1 Model definition

1.1 Data

- $G1, G2$ players' grades of the two rounds
- $auct$ maximum number of sold/bought players during the winter call
- $coins$ number of coins users start the first auction with
- $Pr1, Pr2$ prices of players bought during the first and the winter auction
- $Sc1, Sc2$ players' scores for each day and for each round
- Nk, Nd, Nm, Ns numbers of keepers, defenders, midfielders and strikers to line up, given by the formation chosen by the user

1.2 Variables

- $X1, X2$ user's teams for the two rounds, binary (meaning bought or not bought in the fist auction)
- $Y1, Y2$ lined up players on each day of each round, binary (meaning lined up or not lined up during one day)
- B, S bought and sold players during the winter call, binary (meaning bought or not bought and sold or not sold during the winter auction)

1.3 Indices

- $T1, T2$ number of days in the first and second round, indices of $Y1, Y2, G1, G2, Sc1, Sc2$
- P total number of players, indices of $X1, X2, Y1, Y2, B, S, G1, G2, Pr1, Pr2, Sc1, Sc2$
- $\{k_i\}, \{d_i\}, \{m_i\}, \{s_i\}$ sub indices of P divided by players' roles, indices of $X1, X2, Y1, Y2$ (and theoretically of all variables who have P as index)

1.4 Model

$$\max \quad \sum_{t}^{T1}\sum_{p}^{P} Y1_{pt}G1_{pt} + \sum_{t}^{T2}\sum_{p}^{P} Y2_{pt}G2_{pt} \tag{1}$$

$$\text{s.t.} \quad \sum_{k_i} X1 = \sum_{k_i} X2 = 3 \quad \sum_{d_i} X1 = \sum_{d_i} X2 = 8 \tag{2}$$

$$\sum_{m_i} X1 = \sum_{m_i} X2 = 8 \quad \sum_{s_i} X1 = \sum_{s_i} X2 = 6$$

$$\sum_{p} S = \sum_{p} B \leq auct \tag{3}$$

$$\sum_{p} X1_p * Pr1_p \leq coins \tag{4}$$

$$X2 = X1 - S + B \tag{5}$$

$$B_p * Pr1_p \leq 1 \qquad \forall p \tag{6}$$

$$\sum_{p} \Big(Pr1_p * (X1_p - S_p) + B_p * Pr2_p \Big) \leq coins \tag{7}$$

$$Y1_{pt} \leq X1_p, \quad Y2_{pt} \leq X2_p \qquad \forall t, p \tag{8}$$

$$\sum_{p} Y1_p * Sc1_{pt} \geq 78 \quad \forall t \in T1 \tag{9}$$

$$\sum_{p} Y2_p * Sc2_{pt} \geq 78 \quad \forall t \in T2$$

$$\sum_{k_i} Y1_{pt} = \sum_{k_i} Y2_{pt} = Nk \quad \forall t \in T1, T2 \tag{10}$$

$$\sum_{d_i} Y1_{pt} = \sum_{d_i} Y2_{pt} = Nd \quad \forall t \in T1, T2$$

$$\sum_{m_i} Y1_{pt} = \sum_{m_i} Y2_{pt} = Nm \quad \forall t \in T1, T2$$

$$\sum_{s_i} Y1_{pt} = \sum_{s_i} Y2_{pt} = Ns \quad \forall t \in T1, T2$$

$$X1, X2, B, S \in \{0,1\}^P$$

$$Y1 \in \{0,1\}^{P \times T1}, \quad Y2 \in \{0,1\}^{P \times T2}$$

1.4.1 Explanation

Objective function (1): we want to maximize the scores obtained by lined up players.

Constraints (2): as users, we must buy 3 keepers, 8 defenders, 8 midfielders and 6 strikers.

Constraint (3): we must buy and sell the same number of players during the winter auction. The number of bought/sold players cannot exceed *auct*.

Constraint (4): the sum of the prices of players bought at the first auction must not exceed *coins*.

Constraint (5): the players we own during the second round are the ones we bought at the first auction plus the ones we bought at the winter auction minus the ones we sold at the winter auction.

Constraint (6): during the winter auction we can only buy players that were not bought during the first auction (this is not totally true but it should be in the majority of the cases), so we can only buy players with price equal to 1 in $Pr1$. I'm aware this constraint implies that during the winter call we can buy players that were bought during the first auction by other users at a price equal to one (and possibly not sold during the winter auction) but it does not seem to cause problems to the model.

Constraint (7): the sum of the prices of the players bought during the first auction and the players bought during the winter auction must be less or equal than $coins$. This constraint implies that when we sell a player during the winter auction, we receive an amount of coins equal to the price we paid when we bought him during the first auction. This seems to me the most common and general solution beacusa it does not depend on scores given by fantasy soccer websites.

Constraint (8): the only players that we can line up are the ones that we bought.

Constraints (9): every day we must score at least 78 points (a number that guarantees 3 goals both if you score a goal every 4 or 6 points)

Constraints (10): for all the roles, among all the players that we have, we can only line up the numbers of players given by the formation, for example 3 defenders, 4 midfielders and 3 strikers.