Fisher Matrix Forecast of HI IM FAST

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Detectbility and Fisher Matrix Forecast

The total brightness temperature is given by

$$\delta T(\vec{\theta}_p, \nu_p) = \delta T^S(\vec{\theta}_p, \nu_p) + \delta T^N(\vec{\theta}_p, \nu_p) + \delta T^F(\vec{\theta}_p, \nu_p) \tag{1}$$

Fourier Transfomation

$$< \delta T^{X*}(\vec{q}, y)\delta T^{X'}(\vec{q}', y') >= (2\pi^3)C^X(\vec{q}, y)\delta^2(\vec{q} - \vec{q}')\delta(y - y')\delta_{XX'} \tag{2}$$

For signal:

$$T_b = \bar{T}_b(1 + \delta_{HI}) \tag{3}$$

$$\bar{T}_b = \frac{3}{32}\frac{hc^3 A_{10}}{k_B m_p \nu_{21cm}}\frac{(1+z)^2}{H(z)}\Omega_{HI}(z)\rho_{c,0} \tag{4}$$

$$\delta_{HI}(\vec{k}) = (b_{HI} + f\nu^2)\exp(-k^2\mu^2\sigma_{NL}^2/2)\delta_M(\vec{k}) \quad (5)$$

And

$$< \delta_M^* \delta_M >= (2\pi)^3 P(\vec{k})\delta^3(\vec{k} - \vec{k}') \quad (6)$$

So signal covariance is

$$C^S(\vec{q}, y) = T_b^2(z_i)\frac{P_{tot}}{r^2 r_\nu} \quad (7)$$

$$P_{tot} = F_{RSD}(\vec{k}_\perp, k_\parallel)D_z^2 P(k, z = 0) \quad (8)$$

$$F_{RSD} = (b_{HI} + f\mu^2)^2 \exp(-k^2\mu^2\sigma_{NL}^2) \quad (9)$$

Noise model

$$C^N(q,y) = \frac{T_{sys}^2}{t_{tot}\Delta\nu} U_{bin} \mathcal{L} B_\perp^{-2} B_\parallel^{-1} \tag{10}$$

T_{sys}: system temperature; t_{tot}: total integration time; $U_{bin} = S_{area\Delta\tilde{nu}}$: the volume of an individual redshift bin; S_{area}: survey area; $Delta\tilde{\nu}$: (dimensionless) bandwidth

$$B_\parallel = \exp(-\frac{(y\delta\nu/\nu_{21cm})^2}{16\ln 2}) \tag{11}$$

For single dish

$$\mathcal{L} = \frac{f(\nu)}{N_b N_d} \tag{12}$$

$$B_\perp = \exp(-\frac{(q\theta_B)^2}{16\ln 2}) \tag{13}$$

For interferometers

$$\mathcal{L}B_{\perp}^{-2} = \frac{FOV}{n(u = q/2\pi)} \tag{14}$$

$$n(u) = \frac{N_d(N_d - 1)}{2\pi(u_{max}^2 - u_{min}^2)} \tag{15}$$

Foreground model

$$C^F(\vec{q}, y) = \epsilon^2 \sum_X A_X (\frac{l_p}{2\pi q})^{nx} (\frac{\nu_p}{\nu_i})^{mx} \tag{16}$$

Foreground	A_X/mK^2	n_x	m_x
Extragalactic point sources	57.0	1.1	2.07
Extragalactic free-free	0.014	1.0	2.10
Galactic synchrotron	700	2.4	2.80
Galactic free-free	0.088	3.0	2.15

Fisher Matrix Forecast

$$F_{ij}^{IM} = \frac{1}{2}U_{bin}\int \frac{d^2qdy}{(2\pi)^3}[\partial_i \ln C^T(\vec{q},y)\partial_j \ln C^T(\vec{q},y)] \tag{17}$$

For the uncertainty of power spectrum

$$(\frac{\Delta P_a}{P_a})^2 = [\frac{1}{2}U_{bin}\int_{V_n} \frac{d^2qdy}{(2\pi^3)}(\frac{C^S(\vec{q},y)}{C^T(\vec{q},y)})^2]^{-1} \tag{18}$$

Pipeline of Fisher Forecast: Philip Bull, Pedro G. Ferreira, Prina Patel, and Mario Santos, ApJ 803, 21 (2015) [arXiv:1405.1452][doi:10.1088/0004-637X/803/1/21].

Cosmology model

scale factor $a = \frac{1}{1+z}$,and Hubble Constant $H(t) = \frac{\dot{a}}{a}$

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{19}$$

$$H(a) = \frac{\dot{a}}{a} = H_0\sqrt{(\Omega_c + \Omega_b)a^{-3} + \Omega_{rad}a^{-4} + \Omega_k a^{-2} + \Omega a^{-3(1+\omega)}} \tag{20}$$

$$H^2(a) = H_0^2[\omega_R a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_{DE}\exp\{3\int_a^1 \frac{da'}{a'}[1+\omega(a')]\}] \tag{21}$$

Cosmological Constant

$$\{h, \Omega_b h^2, \omega_D E, \Omega_K, \omega_0, \omega_a, n_s, \sigma_8\} \tag{22}$$

Cosmological Constant

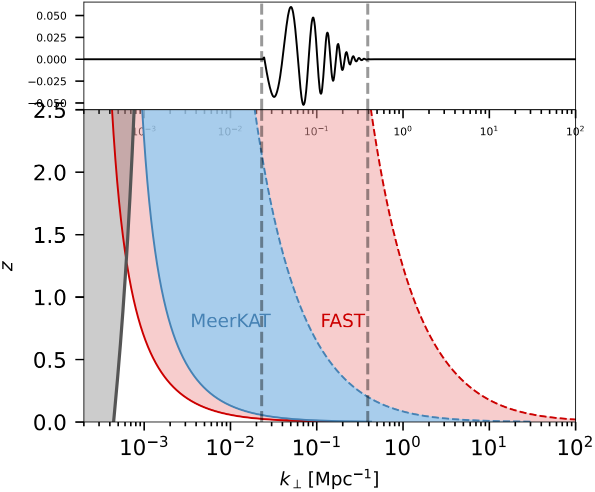
$$\{h, \Omega_b h^2, \omega_D E, \Omega_K, \omega_0, \omega_a, n_s, \sigma_8\} \tag{23}$$

- n_s is the spectral index of the scalar power spectrum($P(k) \sim k^{n_s - 1}$). σ_8 is the amplitude of the power spectrum on the scale of 8 Mpc/h.
- $\omega_a = \omega_0 + (1-a)\omega_a$

HI intensity mapping survey

	FAST	MeerKATb1	MeerKATb2	FASTWB
Total integration time/h	100000	4000	4000	-
Ndish	1	64	64	1
Nbeam	19	1	1	1
Ddish/m	300	13.5	13.5	300
System Temp./mK	20	29	20	60
Total bandwidth/MHz	300	435	520	-
Max frequency/MHz	1350	1015	1420	1050
Survey area/deg^2	20000	4000	4000	-

resolution



0.050
0.025
0.000
−0.025
−0.050
2.5
2.0
1.5
1.0
0.5
0.0
z
MeerKAT
FAST
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
$k_\perp$ [Mpc^{-1}]

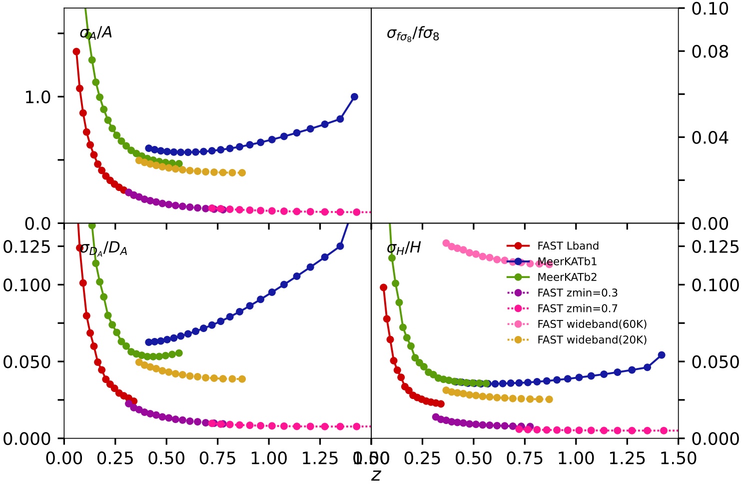


Figure: Fractional errors on $A(z)$, $f\sigma_8(z)$(with a bug), $D_A(z)$, and $H(z)$, as a function of redshift

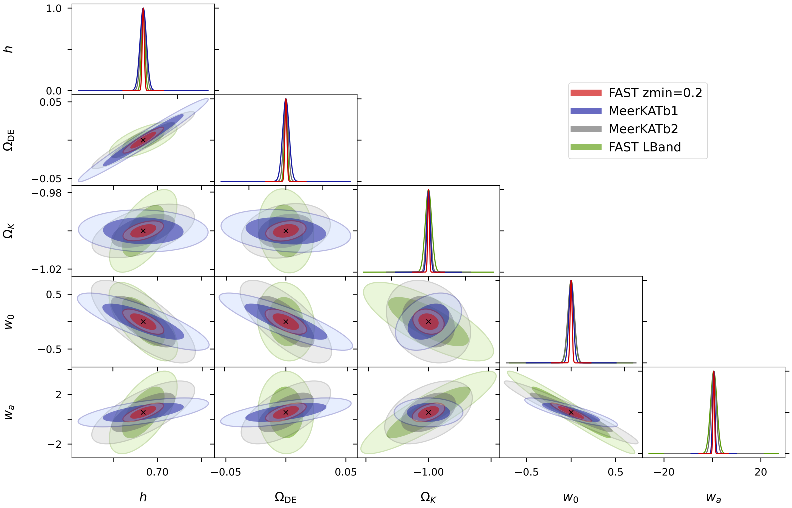
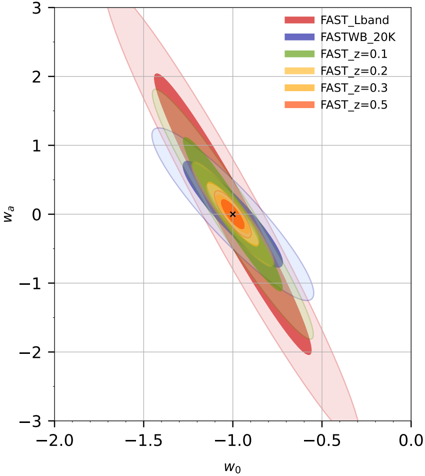
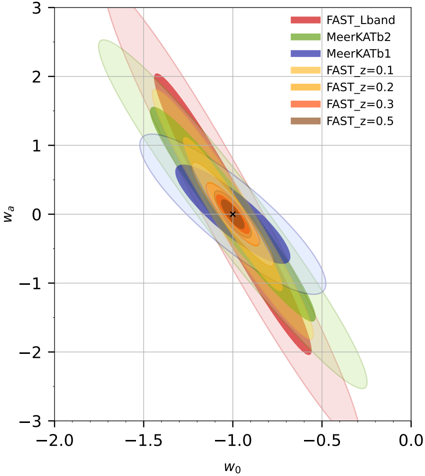


Figure: Forecasts for dark energy and modified growth parameters (without γ)

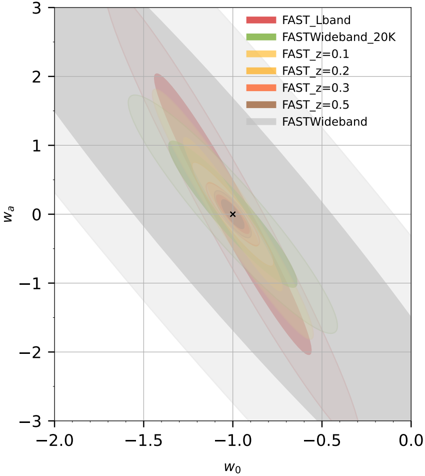


	σ_{ω_0}	σ_{ω_a}
FAST_Lband	0.28	1.33
FAST zmin=0.1	0.182	0.7314
FAST zmin=0.2	0.0927	0.3047
FAST zmin=0.3	0.0603	0.1832
FAST zmin=0.5	0.0416	0.1376
FAST wb(20K)	0.2365	0.6975

FAST: $t_{int} = 48(1+z)s/pix$
total bandwidth=300Hz
$S_{area} = 20000deg^2$



	σ_{ω_0}	σ_{ω_a}
FAST_Lband	0.28	1.33
MeerKAT_b1	0.2103	0.4689
MeerKAT_b2	0.3036	1.0204
FAST zmin=0.1	0.182	0.7314
FAST zmin=0.2	0.0927	0.3047
FAST zmin=0.3	0.0603	0.1832
FAST zmin=0.5	0.0416	0.1376

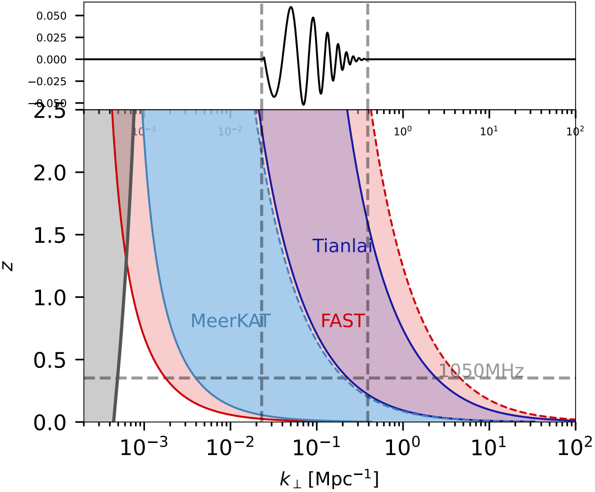


	σ_{ω_0}	σ_{ω_a}
FAST_Lband	0.28	1.33
FAST zmin=0.1	0.182	0.7314
FAST zmin=0.2	0.0927	0.3047
FAST zmin=0.3	0.0603	0.1832
FAST zmin=0.5	0.0416	0.1376
FAST wb(60K)	1.1732	3.5682
FAST wb(20K)	0.2365	0.6975

NEXT

- increase integration time
- decrease S_{area}

Resolution, add Tianlai



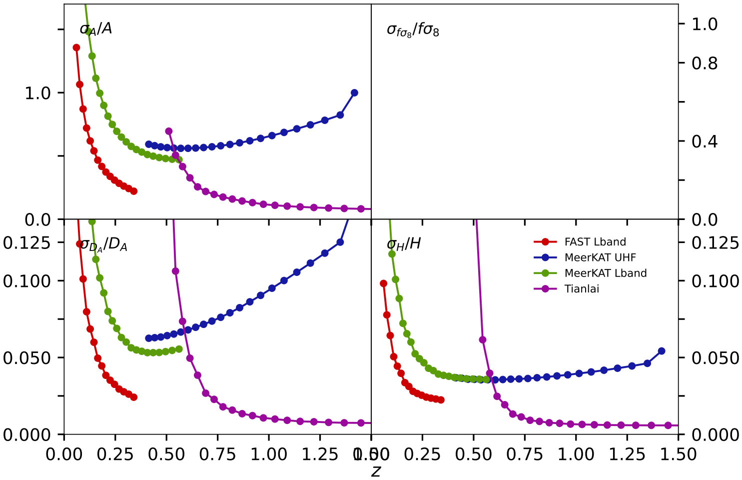


Figure: Fractional errors on $A(z)$, $f\sigma_8(z)$(with a bug), $D_A(z)$, and $H(z)$, as a function of redshift

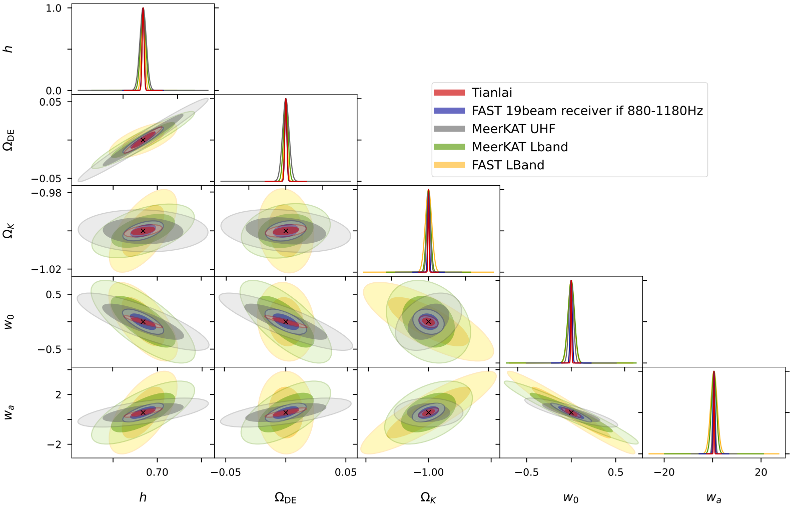


Figure: Forecasts for dark energy and modified growth parameters (without γ)

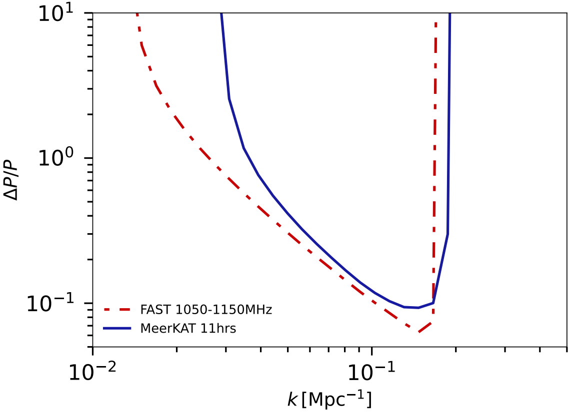


Figure: Fractional constraints on P(k) of FAST and MeerKAT 11hrs wiggleZ field

MeerKAT 11hrs & FAST $200deg^2$ 1050-1150MHz

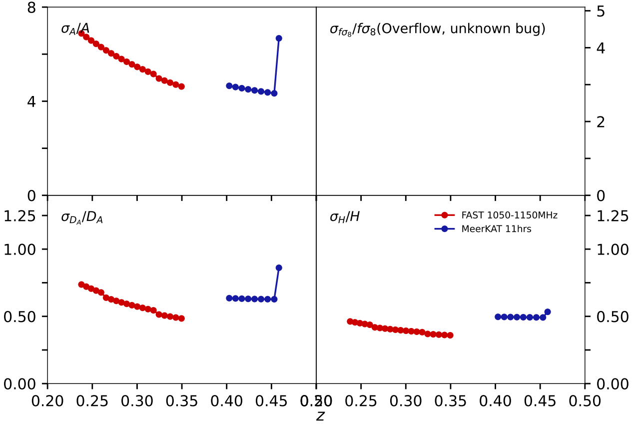


Figure: Fractional errors on $A(z)$, $D_A(z)$, and $H(z)$, as a function of redshift. Assuming a FAST survey with $S_{area} = 200deg$ and $48/(1+z)s/pix$(DF mode scans twice)