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The Effects of Yield Stress and Casing Thickness on Blast Impulse and Fragment Velocity

Michael D. Hutchinson*[a]

Abstract: A previous paper by the author [1] showed how his equation [2], based on the model first proposed by Gurney [3] for the blast impulse from cased munitions, can be modified to allow for the yield stress of the casing metal. This paper [1] also provided validation for this further equation from available experimental data. Further issues regarding the many smaller but significant interactions between the fracturing casing and the expanding gases need to be investigated before it can be said that a complete theory of blast impulse and fragment initial velocity is in place. In this paper, an approximate evaluation will be made of the potential effect and significance of kinetic energy losses, due to dissipation by means of casing strain energy dissipation. As expected, these losses are found to be most significant for thick, low density, high yield stress casings. The effect of finite casing thickness, also not previously considered, has been found to be small.

Keywords: Gurney · Blast impulse · Strain energy · Fragment velocity · Casing thickness

1 Introduction

Previous papers [1,2,4] have discussed the major contribution made by R. W. Gurney [3] to the field of explosion mechanics. These previous derivations, based on Gurney's original energy balance equation, have provided equations that can now form the basis for further developments in the theory of blast impulse from cased munitions. So far, it has been assumed that energy available for conversion to a kinetic form is constant throughout the casing expansion and fracture process until the gases are released. Herein, an estimate is made of the loss of available energy through casing strain dissipation during its expansion under compressive yield stress towards fracture.

The estimates made will be "worst case" i.e. they will assume that, following explosive initiation, the casing metal is strained homogeneously at a constant yield stress towards its failure strain, which will be estimated based on Taylor's criterion [5] as discussed in [6] and applied in [2]. No great accuracy will be claimed for these estimates, since the objective here is to provide an assurance that in most instances the losses via casing strain energy dissipation are small.

2 Calculations

2.1 Dynamic Gurney Energy Balance for Strong Casings

Following Gurney, the charge mass is denoted by C and the casing mass by M. Equation (8) in Ref. [2] set out, in cylindrical geometry, the energy balance at casing fracture between casing metal and expanding gases, where σ_y is the yield stress of the casing metal and P_0 is the initial mean pressure of the detonated explosive gases (about 42% of the Chapman-Jouguet pressure):

$$E=\frac{1}{2}V^2\frac{M}{C}\left(1-\left(\frac{\sigma_y}{P_0}\right)^{(\gamma-1)/\gamma}\right)+\frac{1}{2}V^2\left(\frac{1}{2}+\frac{M}{C}\left(\frac{\sigma_y}{P_0}\right)^{(\gamma-1)/\gamma}\right) \tag{1}$$

As described in Ref. [2], this equation is based on exactly the same assumptions made by Gurney in deriving the initial velocity of casing fragments from exploding munitions. What has been added is Taylor's prediction [5] that the casing will fracture when the internal gas pressure falls below the level needed to continue to force the metal to yield in compressive stress. Through the adiabatic constant, γ, for an assumed ideal gas, the expansion ratio at failure can be shown, as in Equation (5), to be a simple function of the ratio σ_y/P_0. The Gurney energy, E, is the energy per unit charge mass available for conversion to kinetic energy, i.e. a fraction, typically 70%, of the heat of detonation. It has been shown experimentally by Cooper [7] and analytically by Koch, Arnold, and Estermann [8] that $E\approx D^2/18$, where D is the measured detonation velocity.

[a] *M. D. Hutchinson*
Hydrodynamics Technology Centre
AWE
Aldermaston, Reading RG7 4PR, UK
**e-mail: m.hutchinson@imperial.ac.uk*

2.2 Casing Metal Strain Energy

In these previous calculations, the strain energy dissipated in the case material was neglected. At high M/C, strain energy dissipation may become significant and can be roughly quantified as follows. If the casing has density ρ_m, it has volume M/ρ_m. It also has yield stress σ_y, as above. In being expanded from radius R_0 to radius R, the case material experiences natural strain $\log_e(R/R_0)$. The stress-strain curve for the case metal will show an initial rise through the elastic and initial plastic regime, however, assuming as a bounding case that this initial rise is very short; the maximum strain energy E_s deposited in the case material will therefore be simply the product of stress and strain, as follows:

$$E_s = \frac{M\sigma_y}{\rho_m}\ln\left(\frac{R}{R_0}\right) \tag{2}$$

Now, according to Taylor [5], at the point of case fracture, the remaining gas pressure inside the casing equals the casing yield stress. Furthermore, we have, from the ideal gas equation, the following thermodynamic relationships. The perfect gas equation of state is $P=\rho RT$, where R is the gas constant and T the temperature. Denoting the gas specific heats at constant pressure and constant volume by c_p and c_v, the internal energy, $E=c_vT$, also, $R=c_p-c_v$, and, by definition, $\gamma=c_p/c_v$.

Therefore, from the above equations:

$$E = c_v\frac{P}{\rho\Re} = c_v\frac{P}{\rho\left(c_p-c_v\right)} = \frac{P}{\rho(\gamma-1)} \tag{3}$$

and therefore the initial pressure of the unexpanded explosive gases will be:

$$P_0 = E\rho_0(\gamma-1) \tag{4}$$

where ρ_0 is the initial density of the explosive. Thus, by first using Taylor's criterion to equate σ_y to the gas pressure, which was adiabatically expanded from initial radius R_0 to the casing radius at fracture R_f, and then substituting for P_0 from Equation (4):

$$\sigma_y = P_0\left(\frac{R_0}{R_f}\right)^{2\gamma} = E\rho_0(\gamma-1)\left(\frac{R_0}{R_f}\right)^{2\gamma} \tag{5}$$

Finally, substituting from Equation (2) for σ_y in Equation (5), the bounding value for the strain energy dissipated in the casing metal up to its point of fracture will be:

$$E_s = E\left(\frac{\rho_0}{\rho_m}\right)(\gamma-1)\left(\frac{R_0}{R_f}\right)^{2\gamma} M\ln\left(\frac{R_f}{R_0}\right) \tag{6}$$

This energy loss now needs to be divided appropriately between the casing metal and the gases, in proportion to the amount of kinetic energy they would each have otherwise possessed at casing fracture. The potential energy in the gases, in the form of pressure, P(R), is governed purely by adiabatic expansion and is not affected by any losses to casing strain dissipation, so the whole loss is taken by the shared kinetic energy of the gases and the metal.

2.3 Kinetic Energy Robbed from the Gases and the Casing

Separating out the kinetic energy terms, one can identify the first two terms in the right-hand side of the following expanded version of Equation (1), where the initial gas pressure/yield stress ratio has, for now, been converted back to the inverse expansion ratio (R_0/R), as in Ref. [2], and both sides multiplied by C:

$$EC = \frac{1}{2}MV^2\left(1-\left(\frac{R_0}{R}\right)^{2(\gamma-1)}\right) + \frac{1}{4}CV^2\left(1-\left(\frac{R_0}{R}\right)^{2(\gamma-1)}\right) + \frac{1}{2}CV^2\left(\frac{1}{2}+\frac{M}{C}\right)\left(\frac{R_0}{R}\right)^{2(\gamma-1)} \tag{7}$$

So, extracting the necessary ratio from Equation (7), the proportion of the system kinetic energy, which resides in the gas at all expansion radii is therefore:

$$\frac{\frac{1}{4}C}{\frac{1}{2}M+\frac{1}{4}C} = \frac{C}{C+2M} \tag{8}$$

The energy robbed from the gas by case strain energy dissipation is, from Equation (6) and Equation (8):

$$\left(\frac{C}{C+2M}\right)E_s = \left(\frac{C}{C+2M}\right)E\left(\frac{\rho_0}{\rho_m}\right)(\gamma-1)\left(\frac{R_0}{R_f}\right)^{2\gamma} M\ln\left(\frac{R_f}{R_0}\right) \tag{9}$$

Similarly, the kinetic energy which the casing gives up to its own strain energy dissipation is:

$$\left(\frac{2M}{C+2M}\right)E_s = \left(\frac{2M}{C+2M}\right)E\left(\frac{\rho_0}{\rho_m}\right)(\gamma-1)\left(\frac{R_0}{R_f}\right)^{2\gamma} M\ln\left(\frac{R_f}{R_0}\right) \tag{10}$$

2.4 Casing Fragment Modified Initial Velocity

From Equation (10), the modified casing kinetic energy is:

$$\frac{1}{2}MV^2 = EM\frac{2C}{C+2M}\left[1-\frac{M}{C}\left(\frac{\rho_0}{\rho_m}\right)(\gamma-1)\left(\frac{R_0}{R}\right)^{2\gamma}\ln\left(\frac{R}{R_0}\right)\right] \tag{11}$$

As a fraction of the unmodified casing Gurney energy, this is simply:

$$1-\frac{M}{C}\left(\frac{\rho_0}{\rho_m}\right)(\gamma-1)\left(\frac{R_0}{R}\right)^{2\gamma}\ln\left(\frac{R}{R_0}\right) \tag{12}$$

When R reaches the case fracture radius, we can convert R/R_0 back to the yield stress parameter for failure based on Taylor's criterion [5] and take the square root to obtain the calculated loss in casing fragment initial velocity, relative to that predicted by Gurney:

$$\frac{V}{V_G}=\sqrt{1-\frac{M}{C}\left(\frac{\rho_0}{\rho_m}\right)(\gamma-1)\left(\frac{\sigma_y}{P_0}\right)\ln\left(\frac{\sigma_y}{P_0}\right)^{-\frac{1}{2\gamma}}} \tag{13}$$

This calculated loss in initial fragment velocity is shown as a percentage in Figure 1, for $\gamma=3$ and a range of values of both the casing/charge mass ratio M/C and σ_y/P_0.

As Figure 1 shows, losses will be highest for heavier, but less dense casings of stronger alloys. However, the more extreme examples plotted, e.g. munitions with an aluminum alloy casing yields stress $\sigma_y=0.6$ GPa and $M/C=5$, are unlikely to be deployed in practice. In practice, casing velocity reductions are unlikely to exceed 5%, even according to this model, which neglects both the finite rise of the stress-strain curve and the finite casing thickness to arrive at an over-estimate.

Figure 1 shows only the departure from ideal Gurney velocity due to the casing's own strain energy dissipation. Fracture before the casing has received the full available drive from the gases, followed by early gas escape, and may affect the initial fragment velocities more significantly.

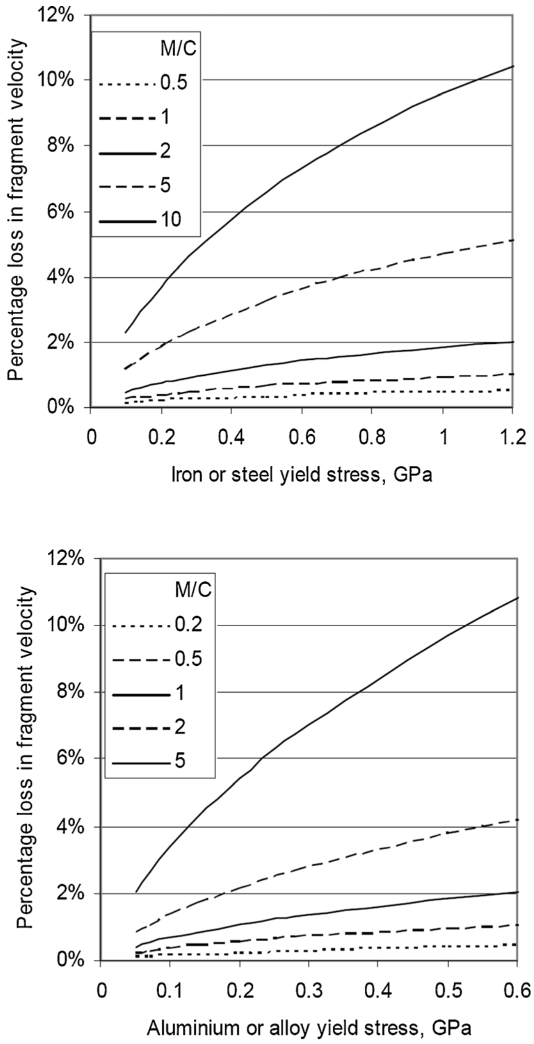

Figure 1. Predicted percentage losses in fragment initial velocity due to strain energy dissipation, both for cylindrical aluminium/alloy casings of yield stress up to 0.6 GPa with $0.2<M/C<5$ and for iron/steel casings, yield stress up to 1.2 GPa with $0.5<M/C<10$.

2.5 Modification to Relative Blast Impulse

Returning to the energy lost by the gases, and similarly applying Taylor's criterion to Equation (9), and dividing by EC to obtain the case strain energy dissipation loss to blast energy equivalence:

$$\left(\frac{C}{C+2M}\right)\frac{E_s}{EC}=\left(\frac{C}{C+2M}\right)\left(\frac{\rho_0}{\rho_m}\right)(\gamma-1)\frac{M}{C}\left(\frac{\sigma_y}{P_0}\right)\ln\left(\frac{\sigma_y}{P_0}\right)^{\frac{-1}{2\gamma}} \tag{14}$$

Now, adding back into Equation (14) the gas potential energy at fracture, based on the ideal gas law, see Equation (5), and the instantaneous gas kinetic energy, (second term on the right-hand side in Equation(7)), the case-modified blast energy equivalence is calculated to be:

$$\frac{E_c'}{E_c}=\left(\frac{\sigma_y}{P_0}\right)^{\frac{\gamma-1}{\gamma}}+\left(\frac{C}{C+2M}\right)\left(\begin{array}{c}1-\left(\frac{\sigma_y}{P_0}\right)^{\frac{\gamma-1}{\gamma}}-\\ \left(\frac{\rho_0}{\rho_m}\right)(\gamma-1)\frac{M}{C}\left(\frac{\sigma_y}{P_0}\right)\ln\left(\frac{\sigma_y}{P_0}\right)^{\frac{-1}{2\gamma}}\end{array}\right) \tag{15}$$

In this last equation, the first term is gas potential energy (as pressure) and within the second term we have gas kinetic energy, from which is being subtracted its proportion of the case strain losses.

Now, if the case fails and the gas is released, then the first term in Equation (15) gets converted to additional kinetic energy as the gas escapes. When the expression (16) for the ratio of modified (I) to bare blast impulse (I_0):

$$\frac{I}{I_0}=\sqrt{\left(\frac{\sigma_y}{P_0}\right)^{\frac{\gamma-1}{\gamma}}+\left(\frac{C}{C+2M}\right)\left(\begin{array}{l}1-\left(\frac{\sigma_y}{P_0}\right)^{\frac{\gamma-1}{\gamma}}-\\ \left(\frac{\rho_0}{\rho_m}\right)(\gamma-1)\frac{M}{C}\left(\frac{\sigma_y}{P_0}\right)\ln\left(\frac{\sigma_y}{P_0}\right)^{\frac{-1}{2\gamma}}\end{array}\right)} \quad (16)$$

is employed in MS-Excel with representative input data (see next paragraph), the case strain energy dissipation is seen to make very little difference, even up to the point, at which the gas kinetic energy is brought nearly to zero.

This is because, at large *M/C*, where case energy dissipation is significant, the gas already has much less kinetic energy in comparison to its potential energy, which is unaffected.

The worked examples in Figure 2 are for a steel casing with a 1.0 GPa yield stress (black lines) and an aluminum alloy casing with an 0.5 GPa yield stress (grey lines), both driven by an explosive with an initial (uniform) pressure of 10 GPa and density of 1800 kg m^{-3}. Full lines are without strain energy dissipation, as calculated in Ref. [1], dotted curves include this using Equation (16). Even at higher *M/C* values, the effect on relative blast impulse of case metal strain energy dissipation is no more than 2.5 %, i.e. still hardly detectable with current experiment techniques.

At high enough values of *M/C* and σ_y/P_0, where the gas kinetic energy falls to zero, the case expansion will be arrested and, while the gas potential energy will remain, it will be in the form of contained gas pressure and not released to create blast.

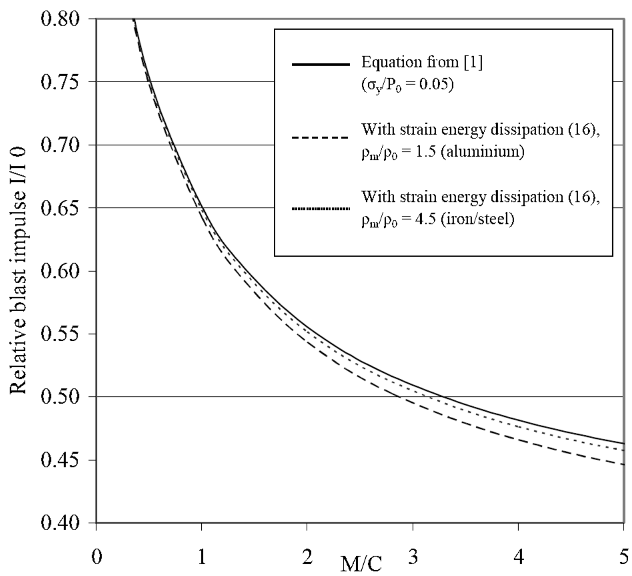

Figure 2. Predictions of relative blast impulse for a steel-cased cylindrical charge with and without an allowance for strain energy deposition in the case metal. Note that a suppressed zero of I/I_0 was used to reveal the small effect of strain energy dissipation.

3 Casing Thickness Effects

3.1 Discussion

The above calculations ignore the thickness of the casing and thus slightly over-estimate the strain energy dissipation, since by geometry the strain in the outer material of a thick casing will be significantly reduced compared to that in contact with the gas. The loss of blast effectiveness due to losses in the casing metal is already small, but the lesser losses in a thick casing will lead to a still higher *M/C* value being required to slow or arrest the expansion.

For an even thicker casing, the elastic limit will not be exceeded in the outer thickness of the casing and gas will not even be able to plastically strain the entire metal of the casing or, in this case, container. In a containment vessel, where there is large air space around the charge, a blast wave can form and impact the vessel inner wall, presenting a different problem for analysis.

3.2 Calculation

A simple, approximate treatment that makes an allowance for the thickness of the casing is as follows. Denoting the initial inner radius of the casing by r_0 and its expanded radius by r, an expression is required for the strain in the metal outside this gas-case interface. Let the initial outer case radius be R_0 and let this expand to R. In cylindrical geometry, the initial explosive and case volumes (per unit length) are $\pi r_0^2 = C/\rho_0$ and $\pi(R_0^2 - r_0^2) = M/\rho_m$. Therefore the ratio of outer and inner casing radii is given, initially, by:

$$\frac{R_0^2}{r_0^2}=1+\frac{M}{\pi r_0^2\rho_m}=1+\frac{M}{C}\frac{\rho_0}{\rho_m}, \quad \text{i.e.}\ \frac{R_0}{r_0}=\sqrt{1+\frac{M}{C}\frac{\rho_0}{\rho_m}} \quad (17)$$

At expansion radius r, $\pi r^2 = C/\rho$, where ρ is the then prevailing explosive density, which in line with Gurney's assumption is assumed to be uniform. Since the same is true at $r=r_0$, $r/r_0=\sqrt{\rho_0/\rho}$ and $\rho=\rho_0 r_0^2/r^2$. The casing material is much less compressible than the gas and therefore its volume can be assumed to be constant, so from Equation (17) at expansion radius r:

$$\frac{R^2}{r^2}=1+\frac{M}{C}\frac{\rho}{\rho_m}=1+\frac{M}{C}\frac{\rho_0}{\rho_m}\frac{r_0^2}{r^2} \quad (18)$$

The strain at radius R is a function of the radial expansion ratio R/R_0, and expanding the square of R/R_0:

$$\frac{R^2}{R_0^2}=\frac{R^2}{r^2}\frac{r^2}{r_0^2}\frac{r_0^2}{R_0^2} \quad (19)$$

From this and from Equation (17) and Equation (18):

$$\frac{R^2}{R_0^2}=\frac{r^2}{r_0^2}\left(1+\frac{M}{C}\frac{\rho_0}{\rho_m}\frac{r_0^2}{r^2}\right)\bigg/\left(1+\frac{M}{C}\frac{\rho_0}{\rho_m}\right) \tag{20}$$

Therefore, the strain at the casing outer surface, at its inner expansion radius r is:

$$\frac{R}{R_0}-1=\frac{r}{r_0}\sqrt{\left(1+\frac{M}{C}\frac{\rho_0}{\rho_m}\frac{r_0^2}{r^2}\right)\bigg/\left(1+\frac{M}{C}\frac{\rho_0}{\rho_m}\right)}-1 \tag{21}$$

or, if $b=\dfrac{M}{C}\dfrac{\rho_0}{\rho_m}$, then

$$\frac{R}{R_0}-1=\frac{1}{r_0}\sqrt{\frac{\left(r^2+br_0^2\right)}{(1+b)}}-1\text{, which simplifies to:} \tag{22}$$

$$\frac{R}{R_0}=\sqrt{\frac{\left(r^2/br_0^2+1\right)}{(1/b+1)}}$$

The term which has unit value in Equation (21) is actually the fraction of case mass being included, which so far has been 1. However, if it is taken to zero then $R/R_0=r/r_0$, i.e. no case mass is included. Therefore, it can be made a variable in order to find the mean strain in the case material. The mean expansion ratio $\bar{R}/R_0$ can then be found by integration:

$$\frac{\overline{R}}{R_0}=\int_0^1\sqrt{\frac{\left(r^2/br_0^2+x\right)}{(1/b+x)}}dx \tag{23}$$

However, while the resulting expressions could be evaluated, since only a small correction factor is being sought, it is possible to approximate by using the median strain, i.e. the strain at a radius within which lies 50% of the case mass, which will be:

$$\frac{\overline{R'}}{R_0}-1=\frac{1}{r_0}\sqrt{\frac{\left(2r^2+br_0^2\right)}{(2+b)}}-1 \tag{24}$$

which leads to an acceptably accurate case thickness correction factor

$$s=\sqrt{\frac{\left(2r^2+br_0^2\right)}{(2+b)}}\text{, where } b=\frac{M}{C}\frac{\rho_0}{\rho_m} \tag{25}$$

This thickness correction coefficient, which replaces the strain at the case inner surface by the median strain (i.e. 50% of the case is strained either more or less than this amount), is placed as follows in Equation (16):

$$\frac{I}{I_0}=\sqrt{\left(\frac{\sigma_y}{P_0}\right)^{\frac{\gamma-1}{\gamma}}+\left(\frac{C}{C+2M}\right)\left(\begin{array}{l}1-\left(\dfrac{\sigma_y}{P_0}\right)^{\frac{\gamma-1}{\gamma}}-\\ \left(\dfrac{\rho_0}{\rho_m}\right)(\gamma-1)\dfrac{M}{C}\left(\dfrac{\sigma_y}{P_0}\right)\ln s\left(\dfrac{\sigma_y}{P_0}\right)^{\frac{-1}{2\gamma}}\end{array}\right)} \tag{26}$$

Again, for a thick aluminum alloy casing where $M/C=5$, $\rho_0/\rho_m=0.23$ and $r_0/r=0.7$ and, if anywhere at all, the effect of casing thickness is likely to be more pronounced: $s\approx\sqrt{1.28/1.57}\approx0.815$. Even in this example, the further case thickness correction from applying this value of s in Equation (26) would be small.

4 Conclusion

According to the above calculations, losses in fragment initial velocity could be significant, i.e. as much as 10%, for thick casings composed of strong alloys. The method above provides over-estimates, which could be corrected downwards, firstly given knowledge of the integral under the actual dynamic stress-strain curve for the relevant casing metal, as a fraction of the simple product of yield stress and fracture strain and secondly by taking into account the fact that a fraction of the strain energy will be elastic in nature and could be returned to the metal.

The effect on blast impulse, due to loss of kinetic energy of the expanding gas within the casing, is less pronounced. This is due to the fact that casings of stronger metal, which dissipate significant strain energy, will also fracture when their internal gas pressure is still high. This residual gas pressure is converted to additional gas kinetic energy, from a source which is unaffected by casing strain dissipation losses.

An approximate calculation of the effect of finite casing thickness on strain energy dissipation shows that even for thick, low density, high yield stress casings, this additional correction will be too small to be worth taking into account.

Acknowledgments

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