Could you clarify these points in the proposed algorithm?

Q1: In item (1) of Definition 3.9, shouldn't we require x_u be not in J?
R1: The Definition 3.9 is based on 3.7 where x_u be not in J has been assumed, we will make it more clear in the paper.

Q2: It is a bit unclear why the adjustment to x_u is defined not based on the auxiliary constraint itself but rather on one other constraint that x_u is involved in (in (2)).
R2: The auxiliary constraint is used to find the auxiliary variable x_u to modify x_t, and x_t will be modified to make the auxiliary constraint as tight as possible. To increase the range that x_t can move, the value of x_u is also modified. The range that x_u can vary is thus limited by another constraint. We will make it clearer in our paper.

Q3: Lines 486--487: "Since the valid range of x_t is an interval, its points are monotone" - this is not clear to me, e.g. why having the valid range as an interval implies that its points are monotone?
R3: If we regard the endpoint of the range value of x_t as a function of x_u, then the values of the points are monotone w.r.t. the change of x_t . We will make it clearer in our paper.

Q4: Line 667: If no decreasing operation is found, it indicates that the algorithm may reach a local minimum - what does that mean to reach a local minimum in this case if the point is infeasible? If it's wrt the score function then what does it mean to be local in the space of the moves?
R4: The "local minimum" here is defined with repect to the score function, meaning there is no move to improve the score function for current solution. Since the score function counts also violated constraints, infeasible solution can also be a "local minimum" in this sense. It is different than the "local minimum" of only the objective function, this comment is very helpful and we will make it more clear in the paper.

Q5: In the Algorithm 1 - what does it mean to update weights if we have not made any move yet? (i.e. why wouldn't they be up-to-date)?
R5: There may be weight-updating even if no move has been made, in this case, it means there is no better solution in the neighborhood of the initial solution w.r.t. the score function, thus we update the weight to guide the search.

Q6: Why to select bound moves in the feasible mode (is the feasibility meant only wrt to the multilinear constraints in the description of the algorithm)?
R6: The feasibility in the description of the algorithm is w.r.t. all constraints, not only the multilinear constraints. We select the bound move in feasible mode to make the objective function decrease as much, although it may cause some constraints violated, the violations are controlled and tend to be eliminated by the score function.

Q7: Minor: the last point in the definition of the weighting scheme in 3.2.1 can be made

clearer by specifying that alpha is the (updated) current assignment

R7: We will make the definition of the weighting scheme in 3.2.1 clearer in our paper.