JMÉNO A PŘÍJMENÍ: LUKÁŠ RUNT

ČÍSLO ÚLOHY: 6.1.4

ZADÁNÍ: Určete T maticí přechodu od báze $f_1, f_2 \dots$ k bázi $g_1, g_2, \dots$ prostoru L a T^{-1} maticí přechodu od báze $g_1, g_2 \dots$ k bázi $f_1, f_2 \dots$

$L = \mathbb{R}_4$

$f_1 = [1,1,1,-3]^T$, $f_2 = [1,1,-3,1]^T$, $f_3 = [1,-3,1,1]^T$, $f_4 = [3,1,1,1]^T$

$g_1 = [1,2,0,0]^T$, $g_2 = [0,1,2,0]^T$, $g_3 = [0,0,2,1]^T$, $g_4 = [2,0,0,1]^T$

ŘEŠENÍ: $T = \left[\hat{g}_1 \mid \hat{g}_2 \mid \hat{g}_3 \mid \hat{g}_4\right]$

f_1 f_2 f_3 f_4 g_1 g_2 g_3 g_4

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 3 & 1 & 0 & 0 & 2 \\ 1 & 1 & -3 & 1 & 2 & 1 & 0 & 0 \\ 1 & -3 & 1 & 1 & 0 & 2 & 1 & 0 \\ -3 & 1 & 1 & 1 & 0 & 0 & 2 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 3 & 1 & 0 & 0 & 2 \\ 0 & 0 & -4 & -2 & 1 & 1 & 0 & -2 \\ 0 & -4 & 0 & -2 & -1 & 2 & 1 & -2 \\ 0 & 4 & 4 & 10 & 3 & 0 & 2 & 7 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 3 & 1 & 0 & 0 & 2 \\ 0 & -4 & 0 & -2 & -1 & 2 & 1 & -2 \\ 0 & 4 & 4 & 10 & 3 & 0 & 2 & 7 \\ 0 & 0 & -4 & -2 & 1 & 1 & 0 & -2 \end{array}\right] \sim$$

- cílem je vytvořit pomocí elementárních úprav maticovo jednotkovou matici

$$\sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 3 & 1 & 0 & 0 & 2 \\ 0 & -4 & 0 & -2 & -1 & 2 & 1 & -2 \\ 0 & 0 & 4 & 8 & 2 & 2 & 3 & 5 \\ 0 & 0 & -4 & -2 & 1 & 1 & 0 & -2 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 3 & 1 & 0 & 0 & 2 \\ 0 & -4 & 0 & -2 & -1 & 2 & 1 & -2 \\ 0 & 0 & 4 & 8 & 2 & 2 & 3 & 5 \\ 0 & 0 & 0 & 6 & 3 & 3 & 3 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 3 & 1 & 0 & 0 & 2 \\ 0 & -4 & 0 & -2 & -1 & 2 & 1 & -2 \\ 0 & 0 & 4 & 8 & 2 & 2 & 3 & 5 \\ 0 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 0 & -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} & \frac{1}{2} \\ 0 & -4 & 0 & 0 & 0 & 3 & 2 & -1 \\ 0 & 0 & 4 & 0 & -2 & -2 & 1 & 1 \\ 0 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 0 & -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & -\frac{3}{4} & -\frac{2}{4} & \frac{1}{4} \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}\right] \sim$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 0 & -1 & -\frac{5}{4} & \frac{1}{4} \\ 0 & 1 & 0 & 0 & 0 & -\frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & -\frac{1}{4} & -\frac{3}{4} & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}\right]$$

$$T = \frac{1}{4} \cdot \begin{bmatrix} 0 & -1 & -3 & 0 \\ 0 & -3 & -2 & 1 \\ -2 & -2 & -1 & 1 \\ 2 & 2 & 2 & 2 \end{bmatrix}$$

T

$$T^{-1} = \left[\hat{f}_1 \mid \hat{f}_2 \mid \hat{f}_3 \mid \hat{f}_4\right]$$

$g_1\ g_2\ g_3\ g_4 \quad f_1\ f_2\ f_3\ f_4$

$$\left[\begin{array}{cccc|cccc} 1&0&0&2&1&1&1&3\\ 2&1&0&0&1&1&-3&1\\ 0&2&1&0&1&-3&1&1\\ 0&0&2&1&-3&1&1&1 \end{array}\right] \quad (-2)$$

$$\sim \left[\begin{array}{cccc|cccc} 1&0&0&2&1&1&1&3\\ 0&1&0&-4&-1&-1&-5&-5\\ 0&2&1&0&1&-3&1&1\\ 0&0&2&1&-3&1&1&1 \end{array}\right] \quad (-2)$$

$$\sim \left[\begin{array}{cccc|cccc} 1&0&0&2&1&1&1&3\\ 0&1&0&-4&-1&-1&-5&-5\\ 0&0&1&8&3&-1&11&11\\ 0&0&2&1&-3&1&1&1 \end{array}\right] \quad (-2)$$

$$\sim \left[\begin{array}{cccc|cccc} 1&0&0&2&1&1&1&3\\ 0&1&0&-4&-1&-1&-5&-5\\ 0&0&1&8&3&-1&11&11\\ 0&0&0&-15&-9&3&-21&-21 \end{array}\right] \quad (:-15)$$

$$\sim \left[\begin{array}{cccc|cccc} 1&0&0&2&1&1&1&3\\ 0&1&0&-4&-1&-1&-5&-5\\ 0&0&1&8&3&-1&11&11\\ 0&0&0&1&\frac{3}{5}&-\frac{1}{5}&\frac{7}{5}&\frac{7}{5} \end{array}\right] \quad (-2)\ (+4)\ (-8)$$

$$\sim \left[\begin{array}{cccc|cccc} 1&0&0&0&-\frac{1}{5}&\frac{7}{5}&-\frac{9}{5}&\frac{1}{5}\\ 0&1&0&0&\frac{7}{5}&-\frac{9}{5}&\frac{3}{5}&\frac{3}{5}\\ 0&0&1&0&-\frac{9}{5}&\frac{3}{5}&-\frac{1}{5}&-\frac{1}{5}\\ 0&0&0&1&\frac{3}{5}&-\frac{1}{5}&\frac{7}{5}&\frac{7}{5} \end{array}\right] \quad \nwarrow T^{-1}$$

$$T^{-1} = \frac{1}{5}\begin{bmatrix} -1&7&-9&1\\ 7&-9&3&3\\ -9&3&-1&-1\\ 3&-1&7&7 \end{bmatrix}$$