JMÉNO A PŘIJMENÍ: LUKÁŠ RUNT

ČÍSLO ÚLOHY: 10.3.1

ZADÁNÍ: Je dána kvadratická forma $\mathcal{H}(x)$. Určete inercii $in(\mathcal{H})$ a definitnost kvadratické formy $\mathcal{H}(x)$.

$\mathcal{H}(x) = -2x_1^2 + x_2^2 + 4x_1x_2$

ŘEŠENÍ: Reálná symetrická matice:

$$A = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -2-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = (-2 + 2\lambda - \lambda + \lambda^2) - 4 = \lambda^2 + \lambda - 6 = (-3-\lambda)(2-\lambda)$$

$D = 25 \qquad \lambda_1 = \frac{-1-\sqrt{25}}{2} = -3$

$\lambda_2 = \frac{-1+\sqrt{25}}{2} = 2$

$in(A) = in(\mathcal{H}(x)) = (\overset{+}{1}, \overset{-}{1}, \overset{0}{0})$

$\Rightarrow$ $\mathcal{H}(x)$ je indefinitní!

$$J = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$$

$\lambda_1 = -3$

$$(A + 3I|0) = \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 1 & 4 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad h_1 = \begin{pmatrix} a \\ b \end{pmatrix}$$

$a + 2b = 0 \Rightarrow a = -2b$

$\Downarrow$

$h_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$\|h_1\| = \sqrt{(h_1, h_1)} = \sqrt{5} \qquad h_1' = \frac{1}{\sqrt{5}}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$\lambda_2 = 2$

$$(A - 2I|0) = \left[\begin{array}{cc|c} -4 & 2 & 0 \\ 2 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad h_2 = \begin{pmatrix} a \\ b \end{pmatrix}$$

$2a - b = 0 \Rightarrow a = \frac{b}{2}$

$\Downarrow$

$h_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\|h_2\| = \sqrt{(h_2, h_2)} = \sqrt{5} \qquad h_2' = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$h_1' \perp h_2' \Leftrightarrow (h_1, h_2) = 0 \qquad \frac{-2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} = 0$ ✓

$$T = \frac{1}{\sqrt{5}}\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\mathcal{H}(x) = x^T \cdot A \cdot x = (x^T \cdot T) \cdot J \cdot (T^T \cdot x) = (T^T \cdot x)^T \cdot J \cdot (T^T \cdot x) = y^T \cdot J \cdot y$$

$$T^T \cdot x = y = \left(\frac{1}{\sqrt{5}} \cdot \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}\right)^T \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{\sqrt{5}} \cdot \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{\sqrt{5}} \cdot \begin{bmatrix} -2x_1 + x_2 \\ x_1 + 2x_2 \end{bmatrix}$$

$$\mathcal{H}(x) = y^T \cdot J \cdot y = \left(\frac{1}{\sqrt{5}} \cdot \left[-2x_1 + x_2, x_1 + 2x_2\right] \cdot \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix} \cdot \frac{1}{\sqrt{5}} \cdot \begin{bmatrix} -2x_1 + x_2 \\ x_1 + 2x_2 \end{bmatrix}\right) =$$

$$= -3 \cdot y_1^2 + 2y_2^2 = \underline{\underline{-3\left(\frac{1}{\sqrt{5}}(-2x_1 + x_2)\right)^2 + 2\left(\frac{1}{\sqrt{5}}(x_1 + 2x_2)\right)^2}}$$