M-啥b二次元3

这题以前在洛谷做过，但又记不起source了，来讲点乱搞的做法——OEIS的快乐

将10, 32输入到OEIS中，得到如下结果

Search: **seq:10,32**

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Sort: relevance | references | number | modified | created Format: long | short | data

A001250 Number of alternating permutations of order n. +30
(Formerly M1235 N0472) 122

1, 1, 2, 4, **10**, **32**, 122, 544, 2770, 15872, 101042, 707584, 5405530, 44736512, 398721962, 3807514624, 38783024290, 419730685952, 4809759350882, 58177770225664, 740742376475050, 9902996106248192, 138697748786275802, 2030847773013704704, 31029068327114173810 (list; graph; refs; listen; history; text; internal format)

OFFSET	0,3
COMMENTS	For n>1, a(n) is the number of permutations of order n with the length of longest run equal 2. Boustrophedon transform of the Euler numbers (A000111). [Berry et al., 2013] - N. J. A. Sloane, Nov 18 2013 Number of inversion sequences of length n where all consecutive subsequences i,j,k satisfy i >= j < k or i < j >= k. a(4) = 10: 0010, 0011, 0020, 0021, 0022, 0101, 0102, 0103, 0112, 0113. - Alois P. Heinz, Oct 16 2019
REFERENCES	L. Comtet, Advanced Combinatorics, Reidel, 1974, p. 261. C. K. Cook, M. R. Bacon, and R. A. Hillman, Higher-order Boustrophedon transforms ..., Fib. Q., 55 (No. 3, 2017), 201-208. F. N. David, M. G. Kendall and D. E. Barton, Symmetric Function and Allied Tables, Cambridge, 1966, p. 262. N. J. A. Sloane, A Handbook of Integer Sequences, Academic Press, 1973 (includes this sequence). N. J. A. Sloane and Simon Plouffe, The Encyclopedia of Integer Sequences, Academic Press, 1995 (includes this sequence).
LINKS	Max Alekseyev and Alois P. Heinz, Table of n, a(n) for n = 0..500 (terms n=1..100 from Max Alekseyev) Max A. Alekseyev, On the number of permutations with bounded run lengths, arXiv:1205.4581 [math.CO], 2012-2013.

原来这叫**交错排列数**，再在Google中键入Number of alternating permutations

找到一篇Paper，

We will sketch three proofs of Theorem 1.1.

First proof. Let $0 \le k \le n$. Choose a k-subset S of $[n] = \{1, 2, \ldots, n\}$ in $\binom{n}{k}$ ways, and set $\bar{S} = [n] - S$. Choose a reverse alternating permutation u of S in E_k ways, and choose a reverse alternating permutation v of $\bar{S}$ in E_{n-k} ways. Let w be the concatenation $u^r, n+1, v$, where u^r denotes the reverse of u (i.e., if $u = u_1 \cdots u_k$ then $u^r = u_k \cdots u_1$). When $n \ge 2$, we obtain in this way every alternating and every reverse alternating permutation w exactly once. Since there is a bijection between alternating and reverse alternating permutations of any finite (ordered) set, the number of w obtained is $2E_{n+1}$. Hence

$$2E_{n+1} = \sum_{k=0}^{n} \binom{n}{k} E_k E_{n-k}, \quad n \ge 1. \tag{1.4}$$

Set $y = \sum_{n \ge 0} E_n x^n / n!$. Taking into account the initial conditions $E_0 = E_1 = 1$, equation (1.4) becomes the differential equation

$$2y' = y^2 + 1, \quad y(0) = 1.$$

The unique solution is $y = \sec x + \tan x$. □

NOTE. The clever counting of both alternating and reverse alternating permutations in the proof of Theorem 1.1 can be avoided at the cost of a little elegance. Namely, by considering the position of 1 in an alternating permutation w, we obtain the recurrence

$$\boxed{E_{n+1} = \sum_{\substack{1 \le j \le n \\ j \text{ odd}}} \binom{n}{j} E_j E_{n-j}, \quad n \ge 1.}$$

This recurrence leads to a system of differential equations for the power series $\sum_{n \ge 0} E_{2n} x^{2n}/(2n)!$ and $\sum_{n \ge 0} E_{2n+1} x^{2n+1}/(2n+1)!$.

于是组合数可以 $\mathcal{O}(n^2)$ 预处理，递推也是 $\mathcal{O}(n^2)$ 的，可以通过此题