# An Efficient Local Search Algorithm for Large GD Advertising Inventory Allocation with Multilinear Constraints

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**Table 1: Results of LS-IMP for solving real-world ad benchmark under different parameter settings. P0 is the parameter in paper.**

| Parameter settings | $t$ | $\zeta$ | Total #win (60s) | Total #win (300s) | Avg #UR (60s) | Avg #FR (60s) |
|---|---|---|---|---|---|---|
| **P0** | **100** | **100** | **860** | **764** | **16.4** | **62.8** |
| P1 | 50 | 10 | 848 | 755 | 15.7 | 61.7 |
| P2 | 75 | 10 | 858 | 754 | 15.8 | 62.2 |
| P3 | 100 | 10 | 854 | 750 | 15.5 | 62.1 |
| P4 | 125 | 10 | 855 | 748 | 15.9 | 61.2 |
| P5 | 150 | 10 | 845 | 745 | 15.2 | 61.8 |
| P6 | 200 | 10 | 846 | 757 | 15.1 | 62.3 |
| P7 | 50 | 50 | 867 | 759 | 16.2 | 64.1 |
| P8 | 75 | 50 | 852 | 774 | 16.3 | 62.8 |
| P9 | 100 | 50 | 857 | 747 | 16.3 | 61.3 |
| P10 | 125 | 50 | 878 | 745 | 16.5 | 64.4 |
| P11 | 150 | 50 | 864 | 762 | 16.7 | 62.1 |
| P12 | 200 | 50 | 859 | 776 | 16.0 | 63.0 |
| P13 | 50 | 80 | 850 | 759 | 16.3 | 62.0 |
| P14 | 75 | 80 | 876 | 769 | 16.6 | 61.9 |
| P15 | 100 | 80 | 847 | 759 | 16.2 | 62.3 |
| P16 | 125 | 80 | 840 | 764 | 16.6 | 62.5 |
| P17 | 150 | 80 | 851 | 744 | 16.8 | 63.1 |
| P18 | 200 | 80 | 848 | 746 | 15.8 | 63.7 |
| P19 | 50 | 100 | 867 | 784 | 16.1 | 62.5 |
| P20 | 75 | 100 | 869 | 753 | 16.1 | 62.4 |
| P21 | 125 | 100 | 873 | 773 | 16.1 | 62.1 |
| P22 | 150 | 100 | 839 | 788 | 15.8 | 63.4 |
| P23 | 200 | 100 | 861 | 757 | 16.5 | 63.8 |
| P24 | 50 | 200 | 861 | 775 | 16.4 | 62.1 |
| P25 | 75 | 200 | 870 | 759 | 16.0 | 62.0 |
| P26 | 100 | 200 | 852 | 740 | 16.3 | 61.2 |
| P27 | 125 | 200 | 852 | 781 | 16.2 | 63.0 |
| P28 | 150 | 200 | 836 | 769 | 16.0 | 61.1 |
| P29 | 200 | 200 | 850 | 778 | 16.2 | 63.3 |
| P30 | 50 | 500 | 855 | 758 | 15.1 | 61.3 |
| P31 | 75 | 500 | 852 | 750 | 15.7 | 62.1 |
| P32 | 100 | 500 | 844 | 756 | 15.1 | 61.6 |
| P33 | 125 | 500 | 852 | 742 | 15.4 | 61.5 |
| P34 | 150 | 500 | 841 | 752 | 15.9 | 61.0 |
| P35 | 200 | 500 | 853 | 750 | 15.6 | 61.7 |

**Table 2: The numeric comparsion in minlplib instances**

| Instance | Gurobi-Exact | Gurobi-Heur | LS-IMP |
|---|---|---|---|
| cutoff = 60s | | | |
| sonet17v4 | 1233700 | **1182604.5** | **1182604.5** |
| sonet18v6 | 3709850 | 3562770 | **3389110** |
| sonet19v5 | 2590910 | 2557780 | **2528144** |
| sonet20v6 | 3747400 | **3311060** | 3480810 |
| sonet21v6 | 8317140 | **7701520** | 7731250 |
| sonet22v4 | 2698220 | **2386630** | 2397132 |
| sonet23v6 | 8411680 | 7474600 | **7197950** |
| sonet24v2 | 5073420 | 6763260 | **5060259** |
| sonet25v5 | 7852600 | 7185020 | **7064664** |
| cutoff = 300s | | | |
| sonet17v4 | 1184730 | **1182604.5** | **1182604.5** |
| sonet18v6 | **3389110** | **3389110** | **3389110** |
| sonet19v5 | 2590910 | 2549380 | **2528144** |
| sonet20v6 | 3658580 | **3311060** | 3390200 |
| sonet21v6 | 7908790 | **7600750** | **7600750** |
| sonet22v4 | 2517840 | **2379970** | 2387835 |
| sonet23v6 | 8411680 | 7150690 | **7078810** |
| sonet24v2 | 3312580 | 3312580 | **3312579** |
| sonet25v5 | 7525050 | 7093900 | **7064664** |

**Table 3: The number of instances which can find the optimal solution.**

| Dataset | Gurobi | | | | LS-IMP | | LS-IMP + Gurobi | |
|---|---|---|---|---|---|---|---|---|
| | Exact | | Heur | | | | | |
| | #sol | #feas | #sol | #feas | #sol | #feas | #sol | #feas |
| cutoff = 60s | | | | | | | | |
| $\mathcal{D}_1$ | 8 | 12 | 10 | 12 | **85** | **179** | 84 | **179** |
| $\mathcal{D}_2$ | 2 | 4 | 9 | 26 | **92** | **166** | 89 | **166** |
| $\mathcal{D}_3$ | 0 | 0 | 13 | 43 | **79** | **178** | 77 | **178** |
| $\mathcal{D}_4$ | 0 | 0 | 4 | 29 | **82** | **171** | 78 | **171** |
| $\mathcal{D}_5$ | 0 | 0 | 8 | 41 | **97** | **176** | 94 | **176** |
| Total | 10 | 16 | 44 | 151 | **435** | **870** | 422 | **870** |
| cutoff = 300s | | | | | | | | |
| $\mathcal{D}_1$ | 50 | 96 | 12 | 17 | 86 | **180** | **90** | **180** |
| $\mathcal{D}_2$ | 51 | 82 | 18 | 44 | 94 | **167** | **101** | **167** |
| $\mathcal{D}_3$ | 35 | 66 | 13 | 56 | 80 | **179** | **85** | **179** |
| $\mathcal{D}_4$ | 32 | 74 | 12 | 43 | 83 | **172** | **87** | **172** |
| $\mathcal{D}_5$ | 31 | 67 | 15 | 53 | 99 | **176** | **103** | **176** |
| Total | 199 | 385 | 70 | 213 | 442 | **874** | **466** | **874** |
| cutoff = 1000s | | | | | | | | |
| $\mathcal{D}_1$ | 100 | 150 | 30 | 81 | 87 | **180** | **113** | **180** |
| $\mathcal{D}_2$ | 99 | 151 | 32 | 91 | 96 | **167** | **110** | **167** |
| $\mathcal{D}_3$ | 79 | 124 | 28 | 87 | 80 | **179** | **95** | **179** |
| $\mathcal{D}_4$ | 83 | 132 | 31 | 71 | 84 | **172** | **90** | **172** |
| $\mathcal{D}_5$ | 95 | 141 | 28 | 90 | 100 | **176** | **106** | **176** |
| Total | 456 | 698 | 149 | 420 | 447 | **874** | **514** | **874** |

**Table 4: The number of Feaible and violated instances in Gurobi with real variable.**

| Dataset | Gurobi | Gurobi-Real | | | LS-IMP |
|---|---|---|---|---|---|
| | #feas | #feas | #vio | #vio_rate | #feas |
| cutoff = 60s | | | | | |
| $\mathcal{D}_1$ | 12 | 0 | 0 | 0 | **179** |
| $\mathcal{D}_2$ | 4 | 0 | 0 | 0 | **164** |
| $\mathcal{D}_3$ | 0 | 0 | 0 | 0 | **172** |
| $\mathcal{D}_4$ | 0 | 0 | 0 | 0 | **170** |
| $\mathcal{D}_5$ | 0 | 0 | 0 | 0 | **175** |
| Total | 16 | 0 | 0 | 0 | **860** |
| cutoff = 300s | | | | | |
| $\mathcal{D}_1$ | 96 | 98 | 19 | 4.6% | **180** |
| $\mathcal{D}_2$ | 82 | 85 | 15 | 4.2% | **167** |
| $\mathcal{D}_3$ | 66 | 66 | 10 | 3.3% | **179** |
| $\mathcal{D}_4$ | 74 | 75 | 12 | 5.3% | **172** |
| $\mathcal{D}_5$ | 67 | 68 | 13 | 6.3% | **176** |
| Total | 385 | 392 | 69 | 4.7% | **874** |
| cutoff = 1000s | | | | | |
| $\mathcal{D}_1$ | 150 | 157 | 22 | 4.3% | **180** |
| $\mathcal{D}_2$ | 151 | 158 | 23 | 4.1% | **167** |
| $\mathcal{D}_3$ | 124 | 133 | 17 | 3.1% | **179** |
| $\mathcal{D}_4$ | 132 | 141 | 17 | 4.9% | **172** |
| $\mathcal{D}_5$ | 141 | 149 | 14 | 5.8% | **176** |
| Total | 698 | 738 | 92 | 4.4% | **874** |

**Table 5: The number of Feaible and violated instances in Gurobi with real variable.**

| Dataset | Gurobi | | Gurobi-Real | | | | | | LS-IMP |
|---|---|---|---|---|---|---|---|---|---|
| | Exact | Heur | Exact | | | Heur | | | |
| | #feas | #feas | #feas | #vio | #vio_rate | #feas | #vio | #vio_rate | #feas |
| cutoff = 60s | | | | | | | | | |
| $\mathcal{D}_1$ | 12 | 12 | 0 | 0 | 0 | 99 | 19 | 4.3% | **179** |
| $\mathcal{D}_2$ | 4 | 11 | 0 | 0 | 0 | 96 | 18 | 3.8% | **164** |
| $\mathcal{D}_3$ | 0 | 16 | 0 | 0 | 0 | 96 | 22 | 6.7% | **172** |
| $\mathcal{D}_4$ | 0 | 5 | 0 | 0 | 0 | 84 | 17 | 3.2% | **170** |
| $\mathcal{D}_5$ | 0 | 10 | 0 | 0 | 0 | 95 | 16 | 6.2% | **175** |
| Total | 16 | 54 | 0 | 0 | 0 | 470 | 92 | 5.0% | **860** |
| cutoff = 300s | | | | | | | | | |
| $\mathcal{D}_1$ | 96 | 17 | 98 | 19 | 4.6% | **189** | 31 | 7.6% | 180 |
| $\mathcal{D}_2$ | 82 | 44 | 85 | 15 | 4.2% | **193** | 36 | 6.5% | 167 |
| $\mathcal{D}_3$ | 66 | 56 | 66 | 10 | 3.3% | **192** | 28 | 7.2% | 179 |
| $\mathcal{D}_4$ | 74 | 43 | 75 | 12 | 5.3% | **191** | 26 | 7.0% | 172 |
| $\mathcal{D}_5$ | 67 | 53 | 68 | 13 | 6.3% | **191** | 27 | 6.8% | 176 |
| Total | 385 | 213 | 392 | 69 | 4.7% | **956** | 148 | 7.0% | 874 |
| cutoff = 1000s | | | | | | | | | |
| $\mathcal{D}_1$ | 150 | 81 | 157 | 22 | 4.3% | **189** | 31 | 7.6% | 180 |
| $\mathcal{D}_2$ | 151 | 91 | 158 | 23 | 4.1% | **193** | 36 | 6.5% | 167 |
| $\mathcal{D}_3$ | 124 | 87 | 133 | 17 | 3.1% | **192** | 28 | 7.2% | 179 |
| $\mathcal{D}_4$ | 132 | 71 | 141 | 17 | 4.9% | **191** | 26 | 7.0% | 172 |
| $\mathcal{D}_5$ | 141 | 90 | 149 | 14 | 5.8% | **191** | 27 | 6.8% | 176 |
| Total | 698 | 420 | 738 | 92 | 4.4% | **956** | 148 | 7.0% | 874 |

**Table 6: The metric in Gurobi with real variable.**

| Dataset | Gurobi | | | | | | Gurobi-Real | | | | | | LS-IMP | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Exact | | | Heur | | | Exact | | | Heur | | | | | |
| | #win | #UR | #FR | #win | #UR | #FR | #win | #UR | #FR | #win | #UR | #FR | #win | #UR | #FR |
| cutoff = 60s | | | | | | | | | | | | | | | |
| $\mathcal{D}_1$ | 12 | 1.7% | 5.8% | 12 | 1.7% | 5.8% | 0 | 0 | 0 | 32 | 5.5% | 15.7% | **179** | **15.5%** | **64.7%** |
| $\mathcal{D}_2$ | 4 | 0.6% | 1.9% | 11 | 1.7% | 5.7% | 0 | 0 | 0 | 26 | 3.0% | 9.6% | **164** | **16.4%** | **60.7%** |
| $\mathcal{D}_3$ | 0 | 0 | 0 | 16 | 2.3% | 9.0% | 0 | 0 | 0 | 30 | 15.0% | 36.0% | **172** | **16.7%** | **62.0%** |
| $\mathcal{D}_4$ | 0 | 0 | 0 | 5 | 1.2% | 5.0% | 0 | 0 | 0 | 21 | 14.0% | 35.0% | **170** | **16.3%** | **61.6%** |
| $\mathcal{D}_5$ | 0 | 0 | 0 | 10 | 1.8% | 4.8% | 0 | 0 | 0 | 19 | 3.0% | 9.8% | **175** | **17.0%** | **62.8%** |
| Total/Avg | 16 | 0.4% | 1.9% | 54 | 1.4% | 4.9% | 0 | 0 | 0 | 128 | 8.1% | 21.2% | **860** | **16.4%** | **62.8%** |
| cutoff = 300s | | | | | | | | | | | | | | | |
| $\mathcal{D}_1$ | 69 | 7.7% | 39.6% | 13 | 1.8% | 7.0% | 67 | 7.5% | 36.7% | 40 | 5.9% | 20.3% | **146** | **17.7%** | **68.8%** |
| $\mathcal{D}_2$ | 61 | 6.0% | 35.5% | 24 | 2.6% | 13.9% | 66 | 6.3% | 38.5% | 35 | 3.7% | 10.9% | **141** | **17.7%** | **62.8%** |
| $\mathcal{D}_3$ | 42 | 5.1% | 22.8% | 17 | 4.3% | 14.7% | 39 | 4.7% | 20.6% | 37 | 16.3% | 38.7% | **159** | **18.1%** | **64.3%** |
| $\mathcal{D}_4$ | 47 | 5.9% | 28.8% | 17 | 2.4% | 11.7% | 53 | 6.9% | 30.8% | 29 | 15.4% | 37.7% | **153** | **16.6%** | **62.5%** |
| $\mathcal{D}_5$ | 42 | 5.3% | 24.7% | 21 | 3.4% | 11.2% | 46 | 5.7% | 24.7% | 31 | 6.4% | 11.1% | **165** | **17.3%** | **65.7%** |
| Total/Avg | 261 | 6.0% | 30.3% | 92 | 2.7% | 11.7% | 271 | 6.2% | 30.2% | 144 | 9.5% | 23.7% | **764** | **17.5%** | **64.8%** |
| cutoff = 1000s | | | | | | | | | | | | | | | |
| $\mathcal{D}_1$ | 115 | 15.3% | 62.3% | 34 | 6.6% | 24.1% | **123** | **18.5%** | 66.7% | 40 | 5.9% | 20.3% | 117 | 17.7% | **68.9%** |
| $\mathcal{D}_2$ | 109 | 14.7% | 61.3% | 40 | 6.3% | 28.4% | **116** | 15.3% | 64.5% | 35 | 3.7% | 10.9% | 112 | **17.7%** | **62.9%** |
| $\mathcal{D}_3$ | 87 | 11.4% | 46.8% | 30 | 7.0% | 25.2% | 89 | 12.7% | 50.6% | 37 | 16.3% | 38.7% | **131** | **18.1%** | **64.4%** |
| $\mathcal{D}_4$ | 92 | 12.1% | 52.6% | 35 | 4.2% | 22.2% | 95 | 13.9% | 55.8% | 29 | 15.4% | 37.7% | **124** | **16.6%** | **62.6%** |
| $\mathcal{D}_5$ | 105 | 14.6% | 55.6% | 33 | 6.8% | 23.8% | 107 | 15.7% | 56.7% | 31 | 6.4% | 11.1% | **130** | **17.4%** | **66.0%** |
| Total/Avg | 508 | 13.6% | 55.7% | 172 | 6.2% | 24.7% | 530 | 14.8% | 58.8% | 172 | 9.5% | 23.7% | **614** | **17.5%** | **65.0%** |

**Table 7: The metric with Lagrange multiple method**

| Dataset | Lagrange multiple method | | | | LS-IMP | | | |
|---|---|---|---|---|---|---|---|---|
| | #feas | #win | #UR | #FR | #feas | #win | #UR | #FR |
| cutoff = 60s | | | | | | | | |
| $\mathcal{D}_1$ | 4 | 0 | 0.4% | 2.1% | **179** | **179** | **15.5%** | **64.7%** |
| $\mathcal{D}_2$ | 4 | 0 | 0.4% | 2.0% | **166** | **166** | **16.4%** | **60.7%** |
| $\mathcal{D}_3$ | 0 | 0 | 0 | 0 | **178** | **178** | **16.7%** | **62.0%** |
| $\mathcal{D}_4$ | 0 | 0 | 0 | 0 | **171** | **171** | **16.3%** | **61.6%** |
| $\mathcal{D}_5$ | 0 | 0 | 0 | 0 | **176** | **176** | **17.0%** | **62.8%** |
| Total/Avg | 8 | 0 | 0.08% | 0.1% | **870** | **870** | **16.4%** | **62.8%** |
| cutoff = 300s | | | | | | | | |
| $\mathcal{D}_1$ | 5 | 1 | 0.5% | 2.5% | **180** | **180** | **17.7%** | **68.8%** |
| $\mathcal{D}_2$ | 7 | 2 | 0.7% | 3.6% | **167** | **167** | **17.7%** | **62.8%** |
| $\mathcal{D}_3$ | 0 | 0 | 0 | 0 | **179** | **179** | **18.1%** | **64.3%** |
| $\mathcal{D}_4$ | 0 | 0 | 0 | 0 | **172** | **172** | **16.6%** | **62.5%** |
| $\mathcal{D}_5$ | 0 | 0 | 0 | 0 | **176** | **176** | **17.3%** | **65.7%** |
| Total/Avg | 12 | 3 | 0.3% | 1.2% | **874** | **874** | **17.5%** | **64.8%** |
| cutoff = 1000s | | | | | | | | |
| $\mathcal{D}_1$ | 6 | 2 | 0.6% | 3.2% | **180** | **180** | **17.7%** | **68.9%** |
| $\mathcal{D}_2$ | 9 | 3 | 1.0% | 6.2% | **167** | **167** | **17.7%** | **62.9%** |
| $\mathcal{D}_3$ | 1 | 0 | 0.1% | 0.4% | **179** | **179** | **18.1%** | **64.4%** |
| $\mathcal{D}_4$ | 0 | 0 | 0 | 0 | **172** | **172** | **16.6%** | **62.6%** |
| $\mathcal{D}_5$ | 1 | 0 | 0.1% | 0.5% | **176** | **176** | **17.4%** | **66.0%** |
| Total/Avg | 17 | 5 | 0.3% | 2.2% | **874** | **874** | **17.5%** | **65.0%** |