Forecasting Report

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1. Data Description and Examination

The variables I used for forecasting project is "U.S. Imports of Goods by Customs Basis from Japan" (IMPJP) which monthly measures the total U.S. imports of good from Japan without seasonal adjustment. This data releases at every early month and are obtained from FRED website (https://fred.stlouisfed.org/series/IMPJP). (The import in July was latest released on 4th September.) It can be transferred to a percent change form and other forms by FRED online tools. The original source of this data is from the U.S. Census Bureau website (https://www.census.gov/foreign-trade/Press-Release/current_press_release/index.html). My project intends to forecast this time series data in September, which will be released on 5th November by U.S. Census Bureau and viewed from the FRED website.

Moreover, I incorporate two explanatory variables in my OLS and VAR forecasting models, which are Japan / U.S. Foreign Exchange Rate (EXJPUS) and Consumer Price Index: Total All Items for the United States (USCPI).

Figure 1 shows that the time series of these variables are relatively difference stationary and we reconfirm their stationarity using unit root tests by adftest-function in MATLAB. Besides, the ACF and PACF of IMPJP in Figure 2 provide us the reason why we use the ARMA model in forecasts.

Also, the time series I used are started from February 1985 to August 2019 (T = 415). I choose the forecasting T0 around 80 so that there are 335 forecasters for calculating MSFE.

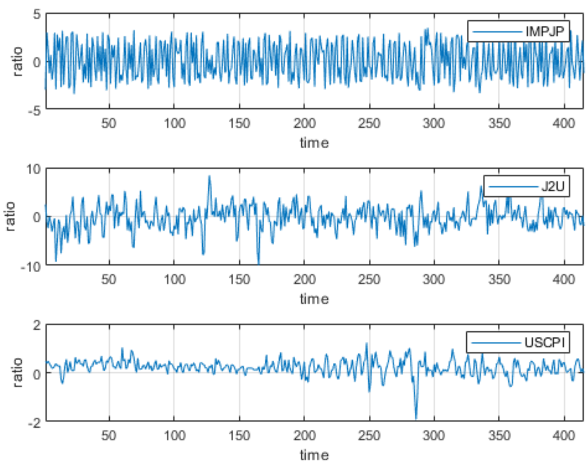


Figure 1: Time series of data

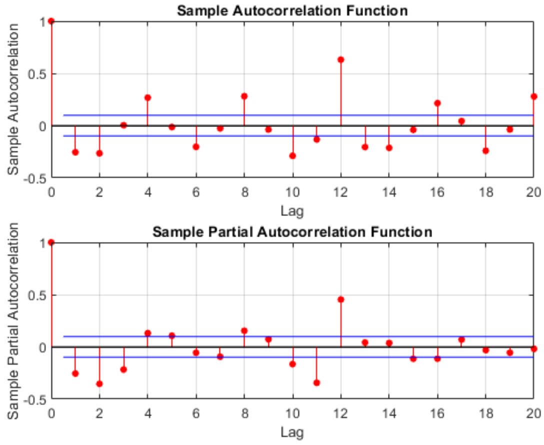


Figure2: autocorrelation coefficient and partial correlation coefficient

2. Model Specification

A. Benchmark

We firstly construct a random walk model as our benchmark. In random walk model, we only use current information for forecasting.

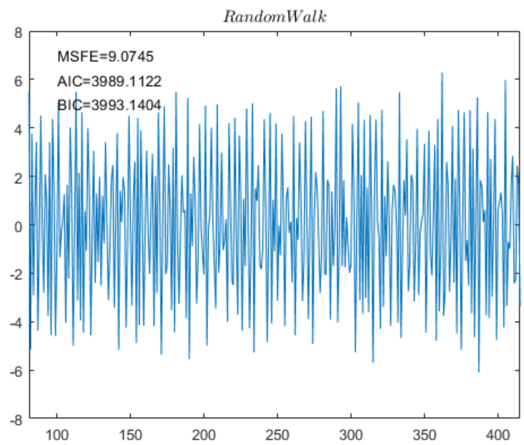


Figure 3

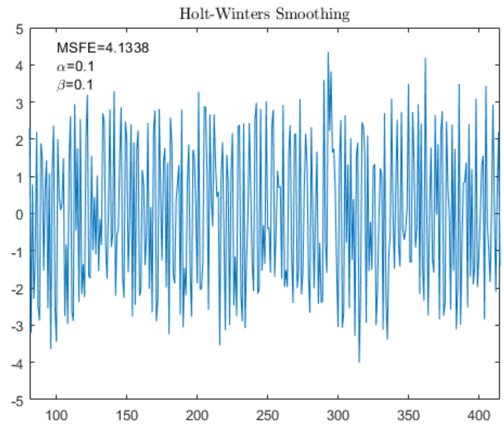
Table 1

Model	AIC	BIC	MSFE
Random walk	3989.1122	3993.1404	9.0745

Figure 3 and Table 1 report the result of our benchmark forecast.

B. Holt-Winters Smoothing

Holt-Winters Smoothing model can smooth the trend and seasonality in our time series, which is based on historical data. By trying and considering that our data has potential seasonality and trends, we only report the smoothing result with two parameters and three parameters, where all the parameters are set to 0.1.



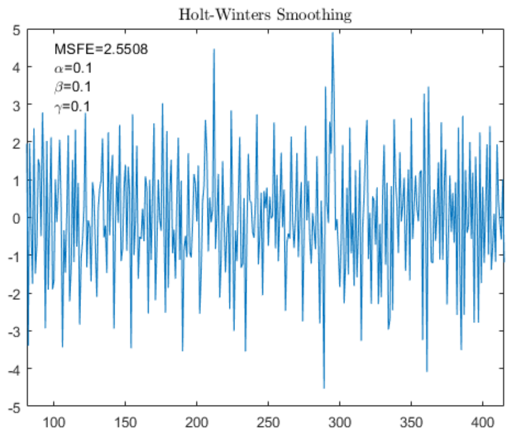


Table2

	Holt-Winters	Holt-Winters with seasonality	Random Walk
MSFE	4.1338	2.5508	9.0745

Table 2 shows that the Holt-winters smoothing model is significantly better than the random walk model in a 1-step forecast. As our expectation, the Holt-winters model with three parameters has a better predicting effect than with two parameters. Because Holt winters model uses more historical information than our benchmark model and seasonal smoothing is considered in the Holt winters model with three parameters.

However, Holt-Winters smoothing only offers point forecasts.

C. Simple OLS

Next, we consider the relationship between our forecasting variable and explanatory variables in simple OLS models for forecasting:

1. Model C1 is the simplest OLS model :

$$JMPJP_t = C + \alpha t$$

   Where *C* is the constant tern and *t* is the time trend.

2. Model C2 incorporates seasonal dummies:

$$JMPJP_t = C + \alpha t + \beta \sum_{i=1}^{12} M_i$$

   Where *M* is monthly dummy.

3. Model C3 incorporates our explanatory variables :

$$JMPJP_t = C + \alpha t + \gamma USCPI + \delta EXJPUS$$

4. In model C4, we only consider data from $JMPJP_{t-j-1}$ *to* $JMPJP_{t-1}$ as our historical information ($j = 80$ in this case) for forecasting.

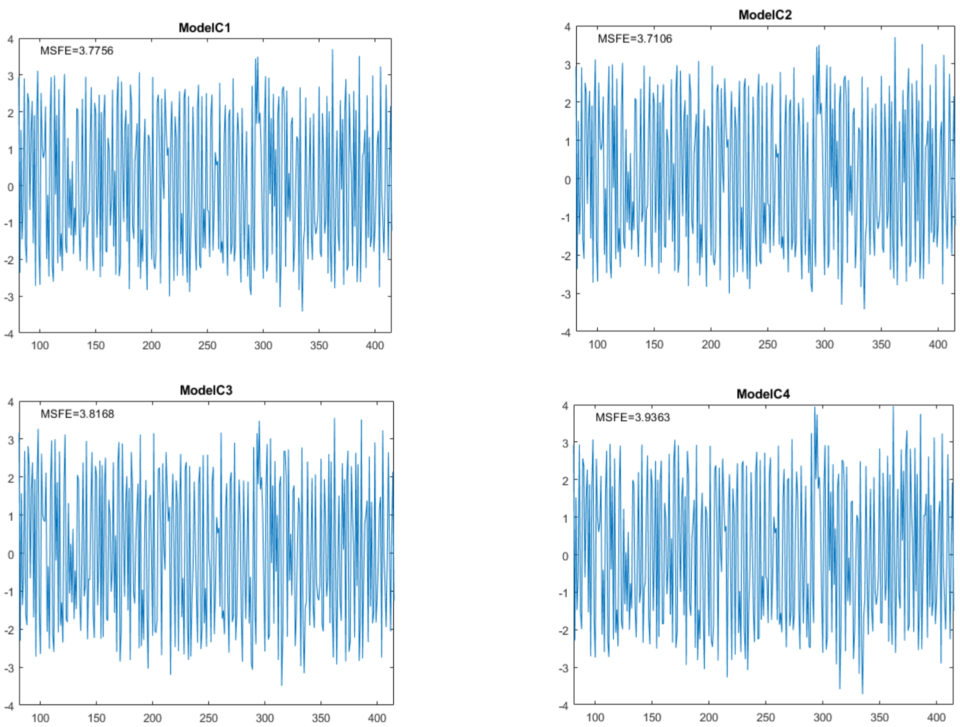


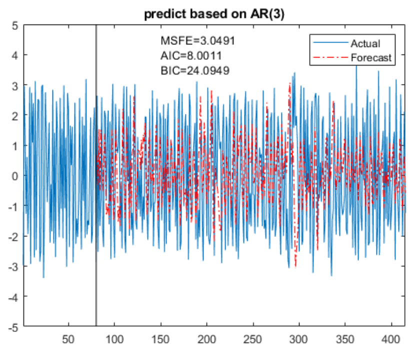
Table 3

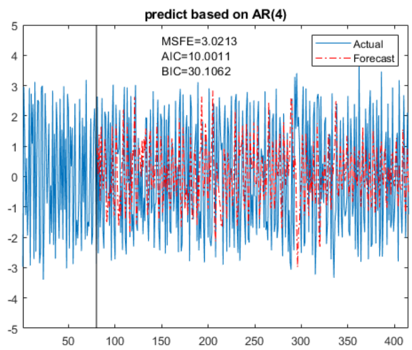
	ModelC1	ModelC2	ModelC3	ModelC4	Random Walk
MSFE	3.7756	3.7106	3.8168	3.9363	9.0745

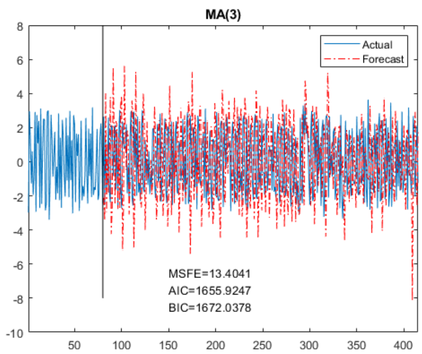
Table 3 shows that the OLS model is always better than our benchmark, Model C2 has the best validity of forecast by adding seasonality dummies. However, we lose some accuracy by adding explanatory variables in our OLS model as the MSFE of Model C3 shows. More importantly, Model C4 performs the worst forecast, one plausible reason for this phenomenon is that IMPJP is highly correlated to its recently historical data.

D. AR(p) and MA(q)

As we mentioned in data description and examination section, the ACF and PACF of IMPJP show that ARMA model is plausible in our forecast. By trying and error, we report the predicting results of AR (3), AR (4), MA (3) and MA (2) in Table 4.







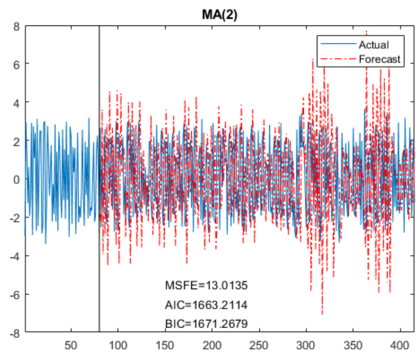


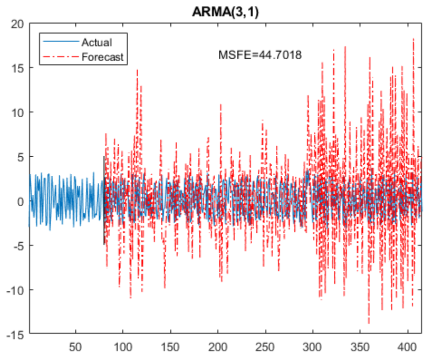
Table 4

	AR(3)	AR(4)	MA(3)	MA(2)	Random Walk
MSFE	3.0491	3.0213	13.4041	13.0135	9.0745
AIC	8.0011	10.0011	1655.9247	1663.2114	3989.1122
BIC	24.0949	30.1062	1672.0387	1671.2679	3993.1404

We can conclude that the AR (3) model is the most optimal model in above table by its AIC, BIC criteria and MSFE. At the same time, the forecasting effect of the AR (4) model is also better than our benchmark. However, the MSFE of all MA models is larger than random walk model. In other words, IMPJP data is highly related to its Lags, specifically, the AR model with 3 Lags has the best forecasting effect.

E. ARMA (p, q)

Although AR (3) model shows a great forecasting effect and MA models perform poorly. We still consider ARMA model as a comparison to the above section. The results in Table 5 show that the ARMA model lacks the validity of 1-step forecast and has a worse forecasting effect than our benchmark. We reconfirm our explanation in the above section.



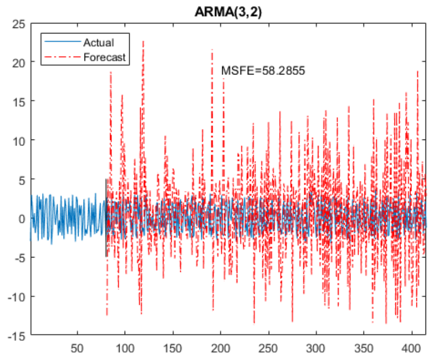


Table 5

	ARMA(3,1)	ARMA(3,2)	Random Walk
MSFE	44.7018	58.2855	9.0745

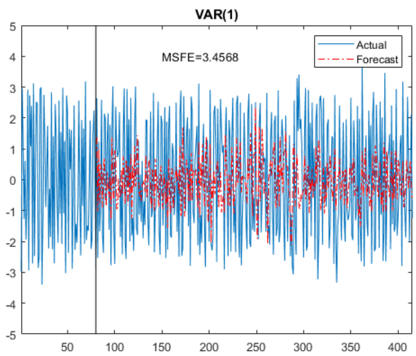
F. VAR (p)

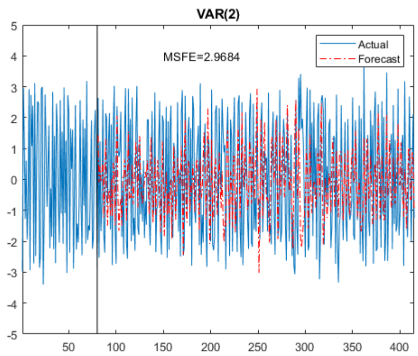
Finally, we consider VAR model with our explanatory variables.

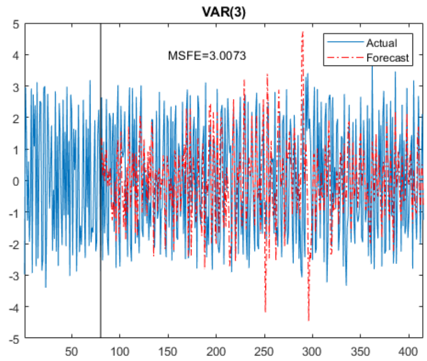
The forecasting results of VAR (1), VAR (2) and VAR (3) model are reported in Table 6. We can conclude that the VAR model has the best forecasting effect with 2 Lags, even though explanatory variables did not contribute better forecasting accuracy in the OLS model.

Table 6

	VAR(1)	VAR(2)	VAR(3)	AR(4)	Random Walk
MSFE	3.4568	2.9684	3.0073	3.0213	9.0745







3. Conclusion

i. IMPJP is difference stationary, so the random walk is a good specification.

ii. MA model performs the worst forecasting effect in our project, the HW model with 3 components, AR models and VAR models have significantly better forecasting effect than our benchmark. Specifically, VAR(2) is the best model in our project:

$$Y_t = C + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \varepsilon_t$$

Where ***Y*** is the set of our forecasting variable and explanatory variables; and

$$\beta_1 = \begin{pmatrix} -0.3747 & -0.0564 & 0.8211 \\ -0.1290 & 0.3066 & 0.1846 \\ 0.0055 & 0.0080 & 0.5772 \end{pmatrix}$$

$$\beta_2 = \begin{pmatrix} -0.3705 & -0.0675 & 0.5657 \\ -0.0978 & -0.0046 & 0.2297 \\ 0.0124 & 0.0021 & -0.1970 \end{pmatrix}$$

$$\boldsymbol{C} = \begin{pmatrix} -0.2371 \\ -0.2020 \\ 0.1305 \end{pmatrix}$$

$$\varepsilon_t = \begin{pmatrix} 2.8371 & -0.3038 & -0.0396 \\ -0.3038 & 5.9963 & -0.0304 \\ -0.0396 & -0.0304 & -0.0718 \end{pmatrix}$$

iii. By comparing the OLS model and AR model, we see that IMPJP is highly autocorrelated, the results imply that the amount of import is influenced by the previous data.

iv. MSFE selects VAR(2) for the best forecasting model, while both AIC and BIC select AR(3) under parsimonious criterion. (Holt-Winters only offers point forecast.)

Table 7: MSFE, AIC, BIC summary

	MSFE	AIC	BIC
Random walk	9.0745	3989.1122	3993.1404
Holt-Winters with Seasonality	2.5508	-	-
AR(3)	3.0491	8.0011	24.0949
AR(4)	3.0213	10.0011	30.1062
VAR(2)	2.9684	-	-
VAR(3)	3.0073	-	-

v. Next period's confidence intervals of prediction, $\hat{y}_{t+1} + 1.96\hat{\sigma}$ at 5% significance level (We only report the results of AR(3), VAR(2) and VAR(3)):

Table 8

	AR(3)	VAR(2)	VAR(3)
Mean	-0.4175	-0.4053	-0.2914
$\hat{\sigma}^2$	3.4267	2.9104	2.6949
95% Confidence interval	(-4.2198, 3.0367)	(-3.7491, 2.9384)	(-3.5089, 2.9262)

vi. Point forecasts for the following year are reported in Table 9.

Table 9

Forecasts for following 12 months

	AR(3)	VAR(2)	VAR(3)
2019-09-01	-0.4175	-0.4053	-0.2914
2019-09-02	-0.4175	0.3173	-0.2879
2019-09-03	-0.4188	-0.2765	-0.2879
2019-09-04	-0.4202	-0.2754	-0.2886
2019-09-05	-0.4218	-0.2744	-0.2879
2019-09-06	-0.4233	-0.2733	-0.2873
2019-09-07	-0.4249	-0.2722	-0.2866
2019-09-08	-0.4264	-0.2712	-0.2860
2019-09-09	-0.4279	-0.2701	-0.2853
2019-09-10	-0.4295	-0.2691	-0.2847
2019-09-11	-0.4310	-0.2681	-0.2841

vii. IMPJP is a time series data with great volatility since the import from a certain country is interacted with many factors, specifically, the U.S. government can influence the volatility of this time series by changing policies. For example, the current Trade War between America and other countries makes our prediction lacks accuracy.

4. Appendix

Code:

```
%%read the data and perpare the variables
clear
clc
[y,txt,raw] = xlsread('IMPJP.xls');
[y2,txt2,raw2] = xlsread('J2U.xls');
[y3,txt3,raw3] = xlsread('USCPI.xls');
T1=length(y);T2=length(y2);T3=length(y3);
T=min([T1,T2,T3]);
y=y(T1-T+1:T1);y2=y2(T2-T+1:T2);y3=y3(T3-T+1:T3);

Time1=cell(T,1);
for i=T1-T+1:T1
    Time1{i-T1+T}=raw{i+11};
end
Time2=cell(T,1);
for i=T2-T+1:T2
    Time2{i-T2+T}=raw2{i+11};
end
Time3=cell(T,1);
for i=T3-T+1:T3
    Time3{i-T3+T}=raw3{i+11};
end

Time1{1}
Time2{1}
Time3{1}
Time1{end}
Time2{end}
Time3{end}

T0=80; %the start of prediction
%%plot the line for y
Y=[y,y2,y3];
names={'IMPJP','J2U','USCPI'};
for i=1:3
subplot(3,1,i)
```

```
plot(Y(:,i))
xlim([1,T])
ylabel('ratio')
xlabel('time')
legend(names{i})
grid on
end

%% UNIT ROOT TEST
[h1,pValue1]= adftest(y)
[h2,pValue2]= adftest(y2)
[h3,pValue3]= adftest(y3)

%% ACF and PACF
figure
subplot(2,1,1);
autocorr(y);
subplot(2,1,2)
parcorr(y);


%% Benchmark: RW
h=1;
k=1;
ytph = y(T0+h:end);
syhat = y(T0: end-h);
MSE_rw = mean((y(2:end)-y(1:end-1)).^2);
AIC_rw = MSE_rw*T + k*2
BIC_rw = MSE_rw*T + k*log(T)
MSFE_rw = mean((ytph - syhat).^2)
figure
plot(T0+h:T,y(T0+h:T)-syhat)
xlim([T0+h,T])
title('$ Random Walk $', 'interpreter','latex')
text(100,7,['MSFE=',num2str(MSFE_rw)])
text(100,6,['AIC=',num2str(AIC_rw)])
text(100,5,['BIC=',num2str(BIC_rw)])
hold off

%% Holt-Winters smoothing
alpha = 0.1; beta = 0.1;
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);
Lt = y(1); bt = y(2) -y(1);
```

```
for t = 2: T-h
%
newLt = alpha*y(t) + (1 - alpha)*(Lt+bt);
%
newbt = beta*(newLt - Lt) + (1 - beta)*bt;
%
yhat = newLt + h*newbt ;
Lt = newLt; bt = newbt; % update Lt and bt
if t>= T0 % store the forecasts for t >= T0
syhat(t-T0+1,:) = yhat;
end
end
time = [T0+h:T]';
MSFE_HW = mean((ytph - syhat).^2)



figure
plot(T0+h:T,y(T0+h:T)-syhat)
xlim([T0+h,T])
title(' Holt-Winters Smoothing', 'interpreter','latex')
text(100,4.5,['MSFE=',num2str(MSFE_HW)])
text(100,4,['\alpha=',num2str(alpha)])
text(100,3.5,['\beta=',num2str(beta)])
hold off

%% Holt Winters S
s=4;
syhat=zeros(T-h-T0+1,1);
ytph=y(T0+h:end);% observed y: T0+h->T
alpha=0.1;beta=0.1; gamma=0.1;%smoothing parameters
St=zeros(T-h,1);
%initizlize
Lt=mean(y(1:s));
bt=0;
St(1:4)=y(1:s)-Lt;
for t=s+1:T-h
   newLt=alpha*(y(t)-St(t-s))+(1-alpha)*(Lt+bt);
   newbt=beta*(newLt-Lt)+(1-beta)*bt;
   St(t)=gamma*(y(t)-newLt)+(1-gamma)*St(t-s);
   yhat=newLt+h*newbt+St(t+h-s);
   Lt=newLt;
   bt=newbt;%updata Lt and bt
   if t>=T0 %store the forecasts for t>=T0
```

```
        syhat(t-T0+1)=yhat;
    end
end
MSFE_HW2=mean((ytph-syhat).^2)

figure
plot(T0+h:T,y(T0+h:T)-syhat)
xlim([T0+h,T])
title('Holt-Winters Smoothing','interpreter','latex')
text(100,4.5,['MSFE=',num2str(MSFE_HW2)])
text(100,4,['\alpha=',num2str(alpha)])
text(100,3.5,['\beta=',num2str(beta)])
text(100,3,['\gamma=',num2str(gamma)])
hold off



%% X: constant, trend
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);
for i = T0:T-h
yt = y(1:i);
%D gfc t = D gfc(1:i);
%D dot t = D dot(1:i);
Xt = [ones(i,1) (1:i)'];
beta = (Xt'*Xt)\(Xt'*yt);
yhat = [1 i+h] * beta;
syhat(i-T0+1) = yhat;
end
MSFE_ols1 = mean((ytph - syhat).^2);

figure
plot(T0+h:T,y(T0+h:T)-syhat)
xlim([T0+h,T])
title('ModelC2')
text(100,3.7,['MSFE=3.7106'])
hold off


%% X: constant, trend s
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);
Month=month(Time1);
Dumy1=(1<=Month&Month<=3);
```

```
Dumy2=(4<=Month&Month<=6);
Dumy3=(7<=Month&Month<=9);
Dumy4=(10<=Month&Month<=12);
for i = T0:T-h
yt = y(1:i);
Xt = [(1:i)' Dumy1(1:i), Dumy2(1:i), Dumy3(1:i), Dumy4(1:i)];
beta = (Xt'*Xt)\(Xt'*yt);
yhat = [i+h Dumy1(i+h) Dumy2(i+h) Dumy3(i+h) Dumy4(i+h)] * beta;
syhat(i-T0+1) = yhat;
end
MSFE_ols2 = mean((ytph - syhat).^2);

figure
plot(T0+h:T,y(T0+h:T)-syhat,'blue')
xlim([T0+h,T])
title('modelC2')
text(100,3.7,['MSFE=',num2str(MSFE_ols2)])
hold off



%% model3: y~constant, trend£¬ J2U, USCPI
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);
for i = T0:T-h
yt = y(1:i);
Xt = [ones(i,1), (1:i)', y2(1:i), y3(1:i)];
beta = (Xt'*Xt)\(Xt'*yt);
yhat = [1 i+h y2(i+h) y3(i+h)] * beta;
syhat(i-T0+1) = yhat;
end
MSFE_ols3 = mean((ytph - syhat).^2);
figure
plot(T0+h:T,y(T0+h:T)-syhat)
xlim([T0+h,T])
title('ModelC3')
text(100,3.7,['MSFE=',num2str(MSFE_ols3)])
% text(150,40,['AIC=',num2str(AIC_rw)])
% text(150,35,['BIC=',num2str(BIC_rw)])
hold off

%% costant+trend
L=T0;
ytph= y(T0+h:end);
```

```
syhat = zeros(T-h-T0+1, 1);
for i = T0:T-h
yt = y(i-L+1:i);
%D gfc t = D gfc(1:i);
%D dot t = D dot(1:i);
Xt = [ones(L,1) (1:L)'];
beta = (Xt'*Xt)\(Xt'*yt);
yhat = [1 L+1] * beta;
syhat(i-T0+1) = yhat;
end
MSFE_ols4 = mean((ytph - syhat).^2);

figure
plot(T0+h:T,y(T0+h:T)-syhat)
xlim([T0+h,T])
title('ModelC4')
text(100,3.7,['MSFE=',num2str(MSFE_ols4)])




%% AR(3)
p=3;
muexist=1;
I=0;
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);
for i = T0:T-h
yt = y(1:i);
[phihat,sigma2hat]=estimate_IAR(I,p,yt,muexist);
yhat = phihat(muexist)+y(i:-1:i-p+1)'*phihat(1+muexist:end);
syhat(i-T0+1) = yhat;
end
MSFE_IAR1= mean((ytph - syhat).^2);
%calculate the AIC and BIC
[AIC,BIC]=AicBic_AR(p,y,muexist,phihat)
%prediciton
[phihat,sigma2hat]=estimate_IAR(I,p,y,muexist);
preyhat_mean =  phihat(muexist)+y(end:-1:end-
p+1)'*phihat(1+muexist:end)
sigma2hat
L_CI=preyhat_mean-1.96*sqrt(sigma2hat)
U_CI=preyhat_mean+1.96*sqrt(sigma2hat)
%picture
```

```
figure
plot(y);
xlim([1,length(y)])
hold on
plot(T0+h:T, syhat,'red-.')
plot([T0,T0],[-5,5],'black')
legend('Actual','Forecast')
title('AR(3)')
text(150,4.5,['MSFE=',num2str(MSFE_IAR1)])
text(150,4,['AIC=',num2str(AIC)])
text(150,3.5,['BIC=',num2str(BIC)])
hold off


%prediciton of the next whole year
p=3;
muexist=1;
I=0;
syhat = zeros(12, 1);
for i = 0:11
yt = [y; syhat(1:i)];
[phihat,sigma2hat]=estimate_IAR(I,p,yt,muexist);
yhat = phihat(muexist)+y(end:-1:end-p+1)'*phihat(1+muexist:end);
syhat(i+1) = yhat;
end



%% AR(4)
p=4;
muexist=1;
I=0;
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);
for i = T0:T-h
yt = y(1:i);
[phihat,sigma2hat]=estimate_IAR(I,p,yt,muexist);
yhat = phihat(muexist)+y(i:-1:i-p+1)'*phihat(1+muexist:end);
syhat(i-T0+1) = yhat;
end
MSFE_IAR2= mean((ytph - syhat).^2);
%calculate the AIC and BIC
[AIC,BIC]=AicBic_AR(p,y,muexist,phihat);
```

```
figure
plot(y);
xlim([1,length(y)])
hold on
plot(T0+h:T, syhat,'red-.')
plot([T0,T0],[-5,5],'black')
legend('Actual','Forecast')
title('AR(4)')
text(150,4.5,['MSFE=',num2str(MSFE_IAR2)])
text(150,4,['AIC=',num2str(AIC)])
text(150,3.5,['BIC=',num2str(BIC)])
hold off


%% MA(3)
p=3;
muexist=1;
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);

for i = T0:T-h
yt = y(1:i);
diff2y=diff(yt,2);
ob_fun=@(thetas)(-loglike_MA(thetas,p,yt,muexist));
thetas=fminsearch(ob_fun,ones(p+1+muexist,1));
yhat=y(i)+thetas(end:-1:end-muexist+1)+diff2y(end:-1:end-
p+1)'*thetas(1:end-1-muexist);
syhat(i-T0+1) = yhat;
end
MSFE_IMA3= mean((ytph - syhat).^2);
%calculate the AIC and BIC
AIC=-2*loglike_MA(thetas,p,y,muexist)+2*(p+muexist);
BIC=-2*loglike_MA(thetas,p,y,muexist)+(p+muexist)*log(T);
%pricture
figure
plot(y);
xlim([1,length(y)])

hold on
plot(T0+h:T, syhat,'red-.')
plot([T0,T0],[-8,8],'black')
legend('Actual','Forecast')
title('MA(3)')
text(150,-6.5,['MSFE=',num2str(MSFE_IMA3)])
```

```
text(150,-7.5,['AIC=',num2str(AIC)])
text(150,-8.5,['BIC=',num2str(BIC)])
hold off


%% MA(2)
p=2;
muexist=0;
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);

for i = T0:T-h
yt = y(1:i);
diff2y=diff(yt,2);
ob_fun=@(thetas)(-loglike_MA(thetas,p,yt,muexist));
thetas=fminsearch(ob_fun,1*ones(p+1+muexist,1));
yhat=y(i)+diff2y(end:-1:end-p+1)'*thetas(1:end-1-muexist);
syhat(i-T0+1) = yhat;
end
MSFE_MA2= mean((ytph - syhat).^2);
% AIC and BIC
AIC=-2*loglike_MA(thetas,p,y,muexist)+2*(p+muexist);
BIC=-2*loglike_MA(thetas,p,y,muexist)+(p+muexist)*log(T);
figure
plot(y);
xlim([1,length(y)])

hold on
plot(T0+h:T, syhat,'red-.')
plot([T0,T0],[-8,8],'black')
legend('Actual','Forecast')
title('MA(2)')
text(150,-5.5,['MSFE=',num2str(MSFE_MA2)])
text(150,-6.5,['AIC=',num2str(AIC)])
text(150,-7.5,['BIC=',num2str(BIC)])
hold off

%% ARIMA(3,1)
diffy=diff(y);
p=3;
q=1;
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);
muexist=1;
```

```
for i = T0:T-h
yt = y(1:i);
diff2y=diff(yt,2);
ob_fun=@(thetas)(-loglike_ARMA(thetas,p,q,yt,muexist));
thetas=fminsearch(ob_fun,0.5*ones(p+q+1+muexist,1));

yhat=y(i)+thetas(end)+diffy(i:-1:i-p+1)'*thetas(1:p)+diff2y(end:-
1:end-q+1)'*thetas(p+1:p+q)';
syhat(i-T0+1) = yhat;
end
MSFE_ARIMA= mean((ytph - syhat).^2);
%calculate AIC and BIC
AIC=-2*loglike_ARIMA(thetas,p,1,q,yt,muexist)+2*(p+q+muexist);
BIC=-2*loglike_ARIMA(thetas,p,1,q,yt,muexist)+(p+q+muexist)*log(T);

figure
plot(y);
xlim([1,length(y)])

hold on
plot(T0+h:T, syhat,'red-.')
plot([T0,T0],[-5,5],'black')
legend('Actual','Forecast','location','northwest')
title('ARMA(3,1)')
text(200,16.5,['MSFE=',num2str(MSFE_ARIMA)])
%%text(200,4,['AIC=',num2str(AIC)])
%%text(200,3.5,['BIC=',num2str(BIC)])
hold off


%% ARIMA(3,2)
diffy=diff(y);
p=3;
q=2;
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);
muexist=1;
for i = T0:T-h
yt = y(1:i);
diff2y=diff(yt,2);
ob_fun=@(thetas)(-loglike_ARMA(thetas,p,q,yt,muexist));
thetas=fminsearch(ob_fun,0.5*ones(p+q+1+muexist,1));
```

```
yhat=y(i)+thetas(end)+diffy(i:-1:i-p+1)'*thetas(1:p)+diff2y(end:-
1:end-q+1)'*thetas(p+1:p+q);
syhat(i-T0+1) = yhat;
end
MSFE_ARIMA= mean((ytph - syhat).^2);
%calculate AIC and BIC
AIC=-2*loglike_ARIMA(thetas,p,1,q,yt,muexist)+2*(p+q+muexist);
BIC=-2*loglike_ARIMA(thetas,p,1,q,yt,muexist)+(p+q+muexist)*log(T);

figure
plot(y);
xlim([1,length(y)])

hold on
plot(T0+h:T, syhat,'red-.')
plot([T0,T0],[-5,5],'black')
legend('Actual','Forecast')
title('ARMA(3,2)')
text(200,19,['MSFE=',num2str(MSFE_ARIMA)])
%%text(200,100,['AIC=',num2str(AIC)])
%%text(200,85,['BIC=',num2str(BIC)])
hold off




%% VAR(1)
% one-head prediction
h=1;
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);
for i = T0:T-h
Yt = Y(1:i,:);
[Bhat,Sigma2]=estimate_VAR(Yt,1);
Yhat=Bhat(:,1)+Bhat(:,2:end)*Yt(end,:)';
yhat = Yhat(1);
syhat(i-T0+1) = yhat;
end
MSFE_VAR1= mean((ytph - syhat).^2);

[Bhat,Sigma2]=estimate_VAR(Y,1);
sigma2hat=Sigma2(1,1);

preyhat_mean = Bhat(:,1)+Bhat(:,2:end)*Yt(end,:)';
```

```
sigma2hat
L_CI=preyhat_mean(1)-1.96*sqrt(sigma2hat)
U_CI=preyhat_mean(1)+1.96*sqrt(sigma2hat)

figure
plot(y);
xlim([1,length(y)])
hold on
plot(T0+h:T, syhat,'red-.')
plot([T0,T0],[-5,5],'black')
legend('Actual','Forecast')
title('VAR(1)')
text(150,4,['MSFE=',num2str(MSFE_VAR1)])
hold off

%% VAR(2)
% one-head prediction
h=1;
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);
for i = T0:T-h
Yt = Y(1:i,:);
[Bhat,Sigma2]=estimate_VAR(Yt,2);
Yhat=Bhat(:,1)+Bhat(:,2:end)*[Yt(end,:)';Yt(end-1,:)'];
yhat = Yhat(1);
syhat(i-T0+1) = yhat;
end
MSFE_VAR2= mean((ytph - syhat).^2);
%prediciton
[Bhat,Sigma2]=estimate_VAR(Y,2);
sigma2hat=Sigma2(1,1);

preyhat_mean = Bhat(:,1)+Bhat(:,2:end)*[Y(end,:)';Y(end-1,:)']
sigma2hat
L_CI=preyhat_mean(1)-1.96*sqrt(sigma2hat)
U_CI=preyhat_mean(1)+1.96*sqrt(sigma2hat)
%picture
figure
plot(y);
xlim([1,length(y)])
hold on
plot(T0+h:T, syhat,'red-.')
plot([T0,T0],[-5,5],'black')
legend('Actual','Forecast')
```

```
title('VAR(2)')
text(150,4,['MSFE=',num2str(MSFE_VAR2)])
hold off



%the prediction of the next whole year
syhat = zeros(12, 1);
Syhat = zeros(12, size(Y,2));
for i = 0:11
Yt = [Y ; Syhat(1:i,:)];
[Bhat,Sigma2]=estimate_VAR(Yt,2);
Yhat=Bhat(:,1)+Bhat(:,2:end)*[Yt(end,:)';Yt(end-1,:)'];
yhat = Yhat(1);
syhat(i+1) = yhat;
end



%% VAR(3)
% one-head prediction
h=1;
ytph = y(T0+h:end);
syhat = zeros(T-h-T0+1, 1);
for i = T0:T-h
Yt = Y(1:i,:);
[Bhat,Sigma2]=estimate_VAR(Yt,3);
Yhat=Bhat(:,1)+Bhat(:,2:end)*[Yt(end,:)';Yt(end-1,:)';Yt(end-2,:)'];
yhat = Yhat(1);
syhat(i-T0+1) = yhat;
end
MSFE_VAR3= mean((ytph - syhat).^2);

[Bhat,Sigma2]=estimate_VAR(Y,3);
sigma2hat=Sigma2(1,1);

preyhat_mean = Bhat(:,1)+Bhat(:,2:end)*[Y(end,:)';Y(end-1,:)';Y(end-
2,:)']
sigma2hat
L_CI=preyhat_mean(1)-1.96*sqrt(sigma2hat)
U_CI=preyhat_mean(1)+1.96*sqrt(sigma2hat)

figure
plot(y);
```

```
xlim([1,length(y)])
hold on
plot(T0+h:T, syhat,'red-.')
plot([T0,T0],[-5,5],'black')
legend('Actual','Forecast')
title('VAR(3)')
text(150,4,['MSFE=',num2str(MSFE_VAR3)])
hold off
```

```
%the prediction of the next whole year
syhat = zeros(12, 1);
Syhat = zeros(12, size(Y,2));
for i = 0:11
Yt = [Y ; Syhat(1:i,:)];
[Bhat,Sigma2]=estimate_VAR(Yt,3);
Yhat = Bhat(:,1)+Bhat(:,2:end)*[Y(end,:)';Y(end-1,:)';Y(end-2,:)'];
yhat = Yhat(1);
syhat(i+1) = yhat;
end
```

```
 Functions
 function [AIC,BIC]=AicBic_AR(p,y,muexist,phihat)
T=length(y);
if(muexist)
   X=ones(T-p,1);
else
   X=[];
end
for i=1:p
   X=[X, y(p-i+1:end-i)];
end
yhat2 = X*phihat;
MSE_IAR1=mean(y(p+1:end)-yhat2).^2;
AIC=MSE_IAR1*(T-p+1)+(p+muexist)*2;
BIC=MSE_IAR1*(T-p+1)+(p+muexist)*log(T-p+1);
```

```
end

function [AIC,BIC]=AicBic_IAR(I,p,y,muexist,phihat)
T=length(y);
diffy=diff(y,I);
if(muexist)
   X=ones(T-p,1);
else
   X=[];
end
for i=1:p
   X=[X, diffy(p-i+1:end-i)];
end
yhat2 = y(p+1:end-1)+X*phihat;
MSE_IAR1=mean(y(p+2:end)-yhat2).^2;
AIC=MSE_IAR1*(T-I-p+1)+(p+muexist)*2;
BIC=MSE_IAR1*(T-I-p+1)+(p+muexist)*log(T-I-p+1);
end

function [phihat,sigma2hat]=estimate_AR(p,y,muexist)
T = length(y);
if(muexist)
   X=ones(T-p,1);
else
   X=[];
end
for i=1:p
   X=[X, y(p-i+1:T-i)];
end
yy=y(p+1:T);
phihat=(X'*X)\X'*yy;
sigma2hat=(yy-X*phihat)'*(yy-X*phihat)/length(yy);

function [phihat,sigma2hat]=estimate_IAR(I,p,y,muexist)
if(I>0)
y=diff(y,I);
end
T = length(y);
if(muexist)
   X=ones(T-p,1);
else
   X=[];
end
for i=1:p
```

```
    X=[X, y(p-i+1:T-i)];
end
yy=y(p+1:T);
phihat=(X'*X)\X'*yy;
sigma2hat=(yy-X*phihat)'*(yy-X*phihat)/length(yy);

function [Bhat,Sigma2]=estimate_VAR(Y,p)

T=size(Y,1);
D=size(Y,2);

X=[];
for t=T:-1:(p+1)
    Xtemp=1;
    for lag=1:p
        for i=1:D
            Xtemp=[Xtemp; Y(t-lag,i)];%,i); Y(t-1); y1(t-2); y2(t-2);
y3(t-2)];
        end
    end
   X=[X, Xtemp];
end
Yt=Y(T:-1:(p+1),:)';
Bhat=Yt*X'/(X*X');

Sigma2=(Yt-Bhat*X)*(Yt-Bhat*X)'/size(Yt,2);
end

function loglikelihood=loglike_ARIMA(thetas,p,i,q,y,muexist)
y=diff(y,i);
sigma2=thetas(p+q+1);
T=length(y);
%%AR(p)
if(muexist)
    if(length(thetas)~=(p+q+2))
        disp('the input is less!')
        return
    end

    X=ones(T-p,1);
    parm1=[thetas(end);thetas(1:p)];
else
    if(length(thetas)~=(p+q+1))
        disp('the input is less!')
```

```
        return
    end
    X=[];
    parm1=thetas(1:p);
end
for i=1:p
    X=[X, y(p-i+1:T-i)];
end
yt=y(p+1:end)-X*parm1;


%%MA(q)
T2=T-p;
Gam = speye(T2) ;
for i=1:q
Gam = Gam+ sparse(i+1:T2,1:T2-i,ones(T2-i,1),T2,T2)*thetas(p+i);
end

Gam2 = Gam*Gam';
loglikelihood = -
T2/2*log(2*pi*sigma2)-.5*log(det(Gam2))-.5*yt'*(Gam2\yt)/sigma2;
end

function loglikelihood=loglike_ARMA(thetas,p,q,y,muexist)
sigma2=thetas(p+q+1);
T=length(y);
%%AR(p)
if(muexist)
    if(length(thetas)~=(p+q+2))
        disp('the input is less!')
        return
    end

    X=ones(T-p,1);
    parm1=[thetas(end);thetas(1:p)];
else
    if(length(thetas)~=(p+q+1))
        disp('the input is less!')
        return
    end
    X=[];
    parm1=thetas(1:p);
end
for i=1:p
```

```
    X=[X, y(p-i+1:T-i)];
end
yt=y(p+1:end)-X*parm1;


%%MA(q)
T2=T-p;
Gam = speye(T2) ;
for i=1:q
Gam = Gam+ sparse(i+1:T2,1:T2-i,ones(T2-i,1),T2,T2)*thetas(p+i);
end

Gam2 = Gam*Gam';
loglikelihood = -
T2/2*log(2*pi*sigma2)-.5*log(det(Gam2))-.5*yt'*(Gam2\yt)/sigma2;
end

function loglikelihood=loglike_IMA(I,thetas,p,y,muexist)
y=diff(y,I);
if(muexist)
    y=y-thetas(end);
end
sigma2 = thetas(p+1);
T = length(y);
Gam = speye(T) ;
for i=1:p
Gam = Gam+ sparse(i+1:T,1:T-i,ones(T-i,1),T,T)*thetas(i);
end
Gam2 = Gam*Gam';
loglikelihood = -T/2*log(2*pi*sigma2)-
1/2*log(det(Gam2))-.5*y'*(Gam2\y)/sigma2;
end

function loglikelihood=loglike_MA(thetas,p,y,muexist)
if(length(thetas)~=p+1+muexist)
    disp('0')
    return
end
if(muexist)
    y=y-thetas(end);
end
sigma2 = thetas(p+1);
T = length(y);
Gam = speye(T) ;
```

```
for i=1:p
Gam = Gam+ sparse(i+1:T,1:T-i,ones(T-i,1),T,T)*thetas(i);
end
Gam2 = Gam*Gam';
loglikelihood = -
T/2*log(2*pi*sigma2)-.5*log(det(Gam2))-.5*y'*(Gam2\y)/sigma2;
end
```