of per column centimeter rates. The advertisement rate of the newspaper are Rs. 800 per cm (col. cm). It has to give an incentive of 15% on the advertising bill to the advertising agents. If the newspaper has a circulation of 12,000 copies, what is the approximate minimum advertising bookings required if the newspaper has to break-even on a particular day. (Assume there is no wastage.)
(a) 300 cc (b) 350 cc (c) 435 cc (d) 400 cc

40. For question 39, if it is known that the newspaper house is unable to recover 20% of its dues, what will be the approximate advertising booking target on a particular day in order to ensure the break-even point?
(a) 375 cc (b) 438 cc
(c) 544 cc (d) 562.5 cc

Hints and Solutions

1. Cost of 750 articles = Rs. 450 → outlay
Hence, SP of 600 articles = Rs. 630. (40% profit on his outlay)
∴ SP of each article = Rs. 1.05.

2. When the manufacturer makes 100 articles, he sells 88.
Revenue = 88 × 7.5 = Rs. 660
But profit being 20% of outlay, total outlay is Rs 5[illegible]

3. Total cost = Type + Running cost of printing machine + paper, ink
= 1000 + 120 × 5 + 540 = 2020
∴ Net sum to be recovered = Rs. 2882

4. Assume CP of goods = 100.
Then marked price = 130.
Revenues = $\frac{1}{2} \times 130 + \frac{1}{4} \times 0.85 \times 130 + \frac{1}{4} \times 0.[illegible] \times 130$

5. If CP = x, then marked price = x + 20% and
Selling price = 0.9x + 184.5
Profit = 184.5 − 0.1x

7. Assume CP = 1 Rs/gram.
Then, he sells 900 grams for Rs. 1080.
While the CP of 900 grams is Rs. 900.

8. He buys 1100 grams for Rs.1000 and sells 900 grams for Rs. 1080.
To calculate profit percentage, either equate money or the goods.

9. He buys 1100 grams for Rs. 900 and sells 900 grams for Rs. 1080.
To calculate percentage profit, either equate the money or equate the goods.

10. Take the LCM of 4 and 3 and assume 24 apples bought at each price.
Ensure that you express either the goods or the money.
12. The loss is assumed by the sale of 20 extra kg of rice.
i.e. CP of 100 kg = SP of 120 kg
13. Profit maximization is usually dependent on revenue maximization, since the cost is constant.
14-15 Solve using options.
16. Difference between original CP and SP is 2%, while the reverse difference is 62.5% of Q.
The only values satisfying these are 60% and 37.5%.
Hence Q = 60%.
17. If CP = 100, original marked price = 115 and new marked price = 126.5. But he now sells at 80% of his marked price.
[illegible] the SP to be 125 for both the calculations.
18. If initial cost price is 100, then initial selling price will be 120. Then, the new cost price is 90, while the new selling price is 132.
19. The successive percentage drops in marked price to get his selling price are 10% each.
20. Rising cost = 0.35 × 12000
Gross profit = 12000 − (1400 + 650 + 252) − 0.35 × 12000
∴ Trading cost = 25% of Gross profit.
∴ Net profit = Gross profit − 25% of Gross profit
= 75% of Gross profit.
and percent profit = $\frac{\text{Net Profit}}{\text{Outlay}} \times 100$
21. Solve using options.
19. Assume Daniel's cost price as 100. Then, Goliath buys it at 80 sells it at 100 and buys back from Bertrides at 90.
20. Buys 1200 when he pays for 1000 and sells 900 when he receives money for 1000.
21-24 Investments are Rs. 35000 for men and Rs. 45000 for women.
The rate of return being given, we can find out the percentage profit overall.
25. Solve through options.
26. It's share is only dependent on the ratio of capital investment.
27. Assume cost = 100 and SP = 120.
Then, when price of petrol is reduced →
Cost = 80 and SP = 100.
28. The number of washing machines plus the number of microwaves = 25.
Unsold items = 3 washing machines + 3 microwaves
Goods sold = 8 washing machines + 12 microwaves
= 40% of total value.
24. If L represents the loss, then 20 − L represents the profit. Hence, L = $\frac{525}{21}$ = 25.
Hence, CP = 400.
25. A sells at 30% profit, while B sells at 30% profit.
26. The required profit will be given by:
Maximum possible selling price − Minimum possible purchase price.
30. Sales of 3000 copies for Rs 180 and 1500 copies for Rs. 200 each.

Hints and Solutions

1–20. Concentrate on creation of the revenue equation and the cost equations separately. Revenue from a journey will depend on

(a) length of journey (over 2 kilometres)
(b) time of waiting.

Besides, the fixed sum reading of Rs. 10 at the start is used up in the rate of 1 Re per 300 metres and/or 1 Rs/minute of waiting.

11. Coca-Cola earns (10 − 3) − (2 + 0.6 + 0.1) = Rs. 4.3 per 300 ml bottle.
Similarly, for 500 ml bottle, the profit is

$$12 - 6.15 = \text{Rs. } 5.85$$

and for 1500 ml bottle, 28 − 13.2 = 14.8 for 1500 ml bottle.
The profit per ml sold has to be maximised.

12. $\frac{\text{Rs. 10 crore}}{4.3} = 2,32,55,814$.

13–14. The earning for one set of 5 bottles = 4.3 × 4 + 5.85 = Rs. 23.05.

15. Maximum profitability = $\frac{\text{Margin per bottle}}{\text{Cost per bottle}}$.

16–18. Profit = Revenue − Expenses.

19–22. Observe the profitability ratios for slots steps of three.

23. Revenue = 8 computers × Rs. 8000 = 12 hours × 30 days

$$\text{Cost} = \text{monthly cost} + \frac{\text{Depreciation}}{12} = \text{Hourly cost}$$

$$= 37 \text{ hours} \times 30 \text{ days}$$

24-28. Will be solved on the same principle as question 23.

29-33. Revenue − expenses = profit/loss

$$\frac{\text{Cost}}{\text{Revenue}} = \text{pc. of revenue}$$

Detailed Solutions & Short Cuts

LOD 1

1. 0.8 × Price = 440 → Price 550
2. 1.12 × Price = 4000 → Price = 4000
3. 240 = 1.25 × cost price → Cost price = 192.0
   Profit in 240 = Rs. 120
   Percentage profit = 120/240 × 100 = 6.25%
7. The CP will be Rs. 5400. Hence at an S.P. of 5703.75 the percentage profit will be 5.625%
9. The buying price is Rs. 4 per dozen, while the sales price is Rs. 12 per dozen – a profit of 33.33%
10. C.P. × 1.2 = 25 → CP = 20.833
    At a selling price of Rs. 22.5, the profit percent 1.667/20.833 = 8%
12. Solve using options. Option (c) gives cost Rs. 175 as the cost of the jeans. Hence, the sale will cost 12% more i.e. 175 + 17.5 = 3.5 = 196.
    This satisfies the total cost requirement of Rs. 171.
13. The formula that satisfies this condition is:
    Loss of x^2/100% (Where x is the continuous profit and loss percentage). Hence, in this case 400/100 = 4% loss
14. Use POG as follows:
    $100 \xrightarrow{\text{loss } 1} 80 \xrightarrow{\text{Profit } 5} 76 \xrightarrow{\text{loss } 1} 68.4$
    Hence, 31.6% is the correct answer.
16. On a marked price of Rs. 90, a discount of 10% would mean a selling price of Rs. 72. Since this represents a 20% profit we get:
    CP × CP = 72 → CP = 60
18. For Rs. 12, he can buy 4 dozen pair of gloves. Hence, for Rs. 24 we can buy 8 pairs of gloves.
20. 180 × 0.8 × x = 135 → x = 0.93
    Which results a 15% discount.
21. In this, there is a 6% loss. This means a cost price of Rs. 1200. For a 16% profit, the selling price should be Rs. 1160.
28. The customer pays Rs. 39 after a discount of 2%. Hence, the list price must be Rs. 40.
    This also means that at a 20% discount, the retailer buys the item at Rs. 32.
    Hence, the profit for the retailer will be Rs. 9 (39 − 32).
29. The labour price accounts for Rs. 400. Hence the profit percentage gives a 20% profit on this amount i.e. Rs. 80.
    Hence, the marked price is Rs. 980.
30. The percentage profit = $\frac{\text{Goods left}}{\text{Goods sold}} \times 100$
    = 10/40 × 100 = 25%
31. In the question, A's investment has to be considered as Rs. 10,000 (the house he puts up for sale).
    He sells at 11,500 and buys back at Rs. 9775. Hence his profit is Rs. 1725.
    Required answer = $\frac{1725}{10000} \times 100 = 17.25$
33. For 12 bricks, he would have paid Rs. 51, and sell them at Rs. 57. This would mean a profit percentage of 11.76%
35. A 10% reduction in price increases the consumption by 11.11% (Refer Table 4.1). But the increase consumption is 6.2 kg.
    Hence, the consumption (original) will be 6.2 × 9 = 55.8 kg.
    Hence, original price = 270/55.8 = Rs.5.
    Hence, reduced price = Rs. 4.5
37. Percentage profit = $\frac{\text{Goods left}}{\text{Goods sold}} \times 100$
    = 10/20 × 100 = 50%
40. B sold the table at 25% profit at Rs. 75. Thus cost price would be given by: CP_B = 1.25 × 75
    B's Cost price = Rs. 60
    We also know that A sold it to B at 20% profit. Thus,
    A's Cost price × 1.2 = 60
    → A's cost price = 50.
42. Net loss = $(20/100)^2$ = 4% of cost price. Thus 48 (total money realized) represents 96% of the cost. Thus, the cost price would be Rs. 5000 and the loss would be Rs. 200.
49. The profit of 14% amounts to Rs. 450. This should also be the actual loss on the second TV.
    Thus, the actual loss = Rs. 450 (10% of C.P.)
    Hence, the CP of the second set = Rs. 4500.

40. 2/3 of the cost price = Rs. 300
Thus cost price = Rs. 900.
and selling price at 30% profit = Rs. 1 080.

41. Original Cost Price = Rs. 5000
New Cost Price = 1.3 × 5000 = Rs. 6500
Price paid by retailer = 1.2 × 5720 = Rs. 6900
Profit percentage = (1800/6500) × 100 = 6.12/13%

42. Assume marked price for both to be 100.
Its selling price = 100 × 0.75 × 0.95 = 71.25
Its selling price = 100 × 0.64 × 0.88 = 73.92
Buying from 'A' is more profitable.

46. The trader pays 800 × 0.85 × 0.85 = Rs. 722

47. Manufacturer's profit percentage
= (220/100) × 100 = 120%

48. Cost price to the watch dealer
= 240 + 10% of 240 = Rs. 275
Desired selling price for 20% profit
= 1.2 × 275 = 330
But 330 is the price after 25% discount on the marked price.
Thus,
Marked price × 0.75 = 330 → MP = 440
Hence, he should mark the item at Rs. 440.

49. Cost for 100 apples = 60 + 15% of 60 = Rs. 69
Selling price @ 20% profit = 1.2 × 69 = Rs. 82.8

45. Profit percent = (100/900) × 100 = 11.11%

LOD II

1. Total money claimed (investment) = 750 × 0.6 = Rs. 450
By selling 600, he should make a 40% profit on the selling. This means that the selling price for 600 should be 1.4 × 450 → Rs. 630
Thus, selling price per article = 630/600 = 1.05
Hence, he sells only 630 articles at this price to avoid recovery = 1.05 × 630 = 661.5
Profit percent (actual) = (211.5/450) × 100 = 47%

4. Total cost (assume) = 100
Recovered amount = 65 + 0.85 × 32.5 + 0.7 × 32.5
= 65 + 27.625 + 22.75 = 115.375
Hence, profit percent = 15.375%

5. Cost price = x
Marked Price = x + 20%
Selling Price = 0.90 × 184.5
Percentage Profit = [1−0.13 × 184.5/x] × 100

= (18450 − 10x)/x

6. She should opt for a straight discount of 30% as that gives her the maximum benefit.

8. While buying
He uses 1100 grams instead of 1000 grams (due to his cheating).
Suppose he bought 1100 grams for Rs. 1000
While selling
He sells only 900 grams when he takes the money for 1 kg.
Now according to the problem he sells at a 20% profit (20% mark up and 10% discount).
Hence his selling price is Rs. 1080 for 900 grams.
To calculate profit percentage, we either equate the goods or the money.
In this case, let us equate the money as follows:
Buying:
1100 grams for Rs. 1000
Hence: 1188 grams for Rs. 1080
Selling: 900 grams for Rs. 1080
Hence, profit% = 288/900 = 32%
(using profit left by goods sold formula)

9. The new situation is
Buying:
1100 grams for Rs. 900
Hence, 1320 grams for Rs. 1080
Selling: 900 grams for Rs. 1080

Profit % = 420/900 × 100 = 46.66%

10. Assume he bought 20 apples each. Net investment = Rs. 5 + Rs. 4 = Rs. 9 for 40 apples. He would sell 40 apples @ (40) × 2% = Rs. 8.888 → Loss of Rs. 0.111 on Rs. 9 investment.
Loss percentage = 1.23%

12. The problem is structured in such a way that you should be able to interpret that if he had sold 120 kg of rice he would recover the investment on 100 kg of rice.

% Loss/Profit = (Goods left − Goods sold)/Goods sold × 100

(1−20/120) × 100 = 16.66% loss

Hence, cost price for Dish is Rs. 11, selling price per kg would be Rs. 9/kg.

13. Comparisons have to be made between:
192 × 34, 160 × 34, 204 × 27 and 110 × 31 for the highest product amongst them.

The highest value of revenue is seen at a price of Rs 100.

11. (c) (i) Using options from question 13. Suppose she had spent Rs 5 at the market complex, she would spend Rs 3 at the uncle's shop. The other condition that she gets 2 sweets less per rupee at the market gives price gets satisfied in this scenario if she had bought 41 chocolates overall. In such a case, her buying would have been 2 per rupee at the market and 4 per rupee at Uncle Sweetie's shop.

Total and cross will show that this condition is not satisfied for any other option combinations.

12. The given situation fits if we take Q as 80% profit and then the loss would be 37.5% (which is 62.5% of Q). Thus, if Rs. 24 is the sent price, the selling price should be $24 \times 1.6 =$ Rs. 38.4

13. Ramu's total discount

6% on 9000	= Rs. 540
5% on 12000	= Rs. 600
2% on 16000	= Rs. 320
Total	= Rs. 1460 or Rs. 36000

Hence, Realised value = 34540

Shyam's Discount

7% on 12000	= 840
6% on 9000	= 540
5% on 16000	= 800
	Rs. 2180 or Rs. 36000

Hence, Realised value = 33820.

The higher profit is for Ramu.

Now, the CP has a mark up of 25% for the Marked price. Thus the CP must have been 28800. (This is 24 by 30000 – 20% of 30000 – PCG thinking)

Thus, the profit % for Ramu would be, (5460×100/28800) → 19% approx.

15. In the case of the given details, the discount for Ramu would have gone down to

4% on 12000 (the second payment) and the second discount would then have been Rs 480 meaning the net price would have risen by Rs 120. (Hence there is a Rs 120 drop in the discount)

2% on 16000 → A reduction of 2% of 16000 in the discount → a reduction of Rs. 320.

Hence, Ramu's profit would have gone up by Rs 440 in all. It would yield his new profit as:

$5460 + 440 =$ Rs. 5920

16. The following working would show the savings:

Ramu's Discounts:

5% on 9000	= Rs. 450
4% on 12000	= Rs. 480
2% on 16000	= Rs. 320
Total	= Rs. 1250 or Rs. 36,500

Shyam's discounts:

6% on 12000	= 720
5% on 9000	= 450
4% on 16000	= 640
Rs. 1760 or Rs. 36000	

Thus, their profits would vary by Rs. 400 (since the cost price is the same).

24. If their CP's had been 1000 each, their respective SP's can be calculated as follows:

$1000 \frac{100+P}{100+20} = 1250$. (for the person calculating profit on the CP)

$1000 \frac{100+P}{100+15} = 1173.33$

(for the person calculating his profit on SP: 25% of SP = 15.33% of CP)

The value of the difference in this case has turned out to be Rs.83.33. This has occurred when we have assumed the CP as 1000. But, we are given a difference of Rs.100.

Use unitary method as follows:

Difference of 83.33 when CP is 1000

Hence, difference of 100 when CP = ?

$\frac{100 \times 1000}{83.33} =$ Rs.1200

25. Find out the total revenue realisation for both the cases.

Case 1: (Old) Total sales revenue = 3000 × 1.5 × 0.75

$Profit_{old}$ = Total sales revenue – 4500

Case 2: (New) Total sales revenue = 3000 × 1.25 × 0.75

$Profit_{new}$ = Total sales revenue – 4500

The ratio of profits will be given by $Profit_{old}$: $Profit_{new}$

26. The successive discounts must have been of 10% each. The required price will be got by reducing it by 10% twice consecutively (use PCG approach for successive changes)

37. The following calculations will show the respective profit.
Primary Cost: 75% of 12000 = 4500
Miscellaneous costs = 2% of 12000 = 240
Gross Profit = 12000 − 4500 − 1400 − 800 − 250 = 5050
Trading tax = 0.25 × 5050 = 1262
Hence, the profit = 4416.
Percentage profit = 4416/14000 = 31.54%
38. If we assume the value of the first cycle as Rs. 900. Then 900 + 96 = 996 should be equal to twice the value of the second cycle. Hence, the value of the second cycle works out to be: 498.
Also 498 + 96 = 594 which is Rs.306 less than 900.
Hence, Option a fits the situation perfectly and will be the correct answer.
Note here that if you had tried to solve this through equations, you would have got stuck for a very long time.
39. David (100) → Goliath (80) → Hercules (100) → Goliath (60)
Hercules loss corresponds to 50 when David buys the laptop for 100.
Hence, Hercules's loss would be Rs.17500 when David buys the laptop for 1,75,000.
40. While purchasing he would take 1250 grams for the price of 1000 grams.
While selling he would sell 900 grams for the price of 1000 grams. Since CP = SP, the profit earned is through the weight manipulations. It will be given by,
Gain (%)/goods sold = 300 × 100/900 = 33.33%
41. An increase in the price by Rs.12 will correspond to 50% of the CP.
Hence, The CP is Rs.24 and initially the book was being sold at Rs.19.2. Hence, if there is an increment of Rs. 4.8 in the selling price, there would be no profit or loss.
42. In the first year, the profit percentage would be:

$$\frac{0.55 \times 5 + 0.45 \times 9}{1} = 0.65\%$$

$$\frac{0.45 \times 8 + 0.45 \times 5}{1} = 0.65$$

43. Since the sales remains unchanged the percentage profit of the village will remain unchanged too.
44. The profit would increase by 10% as there is no change in the percentage profit.
45. $x + 8 + 0.75x + 12 = 4.4 \times 4725 \Rightarrow x = 2700$
Hence, the investment of Rs. 4725 is profit of 40% on an investment of Rs. 2700.
Hence, profit of 1890.
Hence, the marked sales realization is Rs. 6615.
The needed equation would be:
$8x + 22.5x + 12 = 6615$
$\Rightarrow 8x + 22.5x = 6615$
In this expression for LHS to be equal to RHS, we need 32x/3 to be an odd number. This can only happen when x is a multiple of 6 only. Apply your mind. Hence, options 2 & 3 are eliminated automatically.
46. After 2 years, the flat would be worth Rs. 288000, while the land would be worth Rs. 256200. The profit percentage of the power would be given by:

$$(256200/200000) \times 100 = 8.189\%$$

Hence (3).
47. The total investment will be $A + B + C$.
C being 3000, B will be 2250 and A will be 1500.
The total investment is 6750.
Returns to be given as their expectations:
$A = 190$, $B = 337.5$ and $C = 0$.
From this point calculate the total profit, estimate A's and B's expected income and B's share of the profits for managing the business before dividing the profits in the ratio of capital invested.
48. The cost of the toy would be proportional to the price of petrol. So, if initially the cost is 100, the new cost would be 80. Also, initially since the profit is 30%, his revenue would be 130. When he takes 4 passengers instead of 3 his revenue would go up to 160 – and his profit would increase 100% from 30 and revenue 160.
49. Total number of successful prizes = 15
Hence, matching machines = 30
Thus, He sells 80% of tickets at a profit of Rs. 40,000.
Cost of 80% of the goods = 0.8 × 3,00,000 = 2,40,000.
Total revenue received = 2,40,000 + 40,000 = 2,80,000
Hence, loss = Rs. 1000
47. There were 5 prizes (2 + 3x and 2) overall. He sells 5 prizes for a profit of Rs. 1000 each. Hence, profit from prizes sales = Rs. 5000
Thus, profit from revenue sales = Rs. 45000

Gross profit per scooter = $\frac{45000}{15}$ = Rs. 3000

(Since, 15 scooters were sold in all.)

Hence, C.P. of scooter = Rs. 15000

and C.P. of Fridge = Rs. 7500

Total cost = 15000 × 20 + 7500 × 5 = 3,37,500

∴ Total Revenue = 16000 × 15 + 9500 × 5 = 3,09,000

Hence, loss of Rs. 48,500

46. Loss% = $\frac{48,500}{3,37,500} \times 100 = 14.37\%$

ANSWER KEY

Profit and Loss LEVEL 1

1. b
2. b
3. c
4. d
5. a
6. b
7. d
8. a
9. b
10. a
11. d
12. c
13. d
14. b
15. c
16. d
17. b
18. a
19. b
20. c
21. c
22. b
23. c
24. c
25. c
26. d
27. b
28. a
29. c
30. b
31. d
32. d
33. a
34. a
35. b
36. a
37. c
38. c
39. d
40. c
41. a
42. b
43. c
44. b
45. c
46. d
47. a
48. b
49. a
50. c
51. c
52. a
53. b
54. b
55. c
56. a
57. b
58. c
59. b
60. a
61. b
62. c
63. d
64. c
65. b

Profit and Loss LOD II

1. c
2. a
3. b
4. b
5. a
6. a
7. c
8. c
9. d
10. a
11. d
12. b
13. a
14. a
15. a
16. d
17. c
18. a
19. a
20. c
21. a
22. a
23. b
24. c
25. d
26. d
27. b
28. a
29. c
30. a
31. c
32. a
33. a
34. c
35. a
36. b
37. b
38. d
39. d
40. c
41. c
42. c
43. a
44. c
45. a
46. a
47. a
48. a
49. d
50. b

Profit and Loss 1.00 m

1. a
2. c
3. a
4. c
5. c
6. a
7. b
8. a
9. a
10. a
11. b
12. b
13. c
14. c
15. a
16. c
17. a
18. a
19. b
20. a
21. c
22. c
23. c
24. b
25. a
26. c
27. c
28. a
29. c
30. c
31. b
32. a

33. d
34. b
35. c
36. d
37. a
38. a
39. c
40. c

7

INTEREST

INTRODUCTION

The chapter on Interests forms another important topic from the CAT's point of view. However, this relevance/importance is only restricted to the use of interests and its concepts in Data Interpretation.

As far as the use of direct questions is concerned, interests (like Profit and Loss) has been conspicuous by its absence in the last five years.

However, it's relevance to Data Interpretation remains as high as always.

Prior to studying this chapter however, you are required to ensure that a clear understanding of percentages and percentage calculation is a must. The faster you are at percentage calculation, the faster you will be in solving questions of interest.

However, questions of interests are still important for exams like MAT, XAT, ATMA, CET, IRMA and BANK PO exams. Hence, if you are planning to go for the entire spectrum of management exams—this chapter retains its importance in terms of mathematics too.

Questions from LOD I and LOD II of the chapter regularly appear in exams like the Bank PO entrance or others management exams.

CONCEPT OF TIME VALUE OF MONEY

The value of money is not constant. This is one of the principal facts on which the entire economic world is based. A rupee today is not equal to a rupee tomorrow. Hence, a rupee borrowed today cannot be repaid by a rupee tomorrow. This is the basic need for the concept of interest. The rate of interest is used to determine the difference between what is borrowed and what is repaid.

There are two basis on which interests are calculated.

Simple Interest. It is calculated on the basis of a base amount borrowed for the entire period at a particular rate of interest. The amount borrowed is the principal for the entire period of borrowing.

Compound Interest. The interest of the previous year is/are added to the principal for the calculation of the compound interest.

This difference will be clear from the following illustration.

A sum of Rs. 1000 at 10% per annum will have

Simple interest		Compound interest
Rs. 100	First year	Rs. 100
Rs. 100	Second year	Rs. 110
Rs. 100	Third year	Rs. 121
Rs. 100	Fourth year	Rs. 133.1

Note that the previous year's interests are added to the original sum of Rs. 1000 to calculate the interest to be paid in the case of compound interest.

Terminology Pertaining to Interests

The man who lends money is the **Creditor** and the man who borrows money is the **Debtor**.

The amount of money that is initially borrowed is called the **Capital** or **Principal** money.

The period for which money is deposited or borrowed is called **Time**.

[illegible] is called the Total amount or the capital.

The sum of the principal and the interest at the end of any period is called the Amount.

$$\text{Amount} = \text{Principal} + \text{Total Interest}$$

Rate of interest is the rate at which the interest is calculated and is always specified in percentage (%).

SIMPLE INTEREST

[illegible] Rs. 100 is [illegible] Rs. 105 [illegible] "The rate of interest per annum is 5%" means Rs. 5 is the interest on a principal of Rs. 100 for 1 year.

Relation Among Principal, Time, Rate Percent of Interest Per Annum and Total Interest

Let Principal = Rs. P, Time = n years, Rate of interest per annum = $r\%$ and Total interest = Rs. I.

Then
$$I = \frac{P \times r \times n}{100}$$
is total interest.

$$\text{Rate} = \frac{\text{Interest} \times \text{Time} \times \text{Hundred}}{\text{Principal}}$$

Time = Interest × Hundred / Principal × Rate

$$\text{Amount (A)} = P + \frac{P \times r \times n}{100}$$

$$\text{Time} = \left[\frac{\text{Total interest}}{\text{Interest on the Principal for one year}}\right] \text{years}$$

Thus, if we have the total interest as Rs. 300 and the interest per year is Rs. 10, then we can say that the number of years is equal to 30 years.

Note : The rate of interest is generally taken 5% or 6% or 7% or 8% or 9% or 10% in such a case we take the rate 5% or 6% or 7% or 8% or 9% or 10%.

Remark : If the rate of interest is specified in terms of half yearly, we take time in terms of 6 months.

Or, the half yearly rate of interest is 5% the amount of interest. Thus, if the interest is 10% per annum compounded quarterly we have to add interest every 3 months at 2½%.

COMPOUND INTEREST

In many transactions, either the borrower and the lender, in order to settle an account, agree on a certain amount of money to be paid in the future on the basis of specified rate of time. This may be yearly or half yearly or quarterly, with the condition that the payee permits the payment at a certain interval of time be added to the principal or that the total amount at the end of an interval becomes the principal for the next interval. Thus, it is obvious that [illegible].

In such cases, the interest for the first interval is added to the principal for the second interval and so on.

The difference between the amount and the corresponding interval is called the compound interest for the given period.

Formula

Let Principal = P, Time = n years and rate of interest = $r\%$ p.a. and let A be the amount at the end of n years. Then

$$A = P\left(1 + \frac{r}{100}\right)^n$$

Case (2) When compound interest is reckoned half yearly.

If the rate of interest is $r\%$ per annum and is reckoned half yearly for n years.

Then the amount will be $r/2\%$ half yearly and there are $2n$ half years.

Then in this case we get

$$A = P\left(1 + \frac{r/2}{100}\right)^{2n}$$

Case (3) When compound interest is reckoned quarterly.

In this case, rate is $r/4\%$ quarterly and time = $4n$ quarter years.

So, we have

$$A = P\left(1 + \frac{r/4}{100}\right)^{4n}$$

Note : The difference between the compound interest and the simple interest over two years is given by

$$\text{Difference} = P\left(\frac{r}{100}\right)^2$$

DEPRECIATION OF VALUE

The value of a machine or any other which unfortunately wears and tear, decreases with time.

This decrease is called as depreciation.

Thus if V_0 is the value at a certain time and $r\%$ per annum is the rate of depreciation per year, then the value V_t at the end of t years is

$$V_t = V_0\left(1 - \frac{r}{100}\right)^t$$

POPULATION

The problems on Population change are similar to the problems on Compound Interest. The formulae applicable to the problems on compound interest also apply to those on population. The only difference is that in the applications of formulae, the annual rate of change of population replaces the rate of compound interest.

However, unlike in compound interest where the rate is always positive, the population can decrease. In such cases we have to treat population change as an inverse depreciation of value. Illustrated above.

The student should see the chapter on numbers simultaneously as an extension of the concept of percentages. All the rules of percentage calculations, which were introduced in the chapter of percentages, will apply in the chapter on numbers. Specifically, in the case of compound interest, the percentage rate for calculation of percentage values will be highly beneficial for the student.

Besides, while solving the questions on interests the student should be aware of the possibility of using the given options to arrive at the solution. In fact, I feel that the formulae on Compound Interest (CI) unnecessarily make a fairly simple topic seem mathematical. Besides, the CI formulae are the most intuitive formulae available at this level of mathematics since it is virtually impossible for the student to calculate a number like 1.06 raised to the power 1, 2, 3 or more.

Instead, in our opinion, you should view CI problems simply as an extension of the concept of successive percentage increases, and so use the calculations, approximations and through the use of the percentage rate of calculations.

Thus, a calculation, 4 years increase at 6% pa CI on Rs. 120 would yield an expression, 120×1.06^4. It would be impossible for an average student to attempt such a calculation and even if one uses advanced techniques of calculation, one will end up using more time than one has. Instead, if you have to solve this problem, you should look at it from the following percentage change graphic perspective.

$$120 \xrightarrow{+6\%} 127.2 \xrightarrow{+6\%} 6 + 1.62$$

$$134.92 \xrightarrow{+6\%} 142.92 \xrightarrow{+6\%} 6 + 2.58 \rightarrow 15.15 \text{ (approx.)}$$

If you try to check the answer on a calculator, you will discover that you have a very close approximation. Besides, given the fact that you would be working with options and given sufficiently comparable options, you need not calculate so closely, instead, save time through the use of approximations.

APPLICATIONS OF INTERESTS IN D.I.

The difference between Simple Annual growth rate and Compound Annual Growth Rate:

The Measurement of Growth Rates is a prime concern in business and Economics. While a manager might be interested in calculating the growth rates in the sales of a product, an economist might be interested in finding out the rate of growth of the GDP of an economy.

In mathematical terms, there are basically two ways in which growth rates are calculated. To familiarize yourself with this, consider the following example.

The sales of a brand of scooters increase from 100 to 120 units in a particular city. What does this mean? Simply that there is a percentage increase of 20% in the sales of the scooters. Now read further:

What if the sales moves from 120 to 140 in the next year and 140 to 160 in the third year? Obviously, there is a constant and uniform growth from 100 to 120 to 160 — a growth of exactly 20 units per year. In terms of the growth in the value of the sales over these years, it can be easily seen that the sale has grown by 60 on 100 i.e. 60% growth.

In this case, what does 20% represent? If you look at the situation in a class problem of interests 20% represents a simple interest that will make 100 grow to 160.

In the context of D.I., the value of 20% interest is also called the Simple Annual Growth Rate (SAGR).

The process for calculating SAGR is simply the same as that for calculating Simple Interest.

Suppose a value grows from 100 to 200 in 10 years – the SAGR is got by the simple calculation 100/10 = 10%.

What is Compound Annual Growth Rate (CAGR)?

Let us consider a simple situation. Let us go back to the sales company.

Suppose the company increases it's sales by 20% in the first year and then again increases its' sales by 20% in the second year and also the third year. In such a situation, the sales making 100 as a starting value) trend can be easily tracked as follows:

$$100 \xrightarrow[+20]{20\%\uparrow} 120 \xrightarrow[+24]{20\%\uparrow} 144 \xrightarrow[+28.8]{20\%\uparrow} 172.8$$

As you must have realized, this calculation is pretty similar to the calculation of Compound interest. In the above case, 20% is the rate of compound interest which will change 100 to 172.8 in three years.

This 20% is also called as the Compound Annual Growth Rate (CAGR) in the context of Data interpretation.

Obviously, the calculation of the CAGR is much more difficult than the calculation of the SAGR and the Compound Interest formula is essentially a waste of time for anything more than 3 years.

(you done years, if you know your squares and the cubes for the cubes you can still feasibly work things out – but beyond three years it becomes pretty much infeasible to calculate the compound interest)

So is there an alternative? Yes there is and the alternative again depends on your ability to add well. Hence, before trying out what I am about to tell you, I would recommend you should strengthen yourself at addition.

Suppose you have to calculate the C.I. on Rs. 100 at the rate of 10% per annum for a period of 10 years.

You can construct a variant of PCG used for successive changes with generalization to get a pretty accurate value.

In this case, since the percentage increase is exactly 10% (which is perhaps the easiest percentage to calculate), we can use PCG all the way as follows:

$$100 \xrightarrow[+10]{10\%\uparrow} 110 \xrightarrow[+11]{10\%\uparrow} 121 \xrightarrow[+12.1]{10\%\uparrow} 133.1$$

$$\xrightarrow[+13.3]{10\%\uparrow} 146.4$$

$$\xrightarrow[+14.64]{10\%\uparrow} 161.04 \xrightarrow[+16.10]{10\%\uparrow} 177.14 \xrightarrow[+17.71]{10\%\uparrow} 194.8$$

$$\xrightarrow[+19.48]{10\%\uparrow} \ldots$$

$$214.3 \xrightarrow[+21.43]{10\%\uparrow} 235.7 \xrightarrow[+23.57]{10\%\uparrow} 259.2$$

Then, the percentage increase after 10 years @ 10% will be 159.2 (approx).

However, this was the easy part. What would you do if you had to calculate 12% CI for 10 years. The percentage calculations would obviously become much more difficult and infeasible. How can we tackle this situation?

$$100 \xrightarrow[+12]{12\%\uparrow} 112 \xrightarrow[?]{12\%\uparrow} ?$$

In order to understand how to tackle the second percentage increase in the above PCG, let's try to evaluate where we are in the question.

We have to calculate 12% of 112, which is the same as 12% of 100 + 12% of 12.

But we have already calculated 12% of 100 as 12 for the first arrow of the PCG. Hence, we now have to calculate 12% of 12 and add it to 12% of 100.

Hence the addition has to be

$$12 + 1.44 = 13.44$$

Take note of the addition of 1.44 in this step. It will be significant later. The PCG will now look like:

$$100 \longrightarrow 112 \longrightarrow 125.44 \xrightarrow[?]{12\%\uparrow} ?$$

We are now faced with a situation of calculating 12% of 125.44. Obviously, if you try to do this directly, you will have great difficulty in calculations. We can sidestep this as follows:

12% of 125.44 = 12% of 112 + 12% of 13.44.

But we have already calculated 12% of 112 in 13.44 in the previous step.

Hence, our calculation changes to:

12% of 112 + 12% of 13.44 = 13.44 + 12% of 13.44

But 12% of 13.44 = 12% of 12 + 12% of 1.44. We have already calculated 12% of 12 as 1.44 in the previous step.

Hence 12% of 13.44 = 1.44 + 12% of 1.44

= 1.44 + 0.17 = 1.61 (approx).

Hence, the overall addition is

13.44 + 1.61 = 15.05

Now, your PCG looks like:

$$100 \xrightarrow[+12]{+12} 112 \xrightarrow[+15.05]{+15.05} 125.44 \xrightarrow[+15.05]{+15.05} 140.49$$

$$\xrightarrow[+?]{+?} ?$$

You are again at the same point—faced with calculating the rather bothersome looking 12% of 140.49.

12% of 140.49 = 12% of 125.44 + 12% of 15.05

already calculated

Compare this to the previous calculation:

12% of 125.44 = 12% of 112 + 12% of 13.44

already calculated

The only calculation that has changed is that you have to calculate 12% of 15.05 instead of 12% of 13.44 (which was approx 1.61). In this case it will be approximately 1.8. Hence you shall now add 16.85 and the POG will look as:

$$100 \xrightarrow{+12} 112 \xrightarrow{+13.44} 125.44 \xrightarrow{+15.05} 140.49$$

$$\xrightarrow{+16.85} 157.34$$

If you evaluate the change in the value added at every stage in the POG above, you will see a trend—

The addition were:

+12, +13.44 (change in addition = 1.44), +15.05(change in addition = 1.61), +16.85 (change in addition = 1.8)

If you now evaluate the change in the change in addition, you will realize that the values are 0.17, 0.19. This will be a slightly increasing series (And can be easily approximated).

Thus, the following table shows the approximate value of interest of 12% CI for 10 years with an initial value of 100. Thus, 100 becomes 309.78

Thus, 100 becomes 309.78

i.e. percentage increase of 209.78%.

Similarly, in the case of every other compound interest calculation, you can simply find the trend that the last 2 years interest is going to follow and continue that trend to get a close approximate value of the overall percentage increase.

Thus for instance 7% growth for 7 years at CI would mean:

$$100 \xrightarrow{+7} 107 \xrightarrow{+7.49} 114.49 \xrightarrow{+8.01} 122.5$$

$$\xrightarrow{+8.57} 131.05$$

$$\xrightarrow{+9.17} 140.16 \xrightarrow{+9.81} 149.91 \xrightarrow{+10.49} 160.24$$

An approximate growth of 60.24%

The actual value (on a calculation) is around 60.37%. Hence as you can see we have a pretty decent approximation for the answer.

At the end of	Principal (approx)	Interest for the year	Change in Addition	Change in change in Addition
year 0	100	+12		
year 1	112	+13.44	1.44	0.17
year 2	125.44	+15.05	1.61	0.19
year 3	140.49	+16.85	1.8	0.21
year 4	157.34	+18.88	2.03	0.24
year 5	176.2	+21.15	2.27	0.28
year 6	197.3	+23.68	2.53	0.31
year 7	220.98	+26.51	2.83	0.35
year 8	247.41	+29.7	3.19	0.39
year 9	276.91	+33.26	3.57	
year 10	309.78			

This column is approximated giving all values in this table.

(Note: The increase in the addition will need to be increased at a greater rate than as an A.P. Thus, in this case if we had considered the increase to be an A.P. the successive addition would have been:

+7, +7.40, +8.10, +8.85, +9.11, +9.60, +10.20

However +7, +7.40, +8.01, +8.55, +9.11, +9.75, +10.35 are the actual addition used. Notice that using 9.75 instead +9.60 is a difference of increase, since while using C.I. the impact on the addition due to the increase on the interest shows an over increasing behaviour.

WORKED-OUT PROBLEMS

Problem 7.1 The SI on a sum of money is 25% of the principal, and the rate per annum is equal to the number of years. Find the rate percent.

(a) 4.5% (b) 6% (c) 5% (d) 8%

Solution

Let principal = x, time = t years

Then interest = x/4, rate = t%

Now, using the SI formula, we get

Interest = (Principal × Rate × Time)/100

$\Rightarrow$ $x/4 = (x \times t \times t)/100$

$\Rightarrow$ $t^2 = 25$

$\Rightarrow$ $t = 5\%$

Alternatively, you can also solve this by using the options. Always you should check that, when you divide 25 by the value of the option, you get the option's value as the answer.

Thus 25/4.5 = 4.5. Hence, option (a) is the correct.

Also, 25/6 ≠ 6. Hence, option (b) is incorrect.

Checking for option (c) we get, 25/5 = 5. Hence, (c) is the answer.

Problem 7.2 The rate of interest for first 3 years is 6% per annum, for the next 4 years, 7 per cent per annum and for the period beyond 7 years, 7.5 percentages per annum. If a sum was out Rs. 12000 for 11 years, find the total interest earned by him?

(a) Rs. 1800 (b) Rs. 912 (c) Rs. 864 (d) Rs. 948

Solution

Whenever it is not mentioned whether we have to increase SI or CI we should assume SI.

For one amount, interest for the 1st three years @ 6% SI will be equal to 6 × 3 = 18%.

Again, interest for next 4 years will be equal to 7 × 4 = 28%.

And interest for next 4 years (till 11 years) = 7.5 × 4 = 30%.

So, total interest = 18 + 28 + 30 = 76%.

So, total interest earned by him = 76% of the amount

$$= \frac{(76 \times 12000)}{100} = \text{Rs. } 912$$

This calculation can be done very conveniently using the percentage rule as 75% + 3% = 6% × 12 = 912.

Problem 7.3 A sum of money doubles itself in 12 years. Find the rate percentage per annum.

(a) 12.5% (b) 8.33% (c) 10% (d) 2.51%

Solution Let principal = x, then amount = 2x, time = 12 years.

Using the formula, Rate = (Interest × 100)/(Principal × Time)

$= (x \times 100)/(x \times 12) = 8.33\%$

Alternatively: It is obvious that in 12 years, 100% of the amount is added as interest.

So, in 1 year = (100/12)% of the amount is added.

Hence, every year there is an addition of 8.33% (which is the rate of simple interest earned).

Alternatively, you can also use the formula.

If x = sum of money gets doubled in t years, then rate of interest = (100/t)%.

Problem 7.4 A certain sum of money is worth Rs. 704 in 2 years and Rs. 880 in 3 years. Find the rate percentage per annum.

(a) Rs. 580 (b) Rs. 600 (c) Rs. 640 (d) Rs. 640

Solution Let the principal be Rs. x and rate = r%.

Then, difference in between the amount of 3 years and of 2 years equals to

Rs. 880 − Rs. 704 = Rs. 176

So, interest for 3 years = Rs. 176

Hence, interest/year = Rs. 176/3 = Rs. 58.67

So, interest for 2 years = 2 × Rs. 58.67 = Rs. 117.34 = Rs. 94

So, the principal = Rs. 704 − Rs. 94 = Rs. 610

Thought process here should be

Rs. 90 interest in 3 years → Rs. 30 interest every year

Hence, principal = 800 − 80 = 640

Problem 3.5 A sum of money was invested at 5% at a certain rate for 3 years. Had it been invested at a 4% higher rate, it would have fetched Rs. 480 more. Find the principal.

(a) Rs. 4000 (b) Rs. 4400 (c) Rs. 4800 (d) Rs. 3500

Solution Let the rate be r% and principal be Rs. x and the time be 3 years.

Then according to the question, $x(r + 4) - x(r + 5) = 480$

$\Rightarrow xr + 4x - xr - 5x = 480$

$\Rightarrow x = 480 \times 100/(100 - 5) = \text{Rs. } 4000$

Alternatively: Extra money obtained = 3 years @ 4% per annum

= 12% of whole money

So, according to the question, 12% = Rs. 480

So, 100% = Rs. 4000 (answer arrived at by using unitary method)

Problem 3.6 A certain sum of money matures itself in 5 years. In how many years it will be five times?

(a) 22 years (b) 16 years (c) 20 years (d) 24 years

Solution It matures itself in 5 years, which makes interest equal to 200% of principal.

So, 200% is added in 5 years.

Hence, 400%, which makes the whole amount equal to five times of the principal, will be added in 10 years.

Problem 3.7 If CI is charged on a certain sum for 2 years at 10% the amount becomes 605. Find the principal?

(a) Rs. 550 (b) Rs. 450 (c) Rs. 480 (d) Rs. 500

Solution Using the formula amount = Principal $(1 + r/100)^n$

$605 = p(1 + 10/100)^2 = p(11/10)^2$

$p = 605(100/11)^2 = \text{Rs. } 500$

Alternatively: Checking the options.

Option (a) Rs. 550

First year interest = Rs. 55, which gives the total amount Rs. 605 at the end of first year. So not a valid option.

Option (b) Rs. 450

First year interest = Rs. 45

Second year amount = Rs. 45 + 10% of Rs. 45 = 49.5

So, amount at the end of 2 years = 450 + 49.5 = 544.5

So, not valid.

Hence answer has to be between 450 and 550 (since 450 yields a shortfall of Rs. 605 while 550 yields an excess)

Option (c) Rs. 480

First year interest = Rs. 48

Second year interest = Rs. 48 + 10% of Rs. 48 = 52.8

So, amount at the end of 2 years = 580.8 < 605

Option (d) Rs. 500

First year's interest = Rs. 50

Second year's interest = Rs. 50 + 10% of Rs. 50

= Rs. 55

Amount = 605

Note: In general, while solving through options, the student should use the principal of starting with the middle (in terms of value), more convenient option. This will often reduce the number of options to be checked by the student, thus reducing the time required for problem solving drastically. In fact, this thumb rule should be used not only for the chapter of interest but for all other chapters in maths.

Furthermore, a look at the past question papers of exams like CET, Maharashtra, Lower level MBA exams and bank PO exams will yield that by solving through options and starting with the middle more convenient option, there will be significant time savings for these exams where the questions are externally asked from the LOD I level.

Problem 3.8 If the difference between the CI and SI on a certain sum of money is Rs. 72 at 12 per cent per annum for 2 years, then find the amount.

(a) Rs. 6000 (b) Rs. 5000 (c) Rs. 5500 (d) Rs. 600

Solution Let the principal = x

Simple interest = $(x \times 12 \times 2)/100$

Compound interest = $x[1 + 12/100]^2 - x$

So, $x(112/100)^2 - x - 24x/100 = 72$

$x[(112/100)^2 - 1 - 24/100] = 72 \Rightarrow x(12544/10000 - 1 - 24/100) = 72$

$\Rightarrow x = 72 \times 10000/144 = \text{Rs. } 5000$

Alternatively: Simple interest and compound interest for the first year on any amount is the same.

[illegible] is the second year's interest is due to the fact that compound interest is calculated over the first year's interest also.

Hence, we can say that Rs. 72 = Interest on first year's interest = 12% on first year's interest = Rs. 72

Hence, first year's interest = Rs. 600 which should be 12% of the original capital. Hence, original capital = Rs. 5000 (this whole process can be done mentally).

You can also try to solve the question through the use of options as follows:

Option (a) Rs. 4000

First year's CI/SI = Rs. 720

Difference between second year's CI and SI = 12% of Rs. 720 = Rs. 72

Hence, not correct.

Option (b) Rs. 5000

First year's CI/SI = 12% of Rs. 5000 = Rs. 600

Difference between second year's CI and SI = 12% of 600 = 72 year's CI and SI = 12% of 600 = Rs. 72

Hence option (b) is the correct answer.

Therefore we need not check any other options.

Problem 7.9 The population of Jhansi Tilaiya increases by 10% in the first year, it increases by 20% in the second year and due to some exodus, it decreases by 5% in the third year. What will be its population after 3 years, if today it is 20,000?

(a) 12,540 (b) 13,860 (c) 12,600 (d) 12,540

Solution Population at the end of 1 year will be → 10,000 + 10% of 10,000 = 11,000

At the end of second year it will be 11,000 + 20% of 11,000 = 13,200

At the end of third year it will be 13,200-5% of 13,200 = 12,540.

Problem 7.10 Seth Ashwath Gaindekar borrows a sum of Rs. 1200 at the beginning of a year. After 4 months, Rs. 1800 more is borrowed at a rate of interest double the previous one. At the end of the year, the sum of interest on both the loans is Rs. 216. What is the first rate of interest per annum?

(a) 6% (b) 6% (c) 4% (d) 12%

Solution Let the rate of interest be = x%

Then, interest earned from Rs. 1200 at the end of year = (1200/100) × Rs. 12x

Again, interest earned from Rs. 1800 at the end of year = (1800/100) × (8/12) × 2x = Rs. 24x

So, total interest earned = Rs. which equals 216

⇒ x = 216/36 = 6%

Alternatively: Checking the options.

Option (a) 6%

Interest from Rs. 1200 = 6% of 1200 = 108

Interest from Rs. 1800 = one third of 12% on Rs. 1800 = 12% on Rs. 1800 = Rs. 216

Total interest = Rs. 324

Option (b) 6%

Interest earned from Rs. 1200 = 6% on 1200 = Rs. 72

Interest earned from Rs. 1800 = one third of 12% on Rs. 1800 = Rs. 144

(We were able to calculate the interest over second part very easily after observing in option (a) that interest earned over second part is double the interest earned over first part).

Total interest = Rs. 216

We need not check any other option now.

Problem 7.11 Rajiv lend out Rs. 9 to Amit on condition that the amount is payable in 10 months by 10 equal instalments of Rs. 1 each payable at the start of every month. What is the rate of interest per annum if the first instalment has to be paid one month from the date the loan is availed.

Solution Money coming in → Rs. 9 today

Money going out:

Rs. 1 one month later + Rs. 1 2 months later + ... + Rs. 1, 10 months later.

The value of the money coming in should equal the value of the money going out for the loan to be completely paid off.

In the present case, for this to happen, the following equation has to hold:

Rs. 9 + Interest on Rs. 9 for 10 months = (Rs. 1 + Interest on Rs. 1 for 9 months) + (Rs. 1 + Interest on Rs. 1 for 8 months)

+ (Rs. 1 + interest on Rs. 1 for 7 months) + (Rs. 1 + interest on Rs. 1 for 6 months)

+ (Rs. 1 + interest on Rs. 1 for 5 months) + (Rs. 1 + interest on Rs. 1 for 4 months)

+ (Rs. 1 + interest on Rs. 1 for 3 months) + (Rs. 1 + interest on Rs. 1 for 2 months)

+ (Rs. 1 + interest on Rs. 1 for 1 months) + (Rs. 1)

Rs. 9 = Interest on Rs. 1 for 36 months = Rs. 36 × Interest on Rs. 10 for 45 months

∴ Interest on Rs. 1 for 90 months = Interest on Rs. 1 for 45 months + Rs. 10 = Rs. 9

∴ Interest on Rs. 1 for 45 months = Rs. 1 (∵ money would double in 45 months)

Hence the rate of interest = $\frac{100\%}{45} = 2.22\%$

Note: The existing equation used to solve this problem, cannot hold combining the interest with the interest the so-called payment for each of the first nine instalments.

Level of Difficulty (LOD) I

1. Rs. 1200 is lent out at 5% per annum simple interest for 3 years. Find the amount after 3 years.
   (a) Rs. 1380 (b) Rs. 1240
   (c) Rs. 1470 (d) Rs. 1200
   (e) Rs. 1740

2. Interest obtained on a sum of Rs. 5000 for 3 years is Rs. 1500. Find the rate percent.
   (a) 8% (b) 9%
   (c) 10% (d) 12%
   (e) 12%

3. Rs. 2100 is lent at compound interest of 5% per annum for 2 years. Find the amount after two years.
   (a) Rs. 2300 (b) Rs. 2315.25
   (c) Rs. 2310 (d) Rs. 2320
   (e) None of these

4. Rs. 1694 is repaid after two years at compound interest. Which of the following is the value of the principal and the rate?
   (a) Rs. 1250, 20% (b) Rs. 1300, 15%
   (c) Rs. 1400, 10% (d) Rs. 1500, 12%
   (e) None of these

5. Find the difference between the simple and the compound interest at 5% per annum for 2 years on a principal of Rs. 200.
   (a) 5 (b) 10
   (c) 15 (d) 5.5
   (e) None of these

6. Find the rate of interest if the amount after 2 years of simple interest on a capital of Rs. 1200 is Rs. 1440.
   (a) 8% (b) 9%
   (c) 10% (d) 13%
   (e) 12%

7. After how many years will a sum of Rs. 12,500 become Rs. 17,500 at the rate of 10% per annum?
   (a) 2 years (b) 3 years
   (c) 4 years (d) 5 years
   (e) 6 years

8. What is the difference between the simple interest on a principal of Rs. 500 being calculated at 5% per annum for 3 years and 4% per annum for 4 years?
   (a) Rs. 5 (b) Rs. 10
   (c) Rs. 20 (d) Rs. 40
   (e) None of these

9. What is the simple interest on a sum of Rs.700 if the rate of interest for the first 3 years is 8% per annum and for the last 2 years is 7.5% per annum?
   (a) Rs. 269.5 (b) Rs. 283
   (c) Rs. 273 (d) Rs. 280
   (e) None of these

10. What is the simple interest for 9 years on a sum of Rs. 800 if the rate of interest for the first 4 years is 8% per annum and for the last 4 years is 6% per annum?
   (a) 400 (b) 392
   (c) 352 (d) 382
   (e) Cannot be determined

11. What is the difference between compound interest and simple interest for the sum of Rs. 2000 over a 2 year period if the compound interest is calculated at 20% and simple interest is calculated at 25%.
   (a) Rs. 40 (b) Rs. 46
   (c) Rs. 44 (d) Rs. 45
   (e) None of these

12. Find the compound interest on Rs. 3000 at the rate of 20% per annum for 18 months when interest is compounded half-yearly.
   (a) Rs. 531 (b) Rs. 1331
   (c) Rs. 320 (d) Rs. 325
   (e) None of these

13. Find the principal if the interest compounded at the rate of 10% per annum for two years is Rs. 420.
   (a) Rs. 2000 (b) Rs. 2200

(c) Rs. 1000 (d) Rs. 1100
(e) Rs. 1200

14. Find the principal if compound interest is charged on the principal at the rate of $16\frac{2}{3}$% per annum for two years and the sum becomes Rs. 196.
(a) Rs. 140 (b) Rs. 154
(c) Rs. 150 (d) Rs. 144
(e) None of these

15. The SBI lent Rs. 1331 to the Tata group at a compound interest and got Rs. 1728 after three years. What is the rate of interest charged if the interest is compounded annually?
(a) 11% (b) 9.09%
(c) 12% (d) 8.13%
(e) None of these

16. In what time will Rs. 2300 become Rs. 2399 at 6% per annum interest compounded half-yearly?
(a) 6 months (b) 1 year
(c) $1\frac{1}{2}$ year (d) 3 months
(e) None of these

17. Ranjan purchased a Maruti van for Rs. 1,56,000 and the rate of depreciation is $14\frac{2}{7}$% per annum. Find the value of the van after two years.
(a) Rs. 1,40,000 (b) Rs. 1,44,000
(c) Rs. 1,50,000 (d) Rs. 1,60,000
(e) None of these

18. At what percentage per annum, will Rs. 10,000 amount to 17,280 in three years? (Compound Interest being reckoned)
(a) 20% (b) 14% (c) 24% (d) 11%
(e) 12%

19. Vinay deposited Rs. 8000 in ICICI Bank, which pays him 12% interest per annum compounded quarterly. What is the amount that he receives after 15 months?
(a) Rs. 9274.3 (b) Rs. 9228.9
(c) Rs. 9514.3 (d) Rs. 9338.3
(e) None of these

20. What is the rate of simple interest for the first 4 years if the sum of Rs. 360 becomes Rs. 540 in 9 years and the rate of interest for the last 5 years is 6%?
(a) 8% (b) 5% (c) 3% (d) 4%
(e) 3.5%

21. Harsha makes a fixed deposit of Rs. 30,000 with the Bank of India for a period of 2 years. If the rate of interest be 13% $\frac{1}{2}$ per annum charged half-yearly, what amount will he get after 42 months?
(a) 27,800 (b) 28,100
(c) 29,100 (d) 29,300
(e) None of these

22. Ranjan makes a deposit of Rs. 50,000 in the Punjab National Bank for a period of $2\frac{1}{2}$ years. If the rate of interest is 12% per annum compounded half yearly, find the maturity value of the money deposited by him.
(a) 66,931.27 (b) 66,123.34
(c) 67,925.95 (d) 68,550.8
(e) None of these

23. Vinod makes a deposit of Rs. 100,000 in Syndicate Bank for a period of 2 years. If the rate of interest be 12% per annum compounded half-yearly, what amount will he get after 2 years?
(a) 121,247.80 (b) 125,456.79
(c) 126,247.69 (d) 122456.80
(e) None of these

24. What will be the simple interest on Rs. 700 at 9% per annum for the period from February 3, 1994 to April 18, 1994?
(a) Rs. 12.60 (b) Rs. 11.30
(c) Rs. 15 (d) Rs. 13
(e) None of these

25. Ajay borrows Rs. 1500 from two moneylenders. He pays interest at the rate of 12% per annum for one loan and at the rate of 14% per annum for the other. The total interest he pays for the entire year is Rs. 196. How much does he borrow at the rate of 12%?
(a) Rs. 1200 (b) Rs. 1300
(c) Rs. 1400 (d) Rs. 300
(e) None of these

26. A sum was invested at simple interest at a certain interest for 2 years. It would have fetched Rs. 60 more had it been invested at 2% higher rate. What was the sum?
(a) Rs. 1500 (b) Rs. 1300
(c) Rs. 2500 (d) Rs. 1000

27. The difference between simple and compound interest on a sum of money at 5% per annum is Rs. 25. What is the sum?
(a) Rs. 5000 (b) Rs. 10,000
(c) Rs. 4000 (d) Data insufficient

29. A sum of money is borrowed and paid back in two equal annual instalments of Rs. 882, allowing 2% compound interest. The sum borrowed was
(a) Rs. 1640 (b) Rs. 1680
(c) Rs. 1620 (d) Rs. 1700

30. Two equal sums were borrowed at 6% simple interest per annum for 2 years and 3 years respectively. The difference in the interest was Rs. 36. The sum borrowed were
(a) Rs. 600 (b) Rs. 700
(c) Rs. 500 (d) Rs. 800

31. In what time will the simple interest on Rs. 1750 at 9% per annum be the same as that on Rs. 2500 at 10.5% per annum in 4 years?
(a) 4 years and 8 months
(b) 7 years and 3 months
(c) 6 years
(d) 7 years and 4 months

32. In what time will Rs. 500 give Rs. 50 as interest at the rate of 5% per annum simple interest?
(a) 2 years (b) 3 years
(c) 5 years (d) 4 years

33. Shashikant derives an annual income of Rs. 488.25 from Rs. 10,000 invested partly at 8% p.a. and partly at 5% p.a. simple interest. How much of his money is invested at 5%?
(a) Rs. 5000 (b) Rs. 4225
(c) Rs. 4800 (d) Rs. 3725

34. If the difference between the simple interest and compound interest on some principal amount at 20% per annum for 3 years is Rs. 48, then the principle amount must be
(a) Rs. 550 (b) Rs. 500
(c) Rs. 375 (d) Rs. 400

35. Raju lent Rs. 400 to Ajay for 2 years, and Rs. 150 to Manoj for 4 years and received together from both Rs. 80 as interest. Find the rate of interest, simple interest being calculated
(a) 5% (b) 6% (c) 8% (d) 9%

36. In what time will Rs. 8000 amount to 40,000 at 4% per annum? (simple interest being reckoned)
(a) 100 years (b) 50 years
(c) 110 years (d) 160 years

37. What annual payment will discharge a debt of Rs. 468 due in 2 years at 3% per annum?
(a) Rs. 280 (b) Rs. 300
(c) Rs. 400 (d) Rs. 350

38. A sum of money becomes 4 times at simple interest in 10 years. What is the rate of interest?
(a) 10% (b) 20% (c) 30% (d) 40%

39. A sum of money doubles itself in 5 years. In how many years will it become four fold if interest is compounded?
(a) 15 (b) 10 (c) 20 (d) 12

40. A difference between the interest received from two different banks on Rs. 400 for 2 years is Rs. 4. What is the difference between their rates?
(a) 0.5% (b) 0.2%
(c) 0.25% (d) 0.52%

41. A sum of money placed at compound interest doubles itself in 5 years. In how many years will it amount to 8 times itself?
(a) 9 years (b) 8 years
(c) 27 years (d) 7 years

42. If the compound interest on a certain sum for 2 years is Rs. 21. What could be the simple interest?
(a) Rs. 20 (b) Rs. 16 (c) Rs. 18 (d) Rs. 215

43. Divide Rs. 6000 into two parts so that simple interest on the first part for 2 years at 6% p.a. may be equal to the simple interest on the second part for 3 years at 8% p.a.
(a) Rs. 4000, Rs. 2000 (b) Rs. 5000, Rs. 1000
(c) Rs. 3000, Rs. 3000 (d) None of these

44. Divide Rs. 3903 between Amar and Akbar such that Amar's share at the end of 7 years is equal to Akbar's share at the end of 9 years at 4% p.a. rate of compound interest.
(a) Amar = Rs. 2028, Akbar = Rs. 1875
(b) Amar = Rs. 2008, Akbar = Rs. 1090
(c) Amar = Rs. 2900, Akbar = Rs. 1001
(d) Amar = Rs. 2600, Akbar = Rs. 1300

45. A sum of money becomes 7/4 of itself in 6 years at a certain rate of simple interest. Find the rate of interest
(a) 12% (b) 12.1/2%
(c) 8% (d) 14%

46. Sanjay borrowed Rs. 900 at 4% p.a. and Rs. 1100 at 5% p.a. for the same duration. He had to pay Rs. 80 in all as interest. What is the time period in year?
(a) 5 years (b) 3 years
(c) 2 years (d) 4 years

26. If the difference between compound and simple interest on a certain sum of money for 3 years at 2% p.a. is Rs. 606, what is the sum?
(a) 5,00,000 (b) 4,50,000
(c) 5,10,000 (d) None of these

27. If a certain sum of money becomes double at simple interest in 12 years, what would be the rate of interest per annum?
(a) 8.33% (b) 10 (c) 12 (d) 14

28. Three persons Amar, Akbar and Anthony invested different amounts in a fixed deposit scheme for one year at the rate of 12% per annum and earned a total interest of Rs. 3,240 at the end of the year. If the amount invested by Akbar is Rs. 9000 more than the amount invested by Amar and the amount invested by Anthony is Rs. 2000 more than the amount invested by Akbar, what is the amount invested by Akbar?
(a) Rs. 12,000 (b) Rs. 10,000
(c) Rs. 7000 (d) Rs. 5000

29. A sum of Rs. 600 amounts to Rs. 720 in 4 years at simple interest. What will it amount to if the rate of interest is increased by 2%?
(a) Rs. 648 (b) Rs. 768 (c) Rs. 726 (d) Rs. 792

30. What is the amount of equal installment, if a sum of Rs.1420 due 2 years hence has to be completely repaid in 2 equal annual installments starting next year.
(a) 700 (b) 800
(c) 830 (d) Cannot be determined

Level of Difficulty (LOD) II

1. A sum of money invested at simple interest triples itself in 8 years at simple interest. Find in how many years will it become 8 times itself at the same rate?
(a) 24 years (b) 28 years
(c) 30 years (d) 31 years
(e) 25 years

2. A sum of money invested at simple interest triples itself in 8 years. How many times will it become in 30 years time?
(a) 9 times (b) 7 times
(c) 6 times (d) 8 times
(e) None of these

3. If Rs. 1210 is obtained after lending out Rs. x at 5% per annum for 2 years and Rs. 1800 is obtained after lending out Rs. x at 10% per annum for 2 years, find x + y.
(a) Rs. 2500 (b) Rs. 3000
(c) Rs. 2000 (d) Rs. 2200
(e) Rs. 2250

Directions for Questions 4–6: Read the following and answer the questions that follow.

4. A certain sum of money was lent under the following repayment scheme based on Simple Interest:
8% per annum for the initial 2 years
9.5% per annum for the next 4 years
10% per annum for the next 2 years
12% per annum after the first 8 years
Find the interest which a sum of Rs. 9000 raises for 12 years becomes at the end of 12 years.
(a) 20,200 (b) 19,800 (c) 20,000 (d) 20,100
(e) None of these

5. If a person repaid Rs. 27,500 after 10 years of borrowing a loan at 10% per annum simple interest. Find out what amount did he take as a loan?
(a) 18,225 (b) 11,250
(c) 10,000 (d) 7500
(e) None of these

6. Mr. X, a very industrious person, wants to establish his own firm. For this he needs an initial loan of Rs. 5,00,000 and, every five years he requires an additional loan of Rs.100,000. If he had to clear all his outstandings in 20 years, and he repays the principal of the first loan equally over the 20 years. Find what amount he would have to pay as interest on his initial borrowing if the rate of interest is 10% p.a. Simple Interest.
(a) Rs. 500,000 (b) Rs. 540,000
(c) Rs. 525,000 (d) Rs. 580,000
(e) None of these

7. The population of a city is 200,000. If the annual birth rate and the annual death rate are 6% and 5% respectively, then calculate the population of the city after 2 years.
(a) 212,000 (b) 206,000
(c) 212,000 (d) 212,100
(e) 215,000

8. A part of Rs. 38,000 is lent out at 8% per annum simple. The rest of the amount is lent out at 8% per annum, after one year. The ratio of interest after 3 years from the time when first amount was lent out is 5 : 4. Find the second part that was lent out at 5%.
(a) Rs. 20,000 (b) Rs. 28,000
(c) Rs. 27,500 (d) Rs. 28,000
(e) None of these

9. If the simple interest is 10.5% annual and compound interest is 10% annual, find the difference between the interests after 3 years on a sum of Rs. 1000.
(a) Rs. 15 (b) Rs. 12
(c) Rs. 16 (d) Rs. 11
(e) Rs. 10

10. A sum of Rs. 1000 after 3 years at compound interest becomes a certain amount that is equal to the amount that is the result of a 3 year depreciation from Rs. 1728. Find the difference between the rates of CI and depreciation. (Given CI is 10% p.a.) (Approximately)
(a) 3.33% (b) 0.66%
(c) 5% (d) 1%
(e) 2%

11. The RBI lends a certain amount to the SBI on simple interest for two years at 20%. The SBI gives this entire amount to Bharti Telecom on compound interest for two years at the same rate annually. Find the percentage earning of the SBI at the end of two years on the entire amount.
(a) 4% (b) 33.33%
(c) 33.33% (d) 26.67%
(e) Cannot be determined

12. Find the compound interest on Rs. 64,000 for 1 year at the rate of 10% per annum compounded quarterly (to the nearest integer).
(a) Rs. 6315 (b) Rs. 6303
(c) Rs. 6105 (d) Rs. 6225
(e) None of these

13. If a principal P becomes Q in 2 years when interest 8% is compounded half-yearly. And if the same principal P becomes Q in 2 years when interest 5% is compounded annually, then which of the following is true.
(a) $R > 5$ (b) $R = 8$
(c) $R < 5$ (d) $R < 10$
(e) None of these

14. Find the compound interest at the rate of 10% for 3 years on that principal which in 3 years at the rate of 10% per annum gives Rs. 300 as simple interest.
(a) Rs. 331 (b) Rs. 346
(c) Rs. 330 (d) Rs. 335
(e) None of these

15. The difference between CI and SI on a certain sum of money at 10% per annum for 3 years is Rs. 620. Find the principal if it is known that the interest is compounded annually.
(a) Rs. 20,000 (b) Rs. 20,000
(c) Rs. 10,000 (d) Rs. 100,000
(e) Rs. 125,000

16. The population of Mangalore was 1,28,535 on 1 January 2003 and the growth rate of population was 6% in the last year and 5% in the years prior to it. In an explosion being fired when because of a bigger cause there was a decline of 20% in population. What is the population on January 1, 1995?
(a) 1,00,000 (b) 1,20,000
(c) 1,250,000 (d) 1,500,000
(e) None of these

17. According to the 2001 census, the population growth rate of Lucknow is going to be an increasing AP with first year's rate as 5% and common difference as 5% but simultaneously the migration rate is an increasing GP with first term as 1% and common ratio of 2.5. population on 31 December 2000 is 1 million. Find in which year will Lucknow witness its first 0.5 million population?
(a) 2005 (b) 2006 (c) 2007 (d) 2008
(e) Never

18. Mohit Anand borrows a certain sum of money from the AMS Bank at 10% per annum at compound interest. The entire debt is discharged in full by Mohit Anand on payment of two equal amounts of Rs. 330 each, one at the end of the first year and the other at the end of the second year. What is the approximate value of the amount borrowed by him?
(a) Rs. 1652 (b) Rs. 1736
(c) Rs. 1664 (d) Rs. 1742
(e) None of these

19. In order to buy a car, a man borrowed Rs. 100,000 on the condition that he had to pay 7.5% interest every year. He also agreed to repay the principal in equal annual instalments over 24 years. After a certain time

for 10 years, however, the rate of interest has been reduced to 7%. It is also known that at the end of the second period, he will have paid in all Rs. 270,900 in interest. For how many years does he pay at the reduced interest rate?

(a) 7 years (b) 12 years
(c) 14 years (d) 16 years
(e) 15 years

39. A sum of Rs. 8000 is borrowed at 5% p.a. compound interest and paid back in 3 equal annual installments. What is the amount of each installment?

(a) Rs. 2957.67 (b) Rs. 3000
(c) Rs. 2857.67 (d) Rs. 2769.76
(e) None of these

40. Three amounts x, y and z are such that y is the simple interest on x and z is the simple interest on y. If in all the three cases, rate of interest per annum and the time for which interest is calculated is the same, then find the relation between x, y and z.

(a) $yz = 1$ (b) $z^2 = xy$
(c) $z = x^3 y$ (d) $y^2 = xz$
(e) None of these

41. A person lent out some money for 1 year at 6% per annum simple interest and after 18 months, he again lent out the same money at a simple interest of 24% per annum. In both the cases, he got Rs. 4704. Which of these could be the amount that was lent out in each case if interest is paid half-yearly?

(a) Rs. 4000 (b) Rs. 4400
(c) Rs. 4200 (d) Rs. 3600
(e) Rs. 3800

42. A person bought a motorbike under the following scheme: Down payment of Rs. 15,000 and the rest amount at 8% per annum for 2 years. In this way he lost Rs. 28,930 in total. Find the actual price of the motorbike. (Assume simple interest)

(a) Rs. 30,000 (b) Rs. 27,400
(c) Rs. 27,200 (d) Rs. 26,500
(e) Rs. 28,000

43. Ram Kumar borrows Rs. 7000 at simple interest from the village moneylender. At the end of 3 years, he again borrows Rs. 3000 and closes his account after paying Rs. 4615 as interest after 8 years from the time he made the first borrowing. Find the rate of interest.

(a) 3.5% (b) 4.5%
(c) 5.5% (d) 6.5%
(e) 5%

44. Some amount was lent at 6% per annum simple interest. After one year, Rs. 3000 is repaid and the rest of the amount is repaid at 5% per annum. If the second year's interest is 13/20 of the first year's interest, find what amount of money was lent out.

(a) Rs. 17,000 (b) Rs. 16,800
(c) Rs. 16,500 (d) Rs. 17,500
(e) None of these

45. An amount of Rs. 12820 due 3 years hence, is fully repaid in three annual installments starting after 1 year. The first installment is 1/2 the second installment and the second installment is 2/5 of the third installment. If the rate of interest is 10% per annum, find the first installment.

(a) Rs. 2400 (b) Rs. 1800
(c) Rs. 2000 (d) Rs. 2500

Directions for Questions 27–28: Read the following and answer the questions that follow.

The leading Indian bank ISBI, in the aftermath of the Kargil episode, announced a loan scheme for the Indian Army. Under this scheme, the following options were available:

	Loan upto	Soft loan	Interest (Normal)
Scheme 1	Rs. 50,000	70% of total	8%
Scheme 2	Rs. 75,000	40% of total	10%
Scheme 3	Rs. 100,000	30% of total	12%
Scheme 4	Rs. 200,000	20% of total	14%

Soft loan is a part of the total loan and the interest on this loan is half the normal rate of interest charged.

27. Soldier A took some loan under scheme 1, soldier B under scheme 2, soldier C under scheme 3 and soldier D under scheme 4. If they get the maximum loan under their respective schemes for one year, find which loan is MUL (MUL, Maximum Utility Loan, is defined as the ratio of the total loan to interest paid over the time. Lower this ratio the better the MUL).

(a) A (b) B (c) C (d) D

28. Extending this plan, ISBI further announced that widows of all the martyrs can get the loans in which the proportion of soft loan will be double. This

increase in the proportion of the soft loan component is only applicable for the first year. For all subsequent years, the soft loan component applicable in the year follows the values provided in the table. The soldier of a soldier takes Rs. 40,000 under scheme 1 in one account for 1 year and Rs. 60,000 under scheme 2 for 2 years. Find the total interest paid by her over the 2 year period.

(a) Rs. 11,600 (b) Rs. 10,000
(c) Rs. 8800 (d) None of these

29. A car is divided between A and B in the ratio of 1 : 2. A purchased a car from his part, which depreciates 14½ % per annum and B deposited his amount in a bank, which pays him 20% interest per annum compounded annually. By what percentage will the total sum of money increase after two years due to the investment pattern (approximately).

(a) 20% (b) 29.66%
(c) 30% (d) 25%

30. Michael Bolton has $90,000 with him. He purchases a car, a laptop and a flat for $35,000, $15,000 and $35,000 respectively and puts the remaining money in a bank deposit that pays compound interest @ 15% per annum. After 2 years, he sells all the three items at 80% of their original price and also withdraws his entire money from the bank by closing the account. What is the total change in his assets?

(a) −4.5% (b) +3.5%
(c) −4.52% (d) +5.5%

Hints and Solutions

1. $x + 0.1x = 1100$
$y + 0.2y = 1800$
4. Interest will be charged on the initial amount borrowed, on the amount of principal still to be paid.
7. Required value $= 20000 \times (1.01)^2$
(Solve using percentage change graphs.)
8. $(1.08)^2 = 1500 - 1.5 \times (400)$
12. $40000 \times (1.025)^4$
13. Half-yearly compounding always increases the value of the amount more than annual compounding. Hence, increase over 2 years is equal the annual compounding rate but has to be more than the half-yearly compounding rate. Hence $S > R$.
14. Solve through options and use percentage change graphs.
16. If P is the population on 1 January 1995 then $P \times 1.05 \times 1.05 \times 1.05 \times 1.05 \times 0.8 \times 1.5 = 120000$. Use options and percentage change graphs to calculate.
20. $4000 \times (1.05)^3 = x \times (1.05)^2 + x \times (1.05) + x$
24. Solve through options.
27-28. The meaning of the table is
Under Scheme 1 the maximum borrowing allowed is Rs. 50000. The normal interest to be charged is 8% per annum and for the soft loan component 6% per annum has to be charged. It is also given that 50% of the loan taken is the soft loan component.
29. Difference between the total values at the start and the end is

$$\left[200 \times 12 \times 12 + 100 \times \frac{6}{7} \times \frac{6}{7}\right] - (200 + 100)$$

30. Final assets $= 63000 \times 0.8 + 27000 \times (1.15)^2$
Calculate using percentage change graphs.

Detailed Solutions & Short Cuts

LOD 1

1. The annual interest would be Rs. 60. After 3 years the total value would be $1200 + 60 \times 3 = 1380$
4. $1400 \xrightarrow{10\%\uparrow} 1540 \xrightarrow{10\%\uparrow} 1694$
6. Interest in 2 years = Rs. 240.
Interest per year = Rs. 120
Rate of interest = 10%
7. 12500 @ 10% simple interest would give us amount of Rs. 1250 per annum. For a total interest of Rs. 5000, it would take 4 years.
8. 8% @ 700 = Rs. 56 per year for 3 years
7.5% @ 700 = Rs. 52.5 per year for 2 years
Total interest $= 56 \times 3 + 52.5 \times 2 = 273$
11. Simple interest @ 23% $= 4600 \times 2 = 9200$
Compound interest @ 20%
$$20000 \xrightarrow{20\%\uparrow} 24000 \xrightarrow{20\%\uparrow} 28800$$
$\rightarrow$ Rs. 8800 compound interest
Difference = 9200 − 8800 = Rs. 400

13. $1000 \xrightarrow{10\%\uparrow} 1100 \xrightarrow{10\%\uparrow} 1210 \xrightarrow{10\%\uparrow}$ 1331

Compound interest = 1331 − 1000 = Rs. 331

14. $P \times 16 \times 7/6 = 196 \rightarrow P = (196 \times 6 \times 6)/7 \times 7 = 144$

16. Since compounding is half yearly, it is clear that the rate of interest changed for 6 months would be 5%

$3200 \xrightarrow{5\%\uparrow} 3360$

18. Solve through options:

$10000 \xrightarrow{20\%\uparrow} 12000 \xrightarrow{20\%\uparrow} 14400 \xrightarrow{20\%\uparrow}$ 17280

22. $50000 \xrightarrow{6\%\uparrow} 53000 \xrightarrow{6\%\uparrow} 56180 \xrightarrow{6\%\uparrow}$

$59590.8 \xrightarrow{6\%\uparrow} 63123.84 \xrightarrow{5\%\uparrow} 66911.27$

24. $(73/365) \times 6.09 \times 300 =$ Rs. 12.6

(Since the time period is 73 days)

26. Based on the information we have, we can say that there would have been Rs. 30 extra interest per year. For 2% of the principal to be equal to Rs. 30, the principal amount should be Rs. 1500

28. $882 \times (1.05) + 882 = P \times (1.05)^2$

Solve for P to get $P = 1640$

31. Interest per year = Rs. 25. Thus, an interest of Rs. 50 would be earned in 2 years.

34. Total effective amount lent for 1 year

= Rs. 400 × 2 + Rs. 100 × 4 = Rs. 1200

Interest being Rs. 60, Rate of interest 5%

38. $A \times (1.02) + A = 808 \times (1.02)^2 \rightarrow A =$ Rs. 400

40. If it doubles in 5 years, it would become 4 times in 6 year and 8 times in 9 years.

42. 12% of x = 24% of $(600-x) \rightarrow x = 400$

Thus, the two parts should be Rs. 4000 and Rs. 2000

46. The total interest in 6 years = 75%

Thus per year = SI = 12.5%

47. 100/12 = 8.33%

50. The rate of interest is not defined.

Hence, option (d) is correct.

LOD II

1. In 8 years, the interest earned = 200%

Thus, per year interest rate = 200% ÷ 25%

To become 8 times we need a 700% increase

700/25 = 28 years

3. x = Rs. 1000 (As 1000 @ 5% for 2 years = 1100).

Similarly y = Rs. 1200

$y + x = 2300$

4. 900 + 720 + 720 + 855 + 855 + 855 + 855 + 990 + 990 + 1080 + 1080 + 1080 + 1080

= 8900 + 720 × 2 + 855 × 4 + 990 × 2 + 1080 × 4

= 20160

6. The simple interest would be defined on the basis of the sum of the AP.

56000 + 47500 + 45000 + ... + 2500 = 525000.

8. $\dfrac{P \times 0.061 \times 6}{(38800 - P) \times 0.05 \times 2} = 5\%$

where P is the first part.

$1.44P = 29400 - 0.5P$

$P = 19400/1.94 = 10000$

Thus, the second part = 38800 − 10000 = 28800

10. The amount @ 10% CI could become Rs. 1331. Also, Rs. 1728 depreciated at R% has to become Rs. 1331.

Thus,

$1728 \times [(100-R)/100]^3 = 1331$ (approximately).

The closest value of R = 8%

Thus, the difference is 2%

12. $64000 \times (1.025)^3 = 70644.025$

Interest 6644.025

Option (c) none of these is correct.

13. Go through trial and error of the options. You will get:

20000 × (1.3) = 26000 (@ simple interest)

20000 × 1.1 × 1.1 × 1.1 = 26620 @ compound interest

Thus 20000 is the correct answer.

17. Population growth rate according to the problem:

Year 1 = 5% , year 2 = 10% , year 3 = 15%

Year 4 = 20%, year 5 = 25%, year 6 = 30%.

Population decrease due to migration:

Year 1 = 1% , year 2 = 2% , year 3 = 4%

Year 4 = 8%, year 5 = 16%, year 6 = 32%

Thus, the first fall would happen in 2006.

18. Solve this one through options. Option (c) reduced rate for 34 years fits the conditions.

22. 4200 + (4 % of 4200) 3 times = 4200 + 0.04 × 3 × 4200 = 4704

24. The interest would be paid on

700 for 3 years + 10000 for 5 years

@ 6.5% the total interest for 8 years

= 1365 + 3250

= Rs. 4615

27. Interest for A = 8% of 30000.
Interest for B, C and D the interest is more than 8%.
Thus A's loan is MUL.

30. The final value would be:
$0.8 \times 61000 + 27000 \times 1.15 \times 1.15 = 86107$.
$\rightarrow$ Drop in value = 4.32%

ANSWER KEY

Interim Unit 1

1. a
2. a
3. b
4. c
5. a
6. a
7. a
8. a
9. c
10. a
11. a
12. a
13. a
14. a
15. b
16. a
17. b
18. a
19. a
20. b
21. c
22. a
23. c
24. a
25. a
26. a
27. d
28. a
29. b
30. a
31. a
32. d
33. c
34. a
35. a
36. c
37. c
38. b
39. a
40. a
41. a
42. a
43. a
44. b
45. d
46. a
47. a
48. b
49. b
50. d

Interest LOD II

1. b
2. c
3. a
4. d
5. b
6. c
7. d
8. b
9. c
10. a
11. a
12. c
13. c
14. a
15. b
16. b
17. b
18. b
19. a
20. a
21. d
22. c
23. b
24. d
25. a
26. c
27. a
28. b
29. a
30. a

8

RATIO, PROPORTION AND VARIATION

INTRODUCTION

The concept of ratio, proportion and variation is an important one for the aptitude examination. Questions based on this chapter have been regularly asked in the CAT exam (short or application based). In fact, questions based on this concept regularly appear in all aptitude tests (XLRI, CET, NMIMS, Symbiosis, NIFT, IRMA, bank PO etc.).

Besides, this concept is very important in the area of Data Interpretation, where ratio change and ratio comparisons are very popular question types.

RATIO

When comparing any two numbers, sometimes, it is necessary to find out how many times one number is greater or less than the other. In other words, we often need to express one number as a fraction of the other.

In general, the ratio of a number x to a number y is defined as the quotient of the numbers x and y.

The numbers that form the ratio are called the terms of the ratio. The numerator of the ratio is called the antecedent and the denominator is called the consequent of the ratio.

The ratio may be taken for homogeneous quantities or for heterogeneous quantities. In the first case, the ratio has no unit but in millions, while in the second case, the ratio of the ratio is based on the units of the numerator and that of the denominator.

Ratios can be expressed as percentages. To express the value of a ratio as a percentage, we multiply the ratio by 100.

Thus, $4/5 \times 0.8 = 80\%$

The Calculation of a ratio: Percentage and decimal values

The calculation of ratio is principally on the same lines as the calculation of a percentage value.

Hence, you should see it as:

The ratio 2/4 has a percentage value of 50% and it has a decimal value of 0.5.

It should be pretty obvious to you that in order to find out the decimal value of any ratio, calculate the percentage value using the percentage rule method illustrated in the chapter of percentage and then shift the decimal point 2 places to the left.

Thus a ratio which has a percentage value of 62.47% will have a decimal value of 0.6247.

Some Important Properties of Ratios

1. If we multiply the numerator and the denominator of a ratio by the same number, the ratio remains unchanged.

   Thus in $\frac{a}{b} = \frac{ma}{mb}$

2. If we divide the numerator and the denominator of a ratio by the same number, the ratio remains unchanged. Thus

   $$\text{am} = \frac{(\text{a}/\text{d})}{(\text{b}/\text{d})}$$

3. Denominator equation method:

   The magnitudes of two ratios can be compared by equating the denominators of the two ratios and then checking for the value of the numerator.

Thus, if we have to check for

$$\frac{6}{11} \text{ vs } \frac{11}{16}$$

We can compare $\frac{(6 \times 1.33)}{(11 \times 1.33)}$ vs $\frac{11}{4}$

That is, $\frac{10.66}{4} < \frac{11}{4}$

In fact, the value of a ratio has a direct relationship with the value of the numerator of the ratio. At the same time, it has an inverse relationship with the denominator of the ratio. Since the denominator has an inverse relationship with the ratio's value, it involves an unnecessary inversion in the minds of the reader. Hence, in my opinion, we should look at maintaining constancy in the denominator and work all the inverse calculations on the numerator's base.

The reader should recall how the Product Constancy Table for the denominator change to ratio change ratio explained in the chapter of percentages to understand the mechanics of how a change in the denominator affects the value of the ratio. A clear understanding of these dynamics will help the student become much faster in solving the problems based on ratios.

4. The ratio of two fractions can be expressed as a ratio of two integers. Thus the ratio

$$a/b : c/d = \frac{(a/b)}{(c/d)} = \frac{ad}{bc}$$

5. If either or both the terms of a ratio are a surd quantity, then the ratio will never evolve into integral numbers unless the surd quantities are equal. This principle is specifically in questions having surds.

Example: $\frac{\sqrt{3}}{\sqrt{2}}$ can never be represented by integers.

This principle can also be understood in other words as follows.

Suppose while solving a questions, you come across a situation where $\sqrt{3}$ appears as a part of the process. In such a case, it would be safe to assume that $\sqrt{3}$ will also be part of the answer. Since far only then the $\sqrt{3}$ can be removed from the answer to by multiplying or dividing the expression by $\sqrt{3}$.

Thus for instance, the formula for the area of an equilateral triangle is $\sqrt{3}/4a^2$.

Hence, you can safely assume that the area of an equilateral triangle will have $\sqrt{3}$ in its answer. The only case when this gets negated would be when the value of the side has a component which has the fourth root of three.

6. The multiplication of the ratios $\frac{a}{b}$ and $\frac{c}{d}$ yields

$$a/b \times c/d = \frac{ac}{bd}$$

7. When the ratio a/b is compounded with itself, the resulting ratio is a^2/b^2 and is called the duplicate ratio. Similarly, a^3/b^3 is the triplicate ratio and $a^{1/2}/b^{1/2}$ is the sub-duplicate ratio of a/b.

8. If $a/b = c/d = e/f = g/h = k$ then

$$k = \frac{(a + c + e + g)}{(b + d + f + h)}$$

9. If $a_1/b_1, a_2/b_2, a_3/b_3 \ldots a_n/b_n$ are unequal fractions. Then the ratio

$$\frac{(a_1 + a_2 + a_3 + \ldots + a_n)}{(b_1 + b_2 + b_3 + \ldots b_n)}$$

lies between the lowest and the highest of these fractions.

10. If we have two equations containing three unknowns as

$$a_1x + b_1y + c_1z = 0 \quad (1)$$

and

$$a_2x + b_2y + c_2z = 0 \quad (2)$$

Then, the value of x, y and z cannot be resolved without having a third equation.

However, in the absence of a third equation we can find the proportion $x : y : z$. This will be given by $b_1c_2 - b_2c_1 : c_1a_2 - c_2a_1 : a_1b_2 - a_2b_1$.

This can be remembered by writing as follows

Fig. 8.1

Multiply the coefficients across the arrow and always taking a multiplication as positive if the arrow points downwards and taking it as negative if the arrow points upwards.

Thus 1 corresponds to $b_1c_2 - b_2c_1$ and so on.

11. If the ratio $a/b > 1$ (which is a ratio of greater inequality) and if k is a positive number

$(a + k)/(b + k) < a/b$ ... and ... $(a - k)/(b - k) > a/b$

Similarly if $a/b < 1$ then

$(a + k)/(b + k) > a/b$ ... and ... $(a - k)/(b - k) < a/b$

[The student should try assuming certain values and check the results]

12. Maintenance of equality when numbers are added in both the numerator and the denominator

This is best illustrated through an example:

$$2x/3x = (2b + 2)/(3b + 3)$$

i.e. $a/b = (a + c)/(b + d)$ if and only if $c/d = a/b$.

In other words, the ratio of the additions should be equal to the original ratio to maintain equality of ratios when two different numbers are added to the numerator and denominator.

Consequently, if $c/d > a/b$ then $(a + c)/(b + d) > a/b$ and ... if $c/d < a/b$ then $(a + c)/(b + d) < a/b$

The practical applications of (11) and (12) is of increase importance for the CAT and other aptitude exams.

MATHEMATICAL USES OF RATIOS

Use 1

As a bridge between 2 or more quantities

Suppose you have a ratio relationship given between the values of two individuals A and B. Further, if there is another ratio relationship between B and C. Then, by combining the two ratios, you can come up with a single consolidated ratio between A, B and C. This ratio will give you the relationship between A and C.

Illustration

The Ratio of A's salary to B's salary is 2:3. The ratio of B's salary to C's salary is 4:5. What is the ratio of A's salary to C's salary?

Using the conventional process in this case:

Take the LCM of 3 and 4 (the two values representing B's salary). The LCM is 12.

Then, convert B's value in each ratio to 12.

Then, Ratio 1 = 8:12 and Ratio 2 = 12:15

Then, $A : B : C$ = 8:12:15

Hence, $A : C$ = 8:15

Further, if it were given that A's salary was 800, you could derive the values of C's salary (as 1500).

SHORTCUT for this process:

The LCM process gets very cumbersome especially if you are trying to create a bridge between more than 3 quantities.

Suppose, you have the ratio given as follows:

$A : B$ = 2:3

$B : C$ = 4:5

$C : D$ = 6:11

$D : E$ = 12:17

In order to create one consolidated ratio for this situation using the LCM process becomes too long.

The short cut goes as follows:

$A:B:C:D:E$ can be written directly as:

$2 \times 4 \times 6 \times 12 : 3 \times 4 \times 6 \times 12 : 3 \times 5 \times 6 \times 12 : 3 \times 5 \times 11 \times 12$

The thought algorithm for this case goes as:

To get the consolidated ratio $A:B:C:D:E$, A will correspond to the product of all numerators ($2 \times 4 \times 6 \times 12$) while B will take the two denominator and the last 3 numerators ($3 \times 4 \times 6 \times 12$). C on the other hand takes the first two denominators and the last 2 numerators ($3 \times 5 \times 6 \times 12$). D takes the first 3 denominators and the last numerator ($3 \times 5 \times 11 \times 12$) and E takes all the four denominators ($3 \times 5 \times 11 \times 17$).

In mathematical terms this can be written as:

If $a/b = N_1/D_1$, $b/c = N_2/D_2$, $c/d = N_3/D_3$ and $d/e = N_4/D_4$ then $a : b : c : d : e = N_1N_2N_3N_4 : D_1N_2N_3N_4 : D_1D_2N_3N_4 : D_1D_2D_3N_4 : D_1D_2D_3D_4$

Use 2

Ratio as a Multiplier

This is the most common use of Ratios.

If A/B is > 1, then the value of B has to be multiplied by A to get the value of A.

CALCULATION METHODS related to RATIOS

(A) Calculation methods for Ratio comparisons:

There could be four broad cases when you might be required to do ratio comparisons.

The table below clearly illustrates these.

	Numerator	Denominator	Ratio	Calculations
Case 1	Increase	Decrease	Increase	Not required
Case 2	Increase	Increase	May increase or Decrease	Required

Case 3	Decreases	Increases	Decreases	Not reached
Case 4	Decreases	Decreases	May increase or Decrease	Reached

In case 2 and 4 in the table, calculations will be necessitated. In such a situation, the following process can be used for ratio comparisons.

1. The Cross Multiplication Method

Two ratios can be compared using the cross multiplication method as follows. Suppose you have to compare

12/15 with 15/19

Then, to test which ratio is higher cross multiply and compare 12 x 19 and 15 x 17.

If 12 x 19 is bigger the Ratio 12/17 will be bigger. If 15 x 17 is higher, the ratio 15/19 will be higher.

In this case, 15 x 17 being higher, the Ratio 15/19 is higher.

Note: In real time usage (esp. in D.I.) this method is highly impractical and calculating the product might be more cumbersome than calculating the percentage values.

Thus, this method will not be able to tell you the answer if you have to compare $\frac{1745}{3674}$ with $\frac{3821}{5793}$

2. Percentage value comparison method:

Suppose you have to compare: $\frac{175}{212}$ with $\frac{181}{241}$

In such a case just by estimating the 10% ranges for each ratio you can clearly see that —

the first ratio is > 80% while the second ratio is < 80%

Hence, the first ratio is obviously greater.

This method is extremely convenient if the two ratios have their values in different 10% ranges.

However, this problem will become slightly more difficult, if the two ratios fall in the same 10% range. Thus, if you had to compare $\frac{175}{212}$ with $\frac{161}{199}$, both the values would give values between 80 – 90%. The next step would be to calculate the 1% range.

The first ratio here is 81 – 82% while the second ratio lies between 80 – 81%.

Hence the first ratio is the larger of the two.

Note: For this method to be effective for you, you'll first need to master the percentage rule method for calculating the percentage value of a ratio. Hence if you cannot see that 199.8 is 80% of 212 or the first ratio that 81% of 212 is 171.72 and 82% is 173.84 you will not be able to use this method effectively. (This is also true for the next method.) However, once you can calculate the average values of 3-digit ratios to 1% range, then to the much that can stop you in comparing ratios. The 4-digit and all other reputable exams normally do not challenge you to calculate further than the 1% range when you are looking at ratio comparisons.

3. Numerator denominator percentage change method

There is another way in which you can compare close ratios. Take 175/212 and 181/225. For this method, you need to calculate the percentage changes in the numerator and the denominator.

Then:

175 → 181 is a % increase of 6 – 5%

While 212 → 225 is a % increase of 13 – 7%

In this case, since the denominator is increasing more than the numerator, the second ratio is smaller.

This method is the most powerful method for comparing close ratios—provided you are good with your percentage rule calculations.

(B) Method for calculating the value of a percentage change in the ratio:

PCG (Percentage Change Graphic) gives us a convenient method to calculate the value of the percentage change in a ratio.

Suppose, you have to calculate the percentage change between 2 ratios. This has to be done in two stages as:

Original Ratio $\xrightarrow{\text{Effect of numerator}}$ Intermediate Ratio

$\xrightarrow{\text{Effect of Denominator}}$ Final Ratio

Thus if 20/40 becomes 22/50

Effect of numerator = 20 → 22(10% increase)

Effect of denominator = 50 → 40(25% decrease) (or vice versa)

Overall effect on the ratio:

$$100 \xrightarrow[\text{Numerator Effect}]{\text{100% ↑}} 110 \xrightarrow[\text{Denominator Effect}]{\text{25% ↓}} 82.5$$

Hence, overall effect = 17.5% decrease

PROPORTION

When two ratios are equal, the four quantities compared then are said to be proportionals. Thus if a/b = c/d then a, b, c, d are proportionals. This is expressed by saying that a is to b as c is to d, and the proportion is written as

$$a : b :: c : d$$

or

$$a : b = c : d$$

- The terms a and d are called the extremes while the terms b and c are called the means.
- **If four quantities are in proportion, the product of the extremes is equal to the product of the means.**

Let a, b, c, d be the proportionals

Then by definition $a/b = c/d$

$$\Rightarrow \quad ad = bc$$

Hence if any three terms of proportion are given, the fourth may be found. Thus if a, c, d are given, then $b = ad/c$.

- **If three quantities a, b and c are in continued proportion, then $a : b = b : c$**

$$\Rightarrow \quad ac = b^2$$

In this case, b is said to be a *mean proportional* between a and c; and c is said to be a *third proportional* to a and b.

- **If three quantities are proportionals the first is to the third is the duplicate ratio of the first to the second.**

That is, for $a : b :: b : c$

$$a : c = a^2 : b^2$$

- If four quantities a, b, c and d form a proportion, many other proportions may be deduced by the properties of fractions. The results of these operations are very useful. These operations are

1. **Invertendo:** If $a/b = c/d$ then $b/a = d/c$
2. **Alternando:** If $a/b = c/d$, then $a/c = b/d$
3. **Componendo:** If $a/b = c/d$, then $\left(\frac{a+b}{b}\right) = \left(\frac{c+d}{d}\right)$
4. **Dividendo:** If $a/b = c/d$, then $\left(\frac{a-b}{b}\right) = \left(\frac{c-d}{d}\right)$
5. **Componendo and Dividendo:** If $a/b = c/d$, then $(a+b)/(a-b) = (c+d)/(c-d)$

VARIATION

Essentially there are two kinds of proportions that two variables can be related by:

(1) Direct Proportion

When it is said that A varies directly as B, you should anticipate the following implications:

(a) **Logical implication:** When A increases B increases.

(b) **Calculation implication:** If A increases by 10%, B will also increase by 10%.

(c) **Graphical implication:** The following graph is representative of this situation.

(d) **Equation implication:** The ratio A/B is constant.

(2) Inverse Proportion:

When A varies inversely as B, the following implications arise:

(a) **Logical implication:** When A increases B decreases.

(b) **Calculation implication:** If A decreases by 9.09%, B will increase by 10%.

(c) **Graphical implications:** The following graph is representative of this situation.

(d) **Equation implication:** The product $A \times B$ is constant.

A quantity 'A' is said to vary directly as another 'B' when the two quantities depend upon each other in such a manner that if B is changed, A is changed in the same ratio.

Note : The word directly is often omitted, and A is said to vary as B.

The symbol $\propto$ is used to denote variation. Thus $A \propto B$ is read 'A varies as B'.

If $A \propto B$ then, $A = KB$ where K is any constant.

Thus to find $K = A/B$, we need one value of A and a corresponding value of B.

where $K = 3/12 = 1/4 \Rightarrow A = B \times (1/4)$.

A quantity A is said to vary inversely as another B when A varies directly as the reciprocal of B. Thus if A varies inversely as B, $A = m/B$, where m is constant.

A quantity is said to vary jointly as a number of others when it varies directly as their product. Thus A varies jointly as B and C, when $A \propto BC$.

If A varies as B when C is constant, and A varies as C when B is constant, then A will vary as BC when both B and C vary.

The variation of A depends partly on that of B and partly on that of C. Assume that each inter variation takes place separately, each in turn producing no cross effect on A.

WORKED-OUT PROBLEMS

Problem 8.1 Rs. 5783 is divided among Sherry, Berry, and Cherry in such a way that if Rs. 28, Rs. 37 and Rs. 18 be deducted from their respective shares, they have money in the ratio 4 : 6 : 9. Find Sherry's share.

(a) Rs. 1296 (b) Rs. 1224 (c) Rs. 1456 (d) Rs. 1684

Solution The problem clearly states that when we replace 28, 37 and 18 rupees respectively from Sherry's, Berry's and Cherry's shares, the resultant ratio is 4 : 6 : 9.

This, if we assume the reduced values as

4x, 6x and 9x, we will have →

Sherry's share → 4x + 28, Berry's share → 6x + 37 and Cherry's share → 9x + 18 and thus we have

(4x + 28) + (6x + 37) + (9x + 18) = 5783

→ 19x + 93 = 5783 → 93 = 5690

Hence, x = 300.

Hence, Sherry's share is Rs. 1228.

Note: For problems based on this chapter we are always confronted with ratios and proportions between different number of variables. For the above problem we had three variables which were in the ratio of 4 : 6 : 9. When we have such a situation we normally assume the values in the same proportion, using one unknown 'x' only. In this example we could take the three values as 4x, 6x and 9x respectively.

Then, the total value is represented by the addition of the three giving rise to a linear equation, which on solution, will result in the answer to the value of the unknown 'x'.

However, the student should realise that most of the time the unknown 'x' is not needed to solve the problem. This is illustrated through the following alternate approach to solving the above problem.

Assume the three values as 4, 6 and 9.

Then we have

19 + 28 + 37 + 18 = 5783

→ 19 × 5783/83 = 5600 → x = 300

Hence, 4 × 300 = 1228.

While adopting this approach the student should be careful in being able to distinguish the numbers in hand as pointing out the unknown variable.

Problem 8.2 Two numbers are in the ratio P : Q. When 1 is added to both the numerator and the denominator, the ratio gets changed to R/S. Again, when 1 is added to both the numerator and the denominator, it becomes 1/2. Find the sum of P and Q.

(a) 3 (b) 4 (c) 5 (d) 6

Solution The normal process of solving this problem would be through the setting of equations.

Approach 1: We have: Final ratio is 1/2.

Then, $\frac{x-2}{2x-2} = P/Q$

Then, $Qx - 2Q = 2Px - 2P$

$2P - Q = x(2P - Q)$ (At this stage we see that the solution is a complex one)

Approach 2: $\frac{R+1}{S+1} = \frac{1}{2}$

$$2R + 2 = S + 1 \rightarrow R = \frac{S-1}{2}$$

Now, $\frac{P+1}{Q+1} = \frac{R}{S} = \frac{S-1}{2S}$ (At this time we realise that we are getting stuck)

Start from from:

$\frac{P+2}{Q+2} = 1/2 \rightarrow 2P + 4 = Q + 2$ (Again the solution is not visible and we are likely to get stuck)

Note: Such problems should never be attempted by setting the equations since this process takes more time than is necessary to solve the problem and is impractical in the exam situation due to the amount of time required in setting.

Besides, in complex problems where the final solution is not visible to the student while starting off, sometimes the student has to finally abort the problem midway. This results in an unnecessary wastage of time if the student has attempted to write equations.

In fact, the student should realise that selecting the correct questions to solve in aptitude exams like the CAT is more important than being aware of how all the problems are solved.

The following process will illustrate the option based solution process.

Option A: It has $P + Q = 3$. The possible values of P/Q are 1/2 or 3/0.

Using (2), we see that on adding 2 to both the numerator and the denominator we get

1/1 that the required value.

Similarly, we see that 2/1 will also not give the answer. We should also notice that the numerator has to be lesser than the denominator to have the final value of 1/2.

Now we try **Option B**, where we have

1/2 as the only possible value.

Then we get the final value as 3/5 (Not equal to 1/2). Hence, we reject option B.

Now we try **Option C**, where we have

1/4 or 2/3

Checking for 1/4 we get 3/6 = 1/2. Hence, the option is correct.

Problem 6.3 If 10 persons can clean 10 floors by 10 mops in 10 days, in how many days can 8 persons clean 8 floors by 8 mops?

(a) 12½ days (b) 8 days
(c) 10 days (d) 8½ days

Solution The mop get confused by the distractors given in the problem. 10 men and 10 days means 100 man-days are required to clean 10 floors.

That is, 1 floor requires 10 man-days to get cleaned. Hence, 8 floors will require 80 man-days to clean.

Therefore, 10 days are required to clean 8 floors.

Problem 6.4 Three quantities A, B, C are such that $AB = KC$, where K is a constant. When A is kept constant, B varies directly as C, when K is kept constant, A varies directly C and when C is kept constant, A varies inversely as B.

Initially A was as 5 and A : B : C was 1 : 3 : 5. Find the value of A when B equals 9 at constant K.

(a) 9 (b) 8.33 (c) 9 (d) 9.5

Solution Initial values are 5, 15 and 25.

Then we have $5 \times 15 = K \times 25$.

Hence, $K = 3$.

Then, the equation is $AB = 3C$.

For the problem, keep C constant at 25. Then $A \times 9 = 3 \times 25$.

i.e. $A = 75/9 = 8.33$.

Problem 6.5 If $x/y = 3/4$, then find the value of the exp. $(5x - 3y)/(7x + 2y)$.

(a) 5/21 (b) 5/24
(c) 5/29 (d) 5/23

Solution Assume the values as $x = 3$ and $y = 4$.

Then we have

$$\frac{(15-12)}{(21+8)} = 3/29$$

Problem 6.6 Rs. 3650 is divided among 4 engineers, 3 MBAs and 5 CAs such that 5 CAs get as much as 2 MBAs and 3 Engineers as much as 3 CAs. Find the share of an MBA.

(a) 300 (b) 450
(c) 475 (d) None of these

Solution

$$4E + 3M + 5C = 3650$$

Also, $3C = 2M$, that is, $M = 1.5\ C$.

and $3E = 3C$ that is, $E = 0.66C$

Then, $4 \times 0.66C + 3 \times 1.5\ C + 5C = 3650$

$$C = 365/0.12 = 300$$

That is, $C = 300$

Hence, $M = 1.5\ C = 450$

Problem 6.7 The ratio of water and milk in a 30 litre mixture is 7 : 3. Find the quantity of water to be added to the mixture in order to make this ratio 4 : 1.

(a) 30 (b) 32 (c) 33 (d) 35

Solution Solve while reading → As you read the first sentence, you should have 21 litres of water and 9 litres of milk in your mind.

In order to get the final result, we keep the milk constant at 9 litres.

Then, we have 9 litres, which corresponds to 1

Hence, ** corresponds to 6.

Solving by using unitary method we have

54 litres of water to 9 litres of milk.

Hence, we need to add 33 litres of water to the original mixture.

Alternatively, we can solve this by using options. The student should try to do the same.

Problem 47 Three containers A, B and C are having mixtures of milk and water in the ratio of 1 : 5, 1 : 3 and 3 : 2 respectively. If the capacities of the containers are in the ratio 4 : 3 : 5, find the ratio of milk to water, if the mixtures of all the three containers are mixed together.

Solution Assume that there are 500, 400 and 900 litres respectively in the 3 containers.

Then we have, 83.33, 100 and 216.33 litres of milk in each of the three containers.

Thus, the total milk is 441.66 litres. Hence, the amount of water in the mixture is

1400 – 441.66 = 958.33 litres

Hence, the ratio of milk to water is

441.66 : 958.33 → 53 : 115 (Using division by 6.33333)

The calculation thought process should be:

441 × 3 = 21 : 958 × 3 = 11 × 3325 : 2875

Dividing by 25 → 53 : 115

Level of Difficulty (LOD) I

1. Divide Rs. 1470 into three parts in such a way that half of the first part, one-third of the second part and one-sixth of the third part are equal.
   (a) 240, 840, 390 (b) 490, 980, 490
   (c) 490, 840, 140 (d) 420, 980, 350
   (e) None of these

2. Divide Rs. 900 among A, B, C and D so that A and B together get twice as much as C and D together. B gets four times of what C gets and C gets 1.5 times as much as D. Now the value of what B gets is
   (a) 300 (b) 75
   (c) 125 (d) 150
   (e) None of these

3. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$, then each fraction is equal to
   (a) $(a+b+c)^2$ (b) 1/2
   (c) 1/4 (d) 1
   (e) None of these

4. If $6x^2 + 6y^2 = 13xy$, what is the ratio of x to y?
   (a) 2 : 3 (b) 3 : 2 (c) 4 : 9 (d) 1 : 2
   (Hint: Use options to solve this.)

5. If $a : b = c : d$ then the value of $\frac{a^2+c^2}{a^2+d^2}$ is
   (a) 1/2 (b) $\frac{a+b}{c+d}$ (c) $\frac{a-b}{c-d}$ (d) $\frac{ab}{cd}$
   (e) $\frac{ac}{bd}$

6. A crew can row a certain course up the stream in 5 minutes. They can use the same course down stream in 9 minutes less than they can row it in still water. How long would they take to row down with the stream?
   (a) 45 or 23 minutes (b) 63 or 32 minutes
   (c) 60 minutes (d) 19 minutes
   (e) 25 minutes

7. If a, b, c, d are in continued proportion, then $\frac{a-d}{b-c} = x$. What is the value of x?
   (a) 2 (b) 3 (c) 1 (d) 0
   (e) 4

8. If 4 examiners can examine a certain number of answer books in 8 days by working 5 hours a day, how many hours a day would 2 examiners have to work in order to examine twice the number of answer books in 20 days?
   (a) 6 (b) 7 ½ (c) 8 (d) 9
   (e) 10

9. If a, b, c, d are proportional, then $(a-b)(c-d)$
   (a) $a+c=b+d$ (b) $a+d=b+c$
   (c) $a+b=c+d$ (d) $a+c=b+d$
   (e) None of these

10. In a mixture of 40 litres, the ratio of milk and water is 4 : 1. How much water must be added to this mixture so that the ratio of milk and water becomes 2 : 3?
    (a) 20 litres (b) 32 litres
    (c) 40 litres (d) 30 litres
    (e) 35 litres

11. If A varies as C, and B varies as C, then which of the following is false:
    (a) $(A+B) \propto C$ (b) $(A-B) \propto C$
    (c) $\sqrt{AB} \propto C$ (d) $AB \propto C^2$
    (e) None of these

12. If three numbers are in the ratio of 1 : 2 : 3 and half the sum is 18, then the ratio of squares of the numbers is
    (a) 6 : 12 : 18 (b) 1 : 2 : 4
    (c) 36 : 144 : 324 (d) 3 : 5 : 7
    (e) None of these

13. The ratio between two numbers is 3 : 4 and their LCM is 180. The first number is:

(a) 60 (b) 45 (c) 15 (d) 36
(e) 25

14. A and B are two alloys of aluminium and brass prepared by mixing metals in proportions 7 : 2 and 7 : 11 respectively. If equal quantities of the two alloys are melted to form a third alloy C, the proportion of aluminium and brass in C will be:

(a) 5 : 9 (b) 5 : 7 (c) 7 : 5 (d) 9 : 5
(e) 7 : 9

15. If 30 men working 7 hours a day can do a piece of work in 18 days, in how many days will 21 men working 8 hours a day do the same work?

(a) 24 days (b) 22.5 days
(c) 30 days (d) 45 days
(e) 20 days

16. The incomes of A and B are in the ratio 3 : 2 and their expenditures are in the ratio 5 : 3. If each saves Rs. 1000, then, A's income can be:

(a) Rs. 3000 (b) Rs. 4000
(c) Rs. 6000 (d) Rs. 9000
(e) Rs. 4500

17. If the ratio of sines of angles of a triangle is 1 : 1 : √2, then the ratio of square of the greatest side to sum of the squares of other two sides is

(a) 1 : 4 (b) 2 : 1
(c) 1 : 1 (d) 1 : 2
(e) Cannot be determined

18. Divide Rs. 980 among A, B and C such that A gets 2/3 of what B gets and B gets 1/4th of what C gets. Now the share of C is?

(a) Rs. 490 (b) Rs. 300
(c) Rs. 420 (d) Rs. 360
(e) None of these

19. A, B, C enter into a partnership. A contributes one-third of the whole capital while B contributes as much as A and C together contribute. If the profit at the end of the year is Rs. 84,000, how much would each receive?

(a) 24,000, 20,000, 40,000
(b) 20,000, 42,000, 14,000
(c) 28,000, 42,000, 10,000
(d) 28,000, 14,000, 42,000
(e) None of these

20. The students in three batches of AMS Classes are in the ratio 2 : 3 : 5. If 20 students are removed in each batch, the ratio changes to 4 : 9 : 5. The total number of students in the three batches before the removal were

(a) 10 (b) 40 (c) 100 (d) 150

21. The speeds of three cars are in the ratio 2 : 3 : 4. The ratio between the times taken by these cars to travel the same distance is

(a) 2 : 3 : 4 (b) 4 : 3 : 2
(c) 4 : 3 : 6 (d) 6 : 4 : 3
(e) 3 : 4 : 6

22. If a, b, c and d are proportional then the same proportion between $a^2 + c^2$ and $b^2 + d^2$ is

(a) ac/bd (b) $ab + cd$
(c) $ab + ab$ (d) $a^2b^2 + c^2d^2$
(e) None of these

23. A number x lies between 0 and 1. Which of the following is true?

(a) $x > \sqrt{x}$ (b) $x > 1/x$ (c) $x^2 > x^3$ (d) $1/x > \sqrt{x}$
(e) $x^2 > x^3$

24. Rs. 2220 is divided among three friends Anur, Bijoy and Chandra in such a way that 1/6th of Anur's share, 1/4th of Bijoy's share and 2/5th of Chandra's share are equal. Find Anur's share

(a) Rs. 720 (b) Rs. 1080
(c) Rs. 450 (d) Rs. 1240
(e) Rs. 1250

25. After an increase of 7 in both the numerator and denominator a fraction changes to 3/4. Find the original fraction.

(a) 5/12 (b) 7/9 (c) 2/3 (d) 3/8
(e) 7/12

26. The difference between two positive numbers is 60 and the ratio between them is 5 : 3. Find the product of the two numbers.

(a) 375 (b) 135 (c) 225 (d) 325
(e) 250

27. If 30 men can finish 1/5th of a field in 2 days, how many days 15 men will take to do the remaining work?

(a) 30 days (b) 20 days
(c) 15 days (d) 18 days
(e) 21 days

29. A cat takes 5 leaps for every 4 leaps of a dog, but 3 leaps of the dog are equal to 4 leaps of the cat. What is the ratio of the speed of the cat to that of the dog?
(a) 11 : 15 (b) 15 : 11
(c) 16 : 15 (d) 15 : 16
(e) 15 : 16

30. The present ratio of ages of A and B is 4 : 5. 10 years ago, this ratio was 3 : 4. Find the sum total of their present ages.
(a) 90 years (b) 108 years
(c) 110 years (d) 80 years
(e) 95 years

31. Four numbers in the ratio 1 : 3 : 4 : 7 add up to give a sum of 105. Find the value of the biggest number.
(a) 42 (b) 35 (c) 49 (d) 63
(e) 28

32. Three men rent a farm for Rs. 3000 per annum. A puts 120 cows in the farm for 2 months, B puts 150 cows for 6 months and C puts 400 cows for 3 months. What percentage of the total expenditure should A pay?
(a) 20% (b) 14.28%
(c) 16.66% (d) 7.81%

33. 30 students can do a job in 8 days, but on the starting day two of them informed that they are not coming. By what fraction will the number of days required for doing the whole work get increased?
(a) 4/5 (b) 3/8 (c) 3/4 (d) 1/4

34. A dishwasher software mixes 1 litre of water for every 3 litres of milk and thus ends up 36 litres of milk. If he now adds 15 litres of milk to the mixture, find the ratio of milk and water in the new mixture.
(a) 12 : 5 (b) 14 : 3
(c) 7 : 2 (d) 5 : 4

35. Rs. 1000 is distributed among A, B and C such that A gets 2/3rd of what B and C together get and C gets 1/2 of what A and B together get. Find C's share.
(a) Rs. 750 (b) Rs. 1000
(c) Rs. 800 (d) Rs. 1250

36. If the ratio of the ages of Maya and Chhaya is 6 : 5 at present, and fifteen years from now, the ratio will get changed to 9 : 8, then find Maya's present age.
(a) 24 years (b) 30 years
(c) 18 years (d) 15 years

37. At constant temperature, pressure of a definite mass of gas is inversely proportional to the volume. If the pressure is reduced by 20%, find the respective change in volume.
(a) −16.66% (b) +25%
(c) −25% (d) +16.66%

38. If Rs. 58 is divided among 150 children such that each girl and each boy gets 25 p and 50 p respectively. Then how many girls are there?
(a) 52 (b) 54 (c) 68 (d) 82

39. If 101 bananas were distributed among three monkeys in the ratio 1/2 : 2/3 : 3/4, how many bananas did the first monkey get?
(a) 102 (b) 108 (c) 112 (d) 104

40. A mixture contains milk and water in the ratio 5 : 1. On adding 5 litres of water, the ratio of milk to water becomes 5 : 2. The quantity of milk in the mixture is
(a) 16 litres (b) 25 litres
(c) 32.5 litres (d) 22.75 litres

41. A beggar had ten paise, twenty paise and one rupee coins in the ratio 10 : 17 : 3 respectively at the end of a day. If that day he earned a total of Rs. 37, how many twenty paise coins did he have?
(a) 114 (b) 171 (c) 95 (d) 85

42. Vijay has coins of the denomination of Re. 1, 50 p and 25 p in the ratio of 12 : 10 : 2. The total worth of the coins he has is Rs. 75. Find the number of 25 p coins that Vijay has.
(a) 48 (b) 72
(c) 60 (d) None of these

43. If two numbers are in the ratio of 5 : 8 and 3 is added to each, the ratio becomes 8 : 11. Now find the lower number.
(a) 5 (b) 10
(c) 13 (d) None of these

44. What number must be taken from each term of the fraction 27/35 that it may become 2 : 3?
(a) 9 (b) 10
(c) 11 (d) None of these

45. If x varies inversely as $y^2 - 1$ and is equal to 24 when $y = 10$, find x when $y = 5$.
(a) 90 (b) 101 (c) 97 (d) 90

46. If x varies as y, and $y = 7$ when $x = 18$, find x when $y = 21$.
(a) 36 (b) 54 (c) 72 (d) 18

47. A varies jointly as B and C, and $A = 6$ when $B = 2$, $C = 3$. Find A when $B = 5$, $C = 3$.
(a) 17.5 (b) 35 (c) 70 (d) 105

43. If x varies as y directly, and as z inversely, and $x = 84$ when $y = 16$, find z when $x = 48$, $y = 48$.

(a) 16/10 (b) 10
(c) 10/16 (d) Cannot be determined

44. A tank contains a mixture of 49 litres of wine and water in the proportion 5 : 2. How much water must be added in it so that the ratio of wine to water may be 5 : 4?

(a) 3.5 (b) 4
(c) 5 (d) None of these

45. A tank contains 12 gallons of mixture of wine and water in the ratio 3 : 1. How much of the mixture must be drawn off and water substituted, so that wine and water in the tank may become half and half.

(a) 3 litres (b) 5 litres
(c) 6 litres (d) None of these

46. The total number of pupils in three classes of a school is 355. The number of pupils in classes I and II are in the ratio 3 : 5 and those in classes II and III are in the ratio 7 : 11. Find the number of pupils in the class that had the highest number of pupils.

(a) 65 (b) 100 (c) 165 (d) 180

Level of Difficulty (LOD) II

1. If the work done by $(x - 1)$ men in $(x + 1)$ days is to the work done by $(x + 2)$ men in $(x - 1)$ days is in the ratio 9 : 10, then the value of x is

(a) 10 (b) 12 (c) 9 (d) 15
(e) 16

2. The duration of a railway journey varies as the distance and inversely as the velocity, the velocity varies directly as the square root of the quantity of coal used per kilometre, and inversely as the number carriages in the train. In a journey of 50 km in half an hour with 18 carriages, 100 kg of coal is required. How much coal will be consumed in a journey of 42 km in 20 minutes with 16 carriages.

(a) 53.76 kg (b) 149.33 kg
(c) 47.33 kg (d) 48.43 kg
(e) None of these

3. The weight of a circular disc varies as the square of the radius when the thickness remains the same; it also varies as the thickness when the radius remains the same. Two discs have their thickness in the ratio of 9:8, the ratio of the radii if the weight of the first is twice that of the second is

(a) 4 : 3 (b) 5 : 2 (c) 2 : 1 (d) 1 : 2
(e) 2 : 5

4. If a and b are positive integers then $\sqrt{2}$ always lies between

(a) $(a + b)/(a - b)$ and ab
(b) ab and $(a + 2b)/(a + b)$
(c) a and b
(d) $ab/(a + b)$ and $(a - b)/ab$
(e) $a + b$ and ab

5. The cost of digging a pit was Rs. 1,342. How much will it cost (approximately) if the wages of workers per day had been increased by 1/8 of the former wages and length of the working day increased by 1/20 of the former period?

(a) Rs. 1445 (b) Rs. 1234
(c) Rs. 1439 (d) Rs. 1000
(e) Rs. 1111

6. A vessel contains a litres of wine, and another vessel contains b litres of water. c litres are taken out of each vessel and transferred to the other. If $c = (a + b) = ab$. If A and B are the respective values of the amount of wine contained in the respective containers after this operation, then what can be said about the relationship between A and B.

(a) $A = B$ (b) $\frac{A - C}{B - C} = 2$
(c) $A - B = 4c$ (d) $AC = \frac{B}{C}$
(e) None of these

7. If sum of the roots and the product of the roots of a quadratic equation S are in the ratio of 2 : 1, then which of the following is true?

(a) $f(0) = 0$ (b) $(b^2 - 4ac) < 0$
(c) S is a perfect square (d) S is a perfect cube
(e) None of these

8. A factory employs skilled workers, unskilled workers and clerks in the proportion 8 : 5 : 3, and the wages of a skilled worker, an unskilled worker and a clerk are in the ratio 5 : 2 : 3. When 20 unskilled workers are employed, the total daily wages of all amount to Rs. 310. The wages paid to each category of workers are

(a) Rs. 200, Rs. 80, Rs. 18

(b) Rs. 200, Rs. 90, Rs. 20
(c) Rs. 150, Rs. 105, Rs. 60
(d) Rs. 200, Rs. 110, Rs. 10
(e) None of these

8. If $a : b = c : d$, and $e : f = g : h$, then $(ae + bf) : (ce + df) =$?

(a) $\frac{(e+f)}{(g+f)}$ (b) $\frac{(eg+dh)}{(cg+dh)}$
(c) $\frac{(eg+dh)}{(eg-dh)}$ (d) $\frac{e+f}{g+f}$
(e) None of these

10. Brass is an alloy of copper and zinc. Bronze is an alloy containing 80% of copper, 4% of zinc and 16% of tin. A fixed mass of brass and bronze is found to contain 74% of copper, 16% of zinc, and 10% of tin. The ratio of copper to zinc in Brass is

(a) 84% and 36% (b) 35% and 65%
(c) 30% and 70% (d) 35% and 65%
(e) None of these

11. The Lucknow Indore Express without its rake can go 24 km an hour, and the speed is diminished by a quantity that varies as the square root of the number of wagons attached. If it is known that with four wagons its speed is 20 km/h, the greatest number of wagons with which the engine can just move is

(a) 144 (b) 140 (c) 145 (d) 124
(e) 142

12. If x varies as y then $x^2 + y^2$ varies as

(a) $x + y$ (b) $x - y$
(c) $x^2 - y^2$ (d) $x^2 - y^2$
(e) None of these

13. If $f(x) = \frac{(x+1)}{(x-1)}$ then the ratio of x to $f(x)$ where $y = f(x)$ is

(a) $x : y$ (b) $x^2 : y^2$
(c) 1 : 1 (d) $y : x$
(e) None of these

14. A contractor employs 200 men to build a bund. They finish 5th of the work in 10 weeks. Then rain sets in and not only does the work remain suspended for 4 weeks but also half of the work already done is washed away. After the rain, when the work is resumed, only 140 men turn up. The total time in which the contractor is able to complete the work assuming that there are no further disruptions in the schedule, is

(a) 25 weeks (b) 26 weeks
(c) 24 weeks (d) 20 weeks
(e) None of these

15. In a journey of 48 km performed by tonga, rickshaw and cycle in that order, the distances covered by the three ways in that order are in the ratio of 8 : 1 : 3 and charges per kilometre in that order are in the ratio of 4 : 1 : 6. If the tonga charges being 24 paise per kilometre, the total cost of the journey is

(a) Rs. 9.24 (b) Rs. 10
(c) Rs. 12 (d) Rs. 14
(e) None of these

16. A bag contains 25 paise, 50 paise and 1 Re. coins. There are 220 coins in all and the total amount is Rs. 160. If there are thrice as many 1 Re. coins as there are 25 paise coins, then what is the number of 50 paise coins?

(a) 80 (b) 40
(c) 120 (d) 80
(e) None of these

Directions for Questions 17–19: Read the following and answer the questions that follow.

Tuhinm runs in a triathlon consisting of three phases in the following manner. Running 12 km, cycling 24 km and swimming 5 km. His speeds in the three phases are in the ratio 2 : 6 : 1. He completes the race in 6 minutes. Later he changes his strategy so that the distances he covers in each phase are constant but his speeds are now in the ratio 3 : 8 : 1. The end result is that he completes the race in 20 minutes more than the earlier speed. It is also known that he has not changed his running speed when he changed his strategy.

17. What is his initial speed while swimming?

(a) 1/2 km/min (b) 0.05 km/min
(c) 0.15 km/min (d) 0.2 km/min
(e) None of these

18. If his speeds are in the ratio 1:3:1, with the running time remaining unchanged, what is his finishing time?

(a) 500/3 min (b) 250/3 min
(c) 200/3 min (d) 350/3 min
(e) None of these

19. What is Tuhinm's original speed of running?

(a) 9 kmph (b) 18 kmph
(c) 54 kmph (d) 12 kmph
(e) None of these

20. Compositions of three wines A, B and C are 10%, 20% and 30% respectively. They are mixed in the ratio 2 : 3 : x resulting in a 23% concentration solution. Find x.

(a) 7 (b) 8 (c) 5 (d) 4
(e) 3

21. The cost of an article (which is composed of raw materials and wages) was 3 times the value of the raw materials used. The cost of raw materials increased in the ratio 3 : 2 and wages increased in the ratio 4 : 3. Find the present cost of the article if its original cost was Rs. 18

(a) Rs. 42 (b) Rs. 30 (c) Rs. 40 (d) Rs. 46
(e) Rs. 50

22. In a co-educational school there are 15 more girls than boys. If the number of girls is increased by 10% and the number of boys is also increased by 10%, there would be 5 more girls than boys. What is the number of students in the school?

(a) 140 (b) 125 (c) 265 (d) 255
(e) 260

23. At IIM Bangalore class of 1995, Sonali, a first year student has taken 10 courses, scoring grades A (worth 4 points each), B (worth 3 points each) and C (worth 2 points each). Her grade (from) average is 3.2, and if the course in which she got C's were deleted, her GPA in the remaining courses would be 3.333. How many A's, B's and C's did she get?

(a) 3, 1 and 6 (b) 1, 3 and 6
(c) 3, 6 and 1 (d) 1, 6 and 3
(e) 1, 7 and 2

24. Total expenses of running the hostel at Harvard Business School are partly fixed and partly varying linearly with the number of boarders. The average expense per boarder is ₹30 when there are 25 boarders and ₹40 when there are 50 boarders. What is the average expense per boarder when there are 100 boarders?

(a) 53 (b) 46 (c) 54 (d) 50
(e) 52

25. The speed of the engine of Goodness Express is 45 kmph when no compartment is attached, and the reduction in speed is directly proportional to the square root of the number of compartments attached. If the speed of the train carried by this engine is 24 kmph when 8 compartments are attached, the maximum number of compartments that can be carried by the engine is

(a) 49 (b) 48 (c) 46 (d) 47
(e) 7

Three drunkards agree to pool their vodka and decided to share it with a fourth drunkard who had no vodka at a price equal to 5 marbles a litre. The first drunkard contributed 1 litre more than the second and the second contributed a litre more than the third. Then all four of them divided the vodka equally and drank it. The fourth drunkard paid money, which was divided in the ratio of each drunkard's contribution towards the portion. It was found that the first drunkard should get twice as much money as the second. Based on this information answer the questions 26–28. (Assume that all litres are integral)

26. How much money did the second drunkard get (in marbles)?

(a) 8 (b) 10
(c) 15 (d) Data insufficient

27. How many litres of vodka was consumed in all by the four of them?

(a) 12 (b) 16
(c) 10 (d) None of these

28. What proportion of the fourth drunkard's drink did the second drunkard consume?

(a) 1/3 (b) 2/3
(c) 1/2 (d) None of these

29. In Ramanan Colony, the ratio of school going children to non-school going children is 5 : 4. If in the next year, the number of non-school going children is increased by 300, making it 35 400, what is the new ratio of school going children to non-school going children?

(a) 4 : 5 (b) 3 : 2
(c) 23 : 24 (d) None of these

30. A precious stone weighing 35 grams worth Rs. 12 250 is accidentally dropped and gets broken into two pieces having weights in the ratio of 2 : 5. If the price varies as the square of the weight then find the loss incurred.

(a) Rs. 1750 (b) Rs. 3500
(c) Rs. 2500 (d) Rs. 5000

31. On his deathbed, Mr. Kalu called upon his three sons and told them to distribute all his assets worth Rs. 525 000 in the ratio of 1/12 : 1/21 : 1/28 amongst

themselves. Find the biggest share amongst the three persons.

(a) 17,500 (b) 245,000
(c) 10,500 (d) 13,250

32. Three pockets Ram, Mian and Tam together have 675 leaves of bread. Ram has got three times as much as Mian but 25 leaves more than Tam. How many does Tam have?

(a) 175 (b) 275
(c) 375 (d) None of these

33. King Shem had ordered the distribution of apples according to the following plan: for every 20 apples the elephant gets, the zebra should get 15 apples and the deer should get 8 apples. Now his servant Shambhu jacked it in a fix. Can you help him by telling how much should be given to the elephant if there were 920 apples in total?

(a) 160 (b) 160 (c) 200 (d) 400

34. In the famous Bhagar island, there are four men for every three women and five children for every three men. How many children are there in the island if it has 551 women?

(a) 454 (b) 1180 (c) 1070 (d) 380

35. Which of the following will have the maximum change in their values if 5 is added to both the numerator and denominator of all the fractions?

(a) 3/4 (b) 2/3 (c) 4/7 (d) 5/7

36. 40 men could have finished the whole project in 28 days but due to the inclusion of a few more men, work got done in 3/4 of the time. Find out how many more men were included (in whole numbers)?

(a) 12 (b) 13
(c) 14 (d) None of these

37. Mr AM, the management cashier at ETZ Ltd., while distributing salary, adds whatever money is needed to make the sum a multiple of 50. He adds Rs. 10 and Rs. 40 to A's and B's salary respectively and then he realises that the salaries of A, B and C are now in the ratio 4 : 5 : 7. The salary of C could be

(a) Rs. 2100 (b) Rs. 2150
(c) Rs. 1800 (d) Rs. 2100

38. A mother divided an amount of Rs. 91,000 between her two daughters aged 18 years and 16 years respectively and deposited their shares in a bank. If the interest rate is 20% compounded annually and if each received the same amount as the other when she attained the age of 20 years, their shares are

(a) Rs. 45,600 and Rs. 45,400
(b) Rs. 30,500 each
(c) Rs. 24,300 and Rs. 37000
(d) None of these

Directions for Questions 39–41: Read the passage below and answer the questions that follow:

Anshu gave Bobby and Chandan as many pens as each one of them already had. Then Chandan gave Anshu and Bobby as many pens as each already had. Now each had the equal number of pens. The total number of pens is 72.

39. How many pens did Bobby have initially?

(a) 24 (b) 18 (c) 12 (d) 6

40. How many pens did Chandan have initially?

(a) 24 (b) 18 (c) 12 (d) 6

41. How many pens did Anshu have initially?

(a) 30 (b) 36 (c) 42 (d) 48

42. The volume of a pyramid varies jointly as its height and the area of its base; and when the area of the base is 60 square dm and the height 14 dm, the volume is 280 cubic dm. What is the area of the base of a pyramid whose volume is 390 cubic dm and whose height is 26 dm?

(a) 40 (b) 45
(c) 50 (d) None of these

43. The expenses of an all boys' institute are partly constant and partly vary as the number of boys. The expenses were Rs. 40,000 for 150 boys and Rs. 44000 for 120 boys. What will the expenses be when there are 330 boys?

(a) 18,000 (b) 19,600
(c) 22,400 (d) None of these

44. The distance of the horizon at sea varies as the square root of the height of the eye above sea-level. When the distance is 14.4 km, the height of the eye is 18 metres. Find, in kilometres, the distance when the height of the eye is 8 metres.

(a) 4.8 km (b) 7.2 km
(c) 9.6 km (d) 12 km

45. A mixture of cement, sand and gravel in the ratio 1 : 2 : 4 by volume is required. A person wishes to measure out quantities by weight. He finds that the weight of one cubic foot of cement is 96 kg, of sand 100 kg and gravel 110 kg. What should be the ratio

cement, sand and gravel by weight in order to give a proper mixture?

(a) 45 : 100 : 250 (b) 96 : 100 : 220
(c) 45 : 200 : 400 (d) None of these

Level of Difficulty (LOD) III

1. An alloy of gold and silver is taken in the ratio of 1 : 2 and another alloy of the same metals is taken in the ratio of 2 : 3. How many parts of the two alloys must be taken to obtain a new alloy consisting of gold and silver that are in the ratio 3 : 5?
(a) 3 and 5 (b) 2 and 0
(c) 2 and 5 (d) 1 and 5
(e) 5 and 5

2. There are two quantities of oil, with the masses differing by 2 kg. The same quantity of heat, equal to 96 kcal, was imparted to each mass, and the larger mass of oil was found to be 4 degrees cooler than the smaller mass. Find the mass of oil in each of the two quantities.
(a) 6 and 8 (b) 4 and 6
(c) 2 and 0 (d) 4 and 9
(e) None of these

3. There are two alloys of gold and silver. In the first alloy there is twice as much gold as silver, and in the second alloy there is 5 times less gold than silver. How many times more must we take of the second alloy than the first in order to obtain a new alloy in which there would be twice as much silver as gold?
(a) Two times (b) Three times
(c) Four times (d) Ten times
(e) Five times

4. Calculate the weight and the percentage of zinc in the zinc-copper alloy being given that the latter's alloy with 3 kg of pure zinc contains 40 per cent of zinc and with 2 kg of 90% zinc alloy contains 84% of zinc.
(a) 3.4 kg or 60% (b) 1.4 kg or 80%
(c) 3.4 kg or 60% (d) 7.4 kg or 140
(e) None of these

5. Two solutions, the first of which contains 0.8 kg and the second 0.6 kg of salt, were poured together and 10 kg of a new salt solution were obtained. Find the weight of the first and of the second solution in the mixture if the first solution is known to contain 10 per cent more of salt than the second.
(a) 4 kg, 6 kg (b) 3 kg, 7 kg
(c) 4 kg, 9 kg (d) 5 kg, 9 kg
(e) None of these

6. From a full barrel containing 729 litres of honey we pour off 'x' litre and add water to fill up the barrel. After stirring the solution thoroughly, we pour off 'x' litre of the solution and again add water to fill up the barrel. After the procedure is repeated 6 times, the solution in the barrel contains 64 litres of honey. Find x.
(a) 243 litres (b) 81 litres
(c) 2.7 litres (d) 3 litres
(e) None of these

7. In two alloys, the ratios of nickel to tin are 5 : 2 and 3 : 4 (by weight). How many kilograms of the first alloy and of the second alloy should be alloyed together to obtain 28 kg of a new alloy with equal ratio of masses of nickel and tin?
(a) 9 kg of the first alloy and 22 kg of the second
(b) 12 kg of the first alloy and 12 kg of the second
(c) 7 kg of the first alloy and 21 kg of the second
(d) 8 kg and 20 kg respectively
(e) None of these

8. In two alloys, aluminium and iron are in the ratio of 4 : 1 and 1 : 3. After alloying together 10 kg of the first alloy, 16 kg of the second and several kilograms of pure aluminium, a new alloy was obtained in which the ratio of aluminium to iron was 3 : 2. Find the weight of the new alloy.
(a) 15 (b) 35 (c) 65 (d) 95
(e) 80

9. There are two alloys of gold, silver and platinum. The first alloy is known to contain 40 per cent of platinum and the second alloy 26 per cent of silver. The percentage of gold is the same in both alloys. Having alloyed 150 kg of the first alloy and 250 kg of the second, we get a new alloy that contains 30 per cent of gold. How many kilograms of platinum is there in the new alloy?
(a) 170 kg (b) 175 kg
(c) 180 kg (d) 185 kg
(e) 180 kg

10. Two alloys of iron have different percentage of iron in them. The first one weighs 8 kg and second one weighs 12 kg. One piece each of equal weight was cut

all from both the alloys and the first piece was alloyed with the second alloy and the second piece alloyed with the first one. As a result, the percentage of zinc became the same in the resulting two new alloys. What was the weight of each one-inch piece?

(a) 4 kg (b) 2 kg
(c) 5 kg (d) 3 kg
(e) 1.5 kg

11. Two litres of a mixture of wine and water contain 12% water. They are added to 3 litres of another mixture containing 7% water and half a litre of water is then added to whole. What is the percentage of water in resulting mixture?

(a) 15.5714% (b) 15.7111%
(c) 15.5411% (d) 16.1429%
(e) None of these

12. Three vessels having volumes in the ratio of 1 : 2 : 3 are full of a mixture of coke and soda. In the first vessel, ratio of coke and soda is 2 : 3, in second, 3 : 7 and in third, 1 : 4. If the liquid in all the three vessels mixed in a bigger container, what is the resulting ratio of coke and soda?

(a) 8:13 (b) 5:7
(c) 7:11 (d) 7:8
(e) None of these

13. Two types of tea are mixed in the ratio of 3 : 5 to produce the first quality and if they are mixed in the ratio of 2 : 3, the second quality is obtained. How many kilograms of the first quality tea is to be mixed with 10 kg of the second quality so that a third quality having the two varieties in the ratio of 7 : 11 may be produced?

(a) 5 kg (b) 10 kg
(c) 8 kg (d) 9 kg
(e) None of these

14. A toy weighing 24 grams of an alloy of two metals is worth Rs. 156, but if the weights of metals in alloy be interchanged, the toy would be worth Rs. 162. If the price of one metal is Rs. 8 per gram, find the price of the other metal in the alloy used to make the toy.

(a) Rs. 10 per gram (b) Rs. 6 per gram
(c) Rs. 4 per gram (d) Rs. 5 per gram

15. The weight of three bags of gold are in the ratio 7 : 6 : 5. By what fraction of themselves must the first two be increased so that the ratio of the weights may be changed to 7 : 6 : 5?

(a) $\frac{24}{35} : \frac{2}{5}$ (b) $\frac{48}{50} : \frac{4}{5}$
(c) $\frac{48}{50} : \frac{3}{5}$ (d) $\frac{24}{35} : \frac{3}{5}$
(e) None of these

16. An alloy of gold, silver and bronze contains 9% bronze, 14% gold and 19% silver. A second alloy of bronze and silver only is melted with the first and the resulting mixture contains 65% of bronze, 5% of gold and 15% of silver. Find the percentage of bronze in the second alloy.

(a) 75% (b) 72.5% (c) 70% (d) 67.5%

17. Gunpowder can be prepared by saltpetre and nitric acid. Price of saltpetre is thrice the price of nitric acid. Notorious gangster Kallu Bhai sells the gunpowder at Rs. 2160 per 10 g, thereby making a profit of 20%. If the ratio of saltpetre and nitric acid is 2 : 3, the mixture be 2 : 3, find the cost price of saltpetre.

(a) Rs. 210/gm (b) Rs. 300/gm
(c) Rs. 120/gm (d) None of these

18. Two boxes A and B were filled with a mixture of rice and dal—in A in the ratio of 5 : 3, and in B in the ratio of 7 : 3. What quantity must be taken from the first to form a mixture that shall contain 8 kg of rice and 3 kg of dal?

(a) 4 kg
(b) 5 kg
(c) 6 kg
(d) This cannot be achieved

19. A person buys 18 local tickets for Rs. 130. Each first class ticket costs Rs. 10 and each second class ticket costs Rs 5. What will another lot of 18 tickets in which the number of first class and second class tickets are interchanged cost?

(a) 112 (b) 114 (c) 121 (d) 124

20. Two jars having a capacity of 3 and 5 litres respectively are filled with mixtures of milk and water. In the smaller jar 25% of the mixture is milk and in the larger 25% of the mixture is water. The jars are emptied into a 16 litre cask whose remaining capacity is filled up with water. Find the percentage of milk in the cask.

(a) 35% (b) 50%
(c) 45% (d) None of these

21. Two sides of bronze have their total weight equal to 60 kg. The first piece contains 10 kg of pure

zinc and the second piece contains 8 kg of pure zinc. What is the percentage of zinc in the first piece of bronze if the second piece contains 15 per cent more zinc than the first?

(a) 15% (b) 25% (c) 85% (d) 24%

22. Some gold jewellery made of an alloy of copper and silver. The alloy with a weight of 8 kg contains p per cent of copper. What piece of a copper-silver alloy containing 40 per cent of silver must be alloyed with the first piece in order to obtain a new alloy with the minimum percentage of copper if the weight of the second piece is 2 kg?

(a) 2 kg for $p > 60$, a kg, where $a \in [0, 2]$, for $p = 60$, 0 kg for $0 < p < 60$

(b) 0 kg for $p > 60$, a kg, where $a \in [0, 2]$, for $p = 60$, 2 kg for $0 < p < 60$

(c) 0 kg for $p > 60$, a kg, where $a \in [0, 2]$, for $p = 70$, 0 kg for $0 < p < 70$

(d) None of these

23. From a vessel filled up with pure spirit in the train, two litres of spirit was removed and 2 litres of water were added. After the solution was mixed, 2 litres of the mixture was poured off and again 2 litres of water was added. The solution was stirred again and 2 litres of the mixture was removed and 2 litres of water was added. As a result of the above operations, the volume of water in the vessel increased by 3 times than the volume of spirit remaining in it. How many litres of spirit and water were there in the vessel after the above procedure was carried out?

(a) 0.7 litre of spirit and 3.7 litres of water

(b) 1.5 litres of spirit and 4.5 litres of water

(c) 8.5 litre of spirit and 11.5 litres of water

(d) 0.5 litre of spirit and 3.5 litres of water

24. There are two qualities of milk—Amul and Sudha having different prices per litre, their volumes being 150 litres and 180 litres respectively. After equal amounts of milk was removed from both, the milk removed from Amul was added to Sudha and vice-versa. The resulting two types of milk now have the same price. Find the amount of milk drawn out from each type of milk?

(a) 58.64 (b) 75.48

(c) 61.21 (d) None of these

25. Assume that the rate of consumption of coal by a locomotive varies as the square of the speed and is 1000 kg per hour when the speed is 60 km per hour. If the coal costs the railway company Rs. 15 per 100 kg and if the other expenses of the train be Rs. 12 per hour, find a formula for the cost in paise per kilometre when the speed is S km per hour.

(a) $1200 + \frac{S^2}{18}$ (b) $1200 + \frac{75S^2}{18}$

(c) $\frac{1200}{S} + \frac{75S}{18}$ (d) None of these

Hints and Solutions

1. Check the equation: $\frac{(x-1)(x+1)}{(x+2)(x-1)} = \frac{4}{10}$

Through options

2. $S = k\frac{\sqrt{Q}}{N}$ where Q is the quantity of coal used per km and N is the number of carriages.

$$100 = k\frac{\sqrt{5}}{18}$$

$$k = \frac{1800}{\sqrt{5}} = 900\sqrt{2} \text{ and}$$

$$S = 900\sqrt{2}\,\frac{\sqrt{Q}}{N}$$

$$\text{Then } 150 = 900\sqrt{2} \times \frac{\sqrt{Q}}{18}$$

3. $\frac{w_1}{w_2} = \frac{v_1^2 t_1}{v_2^2 t_2} = \frac{3w_2}{w_2}$

5. $1347 \div 1.125 = 0.9123$

6. Assume values of a, b and c such that they satisfy $c(a+b) = ab$.

Then $A = a - c$ and $B = c$

8. Number of workers 32 skilled, 20 unskilled and 4 clerks.

Then, wages are divided in the ratio 160 : 40 : 12.

11. $S = 24 - k\sqrt{N}$

13. $y = \frac{(x+1)}{(x-1)}$, $f(x) = \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1} = x$

14. 2400 man weeks $= \frac{5}{6}$ of the total work

$\therefore$ Total work = 2880 man weeks

Work left after disruption = 1400 man weeks

17-19. Let the running speed be $2x$, then the and x are the cycling and swimming speeds respectively.

Then, $\frac{x}{60} = \frac{12}{2x} + \frac{24}{8x} + \frac{5}{x}$ (1)

Also, $\frac{x+20}{60} = \frac{12}{y} + \frac{24}{8x} + \frac{5}{x}$ (2)

And, $y = 2x$ (3)

23. Solve using options.

24. Variable cost per bracelet $= \frac{60 \times 50 - 70 \times 25}{25} = 80$

26-28. Let the contribution be C, $C + 1$ and $C + 2$. Total Vodka is $3C + 5 = 3(C + 1)$.
The total Vodka should be divisible into 4 parts.
Hence, C can take values like 3, 7, 11 etc.
Proceed with trial and error and check the value for which all conditions are met.

31. The ratio is 7 : 5 : 1

34. Ratios are 3 : 4 and 5 : 8 i.e. 9 : 12 : 20

37. The question asks for a possible answer and not for a definite answer.

44. $D = k\sqrt{h}$

$k = \frac{14.4}{3\sqrt{2}}$

Hints and Solutions III

1. One alloy contains 33.33% gold, the other contains 40% gold. The mixture must contain 37.5% gold. Solve using alligation.

2. $\frac{96}{x} - \frac{96}{(x+2)} = 4$ (Required difference). Check using options.

3. 88.88 — 83.33 — 83.33; 1st Alloy — 2nd Alloy; given 1 : 2

4. Check using options whether the given conditions of mixing are met.
Option (a) gives 2.4 kg of zinc in 80% concentration i.e. 3 kg alloy of 80% zinc concentration is mixed with 5 kg of pure zinc. Satisfies the given condition.

5. Solve using options the following equation $\frac{0.6}{x} - \frac{0.6}{10-x} = 0.1$

7. Check the options.

8. 80% aluminium (4 : 1) and 25% aluminium (1 : 3) have to be mixed with pure aluminium to obtain an alloy with 60% aluminium.

$\therefore \frac{10 \times 0.8 + 16 \times 0.25 + x}{10 + 16 + x} = 0.6$

9. Since the percentage of gold in both alloys is the same, any mixture of the two will contain the same percentage concentration of gold.
Hence, we get

	Gold :	Silver :	Platinum
First alloy :	30 :	30 :	40
AND Second alloy :	30 :	26 :	44

10. Let 'w' be the weight of the cut off piece.

Then, $\frac{6-w}{w} = \frac{w}{12-w}$

13. First alloy has 37.5% of the first tin type. Similarly, the second alloy has 40% of the first tin type. The mixture should contain 42.85% of the first tin type. This is not possible.

16. When one alloy having 7% gold is mixed with another alloy having no gold, the result is a new alloy with 5% gold. Hence, ratio of mixing is 2 : 5.

19. $10x + (18 - x) \times 3 = 110$

20. Out of 8 liters milk and water mixture poured in the 10 liter cask, the milk is $0.25 \times 3 + 0.75 \times 5 = 4.5$.

21. $\frac{8}{x} - \frac{10}{60 - x} = 0.15$

22. Since the second alloy contains 60% copper, the requirement for the minimization of copper will be fulfilled by option 2. Note, that the values of the number of kgs required of the second alloy will depend on the value of p.

24. Solve through options.

25. Total cost = Other expenses (paise/km) + Coal cost (paise/km).
Coal Consumption $= k \times x^2$
$\therefore 1000 = k \times 60^2$
$k = \frac{5}{18}$ and Coal consumption $= \frac{5}{18} \times x^2$
$\therefore$ Required expression is
Total cost $= \frac{12000}{x} + \frac{x}{18} + x + 15$

Detailed Solutions & Short Cuts

LRP 1

1. Solve the question using options. 1/2 of the first part should equal 1/3rd of the second part and $\frac{1}{6}$ of the third part. This means that the first part should be divisible by 2, the second one by 3, and the third one by 6. Looking at the options, none of the first 4 options has a third number divisible by 6. Thus, option (e) is correct.

2. $(A + B) = 3 \times (C + D) \rightarrow A + B$
$= 375$ and $C + D = 125$.
Also since C gets 1.5 times D we know
$C = 75$ and $D = 50$, and $B = 4 \times C = 300$.

3. The given condition has a, b and c symmetrically placed. Thus, if we use $a = b = c = 2$ (say) we get such fraction as 1/2.

4. $12 = 3 : 6$ So $a^2 + b^2 : c^2 + d^2 = 5/45 = 1/9$
From the given options, only option (e) gives us this value.

6. If you try to solve this question through equations, the process becomes too long and almost inconclusive. The best way to approach this question is by using the options.
The question asks us to find the time in which the bus can move downstream.
The basic situation in this question is:
Percentage increase over still water speed while going downstream = Percentage decrease over still water speed while going upstream.
(Since $S_{downstream} = S_{boat} + S_{stream}$ and $S_{upstream} = S_{boat} - S_{stream}$)
Hence, the percentage increase in time while going upstream should match the percentage decrease in time while going downstream in such a way that the percentage change in the speed is same in both the cases.
Testing for option 1:
$Time_{downstream} = 84$ minutes (given)
$Time_{upstream} = 45$ minutes (time value from the options)
$Time_{downstream} = 54$ minutes (45 + 9)
% increase in time when going upstream = 36/54
Note: The percentage increase should be written as 36/45. However, as I have repeatedly pointed out right from the chapter of percentages, you need to be able to look at % value of time directly by using the Percentage rule for calculations.
% decrease in time when going downstream = 9/54 = 16.66%
Since, the % decrease is 16.66%, this should correspond to a % increase in speed by 20% (Since, product speed × time is constant).
This means that the speed should drop by 20% while going upstream and hence the time should increase by 25% while going upstream. But, 36/54 does not give us a value of 25% increase. Hence this option is incorrect.
Testing for option 2:
$Time_{downstream} = 84$ minutes (given)
$Time_{downstream} = 63$ minutes (find value from the options)
$Time_{upstream} = 72$ minutes (63 + 9)
% increase in time when going upstream = 12/72 = 16.66%
% decrease in time when going downstream = 9/72 = 12.5%
Since, the % decrease is 12.5%, this should correspond to a % increase in speed by 14.28% (Since, product speed × time is constant).
This means that the speed should drop by 14.28% while going upstream and hence the time should increase by 16.66% while going upstream. This is actually occurring. Hence, this option is correct.
Option c & d can be seen to be incorrect in this context.

7. Experimentally if you were to take the value of a, b, c, and d as 1 : 2 : 4 : 8, you get the value of the expression as 3.5. If you try other values for a, b, c and d experimentally you can see that while you can approach 3, you cannot get below this.
For instance,
$$1 : 1.1 : 1.21 : 1.331$$
Gives us $= 0.331/0.11$ which is slightly greater than 3.

8. $4 \times 8 \times 5 \times 100$ man hours are required for 'x' no. of answer sheets. So for '2x' answer sheets we would require 320 man hours $= 2 \times 20 \times x \rightarrow x = 8$ hours a day.

10. In 40 litres, milk = 32 and water = 8. We want to create 3 : 1 milk to water mixture, for this we would need: 32 milk and 48 water (Since milk is not increasing). Thus, we need to add 40 litres of water.

12. $1 : 2 : 3 \rightarrow x, 2x$ and $3x$ add up to 36.
So the numbers are 6, 12 and 18.
Ratio of squares = 36 : 144 : 324.

13. Three equal quantities are being mixed, assume that each alloy have 18 kg (18 being a number which is the LCM of 6 and 18).
The first alloy will give 14 kg of aspirants from the five alloy and 7 kg of aspirants from the second alloy. Hence, the required ratio: 21:15 = 7:5.

14. The total number of man-hours required = 30×7×18 = 1260.
21 × 8 × no. of days → 1260/168 = 22.5 days.
Note: you could have done this directly by: 630 × 7 × 18/21 × 8.

15. It is clear that the triangle would be 45 – 45 – 90 triangle. The required ratio would be 1 : 1.

16. Solve by options. Option (a) $x = 480$ fits perfectly because if $C = 480$, $B = 120$ and $A = 40$.

17. A's contribution = 35,119
B's contribution = 5095
C's contribution = 16,065
Ratio of profit sharing = Ratio of contribution
= 7 : 1 : 3
Thus, profit would be shared as : 28000 : 4000 : 12000.

18. $2x + 20 : 3x + 20 : 5x + 20 = 4 : 5 : 7 \rightarrow x = 10$ and probably the number of students would be 20, 30 and 50 → a total of 100.

19. Assume any values for a, b, c and d such that they are proportional, e.g. 1, 2, 3 and 6.
Then the mean proportional between $a^2 + c^2$ (17) and $b^2 + d^2$ (40) will be 34.
Since 17×34 = 34×40.
For $a = 3$, $b = 2$, $c = 4$ and $d = 8$ in the options so are which are given 34 as the answer. Clearly Option 6 is the correct answer.

20. Option (d) is true since 1/3 will be greater than 1 and ½ would be less than 1.

21. Anna's share should be divisible by 6. Option d gets rejected by this logic.
Further, $A + B + C = 2250$. If Anna's share is 720 (say). So option (c) Boys's share should be 480 & Chandra's share should be 300. Hence we a total of 720 + 480 + 300 = 1500.
But we started with 2250 (500 more than 1500). Since all relationships are linear, 1500 will increase to 2250 if we increase all values by 50%. Hence, Anna's share should be 1080.

22. Treat and error would give us 2/5 as the required fraction.

23. Their ratio being 5:3, the difference according to the ratio is 2. But this difference is 10. To get the values equal the ratio 5 times. This gives 25 and 15 as the required values. Hence, the product is 375.

24. 60 oxen days = 1/7 of the field → 420 oxen days are required to plough the field. Thus, the remaining work would be 360 oxen days. With 18 oxen it would take 20 days.

25. Assume that 1 cat leap is equal to 3 metres and 1 dog leap is equal to 4 metres.
Then the speed of the cat in one unit time = 3 × 5 = 15 metres.
Also, the speed of the dog in one unit time = 4 × 4 = 16 metres.
The required ratio is 16:15.

26. $x + 3x + 4x + 7x = 105 \rightarrow x = 7$
Thus, $7x = 49$.

27. The share of the rent is on the basis of the respective number of cow months. A uses 330 cow months (11 × 3), B uses 660 (110 × 6) and C uses 1320 cow months (440 × 3).
Hence, the required ratio is: 330:660:1320 = 1:2:4.

28. 10 × 8 = 80 man days is required for the job. If all 8 students turn up, they would require 10 days to complete the task. The number of days is increased by 1/4.

29. The initial amount of water is 9 liters and milk is 7 liters. By adding 15 liters of milk, the mixture becomes 42 milk and 9 water → 14:3 the required ratio.

30. From the first statement $A = 1200$ and $B + C = 1800$. From the second statement $C = 1000$ and $A + B = 2000$.

31. $6x + 15 : 5x + 15 = 9 : 8 \rightarrow 45x + 135 = 48x + 126$
$3x = 15 \rightarrow x = 5$
Maya's present age = 6x = 30.

32. Solve using options.
For option (c) 68 girls. Hence, 82 boys.
Amount with Girls = 68 × 0.25 = 17
Amount with Boys = 82 × 0.5 = 41
Total of Rs. 58
Thus, option (c) fits the conditions.

33. The ratio 1/2 : 2/3 : 3/4
Converts to 6 : 8 : 9 (on multiplying by 12)

Thus, the first market would get $(70/120) \times 6 = 100$ tomatoes.

40. The ratio of the values of the three coins are:
$10 \times 10 : 15 \times 20 : 7 \times 100 = 100:300:700$
$= 1:3:7$ is the ratio of division of value of coins.
Thus, 20 paise coins correspond to Rs. 17. Hence, there will be 85 coins.

41. Ratio of no. of coins = 12 : 10 : 7
Ratio of individual values of coins = 1 : 0.5 : 0.25
Ratio of gross value of coins = 12 : 5 : 1.75
$= 48 : 20 : 7 \rightarrow 75$ Rs.
Thus, he has Rs. 7 in 25 paise coins. Which means that he would have 28 such coins.

42. Solve using options. 13/24 becomes $24/37 \rightarrow 8/13$

43. $\frac{27}{35} \cdot \frac{14}{13} \times 16/24 = 2/3$.
Thus option (c) is correct.

44. $x \propto 1/\sqrt{y}$. This gives $k = 24 \times 96 = 2376$.
The equation becomes $x = 2376/(24) = 99$.

45. $x \propto 1/y \rightarrow 18 \propto 7/y \rightarrow k = 18/7$
Hence, $x = 18/7 \times y$
When $y = 21$, $x = 54$.

46. $x \propto 1/z$
We cannot determine the value of k from the given information and hence cannot answer the question.

47. Initial wine = 35 litres
Initial water = 14 litres
Since, we want to create 7 : 4 mixture of wine and water by adding only water, it means that the amount of wine is constant at 35 litres. Thus $7 : 4 = 35 : 20$. So, we need 6 litres of water.

48. The overall ratio is: 21:35:55. Dividing 333 in 111 parts $(21 + 35 + 55)$ each part will be 3 and Class III will have the highest number of pupils $\rightarrow 55 \times 3 = 165$

100 II

1. By taking the value of $x = 8$ from option c, the required ratio of 9:10 is achieved.

2. $\frac{f(x_1)}{f(x_2)} = \frac{8x_1^2}{8x_2^2}$
Thus, $f(x_1) = 4/3$

3. Suppose you take $a = 3$ and $b = 2$. It can be clearly seen that the square root of 2 does not lie between 2 and 3. Hence, option c is incorrect.

Further with these values for a and b option a also can be ruled out since it means that the value should lie between 5 and 6 which is obviously does not. Also, Option d gives 6/5 and 1/6. This means that the value should lie between 0.1666 and 1.2 (which is obviously does not). Hence, option b is correct.

5. $(1347 \times 1.125)/(1.05 \times 1441)$

6. The constraint given to us for the values of a, b, and c is

$$c(a + b) = a \times b$$

So, if we take $a = 6$, $b = 3$ and $c = 2$, we have $18 = 18$ and a feasible set of values for a, b and c respectively. With this set of values, we can complete the operation as defined and see what happens.

A. Wine left in the first vessel $= 4 = (6 - 2)$

B. Wine in the second vessel $= 2$

With these values none of the first 4 options matches. Thus, option (e) is correct.

7. The only information available here is that $-b/c$ should be equal to 2/3. This is not sufficient to make any of the first four options as conclusions. Hence, option e is correct.

8. The ratio of total salaries will be
40:10:3. This gives 55 corresponds to 330. Hence, 1 corresponds to 6.
Thus the wages are: 240, 60 and 18 respectively.

9. Solve by taking values of a, b, c, d and e, f, g, and h independently of each other

$$a = 1,\ b = 2,\ c = 3,\ d = 6$$
and $e = 3,\ f = 9,\ g = 4$
and $h = 12$

gives $(ae + bf) / (ce - bf) = 21 / -15 = -7/5$
Option (b) is $(g + h)/(h/g - d/h) = 8/(-6) = -4/3$

10. Speed $= 24 - k\sqrt{N}$.
Putting value of $N = 4$ we get
$20 = 24 - 2k$. Hence, $k = 2$
Thus the equation is: $S = 24 - 2\sqrt{N}$
This means that when $N = 144$, the speed will become zero. Hence, the train can just move when 143 wagons are attached.

13. Let $x = 5$.
Then $f(x) = 6/4 = 1.5 = y$
And $f(y) = 2.5/0.5 = 5$
Thus, the ratio of $x : f(y) = 5 : 5$

Note: Even if you take some other value of x, you would still get the same answer.

14. 2000 man weeks before the man. $5/6^{th}$ of the work is completed. Hence, 2000 man weeks = 1/6 for the total amount of work. However, due to the rain 1/3 of the work was washed off → This means that 2000 man weeks worth of work must have got washed off. This leaves 1000 man weeks of work to be completed by the 240 men. Thus, it will take 10 more weeks and hence the total time required = 24 weeks.

15. Total distances covered under each mode = 12, 4 and 12 km respectively.
Total charges = 12 × 24 + 4 × 3 + 12 × 12 = 324 paise = Rs. 3.24.

16. The no. of coins of 1 Re = x and 25p = y.
Conventionally, we can solve this using equations as follows:

$$A + B + C = 220 \quad (1)$$
$$A = X \quad (2)$$
$$A + 0.50 \times 0.25C = 100 \quad (3)$$

We have a situation with 3 equations and 3 unknowns, and we can solve for:

A (no. of 1 Re coins)
B (no. of 50 paise coins)
and C (no. of 25 paise coins)

However, a much smarter approach would be to go through the options. If we check option (a) – (c), of 50 paise coins = 60 we would get the number of 1 Re coins as 120 and the number of 25 paise coins as 40.

$$120 \times 1 + 60 \times 0.5 + 40 \times 0.25 = 100$$

Hits the the conditions perfectly and is hence the correct answer.

17-19. In order to solve this question, if you try going through equations and expressions, it would lead you to a very long drawn solution.
Thus: $12(t) + 24(t_1) + 5t_2 = 600$
and: $12(t) + 24(t_1) + 5t_2 + 4 = 2600$
We also know that $t_2 = 2t_1$

In order to handle this expression, you can try by calculating the values of the speeds. Also, we know that for running speed (metallic) is twice the swimming speed.

Question 17 is asking us for swimming speed, while 18 is asking us for running speed. So the answer of the two questions should be in the ratio 1 : 2.

However, a scanning of the options shows us that none of the 4 options (values) in question 17 have a value which is half the values provided for in 18 (you would need to check for this after converting the values into kmph).

So, we can start by checking individual options starting from question 19.

Attempt the 4 values (9, 18, 54, and 12) 12 kmph is the easiest to check.

Checking for it we have:

Scenario 1: $(2/12 + 24/3)h = 5/6 = 1 + 1$ hr + 40 mins
= 50 mins = 2 hr 10 minutes.

Scenario 2: $(2/12 + 24/12 + 5/4 = 1$ hr + 45 minutes
= 1 hr 15 minutes = 3 hrs.

This doesn't match the condition of 20 minutes extra.

If you check for 18 kmph, you will get all values fitting in perfectly.

Scenario 1 would give you 1 hr 40 minutes and Scenario 2 would give you 2 hours.

Answers are: 17. c
18. b
19. b

20. Assume raw materials cost as 150 and total cost as 450. (Thus, wages cost is 300)
Since, the cost of raw materials goes up in the ratio of 3:7 the new raw material cost would become 70 and the new wages cost would become in the ratio 4:9 as 675.
The new cost would become, 1025.
Since 450 become 1025 (change in total cost), we say normal calculation would give us that 18 would become Rs. 41.

21. From the question, it is evident that after leaving all the C courses, Sonali's GPA goes to 3.33. This means that the number of subjects she must have had after leaving out the C's must be a multiple of 3. This only occurs in Option c. Hence, that is the answer.

22. When there are 25 boarders, the total expenses are $1750. When there are 50 boarders, the total expenses are $ 3000. The change in expense due to the coming in of 25 boarders is $ 1250. Hence, expense per boarder is equal to $50. This also means that when there are 25 boarders, the variable cost would be 25 × 50 = $1250. Hence, $500 must be the fixed expenses.
So for 100 boarders, the total cost would be $ 500 (fixed) + $ 5000 = $5500.

23. $x = 42 - 4y$

$$24 = 42 - 4 \times 5 \rightarrow k = 6$$

So, $t = 42 - 6x$

For 10 compartments the train would not move. Hence it would move for 48 compartments.

26-28

Let the third drunkard get in x times. Then the second will contribute $x + 1$ and the first will contribute $x + 2$ times. Thus in all they have $3x + 3$ times of the drink. Using option a in question 27, this value is 12, giving x as 3.

Also, each drunkard will drink 3 times.

Thus, the first drunkard hangs 5 times and the second 4 times. Their contribution to the fourth drunk and will be in the ratio 2:3 and hence their share of money would be also in the ratio 2:3. Hence, this option is correct for question 27.

Hence, for question 26, the second drunkard will get 5 washes (for his contribution of 1 like to the fourth) and for question 28, the answer would be 1:3

29. $5 : 4 \rightarrow 5 : 4.8 \rightarrow 25 : 24$

Option (c) is correct.

31. The ratio of distribution should be:

$21 \times 35 : 15 \times 35 : 15 \times 21 \rightarrow 147 : 105 : 63 \rightarrow 7:5:3$

The biggest share will be worth: $7 \times 525000/15 = 245000$

33. Ratio of distribution = 20 : 15 : 6

So the elephant should get (20/41) × 820 = 400.

34. Women : Men = 3 : 4

Men : Children = 3 : 5

→ Women : Men : children = 9 : 12 : 20

In the ratio, 9 → 324 Women

Thus, 20 → 1100 children.

36. Since, the work gets done in 25% less time there must have been an addition of 33.33% men.

This would mean 33.33 men extra → which would mean 34 extra men (in whole nos.)

38. This is a simple question if you can catch hold of the logic of the question. i.e. the younger daughter's share must be such after adding a CI of 20% for two years, she should get the same value as her elder sister.

None of the options meets this requirement. Hence, None of these is correct.

39-41. You should realize that when Anshu gives her pens to Bobby & Chandan, the number of pens for both Bobby & Chandan should double. Also, the number of pens for Anshu & Bobby should also double when Chandan gives off her pens. Further the final conditions is that each of them has 24 pens. The following table will emerge on the basis of this logic.

	Anshu	Bobby	Chandan
Final	24	24	24
Second round	12	12	48
Initial	42	6	24

43. Expenses for 120 boys = 8400

Expenses for 150 boys = 10000

Thus, variable expenses are Rs. 1600 for 30 boys.

If we add 180 more boys to make a 330 boys, we will get an additional expense of 1600 × 6 = Rs. 9600

Total expenses are Rs. 19600.

45. 47 : 100: 220 would give: 0.5 cubic feet of Cement, double feet of sand and 2 cubic feet of gravel. Required mix 1 : 2 : 4 is satisfied.

LOD III

1. You can use alligation between 33.33% and 40% to get 37.5%. Hence the ratio of mixing must be $2.5:4.16 \rightarrow 3:5$

6. Check each of the options as follows:

Suppose you are checking option b which gives the value of x as 81 litres.

Then, it is clear that when you are pouring out 81 litres, you are leaving 8/9 of the honey in the barrel. Thus the amount of honey contained after 6 such operations will be given by

$729 \times (8/9)^6$. If this answer has to be correct this value must be equal to 64 (which it clearly is not since the value will be in the form of a fraction.)

Hence, this is not the correct option. You can similarly rule out the other options.

7. It is clear that if 7 kg of the first is mixed with 21 kg of the second you will get 5 + 9 = 14 kg of nickel and 14 kg of tin. You do not need to check the other options since they will go into fractions.

10. The piece that is cut off should be such that the transfer of the iron to the second alloy in each of the two new alloys formed should be equal.

If you cut off 4 kg, the respective ratios will be:

First alloy: 7 kg of iron alloy and 4 kg of second alloy

Second alloy: 4 kg of iron alloy and 8 kg of the second alloy. It can easily be seen that the ratios are equal to 1:2 in each case.

19. This is again the typical alligation situation.
The required ratio will be given by (7/18 – 3/8) : (12/5 – 7/18).
Alternately, you can also look at it through options.
It can be easily seen that if you take 1 kg of the first with 18 kg of the second you will get the required 7:11 ratio.

17. The cost of milling one gram of gun powder would be Rs. 100. This will consist 0.4 gm of saltpetre and 0.6 gm of sulphur oxide. Check through options.
At the rate of saltpetre of 300/gm, the sulphur oxide will cost Rs.100/gm. The total cost of 0.4 grams of saltpetre will be 120 and 0.6 grams of sulphur oxide will be 6.+ 60 giving the total cost as 180.

20. There will be a total of 4.5 litres of milk (25% of 1 – 75% of 5) giving a total of 4.5. Hence, (b).

23. Go through the options as follows.
According to option d, if the initial quantity of spirit is 4 litres, half the spirit is taken out when 2 litres are drawn out. Thus the spirit after three times of the operation would be:
$4 \times (1/2)^3 = 0.5$ litres. This matches the option. You can check for yourself that the first three options do not work.

ANSWER KEY

Ratio, Proportion and Variation LOD 1

1. a
2. a
3. b
4. a
5. d
6. b
7. b
8. c
9. b
10. c
11. b
12. c
13. b
14. c
15. b
16. c
17. a
18. a
19. b
20. a
21. d
22. b
23. d
24. b
25. c
26. a
27. b
28. d
29. a
30. c
31. b
32. d
33. b
34. b
35. b
36. b
37. c
38. a
39. b
40. d
41. d
42. c
43. c
44. a
45. b
46. b
47. d
48. b
49. d
50. c

Ratio, Proportion and Variation LOD II

1. c
2. a
3. a
4. b
5. a
6. c
7. c
8. a
9. b
10. c
11. c
12. a
13. a
14. c
15. a
16. a
17. a
18. b
19. b
20. c
21. a
22. c
23. c
24. a
25. b
26. c
27. a
28. a
29. c
30. d
31. b
32. b
33. d
34. b
35. c
36. c
37. d
38. d
39. d
40. c
41. c
42. b
43. b
44. c
45. c

Ratio, Proportion and Variation LOD III

1. a
2. a
3. a
4. a
5. a
6. a
7. a
8. b
9. a
10. a
11. a
12. a
13. c
14. b
15. a
16. b
17. b
18. d
19. d
20. c
21. b
22. a
23. d
24. b
25. c

9

TIME AND WORK

INTRODUCTION

The concept of time and work is another important topic for the aptitude exams. Questions on this chapter have been appearing regularly over the past decade in all aptitude exams. On an average, you will have 2–3 questions based on this chapter every year in the CAT.

Theory

In the context of the CAT, you have to understand the following basic concepts of this chapter:

If A does a work in a days, then in one day A does $\rightarrow \frac{1}{a}$ of the work.

If B does a work in b days, then in one day B does $\rightarrow \frac{1}{b}$ of the work.

Then, in one day, if A and B work together, then their combined work is $\frac{1}{a} + \frac{1}{b}$

or

$$\frac{a+b}{ab}$$

In the above case, we take the total work to be done as "1 unit of work". Hence, the work will be completed when 1 unit of work is completed.

For example, if A can do a work in 10 days and B can do the same work in 12 days, then the work will be completed in how many days.

One day's work = $1/10 + 1/12 = (12 + 10)/120$
[Taking LCM of the denominator]
$= 22/120$

Then the number of days required to complete the work is $120/22$.

Note that this is a reciprocal of the fraction of work done in one day. This is a benefit associated with solving time and work through fractions. It can be stated as—the number of time periods required to complete the full work will be the reciprocal of the fraction of the work done in one time period.

ALTERNATIVE APPROACH

Instead of taking the value of the total work as 1 unit of work, we can also look at the total work as 100 per cent work. In such a case, the following rule applies.

If A does a work in a days, then in one day A does $\rightarrow \frac{100}{a}$ % of the work.

If B does a work in b days, then in one day B does $\rightarrow \frac{100}{b}$ % of the work.

Then, in one day, if A and B work together, their combined work is

$$\frac{100}{a} + \frac{100}{b}$$

This is often a very useful approach to look at the concept of time and work because thinking in terms of percentages gives a direct and clear picture of the actual quantum of work done.

What I mean to say is that even though we can deal in either a percentage or a fractional value to solve the problem there will be a thought process difference between the two.

Thinking about work done as a percentage value gives us a better picture of the amount of the work that has been done and the quantum of the work that is to be done. On the other hand, if we think of the work done as a fractional value, the thought process will have to be slightly longer to get a full understanding of the work done.

For instance, we can think of work done as 7/9 or 77.77%. The percentage value makes it clear as to how much quantum is left. The percentage value can be visualised on the number line, while the fractional value requires a mental gymnast to fully understand the quantum.

An additional advantage of the percentage method of solving time and work problems would be the elimination of the need to perform cumbersome fraction additions involving LCMs of denominators.

However, you should realise that this would work only if you are able to handle basic percentage calculations involving standard decimal values. If you have really internalised the techniques of percentage calculations given in the chapter of percentages, then you can reap the benefits for this chapter.

The benefit of using this concept will become abundantly clear by solving through percentages the same example that was solved above using fractions.

Example: If A can do a work in 10 days and B can do the same work in 12 days, then the work will be completed in how many days.

One day's work = 10% + 8.33% = 18.33% (Note, no LCMs required here)

Hence, to do 100% work, it will require: 100/18.33

This can be solved by adding 18.33 mentally to get between 5-6 days. Then, one can go through options and mark the closest answer.

The process of solving through percentages will yield such dividends if and only if you have adequate practice on adding standard percentage values. Thus, 18.33 × 5 = 91.66 should not give you any headaches and should be done while reading for the first time.

Thus a thought process chart for this question should look like this.

If A can do a work in 10 days (→ means 10% work) and B can do the same work in 12 days (→ 8.33% work → 18.33% work in a day ⇒ 5 days 91.66% work → Leaves 8.33% work to be done → which can be done in 8.33/18.33 of a day = 5/11 of a day (since both the numerator and the denominator are divisible by 1.66), then the work will be completed in $5\frac{5}{11}$ days.

The entire process can be done mentally.

The Concept of Negative Work

Suppose, that A and B are working to build a wall while C is working to break the wall. In such a case, the wall is being built by A and B while it is being broken by C. Hence, if we consider the work as the building of the wall, we can say that C is doing negative work.

Example: A can build a wall in 10 days and B can build it in 5 days, while C can completely destroy the wall in 20 days. If they start working at the same time, in how many days will the work be completed.

Solution: The net combined work per day here is:

A's work + B's work − C's work = 10% + 20% − 5% = 25% work in one day.

Hence, the work will get completed (100% work) in 4 days.

The concept of negative work commonly appears as a problem based on pipes and cisterns, where there are inlet pipes and outlet pipes/leaks which are working against each other.

If we consider the work to be filling a tank, the inlet pipe does positive work while the outlet pipe/leak does negative work.

Application of Product Constancy Table to Time and Work

The equation that applies to Time and Work problems is:

$$\text{Work Rate} \times \text{Time} = \text{Work done}$$

This equation means that if the work done is constant, then →

Work rate is inversely proportional to time. Hence, the Product Constancy Table will be directly applicable to time and work questions.

(Notice the parallelism between this formula and the formula of time speed and distance, where again there is product constancy between speed and time if the distance is constant.)

Time is usually in days or hours although any standard unit of time can be used. The unit of time that has to be used in a question is usually decided by the determinants of the unit of work rate.

Here, there are two ways of defining the Work rate.

(a) In the context of situations where individual working efficiencies or individual time requirements are given in the problem, the work rate is defined by the unit Work done per unit time.

In this case, the total work to be done is normally considered to be 1 (if we solve through fractions) or 100% (if we solve through percentages).

Thus, in the solved problem above, when we calculated that A and B together do 18.33% work in a day, this was essentially a statement of the rate of work of A and B together.

Thus the solution proceeded as:

18.33% work per day × No. of days required = 100% work

Giving us: no. of days required = $100/18.33 = 5\frac{5}{11}$

(b) In certain types of problems (typically those involving projects that are to be completed), where a certain category of worker has the same rate of working, the Work rate will be defined as the number of workers of a particular category working on the project.

For instance, questions where all men work at a certain rate, the work rate when 2 men are working together will be double the work rate when 1 man is working alone. Similarly, the work rate when 10 men are working together will be 10 times the work rate when 1 man is working alone.

In such cases, the work to be done is taken as the number of man-days required to finish the work.

Note: the future information that the work to be done can also be measured in terms of the volume of work defined in the context of day-to-day life.

For example, the volume of a wall to be built, the number of people to be transferred, the number of chapatis to be made and so on.

WORK EQUIVALENCE METHOD
(To Solve Time and Work Problems)

The work equivalence method is nothing but an application of the formula:

Work rate × Time = Work done (or work to be done)

Thus, if the work to be done is doubled, the product of work rate × time also has to be doubled. Similarly, if the work to be done increases by 20%, the product of work rate × time also has to be increased by 20%, and so on.

This method is best explained by an example.

A contractor estimates that he will finish the road construction project in 100 days by employing 30 men.

However, at the end of the 50th day, when as per his estimation half the work should have been completed, he finds that only 40% of his work is done.

(a) How many more days will be required to complete the work?

(b) How many more men should he employ in order to complete the work in time?

Solution:

(a) The contractor has completed 40% of the work in 50 days.

If the number of men working on the project remains constant, the rate of work also remains constant. Hence, to complete 100% work, he will have to complete the remaining 60% of the work.

For this he would require 75 more days. (This calculation is done using the unitary method.)

(b) In order to complete the work on time, it is obvious that he will have to increase the number of men working on the project.

This can be solved as:

50 men working for 50 days → 50 × 50 = 2500 man-days

2500 man-days has resulted in 40% work completion. Hence, the total work to be done in terms of the number of man-days is got by using unitary method:

Work left = 60% = 2500 × 1.5 = 3750 man-days

This has to be completed in 50 days. Hence, the number of men required per day is 3750/50 = 75 men.

Since, 50 men are already working on the project, the contractor needs to hire 25 more men.

[Note, this can be done using the percentage change graphic. See problem change. Since, the number of days is constant at 50, the 50% increase in work from 40% to 90% is seen to be met by increasing the number of men. Hence, the number of men to be increased is 50% of the original number of men = 25 men.]

The Specific Case of Building a Wall (Work as Volume of Work)

As already mentioned, in certain cases, the unit of work can also be considered to be in terms of the volume of work, for example, building of a wall of a certain length, breadth and height.

In such cases, the following formula applies:

$$\frac{L_1 B_1 H_1}{L_2 B_2 H_2} = \frac{m_1 t_1 d_1}{m_2 t_2 d_2}$$

where L, B and H are respectively the length, breadth and height of the wall to be built; while m, t and d are respectively the number of men, the amount of time per day and the number of days. Further, the suffix 1 is for the first work situation, while the suffix 2 is for the second work situation.

Consider the following problem:

Example: 20 men working 8 hours a day can completely build a wall of length 200 metres, breadth 10 metres and height 20 metres in 10 days. How many days will 25 men working 12 hours a day require to build a wall of length 400 metres, breadth 10 metres and height of 15 metres.

This question can be solved directly by using the formula above:

$$\frac{L_1 B_1 H_1}{L_2 B_2 H_2} = \frac{m_1 t_1 d_1}{m_2 t_2 d_2}$$

Here: L_1 is 200 metres; L_2 is 400 metres
B_1 is 10 metres; B_2 is 10 metres
H_1 is 20 metres; H_2 is 15 metres
m_1 is 20 men; m_2 is 25 men
d_1 is 10 days; d_2 is unknown
t_1 is 8 hours a day; t_2 is 12 hours a day

Then we get: $200 \times 10 \times 20/400 \times 10 \times 15 = 20 \times 8 \times 10/25 \times 12 \times d_2$

$$d_2 = 3.3333\,6666 = 8 \text{ days}$$

Alternatively, you can also directly write the equation as follows:

$d_2 = 10 \times (400/200) \times (10/10) \times (20/15) \times (20/25) \times (8/12)$

This can be done by thinking of the problem as follows:

The number of days have to be found out in the second case. Hence, on the LHS of the equation write down the unknown, and on the RHS of the equation write down the corresponding known:

$$d_2 = 10 \times \ldots$$

Then, the length of the wall has to be factored in. There are only two options for doing so, viz:

Multiplying by $200/400 (< 1)$, which will reduce the number of days or multiplying by $400/200 (> 1)$, which will increase the number of days.

The decision of which one of these is to be done is made on the basis of the fact that when the length of the wall is increasing, the number of days required will also increase.

Hence, we take the value of the fraction greater than 1 to get:

$$d_2 = 10 \times (400/200)$$

We continue in the same way to get:

No change in the breadth of the wall → hence, multiply by $10/10$ (no change in d_2)

Height of the wall is decreasing → hence, multiply by $15/20 (< 1)$ to reduce d_2)

Number of men working is increasing → hence, multiply by $20/25 (< 1)$ to reduce d_2)

Number of hours per day is increasing → hence, multiply by $8/12 (< 1)$ to reduce the number of days.

The Concept of Efficiency

The concept of efficiency is closely related to the concept of work rate.

When we make a statement saying A is twice as efficient as B, we mean to say that A does twice the work as B in the same time. In other words, we can also understand this as A will require half the time required by B to do the same work.

In the context of efficiency, another statement that you might come across is A is two times more efficient than B. This is the same as A is thrice as efficient as B or A does the same work as B in 1/3rd of the time.

Equating Men, Women and Children This is directly derived from the concept of efficiencies.

Example: 6 men can do a work in 12 days while 20 women can do it in 10 days. In how many days can 12 men and 15 women complete the same work.

Solution: Total work to be done = $6 \times 12 = 96$ man-days or total work to be done = $20 \times 10 = 200$ women-days.

Since, the work is the same, we can equate 96 man-days = 200 women-days.

Hence, 1 man-day = 2.08333 women-days.

Hence, if 12 men and 10 women are working on the work, we get

12 men are equal to 12 × 2.08333 = 25 women

Hence, the work done per day is equivalent to 25 + 15 women working per day.

Thus, in 40 women working per day

Hence, 40 × no. of days = 500 women days

Number of days = 5 days.

WORKED-OUT PROBLEMS

Problem 8.1 *A* can do a piece of work in 10 days and *B* in 12 days. Find how much time they will take to complete the work under the following conditions:

(a) Working together

(b) Working alternately starting with *A*.

(c) Working alternately starting with *B*.

(d) If *B* leaves 2 days before the actual completion of the work.

(e) If *B* leaves 2 days before the substantial completion of the work.

(f) If another person *C* who does negative work (i.e. works against *A* and *B* and can completely destroy the work in 20 days) joins them and they work together all the time.

Solution

(a) 1 day's work for *A* is 1/10 and 1 day's work for *B* is 1/12.

Then, working together the work in one day is equal to

$\frac{1}{10}+\frac{1}{12}=\frac{11}{60}$ of the work. Then working together they need 60/11 days to complete the work. → 5.45 days

Alternately you can use percentage values to solve the above question:

A's work = 10%, *B*'s work = 8.33%. Hence, *A* + *B* = 18.33% of the work in one day.

Hence, to complete 100% work, we get the number of days required = 100/18.33 → 5.55 days.

This can be calculated as

@ 18.33% per day in 5 days, they will come 18.33 × 5 = 91.66%. (The decimal value 0.33 is not difficult to handle if you have internalized the fraction to percentage conversion table of the chapter of percentages).

Work left on the sixth day is 8.33%, which will require 8.33/18.33 of the sixth day.

Since, both these numbers are divisible by 1.66, so 5/11 of the sixth day will be used. → 0.45 of the sixth day is used.

Hence, 5.45 days are required to finish the work.

Note: Although the explanation to the question through percentages seems longer, the student should realize that if the values in the fraction-to-percentage table is internalized by the student, the process of solution through percentage will take much lesser time because we are able to eliminate the need for the calculation of LCMs, which are often cumbersome. (If the numbers in the problem are those that are covered in the fraction to percentage conversion table). In fact, the percentage method allows for solving while reading.

(b) Working alternately: When two people are working alternately the question has to be solved by taking 2 days as a unit of time instead of 1 day.

So in (a) above, the work done in 1 day will be covered in 2 days here.

Thus, in 2 days the work done will be 18.33%. In 10 days it will be 91.66%. On the 11th day *A* works by himself.

But *A*'s work in 1 day is 10%. Therefore, he will require 4/5 of the 11th day to finish the work.

(c) Working alternately starting with *B*: Here, there will be no difference in work completed by the 10th day. On the 11th day, *B* works alone and does 8.33% of the work (which was required to complete the work). Hence, the whole of the 11th day will get used.

(d) If *B* leaves 2 days before the actual completion of the work: In this case, the actual completion of the work is after 2 days of *B*'s leaving. This means, that *A* has worked alone for the last 2 days to complete the work. But *A* does 10% work in a day. Hence, *A* and *B* must have done 80% of the work together (@18.33% per day). Then, the answer can be found by

80/18.33 + 20/10 days.

Note: For calculation of 80/18.33, we can use the fact that the decimal value is a convenient one. If they worked together they would complete 73.33% of the work in 4 days and the work that they would have done on the 5th day would be 6.66%.

At the rate of 18.33% work per day while working together, they would work together for 6.66/18.33 of

the 5th day. Since both the numerator and denominator are divisible by 3 so the above ratio is converted into 4/11 = 0.3636.

Hence, they work together for 4.5636 days after which B leaves and then A completes the work in 2 more days. Hence, the time required to finish the work would be = 6.3636 days.

(c) If B leaves 2 days before the scheduled completion of the work. Completion of the work would have been scheduled assuming that A and B both worked together for completing the work (say, this is x days). Then, the problem has to be viewed as x − 2 days was the time for which A and B worked together. The residual amount of work left (which will be got by 2 days work of A and B together) would be done by A alone at his own pace of work.

Thus we can get the solution by

Number of days required to complete the work = (x)(30)/(0.35x − 2) + 30.66/10

(f) If C joins the group and does negative work, we can see that one day's work of the three together would be

A's work + B's work − C's work = 30% + 8.33% − 4% = 33.33%

Hence, the work will be completed in (100)/(33.33) days.

(Note: This can be calculated by 33.33 × 7 = 33 × 7 + 0.33 × 7 = 99.33

Then, work left = 0.66, which will require half a day more at the rate of 33.33% per day.

Advantage of Solving Problems on Time and Work through Percentages : Students should understand here that most of the times the values given for the number of days in which the work is completed by a worker will be convenient values like: 60 days, 40 days, 30 days, 25 days, 24 days, 20 days, 16 days, 15 days, 12 days, 10 days, 8 days, 6 days, 5 days, 4 days, 3 days and 2 days. All these values for the number of days will yield convenient decimal values. If your fraction in percentage value is normalised, you can use the process of solving while reading by taking the percentage of work done per day process rather than getting delayed by the need to find LCM's while solving through the process of the fraction of work done per day.)

Problem 9.2 A contractor undertakes to build a wall in 50 days. He employs 50 people for the same. However, after 25 days he finds that the work is only 40% complete. How many more men need to be employed to

(a) complete the work in time?

Solution In order to complete the work in time, the contractor has to finish the remaining 60% of the work in 25 days.

Now, in the first 25 days the work done = 50 × 25 = 1250 man-days → 40% of the work.

Hence, work left = 60% of the work = 1875 man-days.

Since 25 days are left to complete the task, the number of people required is 1875/25 = 75 men.

Since 50 men are already working, 25 more men are needed to complete the work.

Thought process should go like: 1250 → 40% of work. Hence, 1875 man-days required to complete the work.

Since there are only 25 days left, we need 1875/25 = 75 men to complete the work.

(b) Complete the work 10 days before time?

For this purpose, we have to do 1875 man-days of work in 15 days. Hence, new = 1875/15 = 125 men.

Problem 9.3 For the previous problem, if the contractor continues with the same workforce

(a) how many days behind schedule will the work be finished?

Solution He has completed 40% work in 25 days. Hence, to complete the remaining 60% of the work, he would require 50% more days (i.e. 37.5 days (Since 60% is 1.5 times of 40%).

Hence, the work would be done 12.5 days behind schedule.

(b) how much increase in efficiency is required from the work force to complete the work in time?

Solution If the number of men working is kept constant, the only way to finish the work in time is by increasing the efficiency so that more work is done every man-day.

This should be mathematically looked at as follows:

Suppose that 1 man-day takes care of 1 unit of work. Then, in the first 25 days, work done = 25 (days) × 50 (men) × 1 (work unit per man-day) = 1250 units of work.

Now, the 1250 units of work is just 40% of the work. Hence, work left = 1875 units of work.

Thus, 10 (days) × 30 (men) × x (work units per man-day) = 100% → x = 1.5

Thus the work done per man-day has to rise from 1 to 1.5, that is, by 50%. Hence, the efficiency of work has to rise by 50%.

Problem 9.4 A is twice as efficient as B. If they complete a work in 30 days find the times required by each to complete the work individually.

Solution When we say that A is twice as efficient as B, it means that A takes half the time that B takes to complete the same work.

Thus, if we denote A's 1 day's work as A and B's one day's work as B, we have

$$A = 2B$$

Now, using the information in the problem, we have: 30 A + 30 B = 100% work

That is, 30 B = 100% work → B = 1.11 % (is the work done by B in 1 day) → B requires **90 days** to complete the work alone.

Now, A = 2B → we have A = 2.22 % → A requires 45 days to do the work alone.

You should be able to solve this mentally with the following thought process while reading for the first time:

$\frac{100}{30} = 3.33\%$, $\frac{3.33}{3} = 1.11\%$. Hence, work done is 1.11% per day and 2.22% per day → 90 and 45 days.

Problem 9.5 A is two times more efficient than B. If they complete a work in 30 days, then find the times required by each to complete the work individually.

Solution Interpret the first sentence as A = 3B and solve according to the process of the previous problem to get the answers. (You should get A takes 40 days and B takes 120 days.)

Level of Difficulty (LOD) I

1. Ria can do 25% of a piece of work in 5 days. How many days will he take to complete the work ten times?
   (a) 150 days (b) 250 days
   (c) 200 days (d) 180 days
   (e) 190 days

2. 6 men can do a piece of work in 12 days. How many men are needed to do the work in 18 days.
   (a) 3 men (b) 6 men (c) 4 men (d) 2 men
   (e) 5 men

3. A can do a piece of work in 20 days and it can be done in 15 days. How long will they take if both work together?
   (a) $9\left(\frac{6}{7}\right)$ days (b) $8\left(\frac{4}{7}\right)$ days
   (c) $9\left(\frac{3}{7}\right)$ days (d) $9\left(\frac{4}{7}\right)$ days
   (e) None of these

4. In question 3 if C, who can finish the same work in 21 days, joins them, then how long will they take to complete the work?
   (a) $8\left(\frac{18}{47}\right)$ days (b) 12 days
   (c) $2\left(\frac{9}{11}\right)$ days (d) $47\left(\frac{6}{18}\right)$ days
   (e) None of these

5. Nisha and Archana can do a piece of work in 30 days and Nisha alone can do it in 42 days. In how many days can Archana do it alone?
   (a) 60 days (b) 30 days
   (c) 30 days (d) 45 days
   (e) 52 days

6. Robo alone can do a piece of work in 10 days. Andy alone can do it in 15 days. If the total wages for the work is Rs. 50. How much should Robo be paid if they work together for the same duration of the work?
   (a) Rs. 30 (b) Rs. 20
   (c) Rs. 50 (d) Rs. 40
   (d) None of these

7. 4 men and 1 woman finish a job in 6 days, and 3 men and 7 women can do the same job in 4 days, then how will 1 man and 1 woman take to do the work?
   (a) $23\left(\frac{2}{3}\right)$ days (b) $25\left(\frac{1}{2}\right)$ days
   (c) $3\left(\frac{1}{3}\right)$ days (d) $32\left(\frac{7}{22}\right)$ days
   (d) None of these

8. If 8 boys and 12 women can do a piece of work in 2 days, in how many days can the work be done by 7 boys and 11 women working together?
   (a) 15 days (b) 10 days

(c) 12 days (d) 8 days
(e) Cannot be determined

8. A can do a piece of work in 10 days and B can do the same work in 20 days. With the help of C they finish the work in 5 days. How long will it take for C alone to finish the work?
(a) 20 days (b) 10 days
(c) 10 days (d) 15 days
(e) 12 days

10. A can do a piece of work in 20 days. He works alone for 5 days and then B finishes it in 10 more days. In how many days will A and B together finish the work?
(a) 8 days (b) 10 days
(c) 12 days (d) 6 days
(e) 18 days

11. A and B undertake to do a piece of work for Rs. 100. A can do it in 5 days and B can do it in 10 days. With the help of C they finish it in 2 days. How much should C be paid for his contribution?
(a) Rs. 40 (b) Rs. 20 (c) Rs. 60 (d) Rs. 30
(e) Rs. 50

12. Twenty workers can finish a piece of work in 30 days. After how many days should 5 workers leave the job so that the work is completed in 35 days?
(a) 5 days (b) 10 days
(c) 15 days (d) 20 days
(e) 25 days

13. Arun and Vinay together can do a piece of work in 7 days. If Arun does twice as much work as Vinay in a given time, how long will Arun alone take to do the work
(a) 6.33 days (b) 10.5 days
(c) 14 days (d) 22 days
(e) 30 days

14. Subhash can copy 50 pages in 10 hours, Subhash and Prakash together can copy 300 pages in 40 hours. In how much time can Prakash copy 30 pages?
(a) 13 h (b) 12 h (c) 11 h (d) 9 h
(e) 14 h

15. A number of men can finish a piece of work in 30 days. If there were 6 men more, the work could be finished in 10 days less. What is the original number of men?
(a) 10 (b) 14 (c) 12 (d) 13
(e) 15

16. Sashi can do a piece of work in 24 days and Rishi can do it in 20 days. They work for 5 days and then Sashi goes away. In how many more days will Rishi finish the work?
(a) 10 days (b) 12 days
(c) 14 days (d) 15 days
(e) None of these

17. Raja can do a piece of work in 10 days, Vicky in 12 days and Tinka in 15 days. They all start the work together, but Raja leaves after 2 days and Vicky leaves 3 days before the work is completed. In how many days is the work completed?
(a) 5 days (b) 6 days
(c) 7 days (d) 8 days
(e) 9 days

18. Sunder can do 1/2 of the work in 8 days while Kalu can do 1/3 of the work in 6 days. How long will it take for both of them to finish the work?
(a) $\frac{88}{17}$ days (b) $\frac{144}{17}$ days
(c) $\frac{72}{17}$ days (d) 6 days
(e) None of these

19. Manoj takes twice as much time as Anjay and thrice as much as Vijay to finish a piece of work. Together they finish the work in 1 day. What is the time taken by Manoj to finish the work?
(a) 6 days (b) 3 days
(c) 2 days (d) 4 days
(e) None of these

20. An engineer undertakes a project to build a road 15 km long in 300 days and employs 45 men for the purpose. After 100 days, he finds only 2.5 km of the road has been completed. Find the (approx.) number of extra men he must employ to finish the work in time.
(a) 43 (b) 45 (c) 55 (d) 60
(e) 60

21. Aparna can do a piece of work in 12 days. Aparna and Amit complete the work together and were paid Rs. 54 and Rs. 81 respectively. How many days must they have taken to complete the work together?
(a) 4 days (b) 4.5 days
(c) 4.8 days (d) 5 days
(e) 4.2 days

22. Raju is twice as good as Vijay. Together, they finish the work in 14 days. In how many days can Vijay alone do the same work?

(a) 75 days (b) 15 days
(c) 21 days (d) 72 days
(e) 30 days

11. In a company [illegible] is a certain number of employees can do a work in 40 days. If 40 more men, 5 more employees, it could be finished in 10 days less. How many employees were there in the beginning?
(a) 20 (b) 30 (c) 25 (d) 15
(e) 40

12. 36 men and 36 women can complete a piece of work in 5 days and 15 men and 24 women can do it in 4 days. [illegible] men and women do the work when they work together?
(a) 1 : 2 (b) 1 : 1
(c) 2 : 1 (d) 3 : 1
(e) 3 : 2

13. A can do a work in 10 days and B can do the same work in 12 days. They work together for 5 days and then A goes away. In how many more days will B finish the work?
(a) 5 days (b) 4 days
(c) 10 days (d) $4\frac{1}{2}$ days
(e) None of these

14. If one working [illegible] can do a work in 16 days, [illegible] many days will 20 men [illegible] the work?
(a) $22\frac{1}{2}$ days (b) 25 days
(c) 15 days (d) None of these

15. Ajay and Vijay undertake to do a piece of work for Rs. 300. Ajay alone can do it in 24 days while Vijay alone can do it in 30 days. With the help of Prabhat they finish the work in 12 days. How much should Prabhat get for his work?
(a) Rs. 20 (b) Rs. 100
(c) Rs. 140 (d) Rs. 80

16. If men could do a piece of work in 10 days. But at the end of 100 days, 10 additional men are engaged, in how many days it will be ready for completion?
(a) 90 days (b) 80 days
(c) 85 days (d) 70 days

17. Ajay, Vijay and Sanjay are working on a piece of work for Rs. 720. Ajay and Vijay together do 1/3 of the work and Vijay and Sanjay do 1/4 of the work. How much should each get?
(a) Rs. 240 (b) Rs. 200
(c) Rs. 245 (d) Rs. 340
(e) Rs. 440

18. Ajay is twice as good a workman as Vijay and together they finish a piece of work in 14 days. In how many days Ajay alone can finish the work?
(a) $21\left(\frac{1}{3}\right)$ days (b) $11\left(\frac{1}{3}\right)$ days
(c) $\frac{14}{3}\left(\frac{1}{3}\right)$ days (d) 28 days
(e) 14 days

19. A fort had enough food for 200 soldiers for 30 days. After 27 days 125 soldiers left the fort. In how many days the food will last for the remaining soldiers?
(a) 12 days (b) 30 days
(c) 9 days (d) 5 days

20. Ajay, Manoj and Sanjay together can complete a piece of work in 20 days. If Ajay can do it alone in 60 days and Manoj in 30 days, in how many days Sanjay alone can do the work?
(a) 40 days (b) 30 days
(c) 20 days (d) 12 days

21. Ajay and Vijay can do a piece of work in 20 days. With the help of Manoj they can finish it in 12 days. How long will Manoj take to finish the work alone?
(a) 84 days (b) 60 days
(c) 75 days (d) 70 days

22. Ashok and Mohan can do a piece of work in 15 days. Mohan and Sohan together do it in 24 days. If Ashok is twice as good a workman as Sohan, in how many days Sohan can do the work?
(a) 15 days (b) 20 days
(c) 30 days (d) 35 days

23. Ajay and Vijay together can do a piece of work in 8 days. Ajay alone does it in 20 days. After how many days Vijay leaves it to Ajay?
(a) 20 days (b) 18 days
(c) 23 days (d) 30 days

24. A cistern is normally filled in 8 hours. However it takes 2 hours extra for a leak to fill the cistern. If the cistern is full, in what time the leak can empty it?
(a) 8 h (b) 5 h (c) 24 h (d) 17 h

25. Pipe A can fill a tank in 20 minutes and pipe B in 30 minutes. If A fills 2 minutes before then B, in how many minutes the tank will be filled?

[illegible] then the time in which A and B will do the work separately will be respectively?

(a) 15 days, 20 days (b) 25 days, 10 days
(c) 10 days, 15 days (d) 22 days, 20 days

40. A can do a work in 18 days. B is 8 days and C is 6 days. A and B start working together and after 3 days C joins them. In how many days will they complete it?

(a) 8 1/2 days (b) 8 days
(c) 8.00 days (d) 9 days

41. A man working 8 hrs a day can finish a work in 11 days. Working 6 hrs a day 10 days, the number of man required to finish the work in 6 days is

(a) 10 (b) 11 (c) 14 (d) 16

42. A certain job was assigned to a group of men to do it in 20 days. But 2 men were not able to do the job and the remaining men took the job in 22 days. The original number of men in group was

(a) 22 (b) 24 (c) 26 (d) 40

43. [illegible] a work in 14 days. [illegible] working. [illegible] work. How many days will it take to complete the remaining work?

(a) 10 days (b) 12 days
(c) 15 days (d) 9 days

44. A takes 5 days more than B to do a certain job and B takes 4 days more than C. A and B together can do the job in the same time as C. How many days A would take to do it?

(a) 10 days (b) 20 days
(c) 15 days (d) 20 days

45. A clerk is usually filled in a letter when he keeps 40 letters of a line in a format. If he decides to fill the line with 50 letters, it is found that 5 less lines are required. How many letters in a line?

(a) 12 (b) 10
(c) 20 (d) None of these

46. If there are two cisterns, a tank is filled in 10 hrs. Only if the tap is 7 hrs and another is 10 hrs. At what time does the tank take to fill?

(a) Water pipe emptying the tank in 30 h
(b) Water pipe emptying the tank in 50 h
(c) Water pipe emptying the tank in 5 h
(d) Pipe the tank in 10 h

47. There are two pipes to a tank. Pipe A is for filling the tank and the B is for emptying the tank. If a tank is filled in 30 hours and B can empty the tank in 15 hours, then how many hours will it take to completely fill a half empty tank?

(a) 30 hours (b) 15 hours
(c) 20 hours (d) 37.50 hours

48. Sitha can do a certain work in 10 days. Rati can do it in 20 days and Chetan can do it in 40 days. They start working in turns with Sitha starting the work on the first day followed by Rati on the second day and by Chetan on the third day and again by Sitha on the fourth day and so on till the work is completed. Find the time taken to complete the work fully?

(a) 10 days (b) 12 days
(c) 17 days (d) 16.5 days

49. A, B and C can do a work in 24 days. A and B together do it in 16 days and A and C together do it in 18 days. Find the time taken by C to do the work alone.

(a) 72 days (b) 96 days
(c) 144 days (d) 120 days

50. There are Pipe A, B and C in a tank. They can fill the tank in 6 hrs, 12 hrs and 24 hrs respectively. At first, all of them are opened simultaneously. Then after 2 hours, pipe C is closed and A and B are working. After the 4th hour, tap B is also closed. The remaining work is done by tap A alone. What the percentage of the work is done by Tap A by itself?

(a) 21% (b) 22%
(c) 78% (d) None of these

51. Two taps are running simultaneously to fill a tank. The first tap could have filled it in 8 hours by itself and the second one by itself it will have filled it in 20 hours. However, unknown to the tank, there was a hole in the tank from the beginning which caused a delay of one hour in the filling of the tank. Find the time in which the tank would empty a filled tank.

(a) 13 hours (b) 20 hours
(c) 25 hours (d) 40 hours

52. A can do a work in 24 days. B can do it in 12 days and C can do it in 48 days. They start working together. A leaves after 8 days and B left after working for 8 days. Many more days A can expect to complete the work alone?

(a) 20 (b) 10 (c) 24 (d) 30

Level of Difficulty (LOD)

1. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]
   (e) None of these

2. According to a plan, a drilling team has to drill in a depth of 270 metres in a certain period. For the first four days the team drilled as per the plan. However, consequently, facing [illegible] work picking underwhelmed as [illegible] to the city is started to drill 5 metres more than the plan every day. Therefore, a day before the planned date they have drilled to a depth of 280 metres. Find more number of drilling and the plan for each day.

   (a) 28 metres (b) 10 metres
   (c) 27 metres (d) 30 metres
   (e) None of these

3. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]
   (e) None of these

4. Two and Tabbo can do a piece of work in 40 and 48 days respectively. They began the work together, but Two leaves after some days and Tabbo finished the remaining work in 23 days. After how many days Two left the work?

   (a) 7 days (b) 8 days
   (c) 9 days (d) 10 days
   (e) 12 days

5. A number [illegible] is [illegible] by 2. Below it is [illegible] 10 and then [illegible] by remaining 11. The first term of it is?

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]
   (e) [illegible]

(Questions for Questions 6-10): Read the following and answer the questions that follow.

A set of 10 pipes [illegible] 30 [illegible] of a [illegible] operates. Another set of 5 pipes [illegible] 30 [illegible] a 3 minutes. A third set of 8 pipes [illegible] 3 [illegible] of the tank is 15 minutes.

6. How many minutes will it take to fill the tank if all the 23 pipes are opened at the same time?

   (a) 4 minutes (b) $7\frac{5}{7}$ minutes
   (c) 8 minutes (d) $11\frac{2}{7}$ minutes
   (e) None of these

7. If only half the pipes of set 2 are closed and the rest of the pipes of set 2 are opened and all other pipes are open, then how will it take to fill the tank?

   (a) 26 minutes (b) 15 minutes
   (c) 7 minutes (d) 8 minutes
   (e) None of these

8. If a pipe is chosen in set 2, and all other pipes are open, then how will it take to fill the tank?

   (a) 3 minutes (b) 6 minutes
   (c) 7 minutes (d) 18 minutes
   (e) 8 minutes

9. If the tank is half full and pipe 8 and set 3 are closed, then [illegible] pipe 2 is [illegible] the tank is [illegible] time of set 2 are closed.

   (a) 12 minutes (b) 30 minutes
   (c) 40 minutes (d) 16 minutes
   (e) 22 minutes

10. If the pipe 8 is added to set 1 and set 2 and set 3 [illegible] by 20%, [illegible] all the pipes are opened at 2:30 p.m., then at what time the tank get filled? (It is initially empty)

    (a) 1:45 p.m. (b) 2:40 p.m.
    (c) 3:30 p.m. (d) 2:30 p.m.
    (e) 3:00 p.m.

11. A [illegible] 30 [illegible] 1. Given as [illegible] 6 days and Bodo [illegible] 5 days. A piece of work takes 15 days if all [illegible]

[illegible] how long would they have to do the job, they said by himself?

(a) [illegible] days (b) [illegible] days
(c) [illegible] days (d) [illegible] days
(e) [illegible] days

10. Two pipes [illegible] 30 in. [illegible] 14 [illegible] in open, [illegible]. The pipes are [illegible] simultaneously and it is found that due to leakage at the bottom of the cistern, it takes 17 minutes more for the cistern to be filled up. When the cistern is full, in what time [illegible] the tank empty?

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
(e) [illegible]

11. A tank holds 150 gallons of water. It [illegible] 1 inches in diameter and 150 in the [illegible] of 7 gallons/min. The [illegible] of the tank is twice the diameter of the other. How many minutes will it take to empty the tank if the [illegible] of water? When the tank is full and the [illegible] is closed? When [illegible] is closed? [illegible] (Assume)

(a) 7.14 min (b) [illegible]
(c) 27 min (d) [illegible]
(e) 15.5 min

12. A pipe of capacity 20 litres [illegible] and a outlet [illegible]. Both are opened [illegible] the tank is filled in 2 minutes. But [illegible] the outlet [illegible] and taps opened the tank never gets filled up. Period of the [illegible] can be [illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
(e) [illegible]

13. [illegible] A [illegible] 60% [illegible] C [illegible] B [illegible] C [illegible] B and C [illegible] 8 days [illegible] 12 days [illegible]

(a) 8 days (b) [illegible] days
(c) [illegible] days (d) [illegible] days
(e) None of these

14. A [illegible] 25% of a certain job in [illegible] days. A [illegible] 15% of the [illegible] job in 8 days and C [illegible] 60% of the work in 12 days. All of them [illegible] for 4 days [illegible] A and C [illegible]. How long will B take for B to complete the remaining work [illegible]?

(a) [illegible] days (b) [illegible] days
(c) [illegible] days (d) [illegible] days
(e) None of these

15. [illegible] suggest that a third of [illegible] B [illegible] simultaneously. During [illegible] the third digger digs as many [illegible] as the second digger more than the first. The third digger's work is 33 days [illegible] the first digger's work is 12 days. How many minutes does the third digger dig per shift?

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
(e) [illegible]

16. A, B and C [illegible] together complete a job in 15 days. However, C [illegible] for the first three days. [illegible] 4 to 7 days [illegible] 8 to 9 days. How many days would it require for the [illegible] to complete the entire work?

(a) [illegible] days (b) [illegible] days
(c) [illegible] days (d) [illegible] days
(e) [illegible] days

17. A and B [illegible] a work together in 5 days. But A worked in every [illegible] and B in half the speed. It could have taken thirty four days to complete the job. How much time would it take for A alone to do the work?

(a) [illegible] days (b) [illegible] days
(c) [illegible] days (d) [illegible] days
(e) [illegible] days

18. Two [illegible] of varying [illegible] which [illegible] when they work together. If for every typed [illegible] the second typist types [illegible] the first typist, they would be able to [illegible] of the work. How many minutes would it take the second typist to complete the typing job working alone?

(a) [illegible] minutes (b) [illegible] minutes
(c) [illegible] minutes (d) [illegible] minutes
(e) [illegible] minutes

19. There were [illegible] 80 [illegible]. They are known to [illegible] 20 [illegible] working together. The [illegible] began working alone and made [illegible] [illegible] the [illegible] more than [illegible]. The remaining [illegible] of the work was done by the [illegible] working together. If each [illegible] of 40 minutes to complete the 80 [illegible]. How many minutes would it take the first [illegible] the [illegible] for a [illegible] the [illegible]?

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) [illegible]

[illegible] If alone (A) does the work in [illegible] and [illegible] working together in (B) days, then (B) would do the same work (A) days more than [illegible]. How many days are the ratio of [illegible] and [illegible] of a [illegible]?

(a) 3 days (b) [illegible]
(c) 5 days (d) [illegible]
(e) 7 days

21. [illegible] of A, B and C [illegible] a certain [illegible] [illegible] [illegible]

Level of Difficulty (LOD)

Direction for Questions 1-10: Study the following table and answer the questions that follow.

Table: The Company has to go through the following steps for the benefit of a new hire

	Expert with experience	Non-expert without experience
1. Design and development	30	40
2. Business analysis	15	20
3. Market survey	6	10
4. Prototyping survey	14	16
5. Marketing and launch	15	20

The profile of the employee's company is

Name	Expert at	Non-Expert at	Related to work at
A	Design and development	All others	Market survey
B	Business analysis	All others	Market survey
C	Market survey and Marketing and launch	All others	Design and development
D	Prototyping	All others	Market survey
E	Market survey	All others	Marketing

1. Given this situation, the minimum number of days in which the company hire launch a new new joining through all the stages is

(a) 48 days (b) 50.5 days
(c) 55 days (d) 60 days
(e) None of these

2. If a staff member who is hired anything is an issue, he contributing left as. The number of days in which the project will get launched will be

(a) 5 days (b) 6 days
(c) 7 days (d) 8 days
(e) None of these

3. If ratio of the three modes of expertise hiring is 8:10:15, the maximum manpower will be recruited by

(a) A (b) C (c) D (d) E
(e) B

4. For question 3, the second highest amount will be to different by

(a) C (b) A (c) D (d) B
(e) E

5. If 5 weeks is the period of the difficulty during new hiring and launch, who will be further staff member the first of them?

(a) A (b) C (c) D (d) E
(e) B

6. If the company decides that the hiring costs will be related proportionately and the experts will be also hired in their respective staffs wise only, and a staff will be hired by a non-expert only if the work is the area of expertise is indifferent, then the overall cost with first be provided to the work will be increase that outstanding and launch cost only by what amount the first three are only considered?

(a) A (b) B (c) C (d) D
(e) E

7. With the previous scenario, the minimum number of days in which the whole project will get successfully completed is affected only some of the cases and is finally is obtained as a result. For the cases in which figures are

(a) 22.5 days (b) 12 days
(c) 24.75 days (d) 25.25 days
(e) None of these

8. For the scenario in question 6, the staffing costing will be the

(a) A (b) Both A and C
(c) C (d) B
(e) Cannot be determined

9. If each staff has an equal proportion of the costs, the total costing for the above situation will be the

(a) D (b) C (c) E (d) B
(e) Cannot be determined

10. The ratio of the costing for the highest staffing points, if the first two positions is in the increasing will be

(a) 49:24:25 (b) 10:15:16
(c) 15:49:9 (d) B
(e) Cannot be determined

Direction for Questions 11-20: Read the following and answer the questions that follow.

A bar contains 3 glasses. But the 1st glass is given. The 2nd is made a single glass in twenty times. Find the number of all the equal sides.

The glass in the 3rd has three times of volume.

Volume = 228 cm³

Height = 1.5 cm

Diameter = 2.25 cm

100 Sepoys can hold 9% of the enemy for one month.
100 Marines can hold 10% of the enemy for 15 days.
50 Rustics can hold 5% of the enemy for one month.

A sepoy eats 1 kg of food per month, a Marine eats 0.5 kg of food per month and a Rustic eats 1 kg of food. (1 tonne = 1000 kg)

The king has to make some decisions based on the least or possible resistance that can be offered to the enemy.

If a king selects a soldier, he will have to feed him for the entire period of the resistance. The king is not obliged to feed a soldier not selected for the resistance.

(Assume that the entire food allocated to a particular soldier for the estimated length of the resistance is ready-tabulated into the king's palace in case a soldier dies and is not available for the other soldiers.)

11. If the king wants to maximize the time for which his resistance holds up, he should
(a) Select all marines (b) Select all Rustics
(c) Select all sepoys (d) None of these

12. Based on existing resources, the maximum number of months for which the Sun's resistance can last is
(a) 5 months (b) 20 months
(c) 7.5 months (d) Cannot be determined

13. If the king makes a decision error, the maximum reduction in the time of resistance could be
(a) 15 months (b) 12.5 months
(c) 14.66 months (d) Cannot be determined

14. If the king estimates that the attackers can last for only 50 months, what should the king do to ensure victory?
(a) Select all marines
(b) Select the marines and the sepoys
(c) Select the Rustics
(d) The king cannot achieve this

15. If a reduction in the ration allocation by 10% reduces the capacity of any soldier to hold off the enemy by 10%, the number of whole months by which the king can increase the life of the resistance by reducing the ration allocation by 10% is
(a) 4 months (b) 2 months
(c) No change (d) This will reduce the time

16. The minimum amount of grain that should be available in the granary to ensure that the fort is not lost (assuming the estimate of the king of 50 months being the duration for which the enemy can last is correct) is
(a) 2000 tons (b) 2500 tons
(c) 5000 tons (d) Cannot be determined

17. If the king made the worst possible selection of his soldiers to offer the resistance, the percentage increase in the minimum amount of grain that should be available in the granary to ensure that the fort is not lost is
(a) 100% (b) 500%
(c) 800% (d) Cannot be determined

18. The difference in the maximum grain required for the second worst choice and the worst choice to ensure that the resistance lasts for 50 months is
(a) 5000 tons (b) 7500 tons
(c) 10000 tons (d) Cannot be determined

19. If the king strategically sends to the feeder fort on the first day of the resistance so that the grain is no longer a constraint, the maximum time for which the resistance can last is
(a) 100 months (b) 150 months
(c) 250 months (d) Cannot be determined

20. If the feeder fort is opened after 6 months and given that the king had made decisions based on food availability being a constraint, then the number of months maximum for which the resistance could last is
(a) 300 months (b) 150 months
(c) 5 months (d) Cannot be determined

Directions for Questions 21–25: Study the following and answer the questions that follow.

A gas cylinder can discharge gas at the rate of 1 cubic metre from burner *A* and at the rate of 2 cubic metres from burner *B* (maximum rates of discharge). The capacity of the gas cylinder is 1000 cc of gas.

The amount of heat generated is equal to 1 kcal per cc of gas.

However, there is wastage of the heat as per table below.

Gas discharge	Loss of heat
0–0.5 cubic metre	10%
0.5–1 cubic metre	20%
1–1.5 cubic metre	25%
1.5 + cubic metre	50%

(Include higher estimates)

21. If both burners are opened simultaneously and the first is opened to 90% of its capacity and the second is opened to 80% of its capacity, the amount

case in which the gas cylinder will be empty (if it was full full at the start) will be:

(a) 240 minutes (b) 400 minutes
(c) 300 minutes (d) None of these

22. The maximum amount of heat with the fastest speed of cooking that can be utilised for cooking will be when

(a) The first burner is opened upto 50% of it's aperture
(b) The second burner is opened upto 25% of it's aperture
(c) Either (a) or (b)
(d) None of these

23. The amount of heat utilised for cooking if a full gas cylinder is burnt by opening the aperture of burner A 100% and that of burner B 50% is

(a) 900 kcal (b) 800 kcal
(c) 750 kcal (d) Cannot be determined

24. For question 23, if burner A had been opened only 25% and burner B had been opened 50%, the amount of heat available for cooking would be

(a) 850 kcal (b) 800 kcal
(c) 750 kcal (d) Cannot be determined

25. For question 24, the amount of time required to finish a full gas cylinder will be

(a) 900 minutes (b) 833.33 minutes
(c) 800 minutes (d) None of these

Hints and Solutions

1. They will at planned rate for 3 days (say x)

$$3x + 5(x + 8) = 280$$

2. Work left = 1/5th of original work. A would have done it in $x/5$ hours.

6–10. First set of two pipes operate at 10%/min (filling)
i.e. Filling done by 1 pipe = 5%/minute
Second set of three pipes operate at 12.5%/minute (filling)
i.e. Filling done by 1 pipe = 2.5%/minute
Set 2 (Emptying) = 9%/minute
Emptying per pipe = 0.625%/minute.

11. $5d = 120 = 150$

13. Since rate of filling or emptying is constant, the emptying rate is 10 gallons/minute.

14. The net flow in the original situation is 3 litres/minutes. The required answer will be such that when it is doubled, the increase in emptying rate (in litres/minute) should be more than 5 litres/minute.

15. Required answer = $\dfrac{\text{work left after 3 days}}{\text{1st's work rate}}$

17. Dish day per day = $\dfrac{324}{6} = 54$ metres

$$x + (x + x) + (x + 2x) = 54$$
$$10x + 2x = 54$$

Use options to solve.

18. Work done on the first three days is 37%. Hence, work done on the next 7 days is 63%.
Since, rate is $A + B$'s work we get
One day's work of $(A + B) = 9\%$
Also, $5A = 4B$.
Hence $A = 4\%$ and $B = 5\%$
B turns out to be the faster worker.

19. Solve through options.

20. Work done by the first typist in 2 minutes = 20%.
Hence, the first typist's work rate = 10% per minute.

22. Solve through options.

23. Solve through options.

24. Let x hours be required to complete the work together.

Then, $\dfrac{1}{(x + 4.5)} + \dfrac{1}{(x + 8)} = \dfrac{1}{x}$. Check the options to see which one fits the equation.

27. $A + B = \dfrac{50\%}{12} = 4.16\%$

(where A and B represent the work per hour of pipes A and B respectively).

Solve using options to see which one fits the remaining conditions of the problem.

For example, if we check option b (4 hrs), then we get that the work of the faster pipe (say A) = 25%. Then B = 16.66%.

Then B was open for 4/2 = 2 hours and A was open for 4/4 = 2 hours.

Work done = 25% × 2 + 16.66% × 2 = 83.33% = $\dfrac{5}{6}$ of work.

Hence, this option is correct.

28. Let the discharges per pipe be A and B litres/hr.

Then $17(A + B) = 425$
i.e. $A + B = 25$

From this point of the solution proceed to check the conditions of the question through options.

29. Difference in times required by the first man (A) and the second man (B) = 3 hours. Also, if t_1 and t_2 are the respective times, then

$$2t_2 - t_1 = 8 \quad \text{and} \quad t_2 - t_1 = 3$$

30. Let the work rate of the four pipes be A, B, C and D respectively.

Then, $D - A = 1.66\%$ and $D + A = 7\%$

Hints and Solutions

1–10. Interpretation of the first row of the first table to the question:

Design and Development requires 30 expert man-days or 90 non-expert man-days.

Hence, work done in 1 expert man-day = 3.33% and work done in 1 non-expert man-day = 1.66%. Further, from the second table, it can be interpreted that: A is an expert at design and development. Hence, his work rate is 3.33% per day and B, D and E are ready to work as non-experts on design and development. hence their work rate is 1.66% per day each. Thus, in 1 day the total work will be

$A + B + D + E = 3.33 + 1.66 + 1.66 + 1.66 = 8.33\%$ work.

Thus, 12 days will be required to finish the design and development phase.

1. $\frac{100}{8.33} + \frac{100}{20.66} + \frac{100}{6.66} + \frac{100}{16.66} + \frac{100}{26.66} = 46.5$

2. Increase of number of days $\rightarrow \frac{100}{10} - \frac{100}{16.66} = 4$ days.

[This happens since the work rate will drop from 16.66% to 10% due to A and C's refusal to work.]

3–4. Find out the work done by each of the 5 workers.

15–20. The resistance offered is equal for 100 members of all types of workers.

15–18. If all sepoys are chosen, the food requirement will be 200 seers/month. The resistance will last for 5 months.

If foreiers are chosen, the food will last for $\frac{2000}{100 \times 3}$ = 7.33 months.

If mantis are chosen, the food will last for $\frac{2000}{100 \times 0.5}$ = 20 months.

Hence, all mantis must be chosen.

19–20. For these questions, since food is no longer a constraint, the constraint then becomes the number of seers. Then, the assumption will be that the seers eaten loses for one month with a loss of either 300 seers, 2000 seers or 1000 footies.

19. Length of resistance = $\frac{200000}{2000} + \frac{100000}{2000} + \frac{10000}{100}$

= 250 months.

20. In 6 months, the resistance will have lost 1700 seers. He would also have lost all other soldiers since he has not fed them.

21. $\frac{500}{0.9 + 1.6} = 200$

22. At 1 co/minute, the loss of heat is 20%. Hence, when 1000 cc of the gas is used, out of the 1000 kcal of heat generated 200 kcal will be lost.

Detailed Solutions & Short Cuts

LOD I

1. He will complete the work in 20 days. Hence he will complete ten times the work in 200 days.
3. A's one day work will be 5%, while B will drink 6 of the work in one day. Hence, their total work will be 11.66% in a day.

   In 8 days they will complete → 11.66 × 8 = 93.3%. This will leave 6.66% of the work. This will now spend 4/3 of the ninth day since in 6.66% of both the numerator and the denominator are divisible by 1.66.
4. A's work = 5% per day

   B's work = 6.66% per day

   C's work = 4% per day.

   Total no. of days = 100/15/66 = 300/47 = 6.38[illegible]
5. $B + A = 10\%$

   $B = 8.33\%$

   Hence $A = 1.66\% \rightarrow 60$ days.
6. The ratio of the wages will be the inverse of the ratio of the number of days required by each to do the work. Hence, the correct answer will be 3:2 → Ans. B
7. 24 man days = 18 women days = 20 man days + 5 women days

→ 4 man days = 10 woman days
→ 1 man day = 2.5 woman days
Total work = 24 man days + 18 woman days = 60 woman days + 18 woman days = 78 woman days.
Hence, 1 man + 1 woman = 3.5 women can do it in 78/3.5 = 156/7 = 22 2/7 days.

8. The data is insufficient, since we only know that the work gets completed in 300 boy days and 300 women days.

9. $A = 10\%$, $B = 5\%$ and Combined work is 20%. Hence, C's work is 5% and will require 20 days.

10. In 5 days, A would do 25% of the work. Hence, B finishes the remaining 75% work in 10 days, we can conclude that B's work is a day = 7.5%
Thus, $(A + B) = 17.5\%$ per day
Together they would take 100/17.5 = 8 days.

11. $A = 20\%$, $B = 10\%$ and $A + B + C = 50\%$. Hence, $C = 20\%$. Thus, in two days, C contributes 40% of the total work and should be paid 40% of the total amount.

12. Total mandays required = 600 man days. If 5 workers leave the job after 'x' days, the total work would be done in 25 days. We have to find the value of 'x' as easily.
$$30 \times x + (25 - x) \times 25 = 600$$
Solving for x, we get
$$30x - 25x + 25 \times 25 = 600$$
$$5x = 25$$
$$x = 15$$

14. Subhash can copy 200 pages in 40 hours (practice in the free sessions). Hence, Prakash can copy 100 pages in 40 hours. Thus, he can copy 30 pages in 30% of the time, i.e. 12 hours.

16. Sonu = 4%, Ritu = 5%. In five days, they do a total of 45% work. Ritu will finish the remaining 55% work in 11 more days.

17. Raju = 10%, Vicky = 8.33% and Tinku = 6.66%. Hence, total work for a day if all three work = 25%. In 2 days they will complete 50% work. On the third day onwards Raju doesn't work. The rate of work will become 15%. Also, since Vicky leaves 3 days before the actual completion of the work, Tinku works alone for the last 3 days (and must have done the last 40% = 3 × 20% work alone). This would mean that Vicky leaves after 80% work is done. Thus, Vicky and Tinku must be doing 20% work together over two days.
Hence, total time required = 2 days (all three) + 2 days (Vicky and Tinku) + 3 days (Tinku alone)

18. Kanhiya requires 16 days to do the work while Kalu requires 18 days to do the work.
$$(1/16 + 1/18) \times n = 1$$
$$\rightarrow n = 288/34 = 144/17$$

20. After 100 days and 6900 mandays, only $1/6^{th}$ of the work has been completed. You can use the product change algorithm of PCG to solve this question.
$1/6 = 45 \times 16.66\%$ of the work. After this you have 200 days (i.e. 100% increase in the time available) while the product $200 \times$ no. of men should come equal to five times times the original product.
$$100 \xrightarrow[\text{100\% increase}]{\text{TIME}} 200 \xrightarrow[\text{x\% required}]{\text{?}} 500$$
This will be got by increasing the no. of men by 150% (300/200).

22. Raju being twice as good a workman as Vijay, you can write the following equation to get the required answer:
$1/8 + 1/22 = 1/14$
Solving will give you that Vijay takes 42 days.

24. 12 × 5 man days = 10 × 3 Boy days
$= 1.1 \times 4$ man days = 24 × 4 Boy days
→ 4 man days = 16 Boy days
1 man day = 2 Boy days.
Required ratio of man's work to boy's work = 2 : 1.

26. Work equivalence method:
$30 \times 5 \times 10 = 20 \times 6 \times x$
Gives the value of x as 20 days

27. Ajay's daily work = 4.166%, Vijay's daily work = 3.33% and the daily work of all the three together is 8.33%. Hence, Pradeep's daily work will be 0.833%. Hence, he will end up doing 10% of the total work in 12 days. This will mean that he will be paid Rs. 20.

29. $A + V = 5 + 1$ (1)
$A + V = 36/24$
$V + C = 8/23$
$\rightarrow A + 2V + C = 25/23$ (2)
(2)−(1) gives us: $V = 4/23$.

30. Interpret the moving statement as: Arunit takes 30 days and Vinay takes 90 days. Hence, the answer will be got by:
$(1/30 + 1/90) * x = 1$

Alternatively, you can also solve using percentages as: 3.33 + 1.11 = 4.44% is the daily work. Hence, the no. of days required is 100/4.44 = 22.5 days.

32. Total work of Anju, Manju and Sanju = 16.66%

Anju's work = 10%

Manju's work = 4.166%

Sanju's work = 2.5%

So Sanju can reap the field in 40 days.

33. Ajay + Vijay = 1/28 And Ajay + Vijay + Mohan = 1/21.

Hence, Mohan = 1/21 − 1/28 = 1/84.

Hence, Mohan will take 84 days to do the work.

36. The rate of filling will be 20% and the net rate of filling (including the leak) is 16.66%. Hence, the leak accounts for 3.33% per hour. Ie. it will take 30 hours to empty the tank.

38. $A + B = 5.55 + 11.11 = 16.66$. In two days, 33.33% of the work will be done. C adds 16.66% of work to that of A and B. Hence, the rate of working will go to 33.33%. At this rate it would take 2 more days to complete the work.

Hence, in total it will take 4 days to complete the entire work.

40. $x \times 20 = (x - 12) \times 32 \rightarrow x = 32$

42. $(A + B)$'s work = C's work.

Also if A takes 'a' days

B would take '$a - 5$' days

and C would take '$a - 9$' days.

Solving through options, option 'c' fits.

A (15 days) → A's work = 6.66%

B (10 days) → B's work = 10%

C (6 days) → C's work = 16.66%

44. Since the net work of the three taps is 10% and the first and second do 20% + 10% = 30%. Hence, the third pipe must be a waste pipe emptying at the rate of 20% per hour. Hence, the waste pipe will take a total of 5 hours to empty the tank.

45. A's work = 10%

B's negative work = 6.66%

$(A + B)$'s work = 3.33%

To fill a half empty tank, they would take 50/3.33 = 15 hours.

47. $(A + B) = 2C$.

Also, $(A + C) = 3B$

$36(A + B + C) = 1$

Solving for C, we get

$36(2C + C) = 1 \rightarrow 108C = 1$

$C = 1/108$

Hence, C takes 108 days.

48. $A + B + C = 10\%$. In the first two hours they will do 30% of the work. Further, for the next two hours work will be done at the rate of 15% per hour. Hence, after 4 hours 60% of the work will be completed, when top B is also closed. The last 32% of the work will be done by A alone. Hence, A has 40% (first 4 days) + 32% = 72% of the work.

50. In 6 days A would do 25% of the work and in 6 days B would do 25% of the work himself. So, C has to complete 50% of the work by himself.

In all C would require 30 days to do 20% of the work. So, he would require 22 more days.

LOD II

1. $25(x - 12) = 21(x + 30)$. Solving this equation, $x = 150$. Hence, the first division harvest 3150 sacks.

3. To solve this question first assume the values of x and y such that $x < y$. If you take x as 10 hours and y as 15 hours, you will get a net work of 3.33% per hour. At this rate it will take 20 hours to fill the tank from one third full. Using this condition try to put these values of x and y into the options to check the values.

For instance option (a) gives the value as $3 + 8 = 15/10 = 45$ which is not equal to 20.

4. $10(1/45 + 1/40) + 25/30 = 1 \rightarrow x = 9$

5. Since A finishes $6/7^{th}$ of the work in 2 hours B would finish 12/7 of the work in 2 hours.

Thus, to do $1/7^{th}$ of the work (which represents the remaining work), B would require $2/12 = 1/6$ hours.

Option (d) is correct.

6–10.

Set X can fill 10% in a minute. Hence, every Pipe in set X can do 1% work per minute. Set Y has a filling capacity of 12.5% per minute or 2.5% per minute for each tap in set Y). Set Z has a capacity of emptying the tank at the rate of 5% per minute and each tap of set Z can empty in the rate of 0.625% per minute.

6. If all the 23 pipes are opened the per minute rate will be:

$10 + 12.5 - 5 = 17.5\%$ → Option b is correct.

7. Set X will do 5 % per minute and Set Y will do 6.25% per minute, while set Z will do 5% per minute

(negative work). Hence, Nitu work will be 6.25% per minute. In 68.4% it will take slightly less than eight minutes and the value will be a fraction. None of the first four options matches this requirement. Hence, the answer will be (e).

12. The 32 minutes extra represent the extra time taken by the pipes due to the leak.

Normal time for the pipes $\rightarrow x = (1/14 + 1/16) = 1$
$\rightarrow x = 112/15 = 7$ hrs 28 minutes.

Thus, with 32 minutes extra, the pipes would take 8 hours to fill the tank.

Thus, $8(1/14 + 1/16) - 8 = (1/L) = 1 \rightarrow 8(L$
$= 8(15/112) - 1$
$1/L = 15/112 - 1/8$
$\rightarrow 1/112$

Thus, $L = 112$ hours.

13. The outlet pipe will empty the tank at a rate which is double the rate of filling (Hence, 10 gallons per minute). If the inlet is shut off, the tank will get emptied of 100 gallons of water in ten minutes.

14. The net inflow when both pipes are opened is 5 litres a minute.

The outlet flow should be such that if its rate is doubled the net inflow rate should be negative or 0. Only an option greater than or equal to '8' would satisfy this condition.

Option (b) is the only possible value.

15. $X \rightarrow 12$ days $\rightarrow 8.33\%$ of the work per day.
$Y \rightarrow 18$ days $\rightarrow 5.55\%$ of the work per day.
$Z \rightarrow 10$ days $\rightarrow 10\%$ of the work per day.
In three days, the work done will be $25 + 16.66 + 10 = 71.66\%$. The remaining work will get done by Y in $28.33/5.55 = 5.1$ days.

[Note: You need to be fluent with your fraction to percentage conversions in order to do well at these kind of calculations.]

16. A takes 9 days to complete the work.
B takes 24 days to complete the work.
C takes 16 days to complete the work.
In 6 days, work done by all three would be:
$6 = (1/9 + 1/24 + 1/16)$

$$= 6 \times \frac{(16 + 6 + 9)}{144} = 154/144$$

$= 11/36$ of the work.

Work left for B would be $5/36$ of the work.
B would require $(5/36) \times 24 = 3.33$ days.

17. The per day digging of all three combined is 54 metres. Hence, their average should be 18. This means that the first should be $18 - x$, the second, 18 & the third $18 + x$.

The required conditions are met if we take the values as 15,18,21 metres for B's first, second and third diggers respectively. Hence, (a) is the correct answer.

18. The equations are:

$$3(A + B + C) = 37/100 = 37\% \text{ of the work.}$$
$$7(A + B) = 63/50 \rightarrow A + B = 9/100 = 9\%$$

(Where A, B and C are 1 day's work of the three respectively).

Further, $3A = 4B$ gives us

$$A = 4\% \text{ and } B = 3\% \text{ work per day}$$

In 3 days $(A + B + C)$ do 37% of the work.

Out of this A and B would do 27% $(= 3 \times 9\%)$ of the work. So, C would do 3.33% of the work per day.

$$\frac{37 - 27}{3}$$

Thus, B is the faster and he would require 30 days to complete the work.

19. $A + B = 20\%$ of the work. Use trial and error with the options to get the answer.

Checking for option (a), $A = 10\%$ and $B = 10\%$. If A doubles his work and B halves his work rate, the total work in a day would become $A = 20$, $B = 5$. This would mean that the total work would get completed in 4 days which is the required condition that needs to be matched if the option is to be correct. Hence, this option is correct.

20. From the condition of the problem and a little bit of trial and error we can see that the first cook worked for 4 minutes and the 2nd and 3rd cooks also worked for 4 minutes. As $8(A + B + C) = 8(A + B + C)$ and we know that $A + B + C = 20$ idlis per minute.

Thus, the first cook make 20 idlis in 4 minutes. To make 160 idlis he would take 32 minutes.

21. $0.5(A + B + C) = 30\%$ of the work.
Means $\rightarrow A$, B and C can do the full work in 1 hour.
Thus, $(A + B + C) = 100\%$.

From this point it is better to solve through options. Option (c) gives the correct answer based on the following thought process.

$B = 30\%$ work per hour, x means C takes 3 hours to complete the work.

Consequently, A would take 3 hours and hence do 33.33% work per hour.

Since, $A + B + C = 100\%$, this gives us B's hourly work rate = 16.66%.

For this option to be correct these rates should match the second instance and the information given there.

According to the second condition

$A + 4B$ should be equal to 100%. Putting $A = 33.33\%$ and $B = 16.66\%$ we see that the condition is satisfied. Hence, this option is correct.

28. Use options for this question as follows.

If discharging delivery is 60, filling delivery will be 16.66% less (this will give a decimal value right at the start and is unlikely to be the answer. Hence, put this option aside for the time being.)

Option (c) gives good values. If discharging delivery = 60, filling delivery will be 50. Also, time taken for discharge of 3600 cu m will be 60 minutes and the time taken for delivery will be 72 minutes (12 minutes more – which is the basic condition of the problem).

30. Let the inlets be A, B, C and D.

$$A + B + C = 8.33\%$$

$$B + C + D = 6.66\%$$

$$A + D = 5\%$$

Thus, $2A + 2B + 2C + 2D = 20\%$

and $A + B + C + D = 10\%$

→ 10 minutes would be required to fill the tank completely.

ANSWER KEY

Time and Work LOD 1

1. c
2. c
3. b
4. a
5. a
6. a
7. a
8. c
9. a
10. a
11. a
12. c
13. b
14. b
15. c
16. a
17. c
18. b
19. a
20. d
21. c
22. a
23. a
24. c
25. a
26. b
27. a
28. c
29. d
30. c
31. d
32. a
33. a
34. b
35. b
36. c
37. c
38. b
39. b
40. a
41. d
42. c
43. a
44. c
45. b
46. d
47. c
48. d
49. b
50. c

Time and Work LOD II

1. a
2. b
3. d
4. c
5. d
6. b
7. e
8. a
9. b
10. d
11. c
12. b
13. a
14. b
15. c
16. b
17. a
18. a
19. a
20. b
21. c
22. d
23. e
24. a
25. a
26. c
27. b
28. c
29. a
30. b

Time and Work LOD III

1. b
2. a
3. b
4. d
5. b
6. d
7. c
8. c
9. d
10. c
11. a
12. b
13. c
14. d
15. c
16. b
17. a
18. b
19. c
20. c
21. c
22. c
23. b
24. a
25. c

10

TIME, SPEED AND DISTANCE AND APPLICATIONS OF TIME, SPEED AND DISTANCE

INTRODUCTION

The concepts underlying the chapter of Time, Speed and Distance (TSD) are amongst the most important for the purpose of the Aptitude section in aptitude exams. The basic concepts of TSD are used in solving questions based on motion in a straight line, Relative motion, circular motion, problems based on trains, problems based on boats, clocks, races etc. Besides, these concepts can also be used for the creation of new types of problems. Your ability to solve these problems will depend only on the depth of your understanding.

Due to the diversity in the possibilities for question setting, this chapter is very important for CAT aspirants. Besides, all other exams based on Aptitude (CET Maharashtra, XLRI, FMS, IIFT, Bank PO) also require the use of TSD. Hence, students are advised to understand the concepts of this chapter clearly.

This chapter is one of the most important chapters in the entire section. Questions based on Time, Speed and Distance are regularly asked in the CAT as well as in all other exams.

The students are advised to clearly understand the concepts contained in this chapter to be comfortable with the problems related to this topic in the examination.

THEORY OF TSD

Concept of Motion and Mathematical Representation of Motion

Motion/movement occurs when a body of any shape or size changes its position with respect to any external stationary point. Thus, when a person travels from city A to city B, we say that he has moved from city A to city B. In general, whenever a body moves with respect to a stationary point, we say that the body has undergone a displacement with respect to the starting point. Thus, for movement with respect to the starting point. Thus, for motion to have occurred, there must have been some displacement with respect to a stationary point on the ground.

The mathematical model that describes motion has three variables, namely: Speed, Time and Distance. The interrelationship between these three is also the most important formula for this chapter, namely:

Speed × Time = Distance (Equation for the description of one motion of one body)

The above equation is the mathematical description of the movement of a body. In complex problems, students tend to get confused regarding the usage of this equation and often end up mixing up the speeds time and distance of different motions of different bodies.

It must be mentioned here that this formula is the cornerstone of the chapter Time, Speed and Distance.

Besides, this formula is also the source of the various formulae applied to the problems on the application of time, speed and distance—to trains, boats and streams, clocks and races, circular motion and straight line motion.

In the equation above, speed can be defined as the rate at which distance is covered during the motion. It is measured in terms of distance per unit time and may have any combination of units of distance and time in the numerator and the denominator respectively. (m/s, km/hour, km/min, km/day, etc.)

When we say that the speed of a body is 5 kmph, we mean to say that the body moves with 5 kmph towards or away from a stationary point on the case may be).

Time is the time duration over which the movement/motion occurs/has occurred. The unit used for measuring time is synchronous with the determinant of the unit used for measuring speed. Thus, if the speed is measured in units of kmph then time is measured in hours.

Distance (d) is the displacement of the body during the motion.

The above equation, as is self-evident, is such that the interrelationship between the three parameters defines the value of the third parameter if two of the three are known. Hence we can safely say that if we know two of the three variables describing the motion, then the motion is fully described and every aspect of it is known.

The Proportionalities Implicit in the Equation $S \times T = D$

The above equation has three implicit proportionality relations each of which has its own context bearing on its solving of time, speed and distance problems.

I. Direct proportionality between time and distance (when the speed is constant) time ∝ distance

Illustration

A car moves for 2 hours at a speed of 25 kmph and another car moves for 3 hours at the same speed. Find the ratio of distance covered by the two cars.

Solution: Since, the speed is constant, we can directly conclude that time ∝ distance.

Hence,
$$\frac{t_A}{t_B} = \frac{d_A}{d_B}$$

Since, the times of travel are 2 and 3 hours respectively, the ratio of distances covered is also 2/3.

Note: This can be verified by looking at the actual distance travelled—being 50 km and 75 km in this case.

II. Direct Proportionality between speed and distance (when the time is constant) speed ∝ distance

(i) *A body travels at S_1 kmph for the first 2 hours and then travels at S_2 kmph for the next two hours.*

Here two motions of one body are being described and between these two motions the time is constant hence speed will be proportional to the distance travelled.

(ii) *Two cars start simultaneously from A and B respectively towards each other with speeds of S_1 kmph and S_2 kmph. They meet at a point C.* Here again, the speed is directly proportional to the distance since two motions are described where the time of both the motions is the same, that is, it is evident here that the first and the second car travel for the same time.

In such a case the following ratios will be valid

$$S_1/S_2 = d_1/d_2$$

Illustrations

(i) A car travels at 30 kmph for the first 2 hours of a journey and then travels at 40 kmph for the next 2 hours of the journey. Find the ratio of the distances travelled at the two speeds.

Solution: Since time is constant between the two motions described, we can use the proportionality between speed and distance.

Hence,
$$d_1/d_2 = s_1/s_2 = 3/4$$

Alternatively, you can also think in terms of percentage as d_2 will be 33.33% higher than d_1 since S_2 is 33% higher than S_1 and time is constant.

(ii) Two cars leave simultaneously from points A and B on a straight line towards each other. The distance between A and B is 100 km. They meet at a point 40 km from A. Find the ratio of their speeds.

Solution: Since time is the same for both the motions described, we have ratio of speed = ratio of distance

$$S_1/S_2 = 40/(100 - 40) = 2/3$$

(iii) Two cars move simultaneously from points A and B towards each other. The speeds of the two cars are 30 mph and 25 mph respectively. Find the meeting point if $d(AB) = 900$ km.

Solution: For the bodies to meet, the time of travel is constant (since the two cars have moved simultaneously).

Hence, speed ratio = distance ratio
= 4:3 = distance ratio

Hence, the meeting point will be 300 km from A and 300 km from B.

4. Inverse proportionality between speed and time (when the distance is constant): Speed ∝ 1/time

(a) *A body travels at S_1 kmph for the first half of the journey and then travels at S_2 kmph for the second half of the journey.* Here two sections of the body are being described and between these two sections the distance travelled is constant. Hence the speed will be inversely proportional to the time travelled for.

(b) Two cars start simultaneously from A and B respectively towards each other. They meet at a point C and reach their respective destinations. If and t_1 and t_2 hours respectively... Here again, the speed is inversely proportional to the time since two sections are described where the distance of both the sections is the same, that is, it is evident here that the time and the second car travel for the distance, viz., AB.

In such a case, the following rules will be valid:

$$S_1/S_2 = t_2/t_1 \quad \text{i.e.} \quad S_1t_1 = S_2t_2 = K$$

Illustrations

(i) A train starts with an accident and moves at 3/4 its original speed. Due to this, it is 20 minutes late. Find the original time for the journey beyond the point of accident.

Solution: Speed becomes 3/4 (Time becomes 4/3)
Extra time = 1/3 of normal time = 20 minutes
Normal time = 60 minutes

Alternatively, from the table on product constancy or the chapter of percentages, we get that a 25% reduction in speed leads to a 33.33% increase in time.

But, 33.33% increase in time is equal to 20 minutes increase in time.

Hence, total time (original) = 60 minutes.

(ii) A body travels half the journey at 20 kmph and the other half at 30 kmph. Find the average speed.

Solution: The shortcut practice is described in the chapter on "Averages". Answer = 24 kmph.

(iii) A man travels from his house to his office at 5 kmph and reaches his office 20 minutes late. If his speed had been 7.5 kmph, he would have reached his office 12 minutes early. Find the distance from his house to his office.

Solution: Notice that here the distance is constant. Hence, speed is inversely proportional to time.

Solving mathematically

$$S_1/S_2 = t_2/t_1 + 32)$$
$$5/7.5 = t_2/t_1 + 32)$$
$$S_1t_1 = 160 = 7.5\ t_2$$
$$t_2 = 160/2.5 = 64 \text{ minutes}$$

Hence, the distance is given by 7.5 × 64/60 = 8 km.

Alternatively, using the Product Constancy Table from the chapter of percentages, if speed increases by 50%, then time will decrease by 33.33%.

But the decrease is equal to 32 minutes.

Hence, original time = 96 minutes and new time = 64 minutes.

Hence, the required distance = 5 × 96/60 km = 8 km

or distance = 7.5 × 64/60 km = 8 km

[*Note:* The entire process can be worked out mentally while reading the problem.]

CONVERSION BETWEEN kmph to m/s

1 km/h = 1000 m/h = 1000/3600 m/s = 5/18 m/s

Hence, to convert x kmph into m/s multiply by 5/18.

Thus, x kmph = $\frac{5x}{18}$ m/s.

And vice versa, y m/s = 18 y/5 km/h. To convert m/s to kmph, multiply by 18/5.

Relative Speed : Same Direction and Opposite Direction

Normally, when we talk about the movement of a body we mean the movement of the body with respect to a stationary place. Hence, there are times when we need to determine the movement and its relationships with respect to a moving position. In such instances, we have to take into account the movement of the body/points with respect to which we are trying to determine relative motion.

Relative movement, therefore, can be viewed as the movement of one body relative to another moving body.

The following formulae apply for the relative speed of two **independent** bodies with respect to each other:

Case I: Two bodies are moving in opposite directions at speeds S_1 and S_2 respectively.
The relative speed is defined as $S_1 + S_2$

Case II: Two bodies are moving in the same direction.

The relative speed is defined as

(a) $S_1 - S_2$ when S_1 is greater than S_2

(b) $S_2 - S_1$ when S_2 is lesser than S_1

In other words, the relative speed can also be defined as the positive value of the difference between the two speeds, that is, $|S_1 - S_2|$.

Motion in a Straight Line

Problems on situations of motion in a straight line are one of the most commonly asked questions in the CAT and other aptitude exams. Hence a proper understanding of the following concepts and their applications to problem solving will be extremely important for the student.

Motion in a straight line is governed by the rules of relative speed enumerated above.

A. **Two or more bodies starting from the same point and moving in the same direction: Their relative speed is $S_1 - S_2$.**

(a) *In the case of the bodies moving to and fro between two points A and B:* The faster body will reach the end first and will turn the second body, on its way back. The relative speed $S_1 - S_2$ will apply till the point of reversal of the faster body and after that the two bodies will start to move in the opposite directions at a relative speed of $S_1 + S_2$. The relative speed governing the movement of the two bodies will alternate between $S_1 - S_2$ and $S_1 + S_2$ everytime any one of the bodies reverses directions. However, if both the bodies reverse their directions at the same instant, there will be no change in the relative speed equation.

In this case, the description of the motion of the two bodies between two consecutive meetings will also be governed by the proportionality between speed and distance since the time of movement between any two meetings will be constant.

Distances covered in this case: For every meeting, the total distance covered by the two bodies will be 2D (where D is the distance between the extreme points). The respective coverage of the distance is in the ratio of the individual speeds.

Thus, for the 9th meeting the total distance covered will be $9 \times 2D = 18D$.

This will be useful for solving problems that require the calculation of a meeting point.

(b) *In the case of the bodies continuing to move in the same direction without coming to an end point and reversing directions:* The faster body will take a lead and will keep increasing the lead and the movement of the two bodies will be governed by the relative speed equation: $S_1 - S_2$.

Here again, if the two bodies start simultaneously, their movement will be governed by the direct proportionality between speed and distance.

B. *Moving in the opposite direction:* **Their relative speed will be initially given by $S_1 + S_2$.**

(a) *In the case of the bodies moving to and fro between two points A and B starting from opposite ends of the path:* The two bodies will move towards each other, meet at a point in between A and B, then move apart away from each other. The faster body will reach its extreme point first followed by the slower body reaching its extreme point later. Relative speed will change every time one of the bodies reverses direction.

The position of the meeting point will be determined by the ratio of the speeds of the bodies (since the 2 movements can be described as having the same constant between them).

Distances covered in the above case: For the first meeting, the total distance covered by the two bodies will be D (the distance between the extreme points). The coverage of the distance is in the ratio of the individual speeds.

Thereafter, as the bodies separate and start coming together, the combined distance to be covered is 2D.

Thus, for the 50th meeting the total distance covered will be $D + 9 \times 2D = 19D$.

This will be useful for the solving of problems that require the calculation of a meeting point.

Illustrations

(1) Two bodies A and B start from opposite ends P and Q of a straight road. They meet at a point 0.4D from P. Find the point of their fourth meeting.

Solution: Since time is constant, we have ratio of speeds as 3 : 2.

Also, total distance to be covered by the one reaching first the fourth meeting is 700. This distance is divided in a ratio of 3 : 2 and thus we have that A will cover 420 and B will cover 280.

The fourth meeting point can then be found out by tracking either A or B's movement. A, having covered a distance of 420, will be at a point 420 from P. This is the required answer.

(ii) A starts walking from a place at a uniform speed of 2 km/h in a particular direction. After half an hour, B starts from the same place and walks in the same direction as A at a uniform speed and overtakes A after 2 hour 48 minutes. Calculate the speed of B.

Solution: Start solving as you read the question. From the first two sentences you see that A is 1 km ahead of B when B starts moving.

This distance of 1 km is covered by B in 4/5 hours (1 hour 48 minutes = 1+4/5 = 9/5 hours).

The equation operationalized here $(S_B - S_A) \times T =$ initial distance

$$(S_B - 2) \times 9/5 = 1$$

Solving, we get $S_B = 2.1$ km/h.

(iii) In the case of the boats, continuing is there in the same direction without coming to an end point and reversing direction. The boats will meet and following their meeting they will start separating and going away from each other. The relative speed will be given by $S_1 + S_2$ initially while approaching each other and thereafter, it will be $S_1 - S_2$ while moving away from each other.

Important: The student is advised to take a closer look and get a closer understanding of these concepts by taking a few examples with absolute values of speed, time and distance. Try to visualize how two bodies separate and then come together. Also, clearly understand the three proportionalities in the equation $d = s \times t$. Since these are very important tools for problem solving.

Concept of Acceleration

Acceleration is defined as the rate of change of speed.

Acceleration can be positive (speed increases) or negative (speed decreases) – also known as deceleration.

The term of acceleration is speed per unit time i.e. m/s^2.

For instance, if a body has an initial speed of 5 m/s and a deceleration of 0.1 m/s^2 it will take 50 seconds to come to rest.

Final speed = Initial speed + Acceleration × Time

Some more examples:

(i) Water flows into a cylindrical beaker at a constant rate. The base area of the beaker is 24 cm^2. The water level rises by 10 cm every second. How quickly will the water level rise in a beaker with a base area of 30 cm^2?

Solution: The flow of water in the beaker is 24 cm^2 × 10 cm/s = 240 cm^3/s.

If the base area is 30 cm^2 then the rate of water level rise will be 240/30 = 8 cm/s.

Note: In case of confusion in such questions the student is advised to use dimensional analysis to determine what to multiply and what to divide.

(ii) A 2 kilowatt heater can heat a given amount of water in 10 minutes. How long will it take for

(a) a less powerful heater of 1.2 kilowatts to heat the same amount of water?

(b) a less powerful heater of 1.2 kilowatts to heat double the amount of water?

Solution:

(a) The heating required to heat the amount of water = 2 × 10 = 20 kilowatt minutes. At the rate of 1.2 kilowatt, this heat will be generated in 20/1.2 minutes = 16.66 minutes.

(b) When the water is doubled, the heating required is also doubled. Hence, heating required = 40 kilowatt minutes. At the rate of 1.2 kilowatt, this heat will be generated in 40/1.2 = 33.33 minutes.

AN APPLICATION OF ALLIGATION IN TIME, SPEED AND DISTANCE

Consider the following situation:

Suppose a car goes from A to B at an average speed of 5 km/h and then comes back from B to A at an average speed of 6 km/h. If you had to find out the average speed of the whole journey, what would you do?

The normal short cut given for this situation gives the average speed as:

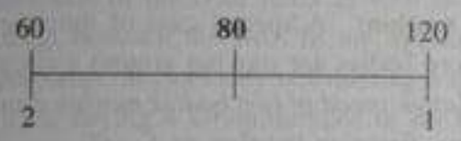
$$\frac{2S_1 S_2}{S_1 + S_2}$$

However, this situation can be solved very conveniently using the process of alligation as explained below:

Since, the two speeds are known to us, we will also know their ratio. The ratio of times for the two parts of the journey will then be the inverse ratio of the ratio of speeds. (Since the distance for the two journeys are equal). The answer will be the weighted average of the two speeds (weighted on the basis of the time travelled at each speed)

The process will become clear through an example:
A car travels at 60 km/h from Mumbai to Poona and at 120 km/h from Poona to Mumbai. What is the average speed of the car for the entire journey.

Solution



The process of alligation, will be used here to give the answer as 80. (*Note:* For the process of alligation, refer to the chapter of Alligation.

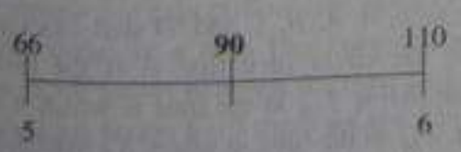
Note here, that since the speed ratio is 1:2, the value of the time ratio used for calculating the weighted average will be 2:1.

What will happen in case the distances are not constant?

For instance, if the car goes 100 km at a speed of 66kmph and 200 km at a speed of 110 kmph, what will be the average speed?

In this case the speed ratio being 6:10 i.e. 3:5 the inverse of the speed ratio will be 5:3. This would have been the ratio to be used for the time ratio in case the distances were the same (for both the speeds). But since the distances are different, we cannot use this ratio in this form. The problem is overcome by multiplying this ratio (5:3) by the distance ratio (in this case it is 1:2) to get a value of 5:6. This is the ratio which has to be applied for the respective weights. Hence, the alligation will look like:

Solution



Thus the required answer is 90 kmph. (The student is advised to check this value through normal mathematical processes.)

APPLICATIONS OF TIME, SPEED AND DISTANCE

Trains

Trains are a special case in questions related to time, speed and distance because they have their own theory and distinct situations.

The basic relation for trains problems is the same: Speed × Time = Distance.

The following things need to be kept in mind before solving questions on trains:

(a) When the train is crossing a moving object, the speed has to be taken as the **relative speed** of the train with respect to the object. All the rules for relative speed will apply for calculating the relative speed.

(b) The distance to be covered when crossing an object whenever a train crosses an object will be equal to: **Length of train + Length of object**

Thus, the following cases will yield separate equations, which will govern the crossing of the object by the train:

For each of the following situations the following notations have been used:

S_T = Speed of train S_O = Speed of object t = time
L_T = Length of train L_O = Length of object

Case I: Train crossing a stationary object without length:

$$S_T \times t = L_T$$

Case II: Train crossing a stationary object with length:

$$S_T \times t = (L_T + L_O)$$

Case III: Train crossing a moving object without length:

- In opposite direction: $(S_T + S_O) \times t = L_T$
- In same direction: $(S_T - S_O) \times t = L_T$

Case IV: Train crossing a moving object with length:

- In opposite direction: $(S_T + S_O) \times t = (L_T + L_O)$
- In same direction: $(S_T - S_O) \times t = (L_T + L_O)$

Note: In order for a train to completely cross a stationary point on the ground, the train has to traverse a distance that is equal to its own length.

This can be visualized by remembering yourself standing on a railway platform and being crossed by a train. You would say that the train starts crossing you when the engine of the train comes in line with you. Also, you would say that you have been crossed by the train when the end of the guard's compartment comes in line with you. Thus, the train would have travelled its own length in crossing you.

Illustrations

(i) A train crosses a pole in 8 seconds. If the length of the train is 200 metres, find the speed of the train.

Solution: In this case, it is evident that the situation is one of the train crossing a stationary object without length. Hence, Case I is applicable here.

Thus, $S_t = 200/8 = 25$ mps $\rightarrow 25 \times \frac{18}{5} = 90$ kmph.

(ii) A train passes a man travelling in another train in the opposite direction in 8 seconds. However, the train requires 25 seconds to cross the same man if the trains are travelling in the same direction. If the length of the first train is 200 metres and that of the train in which the man is sitting is 300 metres, find the speed of the first train.

Solution: Here, the student should understand that the situation is one of the train crossing a moving object without length. Then the length of the man's train is useless or redundant data.

Then applying the relevant formulae after considering the directions of the movements we get the equations:

$$S_t + S_m = 25$$
$$S_t - S_m = 8$$
$$S_t = \frac{33}{2} = \frac{33 \times 18}{2 \times 5} = 59.4 \text{ kmph}$$

Boats and Streams

The problems of boats and streams are also dependent on the basic relation of time, speed and distance. Speed × Time = Distance.

However, as in the case of water the adjustments are made for adverse circumstances on boats and streams are made for adverse circumstances on boats and streams are made for adverse circumstances on boats and streams, which is also called the speed of the boat in still water (S_B).

The boat has a speed of its own, which is also called the speed of the boat in still water (S_B).

Another variable that is used in boats and streams problems is the speed of the stream (S_S).

The speed of the movement of the boat is dependent on whether the boat is moving:

(a) In still water the speed of movement is given by $\rightarrow S_B$.

(b) While moving upstream (or against the flow of the water), the speed of movement is given by $\rightarrow S_B - S_S$.

$S_B - S_S$

(c) While moving downstream (or with the flow of the water), the speed of movement is given by $\rightarrow S_B + S_S$.

$S_B + S_S$

The time of movement and the distance to be covered are to be judged by the context of the problem.

Circular motion. A special case of movement is when two or more bodies are moving around a circular track.

The relative speed of two bodies moving around a circle in the same direction is taken as $S_1 - S_2$.

Also, when two bodies are moving around a circle in opposite direction, the speed of the two bodies is taken as $S_1 + S_2$.

The peculiarity inherent in moving around a circle in the same direction is that when the faster body overtakes the slower body it goes ahead of it. And for every unit time that elapses, the faster body keeps increasing the distance by which the slower body is behind the faster body. However, when the distance by which the faster body is in front of the slower body becomes equal to the circumference of the circle around which the two bodies are moving, the faster body again comes in line with the slower body. This event is called as overlapping or lapping of the slower body by the faster body. We say that the slower body has been lapped or overlapped by the faster body.

First meeting: Three or more bodies start moving simultaneously from the same point on the circumference of the circle in the same direction around the circle. They will meet again in the LCM of the times that the faster body takes in totally overlapping each of the slower ones.

For instance, if A, B, C and D start clockwise from a point A on the circle such that A is the fastest and then we can define T_{AB} as the time in which A completely overlaps B, T_{AC} as the time in which A completely overlaps C and T_{AD} as the time in which A completely overlaps D

then the LCM of T_A, T_B and T_C will be the time in which A, B, C and D will be together again for the first time.

Concurrently or starting point : Three or more bodies start moving concurrently from the same point on the circumference of a circle. In the same direction around the circle. Their meeting at the starting point will occur after a time that is go be the LCM of the times that each of the bodies takes to complete one full round.

For instance, if A, B and C start from a point X on the circle such that T_A, T_B and T_C are the times in which A, B and C respectively cover one complete round around the circle, then they will all meet together at the starting point X the LCM of T_A, T_B and T_C.

Clocks

Problems on clocks are based on the movement of the minute hand and that of the hour hand as well as on the relative movement between the two. In my opinion, it is best to solve problems on clocks by considering a clock to be a circular track having a circumference of 60 km and each kilometre being represented by one minute on the dial of the clock. Then, we can look at the minute hand as a runner running at the speed of 60 kmph while we can also look at the hour hand as a runner running at an average speed of 5 kmph.

Since, the minute hand and the hour hand are both moving in the same direction, the relative speed of the minute hand with respect to the hour hand is 55 kmph, that is, for every hour elapsed, the minute hand goes 55 km (minutes) more than the hour hand.

(Beyond this slight adjustment, the problems of clocks require a good understanding of unitary method. This will be well illustrated through the school example below.)

Important Information

Number of right angles formed by a clock : A clock makes 2 right angles between any 2 hours. Thus, for instance, there are 2 right angles formed between 12 to 1 or between 1 to 2 or between 2 to 3 or between 3 to 4 and so on.

However, contrary to expectations, the clock does not make 48 right angles in a day. This happens because whenever the clock passes between the time period 2–4 or between the time period 8–10 there are not 4 but only 3 right angles.

This happens because the second right angle between 2–3 (or 8–9) and the first right angle between 3–4 (or 9–10) are one and the same, occurring at 3 or 9.

Right angles are formed when the distance between the minute hand and the hour hand is equal to 15 minutes.

Exactly the same situation holds true for the formation of straight lines. There are 2 straight lines in every hour. However, the second straight line between 5–6 (or 11–12) and the first straight line between 6–7 (or 12–1) coincide with each other and are represented by the straight line formed at 6 (or 12).

Straight lines are formed when the distance between the minute hand and the hour hand is equal to either 0 minutes or 30 minutes.

Illustration

At what time between 2–3 p.m. is the first right angle in that time formed by the hands of the clock?

Solution : At 2 p.m. the minute hand can be visualised as being 10 kilometres behind the hour hand (considering the clock dial to be a race track of circumference 60 km such that each minute represents 1 kilometre).

Also, the first right angle between 2–3 is formed when the minute hand is 15 kilometres ahead of the hour hand.

Thus, the minute hand has to cover 25 kilometres over the hour hand.

This can be written using the unitary method:

Distance covered by the minute hand over the hour hand.

55 kilometres _____ in 1 hour

25 kilometres _____ in what time?

→ 5/11 of an hour.

Thus, the first right angle between 2–3 is formed at 5/11 hours past 2 o'clock.

This can be converted into minutes and seconds using unitary method again as:

1 hour _____ 60 minutes

5/11 hours _____ ? minutes

→ 300/11 minutes = 27 (3/11) minutes

1 minute _____ 60 seconds

3/11 minutes _____ ? seconds → (180/11) seconds = 16.36 seconds.

Hence, the required answer is: **2 : 27 : 16.36 seconds.**

Level of Difficulty (LOD) 1

1. The Sahagal Express left Pune at noon sharp. Two hours later, the Deccan Queen started from Pune in the same direction. The Deccan Queen overtook the Sahagal Express at 8 p.m. Find the average speed of the two trains over this journey if the sum of their average speeds is 70 km/h.
   (a) 34.29 km/h (b) 35 km/h
   (c) 50 km/h (d) 12 km/h
   (e) 16 km/h

2. Walking at 3/4 of his normal speed, Amitabh is 10 minutes late in reaching his office. The total time taken by him to cover the distance between his house and his office is
   (a) 40 minutes (b) 60 minutes
   (c) 42 minutes (d) 63 minutes
   (e) 36 minutes

3. Ram and Bharat travel the same distance at the rate of 6 km per hour and 10 km per hour respectively. If Ram takes 30 minutes longer than Bharat, the distance travelled by each is
   (a) 6 km (b) 10 km
   (c) 7.5 km (d) 20 km
   (e) 8 km/h

4. Two trains for Mumbai leave Delhi at 6 a.m. and 6 : 45 am and travel at 100 km/h and 156 km/h respectively. How many kilometres from Delhi will the two trains be together
   (a) 262.4 km (b) 260 km
   (c) 283.33 km (d) 275 km
   (e) None of these

5. Two trains, Calcutta Mail and Bombay Mail, start at the same time from stations Calcutta and Bombay respectively towards each other. After passing each other they take 12 hours and 3 hours to reach Bombay and Calcutta respectively. If the Calcutta Mail is moving at the speed of 48 km/h, the speed of the Bombay Mail is
   (a) 24 km/h (b) 72 km/h
   (c) 36 km/h (d) 96 km/h
   (e) 120 km/h

6. Shyam's house, his office and his gym are all equidistant from each other. The distance between any 2 of them is 4 km. Shyam starts walking from his house in a direction parallel to the road connecting his office and his house and stops when he reaches a point directly east of his office. He then reverses direction and walks till he reaches a point directly south of his office. The total distance walked by Shyam is
   (a) 6 km (b) 9 km (c) 16 km (d) 12 km
   (e) 8 km

7. Lonavala and Khandala are two stations 600 km apart. A train starts from Lonavala and moves towards Khandala at the rate of 25 km/h. After two hours another train starts from Khandala at the rate of 35 km/h. How far from Lonavala will they will meet each other?
   (a) 250 km (b) 300 km
   (c) 279.166 km (d) 475 km

8. Walking at 3/4 of his normal speed, a man takes 2 1/2 hours more than the normal time. Find the normal time.
   (a) 7.5 h (b) 6 h (c) 9 h (d) 12 h
   (e) 10 h

9. Alok walks to a viewpoint and returns to the starting point by his car and thus takes a total time of 6 hours 45 minutes. He would have gained 2 hours by using both ways. How long would it have taken for him to walk both ways?
   (a) 8 h 45 min (b) 7 h 45 min
   (c) 5 h 30 min (d) 6 h 45 min
   (e) None of these

10. Sambhu beats Kalu by 30 metres or 10 seconds. How much time was taken by Sambhu to complete a race of 1200 metres
   (a) 6 min 30 s (b) 5 min 15 s
   (c) 12 min 10 s (d) 2 min 5 s
   (e) None of these

11. What is the time taken by Chanda to cover a distance of 300 km by a motorcycle moving at a speed of 50 m/s
   (a) 10 h (b) 5 h (c) 8 h (d) 6 h
   (e) None of these

12. Without stoppage, a train travels a certain distance with an average speed of 60 km/h, and with stoppage it covers the same distance with an average speed of 40 km/h. On an average, how many minutes per hour does the train stop during the journey?
   (a) 20 min/h (b) 15 min/h

(c) 10 km/h (d) 10 km/h
(e) 12 km/h

13. Rajdhani Express travels 650 km in 5 h and another 440 km in 10 h. What is the average speed of train?
(a) 1540 km/h (b) 108 km/h
(c) 106 km/h (d) 120 km/h
(e) 130 km/h

14. Rajdhani, during his journey, travels for 20 minutes at a speed of 30 km/h, another 30 minutes at a speed of 40 km/h and 1 hour at a speed of 50 km/h and 1 hour at a speed of 60 km/h. What is the average velocity?
(a) 51.18 km/h (b) 60 km/h
(c) 54 km/h (d) 48 km/h
(e) 54 km/h

15. A car travels from A to B at V_1 km/h, travels back from B to A at V_2 km/h and again goes back from A to B at V_3 km/h. The average speed of the car is:
(a) $\frac{2V_1V_2}{V_1+2V_2}$ (b) $\frac{2V_1V_2}{V_1+2V_3}$
(c) $\frac{3V_1V_2}{V_1+2V_2}$ (d) $\frac{3V_1V_2}{V_1+2V_3}$
(e) None of these

16. Narayan Murthy walking at a speed of 20 km/h reaches his college 10 minutes late. Next time he increases his speed by 5 km/h, but finds that he is still late by 4 minutes. What is the distance of his college from his house?
(a) 20 km (b) 6 km (c) 12 km (d) 15 km
(e) None of these

17. Jayshree goes to office at a speed of 6 km/h and returns to her home at a speed of 4 km/h. If she takes 10 hours in all, what is the distance between her office and her house?
(a) 24 km (b) 12 km (c) 10 km (d) 30 km
(e) 45 km

18. A motor car does a journey in 17.5 hours, covering the first half at 30 km/h and the second half at 40 km/h. Find the distance of the journey.
(a) 684 km (b) 600 km
(c) 120 km (d) 540 km
(e) 624 km

19. Raju covers a distance in 40 minutes if he drives at a speed of 60 kilometre per hour on an average. Find the speed at which he must drive it to reduce the time of the journey by 25%?
(a) 80 km/h (b) 70 km/h
(c) 75 km/h (d) 60 km/h
(e) 90 km/h

20. Manish travels a certain distance by car at the rate of 12 km/h and walks back at the rate of 3 km/h. The whole journey took 5 hours. What is the distance he covered in the car?
(a) 12 km (b) 30 km
(c) 15 km (d) 6 km
(e) 18 km

21. A runway passenger counts the telegraph poles on the rail road in 30 minutes. The telegraph poles are at a distance of 50 metres. What will he his count in 3 hours, if the speed of the train is 45 km per hour.
(a) 600 (b) 2500 (c) 3600 (d) 5000
(e) None of these

22. Two trains A and B start simultaneously in the opposite direction from two points A and B and arrive at their destinations 9 and 4 hours respectively after their meeting each other. At what rate does the second train B travel if the first train travels at 50 km per hour
(a) 60 km/h (b) 100 km/h
(c) 120 km/h (d) 80 km/h
(e) None of these

23. Vijay fires two bullets from the same place at an interval of 12 minutes but Raju sitting in a train approaching the place hears the second report 11 minutes 30 seconds after the first. What is the approximate speed of train (if sound travels at the speed of 330 metre per second)?
(a) 440/23 m/s (b) 220/7 m/s
(c) 330/23 m/s (d) 110/23 m/s
(e) Cannot be answered

24. A car driver, driving in a fog, passes a pedestrian who was walking at the rate of 2 km/h in the same direction. The pedestrian could see the car for 9 minutes and it was visible to him up to a distance of 0.6 km. What was the speed of the car?
(a) 30 km/h (b) 15 km/h
(c) 20 km/h (d) 8 km/h
(e) 12 km/h

25. Harsh and Vijay move towards Home starting from BM, Bangalore, at a speed of 40 km/h and 60 km/h respectively. If Vijay reaches Home 200 minutes earlier than Harsh, what is the distance between BM, Bangalore, and Home?

(a) 600 km (b) 400 km
(c) 800 km (d) 200 km
(e) 300 km

26. A journey of 192 km takes 2 hours less by a fast train than by a slow train. If the average speed of the slow train is 16 kmph less than that of the train, what is the average speed of the faster train.
(a) 32 kmph (b) 16 kmph
(c) 12 kmph (d) 48 kmph
(e) 40 kmph

27. A passenger train takes 2 h less for a journey of 300 kilometres if its speed is increased by 5 kmph over its usual speed. Find the usual speed.
(a) 30 kmph (b) 17 kmph
(c) 20 kmph (d) 25 kmph
(e) 24 kmph

28. If Ann had walked 1 kmph faster, he would have taken 10 minutes less to walk 2 kilometers. What is Ann's speed of walking.
(a) 3 kmph (b) 2 kmph
(c) 5 kmph (d) 6 kmph
(e) None of these

29. A plane left half an hour later than the scheduled time and in order to reach its destination 1500 kilometers away in time, it had to increase its speed by 33.33 per cent over its usual speed. Find its increased speed.
(a) 250 kmph (b) 500 kmph
(c) 750 kmph (d) 1000 kmph
(e) 1200 kmph

30. A train moves at a constant speed of 120 kmph for one kilometre and at 40 kmph for the next one kilometre. What is the average speed of the train.
(a) 48 kmph (b) 50 kmph
(c) 60 kmph (d) 60 kmph
(e) None of these

31. A cyclist moving on a circular track of radius 100 metres completes one revolution in 2 minutes. What is the average speed of cyclist (approximately)?
(a) 314 m/minute (b) 200 m/minute
(c) 300 m/minute (d) 400 m/minute
(e) None of these

32. A person travelled a distance of 200 kilometre between two cities by a car covering the first quarter of the journey at a constant speed of 40 km/h and the remaining three quarters at a constant speed of x km/h. If the average speed of the person for the entire journey was 33.33 km/h, what is the value of x.
(a) 55 km/h (b) 60 km/h
(c) 70 km/h (d) 80 km/h
(e) 100 km/h

33. A car travels 1/3 of the distance on a straight road with a velocity of 10 km/h, the next 1/3 with a velocity of 20 km/h and the last 1/3 with a velocity of 60 km/h. What is the average velocity of the car for the whole journey?
(a) 18 km/h (b) 10 km/h
(c) 20 km/h (d) 12 km/h
(e) None of these

34. Two cars started simultaneously toward each other from town A and B, that are 480 km apart. It took the first car travelling from A to B 8 hours to cover the distance and the second car travelling from B to A 12 hours. Determine at what distance from A the two cars met.
(a) 288 km (b) 200 km
(c) 300 km (d) 196 km
(e) None of these

35. Walking at 5/4 of his usual speed, a man is 16 minutes late for his office. The usual time taken by him to cover that distance is
(a) 48 minutes (b) 60 minutes
(c) 42 minutes (d) 62 minutes
(e) 75 minutes

36. A and B travel the same distance at the rate of 8 kilometre and 10 kilometre an hour respectively. A takes 30 minutes longer than B, the distance travelled by B is?
(a) 6 km (b) 10 km
(c) 16 km (d) 20 km

37. Two trains for Pune leave Delhi at 6 a.m. and 6.45 a.m. and travel at 98 kmph and 156 kmph respectively. How many kilometres from Delhi will the two trains meet?
(a) 262.4 km (b) 260 km
(c) 200 km (d) None of these

38. Two trains A and B start from station X to Y 9 and 4 respectively. After passing each other, they take 3 hours and 3 hours to reach Y and X respectively. If train A is moving at the speed of 48 kmph, the speed of train B is
(a) 24 km/h (b) 96 km/h
(c) 21 km/h (d) 20 km/h

40. A and B are two stations 600 km apart. A train starts from A and moves towards B at the rate of 25 km/h. Another train starts from B at the rate of 35 km/h towards A. If they will cross each other

(a) 250 km (b) 300 km
(c) 450 km (d) 475 km

41. A man from a point that is on the circumference of a circle, moves 600 metre in the North direction and then again moves 800 metre East and reaches a point diametrically opposite the starting point. Find the diameter of the circle?

(a) 1000 m (b) 700 m
(c) 800 m (d) 900 m

42. Ram and Shyam run a race of 5000 m. First, Ram gives Shyam a start of 200 m and beats him by 30 s. Next Ram gives Shyam a start of 3 min and is beaten by 1000 metres. Find the time in minutes in which Ram and Shyam can run the race separately

(a) 8, 10 (b) 4, 5
(c) 5, 8 (d) 6, 9

43. A motorboat went downstream for 28 km and immediately returned. It took the boat twice as long to make the return trip. If the speed of the river flow were twice as high, the trip downstream and back would take 672 minutes. Find the speed of the boat in still water and the speed of the river flow.

(a) 8 km/h, 3 km/h (b) 8 km/h, 6 km/h
(c) 6 km/h, 2 km/h (d) 12 km/h, 3 km/h

44. A train requires 7 seconds to pass a pole while it requires 25 seconds to cross a stationary train which is 378 metres long. Find the speed of the train

(a) 75.6 km/h (b) 75.4 km/h
(c) 76.2 km/h (d) 21 km/h

45. A boat sails downstream from point A to point B, which is 10 km away from A, and then returns to A. If the actual speed of the boat (in still water) is 3 km/h, the trip from A to B takes 8 hours less than that from B to A. What must the actual speed of the boat for the trip from A to B to take exactly 100 minutes?

(a) 1 km/h (b) 2 km/h
(c) 3 km/h (d) 4 km/h

46. A boat sails down the river for 10 km and then up the river for 6 km. The speed of the river flow is 1 km/h. What should be the minimum speed of the boat for the trip to take a maximum of 4 hours?

(a) 2 km/h (b) 3 km/h
(c) 4 km/h (d) 5 km/h

46. A man rows 6 km/h in still water. If the river is running at 1 km per hour, it takes him 45 minutes to row to a place and back. How far is the place?

(a) 1.12 km (b) 1.25 km
(c) 1.6875 km (d) 2.5 km

47. A boat goes 30 km upstream in 6 h and a distance of 48 km downstream in 7 h. The speed of the boat in still water is

(a) 5 km/h (b) 5.5 km/h
(c) 6 km/h (d) 6.5 km/h

48. Two trains are running on parallel lines in the same direction at speeds of 40 km/h and 20 km/h respectively. The faster train crosses a man in the second train in 36 seconds. The length of the faster train is

(a) 200 metres (b) 185 metres
(c) 225 metres (d) 210 metres

49. The speed of the boat in still water is 12 km/h and the speed of the stream is 2 km/h. A distance of 8 km, going upstream, is covered in

(a) 1 h (b) 1 h 15 min
(c) 1 h 12 min (d) None of these

50. A boat goes 15 km upstream in 60 minutes. The speed of the stream is 5 km/h. The speed of the boat in still water is

(a) 16.25 km/h (b) 30 km/h
(c) 35 km/h (d) 17 km/h

51. In a stream, B lies in between A and C such that it is equidistant from both A and C. A boat can go from A to B and back in 6 h 30 minutes while it goes from A to C in 9 h. How long would it take to go from C to A?

(a) 3.75 h (b) 4 h
(c) 4.25 h (d) 4.5 h

52. Two trains pass each other on parallel lines. Each train is 100 metres long. When they are going in the same direction, the faster one takes 60 seconds to pass the other completely. If they are going in opposite directions they pass each other completely in 10 seconds. Find the speed of the slower train in km/h

(a) 30 km/h (b) 42 km/h
(c) 48 km/h (d) 60 km/h

53. Vinay runs 100 metres in 20 seconds and Ajay runs the same distance in 25 seconds. By what distance will Vinay beat Ajay in a hundred metre race

(a) 10 m (b) 20 m (c) 25 m (d) 12 m

54. In a 100 m race, Shyam runs at 1.96 m/s. If Shyam gives Rajiv a start of 4 m and still beats him by 12 seconds, what is Rajiv's speed?
(a) 1.11 m/s (b) 0.73 m/s
(c) 1.33 m/s (d) 1 km/h

55. In a game of billiards, A can give B 15 points in 60 and A can give C 20 in 60. How many points can B give C in a game of 60?
(a) 13 (b) 15 (c) 10 (d) 14

56. In a 500 m race, the ratio of speed of two runners Vinay and Shyam is 5 : 4. If Vinay has a start of 120 m then Vinay wins by
(a) 15 m (b) 20 m (c) 25 m (d) 40 m

57. How many seconds will a certain 150 metres long running train take to pass a standing boy?
(a) 10 s (b) 12 s (c) 11 s (d) 14 s

58. Two trains are travelling in the same direction at 80 km/h and 30 km/h respectively. The faster train crosses a man in the slower train in 18 seconds. Find the length of the faster train.
(a) 0.7 km (b) 1 km
(c) 1.5 km (d) 1.4 km

59. Two trains for Howrah leave Muzaffarpur at 8.30 a.m. and 9.00 a.m. respectively and travel at 60 km/h and 70 km/h respectively. How many kilometres from Muzaffarpur will the two trains meet?
(a) 210 km (b) 180 km
(c) 150 km (d) 120 km

60. Without stoppage, a train travels at an average speed of 75 km/h and with stoppages it covers the same distance at an average speed of 60 km/h. How many minutes per hour does the train stop?
(a) 10 minutes (b) 12 minutes
(c) 14 minutes (d) 18 minutes

61. A boat rows 16 km up the stream and 16 km downstream taking 3 h each time. The velocity of the current
(a) 1.1 km/h (b) 1.2 km/h
(c) 1.4 km/h (d) 1.5 km/h

62. Vijay can row a certain distance downstream in 6 h and return the same distance in 9 h. If the stream flows at the rate of 3 km/h, find the speed of Vijay in still water.
(a) 15 km/h (b) 13 km/h
(c) 14 km/h (d) 15 km/h

63. Subhas can row 6 km/h in still water. When the river is running at 1.2 km/h, it takes him 1 hour to row to a place and back. How far is the place?
(a) 2.88 km (b) 2.80 km
(c) 3.12 km (d) 2.76 km

64. A dog is passed by a train in 8 seconds. Find the length of the train if its speed is 36 km/h.
(a) 70 m (b) 80 m
(c) 65 m (d) 90 m

65. A boy man can row upstream at 16 km/h and downstream at 22 km/h. Find the man's rate in still water (in km/h).
(a) 19 (b) 18 (c) 17 (d) 18

66. A man can row 30 km upstream and 44 km downstream in 10 hours. It is also known that he can row 20 km upstream and 55 km downstream in 13 hours. Find the speed of the man in still water.
(a) 4 km/h (b) 6 km/h
(c) 8 km/h (d) 12 km/h

67. In a stream that is running at 2 km/h, a man goes 8 km upstream and comes back to the starting point in 55 minutes. Find the speed of the man in still water.
(a) 20 km/h (b) 22 km/h
(c) 24 km/h (d) 25 km/h

68. A man goes down stream in x km/h and upstream at y km/h. The speed of the boat in still water is
(a) $0.5(x+y)$ (b) $0.5(x-y)$
(c) $x+y$ (d) $x-y$

69. The length of the minutes hand of a clock is 8 cm. Find the distance travelled by its outer end in 15 minutes.
(a) 4π cm (b) 8π cm
(c) 12π cm (d) 16π cm

70. Between 5 a.m. and 3 p.m. of a particular day how many times are the minute and the hour hand together?
(a) 11 (b) 22 (c) 33 (d) 44

71. At what time are the hands of clock together between 2 and 3 p.m.?
(a) $2\frac{10}{11}$ hours past 2 p.m.
(b) $2\frac{9}{11}$ hours past 2 p.m.
(c) 2:10 p.m.
(d) None of these

72. A motorboat went down the river for 14 km and then up the river for 6 km. It took a total of 5 hours for it

entire journey. Find the speed of the river flow if the speed of the boat in still water is 5 km/h.

(a) 1 kmph (b) 1.5 kmph
(c) 2 kmph (d) 3 kmph

73. A motorboat whose speed in still water is 10 km/h went 91 km downstream and then returned to its starting point. Calculate the speed of the river flow if the round trip took a total of 20 hours.

(a) 3 km/h (b) 4 km/h
(c) 6 km/h (d) 8 km/h

74. In a race of 600 metres, Ajay beats Vijay by 60 metres and in a race of 500 metres Vijay beats Anjay by 25 metres. By how many metres will Ajay beat Anjay in a 400 metre race?

(a) 48 m (b) 52 m (c) 56 m (d) 58 m

75. A motorboat whose speed in still water is 15 kmph goes 30 km downstream and comes back in a total 4 hours 30 min. Determine the speed of the stream.

(a) 2 kmph (b) 3 kmph
(c) 4 kmph (d) 5 kmph

Level of Difficulty (LOD) II

1. The J&K express from Delhi to Srinagar was delayed by snowfall for 16 minutes and made up for the delay on a section of 80 km travelling with a speed 10 km per hour higher than its normal speed. Find the original speed of the J&K express (according to the schedule)

(a) 60 km/h (b) 66.66 km/h
(c) 50 km/h (d) 40 km/h
(e) None of these

2. Ayrton Senna had to cover a distance of 60 km. However, he started 6 minutes later than his scheduled time and raced at a speed 1 km/h higher than his originally planned speed and reached the finish at the time he would reach it if he began to race strictly at the appointed time and raced with the assumed speed. Find the speed at which he travelled during the journey described.

(a) 25 km/h (b) 15 km/h
(c) 10 km/h (d) 6 km/h
(e) 24 km/h

3. Amitabh covered a distance of 96 km two hours faster than he had planned to. This he achieved by travelling 1 km more every hour than he intended to cover every 1 hour 15 minutes. What was the speed at which Amitabh travelled during the journey?

(a) 16 km/h (b) 26 km/h
(c) 36 km/h (d) 30 km/h
(e) None of these

4. An urgent message had to be delivered from the house of the Peshwas in Pune to Shivaji who was camping in Bangalore. A horse rider travels on horse back from Pune to Bangalore at a constant speed. If the horse increased its speed by 6 km/h, it would take the rider 4 hours less to cover that distance. And travelling with a speed 6 km/h lower than the initial speed, it would take him 10 hours more than the time he would have taken had he travelled at a speed 6 kmph higher than the initial speed. Find the distance between Pune and Bangalore.

(a) 120 km (b) 600 km
(c) 720 km (d) 750 km
(e) 760 km

5. A pedestrian and a cyclist start simultaneously towards each other from Aurangabad and Paithan which are 40 km apart and meet 2 hours after the start. Then they resumed their trips and the cyclist arrives at Aurangabad 7 hours 30 minutes earlier than the pedestrian arrives at Paithan. Which of these could be the speed of the pedestrian?

(a) 4 km/h (b) 5 km/h
(c) 3 km/h (d) 6 km/h
(e) Cannot be determined

6. Two motorists met at 10 a.m. at the Dadar railway station. After their meeting, one of them proceeded in the East direction while the other proceeded in the North direction. Exactly at noon, they were 60 km apart. Find the speed of the slower motorist if the difference of their speeds is 6 km/h.

(a) 28 km/h (b) 18 km/h
(c) 9 km/h (d) 19 km/h
(e) 20 km/h

7. Two cyclists start simultaneously towards each other from Aurangabad and Ellora, which are 28 km apart. An hour later they meet and keep pedalling with the same speed without stopping. The second cyclist arrives at Ellora 35 minutes later than the first arrives at

hourly speed. Find the speed of the cyclist who started from Kilkee.

(a) 12 km/h (b) 14 km/h
(c) 15 km/h (d) 10 km/h
(e) None of these

8. Two cars start simultaneously from two cities towards each other. The first car covers 8% of the distance between the two cities in 5 hours, the second car covered $\frac{7}{120}$ of the distance in 2 hours 30 minutes. Find the speed (km/h) of the second car if the first car travelled 400 km to the meeting point.

(a) 15 km/h (b) 25 km/h
(c) 42 km/h (d) 35 km/h
(e) 36 km/h

9. A bus left point X for point Y. Two hours later a car left point X for Y and arrived at Y at the same time as the bus. If the car and the bus left simultaneously from the opposite ends X and Y towards each other, they would meet 1.33 hours after the start. How much time did it take the bus to travel from X to Y?

(a) 2 h (b) 4 h (c) 6 h (d) 8 h
(e) 10 h

10. A racetrack is in the form of a right triangle. The length of the legs of the track is 7 km more than the shorter of the legs (both these legs being on a highway). The start and end points are also connected to each other through a side road. The event vehicle for the race took the side road and rode with a speed of 36 km/h and then covered the two sides along the highway during the same time with a speed of 42 km/h. Find the length of the racetrack.

(a) 14 km (b) 10 km
(c) 24 km (d) 36 km
(e) 30 km

11. Two planes move along a circle of circumference 1.2 km with constant speeds. When they move in different directions, they meet every 12 seconds and when they move in the same direction, one plane overtakes the other every 90 seconds. Find the speed of the slower plane.

(a) 0.08 km/s (b) 0.03 km/s
(c) 0.05 km/s (d) 0.02 km/s
(e) None of these

12. Karim, a tourist, leaves Kilkee on a bicycle. Having travelled for 1.5 h at 16 km/h, he makes a stop for 1.5 h and then pedals on with the same speed. Four hours after Karim started, his friend and local guide Rahim leaves Kilkee on a motorcycle and rides with a speed of 28 km/h in the same direction as Karim had gone. What distance will they cover before Rahim overtakes Karim?

(a) 88 km (b) 90.33 km
(c) 93.33 km (d) 96.66 km
(e) None of these

13. A tourist covered a journey partly by foot and partly by tonga. He walked for 90 km and rode the tonga for 10 km. He spent 4 h less on the tonga than on walking. If the tourist had reversed the times he travelled by foot and on tonga, the distances travelled on each part of the journey would be equal. How long did he ride the tonga?

(a) He rode for 6 hours (b) He rode for 4 hours
(c) He rode for 2 hours (d) He rode for 5 hours
(e) None of these

14. Two Indian tourists in the US cycled towards each other, one from point A and the other from point B. The first tourist left point A 6 hrs later than the second left point B, and it turned out on their meeting that he had travelled 12 km less than the second tourist. After

second arrived at A.
faster tourist.

(b) 5 km/h

when he was 2 km short of Noida. If the first pilot jogged as many kilometres as the second and the second as many kilometres as the first, the first one would need 15 min less than the second. Find the distance between Delhi and Noida?

(a) 5 km (b) 15 km (c) 25 km (d) 35 km
(e) 24 km

16. A tank of 4800 m³ capacity is full of water. The discharging capacity of the pump is 10 m³/min higher than its filling capacity. As a result the pump needs 8 min less to discharge the fluid than to fill up the tank. Find the filling capacity of the pump.

(a) 30 m/min (b) 25 m/min
(c) 35 m/min (d) 24 m/min
(e) None of these

17. An ant climbing up a vertical pole ascends 12 metres and then down 5 metres in every alternate hour. If the pole is 63 metres high how long will it take it to reach the top?
(a) 14 hours
(b) 17 hours
(c) 16 hours 35 minutes
(d) 16 hours 40 minutes
(e) None of these

18. Two ports A and B are 500 km apart. Two ships leave A for B such that the second leaves 8 hours after the first. The ships arrive at B simultaneously. Find the time the slower ship spent on the trip, if the speed of one of them is 10 kmph higher than that of the other.
(a) 25 hours (b) 15 hours
(c) 10 hours (d) 20 hours
(e) 24 hours

19. An ant moved for several seconds and covered 3 mm in the first second and 4 mm more in each successive second than in its predecessor. If the ant had covered 1 mm in the first second and 8 mm more in each successive second, then the difference between the path it would cover during the same time and the actual path would be more than 6 mm but less than 30 mm. Find the time for which the ant moved (in seconds)
(a) 2 s (b) 4 s
(c) 6 s (d) 7 s
(e) None of these

20. The Sabarmati Express left Ahmedabad for Mumbai. Having travelled 300 km, which constitutes 60.606 per cent of the distance between Ahmedabad and Mumbai, the train was dropped by a red signal. Half an hour later, the track was cleared and the engine-driver, having increased the speed by 15 km per hour, arrived at Mumbai on time. Find the initial speed of the Sabarmati Express.
(a) 30 kmph (b) 60 kmph
(c) 35 kmph (d) 40 kmph
(e) 48 kmph

21. Two swimmers started simultaneously from the beach out to the south and the other to the east. Two hours later, the distance between them turned out to be 100 km. Find the speed of the faster swimmer, knowing that the speed of one of them was 75% of the speed of the other.
(a) 30 kmph (b) 40 kmph
(c) 45 kmph (d) 60 kmph
(e) 48 kmph

22. A motorcyclist left point A for point B. Two hours later, another motorcyclist left A for B and arrived at B at the same time as the first motorcyclist. Had both the motorcyclists started simultaneously from A and B travelling towards each other, they would have met in 80 minutes. How much time did it take the faster motorcyclist to travel from A to B?
(a) 6 hours (b) 3 hours
(c) 2 hours (d) 4 hours
(e) 5 hours

23. Two horses started simultaneously towards each other and meet each other 5 h 20 min later. How much time will it take the slower horse to cover the whole distance if the first arrived at the place of departure of the second 5 hours later than the second arrived at the point of departure of the first?
(a) 10 hours (b) 8 hours
(c) 15 hours (d) 6 hours
(e) None of these

24. The difference between the times taken by two buses to travel a distance of 350 km is 7 hours 20 minutes. If the difference between their speeds is 5 kmph, find the slower speed.
(a) 35 kmph (b) 30 kmph
(c) 25 kmph (d) 20 kmph
(e) 24 kmph

25. One bad day, at 7 a.m. I started on my bike at the speed of 36 kmph to meet one of my relatives. After I had travelled some distance, my bike went out of order and I had to stop. After resting for 18 minutes, I returned home on foot at a speed of 14 kmph and reached home at 1 p.m. Find the distance from my home at which my bike broke down.
(a) 54 km (b) 63 km
(c) 72 km (d) 66 km
(e) None of these

Direction for Questions 26–30: Read the following data and answer the questions that follow.

Three brothers, Ram, Shyam and Mohan, travelled by road. They all left the college at the same time—12 noon.

The descriptions of the journeys of the three are detailed below:

Travel	Ram	Shyam	Mohan
Phase I	Bus for 2 hours @ 10 mph	Bike for 1 hours @ 10 mph	Bus for 2 hours @ 11 mph
Phase II	Bike for 1.5 hours @ 40 mph	Bus for 1 hours @ 13.3 mph	Bus for 1 hours @ 10 mph
Phase III	Bus for 1 hours @ 13.3 mph	Bus for 4 hours @ 10 mph	Bike for 2 hours @ 30 mph

26. When did Ram overtake Shyam?

(a) 3:15 p.m. (b) 2:22 p.m.
(c) 2:30 p.m. (d) 2:20 p.m.

27. At what distance from the start does Mohan overtake Shyam?

(a) 40 miles (b) 57 miles
(c) 70 miles (d) 80 miles

28. If Ram travelled by bike instead of bus in the last leg of his journey (for the same distance as he had covered by bus), what is the difference in the times of Ram and Mohan to cover 90 miles?

(a) 6 h 30 minutes (b) 10 h 40 minutes
(c) 2 hrs 55 minutes (d) 4 hrs 30 minutes

29. If all of them travelled a distance of 100 miles, who reached first and at what time (minute the last leg was increased to cover 100 miles)?

(a) Mohan at 6 p.m. (b) Ram at 8 p.m.
(c) Shyam at 6 p.m. (d) Mohan at 8 p.m.

30. In the above question, who reached last and at what time?

(a) Ram at 9:30 p.m. (b) Ram at 10 p.m.
(c) Shyam at 9:30 p.m. (d) Shyam at 10 p.m.

31. A motorcyclist rode the first half of his way at a constant speed. Then he was delayed for 5 minutes and therefore, to make up for the lost time he increased his speed by 10 km/h. Find the initial speed of the motorcyclist if the total path covered by him is equal to 50 km.

(a) 36 km/h (b) 46 km/h
(c) 38 km/h (d) 42 km/h

32. Ram Singh and Priyadarshan start together from the same point on a circular path and walk around, each at his own pace, until both arrive together at the starting point. If Ram Singh performs the circuit in 3 minutes 44 seconds and Priyadarshan in 5 minutes 4 seconds, how many times does Ram Singh go around the way?

(a) 8 (b) 13
(c) 15 (d) Cannot be determined

33. Ravi, who lives in the countryside, caught a train he home earlier than usual yesterday. His wife normally drives to the station to meet him. But yesterday he ran on foot from the station to meet his wife on the way. She reached home 12 minutes earlier than he would have done had he waited at the station for his wife. The car travels at a uniform speed, which is 7 times Ravi's speed on foot. Ravi reached home at exactly 6 O' clock. At what time would he have reached home if his wife, forewarned of his plan, had not left the station?

(a) 5 : 48 (b) 5 : 24 (c) 5 : 00 (d) 5 : 36

34. Hemant and Ajay start a two-length swimming race at the same moment but from opposite ends of the pool. They swim in time and at uniform speeds, but Hemant is faster than Ajay. They first pass at a point 18.5 m from the deep end and having completed one length, each rest is allowed to rest on the edge for exactly 45 seconds. After setting off on the return length, the swimmers pass for the second time just 10.5 m from the shallow end. How long is the pool?

(a) 55.5 m (b) 45 m (c) 66 m (d) 49 m

35. Rahim sets out to cross a forest. On the first day he completes 1/40th of the journey. On the second day he covers 2/3rd of the distance travelled the first day. He continues in this manner, alternating the days on which he travels 1/10th of the distance still to be covered with days on which he travels 2/3 of the total distance already covered. At the end of seventh day, he finds that 7½ km more will see the end of his journey. How wide is the forest?

(a) 86¾ km (b) 100 km
(c) 120 km (d) 150 km

36. The metro service has a train going from Mumbai to Pune and Pune to Mumbai every hour, the first one at 6 a.m. The trip from one city to other takes 4½ hours and all trains travel at the same speed. How many trains will you pass while going from Mumbai to Pune if you start at 12 noon?

(a) 6 (b) 10 (c) 9 (d) 13

33. The distance between two towns is x km. A car travelling between the towns covers the first x km at an average speed of y km/h and the remaining distance x y km/h. The time taken for the journey is

(a) $\frac{x}{y} + \frac{(x+y)}{y}$ (b) $\frac{x}{y} + \frac{(x-y)}{y}$

(c) $\frac{x}{y} + \frac{(x-y)}{x}$ (d) $\frac{2x}{y} + (x-y)$

34. Two trains are fired from the same place at a difference of 11 minutes 45 seconds. But a man who is coming towards the place in a train hears the second sound after 18 minutes. Find the speed of train.

(a) 72 km/h (b) 36 km/h
(c) 81 km/h (d) 108 km/h

Direction for Questions 35–40: *Read the following and answer the questions that follow.*

The Kalinga Express started from Puri to Tata at 7 p.m. at a speed of 60 km/h. Another train, Rajdhani Express, started from Tata to Puri at 4 a.m. next morning at a speed of 80 km/h. The distance between Puri to Tata is 800 km.

35. How far from Tata will the two trains meet?

(a) 384 km (b) 156 km
(c) 192 km (d) 134 km

36. At what time will the two trains meet?

(a) 5 : 32 a.m. (b) 5 : 28 a.m.
(c) 5 : 36 a.m. (d) 5 : 44 a.m.

Direction for Questions 41–43: *Read the following and answer the questions that follow.*

A mighty bird is sitting on top of a car. It sees another car approaching it at a distance of 12 km. The speed of the two cars is 60 km/h each. The bird starts flying from the first car and moves towards the second car, reaches the second car and comes back to the first car and so on. If the speed at which the bird flies is 120 km/h then answer the following questions. Assume that the two cars have a crash.

41. The total distance travelled by the bird before the crash is

(a) 6 km (b) 12 km
(c) 18 km (d) None of these

42. The total distance travelled by the bird before it reaches the second car for the second time is

(a) 10.95 km (b) 11.55 km
(c) 12.35 km (d) None of these

43. The total number of times that the bird reaches the front of the second car is (approximately)

(a) 12 times (b) 18 times
(c) Infinite times (d) Cannot be determined

44. A dog sees a cat. It estimates that the cat is 25 leaps away. The cat sees the dog and starts running with the dog in hot pursuit. If in every minute, the dog makes 5 leaps and the cat makes 6 leaps and one leap of the dog is equal to 2 leaps of the cat. Find the time in which the cat is caught by the dog (assume an open field with no trees)

(a) 12 minutes (b) 15 minutes
(c) 12.5 minutes (d) None of these

45. Two people A and B start from P and Q (distance = 10) at the same time towards each other. They meet at a point R, which is at a distance 0.4D from P. They continue to move to and fro between the two points. Find the distance from point P at which the fourth meeting takes place.

(a) 0.8D (b) 0.6D (c) 0.3D (d) 0.4D

46. A wall clock gains 2 minutes in 12 hours, while a table clock loses 2 minutes in 36 hours, both are set right at noon on Tuesday. The correct time when they both show the same time next would be

(a) 12.30 night (b) 12 noon
(c) 1.30 night (d) 12 night

47. Two points A and B are located 48 km apart on the riverbank. A motorboat must go from A to B and return to A as soon as possible. The river flows at 6 km/h. What must be the least speed of the motorboat in still water for the trip from A to B and back again to be completed in not more than six hours (assume that the motorboat does not stop at B)?

(a) 18 km/h (b) 36 km/h
(c) 27 km/h (d) 48 km/h

48. Two ghats are located on a riverbank and are 24 km apart. Leaving one of the ghats for the other, a motorboat returns to the first ghat in 270 minutes, spending 90 min of that time in taking the passengers at the second ghat. Find the speed of the boat in still water if the speed of the river flow is 2.5 km/h?

(a) 16.4 km/h (b) 12.5 km/h
(c) 22.5 km/h (d) 12.5 km/h

49. A train leaves Muzaffarpur for Hajipur at 2.14 p.m. and travels at the rate of 50 km/h. Another train leaves

Hampi for Muzaffarpur at 1.35 p.m. and travels at the rate of 40 km/h. If the distance between Hampi and Muzaffarpur is 390 km, at what distance from Muzaffarpur will the two trains meet?

(a) 200 km (b) 300 km
(c) 250 km (d) 225 km

80. A dog after travelling 50 km meets a worm who constantly has to go slower. He then proceeds at 3/4 of his former speed and arrives at his destination 35 minutes late. Had the meeting occurred 24 km further the dog would have reached its destination 25 minutes late. The speed of the dog is

(a) 48 km/h (b) 56 km/h
(c) 54 km/h (d) 58 km/h

Level of Difficulty (LOD) III

1. Two people started simultaneously towards each other from Siliguri and Darjeeling, which are 60 km apart. They met 5 hours later. After their meeting, the first person, who travelled from Siliguri to Darjeeling, decreased his speed by 1.5 km/h and the other person, who travelled from Darjeeling to Siliguri, increased his speed by 1.5 km/h. The first person is known to arrive at Darjeeling 2.5 hours earlier than the second person arrived at Siliguri. Find the initial speed of the first person.

(a) 6.5 km/h (b) 8 km/h
(c) 7.5 km/h (d) 9 km/h
(e) 8 km/h

2. Two friends, Arun and Nishit, on their last day in college, decided to meet after 20 years on a river. Arun had to sail 42 km to the meeting place and Nishit had to sail 33⅓ per cent less. To arrive at the meeting place at the same time as his friend Nishit, Arun started at the same time as Nishit and sailed with the speed exceeding by 3 km/h the speed of Nishit. Find the speed of Arun.

(a) 10 km/h (b) 14 km/h
(c) 9 km/h (d) Both b and c
(e) None of these

3. Three cars leave Patna for Ranchi after equal time intervals. They reach Ranchi simultaneously and then leave for Vizag, which is 120 km from Ranchi. The first car arrives there an hour after the second car, and the third car, having reached Vizag, immediately reverses the direction and 40 km from Vizag meets the first car. Find the speed of the first car.

(a) 30 km/h (b) 19 km/h
(c) 32 km/h (d) 22 km/h
(e) 36 km/h

4. Two sea trawlers left a sea port simultaneously in two mutually perpendicular directions. Half an hour later the shortest distance between them was 17 km, and another 15 min later, one sea trawler was 10.5 km farther from the origin than the other. Find the speed of each sea trawler.

(a) 36 km/h, 30 km/h (b) 18 km/h, 24 km/h
(c) 30 km/h, 22 km/h (d) 18 km/h, 36 km/h
(e) None of these

5. Shaurya and Arjit take a straight route to the same terminal point and travel with constant speeds. At the initial moment, the positions of the two and the terminal point form an equilateral triangle. When Arjit covered a distance of 80 km, the triangle becomes right angled. When Arjit was at a distance of 120 km from the terminal point, the Shaurya arrived at the point. Find the distance between them at the initial moment assuming that there are integral distances throughout the movements described.

(a) 300 km (b) 240 km
(c) 200 km (d) 225 km
(e) 275 km

6. Mrinalini and Neha travel to Connaught Place along two straight roads with constant speeds. At the initial moment, the positions of Mrinalini, Neha and Connaught Place forms a right triangle. After Mrinalini travelled 30 km, the triangle between the points became equilateral. Find the distance between Mrinalini and Neha at the initial moment if at the time Mrinalini arrived at Connaught Place, Neha had to cover 20 km to reach Connaught Place.

(a) $10\sqrt{3}$ km (b) $12\sqrt{3}$ km
(c) $30\sqrt{5}$ km (d) $20\sqrt{3}$ km
(e) None of these

7. Three cars started simultaneously from Agra to Benares along the same highways. The second car travelled with a speed that was 10 km/h higher than the first car's speed and arrived at Benares 1 hour earlier than the first car. The third car arrived at Benares

33.33 minutes earlier than the first car, travelling half the time at the speed of the first car and the other half at the speed of the second car. Find the total distance covered by these three cars during their journey between Ajmer and Bonsara.

(a) 260 km (b) 400 km
(c) 540 km (d) 840 km
(e) 900 km

8. Three sprinters A, B, and C had to sprint from points P to Q and back again (starting in this order). The time interval between the starting times of the three sprinters A, B and C was 5 seconds each. Thus C started 10 seconds after A, while B started 5 seconds after A. The three sprinters passed a certain point R, which is somewhere between P and Q, simultaneously (one of them having reached point Q yet). Having reached Q and reversed the direction, the third sprinter met the second one 5 m short of Q and met the first sprinter 15 m short of Q. Find the speed of the first sprinter if the distance between PQ is equal to 55 m.
(a) 4 m/s (b) 5 m/s (c) 2 m/s (d) 1 m/s
(e) None of these

9. Two trains start from the same point simultaneously and in the same direction. The first train travels at 40 km/h, and the speed of the second train is 25 per cent more than the speed of the first train. Thirty minutes later, a third train starts from the same point and in the same direction. It overtakes the second train 90 minutes later than it overtakes the first train. What is the speed of the third train?
(a) 20 km/h (b) 40 km/h
(c) 60 km/h (d) 80 km/h
(e) 50 km/h

10. A passenger train left town Alpha for town Beta. At the same time, a goods train left Beta for Alpha. The speed of each train is constant throughout the whole trip. Two hours after the trains met, they were 450 km apart. The passenger train arrived at the place of destination 18 hours after their meeting and the goods train 32 hours after the meeting. How long did it take the passenger train to make the whole trip?
(a) 22 hours (b) 28 hours
(c) 14 hours (d) 22 hours
(e) None of these

11. Two ducks move along the circumference of a circular pond in the same direction and come alongside each other every 54 minutes. If they moved with the same speeds in the opposite directions, they would meet every 9 minutes. It is known that when the ducks moved along the circumference in opposite directions, the distance between them decreased from 54 to 14 feet every 48 seconds. What is the speed of the slower duck?
(a) 20 feet/min (b) 15 feet/min
(c) 30 feet/min (d) 20.83 feet/min
(e) 18.33 feet/min

12. Dev and Nidhi started simultaneously from opposite points X and Y on a straight road, at constant speeds. When Dev had covered 40% of the distance from X to Y, Nidhi was 4 km away from Dev after having crossed Dev. When Nidhi had covered half the way, Dev was 10 km short of the mid point. Find a possible of the time it took Dev to cover the distance from X to Y to the time it took Nidhi to cover the same distance.

(a) $\left[\frac{1+\sqrt{3}}{2}\right]$ (b) $\frac{3\sqrt{2}+2}{2\sqrt{2}+1}$
(c) Either a or b (d) None of these
(e) Data insufficient

13. For question 12 which of these is a possible value of the distance between the points X and Y
(a) $20(2+\sqrt{3})$ (b) $20(1+\sqrt{2})$
(c) 60 km (d) Both (a) and (b)

14. Two ships sail in a fog towards each other with the same speed. When they are 4 km apart, the captains decelerate the engines for 4 minutes with a deceleration rate of 0.1 m/s², and then the ships continue sailing with the speeds attained. For what range of values of the initial speed V_0 will the ships avoid collision?
(a) $0 < V_0 < 30$ m/s (b) $0 < V_0 < 20$ m/s
(c) $0 < V_0 < 50$ m/s (d) None of these
(e) Cannot be determined

15. Three points A, B and C are located at the vertices of an equilateral triangle with sides equal to 168 metres. A donkey called Dinky starts from A to B at 60 metres/hour and at the same time a cow called Moo starts from B to C at 30 metres/hour. In what time after their departure will the distance between the donkey and the cow be the least?
(a) 2 h (b) 3 h (c) 0.5 h (d) 8 h
(e) 4 h

16. A man wants to travel the distance between Aurangabad and Dauladabad, equal to 20 km, at a constant speed. He travelled half the way with the specified speed and stopped for three minutes, to arrive at Dauladabad on time. It had to increase its speed by 10 kmph for the rest of the way. Next time the train stopped half way for five minutes. By what speed must it increase its speed for the remaining half of the distance to arrive at Dauladabad as per the schedule?

(a) 10 kmph (b) 20 kmph
(c) 15 kmph (d) 16 kmph
(e) Cannot be determined

17. Nishit travels from Patna to Kolkata, a distance of 200 km at the speed of 40 kmph. At the same time, Ravi starts from Kolkata at a speed of 20 kmph along a road, which is perpendicular to the road on which Nishit is travelling. When will Nishit and Ravi be closest to each other?

(a) In 1.5 hours (b) In 4 hours
(c) In 2.25 hours (d) In 5 hours
(e) In 6 hours

18. Two towns are at a distance of 240 km from each other. A motorist takes 8 hours to cover the distance if he travels at a speed of V_0 kmph from town A to an intermediate town C, and then continues on his way with an acceleration of x kmph^2. He needs the same time to cover the whole distance if he travels from A to C at V_1 kmph and from C to B at V_1 kmph or from A to C at V_2 kmph and from C to B at V_2 kmph. Find V_0 if the acceleration x is double V_0 in magnitude and $V_1 \neq V_2$.

(a) 15 kmph (b) 10 kmph
(c) 20 kmph (d) 8 kmph
(e) None of these

19. Sailing Ahoy! from Alaska, which is on a highway to Bangor, which is 36 km from the highway. The distance between Alaska and Bangor along a straight line is 34 km. At what point should Sailing must from the highway to reach Bangor in the shortest possible time, if his speed along the highway is 10 kmph and 6 km/h otherwise.

(a) 30 km away from A
(b) 20 km away from A
(c) 14 km away from A
(d) 16 km away from A
(e) None of these

20. An object begins moving at time instant $t = 0$ and t s after the beginning of the motion, attains the distance of t m/s^2. Find the speed of the object t s after the beginning of motion if it is known that the speed of the body varies accordingly to the law $v(t) = t^2 + 2t + 4$ m/s and the object moves along a straight line.

(a) 22 m/s (b) 10 m/s
(c) 30 m/s (d) 15 m/s
(e) Data insufficient

21. For problem 20, find the distance covered by the object in the first 7 seconds if we assume that the speed of the object in a particular second is the speed it attains at the start of the second.

(a) 15 metres (b) 14 metres
(c) 10 metres (d) 5 metres
(e) 8 metres

22. Three ghats X, Y and Z on the Yamuna at Delhi are located on the river bank. The speed of the river flow is 6 kmph in the direction of its flow. Ghat Y being located midway between X and Z. A raft and a launch leave Y at the same time, the raft travelling towards river to Z and the launch travelling to X. The speed of the launch in still water is 5 kmph. On reaching X, the launch reverses its direction and starts to Z. For the range of values of V for which the launch arrive at Z later than the raft.

(a) $8 < V < 24$ kmph (b) $8 < V < 36$ kmph
(c) $8 < V < 20$ kmph (d) $12 < V < 24$ kmph
(e) None of these

23. A pedestrian left point A for a walk, going with the speed of 5 kmph. When the pedestrian was at a distance of 6 km from A, a cyclist followed him starting from A and cycling at a speed 9 kmph higher than that of the pedestrian. When the cyclist overtook the pedestrian, they turned back and returned to A together at the speed of 4 kmph. At what v will the time spent by the pedestrian on his total journey from A to A be the least?

(a) 7 kmph
(b) 6.1 kmph (b) 6 kmph
(c) 5.5 kmph (d) 5.5 kmph
(e) Cannot be determined

24. A cyclist left point A for point B and travelled at the constant speed of 25 kmph. When he covered the distance of 8.33 km, he was overtaken by a car that left point A twelve minutes after the cyclist and trav

...ched at a constant speed time. When the cyclist travelled another 30 km, he encountered the car returning from B. Assume that the car did not stop at point B. Find the distance between A and B.

(a) 46.667 km (b) 41.0833 km
(c) 40.833 km (d) 43.33 km
(e) None of these

25. A robot began moving from point A in a straight line at 8 p.m. with an initial speed of 3 m/s. One second later, the speed of the robot became equal to 4 m/s. Find the acceleration of the robot at the end of the 2nd second if its speed changes by the law $v(t) = v(t^2 + 2t + 1)$

(a) 3 m/s^2 (b) −2 m/s^2
(c) 0 m/s^2 (d) 2 m/s^2
(e) None of these

26. For question 25, the distance of the robot from point A after 6 seconds will be (assuming that for every second the robot travels at a constant speed equal to its starting speed for that second and any acceleration occurs at the start of the next second)

(a) 48 m (b) 36 m (c) 90 m (d) 17 m

27. Two friends Amit and Akshay began moving simultaneously from point A along a straight line in the same direction. Amit started moving at the speed of 5 m/s and moved with an uniform acceleration of 4 m/s^2, while his friend Akshay moved at a uniform speed. The limits for Akshay's speed (S) so that he should first leave Amit behind and then get overtaken by Amit at a distance of 18 m from A are

(a) $5 < S < 9$ (b) $3 < S < 8$
(c) $4 < S < 8$ (d) None of these

28. On the banks of the river Ganges there are two bathing points in Varanasi and Patna. A ship left in the river in Varanasi reaches Patna in 24 hours. However, a motorboat covers the whole way to and fro in exactly 18 hours. If the speed of the motorboat in still water is increased by 40%, then it takes the motorboat 7 hours to cover the same way (from Varanasi to Patna and back again). Find the time necessary for the motorboat to sail from Varanasi to Patna when its speed in still water is not increased.

(a) 3 hours (b) 4 hours
(c) 4.8 hours (d) None of these

29. Two friends started walking simultaneously from points A and B towards each other. 144 minutes later the distance between them was 20% of the original distance. How many hours does it take the faster walker to cover the distance AB if he made eight hours less to travel the distance than his friend (assume all times to be in whole numbers and in hours)?

(a) 5 hours (b) 6 hours
(c) 12 hours (d) 4 hours

30. Two cars left points A and B simultaneously travelling towards each other. 9 hours after their meeting, the car travelling from A arrived at B, and 16 hours after their meeting, the car travelling from B arrived at A. How many hours did it take the slower car to cover the whole distance?

(a) 36 hours (b) 24 hours
(c) 25 hours (d) 28 hours

31. A pedestrian and a cyclist left Nagpur for Bari Bari at the same time. Having reached Bari Bari, the cyclist turned back and met the pedestrian an hour after the start. After their meeting, the pedestrian continued his trip to Bari Bari and the cyclist turned back and also headed for Bari Bari. Having reached Bari Bari, the cyclist turned back again and met the pedestrian 30 mins after their first meeting. Determine what time it takes the pedestrian to cover the distance between Nagpur and Bari Bari.

(a) 1 hour (b) 2 hours
(c) 2.5 hours (d) 3 hours

32. Points A, B and C are at the distances of 120, 104.66 and 112 km from point M respectively. Three people left these points for point M simultaneously: the first person started from point A, the second from B and the third from C. The first person covered the whole way at a constant speed and arrived at M an hour before the second and the third persons (who arrived simultaneously). The third person covered the whole way at a constant speed. The second person, having travelled 32 km at the same speed as the first, stopped for 2 hours. The rest of the way he travelled at a speed that is less than the speed of the third person by the same amount as the speed of the third is less than that of the first. Determine the speed of the first person.

(a) 6 km/h (b) 5 km/h
(c) 4 km/h (d) 3 km/h

33. Two people started simultaneously from points A and B towards each other. At the moment the person who started from A had covered two-thirds of the way, the

other person had covered 2 km less than half the total distance. If it is known that when the person who started from B had covered 1/4 of the way the other person was 3 km short of the mid point. Find the distance between A and B. The speeds of the two people were constant.

(a) $(15 - 2\sqrt{17})$ km (b) $(15 + 5\sqrt{17})$ km

(c) Both a and b (d) $5\sqrt{17} - 5$ km

34. Sohan and Lalan left their house simultaneously. Thirty six minutes later, Sohan met his uncle travelling in their house, while Lalan met the uncle twelve minutes after Sohan. Twenty four minutes after the meeting with Lalan, the uncle rang the door bell at Sohan and Lalan's house. Assume each person travels at a constant speed. Find the ratio of the speeds of Sohan to Lalan to the uncle.

(a) 5 : 2 : 3 (b) 1 : 3 : 2

(c) 3 : 1 : 3 (d) 2 : 1 : 2

35. The distance between two towns Aurangabad and Sohna is 80 km. A bus left Aurangabad and travelled at a constant speed towards Sohna. Thirty minutes later, Deepak Bhagbanwala left Aurangabad in his car towards Sohna. He overtook the bus in thirty minutes and continued on his way to Sohna. Without stopping at Sohna, he turned back and again encountered the bus 40 minutes after he had left Aurangabad. Determine the speed of the bus.

(a) 40 kmph (b) 45 kmph

(c) 50 kmph (d) None of these

36. Rohit left Mahabaleshwar for Nashik at 6 a.m. An hour and a half later Vimal, whose speed was 5 km/h higher than that of Rohit left Mahabaleshwar. At 10 : 30 a.m. of the same day the distance between the two friends was 23 km. Find the speed of Vimal.

(a) 40 kmph (b) 41 kmph

(c) 45 kmph (d) Either b or c

37. Aurangzeb and Babar with their troops left from Delhi and Dadanabad towards each other simultaneously. Each of them marched at a constant speed and, having arrived at their respective points of destination, went back at once. Their first meeting was 34 km from Dadanabad, and the second meeting, eight hours after the first meeting was at a distance of 2 km from Delhi. Find the distance between the Delhi and Dadanabad.

(a) 30 km (b) 25 km

(c) 36 km (d) None of these

38. Two trains left simultaneously point A for point B. The first train covers each kilometre 2 minutes faster than the second. After travelling 30 per cent of the way, the first train stopped for 8 minutes, then the train started for B. The second train stopped for 100 minutes and again started for B. The two trains arrived at B simultaneously. What is the distance between A and B if the second train covered 6 km in 1 hours?

(a) 60 km (b) 70 km

(c) 45 km (d) None of these

39. Amar and Akbar left Bhubaneshwar simultaneously and travelled towards Cuttack. Amar's speed was 10 km/h and that of Akbar was 12 km/h. Half an hour later, Anthony left Bhubaneshwar and travelled in the same direction. Some some hours he overtook Akbar and 80 minutes further on he overtook Amar. Find Anthony's speed.

(a) 18 km/h (b) 24 km/h

(c) 20 km/h (d) 16 km/h

40. Two bodies, moving along a circle in the same direction, meet every 49 minutes. Had they moved at the same speeds in the opposite directions, they would meet every 7 minutes. If, moving in the opposite directions, the bodies are at the distance of 40 m from each other along the arc at time $t = 0$, then at $t = 2$ seconds, their distance will be 26 metres (the bodies do not meet during these 24 seconds). Find the speed of the faster body in metres per minute.

(a) 15 (b) 20

(c) 25 (d) None of these

41. The distance between Varanasi and Lucknow is 75 km. Two buses start from these towns towards each other. They can meet halfway if the first bus leaves 2 hours earlier than the second. If they start simultaneously, they will meet in 4 hours. Find the speed of the buses.

(a) $(7.5(3 + \sqrt{5})$ km/h, $7.5(3 + \sqrt{5})$ km/h

(b) $(7.5(3 + \sqrt{5})$ km/h, $7.5(3 + \sqrt{5})$ km/h

(c) $(7.5(3 - \sqrt{5})$ km/h, $7.5(\sqrt{5} - 1)$ km/h

(d) None of these

42. A road passes through the towns Sangamner and Yeotmal. A cyclist started from Sangamner in the direction of Yeotmal. At the same time, two pedestrians

started from Nominal travelling at the same speed, the first of them towards Sangamner and the other in the opposite direction. The cyclist covered the distance between the towns in half an hour and, continued ahead in the same direction. He overtook the second pedestrian, 1.2 hours after he met the first pedestrian. Determine the time the cyclist spent travelling from Sangamner to the point of the meeting with the first pedestrian (assuming the speeds of the cyclist and the pedestrians to be constant).

(a) 24 min (b) 18 min
(c) 30 min (d) Cannot be determined

43. Two people A and B start moving from P and Q that are 200 km apart, towards each other. A is on a moped and B on foot. They meet at a point R when A gives B a lift to P and returns to his original path to reach Q. On reaching Q he finds that he has taken 2.6 times as normal time. B on the other hand realises that he has saved 40 minutes over his normal travel time. Find the ratio of their speeds.

(a) 3 : 2 (b) 4 : 3
(c) 3 : 4 (d) None of these

44. For question 43, find the speed of the moped.

(a) 160 kmph (b) 180 kmph
(c) 200 kmph (d) None of these

45. Three friends A, B and C start from P to Q that are 100 km apart. A is on a moped while B is riding pillion and C walks on. A takes B to a point R and returns to pick up C on the way and takes him to point Q. B, on the other hand, walks to Q from point R (R is the mid-point between P and Q).

If the ratio of speeds of the three people is 5 : 2 : 3, find where will the last person be when the first person reaches Q.

(a) 12.85 km from Q (b) 12.75 km from Q
(c) 16.66 km from Q (d) 83.33 km from P

46. A motorboat moves from point A to point B and back again, both points being located on the river-bank. If the speed of the boat in still water is doubled, then the trip from A to B and back again would take 20% of the time that the motorboat usually spends in the journey. How many times is the actual speed of the boat higher than the speed of the river flow?

(a) $\sqrt{\frac{3}{2}}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

47. A watch loses 2/3 min during the 24 week and gains 1/3 min during the next week. If it is a Sunday noon, it showed the right time, what time will it show at noon on the Sunday after the next.

(a) 11 : 28 : 24 a.m. (b) 10 : 52 : 48 a.m.
(c) 10 : 52 : 48 a.m. (d) 11 : 38 : 24 a.m.

48. Clock A, B and C strikes every hour. B slows down and takes 2 min longer than A per hour while C becomes faster and takes 2 min less than A per hour. If they strike together at 12 midnight, when will they strike together again.

(a) 10 a.m. (b) 11 a.m.
(c) 9 p.m. (d) 8 p.m.

49. A boat went down the river for a distance of 28 km. It then turned back and returned to its starting point, having travelled a total of 7 hours. On its return trip, at a distance of 17 km from the starting point, it encountered a log, which had passed the starting point at the moment at which the boat had turned downstream. The downstream speed of the boat is

(a) 7 kmph (b) 13 kmph
(c) 16 kmph (d) 10 kmph

50. Two boats go downstream from point X to point Y. The faster boat covers the distance from X to Y 1.5 times as fast as the slower boat. It is known that for every hour the slower boat lags behind the faster boat by 8 km. However, if they go upstream, then the faster boat covers the distance from Y to X in half the time as the slower boat. Find the speed of the faster boat in still water.

(a) 12 kmph (b) 30 kmph
(c) 24 kmph (d) 25 kmph

Hints and Solutions

1. The train takes 16 minutes to travelling faster over a section of 80 km.

Thus, $\frac{80}{S} - \frac{80}{S+30} = \frac{16}{60} = 0.2666$

Check the options.

2. $\frac{60}{S} - \frac{60}{S+1} = \frac{6}{60}$. Check the options to solve.

3. Solve through options.

4. Solve through options.
5. Relative speed (in opposite direction) = 20 km/hr
   Use the options after this.
6. The question is based on Pythagoras' aspects.
8. Assume the distance between the two gas holes is 600 feet. Then, the first ant's speed is 16 feet/hr while the second ant's speed is 14 feet/hr.
   If the first ant covers 800 feet, the second will cover 700 feet (since, distance is proportional to speed). Hence total distance is 1500 feet and required speed is $14 \times 2.5 = 35$ feet/hr.
9. Solve through options.
13. Solve using options as follows.
    Check for option (a) (the rule for 8 hours)
    Then, walking time = 10 hrs
    Then, speed of the tongs = 1.66 km/hr
    And speed of walking = 9 km/hr
    As per the question, if the times are reversed, the distances travelled by each mode will be equal.
    Then, 10×1.66 should be equal to 8×9
    But, this is not so. Hence, move to the next option.
14. Solve using options.
15. Solve using options; the following equations

    Speed of first = $\left(\frac{x-1}{42} \times 60\right)$ km/hr = S_1

    Speed of second = $\frac{x-2}{42} \times 60$ km/hr = S_2

    Then, $\frac{x-2}{S_2} - \frac{x-1}{S_1} = \frac{17}{60}$
19. The first pattern of movement is
    $3 + 7 + 11 + 15 + 19 + 23$
    The second pattern of movement is
    $1 + 9 + 17 + 25 + 33 + 41$
    It is evident that the difference in the net movements between the second pattern and the first pattern is more than 6 min and less than 30 min for a movement of 4 seconds.
20. By increasing speed by 15 kmph, the time reduces by 30 minutes.
22. Given that they meet in 40 minutes, when moving towards each other, the sum of their speeds should be such that they cover 1.25% of the distance per minute (i.e. 75% of the distance per hour).
23. Ratio of times taken in going and coming back is 7 : 10. Also, total time of travel is 325 minutes.
28-30 Chart out the distances covered by the three in each leg of their journeys.
32. The answer would have been 13, if the question had stated the initial directions of their walking. Hence, we cannot determine the answer.
33. Solve on the basis of the speed he would be with a tailwind.
36. The train will meet all trains that start from Pune between 8 a.m. and 4 p.m.
38. Assume speed of sound = 330 m/s.
    Then, distance covered by sound in 45 seconds = distance covered by the trains in 11 minutes.
39-40 Use the concept of relative speed to solve.
41. The bird will fly for the time which the two cars take to approaching one another.
42. Chart the relative movements of the two cars and the bird.
46. The wall clock gains 6 minutes in 36 hours, while the table clock loses 2 minutes in 36 hours. Hence, the differential is 36 hours = 8 minutes. For them to show the same time again, we need a total time difference of 12 hours.
48. Solve through options.

Hints and Solutions

1. The first person who travels from Siliguri to Darjeeling is obviously faster than the second. Hence, reject options (a) and (b). Then, check all the conditions with options (c) and (d) and select the one which satisfies the conditions.

2. Since distance to be travelled by Nishu is $35\frac{5}{7}$% less, his speed will also be $35\frac{5}{7}$% less than Arun's speed. Check the options with the conditions that Arun's speed is 5 kmph higher than that of Nishu.

3. Let S_1, S_2 and S_3 be the speeds of the three cars.
   Then $\frac{120}{S_1} - \frac{120}{S_2} = 1$ hour ... (1)
   It is also known that the speed of the third car is double the speed of the first car.
   With these realisations, check for factors of 120 which can satisfy the equation above.
   (Note that in equations like (1) above, normally the respective values of S_1 and S_2 will be factors of 120.)

5. If the side of the initial equilateral triangle is S_1, then when Arun covers $(S_1 - 120)$ km, Shaurya covers S_1

adjacent. Also, when A(B) covers a distance of 80 kilometres, Rinaiya covers a distance such that the resultant triangle is right angled.

Check these conditions through options.

6. Solve through a process similar to the previous question.

7. If S_1 is the speed of the first car, then $(S_1 + 10)$ will be the second car's speed. If t_1 hours is the time required for the first car, then $(t_1 - 1)$ hours is the time required for the second car in covering the same distance, while

time of the third car is $\left(t_1 + \frac{83.33}{60}\right)$ hours.

Check these conditions through options.

10. The relative speed is 225 km/hr.

11. The ratio of the speeds of the clocks is 30 minutes. Hence, circumference $\times 9 = 30 = 450$ feet and difference of speeds $= \frac{450}{54} = 8.33$.

12. Use options to solve.

16. Through trial and error try to find the ratio of speed of the train, so that the first condition is met.

17. Use options to solve.

18. Let the distance $AC = d$

Then, $\frac{d}{V_b} + \frac{240 - d}{V_1} = \frac{d}{V_b} + \frac{240 - d}{V_2}$

If $V_b = V_1$, then the above condition will be satisfied only if $d = 120$ km.

19. The time required will be expressed by

$$\frac{\text{Distance travelled on highway}}{10} + \frac{\text{Distance travelled on side roads}}{10}$$

This has to be minimised, check the options.

20-21. Acceleration is $\frac{dv}{dt} = 2t + 2b$

At $t = 4$, the acceleration is given to be 5.

Hence, $b = \frac{-5}{2}$

Hence, the velocity equation becomes

$V_{t=4} = V_1 - 5t = 4$

29. Total time taken to cover the entire distance together

$= 144 \times \frac{5}{4} = 180$ minutes $= 3$ hours.

Hence, distance covered per hour $= 93.33\%$ of the road by both of them combined.

Check this condition for all the options.

31. Suppose A and B are the points where the first and the second meetings took place.

The total distance covered by the pedestrian and the cyclist before the first meeting = Twice the distance between Nagpur and Bari Brei.

Total time taken is 1 hour.

Total distance covered by the pedestrian and the cyclist between the two meetings = Twice the distance between A and Bari Brei.

and time taken is half an hour.

Hence, A is the mid-point. This will result in a GP.

32. Solve through options by checking all the conditions given in the question.

33. If $2d$ is the distance between A and B, then

$$\frac{\frac{1}{2} \times 2d}{d - 2} = \frac{d - 1}{1.2 \times \frac{1}{4}}$$

34. In 24 minutes, the night covers the distance for which Lallan requires 48 minutes.

35. Check the options for all the conditions.

36. Total could either be 21 km behind Rishi or 21 km ahead of Rishi.

37. If d is the distance between Delhi and Daulatabad, then you will get the following equation.

$$\frac{d - 14}{14} = \frac{28 + (d - 14)}{(d - 14) + 2}$$

38. Check the conditions through the options.

47. The act time loss is $\frac{1}{3}$% of 156 hours.

48. In the time taken by the boat in traveling $d + (d - 12)$ kms, the log travels 12 km. Let, S_b be the speed of the boat in still water and, S_s be the speed of the stream. Then

$$\frac{12}{S_s} = \frac{20}{(S_b + S_s)} + \frac{8}{(S_b - S_s)} \quad (1)$$

It is also known that,

$$\frac{20}{(S_b + S_s)} + \frac{20}{(S_b - S_s)} = 2 \text{ hours} \quad (2)$$

Solve through options.

Detailed Solutions & Short Cuts

LOD 1

1. The ratio of time for the travel is 4:5 (Analogical to Direct Question). Hence, the ratio of speeds would be

3.4. Since, the sum of their average speeds is 70 kmph, their respective speeds would be 30 and 40 kmph respectively. Use alligation to get the answer = 16.28 kmph.

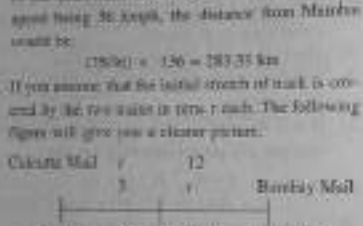
2. When speed goes down to three fourth (i.e., 75%) time will go up to $4/3^{rd}$ or 133.33% of the original time. Since, the extra time required is 16 minutes, it should be equal to $1/3^{rd}$ of the normal time. Hence, the actual time required will be 48 minutes.

3. Since, the ratio of speeds is 3:5, the ratio of times would be 5:3. The difference in the times would be 2 (of course it is the 5:3 ratio contest.) Further, since Ram takes 20 minutes longer, 2 corresponds to 20. Hence, using unitary method, 5 will correspond to 75 and 3 will correspond to 45 minutes. Hence at 10 kmph, Bharat would travel 7.5 km.

4. The train that moves at 6 km would be 75 km ahead of the other train when it starts. Also, the relative speed being 36 kmph, the distance from Mumbai would be:

$$(75/36) \times 136 = 283.33 \text{ km}$$

5. If you assume that the initial stretch of track is covered by the two trains in time t each. The following figure will give you a clearer picture.



From the above figure, we can deduce that,
$t^2 = 12t$.
Hence, $t^2 = 36$, gives us $t = 6$.
Hence, the distance between Calcutta to the meeting point is covered by the Calcutta Mail in 6 hours, while the same distance is covered by the Bombay Mail in 3 hours.
Hence, the ratio of their speeds would be 1:2. Hence, the Bombay Mail would travel at 96 kmph.

6.

From the figure above we see that Shyam would have walked a distance of 4 + 4 + 4 = 12 km. (i.e. P_1P_2, P_2P_3 or G and G to P_4).

7. When the train from Khandala starts off, the train from Lonavala will already have covered 50 km. Hence, 350 km at a relative speed of 60 kmph will take 350/60 hrs. From this, you can get the answer as:

$$50 + (350/60) * 25 = 279.166 \text{ km}.$$

8. When his speed becomes $3/4^{th}$, his time would increase by $1/3^{rd}$. Thus, the normal time = 7.5 hrs. (since increased time = 2.5 hrs).

9. Since he gains 2 hours by driving both ways (instead of walking one way) the time taken for driving would be 2 hours less than the time taken for walking. Hence, he stands to lose another two hours by walking both ways. Hence his total time should be his 45 minutes.

10. Kala's speed = 3 m/s.
For 1200 m, Kala would take 400 seconds and Sambhu would take 10 seconds less. Hence, 390 seconds.

12. Since the train travels at 60 kmph, it's speed per minute is 1 km per minute. Hence, if it's speed with stoppages is 40 kmph, it will travel 40 minutes per hour.

13. Total distance/Total time = 1500/15 = 106 kmph.

15. The average speed would be given by:

$$\frac{3d}{\frac{d}{v_1}+\frac{d}{v_2}+\frac{d}{v_3}} = \frac{3v_1v_2}{2v_1+v_2}$$

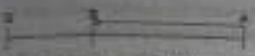
16. By increasing his speed by 25%, he will reduce his time by 20%. (This corresponds to a 6 minute gain in his time for travel—since he goes from late 10 minutes late to only 4 minutes late.) Hence, his time originally must have been 30 minutes. Hence the required distance is 20 kmph × 0.5 hours = 10 km.

17. $d/6 + d/4 = 10 \rightarrow d = 24$ km.

19. To reduce the time of the journey by 25%, he should increase his speed by 33.33% or $1/3^{rd}$. Thus required speed = 80 kmph.

21. In four hours, the train will travel 180 km (180/60 minutes). The number of poles would be 180/0.6 = 300.

22. Is the same question as question No. 5.

32. 

In the above figure, the train travels from A to B in 11.20 minutes.

Suppose, you denote the time at which the first gunshot is heard as $t = 0$. Also, if you consider the travel of the sound of the gunshot is heard at point B at $t = 11.20$ minutes. Also, the second gunshot should reach point B at $t = 12$ minutes. Hence, the sound of the 2nd gunshot would take 30 seconds to travel from B to A.

Thus, $\frac{S_{train}}{S_{sound}} = \frac{t_{sound}}{t_{train}}$

$S_{train} = 330 \times \frac{30}{690} = \frac{330}{23}$ m/s.

34. In 6 minutes, the car goes ahead by 0.6 km. Hence, the relative speed of the car with respect to the pedestrian is equal to 6 kmph.

Hence, the car speed is 8 kmph.

35. At 40 kmph, Harsh would cover (200/60) × 40 km = 400/3 km. = 133.33 km.

This represents the distance by which Vijay would be ahead of Harsh, when Vijay reaches the endpoint mean is essence that Vijay must have travelled for 133.33/20 hours → 6.66 hours

Hence, the distance is 60 × 6.66 = 400 km.

37. The required speed x would be satisfying the equation

$$300/x - 300/y = 5/2$$

Solving for y from the options it is clear that $y = 25$.

38. Solve through options trial and error. For usual speed 5 kmph we have:

Normal time → 2/3 hours = 40 minutes.

39. By increasing the speed by 33.33%, it would be able to reduce the time taken for travel by 25%. But since she is just able to overcome a time delay of 30 minutes, 30 minutes must be equivalent to 25% of the time originally taken. Hence, the original time must have been 2 hours and the original speed would be 750 kmph. Hence, the new speed would be 1000 kmph.

40. The average speed would be given by:

$$\frac{(120 \times 2 + 40 \times 3)}{4} = 80 \text{ kmph}$$

41. Assume a distance of 60 km in each stretch. Use the average speed by the formula. Total distance/ Total time = 180/2.5 = 18 kmph.

35. At 3/4th speed, extra time = 1/3rd of time = 18 minutes.

Normal time = 36 minutes.

37. (72.5/36 × 75). Same logic as the question 4.

39. The distance would get divided in the ratio of speeds (since time is constant). Thus, the distance ratio would be 3 : 7 and required distance = 3/12 × 600 = 250 km.

41. When Ram runs 2000 m, Shyam runs (1000/30)

When Ram runs 1000 m, Shyam runs (2000/1000).

Thus:

$$2000/1000 = \frac{1000 - 30}{2000 - 1000}$$

Solving, we get $t = 8.00$ min.

Thus, Shyam's speed = 400 metres/min and he would take 5 minutes to cover the distance. Option (b) fits.

43. $T = S_1 + S_2$ (1)

$25 = S_1 + S_2 + 375$ (2)

Solving, $S_1 = 21$ metres

$= 21 \times 18/2 = 75.6$ kmph

45. Solve through options. For option (c) at 4 kmph, the man would take exactly 4 hours to cover the distance.

48. $(20 \times 5/18) \times 36 = L_1 \rightarrow L_1 = 200$ m.

50. 17 km upstream in 40 minutes → 25.5/.33 = 41.25 kmph. (upstream speed of the boat)

Thus, still water speed of the boat

= 41.25 + 5 = 46.25 kmph

52. $(S_1 - S_2) \times 60 = 200$

Where S_1 and S_2 are speeds of the frame and chronometer respectively

$\rightarrow S_1 - S_2 = 3.33$

Also, $(S_1 + S_2) \times 30 = 200$

$\rightarrow S_1 + S_2 = 20$

Solving, we get $S_1 = 8.16$ m/s

= 8.16 × 18/5 = 30 kmph

54. Solve using options 1-33 will fits perfectly.

55. Solve using options 1-36 will fits perfectly. When A scores 60 points, B scores 45, and C scores 40.

Thus, when B scores 90, C would score 80. So, B can give C 10 points in 90.

56. When Shyam does 300, Vinay does 375. Since Vinay has a start of 140 m, it means that Vijay only needs to cover 260 m to match the distance.

When Vinay does 300, Shyam would cover 240 m and lose by 20 m. Hence the ratio of their speeds is 3 : 4.

58. $20 \times (5/60) \times 10 = 100$ m = 0.1 km
60. Speed of running of the train = 1.25 km/hr
With stoppage, an effective speed of 60 km/hr means that the time of travel per hour would be $60/1.25 = 48$ minutes.
Thus, the train stops for 12 minutes per hour.
61. Upstream speed = 3.2 km/h
Downstream speed = 6 km/h
Thus, speed of stream = 1.4 km/h
63. Upstream speed = 4.8 km/h
Downstream speed = 7.2 km/h
$d/4.8 + d/7.2 = 1$
Solving we get $d = 2.88$ km
65. Rate in still water = (10 + 21)/2 = 19 km/h
67. $100/x - 2/x + 100/(x + 2) = 55/60 = 11/12$ hours
$x = 22$ fits the expression
69. $(240 \times 2\pi r = 4\pi$ (Since $r = 8$ cm)
70. The hands would be together once in each hour. However, the 12 occurs twice in both 11 to 12 and 12 to 1. Hence, the no. of times = $12 - 1 = 11$
72. $1615 + x) \times 905 - x) = 5$
$x = 2$, fits this equation.
74. When Ajay does 600 metres, Vijay does 540 m. When Vijay does 500 metres, Anjay does 475 m. Thus, Ajay : Vijay : Anjay = 600 : 540 : 513. Thus, Ajay would beat Anjay by (87 × 25) = 38 m in a 600 m race.
75. $30(15 + x) = 50x$ in a 400 m race.
At $x = 5$, the equation is satisfied.

LOD II

1. By travelling at 40 km/h higher than the original speed, the train is able to make up 10 minutes while travelling 80 km.
This condition is only satisfied at an initial speed of 20 (and a new speed of 60 km/h).
2. Solve this question through options. For instance, if he travelled at 25 km/h, his original speed would have been 24 km/h.
The time difference can be seen to be 6 minutes in this case.
$600/24 - 600/25 = 0.1$ hrs = 6 mins. Thus, this is the correct answer.
3. In 1 hours 15 minutes an individual will be able to cover 25% more than his speed per hour. The relationship between the original speed and the new speed is best represented as below:
Original speed $\xrightarrow{\text{25\% increase}}$ speed (as 75 minutes)
$\xrightarrow{\text{25\% increase}}$ New speed
Thus, to go from the new speed to the original speed the process would be:
New speed $\xrightarrow{\text{75\%}}$ Speed per 75 minutes
$\xrightarrow{\text{75\%}}$ Original speed
We need to use this process to check the options. Only the first option satisfies this condition (as a 8 km/h speed would take 6 hours while at 12 km/h it would take 8 hours).
4. The question's structure (and solving) has to be done on the basis of integers. The following equations emerge:
$$\frac{d}{x} - \frac{d}{(x+6)} = 4 \text{ and } \frac{d}{(x-6)} - \frac{d}{(x+6)} = 6$$
Solving these expressions through normal algebraic methods is close to impossible (as the very least would take a huge amount of time.) Instead the question has to be solved using the logic that equal difference in ratios in such a situation can only occur in all the three terms (dx, $dx + 6$) and ($dx - 6$) are integers.
Hence, d should have three divisors which are two apart from each other.
5. The relative speed is 20 km/h. Also, the pickup should take 7.30 hours more than the cycle. Only option (1) the speeds of the two people are 4 and 16 km/h respectively. At this speed the respective times would be 10 hrs and 2.30 hours, giving the required answer.
6.

The distance between the motorcycle will be shown as the hypotenuse. Using the 3,4,5 Pythagorean triplet and the condition that the two speeds are 6 km/h different from each other, you will get the triplet 18,24,30. Hence, the slower motorcycle travelled at 12 km/h.
7. Since the two motorcycles meet after an hour, their combined speed is 28 km/h. Use options to check which

values. Since the speed of the faster cyclist is taken to be a fact to be greater than 14 kmph. Hence only check options > 14 kmph.

9. Since the normal ant covers 20/120 of the distance in 2 hours 30 minutes, we can infer that it covers 8 in (30/4) 7½ of the distance in 1 hours. Thus, in 3 hours both ants together cover 15% of the distance → 5% per hour → they will meet in 20 hours.
Also, ratio of speeds = 8 : 7.
So, the second ant would cover 700 ft to the meeting point in 20 hours and its speed would be 35 ft/hr.

10. In this question consider the total distance as 100%. Hence the sum of their speeds will be 75% per hour. Checking options (c)
If the bus took 6 hours, it would cover 16.66% distance per hour and the car would cover 25% distance per hour. (as it takes 2 hours less than the bus.)
This gives an addition of only 41.66%. Hence, the answer is not correct.
Option (b) is the correct answer.

11. The opposite conditions are met on a Pythagorean triplet 6,8,10. Since the racetrack only consists of the legs of the right triangle the length must be 6 + 8 = 14 km.

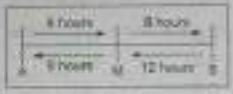
12. The sum of speeds would be 0.08 m/s (relative speed in opposite directions). Also if we go by option (b), the speeds will be 0.05 and 0.05 m/s respectively.
At this speed the overlapping would occur every 80 seconds.

13. When Rohan starts, Karan would have covered 40 km. Also, their relative speed is 12 kmph and the distance between the two would get to 0 in 40/12 = 3.33 hours.
Distance covered = 28 × 3.33 = 93.33 km.

14. Solve this question through options.
For option (c), the condition match since. If he rode for 2 hours (speed = 5 kmph), he would have walked for 9 hours (4 hours more) and his walking speed would be 15 kmph.
If we interchange the times, we get 15 × 2 = 5 × 9.

15. This is a complex trial and error based question and the way you would have to think is this is:

From the figure above, it is clear that A is faster as he takes only x + 2 hours while B has taken x + 9 hours to complete the journey.
Thus, we get (x + 6)(9 + 9)x
Solving for x, we get x = −6 (not possible)
Or x = 12. Putting this value of x in the figure it changes to:



We also get ratio of speeds = 3 : 2 (inverse of ratio of times.)
The next part of the puzzle is to think of the 12 km less travelled by the first person till the meeting point. If the speed of the faster person is 3x, that of the slower person = 2x.
Further

$$12 \times 2x = 6 \times 3x = 12 \text{ km}$$
$$x = 2 \text{ kmph}$$

The speed of the faster runner is 3 × 2 = 6 kmph.

16. Solve using options. The first option you would check for (given the values in the question) would be option (a). This would give that the first jogger would run at 3 min per km, while the second jogger would run at 4 min per km. In the new condition, the first jogger would jog for 13 km while the second jogger would jog for 14 km and their respective times would be 39 mins and 56 minutes. This is consistent with the condition in the question which talks about a difference of 17 minutes in their respective times.

17. Solve this through options i.e. For option (a)

$$480/96 = 480/50 = 16 \text{ minutes}$$

18. The ant would cover 7 × 8 = 56 meters in 16 hours. Further, it would require 7/12 of the 17th hour to reach the top. Thus time required = 16 hours 35 minutes.

19. If the slower ship took 20 hours (option d) the faster ship would take 12 hours and their respective speeds would be 15 and 25 kmph. This satisfies the basic condition in the question.

20. When the signal happened distance left was 150 km. (500/x) − (500/x + 15) = 1/2 hours → x = 60.

21. Since they cover the distance in 80 minutes traveling in opposite directions we infer 100% distance is covered in 80 minutes → 1.25% per minute → 75% per hour.

i.e. their combined distance coverage is 70% per hour.

Hence, we are asked for the time the faster motorcyclist takes, we can pick up this time from the options.

Option	Time for faster motorcyclist	Faster's % coverage per hour	Slower's % coverage per hour
a	6 hours	16.66	58.33
b	5 hours	13.33	41.66
c	2 hours	50	25
d	4 hours	25	30
e	5 hours	20	55

It is clear that options a, b, d and e are not feasible as it is making the faster motorcyclist slower.

Thus, option (c) has to be correct.

Note: You can use the values in option (c) to check the other conditions in problem and see that it works.

23. Since the two homes meet after 200 minutes, they cover 0.5% of the distance per minute (combined) or 30% per hour. This condition is satisfied only if you the slower rider takes 10 hours (thereby covering 10% per hour) and the faster rider takes 5 hours (thereby covering 20% per hour).

24. Solve through options the equation: $350/s - 350/(s + 5) = 2.33$ hours.

$\rightarrow s = 25$.

25. Solve this question through options. The total travel time should be 5 hours 25 minutes.

or $315/(12)$ hours $= 63/12$ hours

$$d/36 + d/14 = 63/12 \rightarrow d = 54.6 \text{ km}$$

Thus, option (c) none of these is correct.

26-28. You can make the following table to chart out the motions of the hare.

Hour	Ram	Shyam	Mohan
1	10	10	5.33
2	20	33.33	6.66
3	40	36.66	10
4	61.66	40	20
5	85	50	30
6	88.33	60	40
	90(6.5)	70(7)	70(7)
		80(8)	100(8)

26. It is evident that Ram would overtake Shyam between 2 and 3. At 2, Shyam is ahead by 13.33 km.

Relative speed between 2 – 3 = 16.66 kmph.

Time required = 13.33/16.66 of the hour

= 4/11 of the hour

= 2 – 22. (approx)

27. Mohan would overtake Shyam after 70 miles.

28. Ram would cover 90 miles in 3 hours 45 minutes.

Mohan would cover 90 miles in 7 hours 40 minutes.

Time difference = 3 hours 55 minutes.

29. Mohan at 8 pm (each of the others would reach later).

30. Ram would reach at 9:30 pm.

31. $25/s - 25/(s + 10) = 1/2$

$s = 30$ km/hr.

32. The respective times are 224 seconds and 364 seconds. They will meet at the starting point in the LCM of these times i.e. 224 × 13. Hence, Ram Singh will cover the circle 13 times.

33. The wife drives for 12 m. earlier than her during on normal days.

Thus, she would have saved 6 minutes each way. Hence, Ravi would have walked for 30 minutes (since his speed is 1/5th of the car's speed).

In effect, Ravi spends 24 minutes extra on the walking (rather than if he had travelled the same distance by car).

Thus, if Ravi had got the car at the station only, he would have saved 24 minutes more and reached at 5 : 36.

35. The distances covered in percentage would be, 100 + 6.66% + 8.33% + 16.66% + 5.833% + 31.666 + 2.0833 = 81.25%

(22.5/18.75) × 100 = 120 km

39-40. In 9 hours, (7 pm to 4 pm) the Kalinga Express would cover 540 km.

Remaining distance = 260 km

Relative speed = 150 kmph.

Time required = 260/150 = 1.733 hour

= 104 minutes.

39. 1.733 × 90 = 156 km.

40. 4 A.M. + 1 hr 44 minutes = 5 : 44 A.M.

41. The total distance the bird would travel would be dependent on the time that the cars crash with each other. Also, the speed of the bird is the same as the relative speed of the cars. Hence, the answer to question 41 will be 12 km.

42. The bird would travel at 120 kmph for 4 + 4/3 + 9 minutes. Or 5.75 minutes. Hence, the answer is 15.75/60 × 120 = 11.55 km.

44. Total distance = 35 dog leaps.

Per minute → dog makes 5 dog leaps.

Per minute → Cat makes 6 cat leaps = 5 dog leaps

Relative speed = 2 dog leaps/minute.

An initial distance of 25 dog leaps would get covered in 12.5 minutes.

40. In 36 hours, there would be a gap of 8 minutes. The two watches would show the same time when the gap would be exactly 12 hours or 720 minutes.

The no. of 36 hour time frames required to create this gap = 720/8 = 90.

Total time = 90 × 36 = 3240 hours. Since this is divisible by 24, the watches would show 12 noon.

41. Solve through options. At 8 kmph The motorboat would take exactly 6 hours.

44. Check through options. Option (d) will give us 14 kmph and 9 kmph as the down stream and up stream speeds. This would mean that the total travel time would be 1.5 hours and 2.33 hours down stream and up stream respectively.

45. The dog loses 1/7th of his normal time from the meeting point. (Thus normal time = 35 × 3 = 105 minutes)

If the meeting occurred 24 km further, the dog loses 25 minutes.

This means that the normal time for the new distance would be 75 minutes. Thus, normally the dog would cover this distance of 24 km in 30 minutes. Thus, normal speed = 48 km/hr.

ANSWER KEY

Time Speed and Distance LOD I

1. a
2. a
3. a
4. a
5. d
6. d
7. c
8. a
9. a
10. a
11. a
12. a
13. c
14. a
15. c
16. c
17. a
18. b
19. d
20. a
21. a
22. c
23. c
24. d
25. b
26. a
27. d
28. c
29. d
30. a
31. a
32. b
33. a
34. a
35. a
36. d
37. d
38. b
39. a
40. a
41. b
42. a
43. a
44. d
45. c
46. a
47. a
48. a
49. d
50. a
51. b
52. a
53. b
54. c
55. c
56. b
57. b
58. a
59. a
60. b
61. a
62. d
63. a
64. b
65. a
66. c
67. b
68. a
69. a
70. a
71. a
72. c
73. a
74. d
75. d

Time Speed and Distance LOD 11

1. c
2. c
3. c
4. b
5. a
6. b
7. b
8. d
9. b
10. a
11. b
12. c
13. c
14. b
15. b
16. a
17. c
18. d
19. b
20. b
21. b
22. c
23. c
24. c
25. c
26. b
27. c
28. c
29. d
30. a
31. c
32. d
33. d
34. b
35. c
36. c
37. a
38. c
39. b
40. d
41. b
42. b
43. c
44. c
45. a
46. b
47. a
48. b
49. c
50. a

Time Speed and Distance LOD III

1. c
2. b
3. a
4. a
5. b
6. c
7. b
8. d
9. c
10. c
11. d
12. b
13. b
14. b
15. a
16. d
17. b
18. c
19. c
20. b
21. b
22. a
23. b
24. c
25. b
26. d
27. a
28. b
29. d
30. d
31. b
32. a
33. c
34. d
35. a
36. d
37. d
38. a
39. a
40. b
41. d
42. b
43. d
44. d
45. a
46. a
47. c
48. b
49. d
50. b

BLOCK REVIEW TESTS

REVIEW TEST ONE

1. A man earns x% on the first Rs. 3,000 of his investment and y% on the rest of his investment. If he earns Rs. 1250 from Rs 7,000 and Rs. 1750 from Rs. 9000 invested, find the value of x.
   (a) 20% (b) 18%
   (c) 25% (d) None of these
2. The price of a television set drops by 30% while the sales of the set goes up by 50%. What is the percentage change in the total revenue from the sales of the set?
   (a) -40% (b) -2% (c) +5% (d) +2%
3. A person John has a certain amount with him goes to the market. He can buy 100 oranges or 80 mangoes. He retains 20% of the amount for petrol expenses and buys 40 apricots and all the balance he purchases oranges. The number of oranges he can purchase is:
   (a) 30 (b) 40 (c) 15 (d) 20
4. A cloth merchant cheats his supplier and his customer to the tune of 20%, while buying and selling cloth respectively. He professes to sell at the cost price but also offers a discount of 20% on cash payment, what is his overall profit percentage?
   (a) 20% (b) 25% (c) 40% (d) 15%
5. I sold two houses for Rs. 50000 each, one at the loss of 20% and the other at the profit of 20%. What is the percentage of loss (-) or profit (+) that resulted from the transaction?
   (a) (+) 20 (b) (-) 4
   (c) (+) 4 (d) (-) 20
6. The cost of a diamond varies directly as the square of its weight. A diamond fell and broke into four pieces whose weights were in the ratio 1:2:3:4. As a result the merchant had a loss of Rs.500000. Find the original price of the diamond.
   (a) Rs. 14 lacs (b) Rs. 20 lacs
   (c) Rs. 18 lacs (d) Rs. 25 lacs
7. Two oranges, three bananas and four apples cost Rs. 21. Three oranges, two bananas and one apple cost Rs. 20. I bought 3 oranges, 3 bananas and 3 apples. How much did I pay ?
   (a) Rs.23.5 (b) Rs.27
   (c) Rs.30 (d) Cannot be determined
8. From each of two given numbers, half the smaller number is subtracted. Of the resulting numbers the larger one is five times as large as the smaller one. What is the ratio of the two numbers?
   (a) 2:1 (b) 3:1
   (c) 5:2 (d) None of these

Directions for Questions 9–10: Answer the questions based on the following information.

A watch dealer incurs an expense of Rs.150 for producing every watch. He also incurs an additional expenditure of Rs. 30,000, which is independent of the number of watches produced. If he is able to sell a watch during the season, he sells it for Rs. 250. If he fails to do so, he has to sell each watch for Rs. 100.

9. If he is able to sell only 1,000 out of 1,500 watches he has made in the season, then he has made a profit of:
   (a) Rs.50,000 (b) Rs.75,000
   (c) Rs.45,000 (d) Rs.60, 000
10. If he produces 2000 watches, what is the number of watches that he must sell during the season (to the nearest 100) in order to break-even, given that he is able to sell all the watches produced?
    (a) 700 (b) 800
    (c) 900 (d) 1,000
11. A stockist wants to make some profit by selling oil. He contemplates about various methods. Which of the following would maximise his profit?
    I. Sell oil at 20% profit.
    II. Use 800 g of weight instead of 1 kg.
    III. Mix 20% impurities in oil and selling it at cost price.
    IV. Increase the price by 10% and reduce weights by 10%.
    (a) I or III (b) II
    (c) II and IV (d) Profits are same
12. A dealer offers a cash discount of 20% and still makes a profit of 20%, when he further allows 160 articles when the customer buys 120. How much percent above the cost price were his sales based?
    (a) 100% (b) 80%
    (c) 75% (d) 66⅔%
13. A man buys sugar at Rs. 600 per litre, adds water to it and then sells it at Rs. 750 per litre. What is the ratio

of the spirit's weight to the weight of the water if the profit in the deal is 37.5%?

(a) 5:1 (b) 10:1

(c) 11:1 (d) None of these

Directions for Questions 14–16: Answer the questions based on the following information.

Anant, on his death bed, keeps half his property for his wife and divides the rest equally among his three sons Rinse, Cutter and Danger. Some years later, Rinse dies, leaving half his property to his widow and half to his brothers, Cutter and Danger together, sharing equally. When Cutter makes his will, he keeps half his property for his widow and the rest he bequeaths to his younger brother Danger. When Danger dies some years later, he keeps half his property for his widow and the remaining 1/4 for his mother. The mother now has Rs.15,75,000.

14. What was the worth of the total property?

(a) Rs.3 crore (b) Rs.6.3 crore

(c) Rs.1.8 crore (d) Rs.2.4 crore

15. What was Cutter's original share ?

(a) Rs.40 lakh (b) Rs. 120 lakh

(c) Rs.60 lakh (d) Rs.30 lakh

16. What was the ratio of the property owned by the widows of the three sons, in the end?

(a) 7:9:13 (b) 8:10:17

(c) 5:7:9 (d) 9:12:21

17. At a bookstore, "MODERN BOOK STORE" is flashed using neon lights. The words are individually flashed at long intervals of $2\frac{1}{2}$, $4\frac{3}{4}$, $5\frac{1}{8}$ seconds respectively, and each word is put off after a second. The least time after which the full name of the bookstore can be read again is:

(a) 46.5 seconds (b) 71.5 seconds

(c) 2144.5 seconds (d) 855 seconds

18. A train approaches a tunnel AB. Inside the tunnel is a cat located at a point that is $\frac{1}{3}^{rd}$ the distance AB measured from the entrance A. When the train whistles, the cat runs. If the cat moves to the entrance of the tunnel, A, the train catches the cat exactly at the entrance. If the cat moves to the exit B, the train catches the cat at exactly the exit. The speed of the train is greater than the speed of the cat by what ratio?

(a) 3 : 2 (b) 4 : 1

(c) 5 : 1 (d) None of these

19. Six artisans working at the same rate completely work of one server in 2.5 hrs. If they start at 11:00 a.m. and one additional technician per hour being added, beginning at 3:00 p.m., at what time the server will be completed?

(a) 6:00 p.m. (b) 7 p.m.

(c) 7:30 p.m. (d) 8:00 p.m.

Directions for Questions 20–21: Answer the questions based on the following information.

A thief, after committing the burglary, started fleeing at 12 noon, at a speed of 60 km/hr. He was then chased by a policeman X. X started the chase, 15 min after the thief had started, at a speed of 65 km/hr.

20. At what time did X catch the thief?

(a) 3:30 p.m. (b) 3 p.m.

(c) 3:15 p.m. (d) None of these

21. If another policeman had started the same chase along with X, but at a speed of 60 km/hr, then how far behind was he when X caught the thief?

(a) 13.75 km (b) 15 km

(c) 23 km (d) 37.5km

22. Two typists undertake to do a job. The second typist begins working one hour after the first. Three hours after the first typist has begun working, there is still 20% of the work to be done. When the assignment is completed, it turns out that each typist has done half the work. How many hours would it take each one to do the whole job individually?

(a) 12 hr and 8 hr (b) 8 hr and 5.6 hr

(c) 10 hr and 8hr (d) 5 hr and 4 hr

23. A man can walk up a moving 'up' escalator in 60s. The same man can walk down this moving 'up' escalator in 90s. Assume that his walking speed is same upwards and downwards. How much time will he take to walk up the escalator, when it is not moving?

(a) 30s (b) 48s (c) 60s (d) 90s

Directions for Questions 24–26: Answer the questions based on the following information.

Boston is 4 hr ahead of Frankfurt and 2 hr behind India. X leaves Frankfurt at 6 p.m. on Friday and reaches Boston the next day. After waiting there for 2 hr, he leaves each at noon and reaches India at 1 a.m. On his return journey he takes the same route as before, but halts at Boston for 1 hr less than his previous halt there. He then proceeds to Frankfurt.

24. If his journey, including stoppage, is covered at an average speed of 180 mph, what is the distance between Frankfurt and India?

(a) 3,600 miles (b) 4,300 miles
(c) 3500 miles (d) Data insufficient

19. If X had started the return journey from India at 2.50 a.m. on the same day that he reached there, after how much time would he reach Frankfurt?
(a) 24 hr (b) 25 hr
(c) 26 hr (d) Data insufficient

20. What is X's average speed for the entire journey (to and fro)?
(a) 170 mph (b) 180 mph
(c) 165 mph (d) Data insufficient

REVIEW TEST TWO

1. A car after travelling 18 km from a point A developed some problem in the engine and the speed became $\frac{4}{5}^{th}$ of its original speed. As a result, the car reached point B 45 minutes late. If the engine had developed the same problem after travelling 36 km from A, then it would have reached B only 30 minutes late. The original speed of the car (in km per hour) and the distance between the points A and B (in km) are
(a) 25,110 (b) 30,150
(c) 20,190 (d) None of these

2. A, B and C individually can finish a work in 6, 8 and 12 hours respectively. They started the work together and after completing the work got Rs.94.60, which they divide the money among themselves. A, B and C will get respectively (in Rs.)
(a) 44.33,17.60 (b) 48.27,24.60
(c) 45.36,19.60 (d) 42.28,24.60

3. Two trains are travelling in opposite directions at uniform speed 60 and 50 km per hour respectively. They take 5 seconds to cross each other. If the two trains had travelled in the same direction, then a passenger sitting in the faster moving train would have overtaken the other train in 18 seconds. The length of the trains in metres are
(a) 112, 75.00 (b) 95.78, 55
(c) 102.78, 50 (d) 102.78, 55

4. Suppose that an equal number of people are born on each day. Find approximately the percentage of the people whose birthday will fall on 29th February.
(a) 0.274 (b) 0.5732
(c) 0.684 (d) None of these

5. A sum of money compounded annually becomes Rs.625 in two years and Rs.675 in three years. The rate of interest per annum is
(a) 7% (b) 8% (c) 6% (d) 5%

6. Every day Asha's husband meets her at the city railway station at 6.00 p.m. and drives her to their residence. One day she left early from the office and reached the railway station at 5.00 p.m. She started walking towards her home, met her husband coming from their residence on the way and they reached home 10 minutes earlier than the usual time. For how long did she walk?
(a) 1 hour (b) 50 minutes
(c) ½ hour (d) 55 minutes

7. Three machines, A, B and C can be used to produce a product. Machine A will take 60 hours to produce a million units. Machine B is twice as fast as Machine A. Machine C will take the same amount of time to produce a million units as A and B running together. How much time will be required to produce a million units if all the three machines are used simultaneously?
(a) 12 hours (b) 10 hours
(c) 8 hours (d) 6 hours

8. Mr. and Mrs. Shah travel from City A to City B and break journey at City C between. Somewhere between City A and City C, Mrs. Shah asks "How far have we travelled?" Mr. Shah replies, "Half as far as the distance from here to city C". Somewhere between City C and City B, exactly 200 km from the point where she asked the first question, Mrs. Shah asks "How far do we have to go?" Mr. Shah replies "Half as far as the distance from City C to here." The distance between Cities A and B is km is
(a) 200 (b) 300 (c) 400 (d) 500

9. A shop sells ball point pens and refills. It used to sell refills for 30 paise each, but there were hardly any takers. When we reduced the price, the remaining refills were sold out enabling the shopkeeper to realize Rs. 19.80. How many refills were sold at the reduced price?
(a) 57 (b) 71 (c) 89 (d) 97

10. Anand and Bharat can cut 5 kg of wood in 20 min, Bharat and Chandra can cut 5 kg of wood in 40 min. Chandra and Anand can cut 5 kg. of wood in 30 min. How much time Chandra will take to cut 5 kg of wood alone?

(a) 120 minutes (b) 48 minutes
(c) 240 minutes (d) 24(07) minutes

11. If 300 soldiers eat 10 tons of food in 200 days, how much will 50 soldiers eat in 20 days? (nearest to 1000 kg)
(a) 5 kg (b) 10 kg
(c) 100 kg (d) 50 kg

12. A servant is paid Rs. 100 plus one shirt for a full year of work. He works for 6 months and gets Rs. 50 plus the shirt. What is the cost of the shirt? (in Rupees)?
(a) 20 (b) 16
(c) 40 (d) 36

13. A man without stopping travels at 60 km per hour and with stoppages at 40 km per hour. What is the time value for stoppages on a route of 300 km?
(a) 10 hours (b) 20 hours
(c) 5 hours (d) 2.5 hours

14. A contractor receives a certain sum every week for paying wages. He uses capital together with the weekly sum received from to pay 45 men for 57 weeks. If he had 60 men and the same wages his capital and weekly sum would suffice for 15 weeks, how many men can be maintained for 28 weeks?
(a) 80 (b) 52 (c) 50 (d) 60

15. A supply of water lasts for 150 days if 15 gallons leak out every day, but only for 100 days if 15 gallons leak out daily. What is the total quantity of water in the supply?
(a) 900 (b) 1125
(c) 1550 (d) 1250

16. If a dealer were to decrease the selling price of his wares by 10% he would double his sale making the same profit as before. In what ratio would his profit decrease if he were to increase his selling price by 10% and thereby halve his sale?
(a) 2:15 (b) 5:4
(c) 1:15 (d) 4:5

17. A sale is 60% of price. One of the 5 taps are removed and a blocking liquid substituted. The process is repeated. Now the ratio of price to frontier is 64:15. What is the full capacity of the can?
(a) 20 litres (b) 60 litres
(c) 40 litres (d) 50 litres

Direction for Questions 18–20: Use the following information.

Kachua Bhaiya started to move from point B towards point A exactly an hour after Jiggly Pup started from A in the opposite direction. Kachua Bhaiya's speed was twice that of Jiggly Puff. When Jiggly Pup had covered one-third of the distance between the points A and B, Kachua Bhaiya had also covered the same distance.

18. The point where the two would meet is
(a) Closer to A
(b) Exactly between A and B
(c) Closer to B
(d) P and Q will not meet at all

19. How many hours would Jiggly Pup take to reach B?
(a) 2 (b) 5 (c) 6 (d) 12

20. How many more hours would Jiggly Pup require to Kachua Bhaiya take to complete his journey?
(a) 4 (b) 5 (c) 6 (d) 7

21. A group of workers was put on a building job. On the second day one worker was withdrawn and each day thereafter one worker was withdrawn from the job. The job was finished when the last worker was withdrawn. Had no worker been withdrawn at any stage, the group would have finished the job in half the time. How many workers were there in the group?
(a) 2 (b) 3 (c) 5 (d) 10

22. A ship leaves on a long voyage. When it is 18 miles from the shore, a seaplane, whose speed is one-third that of the ship, is sent to deliver mail. How far from the shore does the seaplane catch up with the ship?
(a) 24 miles (b) 25 miles
(c) 22 miles (d) 20 miles

23. One man can do as a woman can do in 2 days. A man does one-third the work in a day as a woman. If a house owner hires 30 pairs of hands, men, women and children in the ratio 6:3:2 and pays them a total of Rs. 600 at the end of days work, what must the daily wages of a child be, if the wages are proportional to the amount of work done?
(a) Rs. 14 (b) Rs. 3 (c) Rs. 20 (d) Rs. 7

24. A water tank has three taps A, B and C. A fills the tank in 24 mins, B fills it 8 buckets in 1 hour and C fills 2 buckets in 20 minutes. If all the taps are opened together a full tank is emptied in 2 hours. If a bucket can hold 5 litres of water, what is the capacity of the tank?
(a) 720 litres (b) 240 litres
(c) 180 litres (d) 140 litres

25. A man buys spirit at Rs.60 per litre, adds water and then sells it at Rs.75 per litre. What is the ratio of spirit to water if his profit in the deal is 37.5%?
(a) 9:1 (b) 10:1
(c) 11:1 (d) None of these

REVIEW TEST THREE

1. There is a leak in the bottom of a tank. This leak can empty a full tank in 8 hours. When the tank is full, a tap is opened into the tank which admits 6 liters per hour and the tank is now emptied in 12 hours. What is the capacity of the tank?
   (a) 24.8 liters (b) 36 liters
   (c) 144 liters (d) cannot be determined
2. The winning relay team in a high school sports competition clocked 48 minutes for a distance of 13.2 km. Its runners A, B, C and D maintained speeds of 15 kmph, 16, 17 kmph and 18 kmph respectively. What is the ratio of the time taken by B to that taken by D?
   (a) 5:16 (b) 5:17 (c) 9:8 (d) 8:9
3. Three bells chime at intervals of 18, 24 and 32 minutes respectively. At a certain time they begin to chime together. What length of time will elapse before they chime together again
   (a) 2 hours 24 minutes
   (b) 4 hours 48 minutes
   (c) 1 hours 36 minutes
   (d) 3 hours
4. In a race of 200 meters run, Ashish beats Sunil by 20 meters and Nalin by 40 meters. If Sunil and Nalin are running a race of 100 meters with exactly the same speeds as before, then by how many meters will Sunil beat Nalin?
   (a) 11.11 meters (b) 10 meters
   (c) 12 meters (d) 25 meters
5. A man invests Rs.5000 at a rate of 5% per annum. How much more should he invest at a rate of 8%, so that he can earn a total of 6% per annum?
   (a) Rs.1200 (b) Rs.1400
   (c) Rs.1500 (d) Rs.2000

For the following data for Questions 6–9: Hettabh and Rak Rak are running along a circular course of radius 14 km in opposite directions such that when they meet they reverse their directions as well as they exchange their speeds. After they meet Hettabh will run at the speed of Rak Rak and vice versa. However, this exchange occurs only when they meet outside the starting point. They do not exchange directions or speeds when they meet at the starting point. Initially, the speed of Hettabh is twice the speed of Rak Rak. Assume that they start from M_1 and they first meet at M_2, then at M_3, next M_4, and finally at M_1.

6. What is the shortest distance between M_1 and M_2?
   (a) 22 km (b) 44/2 Km
   (c) 14 km (d) 28 Km
7. What is the shortest distance between M_1 and M_3 along the course?
   (a) 44 km (b) 28/2 km
   (c) 44/2 km (d) 28 Km
8. Which is the point that coincides with M_2?
   (a) M_1 (b) M_3
   (c) M_4 (d) M_1
9. What is the distance travelled by Hettabh when they meet at M_2?
   (a) 154 km (b) 132 km
   (c) 198 km (d) 176 km

Directions for Questions 10–12: A certain race is made up of three stretches A, B and C, each 4 km long, and to be crossed by a certain mode of transport. The following table gives these modes of transport for the stretches, and the minimum and maximum possible speeds (in kmph) over these stretches. The speed over a particular stretch is assumed to be constant. The previous record for the race is six minutes.

Stretch	Mode of transport	Min. Speed	Max. Speed
A	Car	80	120
B	Motor-cycle	60	100
C	Bicycle	20	40

10. Andamman travels at minimum speed by car over A and completes stretch B at the fastest possible speed. At what speed should he cover stretch C in order to break the previous record?
    (a) Max. speed for C
    (b) Min. speed for C
    (c) This is not possible
    (d) None of these
11. Mr. Ram completes the first stretch at the minimum speed and takes the same time for stretch B. He takes 50% more time than the previous record to complete the race. What is Mr. Ram's speed for the stretch C?
    (a) 23.8 kmph (b) 26.66 kmph
    (c) 34.2 kmph (d) None of these
12. Mr. Tarun completes the race at an average speed of 40 kmph. His average speed for the first two stretches is 4 times that for the last stretch. Find his speed over stretch C.
    (a) 40 kmph (b) 24 kmph
    (c) 20 kmph (d) This is not possible

13. After allowing a discount of 11.11% a trader still makes a gain of 20%. At what percent above the cost price does he mark his goods?
(a) 25.56% (b) 35%
(c) 27.22% (d) None of these

14. A dealer buys oil at Rs.100, Rs.80 and Rs.60 per litre. He mixes them in the ratio 5:6:7 by weight and sells them at a profit of 50%. At what price does he sell the dry mix?
(a) Rs.80/litre (b) Rs.110.666/litre
(c) Rs.95/litre (d) None of these

15. An express train travelling at 80 kmph overtakes a goods train twice as long and going at 40 kmph on a parallel track, in 34 seconds. How long will the express train take to cross a station also in long?
(a) 36 sec (b) 42 sec
(c) 27 sec (d) none of these

16. A man earns x% on the first 1000 rupees and y% on the rest of his income. If he earns Rs.300 from Rs.4000 and Rs.600 from Rs.9000 of income. Find x
(a) 30 (b) 15
(c) 25 (d) None of these

17. In the Eastern Hamids minerals, the value of each of a set of gold coins varies as the square of its diameter, if its thickness remains constant and it varies as the thickness, if the diameter remains constant. If the diameters of the two coins are in the ratio 4:3, what should the ratio of their thickness be if the value of the first is 4 times that of the second?
(a) 16:9 (b) 9:4 (c) 9:16 (d) 4:9

A thief after committing a burglary, started fleeing at 12:00 noon at the speed of 60 kmph. He was then chased by a policeman X. X started the chase 15 minutes after the thief had started at a speed of 65 kmph.

18. At what time did X catch the thief?
(a) 3.30 p.m. (b) 3.00p.m.
(c) 3.15 p.m. (d) None of these

19. If another policeman has started the same chase along with X, but at a speed of 60 kmph, then how far behind was he when X caught the thief?
(a) 18.75 km (b) 15 km
(c) 22 km (d) 47.5 km

20. A and B walk from X to Y, a distance of 27 km at 5 kmph and 7 kmph respectively. B reaches Y and immediately turns back meeting A at Z. What is the distance from Y to Z?
(a) 2 km (b) 4.5 km
(c) 3 km (d) 7 km

21. A customer leaves the post office to go to the airport to collect mail. The plane arrives early and the mail is sent on a horse-cart. After half an hour the customer meets the horse-cart, collects the mail and returns to the post office, thus saving 20 minutes. How many minutes early did the plane arrive?
(a) 20 (b) 25
(c) 30 (d) 40

22. In his book on Leonardo da Vinci, Sigmund Freud after a detailed psychoanalysis concluded that Goethe could complete the masterpiece in nine days as he could concentrate evenly but was more persistent as a result of which he could generate 50% more efficiency than Goethe. The number of days it takes Leonardo da Vinci to do the same piece of work that Goethe completes in nine days is:
(a) 4 (1/2) days (b) 6 days
(c) 13 (1/2) days (d) None of these

23. The North south express is a pair of trains between the cities Jammu & Chennai. A train leaves Jammu for Chennai exactly at 12 noon every day of the week. Similarly, there is a train that leaves from Chennai for Jammu on every day of the week at exactly 12 noon. The time required by a train to cover the distance between Chennai & Jammu is exactly 7 days and 7 minutes. Find the number of trains from Chennai to Jammu which a train from Jammu to Chennai will encounter in completing its journey. (Assume all trains run exactly on time)
(a) 7 (b) 8
(c) 14 (d) 15

24. For the questions above, the minimum number of cars that the Indian Railways will have to devote for running this daily service will be:
(a) 16 (b) 32
(c) 30 (d) None of these

25. There are two candles each of the same initial length. The first candle can burn for 24 hours, while the second candle can burn for 16 hours. Both of them are lit at the same time. After sometime, it was found that one of the candles was twice as long as the second. For how long had the candle been burning?
1. 6 hours 2. 8 hours
3. 10 hours 4. 12 hours

REVIEW TEST FOUR

1. The menu in the local cafe at Mumbai Roads is as follows:

Item	Price (Rs.)
Samosa and Coffee	[illegible]
Paratha	[illegible]
Sweet Dish, Samosa and Coffee	[illegible]
Samosa, Coffee and Paratha	[illegible]
Paratha, Sweet Dish, Samosa and Coffee	[illegible]

In the price list, each of the entry in the right hand column stands for a two digit number and each of the different letter stands for a different digit. What is the price of the sweet dish?

(a) Rs. 6 (b) Rs. 13
(c) Rs. 41 (d) Rs. 22

2. Two trains, a passenger train and a freight train, are running in the same direction on parallel railway tracks. The passenger train takes three times as long to pass the freight train as when they are going in opposite directions.

If the trains run at uniform speeds with goods train running at a maximum speed of 25 km/h, what is the speed of passenger train (p) in km/h?

(a) $p = 50$ (b) $p \le 50$ & $p > 0$
(c) $0 \le p \le 50$ (d) $20 \le p \le 50$

3. Ramesh a rice merchant does not think much of ethical practice and neither does he think too much about his reputation, his only aim is to earn as much profit as possible. Which of the following would serve his purpose best?

I. Fixing selling price one-eighth above the cost price.
II. Mix stones (available for free) 12.5% by weight and fixing selling price at cost price.
III. Use an 8.75 kg weight for 10 kg outside and selling at cost price.
IV. Selling 15/16th only by cheating, of the weight kept on balance and fixing the selling price 6.25% above cost price.

(a) I (b) I and II
(c) IV (d) III

Directions for Questions 4-6: A marathon race is held in the Kumaraon to move to Rankhand. The race is held over 4 stretches namely *M*, *N* and *O*. The maximum speed allowed in the stretch *M* is 30 km/hr and the maximum speeds allowed in stretches *N* and *O* are 60 km/hr and 30 km/hr respectively. The minimum speeds allowed over the 3 stretches are 30 km/hr, 40 km/hr and 15 km/hr respectively. Each of the stretches is 4 km in length and speed over a stretch is assumed constant. Current record for the race is 22 min. 30 sec.

4. Ramesh completes the first stretch at minimum speed but is able to maintain the same speed in the second stretch also. If he takes 10% more time than the current record, then speed of Akshay over stretch *O* is approximately

(a) 23 km/hr (b) 17 km/hr
(c) 26.8 km/hr (d) 32 km/hr

5. Amrish covers stretch *M* at minimum speed but completes the next stretch at maximum permissible speed. What should be Andrews' speed in the stretch *O* if he wants aims at breaking the current record?

(a) 30 km/hr
(b) Indeterminate
(c) Breaking Record is not possible
(d) 40 km/hr

6. Steven Miles completes the race at an average speed 30 km/hr. If his average speed over stretches *M* and *N* is 300% more than his speed over stretch *O*, what is his speed over stretch *O*?

(a) 24 km/hr (b) 27 km/hr
(c) 18 km/hr (d) 35 km/hr

7. Ali along with his friend embarked on a boat journey. While making all his preparations for the journey, he checked with the boat manufacturer's data and was glad to learn that the boat was really dependable as it could float even if it was upto 75% full of water (with more water it could sink). Unfortunately, the boat developed a hole during the journey, the hole filling up water @ 2 litre per minute. The boat was also equipped with a drainage pump which could empty a bucket of water in 32 hrs. If Ali figures that this boat a maximum of 12 hrs to reach a safe place before they sink, what is the capacity of the boat?

(a) 1760 litre (b) 1480 litre
(c) 1600 litre (d) 1650 litre

8. The number of votes not cast for Pappu's party increased by $(8\frac{1}{3})$% over those not cast for it in the previous polls in the Farukhabad city corporation where the only other candidate belonged to

the Jhanna Party. If a total of 4,50,000 people voted each from and Pappu's Party lost by a majority one-third votes in bags as that by which it had won in the last polls. How many people voted for Pappu's Party in the current polls?

(a) 150000 (b) 210000
(c) 240000 (d) 180000

Directions for Questions 8–10: On the planet of Krypton (yes the same one from where Superman originated) A small scale businessman owns an ancillary unit that manufactures 6 types of tools namely *A, B, C, D, E* and *F*. These tools for their manufacturing require work on machine *X* and machine *Y* in that order. The time taken on these machines to manufacture one unit of a tool is as follows:

Tool →	A	B	C	D	E	F
Time (hrs) Machine *X*	6	24	10	4	18	22
Machine *Y*	16	20	18	12	6	2

Presently, the businessman has received orders to manufacture one unit of each of the tools. There is no constraint to work on the machines and these machines can be run non-stop.

8. For a particular order from Earth, the tools are to be manufactured in the sequence FEDCAB, what is the minimum time taken to execute the order?

(a) 158 hrs (b) 110 hrs
(c) 130 hrs (d) 120 hrs

10. If a revolutionary technology is imported from the earth, the time required for working on each tool is reduced to half its current time on both machines. With this technology in place, what is the shortest possible time in which the order can be executed?

(a) 65 hrs (b) 55 hrs
(c) 55 hrs (d) None of these

11. A man can climb up a greased pole 30 ft high in 10 seconds (where 10 seconds is used for climbing up and 5 seconds when he is slipping down). If grease causes a downward slip of 2 ft a second, the time taken by the man to climb a non greased pole would be:

(a) 8 seconds (b) 10 seconds
(c) 7.5 seconds (d) 9 seconds

12. A state's GDP during 2018-19 grew by a healthy 6.5%. During the same period service sector growth was 7.0%, average growth for agricultural, manufacturing, and service sector (excluding ITES and IT) was 5.8%. If weightage given to agricultural sector is 24%, to manufacturing sector 42%, and to service sector as a whole 33% (IT sector 13%, and ITES 9%), then what is the approximate growth rate for service sector (excluding ITES and IT)?

(a) 6.4%
(b) 5.9%
(c) 5.0%
(d) 5.8% to 7.0% (depending on weightage)

13. The village of Hara Kiri has two-thirds of its male population married to 80% of its women population. Each of such man is married to exactly two of such women. What is the minimum possible total population of the village (Children are not counted as part of the population unless they are married)?

(a) 1005
(b) 1026
(c) 1080
(d) Cannot be determined/None of these

14.

The figure contains three squares with area of 100 m², 49 m², and 16 m² lying side by side. Rahul wishes to reduce the length *AB* to 19 m by reducing area of only one of the squares without altering its shape, what is the maximum reduction (in %) in the area of that square?

(a) (Approx) 31% (b) 50%
(c) (Approx) 66% (d) 75%

15. Bagal and Sagar practice in the local Gomati Sports stadium daily. One day, they started running around the circular track simultaneously from the same point in opposite directions. Thereafter they met a total of ten times when the practice session was terminated. If they meet at a point diametrically opposite to the starting point when they meet for the second time, what could be ratio of their speed?

(a) 1 : 1 (b) 2 : 1
(c) 1 : 3 (d) Data insufficient

16. The Sohagpur Express and the Gomati Express consisting of 34 and 42 carriages respectively are moving @ 36 kmph in opposite directions. The carriage length (including inter-carriage length) averaged over number of carriages averages 15 m

and is half the engine length. If the trains cross each other on parallel tracks, in how much time will they complete the crossing?

(a) 1 min 20 sec (b) 42 seconds
(c) 38 seconds (d) 40 seconds

17. The selling price and the cost price of an article which is being sold at a profit have increased by r% YOY for a period of 3 years. Which of the following statements summarizes the effect of the inflationary pressure?

(a) The profitability of the article has increased at the same rate r% every year for the 3-year period.
(b) The profit has increased YOY at the same rate of r% while the profitability has remained constant.
(c) The profit has increased YOY at the same rate r% but profitability has increased at a rate which is less than the rate of increase of profit.
(d) The profit has increased YOY at the same rate of r%, while the profitability has decreased over the 3 year period.

18. The Home Computer Company gets a large supply order for assembled computers of two different configurations A and B from the Karnataka government. The supply order for A-type computers being double that of the B-type. A team of hardware experts was assigned the task which worked on A-type computers. For the second day, the team was split into two equal groups: the first continued work on A-type computers and completed the work order in five days. The second group, however could not complete work order for B-type computers on the second day and this work order could only be completed by employing two experts for the whole of the third day. What was the strength of the team if it is known that a B-type computer takes 20% less time to assemble than A-type?

(a) 8 (b) 20
(c) 16 (d) 24

19. In a learning experiment at Stanford University it was found that the square of the inverse of learning time varies directly as the age of a rat in weeks. Also the inverse of learning time and complexity index of the maze are directly related. If the age of the rat in the second trial of the experiment is 40% more than in the first trial and the learning time is 16.67% less, then find change in complexity index of the maze.

(a) Increase of 10% (b) Increase of 5%
(c) Increase of 1% (d) Decrease of 5%

20. Rama is standing at an intersection where two roads intersect at right angles. He observes through his binoculars that two cars start moving towards the intersection at the same time along these two roads, one of them is moving at a speed of 13.333 m/sec and starts at a point 40 km from the intersection. The second car is moving at 10 m/sec and starts at a point 30 km from the intersection. In how many minutes from the start will the distance between the two be minimum?

(a) 85 min (b) 62 min
(c) 50 min (d) (1) or (3)

REVIEW TEST FIVE

Directions for Questions 1–2: Refer to data below. In a motorcycle race, three motorcycles started out. The first motorcycle was 12 km/h faster than the second, while the third was 5 km/h slower than the second. The second motorcycle driven by Ayrton Senna arrived at the finish 12 min after the first motorcycle and 3 minutes before the third one. There were no stops en route.

1. What is the length of the race?

(a) 150 km (b) 72 km
(c) 75 km (d) 90 km

2. What is the speed of the slowest motor cycle?

(a) 75 km/h (b) 90 km/h
(c) 72 km/h (d) 64 km/h

3. Naman Sweets a sweet shop in down town Allahabad manufactures Rosogullas for export. They are currently experimenting with an ideal sugar syrup concentration for their new product.

Originally, the sugar in the syrup is $13\frac{1}{3}$% by weight.

The syrup is heated till 25% of the water in it evaporates. The resultant solution is mixed with a 40% sugar syrup in the ratio 5 : 3. The sugar syrup so obtained is again heated to evaporate $33\frac{1}{3}$% of water in it. Find the strength of this solution assuming the solution never saturates.

(a) 30%
(b) slightly more than 60%
(c) slightly more than 25%
(d) 20%

4. Two sets of boys are playing a match of cricket. It is decided that over the ball falls on the ground from the hands of a side is would get the other side as many points as the number of meters the ball travels before coming to rest and the play won't resume until that happens. It is also known that height of a fall is exactly 150% of that of the bounce for the kind of balls to be used for the match. In the first instance Raghav's team dropped the ball a height of 45 m. How many points does Shreyas's team gain?
(a) 90 (b) 135
(c) 225 (d) None of these

5. Ashish was experimenting with time measurement using a pair of sandglasses. The following pairs of sand glasses are available: sand glass of one pair cannot be combined with sand glass of another pair:
A – 9 min and 12 min
B – 4 min and 7 min
C – 6 min and 7 min
In how many ways can he choose his pair of sand glasses in order to be able to measure exactly 15 minutes?
(a) 3 (b) 2
(c) 1 (d) None

6. In a 4400 meter race around a circular track (in the stadium at IIM Bangalore) having a circumference of 1000 meters, the fastest runner and the slowest runner reach the same point at the end of the 3rd minute, for the first time after the start of the race. All the runners have the same starting point and each runner maintains a uniform speed throughout the race. If the slowest runner runs at half the speed of the fastest runner, what is the time taken by the slowest runner to finish the race?
(a) 40 min (b) 30 min
(c) 20 min (d) 15 min

Directions for Questions 7–8: Answer the questions based on the following information.

Prabhjot made a trip from Hyderabad to Bangalore to attend an interview. He had been travelling at a speed which would have caused him to reach the interview on time. He had a flat tyre on the way and it took 100 min to mend it, the normal 15 min. late for the interview. The distance between Bangalore and Hyderabad is 360 km.

7. If the speed after the flat tyre was 20% more than his original speed, find the normal time taken by Prabhjot to reach the destination from the site of the accident.
(a) 8 hr. 5 min (b) 1 hr. 45 min.
(c) 6 hr. (d) 8 hr. 30 min.

8. If the normal time taken to reach Bangalore after the start was 10 hr. at what distance from Bangalore did the flat tyre occur? (Use data of question 7)
(a) 39 km (b) 221 km.
(c) 160 km (d) Can't be determined.

9. Steven fell in love with Ankita who lived 600 miles away. He decided to propose his beloved and invited her to travel to his place and offered to meet her en route and bring her home. Steven is able to move 4 miles per hour to Ankita's 3 miles per hour. How far will each have travelled upon meeting?
(a) Steven = 270 miles; Ankita = 360 miles
(b) Steven = 360 miles; Ankita = 270 miles
(c) Steven = 400 miles; Ankita = 230 miles
(d) Steven = 450 miles; Ankita = 180 miles

10. The evergreen shrubs at Ramesh's nursery are planted in rows on a square plot of land measuring 2,401 sq. ft. The shrubs are planted in such a manner that the corners of the shrubs are 7 ft. apart and the outer shrubs are planted along the edges of the plot with a shrub at each corner. Ramesh spent $1710 to cover all the costs necessary for raising this crop of the evergreen shrubs. If Ramesh succeeds in selling each shrub for $70, his profit will be what percentage of his total cost?
(a) 100% (b) 50%
(c) 125% (d) 150%

11. Chaitanya is repaid a loan that she had given to Meenakshi such that each year the amount she gets is 15% lesser than what he gave in the previous year. Meenakshi being a rich person, decides to give Chaitanya Rs. 2500 as a lump sum and not the remaining later. Who gains from this agreement and what is the breakeven value, if the initial loan given is Rs. 1000? (Consider no time value of money)

(a) Chennai, Rs. 2000
(b) Mumbai, Rs. 9000
(c) Chennai, Rs. 100
(d) Mumbai, Rs. 10000

12. MTPC Ltd power plant in Dhiyanpur can produce upto 120 MW of power every month. The cost of the power is partly fixed and partly varying. The cost is Rs. 250 million, when 80 MW are produced and it is Rs. 300 million, when 100 MW is produced. On the other hand, the part of selling price per unit goes on decreasing linearly as more power is consumed. The selling rate per unit is Rs. 7.30 at 80 MW sell and it is Rs. 5.30, when 100 MW power is sold. What will be the profit or loss when it produces 110 MW of power and sells it all?

(a) Profit, Rs. 26.5 million
(b) Profit, Rs. 21.5 million
(c) Loss, Rs. 26.5 million
(d) Loss, RS. 21.5 million

13. Divyaja Ananda buys some apples and Chikoos from the market at a rate such that a Chikoo is thirty times costlier than an apple and he sells them each for a Chikoo is thrice the price of an apple. By selling the apple at twice its cost price, he makes 150% profit. Find the proportion of Chikoos to the apples.

(a) 1 : 6 (b) 3 : 4
(c) 1 : 2 (d) Can't be determined

14. Anandhan bets a certain portion of his total money on betting. On a particular day, when he won, the betting rate was 12 : 1 (i.e. if you bet 1 rupee you stand to gain 6 rupees). If you win the bets and his total money became 8 times the initial total money. What is the proportion of his initial total money to the money she put on bet?

(a) 3 : 2 (b) 7 : 8
(c) 8 : 5 (d) None of these

15. On 26th July Mumbai was flooded. Rahubhai Mangrous is in the business of manufacturing umbrella and raincoats. An umbrella can shield 2 people while a raincoat can shield 1 person. People using an umbrella can walk at 1/3rd the speed of a person wearing a raincoat. The cost of an umbrella and cost of the raincoat are in the ratio 2 : 5. Points A and B are 100 m apart and it takes 20 minutes for a person wearing a raincoat to travel the distance. At the maximum how many people will be transported today from A to B, if only Rs. 1000 is available for transporting 20 people? Cost of traveling none is Rs. 2 per man, per head (Assume cost of a raincoat to be Rs. 100.)

(a) 8 (b) 7
(c) 4 (d) None of these

16. A manufacturing job was wrongly estimated to be completed in 10 days by x machines. By deploying 3 extra machines, the job was completed in 12 days. If only one additional machine was used, how many days more than that estimated would it have taken to complete the job?

(a) $\frac{2x-18}{x+3}$ (b) $\frac{2x+13}{x+1}$

(c) $\frac{2x+28}{x+1}$ (d) Cannot be determined

17. A tank in Gauri Apartments is full of water is fitted with pumps of equal capacity. Besides it is also fitted with a certain number of inlet pipes which are always kept open. When the tank is full, 10 pumps of equal capacity empty the tank in 72 hrs., while 15 pumps of the same capacity empty the tank in 6 hrs. In how much time would 25 pumps of the same capacity empty the tank, if the tank is initially full? (All the pipes pouring water into the tank are always open)

(a) 3 hrs. (b) $3\frac{1}{2}$ hrs.

(c) 4 hrs. (d) $3\frac{1}{3}$ hrs.

18. A group of 4 people – Raghav, Geet, Shaurya and Arjit arrive in a car to a friend's house. The group was forced to park around the corner due to lack of parking space. It was raining heavily and the group had only one umbrella. They decided to share it. Two persons would go at a time to the friends house and one of them would come back with the umbrella. The process would be repeated unless all four people reach the friends house. If Raghav, Geet, Shaurya and Arjit take 1, 2, 5 and 10 minutes respectively at their fastest speed to go to the friend's house, what is the minimum time required for all of them to reach the friend's house? (A faster person slows down to accompany a slower one)

(a) 15 mins (b) 17 mins

(c) 12 mins (d) 14 mins

10. At a shooting competition, RATHORE and BINDRA had a bullet between them. BINDRA fired 'a' bullets per min, while RATHORE fired 'b' bullets per min. After time 't', both of them had the same number of bullets. What is the initial number of bullets with RATHORE?

(a) $t - \frac{a+b}{2}$ (b) $t - \frac{t(a-b)}{2}$

(c) $\frac{t-t(a-b)}{2}$ (d) None of these

ANSWER KEY

Review Test Five

1. b
2. c
3. a
4. a
5. b
6. c
7. b
8. b
9. d
10. c
11. b
12. a
13. b
14. d
15. a
16. b
17. b
18. c
19. d
20. c
21. b
22. c
23. b
24. b
25. a
26. a

Review Test Two

1. d
2. a
3. c
4. c
5. b
6. a
7. b
8. d
9. d
10. c
11. c
12. c
13. b
14. c
15. a
16. a
17. c
18. a
19. d
20. c
21. b
22. a
23. d
24. b
25. b

Review Test Three

1. c
2. a
3. b
4. a
5. c
6. b
7. a
8. d
9. a
10. c
11. b
12. a
13. d
14. b
15. c
16. b
17. b
18. c
19. b
20. b
21. d
22. b
23. d
24. a
25. d

Review Test Four

1. b
2. b
3. d
4. a
5. c
6. d
7. a
8. b
9. c
10. d
11. c
12. c
13. a
14. d
15. c
16. b
17. b
18. b
19. b
20. b

Review Test Five

1. d
2. c
3. b
4. c
5. a
6. b
7. a
8. d
9. b
10. c
11. d
12. b
13. a
14. a
15. b
16. a
17. d
18. b
19. d
20. b

BLOCK 4

CHAPTER

* Geometry and Mensuration

...BACK TO SCHOOL

The word GEOMETRY is derived from two words—GEO meaning earth and METRY meaning measurement. Hence, it is quite evident that geometry as a mathematical science developed mainly due to the human need of measuring land masses and distances. The major developments in the fields of Geometry and Mensuration are mainly credited to the ancient Egyptian and Greek civilisations. (In fact, this is one of the reasons why all formulae and theorems primarily carry Greek names.)

Key Geometrical Concepts

Point: The point should be visualised as a singular dot. In physical terms, a point can be defined as a single dot that can be created on a plain paper by a very sharp pencil. It could also be visualised as a singular prick on a piece of paper by a very sharp nail or pin.

Line: Mathematically, lines are defined as a group of points which are straight one after another. All lines are supposed to extend infinitely in two directions.

Segment of a Line

If a part of a line is cut out, we get a segment of a line.

Physically, the closest representation of a segment of a line would be a tight thread or the straight crease of a piece of paper.

Plane: The surface of a smooth wall or a table top is the closest representation of a portion of a plane.

Contd.

Some Geometrical Properties

1. Lines Between Points: Let us take two distinct points in a plane. You can easily verify that an infinite number of lines can be drawn in the plane passing through any one of these two points. However, if you wanted to draw a line which passes through both the points, you will be able to draw only one line. This is an important result to remember.

Property: Given any two distinct points in a plane, there exists one and only one line containing both the points. Alternatively, we can state that two distinct points in a plane determine a unique line.

2. Collinearity of Points: Collinear or non-collinear points are only defined in the context of 3 or more points. Consider the situation of three points. There can only be two cases with respect to these points.

1. All the points lie on the same line. Hence, the points are said to be collinear.
2. All the three points do not lie on the same line. In this case the points are said to be non-collinear.

Note: It is pertinent to note that we could discuss collinear or non-collinear points only if the number of points is more than two. Obviously, if there are only two points they would always lie on one line.

3. Points in Common Between Distinct Lines: In the case of distinct lines, there can only be two cases.

(i) There is one point in common. In such a case, the common point is called as the point of intersection, and the two lines are called as intersecting lines.

(ii) There is no point in common. In such a case, the two lines are non-intersecting and are also called as parallel lines.

The property can thus be stated as:

Property: Two distinct lines in a plane cannot have more than one point in common.

4. Lines and Points:

Property: Given a line and a point in the same plane, such that the point is not on the line, there is one and only one line which passes through the point and is parallel to the given line.

5. Lines:

Property: Two intersecting lines cannot be parallel to the same line.

6. Multiple Lines: There can be four distinct possibilities in such a case.

1. No two lines intersect each other.
2. Some lines intersect.
3. Every line intersects all other lines, but their respective points of intersection are different from each other.
4. Every pair of lines is intersecting and all the points of intersection coincide.

The reader is required to visualize the figures for each of these situations.

The remaining formulae and results for geometry have been given within the chapter. You are required to store in the respective formulae. However, an important note while you are doing this.

Important Note

One of the most underrated issues with respect to this Week is

How does a person move from:

"Cannot Solve Geometry and Mensuration!" to "Can Solve Geometry and Mensuration!"

In fact, this transition should be your sole aim while studying and solving this block of chapters.

As you are already aware, mere knowledge of formulae does not guarantee this movement. Then in that case what should a person do if he/she has to engineer this transition of abilities?

Well, the answer is really quite simple and for understanding the same you need to move out of conventional mathematical study processes. In your mind you need to create an awareness of how to think while solving a geometry/mensuration question. Here are a few tips that will help you through this transition.

1. Some Formulae are More Important than Others: For the CAT (and indeed all other MBA and upscale examinations) your concentration should not be on memorizing complex formulae. A close study of the past trends for questions from this block will show you that the formulae themselves can be graded into the important and the not so important formulae. In fact, if you study over 200 questions from this block that have been asked in the CAT in the past decade or more, you will realise that over 90% of the questions are simply based on a small subset of geometrical formulae and results. This important set of formulae are listed below for your convenience:

Lines: Properties related to intersecting lines and angles formed.

Polygons: Basic properties of regular polygons.

Triangles: Pythagoras theorem, 30-60-90 triangle, 45-45-90 triangle, Sine rule, Properties of Exterior angles of a triangle, equilateral triangle, isosceles triangle.

Quadrilaterals: Areas, perimeters, diagonals and angles between diagonals for all standard quadrilaterals.

Hexagon: A Hexagon can be divided into six equal triangles.

2. Length/Area/Volume Measuring Formulae versus Angle Measuring Formulae: In your mind, divide the formulae into length/area/volume measuring formulae on the one hand and Angle measuring formulae on the other hand. This will be very helpful, since most questions in this block can be clearly segregated as length or angle measuring questions. In such a scenario, in case you have a question requiring angle measurements, you will only expect to think of angle measuring formulae.

Note: There are very few formulae that connect lengths with angles, e.g. Sine rule and Cosine rule in the context of triangles.

3. The Use of Reverse Flowcharting for Solving Questions: All questions in geometry and mensuration ask you to find a particular value. Very often, it makes the question much more convenient to solve if instead of trying to find what is asked, you try to find a value which is related to the required value.

Thus, for example, if in a triangle ABC angle A is given to be 40° & angle B is asked for and you are not getting a clear strategy for getting the value of angle B, then it might be wise to try to derive the value of the angle C. If you can get C, then B will be got by 140-C.

This process is called as reverse flowcharting and is an extremely crucial process for solving questions in geometry.

In other words, it can also be seen to be a deflection of the original question into a more convenient question.

Note: You are advised to concentrate on the special note for solving CAT questions provided prior to the LOD 1 Questions while studying this Block.

PREASSESSMENT TEST

1. In the figure below, $AB = BC$ and $DE = DF = AF = AD = CE$. Then, $\angle DEC$ is approximately

(a) 15°
(b) 20°
(c) 30°
(d) 25°

2. The figure below shows three circles, each of radius 20 and centres at P, Q and R respectively. Further $AB = BC = 12$ and $AF = 14$. What is the perimeter of the triangle PQR?

(a) 107
(b) 116
(c) 111
(d) 122

3. The figure shows a circle which has a diameter AB and a radius 13. If chord CA is 10 cm long, then find the area of ΔABC.

(a) 240 sq.cm
(b) 120 sq.cm
(c) 180 sq.cm
(d) None of these

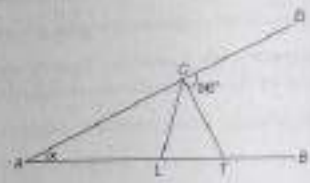
4. The rod AB is 12 metres in length and is tangent to the inner one of the two concentric circles at point C. It is known that the radii of the two circles differ by 6 metres. The radius of the larger circle is

(a) 5 metres
(b) 8 metres
(c) 6 metres
(d) 3 metres

5. Four cities are connected by a road network, as shown in the figure. In how many ways can you start from any city and come back to it without travelling on the same road more than once?

(a) 14
(b) 12
(c) 10
(d) 15

6. In the figure (not drawn to scale) given below, $AL = LC = CT$, and $\angle TCD = 96°$. What is the value of the $\angle LTC$?



(a) 32°
(b) 64°
(c) 84°
(d) Cannot be determined

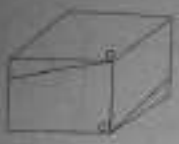
Directions for Questions 7 to 9: Answer the questions on the basis of the information given below.

Consider a cylinder of height h cm and radius $r = 2/\pi$ cm as shown in the figure (not drawn to scale). A string of certain length, when wound on its cylindrical surface, starting at point A and ending at point B, gives a maximum of n turns (in other words, the string's length is the minimum length required to wind n turns).

7. What is the vertical spacing in cm between two consecutive turns?

(a) h/n
(b) h^2/n
(c) h/n^2
(d) Cannot be determined with given information.

8. The same string, when wound on the four external lateral walls of a cube of side n cm, starting at point C and ending at point D, can give exactly one turn (as shown in the figure). The length of the string, in cm, is



(a) $\sqrt{2}\,a$ (b) $\sqrt{11}\,a$
(c) a (d) $\sqrt{13}\,a$

9. In the setup of the previous two questions, how is h related to a?
(a) $h = \sqrt{2}\,a$ (b) $h = \sqrt{13}\,a$
(c) $h = a$ (d) $h = \sqrt{15}\,a$

10. Let $LMNOPQ$ be a regular hexagon. What is the ratio of the area of the triangle LMP to that of the hexagon $LMNOPQ$?
(a) 1/3 (b) 1/2
(c) 2/3 (d) 5/6

11. In the figure below, a circle is inscribed inside a square. In the gap between the circle and the square (at the corner) a rectangle measuring 20 cm × 10 cm is drawn such that the corner A of the rectangle is also a point on the circumference of the circle. What is the radius of the circle in cm?

(a) 30 cm (b) 40 cm
(c) 50 cm (d) None of these

12. In the figure below $ABCDEF$ is a regular hexagon. The $\angle AOF = 90°$ and FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon $ABCDEF$?

(a) 1/12 (b) 1/6
(c) 1/4 (d) 1/3

13. Harry's mopping mop after a lot of accidents by the broken bar spread to form the form below, which is a 30 m by 40 m rectangle. The mopper moves a 1 m wide strip.

If Harry starts at one corner and moves around the lawn towards the centre, about how many times would he go round before he has mowed half the lawn?
(a) 4.3 (b) 4.8
(c) 4.9 (d) 5.0

14. Two sides of a plot measure 32 metres and 24 metres and the angle between them is a perfect right angle. The other two sides measure 25 metres each and the other three angles are not right angles.

What is the area of the plot (in m^2)?
(a) 768 (b) 624
(c) 696.5 (d) 684

15. Euclid has a triangle in mind, its longest side has length 20 and another of its sides has length 10. Its area is 80 sq. cm. What is the exact length of its third side?
(a) $\sqrt{260}$ (b) $\sqrt{290}$
(c) $\sqrt{240}$ (d) $\sqrt{370}$

16. Consider a circle with unit radius. There are seven adjacent vertices $S_1, S_2, S_3, \ldots, S_7$ in the circle such that their total area is 1/16 of the area of the circle. Further, the area of jth sector is twice that of the $(j-1)$th sector, for $j = 2, \ldots, 7$. What is the angle, in radians, subtended by the arc of S_1 at the centre of the circle?
(a) $\pi/508$ (b) $\pi/2040$
(c) $\pi/1016$ (d) $\pi/2032$

17. The figure below shows a set of concentric squares. If the diagonal of the innermost square is 2 units, and if the distance between the corresponding corners of any two successive squares is 1 unit, find the difference between the areas of the sixth and the seventh squares, counting from the innermost square.

(a) $\sqrt{2}$ sq. units (b) 2π sq. units
(c) 2π sq. units (d) None of these

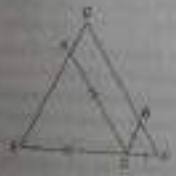
[illegible] In the figure [illegible] inscribed in the circle with center at O. The length of side AB is greater than that of side BC. The ratio of the area of the circle to the area of the rectangle ABCD is $\pi : 6$. The line segment DE bisects AB at E such that $\angle ODE = \angle ADE$. What is the ratio AE : AD?

(a) $1 : \sqrt{6}$ (b) $1 : \sqrt{2}$
(c) $2\sqrt{3} : 1$ (d) $1 : 2$

[illegible] In ΔABC, $\angle B$ is a right angle, $AC = 10$ cm, and D is the mid-point of AC. The length of BD is

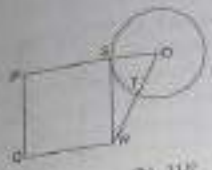
(a) 10 cm (b) 15 cm
(c) 8 cm (d) 5 cm

[illegible] In triangle DEF shown below, points A, B and C are taken on DE, DF and EF respectively such that EC = AC and CF = BC. If angle D = 50 degrees then what is angle ACB in degrees?



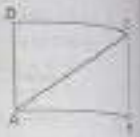
(a) 120 (b) 80
(c) 90 (d) None of these

[illegible] PQRS is a square. SR is a tangent on point S to the circle with center O and PR = OS. Then, the ratio of area of the square to the area of the circle is



(a) 8/7 (b) 11/7
(c) 9/8 (d) 7/11

[illegible] In the adjoining figure, AC = AB = 5 AD and AC − AD = 8. Then, the area of the rectangle ABCD is



(a) 45
(b) 50
(c) 60
(d) Cannot be answered

[illegible] The figure shows the rectangle ABCD with a semicircle and a circle inscribed inside it as shown. What is the ratio of the area of the circle to that of the semicircle?

(a) $(\sqrt{2}-1)^2 : 1$ (b) $2(\sqrt{2}-1)^2 : 1$
(c) $(\sqrt{2}-1)^2 : 2$ (d) None of these

[illegible] A right circular cone of height h is cut by a plane parallel to the base and at a distance h/3 from the top. Then the volumes of the frustum of the cone and the remaining frustum of the cone are in the ratio

(a) 1 : 3 (b) 8 : 19
(c) 19 : 8 (d) 7 : 1

[illegible] Three identical cones with base radius r are placed on their bases so that each is touching the other two. There will be one and only one circle that would pass through each of the three vertices. What can be said about the radius of this circle?

(a) It would be smaller than r
(b) It would be equal to r
(c) It would be larger than r
(d) depends on the height of the cones

Answers (Block 4 Preassessment Test)

1. (d)	2. (a)	3. (b)	4. (a)	5. (b)
6. (c)	7. (a)	8. (b)	9. (c)	10. (b)
11. (c)	12. (a)	13. (c)	14. (d)	15. (b)
16. (d)	17. (b)	18. (a)	19. (c)	20. (b)
21. (c)	22. (c)	23. (d)	24. (c)	25. (c)

11

GEOMETRY AND MENSURATION

The chapters of geometry and mensuration have had their share of questions in the CAT and other MBA entrance exams. For doing well in questions based on this chapter, the student should familiarise himself/herself with the basic formulae and visualisations of the various shapes of solids and two dimensional figures based on this chapter.

The following is a comprehensive collection of formulae based on two dimensional and three-dimensional figures.

For the purpose of this chapter we have divided the theory in two parts

- Part I consists of the geometry and mensuration of two-dimensional figures
- Part II consists of mensuration of three-dimensional figures

Part I: GEOMETRY

INTRODUCTION

Geometry and Mensuration are important areas in the CAT examination. Over the past decade the trend has been that they are around 5 – 8 questions in the section of Mathematics is based on these chapters. Besides, questions from these chapters appear prominently in all major aptitude based exams like MBA's, Bank PO's etc.

Hence, the student is advised to ensure that he/she studies these chapter completely and thoroughly. Skills to be developed while studying and practicing this chapter will be application of formula and visualisation of figures and solids.

The principal skill required for doing well in this chapter is the ability to apply the formulae and theorems.

The following is a comprehensive collection of formulae based on two dimensional figures. The student is advised to memorize the formulae in this chapter so that he is able to solve all the questions based on this chapter.

THEORY

Basic Conversions

A. 1 m = 100 cm = 1000 mm 1 km = 1000 m = 3/8 miles 1 inch = 2.54 cm	B. 1 m = 39.37 inches 1 mile = 1760 yd = 5280 ft 1 nautical mile (knot) = 6080 ft
C. 100 kg = 1 quintal 10 quintal = 1 tonne = 1000 kg 1 kg = 2.2 pounds (approx.)	D. 1 liter = 1000 cc 1 acre = 100 sq m 1 hectare = 10000 sq m

STRAIGHT LINES

Parallel Lines: Two straight lines are parallel if they lie on the same plane and do not intersect however far produced.

Transversal: It is a straight line that intersects two parallel lines. When a transversal intersects two parallel lines then

1. Corresponding angles are equal, that is, in the figure below
   $1 = 5, 2 = 6, 4 = 8, 3 = 7$
2. Alternate interior angles are equal, that is, (Refer figure below)
   $4 = 6, 3 = 5$
3. Alternate exterior angles are equal, that is
   $2 = 8, 1 = 7$
4. Interior angles on the same side of transversal add up to 180°, that is
   $4 + 5 = 1 + 6 = 180°$

POLYGONS

Polygons are plane figures formed by a closed series of rectilinear (straight) segments. **Example:** Triangle, Rectangle.

Polygons can broadly be divided into two types.

(a) *Regular polygons:* Polygons with all the sides and angles equal.

(b) *Irregular polygons:* Polygons in which all the sides or angles are not of the same measure.

Polygons can also be divided as convex or concave polygons.

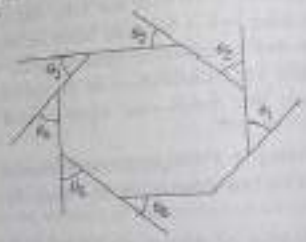
Convex polygons are the polygons in which all the diagonals lie inside the figure otherwise it's a concave polygon.

Polygons can also be divided on the basis of the number of sides they have.

No. of sides	Name of the polygon	Sum of all the angles
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	720°
7	Heptagon	900°
8	Octagon	1080°
9	Nonagon	1260°
10	Decagon	1440°

Properties

1. Sum of all the angles of a polygon with n sides $= (2n - 4)90°$ or $(n - 2)\pi$ Radians $= (n - 2)\,180°$ degrees
2. Sum of all exterior angles $= 360°$.
   i.e. In the figure below:
   $\theta_1 + \theta_2 + \ldots + \theta_n = 360°$
   In general, θ_i = ith exterior angle.
3. No. of sides $= 360°/(\text{ext})$
4. Area $= (nr^2/4) \times \cot(180/n)$; where r = length of side, n = no. of sides.
5. Perimeter $= n \times s$.



TRIANGLES (Δ)

A triangle is a polygon having three sides. Sum of all the angles of a triangle = 180°.

Types 1. Acute angle triangle: Triangles with all the angles acute (less than 90°).

2. Obtuse angle triangle: Triangles with one of the angles obtuse (more than 90°).
   Note: We cannot have more than one obtuse angle in a triangle.
3. Right angle triangle: Triangle with one of the angles equal to 90°.
4. Equilateral triangle: Triangle with all sides equal, the angles in such a triangle are equal to 60°.
5. Isosceles triangle: Triangle with two of its sides equal and consequently the angles opposite the equal sides are also equal.

- Isosceles Triangle: Triangle with none of the sides equal to any other side.

Properties (General)

- Sum of the length of any two sides of a triangle has to be always greater than the third side.
- Difference between the lengths of any two sides of a triangle has to be always lesser than the third side.
- Side opposite to the greatest angle will be the greatest and the side opposite to the smallest angle the smallest.
- The sine rule: $a/\sin A = b/\sin B = c/\sin C = 2R$ (where R = circum radius.)
- The cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

  This is true for all sides and respective angles.

In case of a right $\angle$, the formula reduces to $a^2 = b^2 + c^2$

Since $\cos 90 = 0$

- The exterior angle is equal to the sum of two interior angles not adjacent to it.

$$\angle ACD = \angle BCE = \angle A + \angle B$$

Area

1. Area = 1/2 base × height or 1/2 bh

   Height = Perpendicular distance between the base and vertex opposite to it

2. Area = $\sqrt{s(s-a)(s-b)(s-c)}$ (Hero's formula)

   where $s = \dfrac{a+b+c}{2}$ (a, b and c being the length of the sides)

3. Area = πr (where r is in radius)
4. Area = 1/2 × product of two sides × sine of the included angle

   = 1/2 $ac \sin B$

   = 1/2 $ab \sin C$

   = 1/2 $bc \sin A$

4. Area = $abc/4R$

   where R = circum radius

Congruency of Triangles Two triangles are congruent if all the sides of one are equal to the corresponding sides of another. It follows that all the angles of one are equal to the corresponding angles of another. The notation for congruency is $\left(\cong\right)$.

Conditions for Congruency

1. SAS congruency: If two sides and an included angle of one triangle are equal to two sides and an included angle of another, the two triangles are congruent. (See figure below.)

   Here, $AB = PQ$

   $BC = QR$

   and $\angle B = \angle Q$

   So $\Delta ABC \cong \Delta PQR$

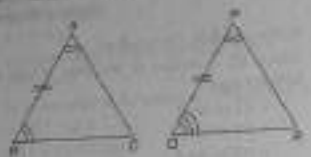
3. *ASA congruency:* If two angles and the included side of one triangle is equal to two angles and the included side of another, the triangles are congruent. (See figure below.)

Then, $\angle A = \angle P$

$\angle B = \angle Q$

and $AB = PQ$

So $\Delta ABC \cong \Delta PQR$



4. *AAS congruency:* If two angles and side opposite to one of the angles is equal to the corresponding angles and the side of another triangle, the triangles are congruent. In the figure below

$\angle A = \angle P$

$\angle B = \angle Q$

and $AC = PR$

So $\Delta ABC \cong \Delta PQR$

5. *SSS congruency:* If three sides of one triangle are equal to three sides of another triangle, the two triangles are congruent. In the figure below

$AB = PQ$

$BC = QR$

$AC = PR$

$\Delta ABC \cong \Delta PQR$

6. *SSA congruency:* If two sides and the angle opposite the greater side of one triangle are equal to the two sides and the angle opposite to the greater side of another triangle, then the triangles are congruent. The congruency doesn't hold if the equal angles lie opposite the shorter side. In the figure below, if

$AB = PQ$

$AC = PR$

$\angle B = \angle Q$

Then the triangles are congruent

i.e. $\Delta ABC \cong \Delta PQR$.

Similarity of triangles Similarity of triangles is a special case where if either of the conditions of similarity of polygons holds the other will hold automatically.

Types of similarity

1. *AAA similarity:* If in two triangles, corresponding angles are equal, that is, the two triangles are equiangular then the triangles are similar.

   Corollary (AA similarity): If two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar. The reason being, the third angle becomes equal automatically.

2. *SSS similarity:* If the corresponding sides of two triangles are proportional then they are similar.

   For ΔABC to be similar to ΔPQR, $AB/PQ = BC/QR = AC/PR$, must hold true.

3. *SAS similarity:* If in two triangles, one pair of corresponding sides are proportional and the included angles are equal then the two triangles are similar.

   $\Delta ABC \sim \Delta PQR$

   If $AB/BC = PQ/QR$ and $\angle B = \angle Q$

   Note: In similar triangles, the following always hold:

   Ratio of medians = Ratio of heights = Ratio of circumradii = Ratio of inradii = Ratio of angle bisectors.

   While there are a lot of methods through which we can prove similarity of triangles, the one thing that all our teachers begin to tell us about similarity is the basic fundamental concept of similarity, i.e. **Two things are similar if they look similar!!**

   If you have been to a toy shop lately, you would have come across models of cars or bikes which are made to

are just like the original—but are made in a different size from the original. Thus you might have seen a toy Morris car which is built in a scale of 1:20 of the original car. The result of this is that the toy car would look very much like the original car except that it is built smaller. Thus if you have two cars rather and are looking exactly like each other, you are mentioned similarity.

We shall use this principle to identify similar triangles. In figure two triangles would be similar simply if they look alike similar.

Thus in the figure below if you write in then the sides OB and PQ the triangles MOB and MPQ will be similar to each other. Simply because they look similar. Of course, the same using the different rules of similarity of triangles will similar will you.

Isosceles Triangles (of side a):

1. $\qquad\qquad$ (∵ $\sin 60 = \sqrt{3}/2$ = side)

$\Rightarrow a = \frac{a\sqrt{3}}{2}$

2. Area = 1/2 (base) × (height) = $\frac{1}{2} \times a \times \frac{a\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^2$

3. If r (the radius) = $\frac{2h}{3} = \frac{a}{\sqrt{3}}$

$\Rightarrow r$ (the radius) = $\frac{h}{3} = \frac{a}{3\sqrt{3}}$

Properties

1. The incenter and circumcenter lies at a point that divides the height in the ratio 2 : 1.
2. The circum radius is always twice the in radius [$R = 2r$].
3. Among all the triangles that can be formed with a given perimeter, the equilateral triangle will have the maximum area.
4. An equilateral triangle in a circle will have the maximum area compared to other triangles inside the same circle.

Isosceles Triangle

Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$

In an isosceles triangle, the angles opposite to the equal sides are equal.

Right Angled Triangle

Pythagoras Theorem In the case of a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. In the figure below, for triangle ABC, $a^2 = b^2 + c^2$.

Area = 1/2 (product of perpendicular sides)

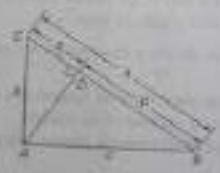
$$\text{Rectangle (area)} = \frac{\text{Height} \times \text{Base}}{2}$$

Area = rs

where r = in radius and $s = (a + b + c)/2$ where a, b and c are sides of the triangle.

$\Rightarrow \qquad 1/2\,bc = rs + b + c/2$

$\Rightarrow \qquad r = (b + c)/a + b + c$



In the triangle ABC,

$$\Delta ABC = \Delta DBA + \Delta DAC$$

(Note: A lot of questions are based on this figure.)

Further, we find the following identities:

1. $\Delta ABC \sim \Delta DBA$

$\Delta BAC \sim \Delta DBA$

$\Rightarrow \qquad AB^2 = DB \times BC$

$\Rightarrow \quad c^2 = pq$

3. $\Delta ABC \sim \Delta DBC$

$\Delta CBC \sim \Delta DAC$

$\Rightarrow \quad BC^2 = DC \times BC$

$\Rightarrow \quad b^2 = qp$

4. $\Delta DBA \sim \Delta DAC$

$DA/DB = DC/DA$

$DA^2 = DB \times DC$

$\Rightarrow \quad AD^2 = pq$

Basic Pythagorean Triplets

→ 3, 4, 5 → 5, 12, 13 → 7, 24, 25 → 8, 15, 17 → 9, 40, 41 → 11, 60, 61 → 12, 35, 37 → 16, 63, 65 → 20, 21, 29 → 28, 45, 53. These triplets are very important since a lot of questions are based on them.

Any triplet formed by either multiplying or dividing one of the basic triplets by any positive real number will be another Pythagorean triplet.

Thus, since 3, 4, 5 form a triplet so also will 6, 8 and 10 as also 3, 3, 4.4 and 3.5.

Similarity of Right Triangles Two right triangles are similar if the hypotenuse and side of one is proportional to hypotenuse and side of another. (RHS—similarity: Right-angle hypotenuse side.)

Important Terms with Respect to a Triangle

1. Median A line joining the mid-point of a side of a triangle to the opposite vertex is called a median. In the figure the three medians are PG, QF and RE where G, E and F are mid-points of their respective sides.

- A median divides a triangle into two parts of equal area.
- The point where the three medians of a triangle meet is called the centroid of the triangle.
- The centroid of a triangle divides each median in the ratio 2 : 1.

i.e. $PC : CG = 2 : 1 = DC : CF = BC : CE$

Important: A median with respect to a median

$\Rightarrow \quad 2 \times (\text{median})^2 + 2 \times (1/2 \text{ the third side})^2$

$= \text{Sum of the squares of other two sides}$

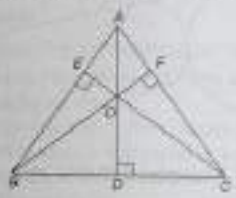
$\Rightarrow \quad 2(PG)^2 + 2 \times \left(\frac{QR}{2}\right)^2$

$= (PQ)^2 + (PR)^2$

2. Altitude/Height A perpendicular drawn from any vertex to the opposite side is called the *altitude* of the triangle. In figure, AD, BF and CE are the altitudes of the triangle.

- All the altitudes of a triangle meet at a point called the orthocentre of the triangle.
- The angle made by any side at the orthocentre and the vertical angle make a supplementary pair (i.e. they both add up to 180°). In the figure below

$\angle A + \angle BOC = 180° = \angle C + \angle AOB$



3. Perpendicular Bisectors A line that is a perpendicular to a side and bisects it is the perpendicular bisector of the side.

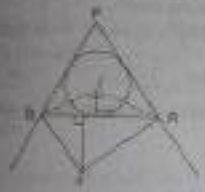
- The point at which the perpendicular bisectors of the sides meet is called the circumcentre of the triangle.
- The circumcentre is the centre of the circle that circumscribes the triangle. There can be only one such circle.
- Angle formed by any side at the circumcentre is twice the vertical angle opposite to the side. This is the property of the circle whereby angles formed by an arc at the centre are twice that of the angle formed

as the same are in the opposite arc. Here we can write as

$\angle QPR = 2 \angle QNR$ (where we consider arc QR and it's opposite arc QPR)

4. Incentre

- The lines bisecting the interior angles of a triangle are the angle bisectors of that triangle.
- The angle bisectors meet at a point called the incentre of the triangle.
- The incentre is equidistant from all the sides of the triangle.



- From the incentre with a perpendicular drawn to any of the sides as the radius, a circle can be drawn touching all the three sides. This is called the incircle of the triangle. The radius of the incircle is known as inradius.
- The angle formed by any side at the incentre is always equal to more than half the angle opposite to the side.

This can be illustrated as $\angle QIR = 90 + 1/2 \angle P$

- If QI and RI be the angle bisectors of exterior angles at Q and R then,

$$QIR = 90 - 1/2 \angle P$$

QUADRILATERALS

Area

(1) Area = 1/2 (product of diagonals) × sine of the angle between them.

If θ_1 and θ_2 are the two angles made between themselves by the two diagonals, we have by the property of intersecting lines $\rightarrow \theta_1 + \theta_2 = 180°$

Then, the area of the quadrilateral $= \frac{1}{2} d_1 d_2 \sin \theta_1 = \frac{1}{2} d_1 d_2 \sin \theta_2$.

(B) Area = 1/2 × diagonal × sum of the perpendiculars to it from opposite vertices $= \frac{d(h_1 + h_2)}{2}$

(C) Area of a circumscribed quadrilateral

$$A = \sqrt{(S-a)(S-b)(S-c)(S-d)}$$

Where $S = \frac{a+b+c+d}{2}$

(where a, b, c and d are the lengths of the sides.)

Properties

1. In a convex quadrilateral inscribed in a circle, the product of the diagonals is equal to the sum of the products of the opposite sides. For example, in the figure below:

$$(a \times c) + (b \times d) = AC \times BD$$

2. Sum of all the angles of a quadrilateral = 360°.

TYPES OF QUADRILATERALS

1. Parallelogram (|| gm)

A parallelogram is a quadrilateral with opposite sides parallel (as shown in the figure below.)

(A) Area = Base (b) × Height (h)

= bh

(B) Area = product of any two adjacent sides × sine of the included angle.

$= ab \sin \theta$

(C) Perimeter = $2(a + b)$

where a and b are any two adjacent sides

Properties

(a) Diagonals of a parallelogram bisect each other.

(b) Bisectors of the angles of a parallelogram form a rectangle.

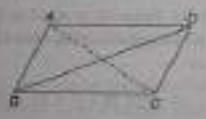
(c) A parallelogram inscribed in a circle is a rectangle.

(d) A parallelogram circumscribed about a circle is a rhombus.

(e) The opposite angles in a parallelogram are equal.

(f) The sum of the squares of the diagonals is equal to the sum of the squares of the four sides in the figure.

$$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$$
$$= 2(AB^2 + BC^2)$$



2. Rectangles

A rectangle is a parallelogram with all angles 90°

(a) Area = Base × Height = $b \times h$

Note: Base and height are also referred to as the length and the breadth in a rectangle.

(b) Diagonal (d) = $\sqrt{b^2 + h^2}$ → by Pythagoras theorem

Properties of a Rectangle

(a) Diagonals are equal and bisect each other.

(b) Bisectors of the angles of a rectangle to parallelogram form another rectangle.

(c) All rectangles are parallelograms but the reverse is not true.

3. Rhombus

A parallelogram having all the sides equal is a rhombus.

(a) Area = 1/2 × product of diagonals × sine of the angle between them.

$= 1/2 \times d_1 \times d_2 \sin 90°$ (Diagonals in a rhombus intersect at right angle)

$= 1/2 \times d_1 d_2$ (since $\sin 90° = 1$)

(b) Area = product of adjacent sides × sine of the angle between them.

Properties

(a) Diagonals bisect each other at right angles.

(b) All rhombuses are parallelograms but the reverse is not true.

(c) A rhombus may or may not be a square but all squares are rhombuses.

4. Square

A square is a rectangle with adjacent sides equal or a rhombus with each angle 90°

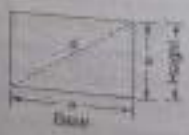
(a) Area = base × height = a^2

(b) Area = 1/2 (diagonal)^2 = $1/2\, d^2$ (square = a rhombus)

(c) Perimeter = $4a$ (a = side of the square)

(d) Diagonal = $a\sqrt{2}$

(e) In radius = $\frac{a}{2}$



Properties

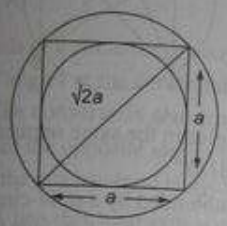
(a) Diagonals are equal and bisect each other at right angles.

(b) Side is the diameter of the inscribed circle.

(c) Diagonal is the diameter of the circumscribing circle.

$\Rightarrow$ Diameter $= a\sqrt{2}$.

Circumradius $= a/\sqrt{2}$



5. Trapezium

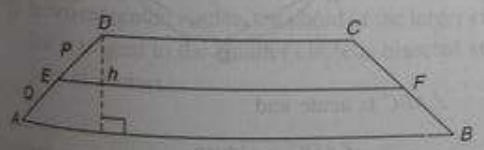
A trapezium is a quadrilateral with only two sides parallel to each other.

(a) Area = 1/2 × sum of parallel sides × height = 1/2 $(AB + DC) \times h$—For the figure below.

(b) Median = 1/2 × sum of the parallel sides (median is the line equidistant from the parallel sides)

For any line EF parallel to AB

$$EF = \frac{\{[P \times (AB)] + [Q \times (DC)]\}}{AD}$$



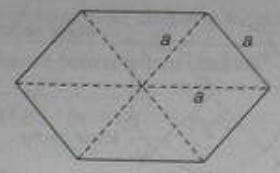
Properties

(a) If the non-parallel sides are equal then diagonals will be equal too.

REGULAR HEXAGON

(a) Area $= [(3\sqrt{3})/2]\,(\text{side})^2$

$$= \frac{3\sqrt{3} \times a^2}{2}$$



(b) A regular hexagon is actually a combination of 6 equilateral triangles all of side 'a'.

Hence, the area is also given by: 6 × side of equilateral triangles $= 6 \times \frac{\sqrt{3}}{4} a^2$

(c) If you look at the figure closely it will not be difficult to realise that circumradius $(R) = a$; i.e the side of the hexagon is equal to the circumradius of the same.

CIRCLES

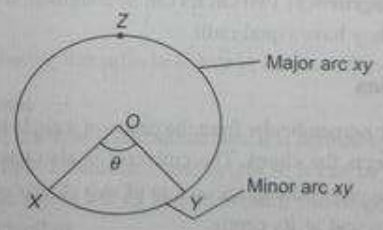
(a) Area $= \pi r^2$

(b) Circumference $= 2\pi r = (r = \text{radius})$

(c) Area = 1/2 × circumference × r

Arc: It is a part of the circumference of the circle. The bigger one is called the *major arc* and the smaller one the *minor arc*.

(d) Length (Arc XY) $= \frac{\theta}{360} \times 2\pi r$



(e) **Sector of a circle** is a part of the area of a circle between two radii.

(f) Area of a sector $= \frac{\theta}{360} \times \pi r^2$

(where θ is the angle between two radii)

$= (1/2)r \times$ length (arc xy)

($\therefore \pi r\theta/180 =$ length arc xy)

$$= \frac{1}{2} \times r \times \frac{\pi r \theta}{360}$$

(g) **Segment:** A sector minus the triangle formed by the two radii is called the segment of the circle.

(h) Area of the sector = Area of the sector − Area ΔOAB =

$$\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times r^2 \sin\theta$$

(i) Perimeter of segment = length of the arc + length of segment AB

$$= \frac{\theta}{360} \times 2\pi r + 2r\sin\left(\frac{\theta}{2}\right)$$

$$= \frac{2\pi r\theta}{180} + 2r\sin\left(\frac{\theta}{2}\right)$$

(j) **Congruency:** Two circles can be congruent if and only if they have equal radii.

Properties

(a) The perpendicular from the centre of a circle to a chord bisects the chord. The converse is also true.

(b) The perpendicular bisectors of two chords of a circle intersect at its centre.

(c) There can be one and only one circle passing through three or more non-collinear points.

(d) If two circles intersect at two points, then the line through the centres is the perpendicular bisector of the common chord.

(e) If two chords of a circle are equal, then the centre of the circle lies on the angle bisector of the two chords.

(f) Equal chords of a circle or congruent circles are equidistant from the centre.

(g) Equidistant chords from the centre of a circle are equal to each other in terms of their length.

(h) The degree measure of an arc of a circle is twice the angle subtended by it at any point on the alternate segment of the circle. This can be clearly seen as follows in figure.

With respect to the arc AB, $\angle AOB = 2\angle ACB$.

(i) Any two angles in the same segment are equal. Thus $\angle ACB = \angle AOB$.

(j) The angle subtended by a semi-circle is a right angle. Conversely, the arc of a circle subtending a right angle at any point of the circle is its alternate segment, a semi-circle.

(k) Any angle subtended by a minor arc in the alternate segment is acute, and any angle subtended by a major arc in the alternate segment is obtuse.

In the figure below

$\angle ABC$ is acute and

$$\angle ADC = 180\text{ degree}$$

Also $\theta_1 = 2\angle B$

And $\theta_2 = 2\angle D$

$$\theta_1 + \theta_2 = 2(\angle B + \angle D)$$

$$= 360^\circ = 2(\angle B + \angle D)$$

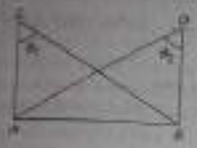
or $\angle B + \angle D = 180^\circ$

or sum of opposite angles of a cyclic quadrilateral = 180°

(l) If a line segment joining two points subtends equal angles at two other points lying on the same side of the line, the four points are concyclic. This is shown in the following figure.

ii. $\theta_1 = \theta_2$

Thus ABCD are concyclic, that is, they lie on the same circle.



(v) Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (or the corresponding centres). The converse is also true.

(vi) If the sum of the opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

Secant: A line that intersects a circle at two points.

Tangent: A line that touches a circle at exactly one point.

(vii) If a circle touches all the four sides of a quadrilateral then the sum of the two opposite sides is equal to the sum of other two.

$$AB + DC = AD + BC$$

(viii) In two concentric circles, the chord of the larger circle that is tangent to the smaller circle is bisected at the point of contact.

Tangents

- Length of direct common tangents is

$$= \sqrt{(\text{Distance between their centres})^2 - (r_1 + r_2)^2}$$

where r_1 and r_2 are the radii of the circles.

$$= \sqrt{(OO')^2 - (r_1 + r_2)^2}$$

- Length of transverse common tangents is

$$= \sqrt{(\text{Distance between their centres})^2 - (r_1 - r_2)^2}$$

$$= \sqrt{(OO')^2 - (r_1 - r_2)^2}$$

ELLIPSE

- Perimeter $= \pi(a + b)$
- Area $= \pi ab$

STAR

Sum of angles of a star $= (2n - 8) \times \pi/2 = (n - 4)\pi$

Part II: MENSURATION

The following formulas hold true in the area of mensuration.

1. Cuboid

A cuboid is a three dimensional box. It is defined by the value of its length l, breadth b and height h. It can be visualized as a room which has its length, breadth and height different from each other.

1. Total surface area of a cuboid $= 2(lb + bh + lh)$
2. Volume of the cuboid $= lbh$

2. Cube of side s

A cube is a cuboid which has all its edges equal i.e. length = breadth = height = s.

1. Total surface area of a cube $= 6s^2$
2. Volume of the cube $= s^3$.

3. Prism

A prism is a solid which can have any polygon at both its ends. Its dimensions are defined by the dimensions of the polygon at its ends and its height.

1. Lateral surface area of a right prism = Perimeter of base × height
2. Volume of a right prism = area of base × height
3. Whole surface of a right prism = Lateral surface of the prism + the area of the two plane ends

4. Cylinder

It is a solid which has both its ends in the form of a circle. Its dimensions are defined in the form of the radius of the base (r) and the height. A gas cylinder is a close approximation of a cylinder.

1. Curved surface of a right cylinder = $2\pi rh$ where r is the radius of the base and h the height
2. Whole surface of a right circular cylinder = $2\pi rh + 2\pi r^2$
3. Volume of a right circular cylinder = $\pi r^2 h$

5. Pyramid

A pyramid is a solid which can have any polygon as its base and its sides converge to a single apex. Its dimensions are defined by the dimensions of the polygon at its base and the length of its lateral edges which lead to the apex. The Egyptian pyramids are examples of pyramids.

1. Slant surface of a pyramid = 1/2 × Perimeter of the base × slant height
2. Whole surface of a pyramid = Slant surface + area of the base
3. Volume of a pyramid = $\frac{\text{area of the base} \times}{3}$ height

6. Cone

A cone is a solid which has a circle at its base and a slanting lateral surface that converges at the apex. Its dimensions are defined by the radius of the base (r), the height (h) and the slant height (l). A structure similar to a cone is used in ice cream cones.

1. Curved surface of a cone = πrl where l is the slant height
2. Whole surface of a cone = $\pi rl + \pi r^2$
3. Volume of a cone = $\frac{\pi r^2 h}{3}$

7. Sphere

It is a solid in the form of a ball with radius r.

1. Surface Area of a sphere = $4\pi r^2$
2. Volume of a sphere = $\frac{4}{3}\pi r^3$

8. Frustum of a pyramid

When a pyramid is cut the left over part is called the frustum of the pyramid.

1. Slant surface of the frustum of a pyramid = 1/2 × sum of perimeter of end × slant height.
2. Volume of the frustum of a pyramid = $\frac{h}{3}(F_1 + F_2 + \sqrt{F_1 F_2})$ = $\frac{h}{3}(F_1 + F_2 + \sqrt{kF_1 F_2})$ where k is the ratio of the areas F_1, F_2 the areas of the ends.

9. Frustum of a cone

When a cone is cut the left over part is called the frustum of the cone.

1. Slant surface of the frustum of a cone = $\pi r_1 l + \pi r_2 l$ where l is the slant height.
2. Volume of the frustum of a cone = $\frac{\pi}{3}(r_1^2 + r_2^2 + r_1 r_2)h$

WORKED-OUT PROBLEMS

Problem 11.1 A right triangle with hypotenuse 10 cm and other two sides of variable length is rotated about a longest side thus giving rise to a solid. Find the maximum possible area of such a solid.

(a) $(250/3)\pi cm^2$ (b) $(400/3)\pi cm^2$
(c) $125\pi cm^2$ (d) None of these

Solution Most of the questions like this that are asked in the CAT will not have figures accompanying them. Drawing a figure takes time so it is always better to visualize an imagination. The beginners can start off by trying to imagine the figure first and trying to solve the problem. They can draw the figure only when they don't arrive at the correct answer and then find out where exactly they went wrong. The key is to spend as much time with the problem as possible trying to understand it fully and analysing the different parts of the same without wasting too much time on it.

Let's now look into this problem. The key here lies in how quickly you are able to visualize the figure and see what is at this.

(i) the triangle has to be an isosceles triangle
(ii) the solid thus formed is actually a combination of two cones.
(iii) the radius of the base has to be the altitude of the triangle to the hypotenuse.

After you have visualized this comes the calculation part of the problem. This is one aspect where you can score over others.

In this question the figure would a somewhat like this (as shown diagram with triangles ABC and ADC representing the cones and AC being the hypotenuse around which the triangle ABC revolves. Now that we know that it is equivalent with AC as the hypotenuse, we realize that ADB has to be an isosceles triangle, which means that BCD is an isosceles triangle too. The next step is to calculate the radius of the base, which is essentially the height of the triangle ABC. To find that, we have to find out AC. We know

$$AC^2 = AB^2 + BC^2$$

triangle is one with the greatest possible area AB must equal to BC that is, $AB = BC = \sqrt{24}$, since $AO = 1/2AC$ = 6 units.

Now take the right angle triangle ABO, BO being the height of triangle AOC. By Pythagoras theorem, $AB^2 = AO^2 + BO^2$, or $BO = 25$ units.

The next step is to find the area of the cone ABD and subtract it with two to get the area of the whole solid

$$\text{Area of the cone } ABDB = \pi(BO)^2 \times AO$$
$$= (BO)\cdot(25) \times 5\pi = (125)\pi$$

Therefore, area solid $ABCD = 2 \times (125\pi) = (250)\pi$

Problem 11.2 A right circular cylinder is to be made out of metal sheet such that the sum of its height and radius is not exceed 9 cm can have an area of maximum.

(a) 54π cm^2 (b) 108π cm^2
(c) 81π cm^2 (d) None of these

Solution Solving this question requires the knowledge of ratio and proportion also. To solve this question, one must know that for $a^2b^3c^5$ to have the maximum value when a, b and c must be in the ratio $2:3:5$. Let us look at this problem. Volume of a cylinder $= \pi r^2 h$.

If we analyse this formula closely, you will find that r and h are the only variable term. So the volume of the cylinder to be maximum, r^2h has to be maximum under the condition that $r + h = 9$ cm. Now, the only way this is possible only when $r : h = 2 : 3$, that is, $r = 6$, $h = 3$. So,

$$\text{Volume of the cone} = \pi r^2 h$$
$$= \pi \times 6^2 \times 3$$
$$= 108\pi$$

Problem 11.3 Given five concentric squares. If the area of the circle inside the smallest square is 77 square units and the distance between the corresponding corners of consecutive squares is 1.5 units, find the difference in the areas of the outermost and innermost squares.

Solution Here again the ability to visualise the diagram would be the key. Once you gain expertise in this aspect, you will be able to see that radius of the circle is equal to the diameter of the innermost square that is

$$\pi r^2 = 77$$
or $$r = 3.5\sqrt{2}$$
or $$2r = 7\sqrt{2}$$

Then the diagonal of the square is 14 sq units.

Which means the diagonal of the fifth square would be $14 + 12$ units $= 26$.

Which means the side of the fifth square would be $26\sqrt{2}$.

Therefore, the area of the fifth square = 1352 sq units

Area of the first square = 98 sq units

Hence, the difference would be 1254 sq. units.

Problem 11.4 A spherical peat of radius 4 cm is to be divided into eight equal parts by cutting it in halves along the same axis. Find the surface area of each of the final pieces

(a) 20π (b) 25π
(c) 24π (d) 10π

Fig. (a)

Solution The peat after being cut will have eight parts each of same volume and surface area. The figure will be somewhat like the above Figure (a) if seen from the top before cutting. After cutting it look something like the Figure (b).

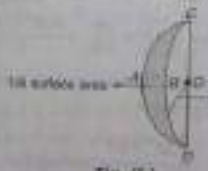


Fig. (b)

Now the surface area of each piece = Area $ACBD$ + 2 (Area $COAD$)

The curved surface is nothing but the arc AB from side pieces which means its surface area is one-eighth the area of the sphere, that is, $\frac{1}{8} \times 4\pi r^2 = \frac{1}{2}\pi r^2$.

Now CDEB can be seen as a semicircle with radius r cm.

Therefore, 2 (Area CDEB) = $2 \times \frac{1}{2}\pi r^2 = \pi r^2$

or surface area of each piece = $(1/2)\pi r^2 + \pi r^2$

$= (3/2)\pi r^2$

$= 3\pi r$

Problem 11.5 A solid metal sphere is melted and smaller spheres of equal radii are formed. 10% of the volume of the sphere is lost in the process. The smaller spheres have a radius, that is 10% the larger sphere. If 10 lines of paint were needed to paint the larger sphere, how many lines are needed to paint all the smaller spheres?

(a) 90 (b) 81 (c) 900 (d) 810

Solution Questions like this require, along with your knowledge of formulas, your ability to form equations. Anyway, it will be some thing like this:

Step 1: Assume values.

Step 2: Find out volume left.

Step 3: Find out the number of small spheres possible.

Step 4: Find out the total surface area of each small sphere as a ratio of the original sphere.

Step 5: Multiply 4 by 10.

Step 1: Let radius of the larger sphere be R and that of smaller ones be r.

Then, volume = $\frac{4}{3}\pi R^3$ and $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(0.9r)^3$ respectively for the larger and smaller spheres.

Step 2: Volume lost due to melting = $\frac{4}{3}\pi R^3 \times \frac{10}{100} = \frac{4\pi R^3}{30}$

Volume left = $\frac{4}{3}\pi R^3 \times \frac{90}{100} = \frac{4\pi R^3 \times 0.9}{3}$

Step 3: Number of small spheres possible = Volume left/Volume of the smaller sphere

$$= \frac{\frac{4}{3}\pi R^3 \times 0.9}{\frac{4}{3}\pi \times (0.9r)^3} = 9^3 = 81$$

Step 4: Surface area of larger sphere = $4\pi R^2$

Surface area of smaller sphere = $4\pi r^2 = 4\pi(0.9r)^2 = \frac{4\pi r^2}{0.81}$

Surface area of all smaller spheres = Number of small spheres × Surface area of smaller sphere

$= (9^3 \times 9) \times (4\pi R^2/81)$

$= 81 \times (4\pi R^2)$

Therefore, ratio of the surface are $\left[\frac{81 \times (4\pi R^2)}{4\pi R^2}\right] = 81$

Step 5:

81 × number of lines = 81 × 10 = 810

Problem 11.6 A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.

(a) 684 cm³ (b) 162 cm³

(c) 432 cm³ (d) 266 cm³

Solution Volume of the cone is given by $\frac{1}{3}\pi r^2 h$

Here, $r = 4.2$ cm; $h = 10.2 - r = 6$ cm.

Therefore the volume of the cone = $\frac{1}{3}\pi \times (4.2)^2 \times 6$ cm³

$= 110.88$ cm³

Volume of the hemisphere = $\frac{1}{2} \times \frac{4}{3}\pi r^3 = 155.232$

Total volume = 110.88 + 155.232 = 266.112

Problem 11.7 A vessel is in the form of an inverted cone. Its depth is 8 cm and the diameter of its top, which is open, is 10 cm. It is filled with water up to the brim. When balls, each of which is a sphere of radius 0.5 cm, are dropped into the vessel, 1/4 of the water flows out. Find the number of balls dropped in the vessel.

(a) 50 (b) 100 (c) 150 (d) 200

Solution In these type of questions, it is just from colour [illegible] that is being sought. You just need to take care [illegible] easy to be lost you don't end up making mistakes [illegible] the distance to be the radius and so forth. The best [illegible] such mistakes is to proceed systematically. For example, in this problem we can proceed thus:

Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{200}{3}\pi\ cm^3$

Volume of all the lead shots = Volume of water that spilled out $= \frac{50}{3}\pi\ cm^3$

Volume of each lead shot $= \frac{4}{3}\pi r^3 = \frac{4}{6}\pi\ cm^3$

Number of lead shots = (Volume of water that spilled out / volume of each lead shot)

$$= \frac{\frac{50}{3}\pi}{\frac{\pi}{6}} = \frac{50}{3} \times 6 = 100$$

Problem 15.8 All you should of a circle of radius 14 cm. The two radii make a right angle at the centre of the circle. Find the area of the minor segment.

(a) 76 sq cm (b) 56 sq cm
(c) 132 sq cm (d) None of these

Solution Area of the minor sector $AOB = \frac{90 \times 14^2}{360}$

$= 154$ sq cm

Area of the triangle $AOB = \frac{14 \times 14}{2}$

$= 98$ sq cm

Area of the segment ACB = Area minor sector AOB − Area of triangle AOB = 56 sq cm

Problem 15.9 A sphere of diameter 12 cm is melted and [illegible] a right circular cone of height 27 cm. Find the diameter of the base of the cone.

(a) 190.70 cm (b) 79.28 cm
(c) 39.64 cm (d) None of these

Solution In questions like this, do not go for complete calculations. Go for as possible, try to cancel out values in the resulting equation.

Volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3$

Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{\pi}{3}r^2(27)$

Now, volume of the cone = volume of the sphere

Therefore, radius of the cone = 10.61 cm

Hence the diameter = **79.28 cm**

Problem 15.10 A chord AB of a circle of radius 5.25 cm makes an angle of 60° at the centre of the circle. Find the area of the minor and minor segments. (Take $\pi = 3.14$)

(a) 108 cm² (b) 42 cm²
(c) 84 cm² (d) None of these

Solution The moment you finish reading this question, it should occur to you that this has to be an equilateral triangle. Once you realise this, the question is reduced to just calculations.

Area of the minor sector $= \frac{300}{360} \times \pi \times 5.25^2$

$= 14.4375$ cm²

Area of the triangle $= \frac{\sqrt{3}}{4} \times 5.25^2 = 11.93$ cm²

Area of the minor segment = Area of the minor sector − Area of the triangle = 2.5 cm²

Area of the major segment = Area of the circle − Area of the minor segment

$= 86.54$ cm² − 2.5 cm² = **84 cm²**

Problem 15.11 A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

(a) 1:2 (b) 2:1
(c) 3:1 (d) None of these

Solution Questions of this type should be solved with ease. By use of one such paper. A good authority over formulas will make things easier.

Volume of the cone $= \frac{\pi r^2 h}{3}$ = Volume of a hemisphere $= \frac{2}{3}\pi r^3$

Height of a hemisphere = Radius of its base

So the question is effectively asking us to find out for

By the formula above we can easily see that $h/r = 2/3$

HOW TO THINK IN GEOMETRY AND MENSURATION

In the back to school section of this block, we have already mentioned that there is very little use of complex and obscure formulae and results while solving questions on this block.

The following is a list of questions (with solutions) of what has been asked in CAT over the past few years. Hopefully you will realise through this exercise, what we are talking about when we say this. For each of the questions given below, try to solve on your own first, before looking at the solution provided.

1. A circle with radius 2 is placed against a right angle. As shown in the figure below, another smaller circle is placed in the gap between the circle and the right angle. What is the radius of the smaller circle?

(a) $1 - 3\sqrt{2}$ (b) $4 - 2\sqrt{2}$

(c) $1 - 4\sqrt{2}$ (d) $6 - 4\sqrt{2}$

Solution: The solution of the above question is based on the following construction.

In the right triangle $OOTP$,

$$OP = (2 - r), OP = (2 - r) \text{ and } OT = 2 + r$$

where r is the radius of the smaller circle.

Using Pythagoras theorem

$$(2 + r)^2 = (2 - r)^2 + (2 - r)^2$$

Solving, we get $r = 6 \pm 4\sqrt{2}$

$6 + 4\sqrt{2}$ cannot be correct since the value of r should be less than 2.

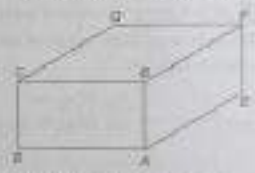
Note: The key to solving this question is in the visualisation of the construction. If you try to use complex formulae while solving, your mind unnecessarily gets cluttered. The key to your thinking in this question is:

(1) Realise that you only have to use length measuring formulae. Hence, put all angle measurement formulae into the back seat.

(2) A quick mental search of the length measuring formulae available for this situation will turn down your mind to the Pythagoras theorem.

(3) The key then becomes the construction of a triangle (right angled of course) where the only unknown is r.

2. ABCDEFGH is a cube. If the length of the diagonals DF, AG & CE are equal to the sides of a triangle, the circumradius of that triangle would be



(a) Equal to the side of the cube

(b) $\sqrt{3}$ times the side of the cube

(c) $1/\sqrt{3}$ times the side of the cube

(d) Indeterminate

Solution: If we assume the side of the cube to be a, the triangle will be an equilateral triangle with side $a\sqrt{2}$ (as per this using Pythagoras theorem). Also, we know that the circumradius of an equilateral triangle is $1/\sqrt{3}$ times the side of the triangle.

Hence, in this case the circumradius would be a—equal to the side of the cube.

(Again the only formula used in this question would be the Pythagoras theorem.)

3. On a semicircle (diameter AB), chord BC is parallel to the diameter AD. Also, $AB = CD = 2$, while $AD = 6$. What is the length of BC?

(a) 7.5 (b) 7
(c) 7.75 (d) None of these

Solution : Think only of length measuring formulae. PYTHAGORAS THEOREM is obvious in this case.

If we can find the value of x, we will get the answer for BC as $(2x - 2)$. Hence, we need to focus our energies on finding the value of x.

The construction shows us two right angled triangles OEC and $DECA$.

In ΔOCE, $OC = 4$ (radius) and $OE = (4 - x)$. Then $(CE)^2 = 16 - x^2$ (Using Pythagoras Theorem)

Then in triangle CED

$$(8 - x^2) + x^2 = 7^2$$

Hence, $x = 0.5$

Then $BC = 8 - 2 \times 0.5 = 7$

4. In the given circle, AC is the diameter of the circle. ED is parallel to AC. $\angle ACB = 65°$, find $\angle DBA$

(a) 35° (b) 55°
(c) 45° (d) 25°

Solution : Obviously this question has to be solved using the angle bisector tools. Further from the figure, it is obvious that we have to use angle measurement tools related to arcs of circles.

Referring to the 65° information in the question above, you will get $\angle EOC = 130°$ (Hence, the angle at the centre of the circle is twice the angle at any point of the circle).

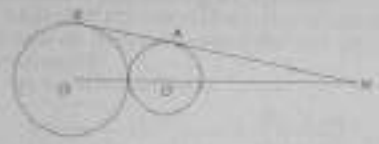
Hence, $\angle AOE = 180 - 130 = 50°$

This will be the same as $\angle EOD$ since the arrow are OE = reverse as CD.

Also, $\angle DEC = 65 = \angle CEBM$

Hence, $\angle DEC = 25°$

Directions for questions 5 – 7: In the figure below, X and Y are circles with centres O and O′ respectively. MAB is a common tangent. The radii of X and Y are in the ratio 4 : 3 and OM = 28 cm.



5. What is the ratio of the length of OA' to that of $O'M'$?

(a) 1 : 4 (b) 4 : 3
(c) 3 : 8 (d) 3 : 4

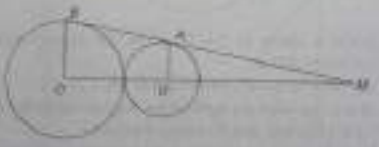
6. What is the radius of circle X?

(a) 2 cm (b) 8 cm
(c) 4 cm (d) 5 cm

7. The length of AM is

(a) $8\sqrt{3}$ cm (b) $16\sqrt{3}$ cm
(c) $12\sqrt{3}$ cm (d) $14\sqrt{3}$ cm

Solution : Construct OW and $O'A$ as shown below.



In the construction it is evident that the two right angled triangles formed are similar to each other ⇒ ΔOBM is similar to $\Delta O'AM$.

Hence, $OM : O'M = 4 : 3$ (since $OB : O'A = 4 : 3$)

Also, $OM = 28$ cm, $\therefore O'M = 21$ cm ⇒ $OO' = 7$ cm. Hence the radius of circle X is 4 cm (Answer to Q. 6)

5. Also, $OO' = 7$ and $O'W = 21$. Hence, required ratio = 1 : 3

7. AM can be found easily using Pythagoras theorem:

$$AM^2 = 24^2 - 3^2 = 432$$

$$AM = \sqrt{432} = 12\sqrt{3}$$

(Note: Only similarity of triangles and Pythagoras theorem was used here)

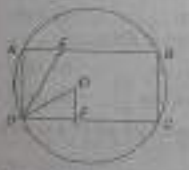
8. In the figure, ABCD is a rectangle inscribed inside a circle with corner O. Side AE = Side BC. The ratio of the area of the circle to the area of the rectangle is $\pi : \sqrt{3}$. Also, $\angle DOC = \angle COE$. Find the ratio AE : AD.

(a) $1 : \sqrt{3}$ (b) $1 : \sqrt{2}$

(c) $2\sqrt{3} : 1$ (d) $1 : 2$

Solution: In my experience, questions involving ratios of length typically involve the use of similar triangles. This question is no different.

Make the following construction



ΔOFD is similar to ΔAED. Hence, the required ratio $AE : AD = OF : FD$.

But $OF = r$, side BC, while $FD = \frac{1}{2}$ side AB.

Hence, we need the ratio of the side of the rectangle $BC : AB$. (This will give the required answer.)

From this point, you can get to the answer through a variety of unconventional thinking.

The ratio of the area of the circle to that of the rectangle is given as $\pi : \sqrt{3}$. Hence, it is obvious that one of the sides has to have a $\sqrt{3}$ component in it. Hence, options 2 and 4 can be rejected. Also the required ratio has to be less than 1, hence, option 1 is correct.

9. Find $\angle BOA$.

(a) 100° (b) 130°

(c) 80° (d) Indeterminate

Solution: Obviously this question has to be solved using angle measurement tools.

In order to measure $\angle BOA$, you could either try to use theorems related to the angle subtended by arcs of a circle or solve using the isosceles ΔBOA.

With this thought in mind let us re-arrange the information in the question.

$\angle OAE = 30°$, hence $\angle BAC = 80°$

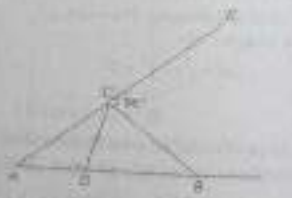
Also, $\angle OCA = (90 - ACE) = 90 - 50 = 40° = \angle OAC$. Hence the triangle OCA is isosceles.

Hence $\angle OAB = 40°$

In isosceles ΔOAB, $\angle OBA$ will also be 40°

Hence $\angle BOA = 180 - 40 - 40 = 100°$

10. In the figure $AD = CD = BC$ and $\angle BCE = 96°$. Find $\angle DBC$.



(a) 32° (b) 34°

(c) 64° (d) Indeterminate

Solution: Can you use angle measuring instruments and the information in the diagram.

From the figure shown, it is clear that

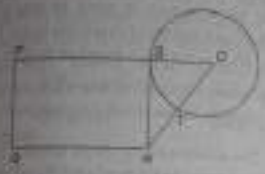
$x + y = 180 - 96 = 84°$

also $4x + y = 180°$

solving we get $x = 32°$

hence $\angle OBC = 2x = 64°$

9. $PQRS$ is a square. SR is a tangent (at point S) to the circle with centre O and $OR = OC$. Then the ratio of area of the circle to the area of the square is



(a) π^2 (b) $11/7$

(c) $3/4$ (d) $7/11$

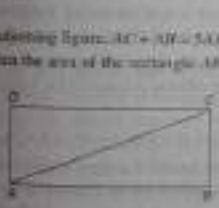
Answer: Looking at the options we can easily eliminate (b) & (d), because in the ratio of the area of the circle & area of the triangle we cannot eliminate π and hence it never should contain π.

Rather the question is asking for the ratio

$\frac{\text{Area of the circle}}{\text{Area of the square}}$ so, π should be in the numerator.

Hence (c)

10. In the adjoining figure, $AC = AB = 5AD$ and $AC = AD + 8$. Then the area of the rectangle $ABCD$ is



(a) 36 (b) 50

(c) 64 (d) Cannot be answered

Answer: Think only of length measuring formulae (Pythagoras Theorem is obvious in this case).

There is no need of forming equations if you have the knowledge of some basic triples like 3, 4, 5; 5, 12, 13 etc.

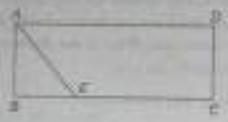
Now looking at the equations given in the question and assuming ΔACD where AD is the height and AC is the hypotenuse we will easily get

$AC = AD + 8 = 5 + 8 = 13$ and

$AC = AB = 13 = 5 \times 2 = 2 \times 5 = 10$

Hence Area of rectangle is length × breadth i.e. $5 \times 12 = 60$

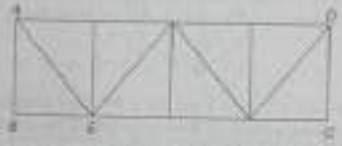
11. In the figure given below, $ABCD$ is a rectangle. The area of the isosceles right triangle $ABE = 9\ cm^2$, $EC = 3BE$. The area of $ABCD$ (in cm^2) is



Solution: The key to solve this question is in the visualization of the construction and the equations.

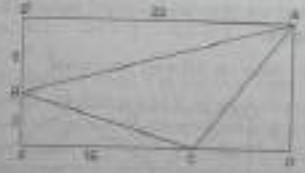
It is given that $EC = 3\ (BE)$ from this we can conclude that the whole side BC can be divided in four equal parts or reconstructed BE.

Now look at this construction



Each part is of equal area or $7\ cm^2$. Hence $7 \times 8 = 56\ cm^2$

14. In the given figure, $EADF$ is a rectangle and ABC is a triangle whose vertices lie on the sides of $EADF$. $AE = 22$, $BE = 6$ $CF = 16$ and $BF = 2$. Find the length of the line joining the mid – points of the side AB and BC



(a) $4\sqrt{2}$ (b) 8

(c) 13 (d) None of these

Answer: Think only of length measuring formulae and triples.

$EA = 22$ and $FC = 16$. So, $CD = 6$

$BF = 8$ So, AD is also 8. Now using the triples 6, 8, 10 based on basic triples 3, 4, 5 we will get that $AC = 10$.

The line joining the midpoints of the sides AB and BC will be exactly half the side AC (using similar triangles).

Hence, 5 is the correct answer.

15. A certain city has a circular wall around it and there will be four gates pointing north, south, east and west. A house stands outside the city, 3 km north of the north gate and it can just be seen from a point nine km east of south gate. What is the diameter of the wall that surrounds the city?

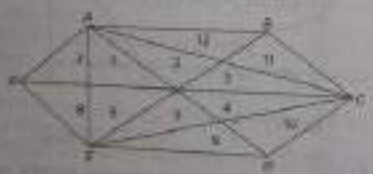
Solution: Make the construction

Given: $AN = 3$, $BC = 9$ and $\angle B$ is $90°$. Now according to conventional method, we have to use tangent theorem to get to the answer, which will be very long.

Instead if we were to use the Pythagorean triplets again we would easily see the (9, 12, 15) triplet which is based on the basic triplet 3, 4, 5. Here $BC = 9$. Hence, $AC = 15$ and $AB = 12$. Hence, the diameter will be $12 - 3 = 9$ km.

16. Let $ABCDEF$ be a regular hexagon. What is the ratio of the area of the triangle ACE to that of the hexagon $ABCDEF$?

Solution: Make the following construction



Now we have to find the ratio $\frac{\text{Area of } \Delta ACE}{\text{Area of } ABCDEF}$.

In order to do so we use the property of a regular hexagon (that it is a combination of 6 equilateral triangles).

We can easily see that we have divided all 6 equilateral triangle into two equal part of the same area.

If we number all the equal areas as 1, 2, 3, ..., 12 as shown in the above construction we will get the answer as

$$\frac{\text{Sum of Area of triangles } 1+2+3+4+5+6}{\text{Sum of Area of triangle } 1+2+3 \ldots +12}$$

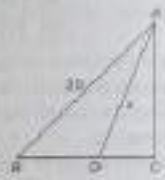
Hence 1/2.

17. Euclid has a triangle in mind. Its longest side has length 20 and another of its side has length 16. Its area is 80. What is the exact length of its third side?

(a) $\sqrt{260}$ (b) $\sqrt{250}$

(c) $\sqrt{240}$ (d) $\sqrt{230}$

Solution: The solution of the above question is hinted in the following construction, where $AB = 20$ and $BD = 16$.



The question is asking for the exact length AD, of map ABD.

Think only of length measuring formulae (Pythagoras theorem is obvious in this case).

If we extend the side BD upto a point C, the length AC will give the Altitude or height of the ΔABD. Then we will get

$\frac{1}{2} b \times h$ as $80 = \frac{1}{2} \times 16 \times h = 80 \Rightarrow h = 10$, i.e. $AC = 10$

And now in ΔABC is a right angled triangle, we can easily get the length of DC as 2, based on the triplet 12, 9, 20.

Now, if $AC = 10$, $DC = 2$, we can easily get the exact length of AD using Pythagoras theorem i.e. $AC = \sqrt{2^2 + 10^2}$

$= \sqrt{260}$

Hence (a).

Note: In this solution we have only used Pythagorean triplet 12, 16, 20 to solve the question. The shortest method for solving this question is through the use of the properties of the triangle. This will lead to a very cumbersome and long solution to this question. For experimental purposes you can try this solution for yourself.

SPECIAL NOTE ABOUT CAT QUESTIONS

The process of solving questions of this block essentially involves three steps:

Step One: Analyse what is asked clearly. While doing so, look at the figure in the question closely.

Step Two: Define clearly what you should focus on trying to find. In many cases it helps to deflect the question so that instead of first finding the deflected answer is more convenient than finding what is originally asked. **The deflection of the specific formulae that are going to be used in solving the question is one of the keys for solving such questions.**

Step Three: Solve the question by using the identified formulae in the correct order.

As is clearly seen in the solutions to original CAT questions given above only core and elementary well known formulae are used in solving questions of geometry and mensuration that appear in CAT. Unfortunately most students have been concentrating on memorising their minds is trying to memorise obscure truly and horrible. The end is that when they sit down to solve elementary questions like those which appear in the CAT, the memorised obscure formulae just confuses their minds. So kids students to not being able to solve easy questions which might essentially be just 2-3 steps long.

My Advice to You would be: Concentrate only on elementary formulae first. 99% of the questions that appear in CAT and other top aptitude tests only focus on the use of core formulae. (In fact, you can see this for yourself in the Infrastructure test and the block review tests given at the end of the block of chapters, where the questions have either appeared in the CAT or are modelled on questions that have appeared in CAT.)

GEOMETRY

Level of Difficulty (LOD) I

A vertical stick 20 m long casts a shadow 10 m long on the ground. At the same time, a tower casts the shadow 50 m long on the ground. Find the height of the tower.

(a) 100 m (b) 120 m
(c) 25 m (d) 200 m
(e) 225 m

2. In the figure, ΔABC is similar to ΔEDC.

If we have AB = 4 cm, ED = 7 cm, CE = 4.2 and CD = 4.8 cm, find the value of CA and CB

(a) 6 cm, 6.4 cm (b) 4.8 cm, 6.4 cm
(c) 5.4 cm, 6.4 cm (d) 3.6 cm, 6.4 cm
(e) 5.4 cm, 6.0 cm

3. The areas of similar triangles ABC and DEF are 144 cm^2 and 81 cm^2 respectively. If the longest side of larger ΔABC be 36 cm, then the longest side of smaller ΔDEF is

(a) 20 cm (b) 28 cm
(c) 27 cm (d) 30 cm
(e) 36 cm

4. Two triangles As have equal angles and their areas are in the ratio 16 : 25. Find the ratio of their corresponding heights

(a) 4:5 (b) 5:4 (c) 3:2 (d) 5:7
(e) None of these

5. The areas of two similar As are respectively 9 cm^2 and 16 cm^2. Find the ratio of their corresponding sides.

(a) 3 : 4 (b) 4 : 3 (c) 2 : 3 (d) 4 : 5
(e) 5 : 4

6. Two poles of height h m and 3h m stand vertically upright on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

(a) 12 cm (b) 14 cm
(c) 13 cm (d) 15 cm
(e) 16 cm

7. The radius of a circle is 9 cm and length of one of its chords is 14 cm. Find the distance of the chord from the centre.

(a) 5.66 cm (b) 6.5 cm
(c) 4 cm (d) 7 cm
(e) None of these

8. Find the length of a chord that is at a distance of 12 cm from the centre of a circle of radius 15 cm.

(a) 9 cm (b) 8 cm
(c) 12 cm (d) 5 cm
(e) 10 cm

9. If O is the centre of circle, find $\angle x$

(a) 39° (b) 30°
(c) 38° (d) 40°
(e) 38°

10. Find the value of $\angle x$ in the given figure.

(a) 120° (b) 130°
(c) 110° (d) 150°
(e) 100°

11. Find the value of x in the figure, if it is given that AC and BD are diameters of the circle.

(a) 60° (b) 45°
(c) 15° (d) 25°
(e) 30°

12. Find the value of x in the given figure.

(a) 2.2 cm (b) 1.6 cm
(c) 3 cm (d) 2.6 cm
(e) 2.5 cm

13. Find the value of x in the given figure.

(a) 16 cm (b) 9 cm
(c) 12 cm (d) 7 cm
(e) 15 cm

14. Find the value of x in the given figure.

(a) 13 cm (b) 17 cm
(c) 18 cm (d) 15 cm
(e) 14 cm

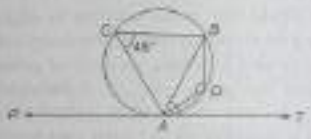
15. ABC is a right angled triangle with BC = 6 cm and AB = 8 cm. A circle with centre O and radius x has been inscribed in ΔABC. What is the value of x?

(a) 2.4 cm (b) 2 cm
(c) 3.6 cm (d) 4 cm
(e) 3 cm

16. In the given figure AB is the diameter of the circle and $\angle BAD = 25°$. Find $\angle TPA$.

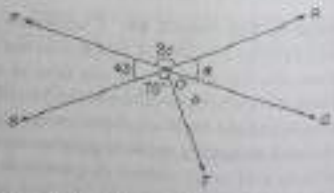
(a) 50° (b) 65°
(c) 70° (d) 45°
(e) 60°

17. In the given figure find $\angle ADB$



(a) 132° (b) 144° (c) 48° (d) 96°
(e) None of these

18. In the given two straight line, PQ and RS intersect each other at O. If $\angle SOT = 75°$, find the value of x, y and z.



(a) $x = 84°$, $y = 24°$, $z = 48°$
(b) $x = 48°$, $y = 20°$, $z = 50°$
(c) $x = 72°$, $y = 24°$, $z = 54°$
(d) $x = 64°$, $y = 38°$, $z = 45°$
(e) $x = 68°$, $y = 20°$, $z = 45°$

19. In the following figure A, B, C and D are the concyclic points. Find the value of x.

(a) 130° (b) 30°
(c) 60° (d) 30°
(e) None of these

2. In the following figure, it is given that O is the centre of the circle and $\angle AOC = 140°$. Find $\angle ABC$.

(a) 100° (b) 120° (c) 110°
(d) 130° (e) 300°

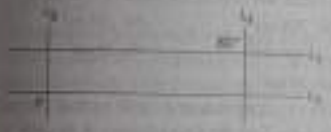
3. In the following figure, O is the centre of the circle and $\angle AOC = 30°$. Find $\angle ACB$.

(a) 60° (b) 120°
(c) 30° (d) 90°
(e) 180°

4. In the following figure, find the value of x.

(a) 40° (b) 25°
(c) 30° (d) 45°
(e) 50°

5. $l_1 \parallel l_2$, in the figure below, what is the value of x.



(a) 80° (b) 100°
(c) 40° (d) 90°
(e) Cannot be determined

6. Find the perimeter of the given figure.

(a) (12 + 3π) cm
(b) (36 + 6π) cm
(c) (36 + 3π) cm
(d) (36 + 9π) cm
(e) (36 + 2π) cm

7. In the figure, AB is parallel to CD and AD is parallel to EF. If AB : BC : CD = 6 : 5 : 2, and it is known that CD + EF = 28, find the value of BE : FG : GH : HI.

(a) 3:1:2:3:4 (b) 3:2:1:3:4
(c) 4:3:2:3:3 (d) 4:3:2:3:1
(e) None of these

MENSURATION

Level of Difficulty (LOD) I

1. In a right angled triangle, find the hypotenuse if base and perpendicular are respectively 360.15 cm and 480.20 cm.
(a) 604.25 cm (b) 600.25 cm
(c) 599.25 cm (d) 600.25 cm
(e) None of these

2. The perimeter of an equilateral triangle is 72.63 cm. Find its height.
(a) 15 metres (b) 24 metres
(c) 18 metres (d) 19 metres
(e) 17 metres

3. The inner circumference of a circular track is 440 cm. The track is 14 cm wide. Find the diameter of the outer circle of the track.
(a) 84 cm (b) 168 cm
(c) 336 cm (d) 37 cm
(e) 38 cm

4. A race track is in the form of a ring whose inner and outer circumferences are 352 metres and 396 metres respectively. Find the width of the track.
(a) 7 metres (b) 14 metres
(c) 14 metres (d) 7.4 metres
(e) None of these

5. The outer circumference of a circular track is 220 metres. The track is 7 metres wide everywhere. Calculate the cost of levelling the track at the rate of 50 paise per square metre.
(a) Rs. 1750.5 (b) Rs. 1111
(c) Rs. 968 (d) Rs. 1386
(e) Rs. 693

6. Find the area of a quadrant of a circle whose circumference is 44 cm.
(a) 77 cm² (b) 38.5 cm²
(c) 19.25 cm² (d) 19.25π cm²
(e) None of these

7. A pit 7.5 metre long, 6 metre wide and 1.5 metre deep is dug in a field. Find the volume of soil removed in cubic metre.
(a) 170 m^3 (b) 101.25 m^3
(c) 810.25 m^3 (d) 67.5 m^3
(e) None of these

8. Find the length of the longest pole that can be placed in an indoor stadium 24 metre long, 18 metre wide and 16 metre high.
(a) 30 metres (b) 25 metres
(c) 34 metres (d) $\sqrt{580}$ metres
(e) 36 metres

9. The length, breadth and height of a room are in the ratio of 3 : 2 : 1. If its volume be 1296 m^3, find its breadth.
(a) 18 metres (b) 18 metres
(c) 16 metres (d) 24 metres
(e) 12 metres

10. The volume of a cube is 216 cm^3. Part of the cube is then melted to form a cylinder of length 8 cm. Find the volume of the cylinder.
(a) 342 cm^3 (b) 216 cm^3
(c) 36 cm^3 (d) 156 cm^3
(e) Data inadequate

11. The whole surface of a rectangular block is 6788 square cm. If length, breadth and height are in the ratio of 4 : 3 : 2, find length.
(a) 26 cm (b) 32 cm
(c) 104 cm (d) 13 cm
(e) 16 cm

12. Three metal cubes with edges 6 cm, 8 cm and 10 cm respectively are melted together and formed into a single cube. Find the side of the resulting cube.
(a) 11 cm (b) 12 cm
(c) 13 cm (d) 24 cm
(e) 20 cm

13. Find curved and total surface area of a cylindrical block of radius 6 cm and height 8 cm.
(a) 60π, 96π (b) 20π, 90π
(c) 48π, 48π (d) 81π, 60π
(e) None of these

14. The volume of a right circular cone is 100 cm^3 and its height is 12 cm. Find its curved surface area.
(a) 130 cm^2 (b) 85 cm^2
(c) 204 cm^2 (d) 65 cm^2
(e) None of these

15. The diameters of two cones are equal. If their curved surface areas and height be in the ratio 5 : 7, find the ratio of their slant heights.
(a) 25 : 7 (b) 25 : 49
(c) 5 : 49 (d) 5 : 7
(e) 7 : 5

16. The curved surface area of a cone is 2376 square cm and its slant height is 18 cm. Find the diameter.
(a) 6 cm (b) 18 cm
(c) 84 cm (d) 12 cm
(e) 16 cm

17. The ratio of radii of a cylinder to a that of a cone is 1 : 2. If their heights are equal, find the ratio of their volumes.
(a) 1 : 3 (b) 2 : 3
(c) 3 : 4 (d) 3 : 2
(e) 3 : 1

18. A silver wire when bent in the form of a square encloses an area of 484 cm^2. Now if the same wire is bent to form a circle, the area of enclosed by it would be
(a) 308 cm^2 (b) 196 cm^2
(c) 616 cm^2 (d) 88 cm^2
(e) 96 cm^2

19. The circumference of a circle exceeds its diameter by 16.8 cm. Find the circumference of the circle.
(a) 12.32 cm (b) 49.28 cm
(c) 58.64 cm (d) 24.64 cm
(e) None of These

20. A bicycle wheel makes 5000 revolutions in moving 11 km. What is the radius of the wheel?
(a) 70 cm (b) 13.5 cm
(c) 17.5 cm (d) 35 cm
(e) 50 cm

21. The volume of a right circular cone is 100π cm^3 and its height is 12 cm. Find its slant height.
(a) 13 cm (b) 16 cm
(c) 9 cm (d) 26 cm
(e) 32 cm

22. The short and the long hands of a clock are 4 cm and 6 cm long respectively. What will be sum of distances travelled by their tips in 4 days? (Take π = 3.14)
(a) 954.56 cm (b) 3018.24 cm
(c) 3909.12 cm (d) 2703.96 cm
(e) None of these

23. The surface areas of two spheres are in the ratio of 1 : 4. Find the ratio of their volumes.

(a) 1 : 2 (b) 1 : 8
(c) 1 : 4 (d) 2 : 1
(e) 4 : 1

6. The outer and inner diameters of a spherical shell are 10 cm and 8 cm respectively. Find the volume of the metal contained in the shell. (Use $\pi = 22/7$)
(a) 409.6 cm^3 (b) 141.95 cm^3
(c) 283.9 cm^3 (d) 478.5 cm^3
(e) None of These

7. The ratio of two spheres are in the ratio of 1 : 2. Find the ratio of their surface areas.
(a) 1 : 2 (b) 2 : 3
(c) 1 : 4 (d) 3 : 4
(e) 4 : 3

8. A sphere of radius r has the same volume as that of a cone with a circular base of radius r. Find the height of cone.
(a) 2r (b) r/3 (c) 4r (d) 12/3r
(e) 3r

9. Find the number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm, required to construct a wall 12 m long, 5 m high and 0.25 m thick, while the sand and cement mortar occupies 5% of the total volume of wall.
(a) 4000 (b) 3000
(c) 1000 (d) 12000
(e) 9000

10. A road that is 7 m wide surrounds a circular path whose circumference is 352 m. What will be the area of the road?
(a) 2816 m^2 (b) 654.5 m^2
(c) 1386 m^2 (d) 5280 m^2
(e) None of these

11. In a storm, 10 cm of rain falls. What will be the volume of water that falls on 1 hectare area of ground?
(a) 700 m^3 (b) 600 m^3
(c) 1000 m^3 (d) 750 m^3
(e) 800 m^3

12. Seven equal cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.
(a) 750 cm^2 (b) 1300 cm^2
(c) 2250 cm^2 (d) 700 cm^2
(e) 900 cm^2

13. An swimming pool measuring 90 m by 40 m, 150 men take a bath. If the average displacement of water by a man is 8 cubic metres, what will be rise in water level?
(a) 30 cm (b) 50 cm
(c) 20 cm (d) 25 cm
(e) 33.33 cm

14. How many metres of cloth 5 m wide will be required to make a conical tent, the radius of whose base is 7 m and height is 24 m?
(a) 55 m (b) 330 m
(c) 220 m (d) 120 m
(e) 130 m

15. Two cones have their heights in the ratio 1 : 2 and the diameters of their bases are in the ratio 2 : 1. What will be the ratio of their volumes?
(a) 4 : 1 (b) 2 : 1
(c) 3 : 2 (d) 1 : 1
(e) 3 : 1

16. A conical tent is to accommodate 10 persons. Each person must have 6 m^2 space to sit and 30 m^3 of air to breath. What will be the height of the cone?
(a) 37.5 m (b) 150 m
(c) 75 m (d) 80 m
(e) None of these

17. A closed wooden box measures externally 10 cm long, 8 cm broad and 6 cm high. Thickness of wood is 0.5 cm. Find the volume of wood used.
(a) 250 cubic cm (b) 365 cubic cm
(c) 330 cubic cm (d) 300 cubic cm
(e) None of these

18. A cuboid of dimension 24 cm × 9 cm × 8 cm is melted and smaller cubes are of side 3 cm is formed. Find how many such cubes can be formed?
(a) 27 (b) 36 (c) 54 (d) 32
(e) 36

19. Three cubes each of volume of 216 m^3 are joined end to end. Find the surface area of the resulting figure.
(a) 504 m^2 (b) 216 m^2
(c) 432 m^2 (d) 480 m^2
(e) None of these

20. A hollow spherical shell is made of a metal of density 4.9 g/cm^3. If its internal and external radii are 10 cm and 12 cm respectively, find the weight of the shell.
(Take $\pi = 3.14$)
(a) 3016 gm (b) 1436.8 gm
(c) 1896.28 gm (d) 3667.1 gm
(e) None of these

21. The largest cone is formed at the base of a cube of side measuring 7 cm. Find the ratio of volume of cone to cube.

(c) 20 : 21 (d) 22 : 21
(e) 21 : 22 (f) 42 : 11
(g) 11 : 42

40. A spherical cannon ball, 28 cm in diameter, is melted and cast into a right circular conical mould the base of which is 35 cm in diameter. Find the height of the cone correct up to two places of decimals.
(a) 8.96 cm (b) 35.84 cm
(c) 5.97 cm (d) 17.92 cm
(e) None of these

41. Find the area of the circle circumscribed about a square each side of which is 10 cm.
(a) 314.28 cm^2 (b) 157.14 cm^2
(c) 196.38 cm^2 (d) 78.57 cm^2
(e) 86.12 cm^2

42. Find the radius of the circle inscribed in a triangle whose sides are 8 cm, 15 cm and 17 cm.
(a) 4 cm (b) 5 cm
(c) 3 cm (d) $2\sqrt{2}$ cm
(e) $3\sqrt{2}$ cm

43. In the given diagram a rope is wound round the outside of a circular drum whose diameter is 70 cm and a bucket is tied to the other end of the rope. Find the number of revolutions made by the drum if the bucket is raised by 11 m.

(a) 38 (b) 2.5 (c) 5 (d) 3.5
(e) 6

44. A cube whose edge is 20 cm long has circle on each of its faces painted black. What is the total area of the unpainted surface of the cube if the circles are of the largest area possible?
(a) 85.71 cm^2 (b) 257.14 cm^2
(c) 314.28 cm^2 (d) 331.33 cm^2
(e) None of these

45. The areas of three adjacent faces of a cuboid are x, y, z. If the volume is V, then V^2 will be equal to
(a) xyz (b) xyz^2 (c) $x^2y^2z^2$ (d) xy^2
(e) xyz

46. In the adjacent figure, find the area of the shaded region. (Use $\pi = 22/7$)

(a) 15.28 cm^2
(b) 62.14 cm^2
(c) 30.57 cm^2
(d) 40.28 cm^2
(e) None of these

47. The diagram represents the area swept by the wiper of a car. With the dimensions given in the figure, calculate the shaded area swept by the wiper.

(a) 402.67 cm (b) 205.14 cm
(c) 81.33 cm (d) 308.16 cm
(e) Cannot be determined

48. Find the area of the quadrilateral ABCD. (Given $\sqrt{3} = 1.73$)

(a) 452 sq units
(b) 269 sq units
(c) 134.5 sq units
(d) 1445 sq units
(e) Cannot be determined

49. The base of a pyramid is a rectangle of sides 15 m and 8 m and its slant height to the shorter side of the base is 24 m. Find its volume.

(a) $156\sqrt{407}$
(b) $78\sqrt{407}$
(c) $312\sqrt{407}$
(d) $234\sqrt{407}$
(e) Data insufficient

50. A wire is looped in the form of a circle of radius 28 m. It is bent again into a square form. What will be the length of the diagonal of the largest square possible thus?
(a) 44 m (b) $44\sqrt{2}$
(c) $176\sqrt{2}$ (d) $88\sqrt{2}$
(e) None of these

GEOMETRY

Level of Difficulty (LOD) III

1. In a triangle ABC, point D is on side AB and point E is on side AC such that $BCED$ is a trapezium. $DE : BC = 1 : 3$. Calculate the ratio of the area of ΔADE and the trapezium $BCED$.

(a) 1 : 4 (b) 9 : 16
(c) 1 : 9 (d) 9 : 25
(e) 3 : 9

2. D, E, F are the mid-points of the sides BC, CA and AB respectively of ΔABC. Determine the ratio of the area of triangle DEF and ABC.

(a) 3 : 4 (b) 1 : 2
(c) 2 : 3 (d) 4 : 3
(e) None of these

3. In the adjoining figure, $ABCD$ is a trapezium in which $AB \parallel DC$ and $AB = 3DC$. Determine the ratio of the area of ΔAOB and ΔCOD.

(a) 9 : 1 (b) 1 : 9
(c) 3 : 1 (d) 1 : 3
(e) None of these

4. A ladder reaches a window that is 8 cm above the ground on one side of the street. Keeping its foot on the same point, the ladder is turned to the other side of the street to reach a window 12 cm high. Find the width of the street if the ladder is 13 m.

(a) 18.2 m (b) 14 m
(c) 14.6 m (d) 12 m
(e) 15 m

5. In the adjoining figure $\angle A = 60°$ and $\angle ABC = 80°$, $\angle BQC$

(a) 40°
(b) 80°
(c) 20°
(d) 60°
(e) None of these

6. The diagram below represents three circular garbage cans, each of diameter 2 m. The three cans are touching as shown. Find, in metres, the perimeter of the rope encompassing the three cans.

(a) $2\pi + 6$
(b) $3\pi + 4$
(c) $4\pi + 6$
(d) $6\pi + 6$
(e) None of these

7. In the figure below, $PQ = QS$, $QR = RS$ and angle $SRQ = 100°$. How many degrees is angle QPS?

(a) 20°
(b) 40°
(c) 25°
(d) 30°
(e) 45°

8. In the figure, $ABDC$ is a cyclic quadrilateral with O as centre of the circle. If $\angle BOC = 130°$, find $\angle BDC$.

(a) 110°
(b) 112°
(c) 100°
(d) 115°
(e) None of these

9. In the given figure, $AD \parallel BC$. Find the value of x.

(a) $x = 8, 9$
(b) $x = 7, 8$
(c) $x = 8, 10$
(d) $x = 7, 10$
(e) None of these

10. In the given figure $\frac{AO}{DC} = \frac{BO}{OB} = \frac{1}{2}$ and $AB = 4$ cm. Find the value of BC above figure, $\frac{AO}{DC} = \frac{BO}{OD} = \frac{1}{2}$ and $AB = 4$ cm. Find the value of BC.

(a) 7 cm
(b) 8 cm
(c) 9 cm
(d) 10 cm
(e) None of these

11. In the above figure, AD is the bisector of $\angle BAC$, $AB = 8$ cm, $AC = 5$ cm and $BD = 3$ cm. Find DC.

(a) 11.5 cm
(b) 13 cm
(c) 14 cm
(d) 4 cm
(e) None of these

12. In a ΔABC, AD is the bisector of $\angle BAC$, $AB = 8$ cm, $BD = 6$ cm and $DC = 3$ cm. Find AC

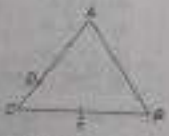
(a) 4 cm (b) 6 cm
(c) 3 cm (d) 5 cm
(e) None of these

13. If ABC is a quarter circle and a circle is inscribed in it and if $AB = 2$ cm, find radius of smaller circle

(a) $\sqrt{2} - 1$
(b) $(\sqrt{2} + 1)/2$
(c) $\sqrt{2} - 1/2$
(d) $1 + 2\sqrt{2}$
(e) None of these

14. ABC is an isosceles triangle. Point D is on AC and point E is on BC, such that $AD = 2CD$ and $CE = EB$. If we draw perpendiculars from D and E to other two sides and find the sum of the length of two perpendicular for each set, that is, for D and E individually and denote them as per (D) and per (E) respectively, then which of the following option will be correct.

(a) Per $(D) >$ per (E)
(b) Per $(D) <$ per (E)
(c) Per $(D) =$ per (E)
(d) None of these
(e) Cannot be determined



15. $ABCD$ is a trapezium in which AB is parallel to DC, $AD = BC$, $AB = 8$ cm, $AE = EF$ and $DF = EC$. If two lines AF and BE are drawn so that area of $AEFC$ is half of $ABCD$. Find DFC (?)

(a) 1/4 (b) 1/3 (c) 2/5 (d) 1/6
(e) 2/3

16. In the given figure, ΔABC and ΔACD are right angled triangles and $AB = x$ cm, $BC = y$ cm, $CD = z$ cm and $x = z$ and x, y and z has minimum integral value. Find the area of $ABCD$

(a) 36 cm^2 (b) 64 cm^2
(c) 24 cm^2 (d) 25 cm^2
(e) 30 cm^2

17. OD, OE and OF are perpendicular bisectors to the three sides of the triangle. What is the relationship between the $\angle BAC$ and the $\angle BOC$?

(a) $m\angle BAC = 180 - m\angle BOC$
(b) $m\angle BOC = 90 + 1/2\, m\angle BAC$
(c) $m\angle BAC = 90 + 1/2\, m\angle BOC$
(d) $m\angle BOC = 3m\angle BAC$
(e) $m\angle BOC = 2m\angle BAC$

18. If two equal circles of radius 5 cm have two common tangents AB and CD which touch the circle at A, C and B, D respectively and if $CD = 24$ cm, find the length of AB.

(a) 27 cm (b) 29 cm
(c) 26 cm (d) 30 cm
(e) None of these

19. If a circle is provided with a diameter of 10 cm, is it possible to divide the circle into ten equal parts

(e) None

(a) Possible when one more measure of 30° is given

(b) Above

(c) Possible if one more measure of 15° is given

(d) None of the above

2. Find the centre of a circle of radius 5 cm. The chord AB subtends an angle of 90° at the centre. Find the area of the shaded portion (approximate value).

(a) 50 cm²

(b) 62.78 cm²

(c) 39.88 cm²

(d) 67.67 cm²

(e) None of these

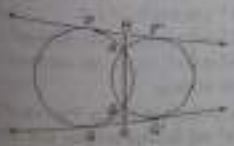
3. Two circles C(O, r) and C′(O′, r′) intersect at two points A and B and O lies on C(O′, r′). A tangent CD is drawn to the circle C(O′, r′) at A. Then

(a) $\angle OAC = \angle OAB$ (b) $\angle OAB = \angle AO'O$

(c) $\angle ADFE = \angle AOB$ (d) $\angle OAC = \angle AOB$

(e) None of these

4. PP′ and QQ′ are two direct common tangents to two circles intersecting at points A and B. The common chord AB produced intersects PP′ in R and QQ′ in S. Which of the following is true?



(a) $RA^2 + RB^2 = AB^2$ (b) $RS^2 = PP'^2 + AB^2$

(c) $RS^2 = PP'^2 + QQ'^2$ (d) $RS^2 = RA^2 + PP'^2$

(e) None of these

5. Two circles touch internally at point P and a chord AB of the circle of larger radius touches the other circle at C and D. Which of the following holds good?

(a) $\angle CPA = \angle DPB$

(b) $2\angle CPA = \angle CPB$

(c) $\angle APX = \angle ABP$

(d) $\angle BPX = 2\angle CPD + \angle CPB$

(e) None of these

6. In a trapezium ABCD, AB ∥ CD and AD = BC. If P is point of intersection of diagonals AC and BD, then all of the following is wrong except

(a) $PA \cdot PB = PC \cdot PD$ (b) $PA \cdot PC = PB \cdot PD$

(c) $PA \cdot AB = PD \cdot DC$ (d) $PA \cdot PD = AB \cdot BC$

(e) None of these

7. All of the following is true except:

(a) The points of intersection of direct common tangents and indirect common tangents of two circles divide the line segment joining the two centres respectively externally and internally in the ratio of their radii.

(b) In a cyclic quadrilateral ABCD, if the diagonal CA bisects the angle C, then diagonal BD is parallel to the tangent at A to the circle through A, B, C, D.

(c) If TA, TB are tangent segments to a circle C(O, r) from an external point T and O′ intersects the circle in P, then AP bisects the angle TAB.

(d) If in a right-angled ABC, BD is the perpendicular on the hypotenuse AC, then

(i) $AC \cdot AD = AB^2$ and

(ii) $AC \cdot BD = BC^2$

(e) All of these are true.

MENSURATION

Level of Difficulty (LOD) III

1. The perimeter of a sector of a circle of radius 5.5 m is 27.5 m. Find the area of the sector.

(a) 30.00 cm² (b) 135.00 cm²

(c) 45 cm² (d) 45.5 cm²

(e) None of these

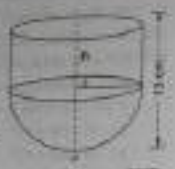
2. The dimensions of a field are 30 m by 0 m. A pit 30 m long, 4.5 m wide and 3 m deep is dug in one corner of the field and the earth removed has been evenly spread

over the remaining area of the field. What will be the rise in the height of field as a result of this operation?

(a) 1.5 m (b) 2 m (c) 3 m (d) 4 m

(e) 5 m

3. A vessel is in the form of a hollow cylinder mounted on a hemispherical bowl. The diameter of the sphere is 14 cm and the total height of the vessel is 13 cm. Find the capacity of the vessel.



(Take $\pi = 22/7$)

(a) $321.33 cm^3$ (b) $1042.67 cm^3$

(c) $1232 cm^3$ (d) $1032.33 cm^3$

(e) None of these

4. The sides of a triangle are 21, 20 and 15 cm. Find the area of the larger triangle into which the given triangle is divided by the perpendicular upon the longest side from the opposite vertex.

(a) $72 cm^2$ (b) $96 cm^2$

(c) $168 cm^2$ (d) $144 cm^2$

(e) $120 cm^2$

5. A circular arm is cylindrical to a height of 4 metres and conical above it. If its diameter is 105 m and the slant height of the conical portion is 53 m, calculate the length of the canvas 5 m wide to make the required tent.

(a) 384 (b) 973.5

(c) 1947 m (d) 1800 m

(e) None of these

6. A steel sphere of radius 4 cm is drawn into a wire of diameter 4 mm. Find the length of wire.

(a) 40.005 mm (b) 42.00 mm

(c) 21.333 mm (d) 34.231 mm

(e) None of these

7. A cylinder and a cone having equal diameter of their bases are placed in the Qutub Minar one on the other, with the cylinder placed in the bottom. If their curved surface areas are in the ratio of 8 : 5, find the ratio of their heights. Assume the height of the cylinder to be equal to the radius of Qutub Minar. (Assume Qutub Minar to be having same radius throughout).

(a) 3 : 4 (b) 3 : 8 (c) 4 : 3 (d) 2 : 3

(e) 3 : 2

8. If the curved surface area of a cone is thrice that of another cone and slant height of the second cone is double that of the first, find the ratio of the area of their base.

(a) 81 : 1 (b) 9 : 1 (c) 3 : 1 (d) 27 : 1

(e) 11 : 1

9. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 12 cm, find the uniform thickness of the cylinder.

(a) 2 cm (b) 3 cm (c) 1 cm (d) 3.5 cm

(e) 4 cm

10. A hollow sphere of external and internal diameters 8 cm and 4 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone.

(a) 4.75 cm (b) 9.5 cm

(c) 19 cm (d) 38 cm

(e) 28.5 cm

11. Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the new cuboid to the sum of the surface areas of the three cubes.

(a) 7 : 9 (b) 49 : 81

(c) 9 : 7 (d) 27 : 23

(e) None of these

12. If V be the volume of a cuboid of dimensions x, y, z and A is its surface, then A/V will be equal to

(a) $x^2y^2z^2$

(b) $1/2\,(1/xy + 1/yz + 1/zx)$

(c) $\frac{2}{3}\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

(d) $1/xyz$

(e) xyz

13. The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 9 a.m. and 9 : 35 a.m.

(a) $183.3 cm^2$ (b) $366.6 cm^2$

(c) $244.4 cm^2$ (d) $188.39 cm^2$

(e) None of these

14. Two circles touch internally. The sum of their areas is $116\pi cm^2$ and distance between their centres is 6 cm. Find the radii of the circles.

(a) 10 cm, 4 cm (b) 11 cm, 4 cm

(c) 9 cm, 5 cm (d) 10 cm, 5 cm

(e) 10 cm, 9 cm

15. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The

height and radius of the cylindrical part are 12 cm and 7 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the top if the height of conical part is 12 cm.

(a) $346 cm^2$ (b) $385 cm^2$
(c) $396 cm^2$ (d) $770 cm^2$
(e) $100 cm^2$

2. A solid wooden toy is in the form of a cone surmounted on a hemisphere. If the radii of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of wood used in the toy.

(a) $305.75 cm^3$ (b) $306.13 cm^3$
(c) $312.32 cm^3$ (d) $333.55 cm^3$
(e) $1635 cm^3$

3. A cylindrical container whose diameter is 12 cm and height is 15 cm, is filled with ice cream. The whole ice cream is distributed to 10 children in equal cones having hemispherical tops. If the height of the conical portion exceeds the diameter of its base, find the diameter of the ice cream cone.

(a) 6 cm (b) 12 cm
(c) 3 cm (d) 18 cm
(e) 15 cm

4. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 7 cm and the diameter of the cylinder is 3 cm. Find the total surface area of the solid. (Use $\pi = 22/7$)

(a) $36.75 cm^2$ (b) $41.0 cm^2$
(c) $44 cm^2$ (d) $43.2 cm^2$
(e) $50 cm^2$

5. A cone, a hemisphere and a cylinder stand on equal base and have the same height. What is the ratio of their volumes?

(a) 1:1:1 (b) 2:3:1:3
(c) 1:2:3 (d) 1:3:2:3
(e) 2:1:1:1

6. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The cost of painting 1 cm^2 of the surface is Rs. 0.05. Find the cost of painting the vessel all over.

(Take $\pi = 22/7$)

(a) Rs. 97.00 (b) Rs. 96.8
(c) Rs. 104 (d) Rs. 96.29
(e) Rs. 102.84

7. A solid is in the form of a right circular cylinder with a hemisphere at one end and a cone at the other end. Their common diameter is 3.5 cm and the heights of conical and cylindrical portion are respectively 6 cm and 10 cm. Find the volume of the solid.

(Use $\pi = 3.14$)

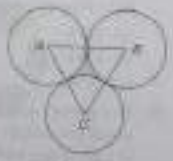
(a) $117 cm^3$ (b) $234 cm^3$
(c) $38.5 cm^3$ (d) $66 cm^3$
(e) None of these

8. In the adjoining figure, ABCD represents a quadrant of a circle of radius 3.5 cm with centre O. Calculate the area of the shaded portion. (Use $\pi = 22/7$)

(a) 25 cm^2 (b) 3.875 cm^2
(c) 9.625 cm^2 (d) 6.125 cm^2
(e) None of these

9. Find the area of the shaded region if the radius of each of the circles is 1 cm.

(a) $2 - \frac{\pi}{2}$
(b) $\sqrt{3} - \pi$
(c) $\sqrt{3} - \frac{\pi}{2}$
(d) $\sqrt{3} - 2\pi$
(e) None of these



10. A right elliptical cylinder full of petrol has its vertical ellipsoid with 2.4 m and the shortest 1.6 m. Its height is 7 m. Find the time required to empty full the tank through a hole of diameter 4 cm if the rate of flow of petrol is 120 litres.

(a) 80 min (b) 90 min
(c) 75 min (d) 70 min
(e) 80 min

11. Find the area of the triangle ABC if

(a) $5(3\sqrt{3} + 2\sqrt{2})$

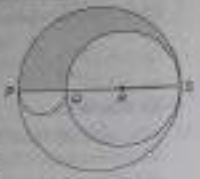
(b) $\frac{3\sqrt{3}(3+3\sqrt{3})}{2}$

(c) $(3\sqrt{3}+2\sqrt{3})$

(d) $(2\sqrt{3})(3+2\sqrt{3})$

(e) None of these

28. $PQRS$ is the diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn with PQ and QR as diameters as shown in the figure alongside. Find the ratio of the area of the shaded region to that of the unshaded region.



(a) 1 : 2 (b) 25 : 121

(c) 7 : 18 (d) 5 : 13

(e) 5 : 12

29. In the right angle triangle PQR, find RN.

(a) 13/60

(b) 13/45

(c) 60/13

(d) 13/59

(e) None of these

30. The radius of a right circular cylinder is increased by 50%. Find the percentage increase in volume.

(a) 120% (b) 29%

(c) 150% (d) 130%

(e) 125%

31. Two persons start walking on a road that diverge at an angle of 120°. If they walk at the rate of 5 km/h and 2 km/h respectively. Find the distance between them after 4 hours.

(a) $4\sqrt{19}$ km

(b) 7 km

(c) 7 km

(d) $8\sqrt{19}$ km

(e) None of these

32. Water flows out at the rate of 10 metres from a cylindrical pipe of diameter 5 mm. Find the time taken to fill a conical tank whose diameter at the surface is 40 cm and depth 24 cm.

(a) 50 min (b) 102.4 min

(c) 30.2 min (d) 25.6 min

(e) 60 min

33. The section of a solid right circular cone by a plane containing vertex and perpendicular to base is an equilateral triangle of side 12 cm. Find the volume of the cone.

(a) 72 cc (b) 144 cc

(c) $72\sqrt{2}$ cc (d) $72\sqrt{3}$ cc

(e) 800 cc

34. Iron weighs 8 times the weight of cork. Find the diameter of an iron ball whose weight is equal to that of a ball of cork 18 cm in diameter.

(a) 4.5 cm (b) 9 cm

(c) 12 cm (d) 15 cm

(e) Cannot be determined

35. In the figure, ABC is a right angled triangle with $\angle B = 90°$, $BC = 21$ cm and $AB = 28$ cm. With AC as diameter of a semicircle and with BC as radius, a quarter circle is drawn. Find the area of the shaded portion correct to two decimal places.

(a) 428.75 cm^2

(b) 857.50 cm^2

(c) 214.37 cm^2

(d) 371.96

(e) None of these

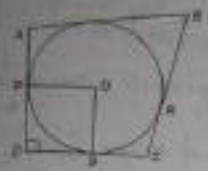
36. Find the perimeter and area of the shaded portion of the adjoining diagram.

(a) 90.8 cm, 414 cm^2 (b) 181.6 cm, 425.7 cm^2

(c) 90.8 cm, 827.4 cm^2 (d) 181.6 cm, 827.4 cm^2

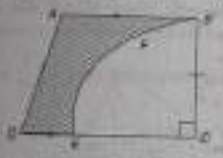
(e) None of these

37. In the adjoining figure, a circle is inscribed in the quadrilateral $ABCD$. Given that $BC = 38$ cm, $AB = 27$ cm and $DC = 25$ cm, and that AD is perpendicular to DC. Find the minimum limit of the radius and the area of the circle.



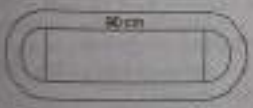
(a) 10 cm, 224 cm^2 (b) 14 cm, 616 cm^2
(c) 14 cm, 216 cm^2 (d) 28 cm, 616 cm^2
(e) None of these

8. From a piece of cardboard, in the shape of a trapezium $ABCD$ and $AB \parallel DC$ and $\angle BCD = 90°$, a quarter circle $(BFEC)$ with C as centre is removed. Given $AB = 10$ cm $= 3.5$ cm and $DE = 2$ cm, calculate the area of the remaining piece of the cardboard. (Take $\pi = 22/7$)



(a) 5.323 cm^2 (b) 5.125 cm^2
(c) 6.125 cm^2 (d) 12.25 cm^2
(e) None of these

9. The inside perimeter of a practice running track with semi-circular ends and straight parallel sides is 312 m. The length of the straight portion of the track is 90 m. If the track has a uniform width of 2 m throughout, find its area.



(a) 5166 m^2 (b) 5802.57 m^2
(c) 636.57 m^2 (d) 1273.14 m^2
(e) 2546.28

10. Find the area of the triangle inscribed in a circle circumscribed by a square made by joining the mid-points of the adjacent sides of a square of side a.

(a) $a^2/16$ (b) $\frac{3\sqrt{3}\,a^2}{16}$
(c) $3a^2(\pi - 1/2)$ (d) $\frac{3\sqrt{3}\,a^2}{32}$
(e) None of these

11. Two goats are tethered to diagonally opposite corners of a field formed by joining the mid-points of the adjacent sides of another square field of side $20\sqrt{2}$. What is the total grazing area of the two goats?

(a) 400 m^2 (b) $50\pi(\sqrt{2} - 1)$ m^2
(c) $100\pi(5 - 2\sqrt{2})$ m^2 (d) $200\pi(2 - \sqrt{2})$ m^2
(e) None of these

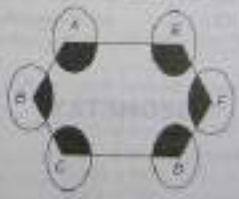
12. The area of the circle circumscribing three circles of unit radius touching each other is

(a) $(\pi/3)(2 + \sqrt{3})^2$ (b) $6\pi(2 + \sqrt{3})^2$
(c) $3\pi(2 + \sqrt{3})^2$ (d) $\left(\frac{\pi}{6}\right)(2 + \sqrt{3})^2$
(e) $\frac{\pi}{4}(2 + \sqrt{3})^2$

13. Find the ratio of the diameter of the circle inscribed in and circumscribing an equilateral triangle to its height.

(a) 1 : 2 : 1 (b) 1 : 2 : 3
(c) 2 : 3 : 4 (d) 3 : 2 : 1
(e) 2 : 1 : 1

14. Find the sum of the areas of the shaded sectors given that $ABCDEF$ is any hexagon and all the circles are of same radius r with different vertices of the hexagon as their centres as shown in the figure.



(a) πr^2 (b) $2\pi r^2$ (c) $5\pi r^2/4$ (d) $3\pi r^2/2$
(e) $4\pi r^2$

15. Circles are drawn with four vertices as the centre and radius equal to the side of a square. If the square is formed by joining the mid-points of another square of side $2\sqrt{6}$, find the area common to all the four circles.

(a) $(\sqrt{3} - 1)(2\sqrt{6})$ (b) $4\pi - 3\sqrt{3}$
(c) $1/2(\pi - 3\sqrt{3})$ (d) $4\pi - (3\sqrt{3} - 1)$
(e) None of these

16. $ABDC$ is a circle and circles are drawn with AO, CD, DO and OB as diameters. Areas E and F are shaded. E/F is equal to

(a) 1 (b) 1/2 (c) 1/3 (d) 1/4
(e) $\pi/2$

49. The diagram shows six equal circles inscribed in equilateral triangle ABC. The circles touch externally among themselves and also touch the sides of the triangle. If the radius of each circle is R, area of the triangle is

(a) $(6+4\sqrt{3})R^2$ (b) $9R^2$
(c) $R^2(12+7\sqrt{3})$ (d) $R^2(9+6\sqrt{3})$
(e) $6R^2$

GEOMETRY

LOD I

1. (a)
When the length of stick = 30 m, then length of shadow = 10 m i.e. in this case length : 2 = shadow with the same angle of inclination of the sun, the length of tower that casts a shadow of 50 m is as 2 : 50 m = 100 m
i.e. height of tower = 100 m

2. (d)
$\Delta ABC \sim \Delta CDE$

Then $\frac{AC}{CD} = \frac{BC}{CE} = \frac{AB}{DE}$

Then $\frac{AC}{2.4} = \frac{4}{3} \Rightarrow AC = 3.4$ cm and

$\frac{BC}{4.2} = \frac{4}{3} \Rightarrow BC = 5.6$ cm

3. (c)
For similar triangles $\Rightarrow$ (Ratio of sides)^2 = Ratio of areas

Then as per question $= \left(\frac{36}{x}\right)^2 = \frac{144}{81}$

(Let the longest side of ΔDEF = x)

$\Rightarrow \frac{36}{x} = \frac{12}{9} \Rightarrow x = 27$ cm

4. (a)
(Ratio of corresponding sides)^2 = Ratio of areas of similar triangle
$\therefore$ Ratio of corresponding sides in this question

$= \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$

5. (a)

Ratio of corresponding sides $= \sqrt{\frac{9}{16}} = \frac{3}{4}$

6. (c)

Let $BC = ED = 6$ m
$AB = AC - BC = 11 - 6 = 5$ m
$CD = BC = 12$ m
Then by Pythagoras theorem

$AC^2 = AB^2 + BE^2 \Rightarrow AE = 13$ m

7. (a)

In the ΔOBC, BC = 7 cm and OC = 9 cm, then with Pythagoras theorem

$OB^2 = OC^2 - BC^2$

$OB = \sqrt{32} = 5.66$ cm (approx.)

4. (d)

In ΔOBC, $OB = 12$ cm, OC = radius = 13 cm. Then using Pythagoras theorem:

$BC^2 = OC^2 - OB^2 = 25$, $BC = 5$ cm

Length of the chord $= 2 \times BC = 2 \times 5 = 10$ cm

5. (d)

$\angle x = 35°$, because angles subtended by an arc at a point on the circumference are equal.

6. (c)

$\angle AOM = 2\angle ABM$ and

$\angle AON = 2\angle ACN$

because angle subtended by an arc at the centre of the circle is twice the angle subtended by it on the circumference on the same segment.

$\angle AON = 80°$ and $\angle AOM = 40°$

$\angle X = \angle AON + \angle AOM$

(∵ vertically opposite angles),

$\angle X = 120°$

7. (c)

AOD and BOD are similar (∵ AC and BD are diagonals). Thus

$\angle ADO = \angle CBO$ (alternate opposite angles)

$\angle X = 30°$

8. (a)

By the rule of tangents, we know:

$x(x + 6) = 16 = 25 + 5x \Rightarrow 11 = 3x \Rightarrow x = 2.2$ cm

13. (b)

By the rule of tangents, we get

$12^2 = (x + 7)x \Rightarrow 144 = x^2 + 7x$

$\Rightarrow x^2 + 7x - 144 = 0 \Rightarrow x^2 + 16x - 9x - 144 = 0$

$\Rightarrow x(x + 16) - 9(x + 16) \Rightarrow x = 9$ or -16

-16 can't be the length, hence this value is discarded, thus, $x = 9$

14. (a)

By the rule of chords, cutting externally, we get

$\Rightarrow (9 + 6)6 = (5 + x)5 \Rightarrow 90 = 25 + 5x \Rightarrow 5x = 65 \Rightarrow x = 13$ cm

15. (b)

Installs = area/semi perimeter

$= 24/12 = 2$ cm

16. (b)

$\angle APB = 90°$ (angle in a semicircle = 90°)

$\angle PBA = 180 - (90 + 25) = 65°$

$\angle TPA = \angle PBA$ (the angle that a chord makes with the tangent, is subtended by the chord on the circumference in the alternate segment).

$= 65°$

17. (a)

$ADBC$ is a cyclic quadrilateral as all its four vertices are on the circumference of the circle. Also, the opposite angles of the cyclic quadrilateral are supplementary.

Therefore, $\angle ADB = 180 - 48° = 132°$

18. (a)

$\angle POS = \angle QOR$ (vertically opposite angles)

So $a = 4b$

$\angle SOT + \angle TOQ + \angle QOR = 180°$

(sum of angles on a line = 180°)

$4b + 2c = 180°$

$84 + 2c = 180°$

$\Rightarrow 2c = 96° \Rightarrow c = 48°$

So $a = 84°$, $b = 21°$, $c = 48°$

19. (b)

$\angle ABC = 180° - 130° = 50°$

(∵ sum of angles on a line = 180°)

$\angle ADC = 180° - \angle ABC = 130°$

(∵ opposite angles of a cyclic quadrilateral are supplementary)

$\angle x = 180° - 130° = 50°$

(∵ sum of angles on a line = 180°)

20. (a)

$\angle ADC = \frac{140}{2} = 70°$ (because the angle subtended by an arc on the circumference is half of what it subtends at the centre). $ABCD$ was cyclic quadrilateral

So $\angle ABC = 180° - 70° = 110°$ (because opposite angles of a cyclic quadrilateral are supplementary)

21. (b)

$OB = OA$ = radius of the circle

$\angle AOB = 180 - (30 + 30)$

(Sum of angles of a triangle = 180°)

$= 120°$

Then $\angle ADB = \frac{120}{2} = 60°$, because the angle subtended by a chord at the centre is twice of what it was subtend at the circumference. Again, $ABCD$ is a cyclic quadrilateral.

So $\angle ACB = 180° - 60° = 120°$ (because opposite angles of cyclic quadrilateral are supplementary).

22. (a)

$\angle BAC = 50°$ (angles subtended by an arc anywhere on the circumference to the same segment are equal)

In ΔBAC, $\angle x = 180° - (110° + 50°) = 80°$

(∵ sum of angles of a triangle = 180°)

23. (d)

As L_1 and L_2 are not ∥ lines, so there can't be any relation between 89° and L_1.

Hence the answer cannot be determined.

24. (d)

Perimeter of the figure $= (10 + 10) + 6 + 3\pi$

$= 26 + 3\pi$

25. (b) $CN : BN = 4 : 3$

So the required answer is $3 : 3 : 2 : 3 : 4$

Option (b) is correct.

LOD II

1. (b)

ΔADE is similar to ΔABC (AAA property)

$ED : BC = 3 : 5$

Area of ΔADE : Area of $\Delta ABC = 9 : 25$

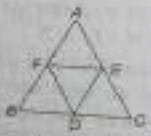
Area of trapezium = area of ΔABC − Area of ΔADE

$= 25 - 9 = 16$

Thus,

Area of ΔADE : Area of trapezium $EDBC = 9 : 16$

2. (a)



The area of a triangle formed by joining the mid-points of the sides of another triangle is always $1/4^{th}$ of the area of the bigger triangle.

So, the ratio is $= 1 : 4$

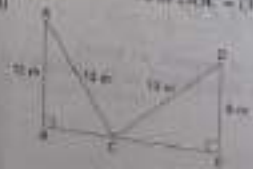
3. (a)

ΔDOC and ΔAOB are similar (by AAA property)

$AB : DC = 3 : 1$

So area of AOB : Area of $DOC = (3 : 1)^2 = 9 : 1$

4. (a)



In ΔABC, $BC = \sqrt{(4)^2 - (2)^2} = 3$

In ΔCDE, $CE = \sqrt{(13)^2 - (8)^2} = \sqrt{105} = 10.2$

Width of street $= BC + CE = 5 + 10.2 = 15.2$ m

7. (c)

$ABCD$ is a cyclic quadrilateral. Therefore

$\angle DCB = 180 - \angle A = 180 - 60 = 120°$

$\angle ABC = 80°$, therefore $\angle BCQ = 180° - 120° = 60°$

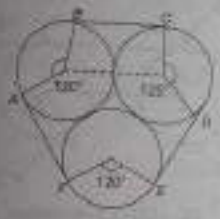
And $\angle CBQ = 180° - 80° = 100°$

(because, sum of angles on a line = 180°)

Then in ΔBCQ, $\angle Q = 180 - (100 + 60) = 20$

(∵ sum of angles of a triangle = 180°)

8. (a)



$\angle AOB = \angle COD = \angle EOF = 120°$

Distance between 2 centres = 2 m

$BC = DE = FA = 2$ m

Perimeter of the figure = $BC + DC + FA$ + circumference of sectors AOB, COD & FOE.

As three equal sectors of 120° = 1 full circle of same radius.

Therefore, perimeter of surface

$= 2\pi r + BC + DE + FA = (2\pi + 6)$ m

9. (c)

In ΔQRS, $QR = RS$, therefore $\angle RQS = \angle RSQ$ (because angles opposite to equal sides are equal).

Then $\angle RQS + \angle RSQ = 180° - 100° = 80°$

$\angle RQS = \angle RSQ = 40°$

$\angle PQS = 180° - 40° = 140°$

(sum of angles on a line = 180°)

Then again $\angle QRS = \angle QSP$

(∵ angles opposite to equal sides are equal)

Then $\angle QRS + \angle QSP = 180° - 140° = 40°$

And $\angle QPS = \angle QSP = 20°$

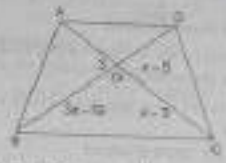
10. (c)

$\angle BOC = 136°$

$\angle BAC = \frac{136}{2} = 68°$ (because angle subtended by an arc anywhere on the circumference is half of the angle it subtends at the centre).

$\angle BDC = 180° - 68° = 112°$ (∵ $ABCD$ is a cyclic quadrilateral and its opposite angles are supplementary)

11. (a)



ΔAOD and ΔBOC are similar (AA property)

Then $\frac{3}{3x - 10} = \frac{x - 5}{x - 3}$

or $3x - 9 = 3x^2 - 15x - 85x + 95$

or $3x^2 - 37x + 104 = 0$

On solving this quadratic equation, we get $x = 8$ or 9

12. (b)

ΔAOD and ΔBOC are similar (AAA property)

$\frac{AO}{OC} = \frac{1}{2}$; therefore $\frac{AO}{BC} = \frac{1}{3} \Rightarrow \frac{4}{BC} = \frac{1}{2}$

or $BC = 8$ cm

13. (b)

In the given figure, ΔABD is similar to ΔADC

Then $\frac{AB}{AD} = \frac{AC}{DC} = \frac{6}{3} = \frac{5}{DC} \Rightarrow DC = 2.5$ cm

14. (a)

Given that AD bisects $\angle CAB$ in ΔABD

Then $\frac{AB}{DB} = \frac{AC}{DC} \Rightarrow \frac{8}{5} = \frac{AC}{3} \Rightarrow AC = 4.8$ cm

12. (a)

Area of the quarter circle $= \frac{\pi r^2}{4} \Rightarrow 0.25\pi$. Going by options, we have to see that the area of the inscribed circle is less than the area of the quarter circle.

Option (b) $\frac{(\sqrt{2}+1)}{2} \Rightarrow$ Area $= (1.5 + \sqrt{2})\pi$

$2.9\pi > 0.25\pi$. Hence discarded.

Option (c) $\sqrt{2} - \frac{1}{2} \Rightarrow$ Area $= \pi\left(2 + \frac{1}{4} - \sqrt{2}\right) \Rightarrow 0.35\pi$

$> 0.25\pi$. Hence discarded.

Option (d) $1 - \sqrt{2} \Rightarrow$ Area $= \pi(1 + 2 - 2\sqrt{2}) \Rightarrow 0.85\pi$

$> 0.25\pi$. Hence discarded.

Option (e) $\sqrt{2} - 1 \Rightarrow$ Area $\pi(2 + 1 - 2\sqrt{2}) \Rightarrow 0.20\pi$

$< 0.25\pi$

Hence this option is correct.

14. (a)

Sum of length of all perpendiculars drawn on the sides of any equilateral triangle is constant. Perpendicular (D) = Perpendicular (E)

15. (b)

In the above question,

$FE = AB = 8$ cm

$\Delta AFN \cong \Delta BEC$; so $DF = EC$

Let $DF = EC = x$

Solving through options e.g. option (b) 1/3, $x = 6$

Then by Pythagoras triplet $AF = 8$

Area of $ABEF = 8 \times 6 = 48$ cm^2

Area of $\Delta AFD + \Delta BEC = 2 \times \frac{1}{2} \times 6 \times 8 = 48$ cm^2

$\Rightarrow$ Area of $ABCD = 48 + 48 = 96$ cm^2. Hence the condition is proved.

16. (a)

Find the individual area of ΔABC and ΔACD. Add them then.

$\frac{1}{2}AB \times BC + \frac{1}{2}AC \times CD = \frac{1}{2}xy + \frac{1}{2}(\sqrt{x^2+y^2})^2$

$\Rightarrow$ replace x, y, z by the lowest integral values.

Here as we are talking about right angles, we have to take the smallest Pythagorean triples, which in this case would be = 3, 4, 5

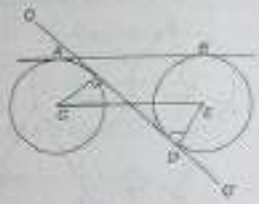
Answer $\Rightarrow$ 36 cm^2

17. (d)

As the point 'O' is formed by the $\perp$ bisects to the three sides of the Δ, so point 'O' is the circumcentre. This means that virtually, points A, B and C are on the circumference of the circle.

Then $m\angle BOC = 2m\angle BAC$ ($\because$ angle subtended by an arc at the centre of the circle is twice the angle subtended at the circumference).

18. (c)



$OC = CFD = 5$ cm (radius)

$CD = 24$ cm

ΔCOE and ΔEOD are similar therefore $OE = OE$ and $CE = ED = 12$ cm

In ΔCOD, $OE^2 = CE^2 + OC^2$

$= 12^2 + 5^2 = 169$

$OE = 13$

$\therefore$ $OO' = OE + EO' = 13 + 13 = 26$ cm

19. (c)

It will always be possible to divide a circle into 360 equal parts, because the sum of angles that can be subtended at the centre of the circle = 360°

20. (d)

Area of the shaded portion = Area of circle − Area of triangle

$\Rightarrow$ Area of circle $= \pi r^2 \Rightarrow \frac{22}{7} \times 5 \times 5 \Rightarrow \frac{22 \times 25}{7}$ cm^2

$= 78.50$ cm^2

Area of triangle $\Rightarrow \frac{1}{2}r^2 \sin\theta \Rightarrow \frac{1}{2} \times 25 \times \sin\theta$

$\Rightarrow \frac{25\sqrt{2}}{4} \Rightarrow 6.25 \times 1.732 \Rightarrow 10.8$

$\therefore$ Area of shaded portion $= 78.50 - 10.8 = 67.7$ cm^2

21. (a)

OB = OA = radius of circle

$\therefore \angle CMO = \angle OBA$

(angles in alternate segments are equal)

Now, if $\angle CMO = \angle OBA$

$\angle CMO = \angle OAB$

option (a) is correct.

2. (b)

3. (d)

Angle EBA = angle $ABF = x$

Angle CPE = angle $CDP = x + y$

Angle CDP is exterior angle of triangle PDB

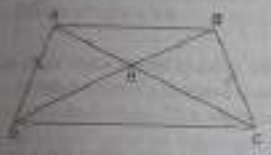
So angle $CDP = DBP + DPB$

$x + y = x + DPB$

$DPB = y$

So angle $CPE = DPB$

4. (b)



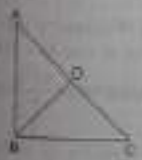
$\Delta APD \approx \Delta BPC$

$$\frac{PA}{PB} = \frac{PD}{PC}$$

$\therefore$ $PA \cdot PC = PB \cdot PD$

Hence (b)

5. (d)



$AC \cdot AD = AB^2$

$AC \cdot AD = BC^2$

Since $\Delta ADC \approx \Delta ADB$

$$\frac{AC}{AB} = \frac{AB}{AD}$$

(Corresponding sides of similar triangle are proportional)

$AC \cdot AD = AB^2$ (i)

Also $\Delta ADC \approx \Delta BDC$ so $\frac{AC}{BC} = \frac{BC}{CD}$

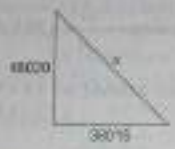
$AC \cdot CD = BC^2$ (ii)

Both conditions of options (c) are found, therefore (d) is the answer.

MENSURATION

LOD I

1. (b)



Let hypotenuse = x cm

Then, by Pythagoras theorem

$x^2 = (48000)^2 + (36015)^2$

$x = 60025$ cm

2. (d)

Let one side of the Δ be = a

Perimeter of equilateral triangle = $3a$

$\therefore$ $3a = 72\sqrt{3} \Rightarrow a = 24\sqrt{3}$ cm

Height = AC; by Pythagoras theorem

$$AC^2 = a^2 - \left(\frac{a}{2}\right)^2$$

$AC = 36$ cm

3. (b)

Let inner radius = A, then $2\pi r = 440$ $\therefore r = 70$

Radius of outer circle = 70 + 14 = 84 cm

$\therefore$ Outer diameter = 2 × Radius = 2 × 84 = 168

4. (a)

Let inner radius = r and outer radius = R

$$\text{Width} = R - r = \frac{396}{2\pi} - \frac{352}{2\pi}$$

$$\Rightarrow \quad (R - r) = \frac{44}{2\pi} = 7 \text{ meters}$$

5. (a)

Let outer radius = R, then inner radius = $r = R - 7$

$2\pi R = 220 \Rightarrow 35$ m

$r = 35 - 7 = 28$ m

Area of track = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2) = 1386$ m^2

Cost of travelling is $= 1386 \times \frac{1}{3} = Rs.\ 462$

6. (b)

Circumference of circle $= 2\pi r = 44$

$\Rightarrow r = 7$ cm

Area of a quadrant $= \frac{\pi r^2}{4} = 38.5\ cm^2$

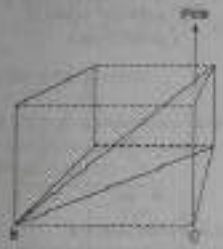
7. (d)

Volume of soil removed $= l \times b \times h$

$= 7.5 \times 6 \times 1.5 = 67.5\ m^3$

8. (c)

The longest pole can be placed diagonally (3 dimensional)



$BC = \sqrt{18^2 + 24^2} = 30$

$AC = \sqrt{30^2 + 16^2} = 34\ m$

9. (a)

Let the common ratio be $= x$

Then, length $= 3x$, breadth $= 2x$ and height $= x$

Then as per question $3x \cdot 2x \cdot x = 1296 \Rightarrow 6x^3 = 1296$

$\Rightarrow x = 6$ m

Breadth $= 2x = 12$ m

10. (c)

Data is inadequate as it's not mentioned that what part of the cube is melted to form cylinder.

11. (b)

Let the common ratio be $= x$

Then, length $= 4x$, breadth $= 3x$ and height $= 2x$

As per question

$2(4x \cdot 3x + 3x \cdot 2x + 2x \cdot 4x) = 8788$

$\Rightarrow 2(12x^2 + 6x^2 + 8x^2) = 8788 \Rightarrow 52x^2 = 8788$

$\Rightarrow x = 13$

Length $= 4x = 52$ cm

12. (b)

The total volume will remain the same, let the side of the resulting cube be $= a$. Then,

$6^3 + 8^3 + 10^3 = a^3 \Rightarrow a = \sqrt[3]{1728} = 12$ cm

13. (a)

Slant height $= l = \sqrt{6^2 + 8^2} = 10$ cm

Then curved surface area $= \pi r l = \pi \times 6 \times 10 = 60\pi$

And total surface area $= \pi r l + \pi r^2 \Rightarrow \pi \times 6 \times 10 + \pi \times 36\pi$

$= 96\pi$

14. (b)

Volume of a cone $= \frac{\pi r^2 h}{3}$

Then, $1188\pi = \frac{\pi r^2 \cdot 12}{3} \Rightarrow r = 9$ cm

Curved surface area $= \pi r l$

$l = \sqrt{h^2 + r^2} \Rightarrow \sqrt{12^2 + 5^2} = 13$

then, $\pi r l = \pi \times 12 \times 5 = 65\pi\ cm^2$

15. (d)

Let the radius of the two cones be $= x$ cm

Let slant height of 1st cone $= 5$ cm and

Slant height of 2nd cone $= 7$ cm

Then ratio of curved surface area $= \frac{\pi \times 5}{\pi \times 7} = 5 : 7$

16. (c)

$$\text{Radius} = \frac{\pi r l}{e l} = \frac{2376}{3.14 \times 18} = 42\ cm$$

Diameter $= 2 \times$ Radius $= 2 \times 42 = 84$ cm

17. (c)

Let the radius of cylinder $= 3(r)$

Then the radius of cone be $= 2(R)$

Then as per question $= \frac{\pi r^2 h}{\frac{\pi R^2 h}{3}} = \frac{3\pi r^2 h}{\pi R^2 h}$

$\Rightarrow \frac{3r^2}{R^2} \Rightarrow 3 : 4$

18. (c)

The perimeter would remain the same in any case

Let one side of a square be $= a$ cm

Then $a^2 = 484 \Rightarrow a = 22$ cm $\therefore$ perimeter $= 4a = 88$ cm

Let the radius of the circle be $= R$ cm

Then $2\pi r = 88 \Rightarrow r = 14$ cm

Then area $= \pi r^2 = 616\ cm^2$

19. (d)

Let the radius of the circle be $= R$

Then $2\pi r - 2r = 16.8 \Rightarrow r = 3.92$ cm

Then $2\pi r = 24.6$ cm

[illegible]

Let the radius of the wheel be $= r$

Then $4400 \times 2\pi r = 1100000$ or $r = 35$ cm

[illegible]

Let the slant height be $= l$

Curved surface $= \pi r l$

Then $l = \frac{\pi r^2 h}{V} \Rightarrow l = \frac{\sqrt{V}}{\sqrt{\pi h}} \Rightarrow \sqrt{\frac{2 \times 1980}{\pi \times 12}} = 5$ cm

$l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = 13$ cm

[illegible]

In 4 days, the short hand covers its circumference

In 2 days, the long hand covers its circumference

$4 \times 24 = 96$ times

So, long will cover a total distance of:

$12 \times 96 = 108 + 12 \times \pi \times 6 \times 96 \Rightarrow 3619.24$ cm

[illegible]

Let the radius of the smaller sphere $= r$

Then, the radius of the bigger sphere $= R$

Let the surface area of the smaller sphere $= 1$

Then, the surface area of the bigger sphere $= 4$

Then, as per question

$$\Rightarrow \frac{4\pi r^2}{4\pi R^2} = \frac{1}{4} \Rightarrow \frac{r}{R} = \frac{1}{2} \Rightarrow R = 2r$$

Ratio of their volumes

$$= \frac{4\pi r^3}{3} : \frac{4\pi r^3}{3} = \frac{3}{4\pi(2r)^3} \Rightarrow 1 : 8$$

[illegible]

Now radius $R = \frac{9}{2} = 4.5$ cm

Outer radius $(R) = \frac{10}{2} = 5$ cm

Volume of metal contained in the shell $= \frac{4\pi R^3 - 4\pi r^3}{3}$

$= \frac{4\pi}{3}(R^3 - r^3)$

$= 110.8$ cm^3

[illegible]

Let smaller radius $(r) = 1$

Then bigger radius $(R) = 2$

Then, as per question

$$\Rightarrow \frac{4\pi r^2}{4\pi R^2} = \left(\frac{r}{R}\right)^2 \Rightarrow \left(\frac{1}{2}\right)^2 = 1 : 4$$

26. (c)

As per question $\Rightarrow \frac{4\pi r^2}{3} = \frac{\pi r^2 h}{3} \Rightarrow h = 4r$

27. (a)

Volume of wall $= 1200 \times 800 \times 25 = 15000000$ cm^3

Volume of cement $= 5\%$ of $15000000 = 750000$ cm^3

Remaining volume $= 15000000 - 750000$

$= 14250000$ cm^3

Volume of a brick $= 25 \times 12.5 \times 7.5 = 2343.75$ cm^3

Number of bricks used $= \frac{14250000}{2343.75} = 6080$

28. (a)

Let the inner radius $= r$

Then $2\pi r = 352$ m. Then $r = 56$

Then outer radius $= r + 7 = 63 = R$

Now, $\pi R^2 - \pi r^2 =$ Area of road

$\Rightarrow \pi(R^2 - r^2) = 2618$ m^2

29. (c)

1 hectare $= 10000$ m^2

Height $= 10$ cm $= \frac{1}{10}$ m

Volume $= 10000 \times \frac{1}{10} = 1000$ m^3

30. (a)

Total surface area of 7 cubes $= 7 \times 6a^2 = 1050$

But on joining end to end, 12 sides will be covered.

So there area $= 12 \times a^2 \Rightarrow 12 \times 25 = 300$

So the surface area of the resulting figure $= 1050 - 300$

$= 750$

31. (a)

Let the rise in height be $= h$

Then as per the question, the volume of water should be equal in both the cases.

Now, $90 \times 40 \times h = 150 \times 8$

$$h = \frac{150 \times 8}{90 \times 40} = \frac{1}{6} \text{ m} = \frac{100}{3} \text{ cm}$$

$= 33.33$ cm

32. (c)

Slant height $(l) = \sqrt{7^2 + 24^2} = 25$ m

Area of cloth required = curved surface area of cone

$= \pi r l = \frac{22}{7} \times 7 \times 25 = 550$ m^2

Amount of cloth required $= \frac{550}{5} = 110$ m

33. (b)

If the ratio of their diameters = 2 : 1, then the ratio of their radii will also be = 2 : 1

Let the radius of the broader cone = 2 and height be = 1

Then the radii of the smaller cone = 1 and height be = 2

$$\text{Ratio of volumes} = \frac{\pi 2^2 \cdot 1}{3} \div \frac{\pi 1^2 \cdot 2}{3}$$

$$\frac{4\pi}{3} \times \frac{3}{2\pi} \Rightarrow 2 : 1$$

34. (c)

Area of base = 6 × 10 = 60 m²

Volume of tent = 30 × 10 = 300 m³

Let the radius be = r, height = h, slant height = l

$$\pi r^2 = 60 \Rightarrow r = \sqrt{\frac{60}{\pi}}$$

$$300 = \frac{\pi r^2 h}{3} \Rightarrow 900 = \pi \cdot \frac{60}{\pi} \cdot h \Rightarrow h = 15 \text{ m}$$

35. (b)

Volume of wood used = External volume − Outer Volume

$\Rightarrow (10 \times 8 \times 6) - (10 - 1) \times (8 - 1) \times (6 - 1)$

$\Rightarrow 480 - (9 \times 7 \times 5) = 165 \text{ cm}^3$

36. (b)

Total volume in both the cones will be equal. Let the number of smaller cubes = x

$$x \cdot 3^3 = 24 \times 9 \times 8 \Rightarrow x = \frac{24 \times 72}{27} = 64$$

37. (a)

Let one side of the cube = x

Then $x^3 = 216 \Rightarrow x = 6$ m

Area of the resultant figure

= Area of all 5 cubes − Area of covered figure

$\Rightarrow 216 \times 5 - (4 \times x^2) \Rightarrow 648 - 144 \Rightarrow 504 \text{ m}^2$

38. (c)

$$\text{Volume of metal used} = \frac{4\pi R^3}{3} - \frac{4\pi r^3}{3}$$

$$= \frac{4\pi}{3}(12^3 - 10^3)$$

$$= 3967.89 \text{ cm}^3$$

Weight = volume × density $\Rightarrow 8.9 \times 3967.89$

$\Rightarrow 34982.28$ gm

39. (c)

Volume of cube = $7^3 = 343 \text{ cm}^3$

$$\text{Radius of cone} = \frac{7}{2} = 3.5 \text{ cm}$$

Height of cone = 7

$$\text{Ratio of volumes} = \frac{\frac{\pi r^2 h}{3}}{343} = \frac{\frac{22 \times 3.5 \times 3.5 \times 7}{7 \times 3 \times 343}}$$

$\Rightarrow 11:42$

40. (b)

The volume in both the cases will be equal. Let the height of cone be = h

$$4 \times \frac{22}{7} \times (14)^2 \times \frac{1}{3} = \frac{22}{7} \times \left(\frac{35}{2}\right)^2 \times \frac{h}{3}$$

$$\Rightarrow \quad 4(14)^2 = h\left(\frac{35}{2}\right)^2 = h$$

$$= \frac{4 \times 14 \times 14 \times 14 \times 2 \times 2}{35 \times 35}$$

$$= h = 35.84 \text{ cm}$$

41. (b)

Diameter of circle = diagonal of square

$$= \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$$

$$\text{Radius} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

Area of circle = $\pi r^2 \Rightarrow 50\pi = 50 \times 3.14 = 157.14 \text{ cm}^2$

42. (c)

Area of triangle = πS; where π = inradius

$$S = \frac{15 + 8 + 17}{2} = 20 \text{ cm}$$

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

$$\Rightarrow \Delta = \sqrt{20(20-15)(20-8)(20-17)}$$

$$\Delta = \sqrt{20 \times 5 \times 12 \times 3} = 60 \text{ cm}^2$$

$$r = \frac{\Delta}{S} = \frac{60}{20} = 3 \text{ cm}$$

43. (c)

Circumference of the circular face of the cylinder = 2π

$$\Rightarrow 2 \times \frac{22}{7} \times \frac{35}{100} = 2.2 \text{ m}$$

Number of revolutions required to lift the bucket 11 m

$$11 \text{ m} = \frac{11}{2.2} = 5$$

$\therefore$ Surface area of the cube $= 6a^2 = 6 \times (20)^2$
$= 2400$

Area of a circle of radius 10 cm $= \pi r^2$
$= 3 \times 8 \times 100$
$= 1885.71$

Remaining area $= 2400 - 1884 = 514.28$

8. (a)

$$x \cdot y \cdot z = 36 \times 36 \times 36 = (36)^3$$

(X) volume of a cubical $= 36a$

or $\quad a^3 = (36)^3 = 216$

9. (c)

Diameter of the circle = diagonal of rectangle

$$= \sqrt{8^2 + 6^2} = 10 \text{ cm}$$

Radius $= \frac{10}{2} = 5$ cm

Area of shaded portion $= \pi r^2 - lb$
$= 3.14 \times 5^2 - 8 \times 6$
$= 30.57 \text{ cm}^2$

10. (a)

Outer Radius $(R) = 14 + 7 = 21$ cm
Inner Radius $(r) = 7$ cm

Area of shaded portion $= \pi R^2 \frac{\theta}{360} - \frac{\pi r^2 \theta}{360}$

$\Rightarrow \frac{\pi\theta}{360}(R^2 - r^2) = 102.67$ cm

11. (c)

Area of quadrilateral = Area of right angled triangle +

Area of equilateral triangle $x = \sqrt{21^2 - 12^2} = 16$

Area of quadrilateral $= \left(\frac{1}{2} \times 16 \times 12\right) + \frac{\sqrt{3}}{4} \times 20 \times 20$
$= 249 \text{ units}^2$

12. (a)

Height $= \sqrt{28^2 - 18^2} = \sqrt{407}$

Volume $= \frac{\text{Area of base} \times \text{height}}{3} \Rightarrow \frac{28 \times 28 \times \sqrt{407}}{3}$

$= 28\sqrt{407}$

13. (c)

The perimeter would remain the same in both cases.

Circumference of circle $= 2\pi r = 2 \times \frac{22}{7} \times 28 = 176$ cm

Perimeter of square = 176

Greatest side possible $= \frac{176}{4} = 44$ cm

Length of diagonal $= \sqrt{44^2 + 44^2} = 62.206$

$$= \frac{44}{2} \cdot \sqrt{2} = 44\sqrt{2}$$

LOD II

1. (c)

Let the angle subtended by the sector at the centre be $= \theta$

Then,

$$5.7 = 5.7 + (2r) = 5.7 + \frac{\theta}{360} \times 2\pi r$$

$$11.4 = \frac{11.4 \times 3.14 \times \theta}{360}$$

or $\quad \frac{\theta}{360} = 0.44$

Area of the sector $= \pi r^2 \frac{\theta}{360} = (3.14) \times (5.7)^2 \times 0.44$
$= 44.92$ approx.

2. (c)

Volume of mud dug out $= 10 \times 4.5 \times 3 = 135 \text{ m}^3$

Let the remaining ground rise by $= h$ m

Then $(120 \times 9) - (10 \times 4.5)h = 135$

$$1155h = 135 \Rightarrow h = 3 \text{ m}$$

3. (b)

Height of the cylinder $= 13 - 7 = 6$ cm

Radius of the cylinder and the hemisphere $= 7$ cm

Volume of the vessel = volume of cylinder + volume of hemisphere

$$\Rightarrow \pi r^2 h + \frac{4\pi r^3}{3 \times 2} \Rightarrow 3.14 \times (7)^2 \times 6 + \frac{4 \times 3.14 \times (7)^3}{3 \times 2}$$

$= 1042.6 \text{ cm}^3$

4. (b)

Let the original triangle be $= \Delta ACD$

Longest side $= AC = 21$ cm

In the right angled ΔABD, by Pythagoras theorem, we get $AB = 5$ and $AD = 12$

Then, $BC = 21 - 5 = 16$

By Pythagoras theorem,

$$AD^2 + AB^2 = BC^2 \Rightarrow AD = 12 \text{ cm}$$

Then, on measurement we convert

Area of the larger $\Delta BDC = \frac{1}{2} \times 16 \times 12 = 96 \text{ cm}^2$

5. (a)

Radius $= \frac{105}{2} = 52.5$ cm

Area of the entire canvas, used for the tent

= Area of cylinder + curved surface of cone

$= 2\pi rh + \pi rl$

$= \pi r(2h + l) = 3.14 \times 52.5 \times (2 \times 5 + \sqrt{52.5^2 + 52.5^2})$

$= 5 \times 1$ (because area of canvas $= l \times b$ also)

$= 1 = 1987$ m

6. (d)

The volume in both the cones would be the same

Thereby $= \frac{\pi r^2 h}{3} = \pi r^2 h$

$$\frac{4 \times 3.14 \times (10 \times 10)^2}{3} = 3.14 \times 2^2 \times h$$

$$\Rightarrow \quad h = \frac{40000}{3} = 13333.33 \text{ mm}$$

7. (b)

As the cylinder and cone have equal diameters. So they have equal area. Let cone's height be h_1 and as per question, cylinder's height be h_2

$$\frac{2\pi r h_2}{\pi r\sqrt{h_1^2 + r^2}} = \frac{4}{3}$$

On solving we get the desired ratio as 3 : 4

8. (a)

Let the slant height of 1st cone $= L$

Then the slant height of 2nd cone $= 3L$

Let the radius of 1st cone $= r_1$

and let the radius of 2nd cone $= r_2$

Then $\pi r_1 L = 5 \times \pi r_2 \times 3L$

$\Rightarrow \quad \pi r_1 L = 15\pi r_2 L \Rightarrow r_1 = 3r_2$

Ratio of area of the base

$$\frac{4\pi r_1^2}{\pi r_2^2} \Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3r_2}{r_2}\right)^2 \Rightarrow 9 : 1$$

9. (c)

Let the internal radius of the cylinder $= r$

Then the volume of sphere = Volume of the cylinder

$$\Rightarrow \quad \frac{4\pi \times 5^3}{3} = \pi h(5^2 - r^2)$$

$$\Rightarrow \quad \frac{500\pi}{3} = 32\pi(25 - r^2)$$

$$\Rightarrow \quad r^2 = 16 \Rightarrow r = 4 \text{ cm}$$

So thickness of the cylinder $= 5 - 4 = 1$ cm

10. (d)

The volume in both the cones would be the same

Let the height of the cone $= h$

Then, external radius $= 6$ cm

Internal radius $= 4$ cm

$$\Rightarrow \frac{4\pi(6^3 - 4^3)}{3} = \frac{\pi \times 4^2 \times h}{3}$$

$$\Rightarrow \quad h = \frac{6^3 - 4^3}{4} \Rightarrow h = \frac{216 - 64}{4} = 38 \text{ cm}$$

11. (a)

Let an side of the cube be $= x$ units

Total surface area of 3 cubes $= 3 \times 6x^2$

$= 18x^2$

Total surface area of cuboid $= 2(6x^2 + 4x^2 + 3x^2) = 26x^2$

$$\text{Ratio} = \frac{18x^2}{26x^2} = 9 : 13$$

12. (c)

$A = 2(xy + yz + zx)$

$V = xyz$

$$A/V = \frac{2(xy + yz + zx)}{xyz} = \frac{2}{z} + \frac{2}{x} + \frac{2}{y}$$

$$\Rightarrow 2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

13. (a)

The entire dial of the clock $= 360°$

Every 5 minutes $= \frac{360}{12} = 30°$

So 35 minutes $= 30 \times 17 = 210$

$$\text{Area} = \pi r^2 = \frac{\theta}{360} = 3.14 \times 100 \times \frac{210}{360}$$

$= 183.3 \text{ cm}^2$

(c) (iii)

Let the radius of the bigger circle = R

Let the radius of the smaller circle = r

Then as per questions, $R - r = 8$

Solving through options, only option (c) satisfies this condition.

(c) (d)

Radius of cylinder, hemisphere and cone = 5 cm

Height of cylinder = 13 cm

Height of cone = 12 cm

$$\text{Surface area of toy} = 2\pi rh + \frac{4\pi r^2}{2} + \pi rl$$

$$l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = 13$$

Then $= (2 \times 3.14 \times 5 \times 13) + (2 \times 3.14 \times 25) + (3.14 \times 5 \times 13) = 770 \text{ cm}^2$

(b) (b)

Height of cone = 10.2 − 4.2 = 6 cm

$$\text{Volume of mould} = \frac{\pi r^2 h}{3} + \frac{4\pi r^3}{3 \times 2}$$

$$= \frac{3.14 \times (4.2)^2 \times 6}{3} + \frac{4 \times 3.14 \times (4.2)^3}{3 \times 2}$$

$$= 296 \text{ cm}^3$$

(b) (a)

$$\text{Volume of ice cream} = \pi r^2 h$$

$$= 3.14 \times (10)^2 \times 1.5$$

$$= 1695.6 \text{ cm}^3$$

$$\text{Volume of 1 cone} = \frac{\pi r^2 \times 24}{3}$$

Then $= 10 \times 3.14 \times r^2 \times 8 = 1695.6$

or $r = 5 \text{ cm (approx.)}$

So diameter $= 2 \times r = 6$ cm

(b) (b)

$$\text{Radius of cylinder and hemispheres} = \frac{7}{2} = 3.5 \text{ cm}$$

Height of cylinder = 19 − (3.5 × 2) = 12 cm

Total surface area of solid $= 2\pi rh + 4\pi r^2$

$= 2 \times 3.14 \times 3.5 \times 12 + 4 \times 3.14 \times (3.5)^2$

$= 418 \text{ cm}^2$

(c) (c)

As they stand on the same base so their radius is also same.

$$\text{Then volume of cone} = \frac{\pi r^2 h}{3}$$

$$\text{Volume of hemisphere} = \frac{2\pi r^3}{3}$$

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\text{Ratio} = \frac{\pi r^2 h}{3} : \frac{2\pi r^3}{3} : \pi r^2 h$$

$$\Rightarrow \frac{h}{3} : \frac{2r}{3} : h$$

$$\Rightarrow h : 2r : 3h$$

Radius of a hemisphere = 3m height

So $h : 2r : 3h = 1 : 2 : 3$

20. (d)

Total surface to be painted

= external surface area + internal surface + surface area of ring area

$= 2\pi R^2 + 2\pi r^2 + 2\pi(R^2 + r^2) + 2\pi(R^2 - r^2)$

Cost of painting

$$= 2 \times 3.14 \times \left[\left(\frac{25}{2}\right)^2 + \left(\frac{24}{2}\right)^2 + \left(\frac{25}{2}\right)^2 - \left(\frac{24}{2}\right)^2\right] \times 0.08$$

$= 2 \times 6.28 \times (12.5)^2 \times 0.08 = \text{Rs. } 98.2$

21. (a)

$$\text{Radius} = \frac{3.5}{2} = 1.75 \text{ cm}$$

$$\text{Volume of solid} = \pi r^2 h + \frac{4\pi r^3}{3} + \frac{2\pi r^3}{3}$$

$$= \pi r^2 \left(h + \frac{4}{3} r + \frac{2}{3} r\right)$$

$$= 3.14 \times (1.75)^2 \times \left(10 + \frac{4}{3} \times 1.75 + \frac{2}{3} \times 1.75\right)$$

$= 171 \text{ cm}^3$

22. (d)

Area of shaded portion = Area of quadrant − Area of triangle

$$= \frac{\pi r^2}{4} - \frac{1}{2} \times 3.5 \times 3 = \frac{3.14 \times (3.5)^2}{4} - 3.5$$

$= 6.1 \text{ cm}^2$

23. (c)

ABC is an equilateral triangle with sides = 2 cm

Area of shaded portion = Area of equilateral triangle − Area of 3 quadrant

$$= \text{i.e. } \frac{\sqrt{3}}{4} a^2 - 3\left(\pi r^2 \cdot \frac{\theta}{360}\right), \theta = 60^\circ \text{ i.e. A, B, C is an equilateral triangle}$$

$$\Rightarrow \frac{\sqrt{3}}{4} \times 2^2 - 3\left(3.14 \times 1 \times \frac{90}{360}\right)$$

$$\Rightarrow \sqrt{3} - \frac{3.14}{2} = \sqrt{3} - \frac{\pi}{2}$$

24. (d)

Volume of the elliptical cylinder

$$= \pi \times \frac{2.4}{2} \times \frac{1.8}{2} \times 7$$

$$= 3.14 \times 1.2 \times 0.9 \times 7 = 9.9\ m^3$$

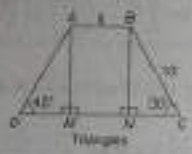
Amount of water emptied per minute

$$= 120 \times 3.14 \times \left(\frac{2}{100}\right)^2$$

Time required to empty half the tank

$$= \frac{4.5}{120 \times 3.14 \times (0.02)^2} = 70\ min$$

25. (c)



AB and DC are the 11 sides

$$\text{Height} = AM = BN$$

$$AB = MN = 4$$

ΔBNC and ΔAMD are right angled triangles

In $\Delta BNC \Rightarrow \sin 30 = \frac{BN}{10} \Rightarrow BN = 5$

Using Pythagoras theorem $NC = \sqrt{10^2 - 5^2} = 5\sqrt{3}$

In ΔADM, $AM = 5$; $\tan 45 = \frac{AM}{DM} = 1 = \frac{5}{DM}$

$\Rightarrow DM = 5$

Area of trapezium $\Rightarrow \frac{1}{2}$ (Sum of 11 sides) × height

$$\Rightarrow \frac{1}{2}(4 + 4 + 5\sqrt{3} + 5) \times 5 = \frac{5(13 + 5\sqrt{3})}{2} \text{ (Answer)}$$

26. (d)

$$PQ = QR = RS = \frac{12}{3} = 4\ cm$$

Area of unshaded region $\Rightarrow \frac{\pi 6^2}{2} + \frac{\pi 4^2}{2}$

$\Rightarrow 18\pi + 8\pi \Rightarrow 26\pi$

Area of shaded region $\Rightarrow \frac{\pi 10^2}{2} - \frac{\pi 4^2}{2}$

$\Rightarrow 50\pi - 8\pi = 108$

Ratio $= \frac{108\pi}{26\pi} \Rightarrow \frac{9}{13} \Rightarrow 9 : 13$

27. (c)

$$QR = \sqrt{5^2 + 12^2} = 13$$

Area of the triangle $= \frac{1}{2} \times b \times h = 30$

$\Rightarrow$ As RS is a $\perp$ drawn to the hypotenuse

So $RS = \frac{2 \times \text{Area}}{\text{Hypotenuse}} = \frac{60}{13}$

28. (d)

Let initial area $= \pi$

Then volume $= \pi r^2 h$

$$\text{New radius} = \pi + \frac{\pi}{2} \Rightarrow \frac{3\pi}{2}$$

$$\text{New volume} = \pi \cdot \frac{9}{4} r^2 h$$

Increased volume $= \frac{5}{4} \pi r^2 h$

$$\text{Percent increase} = \frac{5\pi r^2 h}{4\pi r^2 h} \times 100 = 125\%$$

29. Distance after 4 hours $= AB = C$

$$a = 3 \times 4 = 12;\ b = 2 \times 4 = 8$$

and $\frac{a+b+c}{2} \Rightarrow \frac{12+8+C}{2} \Rightarrow \left(\frac{10+C}{2}\right)$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area} = \frac{1}{2} ab \sin 120^\circ$$

$$\text{Area} \Rightarrow 48 = \frac{\sqrt{3}}{2} = 24\sqrt{3}$$

As per question

$$24\sqrt{3} = \sqrt{\left(10 + \frac{c}{2}\right)\left(\frac{c}{2} - 2\right)\left(2 + \frac{c}{2}\right)\left(10 - \frac{c}{2}\right)}$$

On solving, we get $c = 8\sqrt{10}$ km

(i) (c)

Volume of the cone = $\frac{\pi r^2 h}{3} = \frac{3.14 \times 20 \times 20 \times 25}{3}$

$= 10467 \text{ cm}^3$

Diameter of pipe = 5 cm

Volume of water flowing out of the pipe per minute

$= (\pi r^2 h) = 3.14 \times 196.25 \text{ cm}^3$

Time taken to fill the tank = $\frac{10467}{196.25} = 53.3$ mins

(ii) (b)

The side of the equilateral triangle = diameter of cone

Therefore radius of cone = $\frac{12}{2} = 6$

Height of cone = Height of equilateral triangle be

Height of cone = $\frac{\sqrt{3}a}{2} = 6\sqrt{3}$

Volume of cone = $\frac{\pi r^2 h}{3}$

$= \frac{\pi \times 6^2 \times 6\sqrt{3}}{3} = 72\sqrt{3}\pi \text{ cm}^3$

(iii) (b)

Let the radius of iron ball = r_1

Let the radius of rust ball = r_2

Then, as size weighs 8 times more than Ore

$\frac{4\pi r_1^3}{3} = \frac{8 \times 4\pi r_2^3}{3} = \frac{32}{3}\pi r_2^3$ or $r_1 = 2r_2$

So diameter of iron = $\frac{1}{2}$ diameter of rust

$= \frac{1}{2} \times 18 = 9$ cm

(iv) (a)

Required shaded portion = Area of $\triangle ABC$ + Area of sector BCD − Area of $\triangle ADB$ − sector ACD

$\Rightarrow$ Area of $\triangle ABC = \frac{1}{2} \times (7.5)^2 \times \frac{5}{2} = 48.1 \text{ cm}^2$

$\frac{\angle DBC}{\angle ABC} = \frac{2r}{2r} \Rightarrow \angle DBC = 67.5$ and $\angle DBA = 22.5$

$\Rightarrow$ Area of sector $DC = \left(\pi \times 2r^2 \times \frac{67.5}{360}\right)$

$= \left(\frac{22}{7} \times 2r^2 \times 0.188\right) = 56 \text{ cm}^2$

$\Rightarrow$ Area of $ADB = \left(\frac{1}{2} \times 2r \times 2r\right)$

$= \left(2r \times \frac{1}{2} \times 2r^2 \times \sin 22.5\right) = 3.6 \text{ cm}^2$

Thus area of shaded portion = 48.1 − 56 + 3.6 = 42.6 cm²

34. (i)

AB = radius of semicircle = 10 cm

4 Quadrants of quarter radius = 1 circle of that radius

Area of shaded portion = Area of rectangle − Area of circle

$= (28 \times 20) - (3.14 \times 10^2) = 414 \text{ cm}^2$

$BC = 28 - (10 + 10) = 8$ and $CD = 20 - (10 + 10) = 0$

Perimeter of shaded portion = 28 cm + 2 cm

Answer $\Rightarrow$ 414 cm² = Area and

Perimeter = 60.8

35. (d)

Go through the option.

Only option (d) is correct.

36. (c)

Area of remaining cardboard = Area of trapezium − Area of quadrant

$\Rightarrow$ Area of trapezium = $\frac{1}{2}$ (sum of parallel sides) × height

$= \frac{1}{2} \times (AB + DC) \times BC$

$= \frac{1}{2} \times (3.5 + 3.5) \times 3.5$

$= 3.5 \times 3.5 = 12.25 \text{ cm}^2$

Area of quadrant $= \frac{\pi r^2}{4} \Rightarrow \frac{3.14 \times 3.5 \times 3.5}{4} = 9.6$

$\Rightarrow$ Area of remaining quadrant $= 15.7 - 9.6 = 6.1\ cm^2$

37. (a)

Circumference of the 2 semicircles $= 21 + 21 + 90 + 90$

$= 132$

2 semicircles = 1 circle with equal radius

So $2\pi r = 132 \Rightarrow 2r = \frac{132}{3.14} \Rightarrow 42$ is diameter

Area of track = Area within external border − Area within internal border

$\Rightarrow \pi(42)^2 - 21^2) + 90 \times 84 - 90 \times 4^2$

$\Rightarrow 888 + 360 = 636.3\ m^2$

38. (d)

AB = side of the outermost triangle = a

$AC = CB = a/2$

$BC = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \frac{a}{\sqrt{2}}$

Diameter of circle $= \frac{a}{\sqrt{2}}$, radius $= \frac{a}{2\sqrt{2}}$

O is the centre of the circle. Then $\angle EOF = 120°$

Then Area of $\Delta EOF = \frac{1}{2} EO \cdot OF \cdot \sin 120°$

$\Rightarrow \frac{1}{2} \times \frac{a^2}{8} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}a^2}{32}$

Then area of $\Delta EFG = \frac{3\sqrt{3}a^2}{32}$

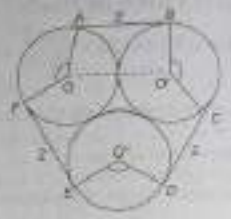
39. (a)

The length of rope of goat $= 10\sqrt{2}$ m

Then the max graze will graze an area = Area of a semicircle with area $10\sqrt{2}$ m

So total area grazed $= \frac{\pi r^2}{2} \Rightarrow 100\pi\ m^2$

40. (a)



In this figure, the sides $AB = CD = FE$ = (distance between 2 rails) = 2 cm

$\angle AOF = \angle BO'C = \angle EO''D = 120°$

So perimeter of the bigger triangle $= (2 + 2 + 2 + 2 \times 3.14 \times 1)$ (because 3 sectors of circles of 120° = 1 full circle of same radius)

Let the radius of bigger circle be = R

Then $2 \times \pi \times R = 12.28 \Rightarrow R = \frac{12.28}{2 \times 3.14} = 2$

Area $= \pi R^2 \Rightarrow 3.14 \times 4 \Rightarrow 13\ cm^2$ (approximately)

Option (a) is closest to the answer.

41. (b)

Let are side of equilateral triangle = a

Then height $= \frac{a\sqrt{3}}{2}$

Area $= \frac{\sqrt{3}}{2} a^2$, $s = \frac{a + a + a}{2} = \frac{3a}{2}$

Diameter of inner circle $= \frac{2 \times \text{Area}}{s}$

$= \frac{\sqrt{3}}{2} a^2 \times \frac{2}{3a} = \frac{a}{\sqrt{3}}$

Diameter of outer circle $= \frac{a^2}{2 \times \text{Area}} = a^2 \times \frac{6}{\sqrt{3}a^2}$

$= \frac{6a}{\sqrt{3}}$

$\text{Ratio} = \frac{a}{\sqrt{3}} : \frac{2a}{\sqrt{3}} : \frac{6a\sqrt{3}}{2} \Rightarrow \text{Ratio} = 1 : 2 : 3$

(c) Ans

Sum of interior angles of a hexagon = 120°

6 arcs with same radius a = 2 full circles of same radius.

So area of shaded region as $2\pi r^2$

(a) Ans

(a) Ans

$AG = GD = DO = OB$ = radius of bigger circle = r (let)

Then area of $(G + F) = \frac{\pi r^2}{2}$

area of $2(G + F) = \pi r^2$. Also area of $2G + F + E = \pi r^2$

i.e. $2G + E + F = 2G + F + E \Rightarrow F = E$

So the ratio of areas E and $F = 1 : 1$

(d) (c)

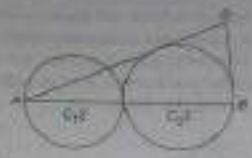
GEOMETRY & MENSURATION

Extra Practice Exercise

1. In the figure given what is the measure of $\angle ACD$

(a) 75° (b) 80°
(c) 90° (d) 105°
(e) Cannot be determined

2. Two circles C_1 and C_2 of radius 2 and 3 respectively touch each other as shown in the figure. If AD and BD are tangents then the length of BD is

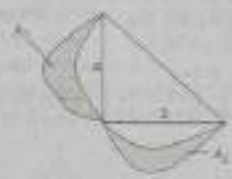


(a) 3/6 (b) 5/6
(c) $\sqrt{(6)}$ (d) 6
(e) None of these

3. If the sides of a triangle measure 13, 14, 15 cm respectively, what is the height of the triangle for the base side 14

(a) 10 (b) 12
(c) 14 (d) 15
(e) None of these

4. A right angled triangle is drawn on a plane such that sides adjacent to right angle are 3 cm and 4 cm. Now three semi circles are drawn taking all three sides of the triangle as diameters respectively (as shown in the figure). What is the area of the shaded regions $A_1 + A_2$



(a) 3π (b) 4π
(c) 5π (d) 6π
(e) None of these

5. A lateral side of an isosceles triangle is 15 cm and the altitude is 8 cm. What is the radius of the circumscribed circle

(a) 9.625 (b) 9.375
(c) 9.5 (d) 9.125
(e) None of these

6. Let a, b, c be the length of the sides of triangle ABC. Given $(a + b + c)(b + c - a) = abc$. Then the value of a will be in between

(a) −1 and 1 (b) 0 and 4
(c) 0 and 1 (d) 0 and
(e) None of these

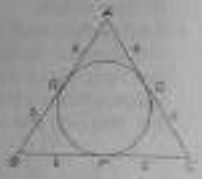
7. In the figure given below (not drawn to scale), A, B and C are three points on a circle with centre O. The chord BC is extended to point T such that AT becomes a tangent to the circle at point A. If ∠CEA = 35° and ∠CAT = 45° calculate x° (∠BAC)

(a) 100° (b) 90°
(c) 110° (d) 45°
(e) Cannot be determined

8. In the given figure

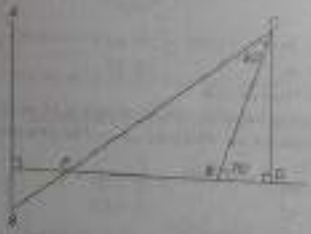
AB = 20
BC = 15
CA = 10

Calculate a, b, c



(a) a = 12, b = 8, c = 7
(b) a = 8, b = 12, c = 7
(c) a = 9, b = 10, c = 15
(d) a = 10, b = 15, c = 9
(e) None of these

9. In the figure given below, AB is parallel (or) to XO. ∠CED = 25° and ∠ECY = 30°. What is the measure of ∠ABC?

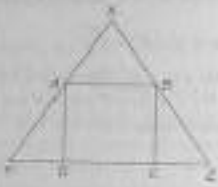


(a) 40° (b) 45°
(c) 35° (d) 30°
(e) 50°

10. The angle between lines L and M measures 35° degrees. If line M is rotated 40° degrees counter clockwise about point P to line M' what is the angle in degrees between lines L and M'

(a) 90° (b) 80°
(c) 75° (d) 60°
(e) None of these

11. In the figure given below, XYZ is a right angled triangle in which ∠Y = 45° and ∠X = 90°. ABCD is a square inscribed in it whose area is 64 cm². What is the area of triangle XYZ?

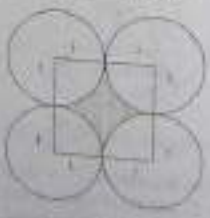


(a) 320 (b) 64
(c) 144 (d) 80
(e) None of these

12. The area of circle circumscribed about a regular hexagon is 144π. What is the area of hexagon?

(a) 306√3 (b) 216√3
(c) 256 (d) 225
(e) None of these

13. Find the area of the shaded portion



(a) 4 − π (b) π − 4
(c) 8 − π (d) π
(e) None of these

4. The numerical value of the product of the three sides which are integers when measured in cm of a right angled triangle having a perimeter of 56 cm is 4200. Find the length of the hypotenuse.

(a) 24 (b) 25 (c) 15 (d) 20

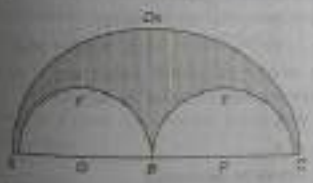
(e) 4

5. In the figure ABDC is a cyclic quadrilateral with O as centre of the circle. Find $\angle ADC$.

(a) 10° (b) 120° (c) 130° (d) 90°

(e) 50°

6. R, O, P are centres of semicircles ATC, ATB & BTC respectively. AC = 12 cm. Find the area of the shaded region.



(a) 9π (b) 18π

(c) 25π (d) 25π

(e) None of these

7. O is the centre of the circle. OP = 5 and OT = 4, and AB = 6. The line PT is a tangent to the circle. Find PB.

(a) 8 cm (b) 10 cm

(c) 7 cm (d) 6 cm

(e) None of these

8. In the figure given below, AB = 16, CD = 12 and CM = 8. Calculate ON.

(a) 8

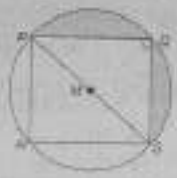
(b) 10

(c) 12

(d) 14

(e) None of these

9. In the figure, M is the centre of the circle. $l(QS) = 10\sqrt{2}$, $l(PR) = l(RS)$ and PR is parallel to QS. Find the area of the shaded region.



(a) $50\pi - 50$ (b) $50\pi - 100$

(c) $150\pi - 100$ (d) $125\pi - 125$

(e) Cannot be determined

10. In the given figure PBC and PEF are straight lines. If AD = AC, b = 70°, c = 40°, the value of a is

(a) 20° (b) 25° (c) 15° (d) 30°

(e) 40°

11. In the given figure, circle AXB passes through 'O' the centre of circle AYB. AX and BX and AY and BY are tangents to the circles AYB and AXB respectively. The value of x° is

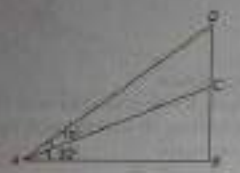
(a) 180° - y°, (b) 180° - 2y°

(c) $\sin 60° = x^2$ (d) $90° - \sin^{-1}(x)$

(e) Cannot be determined

22. In the figure, $AB = x$

Calculate the area of triangle ADC ($\angle B = 90°$)



(a) $\frac{1}{2}x^2 \sin 30°$ (b) $\frac{1}{2}x^2 \cos 30°$

(c) $\frac{1}{2}x^2 \tan 30°$ (d) $\frac{1}{2}x^2(\tan 45° - \tan 30°)$

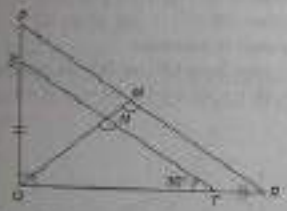
(e) None of these

23. In the given figure $SQ = PR = x$, $QT = h$, $QM \perp PR$, ST is parallel to PR.

$m\angle STQ = 30°$

$m\angle SQT = 90°$

Find QM



(a) $(x + h\sqrt{2})$ (b) $2(x + h)$

(c) $(2x + h)/2$ (d) $(x + 2h)/2$

(e) None of these

24. In the figure given, AB is a diameter of the circle and C and D are on the circumference such that $\angle CAD = 40°$. Find the measure of the $\angle ACD$

(a) 40° (b) 50°

(c) 60° (d) 80°

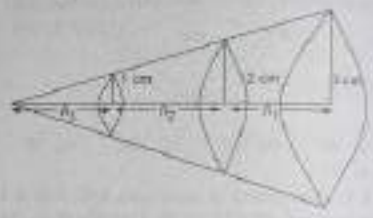
(e) None of these

25. Six solid hemispherical balls have to be arranged one upon the other vertically. Find the minimum total surface area of the cylinder in which the hemispherical balls can be arranged, if the radii of each hemispherical ball is 7 cm.

(a) 3080 (b) 2156

(c) 1232 (d) None of these

(Questions 26 – 27). In the following figure, there is a cone which is being cut and extracted in three segments having heights h_1, h_2 and h_3 and the radius of their bases 1 cm, 2 cm and 3 cm respectively, then



26. The ratio of the volumes of the smallest segment to that of the largest segment is

(a) 1 : 27 (b) 27 : 1

(c) 1 : 19 (d) None of these

27. The ratio of the curved surface area of the second largest segment to that of the full cone is

(a) 2 : 9

(b) 4 : 9

(c) cannot be determined

(d) None of these

28. On a semicircle with diameter AD, Chord BC is parallel to the diameter. Further each of the chords AB and CD has Length 2 cm while AD has length 8 cm. Find the length of BC.

(a) 7.5 cm (b) 7 cm

(c) 7.75 cm (d) Cannot be determined

29. In the given figure, B and C are points on the diameter AD of the circle such that $AB = BC = CD$. Then find the ratio of area of the shaded portion to that of the whole circle.

(a) 2 : 1 (b) 2 : 3
(c) 1 : 2 (d) None of these

2. In the given figure, ABC is a triangle in which AD and AE are equal to BC and AB respectively, the ratio of the area of ΔBED to that of ΔABC is

(a) 1 : 4 (b) 1 : 16
(c) Data inadequate (d) None of these

3. Two identical circles intersect so that their centres, and the points at which they intersect, form a square of side 1 cm. The area in square cm of the portion that is common to the two circles is

(a) $\pi/2$ (b) $\pi/2 - 1$ (c) $\pi/4$ (d) $\sqrt{2} - 1$

4. If the height of a cone is trebled and its base diameter is halved, then the ratio of the volume of the resultant cone to that of the original cone is

(a) 6 : 1 (b) 9 : 2 (c) 12 : 1 (d) 8 : 1

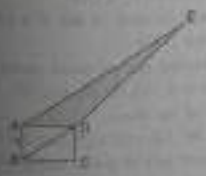
5. Two cylinders of equal volume have their heights in the ratio 1 : 3, then the ratio of their radii is

(a) $\sqrt{3} : 1$ (b) 2 : 1
(c) $\sqrt{3} : \sqrt{2}$ (d) $\sqrt{6} : \sqrt{3}$

6. Through three given non-collinear points, how many circles can pass

(a) 2 (b) 1
(c) Both 1 and 2 (d) None of these

7. In the rectangle $ABCD$, $AB = 2$ and $BD = DE$. Find the area of the shaded region



(a) $\pi/6$ (b) $2/\pi$ (c) $\sqrt{3}/2\pi$ (d) 1

8. In a right angled triangle, the square of the hypotenuse is equal to twice the product of the other two sides. The acute angles of the triangle are

(a) 30° and 60° (b) 30° and 60°
(c) 15° and 75° (d) 45° and 45°

17. Find $\angle ALC$ in $ABCD$

(a) 70 (b) 75 (c) 105 (d) 145

18. If the sides of a triangle are in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ and the perimeter is 52 cm, then the length of the smallest side is

(a) 12 cm (b) 11 cm
(c) 9 cm (d) None of these

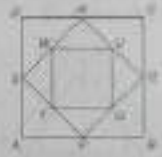
19. The number of distinct triangles with integral valued sides and perimeter as 14 is

(a) 2 (b) 3 (c) 4 (d) 5

20. A polygon has 65 diagonals. Then, what is the number of sides of the same polygon?

(a) 11 (b) 12
(c) 14 (d) None of these

21. $PQRS$ is a square drawn inside square $ABCD$ of side 2a units by joining the midpoints of the sides AB, BC, CD, DA. The square $PQRS$ is drawn inside $PQRS$, where T, U, V, W are the midpoints of SP, PQ, QR and RS. If the process is repeated an infinite number of times the sum of the areas of all the squares will be equal to:



(a) $8a^2$ (b) $4a^2$
(c) $16a^2$ (d) $6a^2/3$

22. Suppose the same thing is done with an equilateral triangle of side a, wherein the mid points of the sides are connected to each other to form a second triangle and the mid points of the sides of the second triangle are connected to form a third triangle and so on as infinite number of times—then the sum of the areas of all such equilateral triangles would be

(a) $3a^2$ (b) $4a^2$
(c) $12a^2$ (d) None of these

23. If in the figure given below $OP = PQ = 28$ cm and OQ, PQ and OP are all joined by semicircles, then the perimeter of the figure (shaded area) is equal to

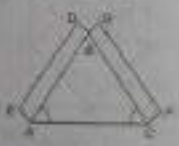
(a) 152 cm (b) 264 cm
(c) 176 cm (d) 88 cm

44. For the question above, what is the shaded area?
(a) 1232 sq. cm (b) 1264 sq. cm
(c) 1252 sq. cm (d) 1188 sq. cm

45. What is the area of the shaded portion? It is given that $\angle YOXY$, $WZ = ZX$, $ZY = 2a$ and $ZX = 2b$.

(a) $\frac{4ab}{2}$ (b) $\frac{3ab}{2}$ (c) $6ab$ (d) $3ab$

46. In the given figure there is an isosceles triangle ABC with angle A = angle C = 50°. $ABDE$ and $BCFG$ are two rectangles drawn on the sides AB and BC respectively, such that $BD = BG = AE = CF$.

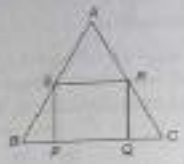


Find the value of the angle DBG.
(a) 80° (b) 130° (c) 100° (d) 140°

47. In the figure, ABE, DCE, BCF and ADF are straight lines. $E = 30°$, $F = 50°$. Find $\angle A$.

(a) 47° (b) 37° (c) 40° (d) 42°

48. ABC is an equilateral triangle. $PQRS$ is a square inscribed in it. Therefore
(a) $AR^2 = BC^2$ (b) $2AR^2 = BC^2$
(c) $3AR^2 = 4BC^2$ (d) $4AR^2 = 3BC^2$



49. Consider the five points comprising the vertices of a square and the intersection point of its diagonals. How many triangles can be formed using these points?
(a) 4 (b) 6 (c) 8 (d) 10

50. In a triangle ABC, the internal bisector of the angle A meets BC at D. If $AB = 4$, $AC = 3$ and $\angle A = 60°$. Then the length of AD is:
(a) $2\sqrt{3}$ (b) $(12\sqrt{3})/7$
(c) $(15\sqrt{3})/8$ (d) $(6\sqrt{3})/7$

Directions for Questions 51–52: Answer the questions based on the following information.

A rectangle $PWSU$ is divided into two smaller rectangles $PQTU$ and $QWST$ by the line QT. $PQ = 40$ cm, $QR = 20$ cm and $RS = 40$ cm. Points A, B, F are within rectangle $PQTU$, and points C, D, E are within the rectangle $QWST$. The closest pair of points among the pairs (A, C), (A, D), (A, E), (F, C), (F, D), (F, E), (B, C), (B, D), (B, E) are 40√2 cm apart.

51. Which of the following statements is necessarily true?
(a) The closest pair of points among the six given points cannot be (F, C).
(b) Distance between A and B is greater than the between F and C.
(c) The closest pair of points among the six given points is (C, D), (D, E) or (C, E).
(d) None of the above.

52. $AB > AF > BF$; $CD > DE > CE$ and $BF = 24\sqrt{5}$ cm. Which is the closest pair of points among all the six given points?
(a) B, F (b) C, D
(c) A, B (d) None of these

8. If ABCD is a square and CDE is an equilateral triangle, what is the measure of $\angle DEB$?

(a) 25° (b) 30° (c) 20° (d) 45°

9. If $\angle BC$, $BD \perp AC$ and CE bisects $\angle C$, $\angle A = 30°$. Then, what is $\angle CED$?

(a) 30° (b) 40° (c) 45° (d) 65°

10. Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then, the ratio of the shorter side to the longer side is:

(a) 1/2 (b) 2/3 (c) 1/4 (d) 3/4

ANSWER KEY

Geometry LOD 1

1. c
2. d
3. c
4. a
5. c
6. c
7. a
8. c
9. a
10. c
11. c
12. a
13. b
14. a
15. b
16. b
17. a
18. a
19. b
20. a
21. b
22. c
23. c
24. c
25. b

1. b
2. a
3. a
4. a
5. c
6. a
7. a
8. a
9. a
10. b
11. b
12. a
13. a
14. a
15. b
16. a
17. a
18. a
19. c
20. d
21. a
22. b
23. a
24. b
25. d

Measurement (LHD)

1. b
2. d
3. b
4. a
5. a
6. b
7. d
8. c
9. a
10. a
11. b
12. b
13. a
14. b
15. d
16. c
17. c
18. c
19. d
20. d
21. a
22. b
23. b
24. b
25. c
26. c
27. a
28. a
29. c
30. a
31. c
32. a
33. b
34. c
35. b
36. b
37. a
38. c
39. c
40. b
41. b
42. c
43. c
44. c
45. c
46. c
47. a
48. b
49. a
50. b

1. c
2. a
3. b
4. b
5. c
6. c
7. b
8. a
9. a
10. d
11. a
12. d
13. a
14. a
15. d
16. b
17. a
18. b
19. a
20. b
21. a
22. a
23. d
24. a
25. c
26. a
27. c
28. a
29. a
30. a
31. b
32. d
33. a
34. a
35. d
36. c
37. c
38. d
39. a
40. a
41. b
42. b
43. a
44. a
45. c

Geometry and Mensuration Extra Practice Exercise

1. b
2. c
3. b
4. c
5. b
6. b
7. c
8. a
9. b
10. b
11. c
12. b
13. a
14. b
15. c
16. a
17. a
18. a
19. b
20. c
21. d
22. d
23. a
24. e
25. b
26. c
27. e
28. b
29. e
30. a
31. b
32. c
33. a
34. d
35. d
36. d
37. b
38. a
39. c
40. d
41. a
42. d
43. c
44. c
45. c
46. a
47. b
48. d
49. c
50. b
51. d
52. d
53. a
54. b
55. d

BLOCK REVIEW TESTS

REVIEW TEST ONE

A piece of paper is in the shape of a right angled triangle. This paper is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. The cut was done in such a manner that there was a 75% reduction in the length of the hypotenuse of the triangle. If the area of the original triangle was 54 square inches before the cut, what is the area (in square inches) of the smaller triangle?

(a) 20.025 (b) 22.815
(c) 23.495 (d) 24.365

2. Consider two different cloth cutting processes. In the first one, 4 square cloth pieces of the same size are cut from a square cloth piece of the size a; then a circle of maximum possible area is cut from each of the smaller squares. In the second process, only one circle of maximum possible area is cut from a square of the same size and the process ends there. The cloth pieces remaining after cutting the circles are scrapped in both the processes. The ratio of the total area of scrap cloth produced in the former to that in the latter is

(a) 3 : 1 (b) $\sqrt{2}$: 1
(c) 2 : 1 (d) $\frac{4a - \pi}{(4/4 - \pi)}$

3. The length of the circumference of a circle equals the perimeter of a triangle of equal sides. This is also equal to the perimeter of a square and also the perimeter of a regular pentagon. The areas covered by the circle, triangle, pentagon and square are x, y, p and s respectively. Then

(a) $x > s > y > p$ (b) $x > y > s > p$
(c) $x > p > s > y$ (d) $x > p > s > y$

4. Three horses are grazing within a semi-circular field. In the diagram given below, AB is the diameter of the semi-circular field with centre at O. Horses are tied up at P, R and S such that PO and RO are the radii of the two semi-circles drawn with centres at P and R respectively. The arcs of the circle touching the two semi-circles for the centre of the circle touching the two semi-circles with diameters AO and OB. The horses tied at P and R can graze within the respective semi-circles and the horse tied at S can graze within the circle centred at S.

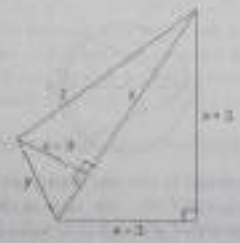
The percentage of the area of the semi-circle with diameter AB that cannot be grazed by the horses is nearest to

(a) 16 (b) 24 (c) 36 (d) 52

5. A certain city has a circular wall around it with four gates pointing north, south, east and west. A house stands outside the city, three km north of the north gate, and it can just be seen from a point one km east of the south gate. What is the diameter of the wall that surrounds the city?

(a) 6 km (b) 9 km
(c) 12 km (d) None of these

6. Based on the figure below, what is the value of x, if $y = 10$.



(a) 20 (b) 11
(c) 32 (d) None of these

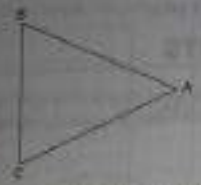
7. What is the number of distinct triangles with integral valued sides and perimeter as 12?

(a) 8 (b) 5 (c) 4 (d) 3

Directions for questions 8-9: Answer the questions based on the following information.

A cow is tethered at point A by a rope. Neither the rope nor the cow is allowed to enter the triangle ABC.

$\angle BAC = 30°$,
$AB = AC = 10$ m.



8. What is the area that can be grazed by the cow if the length of the rope is 8 m?

(a) $136\pi\frac{1}{3}$ sq. m (b) 121π sq. m

(c) 132π sq. m (d) $\frac{176\pi}{3}$ sq. m

9. What is the area that can be grazed by the cow if the length of the rope is 12 m?

(a) $336\pi\frac{1}{3}$ sq. m (b) 121π sq. m

(c) 132π sq. m (d) $\frac{176\pi}{3}$ sq. m

10. AB is the diameter of the given circle, while points C and D lie on the circumference as shown. If AB is 25 cm, AC is 20 cm and BD is 7 cm, find the area of the quadrilateral $ACBD$.

(a) 300 sq. cm (b) 340 sq. cm

(c) 330 sq. cm (d) None of these

11. The value of each of a set of coins varies as the square of its diameter, if its thickness remains constant, varies as the thickness, if the diameter remains constant. If the diameter of two coins are in the ratio 3 : 4, what should be the ratio of their thicknesses be if the value of the second is four times that of the first?

(a) 9 : 16 (b) 4 : 9 (c) 16 : 9 (d) 9 : 4

12. If a, b and c are the sides of a triangle, and $a^2 + b^2 = c^2$ or $b^2 + c^2 = a^2$, then the triangle is:

(a) equilateral (b) isosceles

(c) right-angled (d) obtuse-angled

Directions 13-14: Use the following information: XYZ is an equilateral triangle in which Y is 2 km from X. Sabiha starts walking from Y in a direction parallel to XZ and stops when she reaches a point M directly east of X. He, then, changes direction and walks till he reaches a point N directly south of Z.

13. Then M is

(a) 3 km east and 1 km south of X

(b) 3 km east and $\sqrt{3}$ km north of X

(c) $\sqrt{3}$ km east and 1 km south of X

(d) $\sqrt{3}$ km west and 3 km north of X

14. The total distance walked by Sabiha is:

(a) 7 km (b) 4 km (c) $2\sqrt{3}$ km (d) 6 km

15. The sides of a triangle are 5, 12 and 13 units. A rectangle is constructed, which has an area twice the area of the triangle, and has a width of 10 units. Then, the perimeter of the rectangle is

(a) 30 units (b) 36 units

(c) 32 units (d) None of these

16. The diameter of a hollow cone is equal to the diameter of a spherical ball. If the ball is placed at the base of the cone, what portion of the ball will be inside the cone?

(a) 50% (b) less than 50%

(c) more than 50% (d) 100%

17. The length of a ladder is exactly equal to the height of the wall it is leaning against. If lower end of the ladder is kept on a platform of height 3 feet and the platform is kept 9 feet away from the wall, the upper end of the ladder coincides with the top of the wall. Then the height of the wall is:

(a) 12 feet (b) 15 feet

(c) 18 feet (d) 16 feet

18. A cube of side 12 cm is painted red on all its faces and then cut into smaller cubes, each of side 3 cm. What is the total number of smaller cubes having none of their faces painted?

(a) 16 (b) 8

(c) 12 (d) None of these

19. From a circular sheet of paper with a radius 20 cm, four circles of radius 5 cm each are cut out. What is the ratio of the cut to the uncut portion?

(a) 4 : 5 (b) 4 : 1 (c) 3 : 4 (d) 4 : 3

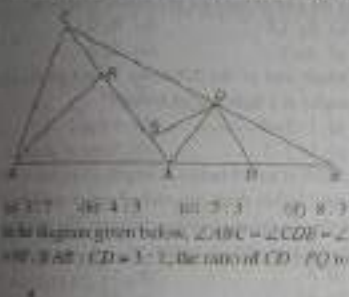
20. Four friends Mani, Sunny, Honey & Funny met from four towns Mindam, Sindam, Mindam and Findam respectively. The four towns are at the four corners of a rectangular rectangle. They meet at a point which falls inside this imaginary rectangle. At that point three of them (Mani, Sunny and Honey) had travelled distances of 30 m, 50 m and 80 m respectively. The minimum distance that Funny could have travelled is nearest to

(a) 40 m (b) 39 m
(c) 24 m (d) Cannot be determined

7. Each side of a given polygon is parallel to either the x or the y axis. A corner of such a polygon is said to be convex if the internal angle is 90° or concave if the internal angle is 270°. If the number of convex corners in such a polygon is 20, the number of concave corners must be

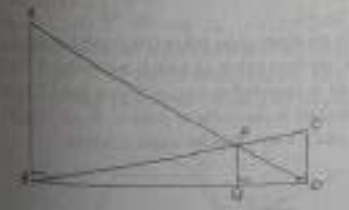
(a) 20 (b) 0 (c) 25 (d) 26

8. In the figure (not drawn to scale) given below, L is a point on AB such that $AL : LB = 4 : 3$. LD is parallel to AC and CD is parallel to CL. In ΔABC, $\angle ABC = 90°$, and in ΔLDC, $\angle CLD = 90°$. What is ratio $AL : LD$?



(a) 3 : 7 (b) 4 : 3 (c) 5 : 3 (d) 8 : 3

9. In the diagram given below, $\angle ABC = \angle CDE = \angle PQD = 90°$. If $AB : CD = 3 : 2$, the ratio of $CD : PQ$ is:



(a) 1 : 0.866 (b) 1 : 0.75
(c) 1 : 0.54 (d) None of the above

10. In the figure given below, $ABCD$ is a rectangle. The area of the isosceles right triangle $ADE = 9\ cm^2$, $BC = 10\ cm$. The area of $ABCD$ (in cm^2) is:

(a) 27 (b) 36 (c) 54 (d) 72

11. The length of the common chord of two circles of radii 14 cm and 30 cm whose centres are 25 cm apart, is (in cm):

(a) 24 (b) 10 (c) 12 (d) 20

REVIEW TEST TWO

1. A square tin sheet of side 24 inches is converted into a box with open top in the following ways: The sheet is placed horizontally. Then, equal sized squares, each of side x inches, are cut from the four corners of the sheet. Finally, the four resulting sides are bent vertically upwards to the shape of a box. If x is an integer, then what value of x maximizes the volume of the box?

(a) 6 (b) 8 (c) 2 (d) 4

2. Let S_1 be a square of side a. Another square S_2 is formed by joining the mid-points of the sides of S_1. The same process is applied to S_2 to form yet another square S_3, and so on. If $A_1, A_2, A_3, \ldots$ be the areas and $P_1, P_2, P_3, \ldots$ be the perimeters of $S_1, S_2, S_3, \ldots$ respectively, then the ratio $\dfrac{A_1 + A_2 + A_3 + \ldots}{P_1 + P_2 + P_3 + \ldots}$ equals:

(a) $a(2(\sqrt{2}+1))$ (b) $a(2(\sqrt{2}-1))$
(c) $a(2(2+\sqrt{2}))$ (d) $a(2(1+\sqrt{2}))$

3. There are two concentric circles such that the area of the outer circle is four times the area of the inner circle. Let A, B and C be three distinct points on the perimeter of the outer circle such that AB and AC are tangents to the inner circle. If the area of the triangle ABC is $\dfrac{9\sqrt{3}}{\pi}$ then what would be the area of the outer circle?

(a) 9π (b) 36π
(c) 12 (d) None of these

4. Four horses are tethered at four corners of a square plot of side 42 metres so that the adjacent horses can just reach one another. There is a small circular pond of area 20 m^2 at the center. Find the ungrazed area.

(a) 378 m^2 (b) 369 m^2 (c) 358 m^2 (d) 348 m^2

5. A square, whose side is 2 metres, has its corners cut away so as to form an octagon without sides equal. Then length of each side of the octagon, in metres is:

(a) $\dfrac{\sqrt{2}}{\sqrt{2}+1}$ (b) $\dfrac{2}{\sqrt{2}+1}$
(c) $\dfrac{2}{\sqrt{2}-1}$ (d) $\dfrac{\sqrt{2}}{\sqrt{2}-1}$

6. A rectangular pool 30 metres wide and 50 metres long is surrounded by a walkway of uniform width. If the area of the walkway is 504 square metres, how wide, in metres, is the walkway?
   (a) 1 (b) 2
   (c) 3 (d) None of these

7. A farmer has decided to build a wire fence along one straight side of his property. For this purpose he has planned to place several fence-posts at 12 m intervals, with posts fixed at both ends of the side. After he bought the posts and wire, he found that the number of posts he had bought was 5 less than required. However, he discovered that the number of posts he had bought would be just sufficient if he spaced them 16 m apart. What is the length of the side of his property and how many posts did he buy?
   (a) 200 m, 15 (b) 200 m, 16
   (c) 240 m, 15 (d) 240 m, 16

8. The figure below shows two concentric circles with centre O. PQRS is a square inscribed in the outer circle. It also circumscribes the inner circle, touching it at points B, C, D and A. What is the ratio of the perimeter of the polygon ABCD to that of the outer circle?

   (a) $4/\pi$ (b) $2\sqrt{2}/\pi$ (c) $2/\pi$ (d) $1/\pi$

9. Four identical coins are placed in a square. For each coin, the ratio of area to circumference is same as the ratio of circumference to area. Then, find the area of the square that is covered by the coins.

   (a) 16π (b) 32π
   (c) 8π (d) Insufficient data

10. The sum of the areas of two circles, which touch each other externally, is 153π. If the sum of their radii is 13, the ratio of the smaller to the larger radius is:
   (a) 1 : 4 (b) 1 : 2
   (c) 1 : 3 (d) None of these

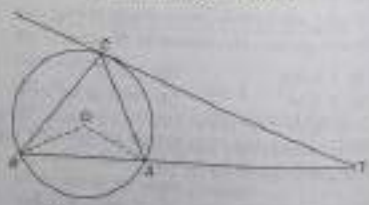
11. In ΔABC, points P, Q and R are the mid-points of sides AB, BC and CA respectively. If area of ΔABC is 40 sq. units, find the area of ΔPQR.
   (a) 10 sq. units (b) $10\sqrt{3}$ sq. units
   (c) 5 sq. units (d) None of these

12. A wooden box open at the top of thickness 0.5 cm, length 21 cm, width 11 cm and height 6 cm is painted on the inside. The expenses of painting are Rs. 140. What is the rate of painting per square centimetre?
   (a) Rs. 0.8 (b) Rs. 0.5
   (c) Rs. 1 (d) Rs. 2

13. Which one of the following cannot be the ratio of angles in a right-angled triangle?
   (a) 1 : 2 : 3 (b) 1 : 1 : 2
   (c) 1 : 3 : 5 (d) None of these

14. A slab of size 10 inches in length, 15 inches in breadth and 5 inches thick was melted and re-solidified in the form of a rod of 10 inches diameter. The length of such a rod, in inches, is nearest to:
   (a) 5 (b) 5.7
   (c) 4.5 (d) 5.5

15. In the figure given below (not drawn to scale), A, B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C. If $\angle ATC = 20°$ and $\angle ACT = 40°$, then the angle $\angle BOA$ is:



   (a) 140°
   (b) 150°
   (c) 160°
   (d) Not possible to determine

16. A vertical tower OP stands at the corner O of a square OBCD. Let x and y denote the height of the tower OP and the side of the square OB respectively. Suppose

$\angle AOB = 90°$, then the relationship between x and y can be expressed as:

(a) $3x^2 = y^2$ (b) $3x^2 = y^3$

(c) $x^2 = 3y^2$ (d) $3x^2 = 2y^2$

2. In a triangle XYZ, $XY = 6$, $YZ = 8$ and $XZ = 10$. A perpendicular dropped from Y, meets the side XZ at M. A circle of radius YM (with center Y) is drawn. If the circle cuts XY and YZ at P and Q respectively, then $QZ : YP$ is equal to:

(a) 1 : 1 (b) 2 : 3 (c) 1 : 4 (d) 8 : 3

3. In the figure below, AB is the chord of a circle with center O. AB is extended to C such that $BC = OB$. The straight line CO is produced to meet the circle at D. If $\angle ACD = 20°$ and $\angle AOD = x$ degrees then the value of x is

(a) 20 (b) 40

(c) 60 (d) None of these

4. The area of the triangle whose vertices are (a, a), $(a + 1, a + 1)$, $(a + 2, a)$ is:

(a) a^2 (b) 1 (c) $2a$ (d) $2^{1/2}$

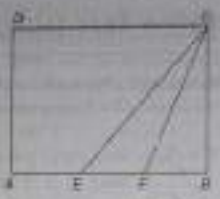
5. In the given figure, AB is the diameter of the circle and points C and D are on the circumference such that $\angle CAD = 40°$ and $\angle CBA = 60°$. What is the measure of $\angle ACD$?

(a) 20° (b) 30° (c) 40° (d) 50°

6. A ladder leans against a vertical wall. The top of the ladder is 12 m above the ground. When the bottom of the ladder is moved 4 m further away from the wall, the top of the ladder rests against the foot of the wall. What is the length of the ladder?

(a) 10 m (b) 13 m (c) 20 m (d) 17 m

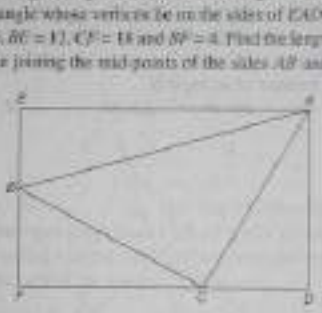
7. In the adjacent diagram, $ABCD$ is a rectangle with $AE = 2EB$ and $CF = 2FB$. What is the ratio of the area of the rectangle to that of the triangle CEF?



(a) 11 : 3 (b) 22 : 3

(c) 11 : 6 (d) None of these

8. In the given figure, $EADF$ is a rectangle and ABC is a triangle whose vertices lie on the sides of $EADF$. $AE = 30$, $BC = 12$, $CF = 18$ and $BF = 4$. Find the length of the line joining the mid-points of the sides AB and BC.



(a) 5 (b) 10

(c) 15 (d) None of these

9. In the figure, points A, B, C and D lie on the circle. $AD = 44$ and $BC = 11$. What is the ratio of the area of $\triangle CDE$ to that of the triangle $\triangle ADE$?

(a) 1 : 4 (b) 1 : 16

(c) 1 : 16 (d) Data insufficient

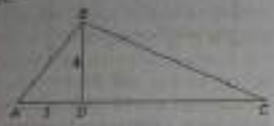
10. In a rectangle, the difference between the sum of the adjacent sides and the diagonal is two-thirds the length of the shorter side. What is the ratio of the shorter to the longer side?

(a) $\sqrt{3} : 2$ (b) $1 : \sqrt{3}$

(c) 2 : 5 (d) 3 : 4

REVIEW TEST THREE

1. In figure, if circle $ABC = 90$, and AD is perpendicular to BC & $BD = 4$ cm and $AD = 3$ cm, what will be the length of BC?



(a) 13 (b) 15 (c) 10 (d) 9

2. In the figure shown, the measure of an angle formed by the intersection of two angles in a triangle ABC is 130 find the measure of an angle B

(a) 40 (b) 45 (c) 50 (d) 80

3. The perimeter of right triangle is 36 and the sum of the square of its sides is 450. The area of the right triangle

(a) 42 (b) 54 (c) 62 (d) 100

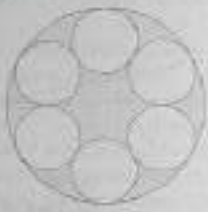
4. The circles are tangent to one another and each circle is tangent to the sides of the right triangle ABC with right angle ABC. If the larger circle has radius 12 and the smaller circle has radius 3, what is the area of the triangle?

(a) 420 (b) 820 (c) 540 (d) 440

5. All the circles are tangent to one another / or the sides of the rectangle. All circles have radius 1. What is the area of the shaded region to the nearest whole area, i.e. the region outside all the circles but inside the rectangle?

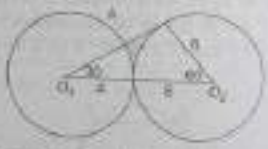
(a) 27 (b) 28 (c) 29 (d) 30

6. Given six congruent circles drawn internally tangent to a circle of a radius 21, each smaller circle is also tangent to each of its adjacent circles. Find the shaded area between the circle and the six smaller circles.



(a) 126π (b) 105π (c) 189π (d) 147π

7. In the figure shown, O_1 and O_2 are centers of the circle O_1A is the circle centers at O_1.



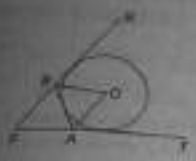
Find the area of the shaded region

(a) $22 - 7\pi$ (b) $24(2) - 7\pi$
(c) $48 - 9\pi$ (d) $24 - \pi 22/3$

8. In the figure, which of the following is correct?

(a) $x = 40$ (b) $x = 70$
(c) $x = 10$ (d) $x = 60$

[illegible] PQR is formed by three tangents to circle O and angle $APB = 80$, then the angle BQA equals



(a) 30 (b) 35 (c) 40 (d) 50

[illegible] PQRS is a cyclic quadrilateral. The angle bisectors of angles P, Q, R and S intersect at A, B, C and D as shown in the figure below. Then these four points form a quadrilateral ABCD is a:

(a) Rectangle (b) square
(c) Rhombus (d) Cyclic quadrilateral

[illegible] In the given figure, O is the center of the circle, angle $AOC = m°$, angle $BAC = n°$, then which of the following is correct?

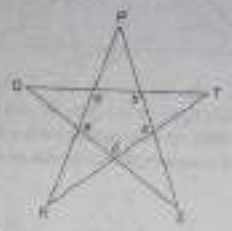
(a) $m + n = 90$ (b) $m + n = 180$
(c) $2m + n = 180$ (d) $m + 2n = 180$

[illegible] Find the area of shaded portion given that the circles with centers O and O' are 8 cm and 16 cm in diameter respectively and ACB is a semi circle

(a) 54π cm^2 (b) 27π cm^2
(c) 36π cm^2 (d) 18π cm^2

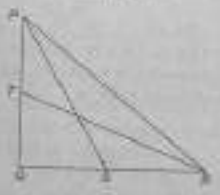
13. There are two spheres and one cube. The cube is inside the bigger sphere and the smaller sphere is inside the cube. Find the ratio of surface area of the bigger sphere to the smaller sphere?
(a) 3 : 1 (b) 2 : 1 (c) 4 : 1 (d) 2 : 1

14. In the adjoining figure, a star is shown. What is the sum of the angles P, Q, R, S and T?



(a) 240 (b) 180
(c) 120 (d) Can't be determined

15. In the figure given below PS & RT are the medians each measuring 4 cm. If triangle PQR is right angled at Q, what is the area of the triangle PQR?



(a) 5.2 (b) 6.4 (d) 6.2 (d) 7.2

16. The area of the largest triangle that can be inscribed in a semi circle whose radius is
(a) $2R^2$ (b) $3R^2$ (c) R^2 (d) $3R^2/2$

17. R is O is the center of the circle having radius OP = r. PQRSTU is a regular hexagon and PNQMRCSOTEUKP is a regular six pointed star

Find the perimeter of hexagon $PQRSTU$.

(a) 12 (b) 9 (c) 6 (d) 3

18. In the given figure ABC is a triangle in which $AD = 3CD$ and E lies on BD. $DE = 2BE$. What is the ratio of area of triangle ABE and area of triangle ABC?

(a) 1/12 (b) 1/8 (c) 1/6 (d) 1/10

19. In the given figure, P and Q are the mid points of AC and AB. Also, $PQ = QR$ and $BQ = AR$, what is the ratio of area of triangle PQR: area of triangle ABC?

(a) 1/2 (b) 2/3 (c) 3/5 (d) 1/3

20. In the given figure, it is given that angle $C = 90$, $AD = DB$, DE is perpendicular to $AB = 20$, and $AC = 12$. The area of quadrilateral $ADEC$ is

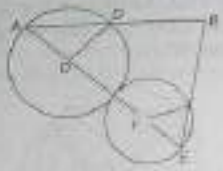
(a) $150\sqrt{2}$ (b) 75
(c) 48 (d) $50\sqrt{2}$

REVIEW TEST FOUR

1. Rectangle $PQRS$ contains 4 congruent rectangles. If the smaller dimension of one of the small rectangles is 4 units, what is the area of rectangle $PQRS$ in square units?

(a) 144 (b) 172 (c) 256 (d) 192

2. If the radii of the circles with centers O and P in diagram below are 4 and 2 units respectively. Find the area of triangle ABC.
Given: angle DOA = angle $EPC = 90$



(a) 36 (b) 42 (c) 18 (d) 48

3. A cube of side 16 cm is painted red on all the faces and then cut into smaller cubes, each of side 4 cm. What is the total number of smaller cubes having none of their faces painted?

(a) 16 (b) 8 (c) 32 (d) 24

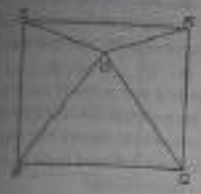
4. Identical regular pentagons are placed together side by side to form a ring in the manner shown. The diagram shows the first two pentagons. How many are needed to make a full ring?

(a) 9 (b) 10 (c) 11 (d) 12

5. Find angle EBC = angle EAW from the given figure given ADF is an equilateral triangle and angle $DCE = 2$

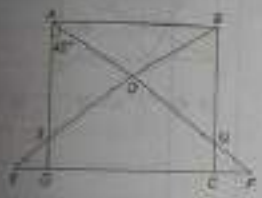
(a) 160° (b) 140 (c) 100 (d) 120

6. PQRS is a square and PORQ is an equilateral triangle. What is the value of angle SOR?



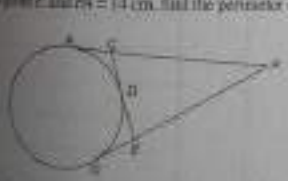
(a) 90 (b) 120 (c) 125 (d) 100

7. In the following figure ABCD is a square, angle BAO = 40°. Find angle BNO.



(a) 80 (b) 60 (c) 30 (d) 40

8. From an external point P, tangents PA and PB are drawn to a circle with center O. If CD is the tangent to the circle at point E and PA = 14 cm, find the perimeter of ΔCPD.



(a) 28 (b) 28 (c) 24 (d) 25

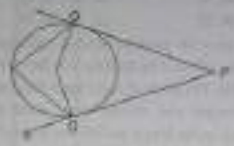
9. A circle is inscribed in an equilateral triangle; the radius of the circle is 2 cm. Find the area of triangle.

(a) $12\sqrt{3}$ (b) $25\sqrt{3}$ (c) $12\sqrt{2}$ (d) $18\sqrt{3}$

10. If interior angle of a regular polygon is 144°, then find no. of sides in this polygon.

(a) 40 (b) 30 (c) 30 (d) 25

11. In the given figure, PQ and PR are two tangents to the circle, whose center is O. If angle QPR = 40°, find angle QOR.



(a) 90 (b) 70 (c) 80 (d) 50

12. In the figure AB∥DE, angle BAD = 30° and angle DEC = 30°, and DE = EC. Then x equals

(a) 90 (b) 65 (c) 75 (d) 30

13. There is an equilateral triangle of side 32 cm. The mid-points of the sides are joined to form another triangle, whose mid-points are again joined to form still another triangle. This process is continued for 'n' number of times. The sum of the perimeters of all the triangles is 180 cm. Find the value of n.

(a) 4 (b) 5 (c) 8 (d) 3

14. There is a circle of diameter AB and radius 26 cm. If chord CA is 10 cm long, find the ratio of area of triangle ABC to the remaining area of circle.

(a) 0.09 (b) 0.30 (c) 0.28 (d) 0.5

15. Ram Singh has a rectangular plot of land of dimensions 30 m × 40 m. He wants to construct a unique swimming pool which is in the shape of an equilateral triangle.

Find the area of the largest swimming pool which he can have?

(a) $300\sqrt{3}$ sq cm (b) $225\sqrt{3}$ sq cm

(c) 300 sq cm (d) $225\sqrt{2}$ sq cm

16. The perimeter of a triangle is 105 cm. The ratio of its altitudes is 3 : 5 : 6. Find the sides of the triangle.

(a) 32, 46, 36 (b) 62, 28, 41

(c) 30, 40, 25 (d) 50, 30, 25

17. Rizwan gave his younger sister a rectangular sheet of paper. He halved it by folding it at the mid point of its longer side. The piece of paper again became a rectangle whose longer and shorter sides were in the same proportion as the longer and shorter sides of the original rectangle. If the shorter side of the original rectangle was 4 cm, find the diagonal of the smaller rectangle?

(a) $3(2)^{1/2}$ (b) $5(2)^{1/2}$

(c) $4(2)^{1/2}$ (d) $2(2)^{1/2}$

18. A rectangular hall, 30 m in length and 75 m in width has to be paved with square tiles of equal size. What is the minimum number of tiles required?

(a) 4 (b) 5 (c) 6 (d) 8

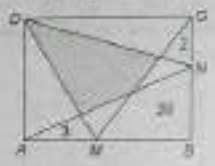
19. In triangle ABC we have angle $A = 100$ degree and $B + C = 40$. The side AB is produced to a point D so that B lies between A and D and $AD = BC$. Then angle BCD is?

(a) 20° (b) 10° (c) 30° (d) 40°

20. The sides of a cyclic quadrilateral are 9, 10, 12 and 14. If one of its diagonals is 14, then find the other diagonal?

(a) 16 (b) 17 (c) 18 (d) 19

21. Segments starting with points M and N and ending with vertices of the rectangle $ABCD$ divide the given figure into eight parts (see the figure). The areas of three parts of the rectangle are indicated in the picture. What is the area of the shaded region?



(a) 25 (b) 40 (c) 29 (d) 30

ANSWER KEY

Review Test One

1. b
2. a
3. c
4. d
5. a
6. b
7. d
8. d
9. c
10. d
11. b
12. a
13. b
14. d
15. c
16. b
17. b
18. b
19. a
20. a
21. c
22. a
23. b
24. d
25. a

Review Test Two

1. d
2. c
3. a
4. c
5. a
6. c
7. d
8. c
9. a
10. a
11. a
12. c
13. c
14. b
15. a
16. b
17. d
18. c
19. b
20. a
21. c
22. b
23. b
24. b
25. b

Review Test Three

1. b
2. d
3. b
4. d
5. d
6. d
7. d
8. d
9. a
10. d
11. a
12. b
13. a
14. b
15. b
16. c
17. c
18. a
19. a
20. d

Review Test Paper

1.	d
2.	a
3.	b
4.	b
5.	c
6.	a
7.	d
8.	b
9.	a
10.	c
11.	b
12.	b
13.	a
14.	c
15.	a
16.	d
17.	a
18.	c
19.	b
20.	c
21.	a

BLOCK 5

CHAPTERS

- Functions
- Inequalities
- Quadratic Equations
- Logarithms

...BACK TO SCHOOL

[illegible]

WHAT APPROACH CHANGE DO YOU NEED?

[illegible]

- [illegible] The X – FYIRe
- [illegible] Lines of points
- [illegible] What is an equation?
- [illegible] = Equations and Functions
- [illegible]

Let us Now Start to Look at These Issues One by One

ISSUE 1: The X – FYIRe: [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

The following figure illustrates this idea:

Fig 1 [illegible]

[illegible]

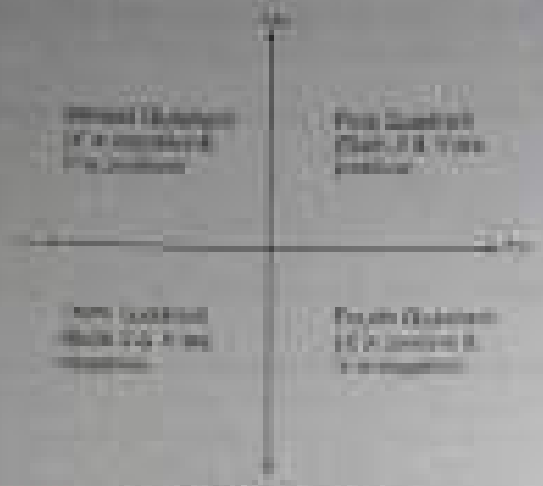


Fig. 3 The $x-y$ plane

Finding a Point on the $x-y$ Plane

[illegible]

ISSUE 2: Lines of Points. [illegible]

ISSUE 3: What Is an Equation? Consider the following:

(a) [illegible]

(b) [illegible]

(c) [illegible]

[illegible]

All Problems Have Solutions

[illegible]

Note: [illegible]

Contd

[illegible]

[illegible] 4: Equations and Functions. [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Other Principles Related to Functions and Equations

[illegible]

We first created a table where we started putting values of x and getting the corresponding value for y. Thus, we got a series of points we drawn in the table above which represented the function $y = 2x - 3$. This representation of the function is called in the analytical representation of the function.

Further, the line representing $y = 2x + 3$ was plotted on the $X - Y$ plane. This is called as the graphical representation of the function.

iii. The intercepts where the graph of the function $y = f(x)$ intersects the x axis represents the solution of the equation $f(x) = 0$.

Thus, if you matched $y = 2x + 3$ table for y equal to $y = -1.5$. At the same time the root or solution of the equation $2x + 3 = 0$ is $x = -1.5$.

This is not accidental but only happens in rare and crazy case. Thus, in general you can see that the point where the graph of a function $y = f(x)$ cuts the x axis is the solution of the equation $f(x) = 0$. [Note: This will be true for a fairly complicated $f(x)$ irrespective of the highest power of 'x' in the expression.]

[As a consequence of this logic that linear functions will cut the x axis only once since linear equations have only one solution.]

c) Quadratic Equations and Quadratic Functions: Let us now practice the equation $x^2 - 5x + 6 = 0$.

If you solve this equation you will get the solution of the equation as two roots of x, viz. at $x = 2$ as well as at $x = 3$.

[Remember, the representation of an equation is the value of the variable of x is a value for which the LHS = RHS in the equation. You can easily see that if both $x = 2$ and $x = 3$ for the expression $x^2 - 5x + 6$ yields a value of 0].

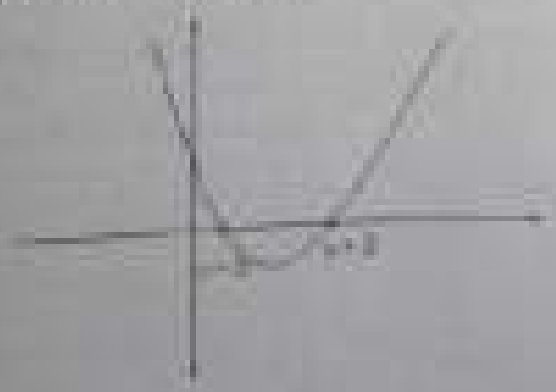
$$\text{At } x = 2,\ f^2 - 5x + 6 = 2^2 - 5 \times 2 + 6 = 0$$
$$\text{At } x = 3,\ f^2 - 5x + 6 = 3^2 - 5 \times 3 + 6 = 0$$

At this point you can partake that the graph of the expression $y = x^2 - 5x + 6$ will cut the x axis at both these points viz. $x = 2$ and $x = 3$.

If we now let us plot $y = x^2 - 5x + 6$, your tabular representation will be

x	5	4	3	2	1	0	1	2	3	4	5
y	20	12	10	20	12	6	2	0	0	2	6

The table of points will yield the graph below:



Naturally, the graph will cut the x axis at $x = 2$ and $x = 3$.

Special Note on Quadratic Equations

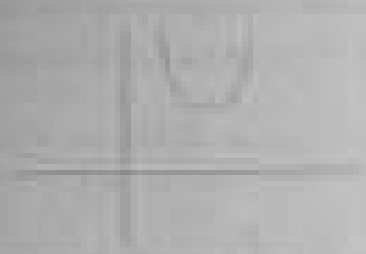
[illegible]

[illegible]

Case 1:

[illegible]

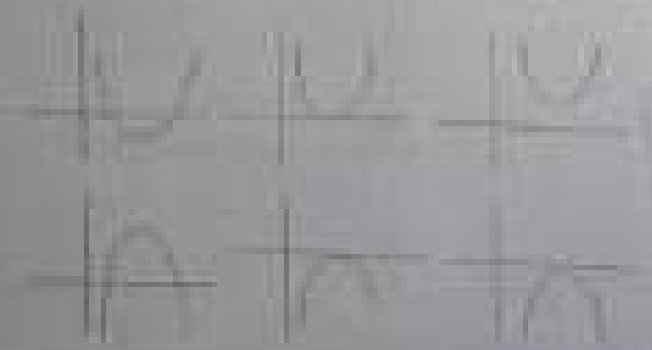
Case 2:



[illegible]

Note: [illegible]

[illegible]

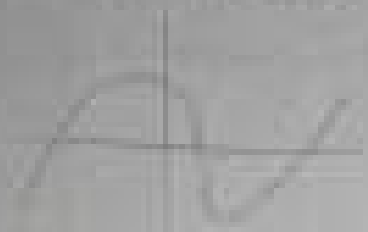


[illegible]

[illegible]

(AB.1) When the Coefficient of x^2 is Positive

[illegible]



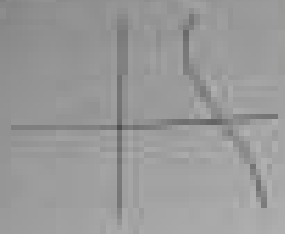
[illegible]

[illegible]

(B) If Was the Coefficient of x^2 is Negative

[illegible]

(b) Cubic function with three real and equal roots.



(c) Cubic function with one real and 2 imaginary roots.

Take an unknown quadratic function with highest power as 5 and try to sketch its graph with the help of the Graph Theorem and its Tips. Following are sketched figures for expressions with:

(a) Degree 5 (i.e. x^5) (4 roots)

(b) Degree 5 (5 roots)

A Key Note

Graphical representations and ability to understand how exponentials will look in the $X - Y$ plot is a very key factor in your merits. Graph sensitivity and self dependence is the crucial best of scripts. However, I would encourage candidates to try to plot the graphs of as many $y = f(x)$ functions as possible imagine. To give you a start, I would recommend a minimum of 100 graphs to be plotted by you every day. Doing so would not be able to save a lot of creative brainstorming graphs.

Lesson 8: How Inequalities Relate to Graphs of Functions?

We have just seen the relationship between a function and its inverse. When a function is increasing for $y \in [0, 1]$, then its inverse is defined on values in the $[0, 1]$ range.

Let's consider the inequality $x^2 - 3x + 4 \geq 0$ and we rewrite it as $f(x) \geq 0$.

Now, if we use f^{-1} $[0, 1]$ we are looking for the x-values of f where the y-axis is greater than 0.

Going back to the graph for f^{-1} $[0, 1]$ we observe that the function is greater than 0 when $x < 3$ or $x > 2$.

Hence, the solution of the inequality f^{-1} $[0, 1]$ is $x \in (-\infty, 2) \cup (3, \infty)$. Interestingly, we understand here that when we are solving an inequality, it is not the solution which is in the domain values of y does the inequality hold true.

[illegible] TEST

Instructions for Questions [illegible]. Answer the questions on the basis of the [illegible] given below.

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

1. [illegible] integer [illegible] such that [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

2. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

3. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

4. What is the sum of [illegible] in the series [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

5. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

6. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

7. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

8. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

9. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

10. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

11. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

12. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

13. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

14. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

Answers (Block-3 Fundamental Tests)

1. (c)	2. (a)	3. (d)	4. (c)	5. (a)
6. (c)	7. (b)	8. (b)	9. (d)	10. (b)
11. (b)	12. (a)	13. (d)	14. (b)	15. (a)
16. (b)	17. (b)	18. (b)	19. (b)	20. (c)
21. (b)	22. (d)	23. (c)	24. (b)	25. (d)

12

FUNCTIONS

Quantities of various characters such as length, area, mass, temperature, volume either have constant values or they vary with the values of other quantities. Such quantities are called constant and variable respectively.

Function is that chapter of mathematics that studies the relations between variable quantities in the process of their change. For instance, with a change in the side of a square, the area of the square also varies. The question of how the change in the side of the square affects the area is answered by a mathematical relationship between the area of the square and the side of the square.

Let the variable x take on numerical values from the set D.

A function is a rule that attributes to every number x from D one definite number y where y belongs to the set of real numbers.

Here, x is called the independent variable and y is called the dependent variable.

The set D is referred to as the *domain of definition* of the function and the set of all values attained by the variable y is called the *range of the function*.

In other words, *a variable y is said to be the value of function of a variable x in the domain of definition D if to each value of x belonging to this domain there corresponds a definite value of the variable y.*

This is symbolized as $y = f(x)$ where f denotes the rule by which y varies with x.

BASIC METHODS OF REPRESENTING FUNCTIONS

Analytical Representation

This is essentially representation through a formula.

This representation could be a uniform formula in the entire domain, for example, $y = 3x^2$

or

by several formulae which are different for different parts of the domain.

Example: $y = 3x^2$ if $x < 0$
and $y = x^2$ if $x > 0$

In analytical representations, the domain of the function is generally understood as the set of values for which the equation makes sense.

For instance, if $y = x^2$ represents the area of a square then we get that the domain of the function is $x > 0$.

Problems based on the analytical representation of functions have been favourite for the XLRI exam and have also become very common in the CAT over the past 3–4 years. Other exams are also moving towards asking questions based on this representation of functions.

Tabular Representation of Functions

For representing functions through a table, we simply write down a sequence of values of the independent variable x and then write down the corresponding values of the dependent variable y. Thus, we have tables of logarithms, trigonometric values and so forth, which are essentially tabular representations of functions.

The types of problems that appear based on tabular representation have been restricted to questions that give a table and then ask the student to trace the appropriate analytical representation or graph of the function based on the table.

Graphical Representation of Functions

This is a very important way to represent functions. The process is to use the coordinates to plot for each value of x from the domain D (if the function is given) the corresponding value of y and then connect the points to a smooth curve. If x is real, the curve will be a continuous curve. If x is integer, the curve will be a discrete curve.

For example, for the function $y = x^2$, we first calculate the values of y for which we need to plot the graph.

Then for any value x in D and get $y \in R$ (assume the point (x, y) is on the graph).

Then for $x = 0$, $y = 0$; for $x = 1$, $y = 1$; for $x = 2$, $y = 4$; for $x = -1$, $y = 1$; for $x = -2$, $y = 4$; and so on.

EVEN AND ODD FUNCTIONS

Even Functions

Let a function $y = f(x)$ be given in a certain interval. The function is said to be even if for any value of x,

$$f(-x) = f(x)$$

Properties of even functions:

(a) The sum, difference, product and quotient of an even function is also an even function.

(b) The graph of an even function is symmetrical about the y-axis.

Moreover, since y is the independent variable, it is symmetric about the y-axis. In other words, if $y = f(x)$ is an even function, then the graph of this function will be symmetrical about the y-axis. Example: $y = x^2$.

Examples of even functions: $y = x^2$, $y = x^4$, $y = \cos(x)$, $y = \cos^2(x)$, $y = x^2 + 1$, $y = x^4 + 3$, $y = |x|$ are all even functions.

The symmetry about the y-axis of an even function is illustrated below.

Odd Functions

Let a function $y = f(x)$ be given in a certain interval. The function is said to be odd if for any value of x

$$f(-x) = -f(x)$$

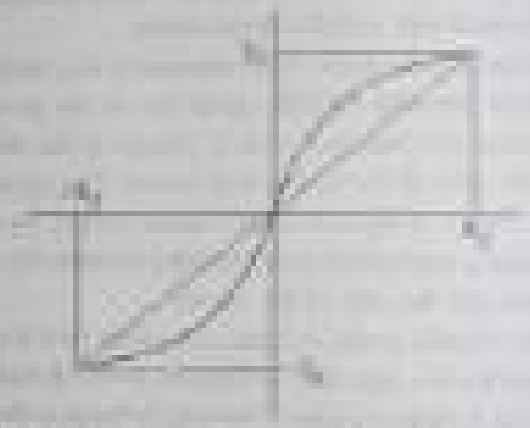
Properties of odd functions:

(a) The sum and difference of an odd function is also an odd function.

(b) The product and quotient of an odd function is an even function.

(c) The graph of an odd function is symmetrical about the origin.

The symmetry about the origin of an odd function is illustrated below.



Examples of odd functions: $y = x^3$, $y = x$, $y = x^5$, $y = \sin(x)$, $y = x^3 + x$.

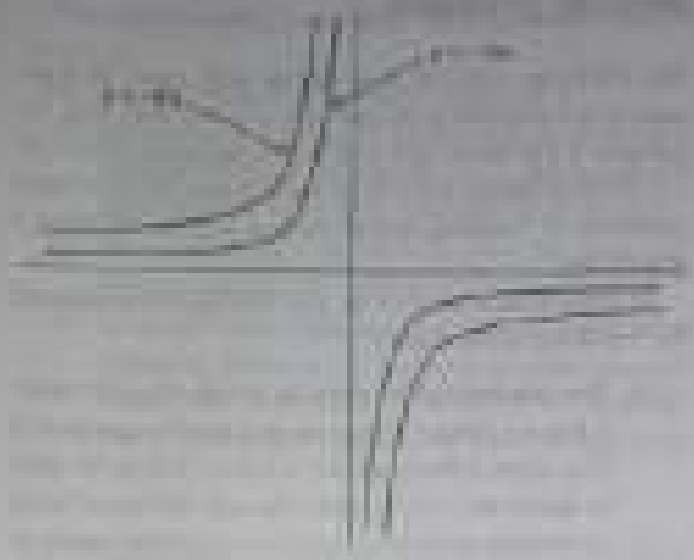
Any of functions could be sums of odd functions and even functions and be represented as the sum of an even function and an odd function.

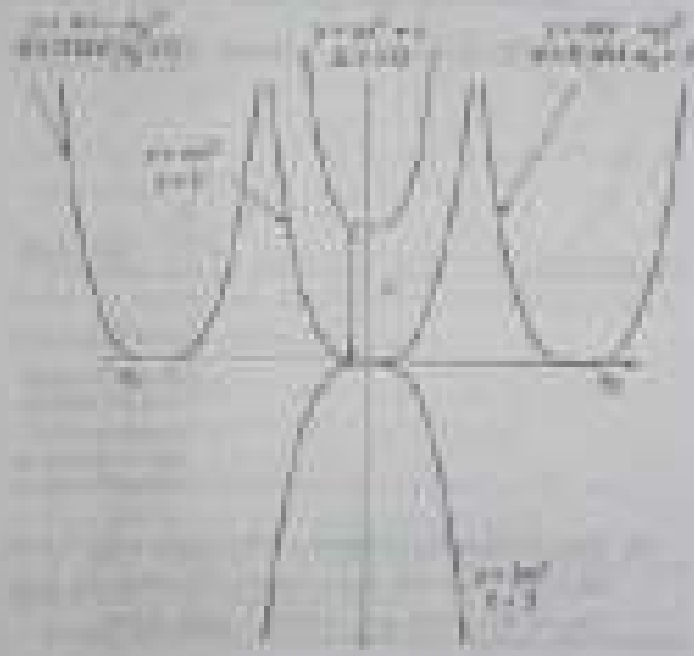
Inverse of a Function

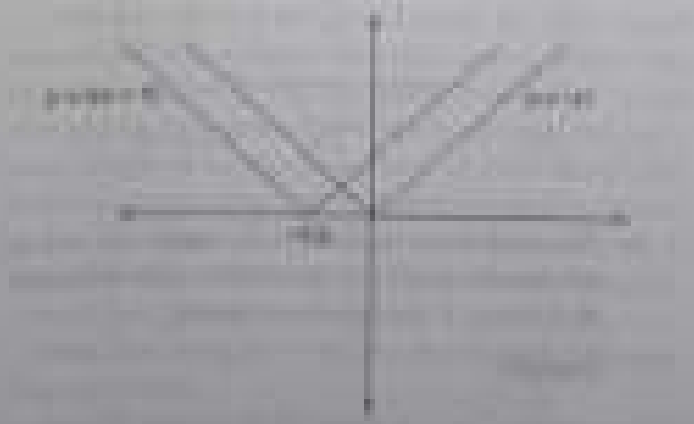
Let there be a function $y = f(x)$, which is defined on the domain X and has a range Y.

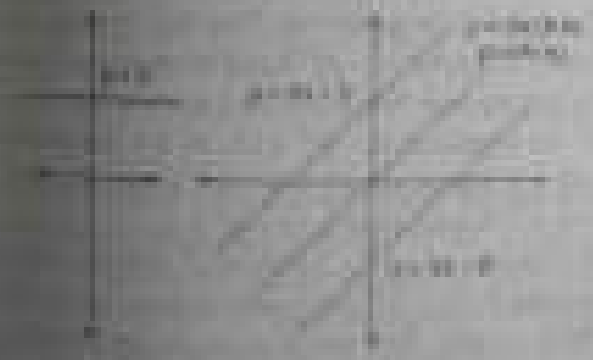
Then, by definition, for every value of the independent variable (or the function) x corresponding to a certain value of the dependent variable y is shown, and the point (x, y) is shown on the graph. Conversely, for any value of y in the range, there is a different value of x in the domain. If $y = f^{-1}(x)$, then for $x = 1$ and $y = 2$, figure for value of $x = 2$ & $y = 1$.

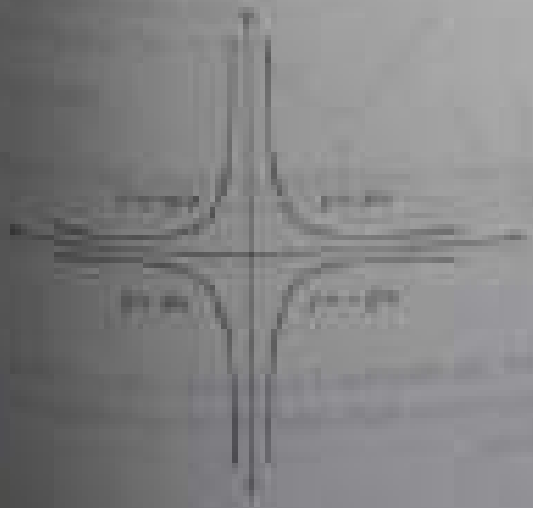
In such a case, the inverse function of the function $y = f(x)$ does not exist.











SHIFTING OF GRAPHS

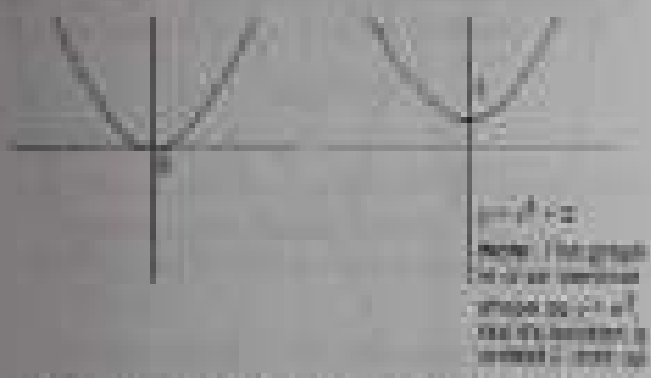
The ability to visualize how graphs shift when the basic mathematical expression is changed is very important skill. For example if you have to visualize the graph of $y = 2x^2 - 1$ it is definitely not a bad idea to your ability to visualize movement of functions out of values. Diagrams of shifts is progressively

In order to be able to do so, you must be familiar with the following general rules:

(i) **The relationship between the graph of $y = f(x)$ and $y = f(x) + c$ (where c represents a positive constant)**
The shape of the graph of $y = f(x) + c$ will be the same as that of the $y = f(x)$ graph. The only difference is that in terms of the last $f(x) + c$ is shifted c units up on the $x - y$ plot.

The following figure will serve to clear this you

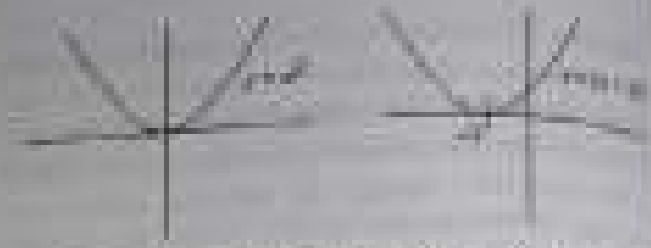
Example: Relationship between $y = x^2$ and $y = x^2 + 2$.



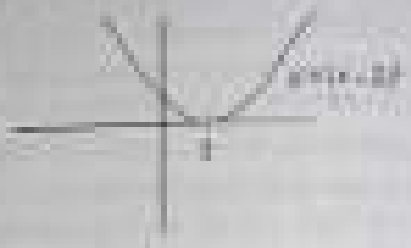
(ii) **The relationship between $y = f(x)$ and $y = f(x - c)$**
The case shifts to the right whereas the latter, the position of the graph gets shifted c units towards

(iii) **The relationship between $y = f(x)$ and $y = f(x + c)$**
The curve graph will get shifted c units to the left (Remember, c was a positive constant)

Example:



(iv) **The relationship between $y = f(x)$ and $y = f(x - c)$**
The curve the graph will get shifted c units to the right on the $x - y$ plane



COMBINING MOVEMENTS

It is best understood through an example:

Visualizing a graph for a function like $x^2 - 4x + 7$

First convert $x^2 - 4x + 7$ into $(x - 2)^2 + 3$

(Note: In order to do this conversion, the key point of the thinking should be on the $-4x$. Your first focus has to be get down a bracket $(x - a)^2$ which on expansion gives $-4x$ as the middle term. When you think this way, you will get $-2x^2$. On expansion $(x - 2)^2 = x^2 - 4x + 4$. But we want $x^2 - 4x + 7$. Hence add $+3$ to $(x - 2)^2$ ensure the expression $x^2 - 4x + 7$ is equivalent to $(x - 2)^2 + 3$)

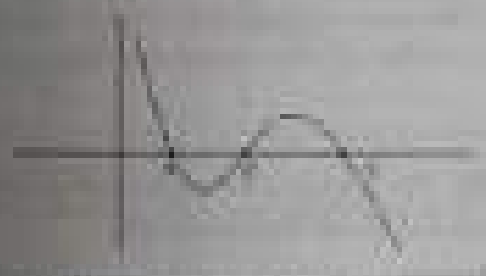
To visualize $(x - 2)^2 + 3$ shift the x^2 graph two units right (as we have to $(x - 2)^2$) and 3 units up (as we want to $+3$) on the $x - y$ plot. This will give you the required plot

Trick for the students: I wished now this to challenge the coverage from so much of how to add and multiply functions (Practically).

Graphical Empirical Process for Solving Inequalities

[illegible]

[illegible]



[illegible]

Matching

Graphical View of the Logarithmic Function

[illegible]

[illegible]

(A) [illegible]

(B) For values of x larger than 1, the logarithmic function is negative. At the same time for $x = 1$, the logarithmic function has a value of 0. Therefore, it is the lower

(C) The value of $\log x$ increases x, when the value of x becomes equal to the square of the base.

(D) As we go further right on the x-axis, the graph keeps increasing. However, the increase becomes more and more gradual and minor the slope of the graph becomes increasingly flatter as we move further to the right.

WORKED-OUT PROBLEMS

Problem 1: Find the domain of the definition of the function $y = \log_{(x-2)}(x^2 - 2x)$

(a) $(-\infty, 2)$ (b) $(-\infty, +\infty)$ except $\{0, 2\}$

(c) $(2, +\infty)$ (d) $(-\infty, 0)$

Solution: For the positive value defined the expression under the radical should be non-negative and the denominator should not be equal to zero.

So $x^2 - 2x > 0$ and $(x^2 - 2x) \neq 0$

or $x(x-2) > 0$ or $x^2 - 2x \neq 0$

So x must be in between 0 and 2 and $x \neq 0, x \neq 2$.

So x will be $(-\infty, +\infty)$ excluding 0 and 2.

In some situations, to solve the above problem, you should check the options as follows.

In fact, for solving all questions on functions, the student should ignore the option based approach.

Often one will find that going through the option-based approach will take you over a significant amount of time. The student should try to memorize hidden properties of various functions so that he/she could be able to eliminate the many wrong options on the basis of a very quick check. The student should develop a hand for discovering these options so that the results automatically becomes the answer. It should also be noted that if any option cannot be eliminated, a reason that it is the correct option.

What I am trying to say will be clear from the following subject process.

... For the question if we check at $x = 1$, the function is defined. However $x = 1$ is not in the order of options and

[illegible]

[illegible]

EXAMPLE 3.3.3 Which of the following is an even function?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

SOLUTION The correct [illegible]

If a function is even it should satisfy the equation $f(x) = f(-x)$.

We now check the four options to see which of them represents an even function.

Checking option (a): $f(x) = x(x^2 + \ldots)$.
Putting $-x$ in the place of x,

$$f(-x) = (-x)(-x)^2 + \ldots$$
$$= -x(x^2 + \ldots) = -f(x)$$

Checking option (b): $f(x) = x^2 + x^4$.
Putting $-x$ in the place of x,

$$f(-x) = (-x)^2 + (-x)^4 = x^2 + x^4 = f(x)$$

So f is the answer.

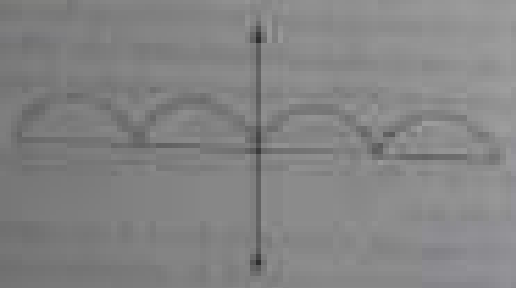
You do not need to go further to check the [illegible]. However, if you feel [illegible] you would [illegible] also in deciding it as follows.

Checking option (c): $f(x) = x + x^3$.
Putting $-x$ in the place of x,

$$f(-x) = -x + (-x)^3 = -x - x^3 = -f(x)$$

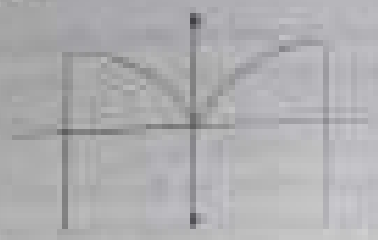
Therefore for [illegible]
[illegible]
[illegible]
[illegible]

EXAMPLE 3.3.4



SOLUTION The graph is symmetrical about the [illegible] axis. So it is the definition of an even function. So $f(x)$.

EXAMPLE 3.3.5

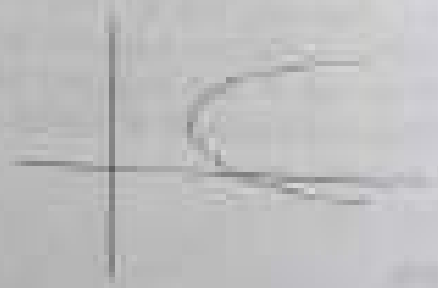


SOLUTION The graph is symmetrical about the y-axis. So it is the definition of an even function. So $f(x)$.

EXAMPLE 3.3.6

SOLUTION The graph is symmetrical about origin. So f is the definition of an odd function. So $f(x)$.

EXAMPLE 3.3.7



SOLUTION The graph is neither symmetrical about [illegible] about origin. So $f(x)$.

EXAMPLE 3.3.8 Which of the following [illegible] is/are odd?

(a) [illegible] (b) [illegible]
(c) [illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) None of these

Solution: For every [illegible] in the minimal, their domains [illegible]
[illegible]
[illegible] the domains of $f(x)$ [illegible] given and $h(x)$,
[illegible] The [illegible] should not be repeat in [illegible]
[illegible] and be all real numbers except [illegible]
[illegible] $\sqrt{x^2}$ [illegible] should be non negative
[illegible] [illegible] will be all positive real numbers.
[illegible] is defined every where.
So [illegible] the domain of these three the same domain.
[illegible] (d) is the correct option.

Question 112 If $f(x) = \sin x$, $g(x) = x$ and $h(x) = x^2$, [illegible] (2)

(a) 1
(b) 2
(c) 0
(d) None of these

Solution: Equation (2) is the same as $f(g(h(2)))$.
[illegible] specific trigonometric function. This means [illegible] the function in [illegible]. Hence $f(2) = \sin(\sin(\sin 2))$
$\sin 2$
[illegible] $\sin(\sin 2) = \sin(1) = 0$. Hence the optimal [illegible] is zero.

Question 113 [illegible] the [illegible] of $f(x)$ Problems 1 1.5 and [illegible]

$$f \circ g = f^{-1} \circ g^{-1} = f^{-1}$$
$$f \circ g = f \circ g = f \circ f^{-1}$$
$$f \circ g = f \circ f = f_{id}$$

Question 114 Find the value of $(1 + 2^2)(1 + 3^2)(1 + 4^2) \ldots (1 + n^2)$ [illegible]

(a) $n!2^n$
(b) $n!2^{n-1}$
(c) $n!2^{n+1}$
(d) None of these

Solution: Such questions, should be solved by the [illegible] method. [illegible] the beginning of expression.
[illegible] Now we get $(1 + 1^2)(1 + 2^2) \ldots (1 + n^2)$

$= (1 + 1^2)$ [illegible] [illegible] that for $n = 1$ right hand side $= 2!$
[illegible] $n = 1$ the expression [illegible] above.
$= 2!2^{1-1} = 2!$

Question 115 Which of the following expressions will give the [illegible] of x and y [illegible]?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

Solution: For arbitrary quantifiers containing a function or the quantifier or and in a function or the optional [illegible] the [illegible] the [illegible] process [illegible] the [illegible] of the variables in the expression and checking for the related optimal [illegible] given a complete [illegible] the expression [illegible] the quantifiers. The [illegible] of this [illegible] of solution is just [illegible] [illegible] [illegible] [illegible]. Therefore, if the expression is not [illegible] and [illegible] [illegible] [illegible]

Level of Difficulty (LOD) 1

1. Find the domain of the definition of the function [illegible]
   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]
   (e) None of these
2. Find the domain of the definition of the function [illegible]
   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]
   (e) None of these
3. Find the domain of the definition of the function [illegible]
   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]
   (e) None of these
4. Find the domain of the definition of the function [illegible]
   (a) All the real values except [illegible]
   (b) [illegible]
   (c) [illegible]
   (d) [illegible]
   (e) None of these
5. Find the domain of the definition of the function [illegible]
   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]
   (e) None of these
6. Find the domain of the definition of the function [illegible]
   (a) [illegible] (b) [illegible]
   (c) [illegible]
   (d) [illegible]
   (e) None of these
7. The values of [illegible] which the function [illegible] is defined are [illegible] are [illegible]
   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]
   (e) None of these
8. The values of [illegible] which the function [illegible] is defined are [illegible]
   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]
   (e) None of these
9. If [illegible] then [illegible] is equal to
   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]
   (e) None of these
10. If [illegible] then the value of [illegible] will be equal to
    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]
    (e) None of these
11. If [illegible] and [illegible] then [illegible] and [illegible] are identical
    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]
    (e) None of these
12. If [illegible] is inverse function, then the graph of [illegible] will be symmetrical about
    (a) [illegible] (b) [illegible]
    (c) Both the axes (d) None of these
13. [illegible] is inverse function, then the graph of [illegible] will be symmetrical about
    (a) [illegible] (b) [illegible]
    (c) Both the axes (d) [illegible]
    (e) None of the axes
14. Which of the following is an even function?
    (a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
    (e) None of these
15. Which of the following is not an odd function?
    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]
    (e) None of these
16. For what value of [illegible] [illegible] will give the minimum value?
    (a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
    (e) [illegible]
17. In the above problem, what will be the minimum value of the function?

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]

(e) [illegible]

[illegible] Find the maximum value of the function $f(x) = [illegible]$

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]

(e) None of these

[illegible] Find the maximum value of the function $f(x) = \log_{[illegible]}(x^2 + 1)$

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]

(e) [illegible]

[illegible] Let f be any function and $f^{-1}(x)$ is defined as (where g)

(a) $\sin(f^{-1}(x)) = \frac{1}{g}$ [illegible]

(b) [illegible] (c) [illegible]

(d) $\frac{1}{[illegible]}$ (e) $\frac{1}{[illegible]}$

(e) None of these

Answer for (Question 21–23) Find the [illegible] and [illegible]

$f(x) = [illegible]$ (a [illegible] number)

$f(2) = 8, f(3) = 1$

[illegible] The value of $f(1)$ is

(a) 9 (b) 5

(c) 1 (d) −9

(e) 8

[illegible] The value of $f(1) + f(4)$ is

(a) 1 (b) −9

(c) 2 (d) 3

(e) −2

[illegible] What is the [illegible] value of $\sum_{x=1}^{[illegible]} f(x)$

(a) 0 (b) −10 (c) +10 (d) 55

(e) [illegible]

[illegible] What will be the domain of the [illegible] of the function $f(x) = \log_{[illegible]}(x)$ for positive values of x?

(a) $[-2, 2]$ (b) $[-\frac{1}{2}, 2]$

(c) $[-2, 0, 2]$ (d) $[-1, 1] - \{-0.5, 0, 1\}$

(e) None of these

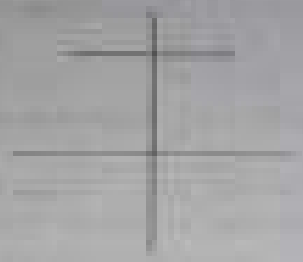
Answer for (Question 25–29)

[illegible]

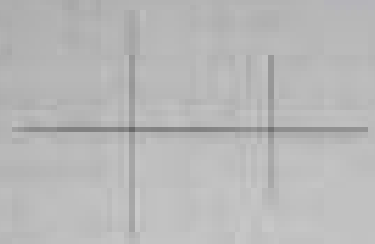
[illegible]

Match the number of real roots of the equation $f(x) = 0$ with the graph that intersects at at least one point of the domain.

26.



27.

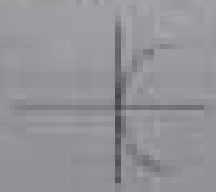


28.

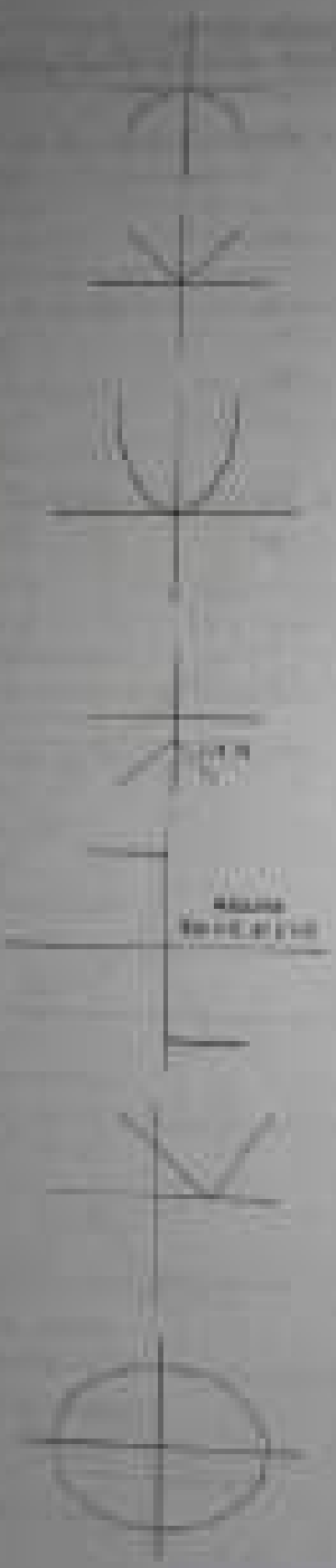
29.

30.

31.



[illegible]



[illegible]

Directions for questions [illegible]: Define the following functions:

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[illegible]

Directions for questions [illegible]: Define the following functions:

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[illegible]

[illegible]

[illegible]

16. The graph of $y = f(x) = 2x^2 + 1$ is obtained from $f(x) = x^2$ as follows
   (a) 1 units to the right and 1 unit down
   (b) 2 units to the left and 1 units down
   (c) 2 units to the left and 1 unit up
   (d) 2 units to the right and 1 unit up

17. If $f(x) = 3x^2$ and $g(x) = 2x^3$, then $(f \circ g)(x) = 6x^3$ is
   (a) Even function (b) Odd function
   (c) Both (d) None of these

18. If $f(x) = x^2$ and $g(x) = \log_2 x$, then $f(x) + g(x) = 0$ is
   (a) Even function (b) Odd function
   (c) Both (d) Neither (a) nor (b)

19. If $f(x) = x^2$ and $g(x) = x^{1/2}$, then $f(x) + g(x) = 0$ is
   (a) Odd function (b) Even function
   (c) Neither (a) nor (b) (d) Both

20. $f(x)$ is any function and $f^{-1}(x)$ is known as inverse of $f(x)$. Then $f^{-1}(x)$ of $f(x) = 2x + 1$ is
   (a) $\frac{1}{x} + 2$ (b) $\frac{1}{(x+2)}$
   (c) $\frac{1}{x} + 0.5$ (d) None of these

21. $f(x)$ is any function and $f^{-1}(x)$ is known as inverse of $f(x)$. Then $f^{-1}(x)$ of $f(x) = e^x$ is
   (a) $-e^x$ (b) e^{-x}
   (c) $\log x$ (d) None of these

22. $f(x)$ is any function and $f^{-1}(x)$ is known as inverse of $f(x)$. Then $f^{-1}(x)$ of $f(x) = x(x-1)$, $x \geq 1$ is
   (a) $x(x+1)$ (b) $\frac{x}{x^2-1}$
   (c) $x(x-1)$ (d) $-x(x+1)$

23. Which of the following functions will have a minimum value at $x = -1$?
   (a) $f(x) = 2x^2 - 4x + 1$ (b) $f(x) = 4x^2 + 4x + 2$
   (c) $f(x) = x^2 + 2x - 6$ (d) None of these

Directions for Questions 24–33:

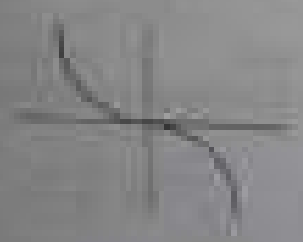
Mark (a) if $f(-x) = -f(x)$

Mark (b) if $f(-x) = f(x)$

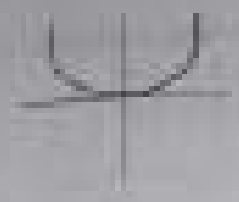
Mark (c) if neither (a) nor (b) is true

Mark (d) if $f(x)$ does not exist at at least one point of the domain.

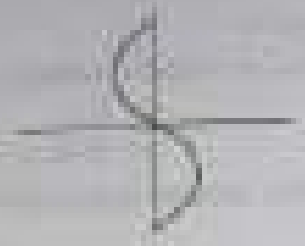
24.



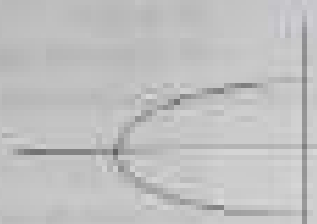
31.



32.

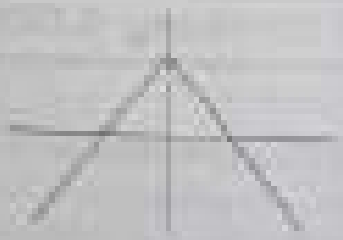


33.

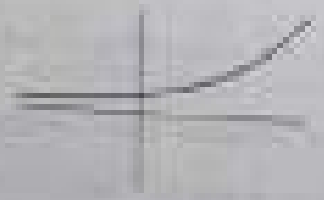


34.

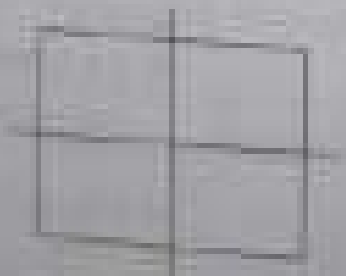
35.

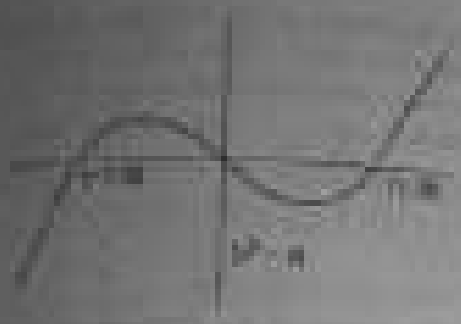


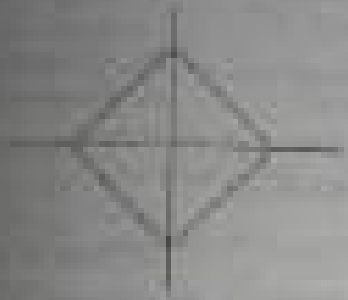
36.



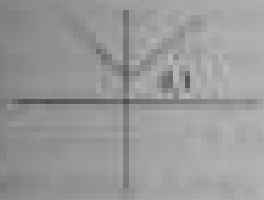
37.







Answer for Questions 13–16: $f(x)$ is represented by the graphs

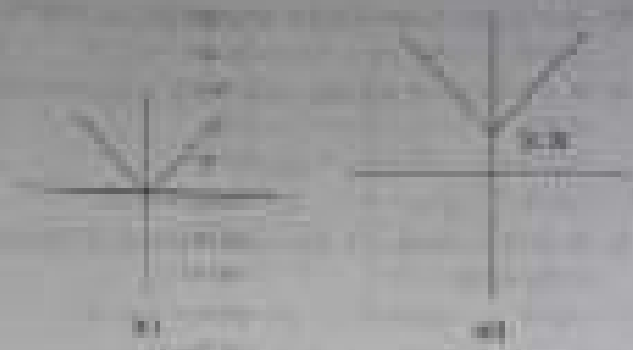


13. Which of the following will represent the function $-f(x)$?

14. Which of the following will represent the function $f(|x|)$?

15. Which of the following will represent the function $|f(x)|$?

16. Which of the following will represent the function $f(x+1)$?



Answer for Questions 17–40: Given the following functions

$$f(x) = 2x^2 + x + 4$$
$$g(x) = 2x^3 + x^2 + 2x + 100$$
$$h(x, y) = 2x^2$$

Find the values of all following expressions:

17. $f(f(1)) + g(f(1)) + h(f(1), f(1))$
    (a) 0 (b) 1000
    (c) 2041 (d) None of these

18. $g(f(1)) + g(g(1)) + h(g(1), 1)$
    (a) 0 (b) 1
    (c) 32 (d) None of these

19. $f(f(1)) + f(g(1)) + h(f(1), f(1))$
    (a) 8 (b) 9
    (c) 32 (d) None of these

20. $f(x+y) + g(x+y) + h(x, y)$
    (a) -4 (b) 8 (c) 2 (d) 0

21. If $f(x) = 1/(1-x)$, then which of the following is correct?
    (a) $f(f(x)) = f(g(x))$
    (b) $f(g(x)) = f(h(x))$
    (c) $g(g(x)) = f(h(x))$
    (d) $f(g(x)) = g(f(x))$

22. If $f(x) = 1/(1-x)$, then the value of $f(f(f(x)))$ is independent of x, and $g(x) > 1$, will be
    (a) 0
    (b) 1
    (c) Depends upon the value of $f(x)$ and $g(x)$
    (d) None of these

Answer for Questions 23–25:

23. $g(x)$ is the remainder when x is divided by 3.
    $f(x)$ is the quotient obtained when x is divided by 3.
    $h(x)$ is the integer just larger than x divisible by 3.

40. If $a=\{1,2,3\}$ then number of [illegible] is [illegible]
(a) 0 (b) [illegible] (c) 2 (d) 3

41. If $a \in N$, $a \neq 1$, then the value of $\int_0^1 \frac{x^a(1-x)^a}{1+x}dx$ is equal to
(a) π (b) 1
(c) 2 (d) None of these

42. If $\cos\theta = \cos\alpha$ and $\sin\theta = \sin\alpha$ then the value of $\int_\alpha^\theta \sin(\theta-\alpha)d\theta$ equals
(a) 0 (b) 4 (c) π (d) π

Directions for Questions 43–45: Read the following passage and answer the questions based on it.

(1) is defined, integer less than or equal to x.
(2) is greatest integer greater than or equal to x.

43. If x is an integer then the value of $[x]+[-x]$ is
(a) 0 (b) 1 (c) -1 (d) 1

44. If x is an integer then $[x]+[x]$ is
(a) An even integer
(b) An odd integer
(c) x
(d) $x+1$

45. What is the value of x if $x \in N$ and $x[x+1] = 1 + 2x^2$
(a) 22 (b) 38 (c) 11 (d) 19

46. If $f(x) = x^2 + 1$ and $g(x) = 1/x^2 + 2$ then the $f+2$ $(f \circ g)(x) = f(g(x))$ is
(a) 1 (b) 2
(c) 4 (d) None of these

47. Then $f(x) = x$ is set $\{f(x) = 0 - 2, 0, 2\}$ $N(x) = \{0\}$ Rule
(a) 2 (b) 3 (c) 3 (d) 4

Level of Difficulty (LOD) III

1. Find the domain of the definition of the function $y = \log_{10}(x-1)^{1/2}$
(a) $-\infty < x < \infty$ (b) $-1 < x < 0$
(c) $0 < x < \infty$ (d) No solution

2. Find the domain of the definition of the function $y = \log_{10}(x-1)^{1/2} + 2x - x^2 + (x^2+x)^{1/2}$
(a) $x = 0$ (b) $(1, \infty)$
(c) $[-1, 1]$ (d) 1

3. Find the domain of the definition of the function $y = \sqrt{-2x^2 + x + 3} + (x^2-1)^{1/2}$
(a) $-1 \le x \le 1$ (b) $x \le 1$ or $x \ge 1$ or $x = 1$
(c) $-2 \le x \le 3$ (d) Everywhere

4. Find the domain of the definition of the function $y = \sqrt{x^2 - 2x - 3}^{1/2} + (8-2x+3x^2)^{1/2}$
(a) $x > 0$ (b) $-1 < x < 1$
(c) x^2 (d) None of these

5. Find the domain of the definition of the function $y = \sin_{10}[1 - \log_{10}(x)]^{1/2}$, $0 < x < \infty$
(a) $[1, 3]$ (b) $[2, 3]$
(c) $[2, 3]$ (d) None of these

6. If $f(x) = 0$, $f(x) = 1$, then $f(x)/f(x) = 0$ is equal to
(a) 10^x (b) -10^x (c) x (d) $-x$

7. If $f(x) = e^x$ and $g(x) = \log_{10}(x)$ then value of $f(x) \circ g(x)$
(a) x (b) 0 (c) 1 (d) x

8. In the above question, find the value of $g(x)$
(a) x (b) 0 (c) 1 (d) x

9. The function x is defined and defined and $g(x) = 0$ is given by
(a) $f = \{x \mid x \in (-\infty, \infty)\}$ (b) $f = \{x \mid x \in (0, \infty)\}$
(c) $f = \{x \mid x \in [0, \infty)\}$ (d) $f = \{x \mid x \in (-1, 1)\}$

10. Which of the following function is the exponential of x?
(a) $f(x) = \log_e x + \sqrt{(x^2-x)}$
(b) $f(x) = \log_e x + \sqrt{(x^2-1)}$
(c) $f(x) = \log_e x - \sqrt{(x^2+1)}$
(d) None of these

11. Which of the following function is not in x of the form?
(a) $f(x) = \log_e(x + \sqrt{x^2+1})$
(b) $f(x) = \log_e x + \sqrt{(x^2-x)}$
(c) $f(x) = \log_e x + \sqrt{(x^2-1)}$
(d) All of these

12. Find $f(x)$ [illegible] $(x^2)^{1/2}$
(a) $(x)(x^2)^{1/2}$ (b) $(x)+(x^2)^{1/2}$
(c) $(x)+(x^2)$ (d) None of these

[illegible]

(a) 1 (b) 0

(c) 4 (d) None of these

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(e) None of these

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

Questions for Questions 28–30: Read the [illegible] and answer [illegible]

[illegible]

[illegible]

(a) [illegible]

(b) [illegible]

(c) [illegible]

(d) [illegible]

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

Questions for Questions 33–34

[illegible]

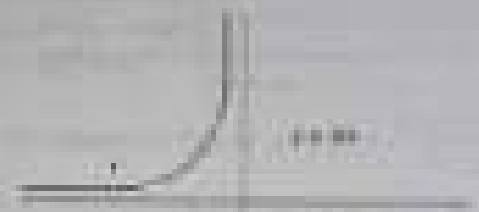
[illegible]

[illegible]

[illegible]

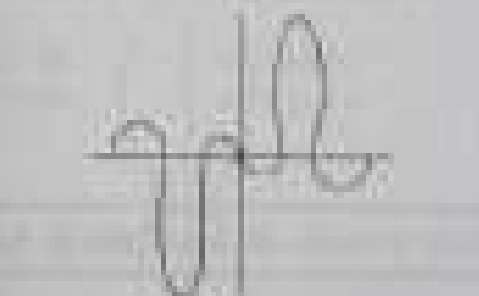
33.

34.

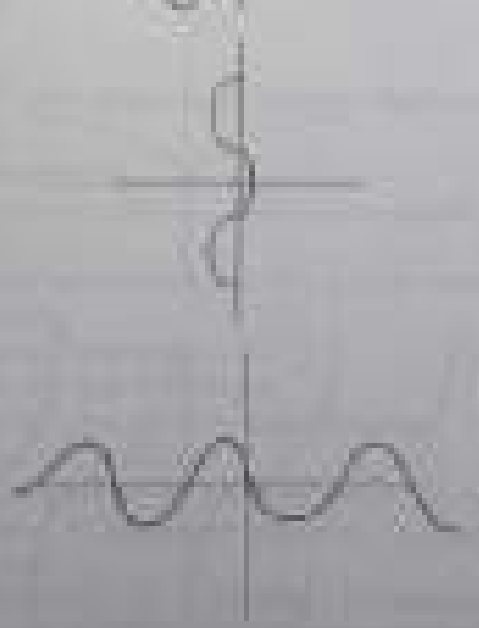


35.

36.

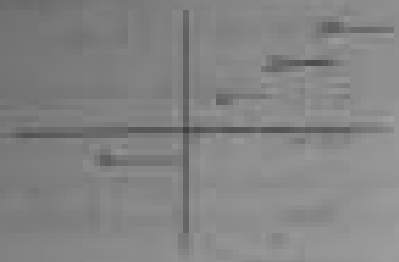


37.



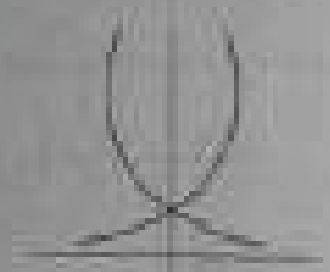
29.

30.



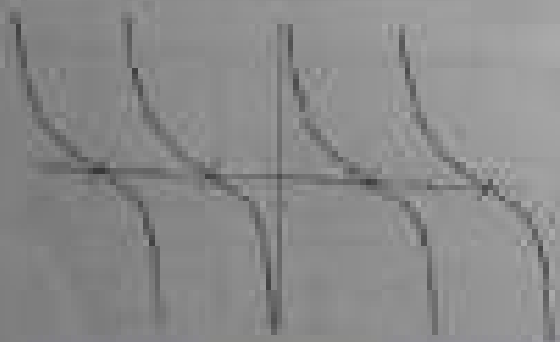
31.

32.

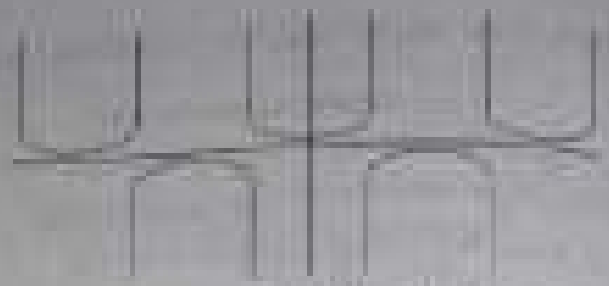


33.

34.



34.



Directions for Questions 35 to 40: Define the functions

$A(x, y, z) =$ Min (max (x, y), max (y, z)), z) $B(x, y, z) =$ Min (max (x, y), max (y, z), max (x, z)) $C(x, y, z) =$ Min (min (x, y), min (y, z), min (x, z)) $D(x, y, z) =$ Min (max (x, y), max (y, z), min (x, z)) $E(x, y, z) =$ Min (max (x, y), min (y, z), max (x, z)) $F(x, y, z) =$ Maximum of x, y and z.

Assume that x, y and z are distinct integers.

35. For what condition will $A(x, y, z) = D$ be equal to Max (x, y, z)?
    (a) When x is maximum (b) When y is maximum
    (c) When z is maximum (d) Either (a) or (b)

36. For what condition will $B(x, y, z)$ be equal to Min (x, y, z)?
    (a) When x is minimum (b) When y is minimum
    (c) Either (a) or (b) (d) Never

37. For what condition will $A(x, y, z)$ not be equal to $B(x, y, z)$?
    (a) $x \neq y \neq z$ (b) $y > z > x$
    (c) $x > y > z > x$ (d) None of these

38. Under what condition will $C(x, y, z)$ be equal to $D(x, y, z)$?
    (a) $x > y > z > x$ (b) $x = y = z$
    (c) Both a and b (d) Never

39. Which of the following will always be true?
    (I) $E(x, y, z)$ will always be greater than Min (x, y, z)
    (II) $E(x, y, z)$ will always be lower than Min (x, y, z)
    (III) $E(x, y, z)$ will never be greater than $B(x, y, z)$
    (a) I only (b) II only
    (c) Both a and b (d) All the three

40. The logical value obtained the following will be
    (a) True/False (b) A/B
    (c) 1/0 (d) Cannot be determined

Directions for Questions 41–45: Suppose x and y are real numbers. Let $f(x, y) = (x + y)$, $g(x, y) = -|x|$, y and $h(x, y) = f(x, y)$

41. Which one of the following is true?
    (a) $h(x, y) = g(x, y) + f(x, y)$ (b) $h(x, y) = f(x, y) - g(x, y)$

[illegible]

[illegible]

[illegible]

7. [illegible] would be decreased if [illegible] increased.

   Hence, [illegible] is the answer.

8. [illegible]

   [illegible] the input values in [illegible] a [illegible] is not defined.

   Hence, [illegible]

9. [illegible]

   [illegible]

10. [illegible]

11. [illegible] [illegible] and [illegible] [illegible] [illegible]

[illegible]

[illegible]

[illegible]

42. The required minimum value would occur at $f(x) = g(x) = 1$.
43. $SQ[R[(a+b)/b]] = SQ[R[17/5]] \Rightarrow SQ[2] = 2$.
44. $Q[[SQ(63)+7]/9] = Q[[8+7]/9] = Q[15/9] = 1$.
45. $Q[[SA(36)+R(18/7)]/2] = Q[(7+4)/2] = Q[11/2] = 5$.
46. $\{x\} - [x] = -1$
47. $\{x\} + [x]$ will always be odd as the values are consecutive integers.
48. At $x = 5.5$, the given equation can be seen to be satisfied as: $6 + 5 = 2 \times 5.5 = 11$.
49. $f(g(t)) - g(f(t)) = f(2.5) = g(6) = 8.25 - 2.166 = 6.0833$.
50. $fog = f(3t+2) = K(3t+2)+1$

$gof = g(kt+1) = 3(kt+1)+2$

$$K(3t+2)+1 = 3(kt+1)+2$$
$$\Rightarrow \quad 2k+1 = 5$$
$$\Rightarrow \quad k = 2.$$

LOD III

1. $x - |x|$ is either negative for $x < 0$ or 0 for $x \geq 0$. Thus, option (d) is correct.
2. The domain should simultaneously satisfy:

$$x - 1 \geq 0, (1-x) \geq 0 \text{ and } (x^2+3) \geq 0.$$

Gives us: $x \geq 1$ and $x \leq 1$

The only value that satisfies these two simultaneously is $x = 1$.

3. For the function to exist, the argument of the logarithmic function should be positive. Also, $(x+4) \geq 0$ should be obeyed simultaneously.

For $\dfrac{(x-5)}{(x^2 - \text{to } x + 24)}$ to be positive both numerator and denominator should have the same sign. Considering all this, we get:

$$4 < x < 5 \text{ and } x > 6.$$

Option (c) is correct.

4. Both the brackets should be non-negative and neither $(x+3)$ nor $(1+x)$ should be 0.

For $(x-3)/(x+3)$ to be non negative we have $x > 3$ or $x < -3$.

Also for $(1-x)/(1+x)$ to be non-negative $-1 < x < 1$. Since there is no interference in the two ranges, option (d) would be correct.

8. $f(f(t)) = f((t-1)/(t+1))$

$$= [(t-1/t+1)-1]/[(t-1)/(t+1)+1]$$
$$= [(t-1)+(-t-1)]/[t-1+t+1]$$
$$= -2/2t = -1/t.$$

9. $fog = f(\log_e x) = e^{\log_e x} = x$.
10. $gof = g(e^x) = \log_e e^x = x$.
11. Looking at the options, one unit right means x is replaced by $(x-1)$. Also, 1 unit down means -1 on the RHS.

Thus, $(y+1) = 1/(x-1)$

12. For none of these is $f(t) = f(-t)$

Thus, option (d) is correct.

13. Option (b) is odd because:

$$\frac{a^{-t}+a^t}{a^t-a^{-t}} = -1 \times \left(\frac{a^{-t}+a^t}{a^{-t}-a^t}\right)$$

Similarly option (c) is also representing an odd function. The function in option (a) is not odd.

14. $f(f(t)) = f[t/(1+t^2)^{1/2}] = t/(1+2t^2)^{1/2}$
15. By trial and error it is clear that at $t = 3$, the value of the function is 19. At other values of 'x' the value of the function is less than 19.
17. Take different values of n to check each option. Each of options a, b and c can be ruled out. Hence, option (d) is correct.

Solution to 18 to 20:

$$f(1) = 0, f(2) = 1,$$
$$f(3) = f(1) - f(2) = -1$$
$$f(4) = f(2) - f(3) = 2$$
$$f(5) = f(3) - f(4) = -3$$
$$f(6) = f(4) - f(5) = 5$$
$$f(7) = f(5) - f(6) = -8$$
$$f(8) = f(6) - f(7) = 13$$

18. It can be seen that $f(x)$ is positive wherever x is even.
19. $f(f(6) = f(5) = -3$.
20. $f(6) - f(8) = 5 - 13 = -8 = f(7)$.
21. Option (b) is not even since $e^x - e^{-x} \neq e^{-x} - e^x$.
22. We have $f(x) \cdot f(1/x) = f(x) + f(1/x)$

$\Rightarrow \quad f(1/x)[f(x)-1] = f(x)$

For $x = 4$, we have $f(1/4)[f(4)-1] = f(4)$

$\Rightarrow \quad f(1/4)[64] = 65$

$\Rightarrow \quad f(1/4) = 65/64 = 1/64 + 1$

This mean $f(x) = x^3 + 1$

For $f(6)$ we have $f(6) = 216 + 1 = 217$.

Directions for Questions 23–34: You essentially have to mark (a) if it is an even function,
mark (b) if it is an odd function,
mark (c) if the function is neither even nor odd.

Also, option (d) would occur if the function does not exist atleast one point of the domain. This means one of two things

[illegible]

[illegible]

Answer to 44–46:

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

I. [illegible] is always positive.

II. [illegible] is always negative.

III. [illegible] is always positive.

[illegible]

Hence, only statement (I) of [illegible] is true.

45. [illegible] is always positive.

46. [illegible]

Option (b) [illegible] is correct.

Directions for Questions 47–49: Use the following analysis

$A(x, y)$ is positive

$B(x, y)$ is negative

$C(x, y)$ is positive

$D(x, y)$ is positive

$E(x, y)$ is positive and even

48. [illegible]

$= C(1, 2) \times 3 \times D(4, \ldots, 4) - (3 + 7 \times (1, \ldots, 4)) \times 5$

$= 16$

49. Nearly each statement is true that (I) and (II) are true.

47. The given expression becomes:

$\text{Min}(\text{max}(5, -7), \text{Mod}(5, -1), (\text{Lcm}(5, 11)) \times \text{Min}(6, -5, 3))$

$= -1$

48. The given expression becomes:

$\text{Mod}(\text{Lcm}(-4, -3), \text{Lcm}(-3, -4))$

We would never be negative.

49. The respective values are

$-8, -7, 132, -8, 11$, and -56.

Hence (I) is second lowest.

50. Let $y = 1, x = 2$ and $z = 3$.

Then $f(x, y, z) = (x$

$= (1 \times 6 - 3)(x(7 + 3 \times 3)) + (3 \times 7 \times 2)$

$= (27 \times (7 + 9)) + (3 \times 7 \times 2)$

$= 216 + 42$

$= 258$

Option (b) $f(1, 2, 3) = 258$ gives the same value.

ANSWER KEY

Exercise 1.(A)1

1.	b
2.	b
3.	a
4.	c
5.	a
6.	d
7.	b
8.	a
9.	c
10.	b
11.	a
12.	a
13.	d
14.	a
15.	a
16.	a
17.	a
18.	a
19.	b
20.	c
21.	b
22.	a
23.	a
24.	b
25.	b
26.	a
27.	a
28.	a
29.	a
30.	d

31.	a
32.	a
33.	a
34.	a
35.	b
36.	c
37.	a
38.	d
39.	d
40.	a
41.	a
42.	a
43.	d
44.	d
45.	a
46.	a
47.	c
48.	d
49.	a
50.	d

Fahrenheit 451 (F)

1.	a
2.	d
3.	c
4.	d
5.	a
6.	a
7.	c
8.	b
9.	a
10.	a
11.	c
12.	c
13.	a
14.	b
15.	a
16.	b
17.	a
18.	b
19.	a
20.	b
21.	b
22.	a
23.	a
24.	a
25.	a
26.	a
27.	b
28.	a
29.	b
30.	a
31.	c
32.	c
33.	a
34.	a
35.	a
36.	c
37.	c
38.	d
39.	c
40.	d
41.	b
42.	a
43.	c
44.	b
45.	c
46.	b
47.	c
48.	b
49.	b
50.	b

13

INEQUALITIES

[illegible]

PROPERTIES OF INEQUALITIES

[illegible]

[illegible]

[illegible]

[illegible] Inequalities

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]
6. [illegible]
7. [illegible]
8. [illegible]
9. [illegible]
10. [illegible]
11. [illegible]
12. [illegible]

Certain Important Inequalities

1. [illegible]
2. [illegible]

[illegible]

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]
6. [illegible]
7. [illegible]
8. [illegible]
9. [illegible]
10. [illegible]
11. [illegible]
12. [illegible]
13. [illegible]
14. [illegible]

Some Important Results

1. [illegible]

[illegible]

What Are Greatest Value of a Product of a Sum of Whose Factors is Constant

[illegible]

Suppose $a_1, a_2, \ldots, a_n$ are positive numbers and $a_1 + a_2 + \cdots + a_n = S$ (constant). Then the product $a_1 a_2 \cdots a_n$ is maximum when $a_1 = a_2 = \cdots = a_n = \frac{S}{n}$.

[illegible]

Suppose $a_1, a_2, \ldots, a_n$ are positive integers, then we $P(a_1, a_2, \ldots, a_n)$ and therefore $a_1^{a_1} \cdots a_n^{a_n}$ where $a_1 + \cdots + a_n$ [illegible] positive integers. Prove

$$\left(\frac{a_1}{a_1}\right)^{a_1} \cdots \left(\frac{a_n}{a_n}\right)^{a_n} \text{ that } a_1 \cdots a_n \le a_1^{a_1} \cdots a_n^{a_n}$$

[illegible]

Solution: [illegible]

$$\text{Adding } \left(\frac{a_1}{a_1}\right)^{a_1} + \cdots + \left(\frac{a_n}{a_n}\right)^{a_n}$$

$$\text{Given } \left(\frac{a_1}{a_1}\right)^{a_1} \cdots \left(\frac{a_n}{a_n}\right)^{a_n} \le \left(\frac{a_1}{a_1}\right)^{a_1} + \cdots + \left(\frac{a_n}{a_n}\right)^{a_n}$$

[illegible]

[illegible]

[illegible]

Suppose $a_1, a_2, \ldots, a_n$ are positive integers and $a_1 + a_2 + \cdots + a_n = S$ (constant). Then the product $a_1 a_2 \cdots a_n$ is greatest when $a_1 = a_2 = \cdots = a_n = \frac{S}{n}$.

[illegible]

Suppose $a_1, a_2, \ldots, a_n$ are positive integers and $a_1 + a_2 + \cdots + a_n = S$ (constant). Then the product $a_1 a_2 \cdots a_n$ is greatest when $a_1 = a_2 = \cdots = a_n = \frac{S}{n}$.

[illegible]

Suppose $a_1, a_2, \ldots, a_n$ are positive integers and $a_1 + a_2 + \cdots + a_n = S$ (constant). Then the product $a_1 a_2 \cdots a_n$ is greatest when $a_1 = a_2 = \cdots = a_n = \frac{S}{n}$.

[illegible]

Suppose $a_1, a_2, \ldots, a_n$ are positive integers and $a_1 + a_2 + \cdots + a_n = S$ (constant). Then the product $a_1 a_2 \cdots a_n$ is greatest when $a_1 = a_2 = \cdots = a_n = \frac{S}{n}$.

[illegible]

Suppose $a_1, a_2, \ldots, a_n$ are positive integers and $a_1 + a_2 + \cdots + a_n = S$ (constant). Then the product $a_1 a_2 \cdots a_n$ is greatest when $a_1 = a_2 = \cdots = a_n = \frac{S}{n}$.

[illegible]

Suppose $a_1, a_2, \ldots, a_n$ are positive integers and $a_1 + a_2 + \cdots + a_n = S$ (constant). Then the product $a_1 a_2 \cdots a_n$ is greatest when $a_1 = a_2 = \cdots = a_n = \frac{S}{n}$.

[illegible]

Suppose $a_1, a_2, \ldots, a_n$ are positive integers and $a_1 + a_2 + \cdots + a_n = S$ (constant). Then the product $a_1 a_2 \cdots a_n$ is greatest when $a_1 = a_2 = \cdots = a_n = \frac{S}{n}$.

[illegible]

Suppose $a_1, a_2, \ldots, a_n$ are positive integers and $a_1 + a_2 + \cdots + a_n = S$ (constant). Then the product $a_1 a_2 \cdots a_n$ is greatest when $a_1 = a_2 = \cdots = a_n = \frac{S}{n}$.

[illegible]

Suppose $a_1, a_2, \ldots, a_n$ are positive integers and $a_1 + a_2 + \cdots + a_n = S$ (constant). Then the product $a_1 a_2 \cdots a_n$ is greatest when $a_1 = a_2 = \cdots = a_n = \frac{S}{n}$.

[illegible]

Definition of Solution of an Inequality

[illegible]

Equivalent Inequalities: Two inequalities are said to be equivalent if any solution of one is also a solution of the other and vice versa.

[illegible]

Notation of Ranges

1. Ranges Where the Ends are Excluded If the value of x is denoted as $(1, 2)$ it means $1 < x < 2$ i.e. x is greater than 1 but smaller than 2.

Similarly, if we denote the range of values of x as $(-\infty, -2) \cup (3, 2)$, this means that the value of x can be denoted as $-\infty < x < -2$ and $3 < x < 2$. This would mean that the inequality is satisfied between the two ranges and is not satisfied outside these two ranges.

Based on the notation write the ranges of x for the following representations

$$(1, +\infty) \cup (-\infty, -7)$$
$$(-\infty, 10) \cup (4, +\infty) \cup (-\infty, 50) \cup (-20, +\infty)$$

2. Ranges Where the Ends are Included

$$(2, 5] \text{ means } 2 \le x \le 5$$

3. Mixed Ranges

$$(5, 21] \text{ means } 5 < x \le 21$$

Solving Linear Inequalities in one Unknown

A linear inequality is defined as an inequality of the form

$$ax + b \; I \; 0$$

where the symbol 'I' represents any of the inequalities $<, >, \le, \ge$.

For instance if $ax + b \le 0$, then $ax \le -b$
$\rightarrow x \le -b/a$ if $a > 0$ and $x \ge -b/a$ if $a < 0$

Example: Solve the inequality $3(x - 2) - 1 > 2(x - 2x + 1)$

$\rightarrow 3x - 7 > 2x - 6 - 4x - 4 \rightarrow 5x > -3$. Hence, $x > -1$

This can be represented in mathematical terms as $(-1, +\infty)$

Example: Solve the inequality $3(x - 1) + 1 > 3 - (1 - 2x)$
$\rightarrow 2x - 1 > 2 + 2x \rightarrow 0x > 3 \rightarrow$ This can never happen. Hence, no solution.

Example: Solve the inequality $2(x - 1) + 1 < 3 - (1 - 2x)$

Given

$$0x < 2.$$

This is true for all values of x.

Example: Solve the inequality $ax > a$.

This inequality has the parameter a that needs to be investigated further.

If $a > 0$, then $x > 1$
If $a < 0$, then $x < 1$

Solving Quadratic Inequalities

A quadratic inequality is defined as an inequality of the form

$$ax^2 + bx + c \; I \; 0 \; (a \ne 0)$$

where the symbol 'I' represents any of the inequalities $<, >, \le, \ge$.

For a quadratic expression of the form $ax^2 + bx + c$, $b^2 - 4ac$ is defined as the discriminant of the expression and is often denoted as D, i.e. $D = b^2 - 4ac$.

The following cases are possible for the value of the quadratic expression:

Case 1: If $D < 0$

1. If $a < 0$ then $ax^2 + bx + c < 0$ for all x
2. If $a > 0$ then $ax^2 + bx + c > 0$ for all x.

In other words, we can say that if D is negative then the values of the quadratic expression takes the same sign as the coefficient of x^2.

This can also be said as

If $D < 0$ then all real values of x are solutions of the inequalities $ax^2 + bx + c > 0$ and $ax^2 + bx + c \ge 0$ for $a > 0$ and have no solution in case $a < 0$.

Also, for $D < 0$, all real values of x are solutions of the inequalities $ax^2 + bx + c < 0$ and $ax^2 + bx + c \le 0$ if $a < 0$ and these inequalities will not give any solution for $a > 0$.

Case 2: $D = 0$

If the discriminant of a quadratic expression is equal to zero, then the value of the quadratic expression takes the same sign as that of the coefficient of x^2 except when $x = -b/2a$ at which point the value of the quadratic expression becomes 0.

We can also say the following for $D = 0$

1. The inequality $ax^2 + bx + c > 0$ has as a solution any $x \ne -b/2a$ if $a > 0$ and has no solution if $a < 0$.
2. The inequality $ax^2 + bx + c < 0$ has as a solution any $x \ne -b/2a$ if $a < 0$ and has no solution if $a > 0$.
3. The inequality $ax^2 + bx + c \ge 0$ has as a solution any x if $a > 0$ and has a unique solution $x = -b/2a$ if $a < 0$.

4. The inequality $ax^2 + bx + c \le 0$ has no solution, if $a \ne 0$ and $a < 0$ and has a unique solution $x = -b/2a$ for $a = 0$.

Case 3: $D > 0$

If x_1 and x_2 are the roots of the quadratic expression then it can be said that:

1. For $a > 0$, $ax^2 + bx + c$ is positive for all values of x outside the interval $[x_1, x_2]$ and is negative for all values of x within the interval (x_1, x_2). Besides for values of $x = x_1$ or $x = x_2$, the value of the quadratic expression becomes zero (By definition of the roots).
2. For $a < 0$, $ax^2 + bx + c$ is negative for all values of x outside the interval $[x_1, x_2]$ and is positive for all values of x within the interval (x_1, x_2). Besides for values of $x = x_1$ or $x = x_2$, the value of the quadratic expression becomes zero (By definition of the roots).

Now we give examples illustrating how quadratic inequalities are solved.

Solve the following inequalities.

Example 1: $x^2 - 5x + 6 > 0$

Solution: (a) The discriminant $D = 25 - 4 \times 6 > 0$ and a is positive (=1), the roots of the quadratic expression are real and distinct: $x_1 = 2$ and $x_2 = 3$.

By the property of quadratic inequalities, we get that the expression is positive outside the interval $[2, 3]$. Hence, the values $x \le 2$ and $x > 3$.

We can also see it as $x^2 - 5x + 6 = (x - 2)(x - 3)$ and the given inequality takes the form $(x - 2)(x - 3) > 0$.

The solutions of the inequality are the numbers $x < 2$ (when both factors are negative and their product is positive) and the numbers $x > 3$ (when both factors are positive and hence, their product is also positive).

Answer: $x < 2$ and $x > 3$.

Example 2: $2x^2 + x + 1 \ge 0$

Solution: The discriminant $D = 1 - 4 \cdot (-2) = 9 > 0$, the roots of the quadratic expression are real and distinct:

$$x_{1,2} = \frac{-1 \pm \sqrt{9}}{2 \cdot (-2)} = \frac{-4 \pm 3}{-4}$$

Hence, $x_1 = -1/2$, and $x_2 = 1$, and consequently, $2x^2 + x + 1 = 2(x + 1/2) \times (x - 1)$. We have

$$-2(x + 1/2)(x - 1) \ge 0 \text{ or } (x + 1/2)(x - 1) \le 0$$

(When dividing both sides of an inequality by a negative number, the sense of the inequality is reversed). The inequality is satisfied by all numbers from the interval

$$[-1/2, 1]$$

Please note that this can also be concluded from the property of quadratic expressions when $D > 0$ and a is negative.

Answer: $-1/2 \le x \le 1$.

Example 3: $2x^2 + x - 1 < 0$

Solution: $D = 1 - 4 \cdot (-2) \cdot (-1) < 0$, the coefficient of x^2 is negative. By the property of the quadratic expression when $D < 0$ and a is negative $-2x^2 + x - 1$ attains only negative values.

Answer: x can take any value.

Example 4: $3x^2 - 4x + 5 < 0$

Solution: $D = 16 - 4 \times 3 \times 5 < 0$, the coefficient of x^2 is positive. The quadratic expression $3x^2 - 4x + 5$ takes on only positive values.

Answer: There is no solution.

Example 5: $4x^2 + 4x + 1 > 0$

Solution: $D = 16 - 4 \times 4 = 0$. The quadratic expression $4x^2 + 4x + 1$ is the square $(2x + 1)^2$, and the given inequality takes the form $(2x + 1)^2 > 0$. It follows that all real numbers x except for $x = -1/2$, are solutions of the inequality.

Answer: $x \ne -1/2$.

Example 6: Solve the inequality $(a - 2)x^2 - x - 1 \ge 0$.

Here, the value of the determinant $D = 1 - 4(-1)(a - 2) = 1 + 4(a - 2) = 4a - 7$.

There can then be three cases:

Case 1: $D < 0 \to a < 7/4$

Then the coefficient of $x^2 \to a - 2$ is negative.

Hence, the inequality has no solution.

Case 2: $D = 0$

$a = 7/4$. Put $a = 7/4$ in the expression and then the inequality becomes

$-(x - 2)^2 \ge 0$. This can only happen when $x = 2$.

Case 3: $D > 0$

Then $a > 7/4$ and $a \ne 2$, then we find the roots x_1 and x_2 of the quadratic expression:

$$x_1 = \frac{1 + \sqrt{4a - 7}}{2(a - 2)} \quad \text{and} \quad x_2 = \frac{1 - \sqrt{4a - 7}}{2(a - 2)}$$

Using the property of quadratic expression's values that $D > 0$ we get

If $a - 2 < 0$, the quadratic expression takes negative values outside the interval $[x_1, x_2]$. Hence, it will take positive values inside the interval (x_1, x_2).

If $a - 2 > 0$, the quadratic expression takes positive values outside the interval $[x_1, x_2]$ and becomes zero for x_1 and x_2.

If $a - 2 = 0$, then we get a straight linear equation $-x - 1 = 0 \Rightarrow x = -1$.

System of Inequalities in One Unknown

Let there be given several inequalities in one unknown. If it is required to find the number that will be the solution of all the given equalities, then the set of these inequalities is called a *system of inequalities*.

The solution of a system of inequalities in one unknown is defined as the value of the unknown for which all the inequalities of the system reduce to true numerical inequalities.

To solve a system of inequalities means to find all the solutions of the system or to establish that there is none.

Two systems of inequalities are said to be equivalent if any solution of one of them is a solution of the other, and vice versa. If both the systems of inequalities have no solution, then they are also regarded to be equivalent.

Example 1: Solve the system of inequalities:

$$3x - 4 < 8x + 6$$
$$3x - 1 > 5x - 4$$
$$11x - 9 \le 15x + 3$$

Solution: We solve the first inequality:

$$3x - 4 < 8x + 6$$
$$-5x < 10$$
$$x > -2$$

It is fulfilled for $x > -2$.

Then we solve the second inequality:

$$3x - 1 > 5x - 4$$
$$-3x > -3$$
$$x < 1$$

It is fulfilled for $x < 1$.

And, finally, we solve the third inequality:

$$11x - 9 \le 15x + 3$$
$$-4x \le 12$$
$$x \ge -3$$

It is fulfilled for $x \ge -3$. All the given inequalities are true for $-2 < x < 1$.

Answer: $-2 < x < 1$.

Example 2: Solve the inequality $\frac{2x-1}{x+1} < 1$

We have $\frac{2x-1}{x+1} - 1 < 0 \rightarrow \frac{x-2}{x+1} < 0$

This means that the fraction above has to be negative. A fraction is negative only when the numerator and the denominator have opposite signs.

Hence, the above inequality is equivalent to the following set of 2 inequalities:

$$x - 2 > 0 \quad \text{and} \quad x - 2 < 0$$
$$\text{and} \quad x + 1 < 0 \qquad x + 1 > 0$$

From the first system of inequalities, we get $x > 2$ or $x < -1$. This cannot happen simultaneously since these are inconsistent.

From the second system of inequalities we get

$$x < 2 \text{ or } x > -1 \text{ i.e. } -1 < x < 2$$

Inequalities Containing a Modulus

Result:

$|x| \le a$, where $a > 0$ means the same as the double inequality

$$-a \le x \le a$$

This result is used in solving inequalities containing a modulus.

Example 1: $|2x - 3| \le 5$

This is equivalent to $-5 \le 2x - 3 \le 5$

i.e.

$$2x - 3 \ge -5 \quad \text{and} \quad 2x - 3 \le 5$$
$$2x \ge -2 \qquad\qquad x \le 4$$
$$x \ge -1$$

The solution is

$$-1 \le x \le 4$$

Example 2: $|1 - x| > 3$

$$|1 - x| = |x - 1|$$

Hence,

$$|x - 1| > 3 \rightarrow x - 1 > 3 \text{ i.e. } x > 4$$

or

$$x - 1 < -3 \text{ or } x < -2$$

Answer: $x > 4$ or $x < -2$

WORKED-OUT PROBLEMS

Problem 13.1 Solve the inequality $\frac{1}{x} < 1$

Solution: $\frac{1}{x} < 1 \Leftrightarrow \frac{1}{x} - 1 < 0 \Leftrightarrow \frac{1-x}{x} < 0 \Leftrightarrow \frac{x-1}{x}$

This can happen only when both the numerator and denominator take the same sign (Why?)

Case 1: Both are positive: $x - 1 > 0$ and $x > 0$ (i.e. $x > 1$)

Case 2: Both are negative: $x - 1 < 0$ and $x < 0$ (i.e. $x < 0$)

Answer: $(-\infty, 0) \cup (1, +\infty)$

Problem 13.2 Solve the inequality $\frac{x}{x+2} \le \frac{1}{x}$

Solution: $\frac{x}{x+2} \le \frac{1}{x} \Leftrightarrow \frac{x}{x+2} - \frac{1}{x} \le 0 \Leftrightarrow \frac{x^2 - x - 2}{x(x+2)}$

$\Leftrightarrow \frac{(x-2)(x+1)}{x(x+2)} \le 0.$

The function $f(x) = \frac{(x-2)(x+1)}{x(x+2)}$ becomes negative when numerator and denominator are of opposite signs.

Case 1: Numerator positive and denominator negative. This occurs only between $-2 < x < -1$.

Case 2: Numerator negative and Denominator Positive: Numerator is negative when $(x-2)$ and $(x+1)$ take opposite signs. This can be got for:

Case 1: $x - 2 < 0$ and $x + 1 > 0$ (i.e. $x < 2$ and $x > -1$)

Case 2: $x - 2 > 0$ and $x + 1 < 0$ (i.e. $x > 2$ and $x < -1$). Cannot happen.

Hence the answer is $-2 < x \le 2$.

Problem 13.3 Solve the inequality $\frac{x}{x-3} \le \frac{1}{x}$

Solution: $\frac{x}{x-3} \le \frac{1}{x} \Leftrightarrow \frac{x}{x-3} - \frac{1}{x} \ge 0 \Leftrightarrow \frac{x^2 - x + 3}{x(x-3)}$

Let the function $f(x) = \frac{x^2 - x + 3}{x(x-3)}$

The numerator being a quadratic equation with $D < 0$ and $a > 0$, we see that it will always be positive for all values of x. (From the property of quadratic inequalities).

Further, for the expression to be negative, the denominator should be negative.

That is, $x^2 - 3x < 0$. This will occur when $x < 3$ and x is positive.

Answer: $(0, 3)$

Suppose $F(x) = (x - x_1)^{k_1}(x - x_2)^{k_2} \ldots (x_n - x_n)^{k_n}$ where $k_1, k_2, \ldots k_n$ are integers. If k_1 is an even number, then the function $(x - x_1)^{k_1}$ does not change sign when x passes through the point x_1 and, consequently, the function $F(x)$ does not change sign. If k_1 is an odd number, then the function $(x - x_1)^{k_1}$ changes sign when x passes through the point x_1 and, consequently, the function $F(x)$ also changes sign.

Problem 13.4 Solve the inequality $(x-1)^2(x+1)^3(x-4) < 0$

Solution The above inequality is valid for

Case 1: $x + 1 < 0$ and $x - 4 > 0$

That is, $x < -1$ and $x > 4$ simultaneously. This cannot happen together.

Case 2: $x + 1 > 0$ and $x - 4 < 0$

That is $x > -1$ or $x < 4$, i.e. $-1 < x < 4$ Answer

Problem 13.5 Solve the inequality $\frac{(x-1)^2(x+1)^3}{x^2(x-2)} \le 0$

Solution The above expression becomes negative when the numerator and the denominator take opposite signs.

That means that the numerator is positive and the denominator is negative or vice versa.

Case 1: Numerator positive and denominator negative. The sign of the numerator is determined by the value of $x + 1$ and that of the denominator is determined by $x - 2$.

This condition happens when $x < 2$ and $x > -1$ simultaneously.

Case 2: Numerator negative and denominator positive. This happens when $x < -1$ and $x > 2$ simultaneously. This will never happen.

Hence, the answer is $-1 \le x < 2$.

Level of Difficulty (LOD) I

Solve the following inequalities:

1. $3x^2 - 3x + 4 \le 0$

(a) $x > 0$ (b) $x \le 0$

(c) All x (d) No solution
(e) None of these

2. $3x^2 - 5x - 6 < 0$
(a) $-0.66 < x < 3$ (b) $x < -0.66$ or $x > 3$
(c) $3 < x < 7$ (d) $-2 < x < 2$
(e) None of these

3. $3x^2 - 3x + 6 < 0$
(a) $0.66 < x < 3$ (b) $-0.66 < x < 3$
(c) $-1 < x < 3$ (d) $-0.66 < x < 0.66$
(e) None of these

4. $x^2 - 3x + 3 > 0$
(a) $x > 0$ (b) $x < 0$
(c) Both (a) and (b) (d) $-\infty < x < \infty$
(e) None of these

5. $x^2 - 14x - 15 > 0$
(a) $x < -1$ (b) $15 < x$
(c) Both (a) and (b) (d) $-1 < x < 15$
(e) None of these

6. $2 - x - x^2 \geq 0$
(a) $-2 \leq x \leq 1$ (b) $-2 < x < 1$
(c) $x < -2$ (d) $x > 1$

7. $|x^2 - 4x| < 5$
(a) $-1 \leq x \leq 5$ (b) $1 \leq x \leq 5$
(c) $-5 \leq x \leq 1$ (d) $-1 < x < 5$
(e) None of these

8. $|x^2 + x| + 5 < 0$
(a) $x < 0$ (b) $x > 0$
(c) All values of x (d) $x > 5$
(e) None of these

9. $|x^2 - 3x| < 6$
(a) $-1 < x < 2$ (b) $-3 < x < 6$
(c) Both (a) and (b) (d) $-1 < x < 6$
(e) None of these

10. $|x^2 - 2x| < x$
(a) $1 < x < 3$ (b) $-1 < x < 3$
(c) $0 < x < 4$ (d) $x > 3$
(e) None of these

11. $|x^2 - 2x - 3| < 3x - 3$
(a) $1 < x < 5$ (b) $-2 < x < 3$
(c) $x > 5$ (d) $2 < x < 5$
(e) None of these

12. $|x^2 - 3x| + x - 2 < 0$
(a) $(1 - \sqrt{5}) > x < (2 + \sqrt{2})$
(b) $0 < x < 5$
(c) $(1 - \sqrt{5}, 2 - \sqrt{2})$
(d) $1 < x < 4$
(e) None of these

13. $x^2 - 3x + 12 < |x - 4|$
(a) $x < 2$ (b) $x > 4$
(c) $2 < x < 4$ (d) $2 \leq x \leq 4$
(e) $1 < x < 4$

14. $x^2 - |5x - 3| - x < 2$
(a) $x > 1 + 2\sqrt{2}$ (b) $x < 3 + 2\sqrt{2}$
(c) $x > -5$ (d) $-5 < x < 3 + 2\sqrt{2}$
(e) None of these

15. $|x - 6| > x^2 - 5x + 9$
(a) $1 \leq x < 3$ (b) $1 < x < 3$
(c) $2 \leq x \leq 5$ (d) $-3 < x < 1$
(e) None of these

16. $|x - 6| < x^2 - 5x + 9$
(a) $x < 1$ (b) $x > 3$
(c) $1 < x < 3$ (d) Both (a) and (b)
(e) $x \geq 1$ & $x \geq 3$

17. $|x - 2| \leq 2x^2 - 4x + 9$
(a) $x > (4 - \sqrt{2})/2$ (b) $x < (5 + \sqrt{5})/2$
(c) Both (a) and (b) (d) $x \geq (4 - \sqrt{2})/2$
(e) None of these

18. $3x^2 - |x - 3| > 9x - 2$
(a) $x < (4 - \sqrt{19})/3$ (b) $x > (4 + \sqrt{19})/3$
(c) Both (a) and (b) (d) $-2 < x < 2$
(e) None of these

19. $x^2 - 15x + 81 > 0$
(a) $x < (5 - \sqrt{57})/2$ (b) $x < (5 + \sqrt{57})/2$
(c) $x > (5 + \sqrt{57})/2$ (d) $x > \frac{(5 - \sqrt{57})}{2}$
(e) Both (a) and (b)

20. $3|x - 1| + x^2 - 2 > 0$
(a) $x > -1$ (b) $x < -1$
(c) $x > 2$ (d) Both (b) and (c)
(e) None of these

21. $|x - 6| > |x^2 - 5x + 6|$
(a) $x < 1$ (b) $x > 3$
(c) $(1 < x < 2)$ (d) Both (a) and (b)
(e) None of these

2. $(x-1)(x-3)(x+2)(x-5) < 0$

(a) $-2 < x < -1$ and $3 < x < 5$

(b) $x < -2$ and $x > 4$

(c) $x < -2$ and $x > 3$

(d) Any of these

(e) None of these

3. $x^2 - 2x - 15 \geq 2x$

(a) $x \geq 2\sqrt{3}$ (b) $x < 3$ or $x \geq \sqrt{5}$

(c) $x \geq 2 + 2\sqrt{3}$ (d) Both (a) and (c)

(e) None of these

4. $(x+1)\sqrt{x^2 - x - 2} \geq 0$

(a) $x \geq 2$ (b) $x \geq 2$

(c) $x \leq -2$ (d) $x \geq 0$

(e) None of these

5. $x^2 - 4x\sqrt{x^2 - x - 2} \geq 0$

(a) $x \leq -1$ (b) $x \geq -1$

(c) $x \geq 2$ (d) (a) and (c)

(e) None of these

6. $\sqrt{\frac{x+1}{1-2x}} > -x$

(a) $0 \leq x$ (b) $x > 2$

(c) Both (a) and (b) (d) $0.5 < x \leq 2$

(e) None of these

7. $\sqrt{\frac{3x-1}{2-x}} > 1$

(a) $0 < x < 2$ (b) $-0.75 < x < 4$

(c) $0.75 < x < 3$ (d) $0 < x < 4$

(e) None of these

8. $\sqrt{x^2 - 10x} > \sqrt{6 - x}$

(a) $4 < x \leq 6$ (b) $x < 4$ or $x > 6$

(c) $x < 4$ (d) $x > 5$

(e) $4 < x < 6$

9. $\sqrt{x^2 - 2x - 3} > 1$

(a) $x \leq -\sqrt{8}$ or $x > 3$ (b) $x \leq x < (\sqrt{8} - 1)$

(c) $x > 1$ (d) Both (c) and (b)

(e) None of these

10. $\sqrt{x^2 - \frac{x+2}{x^2}} < \frac{2}{3}$

(a) $x \in (0, 1) \cup x \leq -1$ or $2 \leq x < 3$

(b) $x \in (0, 1)$

(c) $2 \leq x < 3$

(d) $x \in (0, 1)$ or $x \leq -3$

(e) None of these

11. $2\sqrt{x - 1} < 3$

(a) $x > 1$ (b) $x \geq 1, x \neq 2$

(c) $x < 1$ (d) $1 < x < 5$

(e) None of these

12. $\sqrt{x + 18} < 2 - x$

(a) $x \leq -18$ (b) $x \leq -2$

(c) $x > -2$ (d) $-18 \leq x < -2$

(e) None of these

13. $x > \sqrt{24 + 5x}$

(a) $x < 3$ (b) $3 < x \leq 4.8$

(c) $x \geq 240/5$ (d) $x > 8$

(e) None of these

14. $\sqrt{8x - 32} < x$

(a) $4 < x < 8$ (b) $5.09 \leq x < 8$

(c) $x > 5$ (d) Both (b) and (c)

(e) None of these

15. $\sqrt{x + 7} < x$

(a) $x > 2$ (b) $x > \sqrt{30}/2$

(c) $x \geq (1 + \sqrt{29})/2$ (d) $x > 1 + \sqrt{30}/2$

(e) None of these

16. $\sqrt{2x - 1} < x - 2$

(a) $x < 3$ (b) $x > 5$

(c) $x > 3$ or $x < -5$ (d) $5 < x < 13$

17. $\sqrt{x + 16} < x + 6$

(a) $x < 2$ (b) $x > 3$ or $x > 2$

(c) $x > 2$ (d) $2 < x < 10$

18. $\sqrt{3 - 2x} < 5x - 2$

(a) $0.5 < x$ (b) $x < 2.5$

(c) $0.5 < x < 2.5$ (d) $x > 2.5$

19. $\sqrt{x + 65} < x + 5$

(a) $x < 3$ (b) $x > 1$ or $x > 4$

(c) $x > 5$ (d) $5 < x < 13$

20. $x > \sqrt{x^{x+1} - 3}$

(a) $x > 1$ (b) $x > 4$

(c) $-2 < x < 4$ (d) $4 < x$

41. $x+5<\sqrt{x+21}$

(a) $x>3$ (b) $x\geq 3$
(c) $-3\leq x\leq 3$ (d) $-21<x<3$

42. $\sqrt{2x+11}>x+5$

(a) $x\leq -3$ (b) $-7\leq x<1$
(c) $x>3$ (d) $-7<x<1$

43. $x-3>\sqrt{x-7}$

(a) $2\leq x<(7+\sqrt{5})/2$ (b) $2\leq x$
(c) $x<(7+\sqrt{5})/2$ (d) $x\leq 2$

44. $x-2\geq\sqrt{x+14}$

(a) $-14\leq x<2$ (b) $x\geq -14$
(c) $x\geq 2$ (d) $-14<x<2$

45. $x-1\geq\sqrt{7-x}$

(a) $x\geq 4$ (b) $x<3$
(c) $-33<x<3$ (d) $-33<x<3$

46. $\sqrt{5x-20}>x$

(a) $x<4$ (b) $x<3$
(c) $x\leq 4$ or $x\geq 5$ (d) $4<x<5$

47. $\sqrt{11-3x}>x+1$

(a) $x<2, x<5$ (b) $-3<x<2$
(c) $-2<x<2$ (d) $x<2$

48. $\sqrt{x+2}>x$

(a) $-2\leq x\leq 2$ (b) $-2\leq x$
(c) $x<2$ (d) $x<-2$ or $x>2$

Directions for questions 49–52: Find the largest integral x that satisfies the following inequalities.

49. $\dfrac{x-3}{x^2-9}<0$

(a) $x<-4$ (b) $x<-2$
(c) $x<3$ (d) None of these

50. $\dfrac{2}{x+1}-\dfrac{1}{x^2-x+1}<\dfrac{3-2x}{x^3+1}$

(a) $x<4$ (b) $x<2$
(c) $x<-2$ (d) None of these

51. $\dfrac{x+4}{x^2-9}-\dfrac{2}{x+3}<\dfrac{4x}{3x-x^2}$

(a) $x<4$ (b) $x<3$
(c) $x<-1$ (d) None of these

52. $\dfrac{4x+19}{x+5}<\dfrac{4x-17}{x-3}$

(a) $x<1$ (b) $x<2$
(c) $x<-1$ (d) None of these

53. $(x+1)(x-3)^2(x-5)(x-4)^3(x-2)<0$

(a) $x<1$ (b) $x<-2$
(c) $x<-1$ (d) None of these

Directions for questions 54–60: Solve the following inequalities.

54. $(x-1)(3-x)(x-2)^2>0$

(a) $1<x<3$ (b) $1<x<3$ but $x\neq 2$
(c) $0<x<2$ (d) $-1<x<3$

55. $\dfrac{4x-5}{4x+1}\geq 0$

(a) $-1/4<x<1$ (b) $-2/2<x<1$
(c) $-1\leq x\leq 1$ (d) $-1/4<x<5/4$

56. $\dfrac{2x-3}{3x-7}\geq 0$

(a) $x<3/2$ or $x>7/3$ (b) $3/2<x<7/3$
(c) $x>7/3$ (d) None of these

57. $\dfrac{5}{x-2}<1$

(a) $1<x<5$ (b) $x<2$
(c) $x>5$ (d) $x<2$ or $x>5$

58. $\dfrac{1}{x-1}\geq 2$

(a) $x<1$ (b) $x\geq 1.5$
(c) $-5<x<1$ or $x\geq 1.5$ (d) Both (a) and (b)

59. $\dfrac{4x+3}{2x-5}<6$

(a) $x<2.5$ (b) $x<23/8$
(c) $x>2.5$ (d) $x<2.5$ or $x>23/8$

60. $\dfrac{3x-6}{x+6}<3$

(a) $-6<x<6$ (b) $-6<x<0$
(c) $-6<x<3$ (d) None of these

61. $\dfrac{5x+8}{4-x}<3$

(a) $x<0$ or $x>4$ (b) $0<x<4$
(c) $0\leq x<4$ (d) $x>4$

(c) $\frac{x-4}{x+3} > 2$

(a) $x < -7$ (b) $x < -3$
(c) $-7 < x < -3$ (d) None of these

(d) $\frac{3x-5}{x+1} > 3$

(a) $-17/25 < x < -3/8$ (b) $x > -17/25$
(c) $0 < x < 3/8$ (d) $-17/25 < x < 0$

4. $\frac{x}{x-5} > \frac{3}{2}$

(a) $-3 < x < 5$ (b) $-5 < x < 0$
(c) $-5 \le x \le 5$ (d) $x < -5$ or $x > 5$

5. $x \le \frac{8}{x-5}$

(a) $x \le -4$ (b) $x > 5$
(c) $x \ge 0$ (d) $x \le -1$ or $5 < x \le 8$

6. $\frac{3(x-3)}{x-2} \ge 25(x+2)$

(a) $x < -1/4$ or $x > 2$
(b) $x \le -1/4$ or $2 < x \le 1/6$
(c) $x \ge -1/4$ or $2 < x \le 2/6$
(d) None of these

7. $\frac{x}{x+2} > 3 - x$

(a) $-2 < x < -1$ or $x > 2$
(b) $-2 \le x \le 2$
(c) $-2 < x < -1$
(d) $0 \le x \le 3$

8. $x - 17 \ge \frac{60}{x}$

(a) $x < -3$
(b) $x \ge 20$
(c) $-3 \le x < 0$ or $x \ge 20$
(d) $-3 < x \le 0$ or $x \ge 20$

9. $\frac{\sqrt{x}}{x+1} < x + 1$

(a) $x > 0.5$ (b) $x > 0$
(c) All x (d) $x > -0.5$

10. Find the smallest integral x satisfying the inequality

$$\frac{x-5}{x^2+3x-14} \le 0$$

(a) $x > 4$ (b) $x > -3$
(c) $x < -3$ (d) None of these

Level of Difficulty (LOD) I

1. $x^2 - 3|x| + 6 < 0$

(a) $-3 < x < -2$ (b) $2 < x < 3$
(c) Both (a) and (b) (d) $-3 < x < 3$
(e) $-2 < x < 2$

2. $x^2 - |x| - 2 \ge 0$

(a) $-2 < x < 2$ (b) $x \le -2$ or $x \ge 2$
(c) $x < -2$ or $x > 1$ (d) $-2 \le x \le 1$
(e) None of these

Directions for Questions 3–16: Solve the following polynomial inequalities.

3. $(x-1)(3-x)(x-2)^2 > 0$

(a) $1 < x < 2$ (b) $-1 < x < 3$
(c) $-3 < x < -1$ (d) $1 < x < 3, x \ne 2$
(e) $1 \le x \le 3$

4. $\frac{x^2}{x - x^2 - 1} < 0$

(a) $x > 0$ (b) $x \le 0$
(c) $x \ge 0$ (d) $x < 0$
(e) For all real x

5. $\frac{x^2 - 5x + 6}{x^2 + x - 1} < 0$

(a) $x < 2$ (b) $x > 3$
(c) $2 < x < 3$ (d) $x < 2$ or $x > 3$
(e) None of these

6. $\frac{x^2 + 2x - 3}{x^2 + 1} < 0$

(a) $x < -3$ (b) $-7 \le x \le -3$
(c) $-3 \le x \le 1$ (d) $-7 < x < 1$
(e) None of these

7. $\frac{(x-1)(x+2)^2}{-1-x} < 0$

(a) $x < -1$
(b) $x < -1$ or $x > 1$
(c) $x < -1$ and $x \ne 2$
(d) $x < -1$ or $x > 1$ and $x \ne 2$
(e) None of these

8. $\frac{x^2 + 4x + 4}{3x^2 - x - 1} > 0$

(c) $x < -2$ (d) $x = 1$
(e) $x \neq 2$ (f) All of the above
(g) None of these

4. $x^3 - 3x^2 + 4 < 0$
(a) $-2 < x < 1$
(b) $-2 < x < 2$
(c) $-2 < x < -1$ or $1 < x < 2$
(d) $1 < x < 2$
(e) None of these

10. $x^4 - 3x^2 - 63 \leq 0$
(a) $x \leq -3$ or $x \geq 3$ (b) $-3 \leq x \leq 0$
(c) $0 \leq x \leq 3$ (d) $-3 \leq x \leq 3$
(e) None of these

Directions for Questions 11–67: Solve the following polynomial and quadratic inequalities.

11. $\frac{2x-1}{x^2+3} < 1$
(a) $x < 4$ (b) $1 < x < 4$
(c) $x < 1$ or $x > 4$ (d) $1 < x < 3$
(e) None of these

12. $\frac{x-2}{x^2+1} < -\frac{1}{2}$
(a) $-3 < x < 3$ (b) $x < -3$
(c) $-3 < x < 0$ (d) $-3 < x < 1$
(e) $1 < x < 3$

13. $\frac{x+1}{(x-3)^2} < 1$
(a) $x > 3$ or x is negative
(b) $x > 3$
(c) $x > 3$ or $-25 < x < 0$
(d) x is negative and $x > 2$
(e) $x < 3$

14. $\frac{x^2-7x+12}{2x^2+4x+3} > 0$
(a) $x < 3$ or $x > 4$ (b) $3 < x < 4$
(c) $4 < x < 24$ (d) $0 < x < 3$
(e) $0 < x < 4$

15. $\frac{x^2+6x-7}{x^2+1} \leq 2$
(a) x is negative (b) $x \geq 0$
(c) $x \geq 0$ or $x < 0$ (d) Always
(e) None

16. $\frac{x^4+x^2+1}{x^2-4x-5} < 0$
(a) $x < -1$ or $x > 5$ (b) $-1 < x < 5$
(c) $x < 5$ (d) $-5 < x < -1$
(e) $-4 < x < 6$

17. $\frac{1+3x^2}{2x^2-21x+40} > 0$
(a) $0 < x < 5$ (b) $2.5 < x < 8$
(c) $-8 < x < 8$ (d) $5 < x < 8$
(e) None of these

18. $\frac{1+x^2}{x^2-5x+6} < 0$
(a) $x < 2$ (b) $x > 3$
(c) Both a and b (d) $2 < x < 3$
(e) None of these

19. $\frac{x^4+x^2+1}{x^2-4x-5} > 0$
(a) $-4 < x < 5$ (b) $x < -1$ or $x > 5$
(c) $x \leq -1$ or $x > 5$ (d) $-1 < x < 1$
(e) $1 < x < 5$

20. $\frac{1-2x-3x^2}{3x-x^2-3} > 0$
(a) $x < -1$ or $x > 1/3$ (b) $x < -1$ or $x > 1/3$
(c) $-1 < x < 1/3$ (d) $x < 1/3$
(e) None of these

21. $\frac{x^2-3x+7}{-2x^2+2x+2} > 0$
(a) $x > 0.5$ (b) $x > -0.5$
(c) $-0.5 < x < 5$ (d) $-0.5 < x < 2$
(e) $0.5 < x < 2$

22. $\frac{2x^2-3x-450}{x^2+1} > 1$
(a) $x > -20$ (b) $x < 0$
(c) $x < -20$ (d) $-20 < x < 20$
(e) $0 < x < 50$

23. $\frac{x^2-1}{x^2+x+1} < 1$
(a) $x > -2$ (b) $x > 2$
(c) $-2 < x < 2$ (d) $x < 2$
(e) None of these

7. $\frac{1-2x-3x^2}{3x-x^2-2} \geq 0$

(a) $1 \leq x \leq 2$ (b) $1 < x < 2$
(c) $-3 \leq x \leq 1$ (d) $-3 < x < 1$
(e) None of these

8. $\frac{x}{x^2-3x-4} \geq 0$

(a) $-1 < x < 0$ (b) $4 < x$
(c) both (a) and (b) (d) $-1 < x < 4$
(e) None of these

9. $\frac{x^2+7x+10}{x+3/2} \geq 0$

(a) $-4 \leq x \leq -2$ or $\frac{-3}{2} \leq x < \infty$
(b) $-2 < x < 8$
(c) $x < -2$
(d) $x > -2$
(e) None of these

10. $\frac{3x^2-4x-4}{2x-3} \leq 0$

(a) $x \in (2-\sqrt{22})/3$
(b) $x \geq (2+\sqrt{22})/3$
(c) $(2-\sqrt{22})/3 < x < (2+\sqrt{22})/3$
(d) Both (a) and (b)
(e) None of these

11. $\frac{17-13x-2x^2}{x+3} < 0$

(a) $-8.5 < x \leq -3$
(b) $-17 < x < -3$
(c) $-8.5 < x < -3$ or $x > 1$
(d) $-8.5 < x < 1$
(e) $-17 < x < -3$

12. $\frac{x^2-9}{3x+x^2-24} < 0$

(a) $-3 < x < 3$ (b) $x < -3$ or $x > 3$
(c) $x < -5$ or $x > 8$ (d) $x < -7$ or $x > 7$
(e) None of these

13. $\frac{x+3}{x-2} + \frac{3x+1}{2} \geq 0$

(a) $0 \leq x \leq 5$ (b) $-1 < x \leq 5$
(c) $-3 \leq x \leq 2$ or $x > 5$ (d) $-1 < x < 5$
(e) None of these

14. $2x^2 + \frac{1}{x} > 0$

(a) $x > 0$ (b) $x < -1/2$
(c) Both (a) and (b) (d) $x > -1/2$
(e) None of these

15. $\frac{x^2+x-6}{x^2+6x} \geq 0$

(a) $x < -6$ (b) $-2 \leq x < 0$
(c) $x > 3$ (d) All of these
(e) None

16. $\frac{x^2-3x+8}{x^2-11x+30} < 0$

(a) $x < 5$ or $x > 5$
(b) $2 < x < 4$ or $5 < x < 7$
(c) $2 < x < 3$ or $5 < x < 6$
(d) $2 < x < 3$ or $5 < x < 7$
(e) This is never true

17. $\frac{x^2-8x+7}{4x^2-4x+1} < 0$

(a) $x < 7$ or $x > 7$ (b) $1 < x < 7$
(c) $-7 < x < 1$ (d) $-7 < x < 7$
(e) $-1 < x < 7$

18. $\frac{x^2-36}{x^2-3x+18} < 0$

(a) $-6 < x < 3$ (b) $-6 < x < 6$
(c) $x < -6$ or $x > 3$ (d) $-3 < x < 3$
(e) $-3 < x < 6$

19. $\frac{x^2-4x+9}{5-4x-x^2} \geq 0$

(a) $-5 < x \leq 1$ or $x \neq 3$ (b) $-9 \leq x < 1$ or $x \neq 3$
(c) $-5 < x \leq 1$ or $x = 3$ (d) $-9 \leq x \leq -1$
(e) None of these

20. $\frac{x-1}{x+1} < x$

(a) $x < -1$ (b) $x > -1$
(c) $-1 < x < 1$ (d) For all real values of x
(e) None of these

21. $\frac{1}{x+2} < \frac{1}{x-3}$

(a) $-4.5 < x < -2$ (b) $-4.5 < x < -2$ or $3 < x$
(c) $-4.5 < x < -2$ or $x > 3$ (d) (b) or (c)
(e) None of these

39. $\frac{16x}{x+1} - \frac{9x-30}{x-4} \le 8$

(a) $-1 < x < 1$ or $4 < x < 8$
(b) $-1 < x < 4, 5 < x < 7$
(c) $1 < x < 4$ or $5 < x < 7$
(d) $-1 < x < 1$ or $5 < x < 7$
(e) $1 < x < 7$

40. $\frac{5x^2-2}{4x^2-x-3} < 1$

(a) $x < 1$
(b) $-2 < x < 2$
(c) $-2.5 < x < 1.25$
(d) $(-(1+\sqrt{21})/2 < x < (\sqrt{21}-1)/8$
(e) None of these

41. $\frac{x^2-5x+12}{x^2-4x+5} > 3$

(a) $x > 0.5$
(b) $1/2 < x < 3$
(c) $x < 0.5, x > 3$
(d) $1 < x < 3$
(e) $0.5 < x < 2$

42. $\frac{x^2-3x+24}{x^2-3x+3} < 4$

(a) $x < -1$
(b) $4 < x < 8$
(c) $x < 4$ or $x > 8$
(d) Both (a) and (b)
(e) None of these

43. $\frac{x^2-x}{2x+5} < 3$

(a) $x < -2.5$ or $-2 < x < 8$
(b) $-2.5 < x < -2$
(c) $-2.5 < x < 8$
(d) Both (a) and (b)
(e) None of these

44. $\frac{x^2+1}{4x-3} > 2$

(a) $x > 7$
(b) $x > 7, x < 87$
(c) $0.75 < x < 1, x > 7$
(d) $0.25 < x < 1, x > 7$
(e) $x > 7$

45. $\frac{x^2+2}{x^2-1} < -2$

(a) $-1 < x < 2$
(b) $-1 < x < 1$,
(c) $-1 < x < 0, 0 < x < 1$
(d) $-2 < x < 2$
(e) None of these

46. $\frac{3x-5}{x^2+4x-5} \ge \frac{1}{2}$

(a) $x < -5$
(b) $x > 1$
(c) $-5 < x < 1$
(d) $-5 < x < 5$
(e) None of these

47. $\frac{2x+3}{x^2+x-12} \le \frac{1}{2}$

(a) $-4 < x < -3, 3 < x < 6$
(b) $-4 < x < -3, 0 < x < 6$
(c) $x < -4, -3 < x < 3, x > 6$
(d) $x < -4, x > 6$
(e) None of these

48. $\frac{5-2x}{3x^2-2x-16} < 1$

(a) $x < -\sqrt{7}$
(b) $-2 < x < \sqrt{7}$
(c) $8/3 < x$
(d) All of these
(e) Never

49. $\frac{15-4x}{x^2+x-12} < 4$

(a) $x < -\sqrt{63}/2, -3 < x < \sqrt{63}/2$
(b) $x > 4$
(c) Both (a) and (b)
(d) $x > 4, x < -63/2$
(e) None of these

50. $\frac{1}{x^2-5x+6} \ge 2$

(a) $1 < x < 2, 3 < x < 4$
(b) $1 < x < 4$
(c) $x < 1, x > 3$
(d) $x < 1, x > 4$
(e) $2 < x < 4$

51. $\frac{5-4x}{3x^2-x-4} < 4$

(a) $x < -\frac{\sqrt{7}}{2}$
(b) $-1 < x < \frac{\sqrt{7}}{2}$
(c) $x > 4/3$
(d) All of these

52. $\frac{(x+2)(x^2-2x+1)}{4+3x-x^2} \ge 0$

(a) $x < -2$ or $-1 < x < 4$
(b) $-2 < x < 4$ or $x > 6$

(c) $-2 < x < -1$ or $x > 4$
(d) None of these

$$\frac{8}{2x} + \frac{2}{1-x} < 1$$

(a) $-1 < x < 1$ (b) $x < -1$
(c) $x > 1$ (d) Both (b) and (c)

$$x + 1(x+1)(x-2)x$$

(a) $x < -2$ (b) $-1 < x < 0$
(c) $1/2 < x$ (d) All of these

$$11. \frac{2}{x-1} > \frac{8}{x}$$

(a) $0 \le x \le 1$ (b) $2 \le x \le 3$
(c) $-\infty < x < 1$ (d) Always except (a) and (b)

$$\frac{x^3 - 3x^2 + 2x^2}{x^2 + x - 20} > 0$$

(a) $x < -5$ (b) $1 < x < 2$
(c) $x > 4$ (d) All of these

$$\frac{x-1}{x} - \frac{x+1}{x-1} < 3$$

(a) $-1 \le x \le 0$ (b) $1/2 \le x \le 1$
(c) $0 \le x < 1/2$ (d) Always except (a) and (b)

$$\frac{2x-3}{4(x-8)} \ge \frac{1}{x-1}$$

(a) $x < 0$ (b) $1 < x < 8$
(c) Both (a) and (b) (d) Always except (a) and (b)

$$\frac{2(x-6)}{(x-1)(x-7)} \ge \frac{1}{x-2}$$

(a) $1 < x < 2$ or $7 < x$ (b) $2 < x$
(c) $2 < x < 7$ (d) Both (a) and (c)

$$\frac{-2}{x-3} \le \frac{1}{x+2}$$

(a) $x < -3$ (b) $-2 < x < 3$
(c) All except (a) and (b) (d) Both (a) and (b)

$$\frac{1}{x-2} + \frac{1}{x-1} > \frac{1}{x}$$

(a) $-\sqrt{2} < x < 0$ or $2 < x$
(b) $\sqrt{2} < x$
(c) $1 < x < \sqrt{2}$
(d) Both (a) and (c)

$$\frac{7}{(x-2)(x-5)} + \frac{9}{x-3} + 1 < 0$$

(a) $-6 < x < 4$
(b) $-5 < x < 7$ and $1 < x < 3$
(c) $-5 < x < 1$ and $2 < x < 3$
(d) $x < 1$

$$63. \frac{29}{(x-1)(x-4)} + \frac{10}{x-4} + 1 > 0$$

(a) $x < -2$ (b) $-1 < x < 3$ and $4 < x$
(c) All except (a) and (b) (d) Both (a) and (b)

$$64. \frac{(x-2)(x-4)(x-7)}{(x+2)(x+4)(x+7)} > 1$$

(a) $x < -7$ (b) $x < -7$ and $-4 < x < -2$
(c) $-4 < x < 2$ (d) None of these

$$65. \frac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)} > 1$$

(a) $-3 \le x < -2$ (b) $x < -3$
(c) $-2 < x < -1$ (d) None of these

66. $(x^2 + 3x + 1)(x^2 + 2x - 3) \ge 5$
(a) $x < -4$ or $-2 < x$
(b) $-2 < x < -1$ or $1 < x$
(c) $x \le -4$, $-2 \le x \le -1$, $1 \le x$
(d) $x < -4$ or $1 < x$

67. $(x^2 - x - 1)(x^2 - x - 7) < -5$
(a) $-2 < x$
(b) $-2 < x < -1$ and $3 < x < 4$
(c) $-2 < x < -1$ and $3 < x < 3$
(d) $-2 < x < 0$ and $2 < x < 3$

Directions for Questions 68–92: Solve inequalities involving modulus.

68. $|x^2 - 1| \ge 1 - x$
(a) $-1 \le x \le 0$ (b) $x < -1$
(c) $0 < x$ (d) Always except (a)

$$69. \frac{x^2 - 5x + 6}{|x| + 3} < 0$$

(a) $2 \le x \le 3$ (b) $2 < x$
(c) $1 < x < 3$ (d) $2 < x < 3$

$$70. \frac{x^2 + 6x - 7}{|x + 4|} < 0$$

(a) $-7 < x < -5$ and $-4 < x < 1$
(b) $-7 < x < -5$ and $-4 < x < 0$
(c) $-7 < x < -4$ and $-4 < x < 1$
(d) None of these

$$71. \frac{|x-2|}{x-2} > 0$$

(a) $2 < x < 10$ (b) $3 \le x$
(c) $2 \le x$ (d) $\frac{2}{3} \le x$

72. $\left|\frac{2}{x-4}\right| > 1$

(a) $2 < x < 6, 4 \le x \le 6$
(b) $2 < x \le 6, 4 \le x \le 5$
(c) $2 < x < 4, 4 < x < 6$
(d) $2 < x \le 4, 4 \le x \le 6$

73. $\left|\frac{2x-1}{x-1}\right| > 2$

(a) $1 < x$ (b) $1 < x$
(c) $3/4 < x < 1$ (d) Both (b) and (c)

74. $\left|\frac{x^2-3x-2}{x^2+x+1}\right| < 5$

(a) $x < -2$ (b) $-1 < x$
(c) Always except (b) (d) Both (a) and (b)

75. $\frac{x^2-7|x|+10}{x^2-6x+9} < 0$

(a) $-5 < x \le -2, 2 < x < 5$
(b) $-5 < x < -2, 2 < x < 5$
(c) $-5 < x < -2, 2 < x < 3, 3 < x < 5$
(d) $-5 < x < -2, 3 < x < 5$

76. $\frac{|x+3|+x}{x+2} > 1$

(a) $-5 < x \le -2$ (b) $-2 \le x \le -1$
(c) $-1 < x$ (d) Always except (b)

77. $\frac{|x+3|}{x+2} \le 1$

(a) $-4 < x \le -3$ (b) $-3 < x \le -2$
(c) Always except (b) (d) Both (a) and (b)

78. $\frac{|x+2|+x}{x} < 2$

(a) $-3 < x < 0$ (b) $0 \le x \le 1$
(c) Both (a) and (b) (d) Always except (b)

79. $\frac{1}{|x^2-5|} < \frac{1}{2}$

(a) $x < -5$ and $-1 < x < 3$
(b) $3 \le x \le 5$
(c) $-5 \le x \le -3$
(d) Always except (b) and (c)

80. $\left|\frac{2x-1}{x^2-4}\right| \le 1$

(a) $x \le -4$ and $-1 \le x \le 1$
(b) $4 \le x$
(c) Both of these
(d) None of these

81. $\left|\frac{x^2-5x+4}{x^2-4}\right| \le 1$

(a) $[0 < x < 8/5] \cup [5/2 < x < +\infty]$
(b) $[0, 5/2] \cup [16/5, +\infty]$
(c) $[0, 8/5] \cup [5/2, +\infty]$
(d) $[0, 8/5] \cup [5/2, +\infty]$

82. $\frac{|x-3|}{x^2-5x+6} \ge 2$

(a) $[1/2, 1]$ (b) $[1, 2]$
(c) $[1.5, 2)$ (d) None of these

83. $\frac{x^2-(x)-12}{x-5} \ge 2x$

(a) $-100 < x < 25$ (b) $(-\infty, 3]$
(c) $x \ge 3$ (d) $x < 3$

84. $|x| < \frac{9}{x}$

(a) $x < -3$ (b) $0 < x < 3$
(c) $3 < x < 3, x < -3$ (d) $-\infty < x < 3$

85. $1 + \frac{12}{x^2} < \frac{7}{x}$

(a) $x < -2, 2 < x < 3$ (b) $3 \le x < 4$
(c) Both (a) and (b) (d) None of these

86. $\frac{(x^2-4x+3)}{(x^2+5x+6)} \ge 0$

(a) $-\infty < x < \infty$ (b) $x \ne -3$
(c) $x > -2$ (d) Both (b) and (c)

87. $\frac{x+1}{x-1} > \frac{x+3}{x+1}$

(a) $x < -2$ (b) $0 < x < 3$
(c) $1 < x \le 3, x < -1$ (d) $-\infty < x < 3$

88. $\frac{x-1}{x^2-x-12} \le 0$

(a) $x < -3, 2 < x < 3$ (b) $3 \le x < 4$
(c) Both (a) and (b) (d) None of these

89. $1 < (3x^2-7x+8)(x^2+1) < 2$

(a) $1 < x < 6$ (b) $1 \le x < 6$
(c) $1 < x \le 6$ (d) $1 \le x \le 6$

(a) $f(x) \geq g(x)$, where $f(x) = x - x^2$ & $g(x) = \frac{x^3}{3} - \frac{x^2}{2}$

(b) $f(x) = 3x - 7$

(a) $[2, 5]$ (b) $[2, 3]$

(c) $x = 2, 3$ (d) None of these

4. $f(x) \geq g(x)$ if $f(x) = 10x^3 - 12x^2 + 7x$, $g(x) = 15x^2 - 15x^3$

(a) $[-3, 10]$ (b) $[-\infty, 3]$

(c) $[-3, 7/3]$ (d) $[1, 7/3]$

5. $\frac{1}{x-1} + \frac{1}{x} \leq \frac{2}{x+2}$

(a) $(-2, -1) \cup (2, +\infty)$ (b) $-2 < x < 1$

(c) Both (a) and (b) (d) None of these

Solution for Questions 93–95: Solve the following inequalities

6. $(x+3)\sqrt{x^2 - x - 2} \geq 0$

(a) $x < 2$ (b) $3 \leq x < \infty$

(c) Always except (a) (d) Both (a) and (b)

7. $(x^2 - 1)\sqrt{x^2 - x - 2} \geq 0$

(a) $x < -1$ (b) $2 > x$

(c) Both (a) and (b) (d) None of these

8. $\frac{\sqrt{x^2 - 3}}{x - 2} \geq 0$

(a) $0 \leq x < 2$ (b) $x > 3$

(c) $0 < x < 3$ (d) Both (b) and (c)

Detailed Solutions & Short Cuts

While practically solving inequalities remember the following:

1. The answer to an inequality question is always in the form of a range and represents the range of values where the inequality is satisfied.
2. In the case of all continuous functions, the point at which the range of the correct answer will start, will always be a solution of the same function if it exists as an equation.

   This rule is only broken for non-continuous functions.

   Hence, if you judge that a function is continuous always check the options for LHS = RHS at the starting point of the options.
3. The correct range has to have a symmetric proportion of it has to be the correct answer.

   (a) The inequality should be satisfied for each and every value of the range.

   (b) There should be no value satisfying the inequality outside the range of the correct option.

   Operations on inequalities are always tedious using options and based on (b) (a) and (c) the above are would reject an option as the correct answer if:

   (i) we find even a single value λ satisfying the inequality within the range of a single option;

   (ii) we can reject given options, even if we find a single value satisfying the inequality but not lying within the range of the option under check.

   I will now show you certain solved questions on this pattern of thinking.

LOD 1

1. At $x = 0$, inequality is not satisfied. Thus, option (a) is rejected. Also $x = 0$ is not a solution of the equation. Since, this is a continuous function, the solution can not start from 0. Thus options (c) and (d) are not right. Further, we see for the given function to equal zero with real roots. Hence, option (b) is also rejected.
2. At $x = 0$, inequality is satisfied. Hence, options (b) and (c) are rejected. $x = 3$ gives LHS = RHS, and $x = -0.06$ also does the same. Hence roots of the equation are 3 and -0.06.

   Thus, option (a) is correct.
3. At $x = 0$, inequality is not satisfied.

   Hence, options (b), (c) and (d) are rejected. At $x = 2$, inequality is not satisfied. Hence, option (a) is rejected.

   Thus, option (e) is correct.
4. The given quadratic equation has imaginary roots and is hence always positive.

   Thus, option (d) is correct.
5. At $x = 0$ inequality is not satisfied. Thus option (d) is rejected.

   $x = -1$ and $x = 2/3$ are the roots of the quadratic equation. Thus, option (c) is correct.
6. At $x = 0$, inequality is satisfied.

   Thus, options (c) and (d) are rejected.

At $x = 1$, inequality is satisfied.
Hence, we choose option (a).

7. At $x = 0$ inequality is satisfied, option (b) is rejected.
At $x = 2$, inequality is satisfied, option (c) is rejected.
At $x = 5$, LHS = RHS.
At $x = -1$, LHS = RHS.
Thus, option (d) is correct.

8. At $x = 0$ inequality is satisfied.
Thus, options (a), (b), and (d) are rejected.
Option (c) is correct.

10. At $x = 1$ and $x = 3$ LHS = RHS.
At $x = 2$ inequality is satisfied.
At $x = 0.1$ inequality is not satisfied.
At $x = 2.9$ inequality is satisfied.
At $x = 3.1$ inequality is not satisfied.
Thus, option (a) is correct.

12. The options need to be converted to approximate values before you judge the answer. At $x = 0$, inequality is satisfied.
Thus, option (a) is rejected.
Option (c) is correct.

13. At $x = 0$, inequality is not satisfied, option (a) is rejected.
At $x = 5$, inequality is not satisfied, option (b) is rejected.
At $x = 2$ inequality is not satisfied.
Options (d) and (e) are rejected.
Option (c) is correct.

14. At $x = 0$, inequality is satisfied, option (a) rejected.
At $x = 10$, inequality is not satisfied, option (c) rejected.
At $x = -5$, LHS = RHS.
Also at $x = 5$, inequality is satisfied and at $x = 6$ inequality is not satisfied.
Thus, option (d) is correct.

15. At $x = 2$, inequality is satisfied.
At $x = 0$, inequality is not satisfied.
At $x = 1$, inequality is not satisfied but LHS = RHS.
At $x = 3$, inequality is not satisfied but LHS = RHS.
Thus, option (b) is correct.

Solve other questions of LOD 1 and LOD 2 in the same fashion.

ANSWER KEY

[illegible] 1 CD 1

1. [illegible]
2. a
3. [illegible]
4. d
5. [illegible]
6. [illegible]
7. d
8. [illegible]
9. [illegible]
10. a
11. d
12. a
13. [illegible]
14. d
15. a
16. d
17. [illegible]
18. a
19. [illegible]
20. a
21. [illegible]
22. a
23. [illegible]
24. [illegible]
25. [illegible]
26. [illegible]
27. [illegible]
28. [illegible]
29. [illegible]
30. [illegible]
31. b
32. d
33. d
34. a
35. c
36. b
37. a
38. c
39. c
40. b
41. c
42. b
43. a
44. a
45. b
46. d
47. d
48. a
49. a
50. c
51. b
52. b
53. b
54. b
55. d
56. a
57. d
58. d
59. d
60. d
61. a
62. c
63. a
64. d
65. a
66. c
67. a
68. c
69. b
70. a

Inequalities LEVEL II

1. b
2. a
3. d
4. a
5. c
6. c
7. b
8. a
9. c
10. d
11. c
12. a
13. a
14. a
15. d
16. a
17. b
18. d
19. b
20. a
21. d
22. c
23. a
24. a
25. c
26. a
27. a
28. c
29. b
30. c
31. a
32. d
33. c
34. b
35. a
36. a
37. b
38. b
39. a
40. d
41. b
42. d
43. d
44. c
45. c
46. c
47. c
48. d
49. c
50. a
51. d
52. a
53. d
54. d
55. d
56. d
57. d
58. c
59. c
60. d
61. d
62. c
63. d
64. b
65. d
66. c
67. c
68. d
69. d
70. c
71. d
72. c
73. d
74. d
75. d
76. d
77. c
78. d
79. d
80. c
81. a
82. d
83. d
84. b
85. d
86. d
87. c
88. d
89. d
90. d
91. a
92. a
93. c
94. c
95. b

14

QUADRATIC EQUATIONS

INTRODUCTION

The chapter on Quadratic Equations has not been of high importance for the CAT or any other management entrance examinations. My advice to the reader is to go through the theory of this chapter understanding the basic concepts and memorising the basic rules about the discriminant. At the same time, the student is advised to solve the exercises in this chapter as a practice. Please be informed that during the examination, most questions in this chapter have to be solved using the options.

THEORY

An equation of the form

$$ax^2 + bx + c = 0 \qquad (1)$$

where a, b, c are all real and a is not equal to 0, is a quadratic equation. Then,

$D = b^2 - 4ac$ is the discriminant of the quadratic equation.

If $D < 0$ (i.e. the discriminant is negative) then the equation has no real roots.

If $D > 0$ (i.e. the discriminant is positive) then the equation has two distinct roots, namely,

$$x_1 = \frac{-b + \sqrt{D}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{D}}{2a}$$

and then

$$ax^2 + bx + c = a(x - x_1)(x - x_2) \qquad (2)$$

If $D = 0$, then the quadratic equation has equal roots given by

$$x_1 = x_2 = \frac{-b}{2a}$$

and then

$$ax^2 + bx + c = a(x - x_1)^2 \qquad (3)$$

To represent the quadratic $ax^2 + bx + c$ in form (2) or (3) is to expand it into linear factors.

Properties of Quadratic Equations and Their Roots

(i) If D is a perfect square then the roots are rational and in case it is not a perfect square then the roots are irrational.

(ii) In the case of imaginary roots ($D < 0$) and if $p + iq$ is one root of the quadratic equation, then the other must be the conjugate $p - iq$ and vice versa (where p and q are real and $i = \sqrt{-1}$).

(iii) If $p + \sqrt{q}$ is one root of a quadratic equation, then the other must be the conjugate $p - \sqrt{q}$ and vice versa (where p is rational and $\sqrt{q}$ is a surd).

(iv) If $a = 1, b, c \in I$ and the roots are rational numbers, then the root must be an integer.

(v) If a quadratic equation in x has more than two roots, then it is an identity in x.

The Sign of a Quadratic Expression

Let $f(x) = y = ax^2 + bx + c$ where a, b, c are real and $a \neq 0$, then $y = f(x)$ represents a parabola whose axis is parallel to y axis. For some values of x, $f(x)$ may be positive, negative or zero. Also, if $a > 0$, the parabola opens upwards and for $a < 0$, the parabola opens downwards. This gives the following cases:

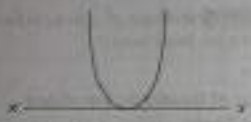
(ii) $a > 0$ and $D < 0$ (The roots are imaginary)

The function $f(x)$ will always be positive for all real values of x. So $f(x) > 0 \ \forall \ x \in R$. Naturally the graph as shown in the figure does not cut the X-Axis.

(iii) When $a > 0$ and $D = 0$ (The roots are real and identical.)

$f(x)$ will be positive for all values of x except at the same where $f(x) = 0$.

So $f(x) \geq 0 \ \forall \ x \in R$. Naturally, the graph touches the X-Axis once.



(iv) When $a > 0$ and $D > 0$ (The roots are real and distinct)

Let $f(x) = 0$ have two real roots α and β ($\alpha < \beta$) then $f(x)$ will be positive for all real values of x which are lower than α or higher than β. $f(x)$ will be equal to zero when x is equal to either α or β.

When x lies between α and β then $f(x)$ will be negative.

Mathematically, this can be represented as

$f(x) > 0 \ \forall \ x \in (-\infty, \alpha) \cup (\beta, \infty)$

and $f(x) < 0 \ \forall \ x \in (\alpha, \beta)$ (Naturally the graph cuts the X axis twice)

(v) When $a < 0$ and $D = 0$ (Roots are real and equal)

$f(x)$ is negative for all values of x except at the same where $f(x) = 0$. ($\therefore f(x) \leq 0 \ \forall \ x \in R$) The graph touches the X axis once.)

(vi) When $a < 0$ and $D > 0$

Let $f(x) = 0$ have two roots α and β ($\alpha < \beta$) then $f(x)$ will be negative for all real values of x that are lower than α or higher than β. $f(x)$ will be equal to zero when x is equal to either of α or β. The graph cuts the X axis twice.

When x lies between α and β then $f(x)$ will be positive.

Mathematically, this can be written as

$f(x) \leq 0 \ \forall \ x \in (-\infty, \alpha) \cup (\beta, \infty)$ and $f(x) > 0 \ \forall \ x \in (\alpha, \beta)$

Sum of the roots of a quadratic equation $= -b/a$.

Product of the roots of a quadratic equation $= c/a$.

WORKED-OUT PROBLEMS

Problem 14.1 Find the value of $\sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}$

(a) 4 (b) 3 (c) 3.5 (d) 2.5

Solution Let $y = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}$

Then, $y = \sqrt{6 + y} \Rightarrow y^2 = 6 + y$

or $(y + 2)(y - 3) = 0 \Rightarrow y = -2, 3$

Hence, $y = -2$ is not admissible

$y = 3$

Alternative: Going through options:

Option (a): For 4 to be the solution, value of the whole expression should be equal to 16. Looking into the expression

and it cannot be equal to 14. So, options (a) cannot be the answer. Option (b) Too 3 to be the solution, the value of the expression should be 9.

So, the expression is $= \sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}$

Let $\sqrt{\ldots} = 2.9$. Hence $= \sqrt{6+2.9+\sqrt{\ldots}} = \sqrt{8.9+\cdots} \simeq 3$

(Since, the remaining part is negligible in value)

Problem 14.2 One of the two students, while solving a quadratic equation in x, copied the constant term incorrectly and got the roots 3 and 2. The other copied the coefficient of x incorrectly and got the roots -6 and 1 respectively. The correct roots are

(a) 3, 2 (b) $-6, 2$ (c) $-6, -1$ (d) $6, -1$

Solution Let α, β be the roots of the equation. Then $\alpha + \beta = 1$ and $\alpha\beta = -6$. So, the equation is $x^2 - 5x - 6 = 0$. The roots of the equation are 6 and -1.

Problem 14.3 If p and q are the roots of the equation $x^2 + px + q = 0$

(a) $p = 1$ (b) $p = 1$ or 0 or $-\frac{1}{2}$

(c) $p = -2$ (d) $p = -2$ or 0

Solution Since p and q are roots of the equation $x^2 + px + q = 0$

$$p^2 + p^2 + q = 0 \quad \text{and} \quad q^2 + pq + q = 0$$
$$\Rightarrow \quad 2p^2 + q = 0 \quad \text{and} \quad q(q + p + 1) = 0$$
$$\Rightarrow \quad 2p^2 + q = 0 \quad \text{and} \quad q = 0 \text{ or } q = -p - 1$$

When we use $q = 0$ and $2p^2 + q = 0$

$\Rightarrow p = 0$

If we use $q = -p - 1$ and $2p^2 + q = 0$

$\Rightarrow 2p^2 - p - 1 = 0 \rightarrow$ which gives us

$p = 1$ or $p = -\frac{1}{2}$

Hence, there can be three values for p

If $q = 1$, $p = 0$, or $p = -\frac{1}{2}$

Problem 14.4 If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then a, b, c are in

(a) AP (b) GP

(c) HP (d) Cannot be determined

Solution Since roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal

$$\Rightarrow \quad b^2(c - a)^2 - 4ac(b - c)(a - b) = 0$$
$$\Rightarrow \quad b^2(c^2 + a^2 - 2ac) - 4ac(bc + ab - c^2 - ab) = 0$$
$$\Rightarrow \quad (bc + ac - 2ab)^2 = 0 \quad \Rightarrow \quad bc + ac - 2ab = 0$$
$$\Rightarrow \quad \frac{1}{a} = \frac{2}{b} + \frac{1}{c} \quad \Rightarrow \quad a, b, c \text{ are in HP}$$

Problem 14.5 The number of roots of the equation

$$x = \frac{2}{(x-1)} + 1 - \frac{2}{(x-1)} \text{ is}$$

(a) 0 (b) 1 (c) 2 (d) infinite

Solution The equation gives $x = 1$.

But $x = 1$ is not admissible because it gives $x - 1 = 0$ which, in turn, makes the whole expression like this, $x = 2/0 = 1 - 2/0$. But $2/0$ gives the value as infinity so, no solution is possible.

Problem 14.6 If the roots of the equation $x^2 - bx + c = 0$ differ by 2, then which of the following is true?

(a) $c^2 = b(c + 1)$ (b) $b^2 = 4c + 4$

(c) $c^2 = b + 4$ (d) $b^2 = 4(c + 2)$

Solution Let the roots be α and $\alpha + 2$

Then $\alpha + \alpha + 2 = b \quad \Rightarrow \quad \alpha = (b - 2)/2 \quad$ (1)

and $\alpha(\alpha + 2) = c \quad \Rightarrow \quad \alpha^2 + 2\alpha = c \quad$ (2)

Putting the value of α from (1) in (2),

$$\left(\frac{b-2}{2}\right)^2 + 2\left(\frac{b-2}{2}\right) = c$$
$$\Rightarrow \quad (b^2 + 4 - 4b)/4 + b - 2 = c$$
$$\Rightarrow \quad b^2 + 4 - 8 = 4c$$
$$\Rightarrow \quad b^2 = 4c + 4$$

$\therefore$ Option (b) is the correct answer.

Problem 14.7 What is the condition for one root of the quadratic equation $ax^2 + bx + c = 0$ to be twice the other?

(a) $b^2 = 4ac$ (b) $2b^2 = 9ac$

(c) $c^2 = 4a + b^2$ (d) $c^2 = 9a + b^2$

Solution

$\therefore \quad \alpha + 2\alpha = -(b/a) \quad$ and $\quad \alpha \times 2\alpha = c/a$

$\Rightarrow \quad 3\alpha = -(b/a) \quad \Rightarrow \quad \alpha = -b/3a$

and $\quad 2\alpha^2 = c/a \quad \Rightarrow \quad 2(-b/3a)^2 = c/a$

$\Rightarrow \quad 2b^2/9a^2 = c/a \quad \Rightarrow \quad 2b^2 = 9ac$

Hence, the required condition is $2b^2 = 9ac$.

Alternative. Assume any equation having two roots as 2 and 4 or any equation having two roots one of which is twice the other.

Where roots are 3 and 4, then equation will be $(x-3)(x-4) = x^2 - 6x + 8 = 0$

Now check the options one by one and you will find only (b) as a suitable option.

Problem 14.8 Solve the system of equations

$$\begin{cases} 1/x + 1/y = 3/2, \\ 8/x^2 + 2/y^2 = 5/4 \end{cases}$$

Solution Let $1/x = u$ and $1/y = v$. We then obtain

$$\begin{cases} u + v = 3/2, \\ u^2 + v^2 = 5/4 \end{cases}$$

From the first equation, we find $v = (3/2) - u$ and substitute this expression into the second equation

$$u^2 + ((3/2) - u)^2 = 5/4 \text{ or } 2u^2 - 3u + 1 = 0$$

where $u_1 = 1$ and $u_2 = 1/2$, consequently, $v_1 = 1/2$ and $v_2 = 1$. Therefore, $x_1 = 1$,

$$y_1 = 2 \quad \text{and} \quad x_2 = 2, \; y_2 = 1$$

Answer: (1, 2) and (2, 1)

Problem 14.9 The product of the roots of the equation $mx^2 + 8x + (2m - 1) = 0$ is -1. Then m is

(a) 1 (b) 1/3 (c) -1 (d) $-1/3$

Solution We have $(2m - 1)/m = -1$ so $m = 1/3$

Level of Difficulty (LOD) I

1. Find the maximum value of the expression

$$\frac{1}{x^2 + 5x + 10}$$

(a) $\frac{15}{2}$ (b) 1 (c) $\frac{4}{15}$ (d) $\frac{1}{4}$

(e) Infinite

2. Find the minimum value of the expression $(x^2 + 8x + 20)$

(a) 4 (b) 2

(c) 28 (d) 36

(e) None of these

3. Find the minimum value of the expression $(p + 1/p)$, $p > 0$.

(a) 1

(b) 0

(c) 2

(d) Depends upon the value of p

(e) None of these

4. If the product of roots of the equation $x^2 - 3(2a + 4)x + a^2 + 14a + 81 = 0$ is unity, then it can take the values as

(a) 3, -6 (b) 10, -8

(c) -10, -8 (d) -10, -6

(e) None of these

5. For the equation $2^{a+3} = 4^{a+2} - 48$, the value of a will be

(a) $\frac{-1}{2}$ (b) -3

(c) -2 (d) 1

(e) None of these

6. The expression $a^2 + ab + b^2$ is _____ for $a < 0$, $b < 0$

(a) $\neq 0$ (b) < 0 (c) > 0 (d) $= 0$

(e) ≤ 0

7. If the roots of equation $x^2 + bx + c = 0$ differ by 2, then which of the following is true?

(a) $b^2c^2 = 4(1 + c)$ (b) $4b + c = 1$

(c) $c^2 = 4 + b$ (d) $b^2 = 4(c + 1)$

(e) None of these

8. If $f(x) = (x + 2)$ and $g(x) = (4x + 5)$, and $h(x)$ is defined as $h(x) = f(x) - g(x)$, then sum of roots of $h(x)$ will be

(a) $\frac{3}{4}$ (b) $\frac{13}{4}$ (c) $\frac{-13}{4}$ (d) $\frac{-3}{4}$

(e) None of these

9. If equation $x^2 + bx + 12 = 0$ gives 2 as its one of the roots and $x^2 + bx + q = 0$ gives equal roots then the value of b is

(a) $\frac{49}{4}$ (b) -8

(c) 4 (d) $\frac{25}{2}$

(e) None of these

10. If the roots of the equation $(a^2 + b^2)x^2 - 2ax + 2ab + (c^2 + d^2) = 0$ are equal then which of the following is true?

(a) $ab = cd$ (b) $ad = bc$

(c) $ab = \sqrt{bc}$ (d) $ab = \sqrt{cd}$

(e) $ab = \sqrt{d}$

17. For what value of k the quadratic equation $x^2 - (k+6)x + 2(2k-1) = 0$ has roots of the roots as half of their product?

(a) 4 (b) -4 (c) 7 (d) 3

(e) 8

18. Two numbers a and b are such that the quadratic equation $x^2 + px + 2b = 0$ has -4 as the root and the product of the roots. Find $a + b$.

(a) 2 (b) -1 (c) 4 (d) -2

(e) 8

19. If α and β are the roots of the Quadratic equation $x^2 - 7x + 1 = 0$ then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$

(a) $\frac{7}{23}$ (b) -2 (c) $\frac{-7}{23}$ (d) -17

(e) 7

20. Find the value of the expression

$$\sqrt{(x+1)(\sqrt{x+1}\sqrt{x-1}+x)}$$

(a) $\frac{1}{2}\left[2\sqrt{(2x-1)}+1\right]$ (b) $\frac{1}{2}\left[\sqrt{4x-1}+1\right]$

(c) $\frac{1}{2}\left[2\sqrt{(2x-1)}-1\right]$ (d) $\frac{1}{2}\left[\sqrt{(4x-1)}-1\right]$

(e) None of these

21. If $x = \sqrt{(7+4\sqrt{3})}$, what will be the value of $\left(x + \frac{1}{x}\right)^2$?

(a) 2 (b) 4 (c) 3 (d) 2

(e) 8

22. If the roots of the equation $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$ are equal then a, b, c, are in

(a) AP (b) GP

(c) HP (d) Cannot be said

(e) None of these

23. If α and β are the roots of the equation $ax^2 + bx + c = 0$ then the equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$ is

(a) $ax^2 + bx + (a+c)x + c = ax^2 + b = 0$

(b) $(x+a)x^2 + bx + ax + c + ax = 0$

(c) $cx^2 + bx + ax + c + (x+a)x^2 = 0$

(d) $ax^2 + bx + ax + c + (x+c)x + ax^2 = 0$

(e) None of these

24. If $x^2 + 4x + 6$ leaves the same remainder 5 when divided by $x - 1$ or $x + 2$ then the values of a and b are respectively

(a) 0 and 4 (b) 3 and 0

(c) 0 and 3 (d) 4 and 0

(e) 3 and 4

25. Find all the values of k for which the equation $x^2 - kx + 1 = 0$ does not possess real roots.

(a) $-1 < k < 1$ (b) $-3 < k < 2$

(c) $-2 < k < 2$ (d) $-1.9 < k < 1.9$

(e) None of these

Directions for Questions 26–28: Read the data given below and solve the questions based on.

If α and β are roots of the equation $x^2 + x - 7 = 0$ then

26. Find $\alpha^2 + \beta^2$

(a) 30 (b) 15 (c) 5 (d) 16

(e) 36

27. Find $\alpha^3 + \beta^3$

(a) 22 (b) -22 (c) 48 (d) 36

(e) None of these

28. For what values of c in the equation $2x^2 - (c^2 + 8c - 11)x + c^2 - 4c = 0$ the roots of the equation would be opposite in signs?

(a) $c \in (0, 4)$ (b) $c \in (-4, 0)$

(c) $c \in (0, 3)$ (d) $c \in (-4, 4)$

(e) None of these

29. The set of real values of x for which the expression $x^2 - 9x + 20$ is negative is represented by

(a) $4 < x < 5$ (b) $4 \le x \le 5$

(c) $x < 4$ or $x > 5$ (d) $-4 < x < 5$

(e) None of these

30. The expression $x^2 + kx + 9$ becomes positive $\forall x \in$ real values of x (given that x is real)?

(a) $k = 6$ (b) $k > 6$

(c) $|k| < 6$ (d) $|k| \le 6$

(e) None of these

Level of Difficulty (LOD) II

1. In the Maths Olympiad of 2020 at Animal Planet, two representatives from the donkey's side, while solving a quadratic equation, committed the following mistakes

(i) One of them made a mistake in the constant term and got the roots as 3 and 4.

(ii) Another one committed an error in the coefficient of x and he got the roots as 12 and 2.

But in the meantime, they realized that they are wrong and they managed to get it right jointly. Find the quadratic equation.

(a) $x^2 + 4x + 14 = 0$ (b) $2x^2 + 7x - 24 = 0$

(c) $x^2 - 14x + 48 = 0$ (d) $3x^2 - 17x + 52 = 0$

(e) None of these

2. If the roots of the equation $a_1x^2 + a_2x + a_3 = 0$ are in the ratio $r_1 : r_2$ then

(a) $r_1 \cdot r_2 \cdot a_1^2 = (r_1 + r_2)^2 a_3^2$

(b) $r_1 \cdot r_2 \cdot a_2^2 = (r_1 + r_2)^2 a_1 \cdot a_3$

(c) $r_1 \cdot r_2 \cdot a_3^2 = (r_1 + r_2)^2 a_1 \cdot a_2$

(d) $r_1 \cdot r_2 \cdot a_3^2 = (r_1 + r_2)^2 a_2 \cdot a_1$

(e) None of these

3. For what value of a do the roots of the equation $2x^2 + 6x + a = 0$, satisfy the conditions $\left(\frac{\alpha}{\beta}\right) + \left(\frac{\beta}{\alpha}\right) < 2$

(a) $a \le 0$ or $a > \frac{9}{2}$ (b) $a \ne 0$

(c) $-1 < a < 0$ (d) $-1 < a < 0$

(e) $-1 < a < 1$

4. For what value of b and c would the equation $x^2 + bx + c = 0$ have roots equal to b and c.

(a) (0, 0) (b) (1, −2)

(c) (1, 2) (d) Both (a) and (b)

(e) None of these

5. The sum of a fraction and its reciprocal equals 85/18. Find the fraction.

(a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{2}{9}$ (d) $\frac{4}{9}$

(e) $\frac{3}{4}$

6. A journey between Mumbai and Pune (192 km apart) takes two hours less by a car than by a truck. Determine the average speed of the car if the average speed of the truck is 16 km/h less than the car.

(a) 48 km/h (b) 64 km/h

(c) 38 km/h (d) 24 km/h

(e) None of these

7. If both the roots of the quadratic equation $ax^2 + bx + c = 0$ lie in the interval (0, 1) then a lies in

(a) (1, 3) (b) (−4, −3)

(c) $(-\sqrt{121}, 0), -\sqrt{8}$ (d) (−1, 3)

(e) None of these

8. If the common factor of $(ax^2 + bx + c)$ and $(bx^2 + ax + c)$ is $(x + 2)$ then

(a) $a = b$, or $a + b + c = 0$

(b) $a = c$, or $a + b + c = 0$

(c) $a = b = c$

(d) $b = c$, or $a + b + c = 0$

(e) None of these

9. If $P = 2^{2/3} + 2^{-2/3}$ then which of the following is true?

(a) $p^3 - 6p - 6 = 0$ (b) $p^3 - 6p + 6 = 0$

(c) $p^3 + 6p - 6 = 0$ (d) $p^3 + 6p + 6 = 0$

(e) $-p^3 - 6p + 6 = 0$

10. If $f(x) = x^2 + 2x - 5$ and $g(x) = 5x + 30$, then the roots of the quadratic equation $g[f(x)]$ will be

(a) −1, −1 (b) 2, −1

(c) $-1 + \sqrt{2}, -1 - \sqrt{2}$ (d) 1, 2

(e) None of these

11. If one root of the quadratic equation $ax^2 + bx + c = 0$ is thrice times the other, find the relationship between a, b and c.

(a) $3b^2 = 16ac$ (b) $b^2 = 8ac$

(c) $(a + c)^2 = 4b$ (d) $(a^2 + c^2)(ac) = \frac{b}{2}$

(e) None of these

12. If $x^2 - 3x + 2$ is a factor of $x^4 - ax^2 + b = 0$ then the values of a and b are

(a) −5, −4 (b) 5, 4

(c) −5, 4 (d) 5, −4

(e) None of these

13. Value of the expression $(x^2 - x + 1)(x - 1)$ cannot lie between

(a) 1, 2 (b) −1, −3 (c) 1, −3 (d) −1, 2

(e) −1, 3

14. The value of p satisfying $\log_2(p^2 + 4p + 12) = 2$ are

(a) 1, −3 (b) −1, −3 (c) −4, 2 (d) −4, −2

(e) 4, 2

15. If $q, r > 0$ then roots of the equation $x^2 + px - r = 0$ are

(a) Both negative

(b) Both positive

(c) Of opposite sign but equal magnitude

(d) Of opposite sign

(e) None of these

4. If the quadratic equations $ax^2 + bx + 2 = 0$ and $x^2 + x + 2 = 0$ have a common root, $2 \neq 1$ then which of the following statements hold true?

(a) $a + b = -3.5$ (b) $ab = 1$

(c) $\frac{a}{b} = \frac{3}{4}$ (d) $a - b = -0.5$

(e) A, B, C (f) B, C, D

(g) A, C, D (h) A, B, D

5. If the expression $ax^2 + bx + c$ is equal to 4 when $x = 0$, gives a remainder 4 when divided by $x + 1$ and a remainder 5 when divided by $x + 2$, then the values of a, b and c are respectively

(a) 1, 1, 4 (b) 2, 2, 4

(c) 3, 3, 4 (d) 4, 4, 4

(e) 5, 5, 4

6. p and q are the roots of the equation $x^2 - px + q = 0$ then

(a) $p = 1, q = -2$ (b) $p = 0, q = 1$

(c) $p = -2, q = 0$ (d) $p = -2, q = 1$

(e) $p = 1, q = 1$

7. Sum of the real roots of the equation $x^2 + |x| - 6 = 0$

(a) Equals to 5 (b) Equals to 10

(c) Equals to -5 (d) Equals

(e) None of these

8. The value of p for which the sum of the squares of the roots of $2x^2 - 2(p - 2)x - p - 1 = 0$ is least is

(a) 1 (b) $\frac{3}{2}$

(c) 2 (d) -1

(e) None of these

9. For what values of p would the equation $x^2 + 2(p - 1)x + p + 5 = 0$ possess at least one positive root?

(a) $p \in (-\infty, -5)$ (b) $p \in (-\infty, -1]$

(c) $p \in (1, \infty)$ (d) $p \in (-5, 1)$

(e) None of these

10. If $a, b \in \{1, 2, 3, 4\}$, then the number of the equations of the form $ax^2 + bx + 1 = 0$ having real roots is

(a) 8 (b) 7

(c) 4 (d) 10

(e) None of these

11. If $a^2 + b^2 + c^2 = 1$, then which of the following cannot be a value of $(ab + bc + ca)$?

(a) 1 (b) 1/2 (c) $-\frac{1}{4}$ (d) -1

(e) $-1/2$

12. If one root of the equation $(r - m)x^2 + (r + m)x + 1$ is double of the other and m lies that the greatest value of m

(a) $\frac{5}{6}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{7}{8}$

(e) $\frac{5}{7}$

13. The set of values of p for which the roots of the equation $3x^2 + 2x + p(p - 1) = 0$ are of opposite sign is

(a) $(-\infty, 0)$ (b) $(0, 1)$

(c) $(1, \infty)$ (d) $(0, \infty)$

(e) $(-\infty, \infty)$

Detailed Solutions & Short Cuts

LOD 1

1. For the given expression to be a rational, the denominator should be minimum. (Since the numerator is the difference two imaginary roots and is always positive) $x^2 - 5x + 10$ should be minimum $x = -2.5$ and be minimum value then $x = -2.5 \Rightarrow 3.75$.
   Hence, required number $= 10/3.75 = 8/3$.
2. Basic concepts.
3. The minimum value of $p + 1/p$ is $p = 1$. The value is 2.
4. The product of the roots is given by $(a^2 + 16a + 81)/a$. Since product is unity we get $a^2 + 16a + 81 = a$.
   Then $a^2 + 15a + 81 = 0$.
   Solving, we get $a = -10$ and $a = -8$.
5. Solve through equations $1.16b = 8.08$ hence $a = 1$.
6. For $a \neq$ negative the given expression will always be positive since a^2, b^2 and ab are all positive.
7. To solve this take any expression whose roots differ by 2.
   Then, $(x - 5)(x - 7) = 0$
   or $x^2 - 12x + 35 = 0$
   In this case $a = 1$, $b = -12$ and $c = 35$.
   We can see that $b^2 = 4c = 16$.
8. $f(x) = 4x^2 + 12x + 15$.
   Sum of roots $= 1/4$.
9. $x^2 + kx + 12 = 0$ has 2 as a root.
   Then $k = -8$.
10. Solve this by comparing each equation to the one and then check whether the given expression has equal roots for the equation under check.
    Then, if we check for equation (a).

$\alpha\beta = b$.

We assume $a = 6$, $b = 6$ & $c = 12$; $c = 2$ is any set of values that satisfies all in (c).

Then $(\alpha^2 + \beta^2)(\beta^2 - 3\alpha\beta + \beta\alpha)(\alpha + \beta)(\alpha^2 + \beta^2) = 0$

$$180 \times -120 \times 20 \neq 0$$

We can see that this has equal roots. Thus, option (b) is a possible answer. The same way if we check for a, c, d and e we see that none of these given as equal roots and can be rejected.

12. $(x + 6)(x + 1)(2x - 1) = x + 6 + 2x - 1 \Rightarrow x = 7$

13. $-3k = -6 \Rightarrow \alpha = 2$;

$2k\alpha = -6$ and $\alpha = 2$

Gives us $k = -1.5$.

$\alpha + k = -1$.

14. $1/\alpha + 1/\beta = (\alpha + \beta)/\alpha\beta$

$= (7/9)/(1/9) = 7$

15. $y = \sqrt{x} + \sqrt{1/x} +$

$\Rightarrow \quad y = \sqrt{x} + y$

$\Rightarrow \quad y^2 = x + y$

$y^2 - y - x = 0$

Solving quadratically, we have option (b) as the root of this equation.

16. The approximate value of $\alpha = \sqrt{13.92} = 3.6$ (approx).

$\alpha + 1/\alpha = 3.6 + 1/3.6$ is closest to 4.

17. Solve by assuming values of a, b, and c in AP, GP and HP to check which satisfies the condition.

18. Assume any equation:

Say $x^2 - 5x + 6 = 0$

The roots are 2, 3.

We are now looking for the equation whose roots are $(2 + 1/3) = 2.33$ and $(3 + 1/2) = 3.5$.

Also $a = 1$, $b = -5$ and $c = 6$.

Put these values in each option to see which gives 2.33 and 3.5 as its roots.

19. Remainder when $x^2 + ax + b$ is divided by $x - 1$ is got by putting $x = 1$ in the expression. Thus, we get

$a + b + 1 = 5$ and

$b - a + 1 = 5$

$\Rightarrow \quad b = 4$ and $a = 0$

20. $b^2 - 4c < 0 \Rightarrow -2 < b < 2$

21. $(\alpha^2 + \beta^2)^2 + (\alpha + \beta)^2 - 2\alpha\beta$

$= (-1)^2 - 2 \times (-7) = 15$

22. $(\alpha^3 + \beta^3) + (\beta^2 - 3 \times \beta(\alpha + \beta)) = (-1)^2 - 3 \times (-7)(-1)$

$= -1 - 21 = -22$

23. For the roots to be opposite in sign, the product should be negative.

$(c/2 - 6)(c/2) < 0 \quad \Rightarrow \quad 0 < c < 4$

The roots of the equation $x^2 - 9x + 20 = 0$ are 4 and 5.

The expression would be negative for $4 < c < 5$.

24. The roots should be imaginary for the expression to be positive.

i.e. $\quad b^2 - 36 < 0$

thus $\quad -6 < b < 6$ or $|b| < 6$.

LOD II

1. From (i) we have sum of roots = 14

and from (ii) we have product of roots = 48.

Option (a) is correct.

2. Assume the equation to be $(x - 1)(x - 2) = 0$

which gives $\alpha_1 = 1$, $\alpha_2 = -3$ and $\alpha_3 = 2$ and $r_1 = 1$, $r_2 = 2$. With this information check the options.

3. $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} < 2 \rightarrow \dfrac{\alpha^2 + \beta^2}{\alpha\beta} < 2$

$\rightarrow \dfrac{(\alpha + \beta)^2 - 2\alpha^2\beta^2}{\alpha\beta} < 2$

Use the formulae for sum of the roots and product of the roots.

4. Solve using options. It can be seen that at $b = 0$ and $c = 0$ the condition is satisfied. It is also satisfied at $b = 1$ and $c = -2$.

5. $2/9 + 9/2 = 85/18$

6. Solve using options. If the car's speed is 48 km/h, the bus's speed would be 32 km/h. The car would take 4 hours and the bus 6 hours.

7. For each of the given options it can be seen that the roots do not lie in the given interval. Thus, option (c) is correct.

8. Using $x = -2$, we get $4a - 2b + c = 4b - 2a + c = 0$.

Thus, $a = b$ and $a + b + c = 0$.

9. Use an approximation of the value of ρ to get the correct option. Such questions are generally not worth solving through mathematical approaches under the constraint of time in the examination.

10. $f(f(x)) = 5x^2 + 10x + 5$

Roots are -1 and -1.

11. Solve using options.

For option a, $3b^2 = 16ac$

We can assume $b = 4$, $a = 1$ and $c = 3$.

Then the equation $ax^2 + bx + c = 0$ becomes

$x^2 + 4x + 3 = 0 \Rightarrow x = -3$ or $x = -1$

which satisfies the given conditions.

10. $x^2 - 3x + 2 = 0$ gives its roots as $x = 1, 2$.
Try these values in the equation and then use the options.

11. Trial and error gives us value as -1 or $x = 0$. If you try some values, you will see that you cannot get a value between -1 and 2 for this expression.

12. $p^2 + 4p + 12 = 0$
or $p^2 + 4p + 3 = 0$
$p = -3$ and -1.

13. The roots would be of opposite sign as the product of roots is negative.

14. Use the value of $x = 1$ in each of the two quadratic equations to get the value of a and b respectively. With these values check the options for their validity.

15. We get $a + a = 4$ (by putting $x = 0$).
Then, at $x = -1$, $a - b + 4 = 4$. So $a - b = 0$.
At $x = -2$, $4a - 2b + 4 = 6 \Rightarrow 4a - 2b = 2 \Rightarrow 2a - 2 = a$
$= 1$. Thus, option (a) is correct.

16. Solve by checking each option for the condition given. Option c gives $x^2 + 2x = 0$ whose roots are -2 and 0.

17. The equation is
$x^2 + 3x + 6 = 0$ and $x^2 - 3x + 6 = 0$
Sum of roots $= -3 + 3 = 0$.

18. We have to minimize $R_1^2 + R_2^2$ or $(R_1 + R_2)^2 - 2R_1R_2$
$= (p - 2)^2 - 2 \times (-(p + 1)/2) = p^2 - 4p + 4 + p + 1$
$= p^2 - 3p + 5$.

This is minimized at $p = 1.5$ or $3/2$.

19. Go through trial and error. At $p = 2$, both roots are negative. So, option (c) is rejected.
At $p = 0$, roots are imaginary. So option (d) is rejected.
At $p = -1$, we have both roots positive and equal.
At $p = -3$, roots are imaginary. Thus, option (b) is correct.

20. $b^2 - 4a \geq 0$ for real roots.
If $b = 1$, no value possible for a.
If $b = 2$, $a = 1$ is possible.
If $b = 3$, a can be either 1 or 2 and if $b = 4$, a can be 1, 2, 3 or 4.
Thus, we have 7 possibilities overall.

21. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$\Rightarrow 1 + 2(ab + bc + ca)$
Since, $(a + b + c)^2 \geq 0 \Rightarrow ab + bc + ca \geq -1/2$
Option (d) is not in this range and is hence not possible.

22. $p(p - 1)/2 < 0$ (Product of roots should be negative).
$\Rightarrow p(p - 1) < 0$
$p^2 - p < 0$
This happens for $0 < p < 1$.
Option (b) is correct.

ANSWER KEY

Quadratic Equation LOD I

1. a
2. a
3. a
4. a
5. d
6. c
7. d
8. c
9. b
10. b
11. c
12. b
13. a
14. b
15. b
16. b
17. c
18. a
19. c
20. b
21. b
22. a
23. b
24. c

1. a
2. b
3. a
4. d
5. a
6. a
7. a
8. a
9. a
10. a
11. a
12. b
13. d
14. b
15. d
16. a
17. a
18. a
19. a
20. b
21. b
22. b
23. d
24. a
25. b

15

LOGARITHMS

INTRODUCTION

Questions based on this chapter are not so frequent in management entrance exams. In exams, most problems that have used the concept of logs have been of an applied nature. However, the aspirants should know the basic concepts of logarithms to ensure against shocks due to surprises in the paper.

While studying this chapter the student should pay particular attention to the basic rules of logarithms as well as develop an understanding of the range of the values of logs.

THEORY

Let a be a positive real number, $a \neq 1$ and $a^x = m$. Then x is called the logarithm of m to the base a and is written as $\log_a m$, and conversely, if $\log_a m = x$, then $a^x = m$.

Note. Logarithm to a negative base is not defined.

Also, logarithm of a negative number is not defined. Hence, in the above logarithmic equation, $\log_a m = x$, and we can say that $m > 0$ and $a > 0$.

Thus $a^x = m \Leftrightarrow x = \log_a m$ and $\log_a m = x \Rightarrow a^x = m$.

In short, $a^x = m \Leftrightarrow x = \log_a m$.

$x = \log_a m$ is called the logarithmic form and $a^x = m$ is called the exponential form of the equation connecting a, x and m.

Two Properties of Logarithms

1. $\log_a 1 = 0$ for all $a > 0$, $a \neq 1$

That is, log 1 to any base is zero

Let $\log_a 1 = x$. Then by definition, $a^x = 1$

But $a^0 = 1 \therefore a^x = a^0 \Leftrightarrow x = 0$.

Hence $\log_a 1 = 0$ for all $a > 0$, $a \neq 1$

2. $\log_a a = 1$ for all $a > 0$, $a \neq 1$

That is, log of a number to the same base is 1

Let $\log_a a = x$. Then by definition, $a^x = a$

But $a^1 = a \therefore a^x = a^1 \Rightarrow x = 1$

Hence $\log_a a = 1$ for all $a > 0$, $a \neq 1$.

Laws of Logarithms

First Law: $\log_a (mn) = \log_a m + \log_a n$

That is, log of product = sum of logs

Second Law: $\log_a (m/n) = \log_a m - \log_a n$

That is, log of quotient

= difference of logs

Note: The first theorem converts a problem of multiplication into a problem of addition and the second theorem converts a problem of division into a problem of subtraction, which are far easier to perform than multiplication or division. That is why logarithms are so useful in all numerical calculations.

Third Law: $\log_a m^n = n \log_a m$

Generalisation

1. $\log (mnp) = \log m + \log n + \log p$
2. $\log (n_1 n_2 n_3 \ldots n_k) = \log n_1 + \log n_2 + \ldots + \log n_k$

Note: Common logarithms. We shall assume that the base $a = 10$ whenever it is not indicated. Therefore, we shall denote

$\log_a x$ by $\log x$ only. The logarithm calculated to base 10 is called common logarithm.

The Characteristic and Mantissa of a Logarithm

The logarithm of a number consists of two parts: the integral part and the decimal part. The integral part is given as the characteristic and the decimal part is called as mantissa.

For example,

$\log 1257 = 3.0128$, the integral part is 3 and the decimal part is .0128; therefore, characteristic = 3 and mantissa = .0128.

It should be remembered that the mantissa is always taken as positive.

Note: Remember the mantissa positive in case the value of the mantissa of a number is negative, subtract 1 from the integral part and add 1 to the decimal part.

Thus, $-3.4328 = -(3 + .4328) = -3 - 0.4328$

$= (-3 - 1) + (1 - 0.4328)$

$= -4 + .5672$

so the mantissa is $= .5672$

Note: The characteristic may be positive or negative. When the characteristic is negative, it is represented by placing a bar on the number.

Thus instead of -4, we write $\bar{4}$

Hence we may write $-4 + .5672$ as $\bar{4}.5672$.

Base Change Rule

The rule states that

(i) $\log_a (bc) = \log_a (b) + \log_a (c)$

It is one of the most important rules for solving logarithms.

(ii) $\log_a (b/c) = \log_a (a) - \log_a (c)$

A corollary of this rule is

(iii) $\log_a (b^c) = c \log_a (b)$

(iv) $\log x$ to the base a^b is equal to $\frac{\log a^x}{b}$

Results on Logarithmic Inequalities

(i) If $a > 1$ and $\log_a x_1 > \log_a x_2$ then $x_1 > x_2$

(ii) If $a < 1$ and $\log_a x_1 > \log_a x_2$ then $x_1 < x_2$

Important conclusions for logarithms

1. The characteristic of common logarithm of any positive number less than 1 is negative.
2. The characteristic of common logarithm of any number greater than 1 is positive.
3. If the logarithm to any base a gives the characteristic n, then we can say that the number of integral positive x is given by $a^{n+1} - a^n$.

   Example: $\log_{10} x = 2.0101\ldots$ then the number of integral values that x can take is given by: $10^{2+1} - 10^2 = 900$. This can be physically verified as follows.

   Log to the base 10 gives a characteristic of 2 for all 3 digit numbers with the lowest being 100 and the highest being 999. Hence, there are 900 integral values possible for x.
4. If $-n$ is the characteristic of $\log_{10} x$, then the number of zeros between the decimal and the first significant number after the decimal is $n - 1$.

   Thus if the log of a number has a characteristic of -3 then the first two decimal places after the decimal point will be zero.

   Thus, the value will be $= 0.00abc$.

WORKED-OUT PROBLEMS

Problem 15.1 Find the value of x if $7^{(2x-4)} \times 9^{(x-1)}$

(a) 6/7 (b) 7/6 (c) 7/4 (d) 8/5

Solution Take the log of both sides, then we get

$$(2x - 4)\log 7 = (2x - 2)\log 9$$
$$= (2x - 2)\log 3^2$$
$$= (2x - 4)\log 3$$

Dividing both sides by $\log 3$, we get

$$(2x - 4)\log 7 = (2x - 4) \qquad (1)$$

Now $(2x - 4) = 2x - 4$ if $x = 4/3$

or if $x = 4/3$

$$2x - 4 = 4x - 4$$

$\Rightarrow \quad 3x = 8x$

$\Rightarrow \quad 3 = 4$

But this is not possible.

Let's take the case of $x < 4/3$

Then $\quad |3x - 4| = 4 - 3x$

Therefore, $\quad 4 - 3x = 8x - 4$ from (1)

$\Rightarrow \quad 3x = 8$

$\Rightarrow \quad x = 8/3$

Problem 15.2 Solve for x

$$\log_{10}x - \log_{10}\sqrt{x} = 2\log_{10}10$$

Solution Now, $\log_{10}\sqrt{x} = 1/2\log_{10}x$

Therefore, the equation becomes

$$\log_{10}x - 1/2\log_{10}x = 2\log_{10}10 \quad (1)$$

$$\Rightarrow \quad 1/2\log_{10}x = 2\log_{10}10$$

Using base change rule $\log_b a = 1/\log_a b$

Therefore, equation (1) becomes

$$1/2\log_{10}x = 2/\log_{10}x$$

$$\Rightarrow \quad (\log_{10}x)^2 = 4$$

$$\Rightarrow \quad \log_{10}x = 2$$

Therefore, $\quad x = 100$

Problem 15.3 If $7^{x+1} - 7^{x-1} = 48$, find x

Solution Take 7^{x-1} as the common term. The equation then reduces to

$$7^{x-1}(7^2 - 1) = 48$$

$$\Rightarrow \quad 7^{x-1} = 1$$

$$\Rightarrow \quad x - 1 = 0 \text{ or } x = 1$$

Problem 15.4 Calculate $\log_2(2/3) + \log_4(9/8)$

$$= \log_2(2/3) + (\log_2(9/8))\log_4 4$$

$$= \log_2(2/3) + 1/2\log_2(9/8)$$

$$= \log_2(2/3) + 1/2\log_2(3/2)$$

$$= \log_2 2/3 + \log_2 3/2 = \log_2 1 = 0$$

Problem 15.5 Find the value of the expression

$$1/\log_2 3 + 2/\log_3 4 - 3/\log_6 8$$

Passing to base 2

we get

$$\log_2 3 + 2\log_3 4 - 3\log_6 (2^3)$$

$$= \log_2 3 + \frac{2\log_2 4}{\log_2 3} - \frac{3\log_2 8}{\log_2 6}$$

$$= 3\log_2 3 - 3\log_2 3$$

$$= 0$$

Problem 15.6 Solve the inequality

(a) $\quad \log_2(x + 3) < 2$

$\Rightarrow \quad 2^2 > x + 3$

$\Rightarrow \quad 4 > x + 3$

$\quad 1 > x$

or $\quad x < 1$

But log of negative number is not possible.

Therefore, $\quad x + 3 \geq 0$

That is, $\quad x \geq -3$

Therefore, $\quad -3 \leq x < 1$

(b) $\quad \log_2(x^2 - 5x + 5) > 0$

$\Rightarrow x^2 - 5x + 5 > 1$

$\Rightarrow x^2 - 5x + 4 > 0$

$\Rightarrow (x - 4)(x - 1) > 0$

Therefore, the value of x will lie outside 1 and 4.

That is, $x > 4$ or $x < 1$.

Level of Difficulty (LOD) I

1. $\log 22700 = ?$

(a) $\log 3.27 + 4$ (b) $\log 3.27 + 2$

(c) $2\log 327$ (d) $100 \times \log 327$

(e) None of these

2. $\log 0.0867 = ?$

(a) $\log 8.67 + 2$ (b) $\log 8.67 - 2$

(c) $\dfrac{\log 867}{1000}$ (d) $-2\log 8.67$

(e) $\log 8.67 - 3$

3. If $\log_{10}2 = .301$ find $\log_{10}125$.

(a) 2.097 (b) 2.301

(c) 2.10 (d) 2.087

(e) None of these

4. $\log_{32} 8 = ?$

(a) 3/5 (b) 3/4 (c) 3/5 (d) 4/5

(e) 5/4

Find the value of x in equations 5–8.

5. $\log_{16}x = 2.5$

(a) 2^{-5} (b) 2^{10} (c) 2^{-14} (d) 2^{20}

(e) 2^{30}

6. $\log_3 x = \frac{1}{2}$

(a) 1 (b) $\sqrt{3}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

(e) $\frac{1}{3}$

7. $\log_{10}(1.77) + \log_{10}(0.024) = ?$

(a) 10 (b) 20 (c) 12 (d) 15

(e) None of these

8. $\log_2 4 + \log_2 16 + \log_2 64 + \log_2 256 = 10$. Then $x = ?$

(a) 4 (b) 7

(c) 8 (d) 5

(e) None of these

9. $\log_{10}(\sqrt{3}) = ?$

(a) 0 (b) 1

(c) 3/4 (d) 1/4

(e) None of these

10. If $\log x + \log(x + 3) = 1$ then the value(s) of x satisfy the solution of the equation

(a) $x + x + 3 = 1$ (b) $x + x + 3 = 10$

(c) $x(x + 3) = 10$ (d) $x(x + 3) = 1$

(e) None of these

11. If $\log_{10} x = 2$, find the value of 10^{3x} in terms of x.

(a) x^3 (b) $3x$

(c) $x + 0.003$ (d) $x + 300$

(e) $3x^2$

12. $3 \log 5 + 2 \log 4 - \log 2 = ?$

(a) 4 (b) 5 (c) 384 (d) 1000

(e) 200

Solve equations 13–25 for the value of x.

13. $\log(5x - 2) = 1$

(a) 1 (b) 2 (c) 4 (d) 6

(e) 5

14. $\log(2x - 3) = 2$

(a) 50 (b) 51.5

(c) 25.75 (d) 26

(e) None of these

15. $\log(x) - x = -1$

(a) 2.16 (b) 7.23 (c) 11 (d) 12.9

(e) 12

16. $\log_{10} x^2 - 5x + 6 = 0$

(a) 3 (b) 1

(c) Both (a) and (b) (d) 1 and 2

(e) None of these

17. $\log 2^x = 3$

(a) 9.97 (b) $3 \log 2$

(c) $3/\log 2$ (d) 0.31

(e) None of these

18. $3^x = 7$

(a) $3/\log 3$ (b) $\log 3$

(c) $3/\log 7$ (d) $\log 7$

(e) None of these

19. $5^x = 10$

(a) $3 \log 3$ (b) $\log 100 \log 2$

(c) $\log 2$ (d) $\frac{\log 5}{2}$

(e) $10 \log 5$

20. Find x, if $(0.01)^x = 2$

(a) $\log 2/2$ (b) $2 \log 2$

(c) $-2 \log 2$ (d) $-\log 2/2$

(e) None of these

21. Find x if $\log x = \log 7.2 - \log 2.4$

(a) 1 (b) 2

(c) 3 (d) 4

(e) None of these

22. Find x if $\log x = \log 1.5 + \log 10$

(a) 12 (b) 6

(c) 10 (d) 15

(e) None of these

23. Find x if $\log x = 2 \log 5 + 3 \log 2$

(a) 50 (b) 100 (c) 150 (d) 180

(e) 200

24. $\log(x - 13) + 3 \log 2 = \log(3x + 1)$

(a) 20 (b) 21

(c) 22 (d) 24

(e) None of these

25. $\log(2x - 5) - \log(3x - 10) = \log x + \log 3$

(a) 10.02 (b) 10.02

(c) 11 (d) 10.93

(e) None of these

Level of Difficulty (LOD) II

1. Express $\log \frac{\sqrt[3]{x^2 y^2}}{\sqrt[5]{x^3 y^3}}$ or $\sqrt[3]{\frac{x^2 y^2}{x^3 y^3}}$ in terms of $\log x$, $\log y$ and

ans –

(a) $\frac{3}{2}\log x = 5\log b - 2\log c$

(b) $\frac{3}{2}\log x = 3\log b - \frac{1}{2}\log c$

(c) $\frac{3}{2}\log x = 5\log b + \frac{1}{2}\log c$

(d) $\frac{3}{2}\log x = 2\log b - \frac{1}{2}\log c$

(e) None of these

2. If $\log 5 = .4771$, find $\log(.81)^{.5} + \log\left(\frac{25}{16}\right)^{.5} + \log 9$

(a) 2.600 (b) .01957

(c) 1.380 (d) 2.302

(e) None of these

3. If $\log 2 = .301$, $\log 3 = .477$, find the number of digits in $(108)^{25}$

(a) 21 (b) 23

(c) 20 (d) 18

(e) None of these

4. If $\log 2 = .301$, find the number of digits in $(125)^{9}$

(a) 29 (b) 30 (c) 28 (d) 62

5. Which of the following options represents the value of $\log_{\sqrt{128}}$ in the base 625?

(a) $\frac{2+\log_5 2}{\log_4 5 - 1}$ (b) $\frac{\log_5 128}{\log_5 5625}$

(c) $\frac{2+\log_5 2}{2(\log_4 5 - 1)}$ (d) Both (b) and (c)

(e) None of these

6-8. Solve for x

6. $\log\frac{25}{24} + 2\log\frac{7}{8} + \log\frac{105}{x} - \log\frac{13}{24} = 0$.

(a) 56 (b) 65 (c) 12 (d) 48

(e) 36

7. $2\log\frac{4}{3} - \log\frac{5}{16} + \log\frac{63}{100} = x$

(a) 7 (b) 14 (c) 9 (d) 5

(e) 4

8. $\log\frac{27}{11} + \log\frac{9}{33} + \log\frac{44}{3} = x$

(a) 0 (b) 1 (c) 2 (d) 3

(e) 4

Questions 9–10: Which one of the following is true

9. (a) $\log_{11} 375 > \log_{13} 419$ (b) $\log_{11} 375 < \log_{13} 375$

(c) $\log_{11} 375 = \log_{13} 419$ (d) Cannot be determined

(e) None of these

10. (a) $\log_{11} 1600 > \log_{13} 1950$

(b) $\log_{11} 1600 < \log_{13} 1950$

(c) $\log_{11} 1600 = \log_{13} 1950$

(d) None of these

(e) Cannot be determined

11. (a) $\frac{\log_2 4096}{3} = \log_4 4096$

(b) $\frac{\log_2 4096}{3} < \log_4 4096$

(c) $\frac{\log_2 4096}{3} > \log_4 4096$

(d) Cannot be determined

(e) None of these

12. $\log\left(\frac{8}{15}\right) + 3\log\frac{25}{24} + 3\log\frac{81}{80} = \log x$, $x = ?$

(a) 2 (b) 3

(c) 0 (d) 1

(e) None of these

If $\log 2 = .3010$ and $\log 3 = .4771$ then find the number of digits in the following.

13. $(6)^{11}$

(a) 25 (b) 22 (c) 23 (d) 20

(e) 28

14. 72^{6}

(a) 17 (b) 20 (c) 18 (d) 15

(e) 10

15. 27^{12}

(a) 38 (b) 37 (c) 36 (d) 35

(e) 30

Questions 16–22: Find the value of the logarithmic expression in the questions below.

16. $\frac{\log\sqrt{27} + \log 8 - \log\sqrt{1000}}{\log 12}$

where $\log_{10} 2 = 0.30103$, $\log_{10} 3 = 0.4771213$

(a) 1.27 (b) 1.57

(c) 2.33 (d) 1.46

(e) None of these

17. $\frac{1}{\log_{12}(abc)} + \frac{1}{\log_{12}(a/c)} + \frac{1}{\log_{12}(b/a)} =$

(a) 1 (b) 2 (c) 3 (d) 4

18. $\log_a a^{\log_b b^{\log_c c^{\log_a a}}} + \log_b b^{\log_c c^{\log_a a^{\log_b b}}} = \log_c c^{\log_a a^{\log_b b^{\log_c c}}}$

(a) 1 (b) a

(c) 0 (d) 2

(e) None of these

18. $\log_{10} x - \log_{10} \sqrt{x} = 2 \log_x 10$

(a) 50 (b) 100 (c) 150 (d) 200

(e) 225

19. $\left(\frac{21}{10}\right)^x = 2$. Then $x = ?$

(a) $\frac{\log 2}{\log 3 + \log 7 - 1}$ (b) $\frac{\log 2}{\log 3 + \log 7 + 1}$

(c) $\frac{\log 3}{\log 2 + \log 7 - 1}$ (d) $\frac{\log 2}{\log 3 - \log 7 + 1}$

(e) None of these

21. $\log (x^3 + 5) = 3 \log (x + 2)$

(a) $\frac{-2 + \sqrt{2}}{2}$ (b) $\frac{-2 - \sqrt{2}}{2}$

(c) Both (a) and (b) (d) $\frac{-2 + \sqrt{2}}{3}$

(e) None of these

22. $(a^4 - 2a^2b^2 + b^4)^{x-1} = (a - b)^{-2} (a + b)^{-2}$

(a) 1 (b) 0

(c) None of these (d) 2

(e) Cannot be determined

23. If $\log_{10} 242 = a$, $\log_{10} 80 = b$ and $\log_{10} 45 = c$, express $\log_{10} 36$ in terms of a, b and c.

(a) $\frac{(c-1)(3c+b-4)}{2}$ (b) $\frac{(c-1)(3c+b-4)}{3}$

(c) $\frac{(c-1)(3c-b-4)}{2}$ (d) $\frac{(c-1)(b-3c-4)}{2}$

(e) None of these

24. For the above problem, express $\log_{10} 66$ in terms of a, b and c.

(a) $\frac{(c-1)(3c+b-4)}{8}$ (b) $\frac{3(a+c)+(2b+-5)}{6}$

(c) $\frac{3(a+c)+(2b-5)}{6}$ (d) $\frac{3(c-1)(3c+b-4)}{6}$

(e) None of these

25. $\log_2 (9 - 2^x) = 10^{\log (3-x)}$. Solve for x.

(a) 0 (b) 3

(c) Both (a) and (b) (d) 0 and 6

(e) None of these

Detailed Solutions & Short Cuts

LOD I

1. $\text{Log } 32700 = \log 3.27 + \log 10000 = \log 3.27 + 4$
2. $\text{Log } 0.0867 = \log (8.67/100) = \log 8.67 - \log 100$
   $\text{Log } 8.67 - 2$
3. $\log_{10} 125 = \log_{10}(1000/8) = \log.1000 - 3\log 2$
   $= 3 - 3 \times 0.301 = 2.097$
4. $\log_{32} 8 = \log 8/\log 32$ (By base change rule)
   $= 3 \log 2/5\log 2 = 3/5.$
5. $\log_{0.5} x = 25 \Rightarrow x = 0.5^{25} = (1/2)^{25} = 2^{-25}$
6. $x = 3^{1/2} = \sqrt{3}.$
7. $\log_{15} 3375 \times \text{Log}_4 1024$
   $= 3 \log_{15} 15 \times 5 \log_4 4 = 3 \times 5 = 15.$
8. The given expression is:
   $\text{Log}_a (4 \times 16 \times 64 \times 256) = 10$
   i.e. $\log_a 4^{10} = 10$
   Thus, $a = 4$.
9. $1/2 \log_{0.25} 5 = [1/(2 \times 4)] \log_5 5 = 1/8.$
10. $\log x (x + 3) = 1 \Rightarrow 10^1 = x^2 + 3x.$
    or $x (x + 3) = 10.$
11. $\log_{10} a = b \Rightarrow 10^b = a \Rightarrow$ By definition of logs.
    Thus $10^{3b} = (10^b)^3 = a^3.$
12. $3 \log 5 + 2 \log 4 - \log 2$
    $= \log 125 + \log 16 - \log 2$
    $= \log (125 \times 16)/2 = \log 1000 = 3.$
13. $10^1 = 3x - 2 \Rightarrow x = 4.$
14. $10^2 = 2x - 3 \Rightarrow x = 51.5$
15. $1/10 = 12 - x \Rightarrow x = 11.9$
16. $x^2 - 6x + 6 = 10^0 \Rightarrow x^2 - 6x + 6 = 1$
    $\Rightarrow x^2 - 6x + 5 = 0$
    Solving gives us $x = 5$ and 1.
17. $x \log 2 = 3$
    $\log 2 = 3/x.$

    Or

    $x = 3/\log 2$

18. $x = 7$ or $\log_x 7 = 1$
Hence $x = 1/\log_7 7$

19. $x = \log_{10}(10) = 1/\log_{10} 5 = 10\log 5$

20. $x = \log_{10} 2 = \log 2/2$

21. $\log x = \log(5/2)$ So $x = \log 5 = x = 1$

22. $\log_2 x = \log 16 \Rightarrow x = 16$

23. $\log x = \log 25 + \log 8 = \log(25 \times 8) = \log 200$

24. $\log(x - 1) + \log 8 = \log(x + 1)$
$\Rightarrow \log(8x - 100) = \log(x + 1)$
$\Rightarrow 8x - 100 = x + 1$
$8x - 101 \Rightarrow x = 21$

25. $\log(2x - 21) + \log 66 = \log 30$
$\Rightarrow (2x - 21)(66 - x) = 30$
$2x - 21 = 330 - 30x$
Hence, $32x = 352 \Rightarrow x = 11$

LOD II

1. $2/3 \log a = 3 \log b = 1/2 \log c$

2. $2\log(81/100) + 2/3 \log(27/10) - \log 9$
$= 2\log 3^4 - \log 100 + 2/3(\log 3^3 - \log 10) - 2\log 3$
$= 2\log 3^4 - \log 100 + 2/3(3\log 3 - 3) - 2\log 3$
Substitute $\log 3 = 0.4771 \Rightarrow -0.0952$

3. Let the number be x
$x = 100^{10}$
$\Rightarrow \log x = 10\log 100$
$\text{Log}\, x = 10\log(2 \times 4)$
$\text{Log}\, x = 10(\log 2 + 2\log 2)$
$\text{Log}\, x = 30(1.43 + 0.602)$
Hence $\log x = 30(2.03) = 20.2$
Thus, x has 21 digits.

4. $\log y = 25\log 125$
$= 25(\log 1000 - 3\log 2) = 25 \times (2.0605)$
$= 52.4$
Hence 53 digits.

5. $0.5\log_{125} 128$
$= 0.5(\log_5 128/\log_5 0.625)$
$= 1/2(\log_5 128/\log_5 0.625)$

$$\frac{\text{Log}_5 128}{2(\log_5 5 - \log_5 8)} = \frac{\text{Log}_5 128}{2(\log_5 5 - 1)}$$

6. $(25/25) \times (49/25) = (x/105) \Rightarrow (25/49) = 1$
$\Rightarrow x = 13$

7. $(169/x) \times (100/x) \times (65/100) = 1$
$\Rightarrow x = 7$

8. Solve in similar fashion.

9. $\text{Log}_{11} 275 < \log_{11} 375$
Because the value of $\text{Log}_{11} 275$ is less than 2 while $\log_{11} 375$ is greater than 2

10. $\log_{11} 1650 > 3$
$\text{Log}_{11} 1950 < 3$
Hence, $\log_{11} 1650 > \log_{11} 1950$

11. $\dfrac{\log_2 4096}{3} = \log_4 4096$

12. $x = (16/15) \times (25/24^2) \times (81/80^2)$
None of these is correct.

13 – 18.
Solve similarly as 3 and 4.

16. $\log(a^2b^3) - (\log b^2 c^3) = \log 1 = 0$

19. $(1/2)\log x = 2\log 10$
$\Rightarrow \log x = 4\log 10$
$\Rightarrow \log x = 4/\log_{10} x (\log x)^2 = 4$
So $\log x = 2$ and $x = 100$.

20. $\log x = \log_{(x+10)} 2$

$$= \frac{\text{Log}\, 2}{\text{Log}\, 2 + \log 10} = \frac{\text{Log}\, 2}{(\text{Log}\, 3 + \log 7 - 1)}$$

21. $6x^2 + 12x + 3 = 0$ or $2x^2 + 4x + 1 = 0$
Solving we get none of these.

25. For $x = 0$, we have LHS
$\text{Log}_3 3 = 3$
RHS: $10^{\log 3} = 3$
We do not get LHS = RHS for other $x = 3$ or $x = 4$.
Thus, option (a) is correct.

ANSWER KEY

Exercise 10.01

1. a
2. b
3. a
4. a
5. a
6. b
7. d
8. a
9. a
10. c
11. c
12. b
13. a
14. b
15. d
16. a
17. c
18. a
19. a
20. d
21. a
22. c
23. b
24. c

Logarithm LOD II

1. a
2. b
3. a
4. a
5. d
6. c
7. a
8. c
9. b
10. a
11. a
12. c
13. b
14. a
15. c
16. d
17. b
18. c
19. b
20. a
21. c
22. b
23. c
24. c
25. a

BLOCK REVIEW TESTS

REVIEW TEST ONE

1. Consider the following two curves in the x-y plane:

$$y = x^2 + x^2 + 3$$
$$y = x^2 + x + 2$$

Which of the following statements is true for $-4 \le x \le 4$?

(a) The two curves intersect once.
(b) The two curves intersect twice.
(c) The two curves do not intersect.
(d) The two curves intersect thrice.

2. Given that $-1 \le x \le 1$ and $-2 \le x \le -0.5$ and $-2 \le y \le 0.5$ and $x = y/x$, then which of the following is necessarily true?

(a) $-0.5 \le x \le 2$ (b) $-4 \le x \le 4$
(c) $-4 \le x \le 0$ (d) $-2 \le x \le 0$

3. Let $g(x) = \max(7 - x, x + 3)$. The smallest possible value of $g(x)$ is:

(a) 4 (b) 4.5
(c) 5 (d) None of the above

4. p, q, r are real numbers such that $p \ne q$, $r \ne 0$. Of the conditions given in the options, which one must p, q and r satisfy so that the following system of linear simultaneous equations has at least one solution?

$$x + 2y - 3z = p$$
$$2x + 6y - 11z = q$$
$$x - 2y + 17z = r$$

(a) $5p - 2q - r = 0$ (b) $3p + 2q + r = 0$
(c) $5p + 2q - r = 0$ (d) $3p - 2q + r = 0$

5. A leather factory produces two kinds of bags, standard and deluxe. The profit margin is Rs 20 on a standard bag and Rs 30 on a deluxe bag. Every bag must be processed on Machine A and on Machine B. The processing times per bag on the two machines are as follows:

	Time required (Hours/bag)	
	Machine A	Machine B
Standard Bag	4	4
Deluxe Bag	5	10

The total time available on machine A is 1000 hours and on machine B is 2500 hours. Among the following production plans, which one meets the machine availability constraints and maximizes the profit?

(a) Standard 150 bags, Deluxe 100 bags
(b) Standard 200 bags, Deluxe 120 bags
(c) Standard 100 bags, Deluxe 200 bags
(d) Standard 150 bags, Deluxe 100 bags

6. The function $f(x) = (x - 2)(x + 1)(x^2 + 15x - x)$, where x is a real number, attains a minimum at

(a) $x = 2.3$ (b) $x = 2.5$
(c) $x = 2.7$ (d) None of the above

7. When the curves $y = \log_{10} x$ and $y = x^{-1}$ are drawn in the x-y plane, how many times do they intersect for positive values of x?

(a) Never (b) Once
(c) Twice (d) More than twice

8. In a coastal village, every year floods destroy exactly half of the huts. After the flood water recedes, twice the number of huts destroyed are rebuilt. The floods occurred consecutively in the last three years, namely 2001, 2002 and 2003. If floods are again expected in 2004, the number of huts expected to be destroyed is:

1. less than the number of huts existing at the beginning of 2001.
2. less than the total number of huts destroyed by floods in 2001 and 2003.
3. less than the total number of huts destroyed by floods in 2002 and 2003.
4. more than the total number of huts built in 2001 and 2002.

9. If x and y are integers then the equation $5x + 10y = 64$ has

(a) no solution for $x < 200$ and $y < 0$
(b) no solution for $x > 250$ and $y > -100$
(c) a solution for $250 < x < 300$
(d) a solution for $-50 < y < -36$

Directions for Questions 10 and 11: The graph given below represents a quadratic expression, $f(x) = ax^2 + bx + c$. The minimum value of $f(x)$ is -2 and it occurs at $x = 2$.

x_1 and x_2 are such that $x_1 + x_2 + 2x_1x_2 = 0$.

(b) What is the nature of the roots of the equation $f(x) = 0$?
(a) Complex conjugate
(b) Rational
(c) Irrational
(d) Cannot be determined

12. Find the value of the function at $x = 1$
(a) 2 (b) -1
(c) 3 (d) none of these

13. Let f denote the infinite sum $1 + 2x + 3x^2 + 4x^3 + 5x^4 + \ldots$, where $|x| < 1$ and the coefficient of x^{n-1} is $(n + 1)$, $(n = 1, 2, \ldots)$. Then f equals
(a) $(2 - x)(1 - x)^2$ (b) $(2 - x)(1 + x)^2$
(c) $(2 + x)(1 - x)^2$ (d) $(2 + x)(1 + x)^2$

14. Sachin was asked to calculate the arithmetic mean of ten positive integers each of which had two digits. By mistake he interchanged the two digits in one of the ten integers. As a result, his answer for the arithmetic mean became 1.8 more than it should have been. Based on this information, find the value of $|x - y|$.
(a) 5 (b) 2 (c) 3 (d) 4

15. Suppose for any real number x, $[x]$ denotes the greatest integer less than or equal to x. Let $f(x, y) = [x] + [y] + [x + y]$ and $F(x, y) = [2x] + [2y]$. Then it's impossible to find any two positive real numbers x and y for which
(a) $f(x, y) \in R$ (b) $f(x, y) \in R$ (c) $F(x, y) \in R$ (d) $F(x, y) \in R$

16. If x, y and z are natural numbers such that $x^{y^z} = y^{z^x} = z^{x^y}$, then which one of the following is true
(a) $\min(x, y, z) = 1$ (b) $\max(x, y, z) = 1$
(c) $\min(x, y, z) = 2$ (d) None of these

17. If $pqr = 1$, the value of the expression $(1 + p + q^{-1}) + (1 + q + r^{-1}) + (1 + r + p^{-1})$ is
(a) $p + q + r$ (b) $(p + q + r)$
(c) 1 (d) $p^{-1} + q^{-1} + r^{-1}$

17. Sanjay's shop sells varieties of different dress. Each variety is sold at the price of Rs. 3 per variety up to 200 varieties. Beyond 200, for every 20 additional varieties, the price of the whole lot goes down by 10 paise per variety. What should be the maximum size of the lot in order to maximize revenue
(a) 280 (b) 300
(c) 400 (d) None of these

18. Let x, y and z be distinct integers x and y are odd and positive, and z is even and positive. Which one of the following statements cannot be true?
(a) $(x - y)^2 z$ is even (b) $(x - y)^2 z$ is odd
(c) $(x - y) z$ is odd (d) $(x - y)^2 z$ is even

Directions for Questions 19 to 21: Answer these questions on the basis of the information given below.

x, y and z are three real numbers satisfying the following conditions

$$x + y + z = 10$$
$$x^3 + y^3 + z^3 = 62$$
$$x^2 + y^2 + z^2 = 340$$

19. Find the value of $(xy + yz + zx)$.
(a) 21 (b) 37 (c) 39 (d) 35

20. Find the value of xyz
(a) 30 (b) 35 (c) -15 (d) -30

21. Find the value of $x^4 + y^4 + z^4$
(a) 1925 (b) 1922
(c) 2921 (d) None of these

22. Every ten years, the Indian government counts all the people living in the country. Suppose that the director of the census has reported the following data on two neighbouring villages Chota Hazri and Moti Hazri:

Chota Hazri has 4,920 fewer males than Moti Hazri.
Moti Hazri has 6,020 more females than males.
Chota Hazri has twice as many females as males.
Chota Hazri has 2,910 fewer females than Moti Hazri.
What is the total number of males in Chota Hazri?
(a) 10284 (b) 14174
(c) 3652 (d) 10854

23. At a certain fast food restaurant, Brian can buy 3 burgers, 7 shakes, and one order of fries for Rs. 120 exactly. At the same place, it would cost Rs. 164.50 for 4 burgers, 10 shakes, and one order of fries. How much would it cost for an ordinary meal of one burger, one shake, and one order of fries?
(a) Rs. 31 (b) Rs. 41
(c) Rs. 21 (d) Cannot be determined

24. If $x > 7$ and $y < -2$, then which of the following statements is true?
(a) $(x + 4)(y) > 1$ (b) $x/y < -4/2$
(c) $-4x < 2y$ (d) None of these

25. For a difference sequence, from the third term onwards, each term in the sequence is the sum of the previous two terms in the sequence. If the difference is square of several, and each series of this sequence is 512, what is tenth term of this sequence?
(a) 341 (b) 36
(c) 121 (d) Cannot be determined

REVIEW TEST TWO

1. Let x be two positive numbers such that $x + y = 1$.

Then the minimum value of $\left(x + \frac{1}{x}\right)^2 + \left(y + \frac{1}{y}\right)^2$ is

(a) 0 (b) 25 (c) 12.5 (d) 12.5

2. Disha and Kushal attempted to solve the quadratic equation. Disha made a mistake in writing down the constant term. He ended up with the roots (4, 3). Kushal made a mistake in writing down the coefficient of x. He wrote as (3, 2). What will be the exact roots of the original quadratic equation?

(a) (6, 1) (b) (−2, −4) (c) (4, 3) (d) (−4, −3)

3. A change making machine contains 1 rupee, 2 rupee and 5 rupee coins. The total number of coins is 500. The amount is Rs. 900. If the number of 1 rupee coins and the number of 2 rupee coins are interchanged, the total value goes by Rs. 40. The total number of 5 rupee coins is

(a) 100 (b) 120 (c) 80 (d) 150

4. If $x < 2$ and $x > -1$, then which of the following statements is necessarily true?

(a) $x < -2$ (b) $-x < 2$

(c) $x \geq -2$ (d) $-x > 2$

Answer for Questions 5–7: A, B, C and D collected coins on coins taken by the given pattern.

Twelve boy collected 100 coins.

Two girl of them collected an equal number of coins.

Two girl of them collected at least 10 coins.

Boy of them collected the same number of coins.

5. The maximum number of coins collected by any one of them cannot exceed

(a) 52 (b) 80

(c) 92 (d) None of these

6. If A collected 54 coins, then the difference in the number of coins between the one who collected maximum number of coins and the one who collected the second highest number of coins must be at least

(a) 32 (b) 24

(c) 30 (d) None of these

7. If A collected 54 coins and B collected two more coins than what the number of coins collected by C, then the number of coins collected by B could be

(a) 38 (b) 20 (c) 36 (d) 22

Directions (Q8 – 9): A, B and C are three numbers. Let

@ (A, B) = Average of A and B.

(A, B) = Product of A and B, and

$ (A, B) = The result of dividing A by B.

8. The sum of A and B is given by:

(a) $(A@(A, B))$ (b) $(A#(A, B))$

(c) $(B@(A, B))$ (d) $(B#(A, B))$

9. Average of A, B and C is given by:

(a) $(A@(A@(A, B), C))$

(b) $(A@(A@(A, B), C))$

(c) $(A@(B@(A, C), C))$

(d) $(A@(A@(B, A), C))$

Directions (Q10 – 11): The real numbers x and y, let $f(x, y)$ = Positive square root of $(x + y)$, if $(x + y)^{1/2}$ is real = 0, + x^2, otherwise $g(x, y) = (x + y)^2$, if $(x + y)^{1/2}$ is real = $-(x + y)$ otherwise

10. Which of the following expressions yields a positive value for every pair of non zero real numbers (x, y)?

(a) $f(x, y) + g(x, y)$ (b) $f(x, y) - |g(x, y)|^2$

(c) $g(x, y) - |f(x, y)|^2$ (d) $f(x, y) + g(x, y)$

11. Under which of the following conditions is $f(x, y)$ necessarily greater than $g(x, y)$?

(a) Both x and y are less than 1

(b) Both x and y are positive

(c) Both x and y are negative

(d) $y > x$

Directions (Q 12–14): For three distinct real numbers x, y and z. Let

$f(x, y, z)$ = max (max (x, y), max (y, z), max (z, x))

$g(x, y, z)$ = max (min (x, y), max (y, z), min (z, x))

$h(x, y, z)$ = min (min (x, y), max (y, z), min (z, x))

$j(x, y, z)$ = min (max (x, y), min (y, z), min (z, x))

$m(x, y, z)$ = min (x, y, z)

$n(x, y, z)$ = max (x, y, z)

12. Which of the following is necessarily greater than 1?

(a) $[h(x, y, z) - f(x, y, z)]/(n(x, y, z))$

(b) $j(x, y, z)/h(x, y, z)$

(c) $f(x, y, z)/g(x, y, z)$

(d) $[f(x, y, z) + h(x, y, z)] - g(x, y, z)]/(n(x, y, z))$

13. Which of the following expressions is necessarily equal to 1?

(a) $[f(x, y, z) - m(x, y, z)]/[n(x, y, z) - m(x, y, z)]$

(b) $m(x, y, z) - [f(x, y, z)] - f(x, y, z)]/[n(x, y, z) - m(x, y, z)]$

(c) $[f(x, y, z) - g(x, y, z)]/h(x, y, z)$

(d) $[f(x, y, z) - h(x, y, z)]/[n(x, y, z)]$

14. Which of the following expressions is an identity?

(a) $[f(x, y, z) - h(x, y, z)]/[g(x, y, z) - j(x, y, z)]$

(d) $f(x, y, z) = f(x, y, z) + f(x, y, z) + f(x, y, z)$ $f(f(x, y, z), y, z)$
$+ f(x, y, z) - f(x, y, z) - f(x, y, z)$
(e) $f(f(x, y, z) - f(x, y, z)) f(f(x, y, z) - f(x, y, z))$
(f) $f(f(x, y, z) - f(x, y, z)) f(f(x, y, z) - g(x, y, z))$.

15.

x	1	2	3	4	5	6
y	4	8	16	28	44	56

In the above table, the variable chosen constants a, b and c, which one of the following best describes the relation between y and x?

(a) $y = ax + bx$ (b) $y = ax + bx + cx^2$
(c) $y = ax^{c+bx}$ (d) None of these

16. If $x_0 = 1$ and $x_{n+1} = 2x_n + 5$, $n = 1, 2, \ldots$ then x_{100} is equal to

(a) $(5 \times 2^{99} - 6)$ (b) $(5 \times 2^{99} + 6)$
(c) $(6 \times 2^{99} + 5)$ (d) $(6 \times 2^{99} - 5)$

17. Let x, y and z be distinct integers that are odd and positive. Which one of the following statements cannot be true?

(a) $(x)^2$ is odd
(b) $(x - y)^2$ is even
(c) $(x + y - z)^2$ $(x + y)$ is even
(d) $(x - y)(y + z)$ $(x + y - z)$ is odd

18. Let S be the set of prime numbers greater than or equal to 2 and less than 100. Multiply all elements of S. With how many consecutive zeros will the product end?

(a) 1 (b) 4 (c) 5 (d) 0

19. Each of the numbers $x_1, x_2, \ldots, x_n$, $n \geq 4$, is equal to 1 or −1. Suppose

$x_1x_2x_3 + x_2x_3x_4 + x_3x_4x_5 + \ldots + x_{n-2}x_{n-1}x_n = 0$
$x_1 + x_{n-1}x_2x_3 + x_2x_3x_4 = 0$, then

(a) n is even
(b) n is odd
(c) n is an odd multiple of 3
(d) n is prime

Recursive function: $f(n) = (n) + n$, if n is non-negative
$= 1 + n$, if n is negative $f^{(n)}(n) = f(f^{(n-1)}(n))$ $(n \geq 2)$.

20. What is the value of the product $f(3)$ $f^{(2)}(3)$ $f^{(3)}(3)$ $f^{(4)}(3)$?

(a) 1101 (b) 3
(c) 1100 (d) None of these

21. r is an integer ≥ 2. Then, what is the value of $f^{(r-1)}(-1)$ $+ f^{(r-2)}(-1) + f^{(r-3)}(-1) + \ldots + f(-1)$?

(a) −1 (b) 0
(c) 1 (d) None of these

22. The set of all positive integers is the union of two disjoint subsets $\{f(1), f(2), \ldots, f(n), \ldots\}$ and $\{g(1), g(2), \ldots, g(n), \ldots\}$ where $f(1) < f(2) < \ldots < f(n) < \ldots$ and $g(1) < g(2) < \ldots < g(n) < \ldots$ and $g(n) = f(f(n)) + 1$ for all $n \geq 1$.

What is the value of $g(1)$?

(a) 0 (b) 2
(c) 1 (d) cannot be determined

Direction for Question (23-25): There are three bottles of water, A, B, C, whose capacities are 10X, 6L, and 4L respectively. For transferring water from one bottle to another and to drain out the bottles, there exists a piping system. The flow through these pipes is computer controlled. The computer that controls the flow through these pipes can be fed with three types of instructions, as explained below.

Instruction type	Explanation of the instruction
Fill(X, Y)	Fill bottle labeled X from the water in bottle labeled Y where the remaining capacity of X is less than or equals to the amount of water in Y
Empty (X, Y)	Empty out the water in bottle labeled X into bottle labeled Y, where the amount of water in X is less than or equals to remaining capacity of Y.
Drain (X)	Drain out all the water contained in bottle labeled X.

Initially, A is full with water, and B and C are empty.

23. After executing a sequence of three instructions, bottle A contains two liters of water. The first and the third of these instructions are shown below.

First instruction: FILL (C, A)
Third instructor: FILL (C, A)

Then, which of the following statements about the instructions is true?

(1) The second instruction is FILL(B, A)
(2) The second instruction is EMPTY(C, B)
(3) The second instruction transfers water from B to C
(4) Cannot be determined

24. Consider the same sequence of three instructions as the same initial state mentioned above. Three more instructions are added at the end of the above sequence to have A contain 8L of water. In this total sequence of six instructions, the fourth one is DRAIN (A). This is the only DRAIN instruction in the entire sequence. At the end of the execution of the above sequence, how much water is contained in A?

(a) 2L (b) 4L
(c) 6 (d) None of these

25. For the question above, what is the amount of water contained in B?

(a) 2L (b) 4L
(c) 6 (d) None of these

REVIEW TEST THREE

Question for Questions 1–3: These are based on the functions defined below

$Q(a, b)$ = Quotient when a is divided by b

$R(a, b)$ = Remainder when a is divided by b

$R(a, b) = a^2 b^2$

$D(a, b) = \sqrt{2a - 1}(b - 1)$

1. $Q(3, 10) - 7 = 0$

(a) $Q(3, R(3, 10))$ (b) $R^2(3, 10) + Q(3, 10)$

(c) $R(3, 10)/2$ (d) $\frac{1}{2}(R(2, 3) + 3Q(17, 2a))$

2. $Q(a, b)$ is same as

(a) $Q(a, R(a))$ (b) $\sqrt{R(a, b) - 1}$

(c) $R(a + 1), (b - 1)$ (d) $\sqrt{R(a, 1) - 1}\sqrt{R(a, 1)}$

3. Which of the following relations cannot be false?

(a) $R(a, b) = R^2(a, b) \cdot Q(a, b)$

(b) $a^2 \cdot Q(a, b) + b^2 \cdot R^2(a, b)$

(c) $a + R^2(a, b) + a \cdot Q(a, b)$

(d) $Q(a, b) + R(a, b) \cdot R^2(a, b)$

Question for Questions 4–7: Answer the questions based on the following information

$N(a, b)$ = least of a and b

$M(a, b)$ = greatest of a and b

$W(a)$ = absolute value of a

4. Find the value of $\{+M\{x + N(-W(x, y))\}, N\{y + W(M(x, N(x)))\}\}$ given that $x = 2$ and $y = -3$.

(a) 4 (b) 1 (c) 2 (d) 3

5. Given that $x > 0$, then the relation $M\{N(x), W(x, y)\} = W\{x, N(W(x, y))\}$ does not hold if

(a) $x > 0, y > 0, (x + y > x)$

(b) $x > 0, y < 0, (y + y > x)$

(c) $y > 0, x > 0$

(d) $x < 0, y > 0$

6. Which of the following must be correct for $x, y > 0$

(a) $N(W(x, y)) = W(N(x), N(y))$

(b) $N(W(x, y)) = W(N(x), N(y))$

(c) $N(W(x, y)) = W(N(x), W(x))$

(d) $N(W(x, y)) = W(N(x), W(x))$

7. For what value of x is $W(x^2 + 2x, x + 3) = 0$?

(a) $-2 < x < 2$

(b) $-2 < x < 0$

(c) $x < -2$

(d) Both (2) and (3)

8. It is given that $(a^{n-1} + a^{n-2}b^2 + \ldots + b^{n-1})(p - q) = 0$, where p and $q \neq 0$ and a is odd then $\frac{(a^n - b^n)(a + b)}{(a^n + b^n)(a - b)} =$?

(a) 1 (b) -1 (c) $\frac{a}{b}$ (d) 0

9. If $f = \frac{1}{\log_3 x} + \frac{1}{\log_{x+3} x}$, which of the following is true?

(a) $f > 4$ (b) $2 < f < 4$

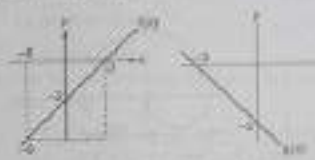
(c) $1 < f < 2$ (d) $0 < f < 1$

10. If $pq + qy = rq + ry$, and $p, r, q, y > 0$ and $(r + y) < p$, then which of the following must be true?

(a) $\frac{p}{q} > \frac{r}{y}$ (b) $p - q > r - y$

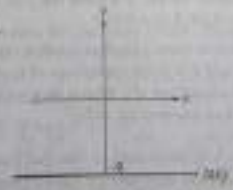
(c) $p + q > r + y$ (d) $p + q = r + y$

Question for Questions 11–13: $f(x)$ and $g(x)$ are defined by the graphs shown below:



Each of the following questions has a graph of function $h(x)$ with the answer choices expressing $h(x)$ in terms of variation of $f(x)$ or/and $g(x)$. Choose the alternative that exactly represents the relationship approximately.

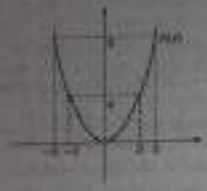
11.



(a) $h(x) = f(x)$ (b) $-1.5f(x) + 1.5g(x)$

(c) $1.5f(x) + 1.5g(x)$ (d) None of these

12.

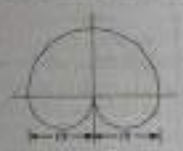


(a) $9 - f(x)g(x)$
(b) $[f(x) + g(x) + 4]^2 + [f(x) - 2]^2 + [g(x)]$
(c) $f(x) - g(x) + 4]^2 - 2$
(d) $f(x)g(x) - 9$

13. (a) $2f(x) + g(x)$ (b) $f(x) + 2g(x) - 9$
(c) $\frac{3}{2}f(x) + \frac{g(x)}{2}$ (d) None of these

14. If $p = q = r$ and $\frac{p}{q} = \frac{q}{p} = \left(\frac{1}{a} + \frac{1}{c}\right)k$?
(a) 1 (b) 1/2 (c) 3/4 (d) 2

15. What could be the equation of the following curve?



(a) $(a^2 + b^2)^2 = a^2(a^2 + b^2)$
(b) $a^2 + b^2 - ab(a^2 + b^2) = a^2(a^2 + b^2)$
(c) $a^2 + b^2 - ab = a^2(a^2 + b^2)$
(d) None of these

Directions for Questions 16–18: It is given that $f(x) = p^x$, $g(x) = (-p)^x$, $h(x) = (1/p)^x$, $k(x) = (-1/p)^x$

16. Think of a situation where a function is odd or even, if the function is odd it is given a weightage of -1, otherwise, it is given a weightage of 0. What is the result if the weightages of four functions are added?
(a) 2 (b) 0 (c) 1 (d) −1

17. If 'p' and 'x' are both whole numbers other than 0 and 1, which of the functions must have the highest value?
(a) $g(x)$ only (b) $f(x)$ only
(c) $g(x)$ and $h(x)$ both (d) $h(x)$ and $k(x)$ both

18. Which of the following is true if 'p' is a positive number and 'x' is a real number?
(a) $f(x) - h(x)$ / $f(x) - k(x)$ is always positive
(b) $f(x)$, $k(x)$ is always negative
(c) $f(x)$, $h(x)$ is always greater than one
(d) $g(x)$, $k(x)$ could even outside the real domain

19. If x, and $y \geq 1$ and belong to set of integers then which of the following is true about the function $f(x, y)$?
(a) The function is odd if 'x' is even and 'y' is odd.
(b) The function is odd if 'y' is odd and 'y' is even.
(c) The function is odd if 'x' and 'y' both are odd.
(d) The function is even if 'x' and 'y' both are even.

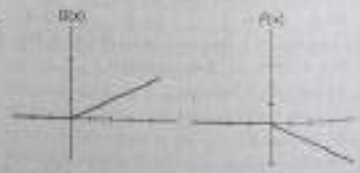
20. If $f(a, b)$ = remainder left upon division of b by a, then the maximum value for $f(f(a, b), f(a + 1, b + 1))$ is to 0 is ..... (b and a are co-primes)
(a) $a - 1$ (b) a (c) 0 (d) 1

REVIEW TEST FOUR

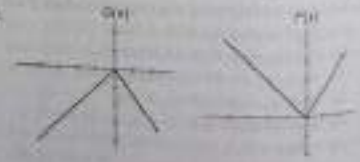
1. The number of solutions of $\frac{\log 5 + \log(x^2 - 1)}{\log(x - 2)} = 2$ is
(a) 3 (b) 2
(c) 1 (d) None of these

Information for 2–3: Graphs below are two graphs labeled F(x) and G(x). Compare the graphs and give the answer in accordance to the options given below.
(a) $F(-x) = G(x) + c$?
(b) $F(-x) = -G(x) - c$?
(c) $F(-x) = -G(x) + c$?
(d) $2F(-x) = +G(x) - c$?

2.



3.



6. If x is a natural number, which of the following statements is/are true?
   (a) $(x+1)(x^2+1)$ is odd
   (b) x^2+3x+6 is even
   (c) x^2-2x is even
   (d) $x^2(x^2+x)+1$ is odd

7. In the figure below, equation of the line PQ is

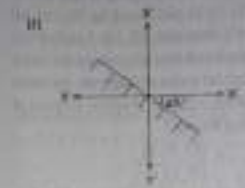
   (a) $x+y=120$ (b) $2x+y=120$
   (c) $x+2y=120$ (d) $2x+y<120$

8. For which of the following functions is $\frac{f(ab)-f(b)}{a-b}$ constant for all the numbers 'a' and 'b', where $a \neq b$?
   (a) $f(x)=kx+7$ (b) $f(x)=x+x^2$
   (c) $f(x)=\cos x$ (d) $f(x)=\log_e x$

9. Given that $f(a,b,c)=\frac{a+b+c}{3}$ then
   (a) $f(a,b,c) \geq \frac{|a|+|b|+|c|}{3}$
   (b) $f(a,b,c) \leq \max(a,b,c)$
   (c) $f(a,b,c) \geq \frac{3a+b+c}{3}$
   (d) $f(a,b,c) \leq \frac{|a|+|b|+|c|}{3}$

10. Given two variables 'x' and 'y'. The values of the variables are $x=\frac{1}{a+b}$ and $y=\frac{3}{c+a}$. Find the value of the expression $\frac{3x}{y}-1$.
   (a) $\frac{2(a+b)^2}{(a+c)(a+1)}$ (b) $\frac{3(a+b)}{2ab+ac}$
   (c) $\frac{2}{3(a+c)(a+b)}$ (d) None of these

11. If $y=\frac{(x-1)(x-3)}{(2x+1)(x-3)}$ the maximum value that 'y' can attain is
   (a) 1 (b) 2 (c) 21 (d) 12

12. 

   Refer to the graph. What does the shaded portion represent?
   (a) $x+y \leq 0$ (b) $x \leq y$
   (c) $y+12x \geq 1$ (d) $x+y \geq 0$

13. If a, b, c are positive numbers and it is known that $a^2+b^2+c^2=8$ then
   (a) $a^3+b^3+c^3 \leq 16\sqrt{\frac{2}{3}}$
   (b) $a^3+b^3+c^3 \geq 64$
   (c) $a^3+b^3+c^3 \geq 16\sqrt{\frac{2}{3}}$
   (d) $a^3+b^3+c^3 \leq 64$

14. Find the value of the expression given in terms of 'a(c)' alike 'x' and 'y'.

$$\frac{(x^2+(a-c)x-ac)(x^2-ax-bx+ab)(x+c)}{(x^2-a^2)(x^2-bx^2-c^2x+bc^2)}$$

   (a) $\frac{(x-b)(x-c)}{(x-a)}$ (b) $\frac{(x+a)(x+c)}{(x-b)}$
   (c) $\frac{(x+a)(x-b)}{(x-c)}$ (d) None of these

Direction for Questions 15–16: These questions are based on the relation given below:
$f'(x)=f''(x-1)$ where $x \geq 1$ (integer values only) and $f''(x)=2x$ if 'x' is positive or $f''(x)=16(x^2+1)$ otherwise.

15. What is $f''(x-1)$?
   (a) 0 (b) 1
   (c) 2 (d) Indeterminate

16. What is the value of $f'(x+1)$?
   (a) 1 (b) x (c) 2x (d) 2

17. Roman devised an equation to denote distance of a Halley's comet (x) in the form of a quadratic equation. Distance is given by solution of quadratic $x^2+bx+c=0$. To determine constants of the above equation for

[illegible] conducted by Annie. Based on the data of the [illegible] total of [illegible] is 1, 2, 3 and based on the second table (Annie [illegible] statement (s)), (a) Least [illegible] is [illegible] (b) [illegible] of data pass statement (s) [illegible] of students" while more than of data pass statement (s) [illegible] value of statement (s). What is the sum of actual [illegible] of Ticket's Counter [illegible] Annie?

(a) 11.3 (b) 15.3

(c) 16.9 (d) 15.75

10. a, b^2 and c^3 are three real numbers. If $a+b+c=0$ the following statements is/are always true?

(a) $a+b+c=-3abc$

(b) $a^2+b^2+c^2=2(ab+bc+ca)$

(c) $a^3+b^3+c^3=3abc$

(d) $a^2b+b^2c+c^2a=-a^3$

(a) Only (a) (b) Only (b) and (c) are true

(c) Only (c) is true (d) None is true

11. If we have $\int g(x)\,dx=g(x)+c$ $\int f(x)\,dx$, then which of the following is true?

(a) $\int f(x)\{f(x)+g(x)\}\,dx=\{f(x)+g(x)\}^2+c$

(b) $\int f(x)\{f(x)+g(x)\}\,dx=\{f(x)+g(x)\}^2/2+c$

(c) $\int f(x)\{f(x)+g(x)\}\,dx=f(x)\{f(x)+g(x)\}+c$

(d) $\int f(x)\{f(x)+g(x)\}\,dx=f(x)\{f(x)+g(x)\}/2+c$

12. If $y=\frac{x^2-1}{x^2+1}$ and $f(x)=\frac{x^2+1}{(x+1)^2}$, what is $f\left(\frac{1}{y+1}\right)$?

(a) $\frac{1}{x^2}$ (b) $\frac{1000}{1+x^2}$

(c) $\frac{x^2}{x^2+1}$ (d) 1

13. The [illegible] $\lim_{x\to\infty}\frac{\ln x}{x}$, $\lim_{x\to\infty}\frac{\ln x}{x^2}$, $\lim_{x\to 0}\frac{\ln x}{x}$ and $\lim_{x\to 0}\frac{\ln x}{x^2}$ are

(a) 1 (b) 0 (c) 0/0 (d) 2

14. The number of solutions of $\frac{\log_2(x^2+\log_2(x^2+4))}{\log_2(x-2)}=2$ is

(a) 10 (b) 5

(c) 1 (d) None of these

REVIEW TEST FIVE

Directions for the Questions 1–6: Refer to [illegible] given below and answer the questions.

[illegible] $\frac{m}{n}=\frac{3}{2}\frac{4}{3}=\frac{3}{2}$ and $1+\frac{m+1}{n+2}$, answer the following [illegible] below (a) to (d).

1. [illegible]

(a) [illegible] (b) $\left\{\frac{1}{3}, 1, 2\right\}$

(c) $\left\{2, \frac{1}{3}, 1\frac{1}{3}\right\}$ (d) $\{1, 2, 3\}$

2. [illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

3. If [illegible], what is the [illegible] value of [illegible] [illegible] highest value of [illegible]?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

4. Find the [illegible] of [illegible]

(a) 2 (b) [illegible]

(c) [illegible] (d) 3

5. If [illegible] and [illegible] [illegible]

(a) [illegible] decreases, [illegible] decreases

(b) As [illegible] increases, [illegible] decreases

(c) As [illegible] increases, [illegible] increases

(d) [illegible] increases, [illegible] increases

6. If [illegible] are real functions defined by [illegible] and [illegible], then [illegible] is equal to

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

7. If [illegible] and [illegible] are the roots of the equation [illegible] [illegible], the value of [illegible] is

(a) 7 (b) 7 (c) 16 (d) 16

8. Given that [illegible] and [illegible] are positive real numbers satisfying [illegible], what is the maximum value of

$$\sqrt{\left(1+\frac{1}{x^2}\right)}+\sqrt{\left(1+\frac{1}{y^2}\right)}$$

(a) 8 (b) 16 (c) 40 (d) 4

9. Let [illegible] be the sum of positive integer [illegible] [illegible] [illegible]

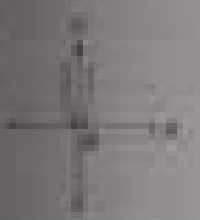
[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) None of these

Questions [illegible] (Questions [illegible]). The following questions are based on the graph of quadratic polynomial $y = f(x) = ax^2 + bx + c$. Mark the appropriate alternative according to given conditions. (Take all data and information from the graph.)

The above graph represents the equation

(a) $f(x) = 0$, $a, b, c > 0$ (b) $f(x) = 0$, $a > 0$
(c) $f(x) = 0$, $a < 0$, $b, c > 0$ (d) $f(x) = 0$, $a < 0$, $b, c < 0$



The above graph represents the equation

(a) $f(x) = 0$, $a, b, c < 0$ (b) $f(x) = 0$, $a > 0$, $b, c < 0$
(c) $f(x) = 0$, $a < 0$, $b, c > 0$ (d) None of these

[illegible]

Suppose a [illegible]

According to the passage, the [illegible] $f(x) = 0$ [illegible] $f(x) = 0$. Also also gives [illegible] $f(x)$. Which of the following [illegible] $f(x)$ [illegible]

[illegible]

"Does the [illegible] to see the graph of the equation [illegible] $f(x) = 0$ [illegible] $f(x) = 0$ [illegible] $f(x) = 0$ [illegible] the following is an [illegible] and provides [illegible] Which of the following [illegible] Number of [illegible] [illegible]

[illegible] is correct the question is simple element. Can you answer the question in simple element? Can you answer [illegible] questions?

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) None of these

Directions for Questions [illegible] (Solve). [illegible]

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

17. Find the value of the following [illegible]

[illegible]

(a) 3 (b) 2
(c) [illegible] (d) None of these

18. Find the value of the following [illegible]

[illegible]

(a) 17 (b) 36 (c) 48 (d) 62

19. Given that [illegible] are 1, 2, 3, [illegible]

[illegible]

(a) 600 (b) 420
(c) 480 (d) None of these

20. We have [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]

21. [illegible] $x^2 - 2mx + 2 = 0$, [illegible] the set A of points on the curve [illegible] the tangent to the curve is horizontal and the set B of points on the curve at which the tangent to the curve is vertical are respectively

(a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

22. If [illegible] and [illegible] then the real [illegible]

[illegible] values of x such that [illegible] and [illegible] is represented by the inequalities

(a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

ANSWER KEY

Section Test (Set)

1	d
2	b
3	a
4	a
5	a
6	a
7	b
8	a
9	a
10	a
11	b
12	a
13	b
14	a
15	a
16	a
17	b
18	a
19	c
20	a
21	a
22	c
23	a
24	a
25	a

Review Test Three

1. d
2. a
3. c
4. a
5. d
6. c
7. a
8. a
9. c
10. a
11. c
12. a
13. d
14. d
15. a
16. b
17. b
18. a
19. b
20. c

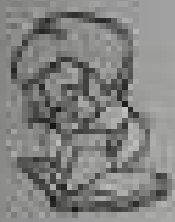
Rough Sol Fm

1	x
2	x
3	x
4	x
5	x
6	x
7	x
8	x
9	x
10	x
11	x
12	x
13	x
14	x
15	x
16	x
17	x
18	x
19	x
20	x

BLOCK 6

CHAPTERS

- Permutations and Combinations
- Probability



...BACK TO SCHOOL

[illegible]

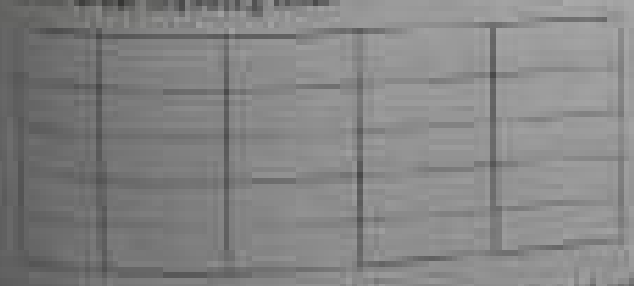
Category A: [illegible] are convenient in solving questions of this block, since they [illegible] the underlying concepts well.

Category B: Students who are not able to make questions in the block of chapters since they are not convenient with the counting tools and techniques in this block.

Belonging to the second category of students, the main thing you should realize is that fundamental concepts [illegible] contemporary school techniques of Permutations and Combinations. Once you are through with this, you should have no problem in understanding concepts in the Probability. Do remember that the very conclusions in the Permutations & Combinations (P&C) and Probability – do not [illegible] this block. However, before you start looking at these counting methods, right at the outset, I would want [illegible] any negative experiences you might have had with learning formulas P&C and Probability. For [illegible] you are a member of the second category of students, you are advised to read on:

Refer to the second category of students, you are advised to read on:

[illegible] the following table:



[illegible]

[illegible]

THE TOOL BOX APPROACH TO P&C

[illegible]

The Twelve Counting Situations and Their Tools

Tool No. 1: The ^nC_r tool – [illegible]

Example: [illegible]

Tool No. 2: [illegible]

Example: [illegible]

Tool No. 3: The P^n tool – [illegible]

[illegible]

[illegible]: The 2^n rule.—[illegible]

[illegible]: The [illegible] rule.—[illegible]

[illegible]: The $^{n+1}C_{n-1}$ rule.—[illegible]

[illegible]: The $^{n}C_{n-1}$ rule.—[illegible]

[illegible]: The [illegible] rule.—[illegible]

[illegible]: [illegible].—[illegible]

[illegible]: The AND rule.—[illegible]

[illegible]: The OR rule.—[illegible]

Read this 2: **Do I Probe Permutation Function = (2^Dim)?** — Two [illegible] the number of ways in which a [illegible] can be placed around a circle. The trend is indeed the value of the function by 2 — the reason lies in the fact that in a circle there is no distinct starting point.

Read this 3: **The Circular Permutation Function does not have to be distinguished between clockwise and anti-clockwise arrangements** [illegible] Thus, given the number of ways in which 5 students can sit for photos around a circle, there is no distinction between clockwise and anti-clockwise arrangements. In such a case the number of circular permutations are just divided by 2.

Two More Issues

(1) What about the 'N' issue? — The 'N' has a most to do with [illegible] the number of ways in which 5 students can sit for photos around a circle, the number of ways in which 5 students can sit for photos around a circle, the number of ways in which 5 students can sit for photos around a circle. For this we can consider the difference between the 'N' and the 'N-1' tools.

$$^{N}C_{r} \text{ and } ^{N-1}C_{r} \text{ where } r \in [1, \text{min}(N, N-1)]$$

A closer look would reveal that the relationship between the 'N' and the '$^{N-1}C_{r}$' tools can be represented as:

$$^{N}C_{r} = {}^{N-1}C_{r} + r$$

Read this 1: If we have to arrange r things out of n things, then simply select r things amongst n things and we find it has the arrangement of the selected r items amongst themselves. This will result in the same as the '$^{N}C_{r}$' tool.

Hence, for a question involving arrangement of r out of n items, the tool to be used is the '$^{N}C_{r}$' tool. While the question only requires the selection of r items from n items (but not r), then the '$^{N-1}C_{r}$' tool is the appropriate tool to be used.

(2) Your Sure Permutations and Combinations and Probability in English language characters rather than Maths examples: One of the key discoveries in getting comfort in this block is that after learning the basic rules involving characters and then moving on to optional choices you need to be comfortable in the manner in the chapter on Permutations & Mathematics and rather spend time in questions in English. Thus, while solving 1 question in these chapters from most commonness and others should be the concerning the question language rather than mere logistics.

What do I mean by this?

Consider this:

Question Language: "In how many ways can a committee of 3 men and 2 women be formed from a group of 5 men and 3 women, if one woman and one man refuse to work together?"

Now consider:

Rewrite the question, we create the mirror language in your mind:

Mirror Language: Select one random set of 5 MWWMM out of the 5 M and 3 W. Find the probability of a MW group in the most efficient mirror language formation.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

1. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

2. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

3. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

4. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

5. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

6. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) None of these

7. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) None of these

8. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) None of these

9. [illegible]

   (a) [illegible] (b) [illegible]
   (c) [illegible] (d) [illegible]

10. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

11. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

12. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

13. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

14. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

Direction for Questions 15-17: Read the [illegible] and select the appropriate [illegible]

[illegible] cricket matches each with New Zealand [illegible] South Africa. In any match, the probability of Africa [illegible] New Zealand is given below:

[illegible]	[illegible]	[illegible]	[illegible]
Probability	[illegible]	[illegible]	[illegible]
Points	2	[illegible]	1

[illegible] of all the matches are independent of each other.

15. What is the probability of India getting 4 total points in the tournament [illegible] of New Zealand [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

16. What is the probability of South Africa getting a total of [illegible] between South Africa and New Zealand [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

17. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) None of these

18. The number of ways in which a chord can [illegible] 8 points and of 9 chords, if 3 matrices are [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

19. What is the value of [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) None of the [illegible]

20. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

21. [illegible]

    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

[illegible] Now, many ways in which the [illegible] can be formed?

(a) [illegible] (b) [illegible]

(c) [illegible] (d) None of these

[illegible] Each question contains [illegible] statements, each of which may be true or false. In how many ways can a student tick the [illegible] statements such that exactly 3 statements are true?

(a) $^{10}C_3 \times {}^{7}P_3 \times 9$ (b) $^{10}C_3 \times {}^{7}C_3 \times 8$

(c) $^{10}C_3 \times {}^{7}C_3 \times 9$ (d) $^{10}C_3 \times {}^{7}P_3 \times 8$

[illegible] A question paper contains 14 true statements, 8 false statements and 3 ambiguous statements. Each question carries 4 marks for a correct answer, 2 marks for a wrong answer and 1 mark for an ambiguous statement. In how many ways can a student tick the statements such that exactly 30 marks are obtained? In how many ways can a student tick the statements such that exactly 30 marks are obtained by attempting exactly 15 questions?

(a) 78 (b) [illegible]

(c) 81 (d) 83

[illegible] A student is asked to tick at least a [illegible] [illegible] statements (true or false). If the total number of ways in which he can tick at least one [illegible] is 63, find the value of n.

(a) 8 (b) [illegible]

(c) 5 (d) 6

[illegible] A fair coin and a normal unbiased die are tossed. Let the [illegible] be the [illegible] of the coin and [illegible] be the [illegible] of the [illegible] of the die. Let [illegible] be the [illegible] of the [illegible] of the [illegible] of

16

PERMUTATIONS AND COMBINATIONS

[illegible]

[illegible]

[illegible] a number of permutations corresponding to n [illegible] is given.

$$^{n}P_{r} = \frac{n!}{(n-r)!}, \quad n \geq r$$

[illegible] are most

(a) [illegible] and numbers [illegible] middle letters and digits repetition.

(b) [illegible] [illegible]

(c) Selection of [illegible] of a cricket team of 11 from 15 members.

(d) Forming [illegible] arrangements in [illegible] place the events, making [illegible] making [illegible] letters [illegible] Finding [illegible] [illegible] etc.

[illegible] and [illegible] can be seen through the Business [illegible]

[illegible] Suppose [illegible] $P(A \cap B)$ [illegible] A [illegible] B [illegible]

[illegible] $A \cap B$ [illegible] AB [illegible] AC [illegible] BC [illegible] [illegible] [illegible]

Problem: [illegible]

Solution: [illegible]

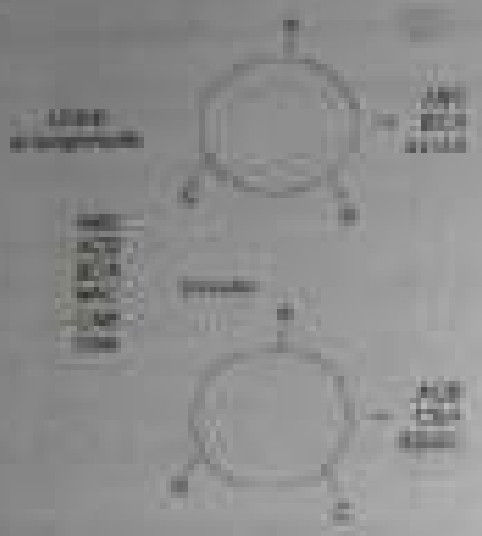
Problem: [illegible]

Solution: [illegible]

Problem: [illegible]

Solution: [illegible]

[illegible]



In fixed form arrangements, there [illegible] only circular arrangements. This happens because there is no unique place starting point in a circular arrangement, i.e. the starting point is not relevant.

Generalizing the above process, total distinct arrangements is $\frac{1}{n}\times n!$ ways. (Total No. $\frac{1}{n}$)

Important Results:

1. Number of ways of arranging n people on a circle is $(n-1)!$ (Circular arrangement is $(n-1)!$)
2. When clockwise and anti clockwise arrangements are different then number of circular arrangements of n different things is $\frac{1}{2}(n-1)!$ (Total No. $\frac{1}{2}$)

   Example: Number of necklaces with different beads. The same arrangement when looked at from the opposite side, becomes anticlockwise.
3. Number of selections of r consecutive things out of n things in a circle

   If $r \le \frac{n}{2}$ then $\frac{1}{r}\times n$

   If $r > \frac{n}{2}$ then $\frac{1}{n-r}\times n$

   Some more results:
4. Number of arrangements of n things $x_1, x_2, \ldots, x_n$ in a circle $x_1 \to x_2 \to \ldots \to x_n \to x_1$

   Illustration: Find the number of circles on the 6 points A, B, C, D, E, F.

   Solution: $\frac{1}{6}\times 6! = 120$

   Exercise: Number of circles that can be formed by 5 points A, B, C, D, E is $\frac{1}{5}\times 5! = 24$

5. Number of ways making the number variations of n of n things taken r at a time is $\frac{1}{r}\times {}^nP_r$ (Shows general way of finding the number of r $\frac{1}{r}\times$)

   Illustration: Find the number of ways in the case of 6 [illegible]

   Solution: $\frac{1}{3}\times {}^6P_3 = \frac{1}{3}\times 6\times 5\times 4 = 40$

   Exercise: Number of $\frac{1}{3}\times {}^nP_3 = \frac{1}{3}\times n(n-1)(n-2)$ [illegible]

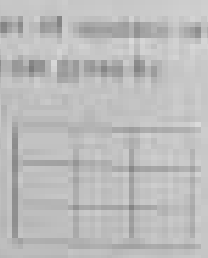
   (Shows general way of finding the number of r $\frac{1}{r}\times$)

   Illustration: Find the number of ways in the case of 3 [illegible] $\frac{1}{3}\times$

   Solution: $\frac{1}{3}\times 3\times 2\times 1 = 2$
6. Number of ways of arrangements of n things $x_1, x_2, x_3, \ldots, x_n$ in a circle $x_1 \to x_2 \to x_3 \to \ldots \to x_n \to x_1$

   Number of bracelets in a circle of n things $x_1, x_2, x_3, \ldots, x_n$ (This includes the reverse arrangements).

   Thus the number of necklaces and bracelets in the Schedule types are given by



   Number of necklaces $= \frac{1}{2}\times(n-1)! = \frac{1}{2}\times n = \frac{1}{2}\times\sum_{r=1}^{n} r^1$

   Number of bracelets $= \frac{1}{2}\times(n-1)! + \frac{1}{2}\times n = \frac{1}{2}\times n + \frac{1}{2}\times n = \frac{1}{2}\times\sum_{r=1}^{n} r^1 + \frac{1}{2}\times\sum_{r=1}^{n} r^1$

   Bracelets

   A bracelet has beads of two kind and a necklace.

   The number of necklaces is given by $\frac{1}{2}\times n = \frac{1}{2}\times(n-1)! + \frac{1}{2}\times(n-1)! = \frac{1}{2}\times 2(n-1)! = \frac{1}{2}\times(n-1)!$ and $\frac{1}{2}\times(n-1)! = \frac{1}{2}\times(n-1)!$ [illegible] n.

   The number of necklaces is given by $\frac{1}{2}\times(n-1)! = \frac{1}{2}\times(n-1)!$ [illegible] $\frac{1}{2}\times(n-1)!$

WORKED-OUT PROBLEMS

[illegible]

PROBLEM 1.1 [illegible]

Solution The answer will be given by $^{n}P_{r}$.

PROBLEM 1.2 How many 5-digit numbers can be formed using digits 1, 2, 3, 4 and 5.

Solution Starting from the left, we have 5 choices for the first digit, 4 for the second, 3 for the third, 2 for the fourth and 1 for the last digit. Hence the number of such numbers is $5 \times 4 \times 3 \times 2 \times 1 = 120$.

PROBLEM 1.3 [illegible] The arrangement is not circular. Hence the number of such arrangements is $^{5}P_{5} = 5! = 120$.

Solution For arranging the 7 letters keeping P and Q together, we have to treat PQ as one letter. Hence we have to find the number of arrangements of the 6 letters R, A, N, D, O, M, PQ. This can be done in $^{6}P_{6} = 6! = 720$ ways.

Now, in each of these arrangements, P and Q can be interchanged. This can be done in $^{2}P_{2} = 2! = 2$ ways. Hence, the required number is $6! \times 2! = 1440$.

Ask for the number Given a word, how many words can be formed using all the letters of the word, keeping vowels together?

Hint Treat the vowels as one letter and the consonants as another letter. Find the number of arrangements of these two letters. Multiply this by the number of arrangements of the vowels among themselves and the number of arrangements of the consonants among themselves.

PROBLEM 1.4 [illegible]

Solution [illegible] Hence, the number of such words is $^{5}P_{5} \times 2! = 240$.

Ask for the number How many words can be formed using all the letters of the word ENGINEER, such that no two vowels are adjacent? (Ans: 48 × 3!)

PROBLEM 1.5 In how many ways can the letters of the word SOLDIER be arranged so that all the vowels are together or [illegible]?

Solution There are 6 vowels and 7 consonants in SOLDIER. If these vowels are kept together, we have to consider AEIO as one letter. Then the problem transforms into arranging the 8 letters among themselves in $8!$ ways. Now, we have to look at the internal arrangement of the 4 vowels taking permutations of vowels.

Hence the answer is $8! \times 4!$.

PROBLEM 1.6 [illegible] Hence, the number of such ways is $^{4}P_{2} \times 2! = 24$.

Ask for the number How many words can be formed using all the letters of the word ENGINEER, such that no two vowels are adjacent? (Ans: 4! × 3!)

PROBLEM 1.7 In how many ways can the letters of the word SOLDIER be arranged so that all the vowels are together or [illegible]?

Solution There are 6 vowels and 7 consonants in SOLDIER. If these vowels are kept together, we have to consider AEIO as one letter. Then the problem transforms into arranging the 8 letters among themselves in $8!$ ways. Now, we have to look at the internal arrangement of the 4 vowels taking permutations of vowels.

Hence the answer is $8! \times 4!$.

PROBLEM 1.8 [illegible] Hence, the number of such ways is $^{4}P_{2} \times 2! = 24$.

Ask for the number How many words can be formed using all the letters of the word ENGINEER, such that no two vowels are adjacent? (Ans: 4! × 3!)

PROBLEM 1.9 [illegible] Hence, the number of such ways is $^{4}P_{2} \times 2! = 24$.

Ask for the number How many words can be formed using all the letters of the word ENGINEER, such that no two vowels are adjacent? (Ans: 4! × 3!)

PROBLEM 1.10 [illegible] Hence, the number of such ways is $^{4}P_{2} \times 2! = 24$.

Ask for the number How many words can be formed using all the letters of the word ENGINEER, such that no two vowels are adjacent? (Ans: 4! × 3!)

PROBLEM 1.11 [illegible] Hence, the number of such ways is $^{4}P_{2} \times 2! = 24$.

Ask for the number How many words can be formed using all the letters of the word ENGINEER, such that no two vowels are adjacent? (Ans: 4! × 3!)

PROBLEM 1.12 [illegible] Hence, the number of such ways is $^{4}P_{2} \times 2! = 24$.

Ask for the number How many words can be formed using all the letters of the word ENGINEER, such that no two vowels are adjacent? (Ans: 4! × 3!)

Solution: [illegible]

PROBLEM 1.2.2 [illegible]

Solution: [illegible]

PROBLEM 1.2.3 [illegible]

Solution: [illegible]

PROBLEM 1.2.4 [illegible]

Solution: [illegible]

PROBLEM 1.2.5 [illegible]

Solution: [illegible]

PROBLEM 1.2.6 [illegible]

Solution: [illegible]

PROBLEM 1.2.7 [illegible]

Solution: [illegible]

PROBLEM 1.2.8 [illegible]

Solution: [illegible]

PROBLEM 1.2.9 [illegible]

Solution: [illegible]

PROBLEM 1.2.10 [illegible]

Solution: [illegible]

PROBLEM 1.2.11 [illegible]

Solution: [illegible]

PROBLEM 1.2.12 [illegible]

Solution: [illegible]

PROBLEM 1.2.13 [illegible]

Solution: [illegible]

PROBLEM 1.2.14 [illegible]

Solution: [illegible]

PROBLEM 1.2.15 [illegible]

Solution: [illegible]

Example 15.15 Find the number of diagonals and triangles in a plane polygon.

Solution: A diagonal of a polygon is a line segment joining any two of its vertices. This can be done in $^nC_2$ ways. But out of these $^nC_2$ lines, n are the sides of the polygon.

Thus, the number of diagonals in a plane polygon is given by $^nC_2 - n$.

Triangles are formed by selecting any three of the n vertices of the polygon. This can be done in $^nC_3$ ways.

Example 15.16 Suppose a group of 10 people is to be divided into 5 groups of 2 people each. In how many ways this can be done?

Solution: If all 10 people were non-collinear then the number of ways would have been $^{10}C_2$. However, in this case, $^{10}C_2$ has overcounting since the 2 members within each group are indistinguishable. Thus, the number of ways in which 10 people can be divided into 5 groups of 2 people each is given by $^{10}C_2 \div 2!$.

Hence, the required answer is $^{10}C_2 \div 2! \times {}^8C_2 \div 2! \times {}^6C_2 \div 2! \times {}^4C_2 \div 2! \times {}^2C_2 \div 2!$

Example 15.17 For the above situation, how many ways are there to divide 10 people into 5 groups of 2 people each?

Solution: The answer will be given by $^{10}C_2 \div 5!$

Level of Difficulty (LOD)

1. How many numbers of 5 digits can be formed with the digits 0, 2, 4, 6, 8 (repetition of digits not allowed)?
   (a) 120 (b) 120 (c) 48 (d) 120
   (e) 48

2. How many numbers between 1000 and 5000 can be formed with the digits 0, 1, 2, 3, 4, 5, 6 (repetition of digits not allowed)?
   (a) 30 (b) 120 (c) 180 (d) 60
   (e) 180

3. In how many ways can a person write a combination of 10 of the 15 of the letters of the word [illegible] taken all at a time?
   (a) 60 (b) 60
   (c) 20 (d) [illegible]
   (e) None of these

4. In how many ways can 7 Indians, 3 Bangladeshi and 4 Bhutanese be seated in a line so that all persons of the same nationality sit together?
   (a) 81 (b) [illegible]
   (c) [illegible] (d) 18
   (e) None of these

5. A thousand [illegible] can go from Allahabad to Patna and 4 ways to go from Patna to Calcutta. In how many ways can one travel from Allahabad to Kolkata via Patna?
   (a) 24 (b) [illegible]
   (c) [illegible] (d) [illegible]
   (e) None of these

6. How many distinguishing permutations can be made from the letters DAD, DAN, BAD, BAN and BAT? All White counters are to be placed through B AT or BAD, A, DA on the other hand can go in one way through the DAD, BAD and BAT but are forbidden to [illegible] Counters, then how [illegible] Counters, Four counters, Four and Three. Find the value of [illegible] is [illegible] 10 but can make 6 in DAD, [illegible] of [illegible] can make 9 in the DAD?
   (a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
   (e) [illegible]

7. How many straight lines can be formed from 8 non-collinear points on the 8-gon?
   (a) 24 (b) 28
   (c) 48 (d) 24
   (e) None of these

8. If $^nC_4 = {}^nC_6$ find the n
   (a) 14 (b) 12 (c) 10 (d) 10
   (e) 11

9. In how many ways can the letters of the word [illegible] be arranged?
   (a) 119 (b) 120 (c) 180 (d) 24
   (e) 70

10. In how many ways can the letters of the word EQUAL be arranged?
    (a) 60 (b) 60
    (c) 20 (d) 120
    (e) None of these

11. A [illegible] can be formed from the word EQUAL, in how many ways?

(a) 40 (b) 120
(c) 150 (d) 30
(e) None of these

4. In the above question, how many words would be there which would begin with the letter R?
(a) 24 (b) 32 (c) 48 (d) 18
(e) 30

5. In question 1, how many words will have both P and Q?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

6. In $^{8}P_{3} = 56$, find n.
(a) 7 (b) 8 (c) 9 (d) 5
(e) 6

7. If $^{n}P_{4} = 56$, find n.
(a) 4 (b) 6 (c) 5 (d) 8
(e) 7

8. How many numbers of four digits can be formed with the digits 1, 2, 3, 4 (repetition of digits is not allowed)?
(a) 16 (b) 24 (c) 64 (d) 18
(e) 32

9. How many numbers of four digits can be formed with the digits 1, 2, 3, 4 (repetition of digits is allowed)?
(a) 16 (b) 256 (c) 256 (d) 512
(e) 144

10. How many numbers between 100 and 1000 can be formed with the digits 1, 2, 3, 4 (repetition of digits is allowed)?
(a) 6 (b) 8 (c) 7 (d) 16
(e) 48

11. For the Mission week 18 boys or girls members can be formed with the same digits. If repetition of digits is allowed?
(a) 48 (b) 63 (c) 12 (d) 14
(e) 18

12. If $(2n+1)P_{n-1} : (2n-1)P_{n} = 7 : 11$, find n.
(a) 4 (b) 6 (c) 3 (d) 1
(e) 5

13. If $^{n}P_{5} : {}^{n-1}P_{4} = 27 : 4$, then $n =$
(a) 8 (b) 10 (c) 9 (d) 6
(e) 4

14. [illegible] in how many ways can the letters be arranged so that the vowels occupy the odd places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

15. In how many ways can the letters be arranged so that the vowels occupy the even places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

16. In how many ways can the letters be arranged so that the vowels occupy the odd places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

17. The minimum fees to be paid by a student for 20 hours boxing is Rs. 1000. If the student will be charged Rs. 50 for every extra hour, then what is the minimum fee to be paid by a student for 25 hours boxing?
(a) 1000 (b) 1250 (c) 1500 (d) 1750
(e) 1800

18. In how many ways can the letters be arranged so that the vowels occupy the odd places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

19. In how many ways can the letters be arranged so that the vowels occupy the even places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

20. In how many ways can the letters be arranged so that the vowels occupy the odd places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

21. In how many ways can the letters be arranged so that the vowels occupy the even places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

22. In how many ways can the letters be arranged so that the vowels occupy the odd places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

23. In how many ways can the letters be arranged so that the vowels occupy the even places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

24. In how many ways can the letters be arranged so that the vowels occupy the odd places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

25. In how many ways can the letters be arranged so that the vowels occupy the even places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

26. In how many ways can the letters be arranged so that the vowels occupy the odd places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

27. In how many ways can the letters be arranged so that the vowels occupy the even places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

28. In how many ways can the letters be arranged so that the vowels occupy the odd places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

29. In how many ways can the letters be arranged so that the vowels occupy the even places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

30. In how many ways can the letters be arranged so that the vowels occupy the odd places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

31. In how many ways can the letters be arranged so that the vowels occupy the even places?
(a) 1 (b) 6 (c) 12 (d) 3
(e) 30

(a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

[illegible] A different [illegible] are given. In how many ways a [illegible] can be chosen to submit [illegible] Word [illegible] [illegible]?

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[illegible] (a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

[illegible]

Level of Difficulty (LOD) II

[illegible]

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(e) None of these

[illegible]

(a) 20 (b) 11 (c) 5 (d) 3

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(e) None of these

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(e) None of these

[illegible]

(a) 30 (b) 11

(c) [illegible] (d) [illegible]

(e) None of these

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(e) None of these

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(e) None of these

[illegible]

(a) [illegible] (b) [illegible]

(c) [illegible] (d) [illegible]

(e) None of these

[illegible]

(a) 100 (b) 100

(c) [illegible] (d) [illegible]

(e) None of these

[illegible]

(a) [illegible] (b) [illegible]

29. In question 27, find the number of ways in which only two [illegible] get in the wrong envelopes?

(a) 4 (b) 2 (c) 6 (d) 3

30. A coin is tossing between three [illegible] Heads. In how many ways can the four consecutive tosses Tails and Heads, if a person can start on Heads between Tails and Heads, in how many ways can the tosses be arranged? [illegible] the number of tosses at any stage? 2 people do not buy the same [illegible]

(a) $^{4}C_{2}$ (b) $^{4}C_{1}$ (c) $^{4}C_{3}$ (d) $^{4}C_{4}$

31. There are three pairs of black shoes and five pairs of white shoes. They do not get [illegible] and shoes are always [illegible]. In [illegible] pairs of black shoes are taken out, what is the number of [illegible] required to be taken out?

(a) 12 (b) 11 (c) 7 (d) 10

32. In the [illegible], what is the [illegible] number of [illegible] required to be [illegible] get at least 2 pairs of [illegible] [illegible]?

(a) 27 (b) 7 (c) 3 (d) 10

33. In how many ways one would arrange [illegible] such that [illegible] are always [illegible]?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

34. How many 6-digit numbers have all [illegible] digits odd or all even?

(a) 8,250 (b) 8,125
(c) [illegible] (d) None of these

35. How many 6-digit numbers have at least [illegible] digit?

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

36. How many 3-digit numbers have at most 2 equal digits?

(a) $9 \times {}^{9}P_{2} + 9$ (b) $9 \times 9^{2} + 81$
(c) 9×9^{2} (d) None of these

37. On a straight AB, on one side of AB 7 points are marked. 8 points are marked on the side AB and 5 points are marked on the side AB (none of the points being the vertices of the triangle). How many triangles can be made by using these points?

(a) 96 (b) 175
(c) 280 (d) None of these

38. If we have to make 4 boys in a row 7 girls in a row, a round table, then the number of different relative arrangements of boys and girls [illegible]

[illegible] (a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

39. If we have to make 4 boys in a row 7 girls in a row, a round table, then the number of different relative arrangements of boys and girls [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

40. In the Lucky Drawing in Mumbai there are 12 prizes are granted. Two people get one prize [illegible] on the ground floor. The 6th floor has [illegible] floor. [illegible] people [illegible] the 12 in a [illegible] [illegible] (Consider they [illegible] in a Grand Draw.)

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

Directions for Questions 41–45: [illegible]

41. [illegible]

(a) 220 (b) 120 (c) 100 (d) 150

42. [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) None of these

43. [illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]

44. [illegible]

(a) [illegible] (b) 2187
(c) 3007 (d) 700

45. [illegible]

(a) 720 (b) [illegible] (c) 2160 (d) [illegible]

46. How many 3-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 and 7 so that the digits should not repeat and the second last digit is even?

(a) 720 (b) 720 (c) 1000 (d) [illegible]

47. How many 3-digit numbers that do not contain [illegible] digits can be made by choosing the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0?

Level of Difficulty (LOD) III

[illegible]

[illegible]

[illegible]

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

Directions for Questions 17-18: Read the passage below and answer the questions.

[illegible]

(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible]
(e) [illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]
(e) None of these

[illegible] How many different 5-digit numbers are there the sum of whose digits are odd?

(a) 45×10^4 (b) 34×10^4
(c) [illegible] (d) None of these

[illegible] How many 6-digit numbers contain exactly 4 different digits?

(a) 4536 (b) 2594400
(c) [illegible] (d) None of these

[illegible] How many numbers smaller than $2 \cdot 10^4$ and are divisible by 5 can be written by means of the digits 0, 1 and 2 (include single digit and double digit numbers)?

(a) 4040 (b) 4050 (c) 4070 (d) 4080

[illegible] In white and six black balls of the same size are placed and among ten urns so that there is at least one ball in each urn. What is the number of different distributions of the balls?

(a) 25000 (b) 26,250 (c) 36,250 (d) 13,125

[illegible] A bouquet has to be formed from 18 different flowers so that it should contain not less than three flowers. How many ways are there of doing this is?

(a) 524,288 (b) 262,144
(c) 245,872 (d) None of these

[illegible] How many different numbers which are smaller than $2 \cdot 10^4$ can be formed using the digits 1 and 2 only?

(a) 96 (b) 94 (c) 92 (d) 126

[illegible] How many distinct 6-digit numbers are there having 3 odd and 3 even digits?

(a) 9! (b) $(5!)^2 \times 4!/3!$
(c) 20250 (d) None of these

[illegible] How many 8-digit numbers are there the sum of whose digits is even?

(a) 1400 (b) $4 \cdot 5^7$
(c) $45 \cdot 10^6$ (d) None of these

Solution for Questions 37 and 38. In a chess tournament [illegible] participating and every participant [illegible] with the other participants. The number of [illegible] played among themselves exceeded the [illegible] that the men played with the women by 66. [illegible] of games that the men played with the women be 66.

37. The number of participants in the tournament were?

(a) 12 (b) 13 (c) 15 (d) 11

38. The total number of games played in the tournament was?

(a) 132 (b) 143 (c) 156 (d) 210

39. There are 3 matches of electric and each have their respective [illegible]. If you are asked to put the correct cap on the [illegible] boxes, how many ways are there so that not a single cap is on the correct bottle?

(a) [illegible] (b) $3^3 - 1$
(c) 3^3 (d) None of these

40. Amartya Bhaiyya has forgotten the telephone number of his best friend Abhijit Roy. All he remembers is that the number had 6-digits and ended with an odd number and had exactly one 4. How many possible numbers does Amartya have to try to be sure that he gets the correct number?

(a) $104 \cdot 5^5$ (b) $111 \cdot 6^5$
(c) $300 \cdot 6^5$ (d) $264 \cdot 6^5$ is

41. In question 40, if Amartya is reminded by his friend Shreya that apart from what he remembered there was the additional fact that the last digit of the number was not repeated under any circumstances, then how many possible numbers does Amartya have to try to be sure that he gets the correct number?

(a) $[illegible] + 72 \cdot 6^5$ (b) 8 [illegible]
(c) $36 \cdot 6^5 + 1 \cdot 96$ (d) $36 \cdot 6^5 + 8 \cdot 6^5$

42. How many natural numbers are more than 4306 can be formed with the digits 0, 1, 2, 3, 4 (if repetitions are allowed)?

(a) 374 (b) 530 (c) 525 (d) 549

43. How many natural numbers less than 4300 can be formed with the digits 0, 1, 2, 3, 4 (if repetitions are not allowed)?

(a) 110 (b) 156 (c) 154 (d) 119

44. How many even natural numbers divisible by 5 can be formed with the digits 0, 1, 2, 3, 4, 5, 6 (if repetitions of digits are allowed)?

(a) 1957 (b) 1956 (c) 1126 (d) 1255

45. There are 100 articles numbered $a_1, a_2, a_3, a_4, \ldots, a_{100}$. They are arranged in all possible ways. How many arrangements would be there in which a_{99} is always to be before a_{50}?

(a) $50! \times 99!$ (b) $50! \times 98!$
(c) $49! \times 99!$ (d) $49! \times 98!$

46. The letters of the word PASTE are written in all possible orders and these words are written out as in a dictionary. Then the rank of the word SPATE is

(a) 102 (b) 96 (c) 98 (d) 448

47. The straight lines l_1, l_2, l_3 are in a parallel and lie in the same plane. A total number of l_1 points on l_1, B points on l_2 and C points on l_3 are used to produce triangles. What is the maximum number of triangles formed?

(a) $^{n+m+1}C_2 = {}^{n}C_2 + {}^{m}C_2 + 1$
(b) $^{n+m+1}C_2$
(c) $^{n+m+1}C_2 + 1$
(d) $^{n+m+1}C_2 = {}^{n}C_2 + {}^{m}C_2 + {}^{n}C_1 + {}^{m}C_1$

40. The sides AB, BC, CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The total number of triangles that can be constructed by using these points as vertices are
(a) 205 (b) 200 (c) 395 (d) 500

41. A library has 20 copies of CAGE, 12 copies each of RAGE Part 1 and Part 2, 5 copies of PAGE Part 1, Part 2 and Part 3 and single copy of SAGE, DAGE and MAGE. In how many ways can these books be divided?
(a) $(20!)(20!)(12!)^2(5!)^3$ (b) 62
(c) $(20!)(12!)^2$ (d) $(20!)(20!)(12!)^2(5!)^3$

36. The AMS MOCK CAT test CATALYST-19 consists of four sections. Each section has a maximum of 42 marks. Find the number of ways in which a student can qualify in the AMS MOCK CAT if the qualifying marks is 40
(a) 38,546 (b) 6546
(c) 6666 (d) None of these

Hints and Solutions II

1. Number of even numbers = 6 × 6 × 6 × 5
2. We need to divide of this as: Number with two digits or numbers with one digit or number with one digit.

0, 1, 2, 3, 4, 5, 6

Numbers with 2 digits

Numbers ending in zero: $^{7}C_2 = 3(2!) = 15$
Numbers Ending in 5 and
(a) Starting with 0: $^{5}C_2 = 2! \times 10$
(b) Not starting with 0: $^{5}C_2$ (0 can't be not allowed) = 4

Number with 1 digit or no digit

Numbers ending in 0: $^{6}C_2 \times 2! = 120$
Numbers ending in 5: $^{6}C_2 \times {}^{5}C_1 \times 2! = 300$

Thus a total of 749 numbers.

3. First arrange 4 pups in 6 places in 6! ways. This will leave us with 7 places for 4 cats. Answer = 6! × $^{7}C_4$.
4. Arrangement of M, A, N, A, E, M, E, N, T is $\frac{9!}{2! \times 2! \times 2! \times 2!}$
5. For four places we have following number of arrangements:
9 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2
6. For a rectangle, we need two pair of parallel lines which are perpendicular to each other. We need to select two parallel lines from 'x' lines and 2 parallel lines from 'y' lines. Hence required number of parallel lines is $^{x}C_2 \times {}^{y}C_2$.
7. From 8 people we have to arrange 3 people or 5 in which three are similar $\frac{8!}{3!}$ or $\frac{8C_3 \times 5!}{3!}$
8. $\frac{4C_3 \times 4C_3 \times 5!}{3!} + \frac{4C_3 \times 4C_3 \times 5!}{3!} = 4C_3 \times 4C_3 \times 2! \times 2!$
10. $^{7}C \times {}^{7}C \times 4! = 49 \times 6040$
12. The required answer will be given by counting the total number of registration numbers starting with DL-5A to DL-5R and the number of registration numbers starting with DL-5S that have to be counted.
13. Out of 100 balls arrange 99 balls (except n_{50}) amongst themselves. Now put n_{50} just before n_{99} in the above arrangement.
14. $^{n}C_2 = 15$.
15. We need to arrange N people on M chairs, 5 people on another set of M chairs and the remaining people on the remaining chairs. $^{M}P_N \times {}^{M}P_5 \times {}^{2M-N-5}P_{2M-N-5}$
16. Each group will consist of m things. This can be done in $^{mn}C_m \times {}^{mn-m}C_m \times {}^{mn-2m}C_m \ldots {}^{m}C_m$

$$= \frac{(mn)!}{(mn-m)!\,m!} \cdot \frac{(mn-m)!}{(mn-2m)!\,m!} \cdots \frac{m!}{0!\,m!} = \frac{(mn)!}{(m!)^n}$$

Divide this by $n!$ since arrangements of the n groups among themselves is not required.

Required number of ways $= \frac{(mn)!}{(m!)^n \cdot n!}$

17. Number of ways of selecting 5 different letters $= {}^{7}C_5 = 21$
Number of ways of selecting 2 similar and 3 different letters: $^{7}C_1 \times {}^{6}C_3 = 20$
Number of ways of selecting 2 similar letters + 2 same similar letters and 1 different letter $= {}^{7}C_1 \times {}^{6}C_2 = 15$
Number of ways of selecting 3 similar letters and 2 different letters $= {}^{7}C_1 \times {}^{6}C_2 = 15$
Number of ways of selecting 3 similar letters and another 2 other similar letters $= {}^{7}C_1 \times {}^{6}C_1 = 6$
Number of ways of selecting 4 similar letters and 1 different letter $= {}^{7}C_1 \times {}^{6}C_1 = 6$

number of ways of selecting 5 similar letters = ${}^5C_1$
= 1

Total number of ways = 1 + 36 + 18 + 18 + 9 + 9 + 1
= 91

(c) Divide 2n + 1 articles in two groups.

(i) n identical articles and the remaining

(ii) 2n + 1 non-identical articles

(i) will select articles in two steps. Some from the first group and the rest from the second group.

Number of articles from first group	Number of articles from second group	Number of ways
0	n	$1 \times {}^{2n+1}C_n$
1	n − 1	$1 \times {}^{2n+1}C_{n-1}$
2	n − 2	$1 \times {}^{2n+1}C_{n-2}$
3	n − 3	$1 \times {}^{2n+1}C_{n-3}$
...	...	.....
n − 1	1	$1 \times {}^{2n+1}C_1$
n	0	$1 \times {}^{2n+1}C_0$

Total number of ways = ${}^{2n+1}C_n + {}^{2n+1}C_{n-1} + {}^{2n+1}C_{n-2}$
+ ... ${}^{2n+1}C_1 + {}^{2n+1}C_0 = \frac{2^{2n+1}}{2} = 2^{2n}$

(ii) We have four options for every place including the left most.

So the total number of numbers = 4 × 4 × 4 × ... = 4^{10}.

We have to consider only positive numbers, so we don't consider one number in which all ten digits are zeros.

(iii) Total number of attempts = 10^4 out of which one is correct.

(iv) For odd places, the number of arrangements = $\frac{4!}{2!2!}$

For even places, the number of arrangements = $\frac{5!}{2!}$

Hence the total number of arrangements = $\frac{4! \times 5!}{2! \times 2! \times 2!}$

(v) The number would be of the form $\underline{\ }\underline{\ }\underline{\ }\underline{\ }\underline{\ }\underline{\ }\underline{\ }\underline{\ }\underline{\ }$.

The 5 missing digits have to be formed using the digits 0, 1, 2, 3, 4, 5, 7, 8, 9 without repetition.

Hence ${}^9C_5 = 126$

(vi) 1 to 9 : $9 \times {}^9C_1 = {}^9C_1 = 9 \times 36 = 648$

Now 2 to ${}^9C_1 \times {}^9C_2 = 18 \times 28 = 504$

$3n = 18\,{}^9C_2 = {}^9C_2 = 36 \times 8 = 648$

$4n = 9\,{}^9C_3 = {}^9C_3 = 70 \times 1 = 70$

Total = 1296

(24) Solve this by dividing the solution into

3 women and 1 man or

2 women and 2 men or

1 woman and 3 men or

0 women and 4 men

This will give us:

${}^4C_3 \times {}^7C_1 + {}^4C_2 \times {}^7C_2 + {}^4C_1 \times {}^7C_3 + {}^4C_0 \times {}^7C_4$

= 448 + 294 + 448 + 70 = 1260

(25) For 1 to 9 we require 9 digits

For 10 to 99 we require 90 × 2 digits

For 100 to 150 we require 51 × 3 digits

(26) Select any three places for A, B and C. They need to arrange among themselves as A would always come before B and B would come before C.

The remaining 5 people have to be arranged in 5 places.

Thus, ${}^7C_3 \times 3! = 35 \times 120 = 4200$

Or

$$\frac{8!}{3! \times 5!} \times 3! = 4200$$

(27) Total number of choices = 47 out of which only one will be right.

(28) At least two letters have to interchange their places for a wrong choice.

(29) Select any two letters and interchange them ${}^7C_2$.

(30) ${}^7C_4$ (refer to school example 16.14).

(31) For one pair of black shoes we require one left black and one right black. Consider the worst case situation.

7LB + 5LW + 5RW = 17 shoes

7RB + 5LW + 5RW = 17 shoes

(32) For one pair of correct shoes, one of the possible combinations is 7LB + 5LW + 1RB (B or W) = 13

Some other cases are also possible with at least 13 shoes.

(33) The first rook can be placed in any of the 64 squares and the second rook will then have only 49 places so that they are not attacking each other.

(34) When all digits are odd

5 × 5 × 5 × 5 × 5 = 5^5

When all digits are even

4 × 5 × 5 × 5 × 5 × 5 × 5 = 4 × 5^6

5^5 + 4 × 5^6 = 20125

35. All six digit numbers – Six digit numbers with only odd digits

$= 900000 - 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 884375$.

36. "Total number of all 10 digit numbers – Total number of all 10 digit numbers with no digit repeated" will give the required answer.

$= 9 \times 10^9 - 9 \times {}^9P_9$

37. There will be two types of triangles.

The first type will have its vertices on the three sides of the ΔABC.

The second type will have two of its vertices on the same side and the third vertex on any of the other two sides.

Hence, the required number of triangles

$= 6 \times 5 \times 5 + {}^6C_2 \times 8 + {}^5C_2 \times 9 + {}^5C_2 \times 11$

$= 90 + 120 + 90 + 55$

$= 355$

38. First step – arrange 7 boys around the table according to the circular permutations rule, i.e., in $6!$ ways.

Second step – now, we have 7 places and have to arrange 7 girls in these places. This can be done in ${}^7P_7$ ways. Hence, the total number of ways $= 6! \times 7!$

39. $2 \times 11 \times 7!$ (Note: we do not need to use circular arrangements here because the seats are numbered.)

40. We just need to select the floors and the people who get down at each floor.

The floors selection can be done in ${}^7C_3$ ways.

The people selection is ${}^7C_3 \times {}^4C_1$

Also, the floors need to be arranged using $3!$

Thus, ${}^7C_3 \times {}^4C_1 \times {}^4C_1 \times 3!$ or ${}^7P_3 \times {}^4C_1 \times {}^4C_1$

41. To arrange a surgeon and an assistant we have ${}^{40}P_2$ ways.

42. To arrange a surgeon and 4 assistants we have or $40 \times {}^{39}P_4$ ways.

43. Give one marble to each of the six children. Then the remaining 4 identical marbles can be distributed amongst the six children in ${}^{(4+6-1)}C_{(6-1)}$ ways.

44. Since it is possible to give no objects to one or two of them we would have 7 choices for giving each item.

Thus, $3 \times 5 \times 5 \times 5 \times 5 \times 5 = 2187$

45. For an even number the unit digit should be either 2, 4 or 6. For the other five places we have six digits. Hence, the number of six digit numbers $= {}^6P_5 \times 3 = 2160$.

46. Visualize the number as:

_ _ _ _ _ _ _ _ _ _

This number has to have the last two digits even. Thus, ${}^5C_2 = 2^5$ will fill the last 2 digits.

For the remaining places, ${}^7C_5 \times 4!$

Thus, we have ${}^7C_5 \times 4! \times {}^5C_2 \times 2^5 = 720$

47. ${}^8C_4 \times 51 = 15120$

48. ${}^5C_1 \times 7 \times 7 \times 7 \times {}^6C_1 \times 7^3$

49. Select the two positions for the two 7's. After that the remaining 7 places have to be filled using either 1 or 2. Thus, ${}^7C_1 = 2^6$

50. ${}^7C_4 \times {}^6C_3 \times 31 = 630$

Hints and Solutions

1. ${}^{10}C_3 \times 6! = 10!/4!$
2. For drawing a circle we need 3 non collinear points. This can be done in:

$${}^7C_3 + {}^7C_2 \times {}^5C_1 + {}^7C_1 \times {}^5C_2 = 35 + 21 + 63 \times 10$$

3. The odd digits have to occupy even positions. This can be done in $\frac{4!}{2!2!} = 6$ ways.

The other digits have to occupy the other positions. This can be done in $\frac{5!}{3!2!} = 10$ ways.

Hence total number of rearrangements possible $= 6 \times 10 = 60$

4. The number of straight lines is n_{C_2} out of which there are n sides. Hence, the number of diagonals is $n_{C_2} - n$.
5. $n_{C_2} - n = 54$
6. We cannot take 0 since the smallest digit must be placed at the left most place. We have only 9 digits from which to select the numbers. First select any number of digits. Then for any selection there is only one possible arrangement where the required condition is met. This can be done in ${}^9C_1 + {}^9C_2 + {}^9C_3 + \ldots + {}^9C_9$ ways $= 2^9 - 1 = 511$ ways.

But we can take numbers which have only one digit hence the required answer is $511 + 9$.

7. 300 runs can be scored by scoring only fours or through a combination of fours and sixes.

Possibilities are 50×4, $47 \times 4 + 2 \times 6$, $44 \times 4 + 4 \times 6$ ... A total of 17 ways.

8. Of the total arrangements possible (87) exactly half would have 1 before 0. Thus, $87/2 = 360$.

[illegible] total number of permutations without any restrictions − Number of permutations having the '[illegible]' pattern + Number of permutations having the '[illegible]' pattern + Number of permutations having both the '[illegible]' and '[illegible]' patterns.

10. A and B can occupy the first and the ninth places, the second and the tenth places, the third and the eleventh places and so on... This can be done in 16 ways. A and B can be arranged in 2 ways.
All the other 24 alphabets can be arranged in 24! ways.
Hence the required answer = 2 × 16 × 24!

11. Just arrange the two sisters around a circle in such a way that there will be one seat vacant between them. This can be done in 2! ways (since the arrangement of the sisters is not a circular plan.)
Then, the other 18 people can be arranged in 18 seats in 18! ways.

12. $^{9}C_{3} \times {}^{6}C_{1} + {}^{9}C_{2} \times {}^{6}C_{2} = 504 + 270 = 774$

13. A chess board consists 9 parallel lines × 9 parallel lines. For a rectangle we need to select 2 parallel lines and two other parallel lines that are perpendicular to the first set. Hence, $^{9}C_{2} \times {}^{9}C_{2}$

P.D. Small or direct formulae

14. This is a direct simple Fibonacci question. Online for a correct. Refer to result no. 6 in Important Results 2.

15. $(1 + 2 + 3 + \ldots + 10)(1^2 + 2^2 + 3^2 + \ldots + 10^2)$

16. Based on simple counting and coding to the conditions given in the passage.

17. $^{9}C_{3} + 3 \times 2 + {}^{9}C_{2} \times 6 = 84 + 6 + 36 = 126$

18. Probability to win the toss = 7/8
Hence, the required ratio is 7/8: 1/8.

19. The partner (B) occupies one seat and thus needs $^{9}C_{3}$ ways = 1176.
Required percentage increase = 977.8%

20. First select six people out of 12 for the first row. The other six automatically get selected for the second row. Arrange the two rows of people amongst themselves. Besides, the papers can be given in the pattern of 121212 or 212121. Hence the answer is $2 \times {}^{12}C_{6} \times 6! \times 6!$.

21. The difference in this question about the previous question is the number of ways in which the papers can be distributed. This can be done by either distributing three different papers in the first three places, keeping these different papers in the first three places of each row or by repeating the same papers in the first and the third places.

22. Required permutations = Total permutations with no restrictions − permutations with the conditions which we do not have to count.

23. We have to count natural numbers which have natural numbers of 4 digits. The required answer will be given by: Number of single digit numbers + Number of two digit numbers + Number of three digit numbers + Number of four digit numbers.

24. Let the three people be A, B and C.
If 1 person gets no objects, the 7 objects must be distributed such that each of the other two get 1 object at least.
This can be done as 6 & 1, 5 & 2, 4 & 3 and their rearrangements.
The answer would be

$$^{7}C_{6} + {}^{7}C_{5} + {}^{7}C_{4} = 7 + 21 = 28$$

Also, two people getting no objects can be done in 3 ways.
Thus, the answer is 28 × 3 = 84.

25. If only one gets 1 object:
The remaining can be distributed as (4,0), (4,2), (3,3).

$$^{7}C_{4} \times {}^{3}C_{0} \times {}^{3}C_{2} \times {}^{5}C_{2} \times {}^{3}C_{3} + {}^{7}C_{3} \times {}^{4}C_{2} \times 3(2!)$$
$$= 42 + 630 + 420 = 1092$$

If 2 people get 1 object each:

$$^{7}C_{5} \times {}^{5}C_{2} \times {}^{3}C_{3} \times 3(3!) = 126$$

Thus, a total of 1218.

26. Natural numbers which consist of the digits 1, 2, 3, 4 and 5 and are divisible by 4 must have either 12, 24, 32 or 52 in the last two places. For the other two places we have to arrange three digits in two places.

27. No. of 1 digit nos = 9
No. of 2 digit nos = 81
No. of 3 digit nos = 9 × 9 × 8 = 648
No. of 4 digit nos = 9 × 9 × 8 × 7 = 4536
Total nos = 9 + 81 + 648 + 4536 = 5274

28. If the two digits are a and b then 6 digit numbers can be formed in the following patterns
aaabb, aaabb or aabbb.
You will have to take two situations in each of the cases, first when the two digits are non zero digits and second when the two digits are zero.

29. For the total of the digits to be odd one of the following has to be true:
The number should contain 1 odd + 6 even or 3 odd + 4 even or 5 odd + 2 even or 7 odd digits. Count each case separately.

35. ${}^{10}C_0 + {}^{9}C_1 + {}^{8}C_2 + \ldots + {}^{1}C_9 + {}^{0}C_{10}$
$= {}^{1}C_0 + {}^{1}C_1 + \ldots + {}^{1}C_{10} - {}^{1}C_0 = {}^{2}C_1 + {}^{3}C_2 + {}^{4}C_3 + {}^{5}C_4$
$= 2^{10} - 2 = 10 + 135 + 210$
$= 36,158$

36. Total number of n digit numbers having 3 odd and 3 even digits (including zero in the left most place) $= 5^3 \times 5^3$.

From this subtract the number of 3 digit numbers with 2 even digits and 2 odd digits (to take care of the zero counting due to zero)

36. There will be 5 types of numbers, viz. numbers which have

All eight digits even or six even and two odd digits or four even and four odd digits or two even and six odd digits or all eight odd digits. This will be further solved as below:

Eight even digits $\rightarrow 5^4 - 5^3 = 4 \times 5^3$

Six even and two odd digits $\rightarrow$
when the left most digit is even $\rightarrow 4 \times {}^{7}C_5 \times 5^5 \times 5^2$
when the left most digit is odd $\rightarrow 5 \times {}^{7}C_6 \times 5^6 \times 5^1$

Four even and four odd digits $\rightarrow$
when the left most digit is even $\rightarrow 4 \times {}^{7}C_3 \times 5^5 \times 5^3$
when the left most digit is odd $\rightarrow 5 \times {}^{7}C_4 \times 5^4 \times 5^4$

Two even and six odd digits $\rightarrow$
when the left most digit is even $\rightarrow 4 \times {}^{7}C_1 \times 5 \times 5^6$
when the left most digit is odd $\rightarrow 5 \times {}^{7}C_2 \times 5^5 \times 5^5$

Eight odd digits $\rightarrow 5^8$

37-38. Solve through options.

39. This question is based on a formula. The condition is that 'n' things (each thing belonging to a particular place) have to be distributed in 'n' places such that no particular thing is arranged in its correct place.

$n! - \frac{n!}{1!} + \frac{n!}{2!} - \frac{n!}{3!}$. Sign of the terms will be alternate and the last term will be $\frac{n!}{n!}$.

However, this can also be solved through logic.

40. The possible cases for counting are:
Number of numbers when the same digit is nine + the number of numbers when neither the same digit nor the left most is nine + number of numbers when the left most digit is nine.

41. The condition is that we have to count the number of natural numbers not more than 4500.
The total possible numbers with the given digits $= 5 \times 5 \times 5 \times 5 = 625 - 1 = 624$.

Subtract from this the number of natural numbers greater than 4500 which can be formed from the given digits $= 1 \times 2 \times 5 \times 5 - 1 = 49$.
Hence, the required number of numbers $= 624 - 49 = 575$.

43. The required answer will be given by
The number of one digit natural number + number of two digit natural numbers + the number of three digit natural numbers + the number of four digit natural number starting with 1, 2, or 3 + the number of four digit natural numbers starting with 4.

46. The following words will appear before SPATE. All words starting with A + All words starting with E + All words starting with P + All words starting with SA + All words starting with SE + SPAET

47. For the maximum possibility assume that no three points other than given in the question are in a straight line.
Hence, the total number of Δ's $= A + B + {}^{7}C_3 - {}^{4}C_3 - {}^{4}C_3 - {}^{3}C_3$

48. Use the formula $\frac{n!}{p!q!r!}$.
$n(E) = 7 \times 6 \times 3 \times 4 \times 3 \times 2 \times {}^{3}C_2$
$n(S) = 7^7$
$n(E) = 8 \times 7 + 8 \times 7 = 112$
$n(S) = 64C_2 = \frac{2 \times 8 \times 7 \times 2}{64 \times 63} = \frac{1}{18}$

49. Use the formula $\frac{n!}{p!q!r!}$

Detailed Solutions & Short Cuts

LOD I

1. $5 \times 4 \times 3 = 60$
2. The number should start with 2. Hence, we need 4 digit numbers starting with 2.
   Thus, the number 2 ___ has to be formed using 5, 1, 3, 4, 5, 6, and 7 i.e., $7 \times 6 \times 5 = 210$
3. The first card can be seen in 4 ways as also each of the other 5 cards. Thus 4^6.
4. Each prize can be given in 6 ways. Thus, 6^6.
5. $21 \times 31 \times 61 \times 31$ (representing the arrangements of the Indians amongst themselves, the Pakistanis amongst themselves and the Dutch amongst themselves and the

(for the arrangements of the three groups amongst themselves)

4. $n = 20$

5. As if Dan can make it to an IIM in 2 ways, while a CA has 3 ways of doing it.

6. ${}^nC_3 = 28$

7. As ${}^nC_4 = {}^nC_{n-3}$, we get $n = 11$ as ${}^{11}C_4 = {}^{11}C_7$

8. $P = 120$

9. $4(2^n - 1) = 60$

In this case 5!/2! represents the no. of arrangements in Paris because we have two A's.

We subtract 1, since we need rearrangements and we should not count the original word.

2. P____ gives us $4!/2! = 12$.

As the blanks are to be filled using A, T, N and A only.

3. P____ T and the blanks are to be filled using A, N and A. Thus, $3!/2! = 3$.

4. Solve through options.

$${}^7C_3 = 70$$

5. Solve through options.

$${}^9P_3 = 720$$

6. $3 \times 3 \times 2 \times 1 = 18$. The first digit cannot have 0 and hence can be filled in 3 ways. There we would have 3 more ways of filling the second digit, 2 ways for the third digit and 1 way for the last digit.

7. The first digit cannot have 0 and hence can be filled in 3 ways. Similarly, the second, third and fourth digits have to be filled using any of 0, 1, 2, or 3. Thus, using the imp rule, we have $3 \times 4 \times 4 \times 4 = 192$.

8. The numbers can be

No. of the form	2____	3____	10____
Number of numbers	$3 \times 3 = 9$ numbers	$3 \times 3 = 9$ numbers	$2 = 12$ numbers

Thus, a total of 14 numbers.

9. The numbers can be

No. of the form	2___	3___	10___	11___
Number of numbers	$3 \times 4 = 12$	$4 \times 4 = 16$	$2 \times 1 = 2$ numbers	$2 \times 4 = 16$

Looks like the numbers that we need to remember for the three digit number starting with 2 we should not count the number 200. Thus, there are only 15. 3-digit numbers starting with 2.

So, the correct answer is $12 + 16 + 16 + 16 = 62$.

10. Solve by trial and error using the values of n from the options. The equation is satisfied at $n = 7$ as

$${}^7P_4 \cdot {}^7P_3 = 5 \cdot 10$$

21. Solve by trial and error using the various values for r given in the options. At $r = 5$, the given condition is met.

22. The number of parties he would hold would be ${}^nC_r$ (where n is the number of friends he invites to a party). But ${}^nC_r$ to be maximum, r should be half of 20 i.e. 10.

23. Use ${}^nC_r = {}^nC_{n-r} = {}^nC_1$

24. The digits to be used are 1, 3, 5, 7 and 9.
The number of numbers $= 5 \times 5 \times 5 \times 5 = 625$

25. ${}^nC_1 = {}^nC_2 = 190$

26. ${}^nC_1 \times {}^nC_2 = 20$

27. There are 4 consonants and 1 vowel in the word ALLAHABAD. Thus, $4 \times 1 = 4$.

28. ${}^nC_3 \times {}^nC_2 = {}^nC_4 = 70 \{(1/2)\}$

29. ${}^nC_1 \times {}^nC_2 \times {}^nC_3 \times {}^nC_4 = 13 \times 12 \times 11 \times 10$

30. PERMUTATION has 11 alphabets out of which the 2 P's are identical.
Thus $(11!/2!) = 1$

31. ${}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + {}^nC_5 = {}^nC_6$

32. All arrangements – Boy and Girls always together = $12! - 2 \times 11!$

33. E, D, A, I and O have to be together. So effectively we have 4 letters Q, T, N and the one group letter considering all the five vowels.
Thus, the required answer is $4! \times 5!$

34. ${}^nC_1 \times {}^nC_2 \times {}^nC_3 = 72$

35. $6 \times 5 \times 4 \times 3 = 360$. (Note: For make which repetition numbers, a 4-digit number can start with 0.)

36. No of 5 digit nos: $5 \times 4 \times 3 \times 2 \times 1 = 120$
No of 4 digit nos: $5 \times 4 \times 3 \times 2 = 480 = 120$
No of 3 digit nos: $5 \times 4 \times 3 = 60$
No of 2 digit nos: $5 \times 4 = 20$
No of single digit nos: 5
Thus, 325 such numbers are possible.

37–39

Divide the points into two groups as 10 are collinear points and 7 collinear points.

37. ${}^7C_3 + {}^7C_2 \times {}^7C_1 + 1 = 280$
Alternately: ${}^{17}C_3 - {}^{10}C_3 + 1 = 280$

38. ${}^7C_4 + {}^7C_3 \times {}^7C_1 + {}^7C_2 \times {}^7C_2 = 2261$

39. ${}^7C_5 + {}^7C_4 \times {}^7C_1 + {}^7C_3 \times {}^7C_2 = 5880 + 5642 + 3213$
$= 14735$

40. ${}^7C_3 \times 9! = 30240$

41. ${}^7C_2 \times {}^7C_3 \times 9! = 67200$

42. ${}^7C_3 \times 11! = 1800 \times 11!$

43. ${}^7C_2 = 495$

44. He can make $^{9}C_{3}$ unique groups.
45. Since, one of the 3 member group is the head boy, he has to select 2 from the remaining 29. Thus, $^{29}C_{2}$.
46. P_____ W. The blanks have to be filled using P, C, L, L, A (and I).
    This can be done in $5!$ Ways.
47. $(30)(1)=40=N$
48. The number should necessarily start with 2, as it is not a possible first digit of the 7 digit number.
    Thus, the number is of the form: 3_____ and the blanks have to be filled using two 3's and four 0's. This can be done in $6!/4!=20=15$ ways.
49. $^{8}C_{3}\times{}^{7}C_{2}\times{}^{6}C_{2}\times 5!=2160$
50. $^{n}C_{3}=45\rightarrow n=10$

ANSWER KEY

Permutations and Combinations LODT

1. a	31. a
2. b	32. d
3. b	33. b
4. c	34. b
5. c	35. d
6. a	36. c
7. b	37. c
8. a	38. b
9. a	39. d
10. b	40. d
11. d	41. c
12. b	42. a
13. a	43. c
14. b	44. b
15. c	45. b
16. d	46. c
17. d	47. d
18. d	48. a
19. b	49. c
20. c	50. b
21. a	
22. d	
23. a	
24. b	
25. c	
26. a	
27. a	
28. a	
29. a	
30. b	

Permutations and Combinations LOGIN

1. a
2. b
3. c
4. b
5. d
6. b
7. b
8. d
9. b
10. c
11. a
12. d
13. c
14. b
15. b
16. c
17. b
18. d
19. a
20. c
21. c
22. c
23. d
24. a
25. d
26. a
27. b
28. d
29. c
30. a
31. d
32. c
33. b
34. b
35. a
36. b
37. b
38. b
39. a
40. b
41. d
42. a
43. d
44. b
45. c
46. a
47. d
48. c
49. a
50. b

Permutations and Combinations LOD III

1. a
2. b
3. d
4. b
5. c
6. c
7. b
8. b
9. a
10. c
11. d
12. b
13. c
14. d
15. c
16. a
17. c
18. b
19. d
20. b
21. a
22. b
23. b
24. c
25. b
26. c
27. c
28. b
29. a
30. b
31. c
32. b
33. d
34. a
35. c
36. c
37. b
38. a
39. d
40. c
41. a
42. c
43. b
44. a
45. c
46. b
47. d
48. c
49. d
50. c

17

PROBABILITY

CONCEPT AND IMPORTANCE OF PROBABILITY

Probability is one of the most important mathematical concepts that we encounter across in our day-to-day life. Particularly important in business and economic situations, probability is also used by us in our personal lives. For a lot of students who are not in touch with Mathematics after their seventh/eth classes, this chapter, along with permutations and combinations, is seen as an indication that 12th standard Mathematics appears in the MBA entrance exams. This leads to students taking negative approach while tackling/preparing for the Mathematics section. Students are advised to remember that the Math asked in MBA entrance is mainly logical while studying the chapter.

As I set out to explain the basics of this chapter, I intend to improve your concepts of probability to such an extent that you feel in total control of this topic.

For those who are reasonably strong, my advice would be to use this chapter both for revisiting the basic concepts as well as for extensive practice.

Probability means the chance of the occurrence of an event. In layman terms, we can say that it is the likelihood that something that is defined as the event will or will not occur. Thus probabilities can be estimated for each of the following events in our personal lives:

(a) the probability that an individual student of B.Com will clear the CAT,

(b) The chance that a candidate chosen at random will clear an interview,

(c) The chance that you will win a game of Dish in cards if you have a trio of twos in a game where four people are playing,

(d) The likelihood of India's winning the football World Cup before 2010,

(e) The probability that a bulb will fuse on it's first day of operation, and so on.

The knowledge of these estimations helps individuals decide on the course of action they will take in their day-to-day life. For instance, your estimation/ judgement of the probability of your chances of winning the card game in Event-c above will influence your decision about the amount of money you will be ready to invest in the stakes for the particular game. The application of probability to personal life helps in improving our decision making.

However, the use of probability is much more varied and has far reaching influence on the world of economics and business. Some instances of these are:

(a) the estimation of the probability of the success of a business project,

(b) the estimation of the probability of the success of an advertising campaign in boosting the profits of a company,

(c) the estimation of the probabilities of the death of a 25-year old man in the next 10 years and that of the death of a 55-year old man in the next 10 years leading to the calculation of the premiums for life insurance.

of the estimation of the probability of the interview in the earlier case of the share of a company, and so on.

INVOLVING FACTORS FOR REAL-LIFE ESTIMATION OF PROBABILITY

In cases involving an over-estimation of how the probability of the event occurrence. For instance, if we estimate the probability of India winning the 2007 World Cup on the basis of certain expectations of our team, then the results will definitely improve if we know that Sachin Tendulkar will score 100 runs in that particular World Cup.

Now we come towards the mathematical aspects of the factor we underlying factor that occurs in every question. Probability is that whenever one is asked the question, what the probability? the immediate question that arises in one's mind is the probability of what?

The answer to this question is the probability of the event.

So EVENT is the cornerstone or the foundation of probability theory, the first objective while trying to solve any event a probability is to define the event.

The event whose probability is to be found out, is decided by the question and the task of the student is trying to solve the problem is to define it.

In general, the student can either define the event narrowly or broadly. Narrow definitions of events are the building blocks of any probability problem and whenever there is a study about a problem, the student is advised to get into the minutest basis of the event definition.

The difference between the narrow and broad definitions of event can be explained through an example:

Example: What is the probability of getting a number greater than 2 in a throw of a normal unbiased dice having 6 faces?

The broad definition of the event here is getting a number greater than 2 and this probability is given by 4/6. However, the event can also be broken down into its more basic definition as:

The event is defined as getting 3 or 4 or 5 or 6. The individual probabilities of each of these are 1/6, 1/6, 1/6 and 1/6 respectively.

Hence, the required probability is 1/6 + 1/6 + 1/6 + 1/6 = 4/6 = 2/3.

Although in this example it seems highly trivial, the narrow event definition approach is very effective in solving tricky problems on probability.

In general, event definition means breaking up the event in the most basic building blocks, which have to be connected together through the two logical conjunctions—AND and OR.

The Use of the Conjunction *AND* (Tool No. 9)

Refer Back to school section. Whenever we use AND as the natural conjunction joining two separate parts of the event definition, we replace the AND by the multiplication sign.

Thus, if *A* AND *B* have to occur, and if the probability of their occurrence are *P(A)* and *P(B)* respectively, then the probability that *A* AND *B* occur is got by connecting *P(A)* AND *P(B)*. Replacing the AND by multiplication sign we get the required probability as:

$$P(A) \times P(B)$$

Example: If we have the probability of *A* hitting a target as 1/3 and that of *B* hitting the target as 1/2, then the probability that both hit the target if one shot is taken by both of them is get by

Event Definition: *A* hits the target AND *B* hits the target

$$\rightarrow P(A) \times P(B) = 1/3 \times 1/2 = 1/6$$

(Note that above we use the conjunction AND in the definition of the event here, we multiply the individual probabilities that are connected through the conjunction AND.)

The Use of the Conjunction *OR* (Tool No. 10)

Refer Back to school section. Whenever we use OR as the natural conjunction joining two separate parts of the event definition, we replace the OR by the addition sign.

Thus, if *A* OR *B* have to occur, and if the probability of their occurrence are *P(A)* and *P(B)* respectively, then the probability that *A* OR *B* occur is got by connecting

P(A) OR *P(B)*. Replacing the OR by addition sign, we get the required probability as:

$$P(A) + P(B)$$

Example: If we have the probability of *A* winning a race as 1/3 and that of *B* winning the race as 1/2, then the probability that either *A* or *B* win a race is get by

Event Definition: *A* wins OR *B* wins.

$$\rightarrow P(A) + P(B) = 1/3 + 1/2 = 5/6$$

(Note that since we use the conjunction OR in the definition of the event here, we add the individual probabilities that are connected through the conjunction OR.)

Combination of *AND* and *OR*

In two dice tossed dices, what is the chance that the sum of the numbers is not less than 10.

Event Definition: (The sum of the numbers is not less than 10) (is equal to 10 OR 11 OR 12)

Which can be done by
(4 AND 6) OR (5 AND 5) OR (5 AND 5) OR (6 AND 5) OR (5 AND 6) OR (6 AND 6)
that is, $1/6 \times 1/6 + 1/6 \times 1/6 + 1/6 \times 1/6 + 1/6 \times 1/6 + 1/6 \times 1/6 + 1/6 \times 1/6 = 6/36 = 1/6$

The instruction is that no matter how complicated the problem on probability is, it can be broken up into its summary parts, which can be connected by AND's and OR's to get the event definition.

Once the event is defined, the probability of each summary event within the broad event is calculated and all the summary events are connected by Multiplication (for AND) or by Addition (for OR) to get the final solution.

Example: In a tour game, match between Kasparov and Anand, the probability that Anand wins a particular game is 3/5 and that of Kasparov winning a game is 3/5. Assuming that there is no probability of a draw in an individual game, what is the chance that the match is drawn (Score is 2-2).

For the match to be drawn, 2 games have to be won by each of the players. If 'A' represents the event that Anand wins a game and 'K' represents the event that Kasparov wins a game, the event definition for the match to end in a draw can be described as: (The student is advised to look at the use of summary event definition.)

(AAKKKK) OR (AKKKAK) OR (AKKAKA) OR (KAKAKA) OR (KKAKKA) OR (KKKKAK)

This further translates into

$$(3/5)^2(3/5)^2 + (2/5)^2(3/5)^2 + (2/5)^2(3/5)^2 + (2/5)^2(3/5)^2 + (2/5)^2(3/5)^2 + (2/5)^2(3/5)^2 = (36/625) \times 6 = 216/625$$

After a little bit of practice, you can also think about the directly as

$$^4C_2 \times (2/5)^2 \times {}^4C_2 \times (3/5)^2 = 6 \times 4 \times 36/625 = 216/625$$

Where, $^4C_2$ gives us the number of ways in which Anand can win two games and $^4C_2$ gives us the number of ways in which Kasparov can win the remaining 2 games (obviously, only one)

BASIC FACTS ABOUT PROBABILITY

For every event that can be defined, there is a corresponding non-event, which is the opposite of the event. The relationship between the event and the non-event is that they are mutually exclusive, that is, if the event occurs then the non-event does not occur and vice versa.

The event is denoted by E, the number of ways in which the event can occur is defined as $n(E)$ and the probability of the occurrence of the event is $P(E)$.

The non-event is denoted by E', the number of ways in which the non-event can occur is defined as $n(E')$ which the probability of the occurrence of the event is $P(E')$.

The following relationships hold true with respect to the event and the non-event.

$n(E) + n(E') =$ sample space representing all the possible events that can occur related to the activity

$$P(E) + P(E') = 1$$

This means that if the event does not occur, then the non-event occurs.

$$\rightarrow P(E') = 1 - P(E)$$

This is often very useful for the calculation of probabilities of events where it is easier to calculate and count the non-event rather than the event.

Illustration

The probability that you get a total more than 3 in a throw of 2 dice.

Here, the event definition will be a long and tedious task which will involve long counting. Hence, it would be more convenient to define the non-event and count the same.

Therefore, here the non-event will be defined as

A total not more than 3 $\rightarrow$ 2 or 3 $\rightarrow$ (1,1) OR (1,2) OR (2,1) $\rightarrow 1/36 + 1/36 + 1/36 = 3/36 = 1/12$.

sometimes a source of confusion especially for students not comfortable in mathematics. Take care while defining the events. Because of a very like or some definition. Tossing a fair coin or a fair dice does not translate into a coin value of 0 or 10 but instead into the sum events of 5, 10.

SOME IMPORTANT CONSIDERATIONS WHILE DEFINING EVENT

Random Experiment An experiment whose outcomes are to be among a set of events that are completely known in advance and whose outcome is unknown is a random experiment. (e.g. Throwing of a dice, tossing of a coin). Most questions in probability are based on random experiments.

Sample Space This is defined as the universe of a random experiment and denotes the set representing all the possible outcomes of the random experiment. (e.g. Sample space when a coin is tossed is {Head, Tail}. Sample space when a dice is thrown is {1, 2, 3, 4, 5, 6}.)

Event The set representing the desired outcome of a random experiment is called the event. Note that the event is a subset of the sample space.

Non-event The outcome that is opposite to the event outcome is the non-event. Note that if the event occurs the non-event does not occur and vice versa.

Impossible Event An event that can never occur is an impossible event. The probability of an impossible event is 0. (e.g. Probability of the occurrence of 7 when a dice with numbers 1–6 is thrown).

Mutually Exclusive Events A set of events is mutually exclusive when the occurrence of any one of these events rules out the other events cannot occur. (Head appears on a coin, tail cannot appear and vice versa.)

Equally Likely Events If two events have the same probability or chance of occurrence they are called equally likely events. (In a throw of a dice, the chance of 1 showing up is equal to 2 is equal to 3 is equal to 4 is equal to 5 is equal to 6 appearing on the dice.)

Exhaustive Set of Events A set of events that includes all the possibilities of the sample space is said to be exhaustive set of events. (e.g. In a throw of a dice the number is less than three or more than or equal to three.)

Independent Events An event is described as such if the occurrence of an event has no effect on the probability of the occurrence of another event. (If the first child of a couple is a boy, there is no effect on the chances of the second child being a boy.)

Conditional Probability It is the probability of the occurrence of an event A given that an event B has already occurred. This is denoted by $P(A|B)$. (E.g. The probability that in two throws of a dice we get a total of 7 as above, given that in the first throw of the dice the number 5 has occurred.)

The Concept of Odds For and Odds Against

Sometimes, probability is also stated in terms of odds for and odds against an event.

Odds in favour of an event E is defined as $\frac{P(E)}{P(\bar{E})}$

Odds against an event is defined as $\frac{P(\bar{E})}{P(E)}$

Expectation: The expectation of an individual is defined as Probability of winning × Reward of winning.

Illustration: A man holds 10 tickets of the 500 tickets in a lottery. If the reward for the winning ticket is Rs. 1000, find the expectation of the man.

Solution: Expectation = Probability of winning × Reward of winning = $\frac{10}{500} \times 1000 =$ Rs. 20.

ANOTHER APPROACH TO LOOK AT THE PROBABILITY PROBLEMS

The probability of an event is defined as

$$\frac{\text{Number of ways in which the event occurs}}{\text{Total number of outcomes possible}}$$

This states that the probability of any event can be got by counting the numerator and the denominator independently.

Hence, from this approach, the convenience exists in counting the numerator and the denominator.

Thus for the example used above, the probability of a number = 3 appearing on a dice is

$$\frac{\text{Number of ways in which the event occurs}}{\text{Total number of outcomes possible}} = \frac{4}{6}$$

The counting is done through any of

(a) The physical counting as illustrated above,
(b) The use of the concept of permutations,
(c) The use of the concept of combinations,
(d) The use of the MNP rule.

(Refer to the chapter on Permutations and Combinations to understand 4C2 and 4P2 above.)

WORKED-OUT PROBLEMS

Problem 17.1 In a throw of two dice, find the probability of getting one prime and one composite number?

Solution The probability of getting a prime number when a dice is thrown is 3/6 = 1/2. (This occurs when we get 2, 3 or 5 out of a possibility of getting 1, 2, 3, 4, 5 or 6.)

Similarly, in a throw of a dice, there are only 2 possibilities of getting composite numbers viz., 4 or 6 and this gives a probability of 1/3 for getting a composite number.

Now, let us look at defining the event. The event is—getting one prime and one composite number.

This can be got as:

The first number is prime and the second is composite OR the first number is composite and the second is prime.

$$= (1/2) \times (1/3) + (1/3) \times (1/2) = 1/3$$

Problem 17.2 Find the probability that a leap year chosen at random will have 53 Sundays.

Solution A leap year has 366 days. 52 complete weeks will have 364 days. The 365th day can be a Sunday (Probability = 1/7) OR the 366th day can be a Sunday (Probability = 1/7). Answer = 1/7 + 1/7 = 2/7.

Alternatively, you can think of this as: The favourable events will occur when we have Saturday and Sunday or Sunday and Monday in the 365th and 366th days respectively. (i.e. 2 possibilities of the event occurring). Besides, the total number of ways that can happen are: Sunday and Monday OR Monday and Tuesday ... OR Friday and Saturday OR Saturday and Sunday.

Problem 17.3 There are two bags containing white and black balls. In the first bag, there are 6 white and 8 black balls and in the second bag, there are 4 white and 7 black balls. One ball is drawn at random from any of these two bags. Find the probability of this ball being black.

Solution The event definition here is: 1st bag and black ball OR 2nd Bag and Black Ball. The chances of picking up either the 1st OR the 2nd Bag are 1/2 each.

Besides, the chance of picking up a black ball from the first bag is 8/14 and the chance of picking up a black ball from the second bag is 7/11.

Thus, using these values and the AND's and OR's we get

$$(1/2) \times (8/14) + (1/2) \times (7/11) = (8/14) + (7/22) = (88 + 98)/(308) = 164/308 = 41/77$$

Problem 17.4 The letters of the word LUCKNOW are arranged among themselves. Find the probability of always having NOW in the word.

Solution The required probability will be given by the equation

= No. of words having NOW/Total no. of words

= 3!/7! = 1/42 [See the chapter of Permutations and Combinations to understand the logic behind these values.]

Problem 17.5 A person has 3 children with at least one boy. Find the probability of having at least 2 boys among the children.

Solution The event is occurring under the following situations:

(a) Second is a boy and third is a girl OR
(b) Second is a girl and third is a boy OR
(c) Second is a boy and third is a boy.

This will be represented by: (1/2) × (1/2) + (1/2) × (1/2) + (1/2) × (1/2) = 3/4

Problem 17.6 Out of 13 applicants for a job, there are 5 women and 8 men. Two persons are to be selected for the job. The probability that at least one of the selected persons will be a woman is

Solution The required probability will be given by:

First is a woman and Second is a man OR
First is a man and Second is a woman OR

first is a woman and Second is a woman

$$= (3/13) \times (9/12) + (10/13) \times (8/12) + (3/11) \times (4/12)$$
$$= 100/156 = 25/39$$

Alternatively, we can define the non-event as: There are two men and no women. Then, probability of the non-event is

$$(4/13) \times (3/12) = 50/156$$

and

$$P(E) = (1 - 50/156) = 106/156 = 25/39$$

[Note: This is a case of probability calculation where repetition is not allowed.]

Problem 17.7 The probability that *A* can solve the problem is 2/3 and *B* can solve it is 3/4. If both of them attempt a problem, then what is the probability that the problem gets solved.

Solution The event is defined as:

Either the problem ... AND ... *B* does not solve the problem

OR

Either *A* solves the problem ... AND *B* solves the problem

OR

Either the problem ... AND ... *B* solves the problem.

Numerically, this is equivalent to:

$$(2/3) \times (1/4) + (1/3) \times (3/4) + (2/3) \times (3/4)$$
$$= (2/12) + (3/12) + (6/12) = 11/12$$

Problem 17.8 Six positive numbers are taken at random and are multiplied together. Then what is the probability that the product ends in an odd digit other than 5.

Solution The event will occur when all the numbers ending in 1, 3, 7 or 9.

If we take numbers between 1 to 10 (both inclusive), we will have a positive occurrence if each of the six numbers selected are either 1, 3, 7 or 9.

The probability of any number selected being either of these is 4/10 (4 positive events out of 10 possibilities). Now, if we try to take numbers between 1 to 20, we will have the probability of 8/20 = 4/10. Hence, we can extrapolate up to infinity and say that the probability of any number selected ending in 1, 3, 7 or 9 is as so fulfill the requirement is 4/10.

Hence, answer = $(0.4)^6$

Problem 17.9 The probability that Arjit will solve a problem is 1/5. What is the probability that he solves at least one problem out of ten problems?

Solution The non-event is defined as:

He solves no problem i.e. he doesn't solve the first problem and he doesn't solve the second problem ... and he doesn't solve the tenth problem.

Probability of non-event = $(4/5)^{10}$

Hence, probability of the event is $1-(4/5)^{10}$

Problem 17.10 A carton contains 25 bulbs, 8 of which are defective. What is the probability that if a sample of 4 bulbs is chosen, exactly 2 of them will be defective?

Solution The probability that exactly two bulbs are defective and exactly two are not defective will be given by $(^8C_2) \times (8/25) \times (17/24) \times (17/23) \times (16/22)$

Problem 17.11 Out of 40 consecutive integers, two are chosen at random. Find the probability that their sum is odd.

Solution Forty consecutive integers will have 20 odd and 20 even integers. The sum of 2 chosen integers will be odd, only if:

(a) First is even and Second is odd OR

(b) First is odd and Second is even

Mathematically, the probability will be given by:

P(First is even) × P(Second is odd) + P(First is odd) × P(Second is even)

$= (20/40) \times (20/39) + (20/40) \times (20/39)$

$= 2/3 \times 20/39 = 40/39 = 20/39$

Problem 17.12 An integer is chosen at random from the first 100 integers. What is the probability that this number will not be divisible by 5 or 8?

Solution For a number from 1 to 100 to be divisible by 5 or 8, we need to remove all the numbers that are divisible by 5 or 8.

Thus, we remove 5, 8, 10, 15, 16, 20, 24, 25, 30, 32, 35, 40, 45, 48, 50, 55, 56, 60, 64, 65, 70, 72, 75, 80, 85, 88, 90, 95, 96, and 100.

i.e. 30 numbers from the 100 are removed.

Hence, answer is 70/100 = 7/10 (required probability)

Alternatively, we could have counted the numbers in number of numbers divisible by 5 + number of numbers divisible by 8 – number of numbers divisible by both 5 or 8.

$$= 20 + 12 - 2 = 30$$

Problem 17.37 From a bag containing 8 green and 5 red balls, three are drawn one after the other. Find the probability of all three balls being green if

(a) the balls drawn are replaced before the next ball is picked

(b) the balls drawn are not replaced

Solution

(a) When the balls drawn are replaced, we can see that the number of balls available for drawing remains the same for every draw. This means that the probability of a green ball appearing in the first draw and a green ball appearing in the second draw as well as one appearing in the third draw are equal to each other.

Hence answer to the question above will be:

$$\text{Required probability} = \frac{8}{13} \times \frac{8}{13} \times \frac{8}{13} = (8/13)^3$$

(b) When the balls are not replaced, the probability of drawing any color of ball for every fresh draw changes. Hence, the answer here will be:

$$\text{Required probability} = \frac{8}{13} \times \frac{7}{12} \times \frac{6}{11}$$

Level of Difficulty (LOD) I

1. In throwing a fair dice, what is the probability of getting the number '7'?

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$

(e) $\frac{1}{8}$

2. What is the chance of drawing a number greater than 8 with an ordinary dice whose faces are numbered from 1 to 6.

(a) $\frac{1}{3}$ (b) $\frac{5}{6}$ (c) $\frac{1}{9}$ (d) $\frac{1}{12}$

(e) $\frac{1}{6}$

3. Find the chance of drawing at least one ace in a single throw with two dice.

(a) $\frac{5}{12}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{11}{36}$

(e) $\frac{13}{36}$

4. Find the chance of drawing 2 blue balls in succession from a bag containing 5 red and 7 blue balls, if the balls are not being replaced.

(a) $\frac{5}{13}$ (b) $\frac{21}{64}$ (c) $\frac{7}{22}$ (d) $\frac{21}{44}$

(e) $\frac{25}{84}$

5. From a pack of 52 cards, two are drawn at random. Find the chance that one is a king and the other a queen.

(a) $\frac{8}{663}$ (b) $\frac{1}{6}$ (c) $\frac{1}{9}$ (d) $\frac{1}{12}$

(e) $\frac{1}{8}$

6. Ace card is picked up at random from a pack of 52 cards. Find the probability that it is

(i) a spade

(a) $\frac{1}{8}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

(e) $\frac{1}{6}$

(ii) a king or queen

(a) $\frac{3}{13}$ (b) $\frac{7}{13}$ (c) $\frac{1}{52}$ (d) $\frac{1}{104}$

(e) $\frac{1}{172}$

(iii) 'a spade' or 'a king' or 'a queen'

(a) $\frac{21}{52}$ (b) $\frac{5}{13}$

(c) $\frac{19}{52}$ (d) $\frac{13}{52}$

(e) None of these

7. Three coins are tossed. What is the probability of getting

(i) 2 Tails and 1 Head

(a) $\frac{3}{4}$ (b) $\frac{3}{8}$ (c) $\frac{2}{3}$ (d) $\frac{1}{6}$

(e) $\frac{5}{9}$

(ii) 1 Tail and 2 Heads

(c) $\frac{3}{8}$ (d) 1 (e) $\frac{2}{3}$ (f) $\frac{3}{4}$

(g) $\frac{2}{11}$

8. Three coins are tossed. What is the probability of getting

(i) neither 3 Heads nor 3 Tails?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

(e) $\frac{3}{8}$

(ii) three heads

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(e) $\frac{1}{9}$

9. For the above question, the probability that there is at least one tail is:

(a) $\frac{7}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{8}$ (d) $\frac{1}{2}$

(e) $\frac{1}{4}$

10. Two fair dice are thrown. Find the probabilities of getting

(i) a number divisible by 2 or 4

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(e) $\frac{1}{4}$

(ii) a number divisible by 2 and 4

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{5}{3}$

(e) $\frac{5}{2}$

(iii) a prime number less than 8

(a) $\frac{11}{12}$ (b) $\frac{1}{12}$ (c) $\frac{1}{4}$ (d) $\frac{13}{36}$

(e) $\frac{5}{18}$

11. A bag contains 5 green and 2 white balls. Two balls are drawn from the bag in succession without replacement. What is the probability that

(i) both are white?

(a) $\frac{1}{5}$ (b) $\frac{5}{11}$ (c) $\frac{7}{11}$ (d) $\frac{7}{18}$

(e) $\frac{7}{22}$

(ii) they are of different colour?

(a) $\frac{7}{18}$ (b) $\frac{7}{8}$

(c) $\frac{11}{18}$ (d) $\frac{7}{11}$

(e) None of these

12. 100 students appeared for two examinations. 60 passed the first, 50 passed the second and 30 passed both. Find the probability that a student selected at random has failed in both the examinations.

(a) $\frac{1}{5}$ (b) $\frac{1}{7}$ (c) $\frac{3}{5}$ (d) $\frac{5}{6}$

(e) $\frac{5}{8}$

13. What is the probability of throwing a number greater than 2 with a fair dice?

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) 1 (d) $\frac{3}{5}$

(e) $\frac{3}{4}$

14. Three cards numbered 2, 4 and 8 are put into a box. If a card is drawn at random, what is the probability that the card drawn is

(i) a prime number?

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{4}{5}$ (d) $\frac{5}{7}$

(e) $\frac{4}{7}$

(ii) an even number?

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$

(e) $\frac{4}{5}$

(iii) an odd number?

(a) 1 (b) 0 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(e) $\frac{3}{4}$

15. Two fair coins are tossed. Find the probability of obtaining

(i) 2 Heads

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

(e) $\frac{3}{4}$

(ii) 1 Head and 1 Tail

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(e) $\frac{1}{3}$

(iii) 2 Tails

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

(e) $\frac{3}{4}$

16. In rolling two dices, find the probability that

(i) there is at least one '5'

(a) $\frac{11}{36}$ (b) $\frac{22}{36}$

(c) $\frac{15}{36}$ (d) $\frac{29}{36}$

(e) None of these

(ii) the sum is 5

(a) $\frac{1}{4}$ (b) $\frac{1}{9}$ (c) $\frac{1}{2}$ (d) $\frac{1}{6}$

(e) $\frac{1}{5}$

17. From a bag containing 4 white and 5 black balls a man draws 3 at random. What are the odds against these being all black?

(a) $\frac{8}{11}$ (b) $\frac{55}{5}$ (c) $\frac{11}{13}$ (d) $\frac{13}{11}$

(e) $\frac{2}{11}$

18. Amit throws three dice in a special game of Ludo. If it is known that he needs 15 or higher in this throw to win then find the chance of his winning the game.

(a) $\frac{5}{54}$ (b) $\frac{15}{216}$ (c) $\frac{11}{216}$ (d) $\frac{15}{216}$

(e) $\frac{19}{216}$

19. Find out the probability of forming 187 or 215 with the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 when only numbers of three digits are formed and when

(i) repetitions are not allowed

(a) $\frac{12}{504}$ (b) $\frac{18}{504}$ (c) $\frac{5}{504}$ (d) $\frac{24}{504}$

(e) $\frac{25}{504}$

(ii) repetitions are allowed

(a) $\frac{5}{729}$ (b) $\frac{8}{729}$ (c) $\frac{11}{729}$ (d) $\frac{4}{729}$

(e) $\frac{5}{729}$

20. In a horse race there were 18 horses numbered 1–18. The probability that horse 1 would win is 1/6, that 2 would win is 1/10 and that 3 would win is 1/8. Assuming that a tie is impossible, find the chance that one of the three will win.

(a) $\frac{47}{120}$ (b) $\frac{119}{120}$

(c) $\frac{11}{120}$ (d) $\frac{3}{8}$

(e) None of these

21. Two balls are to be drawn from a bag containing 8 gray and 3 blue balls. Find the chance that they will both be blue.

(a) $\frac{3}{8}$ (b) $\frac{3}{55}$ (c) $\frac{11}{15}$ (d) $\frac{14}{45}$

(e) $\frac{16}{45}$

22. Two fair dice are thrown. What is the probability of

(i) throwing a double?

(a) $\frac{1}{6}$ (b) 1

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

(e) None of these

(ii) the sum is greater than 10

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{1}{12}$

(e) None of these

(iii) the sum is less than 10?

(a) $\frac{2}{9}$ (b) $\frac{3}{8}$ (c) $\frac{2}{5}$ (d) $\frac{2}{3}$

(e) $\frac{4}{5}$

23. In a certain lottery the prize is Rs. 1 crore and 500 tickets have been sold. What is the expectation of a man who holds 10 tickets?

(a) Rs. 20,000 (b) Rs. 25,000

(c) Rs. 30,000 (d) Rs. 15,000

(e) None of these

[illegible] letters are randomly chosen from the word EVASE. Find the probability that the letters are E and A.

(a) $\frac{1}{5}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$

(e) $\frac{1}{4}$

Passage for Questions 25–27: Read the following passage and answer the questions based on it.

The Bangalore office of Infosys has 1200 executives. Of them, 780 subscribe to the *Time* magazine and 650 subscribe to the *Economist*. Each executive may subscribe to either one or the *Economist* or both. If an executive is picked at random, answer questions 25–27.

25. What is the probability that

(i) he has subscribed to the *Time* magazine

(a) $\frac{11}{15}$ (b) $\frac{11}{12}$ (c) $\frac{7}{15}$ (d) $\frac{7}{11}$

(e) $\frac{7}{9}$

(ii) he has subscribed to the *Economist*

(a) $\frac{13}{21}$ (b) $\frac{13}{20}$ (c) $\frac{13}{24}$ (d) $\frac{17}{30}$

(e) $\frac{13}{30}$

(iii) he has subscribed to both magazines

(a) $\frac{23}{40}$ (b) $\frac{13}{40}$ (c) $\frac{17}{20}$ (d) $\frac{4}{20}$

(e) $\frac{7}{20}$

27. If among the executives who have subscribed to the *Time* magazine, an executive is picked at random. What is the probability that he has also subscribed to the *Economist*?

(a) $\frac{3}{8}$ (b) $\frac{5}{8}$

(c) $\frac{2}{3}$ (d) $\frac{1}{8}$

(e) None of these

28. A bag contains four black and five red balls. If three balls from the bag are chosen at random, what is the chance that they are all black?

(a) $\frac{2}{21}$ (b) $\frac{1}{28}$ (c) $\frac{2}{25}$ (d) $\frac{1}{6}$

29. If a number of two digits is formed with the digits 2, 3, 5, 7, without repetition of digits, what is the probability that the number formed is 35?

(a) $\frac{1}{10}$ (b) $\frac{1}{20}$ (c) $\frac{2}{11}$ (d) $\frac{3}{11}$

30. From a pack of 52 playing cards, three cards are drawn at random. Find the probability of drawing a king, a queen and jack.

(a) $\frac{16}{5525}$ (b) $\frac{8}{13^3}$

(c) $\frac{1}{13^3}$ (d) $\frac{1}{19^3}$

31. A bag contains 20 balls marked 1 to 20. One ball is drawn at random. Find the probability that it is marked with a number multiple of 5 or 7.

(a) $\frac{3}{10}$ (b) $\frac{7}{10}$ (c) $\frac{1}{11}$ (d) $\frac{2}{3}$

32. A group of investigators took a fair sample of 1977 children from the general population and found that there are 1000 boys and 977 girls. If the investigators claim that their research is so accurate that the sex of a new born child can be predicted based on the ratio of the sample of the population, then what is the expectation in terms of the probability that a new child born will be a girl?

(a) $\frac{241}{250}$ (b) $\frac{250}{253}$

(c) $\frac{8}{39}$ (d) $\frac{243}{493}$

33. A bag contains 5 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that both are black?

(a) $\frac{1}{8}$ (b) $\frac{7}{40}$ (c) $\frac{12}{40}$ (d) $\frac{13}{40}$

34. A bag contains 8 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that

(i) all the three balls are of the same colour

(a) $\frac{17}{240}$ (b) $\frac{8}{21}$

(c) $\frac{21}{304}$ (d) None of these

(ii) all the three balls are blue

(a) $\frac{8}{21}$ (b) $\frac{56}{30}$ (c) $\frac{7}{100}$ (d) $\frac{12}{51}$

35. If $P(A) = 3/5$, $P(B) = 1/2$, $P(A \cap B) = 1/10$ then find $P(A' \cap B')$

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

36. A and B are two candidates seeking admission to the IIMs. The probability that A is selected is 0.5 and the probability that both A and B are selected is at most 0.3. Is it possible that the probability of B getting selected is 0.9?

(a) No (b) Yes

(c) Either (a) or (b) (d) Can't say

37. The probability that a student will pass in Mathematics is 3/5 and the probability that he will pass in English is 1/3. If the probability that he will pass in both Mathematics and English is 1/8, what is the probability that he will pass in at least one subject?

(a) $\frac{47}{120}$ (b) $\frac{47}{130}$ (c) $\frac{53}{120}$ (d) $\frac{120}{29}$

38. The odds in favour of standing first of three students Amit, Vikas and Vivek appearing at an examination are 1 : 2, 2 : 3 and 1 : 7 respectively. What is the probability that either of them will stand first (assume that a tie for the first place is not possible).

(a) $\frac{108}{178}$ (b) $\frac{122}{184}$ (c) $\frac{5}{168}$ (d) $\frac{125}{148}$

39. A, B, C are three mutually exclusive and exhaustive events associated with a random experiment. Find P(A) if it is given that P(B) = 3/2 P(A) and P(C) = 1/2 P(B).

(a) $\frac{4}{13}$ (b) $\frac{7}{3}$ (c) $\frac{12}{13}$ (d) $\frac{1}{13}$

40. A and B are two mutually exclusive events of an experiment. If P(A) = 0.65, P(A ∪ B) = 0.95 and P(B) = p, find the value of p.

(a) 0.25 (b) 0.3 (c) 0.1 (d) 0.2

41. A bag contains 4 white and 2 black balls. Another contains 3 white and 5 black balls. If one ball is drawn from each bag, find the probability that

(i) both are white

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

(ii) both are black

(a) $\frac{3}{24}$ (b) $\frac{4}{24}$ (c) $\frac{3}{12}$ (d) $\frac{5}{24}$

(iii) one is white and one is black

(a) $\frac{13}{24}$ (b) $\frac{11}{24}$ (c) $\frac{11}{21}$ (d) $\frac{1}{2}$

42. The odds against an event is 3 : 5 and the odds in favour of another independent event is 7 : 5. Find the probability that at least one of the two events will occur.

(a) $\frac{13}{96}$ (b) $\frac{69}{96}$ (c) $\frac{71}{96}$ (d) $\frac{13}{96}$

43. Kamal and Monica appeared for an interview for two vacancies. The probability of Kamal's selection is 1/5 and that of Monica's selection is 1/3. Find the probability that only one of them will be selected.

(a) $\frac{2}{3}$ (b) $\frac{1}{5}$ (c) $\frac{8}{9}$ (d) $\frac{2}{5}$

44. A husband and a wife appear in an interview for two vacancies for the same post. The probability of husband's selection is (1/7) and that of the wife's selection is 1/5. What is the probability that

(i) both of them will be selected?

(a) $\frac{1}{35}$ (b) $\frac{2}{35}$ (c) $\frac{3}{35}$ (d) $\frac{1}{5}$

(ii) one of them will be selected?

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$

(iii) none of them will be selected?

(a) $\frac{24}{35}$ (b) $\frac{20}{35}$ (c) $\frac{23}{35}$ (d) $\frac{2}{5}$

(iv) at least one of them will be selected?

(a) $\frac{12}{35}$ (b) $\frac{11}{35}$ (c) $\frac{10}{35}$ (d) $\frac{1}{5}$

Level of Difficulty (LOD) II

1. Two fair dices are thrown. Given that the sum of the dice is less than or equal to 4, find the probability that only one dice shows two.

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

(e) $\frac{1}{5}$

2. A can hit the target 3 times in 6 shots, B 2 times in 6 shots and C 4 times in 6 shots. They fire a volley. What is the probability that at least 2 shots hit?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

(e) $\frac{3}{4}$

3. There are two bags, one of them contains 5 red and 7 white balls and the other 3 red and 12 white balls. One ball is to be drawn from one or the other of the two bags. Find the chance of drawing a red ball.

(a) $\frac{29}{120}$ (b) $\frac{30}{120}$

(c) $\frac{41}{120}$ (d) $\frac{29}{120}$

(e) None of these

4. In two bags there are to be put altogether 5 red and 12 white balls, neither bag being empty. How must the balls be divided so as to give a person who draws one ball from either bag

(i) the least chance of drawing a red ball?

(a) $\frac{3}{24}$ (b) $\frac{5}{32}$ (c) $\frac{7}{32}$ (d) $\frac{3}{16}$

(e) $\frac{1}{16}$

(ii) the greatest chance of drawing a red ball?

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{5}{8}$ (d) $\frac{1}{2}$

(e) $\frac{5}{7}$

5. If 5 coins are tossed, what is the chance that one and only one will turn up Head?

(a) $\frac{1}{16}$ (b) $\frac{3}{32}$ (c) $\frac{5}{32}$ (d) $\frac{1}{32}$

(e) $\frac{5}{32}$

6. What is the chance that a leap year, selected at random, will contain 53 Sundays?

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{1}{7}$ (d) $\frac{4}{7}$

(e) $\frac{5}{7}$

7. Out of all the 2-digit integers between 1 to 200, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7?

(a) $\frac{11}{90}$ (b) $\frac{13}{90}$ (c) $\frac{55}{90}$ (d) $\frac{77}{90}$

(e) $\frac{79}{90}$

8. A child is asked to pick up 2 balloons from a box containing 10 blue and 15 red balloons. What is the probability of the child picking, at random, 2 balloons of different colours?

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{5}$ (d) $\frac{1}{9}$

(e) $\frac{2}{9}$

9. Tom and Dick are running in the same race, the probability of their winning are 1/9 and 1/12 respectively. Find the probability that

(i) either of them will win the race

(a) $\frac{7}{36}$ (b) $\frac{3}{36}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(e) $\frac{7}{9}$

(ii) neither of them will win the race

(a) $\frac{7}{10}$ (b) $\frac{3}{10}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

(e) $\frac{4}{5}$

10. Two dices are thrown. If the total on the faces of the two dices are 6, find the probability that there are two odd numbers on the faces?

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{5}{6}$ (d) $\frac{3}{5}$

(e) $\frac{7}{9}$

11. Amarnath appears in an exam that has 4 subjects. The chance he passes an individual subject's test is 0.8. What is the probability that he will

(i) pass in all the subjects?

(a) 0.8^2 (b) 0.3^4

(c) 0.7^3 (d) 0.8^4

(e) None of these

(ii) fail in all the subjects?

(a) 0.4^4 (b) 0.2^4

(c) 0.5^4 (d) 0.4^3

(e) None of these

(iii) pass in at least one of the subjects?

(a) 0.99984 (b) 0.9984

(c) 0.9600 (d) 0.0064

(e) None of these

12. A box contains 2 tennis, 5 cricket and 4 squash balls. Three balls are drawn in succession with replacement. Find the probability that

(i) all are cricket balls

(a) $\frac{1}{27}$ (b) $\frac{2}{27}$ (c) $\frac{25}{27}$ (d) $\frac{1}{9}$

(e) $\frac{1}{8}$

(ii) the first is a tennis ball, the second is a cricket ball, the third is a squash ball

(a) $\frac{8}{243}$ (b) $\frac{5}{243}$ (c) $\frac{4}{243}$ (d) $\frac{7}{243}$

(e) $\frac{11}{243}$

(iii) all three are of the same type

(a) $\frac{11}{81}$ (b) $\frac{1}{9}$ (c) $\frac{12}{81}$ (d) $\frac{15}{81}$

(e) $\frac{17}{81}$

(b) With the data in the above question, answer the questions when the balls are drawn in succession without replacement.

(i)

(a) $\frac{5}{84}$ (b) $\frac{3}{84}$

(c) $\frac{8}{84}$ (d) $\frac{11}{84}$

(e) None of these

(ii)

(a) $\frac{8}{21}$ (b) $\frac{4}{21}$ (c) $\frac{5}{21}$ (d) $\frac{2}{7}$

(e) $\frac{3}{8}$

(iii)

(a) $\frac{5}{84}$ (b) $\frac{1}{84}$ (c) $\frac{5}{83}$ (d) $\frac{7}{84}$

(e) $\frac{11}{84}$

14. In the AMS library, there are 8 books by Stephen Covey and 1 book by Vinay Singh is shelf A. At the same time, there are 5 books by Stephen Covey in shelf B. One book is moved from shelf A to shelf B. A student picks up a book from shelf B. Find the probability that the book by Vinay Singh

(i) is still in shelf A.

(a) $\frac{1}{3}$ (b) $\frac{8}{9}$

(c) $\frac{3}{4}$ (d) $\frac{5}{9}$

(e) None of these

(ii) is in shelf B.

(a) $\frac{5}{54}$ (b) $\frac{4}{54}$

(c) $\frac{8}{54}$ (d) $\frac{6}{57}$

(e) None of these

(iii) is taken by the student.

(a) $\frac{5}{54}$ (b) $\frac{1}{54}$

(c) $\frac{2}{27}$ (d) $\frac{5}{54}$

(e) None of these

13. The ratio of number of officers and ladies in the Scorpion Squadron and in the Gunners Squadron are 3 : 1 and 2 : 5 respectively. An individual is selected to be the chairperson of their association. The chance that the individual is selected from the Scorpions is 2/3. Find the probability that the chairperson will be an officer

(a) $\frac{25}{42}$ (b) $\frac{13}{42}$ (c) $\frac{11}{42}$ (d) $\frac{5}{42}$

(e) $\frac{7}{42}$

16. A batch of 50 transistors contains 3 defective ones. Two transistors are selected at random from the batch and put into a radio set. What is the probability that

(i) both the transistors selected are defective?

(a) $\frac{4}{1225}$ (b) $\frac{3}{1225}$

(c) $\frac{124}{1224}$ (d) $\frac{232}{1224}$

(e) None of these

(ii) only one is defective?

(a) $\frac{141}{1225}$ (b) $\frac{121}{1225}$

(c) $\frac{123}{1224}$ (d) $\frac{124}{1224}$

(e) None of these

(iii) neither is defective?

(a) $\frac{1082}{1224}$ (b) $\frac{1081}{1224}$

(c) $\frac{1081}{1224}$ (d) $\frac{1082}{1225}$

(e) None of these

17. The probability that a man will be alive in 35 years is $\frac{2}{5}$ and the probability that his wife will be alive is $\frac{3}{7}$. Find the probability that after 35 years

(i) both will be alive.

(a) $\frac{2}{35}$ (b) $\frac{9}{35}$ (c) $\frac{6}{35}$ (d) $\frac{1}{35}$

(e) $\frac{3}{35}$

(ii) only the man will be alive

(a) $\frac{12}{35}$ (b) $\frac{11}{35}$ (c) $\frac{23}{35}$ (d) $\frac{1}{35}$

(e) $\frac{4}{35}$

(iii) only the wife will be alive

(a) $\frac{2}{35}$ (b) $\frac{3}{35}$ (c) $\frac{6}{35}$ (d) $\frac{1}{35}$

(e) $\frac{11}{35}$

(iv) at least one will be alive

(a) $\frac{22}{35}$ (b) $\frac{12}{35}$ (c) $\frac{11}{35}$ (d) $\frac{1}{35}$

(e) $\frac{2}{35}$

8. A speaks the truth 3 out of 4 times, and B 5 out of 6 times. What is the probability that they will contradict each other in stating the same fact?

(a) $\frac{2}{9}$ (b) $\frac{1}{3}$

(c) $\frac{3}{4}$ (d) $\frac{1}{2}$

(e) None of these

9. A party of 'n' persons sit at a round table. Find the odds against two specified persons sitting next to each other.

(a) $\frac{n+1}{2}$ (b) $\frac{n-3}{2}$

(c) $\frac{n+3}{2}$ (d) $\frac{n+2}{3}$

(e) None of these

10. If 3 whole numbers are taken at random and multiplied together, what is the chance that the last digit in the product is 1, 3, 7 or 9?

(a) $\frac{45}{625}$ (b) $\frac{32}{625}$ (c) $\frac{16}{625}$ (d) $\frac{33}{625}$

(e) $\frac{17}{625}$

11. In one throw with a pair of dices, what is the chance of throwing a double twice?

(a) $\frac{31}{216}$ (b) $\frac{29}{216}$ (c) $\frac{35}{216}$ (d) $\frac{43}{216}$

(e) $\frac{41}{216}$

12. A life insurance company insured 25,000 young boys, 15,000 young girls and 16,000 young adults. The probability of death within 10 years of a young boy, young girl and a young adult are 0.02, 0.03 and 0.15 respectively. One of the insured persons dies. What is the probability that the dead person is a young boy?

(a) $\frac{36}{365}$ (b) $\frac{25}{166}$ (c) $\frac{28}{163}$ (d) $\frac{30}{165}$

(e) $\frac{32}{163}$

13. Three groups of children contain 3 girls and 1 boy, 2 girls and 2 boys, and 1 girl and 2 boys respectively. One child is selected at random from each group. The probability that the three selected consists of 1 girl and 2 boys is

(a) $\frac{5}{9}$ (b) $\frac{1}{3}$ (c) $\frac{5}{8}$ (d) $\frac{2}{3}$

(e) $\frac{3}{2}$

14. A locker at the world famous WTC building can be opened by dialing a fixed three digit code (between 000 and 999). Dan, a mercenary, only knows that the number is a three digit number and has only one six. Using this information he tries to open the locker by dialing three digits at random. The probability that he succeeds in his endeavour is?

(a) $\frac{1}{243}$ (b) $\frac{1}{900}$ (c) $\frac{1}{1000}$ (d) $\frac{1}{216}$

(e) $\frac{3}{216}$

15. In a bag there are 12 black and 6 white balls. Two balls are chosen at random and the first one is found to be black. The probability that the second one is also black is:

(a) $\frac{11}{17}$ (b) $\frac{12}{17}$

(c) $\frac{13}{14}$ (d) $\frac{5}{17}$

(e) None of these

16. In the above question, what is the probability that the second one is white?

(a) $\frac{2}{17}$ (b) $\frac{6}{17}$ (c) $\frac{7}{17}$ (d) $\frac{1}{17}$

17. A fair dice is tossed six times. Find the probability of getting a third six on the sixth throw.

(a) $\frac{{}^5C_2\,5^3}{6^5}$ (b) $\frac{{}^5C_2\,5^4}{6^5}$

(c) $\frac{{}^7C_3 x^3}{x^7}$ (d) $\frac{{}^7C_3 x^3}{y^3}$

28. In shuffling a pack of cards, four are accidentally dropped. Find the chance that the dropped cards should be one from each suit?

(a) $\frac{{}^{13}C}{{}^{52}C_4}$ (b) $\frac{{}^{13}C^4}{{}^{52}C_4}$

(c) $\frac{{}^{13}C}{{}^{52}C_3}$ (d) $\frac{{}^{13}C}{{}^{52}C_1}$

29. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with these vertices is equilateral is

(a) $\frac{1}{10}$ (b) $\frac{2}{10}$ (c) $\frac{1}{5}$ (d) $\frac{4}{10}$

30. There are 5 red shoes and 4 blue shoes in a box. They have got all mixed up with each other. What is the probability of getting a matched pair if two shoes are taken at random?

(a) $\frac{8}{9}$ (b) $\frac{4}{9}$ (c) $\frac{2}{9}$ (d) $\frac{5}{9}$

31. A person draws a card from a pack of 52, replaces it and shuffles it. He continues doing it until he draws a heart. What is the probability that he has to make 3 trials?

(a) $\frac{9}{64}$ (b) $\frac{3}{64}$ (c) $\frac{5}{64}$ (d) $\frac{1}{64}$

32. For the above problem, what is the probability if he does not replace the cards?

(a) $\frac{234}{1300}$ (b) $\frac{221}{1326}$

(c) $\frac{217}{1300}$ (d) $\frac{234}{1300}$

33. An event E can happen with probability P, and event F can happen with probability P'. What is the probability that exactly one of them happens?

(a) $P + P' - 2PP'$ (b) $2PP' - P' + P$

(c) $P' - P' + 2PP'$ (d) $2P'P - P' + P$

34. In the above question, what is the probability that at least one of them happens?

(a) $P' + P' + PP'$ (b) $P + P' - PP'$

(c) $2PP' - P' - P$ (d) $P + P' - 2PP'$

35. Find the probability that a year chosen at random has 53 Mondays?

(a) $\frac{5}{28}$ (b) $\frac{2}{28}$ (c) $\frac{1}{28}$ (d) $\frac{3}{28}$

36. There are four machines and it is known that exactly two of them are faulty. They are tested one by one in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{5}{6}$

37. For the above question, the probability that exactly 3 tests will be required to identify the 2 faulty machines is

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

38. Seven white balls and three black balls are randomly placed in a row. Find the probability that no two black balls are placed adjacent to each other.

(a) $\frac{7}{15}$ (b) $\frac{2}{15}$ (c) $\frac{3}{7}$ (d) $\frac{2}{7}$

39. A fair coin is tossed repeatedly. If Head appears on the first four tosses then the probability of appearance of tail on the fifth toss is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{2}{5}$

40. The letters of the word 'article' are arranged at random. Find the probability that the vowels may occupy the even places.

(a) $\frac{2}{15}$ (b) $\frac{1}{35}$ (c) $\frac{3}{36}$ (d) $\frac{2}{54}$

41. What is the probability that four Ss come consecutively in the word MISSISSIPPI?

(a) $\frac{4}{165}$ (b) $\frac{2}{165}$ (c) $\frac{3}{165}$ (d) $\frac{1}{165}$

42. Eleven books, consisting of five Engineering books, four Mathematics books and two Physics books, are arranged in a shelf at random. What is the probability that the books of each kind are all together?

(a) $\frac{3}{1155}$ (b) $\frac{2}{1155}$

(c) $\frac{5}{1155}$ (d) $\frac{1}{1155}$

43. Three students appear at an examination of Mathematics. The probability of their success are 1/3, 1/4, 1/5 respectively. Find the probability of success of at least two.

(a) $\frac{1}{6}$ (b) $\frac{2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

44. A bag contains 8 white and 6 red balls. Four balls are drawn at random. What is the probability that two of them are red and 2 are white?

(a) $\frac{15}{24}$ (b) $\frac{15}{32}$ (c) $\frac{16}{24}$ (d) $\frac{15}{16}$

[illegible] A coin is to be constructed such that it gives and a head and the probability that the coin may have 1 girls.

(a) $\frac{4}{11}$ (b) $\frac{3}{24}$ (c) $\frac{8}{14}$ (d) $\frac{2}{11}$

6. 3 persons are seated around a round table. What is the probability that two particular persons sit together?

(a) $\frac{2}{11}$ (b) $\frac{1}{6}$ (c) $\frac{3}{11}$ (d) $\frac{3}{13}$

7. Six boys and six girls sit in a row randomly. Find the probability that all the six girls sit together.

(a) $\frac{1}{12}$ (b) $\frac{1}{132}$ (c) $\frac{1}{5!6!}$ (d) $\frac{1}{6!}$

8. From a group of 7 men and 4 women a committee of 3 persons is formed. What is the probability that the committee will consist of exactly 2 women?

(a) $\frac{2}{11}$ (b) $\frac{3}{11}$ (c) $\frac{4}{11}$ (d) $\frac{5}{11}$

9. A bag contains 5 red, 4 green and 3 black balls. If three balls are drawn out of it at random, find the probability of drawing exactly 2 red balls.

(a) $\frac{7}{22}$ (b) $\frac{10}{33}$ (c) $\frac{7}{12}$ (d) $\frac{7}{11}$

10. A bag contains 100 tickets numbered 1, 2, 3, ..., 100. If a ticket is drawn out of it at random, what is the probability that the ticket drawn has the digit 2 appearing on it?

(a) $\frac{19}{100}$ (b) $\frac{24}{100}$ (c) $\frac{25}{100}$ (d) $\frac{27}{100}$

Level of Difficulty (LOD) II

1. The deck of a pack of 52 cards one is lost, from the remainder of the pack, two cards are drawn and are found to be hearts. Find the chance that the missing card is a spade.

(a) $\frac{1}{52}$ (b) $\frac{13}{52}$ (c) $\frac{10}{49}$ (d) $\frac{10}{51}$

(e) $\frac{13}{51}$

2. A game of chance takes place for a stake of Rs. 11, which is to be won by the player who first throws a six. The game ends when the stake is won by A or B. If A has the first throw, what are their respective expectations?

(a) 2 and 9 (b) 6 and 5
(c) 11 and 0 (d) 10 and 1
(e) 9 and 2

3. 3 numbers marked 1, 2, 3 are placed in a bag and one of them is withdrawn and replaced. The operation being repeated three times, what is the chance of obtaining a total of 6 in these three operations?

(a) $\frac{11}{27}$ (b) $\frac{7}{27}$ (c) $\frac{1}{27}$ (d) $\frac{2}{14}$

(e) $\frac{5}{14}$

4. A speaks the truth 3 times out of 4, B 5 times out of 6. They both assert that a white ball is drawn from a bag containing 6 balls, all of different colours. Find the probability of the truth of the assertion.

(a) $\frac{12}{49}$ (b) $\frac{3}{19}$

(c) $\frac{21}{40}$ (d) $\frac{23}{40}$

(e) None of these

5. In a shirt factory, processes A, B and C respectively manufacture 25%, 35% and 40% of the total shirts. Of their respective productions 5%, 4% and 2% of the shirts are defective. A shirt is selected at random from the production of a particular day. If it is found to be defective, what is the probability that it is manufactured by the process C?

(a) $\frac{16}{69}$ (b) $\frac{25}{69}$ (c) $\frac{18}{44}$ (d) $\frac{23}{44}$

(e) $\frac{27}{44}$

6. A pair of die dice are rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7 is?

(a) 0.45 (b) 0.4 (c) 0.5 (d) 0.6

(e) 0.7

7. For the above problem, the probability of 7 coming before 5 is

(a) $\frac{7}{5}$ (b) 0.55 (c) 0.4 (d) 0.5

(e) 0.7

8. For the above problem, the probability of 4 coming before either 5 or 7 is

(a) $\frac{5}{43}$ (b) $\frac{7}{44}$ (c) $\frac{14}{18}$ (d) $\frac{12}{13}$

(e) $\frac{10}{43}$

9. The probability of a bomb hitting a bridge is 1/2 and two direct hits are needed to destroy it. The least number of bombs required so that the probability of the bridge being destroyed is greater than 0.9 is

(a) 7 bombs (b) 5 bombs
(c) 8 bombs (d) 6 bombs
(e) 9 bombs

10. What is the probability of the destruction of the bridge if only 5 bombs are dropped?

(a) 81.25% (b) 81.25%
(c) 46.25% (d) 35.25%
(e) 31.25%

11. Sanjay writes a letter to his friend from IIT, Kanpur. It is known that one out of 'n' letters that are posted does not reach its destination. If Sanjay does not receive the reply to his letter, then what is the probability that Kanpur did not receive Sanjay's letter? It is certain that Kanpur will definitely reply to Sanjay's letter if he receives it.

(a) $\frac{n}{(2n-1)}$ (b) $\frac{n-1}{n}$
(c) $\frac{1}{n}$ (d) $\frac{(n-1)}{2n}$
(e) None of these

12. A word of 6 letters is formed from a set of 16 different letters of the English alphabet (with replacement). Find out the probability that exactly 2 letters are repeated.

(a) $\frac{225 \times 224 \times 156}{16^6}$ (b) $\frac{15!/60}{16^6}$
(c) $\frac{15 \times 224 \times 156}{16^6}$ (d) $\frac{15 \times 224}{16^6}$
(e) None of these

13. A number is chosen at random from the numbers 10 to 99. By seeing the number, a man will sing if the product of the digits is 12. If he chooses three numbers with replacement, then the probability that he will sing at least once is:

(a) $1-\left(\frac{43}{45}\right)^3$ (b) $\left(\frac{43}{45}\right)^3$
(c) $1-\frac{68 \times 86}{90^3}$ (d) $\frac{68 \times 86}{90^3}$
(e) None of these

14. In a bag, there are six black, eight white and two red balls. Three balls are chosen at random and one is found to be black. The probability that the rest two are white is. Find the probability that the remaining two balls are white:

(a) $\frac{8}{23}$ (b) $\frac{4}{33}$
(c) $\frac{10 \times 8 \times 7}{23 \times 22 \times 21}$ (d) $\frac{4}{21}$
(e) $\frac{5}{23}$

15. In the above question, find the probability that the remaining two balls are red.

(a) $\frac{10}{231}$ (b) $\frac{12}{231}$
(c) $\frac{13}{363}$ (d) $\frac{14}{231}$
(e) None of these

16. Ten tickets are numbered 1, 2, 3 ..... 10. Six tickets are selected at random one at a time with replacement. The probability that the largest number appearing on the selected tickets is 7 is

(a) $\frac{(7^6-1)}{10^6}$ (b) $\frac{7^6-6^6}{10^6}$
(c) $\frac{6^6}{10^6}$ (d) $\frac{7^6+6^6}{10^6}$
(e) None of these

17. A bag contains 15 tickets numbered 1 to 15. A ticket is drawn and replaced. Then one more ticket is drawn and replaced. The probability that first number drawn is even and second is odd is

(a) $\frac{56}{225}$ (b) $\frac{28}{178}$
(c) $\frac{82}{289}$ (d) $\frac{60}{225}$
(e) None of these

18. Six blue balls are put in three boxes. The probability of putting balls in the boxes in equal numbers is

(a) $\frac{1}{21}$ (b) $\frac{2}{9}$ (c) $\frac{1}{28}$ (d) $\frac{1}{9}$
(e) $\frac{1}{4}$

19. AMS employs 8 professors on their staff. Their respective probabilities of remaining in employment for 10 years are 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9. The probability that after 10 years is least 6 of them still work in AMS is

(a) 0.39 (b) 1.22
(c) 0.9 (d) 0.16
(e) None of these

22. A person draws a card from a pack of 52, replaces it and shuffles it. He continues doing so until he draws a heart. What is the probability that he has to make at least 3 draws?

(a) $\frac{3}{17}$ (b) $\frac{8}{19}$ (c) $\frac{2}{17}$ (d) $\frac{6}{13}$

(e) $\frac{11}{16}$

23. Mitra, the largest white goods producer in India, uses a quality check scheme on produced items before they are sent into the market. The plan is as follows. A set of 20 articles is inspected and 4 of them are chosen at random. If any one of them is found to be defective then the whole set is put under 100% screening again. If no defectives are found, the whole set is sent into the market. Find the probability that a box containing 4 defective articles will be sent into the market?

(a) $\frac{364}{969}$ (b) $\frac{364}{663}$

(c) $\frac{36}{969}$ (d) $\frac{546}{969}$

(e) $\frac{341}{969}$

24. In the above question, what is the probability that a box containing only one defective will be sent back for screening?

(a) $\frac{2}{3}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

(e) $\frac{4}{5}$

25. If the integers m and n are chosen at random from 1 to 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

(e) $\frac{7}{8}$

26. Three numbers are chosen at random without replacement from $\{1, 2, 3, \ldots, 10\}$. The probability that the median number is 5 or the maximum number is 7 is

(a) $\frac{12}{35}$ (b) $\frac{11}{40}$ (c) $\frac{13}{35}$ (d) $\frac{12}{44}$

(e) $\frac{24}{35}$

27. An unbiased dice with face values 1, 2, 3, 4, 5 and 6 is rolled four times. Out of the 4 face values obtained, find the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5.

(a) $\frac{16}{81}$ (b) $\frac{16}{97}$

(c) $\frac{16}{80}$ (d) $\frac{16}{47}$

(e) None of these

28. Three faces of a dice are yellow, two faces are red and one face is blue. The dice is tossed three times. Find the probability that the colours yellow, red and blue appear in the first, second and the third toss respectively.

(a) $\frac{5}{18}$ (b) $\frac{1}{12}$ (c) $\frac{1}{9}$ (d) $\frac{1}{36}$

29. If from each of three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random then the probability that 2 white and 1 black ball will be drawn is

(a) $\frac{13}{32}$ (b) $\frac{17}{32}$ (c) $\frac{13}{24}$ (d) $\frac{7}{32}$

30. Probabilities that Rajesh passes in Math, Physics and Chemistry are p, q and r respectively. Of these subjects Rajesh has a 50% chance of passing in at least two, 50% chance of passing in at least two and 40% chance of passing in exactly two. Find which of the following is true?

(a) $p + q + r = \frac{19}{20}$ (b) $p + q + r = \frac{17}{20}$

(c) $pqr = \frac{3}{20}$ (d) $pqr = \frac{1}{8}$

31. There are 5 envelopes corresponding to 5 letters. If the letters are placed in the envelopes at random, what is the probability that all the letters are not placed in the right envelopes?

(a) $\frac{119}{120}$ (b) $\frac{59}{60}$

(c) $\frac{77}{79}$ (d) $\frac{4}{5}$

32. For the above question what is the probability that no single letter is placed in the right envelope.

(a) $\frac{13}{24}$ (b) $\frac{11}{24}$

(c) $\frac{13}{23}$ (d) $\frac{8}{13}$

31. An urn contains four tickets having numbers 112, 121, 211, 222 written on them. If one ticket is drawn at random and A_i ($i = 1, 2, 3$) be the event that the ith digit from left of the number on ticket drawn is 1, which of these can be said about the events A_1, A_2 and A_3?

(a) They are mutually exclusive

(b) A_1 and A_2 are not mutually exclusive or A_2

(c) A_1 and A_3 are mutually exclusive

(d) Both b and c

32. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will get an electric contract is 5/9. If the probability of getting at least one contract is 4/5, what is the probability that he will get both the contracts?

(a) $\frac{15}{45}$ (b) $\frac{15}{45}$ (c) $\frac{12}{35}$ (d) $\frac{11}{25}$

33. If $P(A) = 0.7$, $P(B) = 1/2$ and $P(A' \cap B') = 1/3$, then are A and B are mutually exclusive events?

(a) No (b) Yes

(c) Either yes or no (d) Cannot be determined

34. Six boys and six girls sit in a row at random. Find the probability that the boys and girls sit alternately.

(a) $\frac{1}{132}$ (b) $\frac{1}{462}$ (c) $\frac{1}{673}$ (d) $\frac{1}{234}$

35. A problem on mathematics is given to three students whose chances of solving it are 1/2, 1/3 and 1/4 respectively. What is the chance that the problem will be solved?

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

36. A and B throw a pair of dice alternately. A wins if he throws 6 before B throws 5 and B wins if he throws 5 before A throws 6. Find B's chance of winning if A makes the first throw.

(a) $\frac{5}{2}$ (b) $\frac{5}{12}$ (c) $\frac{1}{3}$ (d) $\frac{5}{11}$

37. Two persons A and B toss a coin alternately till one of them gets head and wins the game. Find B's chance of winning if A makes the coin toss.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) None of these

38. A bag contains 3 red and 4 white balls and another bag contains 4 red and 3 white balls. A die is cast and if the face 1 or 3 turns up, a ball is taken from the first bag and if any other face turns up, a ball is taken from the second bag. Find the probability of drawing a red ball.

(a) $\frac{11}{28}$ (b) $\frac{12}{21}$ (c) $\frac{2}{11}$ (d) $\frac{11}{21}$

39. Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, and 1 girl and 3 boys. One child is selected at random from each group. The probability that the three selected contain of 1 girl and 2 boys is:

(a) $\frac{13}{32}$ (b) $\frac{12}{32}$ (c) $\frac{15}{32}$ (d) $\frac{11}{32}$

40. The probabilities of A, B and C solving a problem are $\frac{1}{3}$, $\frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously. Find the probability that exactly one of them will solve it.

(a) $\frac{26}{63}$ (b) $\frac{21}{56}$ (c) $\frac{42}{63}$ (d) $\frac{28}{32}$

41. A bag contains 5 black and 7 red balls. A ball is taken out of the bag and is not returned to it. If this process is repeated three times, then what is the probability of drawing a black ball in the next draw of a ball?

(a) 0.7 (b) 0.625

(c) 0.3 (d) None of these

42. For question 41, what is the probability of drawing red ball?

(a) 0.375 (b) 0.6 (c) 0.3 (d) 0.70

43. One bag contains 5 white and 4 black balls. Another bag contains 7 white and 9 black balls. A ball is transferred from the first bag to the second and then a ball is drawn from the second bag. Find the probability that the ball drawn is white.

(a) $\frac{7}{18}$ (b) $\frac{5}{9}$

(c) $\frac{4}{9}$ (d) $\frac{11}{18}$

44. V Anand and Gary Kasparov plays a series of 3 chess games. The probability that V Anand wins a game is 2/7 and the probability of Kasparov winning a game is 3/5. There is no probability of a draw. The series will be won by the person who wins 3 matches. Find the probability that Anand wins the series. (The series ends the moment when any of the two wins 3 matches.)

(a) $\frac{992}{3125}$ (b) $\frac{273}{625}$

(c) $\frac{1023}{3125}$ (d) $\frac{1081}{3125}$

4. There are 10 pairs of socks in a cupboard from which 4 individual socks are picked at random. The probability that there is at least one pair is?

(a) $\frac{165}{323}$ (b) $\frac{90}{323}$

(c) $\frac{158}{323}$ (d) $\frac{165}{323}$

5. A fair coin is tossed 10 times. Find the probability that six Heads do not occur consecutively?

(a) $\frac{1}{2^4}$ (b) $\frac{1}{2^5}$

(c) $\frac{1}{2^6}$ (d) None of these

6. In a room there are 7 persons. The chance that two of them were born on the same day of the week is

(a) $\frac{1080}{7^7}$ (b) $\frac{2160}{7^6}$

(c) $\frac{540}{7^6}$ (d) None of these

7. In a hand at a game of bridge what is the chance that no 4 kings are held by a specified player?

(a) $\frac{40}{{}^{50}C_{12}}$ (b) $\frac{40}{{}^{52}C_{13}}$

(c) $\frac{20}{{}^{50}C_{12}}$ (d) $\frac{22}{{}^{52}C_{13}}$

8. One hundred identical coins each with probability P of showing up Heads are tossed once. If $0 < P < 1$ and the probability of Heads showing on 50 coins is equal to that of Heads showing on 51 coins, then value of P is

(a) $\frac{1}{21}$ (b) $\frac{49}{101}$

(c) $\frac{50}{101}$ (d) $\frac{51}{101}$

9. Two small squares on a chess board are chosen at random. Find the probability that they have a common side.

(a) $\frac{1}{32}$ (b) $\frac{1}{28}$

(c) $\frac{2}{31}$ (d) $\frac{3}{14}$

Hints and Solutions

1. The possible outcomes are:

$(1, 3), (1, 2), (2, 1), (2, 3), (3, 1), (3, 2)$

Out of six cases, in two cases there is exactly one '2'. Thus, the correct answer is $2/6 = 1/3$.

2. Event definition is A iff, B iff A iff and C iff A OR are two of the three hits.

3. The event can be defined as:
First bag is selected and red ball is drawn.

$1/2 \times 5/12 + 1/2 \times 3/15 = 5/24 + 1/30 = 17/120$

4. (a) For the least chance of drawing a red ball the distribution has to be 5 Red + 1 white in one bag and 7 white in the second bag. This gives us

$$\frac{1}{2} \times \frac{5}{10} + \frac{1}{2} \times 0 = \frac{5}{20}$$

(b) For the greatest chance of drawing a red ball the distribution has to be 1 Red in the first bag and 6 red + 12 white balls in the second bag. This gives us

$$\frac{1}{2} \times 1 + \frac{1}{2} \times \frac{6}{18} = \frac{4}{18} = \frac{5}{9}$$

5. One head and seven tails would have eight positions where the head can come.
Thus, $8 \times (1/2)^7 = (1/32)$

6. A leap year has 366 days – which means 52 completed weeks and 2 more days. The last two days can be (Monday, Monday) or (Monday, Tuesday) or ..... (Saturday, Sunday)
2 occur in out of 7 have a 53rd Sunday.
Thus, 2/7 is the required answer.

7. The count of the event will be given by
(The number of all 7 digit integers the number of all 7 digit integers divisible by 7).

8. Blue and Red or Red and Blue

$= (10/25 \times 1/15/24) + (15/25 \times 10/24) = (1/3)$

9. (i) $3/5 \times 3/4 = 7/10$
(ii) $1 - (7/10) = 3/10$

10. A point of six can be made in any of the following ways
$(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)$

11. The event definitions are:
    (a) Passes the first AND Passes the second AND Passes the third AND Passes the fourth
    (b) Fails the first AND Fails the second AND Fails the third AND Fails the fourth
    (c) Fails all is the non-event.
12. (i) First is cricket and second is cricket and third is cricket $\rightarrow (3/9)\times(2/9)\times(2/9) = (1/27)$
    (ii) $(2/9)\times(3/9)\times(4/9) = (8/243)$
    (iii) All are cricket or all are tennis or all are squash
    $(3/9)^3 + (2/9)^3 + (4/9)^3 = (99/729) = (11/81)$
13. (i) $(3/8)\times(2/8)\times(1/7) = 1/84$
    (ii) $(2/8)\times(3/8)\times(4/7) = (1/28)$
    (iii) $(3/8)\times(2/8)\times(1/7) + 6 + (4/8)\times(3/8)\times(2/7)$
    $30/56 = 5/14$
14. The event definitions are
    (i) The book transferred is by Stephen Covey
    (ii) The book transferred is by Stephen Covey AND The book picked up is by Stephen Covey
    (iii) The book transferred is by Stephen Covey AND The book picked up is by Vinay Singh.
15. $(2/5)\times(3/6) + (1/5)\times(2/7) = (1/2) + (2/35) = (25/42)$
16. (i) $(3/50)\times(2/49) = (3/1225)$
    (ii) $(3/50)\times(47/49) + 47/50\times 3/49 = (141/1225)$
    (iii) $(47/50)\times(46/49)\times(1081/1225)$
17. (i) $(3/5)\times(3/7) = (9/35)$
    (ii) $(3/5)\times(4/7) = (12/35)$
    (iii) $(2/5)\times(3/7) = (6/35)$
    (iv) $(3/5)\times(4/7) + (2/5)\times(3/7) + (3/5)\times(3/7) = 27/35$
18. They will contradict each other if: A is true and B is false or A is false and B is true.
    $(2/3)\times(1/4) + (1/3)\times(3/4) = 1/3$
19. For the counting of the number of events, think of it as a circular arrangement with n – 1 people (by considering the two specified persons as one). This will give you $n(E) = (n-2)!\times(2)!$
20. The whole number selected can only be 1, 3, 7 or 9 and cannot contain 2, 4, 6, 8, 0 or 5.
21. ${}^{7}C_4 = (6/36)^2\times(30/36)^2\times 6 = (1/36)\times(25/36) = 25/216$
22. The required probability will be given by the expression

    $$\frac{\text{The number of young boys who will die}}{\text{The total number of people who will die}}$$

23. Girl and Boy and Boy or Boy and Girl and Boy
    Or
    Boy and Boy and Girl
    $= (2/4)\times(2/3)\times(2/2) + (1/4)\times(2/3)\times 2/2$
    $+ (1/4)\times(2/3)\times(1/2) = (1/6) + (1/6) = 1/3$
24. $n(E) = 1$
    $n(S) = {}^{9}C_3 \times 8\times 6\times 4 = 243$
25. 11/17 (If the first one is black, there will be 11 black balls left out of 17)
26. 6/17
27. There must have been two sixes in the first five throws. Thus, the answer is given by:

    $${}^{5}C_2\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}$$

28. $$\frac{{}^{14}C_1\times{}^{12}C_1\times{}^{11}C_1\times{}^{10}C_1}{{}^{50}C_4}$$
29. There will be ${}^{n}C_3$ triangles formed overall. Out of these remember the number of equilateral triangles.
30. $(3/9)\times(4/8)\times(4/9)\times(3/8) = 12/72 = 4/9$
    As matched shoes means both red or both black.
31. The event definition here is that first is not a heart and second is not a heart and third is a heart $= (3/4)\times(3/4)\times(1/4) = 9/64$
32. $(9/45)\times(8/44)\times(1/39) = 2/(1/39)$
33. The event definition will be: Event X happens and Y doesn't happen Or, Y happens and X does not happen
34. X happens and Y does not happen or X does not happen and Y happens or X happens and Y happens
    $P = (1 - P_1)\times P_2 + P_1\times(1 - P_2) + P_1\times P_2 = P_1 + P_2 - P_1P_2$
35. Same logic as question 8.
36. $(2/6)\times(1/3)\times 3/6$ (Faulty and faulty)
37. Faulty and not Faulty and Faulty or Not Faulty and Faulty and Faulty $= (2/6)\times(2/3)\times(1/2) + (2/6)\times(1/3)\times(1/2) = 1/2$.
38. When you put the 7 balls with a gap between them, it is true, you will have 8 spaces.
39. The appearance of head or tail on a toss is independent of previous occurrences.
    Hence, 1/2.
40. $\frac{10\times 4!}{5!} = 1728$
41. $$p = \frac{\text{No. of arrangements with four to together}}{\text{Total No. of arrangements}}$$
    $$= \frac{[(8)!\times 2!]}{[(10)!\times 4!\times 2!]}$$
    $= 8!/(10\times 9\times 4!) = 24/90 = 4/15$.

12. $\frac{(4!+4!+2!+3!)}{11!} = \frac{24\times2\times6}{11\times10\times9\times8\times7\times6} = 1/1188$

13. $C(10,4)\times C(6,4)\times C(5,4)+C(10,4)\times C(6,3)\times C(4,4)+C(10,3)\times C(7,4)\times C(4,4)+C(10,3)\times C(7,3)\times C(4,3)+C(10,3)\times C(7,2)\times C(5,3)+C(10,2)\times C(8,3)\times C(5,3)$

$= 5040 = 14!$.

14. $^{12}C_4\times[(8/12)\times(7/11)\times(6/10)\times(4/9)\times(3/8)] = 1/495$.

15. There can be three girls and one boy.

16.

$$P = \frac{\text{Total no. of ways in which two people sit together}}{\text{Total No. of ways}}$$

$= (10!\times2!)/11!$

17. Consider the 6 girls to be one person. Then the number of arrangements satisfying the condition is given by $n(E) = 7!\times6!$

18. $^{12}C_4\times[(5/11)\times(6/10)\times(3/9)\times(4/8)\times(4/7)\times(2/6)]$

$= 5/11$.

19. The exam definition is Red AND Red AND Not Red OR Red AND Not Red AND Red OR Not Red AND Red AND Red.

20. The numbers having 2 in them are: 2, 12, 22, 32 ... 92 and 21, 23, 24, 25 ... 29. Hence, $n(E) = 19$

Hints and Solutions III

1. The problem has to be treated as if we are selecting the first card out of the 50 remaining cards.
13 of these are spades.
Hence, 1/50.

2.

	Chance that A will win	Chance that B will win
First throw	$\frac{1}{6}$	$\frac{5}{6}\times\frac{1}{6}$
Second throw	$\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$	$\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$
Third throw	$\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$	$\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$

The chance that A will win is an infinite GP with $a = \frac{1}{6}$

and $r = \frac{25}{36}$

Similarly, the chance that B will win is an infinite GP with $a = \frac{5}{36}$ and $r = \frac{25}{36}$

3. A total of 6 can be obtained by either (1 + 2 + 3) or by (2 + 2 + 2)

4. A is true and B is true = (3/10)×(2/10) = (2/40)

5. The required probability will be given by:

$$\frac{\text{Percentage of defective items C}}{\text{Percentage of defective items A, B and C}}$$

6. and 7.
We do not have to consider any sum other than 5 or 7 occurring.
A sum of 5 can be obtained by any of (4 + 1, 3 + 2, 2 + 3, 1 + 4)
Similarly a sum of 7 can be obtained by any of (6 + 1, 5 + 2, 4 + 3, 3 + 4, 2 + 5, 1 + 6)
For 6: $n(E) = 4$, $n(S) = 6\times4$
$P = 1/4$
For 7: $n(E) = 6$
$n(S) = 6\times4$ $P = 0.6$

8. The number of ways for a sum of 4 = 5 i.e. (3 + 1, 2 + 2, 1 + 3)

9. Try to find the number of ways in which 0 or 1 bomb hits the bridge if n bombs are thrown.
The required value of the number of bombs will be such that the probability of 0 or 1 bomb hitting the bridge should be less than 0.1.

10. The required probability will be given by: $1 - {}^nC_0 + {}^nC_1\left(\frac{1}{2}\right)^n$ for $n = 5$

11. The required answer will be given by:

P(Kaun does not receive the letter) + P(Kaun replied and Sattar did not receive the reply)

12. $n(E) = {}^5C_2 = 10 = 15 + 84 + 13 + 12$
$n(S) = 16^2$

13. The number of events that the condition that he will sing = 4 (34, 45, 36, 60)
The number of events in the sample = 60
Probability that he will sing at least once = 1 − Probability that he will not sing.

14. The number of ways in which 6 tickets can be selected from 10 tickets is 10^6.
The number of ways in which the selection can be done so that the condition is satisfied = $10^6 - 8^6$.

15. In the first draw we have 7 even tickets out of 15 and in the second we have 8 odd tickets out of 15.

Thus, $(2/3)\times(8/13) = 56/273$

16. The total number of ways in which the 6 identical balls can be distributed amongst the three boxes such that each box can get 0, 1, 2, 3, 4, 5 or 6 balls is given by the formula, $^{n+r-1}C_r$ where n is the number of identical balls and r is the number of boxes. This will give the sample space.

19. Boxes definition:
Any six of them work AND four leave OR any seven work AND three leave OR any eight work AND two leave OR any nine work AND one leaves OR All ten work.

20. "To make at least 3 wins to draw a boxer" implies that he didn't get a loser in the first two rounds.

21. A box containing 4 defectives would get sent to the market if all the 4 articles selected are not defective.
Thus, $^4C_4/^{20}C_4 = 364/969$

22. If the box contains only 1 defective, it would be sent back if the defective is one amongst the 4 selected for testing.
To ensure one of the 4 is selected, the no. of ways is $^1C_1$ while the total no. of selections of 4 out of 20 is $^{20}C_4$.
Thus, $^1C_1/^{20}C_4 = 1/5$.

23. For divisibility by 5 we need the units digit to be either 0 or 5.
The units digit in the powers of 7 follow the pattern – 7, 9, 3, 1, 7, 9, 3, 1, 7, 9 ....
Hence, divide 1 to 100 into four groups of 25 elements each as follows.
$A = 1, 5, 9, \ldots \longrightarrow 25$ elements
$B = 2, 6, 10, \ldots \longrightarrow 25$ elements
$C = 3, 7, 11, \ldots \longrightarrow 25$ elements
$D = 4, 8, 12, \ldots \longrightarrow 25$ elements
Check the combination values of m and n so that $7^m + 7^n$ is divisible by 5.

26. Minimum (or Maximum?)
Minimum 3: $= {}^{15}C_3/^{20}C_3 = 21/120$
Minimum 7: $= {}^{15}C_7/^{20}C_7 = 15/120$
Note: The logic for this can be explained for minimum 3 condition as, Since the minimum value has to be 3, the remaining 2 numbers have to be selected from 4 to 20. This can be done in $^{17}C_2$ ways.

25. For the event to occur, the dice should show values from 2, 3, 4 or 5. This is similar to selection with repetition.

26. Yellow and Red and Blue $= (3/6)\times(2/6)\times(1/6) = (6/216)$
$= 1/36$.

27. Two white and one black can be obtained only through the following three sequences:
Ball drawn from A and B are white and the ball drawn from C is Black. or
Ball drawn from A and C are white and the ball drawn from B is Black. or
Ball drawn from B and C are white and the ball drawn from A is Black.

28. At least one means (exactly one + exactly two + exactly three)
At least two means (exactly two + exactly three)
The problem gives the probabilities for passing in at least one, at least two and exactly two.

29. All four are not in the correct envelopes means that at least one of them is in a wrong envelope. A little consideration will show that one letter being placed in a wrong envelope is not possible, since it will have to be interchanged with some other letter.
Since, there is only one way to put all the letters in the correct envelopes, we can say that the event of not all four letters going into the correct envelopes will be given by

$$5! - 1 = 119$$

30. $n(E) = 48$
$n(S) = 120$

31. $A_1 = (1,12 \text{ or } 12,1)$
$A_2 = (1,12 \text{ or } 2,11)$
$A_3 = (1,2) \text{ or } 2,1)$
Option (b) is correct.

32.

From the venn diagram we get:
$(2/3) + (5/9) - x = 4/5 \rightarrow x = (2/3) + (5/9) - (4/5) = 19/45$

33.

Also $P(A \cup B) = 1 - P(A' \cap B')$
$(3/7) + (1/2) - x = 13/14 \rightarrow x = 0$

Thus, there is no intersection between A and B as $P(A \cap B) = 0 \neq 0$. Hence, A and B are mutually exclusive.

6. The required probability will be given by $\frac{2 \times 6! \times 6!}{12!}$.

7. The sure event in this case is that the problem is not solved.

8. Similar to Question No. 2 of LOD 3.

9. The event definition will be:
The first bag is selected AND a red ball is selected. OR
The second bag is selected AND a red ball is selected.

10. The event definition is:
A girl is selected from the first group and one boy each is selected from the second and third groups. OR A girl is selected from the second group and one boy each are selected from the first and third groups. OR A girl is selected from the third group and one boy each is selected from the first and second groups.

11. The event definition will be:
A solves the problem and B and C do not solve the problem. OR B solves the problem and A and C do not solve the problem. OR C solves the problem and A and B do not solve the problem.

12. The three balls that are taken out can be either 3 black balls or 2 black and 1 red ball or 1 black and 2 red ball or 3 red balls.
Each of these will give their own probabilities of drawing a black ball.

13. 1 Black and 4* is Red or 2 Blacks and 1 Red and 4* is Red or 1 Black and 2 Reds and 4* is Red

$$= (5/8) \times (4/7) \times (3/6) \times (3/5) + {}^3C_1 \times (5/8) \times (4/7) \times (3/6) \times (2/5) + {}^3C_2 \times (5/8) \times (3/7) \times (2/6) \times (1/5)$$

$$= \frac{180 + 360 + 90}{1680} = 630/1680 = 3/8 = 0.375$$

14. The event definition would be: Ball transformed is white and white ball drawn

Or

Ball transformed is black and white ball is drawn

The answer will be given by:

$$(5/9) \times (8/17) + (4/9) \times (7/17) = 68/153 = 4/9$$

15. Solve this on a similar pattern to the example given in the theory of this chapter.

16. Required probability $= 1 -$ probability that no gain is achieved.

17. We can have a maximum of 8 heads.
For 0 heads $\rightarrow P(E) = 1/(2^8) = 1$
For 1 heads $\rightarrow P(E) = 1/(2^8) = 1$

For 2 heads and the three not to occur consecutively we will need to see the possible distribution of 6 tails and 2 heads.

Since the 2 heads do not need to occur consecutively, this would be given by (All – 2heads together) $\rightarrow {}^{(8)}C_2 - 6$

$$P(E) = \frac{{}^{(8)}C_2 - 6}{2^8}$$

Solving in this fashion, we would get 1/2[8].

18. P of heads showing on 50 coins $= {}^{100}C_{50} \times P^{50}(1 - P)^{50}$
P of heads showing on 51 coins $= {}^{100}C_{51} \times P^{51}(1 - P)^{49}$
Both are equal.

$${}^{100}C_{50} \times P^{50}(1 - P)^{50} = {}^{100}C_{51} \times P^{51}(1 - P)^{49}$$

$$\text{or } \frac{100 \times 99 \times \ldots \times 52 \times 51}{1 \times 2 \times \ldots \times 49 \times 50} \times (1 - P)$$

$$= \frac{100 \times 99 \times \ldots \times 53 \times 52}{1 \times 2 \times \ldots \times 48 \times 49} \times P$$

$$\text{or } \frac{51}{50} \times (1 - P) = P$$

$$\text{or } 51 \times (1 - P) = 50P$$

$$\text{or } 51 - 51P = 50P$$

$$\text{or } 51 = 101P$$

$$\therefore P = \frac{51}{101}$$

19. The common side could be horizontal or vertical. Accordingly, the number of ways the event can occur is

$$n(E) = 8 \times 7 + 8 \times 7 = 112$$

$$n(S) = 6!C_2$$

$$= \frac{2 \times 6 \times 5 \times 2}{6! \times 1!} = \frac{1}{18}$$

Detailed Solutions & Short Cuts

LOD 1

1. Out of a total of 6 in a dice, 3 is our possibility $\rightarrow 1/6$.

2. 3 or 6 out of a sample space of

$$1, 2, 3, 4, 5 \text{ or } 6 = 3/6 = 1/2$$

3. Possible events is
(1 and 1) or (1 and 2) or (1 and 3) or (1 and 4) or (1 and 5) or (1 and 6) or (1 and

5 or (1 and 4) or (2 and 3) or (3 and 2) or (4 and 1) or (5 and 1) or (6 and 1)

Total 11 out of 36 possibilities = 11/36

4. Event definition: First is blue and second is blue = 3/12 × 3/11 = 3/22.

5. Knave and queen or Queen and Knave = 4/52 × 4/51 + 4/52 × 4/51 = 8/663

6. (i) 13/52 = 1/4

(ii) 4 kings and 4 queens out of 52 cards.

Thus, 8/52 = 2/13.

(iii) 13 spades + 3 kings → 16/52

7. (i) Event definition is: T and T and H or T and H and T or H and T and T = 3 × 5/8 = 3/8.

(ii) Same as above = 3/8.

8. (i) Probability of 3 heads = 1/8

Also, Probability of 3 tails = 1/8.

Required probability = 1 − (1/8 + 1/8) = 6/8 = 3/4.

(ii) H and H and H = 1/8.

9. At least one tail is the non-event for all heads.

Thus, P (at least 1 tail) = 1 − P(all heads)

= 1 − 1/8 = 7/8.

10. (i) Positive outcomes are 2(1 way), 4(3 ways), 6(5 ways) 8(5 ways), 10(3 ways), 12 (1 way).

Thus, 18/36 = 1/2

(ii) Positive outcomes are: 4, 8 and 12

4 (3 ways), 8(5 ways) and 12(1 way)

Gives us 9/36 = 1/4.

(iii) Positive outcomes are 2(1 way), 3(2 ways), 5(4 ways), 7(6 ways). Total of 13 positive outcomes out of 36.

Thus, 13/36.

11. (i) First is white and second is white 7/10 × 6/9 = 7/15.

(ii) White and Green or Green & White

3/10 × 7/9 + 3/10 × 7/9

42/90 = 7/15.

12.

From the figure it is evident that 40 students passed at least 1 exam. Thus, 20 failed both and the required probability is 20/100 = 1/5.

13. 3 or 4 or 5 or 6 = 4/6 = 2/3

17. Odds against an event = $\frac{p(E^c)}{p(E)}$

In this case, the event is: All black, i.e., First is black & second is black & third is black.

P(E) = 5/9 × 4/8 × 3/7 = 60/504 = 5/42.

Odds against the event = 37/5.

18. Event definition is: 15 or 16 or 17 or 18.

15 can be got as: 5 and 5 and 5 (one way)

Or

6 and 5 and 4 (6 ways)

Or

6 and 6 and 3 (3 ways)

Total 10 ways.

16 can be got as: 6 and 6 and 4 (3 ways)

Or

6 and 5 and 5 (3 ways)

Total 6 ways.

17 has 3 ways and 18 has 1 way of appearing.

Thus, the required probability is: (10 + 6 + 3 + 1)/216 = 20/216 = 5/54.

19. (i) Positive outcomes = 2 (187 or 215).

Total outcomes = 9 × 8 × 7

Required probability = 2/504 = 1/252

(ii) $\frac{2}{9 \times 9 \times 9} = 2/729.$

20. 1/6 + 1/10 + 1/8 = 47/120

23. Expectation = Probability of winning × Reward of winning = (10/5000) × 1 crore = 1 crore/500 = 20000

24. $1/^{6}C_{2} = 1/6$

25.

(i) 800/1200 = 11/18

(ii) 600/1200 = 13/24

26. 180/300 = 11/40

27. 190/880 = 3/8

28. Black and Black and Black = 4/9 × 3/8 × 2/7

= 24/504 = 1/21

29. $1/^{5}P_{2} = 1/20.$

31. Positive Outcomes are: 5, 7, 10, 14, 15 or 20

Thus, 6/30 = 3/10.

32. 97/1972 = 245/497.

9. $P(\text{Both are selected}) = P(A) \times P(B)$

Since $P(B) = 0.5$, we get

$$0.3 \times 0.5 = 0.6$$

The maximum value of $P(B) = 0.6$.

Thus $P(B) = 0.9$ is not possible.

11.

$(19/40) \quad 1/8 \quad 5/24$

Within $(19/40) + 1/8 = 5/24 = 37/120$

12. $P(\text{Red}) = 1/3$

$P(\text{Black}) = 2/7$

$P(\text{Pink}) = 1/8$.

Required Probability $= 1/3 \times 2/7 \times 1/8 = 12/168$

42. $P(E_1) = 3/8$

$P(E_2) = 7/12$

Event definition is: E_1 occurs and E_2 does not occur or E_1 occurs and E_2 occurs or E_1 occurs and E_2 does not occur.

$(3/8) \times (5/12) + (3/8) \times (7/12) + (5/8) \times (7/12) = 71/96$.

43. Kamal is selected and Monica is not selected or Kamal is not selected and Monica is selected $\rightarrow (1/5) \times (4/5) + (2/3) \times (1/5) = (6/15) = (2/5)$

44. (i) $1/5 \times 1/7 = 1/35$

(ii) $(1/5) \times (6/7) + (4/5) \times (1/7) = 10/35 = 2/7$.

(iii) $(4/5) \times (6/7) = 24/35$

(iv) Both selected or 1 selected $= 10/35 + 2/7 = 11/35$

ANSWER KEY

Probability LOD II

1. b
2. a
3. d
4. a
5. a
6(i) c
6(ii) b
6(iii) c
7(i) b
7(ii) a
8(i) d
8(ii) a
9. b
10(i) a
10(ii) b
10(iii) d
11(i) d
11(ii) a
12. a
13. a
14(i) b
14(ii) a
14(iii) b
15(i) d
15(ii) a
15(iii) b
15(iv) a
15(v) b
15(vi) b
17. b
18. a
19(i) c
19(ii) a
20. a
21. b
22(i) a
22(ii) d
22(iii) a
23. a
24. d
25(i) a
25(ii) c
26. b
27. d
28. c
29. b
30. a
31. a
32. d
33. b
34(i) b
34(ii) c
35. d
36. a
37. a
38. d
39. c
40. b
41(i) c
41(ii) d
41(iii) a
42. c
43. a
44(i) a
44(ii) c
44(iii) a
44(iv) b

Probability 1.00 (1)

1. d
2. a
3. a
4(a) b
4(b) c
5. d
6. a
7. d
8. a
9(a) a
9(b) b
10. d
11(a) a
11(b) b
11(c) b
12(a) a
12(b) a
12(c) a
13(a) b
13(b) c
13(c) c
14(a) b
14(b) c
14(c) b
15. a
16(a) b
16(b) a
16(c) b
17(a) c
17(b) a
18. b
19. b
20. c
21. b
22. b
23. a
24. c
25. a
26. b
27. b
28. a
29. a
30. b
31. a
32. c
33. a
34. b
35. a
36. b
37. c
38. a
39. b
40. b
41. a
42. d
43. a
44. b
45. d
46. a
47. b
48. a
49. c
50. a

Probability (Ch 9) (II)

1. a
2. b
3. b
4. c
5. a
6. b
7. a
8. a
9. a
10. b
11. a
12. a
13. a
14. b
15. a
16. b
17. a
18. c
19. a
20. a
21. a
22. b
23. a
24. b
25. a
26. d
27. a
28. b
29. a
30. b
31. b
32. a
33. b
34. b
35. b
36. d
37. a
38. d
39. b
40. b
41. a
42. a
43. c
44. a
45. b
46. b
47. b
48. a
49. d
50. b

BLOCK REVIEW TESTS

REVIEW TEST ONE

1. 16 guests have to be seated, half on each side of a long table. 4 particular guests desire to sit on one particular side and 3 others on the other side. Determine the number of ways in which the seating arrangements can be made

(a) ${}^{16}P_8 \times (8!)^2$ (b) ${}^{16}C_8 \times (8!)^2$
(c) ${}^{16}P_8 \times (8!)^2$ (d) None of these

2. If n parallel lines in a plane are intersected by a family of p parallel lines, find the number of parallelograms that can be formed.

(a) $n^2 \times p^2$ (b) $n^{n-1} \times p^{p-1}$
(c) ${}^nC_2 \times {}^pC_2$ (d) None of these

3. A father with eight children takes them at a time to the zoological garden, as often as he can without taking the same three children together more than once. How often will he go and how often will each child go?

(a) ${}^8C_3$, ${}^7C_2$ (b) ${}^8C_3$, ${}^7C_3$
(c) ${}^8P_3$, ${}^7C_2$ (d) ${}^8P_3$, ${}^7C_3$

4. A candidate is required to answer 7 questions out of 12 questions which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. In how many different ways can he choose the 7 questions?

(a) 360 (b) 520
(c) 700 (d) None of these

5. Find the sum of all 5 digit numbers formed by the digits 1, 3, 5, 7, 9 when no digit is being repeated.

(a) 4444400 (b) 8888800
(c) 1111100 (d) 6666600

6. Consider a polygon of n sides. Find the number of triangles, none of whose sides is the side of the polygon.

(a) ${}^nC_3 - 2n = n \times (n-4)/6$
(b) $n(n-4)(n-5)/3$
(c) $n(n-4)(n-5)/6$
(d) $n(n-1)(n-2)/3$

7. The number of 4 digit numbers that can be formed using the digits 0, 2, 3, 5 without repetition is

(a) 18 (b) 20 (c) 24 (d) 30

8. Find the total number of words that can be made by using all the letters from the word MACHINE, using each only once.

(a) 7! (b) 3020 (c) 6080 (d) 5!/2

9. What is the total number of words that can be made by using all the letters of the word REKHA, using each letter only once?

(a) 240 (b) 4! (c) 124 (d) 5!

10. How many different 5 digit numbers can be made from the first 5 natural numbers, using each digit only once?

(a) 240 (b) 4! (c) 124 (d) 5!

11. There are 7 seats in a row. Three persons take seats at random. What is the probability that the middle seat is always occupied and no two persons are sitting on consecutive seats?

(a) 3/70 (b) 14/35 (c) 8/70 (d) 4/35

12. Let $N = 3^x$, where x is any natural no. What is the probability that the unit digit of N is 3?

(a) 1/4 (b) 1/3 (c) 1/5 (d) 1/2

13. Find the probability of drawing one ace in a single draw of one card out of 52 cards.

(a) $52/52 = 1$ (b) 1/4
(c) 1/52 (d) 1/13

14. In how many ways can a committee of 4 persons be made from a group of 10 people?

(a) 10!/4! (b) 210
(c) 10!/6! (d) None of these

15. In Question 14, what is the number of ways of forming the committee, if a particular member must be there in the committee?

(a) 12 (b) 84
(c) 9!/3! (d) None of these

16. A polygon has 54 diagonals. The number of sides of this polygon are

(a) 12 (b) 84
(c) 3.3! (d) 4.4!

17. An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it, the probabilities of hitting the plane at first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane?

(a) 0.9964 (b) 0.9976
(c) 0.8024 (d) 0.2548

18. 7 white balls and 3 black balls are placed in a row at random. Find the probability that no two black balls are adjacent.

(a) 2/15 (b) 7/15 (c) 8/15 (d) 4/15

19. The probability that A can solve a problem is 5/16 and that B can solve it 1/3. If both of them attempt to solve the problem, what is the probability that the problem can be solved?
(a) 1/5 (b) 1/4 (c) 1/3 (d) 4/7

20. The sides AB, BC, CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. Find the number of triangles that can be constructed using these points as vertices.
(a) 180 (b) 105 (c) 205 (d) 240

21. From 8 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done if there is no restriction in its formation?
(a) 256 (b) 246 (c) 252 (d) 260

22. From 4 officers and 8 jawans in how many ways can 6 be chosen to include exactly one officer?
(a) $^{12}C_6$ (b) 1296 (c) 1344 (d) 224

23. From 4 officers and 8 jawans in how many ways can 6 be chosen to include atleast one officer?
(a) 988 (b) 924
(c) 896 (d) none of these

24. Two cards are drawn one after another from a pack of 52 ordinary cards. Find the probability that the first card is an ace and the second drawn is an honour card if the first card is not replaced while drawing the second.
(a) 23/52 (b) 12/51
(c) 19/61 (d) None of these

25. The probability that Andrews will be alive 15 years from now is 3/5 and that Bill will be alive 15 years from now is 7/10. What is the probability that both Andrews and Bill will be dead 15 years from now?
(a) 33/150 (b) 24/150
(c) 49/150 (d) 34/150

REVIEW TEST TWO

1. A group consists of 100 people; 25 of them are women and 75 men. 20 of them are rich and the remaining poor. 40 of them are employed. The probability of selecting an employed rich woman is
(a) 0.05 (b) 0.04 (c) 0.02 (d) 0.08

2. Out of 17 job applicants, there are 5 boys and 8 men. It is decided to choose 2 applicants for the job. The probability that at least one of the selected applicants will be a boy is
(a) 5/13 (b) 14/39 (c) 25/39 (d) 10/13

3. Four dogs and three pups stand in a queue. The probability that they will stand in alternate positions is
(a) 1/34 (b) 1/35 (c) 1/17 (d) 1/68

4. Asha and Vidya play a number game where each is asked to select a number from 1 to 5. If the two numbers match, both of them win a prize. The probability that they will not win a prize on a single trial is
(a) 1/25 (b) 24/25
(c) 2/25 (d) None of these

5. The number of ways in which 6 British and 5 French are to be seated at a round table if no two French are to sit together is given by
(a) $6! \times 5!$ (b) $5! \times 4!$ (c) 30 (d) $7! \times 5!$

6. A cricket team of 11 players is to be formed from 20 players including 6 bowlers and 3 wicketkeepers. Find the number of ways in which a team can be formed having exactly 4 bowlers and 2 wicketkeepers.
(a) 20700 (b) 6930 (c) 3070 (d) 30

7. Three boys and three girls are to be seated around a circular table. Among them one particular boy Rohit does not want any girl neighbour and one particular girl Shreya does not want any boy neighbour. How many such arrangements are possible?
(a) 5 (b) 6 (c) 4 (d) 2

8. Words with five letters are formed from ten different letters of an alphabet. Then the number of words which have at least one letter repeated is
(a) 39670 (b) 30750
(c) 69760 (d) 99548

9. Sunil and Kapil toss a coin alternatively till one of them gets a head and wins the game. If Sunil starts the game, the probability that he (Sunil) will win is:
(a) 0.66 (b) 1
(c) 0.33 (d) None of these

10. The number of parallelograms that can be formed if 7 parallel horizontal lines intersect 6 parallel vertical lines is
(a) 42 (b) 294
(c) 315 (d) None of these

11. $1.3.5 \ldots (2n-1)/2.4.6 \ldots (2n)$ is equal to:
(a) $(2n)!/2^n(n!)^2$ (b) $(2n)!/n!$
(c) $(2n-1)!/(n-1)!$ (d) 2^n

12. How many four-digit numbers, each divisible by 4, can be formed using the digits 1, 2, 3, 4 and 5 (repetition allowed)?
(a) 300 (b) 180 (c) 125 (d) 75
(e) 36

14. A student is to answer 10 out of 13 questions in a test such that he/she must choose at least 4 from the first five questions. The number of choices available to him is

(a) 140 (b) 280 (c) 196 (d) 346

15. The number of ways in which a committee of 7 ladies and 4 gentlemen can be appointed from a meeting consisting of 8 ladies and 7 gentlemen, if Mrs. Pushkar refuses to serve in a committee if Mr. Modi is its member, is

(a) 1960 (b) 7240

(c) 2540 (d) None of these

16. A room has 5 lamps. From a collection of 10 light bulbs of which 6 are not good, a person selects 3 at random and puts them in a socket. The probability that he will have light is

(a) 5/6 (b) 1/2

(c) 1/6 (d) None of these

17. Two different series of a question booklet for an aptitude test are to be given to twelve students. In how many ways can the students be placed in two rows of six each so that there should be no identical series side by side and that the students sitting one behind the other should have the same series?

(a) $2 \times {}^6C_6 \times (6!)^2$ (b) $6! \times 6!$

(c) $7! \times 7!$ (d) None of these

18. The letters of the word PROMISE are arranged so that no two of the vowels should come together. The total number of arrangements is

(a) 48 (b) 1440 (c) 7 (d) 1008

19. Find the remainder left after dividing $1! + 2! + 3! + \ldots + 1000!$ by 7

(a) 0 (b) 5 (c) 21 (d) 34

20. In the McGraw Hill Mindworkers mock test paper, there are two sections, each containing 4 questions. A candidate is required to attempt 5 questions but not more than 3 questions from any section. In how many ways can 5 questions be selected?

(a) 24 (b) 48 (c) 32 (d) 96

21. A bag contains 10 balls out of which 2 are pink and rest are orange. In how many ways can a random sample of 6 balls be drawn from the bag so that at the most 2 pink balls are included in the sample and no sample has all the 6 balls of the same colour?

(a) 100 (b) 108 (c) 200 (d) 120

REVIEW TEST THREE

1. There are one grandfather, 3 sons and daughters and 4 grandchildren in a family. They are to be seated in a row for dinner. The grandchildren want to occupy the 4 seats at each end and the grandfather refuses to have a grandchild on either side of him. Find the number of ways in which the family can be made to sit.

(a) 11760 (b) 11520

(c) 21840 (d) None of these

2. A bag of 200 electric switches contains 10 defective switches. One switch is taken out at random from the bag. Find the probability that the switch drawn is (i) defective (ii) non-defective?

(a) (i) 2/25 (ii) 23/25 (b) (i) 4/25 (ii) 21/25

(c) (i) 3/25 (ii) 22/25 (d) (i) 1/25 (ii) 24/25

3. In a group photograph of the Patna Women's College, all the seven teachers should sit in the first row and all the twenty students should sit in the second row. The two corners of the second row are reserved for the two tallest students, interchangeably only between them and if the middle seat of the front row is reserved for the principal. Find the number of possible arrangements

(a) $7!20! \times 18!$ (b) $14!20! \times 18!$

(c) $13!20! \times 18!$ (d) None of these

4. In a certain town, all telephone numbers have six digits, the first two digits always being 53 or 54 or 56 or 82 or 84. The number of telephone numbers having all the six digits distinct is

(a) 840 (b) 920

(c) 720 (d) None of these

5. Three groups X, Y and Z are contesting for a position on the Board of Directors of a company. The probabilities of their winning are 0.5, 0.3 and 0.2 respectively. If the group X wins, then the probability of introducing a new product is 0.7 and the corresponding probabilities for group Y and Z are 0.6 and 0.5 respectively. The probability that the new product will be introduced is

(a) 0.52 (b) 0.74

(c) 0.61 (d) None of these

6. A computer hardware firm manufactures two parts of the computer—CPU and Monitor. In the process of manufacture of the CPU, 9 out of 100 are likely to be defective.

Similarly 3 out of 100 are likely to be defective in the process of manufacture of bicycles. The probability that the selected pair will not be defective is:

(a) 0.9605 (b) 0.9645
(c) 0.9245 (d) None of these

7. There are 8 applicants for the post of the Indian cricket captain and one of them is to be selected by the votes of the 5 wise men (also called as the selectors). The number of ways in which the votes can be given is:

(a) 1296 (b) 1792
(c) 1024 (d) None of these

8. In a Football Tournament, there were 171 matches played. Every two teams played one match with each other. The number of teams participating in the Tournament is:

(a) 18 (b) 19 (c) 17 (d) 16

9. Seven points lie on a circle. How many chords can be drawn by joining these points?

(a) 22 (b) 21 (c) 20 (d) 24

10. Ten different letters of an alphabet are given. Words with 5 letters are formed from these given letters. Find the number of words which have at least one letter repeated.

(a) 99750 (b) 30240
(c) 30750 (d) None of these

11. There are eight chairs marked A to H. Two girls and three boys wish to occupy one chair each. First, the girls choose the chairs from amongst the chairs marked A to D, then the boys selected the chairs from amongst the remaining, marked E to H. The number of possible arrangements is:

(a) $^6C_2 \times 4C_3$ (b) $^4P_2 \times {}^4P_3$
(c) $^4C_2 \times {}^4P_3$ (d) $^4C_2 \times {}^4C_3$

12. Professor Hatter of poker wants to play poker every night for the maximum number of nights possible. The key constraint for him is that he has only 8 friends who are willing to play with him and he wants to hold poker games with distinct groups of friends. His mother has put the condition that in case the same group is repeated on any day, she would stop his poker games and make him study—something that he should be doing anyway. Being a mathematically oriented person, Professor Hatter takes the maximum number of nights on which he can play poker without violating his mother's condition. What is the value that he has calculated?

(a) 70 (b) 105 (c) 140 (d) 131

13. A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways this can be done is:

(a) 216 (b) 240 (c) 3125 (d) 600

14. There are 6 equi-distant points A, B, C, D, E and F marked on a circle with radius M. How many convex polygons of distinctly different areas can be drawn using these points as vertices?

(a) $^6P_6$ (b) 1
(c) 55 (d) None of these

15. Five men A, B, C, D and E occupy seats in a row such that C and D are next to each other. In how many possible ways can these five men sit?

(a) 24 (b) 48
(c) 72 (d) None of these

16. The number of zero at the end of 60! is

(a) 12 (b) 14 (c) 16 (d) 18

17. A department had 8 male and female employees each. A project team involving 3 male and 3 female members needs to be selected from the department employees. How many different project teams can be formed?

(a) 112896 (b) 5136
(c) 720 (d) 112

18. From a group of 7 men and 6 women 5 persons are to be chosen to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

(a) 756 (b) 735
(c) 561 (d) None of these

19. The number of different ways that the letters of the word STALLION can be arranged so that the vowels always come together, is:

(a) 2160 (b) 720 (c) 4320 (d) 600

20. If a polygon has 54 diagonals, then find the number of its sides.

(a) 13 (b) 12 (c) 11 (d) 10

ANSWER KEY

Review Test Five

1. b
2. c
3. b
4. a
5. d
6. a
7. c
8. a
9. d
10. d
11. d
12. c
13. d
14. b
15. b
16. c
17. b
18. b
19. c
20. c
21. d
22. c
23. d
24. b

Review Test Two

1. a
2. c
3. b
4. b
5. b
6. a
7. a
8. c
9. c
10. a
11. a
12. c
13. a
14. d
15. d
16. b
17. b
18. b
19. b
20. b

Answer Sheet Three

Personal Details

1. d
2. a
3. b
4. a
5. c
6. a
7. d
8. a
9. b
10. b
11. a
12. a
13. a
14. b
15. b
16. b
17. b
18. a
19. a
20. b

BLOCK

BLOCK 7

MISCELLANEOUS CHAPTERS

- Set Theory
- Coordinate Geometry

18

SET THEORY

SET THEORY

A set is defined as a collection of objects united by a common rule. It can be also defined as 'a well-defined collection of objects', which can be interpreted as a rule on which all the members of the set are associated with.

In the CAT, Set Theory is significant from the angle of that questions asked on set theory. Hence, understanding the basic ground rules of sets and their operations is integral to the very important chapters of Percentages, Probability and Permutations and Combinations. Besides, a lot of puzzle-based questions can be formed on the basis of the concept of sets. These have been placed in either the QA or DI sections of the CAT.

For other examinations like the XLRI, CET, FMS, Bank PO (especially SBI PO) and so on, the chapter is again significant because of the reasons elucidated above.

Different Ways of Representing a Set

1. Roster Method A set is represented by all the elements within under braces separated by commas.

Example: $A = \{1, 2, 3, 4, 5\}$

2. Set Builder Method A set is represented by a characteristic property of all the elements of the set and written as

$A = \{x \mid x, C(x) \text{ holds}\}$

$A = \{x : x, C(x) \text{ holds}\}$, where $C(x)$ is the property with which we can associate all the elements of set A.

Example: $A = \{x \in N \mid x < 7\}$, which can be written with Roster method as $A = \{1, 2, 3, 4, 5, 6\}$

Types of Set

1. Empty or Null Set A set having no element and it is denoted as ϕ or $\{\ \}$.

Example: A set of even prime numbers excluding 2.

2. Singleton Set A set having only one element.

Example: A = Set of even prime numbers.

3. Finite Set A set whose elements can be counted and get terminated at some point, say 'n'.

Cardinal number of a finite set: In the above explanation, 'n' is known as cardinal number of that set and is denoted by $n(A)$.

Example: A = Set of positive odd numbers below 100.

4. Infinite Set A set whose elements cannot be counted.

Example: A = Set of all irrational numbers between 1 and 2.

or a set of all odd numbers below hundred.

5. Equal Sets Two sets are said to be equal sets if all the elements of set A are included in set B and all the elements of set B are included in set A.

If two sets A and B are equal then it is represented by $A = B$ and if A and B are not equal then it is written as

$A \neq B$, that is, all the elements of set A are not included in set B and all the elements of set B are not included in set A.

Example: $A = \{a, b, c\}$ and $B = \{c, b, a\}$ are equal sets.

Hence, in this case we can write Set A = Set B or simply $A = B$.

6. Subsets Set A is said to be the subset of another set B if all the elements of set A are included in set B.

Set A is a subset of set B is shown by $A \subseteq B$.

We can see now that every element of set A is a member of set B.

Example: If $A = \{a, b, c\}$ and $B = \{a, b, c, d, e\}$ then $A \subseteq B$ or A is a subset of B.

Some important results on subsets

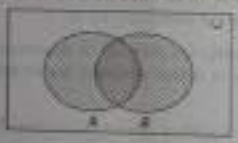
1. Every set is a subset of itself.
2. Every set has an empty set as its subset.
3. Total number of subsets of a set having n elements is 2^n.

7. Universal Set A set which contains all the sets in a given context is a Universal set.

Example: When we are using sets containing natural numbers, N is the universal set.

If $A = \{a, b, c\}$, $B = \{b, c, d\}$, $C = \{c, d\}$

Then we can take $U = \{a, b, c, d\}$ as universal set.



8. Power Set The collection of all the subsets of a set is known as the power set of that set.

If A is the set, then a set containing all the subsets of the A is known as the Power set of A. It is denoted by $P(A)$.

Let $A = \{1, 2\}$, then the number of subsets of this set will be 2^2 and the subsets are $\{1, 1\}$, $\{2\}$ and $\{1, 2\}$ and the set containing all these four sets is known as Power set represented as $P(A)$.

9. Venn Diagrams Swiss mathematician Euler first gave the idea of representing sets by diagrams. Later on, British mathematician Venn brought this into practice. So it is known as Euler-Venn diagram or simply Venn diagram. In this way of representing sets, we use a closed curve, generally a circle, to denote sets and their operations.

Operations on Sets

1. Union of Sets If two sets are A and B, then union of A and B is defined as the set that have all the elements which belong to either A or B or both.

It is represented by $A \cup B$.

2. Intersection of Sets If two sets are A and B, then intersection of A and B is defined as the set that have all the elements which belong to both A and B.

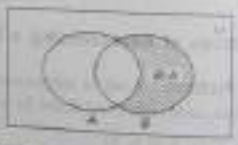
It is represented by $A \cap B$.

Example: Find $A \cup B$ and $A \cap B$ if $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$
$A \cap B = \{2, 4\}$

3. Disjoint Sets Two sets are said to be disjoint if $A \cap B = \emptyset$, that is, not a single element is common to both of these two sets.

Example: If A = {Set of all odd numbers} and B = {Set of all even numbers} then set A and set B are Disjoint sets.



4. Difference of Sets For two sets A and B, $A - B$ is the set of all those elements of A that do not belong to B.

Similarly, $B - A$ is the set of all those elements of B that do not belong to A.

Example: $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 6\}$, then $A - B = \{1, 3, 5, 6\}$ and $B - A = \{\emptyset\}$

5. Complement of a Set If U is the universal set and set A is such that $A \subset U$ then complement of the set A is defined as $U - A$ and represented as A' or A^c.

Example: U = {set of all prime numbers} and A = {set of all even prime numbers}

then $U - A$ = {set of all odd prime numbers}

ALGEBRAIC LAWS IN SET THEORY

1. Commutative Laws For any two sets A and B,

$$A \cup B = B \cup A \quad \text{and} \quad A \cap B = B \cap A$$

2. Associative Laws For any three sets A, B and C,

$(A \cup B) \cup C = A \cup (B \cup C)$ and

$A \cap (B \cap C) = (A \cap B) \cap C$

3. Distributive Laws For any three sets A, B and C,

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. Identity Laws

$A \cup \phi = A$, where ϕ is the null set

$A \cap U = A$, where U is the universal set

5. De-Morgan's Law If A and B are two sets then

$(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

6. $A \cup A = A$ and $A \cap A = A$

Some Important Results

If A and B are any two sets, then

1. $A - B = A \cap B'$
2. $B - A = B \cap A'$
3. $(A - B) \cup B = A \cup B$
4. $(A - B) \cap B = \phi$
5. $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

Some Important Results on Numbers of Elements in Sets

If A, B and C are finite sets then

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. $n(A \cup B) = n(A) + n(B)$ if set A and set B are two disjoint sets
3. $n(A - B) = n(A) - n(A \cap B)$
4. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
5. $n(A' \cup B') = n(U) - n(A \cap B) = n(A \cap B)'$
6. $n(A' \cap B') = n(A \cup B) = n(U) - n(A \cup B)$

WORKED OUT PROBLEMS

Problem 18.1 At the quarterly monday party of Sherry, a baby boy, 40 persons chose to kiss him and 25 chose to shake hands with him. 10 persons chose to both kiss him and shake hands with him. How many persons turned out at the party?

(a) 25 (b) 75 (c) 55 (d) 25

Solution Let $n(A)$ = No. of persons who kissed Sherry = 40

$n(B)$ = No. of persons who shake hands with Sherry = 25

and $n(A \cap B)$ = No. of persons who shook hands with Sherry and kissed him both = 10

Then using the formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$n(A \cup B) = 40 + 25 - 10 = 55$$

Task for the student: Try to solve through Venn diagram.

Directions for Questions 2-5: Refer to the data below and answer the questions that follow.

In an examination 65% passed in Math, 48% passed in Physics and 52% passed in Chemistry. Only 8% students passed in all the three. 14% passed in Math and Physics and 21% passed in Math and Chemistry and 28% passed in Physics and Chemistry. Number of students who took the exam is 300.

Let sets P, and C and set M denote the students who passed in Physics, Chemistry and Math respectively.

Problem 18.2 How many students passed in Math only?

(a) 16 (b) 22 (c) 48 (d) 80

Problem 18.3 Find the ratio of students passing in Math only to the students passing in Chemistry only?

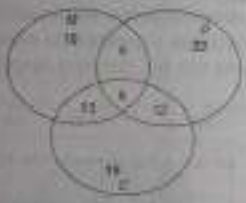
(a) 16 : 33 (b) 20 : 33
(c) 16 : 39 (d) 33 : 40

Problem 18.4 What is the ratio of the number of students passing in Physics only to the students passing in either Physics or Chemistry or both?

(a) 56/46 (b) 22/80
(c) 46/92 (d) None of these

Problem 18.5 A student is declared pass in the exam only if he/she clears any two subjects. The number of students who were declared passed in this exam is?

(a) 33 (b) 66 (c) 16 (d) 78



Solution Let P denote Physics, C denote Chemistry and M denote Math.

% of students who passed in P and C only is given by

% of students who passed in P and C – % of students who passed all three = 20% – 8% = 12%

% of students who passed in P and M only is given by

% of students who passed in P and M – % of students who passed all three = 14% – 8% = 6%

% of students who passed in M and C only is

% of students who passed in C and M – % of students who passed all three = 21% – 8% = 13%

So % of students who passed in P only is given by

Total no. passing in P – No. passing in P and C only – No. passing P and M only – No. passing in all the three →

$$40\% - 12\% - 6\% - 8\% = 22\%$$

% of students who passed in M only is

Total no. passing in M – No. passing in M and C only – No. passing P and M only – No. passing in all three →

$$40\% - 13\% - 6\% - 8\% = 16\%$$

% of students who passed in Chemistry only is

Total no. passing in C – No. passing in P and C only – No. passing C and M only – No. passing in all three →

$$52\% - 12\% - 13\% - 8\% = 19\%$$

18.2 Solution The number of students passing only in Math is 16% of 200 = 32 students.

18.3 Solution Ratio of M only to C only is 16 : 19.

18.4 Solution P only can be seen to be 22%, while either P or C or both is given by P only + C only + P and C only + P and C and M.

Hence the required ratio is $\frac{22}{80}$.

18.5 Solution The % of students who passed any two of the three subjects can be seen from the various intersections as:

$$6 + 8 + 13 + 12 = 39\% \rightarrow 78 \text{ students}$$

Level of Difficulty (LOD) I

1. In the AMS club, all the members participate either in the Tombola or the Fete. 430 participate in the Fete, 350 participate in the Tombola and 220 participate in both. How many members does the club have?
(a) 410 (b) 330
(c) 440 (d) None of these
2. There are 20,000 people living in Bedroom Colony, Gurgaon. One of them 9000 subscribe to Star TV Network and 12,000 to Zee TV Network. If 4000 subscribe to both, how many do not subscribe to any of the two?
(a) 3000 (b) 2000 (c) 1000 (d) 4000

Direction for Questions 1–6: Refer to the data below and answer the questions that follow.

Let us, there were 3 sections in the Cutoffs, a mock cut off paper. Out of them, 35 students cleared the cut-off in section 1, 34 students cleared the cut-off in section 2, and 32 cleared the cut-off in section 3. 10 students cleared the cut-off in section 1 and section 2, 8 cleared the cut-off in section 2 and section 3, 9 cleared the cut-off in section 1 and section 3. The number of people who cleared each cut-off above was equal and was 21 for each section.

1. How many cleared all the three sections?
   (a) 5 (b) 6 (c) 5 (d) 7
2. How many cleared only one of the three sections?
   (a) 23 (b) 62 (c) 42 (d) 52
3. The ratio of the number of students clearing the cut-off in one or more of the sections to the number of students clearing the cut-off in section 1 alone is?
   (a) 79/23 (b) 15/23
   (c) $\frac{79}{63}$ (d) None of these

Direction for Questions 4–7: Refer to the data below and answer the questions that follow.

In the Indian athletic squad sent to the Sydney Olympics, 7 athletes were in the relay team, 26 were in the pole-vault team and 29 were in the marathon team. 14 athletes are able just to relay and pole-vault, 12 can take part in hurdles and marathon, 15 can take part in pole-vault and marathon and 8 can take part in all the three games.

4. How many players are there in all?
   (a) 35 (b) 45
   (c) 49 (d) None of these
5. How many were in the marathon team only?
   (a) 10 (b) 14 (c) 13 (d) 15

Direction for Questions 5–8: Refer to the data below and answer the questions that follow.

In a test in which 120 students appeared, 90 passed in Maths, 85 passed in Sociology and 75 passed in Political Science. 30 students passed in only one subject and 45 failed in only one. Five students passed in no subject.

6. How many students passed in all the three subjects?
   (a) 25 (b) 30
   (c) 10 (d) Data insufficient
7. Find the number of students who passed in at least two subjects.

   (a) 35 (b) 45
   (c) 50 (d) Data insufficient

Directions for Questions 10–13: Refer to the data below and answer the questions that follow.

3% of the passengers who boarded Guwahati New Delhi Rajdhani Express on 20 February 2002, do not like coffee, tea and ice-cream and 10% like all the three. 20% like coffee and tea, 25% like ice-cream and coffee and 25% like ice-cream and tea. 55% like coffee, 50% like tea and 70% like ice-cream.

10. The passengers who like only coffee is greater than the passengers who like only ice-cream by
    (a) 50% (b) 100% (c) 75% (d) 0
11. The percentage of passengers who like both tea and ice-cream but not coffee is
    (a) 15 (b) 5 (c) 10 (d) 25
12. The percentage of passengers who like at least 2 of the 3 products is
    (a) 40 (b) 45 (c) 30 (d) 60
13. If the number of passengers is 100, then the number of passengers who like ice-cream only is
    (a) 10 (b) 16 (c) 27 (d) 36

Directions for Questions 14–21: Refer to the data below and answer the questions that follow.

In a survey among students of all the IIMs, it was found that 40% preferred coffee, 54% liked tea and 64% smoked. Of the total, 28% liked coffee and tea, 32% smoked and drank tea and 30% smoked and drank coffee. Only 6% did none of these. If the total number of students is 2000 then

14. The ratio of the number of students who like only coffee to the number who like only tea is
    (a) 5 : 3 (b) 8 : 9
    (c) 2 : 3 (d) 3 : 2
15. The number of students who like coffee and smoking but not tea is
    (a) 600 (b) 240 (c) 280 (d) 360
16. The percentage of those who like coffee or tea but not smoking among those who like at least one of these is
    (a) More than 30 (b) Less than 30
    (c) Less than 25 (d) None of these
17. The percentage of those who like at least one of these is
    (a) 100 (b) 90 (c) 94 (d) 94

16. The two sense having the ratio 3 : 2 are
    (a) Tea only and tea and smoking only
    (b) Coffee and smoking only and tea only
    (c) Coffee and tea but not smoking and smoking but not coffee and tea
    (d) None of these

17. The number of persons who like coffee and smoking only and the number who like coffee only bear a ratio
    (a) 1 : 2 (b) 2 : 2
    (c) 3 : 4 (d) 2 : 1

18. Percentage of those who like tea and smoking but not coffee is
    (a) 14 (b) 14.8
    (c) Less than 14 (d) More than 15

19. 40 monkeys went to a picnic. 25 monkeys chose to urinate coca while 20 chose to urinate mirinda. How many chose to urinate both mirinda and coca?
    (a) 10 (b) 13 (c) 5 (d) 20

Directions for Questions 20–25: Refer to the data below and answer the questions that follow.

In the CBSE Board Exams last year, 55% passed in Biology, 65% passed in English, 60% in Social Studies, 24% in Biology and English, 35% in English and Social Studies, 27% in Biology and Social Studies and 5% in none.

20. Percentage of passes in all subjects is
    (a) 9.8 (b) 12 (c) 7 (d) 10

21. If the number of students in the class is 300, how many passed in only one subject?
    (a) 48 (b) 46
    (c) More than 50 (d) Less than 40

22. If the number of students in the class is 300, what will be the % change in the number of passes in only two subjects, if the original number of students is 200?
    (a) More than 50% (b) Less than 50%
    (c) 50% (d) None of these

23. What is the ratio of percentage of passes in Biology and Social Studies but not English in relation to the percentage of passes in Social Studies and English but not Biology?
    (a) 3 : 5 (b) 7 : 5
    (c) 4 : 5 (d) None of these

Directions for Questions 26–30: Refer to the data below and answer the questions that follow.

In the AMS Quiz held last year, participants were free to choose their respective areas from which they were asked questions. Out of 600 participants, 224 chose Mythology questions, 192 of 600 participants, 224 chose Mythology questions, 192 chose Sports and 116 chose Science, 64 chose both Sports and Science, 80 chose Mythology and Sports, 40 chose Mythology and Science and 24 chose all the three areas.

26. The percentage of participants who did not choose any area is
    (a) 33.33% (b) 30.25%
    (c) 37.48% (d) 27.27%

27. Of those participating, the percentage who chose only one area is
    (a) 60% (b) More than 60%
    (c) Less than 60% (d) More than 70%

28. Number of participants who chose at least two areas is
    (a) 112 (b) 24
    (c) 136 (d) None of these

29. Which of the following areas chosen a ratio of 1 : 3?
    (a) Mythology and Science but not Sports; Mythology only
    (b) Mythology and Sports but not Science; Science only
    (c) Science; Sports
    (d) None of these

30. The ratio of students choosing Sports and Science but not Mythology to Science but not Mythology and Sports is
    (a) 2 : 3 (b) 1 : 4
    (c) 1 : 3 (d) 1 : 2

Directions for Questions 31–35: Refer to the data below and answer the questions that follow.

The table below gives the distribution of students according to professional courses.

Courses	Students			
	English		Math	
	Males	Females	Males	Females
Part-time MBA	30	10	50	10
Full-time MBA only	120	8	16	0
CA only	90	10	12	3
Full-time MBA and CA	70	2	6	4

32. What is the percentage of Math students over English students?

(a) 50.4 (b) 68.4
(c) 29.4 (d) None of these

33. The average number of Symbols in all the courses is

(a) Less than 12
(b) Greater than 12
(c) 12
(d) None of these

34. The ratio of the number of girls to the number of boys is

(a) 5 : 39 (b) 11 : 74
(c) 1 : 72 (d) None of these

35. The percentage increase in students of full-time MBA side over CA only is

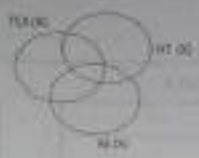
(a) Less than 20 (b) Less than 25
(c) Less than 30 (d) More than 30

36. The number of students doing full-time MBA or CA is

(a) 320 (b) 80
(c) 160 (d) None of these

Instructions for Questions 36–40: Refer to the data below and answer the questions that follow.

A newspaper agent sells TOI, HT and IN in equal number to 302 persons. Seven get HT and IN, twelve get TOI and IN, nine get TOI and HT and three get all the three newspapers. The details are given in the Venn diagram.



36. How many get only one paper?

(a) 280 (b) 327
(c) 108 (d) None of these

37. What percent get TOI or HT but not IN

(a) More than 65% (b) Less than 60%
(c) 2.99% (d) None of these

38. Number of persons buying TOI and HT only, TOI and IN only and HT and IN only are in the ratio of

(a) 6 : 4 : 9 (b) 6 : 9 : 4
(c) 4 : 9 : 6 (d) None of these

39. The difference between the number reading HT and IN only and HT only is

(a) 72 (b) 78 (c) 82 (d) None of these.

40. A group of 78 people watch Zee TV, Star Plus or Sony. Of these, 56 watch Zee TV, 48 watch Star Plus and 32 watch Sony. If 14 people watch both Zee TV and Star Plus, 20 people watch both Star Plus and Sony, and 12 people watch both Sony and Zee TV. Find the ratio of the number of people who watch only Zee TV to the number of people who watch only Sony.

(a) 9 : 4 (b) 3 : 2 (c) 7 : 3 (d) 7 : 4

ANSWER KEY

For Theory (Gr.)

1.	a
2.	a
3.	a
4.	a
5.	a
6.	a
7.	a
8.	a
9.	a
10.	b
11.	a
12.	a
13.	b
14.	a
15.	b
16.	a
17.	d
18.	a
19.	a
20.	c
21.	b
22.	c
23.	a
24.	a
25.	a
26.	c
27.	a
28.	a
29.	c
30.	d
31.	d
32.	b
33.	d
34.	c
35.	a
36.	a
37.	a
38.	b
39.	b
40.	a

19

COORDINATE GEOMETRY

From the CAT point of view, Coordinate Geometry is not a very significant chapter. Basically, applied questions are asked in the form of rudimentary representations requiring the usage of the concepts learned. Moreover, it is meant to get through the basics and fundamentals. Consequently, the good effort we want to be presented for successful completion. Unprecedentedly, angle is called land in the formulas and concepts contained in this chapter. Moreover, the student will have an improved understanding of the graphical representation of functions if he/she is given through constructive geometry.

The student also faces two problems in this chapter, one is the wrong LCD (Concept of getting [illegible]) [illegible]

CARTESIAN COORDINATE SYSTEM

Rectangular Coordinate Axes

1. [illegible] [illegible] be two mutually perpendicular lines [illegible] by point O in the plane of the paper. Point O is called the origin. The line X'OX is called the x-axis or [illegible] and the line Y'OY is known as the y-axis or axis of y. [illegible] together are called the coordinate axes or the axes of coordinates.

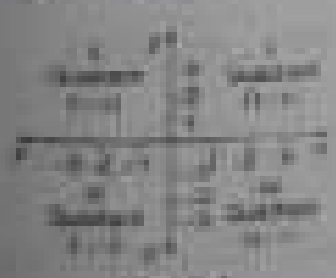


Fig. 19.1

And point can be represented in the plane determined by the coordinate axes by introducing the x and y coordinates. The x coordinate of the point is the [illegible] of the x-axis while the y coordinate of the point is the [illegible] of the y-axis.

1. Distance Formula: If two points P and Q are in the plane and are represented by the points (x_1, y_1) and (x_2, y_2) on the x-y plane (cartesian plane), then the distance between the points P and Q is $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Illustration:

Question 1: Find the distance between the points [illegible] and [illegible].

Answer: Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{[illegible]}$$

$$= [illegible]$$

2. Section Formula: If any point (x, y) divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ internally.

Proof: [illegible]

Fig. 19.2

If any point (x, y) divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m:n$ internally,

then $x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

Illustration

Question 2: Find the point which divides the line joining the points $A(-2, 3)$ and $B(3, 7)$ in the ratio $2:3$ internally.

Answer: $A \equiv (x_1, y_1) = (-2, 3)$, $B \equiv (x_2, y_2) = (3, 7)$, $m = 2$, $n = 3$

$x = \frac{mx_2 + nx_1}{m+n} = \frac{2(3) + 3(-2)}{2+3} = \frac{6-6}{5} = 0$

$y = \frac{my_2 + ny_1}{m+n} = \frac{2(7) + 3(3)}{2+3} = \frac{14+9}{5} = 23/5$

6. Area of a Triangle – The area of a triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by

$$\frac{|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|}{2}$$

Fig. 19.2

(Note: Since the area cannot be negative, we have to take the positive value given by the above formula.)

Corollary: If one of the vertices of the triangle is at the origin and the other two vertices are $A(x_1, y_1)$, $B(x_2, y_2)$,

then the area of triangle $= \left|\frac{x_1 y_2 - x_2 y_1}{2}\right|$

Illustration

Question 3: Find the area of the triangle $A(-1, 4)$, $B(-4, 5)$ and $C(-3, -2)$.

Answer: Area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|(-1)(5+2) + (-4)(-2-4) + (-3)(4-5)|$

$= \frac{1}{2}|-7 + 24 + 3| = \frac{1}{2}(14) = 7$ square units.

7. Centre of Gravity or Centroid of a Triangle – The centroid of a triangle is the point of intersection of its medians. The four joining the vertex to the middle point of the opposite side. Centroid divides the medians in the ratio $2:1$. In other words, the CG is the centroid and the point at which the whole weight of the triangle is considered.

Formula: If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the coordinates of the vertices of a triangle, then the coordinates of the centroid $G(x, y)$ are given by

$$x = \frac{x_1 + x_2 + x_3}{3} \text{ and } y = \frac{y_1 + y_2 + y_3}{3}$$

Fig. 19.4

Illustration

Question 4: Find the centroid of the triangle whose vertices are $(3, 5)$, $(1, 3)$ and $(5, 2)$.

Answer: If coordinates $= \left(\frac{3+1+5}{3}, \frac{5+3+2}{3}\right) = (3, 10/3)$

Centroid $= \left(\frac{3+1+5}{3}, \frac{5+3+2}{3}\right) = (3, 10/3)$

8. In-centre of a Triangle – The centre of the circle that touches the sides of a triangle is called its incentre. In other words, if the three sides of the triangle are tangents to the circle then the centre of the circle is known as the incentre of the triangle.

The incentre is the point of intersection of the internal bisectors of the angles of the triangle. The distance of the incentre from the sides of the triangle is the same and this distance is called the inradius of the triangle.

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the coordinates of the vertices of a triangle, then the coordinates of the incentre are

$$x = \frac{ax_1 + bx_2 + cx_3}{(a+b+c)} \text{ and } y = \frac{ay_1 + by_2 + cy_3}{(a+b+c)}$$

where $BC = a$, $AC = b$ and $AB = c$

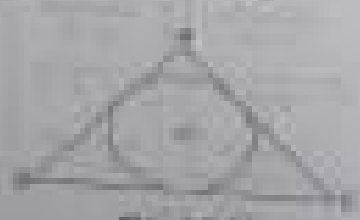


Fig. 19.5

[illegible]

Fig. 10.6

E. Circumcentre of a Triangle: [illegible]

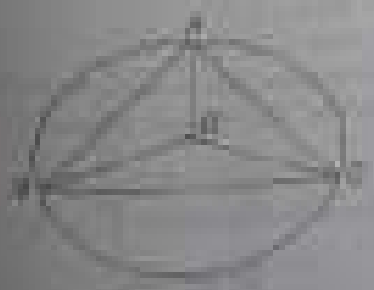


Fig. 10.7

Illustration

[illegible]

F. Orthocentre of a Triangle: [illegible]

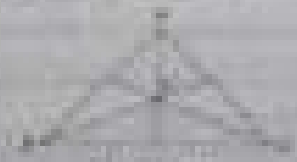


Fig. 10.8

Illustration

[illegible]

[illegible] (or [illegible]) to the straight whose equation is [illegible] and we have the coordinates [illegible] and [illegible] is [illegible] [illegible] of P.

Solving these equations simultaneously, we get coordinates of A, B and C as [illegible], [illegible] and [illegible] respectively.

(i) [illegible] passing through A and (ii) and perpendicular to [illegible].

[illegible] equation is [illegible]

The equation of any line perpendicular to [illegible] is [illegible] represented by [illegible]

The line will pass through [illegible] if [illegible]

[illegible]

So the equation of BC is [illegible]

Solving the equations of AD and BE we get [illegible]

Hence the coordinates of the orthocenter are [illegible].

9. COLINEARITY OF THREE POINTS: Three given points A, B and C are said to be collinear (lie on the same straight line), if any of the following conditions exists

(i) Area of triangle formed by these three points is zero

(ii) Slope of AB = Slope of AC

(iii) Any one of the three points (say B) lies on the straight line joining the other two points (here A and B).

Fig. 10.9

Illustration

Question 8: Select the right option the points (1, −2), (3, 0) and (6, 3) are

(a) Collinear (b) Vertices of a triangle

(c) Vertices of a rectangle (d) None of these

Answer: We can use either of the three methods to check whether the points are collinear.

For the area calculation we use the formula

Let A, B, C are the given vertices coordinates are (1, −2), (3, 0) and (6, 3)

$$\text{Slope of } AC = \frac{\ldots}{\ldots}$$

$$\text{Slope of } AB = \frac{\ldots}{\ldots}$$

So the straight line joining the points A, B and C is collinear.

Hence, (a) is the answer.

(If you have not understood this form, you may [illegible] and then [illegible] the concept.)

Alternative: Draw the points (1, −2), (3, 0) and (6, 3) on a graph paper. This will give you an idea [illegible] the number of points. In the above question, plot [illegible] is in the first quadrant. For [illegible] and point [illegible] is directly opposite to the point [illegible]. So, the third point [illegible] and the fourth point [illegible] is the middle of the straight line joining the points [illegible] and [illegible].

You can check this by assuming any value for 'x' and 'y'.

Also, you can use the method for solving any [illegible] involving points and triangles with the help of [illegible]. Here [illegible] you should be [illegible] straight [illegible] the points [illegible] is [illegible] of [illegible] points which lie [illegible]

8. Slope of a Line: The slope of a line joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is denoted by m and is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$ [illegible] $x_1 \neq x_2$ [illegible] $x_1 = x_2$ the slope is undefined. The angle θ is called the angle of inclination of the line and is measured in the anti clockwise direction from the positive direction of the axis of x.

Fig. 10.10

Illustration

Question 9: Find the equation of a straight line passing through (2, −3) and having a slope of 1 and

Answer: Here slope = 1

And point given is (2, −3)

We are [illegible] slope formula for finding the equation of straight line. This formula is given by

$$y - y_1 = m(x - x_1)$$

So equation of the line will be $y - (-3) = 1(x - 2)$

$\Rightarrow$ $y + 3 = x - 2$

$\Rightarrow$ $y = x - 5$

(d) Different Forms of the Equation of a Straight Line

(i) General Form: The general form of the equation of a straight line is $ax + by + c = 0$ or $ax + by = -c$.

Given degree equations in x and y. Where a, b and c are constants (all a, b are not simultaneously equal to zero).

In this equation, slope of the line is given by $-\frac{a}{b}$.

The general form of this given by $y = mx + c$ where m is the slope and c is the intercept on the y-axis.

In this equation, slope of the line is given by m.

(ii) Line Parallel To The X-Axis: The equation of a straight line parallel to the x-axis and at a distance k from it is given by $y = k$.

Hence, the equation of the x-axis is $y = 0$.

(iii) Line Parallel To Y-Axis: The equation of a straight line parallel to the y-axis and at a distance k from it is given by $x = k$.

Hence, the equation of y-axis is $x = 0$.

(iv) Slope-Intercept Form: The equation of a straight line passing through the point $A(x_1, y_1)$ and having a slope m is given by:

$$(y - y_1) = m(x - x_1)$$

(v) Two Points Form: The equation of a straight line passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

So slope $= \left[\frac{y_2 - y_1}{x_2 - x_1}\right]$

(vi) Intercept Form: The equation of a straight line cutting intercepts a and b on the axis of x and y respectively is given by:

$$\frac{x}{a} + \frac{y}{b} = 1$$

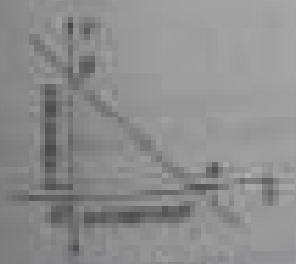


Fig. 10.11

If a straight line cuts a pair of axis the points $(a, 0)$ and $(0, b)$ are known as the intercepts of the line on a pair of axis respectively.

11. Perpendicularity and Parallelism

Condition for two lines to be parallel: Two lines are said to be parallel if their slopes are equal.

For this to happen, ratio of coefficients of x and y in both the lines should be equal.

In general form, this can be stated as: two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = k$ (say), where k is a constant.

Illustration

Question: Are which of the lines represented by the following equations are parallel to each other?

I. $3x + 2y = 5$ II. $2x - 3y = 8$

III. $x + 2y = 6$ IV. $2x + 4y = 9$

(a) I and II (b) II and III (c) I and III (d) I and IV

Answer: (c) Through the equations and check which of the two lines given will satisfy the condition for two lines to be parallel. It will be obvious that option c is correct, that is, the line $3x + 2y = 5$ is parallel to the line $x + 2y = 6$.

Question 11: Find the equation of a straight line parallel to the straight line $2x - 3y = 7$ and passing through the point $(1, -5)$.

Answer: Equation of the line parallel to $2x + 3y = 7$ will be of the form $2x + 3y = k$.

The line passes through $(1, -5)$, so this point will satisfy the equation of straight line $2x + 3y = k$. So, $2 \times 1 + 3 \times (-5) = k$ or $k = -13$.

Hence, equation of the required straight line will be $2x + 3y = -13$.

Condition for two lines to be perpendicular: Two lines are said to be perpendicular if product of the slopes of the lines is equal to -1.

For this to happen, the product of the coefficients of x and y in the product of the coefficients of x should be equal to zero.

Illustration

Question 12: Which of the following two lines are perpendicular?

[illegible]

[illegible]

Question 13. Find the equation of a straight line passing through the origin and by $4x + y = 7$ and making 30° with the positive x-axis.

Answer: Equation of the two perpendicular lines $4x + y = 7$ and one of the lines $x + 4y = 0$.

Now line passes through $(0, 0)$ so the point satisfies the equation of straight line $4x + y = 0$. So $4 \times 0 + 0 = 0$ and $k = 0$.

Hence, equation of required straight line will be $x - 4y = 0$.

12. Length of Perpendicular or Distance of a point from a line

The length of perpendicular from a given point (x_1, y_1) to the line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Corollary

(a) Distance between two parallel lines.

If two lines are parallel, the distance formula (here) will always be the same.

When two straight lines are parallel (where equations are $ax + by + c = 0$ and $ax + by + c_1 = 0$), then the distance between them is given by $\frac{|c - c_1|}{\sqrt{a^2 + b^2}}$

(b) The length of the perpendicular from the origin to the line $ax + by + c = 0$ is given by $\frac{|c|}{\sqrt{a^2 + b^2}}$

Illustrations

Question 14. Two sides of a square lie on the lines $x + y = 2$ and $x + y = 4$. Find the area of the square if each side is [illegible] unit.

Answer: Obviously, the sides are formed by the parallel lines, will be the sides of the square.

To calculate, it does not form of finding the distance of a point from a line. We will have to find out a point at which [illegible]

[illegible] the other line, and drop a perpendicular from any point to the other line, the distance will be the side of the square.

To find the point at which the equation of the line $x + y = 2$ cuts the x-axis, we will put $y = 0$ and from equation $x = 2$.

Hence $(x = 2, y = 2)$ is the coordinates of the point which is one vertex of the square.

Now, any point (x, y) and the equation of line in above perpendicular to it is given by $x + y = 2$.

$$\text{Distance} = \frac{|2 + 2 - 2|}{\sqrt{1^2 + 1^2}} = \frac{2}{\sqrt{2}}$$

$$\text{Area} = \frac{2}{\sqrt{2}} = 2$$

Alternative: Since the points on the right side are of the form of diagonal as 2 units, so length of one side is $2/\sqrt{2}$ unit, therefore the area will be 2 sq units.

Alternative: You can alternate the formula for the distance between two parallel lines as

$$\frac{|c_1 - c_2|}{\sqrt{1^2 + 1^2}} = \frac{2}{\sqrt{2}}$$

13. Change of Axes

If origin (h, k) is shifted to $(0, 0)$ (i.e. the coordinates of the point (x, y) transformed to (X, Y) [illegible] the new axes $(x = 2, y = 3)$ and $(x = 3, y = 4)$.

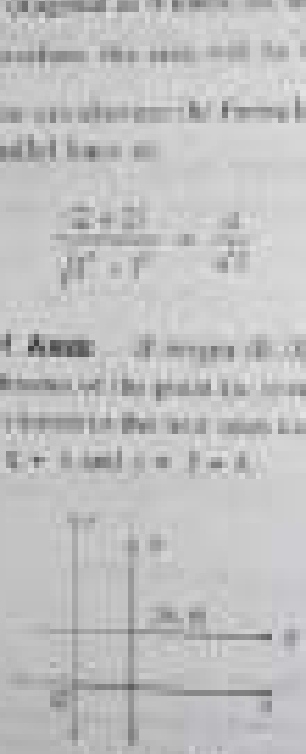


Fig. 1.6.12

Illustration

Question 15. If origin (h, k) is shifted to $(2, 3)$, what will be the coordinates of the point in the new axes which are represented by $(5, 3)$ in the old axes?

Answer: $(3, 0)$ be the coordinates of the point in the new axes.

(ii) point of Intersection of two lines. Point of intersection of two lines can be obtained by solving the equations of corresponding equations.

(i) Important Result

If all the three vertices of a triangle have integral coordinates, then that triangle cannot be an Equilateral triangle.

Level of Difficulty (LOD) I

1. Find the distance between the points (3, 4) and (5, 6).

(a) $\sqrt{5}$ (b) $2\sqrt{2}$ (c) $2\sqrt{3}$ (d) $4\sqrt{5}$

2. Find the distance between the points (3, 2) and (6, 6).

(a) $\sqrt{25}$ (b) $\sqrt{21}$ (c) $\sqrt{25}$ (d) $\sqrt{5}$

3. Possible value of p if the distance between the points (8, p) and (4, 7) is 5.

(a) 4 (b) 0

(c) Both (a) and (b) (d) None of these

4. Find the value of x if the distance between the points (5, 7) and the origin is 7 units.

(a) 2 (b) $\sqrt{2}$

(c) Both a and b (d) None of these

5. Find the total length of the line segment made by joining the points (3, 2) and (6, 6).

(a) $\left(\frac{9}{2}, 4\right)$ (b) $\left(\frac{3}{2}, 3\right)$

(c) $\left(\frac{9}{2}, \frac{3}{2}\right)$ (d) $\left(\frac{9}{2}, \frac{9}{2}\right)$

6. What angle is the mid point of the line segment joining the points (2, 3) and (1, 1). Find the value (Use π)

(a) $(2, 3)$ (b) $(-2, 3)$

(c) $(2, -3)$ (d) $(2, -3)$

7. Find the points that divide the line segment joining (3, 7) and (1, 1) in the ratio 2 : 1 respectively.

(a) $(1, 2)$ (b) $(-1, 2)$

(c) $(1, 2)$ (d) $(1, 2)$

8. In what ratio does the x-axis divide the line segment joining the points $(2, -3)$ and $(5, 6)$?

(a) 1 : 1 (b) 1 : 2 (c) 1 : 4 (d) 1 : 3

9. How many squares are possible if four of the vertices of a quadrilateral are (1, 3) and (3, 1)?

(a) 1 (b) 2 (c) 3 (d) 4

10. If the lines of (x, y) intersect orthogonally the line $x + y = 1$ and passing through $(2, 3)$ and $(4, 5)$ then find the value of k when will be the coordinates of (2)

(a) $(-5, 3)$ (b) $(5, -3)$

(c) $(5, 3)$ (d) $(-5, 3)$

11. Find the coordinates of the points that trisect the line segment joining $(1, -2)$ and $(3, 6)$.

(a) $\left(\frac{7}{3}, 2\right)$ (b) $\left(\frac{7}{3}, 2\right)$

(c) Both (a) and (b) (d) None of these

12. Find the coordinates of the point that divides the line segment joining the points $(6, 1)$ and $(-3, 5)$ in the ratio 1 : 2 externally.

(a) $\left(-9, \frac{13}{3}\right)$ (b) $\left(-9, \frac{13}{3}\right)$

(c) $\left(\frac{15}{3}, \frac{14}{3}\right)$ (d) $\left(\frac{-15}{3}, \frac{-14}{3}\right)$

13. In question 12 given, find the coordinates of the point R is divides the points externally.

(a) $(18, -9)$ (b) $(-18, -9)$

(c) $(-19, 9)$ (d) $(-9, -1)$

14. In what ratio is the line segment joining $(-1, 7)$ and $(4, -3)$ divided by the point $(2, -5)$?

(a) 2 : 3 (b) 2 : 1 (c) 2 : 5 (d) 1 : 5

15. In question 14. Find the value of (k) (ratio)

(a) Internal (b) External

(c) Both a and b (d) None of these

16. In what ratio is the line segment joining by the points $(7, 5)$ and $(-4, 5)$ divided by the x-axis?

(a) 2 : 7 (b) 5 : 3 (c) 2 : 5 (d) 7 : 5

17. What is the nature of the ratio in the above question?

(a) Internal (b) External

(c) Could be both (d) None of these

18. If the coordinates of the mid points of the line segment joining the points $(2, 1)$ and $(x, -3)$ is (x, y) then the coordinate between x and y can be found out by

(a) $3x + 2y = 5$ (b) $3x + y = 5$
(c) $3x - 2y = 5$ (d) $3x - 3y = 5$

18. Points $A(-3, 1)$, $B(-2, 2)$ and $C(-1, -1)$ are joined to form a triangle. What are its vertices?
(a) Scalene (b) Parallelogram
(c) Square (d) Rectangle

19. Points $A(-3, 1)$, $B(-2, 2)$, $C(-1, -1)$ and $D(-2, -1)$ are joined to form a quadrilateral. What will be its shape?
(a) Rhombus (b) Parallelogram
(c) Square (d) Rectangle

20. What will be the centroid of a triangle whose vertices are $(2, 4)$, $(6, 4)$ and $(2, 10)$?
(a) $\left(\frac{7}{2}, \frac{8}{3}\right)$ (b) $(3, 5)$
(c) $\left(\frac{10}{3}, \frac{8}{3}\right)$ (d) $(1, 4)$

22. The distance between the lines $4x + 3y = 11$ and $8x + 6y = 30$ is
(a) 4 (b) $\frac{5}{10}$ (c) $\frac{3}{4}$ (d) 20.

23. If the mid-point of the line joining $(3, 4)$ and $(x, 2y)$ is (x, y) then $2x + 2y + x + y = 0$ (has value) (d) for the value of y^2

(a) 1 (b) $\frac{-1}{4}$ (c) 16 (d) $\frac{11}{2}$

24. Find the third vertex of the triangle whose two vertices are $(-3, 1)$ and $(6, -2)$ and the centroid is the origin
(a) $(3, 3)$ (b) $\left(\frac{-1}{2}, \frac{1}{2}\right)$
(c) $(3, 1)$ (d) $(6, 6)$

25. Find the area of the triangle whose vertices are $(1, 4)$, $(-7, 6)$ and $(5, -4)$.
(a) 20 (b) 10 (c) 18 (d) 36

26. Find the area of the triangle whose vertices are $(a, b + c)$, $(a, b - c)$ and $(-a, c)$
(a) $2ac$ (b) $2ab$
(c) $b + c + a$ (d) $(a + b - c)$

27. The number of lines that are parallel to $x = 2$ and $y = 3$ and have an intercept of length 5 between the coordinate axes is
(a) 1 (b) 2 (c) 3 (d) Infinite

28. Which of the following three points represent a straight line?
(a) $\left(\frac{-1}{2}, 3\right)$, $(-6, 8)$ and $(-8, 10)$
(b) $\left(\frac{-1}{2}, 3\right)$, $(-6, 8)$ and $(-8, 9)$
(c) $\left(\frac{1}{2}, 3\right)$, $(-6, 8)$ and $(-8, 9)$
(d) $\left(\frac{-1}{2}, 3\right)$, $\left(\frac{3}{2}, 6\right)$ and $(0, 3)$

30. Which of the following will be the equation of a straight line that is parallel to the y-axis at a distance of 4 units from it?
(a) $x = 4$ (b) $y = 4$
(c) $y = -4$ (d) None of these

31. Which of the following will be the equation of a straight line parallel to the y-axis at a distance of 5 units to the left?
(a) $x = -5$ (b) $x = 5$
(c) $y = -5$ (d) $x = -y$

32. What can be said about the equation of the straight line $x = 7$?
(a) It is the equation of a straight line at a distance of 7 units towards the right of the y-axis.
(b) It is the equation of a straight line at a distance of 7 units towards the left of the y-axis.
(c) It is the equation of a straight line at a distance of 7 units below the x-axis.
(d) It is the equation of a straight line at a distance of 7 units above the x-axis.

33. What can be said about the equation of the straight line $y = 8$?
(a) It is the equation of a straight line at a distance of 8 units below the y-axis.
(b) It is the equation of a straight line at a distance of 8 units above the y-axis.
(c) It is the equation of a straight line at a distance of 8 units towards the right of the y-axis.
(d) It is the equation of a straight line at a distance of 8 units towards the left of the y-axis.

34. Which of the following straight lines passes through the origin?
(a) $x + y = 8$ (b) $x^2 + y^2 = 0$
(c) $x + y = 5$ (d) $x = 4y$

34. What will be the point of intersection of the straight lines $2x + 3y = 6$ and $2x + 4y = 10$?

(c) $\left(\frac{13}{9}, \frac{4}{9}\right)$ (d) $\left(\frac{-11}{9}, 4\right)$

(e) $\left(\frac{-2}{9}, \frac{-2}{9}\right)$ (f) $\left(4, \frac{-2}{9}\right)$

2. If $P(6, 7)$, $Q(-2, 3)$ and $R(4, -2)$ be the vertices of a triangle, then which of the following is not a point contained in the triangle?
(a) (4, 3) (b) (3, 3)
(c) (4, 2) (d) (6, 1)

3. What will be the reflection of the point (4, 5) in the second quadrant?
(a) (−4, −5) (b) (−4, 5)
(c) (4, −5) (d) None of these

4. What will be the reflection of the point (4, 5) in the first quadrant?
(a) (−4, −5) (b) (−4, 5)
(c) (4, −5) (d) None of these

5. What will be the reflection of the point (4, 5) in the second quadrant?
(a) (−4, −5) (b) (−4, 5)
(c) (4, −5) (d) None of these

6. If the origin gets shifted to (2, 2), then what will be the new coordinates of the point (4, −2)?
(a) (−2, 4) (b) (2, 4)
(c) (4, 2) (d) (2, −4)

7. What will be the length of the perpendicular drawn from the point (4, 5) upon the straight line $3x + 4y = 10$?
(a) $\frac{17}{5}$ (b) $\frac{32}{5}$ (c) $\frac{22}{5}$ (d) $\frac{42}{5}$

Level of Difficulty (LOD) II

1. Find the area of the quadrilateral the coordinates of whose angular points taken in order are (1, 7), (3, 4), (5, −2) and (4, −7).
(a) 20.5 (b) 41 (c) 82 (d) 44.5

2. Find the area of the quadrilateral the coordinates of whose angular points taken in order are (−1, 6), (−5, −5), (5, −6) and (2, 6).
(a) 44 (b) 96 (c) 192 (d) 72

3. Two vertices of a triangle are (5, −1) and (−2, 5). If the orthocentre of the triangle is the origin, what will be the coordinates of the third point?
(a) (4, 7) (b) (−4, 7)
(c) (−4, −7) (d) (4, −7)

4. Find the equation of the straight line passing through the origin and the point of intersection of the lines $x/a + y/b = 1$ and $x/b + y/a = 1$.
(a) $y = x$ (b) $y = -x$
(c) $y = 3x$ (d) $y = -3x$

5. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (−3, 1) and (1, 1). Which of the following may be an equation which represents any of the other three straight lines?
(a) $7x - 4y = 5$ (b) $7x - 4y + 5 = 0$
(c) $y + 1 = 0$ (d) $4x + 7y = 3$

6. The points $(p-1, p+2)$, $(p, p+3)$, $(p+1, p)$ are collinear for
(a) $p = 0$ (b) $p = 1$
(c) $p = -1/2$ (d) Any value of p

7. The straight line joining (1, 2) and (2, −3) is perpendicular to the line joining (6, 2) and (4, p). What will be the value of p?
(a) −1 (b) 1
(c) 3 (d) None of these

8. What will be the length of the perpendicular drawn from the point (−3, −4) to the straight line $12(x + 6) = 5(y - 2)$?
(a) $5\left(\frac{4}{13}\right)$ (b) $5\left(\frac{1}{13}\right)$
(c) $3\left(\frac{2}{13}\right)$ (d) $3\left(\frac{1}{13}\right)$

9. The area of the triangle with vertices at $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$ is
(a) 0 (b) $a + b + c$
(c) $a^2 + b^2 + c^2$ (d) 1

10. Find the distance between the two parallel straight lines $y = mx + c$ and $y = mx + d$? (Assume $c > d$)
(a) $\left(\frac{(c-d)}{(1+m^2)^{\frac{1}{2}}}\right)$ (b) $\left(\frac{(d-c)}{(1+m^2)^{\frac{1}{2}}}\right)$
(c) $\left(\frac{d}{(1+m^2)^{\frac{1}{2}}}\right)$ (d) $\left(\frac{-d}{(1+m)^{\frac{1}{2}}}\right)$

11. What will be the equation of the straight line that passes through the intersection of the straight lines $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ and is perpendicular to the straight line $2x - 4y = 5$?

(a) $8x + 5y = \frac{12}{7}$ (b) $4x + 7y = \frac{84}{17}$

(c) $8x - 5y = \frac{62}{17}$ (d) $8x + 6y = \frac{58}{17}$

12. In question 11, find the equation of the straight line if it is parallel to the straight line $3x + 4y = 5$?

(a) $12x + 16y = \frac{58}{17}$ (b) $3x + 4y = \frac{58}{17}$

(c) $6x + 8y = \frac{58}{17}$ (d) None of these

13. The orthocenter of the triangle formed by the points (0, 0), (8, 0) and (4, 6) is

(a) $\left(4, \frac{8}{3}\right)$ (b) (5, 4) (c) (4, 3) (d) $\left(3, \frac{3}{2}\right)$

14. The area of a triangle is 5 square units, two of its vertices are (2, 1) and (3, −2). The third vertex lies on $y = x + 3$. What will be the third vertex?

(a) $\left(\frac{5}{3}, \frac{14}{3}\right)$ (b) $\left(\frac{7}{3}, \frac{13}{3}\right)$

(c) (3, 4) (d) (1, 2)

15. The equations of two equal sides AB and AC of an isosceles triangle ABC are $x + y = 5$ and $7x - y = 3$ respectively. What will be the equation of the side BC if area of triangle ABC is 5 square units.

(a) $x + 2y - 1 = 0$ (b) $x - 3y + 1 = 0$

(c) $2x - y = 5$ (d) $x + 2y = 5$

16. Three vertices of a rhombus, taken in order are (2, −1), (5, 0) and (−2, 3). Find the fourth vertex.

(a) (3, 2) (b) (−3, −2)

(c) (−3, 2) (d) (3, −2)

17. Four vertices of a parallelogram taken in order are (−3, −1), (a, b), (3, 5) and (4, 3). What will be the ratio of a to b?

(a) 4 : 1 (b) 1 : 2 (c) 1 : 3 (d) 3 : 1

18. What will be the new equation of straight line $3x + 4y = 6$ if the origin gets shifted to (3, −4)?

(a) $3x + 4y = 3$ (b) $4x - 3y = 4$

(c) $3x + 4y + 1 = 0$ (d) $3x + 4y - 13 = 0$

19. What will be the value of p if the equation of straight line $2x + 3y = 4$ gets changed to $2x + 3y = p$ after shifting the origin to (3, 3)?

(a) 16 (b) −12 (c) 12 (d) 10

20. A line passing through the points $(x, 2x)$ and $(-2, 3)$ is perpendicular to the line $4x + 3y + 5 = 0$. Find the value of x?

(a) −14/3 (b) 18/5

(c) 14/3 (d) −18/5

Level of Difficulty (LOD) III

1. The area of a triangle is 5 square units. Two of its vertices are (2, 1) and (3, −2). The third vertex lies on $y = x + 3$. What will be the third vertex?

(a) (4, −7) (b) (4, 7)

(c) (−4, −7) (d) (−4, 7)

2. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (−3, 1) and (1, 1). Which of the following is not an equation of the other three straight lines?

(a) $14x - 8y = 6$ (b) $7x - 4y = -25$

(c) $8x + 7y = 11$ (d) $14x - 8y = 30$

3. The area of triangle formed by the points $(p, 2-2p)$, $(1-p, 2p)$ and $(-4-p, 6-2p)$ is 70 units. How many integral values of p are possible?

(a) 2 (b) 3

(c) 4 (d) None of these

4. What are the points on the axis of x whose perpendicular distance from the straight line $ax + yby = 1$ is p?

(a) $\frac{p}{q}\left[q - \sqrt{(p^2 + q^2)}\right], 0$

(b) $\frac{p}{q}\left[q - \sqrt{(p^2 + q^2)}\right], 0$

(c) Both (a) and (b)

(d) None of these

5. If the medians PT and RS of a triangle with vertices P (0, b), Q (0, 0) and R (a, 0) are perpendicular to each other, which of the following satisfies the relationship between a and b?

(a) $4b^2 = a^2$ (b) $2b^2 = a^2$

(c) $a = -2b$ (d) $a^2 + b^2 = 0$

6. The point of intersection of the lines $ax + yb = 1$ and $xb + ya = 1$ lies on the line

(a) $x + y = 1$ (b) $x + y = 0$

(c) $x - y = 1$ (d) $x - y = 0$

1. PQR is an isosceles triangle. If the coordinates of the base are $Q(1, 3)$ and $R(-2, 7)$, then the coordinates of the vertex P can be

(a) $\left(4, \frac{9}{2}\right)$ (b) $(2, 5)$

(c) $\left(\frac{5}{6}, 8\right)$ (d) $\left(\frac{1}{3}, 2\right)$

2. The extremities of a diagonal of a parallelogram are the points $(3, -4)$ and $(-6, 5)$. If the third vertex is the point $(-2, 1)$, the coordinates of the fourth vertex is

(a) $(1, 0)$ (b) $(-1, 0)$

(c) $(-1, 1)$ (d) $(1, -1)$

3. If the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear then which of the following is true?

(a) $\frac{1}{a} + \frac{1}{b} = 2$ (b) $\frac{1}{a} - \frac{1}{b} = 1$

(c) $\frac{1}{a} - \frac{1}{b} = 2$ (d) $\frac{1}{a} + \frac{1}{b} = 1$

4. If P and Q are two points on the line $3x + 4y = -15$, such that $OP = OQ = 9$ units, the area of the triangle POQ will be

(a) $18\sqrt{2}$ sq units (b) $3\sqrt{2}$ sq units

(c) $6\sqrt{2}$ sq units (d) $15\sqrt{2}$ sq units

5. If the coordinates of the points A, B, C and D are $(0, 3)$, $(-3, -5)$, $(4, -2)$ and $(a, 3a)$ respectively and if the ratio of the area of triangles ABC and DBC is $2 : 3$, then the value of a is

(a) $\frac{-9}{2}$ (b) $\frac{9}{2}$ (c) $\frac{-23}{36}$ (d) $\frac{23}{18}$

6. The equations of two equal sides AB and AC of an isosceles triangle ABC are $x + y = 5$ and $7x - y = 3$ respectively. What will be the length of the intercept cut by the side BC on the y-axis?

(a) $\frac{2}{3}$ (b) 8

(c) 1.5 (d) No unique solution

7. A line is represented by the equation $4x + 5y = 6$ in the coordinate system with the origin $(0, 0)$. You are asked to find the equation of the straight line perpendicular to this line that passes through the point $(1, -2)$ (which is in the coordinate system whose origin is at $(-2, -2)$)

(a) $5x - 4y = 11$ (b) $5x - 4y = 13$

(c) $5x + 4y = -5$ (d) $5x - 4y = 7$

8. $P(3, 1)$, $Q(6, 5)$ and $R(x, y)$ are three points such that the angle PRQ is a right angle and the area of $\Delta PRQ = 7$. The number of such points R that are possible is

(a) 1 (b) 2 (c) 3 (d) 4

9. Two sides of a square lie on the lines $x + y = 1$ and $x + y + 2 = 0$. What is its area?

(a) $\frac{11}{2}$ (b) $\frac{9}{2}$ (c) 5 (d) 4

10. Find the value of k if the straight line $2x + 3y + 4 + k(6x + y + 12) = 0$ is perpendicular to the line $7x + 3y = 4$.

(a) $\frac{-11}{37}$ (b) $\frac{-29}{37}$

(c) $\frac{10}{37}$ (d) None of these

11. If p is the length of the perpendicular from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$, then which of the following is true?

(a) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{a^4}$ (b) $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$

(c) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (d) None of these

12. How many points on $x + y = 4$ are there that lie at a unit distance from the line $4x + 3y = 10$?

(a) 1 (b) 2

(c) 3 (d) None of these

13. What will be the area of the rhombus $ax + by + c = 0$?

(a) $\frac{3c^2}{ab}$ (b) $\frac{4c^2}{ab}$ (c) $\frac{2c^2}{ab}$ (d) $\frac{c^2}{ab}$

14. The coordinates of the mid-points of the sides of a triangle are $(4, 2)$, $(3, 3)$ and $(2, 2)$. What will be the coordinates of the centroid of the triangle?

(a) $\left(3, \frac{7}{3}\right)$ (b) $\left(-3, \frac{-7}{3}\right)$

(c) $\left(3, \frac{-7}{3}\right)$ (d) $\left(-3, \frac{7}{3}\right)$

Hints and Solutions

1. Use the area of a triangle formula for the two parts of the quadrilateral separately and then add them.

[illegible]

(ii) [illegible] the slope of any two of the 3 points should be equal to the slope of any other two points. OR

(iii) The area of the triangle formed by the three points should be equal to zero.

Solve using options.

7. Write the equation of the straight lines and then use the given.

10. Point of intersection of [illegible] and [illegible] with [illegible] is [illegible].

Apply the formula for the distance of a point to a straight line.

10. Find the points of intersection of the lines and then put the coordinates of the point into the equation [illegible] which is perpendicular to the equation of straight line [illegible] to find out [illegible].

11. Determine the point of intersection of diagonals of a triangle and internally divides the triangle. This formed by joining vertices and the orthocentre in the ratio 2 : 1.

Let the vertices of the triangle be [illegible], [illegible], [illegible], [illegible].

The equation of the altitude through [illegible] and perpendicular to [illegible] and similarly the equation of altitude through [illegible] and perpendicular to [illegible]. Now find the point of intersection of these two straight lines.

12. Use the equation

Alternative: Form the points to the [illegible] equations and then use the straight primary formula to calculate the point using the equation.

13. Name the points and then check with the options.

Alternative: Find out the point of intersection with the help of options and then use the formula for area of [illegible].

14. Length of any side of a triangle formed by the coordinates of opposite vertices of a parallelogram are equal.

15. If the origin gets changed to [illegible] from [illegible] then

Old [illegible] coordinate = New [illegible] coordinate + [illegible]

Old [illegible] coordinate = New [illegible] coordinate + [illegible]

20. Equation of any straight line perpendicular to the line [illegible] will be of the form of [illegible] where [illegible] is any constant.

Now write the equation of the straight line with the given line points and then options.

Hints and Solutions

1. Verify that the equation of the [illegible] lie on the intersection of straight lines [illegible]. Add these equations check the options, if divided by [illegible] the overall coefficient regarding sum of the angles.

2. Use the options.

3. Use the formula of area of a quadrilateral which is half of a quadratic equation. Now solve the quadratic equation to get the number of straight lines in the form.

4. Use the formula of distance of a point from the straight line using the options.

5. Use the options to find the length using the formula known.

6. Make the slope of any two points equal to the slope of any other two points.

Steps : Difference of 1 coordinate/Difference of 1 coordinate.

9. Same the points as mentioned coordinate system.

11. Length of the square take for [illegible] and along the hypotenuse of [illegible] the distance between two [illegible] line.

12. Use the formula (perpendicular distance of a point from a straight line)

ANSWER KEY

Coordinate Geometry (L01)

1	a
2	a
3	a
4	a
5	a
6	a
7	a
8	a
9	a
10	a
11	a
12	a
13	a
14	a
15	a
16	a
17	a
18	a
19	a
20	a
21	a
22	a
23	a
24	a
25	a
26	a
27	a
28	a
29	a
30	a
31	a
32	a
33	a
34	a
35	a
36	a
37	a
38	a
39	a
40	a

Cocktail: Gamma EOD II

1.	a
2.	a
3.	a
4.	a
5.	a
6.	b
7.	b
8.	b
9.	a
10.	a
11.	c
12.	c
13.	c
14.	b
15.	c
16.	b
17.	c
18.	c
19.	b
20.	a

Mock Test Papers

[illegible]

- You should take these tests only after you are through with your preparation of the entire book.
- Each question in these tests carries a weightage of 4/4 and –1.
- Take each of these tests in a normal time frame of 60 minutes (allow credit to the time limit).
- A percentage of correct answers (i.e. if you attempt 45 out of 60 questions and correctly answer 36 of them, this would give you a high 80% percentile in the exam. Note that in case you score more than 100 but have made 5-6 errors in doing so, your percentile would drop to around 90). Hence, the higher your percentage accuracy, the more the exam is going to reward you.
- Ideally:

  The ideal 100% accuracy can result in a percentile of 99+.

  The ideal 0 errors should give overall 90 percentile.

[illegible]

1

MOCK TEST PAPER—1

[illegible] W1 and W2 say, [illegible] by Priyanka and [illegible]. Priyanka takes 80% [illegible] W1 and W2 [illegible] that the ratio is 4:3 regardless of the [illegible]. How many pairs were [illegible] W1 and W2 [illegible] that the ratio is 4:3 regardless of the [illegible] W1 [illegible] that the ratio is 4:3 regardless of the [illegible]?

(1) 12% (2) 10%

(3) 2% (4) 30%

7. The value of the expression $(x^2 - x + 1)(x^2 + x + 1)$ [illegible] is between?

(1) [illegible] (2) [illegible]

(3) [illegible] (4) [illegible]

8. What is the minimum value of the function $x + \sin x$ [illegible]?

(1) 0 (2) 1 (3) 2 (4) 3

9. How many integral values for the set $\{x : x$ satisfies [illegible] for the expression $(x - 4)(x + 1) - 2(x + 5)$ [illegible]?

(1) 6 (2) 14 (3) 12 (4) 24

10. A book contains 10 chapters. Each chapter has a different number of pages (each under 30). The first chapter starts on page 1 and each chapter starts on a new page. What is the largest possible number of pages in the book, if each chapter starts on an odd page number?

(1) 18 (2) 28 (3) 30 (4) 31

11. How many even four digit integers, more than [illegible] can be made using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 [illegible] in a way [illegible] repeating digits?

(1) 720 (2) 450

(3) 612 (4) 432

12. An unlimited number of coupons bearing the digits 1, 2 and 3 are available. What is the possible number of ways of choosing a set of three coupons so that they can not be used to form the number 123?

(1) 15 (2) 10 (3) 21 (4) 36

13. How many real solutions exist for the equation $3^x - 2x - 1 = 0$?

(1) 4 (2) 3 (3) 2 (4) 1

14. The number of ordered pairs (x, y) satisfying $\log_2(x - 1)(2y - 1) = 2$, where x is an integer and $y \in (1, 3)$ is equal to

(1) 2 (2) 3 (3) 2 (4) 1

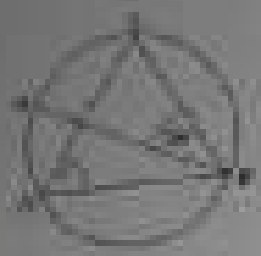
15. Ten schools play against each other in a games competition tournament. Each school is represented by a number. Every game is a doubles game, and every possible pair from the ten schools must play one game against every possible pair from the second school. How many games will each student play?

(1) 198 (2) 180 (3) 90 (4) 364

16. Consider the set $S = \{1 : x \in \{1, 2\} \cup \{3, 4\}, x \in \{5, 6\}\}$. The set $S = \{S_1, S_2, S_3, \ldots, S_{96}\}$. How many of these sets do not contain any 7 or any integral multiple of 7?

(1) 12 (2) 144 (3) 144 (4) 108

17. In the figure below, pole [illegible] passed [illegible] $(x, y, z \in \{a, b\})$. Chord AB is a diameter of this circle. The measure of angle ABC = 51°. Det [illegible] of angle BAC =



(1) [illegible] (2) [illegible] (3) [illegible] (4) [illegible]

(1) There are two arithmetic progressions $A_1, A_2, A_3, \ldots A_n$ whose first terms are 5 and 8 respectively and whose common differences are 3 and 8 respectively. How many terms of the series are common to the first 2 terms of A_1 and A_2 if the sum of the nth terms of A_1 and A_2 is equal to [illegible]?

(1) 18 (2) 19
(3) 17 (4) 21

(2) Shweta Shukla travels from Jaipur to Lucknow at a speed of 120 Kmph and returns to Delhi at a speed of 30 Kmph. She again leaves for Lucknow immediately at a speed of 10 Kmph and goes back to Delhi at a speed of 90 Kmph. What is her average speed for the entire journey?

(1) 54 Kmph (2) 48 Kmph
(3) 50 Kmph (4) 52 Kmph

(3) [illegible] the last minute period is never a distance of 60 km. Alternately the minute period gains 100 km every. The average of prime km per minute proportional to the velocity of the car. What is the minute time that Tiwari will arrive at the prime [illegible]?

(1) 1912 pm (2) 1911 pm
(3) 1910 pm (4) 1915 pm

(4) The number x is defined as the sum of the digits of the number x, and x is the product the digits of the number x. Let n be a number in the number of natural numbers x which satisfy the equation $x + x = x$ and let 'S' be defined as the number of natural numbers x which satisfy the number $x + x + x = x$. Which of the following statements about n and S is/are correct?

(1) $n \in E$ (2) $n \in B$
(3) $n \in B$ (4) $n \in B + n$
(5) $n \in B$ is only (6) $n \in B$ is only
(7) Already (8) $n \in B$ is only

(5) The letters of the word HASTE are written in all possible orders and these words are written in a dictionary. What the dictionary rank of the word HATE is?

(1) 51 (2) 56 (3) 58 (4) 50

(6) Students A, B and C were posted in a mathematics course. 25 computers subject is used one of the days. Answer from who did not solve A, but [illegible] solved B or C. The number [illegible] solved [illegible] times. Let the number solving [illegible] times. The number solving just B equal to the number solving just [illegible]. How many solved just C?

(1) 7 (2) 4
(3) 6 (4) cannot be determined

(7) x is an integer belonging to $\{1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots, 12\}$. How many possible values of x can you get such that x is an integer?

(1) 12 (2) 48 (3) 42 (4) 44

(8) Five friends Anna, Anu, Anuchka, [illegible] and Anuj buy lottery tickets having numbers 2, 6, 8, 9 and 10 respectively. Anna exchanges her ticket with Mukesh, Mukesh with Anuchka, Anuchka with Anuj and Anuj with Anu. Anu does not exchange her ticket. The three remaining exchanges do not [illegible] difference between the cubes of numbers of two pick-outs proves is equal to 2. After the fourth exchange, the difference in their squares is equal to

(1) 3 (2) 2
(3) 4 (4) Cannot be determined

Answer Key

1	(1)	2	(3)	3	(2)	4	(3)	5	(2)
6	(2)	7	(3)	8	(2)	9	(1)	10	(1)
11	(2)	12	(2)	13	(2)	14	(4)	15	(2)
16	(4)	17	(2)	18	(1)	19	(4)	20	(2)

Detailed Solutions & Short Cuts

Question 1: LRD 2

[illegible]

[illegible]

Question 3: LOD 3

[illegible]

Solution 3: LOD 3

[illegible]

Solution 4: LOD 2

[illegible]

Solution to Question 5: LOD 2

[illegible]

Solution to Question 6: LOD 2

[illegible]

Solution 7: LOD 3

[illegible]

Solution 8: LOD 3

[illegible]

Solution 9: LOD 3

[illegible]

Case 2: [illegible]

[illegible] Hence the [illegible] (2) is the right answer.

Solution 10: LOD 2

[illegible] $\Rightarrow 2^n = 256$. Thus, a total of 256 people are seen in them. [illegible] Each of the 16 players would play in the semi-[illegible] Hence, choice (1) is the right answer.

Solution 11: LOD 2

In this case you would get the sum that (1, 2, 3, 4, 5, 6), (1, 5, 6, 1) (1, 11), (10, 10) (1, 21, 10, 31) ... 41 are the [illegible] Progression as 1, 5, 21, ... The last term is [illegible] 1997. The [illegible] 1, 5, 21, 121 ..., 1441 ... And the [illegible] $(1997 - 1)/5 + 1 = 399$ terms.

Solution 12: LOD 1

[illegible] $\Rightarrow \angle C = 90°$. But $\angle ABC$ is [illegible] Hence, choice (1) is the right answer.

Solution 13: LOD 2

For [illegible]

[illegible]

or [illegible]

so, we have, we [illegible]

[illegible]

[illegible]

[illegible] Hence, choice (1) is the right answer.

Solution 14:

[illegible] Hence, the average speed is given by

[illegible]

Hence, choice (4) is the right answer.

Solution 15: LOD 2

[illegible] Hence, choice (2) is the right answer.

Solution 16: LOD 3

[illegible] $\Rightarrow 44$ and 57. Hence, $k = 3$.

Hence, choice (3) is the right answer.

Solution 17: LOD 2

[illegible] alphabetical order. When we form 5 letters [illegible] Hence, the interesting number of expressions is the [illegible] would hold.

[illegible]

Thus, we have [illegible]

Hence, the 50th word [illegible] Hence, choice (3) is the right answer.

Solution 18: LOD 2

[illegible] Hence, choice (1) is the right answer.

Solution 19: LGO 3

[illegible]

Thus there would be a total of [illegible] such integral values. With, the value of [illegible]

Solution 20: LGO 1

Name	Odd	Even	Odd+Odd	Odd+Even	Even+Even
1st exchange	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2nd exchange	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3rd exchange	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4th exchange	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5th exchange	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Every fifth table, it is clear that the difference between Even and Odd. Answer is [illegible] table above is constant after the 4th exchange and equal to [illegible]. After the 4th exchange Odd is equal to [illegible].

2

MOCK TEST PAPER—2

1. In the diagram, all triangles are equilateral. If $AB = 16$, then the total area of all the black triangles is

(1) $256\sqrt{3}$ (2) $256\sqrt{2}$ (3) $384\sqrt{3}$ (4) $192\sqrt{3}$

2. In the figure below, $BC = AC = 1$ and $CD = CE = x$. If the area of triangle ABC is equal to 2 then the area of the quadrilateral ADEB is equal to ...

(1) $\frac{5}{2}$ (2) $\frac{7}{4}$

(3) $\frac{5}{3}$ (4) $\frac{13}{3}$

3. Find the number of zeroes in

$(50!)^{50!}(49!)^{49!}(48!)^{48!}(47!)^{47!}\ldots(2!)^{2!}(1!)^{1!}$

(1) 600 (2) 600

(3) 610 (4) 620

4. The case of an 8-member rowing team is to be chosen from 12 men $(M_1, M_2, \ldots, M_{12})$ and 8 women $(W_1, W_2, \ldots, W_8)$. Therefore, two of people on each side are to be one women and one men. Further, it is also known that on the right side of the boat (viz. port side facing forward) W_1 and M_1 must be selected while on the left side of the boat M_2, M_3 and M_4 must be selected. Find the number of ways in which the rowing team can be arranged?

(1) 1996 (2) 1296

(3) 1120 (4) 720

5. In a community of 20 families, every family is expected to have the number of children as an Arithmetic Progression with a common difference of 2. Starting with 2 children in the first family, each one in every child in the family truly hopes to experience the same number of girls and boys as children. In the family, unlikely a community contribution, no such child the happy but. What will be the total number of girl cases collected in the community?

(1) 1140 (2) 1120

(3) 1080 (4) 1020

6. A boy plays a game in which he plays chess against the worker Tom, but the sum of 2 or more consecutive integers is a prime (e.g. $1001 = 499 + 500$). In how many different ways can he do so?

(1) 5 (2) 6 (3) 7 (4) 8

7. Find the remainder when 2^{2000} is divided by 124

(1) 126 (2) 30 (3) 62 (4) 28

[illegible] Arkadiev [illegible] Fermat [illegible] and [illegible] which have exactly one of their digits as 7?

(1) 280 (2) 1466
(3) 320 (4) 720

[illegible] is a function which is defined at [illegible] as follows:

$$f(x)=\begin{cases} 2x-4 & [illegible] \\ 2x+3 & [illegible] \\ 2x-3 & [illegible] \end{cases}$$

find the value of $f(7)$?

(1) -20 (2) -12 (3) -7 (4) -16

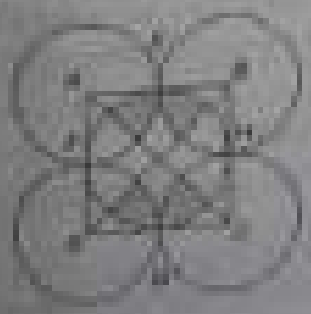
[illegible] game board is [illegible] square in size [illegible] on a fixed pattern. First, he draws a line of one long. Then he constructs a square with the line as its diagonal. He then draws another square with a side of the first square as its diagonal. This process is repeated 7 times. [illegible] the perimeter of the 7th square is less than $\frac{5}{12\cdot 2^{6}}$ [illegible]. What is the least value of n?

(1) 20 (2) 18 (3) 19 (4) 17

[illegible] the world chess [illegible] of table tennis players [illegible] best of 1 game matches. In every game, the [illegible] who wins 21 points wins. [illegible] alternate between the 2 players after every five points. A player can score points both during his service and his opponent's service. Ronak beat Cheng Min by 21-18 in a game. 36 of the 77 points played were won by the player serving. Who served first?

(1) Ronak
(2) Cheng Min
(3) Either could have served first
(4) Indeterminate

[illegible] A square of side 12 m is drawn and a circle is inscribed in it. Now with



[illegible] circle is drawn, its center is taken to be a point passing through the midpoints of the two sides of the square, which meet at the vertex. Find the area of the shaded portion. ($\pi = 3.14$)

(1) 104.4 m² (2) 54.4 m²
(3) 75.4 m² (4) 114.4 m²

[illegible] A bridge of the Nakshatra [illegible] has the [illegible] to [illegible] a bridge on the [illegible]. The bridge is [illegible] if it is determined which exactly [illegible] [illegible] 27 is determined to [illegible] the following [illegible] (1) Sudipta Mishra [illegible] (2) [illegible] [illegible] for the bridge to be [illegible] (3) [illegible] that when [illegible] bridge's [illegible] the weight the [illegible] of the [illegible] the bridge to [illegible]. Weather conditions and [illegible] [illegible] [illegible] to use the bridge [illegible] years. How many [illegible] must be used in [illegible] to be 95% sure that the bridge will be determined?

(1) 7 (2) 6 (3) 9 (4) 8

[illegible] In a [illegible] of the Nakshatra [illegible] the [illegible] [illegible] to [illegible] [illegible] [illegible]

(1) [illegible] (2) [illegible]
(3) [illegible] (4) [illegible]

[illegible] The sum of [illegible] angles [illegible] A, B and C [illegible] points [illegible] at these respectively. The total number of triangles that can be formed using any of these points

(1) [illegible] (2) [illegible] (3) [illegible] (4) [illegible]

[illegible] The sum of [illegible] angles [illegible] A, B and C [illegible] points [illegible] marked on these respectively. The total number of triangles that can be formed using any of these points [illegible] the [illegible] would be?

(1) [illegible] (2) [illegible] (3) [illegible] (4) [illegible]

[illegible] The number of flights [illegible] between [illegible] and [illegible] (including [illegible] and [illegible]) [illegible] from [illegible] flights equal to at most 5 [illegible] equal to

(1) [illegible] (2) [illegible]
(3) [illegible] (4) [illegible]

[illegible] The [illegible] of [illegible] in which [illegible] [illegible] of [illegible] can be [illegible] [illegible] 1, 2, 3, ..., 34 is

(1) [illegible] (2) [illegible] (3) [illegible] (4) [illegible]

Answer Key

1. (3)	2. (4)	3. (4)	4. (1)	5. (1)
6. (3)	7. (4)	8. (3)	9. (4)	10. (2)
11. (1)	12. (3)	13. (1)	14. (4)	15. (2)
16. (4)	17. (4)	18. (1)	19. (1)	20. (4)

Detailed Solutions & Short Cuts

Solution 1: LOD 1

In the diagram, there are 27 [illegible] triangles. In the [illegible] diagram, we [illegible] the [illegible] triangles, there would be [illegible] + [illegible] + [illegible] +

[illegible] Chang Mai [illegible] the following table [illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

Case (ii) : In Kanchanaburi Chang Mai [illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

Solution 12: LOD 1

[illegible]

Solution: 13: LOD 3

[illegible]

Solution 14: LOD 3

[illegible]

[illegible]

Solution 18: [illegible]

[illegible]

Solution 19: [illegible]

The [illegible] of the angle GOF has to be 90° since it [illegible] half of the 180° angle. Hence, the required answer has to be option 4.

Solution 20: [illegible]

You can form triangles by taking 1 point from each side, or by taking 2 points from any 1 side and the third point from either of the other two sides.

[illegible]

Solution 21: [illegible]

In order to solve this question, you will first need to solve [illegible] the number of triangles [illegible] A, B or C [illegible]. Further, we can solve the following way: You can form triangles by taking 1 point from each side, or by taking 2 points from any 1 side and the third point from either of the other two sides.

[illegible]

Now, we need to consider situations where we are either taking one of the vertices of the triangle ABC or taking two vertices from the triangle. Thus, considering 2 points on AB, 1 on AC and 1 on BC, the following possibilities emerge: [illegible] $= 8 \times 5 + 6 \times 4 \times 3 + 8 \times 6 = 56 + 36 + 28 = 124$

[illegible]

Taking $B = {}^{8}C_2 \times 4 \times 3 = 4 \times 4 \times 6 \times 3 \times 8 = 864$

Taking $C = {}^{4}C_2 \times 6 \times 5 = 4 \times 6 \times 6 \times 5 \times 3 = 360$

Taking $AB = 8$

Taking $AC = 11$

Taking $BC = 11$

Thus the total number of ways is $427 + 54 + 864 + 360 + 8 + 11 + 11 = 1735$.

Solution 22: [illegible]

Numbers having 2 digits equal: [illegible] $\Rightarrow$ [illegible] $\Rightarrow$ [illegible] numbers. Numbers with 3 digits equal: [illegible] $\Rightarrow$ [illegible] $\Rightarrow$ 9 numbers.

Total of 36 numbers.

Similarly, numbers starting with 3 and having 3 digits equal would be 18.

numbers starting with 4 and having 3 digits equal would be 36.

numbers starting with 5 and having 3 digits equal would be 36.

numbers starting with 6 and having 3 digits equal would be 36.

Total numbers having 3 digits equal $= 216$

Numbers having 2 digits unequal: [illegible] number of numbers that are starting with 2 and having 2 digits equal are twice multiply the same by 3 to get the answer. In order to find the number of numbers starting with 2 and having 2 digits equal are [illegible] the same time.

(i) Numbers starting with 2 and having 2 unequal digits [illegible]

OR

(ii) Numbers starting with 2 and having 2 unequal digits which are not equal [illegible]

$${}^{5}C_2 \times {}^{5}C_2 \times 2! = 216$$

OR

(iii) Number of numbers starting with 2 and having unequal digits which are same with a known digit [illegible] which has been used in 2 digits [illegible]

$${}^{9} \times {}^{5}C_2 \times {}^{4}C_2 = 216$$

Thus, the total number of such numbers is [illegible]

Totally: $216 + 3 \times 216 + 3 \times 216 = 1944$

[illegible] difference = 1. Then (1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6). Then will be 12 combinations.

[illegible] difference = 2. Then your list [illegible] (1, 3, 5), (1, 4, 6), (2, 4, 6), (2, 5, 8), (3, 5, 7), (3, 6, 9).

[illegible] = 3, 18 combinations are possible.

If we take [illegible] = 0, 36 combinations are possible. Then every two combinations differ by 1: (1, 1, 2), (2, 2, 4), (3, 3, 6), (4, 4, 8), (5, 5, 10), (6, 6, 12).

[illegible] total of 36 combinations.

$$= 2 \times 3 + 18 \times 4 + \ldots + 4 \times 2$$

$$= 6 + 18 \times 4 + \ldots + 8$$

$$= 1 + \frac{37 \times 37}{2} = 666$$

3

MOCK TEST PAPER—3

1. Find the number of common terms to the series $a, a+d, a+2d, \ldots$ and $a, a+r, a+r^2, \ldots$ where $a = 1$, $d = 2$, $r = 3$, n and m are the greatest integer less than or equal to x.

   (1) 0 (2) 1 (3) 2 (4) 3

2. The number of subsets of set of positive integers $\{x_1, x_2, x_3, \ldots, x_n\}$ such that $\frac{x_1}{x_2} + \frac{x_2}{x_3} + \frac{x_3}{x_4} + \ldots + \frac{x_{n-1}}{x_n} \le 1$

   (1) 0 (2) 2 (3) 3 (4) 4

3. Let y be a function such that for each $x \in (0, 1)$, $f(x) = \int_0^x \sin(\pi(x+t))\, dt - \int_0^x \frac{f(t)}{f(1-t)} \cos(\pi t)\, dt$ for $x \in (0, 1)$.

   Then find $f(1/2) = 1$ and $f(1) = 0$, then $f(x) + f(1-x) =$

   (1) cannot be negative
   (2) has minimum value 2/3
   (3) has maximum value 2/3
   (4) none of these

4. $\lim_{n\to\infty} \frac{1^2}{n^3} + \frac{2^2}{n^3} + \frac{3^2}{n^3} + \ldots + \frac{n^2}{n^3} = \frac{\sin(\pi^2)}{\cos(\pi)}$ and $\lim_{n\to\infty} \frac{1^3}{n^4} + \frac{2^3}{n^4} + \frac{3^3}{n^4} + \ldots + \frac{n^3}{n^4} = \frac{\cos(\pi^2)}{\sin(\pi)}$. The weight consist to n, is

   (1) 500 (2) 300
   (3) 100 (4) 100

5. Which of the following statements is false?

   (1) The product of three consecutive even numbers can be written as 48.
   (2) (n) of n (n) is same as (n) of (n) of n.
   (3) The interval along which $\sin(\sin x)$ is increasing is $[-\frac{\pi}{2}, \frac{\pi}{2}]$ where x lies in a perfect square.
   (4) The numbers $(101)_2$, $(202)_3$, $(303)_4$, $(404)_5$ ... and so on when converted to decimal system are in an arithmetic progression.

6. Find the weight of the wooden wheel of cylinder of height 40 cm and a inner diameter of $\frac{1}{2}$ cm. The wheel makes exactly four complete turns and the cylinder slide to two sides touch the cylinder top and bottom.

   (1) 200 gm (2) 800 gm
   (3) 600 gm (4) Cannot be determined

7. In Coorg, the production of tea is three times the production of coffee. If 5 percent more tea and 5 percent more coffee were produced, the agricultural output would be 5 percent more. But if 5 percent more tea and 5 percent more coffee were produced the aggregate output produced would be 5 percent more. What is the ratio $x : y$?

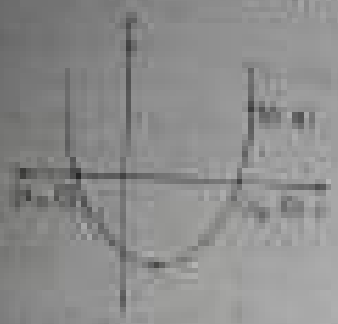
   (1) 1 : 2 (2) 1 : 3
   (3) 2 : 1 (4) 3 : 1

8. A girl wished to purchase 10 roses for a rupee, or 100 for one rupee. The shopkeeper offered to give the remaining 10 flowers free if she gave him a rupee of Rs. 1. This would have resulted in a gain of 33 paise per dozen for her. How many roses do she wish to buy initially?

   (1) 5 (2) 6 (3) 8 (4) 9

[illegible] (b) 6 (d) 7

[illegible] Directions for [illegible]: The graph given below represents quadratic expression, $f(x) = ax^2 + bx + c$. The numerical values of $f(x)$ for $x = -2, -1, 0, 1, 2$ [illegible]



[illegible] are such that $x_1 = x_2 + 2$ [illegible]

[illegible] What is the nature of the roots of the equation [illegible]

(1) Complex conjugates
(2) Rational
(3) Irrational
(4) Cannot be determined

[illegible] Find the value of the [illegible] at $x = 0$

(a) 2 (b) −1
(c) 3 (d) None of these

[illegible] How many numbers are there between 100 and 1000 which have exactly one of their digits as 7?

(1) 300 (2) 1000
(3) 252 (4) 729

[illegible] A group of [illegible] was employed to make 360 [illegible]. Every day after the first, it was noticed that the previous day more pot [illegible] than the previous day. The [illegible] that during the latter part of the period, the number of [illegible] started to decrease. 3 days were required to make the work.

What was the number of [illegible] employed on the third day?

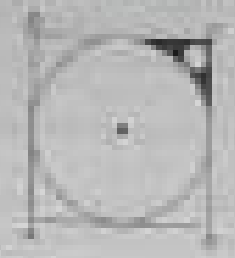
(1) 15 (2) 180 (3) 30 (4) 120

[illegible] A group of [illegible] employed to make 360 [illegible]. Every day after the first, it [illegible] that the previous day more pot [illegible] than the previous day. The [illegible] that during the latter part of the period, the number of [illegible] started to decrease. 3 days were required to [illegible] the work.

What was the number of [illegible] employed on the third day?

(1) 15 (2) 20 (3) 30 (4) 9

[illegible] In the figure given below the radius of the circle is [illegible] cm. Then the area of shaded region will be



(1) $5 - \frac{1}{2}(\sqrt{2} - 1)\pi^2$

(2) $\frac{1}{4}(4 - \pi) + \frac{1}{2}(\sqrt{2} - 1)\pi^2$

(3) $\frac{1}{4}(4 - 3\pi) + \frac{1}{2}(\sqrt{2} - 1)\pi^2$

(4) Data insufficient

[illegible] Two [illegible] group [illegible] students. The [illegible] rules the up [illegible] function [illegible] up [illegible] 100 [illegible] is [illegible] rounded. The [illegible] down [illegible] [illegible] the [illegible] 100. How many [illegible] down for [illegible]?

(1) 70 (2) 60 (3) 80 (4) 20

[illegible] How many [illegible] way the numbers 1, 2, 3, 4, 5, 6 [illegible] such that [illegible] have a [illegible] [illegible]?

(1) 0 (2) 12 (3) 15 (4) [illegible]

[illegible] Let $f(x)$ denote the [illegible] of the [illegible] [illegible] [illegible] [illegible] [illegible]

(1) 18 (2) 30 (3) 60 (4) 9

[illegible] A [illegible] teacher [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] a random sample of 51 fish and tags it and releases it back into the water. The number (51) of the fish in the sample is the number that is normal prior and that will of the fish in the lake that have been tagged by both years. What was the estimate of the number of fish in the lake on the initial test?

(a) 100 (b) 200 (c) 300 (d) 750

8. Mighty Sachin's probability of hitting 100 basketball shots is [illegible] consecutive, which never exceeds a total. To hold him to a point, Sachin is hit a [illegible] and was [illegible] times. If Sachin [illegible] that randomly, what is the [illegible] probability of winning [illegible] (in [illegible])?

(a) $\frac{1}{10}$ (b) $\frac{1}{12}$ (c) $\frac{2}{9}$ (d) [illegible]

Answer Key

1. (a)	2. (d)	3. (b)	4. (b)	5. (c)
6. (c)	7. (a)	8. (c)	9. (d)	10. (b)
11. (d)	12. (b)	13. (c)	14. (a)	15. (d)
16. (c)	17. (c)	18. (b)	19. (c)	20. (b)

Detailed Solutions & Short Cuts

Solution 1: LOD 2

Since a prime factor [illegible] $\frac{7x}{25}$ must be an integer.

[illegible] 90, 72, 90, 108 and 171 [illegible] 36 and 72 [illegible] the given number.

Solution 2: LOD 2

$$\frac{P}{W} = \frac{3}{4} = \frac{1}{\frac{W}{P}} = \frac{W+P}{W} = \frac{7}{12} \Rightarrow 75,000 = 7(16+x) = 112$$

Adding 112 [illegible] $\Rightarrow 112 + 112 = 224$ [illegible] $\Rightarrow 224 - 16 = 208$

[illegible] 1, 2, 4, 8, 16, 32, 64, 128, 256, 512. Hence [illegible] of the 11 [illegible] the given [illegible].

Solution 3: LOD 2

The given progression [illegible] $\frac{1}{3}$. Given that [illegible] $= 3$. So the [illegible] is $f(x) = 3^x$

$$\therefore f(x) + f(-x) = 3^{x} + 3^{-x}$$

$$= \frac{1}{3}(3^x + 3^{-x}) \geq \frac{1}{3} \cdot 2\sqrt{3^x \cdot 3^{-x}}$$

$$\left[\because AM \geq GM\right]$$

$$\Rightarrow 3^x + 3^{-x} \geq 2/3$$

Hence the minimum value of $3^x + 3^{-x} = 2/3$.

Solution 4: LOD 3

$$x = 1 + \frac{1^2}{1} + \frac{2^2 - 1^2}{2} + \frac{3^2 - 2^2}{3} + \frac{4^2 - 3^2}{4} + \ldots$$

$$\frac{100^2 - 100^2}{100} = \frac{100^2}{100}$$

Since $\frac{(n+1)^2 - n^2}{2n+1} = \frac{2n+1}{2n+1} = 1$ for $n = 1, 2, 3, \ldots$ [illegible]

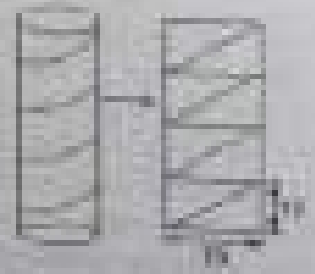
$$x = 1 + \underbrace{1 + 1 + \ldots + 1}_{100} = \frac{100^2}{100} = 100 \Rightarrow 100 + 1 = 101$$

Hence, $x = 101$.

Solution 5: LOD 1

The [illegible] x^2, y^2, z^2, x^3 [illegible] are not in an arithmetic progression. Only other statement always holds true.

Solution 6: LOD 2



The base circumference $= \frac{22}{7} \times \frac{56}{11} = 16$ cm. The [illegible] cylinder here $= \sqrt{16^2 + 12^2} = 20$ cm. Hence [illegible] length $= 84$ cm.

Solution 7: LOD 4

[illegible]

Solution 8: LOD 3

[illegible]

Solution 9: LOD 1

[illegible]

Solution 10: LOD 2

[illegible]

Solution 11: LOD 1

[illegible]

Solution 12: LOD 1

Solution 13: LOD 2

[illegible]

Solution 14: LOD 2

[illegible]

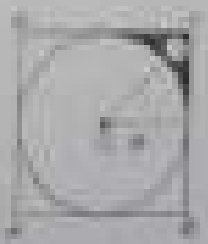
Solution 15: LOD 1

[illegible]

Solution 16: LOD 2

[illegible]

Solution 17: LOD 3



[illegible]

$$\Rightarrow (x+1)(x+4)(x^2-x+1) = \frac{1}{2}\left[(x+1)+\frac{1}{2}\sqrt{3}(x+1)-(x+1)\sqrt{3}\right]$$

$$\Rightarrow (x+1)(x^2-3x+4) = \frac{1}{2}(x+1)(1+\sqrt{3}+1-\sqrt{3})$$

$$\Rightarrow (x+1)(x^2-3x+4) = \frac{1}{2}(x+1)(2)$$

$$\Rightarrow (x+1)\left(x+\frac{3}{2}\right)\left(x-\frac{1}{2}\sqrt{3}\right)\left(x+\frac{1}{2}\sqrt{3}\right)$$

$$\Rightarrow \frac{1}{2}(x+1)(2x+3)(2x-\sqrt{3})(2x+\sqrt{3})$$

Solution 16: LOD 2

Some need to save the other team is much the way. So up [illegible] has caused her. Divided 50 steps, because she [illegible] length of the escalator is 40 steps and the back 10 steps [illegible]. Therefore, in the same time the slower escalator pushed the younger sister back 40 steps. To make up this setback and reach the original 40 steps separating her from the top, the younger sister must make a total of 50 steps.

Solution 17: LOD 2

Each of the numbers from 1 to 9 can only have two possible neighbors to avoid sum including 1 or 2 as multiples. Two neighbors with 2 or 5, 10 are some the numbers 1, 4, 7, 2, 5, 8 and 9 so the order is a clig. Starting 9, number will be said to 10. The positive neighbors. Therefore, to make the six numbers successfully we can only start at any point along the ring (9 choices) and the number to they appear around the ring is only determined (1 choices) for a total of 12 possible ordering.

Solution 18: LOD 2

$$\sin 5\pi/7 + \sin 3\pi/7 = 2\sin(\pi/7)\cos 2\pi/7 = 2\sin \quad \cos 2^{\pi/7} \cos 4\pi/7 = \sin$$

Solution 19: LOD 2

Suppose there are N Fisher normally. Then after 4 months, $N + 75\%N/50\% = 150\%$ Fishes, and $M + 75\% = 87$ [illegible]. Therefore $4N = 150\%N = 2/3$, so $N = 600$.

Solution 20: LOD 2

If I choose a door that $2 \le k \le$ made with probability p where p is between 0 and 1/3, then Michael Fisher k [illegible] door and with probability $1/p$. Hence, the probability that I make a door that Fisher subsequently moves is $p \cdot \frac{1}{3p} = \frac{1}{3}$. The graph of this function is a parabola which [illegible] this when $p = 0$ and $p = \frac{1}{3}$. By symmetry, the [illegible] maximum is achieved between the two, at $p = \frac{1}{6}$.

Thus, the best chance of being at $\frac{1}{6}$ [illegible] the game is $\frac{1}{12}$.

MODEL TEST PAPER
(BASED ON THE LATEST ONLINE PATTERN)

Instructions:

Time: 2hr 15min

This test contains 5 sections having 20 questions each. In order to reach maximum score, you would need to successfully get a total 100% accuracy. However an accuracy of 80% you would need about 80 answers with about 70 correct questions.

SECTION I

QUANTITATIVE APTITUDE

1. [illegible] gave his younger sister a penny on every day. He [illegible] a formula which gives a penny on the 1st day, 2 the second, 100 on a third, second gave another penny [illegible] of the second, 122 on 3. He asked her to [illegible] Help her find the number.

(a) [illegible] (b) 34
(c) [illegible] (d) 48

2. [illegible] to write all the two-digit numbers such that [illegible] are written one after the other in a [illegible] how many digits are there in the resulting number?

(a) 18 (b) 72
(c) 36 (d) None of these

3. [illegible] of length [illegible] is [illegible] [illegible] and shall be in the shape of an equilateral triangle. Find the sum of the largest volumes and which [illegible] and hence?

(a) [illegible] cm
(b) [illegible] cm
(c) [illegible] cm (d) None of these

4. [illegible] 15% of the facilities are of conditioners, 17% are cooler and 75% are generators. What is the minimum percentage of facilities that contain the three items?

(a) 10 (b) 25
(c) 20 (d) 35

Answer for Questions 5 and 6

[illegible] on the following information

[illegible] for sale held at the Annual College Sports Fest [illegible] Adeyo starts [illegible] Adeyo's. When Yoshpal leaves at [illegible] Adeyo's speed by 2 more than that of Yoshpal [illegible] Adeyo and Yoshpal [illegible] Yoshpal [illegible] Adeyo and Yoshpal [illegible] Yoshpal [illegible] Adeyo

5. The speed of Adeyo and Yoshpal is

(a) 7 km/hr (b) 5.5 km/hr
(c) 2 km/hr (d) 1 km/hr

6. The speed of Yoshpal is

(a) 2.6 km/hr (b) [illegible]
(c) 3.1 km/hr (d) 4 km/hr

7. The value of the expression [illegible] is equal to

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

8. A book contains 20 chapters. Each chapter has [illegible] and hundred of pages (say [illegible]). In the first chapter [illegible] in page 1 and each chapter [illegible]. What is the largest possible number of chapters that can begin on odd page numbers?

(a) 19 (b) 17
(c) 18 (d) 11

9. How many positive two digit integers in the sequence [illegible] are divisible by 3 or 7, or 9 or 11, or 13 or 17?

(a) 428 (b) 416
(c) 410 (d) 322

10. The number of ordered pairs [illegible] satisfying the [illegible] is

(a) 2 (b) 3
(c) 5 (d) 1

11. Two schools play against each other in a game [illegible] each school [illegible] every group of students [illegible] the (ith) [illegible] every student [illegible] every student gets [illegible] the [illegible] group [illegible] each other play?

(a) 156 (b) 180
(c) 187 (d) 360

12. [illegible] and 1 are available. What is the possible number of [illegible] of these numbers [illegible] and is odd the number [illegible]?

(a) 12 (b) 8
(c) 15 (d) 26
(e) 21

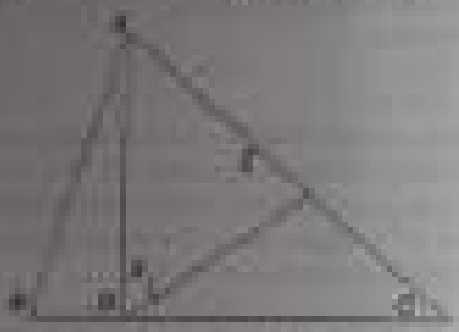
13. A teacher [illegible] of [illegible] [illegible] [illegible]

[illegible] for the number "Rajib found the [illegible] sum. What was the number?

(a) 1 (b) 6
(c) 8 (d) 9

17. In figure (ab = ac, angle BAD = 60°, and AD = AE). Then x equals:

(a) 15 (b) 20
(c) 30 (d) None of these



18. The perimeter of a triangle is 100 cm. The ratio of its altitudes is 2 : 3 : 4. Find the sides of the triangle.

(a) 22, 26, 27 (b) 18, 27, 28
(c) 24, 25, 25 (d) 20, 30, 25

19. There are two spheres and one cube. The cube is inside the bigger sphere and the smaller sphere is inside the cube. Find the ratio of surface areas of the bigger sphere to the smaller sphere.

(a) 2 : 1 (b) 2 : 1
(c) 4 : 1 (d) 2 : 1

17. Adjusting the clock to get rid of the problem [illegible] a pocket watch [illegible] regulator. It proved [illegible] costing a cost of Rs. 10,000 [illegible] price [illegible] a part of advertisement and [illegible] profit. It allowed a discount of 20% on the [illegible] price and gave one extra watch when a watch [illegible] purchased at retail. Could all the watches be [illegible] the published price (Rs. 11,25). Find the gain or [illegible] percentage for the tak.

(a) 10.5% gain (b) 11.5% gain
(c) 5% loss (d) None of these

18. What is the maximum value of the form [illegible] $4x + 6y$?

(a) 12 (b) 10
(c) 11 (d) 8

19. How many integer values for the ar[illegible] satisfy the inequation $-4 < |y - 2| < 5$?

(a) 10 (b) 14
(c) 12 (d) 16

20. How many real solutions exist for the equation $x^2 - 2 = |x + 1|$?

(a) 2 (b) 3
(c) 5 (d) 1

SECTION II

[illegible]

Questions 31 and 32

[illegible]

- [illegible]
- [illegible]
- [illegible]
- [illegible]
- [illegible]

[illegible]

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

[illegible]

I. [illegible]
II. [illegible]
III. [illegible]

(a) I only (b) [illegible]
(c) I and II only (d) [illegible]

Questions for questions 33 and 34

[illegible]

- [illegible]
- [illegible]
- [illegible]

[illegible]

- [illegible]
- [illegible]
- [illegible]

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

[illegible]

(a) [illegible] (b) [illegible]
(c) [illegible] (d) [illegible]

Questions for Questions 35 & 36

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]
6. [illegible]
7. [illegible]
8. [illegible]
9. [illegible]
10. [illegible]
11. [illegible]
12. [illegible]
13. [illegible]
14. [illegible]
15. [illegible]
16. [illegible]
17. [illegible]
18. [illegible]
19. [illegible]
20. Who was the presidential candidate [illegible]?
    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]
21. What was the total number of points [illegible] in the [illegible] round?
    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) None of these
22. Who was the Sergeant?
    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]
23. [illegible]
    (a) [illegible]
    (b) [illegible]
    (c) [illegible]
    (d) [illegible]
    (e) [illegible]
    (f) [illegible]
    (g) [illegible]
    (h) [illegible]
24. How many contestants had a cancelled loan in the first round?
    (a) [illegible] (b) [illegible]
    (c) [illegible] (d) [illegible]

[illegible] (Questions 11 and 12)

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

11. [illegible] for the [illegible] and [illegible] for the [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]?
    (i) [illegible]
    (ii) [illegible]
    (iii) [illegible]
    (iv) [illegible]
    (v) [illegible]
    (vi) [illegible]
    (vii) [illegible]
    (viii) [illegible]
    (ix) [illegible]
    (x) [illegible]
    (xi) [illegible]
    (xii) [illegible]
    (xiii) [illegible]
    (xiv) [illegible]
    (xv) [illegible]
    (xvi) [illegible]
    (xvii) [illegible]
    (xviii) [illegible]
    (xix) [illegible]
    (xx) [illegible]
    (xxi) [illegible]
    (xxii) [illegible]
    (xxiii) [illegible]
    (xxiv) [illegible]
    (xxv) [illegible]
    (xxvi) [illegible]
    (xxvii) [illegible]
    (xxviii) [illegible]
    (xxix) [illegible]
    (xxx) [illegible]
    (xxxi) [illegible]
    (xxxii) [illegible]
    (xxxiii) [illegible]
    (xxxiv) [illegible]
    (xxxv) [illegible]
    (xxxvi) [illegible]
    (xxxvii) [illegible]
    (xxxviii) [illegible]
    (xxxix) [illegible]
    (xl) [illegible]
    (xli) [illegible]
    (xlii) [illegible]
    (xliii) [illegible]
    (xliv) [illegible]
    (xlv) [illegible]
    (xlvi) [illegible]
    (xlvii) [illegible]
    (xlviii) [illegible]
    (xlix) [illegible]
    (l) [illegible]
    (li) [illegible]
    (lii) [illegible]
    (liii) [illegible]
    (liv) [illegible]
    (lv) [illegible]
    (lvi) [illegible]
    (lvii) [illegible]
    (lviii) [illegible]
    (lix) [illegible]
    (lx) [illegible]
    (lxi) [illegible]
    (lxii) [illegible]
    (lxiii) [illegible]
    (lxiv) [illegible]
    (lxv) [illegible]
    (lxvi) [illegible]
    (lxvii) [illegible]
    (lxviii) [illegible]
    (lxix) [illegible]
    (lxx) [illegible]
    (lxxi) [illegible]
    (lxxii) [illegible]
    (lxxiii) [illegible]
    (lxxiv) [illegible]
    (lxxv) [illegible]
    (lxxvi) [illegible]
    (lxxvii) [illegible]
    (lxxviii) [illegible]
    (lxxix) [illegible]
    (lxxx) [illegible]
    (lxxxi) [illegible]
    (lxxxii) [illegible]
    (lxxxiii) [illegible]
    (lxxxiv) [illegible]
    (lxxxv) [illegible]
    (lxxxvi) [illegible]
    (lxxxvii) [illegible]
    (lxxxviii) [illegible]
    (lxxxix) [illegible]
    (xc) [illegible]
    (xci) [illegible]
    (xcii) [illegible]
    (xciii) [illegible]
    (xciv) [illegible]
    (xcv) [illegible]
    (xcvi) [illegible]
    (xcvii) [illegible]
    (xcviii) [illegible]
    (xcix) [illegible]
    (c) [illegible]
    (ci) [illegible]
    (cii) [illegible]
    (ciii) [illegible]
    (civ) [illegible]
    (cv) [illegible]
    (cvi) [illegible]
    (cvii) [illegible]
    (cviii) [illegible]
    (cix) [illegible]
    (cx) [illegible]
    (cxi) [illegible]
    (cxii) [illegible]
    (cxiii) [illegible]
    (cxiv) [illegible]
    (cxv) [illegible]
    (cxvi) [illegible]
    (cxvii) [illegible]
    (cxviii) [illegible]
    (cxix) [illegible]
    (cxx) [illegible]
    (cxxi) [illegible]
    (cxxii) [illegible]
    (cxxiii) [illegible]
    (cxxiv) [illegible]
    (cxxv) [illegible]
    (cxxvi) [illegible]
    (cxxvii) [illegible]
    (cxxviii) [illegible]
    (cxxix) [illegible]
    (cxxx) [illegible]
    (cxxxi) [illegible]
    (cxxxii) [illegible]
    (cxxxiii) [illegible]
    (cxxxiv) [illegible]
    (cxxxv) [illegible]
    (cxxxvi) [illegible]
    (cxxxvii) [illegible]
    (cxxxviii) [illegible]
    (cxxxix) [illegible]
    (cxl) [illegible]
    (cxli) [illegible]
    (cxlii) [illegible]
    (cxliii) [illegible]
    (cxliv) [illegible]
    (cxlv) [illegible]
    (cxlvi) [illegible]
    (cxlvii) [illegible]
    (cxlviii) [illegible]
    (cxlix) [illegible]
    (cl) [illegible]
    (cli) [illegible]
    (clii) [illegible]
    (cliii) [illegible]
    (civ) [illegible]
    (clv) [illegible]
    (clvi) [illegible]
    (clvii) [illegible]
    (clviii) [illegible]
    (clix) [illegible]
    (clx) [illegible]
    (clxi) [illegible]
    (clxii) [illegible]
    (clxiii) [illegible]
    (clxiv) [illegible]
    (clxv) [illegible]
    (clxvi) [illegible]
    (clxvii) [illegible]
    (clxviii) [illegible]
    (clxix) [illegible]
    (clxx) [illegible]
    (clxxi) [illegible]
    (clxxii) [illegible]
    (clxxiii) [illegible]
    (clxxiv) [illegible]
    (clxxv) [illegible]
    (clxxvi) [illegible]
    (clxxvii) [illegible]
    (clxxviii) [illegible]
    (clxxix) [illegible]
    (clxxx) [illegible]
    (clxxxi) [illegible]
    (clxxxii) [illegible]
    (clxxxiii) [illegible]
    (clxxxiv) [illegible]
    (clxxxv) [illegible]
    (clxxxvi) [illegible]
    (clxxxvii) [illegible]
    (clxxxviii) [illegible]
    (clxxxix) [illegible]
    (cxc) [illegible]
    (cxci) [illegible]
    (cxcii) [illegible]
    (cxciii) [illegible]
    (cxciv) [illegible]
    (cxcv) [illegible]
    (cxcvi) [illegible]
    (cxcvii) [illegible]
    (cxcviii) [illegible]
    (cxcix) [illegible]
    (cc) [illegible]
    (cci) [illegible]
    (ccii) [illegible]
    (cciii) [illegible]
    (cciv) [illegible]
    (ccv) [illegible]
    (ccvi) [illegible]
    (ccvii) [illegible]
    (ccviii) [illegible]
    (ccix) [illegible]
    (ccx) [illegible]
    (ccxi) [illegible]
    (ccxii) [illegible]
    (ccxiii) [illegible]
    (ccxiv) [illegible]
    (ccxv) [illegible]
    (ccxvi) [illegible]
    (ccxvii) [illegible]
    (ccxviii) [illegible]
    (ccxix) [illegible]
    (ccxx) [illegible]
    (ccxxi) [illegible]
    (ccxxii) [illegible]
    (ccxxiii) [illegible]
    (ccxxiv) [illegible]
    (ccxxv) [illegible]
    (ccxxvi) [illegible]
    (ccxxvii) [illegible]
    (ccxxviii) [illegible]
    (ccxxix) [illegible]
    (ccxxx) [illegible]
    (ccxxxi) [illegible]
    (ccxxxii) [illegible]
    (ccxxxiii) [illegible]
    (ccxxxiv) [illegible]
    (ccxxxv) [illegible]
    (ccxxxvi) [illegible]
    (ccxxxvii) [illegible]
    (ccxxxviii) [illegible]
    (ccxxxix) [illegible]
    (ccxl) [illegible]
    (ccxli) [illegible]
    (ccxlii) [illegible]
    (ccxliii) [illegible]
    (ccxliv) [illegible]
    (ccxlv) [illegible]
    (ccxlvi) [illegible]
    (ccxlvii) [illegible]
    (ccxlviii) [illegible]
    (ccxlix) [illegible]
    (ccl) [illegible]
    (ccli) [illegible]
    (cclii) [illegible]
    (ccliii) [illegible]
    (cclv) [illegible]
    (cclvi) [illegible]
    (cclvii) [illegible]
    (cclviii) [illegible]
    (cclix) [illegible]
    (cclx) [illegible]
    (cclxi) [illegible]
    (cclxii) [illegible]
    (cclxiii) [illegible]
    (cclxiv) [illegible]
    (cclxv) [illegible]
    (cclxvi) [illegible]
    (cclxvii) [illegible]
    (cclxviii) [illegible]
    (cclxix) [illegible]
    (cclxx) [illegible]
    (cclxxi) [illegible]
    (cclxxii) [illegible]
    (cclxxiii) [illegible]
    (cclxxiv) [illegible]
    (cclxxv) [illegible]
    (cclxxvi) [illegible]
    (cclxxvii) [illegible]
    (cclxxviii) [illegible]
    (cclxxix) [illegible]
    (cclxxx) [illegible]
    (cclxxxi) [illegible]
    (cclxxxii) [illegible]
    (cclxxxiii) [illegible]
    (cclxxxiv) [illegible]
    (cclxxxv) [illegible]
    (cclxxxvi) [illegible]
    (cclxxxvii) [illegible]
    (cclxxxviii) [illegible]
    (cclxxxix) [illegible]
    (ccc) [illegible]
    (ccci) [illegible]
    (cccii) [illegible]
    (ccciii) [illegible]
    (ccciv) [illegible]
    (cccv) [illegible]
    (cccvi) [illegible]
    (cccvii) [illegible]
    (cccviii) [illegible]
    (cccix) [illegible]
    (cccx) [illegible]
    (cccxi) [illegible]
    (cccxii) [illegible]
    (cccxiii) [illegible]
    (cccxiv) [illegible]
    (cccxv) [illegible]
    (cccxvi) [illegible]
    (cccxvii) [illegible]
    (cccxviii) [illegible]
    (cccxix) [illegible]
    (cccxx) [illegible]
    (cccxxi) [illegible]
    (cccxxii) [illegible]
    (cccxxiii) [illegible]
    (cccxxiv) [illegible]
    (cccxxv) [illegible]
    (cccxxvi) [illegible]
    (cccxxvii) [illegible]
    (cccxxviii) [illegible]
    (cccxxix) [illegible]
    (cccxxx) [illegible]
    (cccxxxi) [illegible]
    (cccxxxii) [illegible]
    (cccxxxiii) [illegible]
    (cccxxxiv) [illegible]
    (cccxxxv) [illegible]
    (cccxxxvi) [illegible]
    (cccxxxvii) [illegible]
    (cccxxxviii) [illegible]
    (cccxxxix) [illegible]
    (cccxl) [illegible]
    (cccxli) [illegible]
    (cccxlii) [illegible]
    (cccxliii) [illegible]
    (cccxliv) [illegible]
    (cccxlv) [illegible]
    (cccxlvi) [illegible]
    (cccxlvii) [illegible]
    (cccxlviii) [illegible]
    (cccxlix) [illegible]
    (cccl) [illegible]
    (ccccli) [illegible]
    (ccccliii) [illegible]
    (ccccliv) [illegible]
    (ccccliv) [illegible]
    (ccccliv

[illegible] (Q.Nos. [illegible])

Each problem consists of a statement and two statements [illegible] statements A and B, giving some information. You have to select the correct answer from (1) to (4) according to the rules of the Integrated Test statement to answer the question. Mark your answer as:

Option (a): If the question can be answered by using only one of the statements alone but cannot be answered by using the other statement alone.

Option (b): If the question can be answered by using either statement alone.

Option (c): If the question can be answered by using both the statements together but cannot be answered by using either statement alone.

Option (d): If the question cannot be answered even by using both the statements together.

26. At Zola Zola in the seventh Century of the calendar, [illegible] AA, AB, AC, AD, AE, AF, AG, AH and AI, how many boy pairs are there? Can we know that the concept of pair change in Zola Zola to the pair in India also?
    A. The ratio of AB to AH is 4 to 3 [illegible]
    B. Number of pairs AB, AD and AH is [illegible] to 100 A [illegible] and [illegible] during a group activity there are some male and female students in a group of [illegible] students. How many boys are there?
    A. If all the females are to be asked together in groups of [illegible] for statement A only, [illegible] the number of boys of [illegible] the [illegible] are to asked together.
    B. The ratio of the males to the females is [illegible] 11:12.
28. [illegible] of the [illegible] students in DM Public School [illegible] graduate?
    A. There are [illegible] graduate [illegible] the student in DM Public.
    B. The number of Non-[illegible] graduate is [illegible] Bachelor [illegible] the [illegible] number of students.
30. [illegible] the High school [illegible] in which [illegible] girls [illegible] giving [illegible] to [illegible] question. "What is the [illegible] of the [illegible] number?"
    (a) The ratio of the number of the [illegible] students [illegible] to 44.
    (b) The ratio of the qualifying [illegible] to [illegible].
31. How [illegible] in the [illegible] a [illegible] of the [illegible] [illegible] [illegible]

SECTION III

VERBAL ABILITY & READING COMPREHENSION

Directions for Q. 1 to 15

Select the best (or nearly) synonym for the underscored word.

1. "[illegible] the origins of 'the caste?'" asserted by one reading in his autobiography. Years later he made much more [illegible] to suffer in one sitting. Taking the last couple of hours every day had usually [illegible] a couple of unknown trips to [illegible]. A party of [illegible] gave [illegible] Gangotri Khola's [illegible]. He [illegible] whether the Mungo [illegible] had [illegible] a [illegible] [illegible] wrong?' Life, [illegible] [illegible] [illegible]

[illegible]

[illegible]

[illegible]

(c) [illegible]

(d) [illegible]

(e) [illegible]

(f) [illegible]

(g) [illegible]

(h) [illegible]

(i) [illegible]

(j) [illegible]

(k) [illegible]

(l) [illegible]

(m) [illegible]

(n) [illegible]

(o) [illegible]

(p) [illegible]

(q) [illegible]

(r) [illegible]

(s) [illegible]

(t) [illegible]

(u) [illegible]

(v) [illegible]

(w) [illegible]

(x) [illegible]

(y) [illegible]

(z) [illegible]

(aa) [illegible]

(bb) [illegible]

(cc) [illegible]

(dd) [illegible]

(ee) [illegible]

(ff) [illegible]

(gg) [illegible]

(hh) [illegible]

(ii) [illegible]

(jj) [illegible]

(kk) [illegible]

(ll) [illegible]

(mm) [illegible]

(nn) [illegible]

(oo) [illegible]

(pp) [illegible]

(qq) [illegible]

(rr) [illegible]

(ss) [illegible]

(tt) [illegible]

(uu) [illegible]

(vv) [illegible]

(ww) [illegible]

(xx) [illegible]

(yy) [illegible]

(zz) [illegible]

(aaa) [illegible]

(bbb) [illegible]

(ccc) [illegible]

(ddd) [illegible]

(eee) [illegible]

(fff) [illegible]

(ggg) [illegible]

(hhh) [illegible]

(iii) [illegible]

(jjj) [illegible]

(kkk) [illegible]

(lll) [illegible]

(mmm) [illegible]

(nnn) [illegible]

(ooo) [illegible]

(ppp) [illegible]

(qqq) [illegible]

(rrr) [illegible]

(sss) [illegible]

(ttt) [illegible]

(uuu) [illegible]

(vvv) [illegible]

(www) [illegible]

(xxx) [illegible]

(yyy) [illegible]

(zzz) [illegible]

(aaaa) [illegible]

(bbbb) [illegible]

(cccc) [illegible]

(dddd) [illegible]

(eeee) [illegible]

(ffff) [illegible]

(gggg) [illegible]

(hhhh) [illegible]

(iiii) [illegible]

(jjjj) [illegible]

(kkkk) [illegible]

(llll) [illegible]

(mmmm) [illegible]

(nnnn) [illegible]

(oooo) [illegible]

(pppp) [illegible]

(qqqq) [illegible]

(rrrr) [illegible]

(ssss) [illegible]

(tttt) [illegible]

(uuuu) [illegible]

(vvvv) [illegible]

(wwww) [illegible]

(xxxx) [illegible]

(yyyy) [illegible]

(zzzz) [illegible]

(aaaaa) [illegible]

(bbbbb) [illegible]

(ccccc) [illegible]

(ddddd) [illegible]

(eeeee) [illegible]

(fffff) [illegible]

(ggggg) [illegible]

(hhhhh) [illegible]

(iiiii) [illegible]

(jjjjj) [illegible]

(kkkkk) [illegible]

(lllll) [illegible]

(mmmmm) [illegible]

(nnnnn) [illegible]

(ooooo) [illegible]

(ppppp) [illegible]

(qqqqq) [illegible]

(rrrrr) [illegible]

(sssss) [illegible]

(ttttt) [illegible]

(uuuuu) [illegible]

(vvvvv) [illegible]

(wwwww) [illegible]

(xxxxx) [illegible]

(yyyyy) [illegible]

(zzzzz) [illegible]

(aaaaaa) [illegible]

(bbbbbb) [illegible]

(cccccc) [illegible]

(dddddd) [illegible]

(eeeeee) [illegible]

(ffffff) [illegible]

(gggggg) [illegible]

(hhhhhh) [illegible]

(iiiiii) [illegible]

(jjjjjj) [illegible]

(kkkkkk) [illegible]

(llllll) [illegible]

(mmmmmm) [illegible]

(nnnnnn) [illegible]

(oooooo) [illegible]

(pppppp) [illegible]

(qqqqqq) [illegible]

(rrrrrr) [illegible]

(ssssss) [illegible]

(tttttt) [illegible]

(uuuuuu) [illegible]

(vvvvvv) [illegible]

(wwwwww) [illegible]

(xxxxxx) [illegible]

(yyyyyy) [illegible]

(zzzzzz) [illegible]

[illegible]

[illegible]

[illegible]

ANSWER KEY

1. (B)	2. (B)	3. (B)	4. (B)	5. (A)	31. (A)	32. (A)	33. (B)	34. (D)	61. (C)	62. (D)	63. (B)	64. (C)	91. (A)	92. (A)	93. (B)	94. (C)
6. (B)	7. (C)	8. (B)	9. (B)	10. (B)	35. (C)	36. (D)	37. (D)	38. (D)	65. (D)	66. (D)	67. (D)	68. (D)	95. (D)	96. (D)	97. (D)	98. (C)
11. (B)	12. (A)	13. (A)	14. (A)	15. (B)	39. (B)	40. (B)	41. (B)	42. (B)	69. (B)	70. (B)	71. (B)	72. (B)	99. (B)	100. (B)	101. (B)	102. (C)
16. (C)	17. (A)	18. (A)	19. (A)	20. (A)	43. (D)	44. (D)	45. (D)	46. (D)	73. (D)	74. (D)	75. (D)	76. (D)	103. (D)	104. (D)	105. (D)	106. (D)
21. (B)	22. (B)	23. (B)	24. (B)	25. (B)	47. (D)	48. (D)	49. (D)	50. (D)	77. (D)	78. (D)	79. (D)	80. (D)	107. (D)	108. (D)	109. (D)	110. (D)

ANSWERS WITH SOLUTIONS

QUESTIONS ANSWERED

[illegible]

[illegible]

[illegible]

[illegible]

(b) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(c) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(d) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(e) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(f) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(g) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(h) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(i) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(j) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(k) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(l) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(m) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(n) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(o) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(p) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(q) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(r) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(s) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(t) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(u) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(v) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(w) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(x) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(y) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(z) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(aa) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ab) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ac) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ad) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ae) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(af) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ag) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ah) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ai) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(aj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ak) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(al) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(am) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(an) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ao) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ap) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(aq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ar) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(as) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(at) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(au) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(av) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(aw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ax) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ay) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(az) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ba) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bd) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(be) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bf) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bi) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bl) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bo) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(br) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bs) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bt) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bu) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bx) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(by) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(bz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ca) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cd) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ce) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cf) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ch) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ci) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ck) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cl) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(co) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cs) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ct) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cu) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cx) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cy) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(cz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(da) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(db) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dd) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(de) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(df) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(di) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dl) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(do) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ds) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dt) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(du) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dx) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dy) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(dz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ea) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(eb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ec) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ed) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ee) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ef) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(eg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(eh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ei) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ej) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ek) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(el) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(em) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(en) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(eo) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ep) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(eq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(er) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(es) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(et) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(eu) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ev) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ew) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ex) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ey) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ez) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fa) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fd) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fe) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ff) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fi) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fl) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fo) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fs) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ft) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fu) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fx) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fy) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(fz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ga) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gd) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ge) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gf) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gi) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gl) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(go) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gs) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gt) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gu) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gx) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gy) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(gz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ha) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hd) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(he) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hf) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hi) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hl) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ho) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hs) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ht) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hu) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hx) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hy) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(hz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ia) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ib) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ic) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(id) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ie) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(if) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ig) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ih) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ii) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ij) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ik) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(il) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(im) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(in) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(io) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ip) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(iq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ir) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(is) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(it) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(iu) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(iv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(iw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ix) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(iy) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(iz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ja) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jd) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(je) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jf) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ji) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jl) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jo) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(js) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jt) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ju) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jx) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jy) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(jz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ka) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kd) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ke) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kf) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ki) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kl) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(km) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ko) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ks) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kt) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ku) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kx) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ky) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(kz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(la) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ld) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(le) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lf) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(li) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ll) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ln) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lo) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ls) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lt) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lu) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lx) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ly) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(lz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ma) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(md) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(me) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mf) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mi) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ml) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mo) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ms) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mt) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mu) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mx) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(my) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(mz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(na) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nd) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ne) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nf) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ng) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ni) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nl) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(no) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(np) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ns) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nt) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nu) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nx) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ny) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(nz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(oa) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ob) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(oc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(od) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(oe) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(of) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(og) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(oh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(oi) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(oj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ok) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ol) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(om) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(on) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(oo) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(op) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(oq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(or) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(os) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ot) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ou) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ov) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ow) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ox) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(oy) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(oz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pa) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pd) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pe) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pf) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ph) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pi) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pl) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(po) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ps) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pt) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pu) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(px) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(py) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(pz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qa) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qd) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qe) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qf) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qi) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ql) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qo) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qs) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qt) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qu) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qv) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qw) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qx) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qy) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(qz) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ra) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rb) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rc) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rd) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(re) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rf) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rg) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rh) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ri) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rj) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rk) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rl) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rm) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rn) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(ro) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rp) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rq) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rr) Is there some value of x for which the expression [illegible] is undefined?

[illegible]

(rs) Is there some value of x for which the expression [illegible] is undefined?

[illeg

[illegible]

Solution 21 & 22

[illegible]

[illegible]

[illegible]

Solution for Questions 27 to 30

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

How to Prepare for

Quantitative Aptitude for the CAT

COMMON ADMISSION TEST

FOURTH EDITION

How to Prepare for Quantitative Aptitude for CAT is an indispensable textbook for aspirants preparing for the important segment of the CAT. It is also extremely useful for those preparing for similar prestigious MBA or equivalent examinations like XAT, IIFT, FMS, etc. Written and presented in an easy-to-understand style, it is an authoritative guide to this segment. The book has been revised and updated in the context of the CAT pattern (which first 2009 [illegible]

SOME SALIENT FEATURES

- [illegible]
- [illegible]
- [illegible]
- [illegible]

BOOK OUTLOOK

- [illegible]
- [illegible]
- [illegible]
- [illegible]

Related books of interest

[illegible]

[illegible]

[illegible]

[illegible]

The McGraw-Hill Companies

McGraw Hill Education

[illegible]