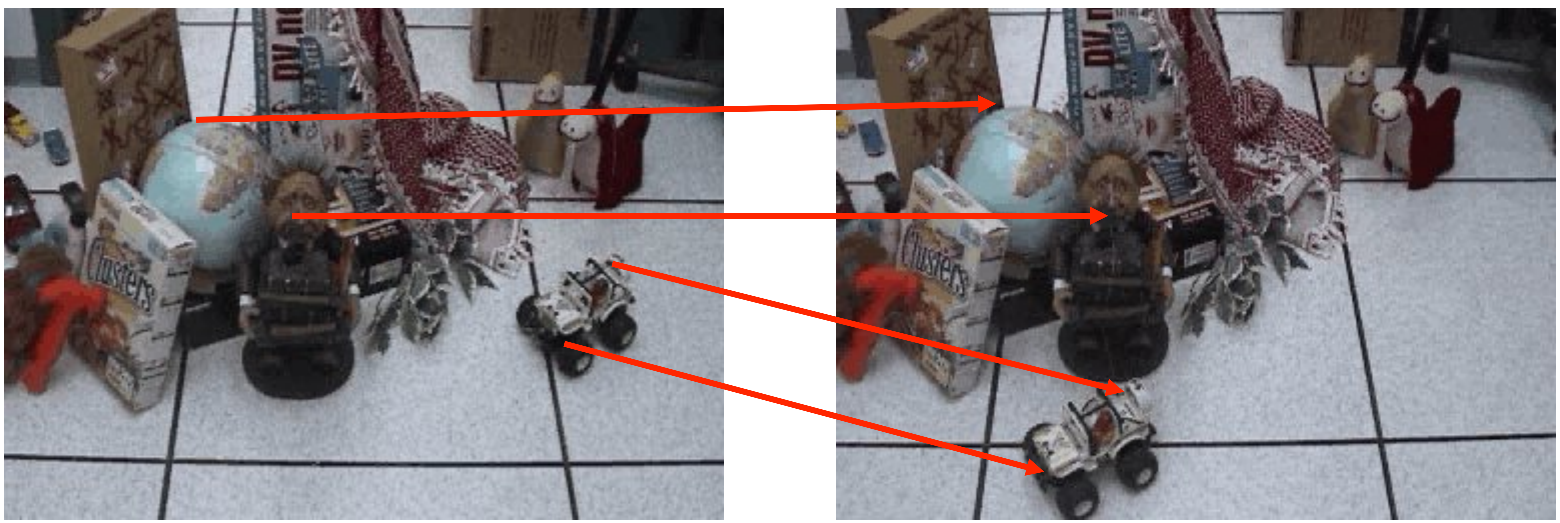
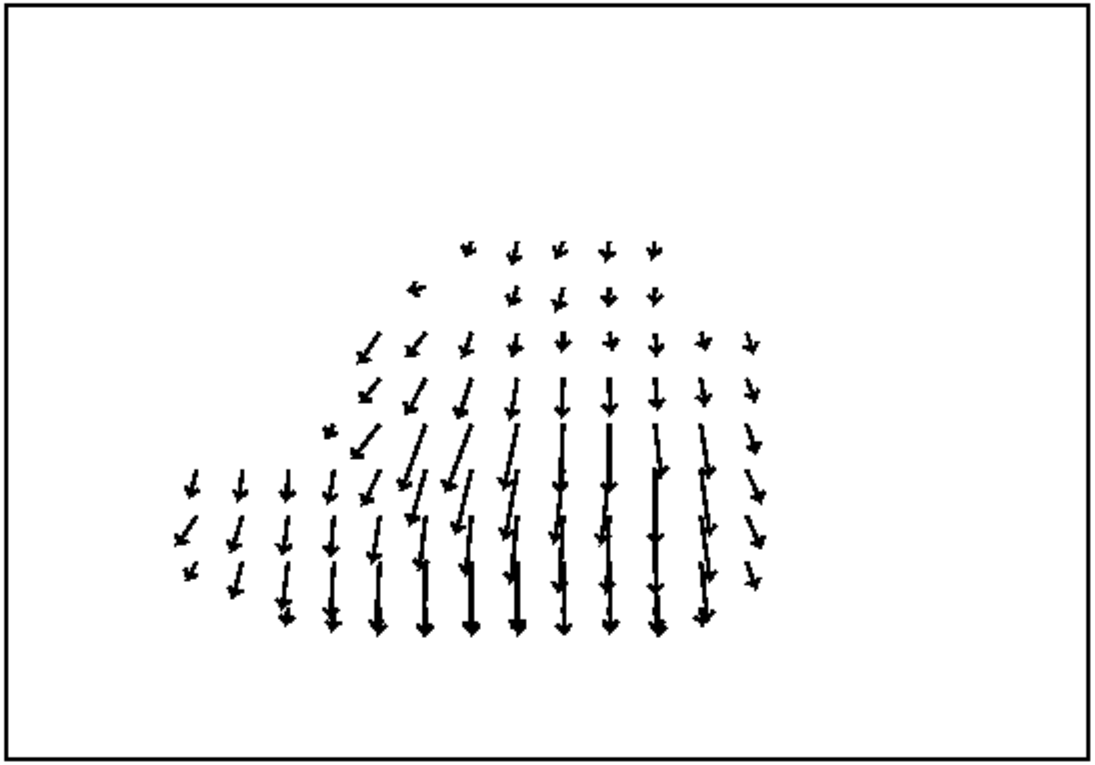
Optical Flow

Where did each pixel in image 1 go to in image 2



Optical Flow



Pierre Kornprobst's Demo

Introduction

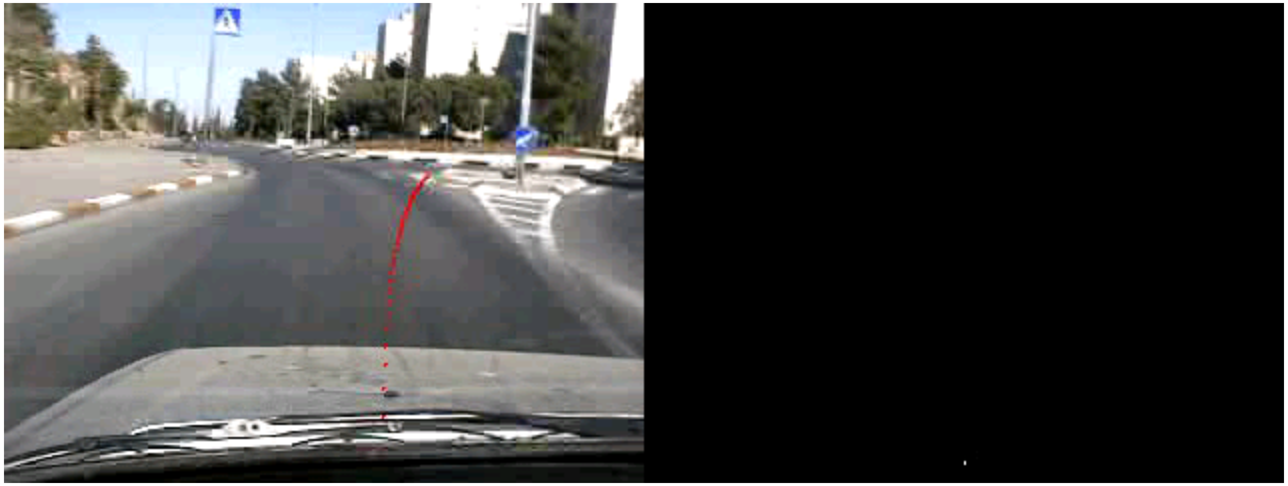
- Given a video sequence with camera/objects moving we can better understand the scene if we find the motions of the camera/ objects.

Scene Interpretation

- How is the camera moving?
- How many moving objects are there?
- Which directions are they moving in?
- How fast are they moving?
- Can we recognize their type of motion (e.g. walking, running, etc.)?

Applications

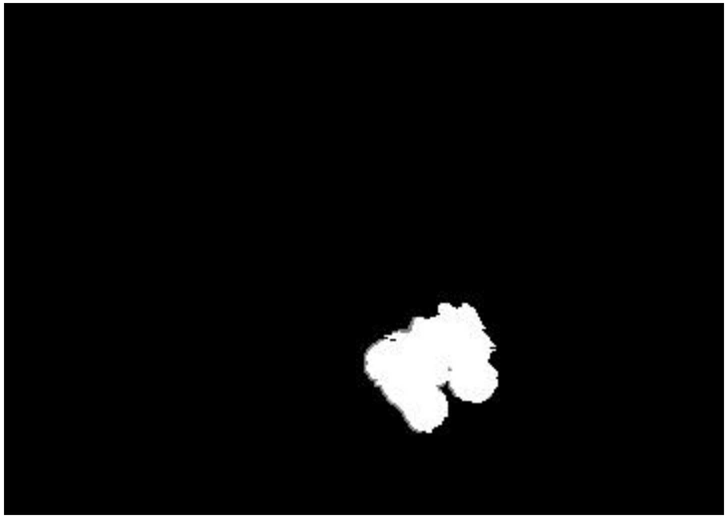
- Recover camera ego-motion.



Result by MobilEye (www.mobileye.com)

Applications

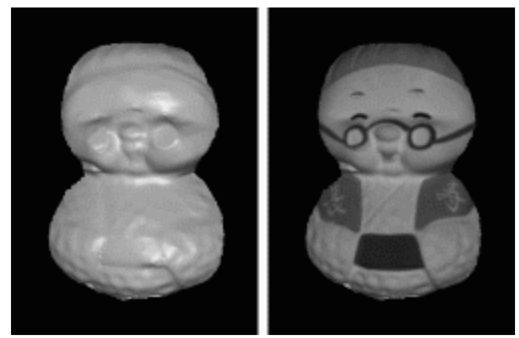
- Motion segmentation



Applications

- Structure from Motion

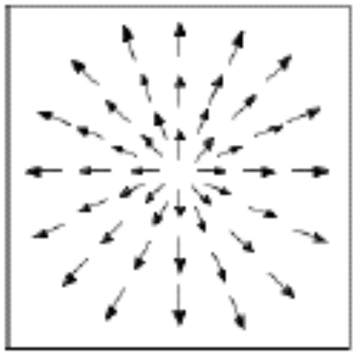
Input



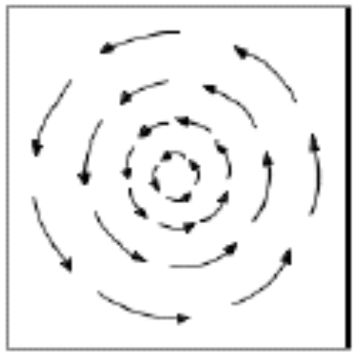
Reconstructed shape

Result by: L. Zhang, B. Curless, A. Hertzmann, S.M. Seitz
"Shape and motion under varying illumination: Unifying structure from motion, photometric stereo, and multi-view stereo" ICCV' 03

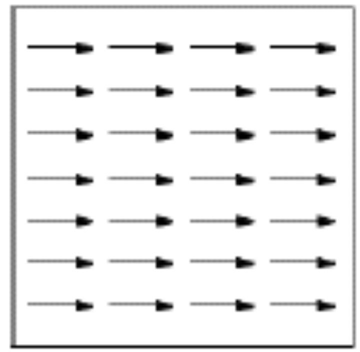
Examples of Motion fields



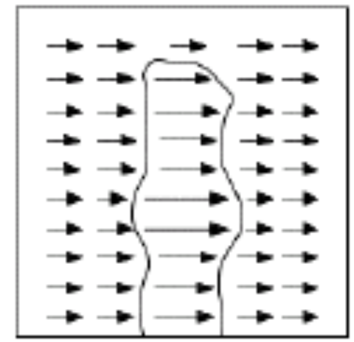
Forward motion



Rotation



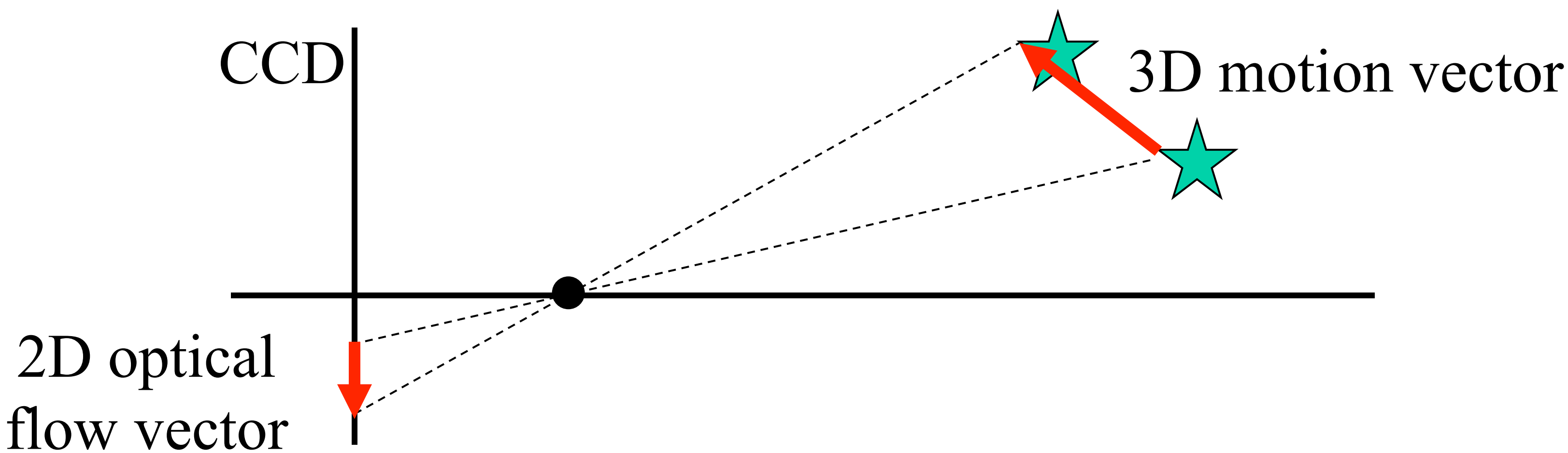
Horizontal translation



Closer objects appear to move faster!!

Motion Field & Optical Flow Field

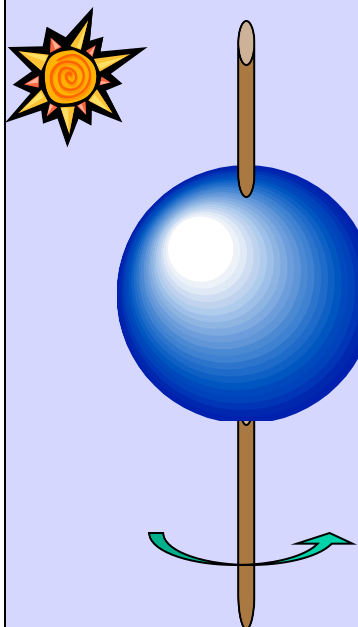
- Motion Field = Real world 3D motion
- Optical Flow Field = Projection of the motion field onto the 2d image



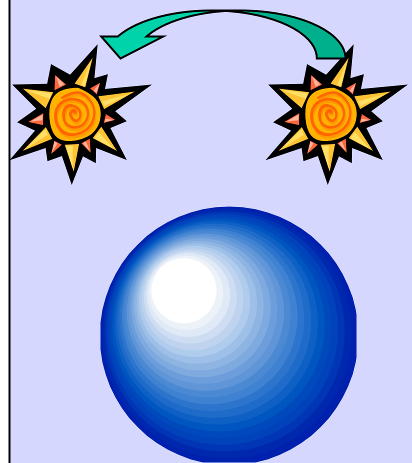
$$\vec{u} = (u, v)$$

When does it break?

The screen is stationary yet displays motion



Homogeneous objects generate zero optical flow.



Fixed sphere. Changing light source.

Non-rigid texture motion

The Optical Flow Field

Still, in many cases it does work....

- Goal:
  Find for each pixel a velocity vector $\vec{u} = (u, v)$ which says:
  - How quickly is the pixel moving across the image
  - In which direction it is moving

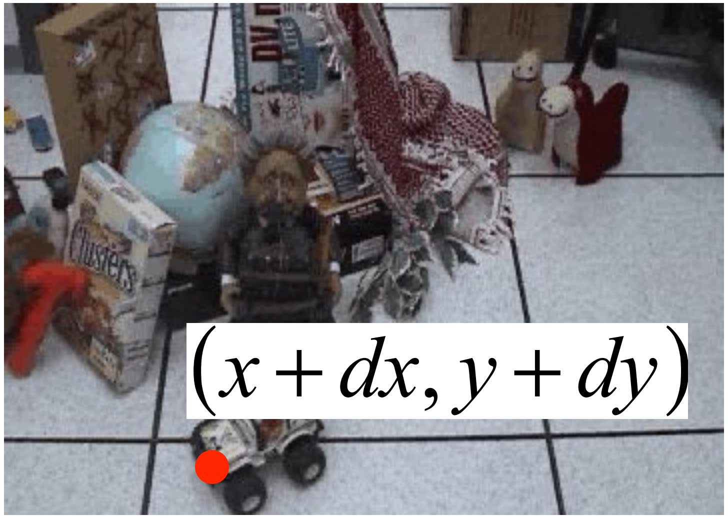
How do we actually do that?

Estimating Optical Flow

- Assume the image intensity I is constant

Time = t

Time = $t+dt$



$$I(x, y, t) = I(x+dx, y+dy, t+dt)$$

Brightness Constancy Equation

$$I(x,y,t) = I(x+dx, y+dy, t+dt)$$

First order Taylor Expansion

$$= I(x,y,t) + \frac{\partial I}{\partial x}dx + \frac{\partial I}{\partial y}dy + \frac{\partial I}{\partial t}dt$$

Simplify notations:

$$I_x dx + I_y dy + I_t dt = 0$$

Divide by *dt* and denote:

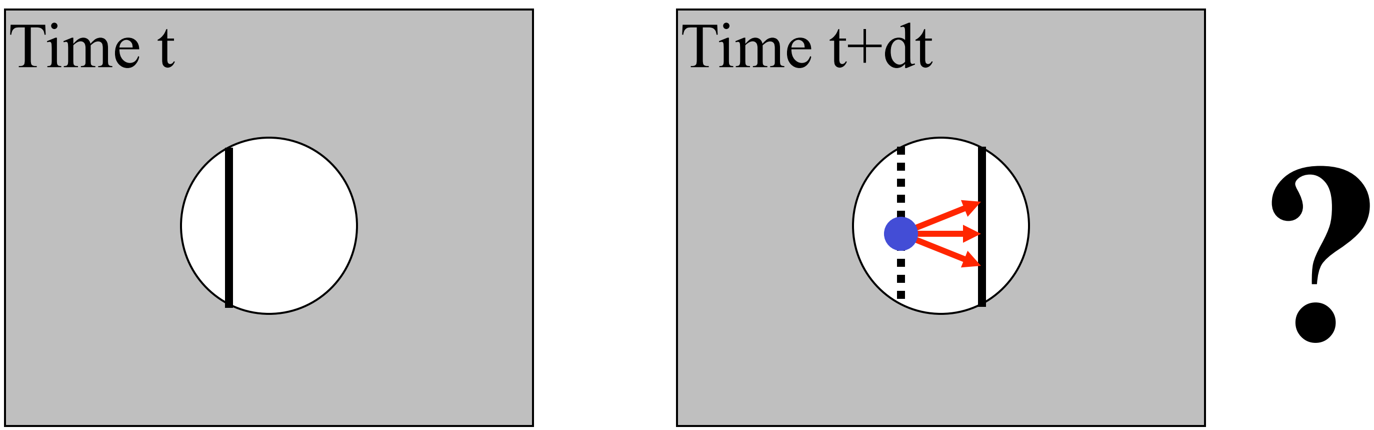
$$u = \frac{dx}{dt} \quad v = \frac{dy}{dt}$$

$$\boxed{I_x u + I_y v = -I_t}$$

<u>Problem I</u>: One equation, two unknowns

Problem II: "The Aperture Problem"

- For points on a line of fixed intensity we can only recover the normal flow

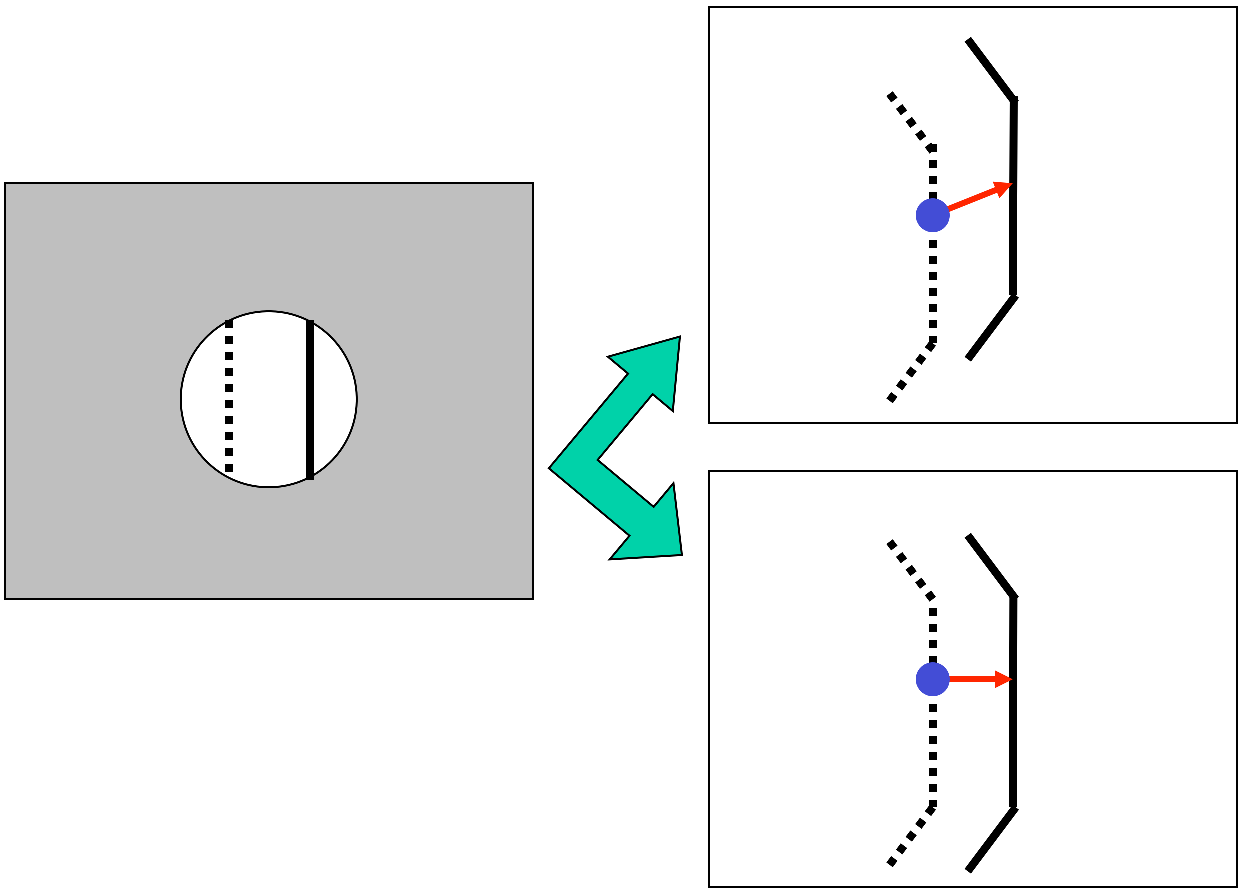


Where did the blue point move to?

We need additional constraints

Use Local Information

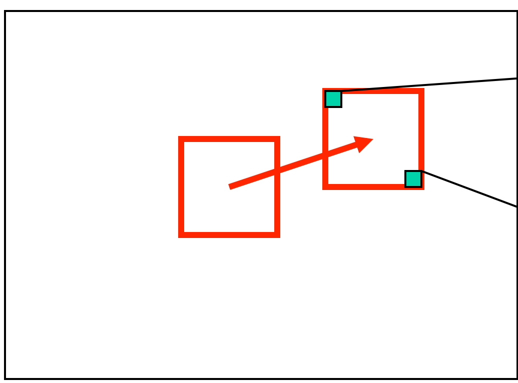
Sometimes enlarging the aperture can help



Local smoothness
Lucas Kanade (1984)

$$I_x u + I_y v = -I_t \quad \Rightarrow \quad \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -I_t$$

Assume constant (u,v) in small neighborhood



$$\begin{bmatrix} I_{x1} & I_{y1} \\ I_{x2} & I_{y2} \\ \vdots & \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_{t1} \\ I_{t2} \\ \vdots \end{bmatrix}$$

$$\boxed{A\vec{u} = b}$$

Lucas Kanade (1984)

Goal: Minimize $\left\| A\vec{u} - b \right\|^2$

Method: Least-Squares

$$A\vec{u} = b$$

$$\Downarrow$$

$$\underbrace{A^T A}_{2x2}\ \underbrace{\vec{u}}_{2x1} = \underbrace{A^T b}_{2x1}$$

$$\Downarrow$$

$$\boxed{\vec{u} = \left(A^T A\right)^{-1} A^T b}$$

How does Lucas-Kanade behave?

$$\boxed{\vec{\mathrm{u}} = \left(A^T A\right)^{-1} A^T b}$$

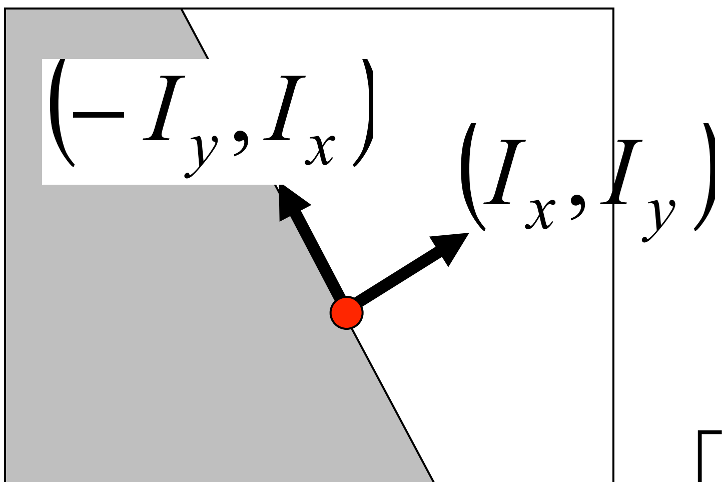
$$A^T A = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

We want this matrix to be invertible.

i.e., no zero eigenvalues

How does Lucas-Kanade behave?

- Edge ➔ $A^T A$ becomes singular



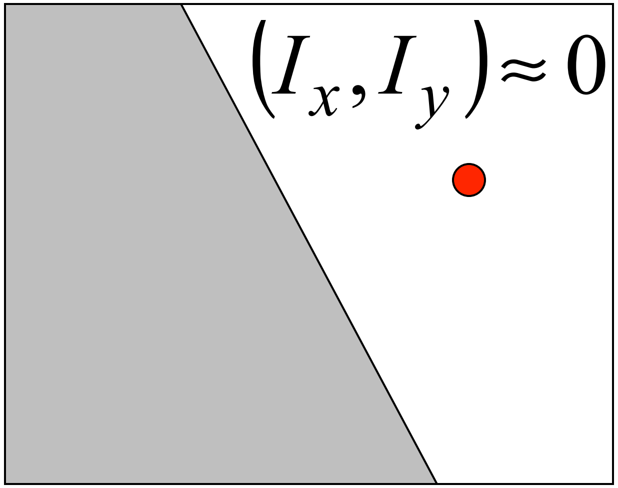
$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} -I_y \\ I_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

⇓

$\begin{bmatrix} -I_y \\ I_x \end{bmatrix}$ is eigenvector with eigenvalue 0

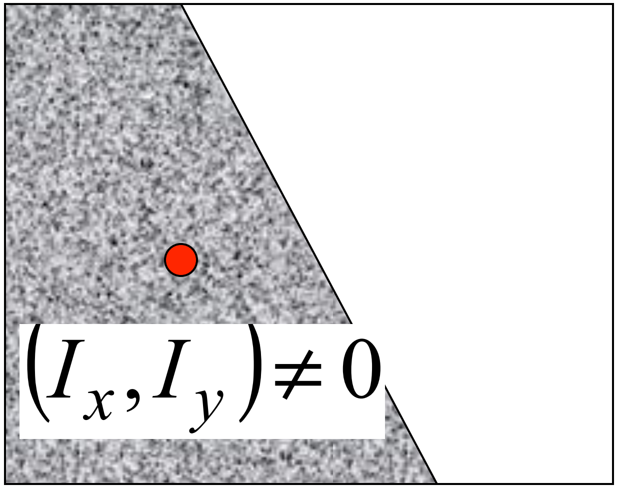
How does Lucas-Kanade behave?

- Homogeneous ➔ $A^T A \approx 0$ ➔ 0 eigenvalues



How does Lucas-Kanade behave?

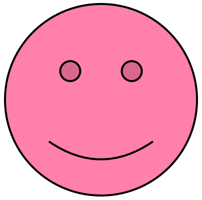
- Textured regions ➔ two high eigenvalues



How does Lucas-Kanade behave?

- Edge ➔ $A^T A$ becomes singular 

- Homogeneous regions ➔ low gradients
  $A^T A \approx 0$ 

- High texture ➔ 

Other break-downs

- Brightness constancy is **not** satisfied

  Correlation based methods

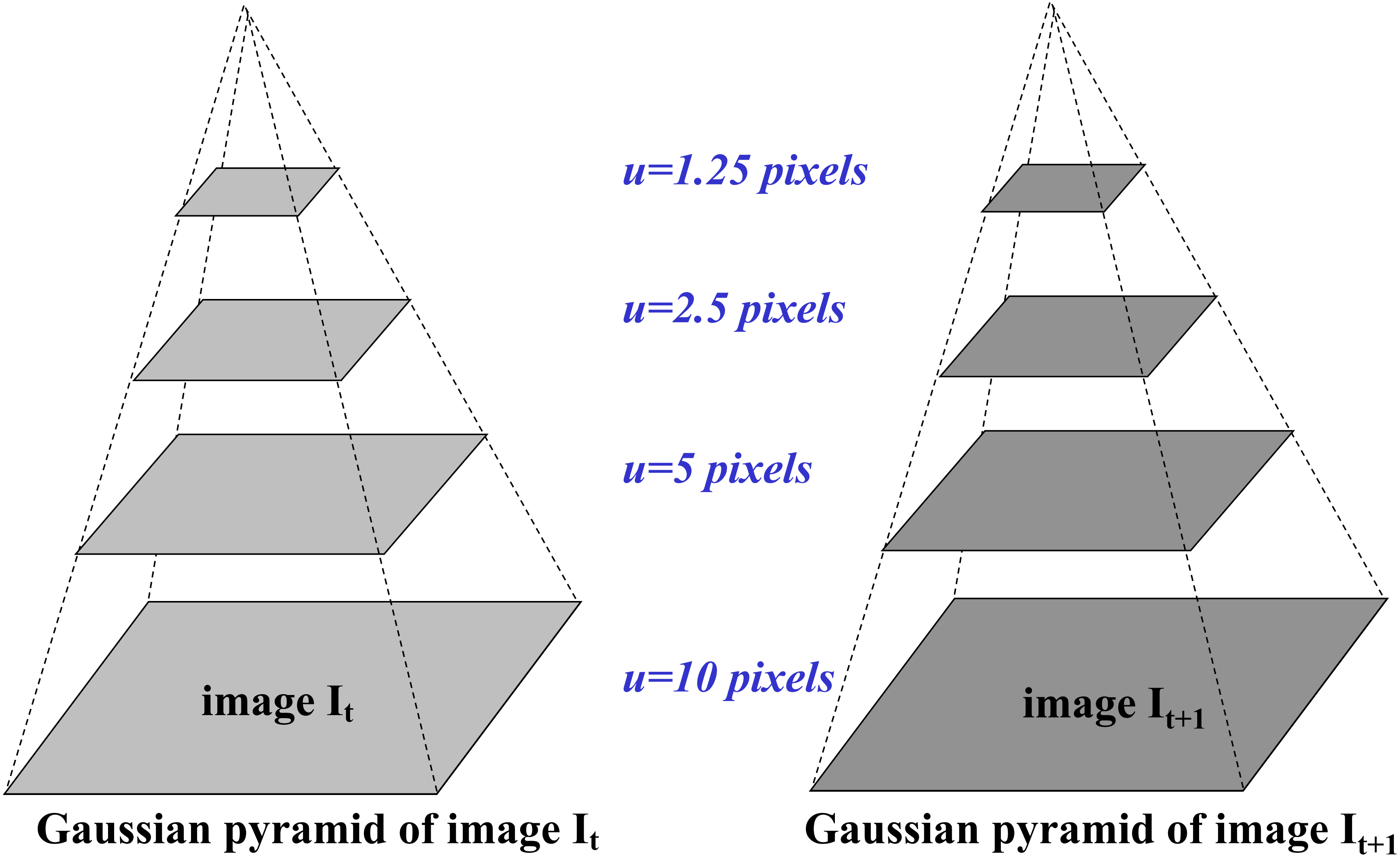
- A point does **not** move like its neighbors
  - what is the ideal window size?

  Regularization based methods

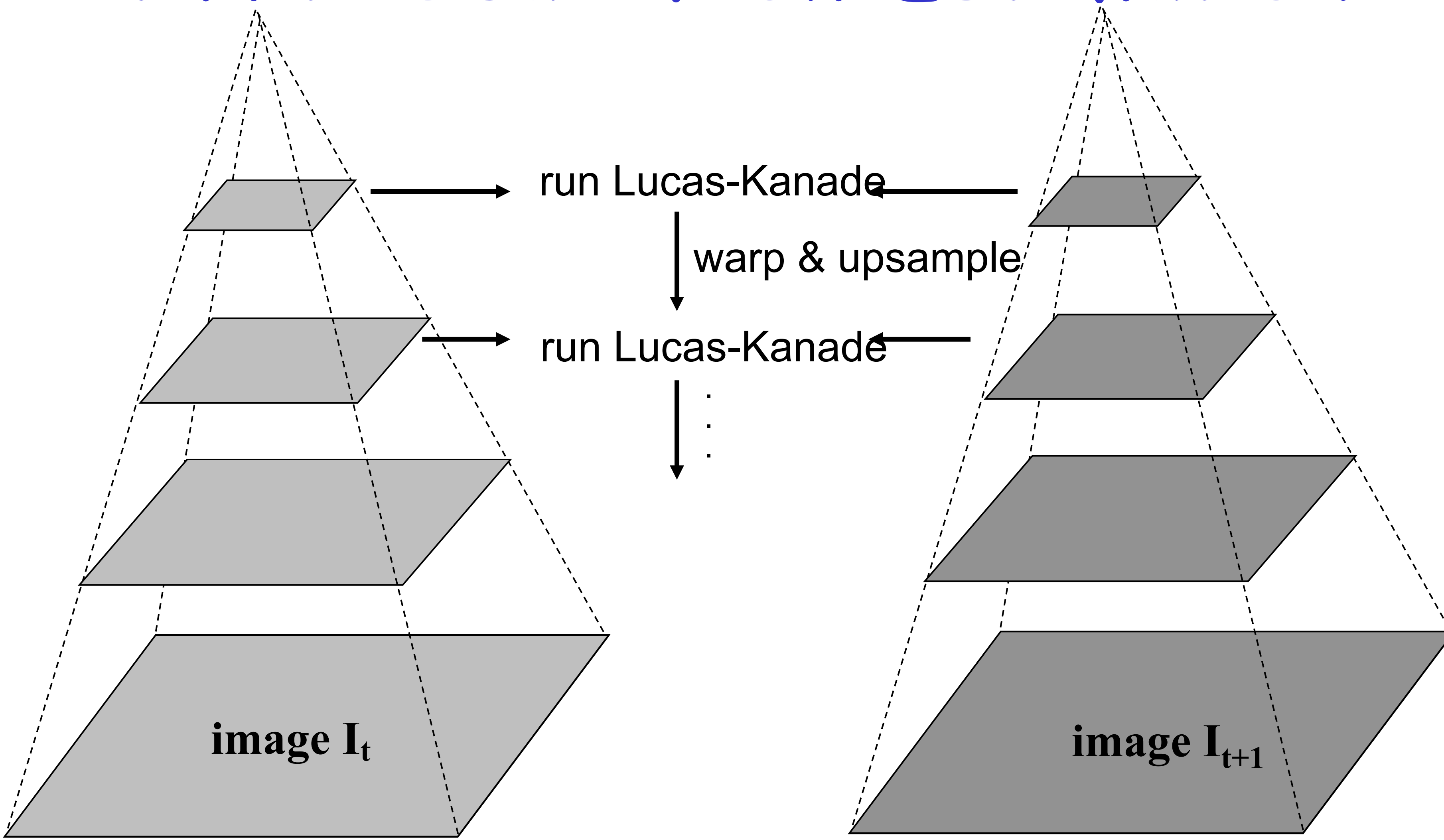
- The motion is **not** small (Taylor expansion doesn't hold)

  Use multi-scale estimation

Multi-Scale Flow Estimation



Multi-Scale Flow Estimation



Gaussian pyramid of image I_t **Gaussian pyramid of image I_{t+1}**

Examples: Motion Based Segmentation

Input

Segmentation result

Examples: Motion Based Segmentation

Input

Segmentation result

Other break-downs

- Brightness constancy is **not** satisfied

  → Correlation based methods

- A point does **not** move like its neighbors
  - what is the ideal window size?

  → Regularization based methods

- The motion is **not** small (Taylor expansion doesn't hold)

  → Use multi-scale estimation

Regularization
Horn and Schunk (1981)

Add global smoothness term

Smoothness error: $$E_s = \iint_D \left(u_x^2 + u_y^2\right) + \left(v_x^2 + v_y^2\right) dx\, dy$$

Error in brightness constancy equation $$E_c = \iint_D \left(I_x u + I_y v + I_t\right)^2 dx\, dy$$

Minimize: $E_c + \lambda E_s$

Solve by calculus of variations

Robust Estimation
Black & Anandan (1993)

Regularization can over-smooth across edges

⬇

Use "smarter" regularization

Minimize:

$$\iint_D \underbrace{\rho_1\left(I_x u + I_y v + I_t\right)}_{\text{Brightness constancy}} + \lambda \left[\underbrace{\rho_2\left(u_x, u_y\right)\rho_2\left(v_x, v_y\right)}_{\text{Smoothness}}\right] dx\, dy$$

Examples: Motion Based Segmentation

Input

Segmentation result

Affine Motion

For panning camera or planar surfaces:

$$u = p_1 + p_2 x + p_3 y$$

$$v = p_4 + p_5 x + p_6 y$$

$$I_x(p_1 + p_2 x + p_3 y) + I_y(p_4 + p_5 x + p_6 y) = -I_t$$

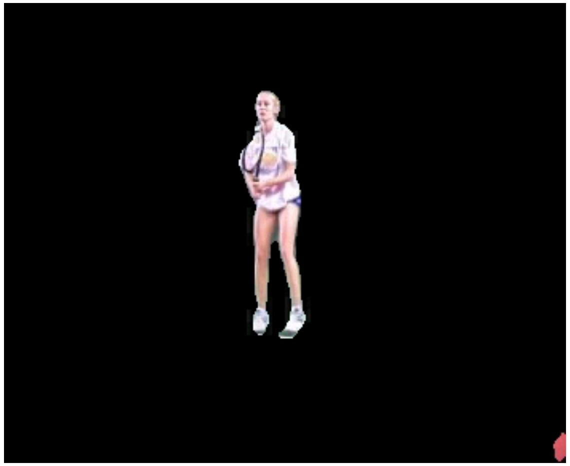
$$\begin{bmatrix} I_x & I_x x & I_x y & \vert & I_y & I_y x & I_y y \end{bmatrix} \vec{\mathrm{p}} = -I_t$$

Only 6 parameters to solve for ➔Better results

Segmentation of Affine Motion

=

+



Input

Segmentation result

Panoramas

Input

Motion estimation by Andrew Zisserman’s group

Stabilization

Result by: L.Zelnik-Manor, M.Irani
“Multi-frame estimation of planar motion”, PAMI 2000

Layered Representation

For scenes with multiple affine motions

Estimate dominant motion parameters

Reject pixels which do not fit

Convergence

Restart on remaining pixels

Some Results

Nebojsa Jojic and Brendan Frey,
"Learning Flexible Sprites in Video Layers“, CVPR 2001.

Action Recognition

- A bit more fun

"Recognizing Action at a Distance"

A.A. Efros, A.C. Berg, G. Mori, J. Malik

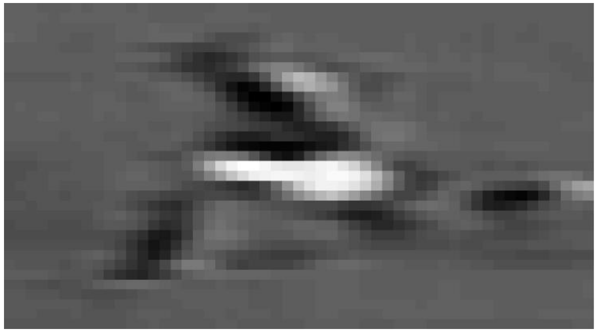
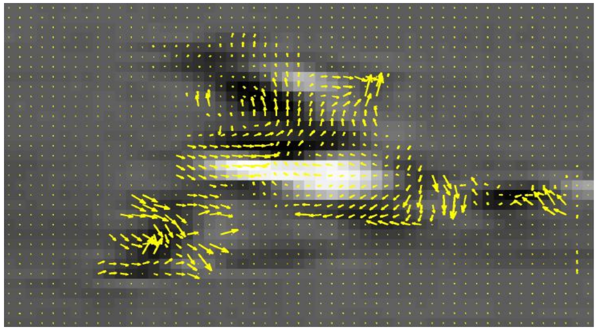


Image frame



Optical flow $F_{x,y}$

Use optical flow as a template for frame classification

"Recognizing Action at a Distance"

A.A. Efros, A.C. Berg, G. Mori, J. Malik

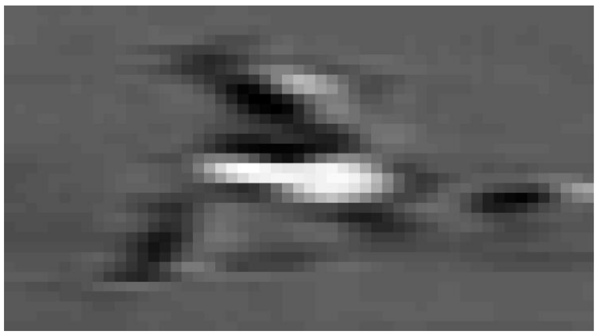
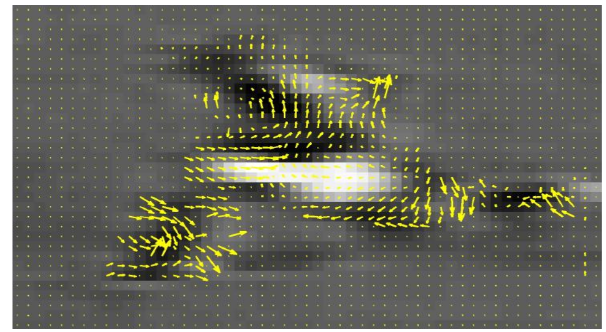
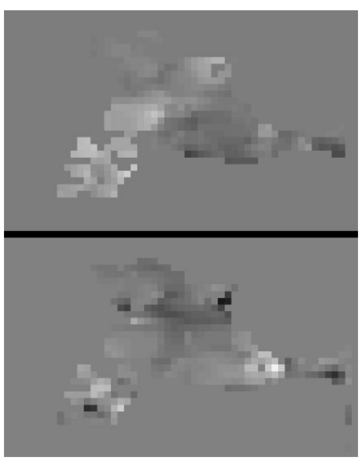


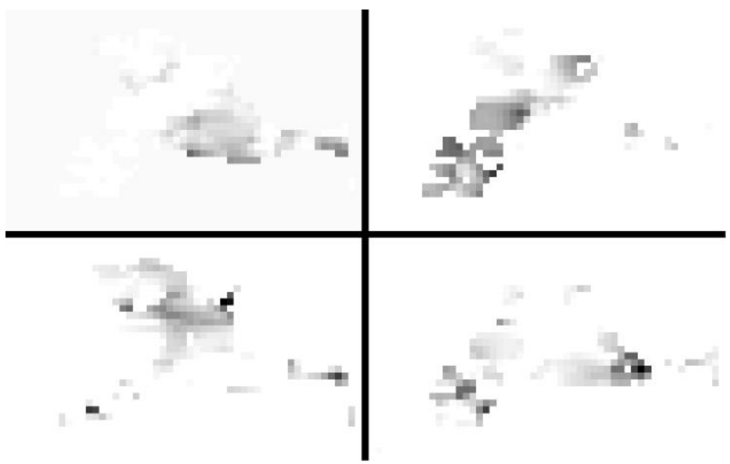
Image frame



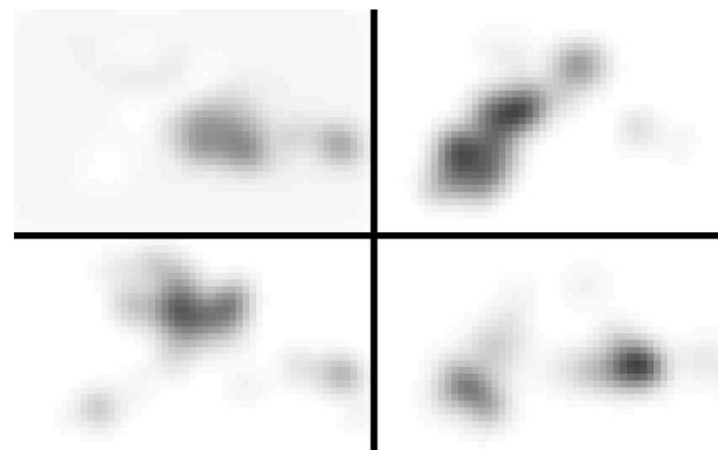
Optical flow $F_{x,y}$



F_x, F_y



$F_x^-, F_x^+, F_y^-, F_y^+$



blurred $F_x^-, F_x^+, F_y^-, F_y^+$

"Recognizing Action at a Distance"

A.A. Efros, A.C. Berg, G. Mori, J. Malik

Database:

Test sequence:

For each frame in test sequence find closest frame in database

References on Optical Flow

Lucas-Kanade method:

- B.D. Lucas and T. Kanade "***An Iterative Image Registration Technique with an Application to Stereo Vision***" IJCAI '81 pp. 674-679

- S. Baker and I. Matthews "***Lucas-Kanade 20 Years On: A Unifying Framework***" IJCV, Vol. 56, No. 3, March, 2004, pp. 221 - 255. http://www.ri.cmu.edu/projects/project_515.html (papers + code)

Regularization based methods:

- B. K. P. Horn and B. Schunck, "***Determining Optical Flow***," Artificial Intelligence, 17 (1981), pp. 185-203

- Black, M. J. and Anandan, P., "***A framework for the robust estimation of optical flow***", ICCV' 93, May, 1993, pp. 231-236 (papers + code)

Comparison of various optical flow techniques:

Barron, J.L., Fleet, D.J., and Beauchemin, S. "***Performance of optical flow techniques***". IJCV, 1994, 12(1):43-77

Layered representation (affine):

James R. Bergen P. Anandan Keith J. Hanna Rajesh Hingorani "***Hierarchical Model-Based Motion Estimation***" ECCV' 92, pp. 237-- 252