# Appendix of Local Search for Integer Quadratic Programming

**Anonymous submission**

## Appendix A: Detailed Calculation of Inequality Exploration Move Operator

In this section, we discuss the inequality exploration in different situations. For any $x_j \in \mathcal{S}$ and its associated constraint:

$$con_i = H(i, x_j) \cdot x_j + I(i, x_j) \leq c_i.$$

$$\Theta(x_j) = W \cdot x_j^2 + K(x_j) \cdot x_j.$$

$$\xi = \frac{K(x_j)}{-2W}$$

The feasible domain of $x_j$ in $con_i$ is denoted as $\mathcal{D}$ , $\mathcal{D}$ is :

$$\mathcal{D} = \begin{cases} [x_0, +\infty] & \text{if } A = 0, \alpha(H(I, x_j)) < 0, \\ [-\infty, x_0] & \text{if } A = 0, \alpha(H(I, x_j)) > 0, \\ [x_1, x_2] & \text{if } A > 0, \Delta \neq 0 \\ [-\infty, x_1] \cup [x_2, +\infty] & \text{if } A < 0, \Delta \neq 0, \end{cases} \tag{1}$$

We determine a candidate value $x^{\text{min}}$ based on the specified conditions $\text{con}_i$ and $\theta(x_j)$, where $x^{\text{min}}$ is the value in $\mathcal{D}$ that minimizes $\Theta(x)$ (more detailed calculations of $x^{\text{min}}$ are provided in the appendix). This candidate value $x^{\text{min}}$ is then used to find the nearest feasible integer within the domain $\mathcal{D}$ to apply the $iem(x_j, \text{con}_i, \alpha)$ operator. Specifically, $x^{\text{min}}$ is calculated by these four forms of:

### I. Linear constraint, Linear objective function

If $con_i$ is a Linear constraint with $\alpha(H(I, x_j)) = 0$ and $\Theta(x_j)$ is a linear objective function with $W = 0$. $x_{min}$ is:

$$x_{min} = \begin{cases} x_0, & \text{if } D = [x_0, +\infty], \alpha(K(I, x_j)) < 0 \\ x_0, & \text{if } D = [-\infty, x_0], \alpha(K(I, x_j)) < 0 \\ \alpha(x_j), & \text{otherwise.} \end{cases} \tag{2}$$

### II. Quadratic constraint, Linear objective function

If $con_i$ is a Quadratic constraint with $\alpha(H(I, x_j)) \neq 0$ and $\Theta(x_j)$ is a linear objective function with $W = 0$. $x_{min}$ is:

$$x_{min} = \begin{cases} x_2, & \text{if } D = [x_1, x_2] \text{ and } \alpha(K(I, x_j)) < 0, \\ x_1, & \text{if } D = [x_1, x_2] \text{ and } \alpha(K(I, x_j)) > 0, \\ \alpha(x_j), & \text{otherwise.} \end{cases} \tag{3}$$

### III. Linear constraint, Quadratic objective function

If $con_i$ is a Linear constraint with $\alpha(H(I, x_j)) = 0$ and $\Theta(x_j)$ is a Quadratic objective function with $W \neq 0$. $x_{min}$ is:

$$x_{min} = \begin{cases} x_0, & \text{if } D = [x_0, +\infty], W > 0, \xi \leq x_0, \\ \xi, & \text{if } D = [x_0, +\infty], W > 0, \xi > x_0, \\ \xi, & \text{if } D = [-\infty, x_0], W > 0, \xi \leq x_0 \\ x_0, & \text{if } D = [-\infty, x_0], W > 0, \xi > x_0 \\ \alpha(x_j), & \text{otherwise.} \end{cases} \tag{4}$$

### IV. Quadratic constraint, Quadratic objective function

If $con_i$ is a Quadratic constraint with $\alpha(H(I, x_j)) \neq 0$ and $\Theta(x_j)$ is a Quadratic objective function with $W \neq 0$.

If $D = [-\infty, x_1] \cup [x_2, +\infty]$, $x_{min}$ is:

$$x_{min} = \begin{cases} \xi, & \text{if } W > 0, \xi < x_1 \text{ or } \xi > x_2, \\ x_1, & \text{if } W > 0, x_1 \leq \xi \leq x_2, |x_1 - \xi| \leq |x_2 - x_i| \\ x_2, & \text{if } W > 0, x_1 \leq \xi \leq x_2, |x_1 - \xi| > |x_2 - x_i| \\ \alpha(x_j), & \text{otherwise.} \end{cases} \tag{5}$$

if $D = [x_1, x_2]$, $x_{min}$ is

$$x_{min} = \begin{cases} x_1, & \text{if } W > 0, \xi < x_1 \\ x_2, & \text{if } W > 0, \xi > x_1 \\ \xi, & \text{if } W > 0, x_1 \leq \xi \leq x_2 \\ x_1, & \text{if } W < 0, x_1 \leq \xi \leq x_2, |x_1 - \xi| > |x_2 - x_i| \\ x_2, & \text{if } W < 0, x_1 \leq \xi \leq x_2, |x_1 - \xi| \leq |x_2 - x_i| \\ x_1, & \text{if } W < 0, \xi > x_2 \\ x_2, & \text{if } W < 0, \xi < x_1 \\ \alpha(x_j), & \text{otherwise.} \end{cases} \tag{6}$$

## Appendix B: Repetitive running with randomness

For each instance, we calculated the average best-found objective function value (*AVG*) and the standard deviation (*STD*) of the best-found solutions obtained from the 10 different seeds. The coefficient of variation(*COV*) for each instance is determined by *AVG*/*STD*, with a lower value indicating greater stability. The experimental results are summarized in Table 1.

Table 1: Experiment of LS-IQCQP using different seeds, where *#NRGV* represents the number of instances within each range of the *COV*.

| Time Limit | $\#NRGV$ | | | |
|---|---|---|---|---|
| | [0,0.01] | [0.01,0.1] | [0.1,0.5] | [0.5,+$\infty$] |
| 10 Seconds | **133** | 48 | 29 | 11 |
| 60 Seconds | **129** | 54 | 25 | 13 |
| 300 Seconds | **124** | 53 | 29 | 15 |

## Appendix C: Complete results of Sensitivity analysis on the parameters

We tested the algorithm with various parameter settings, as detailed in Table 3.

Table 2: Results of LS-IQCQP under different parameter settings. P12 is the parameters ultimately used by LS-IQCQP.

| Parameter settings | $t$ | $\zeta$ | Total #win (10s) | Total #win (60s) | Total #win (300s) |
|---|---|---|---|---|---|
| P0 | 50 | 50 | 175 | 177 | 180 |
| P1 | 50 | 80 | 177 | 174 | 178 |
| P2 | 50 | 100 | 177 | 176 | 177 |
| P3 | 50 | 200 | 173 | 173 | 180 |
| P4 | 50 | 300 | 172 | 180 | 176 |
| P5 | 80 | 50 | 178 | 171 | 179 |
| P6 | 80 | 80 | 174 | 177 | 174 |
| P7 | 80 | 100 | 180 | 178 | 182 |
| P8 | 80 | 200 | 174 | 171 | 175 |
| P9 | 80 | 300 | 175 | 178 | 178 |
| P10 | 100 | 50 | 174 | 177 | 178 |
| P11 | 100 | 80 | 180 | 180 | 180 |
| **P12** | **100** | **100** | **177** | **176** | **178** |
| P13 | 100 | 200 | 178 | 174 | 175 |
| P14 | 100 | 300 | 173 | 180 | 177 |
| P15 | 200 | 50 | 179 | 179 | 177 |
| P16 | 200 | 80 | 177 | 171 | 176 |
| P17 | 200 | 100 | 179 | 175 | 176 |
| P18 | 200 | 200 | 178 | 174 | 175 |
| P19 | 200 | 300 | 172 | 177 | 174 |
| P20 | 300 | 50 | 177 | 176 | 177 |
| P21 | 300 | 80 | 179 | 177 | 178 |
| P22 | 300 | 100 | 178 | 180 | 176 |
| P23 | 300 | 200 | 179 | 175 | 178 |
| P24 | 300 | 300 | 174 | 176 | 177 |