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Replacing the Equations of Fano and Fisher for Cased Charge Blast Impulse – III – Yield Stress Method

Michael D. Hutchinson*[a]

Abstract: Previous papers by the author [1–3] pointed to a discovery by Fisher [4] that an equation by Fano [5], when used to predict blast impulse from cased munitions, did not fit the available data. These previous papers showed that an alternative equation for casing-modified blast impulse could be derived directly from an equation by Gurney [6] for the kinetic energy balance between the mass of casing metal and the mass of explosive gases. However, this equation was derived for very ductile casings that are accelerated to their ideal Gurney velocity before they fracture. A previous paper [3] showed how the finite fracture strain of real casing metal can be taken into account in determining the relative blast impulse from a cased charge. This paper shows how, based on previous work by Taylor [7], the equation in [2] can instead be modified to allow for the yield stress of the casing metal and provides validation for this further equation from available experimental data.

Keywords: Gurney · Impulse · Cased · Blast · Equations

1 Introduction

Previous papers [2,3] have discussed both the major contribution made by Gurney [6] to the field of explosion mechanics and the attempts of Fano [5] and Fisher [4] to extend Gurney's model to derive an equation to predict relative blast impulse from a cased charge. A previous derivation by the author, based on Gurney's original kinetic energy equation, provides a better equation that forms the basis for further developments in the theory of blast impulse from cased munitions. Here, specifically, a method is shown which can be used where the yield stress of the casing metal is known. This has advantages over the previously published [3] method where failure strain is known, since yield stress is much less rate-dependent and can therefore be the subject of useful estimates prior to any testing.

In the author's first two papers [1,2], a previous convention of referring to "mass of explosive equivalent for blast" in connection with cased charges was followed. However, it was found that at least one researcher had been led into confusion by this terminology. Therefore, in the previous paper [3], all that was referred to was the simple ratio I/I_0 of predicted blast impulse I from a cased charge to that (I_0) measured from the same charge without a casing.

2 Calculations

2.1 Author's Equation for Ductile Casings

Following Gurney, the charge mass is denoted by C and the casing mass by M. Equation (12) in Ref. [2] for the blast impulse for a cased charge, at the same scaled distance from the same charge bare, as a fraction of the blast impulse from that bare charge, is now written as:

$$\frac{I}{I_0} = \sqrt{C/(C+2M)} \tag{1}$$

As described in [2], this equation is based on exactly the same assumptions as made by Gurney in deriving the initial velocity of casing fragments from exploding munitions. For clarity, it is repeated here that Gurney's specific energy E relates *only* to the energy available to do work and is *not* equal to the heat of detonation, H. Based on data collated by Dobratz and Crawford [8], E lies in the range $0.62H < E < 0.76\,H$, the higher values being for more powerful explosives.

2.2 Change in Kinetic Energy Partition with Casing Expansion

In order to allow for the fracture of the casing, before it reaches its full Gurney radial velocity, the dependency of that velocity on expansion radius R must first be derived. As in Ref. [2], the assumption will be made here that the explosive gases can be treated as an ideal gas with a relatively high adiabatic constant, $\gamma = 3$. A paper by Koch,

[a] *M. D. Hutchinson*
Hydrodynamics Department
AWE Aldermaston
Reading RG7 4PR, United Kingdom
**e-mail: mike.hutchinson@awe.co.uk*

Arnold and Estermann [9] provides much of the necessary treatment and derives the following equation for casing radial velocity as a function of the radius $R(t)$ at time, t:

$$V(t) = \sqrt{(2/\gamma-1)(P_0/\rho_0)/((n/n+2)+(M/C))} \cdot [1-(R_0/R(t))^{n(\gamma-1)}]^{1/2} \tag{2}$$

Here n is a geometry factor, R_0 the initial casing radius, ρ_0 the initial explosive density and P_0 "the initial pressure in the homogeneous detonation products" and, as the researchers [9] point out, P_0 is *not* equal to the Chapman-Jouget pressure, P_{CJ}. The equation $P_0 = P_{CJ}(\gamma/(\gamma+1))^{\gamma}$, based on a derivation in Ref. [9], shows the relationship between P_0 and P_{CJ}, and leads to values $0.44P_{CJ} > P_0 > 0.40\ P_{CJ}$ for $3 < \gamma < \mathrm{Mk} > 5$. As the final term in square brackets in Equation (2) goes to zero, the ideal Gurney velocity is reached, which makes it possible to write the instantaneous casing velocity as a product of this velocity and the function in Equation (2) of the dynamically expanding casing radius, R.

$$V^2(R) = (1-(R_0/R)^{2(\gamma-1)})V^2 \tag{3}$$

where V is the ideal Gurney velocity and the expanding casing is cylindrical ($n=2$).

Gurney's original kinetic energy balance equation [6] for cylindrical cased charges can be written in the following slightly rearranged form (Equation (4)), which still shows clearly the casing kinetic energy component as the first term and the gas component as the second term in brackets:

$$E = 1/2((M/C)+1/2)V^2 \tag{4}$$

Combining Equation (3) and Equation (4), it was shown in Ref. [3] that the following expression can be obtained in which the terms on the right-hand side represent the partition of energy between first the casing and second the gases at any radius, R:

$$E = 1/2\ V^2(M/C)(1-(R_0/R)^{2(\gamma-1)}) + 1/2\ V^2(1/2+(M/C)(R_0/R)^{2(\gamma-1)}) \tag{5}$$

In Equation (5), it can be seen that when, in this model first proposed by Gurney, $R = R_0$, the left hand (casing) kinetic energy term is zero, but as R increases, energy is fed from the gases to the casing and when $R >> R_0$, $R_0/R \rightarrow$ zero and Equation (5) simplifies back to Gurney's original Equation (4).

2.3 Taylor's Criterion for Early Casing Fracture

Initially, when the explosive is converted into gases at very high pressure, this pressure far exceeds the yield stress of the casing metal and this is compressed against its own inertia in such a way that it thins sufficiently to increase its radius while maintaining its volume. In 1944, Taylor [7] proposed that, when the pressure of gases within the expanding casing dropped below the yield stress of the casing metal, then according to the shear stress criterion established by Tresca, the gases would no longer be able to compress the metal in shear. The only further means for the casing to respond, now more to its own radially expanding inertia, was to fracture and become an envelope of fragments separated by widening gaps through which the gases could escape, reducing the remaining drive pressure to the fragmented casing.

Therefore, remaining with the above assumptions of ideal gas behaviour and uniform gas pressure, it is possible to write the following equation for the gas pressure at expansion radius, R:

$$P(R)/P_0 = (R_0/R)^{2\gamma} \tag{6}$$

If the casing fractures at expansion radius, R_f, then according to Taylor $P(R_f) = \sigma_y$, the yield stress of the metal and therefore, from Equation (6):

$$\sigma_y/P_0 = (R_0/R)^{2\gamma} \tag{7}$$

Now it is possible to replace R_0/R in Equation (5) and obtain the following new expression for the partition of energy between casing and gases at the failure radius, R_f:

$$E = 1/2\ V^2(M/C)(1-(\sigma_y/P_0)^{(\gamma-1)/\gamma}) + 1/2\ V^2(1/2+(M/C)(\sigma_y/P_0)^{(\gamma-1)/\gamma}) \tag{8}$$

In Refs. [2] and [3], a ratio was taken between the kinetic energy in the gases from a cased charge and that in those from bare charge, which is the whole of the available work energy E. Based on the right-hand term in Equation (8), a similar ratio can now be taken for the situation where the casing releases the gases at a finite expansion radius, where E_C is the final energy remaining with the gases from a cased charge:

$$E_C/E = (1/2+(M/C)(\sigma_y/P_0)^{(\gamma-1)/\gamma})/((1/2+(M/C)) \tag{9}$$

As argued in Ref. [2], the impulse from the gases is proportional to their scalar velocity, which can be found from the square root of the kinetic energy which they have acquired. Taking the square root of both sides of Equation (9), the equation for blast impulse modified by the presence of a strong casing metal is:

$$I/I_0 = \sqrt{(1/2+(M/C)(\sigma_y/P_0)^{(\gamma-1)/\gamma})/((1/2+(M/C))} \tag{10}$$

Now, if the yield stress of the case metal can be estimated, then Equation (10) can provide a sufficiently accurate estimate of the cased charge blast impulse that can be compared with available experimental data from the firing of that type of cased charge.

3 Experimental Data

3.1 AWE Archive Experimental Data

Some AWE archive experimental data, recently released to the public domain, were obtained from a series of experiments with bare and cased cylindrical charges dating from 1970. These experiments were conducted at AWE's former outstation at Foulness and were reported internally. The explosive fills were 80 mm in diameter and composed of 88% RDX and 12% wax. The casings were of mild steel, tensile strength (roughly equivalent to yield stress) of 26.3 tons in^{-2} (about 0.4 GPa) and their thickness varied from 0.125 in (3.18 mm) to 0.75 in (19.1 mm).

From the side-on blast pressure data obtained, it was possible to extract relative side-on blast impulses. Using the relationships in Ref. [9], it has also been possible to estimate, from its measured detonation velocity, that the ideal uniform initial gas pressure of the explosive was approx. 11 GPa. The following side-on blast impulses, relative to case thickness and the ratio *C/M*, were in [2] normalised to a single scaled distance, based on the assumption that blast impulse was approximately dependent on the inverse of distance (quoted in feet) from the charge, i.e. 1/ft (Table 1):

3.2 Data Comparisons with Analytical Predictions

Given the scatter evident in the data from which they have been derived, these normalised experimental points cannot be considered accurate to less than $\pm 5\%$ at best. They are compared, in Figure 1, to predictions from Equation (10) with $P_0 = 11$ GPa, $\sigma_y = 0.4$ GPa, $\gamma = 3$ and $s = 1$, i.e. ignoring any possible casing thickness effect:

As the expression $\sqrt{(C/(C+2M))}$, on the right-hand side of Equation (1), represents the ideal Gurney situation of a perfectly ductile casing, cased charge relative blast impulses cannot, in his model, fall below the relevant value for this expression (i.e. below the 'Gurney Line' in Figure 1). Therefore, in Figure 1, to make a clear divide between allowed and non-allowed parameter space, both the experimental points and the values from Equation (10) have been plotted against $\sqrt{(C/(C+2M))}$, rather than *C/M*. This also has the advantage of making all data plots closer to straight lines.

Table 1. Normalised side-on blast impulse data, relative to casing thickness and mass, from experiments reported internally in 1970 by Bishop and James.

Casing thickness [in]	Casing thickness [mm]	Charge/Casing mass ratio [C/M]	$\sqrt{(C/(C+2M))}$	Relative side-on impulse
0	0		1	1
0.125	3.175	1.180	0.609	0.68
0.25	6.35	0.557	0.467	0.53
0.5	12.7	0.264	0.341	0.43
0.75	19.05	0.165	0.276	0.40

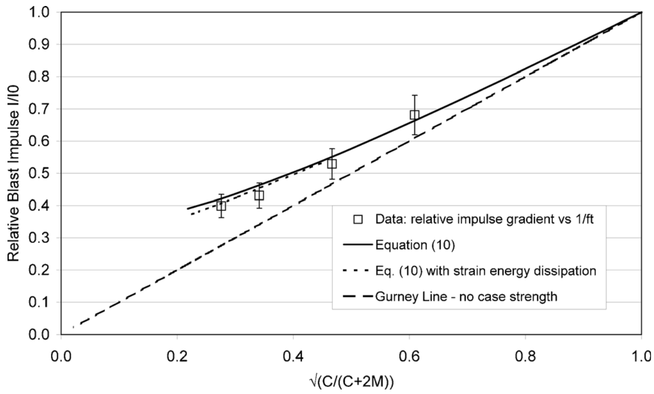

Figure 1. Comparison of the relative blast impulses derived from Bishop and James cased charge data with the predictions of Equation (10) plotted against the Gurney parameter $\sqrt{(C/(C+2M))}$.

Good agreements between the predictions of Equation (10) and the available data can be seen. Figure 1 also shows a dotted line, representing a prediction in which Equation (10) has been modified to include the effect of energy loss in straining the casing metal. This effect appears to be small, however, this topic, together with a consideration of the effect of finite casing thickness, will expanded on in a further submission to this journal.

4 Conclusions

Equation (10) will therefore give sufficiently accurate estimates of cased charge relative blast impulse for nearly all practical purposes. The process of replacing the equations of Fisher and Fano commenced in the author's previous papers [2,3] is now complete and the use of even modified versions of Fisher's equation, e.g. Equation (11), or indeed any similar empirical equations, is no longer necessary, so long as the casing metal yield stress can be estimated.

$$\frac{C_{EB}}{C} = 0.4 + \frac{0.6}{1 + (M/C)} \tag{11}$$

While both the original Fisher equation and its modifications have effectively been normalised to experimental data, priority must be given to discouraging the use of Fano's equation to predict blast effects from cased charges, as the estimates it gives could be dangerously low.

Neither Fano's, nor Fisher's, nor the equations in this paper are applicable to highly oxygen-deficient explosives, such as TNT or aluminised formulations, due to their large after-burn contribution to blast pressure. For a derivation of the applicable equations for relative blast impulse from aluminised etc. explosives, the reader is referred to a recent conference paper [10] by Hutchinson, Locking and Flynn.

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