Rules

Definition

$y = \log_b x$ if and only if $x = b^y$

1. $3 = \log_2 8 \mid 8 = 2^3$

Inverse Property

$\log_b b^x = x$ & $b^{\log_b x} = x$

1. $\log_2 2^3 = 3$
2. $2^{\log_2 3} = 3$

Change Base

$\log_a v = \log_b v / \log_b a$

1. $\log_{100} 10 = \log_{10} 10 / \log_{10} 100$

$100^x = 10$

$x = 1/2$

$\log_{10} 10 / \log_{10} 100 = 1/2$

$10^x = 10$

$x = 1$

$10^x = 100$

$x = 2$

Product / Quotient Property

$\log_b A + \log_b B = \log_b A*B$

$\log_b A - \log_b B = \log_b A/B$

1. $\log_2 4 + \log_2 8 = \log_2 32$
   a. $2 + 3 = 5$
2. $\log_3 27 - \log_3 9 = \log_3 3$
   a. $3 - 2 = 1$

Power Property

$\log_b A^P = P * \log_b A$

1. $\log_2 4^2 = 2*\log_2 4$
   a. $\log_2 16 = 2*\log_2 4$
   b. $4 = 2*2$

Logarithms and Exponentials

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There are usually 2 things that are confusing about logarithms.

1. What exactly the logarithm does
2. The difference between exponentials and polynomials

Formal Definition

The logarithm of a positive real number *x* with base *b* is the exponent by which b must be raised to yield *x*.

The difference between exponentials and polynomials

Exponentials and polynomials often get mixed up because they both utilize power.

Polynomials change the **base** of the power. (**x^2**)
But exponentials change the **exponent** of the power. ($2^{\mathbf{x}}$)

Therein lies the difference between roots and logs, logs return the **exponent** while roots return the **base**.

x	**$y = 2^x$**	**x**	**$y = \log_2 x$**
0	1	1	0
1	2	2	1
2	4	4	2

What the Logarithm does

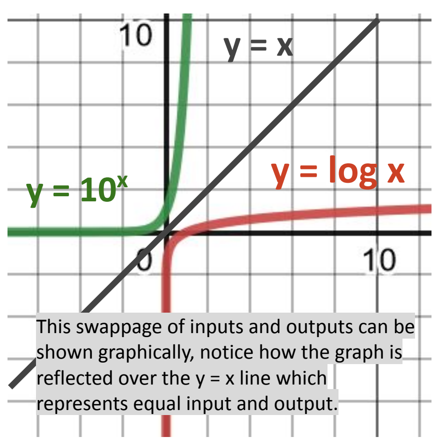
A more concise definition than the formal one would be that the logarithm is the inverse function of $y = b^x$.

Inverse functions **"undo"** what the original function "did". In other words it returns the <u>input</u> from the <u>output</u>.

Ex.
$f(x) = x + 2$ (original function)
$f(x)^{-1} = x - 2$ (inverse function)
$f^{-1}(f(3)) = 3 \longrightarrow (3 + 2) - 2 = 3$

The logarithm will return *x* (the input) given *y* and *b* (the result and unknowns) from the equation $y = b^x$

Put simply, Log is to the exponential (b^x) what subtraction is to addition.



This swappage of inputs and outputs can be shown graphically, notice how the graph is reflected over the y = x line which represents equal input and output.

Properties of Logarithms

Product Property

Use when simplifying

$\log a + \log b = \log a * b$

1. $\log_2 4 + \log_2 8 = \log_2 32$
   a. $2 + 3 = 5$

Quotient Property

Use when simplifying

$\log a - \log b = \log a / b$

1. $\log_3 27 - \log_3 9 = \log_3 3$
   a. $3 - 2 = 1$

Power Property

Use when simplifying

$\log a^b = b * \log a$

1. $\log_2 4^2 = 2*\log_2 4$
   a. $\log_2 16 = 2*\log_2 4$
   b. $4 = 2*2$

Inverse Property

Allows simplification when log or exponential is applied to both sides of an equation

$\log_b b^x = x$ & $b^{\log_b x} = x$

$\log_b b^x = x$ allows

$4 = 2^{x + 3}$

$\log_2 4 = x + 3$

$x + 3 = 2$

$x = -1$

$b^{\log_b x} = x$ allows

$\log_2(x + 3) = 2$

$x + 3 = 2^2$

$x + 3 = 4$

$x = 1$

Change of Base Formula

Use when your calculator doesn't support anything other than natural or common log.

$\log_b x = \log x / \log b$

1. $\log_2 10 = \log 10 / \log 2$

Additional Examples

Example 1

$\log_2(x / 3) = \log_2 3$

$\log_2 x - \log_2 2 = \log_2 3$ → *Quotient Property*

$\log_2 x = \log_2 2 + \log_2 3$ → *Algebra*

$\log_2 x = \log_2 6$ → *Product Property*

$x = 6$ → *Inverse Property*

Example 2

$3^x = \log_2 8^2$

$3^x = 2 * \log_2 8$ → *Power Property*

$3^x = 6$ → *Evaluate*

$x = \log_3 6$ → *Inverse Property*

$x = \log 6 / \log 3$ → *Change of Base Formula*

Example 3

$7^{\log_7 7} - \log_7 7^7$

$7^{\log_7 7} - 7 * \log_7 7$ → *Power Property*

$7 - 7 * 1$ → *Inverse Property*

0 → *Evaluate*