JMÉNO A PŘÍJMENÍ: LUKÁŠ RUNT

ČÍSLO ÚLOHY: 8.2.6

ZADÁNÍ: K matici A určete Jordanův kanonický tvar J a matici T. Ověřte, že platí $A = TJT^{-1}$

$$A = \begin{bmatrix} 8 & -2 & 0 \\ 0 & 8 & -8 \\ -4 & 2 & 4 \end{bmatrix}$$

ŘEŠENÍ: · Vlastní čísla: $|A - \lambda I|$

$$\begin{vmatrix} 8-\lambda & -2 & 0 \\ 0 & 8-\lambda & -8 \\ -4 & 2 & 4-\lambda \end{vmatrix} = \begin{vmatrix} 8-\lambda & -2 & 0 \\ 0 & 8-\lambda & -8 \\ 4-\lambda & 0 & 4-\lambda \end{vmatrix} = (4-\lambda)\begin{vmatrix} 8-\lambda & -2 & 0 \\ 0 & 8-\lambda & -8 \\ 1 & 0 & 1 \end{vmatrix} =$$

$$= (4-\lambda)\begin{vmatrix} 8-\lambda & -2 & \lambda-8 \\ 0 & 8-\lambda & -8 \\ 1 & 0 & 0 \end{vmatrix} = (4-\lambda)\cdot 1 \cdot (-1)^4 \cdot \begin{vmatrix} -2 & \lambda-8 \\ 8-\lambda & -8 \end{vmatrix} =$$

$$= (4-\lambda)\cdot[16-(8\lambda-64-\lambda^2+8\lambda)] = (4-\lambda)\cdot(\lambda^2-16\lambda+80)$$

$D = -64$

$\lambda_2 = \frac{16+\sqrt{64}\,i}{2} = 8+4i$

$\lambda_3 = \frac{16-\sqrt{64}\,i}{2} = 8-4i$

$\lambda_1 = 4$

$\lambda_2 = 8+4i$

$\lambda_3 = 8-4i$

Vlastní vektory:

$\lambda_1 = 4$

$$\left[\begin{array}{ccc|c} 4 & -2 & 0 & 0 \\ 0 & 4 & -8 & 0 \\ -4 & 2 & 0 & 0 \end{array}\right] \sim \begin{array}{c} /:2 \\ /:2 \\ \end{array}\left[\begin{array}{ccc|c} 4 & -2 & 0 & 0 \\ 0 & 4 & -8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & -1 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$2x - y = 0 \Rightarrow x = \frac{y}{2}$

$y - 2z = 0 \Rightarrow z = \frac{y}{2}$

$$h_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$\lambda_2 = 8+4i$

$$\left[\begin{array}{ccc|c} -4i & -2 & 0 & 0 \\ 0 & -4i & -8 & 0 \\ -4 & 2 & -4-4i & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} -4i & -2 & 0 & 0 \\ 0 & -4i & -8 & 0 \\ 0 & 2-2i & -4-4i & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} -2i & -1 & 0 & 0 \\ 0 & -i & -2 & 0 \\ 0 & 1-i & -2-2i & 0 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} 2i & -1 & 0 & 0 \\ 0 & -i & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{aligned} -2ix - y &= 0 \Rightarrow x = -\frac{y}{2i} \\ -iy - 2z &= 0 \Rightarrow z = -\frac{iy}{2} \end{aligned}$$

$$h_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2i \\ 1 \end{bmatrix}$$

$$\lambda_3 = 8 - 4i$$

$$\begin{array}{c} \\ \\ -i \end{array} \left[\begin{array}{ccc|c} 4i & -2 & 0 & 0 \\ 0 & 4i & -8 & 0 \\ -4 & 2 & -4+4i & 0 \end{array}\right] \sim \begin{array}{c} /:2 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 4i & -2 & 0 & 0 \\ 0 & 4i & -8 & 0 \\ 0 & 2+2i & -4+4i & 0 \end{array}\right] \sim \begin{array}{c} \\ \\ (-1+i) \end{array} \left[\begin{array}{ccc|c} 2i & -1 & 0 & 0 \\ 0 & i & -2 & 0 \\ 0 & 1+i & -2+2i & 0 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} 2i & -1 & 0 & 0 \\ 0 & i & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{aligned} 2ix - y &= 0 \Rightarrow x = \frac{y}{2i} \\ iy - 2z &= 0 \Rightarrow z = \frac{iy}{2} \end{aligned}$$

$$h_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -2i \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 8+4i & 0 \\ 0 & 0 & 8-4i \end{bmatrix} \qquad T = \begin{array}{c} \begin{array}{ccc} h_1 & h_2 & h_3 \\ \downarrow & \downarrow & \downarrow \end{array} \\ \begin{bmatrix} 1 & -1 & -1 \\ 2 & 2i & -2i \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

Ověření $A = T \cdot J \cdot T^{-1}$:

$$T^{-1} = \frac{1}{\det T} \cdot [T_{ij}]^T$$

$$\det T = \begin{vmatrix} 1 & -1 & -1 \\ 2 & 2i & -2i \\ 1 & 1 & 1 \end{vmatrix} = 2i - 2 + 2i + 2i + 2i + 2 = 8i$$

$$T_{11} = (-1)^2 \cdot \begin{vmatrix} 2i & -2i \\ 1 & 1 \end{vmatrix} = 1 \cdot (2i + 2i) = 4i$$

$$T_{12} = (-1)^3 \cdot \begin{vmatrix} 2 & -2i \\ 1 & 1 \end{vmatrix} = -1 \cdot (2 + 2i) = -2 - 2i$$

$$T_{13} = (-1)^4 \cdot \begin{vmatrix} 2 & 2i \\ 1 & 1 \end{vmatrix} = 1(2-2i) = 2-2i$$

$$T_{21} = (-1)^3 \cdot \begin{vmatrix} -1 & -1 \\ 1 & 1 \end{vmatrix} = -1 \cdot (-1+1) = 0$$

$$T_{22} = (-1)^4 \cdot \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1 \cdot (1+1) = 2$$

$$T_{23} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = -1 \cdot (1+1) = -2$$

$$T_{31} = (-1)^4 \begin{vmatrix} -1 & -1 \\ 2i & -2i \end{vmatrix} = 1 \cdot (2i+2i) = 4i$$

$$T_{32} = (-1)^5 \cdot \begin{vmatrix} 1 & -1 \\ 2 & -2i \end{vmatrix} = -1(-2i+2) = -2+2i$$

$$T_{33} = (-1)^6 \cdot \begin{vmatrix} 1 & -1 \\ 2 & 2i \end{vmatrix} = 1(2i+2) = 2+2i$$

$$T^{-1} = \frac{1}{8i} \cdot \begin{bmatrix} 4i & 0 & 4i \\ -2-2i & 2 & -2+2i \\ 2-2i & -2 & 2+2i \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{4}+\frac{1}{4i} & -\frac{i}{4} & \frac{1}{4}+\frac{i}{4} \\ -\frac{1}{4}-\frac{1}{4}i & \frac{i}{4} & \frac{1}{4}-\frac{i}{4} \end{bmatrix}$$

$$A = T \circ J \circ T^{-1} = \begin{bmatrix} 1 & -1 & -1 \\ 2 & 2i & -2i \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & 0 \\ 0 & 8+4i & 0 \\ 0 & 0 & 8-4i \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{-1+i}{4} & \frac{-i}{4} & \frac{1+i}{4} \\ \frac{-1-i}{4} & \frac{i}{4} & \frac{1-i}{4} \end{bmatrix}$$

$$T \cdot J = \begin{bmatrix} 4 & -8-4i & -8+4i \\ 8 & -8+16i & -8-16i \\ 4 & 8+4i & 8-4i \end{bmatrix}$$

$$T \cdot J \cdot T^{-1} = \begin{bmatrix} 8 & -2 & 0 \\ 0 & 8 & 8 \\ -4 & 2 & 4 \end{bmatrix} = A$$