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MSc Banking & Finance

The Effects of Instability on Memory

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Abstract

A heuristic switching model (HSM) is used to investigate the predominance of long- or short-memory time series forecasting rules among simulated agents as the macroeconomic time series fluctuates in stationarity and volatility. The time series in the research consist of inflation for The Netherlands and Germany in the period from 1987 to 2017.

The research will examine how agent decisions are shaped in a setting that relaxes the classical assumptions of rational expectations and homogeneity. This expands current knowledge by investigating the forecasting behaviour of agents in the context of macroeconomic data, and its dynamics of stationarity and volatility, using a heuristic switching adaptive learning model outside of a laboratory experiment setting using real-world data.

The study proves that the Dutch and German time series can mathematically be classified as $I(d)$ processes. As a result, traditional accuracy metrics for time series prove inconclusive for the evaluation of the performance of the individual forecasting heuristics, due to distortion by the strong presence of instability and volatility in the series over time. To compensate, fractions of impact for each heuristic in the HSM are calculated and evaluated for each time period over the time horizon along with any significant political or financial (policy) events. Generally, short memory prediction rules prevail during unstable periods and retain overall higher levels of usage throughout the simulation. Long memory heuristics, on the other hand, tend to prevail in periods of strong instability and volatility. Heuristic impact is also dependent on the nature of the time series. The German time series displays the highest long memory parameter relative to the Dutch time series, hence long memory heuristics retain higher impact in the German model.

Keywords: Adaptive learning, heterogeneous beliefs, heuristics, bounded rationality, heuristic switching model, fractionally integrated processes, macroeconomic time series, long memory parameter, econometrics, machine learning.

Contents

Chapter 1

Introduction

Past research and modelling has had a common tendency to rely on classical assumptions. Rational choice theory, considering agents of a (to a certain extent) homogeneous nature, has always more or less been the norm after publication of seminal work such as *Rational Expectations and the Theory of Price Movements* (Muth, 1961) and *Expectations and the Neutrality of Money* (Lucas, 1972). Following the doctrine of rational expectations, market realisations and individual expectations overlap to a large extent. Furthermore, markets are considered efficient in the sense that they reflect economic fundamentals in their price realisations (Fama, 1970) and (Samuelson et al., 1965). One of the most well-known arguments in defence of the school of thought of rational expectations is that agents who do not adhere to rational expectations will be trivialised by agents who do, for their relative profits will be higher. Subsequently, they will perish to the competition (Friedman, 1953).

Alongside the idea of rationality, however, there has been a group of economists who placed value in market psychology as a driving factor. Bounded rationality, or decision making using simple rules of thumb (i.e. heuristics), alleviates the issue of the strong computational and information requirements placed upon pure rational expectations (Simon, 1957). When introducing a degree of uncertainty, laboratory experiments have shown that agents depart from rational expectations in favour of heuristics (Tversky & Kahneman, 1975) and (Kahneman, 2003) and (Camerer & Fehr, 2006).

In this study, the aim will be to investigate the behaviour of heterogeneous agents in forecasting a macroeconomic time series inflation under heterogeneous expectations, i.e. different forecasting rules within the same framework. Data will be used for The Netherlands and Germany with a time horizon for the inflation forecasting stretching from May 1987 to March 2017. The research will be executed using a Heuristic Switching Model (HSM) (Anufriev & Hommes, 2012), featuring various short- and long-memory prediction heuristics. The HSM is an evolutionary model for the explanation of different types of aggregate agent (forecasting) behaviour. In the model, agents have a small set of forecasting heuristics available to them. Over time, agents switch to better performing heuristics as performance is updated evolutionary across model time period iterations.

The focus in this research will be on investigating whether a long or a short memory forecasting rule is predominant when forecasting the time series, as it becomes more stable or unstable. Furthermore, it shall be determined whether or not differences or similarities occur in the process between the German and Dutch time series. The effects of the time-series' long-memory parameters will also be evaluated. The research will shed light on how agent decisions are formed in an environment that relaxes the classical assumptions of rational expectations and homogeneity. It will contribute to the existing literature by investigating agent forecasting behaviour in a macroeconomic setting with respect to the stability or instability of the time series in a heuristic switching model, outside of a laboratory environment using real-world data that considers macroeconomic shocks brought about by financial policy and political events.

Chapter 2

Literature Review

In *A Rational Route to Randomness* (Brock & Hommes, 1997), the authors develop a concept of adaptively rational equilibrium (ARE). Agents generate predictions by choosing from a finite set of predictor or expectation functions. All of these are functions of past observation, accompanied by a publicly available performance measure. Choice between different predictor rules is made by agent assessment of the performance measures of the predictors. As an example, the authors use a cobweb style demand-supply model, which allows agents the choice between rational expectations and naive expectations. This model is an analytically tracable example of adaptive rational equilibrium dynamics (ARED), wherein market equilibrium dynamics are tethered to learning strategies and prediction choice. With a positive cost for the use of rational expectations in an unstable market, high intensity of choice to switch predictors results in irregular equilibria prices, converging to a strange attractor. The authors explain the irregularity of the equilibria thought the existence of a homoclinic orbit, associated with complex dynamic phenomena. They argue local instability and global complex dynamics may exist as a result of the existence of a fully rational notion of equilibrium. This research, with its findings as well as its limits, is seminal work for a lot of future research. The ARED principle incorporated in the cobweb model is the foundation of the Heuristic Switching Model.

In *Evolutionary Selection of Individual Expectations and Aggregate Outcomes in Asset Pricing Experiments* (Anufriev & Hommes, 2012) it is shown by laboratory learning-to-forecast experiments on asset pricing that fully rational behaviour is absent in human subjects. Instead, simple prediction heuristics are used. Also, participants coordinate their price forecasting activity, without being able to interact with each other in any way, by observing aggregate pricing (i.e. market) outcomes. Consequently, prediction heuristics are chosen based on their past performance. The Heuristic Switching Model (HSM), which is the model containing the algorithms enabling agent switching behaviour among heuristics, was developed by the authors to simulate these dynamics. The research finds that performance-based evolutionary selection among simple forecasting heuristics is able to explain individual forecasting behaviour. Three different aggregate outcomes occurred from laboratory forecasting experiments: slow monotonic price convergence, oscillatory dampened price fluctuations and persistent price oscillations. The author's HSM, simultaneously explains aggregate outcomes in all forecasting sessions as well as outperforms the best homogeneous model in almost all sessions. The model is the first learning model able to explain different time series patterns in the same experimental environment. While the authors approach is similar to some learning models (i.e. reinforcement learning models) from Game Theory, the conditions are more complex. The agents in the HSM do not have well-defined strategies, nor do they have knowledge of the pay-off matrix. The strategies in the HSM are state dependent and mutating over time in a matter displaying path-dependence.

In *Individual expectations and aggregate macro behavior* (Assenza, Heemeijer, Hommes, & Massaro, 2013) the authors conduct laboratory experiments with human subjects to study individual expectation formation and the interaction with monetary policy, within a standard New Keynesian model. Individuals forecast both the output gap and the inflation. The results yield three aggregate outcomes: convergence to an equilibrium, persistent oscillatory behaviour and oscillatory convergence. A heterogeneous expectation model is fitted to the experimental data. The model employs performance-based evolutionary selection among heterogeneous forecasting heuristics. The authors find that an interest rate rule that reacts more than point for point to inflation exhibits stabilising effects in the author's experimental economies. Convergence, however, is quite slow due to the evolutionary learning of the model.

In *Behavioral heterogeneity in US inflation dynamics* (Cornea, Hommes, & Massaro, 2013) the authors develop and estimate a behavioural model of inflation dynamics coupled with monopolistic competition. Furthermore, a staggered price setting and heterogeneous firms are incorporated. The framework incorporates two groups of price setters; fundamentalists and naive. The fundamentalists are forward-looking, i.e. assume a present-value relationship between inflation and real marginal costs. The naive price setters are backward-looking, using naive expectations to forecast future inflation. Agents switch between forecasting heuristics based upon their recent relative forecasting performance. The results of the estimations are in support of the evolutionary switching mechanism and behavioural heterogeneity. The authors provide evidence for the presence of a substantial time variation in the weights of forward-looking and backward-looking behaviour. On average, the majority of the firms appear to use the simple naive heuristic. Although, the market goes through phases wherein the naive agents or the fundamentalists are dominant.

In *The danger of inflating expectations of macroeconomic stability: heuristic switching in an overlapping-generations monetary model* (Brazier, Harrison, King, Yates, et al., 2008) the authors have developed an overlapping-generations model, which assumes homogeneous agents form expectations about inflation by choosing amongst a set of two heuristics. Moreover, the implications of monetary policy are considered. One heuristic is based on lagged inflation (naive expectations), whereas the other is based on inflation targets announced by the Central Bank. Heuristic switching occurs based on the past performance of both heuristics. Due to the imperfect assessment of heuristic performance by agents, rational expectations are relaxed. The paper concludes that, in an economy where agents choose among different forecasting heuristics, fluctuations in the variance of inflation are generated. The research also found that monetary policy does not have direct leverage over inflation expectations when heuristics are used. Furthermore, the introduction of an inflation target heuristic reduced inflation volatility and improvement macroeconomic outcomes to some extent.

This research shall build upon aforementioned research and will make a contribution to existing literature by taking the theory outside of a laboratory environment. The research will test the heuristic switching model approach using real-world time series: Dutch and German inflation. As mentioned previously, and in contrast to previous literature, this research will relax assumptions with respect to agent homogeneity and will model agent forecasting behaviour using real-world data (considering the influence of macroeconomic shocks brought about by financial policy and political events) instead of contained laboratory experiments. Additionally, the research will investigate the implications of time series (in)stability, volatility and long-memory parameter effects.

Chapter 3

Data

3.1 Data

The types of data used in this research can be categorised into two types. The first type is simulated data. This type of data will be obtained from the Heuristic Switching Model's output. The core algorithms for the Heuristic Switching Model have been written in *MATLAB* code.

The second type of data is year-over-year CPI (Consumer Price Index) adjusted for seasonality. This data serves as a base for the forecast of the HSM and will mostly simply be referred to as inflation throughout the research. The data has been obtained from the Global Economic Monitor of World Bank in Washington, for The Netherlands and Germany at high frequency (i.e. monthly). The time series range from May 1987 to March 2017 for both countries. Figure 3.1 contains a visualisation of both the time series. Summary statistics for the inflation time series data are shown in table 3.1.

Mean inflation over the observed time period differs by approximately 0.01 percentage points between Germany and The Netherlands. The Netherlands, on average, has a higher inflation rate. The minimum inflation values for both countries are separated by about 0.87 percentage points. The Netherlands has the lowest documented inflation rate in the time series. The maxima are further apart. The German maximum is approximately 1.9 percentage points above the Dutch maximum. The Dutch median inflation rate is about 0.35 percentage points above the German median inflation rate. The Dutch time series exhibits a slight skewness to the left. Germany, on the other hand, displays a stronger and right-oriented skewness. Although both time series are leptokurtic, the kurtosis value for the German time series is more than 1.6 times as high as the value for The Netherlands.

Inflation Vector Summary Statistics

	Mean	Std Dev	Min	Max	Median	Skewness	Kurtosis
The Netherlands	1.936	(0.994)	-1.273	4.461	1.984	-0.036	3.106
Germany	1.839	(1.230)	-0.404	6.358	1.639	1.263	5.096
Observations	359						

Table 3.1: Summary Statistics for Inflation Vectors

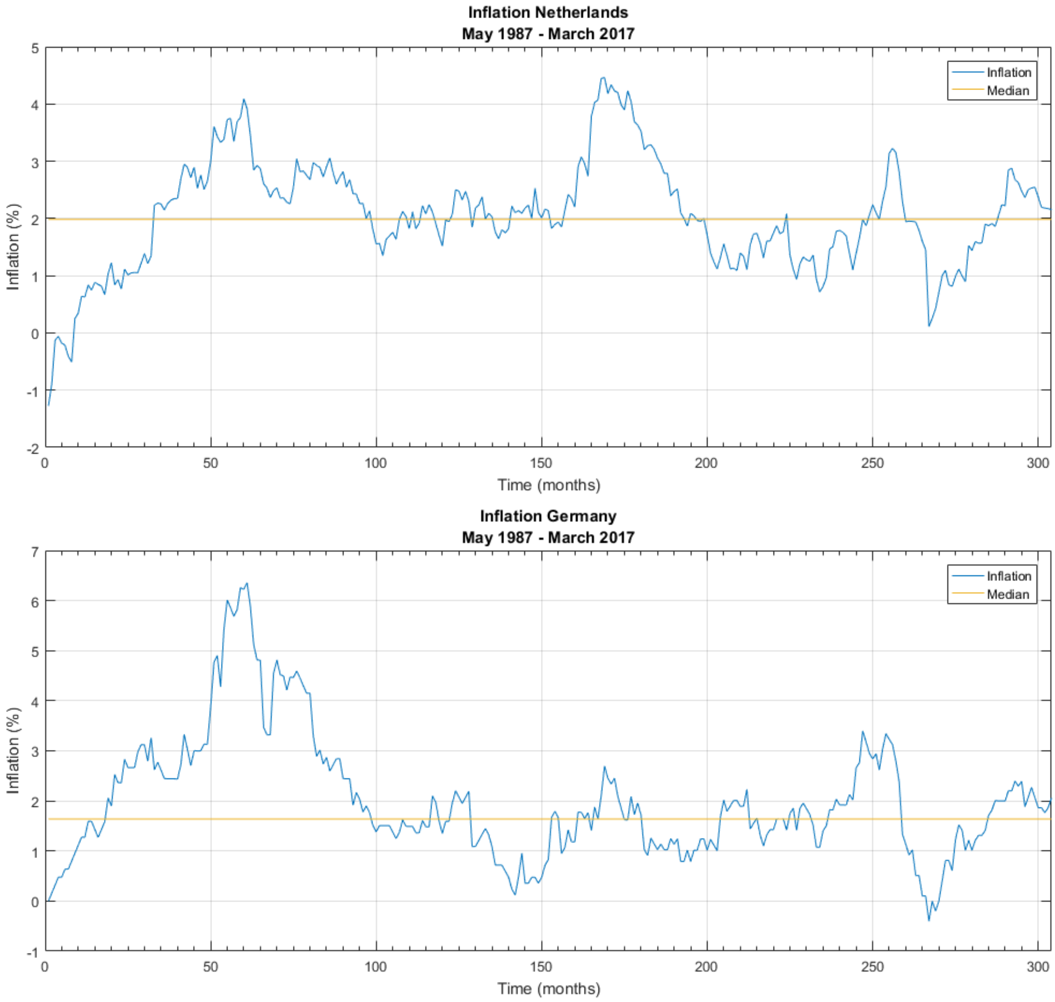

Figure 3.1: Inflation vectors

Chapter 4

Methodology

4.1 Model

This paper uses a Heuristic Switching Model, which has been popularised by Mikhail Anufriev and Cars Hommes in their 2012 paper *Evolutionary Selection of Individual Expectations and Aggregate Outcomes in Asset Pricing Experiments.* (Anufriev & Hommes, 2012)

The model used in this research adheres to the stylised facts found by Anufriev & Hommes in their research. The predictions of agents are based on past observations of the data, following simple forecasting heuristics. Individual evolutionary learning within the model results in the switching from one forecasting heuristic to another, based on observed performance of said heuristics. Furthermore, every forecasting sessions demonstrates a form of agent coordination on a heuristic. Coordination differs across sessions. Finally, it should be noted that the coordination among agents is not perfect and, therefore, a degree of heterogeneity of the heuristics persists throughout the time periods. The model is built in compliance with the Adaptive Belief Scheme (Brock & Hommes, 1997). However, the memory is present in the asynchronous strategy updating and the fitness measure (Anufriev & Hommes, 2012).

This section will start with the mathematical substance and intricacies of the Heuristic Switching Model. Subsequently, the set of available forecasting heuristics for agents shall be defined and explained. Lastly, the initialisation of the evolutionary base of the HSM shall be covered.

4.2 Mathematical Foundation

Let $\mathcal{H}$ denote a set of H heuristics. Every heuristic $h \in \mathcal{H}$ yields a point prediction for the data, ω_{t+1}. Underlying the predictions is a deterministic function $f_h(\cdot)$, reflecting the pool of available information:

$$\omega^e_{h,t+1} = f_h(\omega_{t-1}, \omega_{t-2}, ... \; ; \; \omega^e_{h,t}, \omega^e_{h,t-1}, ...) \tag{4.1}$$

In evolutionary models, the mean forecast $\bar{\omega}^e_{t+1}$ is a population-weighted mean of all generated forecasts of the heuristics in set $\mathcal{H}$. Each heuristic in $\mathcal{H}$ has a weight $n_{h,t}$ associated with it. This represents the impact of the specific heuristic. The impact of a heuristic is evolutionary and dependant on past relative performance $U_{h,t-1}$ of the H heuristics. When a heuristic is more successful at prediction relative to others its impact increases, thus attracting more users.

$$\bar{\omega}^e_{t+1} = \sum_{h=1}^{H} n_{h,t} \, \omega^e_{h,t+1} \tag{4.2}$$

The impact of each heuristic h from set $\mathcal{H}$ is updated consistent with a discrete choice model with asynchronous updating (equation 4.3), in which Z_{t-1} is a normalisation factor to reduce the probability function to a probability density function adhering to $\int_{-\infty}^{\infty} \varphi(x)\, dx = 1$. This process is derived from the cobweb model with heterogeneous beliefs (Brock & Hommes, 1997), which is shown in appendix A. To realise the iterations of asynchronous updating over all time periods in the data, the epsilon naught of the machine of *MATLAB* has been used. The value of this machine epsilon, under Prof. Higham's $b^{-(p-1)}$, is as stated in equation 4.5 (Higham, 2002).

The parameter δ describes the degree of inertia in the updating of the agents' preferred heuristic, or the mean per period fraction of individuals who stick to their current heuristic of choice. In the extreme case of $\delta = 1$, heuristic impact remains unchanged, regardless of past performance. Consistent with former research, for $0 \leq \delta \leq 1$ not all agents choose to update their prediction heuristic in each period nor simultaneously (Hommes, Huang, & Wang, 2005) and (Diks & Van Der Weide, 2005).

The intensity of choice is represented by parameter $\beta \geq 0$. This parameter measures the sensitivity of agents with respect to difference in heuristic performance. Higher values of β will ensure agents will switch to more successful heuristics faster. At the lower bound extreme $\beta = 0$, equation 4.3 will revert to an equal distribution independent on prior performance of heuristics. For $\beta = \infty$, the fraction of agents who choose to update, i.e. $(1 - \delta)$, will all switch to the most successful heuristic.

$$n_{h,t} = \delta\, n_{h,t-1} + (1-\delta)\frac{e^{\beta\, U_{h,t-1}}}{Z_{t-1}} \tag{4.3}$$

$$Z_{t-1} = \sum_{h=1}^{H} e^{\beta\, U_{h,t-1}} \tag{4.4}$$

$$\epsilon_0 = 2.2204 \cdot 10^{-16} \tag{4.5}$$

The performance measure of a heuristic h is reliant on its squared forecasting error. The measure is evolutionary across time periods and available up to time $t-1$ in every time period. For the performance measure U of heuristic h with autoregressive values ω from the time series and memory strength parameter η, let:

$$U_{h,t-1} = -(\omega_{t-1} - \omega^e_{h,t-1})^2 + \eta\, U_{h,t-2} \tag{4.6}$$

The memory parameter of the agents, which expresses the relative weight assigned to previous errors of past heuristics H, is characterised by parameter $0 \leq \eta \leq 1$. For instance in the extreme case of $\eta = 0$, heuristic impact is completely determined by $t-1$ forecasting errors.

4.3 Heuristics

The agents in the HSM can choose from a set of forecasting heuristics to predict the next period's data point. These heuristics are of the short- or long-memory type and differ in degree of sophistication.

Adaptive Heuristic and Naive Heuristic:

$$\omega^e_{t+1} = w\, \omega_{t-1} + (1-w)\, \omega^e_t \tag{4.7}$$

$$w = 1 \equiv \left(\omega^e_{t+1} = \omega_{t-1}\right) \tag{4.8}$$

The adaptive heuristic (ADA) is an adaptive expectations short-memory forecasting rule. The parameter w determines the emphasis placed upon the previously observed data value and the forecast value for the current period. The unique case of $w = 1$, as shown in equation 4.8, reverts the equation to *naive expectations*. The naive heuristic is another short memory forecasting rule that simply uses the last data value as forecast for $t+1$.

Trend-Following Heuristic:

$$\omega_{t+1}^{e} = \omega_{t-1} + \gamma\left(\omega_{t-1} - \omega_{t-2}\right) \tag{4.9}$$

The trend-following heuristics use the $t-1$ value as a base and adjusts the forecast in the direction of the last mutation. The parameter γ measures the propensity of the adjustment. Two variations of the long-memory trend-following heuristic have been used; a weak trend-following heuristic (WTF) and a strong trend-following heuristic (STF). The former uses $\gamma = 0.4$, whereas the latter uses a value of $\gamma = 1.3$.

Anchoring and Adjustment Heuristic:

$$\omega_{t+1}^{e} = \frac{1}{2}(\omega^{f} + \omega_{t-1}) + (\omega_{t-1} - \omega_{t-2}) \tag{4.10}$$

The anchoring and adjustment rule is a more advanced long-memory prediction heuristic, which employs an anchor to extrapolate the last mutation (Tversky & Kahneman, 1975). The anchor, $\frac{1}{2}(\omega^{f} + \omega_{t-1})$, describes the long-run level of the underlying data. The ω^{f} parameter embodies the fundamental value of the data vector. The make this heuristic suitable for use in this paper, a proxy has been generated for it.

Learning Anchoring and Adjustment:

$$\omega^{f} \sim \left(\overline{\omega}_{t-1} = \frac{1}{t}\sum_{j=0}^{t-1}\omega_{j}\right) \tag{4.11}$$

$$\omega_{t+1}^{e} = \frac{1}{2}(\overline{\omega}_{t-1} + \omega_{t-1}) + (\omega_{t-1} - \omega_{t-2}) \tag{4.12}$$

In order to apply an anchoring and adjustment heuristic to the data, a proxy has been generated for the ω^{f} parameter, as shown in 4.11. The proxy is the average of the observed values in the data vector up to and including the $t-1$ value. The value of the proxy changes with each period iteration of the HSM. Therefore, the first term of the heuristic in equation 4.12, $\frac{1}{2}(\overline{\omega}_{t-1} + \omega_{t-1})$, has two mutating variables, thus becoming a *learning anchor*. Consequently, the heuristic in equation 4.12 is called the learning anchoring and adjustment heuristic (LAA).

4.4 Initialisation

In order to initialise the model, all heuristics in set $\mathcal{H}$ need to be able to generate their first predictions. A time length vector $\mathcal{T}$ has been utilised to do so. Furthermore, there needs to be an initial heuristic impact distribution, adhering to equation 4.13. Intuitively, this sums up to a value of 1.

$$\{n_{h,0}\} \sim 1 \leq h \leq H \tag{4.13}$$

Moreover, the heuristic performance measures for each h need to be equalised at zero for the first period when they are due to be updated. This is shown in equations 4.14 and 4.15.

$$U_{h,t} = 0 \tag{4.14}$$

$$t < \mathcal{T} + 2 \tag{4.15}$$

Building upon the previous declarations, the heuristics' $\mathcal{T}+2$ forecasts are calculated. Next, $\omega_{\mathcal{T}+1}$ is computed using the initialisation initial heuristic impacts. Subsequently, the forecasts are updated in the next period iteration of the model. The initialisation of the HSM comes to a conclusion with the computation of $\omega_{\mathcal{T}+2}$.

Now, with a well-defined evolutionary base of the model, performance measures are updated as in equation 4.6 and heuristic impacts are refreshed as shown in equation 4.3. Finally, all the new predictions, as made by the heuristics, are derived by the deterministic function 4.1 and the population-weighted average of the heuristics (equation 4.2) is re-evaluated. All heuristics and parameters in the HSM algorithm are based on prior research mentioned in this section and are as defined in table 4.1, unless mentioned otherwise (Anufriev & Hommes, 2012).

Heuristics and Initial Model Parameters

ADA	Adaptive Heuristic	$\omega^e_{1,t+1} = 0.65\,\omega_{t-1} + 0.35\,\omega^e_{1,t}$
WTF	Weak Trend Following Heuristic	$\omega^e_{2,t+1} = \omega_{t-1} + 0.4\,(\omega_{t-1} - \omega_{t-2})$
STF	Strong Trend Following Heuristic	$\omega^e_{3,t+1} = \omega_{t-1} + 1.3\,(\omega_{t-1} - \omega_{t-2})$
LAA	Learning Anchoring and Adjustment Heuristic	$\omega^e_{4,t+1} = 0.5\,(\overline{\omega}_{t-1} + \omega_{t-1}) + (\omega_{t-1} - \omega_{t-2})$
β	Intensity of Choice	$\beta = 0.4$
η	Agent Memory Parameter	$\eta = 0.7$
δ	Heuristic Update Inertia	$\delta = 0.9$

Table 4.1: Heuristics and Parameters used in the Heuristic Switching Model

4.5 Stationarity & Memory

The long memory parameter of the Dutch and German time series is investigated through various steps. The methodology applied to assess the series' stationarity, and whether or not the series can be classified as $I(d)$ processes (i.e. fractionally integrated process) (appendix D), is as follows.

First, the stationarity is explored through the Augmented Dickey-Fuller test (Dickey & Fuller, 1979), the Phillips-Perron test (Phillips & Perron, 1988) and the Kwiatkowski-Phillips-Schmidt-Shin test (Kwiatkowski, Phillips, Schmidt, & Shin, 1992). Subsequently, the spectral density of the series is estimated. To accomplish this, the original signals are subjected to a Fourier transform and plotted in a periodogram (equation 4.16 and 4.17) (Auger & Flandrin, 1995). Finally, autocorrelation functions are calculated, plotted and evaluated. A gradual decay in the autocorrelation function is typical for an $I(d)$ process (Boutahar, Marimoutou, & Nouira, 2007).

The memory parameters for the series themselves are estimated using the Geweke & Porter-Hudak (GPH) (Geweke & Porter-Hudak, 1983), semi-parametric (*sperio*) and wavelet techniques. The Wavelet estimator uses maximal overlap discrete wavelet transform (*modwt*) (Percival & Walden, 2006) with the *Haar* filter type (Haar, 1910).

$$\hat{P}(f) = \frac{\Delta t}{N} \left| \sum_{n=0}^{N-1} x_n e^{i2xfn} \right|^2, \quad -\frac{1}{2}\Delta t < f \leq \frac{1}{2}\Delta t \tag{4.16}$$

$$\sigma^2 = \int_{-\frac{1}{2}\Delta t}^{\frac{1}{2}\Delta t} P(f)\ df \tag{4.17}$$

Chapter 5

Results

This chapter will discuss the results of the Heuristic Switching Model's simulation. The heuristics' predictions are shown in section 5.1. In this part, each prediction series and the corresponding fit will be briefly qualitatively discussed. Section 5.4 covers all the influential political, financial and (European) policy events that have occured in the time horizon of this research. These events affect the inflation input vectors of the HSM and will therefore influence the switching process. The switching process of the HSM, and the associated heuristic impacts, are considered in section 5.5. The reaction of the HSM to the influence of the events discussed in section 5.4 will also be discussed here. Furthermore, diagnostics and robustness of the fits of the different heuristics are shown and deliberated in sections 5.2 and 5.3. Subsequently, the time series' stability is explored and quantified in section 5.6. Finally, the memory parameter is estimated using various techniques in section 5.7.

5.1 Predictions

Adaptive Heuristic The predictions of the *adaptive heuristic* (ADA) for both Dutch and German inflation rates are shown in figure 5.1. Due to the behaviour of this heuristic, the overall trend of the forecast is correct. The predictions, however, are systematically lagging behind the actual values. The magnitude of this effect depends on the (in)stability and volatility of the underlying time-series. Periods with increased instability and volatility of the time series result in increasing prediction errors. This effect becomes apparent by comparing the Dutch and German graphs in figure 5.1. The German series is inherently more unstable and more volatile (3.1), hence displays the effect to a larger extent.

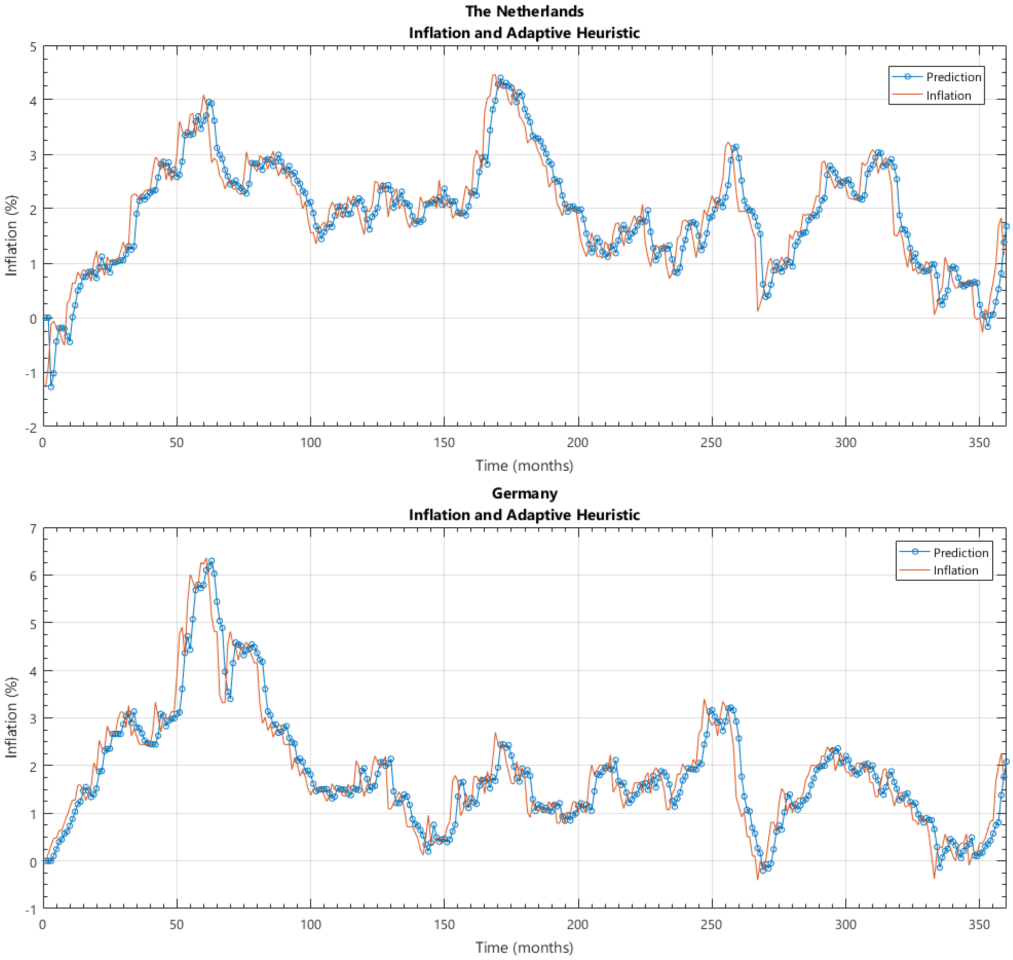

Figure 5.1: Inflation predictions of the Adaptive Heuristic

Weak Trend Following Heuristic Figure 5.2 displays the forecasts of the *weak trend following heuristic* for Germany and The Netherlands. This heuristic also considers the last first-differenced predictions with a propensity for adjustment of 0.4 (i.e. $\gamma = 0.4$). As can be seen from the figure, the fit of the predictions is less lagged than is the case with ADA. However, the predictions are somewhat more volatile. Specifically, in periods where the series becomes more unstable.

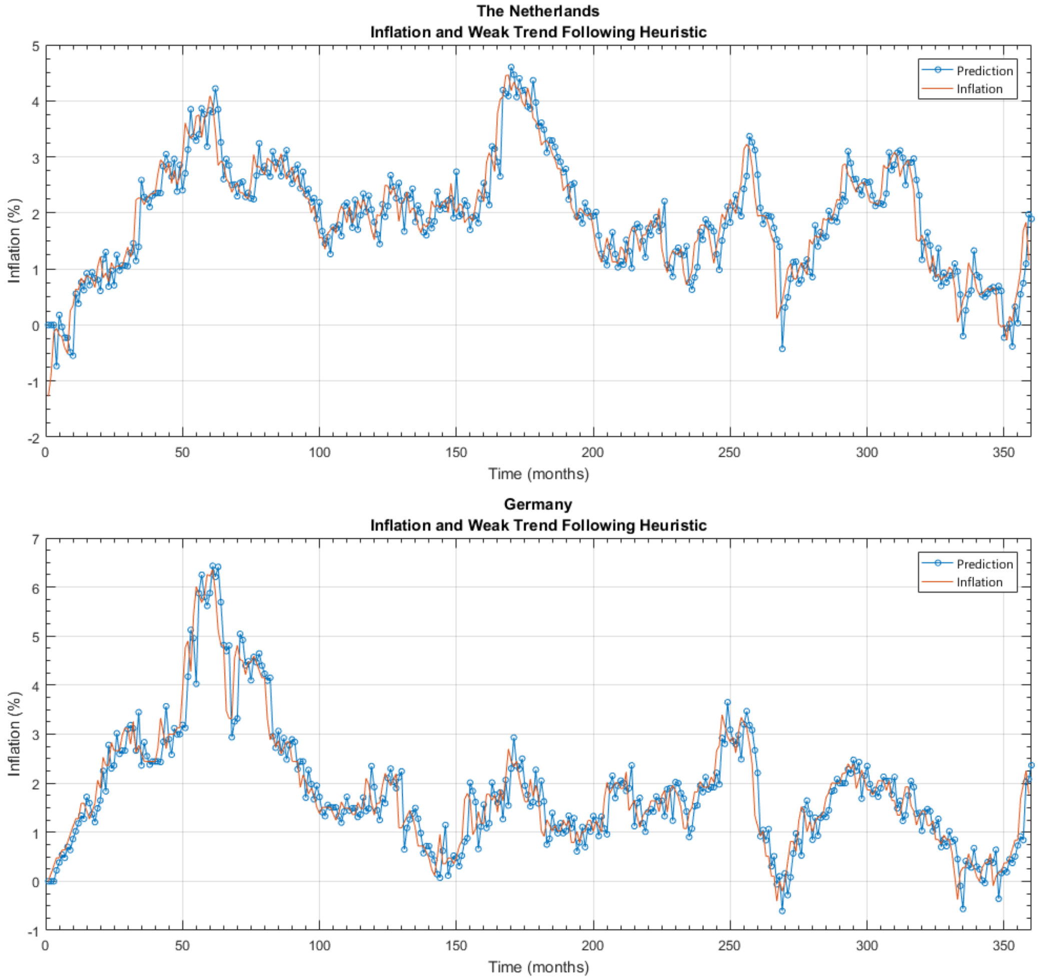

Figure 5.2: Inflation predictions of the Weak Trend Following Heuristic

Strong Trend Following Heuristic The *strong trend following heuristic* is mathematically the same as its weak counterpart. The difference is that it uses a propensity for adjustment of 1.3 (i.e. $\gamma = 1.3$), meaning the last first-differenced predictions are relatively valued more profoundly. As can be seen in figure 5.3, the effect is conspicuous. Predictions are more volatile than was the case with the weak variant. The magnifying effect of the non-stationarity of the series on the predictions seems also more pronounced, to the detriment of its accuracy.

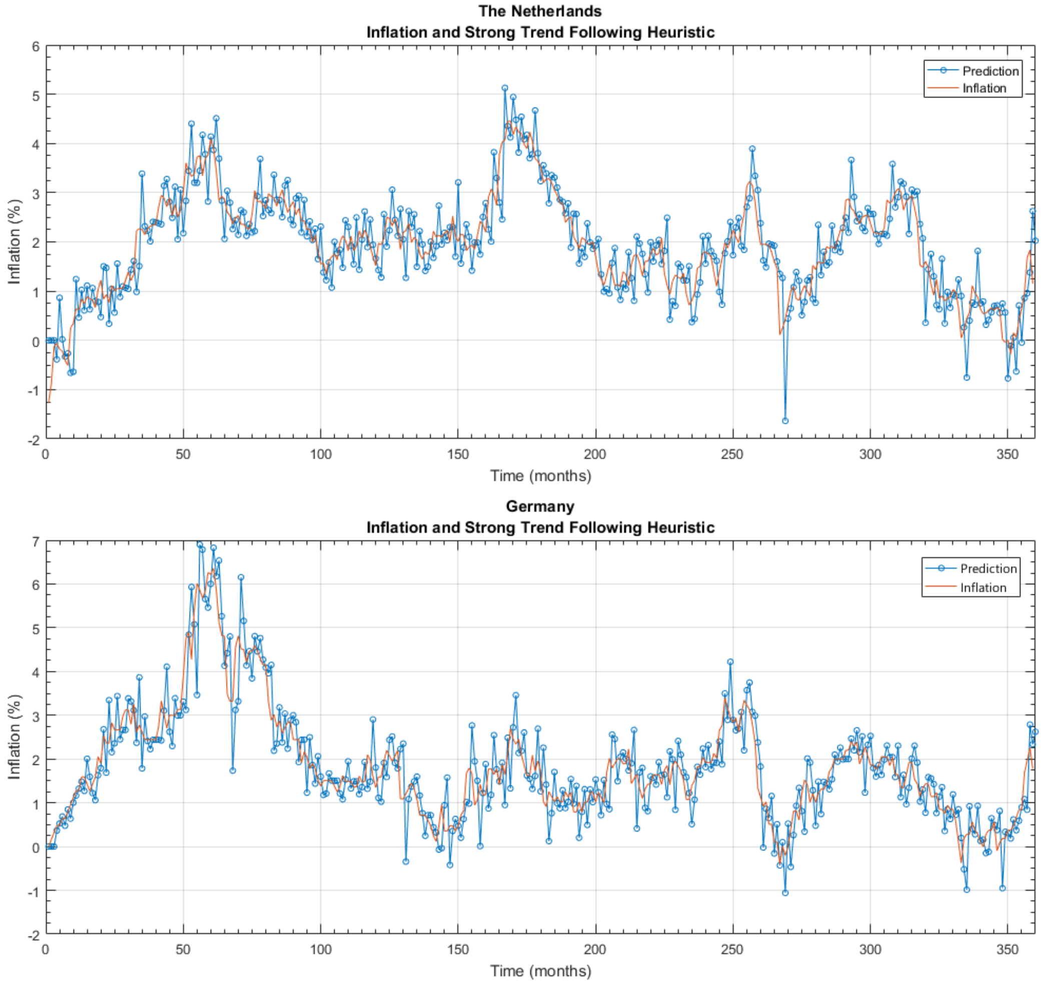

Figure 5.3: Inflation predictions of the Strong Trend Following Heuristic

Learning Anchoring and Adjustment Heuristic The predictions of the *learning anchoring and adjustment heuristic*, the most advanced heuristic, appear to have an entirely new effect. As can be seen in figure 5.4, this heuristic dampens the effects of time-series instability in its predictions, as a result of the mathematical definition of the learning anchor. While this may yield interesting results in stable stable series, it does not for the largely unstable series in this research. This appears to be the least accurate heuristic.

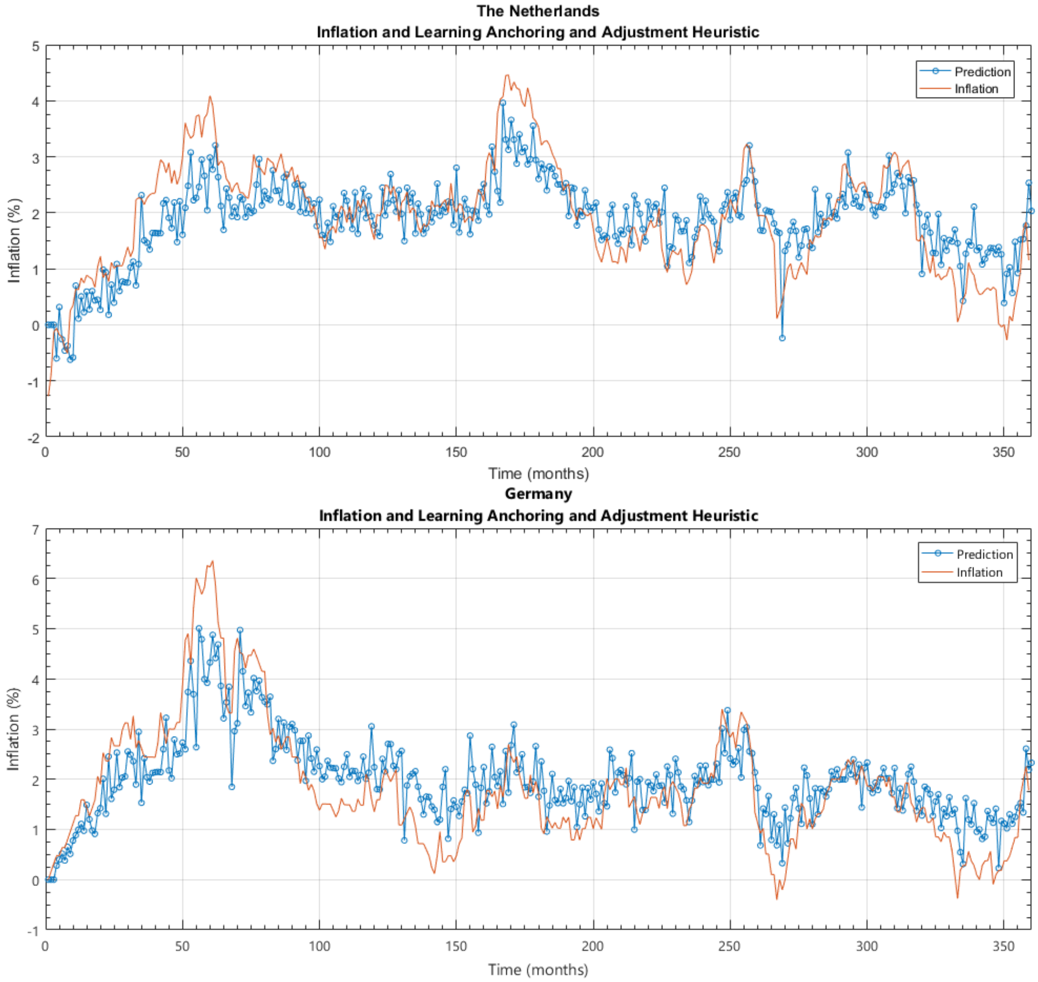

Figure 5.4: Inflation predictions of the Learning Anchoring and Adjustment Heuristic

Naive Heuristic For the sake of comparison of the fit and robustness of the other heuristics, a *naive* heuristic has also been used to forecast the Dutch and German inflation series. This heuristic simply transforms the series $t+1$ to obtain the forecast, as shown in equation 4.8. The results are shown in figure 5.5. This heuristic is not a switching choice for agents in the actual heuristic switching model.

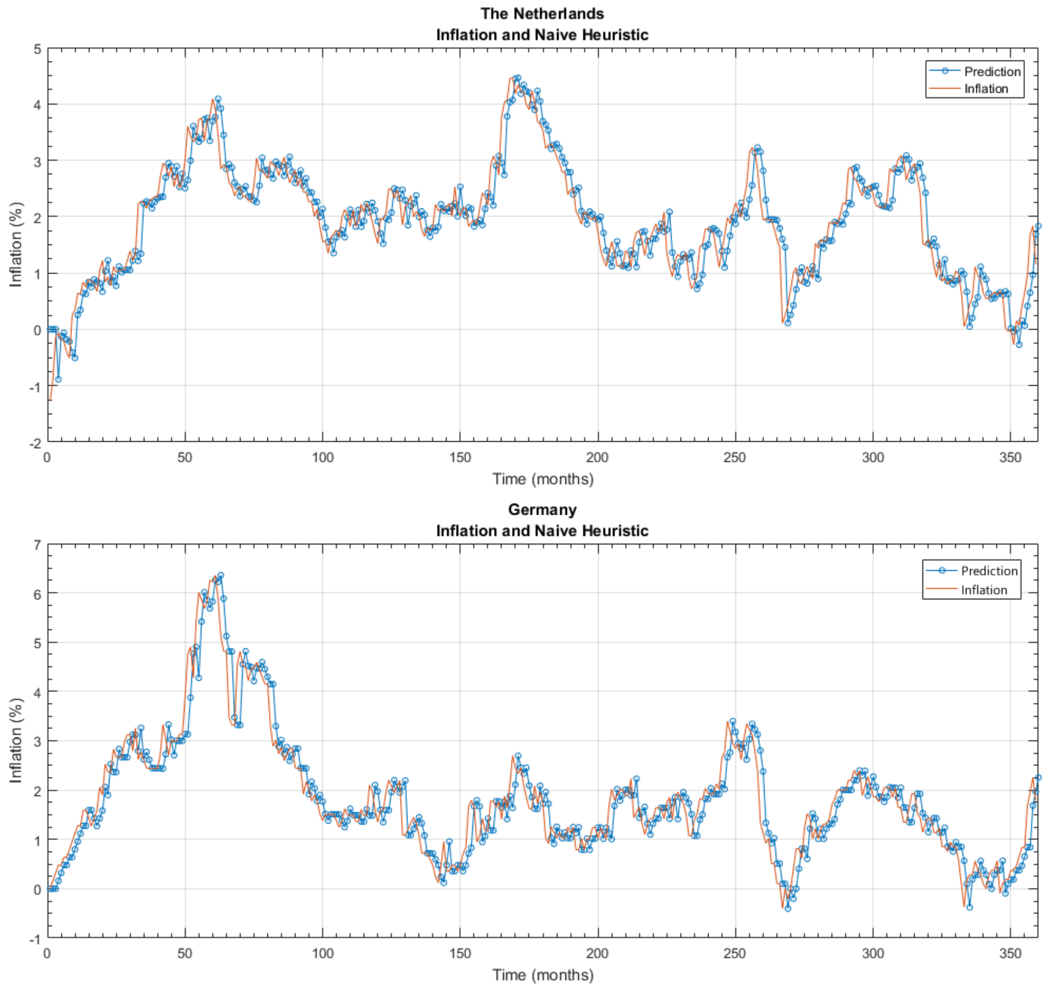

Figure 5.5: Inflation predictions of the Naive Heuristic

5.2 Diagnostics

The prediction errors of each heuristic for The Netherlands and Germany are shown in figure 5.6 and figure 5.7 respectively. These graphs tell a similar story to what has already been observed in the heuristic prediction graphs in section 5.1.

Overall, for both series, errors increase in periods where the time-series are relatively more unstable. Furthermore, the prediction errors for the German time-series are higher across the board, relative to the prediction errors of The Netherlands. For the Dutch inflation series, generally the ADA and WTF heuristic seem to yield the smallest prediction errors. At first glance, the STF and LAA heuristics even seem to underperform relative to the naive heuristic. However, a closer look reveals that both these heuristics have their strengths under certain specific circumstances. In periods of extreme time-series instability and sign-reversal (i.e. inflation to deflation), the STF and LAA heuristics shine. This can be observed in both Dutch and German time-series. The learning anchor of the LAA heuristic acts as a cushion to volatility, as well as instability, whereas the strong trend interpolation of the STF heuristic mediates volatility. Superiority of each heuristic appears is situation- and path-dependent in the HSM. The next sections aim to quantify these findings.

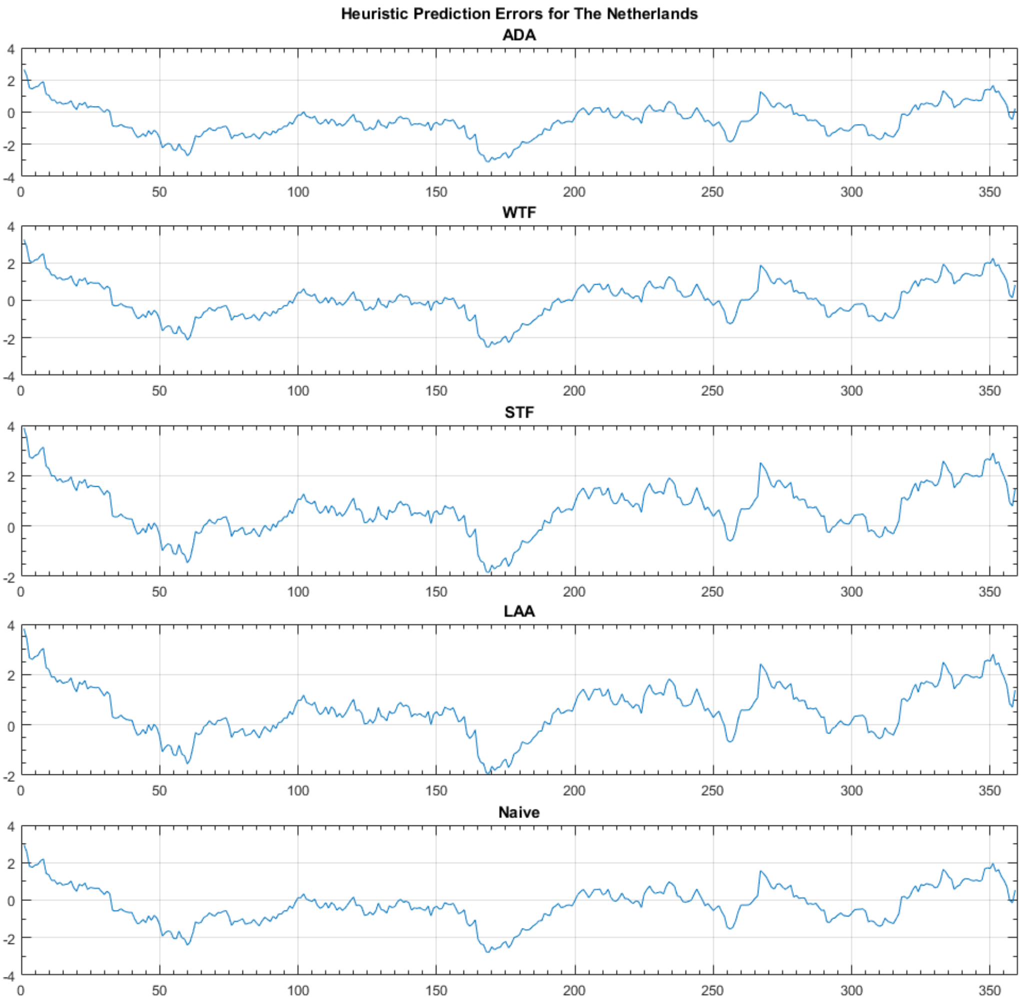

Figure 5.6: Prediction errors of heuristics over time for The Netherlands

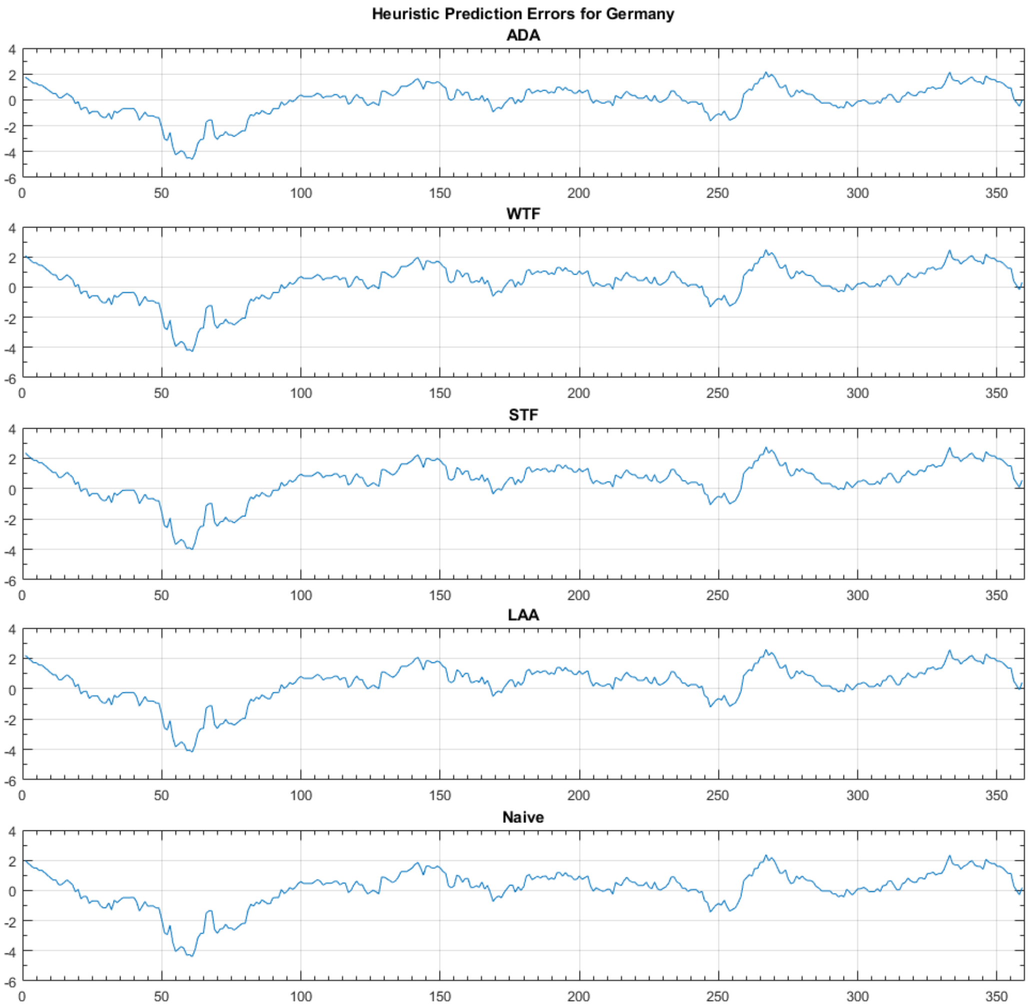

Figure 5.7: Prediction errors of heuristics over time for Germany

5.3 Robustness

Forecast accuracy statistics for all prediction heuristics of the Dutch and German time-series are shown in table 5.1.

For The Netherlands, the MSE of the weak trend following heuristic is the only one lower than the naive heuristic. STF has the highest MSE. The RMSE, the square root of the variance of the residuals, is lowest for the naive heuristic. WTF is, again, the runner-up. LAA has the highest RMSE. The MAE explains how big of an error can be expected of the forecast on average. It is lowest for Naive, closely followed by ADA and WTF. The MAE does not convey the relative size of the error with respect to the two time-series. To alleviate that, the MAPE provides a metric of the mean absolute error in percentage terms. That way, the German and Dutch series can be compared. The lowest MAPE is 0.308% for the naive heuristic. LAA and STF perform much worse. Second and third best are ADA and WTF, with 0.518% and 0.564% respectively. Theil's U compares the obtained forecasts to the results that would have been obtained by forecasting with minimum historical time-series data. According to this statistic, STF has the best fit followed by WTF and Naive.

For Germany, the MSE is lowest for ADA. However, Naive and WTF are close to that value. The RMSE is lowest for Naive, closely followed by ADA and WTF. Exactly the same is true for MAE and MAPE. According to Theil's U, all forecasts made by the heuristics for the German series are worse than a minimum-data forecast. Naive is considered the best of these, followed by WTF and ADA.

Judging from these statistics, the fit of the forecasts seems rather poor and not very robust. However, what should be noted is that these statistics take the entire forecast horizon into account when computed. This means that strengths of specific heuristics during periods of increased instability and/or volatility of the series are not properly taken into consideration. Of the actual HSM heuristics, both the adaptive heuristic and the weak trend following heuristic seem to perform rather well in forecasting either the Dutch or German inflation series.

Heuristic Forecast Accuracy Statistics[1]

The Netherlands

	ADA	WTF	STF	LAA	Naive
MSE	1.294	0.987	1.460	1.343	1.047
RMSE	0.401	0.396	0.507	0.603	0.384
MAE	0.299	0.304	0.402	0.474	0.287
MAPE (%)	0.518	0.564	1.260	0.963	0.308
Theil's U	1.256	0.850	0.718	4.339	0.926

Germany

	ADA	WTF	STF	LAA	Naive
MSE	1.513	1.568	1.757	1.629	1.5249
RMSE	0.453	0.456	0.594	0.733	0.439
MAE	0.330	0.339	0.449	0.580	0.320
MAPE (%)	0.283	0.290	0.382	0.596	0.275
Theil's U	1.755	1.695	2.164	3.543	1.661

Table 5.1: Heuristic forecast accuracy statistics of each heuristic in the model, per country

[1]Abbreviations: MSE (Mean Squared Error) , RMSE (Root Mean Square Error), MAE (Mean Absolute Error), MAPE (Mean Absolute Percentage Error), Theil's U Statistic. Details and derivations in appendix B.

5.4 Seminal Events

The dynamics of the observed periods of inflation for Germany and The Netherlands have been prone to influence by external political and financial events, as well as (European) policy developments. A comprehensive breakdown of all seminal events that have occurred over the period can be found in table 5.2 and 5.3 for Germany and in table 5.4 and 5.5 for The Netherlands.

The table shows the year(s) in which the specific events occur and the months in the inflation vector which correspond with these years. This is useful to find the specific period in all visualisations in this research, e.g. figure 5.8. Furthermore, each described event is also given an event identifier for easy reference in section 5.5. Lastly, there is a succinct description of the specific period and its events, and an account of the event's effects on the inflation of the country.

Section 5.5 shall employ the collection of events in this section to investigate the impact of these influential events on the switching process of the HSM.

Seminal Events in Germany

Years of event	Corresponding months in time series	Event ID	Description of period and events	Effect on inflation
1987 - 1990	1 - 44	G.I	This period encapsulates all events from the first visit of the East-German leader to West-Germany to the tearing down of the Berlin Wall and the reunification of Germany.	Steady increase, smoothing out near the end of the period.
1993	69 - 80	G.II	Germany joins the Maastricht Treaty on European Union.	Increase leading up to the event, followed by a sharp decrease.
1994 - 1998	81 - 140	G.III	Russian and Allied troops leave Berlin; German troops allowed to take part in peacekeeping operations outside Nato; Kohl replaced as CDU leader by Wolfgang Schäuble; compensation fund for former slaves of the Nazi regime announced.	Relatively steady, decrease near the end of the period.
1999 - 2002	141 - 188	G.IV	Gerhard Schröder elected chancellor; CDU Kohl funding scandal, Schäuble resigns and Angela Merkel takes over; Treaty to phase out nuclear energy in the next 20 years; In 1999, the idea of the Euro is introduced; Immigration bill for non-EU labour pushed through parliament; tax cuts delayed due to national catastrophe (torrential rains).	Initial volatile increase, decrease from half of 2001.
2002	177 - 188	G.V	The Euro officially replaces the Deutsche Mark (DM) in January.	Slight decrease, relatively stable period.
2003 - 2005	189 - 224	G.VI	Gerhard Schröder (SPD) re-elected; Protests due to plans to cut unemployment benefits and labour reforms; EU constitution ratified (2005); Angela Merkel of CDU becomes chancellor in a coalition involving SPD and CDU.	Increase in inflation over the period.
2006 - 2008	225 - 260	G.VII	Governance reforms to speed up decision-making; Unemployment falls below 4 million; Onset of the financial crisis (Lehman Brothers has collapsed): $68 billion plan to bail out Hypo Real Estate; EU government pledges up to 1.8 trillion euros to shore up the battered financial sectors; Germany frees up 500 billion euros in loan guarantees and capital to fortify the banking sector; Germany is officially in recession.	Steady increase over the period, huge plummet at the onset of the financial crisis era in 2008.

Table 5.2: Seminal inflation-affecting events in the German inflation time series.

Continued: **Seminal Events in Germany**

Years of event	Corresponding months in time series	Event ID	Description of period and events	Effect on inflation
2009 - 2012	261 - 308	G.VIII	In 2009, Greece's account unravels, revealing substantial budget deficit; Parliament approves $63 billion stimulus package; Germany climbs out of recession with a 0.3% growth; Merkel re-elected; German economy shrank by 5% in 2009, due to a slump in investment and export; In 2010, financial crisis spreads, austerity measures are adopted and Greece and Ireland are bailed out; Parliament approves 22.4 billion euros of bailout contribution for Greece; German economy grew by 3.6% in 2010, as export levels recovered; Merkel backs second bailout for Greece; German economic growth ground to a halt in Q2, 2011; Financial crisis deepens and a permanent rescue fund is planned as European leaders look for a resolution.	Inflation increases steadily over the period, but levels off near the end.
2012 - 2017	309 - 359	G.IX	In 2012, efforts are made to contain the crisis (European Stability Mechanism Treaty signed); Countries and banks struggle to get control of their balance sheets; The German economy remained under pressure during 2012 and 2013. However, in 2014 growth started to pick up again. This trend persisted throughout 2015. In 2016, Germany was the world's fastest growing advanced economy. In 2017, the German economy ran a surplus and sustained its growth once again.	Inflation decreases strongly but the fall starts to level off nearing month 350. Subsequently, there is a sharp rebound.

Table 5.3: Continued: Seminal inflation-affecting events in the German inflation time series.

Seminal Events in The Netherlands

Years of event	Corresponding months in time series	Event ID	Description of period and events	Effect on inflation
1987 - 1993	1 - 80	N.I	Signing of the Maastricht Treaty (1992): European Economic Community formed. Prior events: In 1975, Surinam achieved independence which caused an influx of Surinamese immigrants. Furthermore, in 1980, Queen Juliana abdicates and Crown Princess Beatrix ascends the throne.	Inflation increases up to approximately month 60. Subsequently, inflation decreases followed by a small rebound.
1994	81 - 92	N.II	Wim Kok, head of the labour party, elected prime minister at the head of a three-party coalition.	Inflation remains stable.
1995 - 2001	93 - 176	N.III	Floods in 1995 lead to a national state of emergency; In 1998, Wim Kok is re-elected as prime minister; In 1999, the idea of the Euro is introduced; In 2000, euthanasia is legalised; In 2001, homosexual marriage is legalised.	Inflation falls, then slightly rebounds. Inflation remains stable but somewhat volatile. Near the end, a strong increase.
2002	177 - 188	N.IV	In January, the Euro officially replaces the Dutch Guilder.	Leading up to this period, inflation spiked. Afterwards, inflation decreased up to about month 200.
2003 - 2005	189 - 224	N.V	In 2002, Wim Kok resigns due to Srebrenica massacre report; Popular anti-immigration party leader Pim Fortuyn is murdered; Balkenende becomes prime minister, but government collapses three months later; In 2003, Balkenende becomes prime minister for a second term; In 2004, The Queen's mother Juliana passes away; Film-maker Theo van Gogh was murdered by an Islamic terrorist; In 2005, the EU constitution is rejected by referendum.	Inflation slightly falls and remains relatively stable.

Table 5.4: Seminal inflation-affecting events in the Dutch inflation time series.

<u>Continued:</u> Seminal Events in The Netherlands

Years of event	Corresponding months in time series	Event ID	Description of period and events	Effect on inflation
2006 - 2009	225 - 272	N.VI	In 2006, additional Dutch troops sent to Afghanistan; Balkenende forms temporary government after coalition collapses due to immigration matters; In 2007, Balkenende becomes head of three-party centrist coalition; In 2008, the onset of the financial crisis looms as Lehman Brothers collapses and the national housing bubble bursts;	Inflation initially stable, but climbs near the 250 month mark. Subsequently, inflation sharply decreases around month 255, but recovers onwards from month 265.
2009 - 2011	273 - 296	N.VII	In 2009, seven civilians killed in failed attack on the royal family; Right-wing Freedom Party of Geert Wilders comes second in country's European elections; Greece's accounting unravels, revealing a substantial budget deficit; In 2010, government collapses following dispute over troops in Afghanistan; Financial crisis spreads, austerity measures are adopted and Greece and Ireland are bailed out; Troop withdrawal from Afghanistan commences; The Netherlands' Antilles is dissolved; In 2011, financial crisis deepens and a permanent rescue fund is planned as European leaders look for a resolution.	Inflation increases up to a plateau around month 290, where it remains relatively stable.
2012 - 2014	297 - 332	N.VIII	In 2012, efforts are made to contain the crisis (European Stability Mechanism Treaty signed); Countries and banks struggle to get control of their balance sheets; Prime minister Mark Rutte resigns after right-wing coalition refuses to support an austerity budget; Mark Rutte wins elections (Liberals and Labour) and warns tough austerity measures will be required; In 2014, flight MH17 is shot down over Ukraine.	Inflation increases between months 305 and 310. Around month 315, strong deflation begins.
2015 - 2017	333 - 359	N.IX	The Dutch economy is gradually entering a period of recovery; In 2017, Netherlands denounces Turkish manipulation of expatriates, causing political tension; Mark Rutte forms coalition after 225 days of formation talks; Housing prices are steadily rebounding to 2008 levels and there is some talk of a new looming housing bubble.	Inflation plummets up to month 350 when a sharp rebound occurs.

Table 5.5: Continued: Seminal inflation-affecting events in the Dutch inflation time series.

5.5 Heuristic Impact

The accuracy and robustness statistics, discussed previously in section 5.3, were ambiguous or at times outright not in favour of the performance of the HSM's heuristics. However, these statistics have been computed using the full set of time-series prediction data and may therefore omit important information about performance during specific volatile and/or unstable episodes in the time-series.

To investigate the actual heuristics' presence in the model (i.e. the usage by the agents), based on the heuristics' performance measures in the HSM, the heuristic impacts over time are shown in figure 5.8. The vector of inflation values has been rescaled (i.e. values have been multiplied by $\frac{1}{10}$ to bring them in the same order of magnitude along the scales) and plotted alongside these impacts to make identification of the performance during volatile and unstable periods easier. A visualisation of the evolution of heuristic impacts in the HSM is also available as a stacked density plot in appendix C.

As can be seen in figure 5.8, all heuristics start out with an equal impact of 0.25. Subsequently, as the HSM starts learning and heuristic performance measures are updated, the impacts start to evolve. The general tendency appears to be for short-memory heuristics to prevail in volatile, unstable periods of inflation: The higher the degree of volatility and instability, the more likely ADA is to be used relative to WTF. In stable periods of deflation, the long-memory heuristics become more popular. During unstable periods of volatile dynamics, the LAA heuristic gains ground. During periods of deflation that display a lower degree of instability and volatility, the STF heuristic is employed relatively more often. Detailed descriptions per country and event identifier (detailed in table 5.2, 5.3, 5.4 and 5.5) are shown below.

Germany

G.I The HSM's learning algorithm commences. The short memory rules (ADA and WTF) gain in popularity during this period. Their heuristic impacts increase towards the .3 mark. WTF starts out more popular than ADA, but decreases in relative popularity near the end of the period. The long memory rules are not favoured in this period: LAA even less so than STF. Over the course of this period with somewhat volatile growth, agents appear to favour short memory predictors.

G.II The events leading up to this period (Maastricht Treaty) led to a large spike in volatility and instability in the series. As a result, ADA quickly becomes a popular predictor. Near the end of this period, the agents get used to the new dynamics of the series and start switching to WTF to retain protection against instability and volatility, while also gaining some more advanced prediction power. The long memory predictors are also observed to gain in popularity over this period of deflation and decrease in instability: LAA heuristic impact converges to STF heuristic impact at initial levels.

G.III During this period, the series is relatively stable and displays relatively low volatility. Initially, inflation is stable, followed by inflation and lastly deflation. Over the course of this period, STF slightly gains in popularity. However, the long memory rules remain overshadowed by the short memory rules due to the influence of cached volatility in the algorithm.

G.IV Due to the new influx of volatility and instability, short memory heuristics' fraction (especially ADA) increases again. Later on, influence by steady growth and deflation, the LAA heuristic increases in usage.

G.V After the introduction of the Euro in Germany, the STF heuristic overtakes the LAA heuristic due to the stability of the period.

G.VI A stable, but rather volatile period. Short memory: ADA gains popularity. Long memory: STF is overtaken by LAA again.

G.VII A unique period, due to the onset of the financial crisis near the end. An unstable period of rising inflation up to the end of the period. The dynamics of the inflation series leading up to the crisis era cause distortion in the model: Agents are uncertain what heuristic will be most reliable to forecast due to the esoteric dynamics. As a result, all heuristic fractions converge to a position wherein there occurs indifference between short and long memory rules. Eventually, short memory predictors only hold a very slight advantage in fraction of usage.

G.VIII Extreme instability and deflation. The long memory predictors gain ground; first LAA, subsequently STF. As soon as reversal occurs, WTF and ADA start their climb to dominance once again. However, WTF remains superior to ADA throughout the entire period apart from the final months.

G.IX A period of volatile deflation with a sharp spike of inflation during the last ten months. During the first phase, short memory heuristics dominate more or less equally. In the last ten months, long memory heuristics, especially LAA, gain ground at the expense of short memory heuristics.

The Netherlands

N.I This period displays a volatile and unstable period of inflation up to around month 60. In this time horizon, short memory predictors (ADA and WTF) take the lead immediately. STF mostly remains at baseline level, while LAA continues to lose popularity. The remainder of the period consists of deflation, followed by a rebound. During this time, LAA climbs back to around .2 fraction of usage at the cost of the usage of the short memory heuristics.

N.II A period of stable inflation. LAA features a very slight increase in usage at the expense of WTF. Aside from that, heuristic impact remains more or less equal.

N.III A time of deflation marks the start of this period, causing an increase in the fraction of usage of LAA. Subsequently, a substantial spike in inflation occurs due to the anticipation of the formal introduction of the Euro, resulting in the popularity of short memory heuristics relative to long memory. However, this period is preceded by a time of volatile but relative stability. During that period, heuristic impact evolution has been minimal.

N.IV Deflation occurs after the formal introduction of the Euro. Short memory rules remain most popular, although STF and LAA start gaining as the period drags on.

N.V Around month 200, deflation reaches is limit. Onwards from here, a period of volatile inflation begins. Initially, the convergence of the heuristics towards baseline impact levels continues. However, at the end of the period LAA and ADA are slightly gaining, while WTF and STF are declining.

N.VI A volatile and unstable period, due to the onset of the financial crisis in 2008. Inflation climbs until the 255 month mark, after which it sharply falls until around the 265 month mark. The dynamics of the inflation series leading up to the crisis era cause distortion in the model: Agents are uncertain what heuristic will be most reliable to forecast due to the esoteric dynamics. As a result, all heuristic fractions converge to a position wherein there occurs indifference between short and long memory rules. During the sharp deflation, LAA takes the lead. After the rebound near the end of the period, however, the short memory rules can be seen to ascend to their throne once again.

N.VII This period is marked by a steady climb in inflation. Short memory heuristic retain the largest impact in the HSM, however ADA and WTF perform almost evenly. The same is true for the long memory heuristics: STF and LAA perform almost equally well.

N.VIII Around period 315, inflation rises to its peak. Subsequently, there is a period of strong deflation. Moreover, the resulting dynamics of the inflation series once again cause confusion among the agents in the model, causing the heuristic impacts to converge. This is most likely due to the repercussions of the steps taken by the European Union to contain the financial crisis. After the convergence, WTF and LAA shortly take the lead. After that, the traditional hierarchy (i.e. short memory beats long memory) becomes re-established: LAA keeps declining until it is overtaken by STF, while WTF takes the lead followed by ADA.

N.IX The last period features a time deflation followed by a rebound, then there is a period of stability followed again by deflation. Afterwards, around the 350 month mark, strong inflation occurs again. Throughout this entire time horizon, ADA and WTF are gaining and perform almost equally well. STF is also gaining. Intuitively, LAA declines. Finally, the last months of the series show a sudden reversal to deflation, causing LAA to gain at the expense of the short memory rules.

Due to the large degree of instability and volatility in the time series, the short memory heuristics are shown to be inherently more popular among the agents than the long memory heuristics. The popularity of short and long memory heuristics in the HSM displays mirrored behaviour relative to each other; when short memory gains usage, long memory's usage diminishes and vice versa. Generally, the heuristic impacts of the short memory rules (ADA and WTF) remain above their baseline level (i.e. $U_{h,t=0} = 0.25$) throughout the HSM algorithm's time horizon. Conversely, the heuristic impacts of the long memory rules fluctuate around the baseline level in the case of STF, or below the baseline level for LAA.

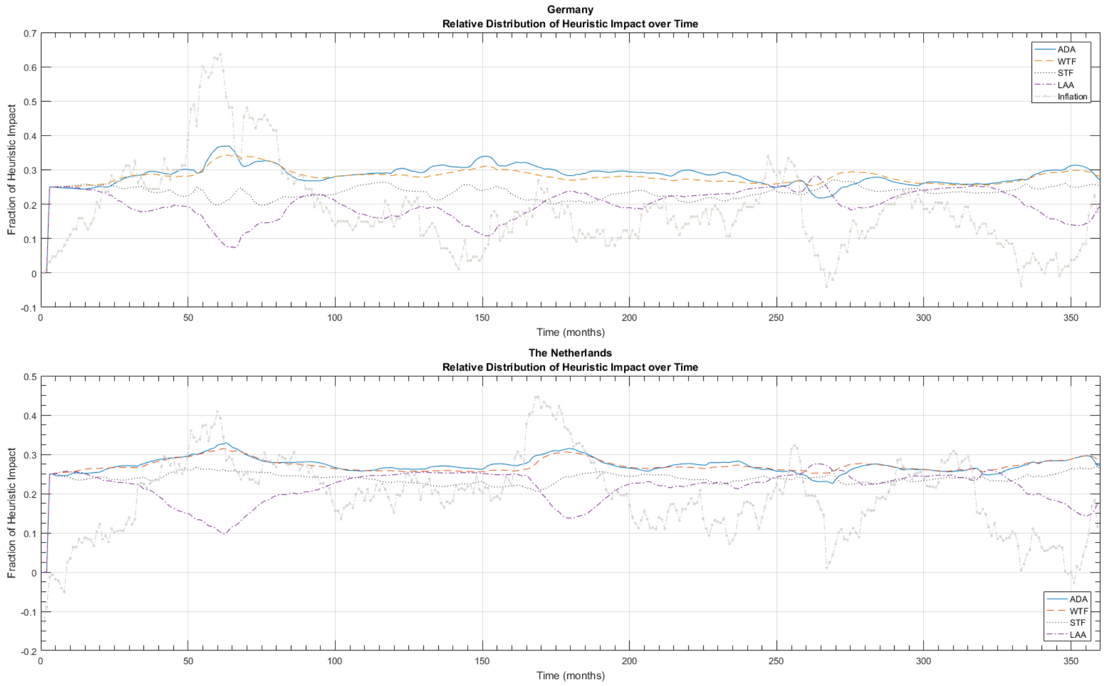

Figure 5.8: The distribution of relative heuristic impact over time for The Netherlands and Germany with a rescaled, arbitrary inflation series for reference.

5.6 Stationarity

In order to test whether or not the inflation series in the HSM can be considered stationary, three specific tests have been performed: The Augmented Dickey-Fuller (ADF) test (Dickey & Fuller, 1979), the Phillips-Perron (PP) test (Phillips & Perron, 1988) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS[2]) test (Kwiatkowski et al., 1992).

The first test, the Kwiatkowski-Phillips-Schmidt-Shin test in table 5.6 and table 5.7, shows that both the Dutch and German inflation series are level and trend stationary at the 1% significance level.

The Netherlands

	KPSS	Truncation lag parameter	p-value
Trend	0.437***	4	0.01
Level	0.799***	4	0.01

Table 5.6: Kwiatkowski-Phillips-Schmidt-Shin tests on the Dutch inflation series.

Germany

	KPSS	Truncation lag parameter	p-value
Trend	0.254***	4	0.01
Level	1.820***	4	0.01

Table 5.7: Kwiatkowski-Phillips-Schmidt-Shin tests on the German inflation series.

The ADF tests, of which the results are shown in table 5.8, already paint another picture. According to this test, the inflation series for the Netherlands is stationary (significant at $\alpha = 0.05$) but the German series is not. The German time series can be considered stationary at $\alpha = 0.1$ with this test, however that is not sufficiently robust.

	Dickey-Fuller	Lag order	p-value
The Netherlands	-3.903**	7	0.014
Germany	-3.409*	7	0.053

Table 5.8: Augmented Dickey-Fuller tests on the Dutch and German inflation series.

The final test for stationarity, the PP test (table 5.9), shows that both the Dutch and German time series are non-stationary. Stationarity can be proven at the 10% significance level with this test, however this is not enough power to provide solid statistical evidence in this case.

	Dickey-Fuller Z(α)	Truncation lag parameter	p-value
The Netherlands	-20.379*	5	0.065
Germany	-18.586*	5	0.092

Table 5.9: Phillips-Perron tests on the Dutch and German inflation series.

The contradiction in the results of the stationarity tests show that the inflation series cannot be truly considered either an $I(0)$ or an $I(1)$ process, and the reality is somewhere in between the two. To further investigate whether or not the series can be classified as an $I(d)$ process, estimates of the spectral density of the series

[2]Critical values for KPSS: 0.216 (1%), 0.146 (5%), and 0.119 (10%)

are considered in the form of a periodogram. These are shown in figure 5.9 for the Dutch series and in figure 5.10 for the German series.

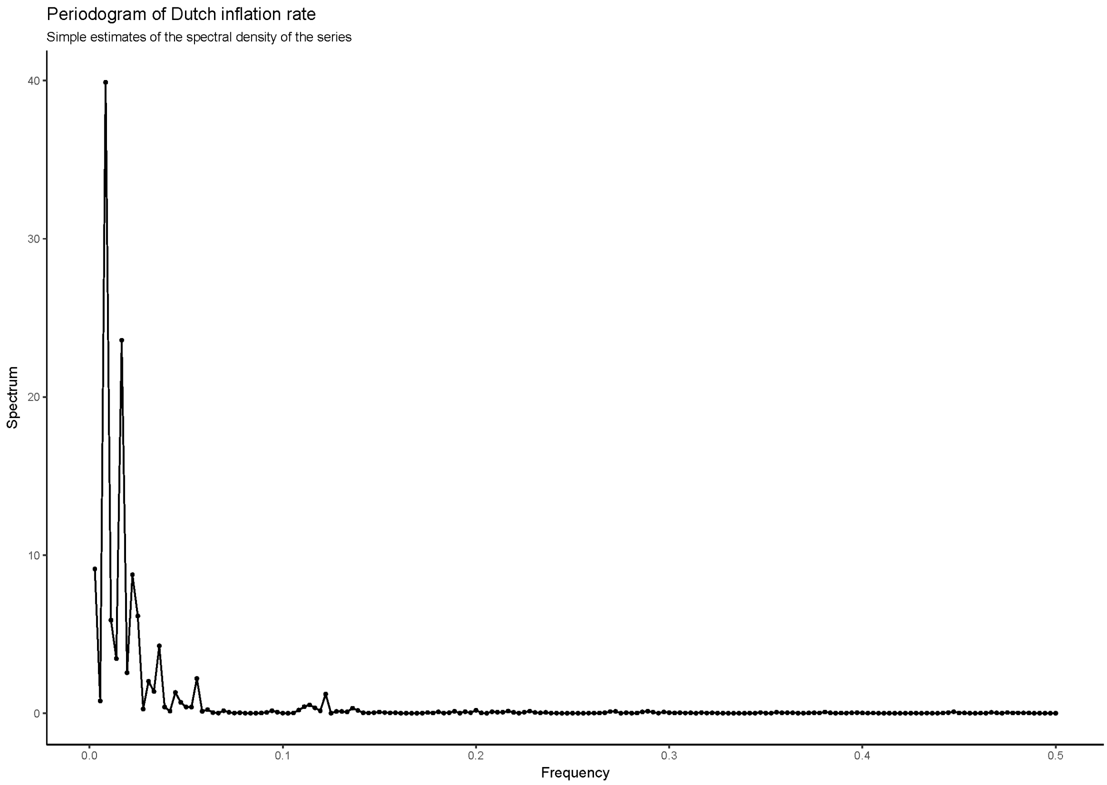

Figure 5.9: Periodogram of the Dutch inflation series.

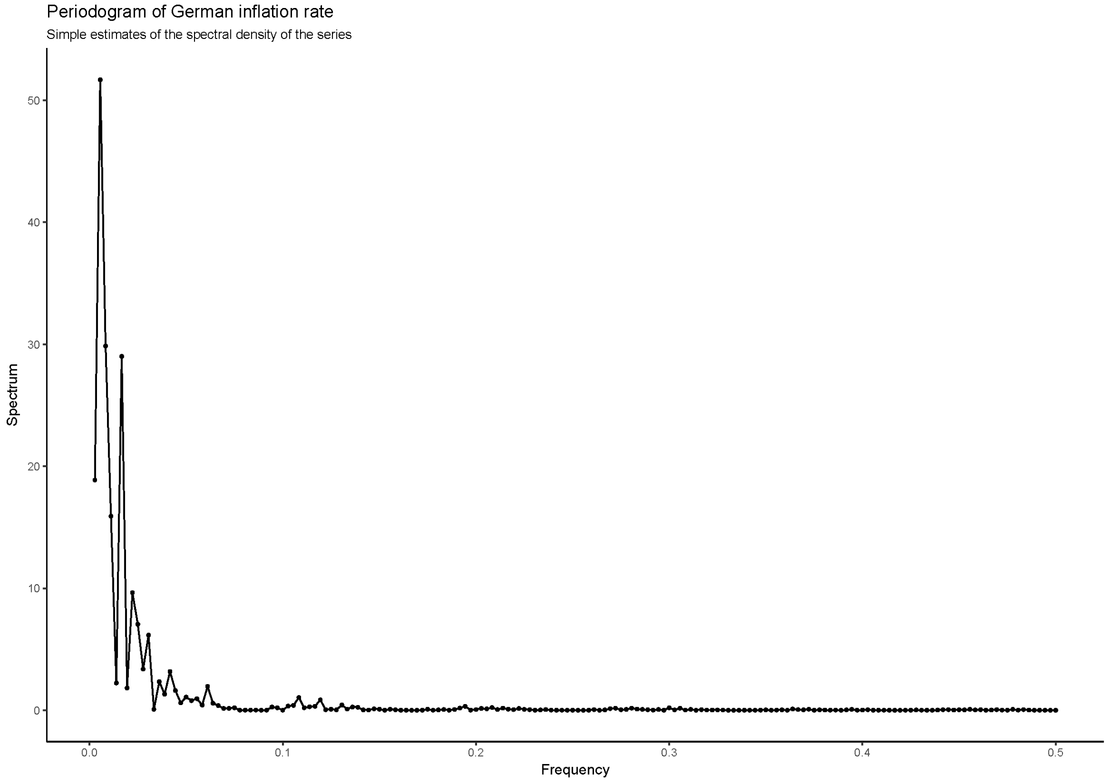

Figure 5.10: Periodogram of the German inflation series.

Both figures display a typical peak at the zero frequency levels, after which they quickly tail off to small spectrum values. This behaviour is typical for an $I(d)$ process (Boutahar et al., 2007).

The final piece of evidence is sought in plots of the autocorrelation function (ACF) of the differentiated series data, using the approximated binomial expression of the long-memory filter and a preliminary estimate of the memory parameter in the ARFIMA$_{(p,d,q)}$ model (appendix D). These plots are shown in figure 5.11.

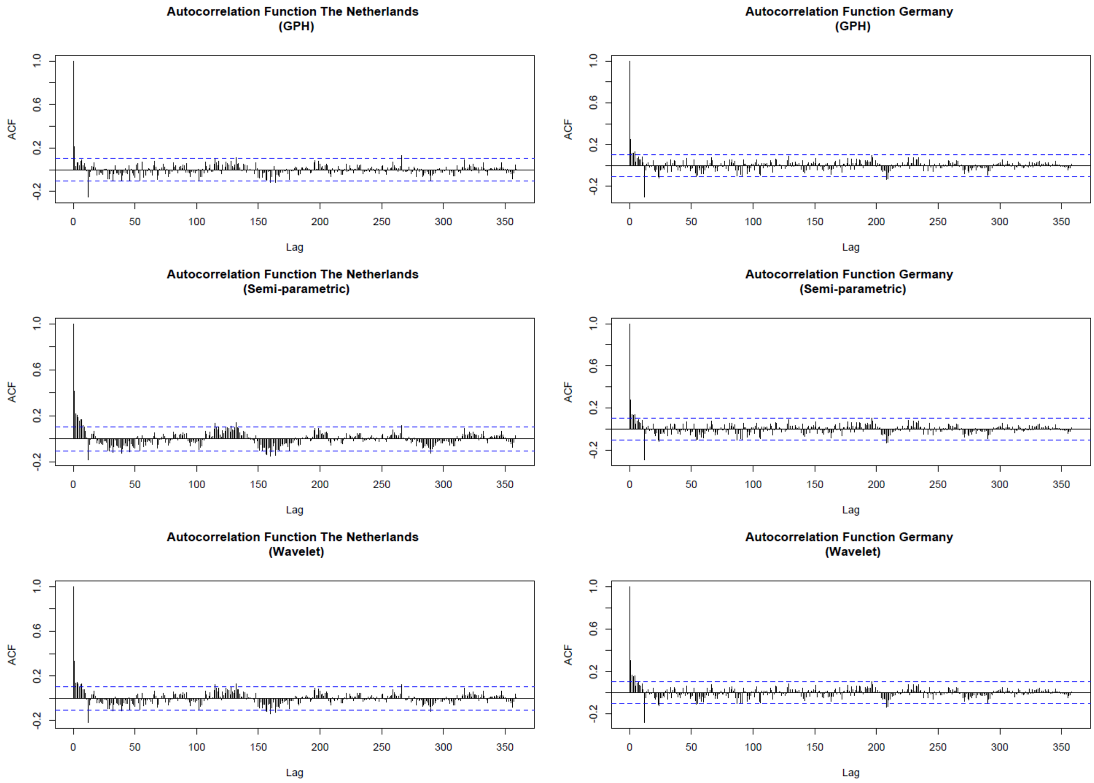

Figure 5.11: Plots of the autocorrelation functions of the differentiated time series data, using the approximated binomial expression of the long-memory filter and the (indicated) estimate of the memory parameter in the ARFIMA$_{(p,d,q)}$ model.

The plots show an initial peak and, subsequently, a slow decay of the ACF. This is typical behaviour for an $I(d)$ process (Boutahar et al., 2007). Depending on the preliminary estimator of the memory parameter, and the underlying time series, a more pronounced tailing-off motion and stronger persistent significant lag behaviour can be observed in the graphs.

The inflation time series used in the HSM in this research display contradictory results in common econometric stationarity tests, have a peak in spectral density at zero frequency in the periodograms and display a slow decay in the ACF over time. There is sufficient econometric evidence to assume the time series can be classified as $I(d)$ processes.

5.7 Memory Parameter

The long memory parameters (d) of the Dutch and German time series are estimated using the Geweke & Porter-Hudak (GPH) (Geweke & Porter-Hudak, 1983), semi-parametric (*sperio*) and wavelet techniques. The resulting estimates for d are shown in table 5.10 and 5.11.

The Netherlands

	Estimate d	Standard deviation	Standard error
GPH	0.795	0.194	0.201
Semi-parametric	0.624	0.083	0.079
Wavelet	0.692	0.072	0.004

Table 5.10: Estimates and statistics of the estimation procedure of the memory parameter of the Dutch inflation series.

Germany

	Estimate d	Standard deviation	Standard error
GPH	0.783	0.194	0.134
Semi-parametric	0.764	0.083	0.065
Wavelet	0.739	0.063	0.003

Table 5.11: Estimates and statistics of the estimation procedure of the memory parameter of the German inflation series.

For all estimation passes of the memory parameter for both in countries in table 5.10 and 5.11: $d > \frac{1}{2}$. Therefore, both series are unstable long-memory processes. Moreover, both series are invertible; the signals can be inverted into a representation of past observations, i.e. MA to AR. This is the case because $d > -\frac{1}{2}$ holds for both series across all estimation passes.

The wavelet estimation yields the lowest relative error and standard deviation for both series. Using this estimate as baseline, the long-memory parameter for Germany is higher with 0.739 compared to The Netherlands with 0.692.

Chapter 6

Conclusions and Limitations

6.1 Conclusions

The research has shown that short-memory prediction rules (i.e. the adaptive heuristic and, to some extent, the weak trend-following heuristic) tend to prevail in forecasting the macroeconomic series during unstable periods, whereas long-memory prediction rules (the learning anchoring and adjustment heuristic and strong trend-following heuristic) tend to rise to popularity during periods of severe instability and volatility in the series. However, due to the substantial degree of volatility and instability in the dynamics of the time series of both countries, the short memory heuristics have proven to retain a higher impact in the model over time relative to their long memory counterparts. Furthermore, the impact (i.e. popularity) of both heuristic memory types exhibit mirrored behaviour: When long memory gains a higher impact, the impact of short memory diminishes (and vice versa). These conclusions can be generalised to some extent, as they apply to both the Dutch and the German inflation series despite differences in fundamentals.

It should be noted, however, that heuristic impact also depends on the nature of the macroeconomic series on which the forecast is made. Long-memory parameters for both the Dutch and the German time-series have been investigated in this research. It can be concluded that both series are unstable long-memory processes. However, the implied long-memory parameter estimated by wavelet estimation is higher for Germany (0.739) compared to The Netherlands (0.692). Thus, the German time-series exhibits a larger degree of instability. As a result, the long-memory prediction rules are relatively more popular in the HSM of the German time-series.

6.2 Limitations & Suggestions

The idea of modelling heterogeneous agent forecasting behaviour under bounded rationality through a heuristic switching model is an interesting concept. However, by implementing this concept out of a contained laboratory experiment environment using real-world data, like in this research, new difficulties and limitations have arisen that were previously not an issue. Real-world data originates from the collective decision-making of a world of agents instead of merely the few constraints of a laboratory experiment. Therefore, the simple prediction rules in this research fail to provide truly satisfactory fits. Moreover, prediction accuracy fluctuates substantially as a result of changes in volatility and stability in the underlying time-series. In short, the agents in the HSM are unrealistically uninformed about the environment of the time-series they want to forecast and need to be provided with more information in a realistic way that adheres to bounded rationality.

In order to address these issues in further research, there are two possible avenues I would like to suggest. The first is to incorporate the concepts of genetic algorithms (GA) in the heuristic switching model in the context of real-world data. A GA is an evolutionary algorithm that is closely inspired by the process of natural selection. The first steps for such endeavours have already been made in *Individual expectations and aggregate macro behaviour* (Assenza et al., 2013) and *Genetic algorithm learning in a New Keynesian macroeconomic setup* (Hommes, Makarewicz, Massaro, & Smits, 2017).

The second, and more interesting suggestion, would be to use a full-blown machine learning approach that departs entirely from the classic short- and long-memory prediction rules. To accomplish this, I would suggest programming a new HSM in *R* or *Python* instead of *Matlab* and to use the *caret* and/or *keras* libraries to respectively train a support vector machine or (preferably) a neural network for the HSM's prediction rules. By demarcating training data for these algorithms that is small in size, or to which (red) noise has been added, one can still simulate the bounded rationality of the agents without depriving them of being 'logically' informed about their macroeconomic environment.

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Appendices

Appendix A

Origin of the Cobweb Model

The cobweb model with heterogeneous beliefs (Brock & Hommes, 1997).

$$D(p_{t+1}) = \sum_{j=1}^{K} n_{j,t}\Big(p_t, \mathscr{H}\big(\vec{P}_{t-1}\big)\Big)\Big(H_j\big(\vec{P}_t\big)\Big) \tag{A.1}$$

$$n_{j,t+1} \equiv n_{j,t+1}\Big(p_{t+1}, \mathscr{H}\big(\vec{P}_t\big)\Big) = \frac{\exp(\beta\, U_{j,t+1})}{Z_{t+1}} \tag{A.2}$$

$$Z_{t+1} \equiv \sum_{j=1}^{K} e^{\beta\, U_{j,t+1}} \tag{A.3}$$

Appendix B

Forecast Accuracy Statistics

The section below gives an overview of the forecast accuracy statistics used in this research. Each statistic has its own unique benefits and drawbacks. Therefore, they are used as a set to complement each other.

Mean Squared Error The MSE measures the average of squares of the errors (or deviations). It is a risk function, expressing the expected value of the quadratic loss. The difference occurs due to stochastic elements, or because the estimator omits information that could enhance the estimation process (Lehmann & Casella, 2006). The MSE of an estimator $\hat{\theta}$ with respect to an unknown parameter θ is as shown in equation B.2. The formula incorporated in the research algorithm is as displayed in equation B.2.

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(\hat{Y}_i - Y_i)^2 \tag{B.1}$$

$$\text{MSE}(\hat{\theta}) = \mathbb{E}\Big[(\hat{\theta} - \theta)^2\Big] \tag{B.2}$$

Root Mean Square Error The RMSE, as shown in equation B.3, is a measure of prediction accuracy that measures the differences between model predictions and actual values. It represents the sample standard deviation of differences between predicted and observed values (Hyndman & Koehler, 2006).

$$\text{RMSE}(\hat{\theta}) = \sqrt{\text{MSE}(\hat{\theta})} = \sqrt{\mathbb{E}\Big((\hat{\theta} - \theta)^2\Big)} \tag{B.3}$$

Mean Absolute Error The MAE (equation B.4) is the average absolute distance between X and Y, i.e. the observed value and the actual value. The lack of squared distances, which hinder the interpretation of other metrics described in this appendix, are not present in the MAE (Willmott & Matsuura, 2005).

$$\text{MAE} = \frac{\sum_{i=1}^{n} |y_i - x_i|}{n} = \frac{\sum_{i=1}^{n} |e_i|}{n} \tag{B.4}$$

Mean Absolute Percentage Error The MAPE (equation B.5) is a prediction accuracy measure that expresses accuracy as a percentage. It does have a common bias to systematically select prediction methods with too low forecasts, relative to the actual values (Tofallis, 2015).

$$\text{MAPE} = \frac{100}{n} \sum_{i=1}^{n} \left| \frac{A_t - F_t}{A_t} \right| \tag{B.5}$$

Theil's U Statistic The Theil's U forecast accuracy statistic, shown in equation B.7, is based on the concept of information entropy. The statistic can be considered as the quotient of the RMSE of the forecast and the RMSE of the naive model (i.e. NF1, equation B.6). For a value of 1, the chosen method of forecasting is no more accurate than the naive forecasting model. For values exceeding 1, the forecasting method is less accurate than a naive forecast. Intuitively, values less than 1 imply a better accuracy in the forecasting method with respect to a naive rule, with $U = 0$ implying a perfect fit (Theil, 1966).

$$y_{t+1} = y_t \tag{B.6}$$

$$\text{Theil's U} = \frac{\sum \hat{u_t^2}}{\sum (y_t - y_{t-1})^2} \tag{B.7}$$

Or, as implemented in many econometric software packages and programming languages, the coefficient is defined as in equation B.8.

$$U = \frac{\sqrt{\sum_{t=N-m+1}^{N} (x_t - \hat{x}_{t-1}(1))^2}}{\sqrt{\sum_{t=N-m+1}^{N} x_t^2} + \sqrt{\sum_{t=N-m+1}^{N} \hat{x}_{t-1}(1)^2}} \tag{B.8}$$

Appendix C

Heuristic Impact Density

The impact distribution of each heuristic shown as a stacked density area plot across time for The Netherlands and Germany.

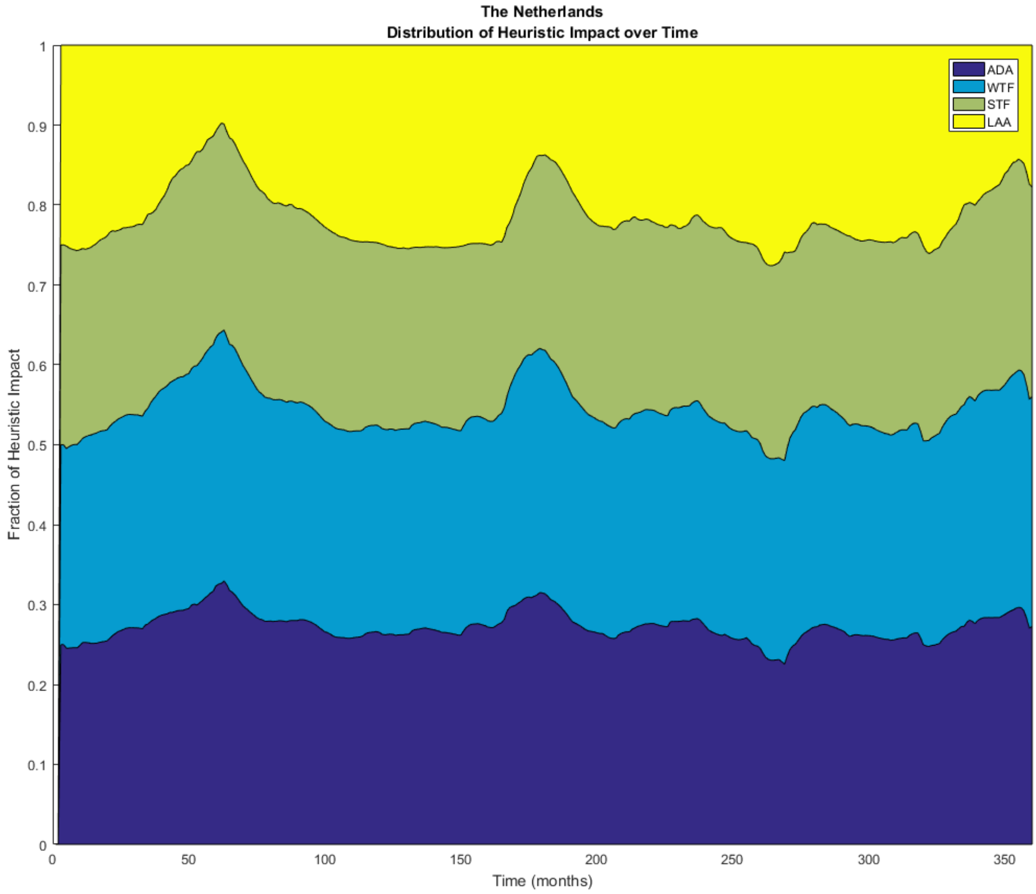

Figure C.1: The distribution of heuristic impact over time for The Netherlands

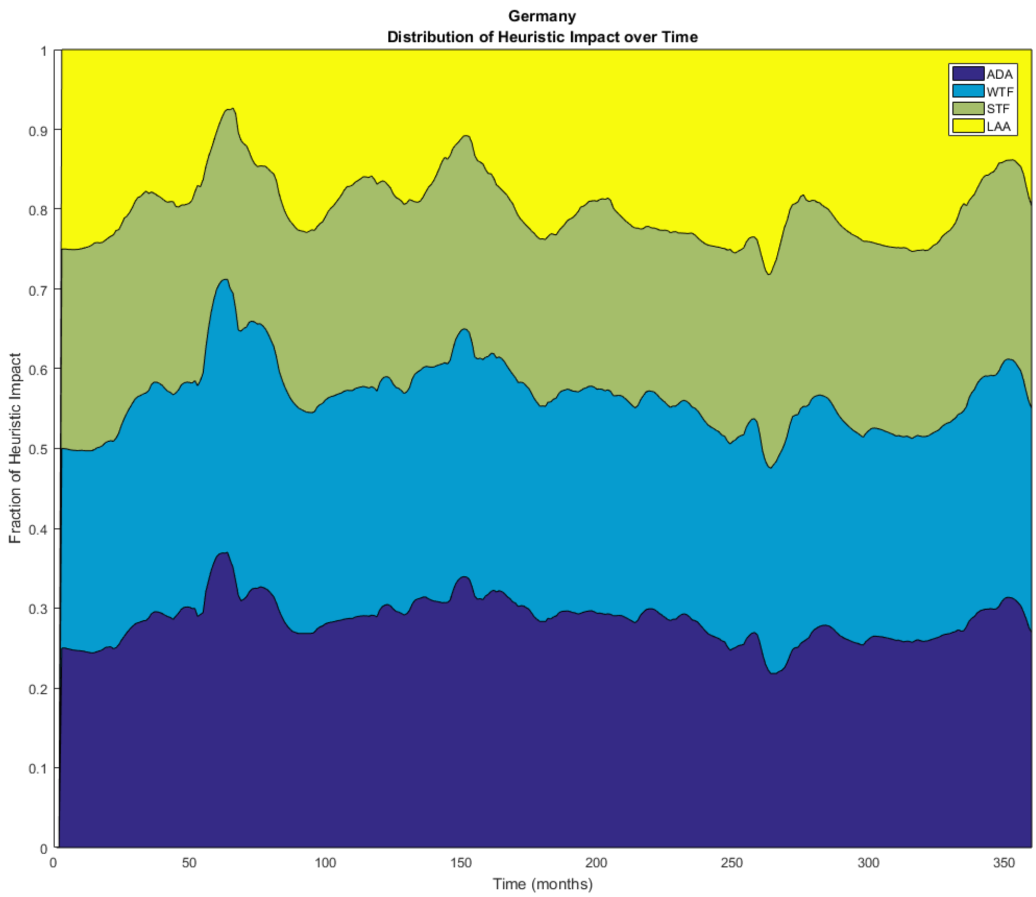

Figure C.2: The distribution of heuristic impact over time for Germany

Appendix D

Models of Long-Memory

ARFIMA$_{(p,d,q)}$ Process The ARFIMA (autoregressive fractionally integrated moving average) model has been put forward by (Granger & Joyeux, 1980) and (Hosking, 1981) to describe time series with long memory behaviour. The d parameter in the ARFIMA$_{(p,d,q)}$ process is able to take on real values. As described in *Estimation Methods of the Long Memory Parameter: Monte Carlo Analysis and Application*, $\{X_t\}$ can be categorised as an ARFIMA$_{(p,d,q)}$ process for (Boutahar et al., 2007):

$$\theta(B)\epsilon_t = \phi(B)(1-B)^d(X_t - \mu) \tag{D.1}$$

$$\phi(B) = 1 - \phi_1 B - \ldots - \phi_p B^p \tag{D.2}$$

$$\theta(B) = 1 + \theta_1 B + \ldots + \theta_q B^q \tag{D.3}$$

$$d \in \Re,\ \mu = E(X_t),\ \epsilon_t \sim \text{i.i.d.}(0, \sigma_\epsilon^2) \tag{D.4}$$

$$(1-B)^d = \sum_{k\geq 0} b_k(d) B^k \tag{D.5}$$

$$b_k(d) = \frac{\Gamma(k-d)}{\Gamma(k+1)\Gamma(-d)} \tag{D.6}$$

In the equations above, B is a lag operator and $\Gamma\cdot$ is the gamma function. The spectral density of the process $\{X_t\}$ can be described by:

$$f(\lambda) = \frac{\sigma_\epsilon^2 \mid \theta(e^{i\lambda}) \mid^2}{2\pi \mid \phi(e^{i\lambda}) \mid^2} \mid 1 - e^{i\lambda} \mid^{-2d} \tag{D.7}$$

$$\text{for} - \pi < \lambda < \pi \tag{D.8}$$

Stationarity and long-memory

- $\{X_t\}$ can be considered invertible for values of $d > -\frac{1}{2}$.
- $\{X_t\}$ can be considered stationary for values of $d < \frac{1}{2}$.
- Mean reversion is more pronounced, $\{X_t\}$ is anti-persistent and $\rho(k)$ decays more quickly in the case of $-\frac{1}{2} < d < 0$ compared to $0 < d < \frac{1}{2}$ (Mandelbrot & Van Ness, 1968).
- $\{X_t\}$ is a stable long-memory process with a hyperbolically decaying ($\lim_{t\to\infty} = 0$) autocorrelation function for values of $0 < d < \frac{1}{2}$.