Exercises: Swarm Intelligence

1. Consider an illustrative example of a PSO system composed of three particles. Consider the following update rule for each particle i and dimension d:

$$v(i;d) = wv(i;d) + r_1(x^*(i;d) - x(i;d)) + r_2(x^*(d) - x(i;d)).$$

   To facilitate calculation, we will ignore the fact that r_1 and r_2 are random numbers and fix them to 0.5 for this exercise. The space of solutions is the two dimensional real valued space and the current state of the swarm is as follows

   - Position of particles: $x_1 = (5,5)$; $x_2 = (8,3)$; $x_3 = (6,7)$;
   - Individual best positions: $x_1^* = (5,5)$; $x_2^* = (7,3)$; $x_3^* = (5,6)$;
   - Social best position: $x^* = (5,5)$;
   - Velocities: $v_1 = (2,2)$; $v_2 = (3,3)$; $v_3 = (4,4)$.

   (a) What would be the next position of each particle after one iteration of the PSO algorithm with $w = 2$?

   (b) And using $w = 0.1$?

   (c) Explain what is the effect of the parameter w.

   (d) Give an advantage and a disadvantage of a high value of w.

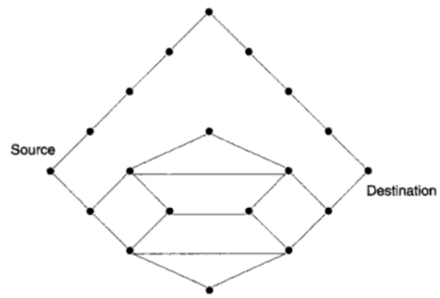
2. Consider a particle swarm consisting of a single member. How would it perform in a trivial task such as the minimization of $f(x) = x^2$ when $w < 1$?

3. Implement the PSO algorithm for clustering described in "Van der Merwe, D. W., and Andries Petrus Engelbrecht. "Data clustering using particle swarm optimization." Evolutionary Computation, 2003. CEC'03. The 2003 Congress on. Vol. 1. IEEE, 2003." (see also swarm intelligence slides). Implement the k-means clustering.

   Apply and compare the performance of the two algorithms in terms of quantization error on Artificial dataset 1 and Iris dataset (the latter available at UCI ML repository, see `https://archive.ics.uci.edu/ml/datasets/iris`). *In both algorithms, use the true number of clusters as value of the parameter for setting the number of clusters.*

4. The figure shows an example from the ACO book by Dorigo and Stuetzle. What results do you expect for an ant colony algorithm that does not use tabu lists (except for inhibition of immediate return to the previous node)?

5. Assume that ants are allowed to lay pheromone on a path at every time step, so that the pheromone update rule is applied at each time step. Come up with a combination local/global updating scheme that encourages exploration and exploitation - consider which parameters influence this.



6. The goal of this exercise is to practice with an implementation of ACO, so please do not use other (possibly better) methods for solving this problem.

   (a) Code an ACO to solve Sudoku's. First, you need to think how to represent Sudoku: there are three conditions an optimal solution must fulfill, each line must contain all integers $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ once and only once, the same applies to columns and 3x3 subgrids. You can fix one of them so that you will prevent violations. The ACO will have 9x9x9 pheromone matrix, where you update pheromones for the values appearing in the best solutions inversely proportionally to the fitness value. You have to consider also how the old pheromones will fade.

   (b) Consider the benchmark Sudoku problems are available at `http://lipas.uwasa.fi/~timan/sudoku/`. Run your ACO on the following benchmark instances `s10a.txt, s10b.txt`, `s11a.txt, s11b.txt`. Report and discuss the results.