

## Question2:

About the integrality constraints.

## Response:

The integrality constraints are from the original problem that the allocation quantities are indeed integral. The number of impressions ranges from hundreds to thousands in ideal assumptions. However, there are always some supply nodes with impression  $< 10$  in real allocation which can not be ignored. Still, we conducted experiments on solving the continuous relaxation of the problem as in tables 5 and 6 in <https://github.com/LS-IMP/appendix/blob/main/appendix.pdf>, there are many instances that fail to rounding the continuous solutions into feasible integral solutions. Moreover, we found Gurobi find very few feasible solutions of the continuous relaxation (20/1000) in the business time limit of 1 min (v.s. 860/1000 integral solution by our algorithm). Thus, the integrality constraint should not be the main cause of differences in efficiency, and continuous relaxation is already challenging for Gurobi. We also conducted experiments on business metrics for rounded solutions of continuous relaxation by gurobi. As for detailed results, please kindly refer to table 7.

## Question3:

About the optimal solving by Gurobi.

## Response:

We added experiments on instances that Gurobi can solve optimally(10 hours time limit) and compare our solution in business timeframe. As presented in table 4 in Q2 link, the results of #opt show that solver can get the same solution with optimal solution for significantly more instances compared to Gurobi. If we combine our algorithm with Gurobi, the results are slightly better. This shows that our algorithm outperforms Gurobi in solution quality within short timeframe in GD advertisement scenarios, and our algorithm can be used to improve the performance of Gurobi.

## Question4:

About the Lagrangian relaxation.

## Response:

Note that the focus ratio are from client's requirements and is in original problem, if we do the Lagrangian relaxation, it is highly possible that the solution is not feasible for the original problem. To explore this method, we conducted the experiment in

table 8 in Q2 link, which shows that the Lagrangian relaxation method is not competent to solve our problem. The time to solve the subproblem is still long regarding to the problem's size and the multilinear objective function. Note that the algorithm design in online advertising must consider the large data size and business timeframe, according to the experiments, our algorithm has much higher performance than the Lagrangian relaxation in business settings.

### **Question5:**

About the knowledge of polyhedral combinatorics.

### **Response:**

we have knowledge of the correspondence on Totally Unimodular matrix and integer polyhedron. But the property relies on whether the optimal solution is on extremes of the polyhedron and there is no evidence for multilinear objective function to have this property since it is not convex.

We thank the reviewer for raising these questions. They are very helpful and we will include the discussions in the updated version of our paper.

### **Concern1:**

About Dataset and Model.

### **Response:**

Due to the large size of the dataset(100GB) and for company privacy, we did not open-source it when submitting the paper. However, after anonymizing and compressing the data through the company's processes, we have now made it available on <https://mega.nz/file/sOIk0byD#ahjRxeZJJjKF6IKK0q-r0nqIT8R7ylg8-k24xitSX2k>. Please kindly refer to it. We model the problem as paper described and put them into .lp file, which as both input for our solver and gurobi.

### **Concern2:**

The IMP model encompasses only three types of constraints: supply, demand, and Focus Ratio. Other relevant constraints, such as budget limitations that are crucial to guaranteed delivery advertising problems, are omitted.

### **Response:**

As the previous papers mentioned in Q1 about the GD advertising problem. It is classical to address the GD Allocation problem on supply and demand, while the budget is the concepts in other online ad scenarios. Even though, the constraint which our algorithm can handle is not limited to supply and demand. It could be easily extended to solve other linear or multilinear constraints.

**Concern3:**

On the sampling w.r.t populations.

**Response:**

We have taken uniformly random samplings among all populations. We will supplement it in updated version.

**Concern4:**

When supply < demand:

**Response:**

Since the problem is **Guaranteed** Delivery, if supply < demand, our preprocessing system will filter this request, so we don't consider this case.