

Appendix of Local Search for Integer Quadratic Programming

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Appendix A: Detailed Calculation of Inequality Exploration Move Operator

In this section, we discuss the inequality exploration in different situations. For any $x_j \in \mathcal{S}$ and its associated constraint:

$$con_i = H(i, x_j) \cdot x_j + I(i, x_j) \leq c_i.$$

$$\Theta(x_j) = W \cdot x_j^2 + K(x_j) \cdot x_j.$$

$$\xi = \frac{K(x_j)}{-2W}$$

The feasible domain of x_j in con_i is denoted as \mathcal{D} , \mathcal{D} is :

$$\mathcal{D} = \begin{cases} [x_0, +\infty] & \text{if } A = 0, \alpha(H(I, x_j)) < 0, \\ [-\infty, x_0] & \text{if } A = 0, \alpha(H(I, x_j)) > 0, \\ [x_1, x_2] & \text{if } A > 0, \Delta \neq 0 \\ [-\infty, x_1] \cup [x_2, +\infty] & \text{if } A < 0, \Delta \neq 0, \end{cases} \quad (1)$$

We determine a candidate value x^{\min} based on the specified conditions con_i and $\theta(x_j)$, where x^{\min} is the value in \mathcal{D} that minimizes $\Theta(x)$ (more detailed calculations of x^{\min} are provided in the appendix). This candidate value x^{\min} is then used to find the nearest feasible integer within the domain \mathcal{D} to apply the $iem(x_j, con_i, \alpha)$ operator. Specifically, x^{\min} is calculated by these four forms of:

I. Linear constraint, Linear objective function

If con_i is a Linear constraint with $\alpha(H(I, x_j)) = 0$ and $\Theta(x_j)$ is a linear objective function with $W = 0$. x_{min} is:

$$x_{min} = \begin{cases} x_0, & \text{if } D = [x_0, +\infty], \alpha(K(I, x_j)) < 0 \\ x_0, & \text{if } D = [-\infty, x_0], \alpha(K(I, x_j)) < 0 \\ \alpha(x_j), & \text{otherwise.} \end{cases} \quad (2)$$

II. Quadratic constraint, Linear objective function

If con_i is a Quadratic constraint with $\alpha(H(I, x_j)) \neq 0$ and $\Theta(x_j)$ is a linear objective function with $W = 0$. x_{min} is:

$$x_{min} = \begin{cases} x_2, & \text{if } D = [x_1, x_2] \text{ and } \alpha(K(I, x_j)) < 0, \\ x_1, & \text{if } D = [x_1, x_2] \text{ and } \alpha(K(I, x_j)) > 0, \\ \alpha(x_j), & \text{otherwise.} \end{cases} \quad (3)$$

III. Linear constraint, Quadratic objective function

If con_i is a Linear constraint with $\alpha(H(I, x_j)) = 0$ and $\Theta(x_j)$ is a Quadratic objective function with $W \neq 0$. x_{min} is:

$$x_{min} = \begin{cases} x_0, & \text{if } D = [x_0, +\infty], W > 0, \xi \leq x_0, \\ \xi, & \text{if } D = [x_0, +\infty], W > 0, \xi > x_0, \\ \xi, & \text{if } D = [-\infty, x_0], W > 0, \xi \leq x_0 \\ x_0, & \text{if } D = [-\infty, x_0], W > 0, \xi > x_0 \\ \alpha(x_j), & \text{otherwise.} \end{cases} \quad (4)$$

IV. Quadratic constraint, Quadratic objective function

If con_i is a Quadratic constraint with $\alpha(H(I, x_j)) \neq 0$ and $\Theta(x_j)$ is a Quadratic objective function with $W \neq 0$.

If $D = [-\infty, x_1] \cup [x_2, +\infty]$, x_{min} is:

$$x_{min} = \begin{cases} \xi, & \text{if } W > 0, \xi < x_1 \text{ or } \xi > x_2, \\ x_1, & \text{if } W > 0, x_1 \leq \xi \leq x_2, |x_1 - \xi| \leq |x_2 - x_i| \\ x_2, & \text{if } W > 0, x_1 \leq \xi \leq x_2, |x_1 - \xi| > |x_2 - x_i| \\ \alpha(x_j), & \text{otherwise.} \end{cases} \quad (5)$$

if $D = [x_1, x_2]$, x_{min} is

$$x_{min} = \begin{cases} x_1, & \text{if } W > 0, \xi < x_1 \\ x_2, & \text{if } W > 0, \xi > x_1 \\ \xi, & \text{if } W > 0, x_1 \leq \xi \leq x_2 \\ x_1, & \text{if } W < 0, x_1 \leq \xi \leq x_2, |x_1 - \xi| > |x_2 - x_i| \\ x_2, & \text{if } W < 0, x_1 \leq \xi \leq x_2, |x_1 - \xi| \leq |x_2 - x_i| \\ x_1, & \text{if } W < 0, \xi > x_2 \\ x_2, & \text{if } W < 0, \xi < x_1 \\ \alpha(x_j), & \text{otherwise.} \end{cases} \quad (6)$$

Appendix B: Repetitive running with randomness

For each instance, we calculated the average best-found objective function value (*AVG*) and the standard deviation (*STD*) of the best-found solutions obtained from the 10 different seeds. The coefficient of variation(*COV*) for each instance is determined by AVG/STD , with a lower value indicating greater stability. The experimental results are summarized in Table 1.

The results of LS-IQCQP under different seed settings is in Table 2, and we compare the number of win and feas instances with state-of-the-art IQP solvers using the average result over 20 runs under different seed settings in Table 4.

Table 1: Experiment of LS-IQCQP using different seeds, where *#NRGV* represents the number of instances within each range of the *COV*.

Time Limit	#NRGV			
	[0,0.01]	[0.01,0.1]	[0.1,0.5]	[0.5,+∞]
10 Seconds	133	48	29	11
60 Seconds	129	54	25	13
300 Seconds	124	53	29	15

Table 2: Results of LS-IQCQP under different seed settings. Default seed 0 is used by LS-IQCQP.

seed settings	LS-IQCQP:10s		LS-IQCQP:60s		LS-IQCQP:300s	
	#feas	#win	#feas	#win	#feas	#win
Seed: 0	221	177	221	176	221	178
Seed: 1	221	175	221	174	221	180
Seed: 2	221	177	221	177	221	181
Seed: 3	221	180	221	174	221	175
Seed: 4	221	179	221	175	221	172
Seed: 5	221	173	221	178	221	177
Seed: 6	221	179	221	177	221	175
Seed: 7	221	172	221	175	221	180
Seed: 8	221	178	221	171	221	180
Seed: 9	221	175	221	172	221	174
Seed: 10	221	177	221	171	221	176
Seed: 11	221	175	221	174	221	180
Seed: 12	221	177	221	177	221	181
Seed: 13	221	180	221	174	221	175
Seed: 14	221	179	221	175	221	172
Seed: 15	221	173	221	178	221	177
Seed: 16	221	179	221	177	221	175
Seed: 17	221	172	221	175	221	180
Seed: 18	221	178	221	171	221	180
Seed: 19	221	175	221	172	221	174

Appendix C: Complete results of Sensitivity analysis on the parameters

We tested the algorithm with various parameter settings, as detailed in Table 3.

Table 3: Results of LS-IQCQP under different parameter settings. P27 is the parameters ultimately used by LS-IQCQP.

Parameter settings	t	ζ	Total #win (10s)	Total #win (60s)	Total #win (300s)
P0	10	10	170	172	175
P1	10	50	157	154	158
P2	10	80	158	153	162
P3	10	100	153	153	160
P4	10	200	152	160	156
P5	10	300	158	151	159
P6	10	500	169	172	169
P7	10	1000	175	173	177
P8	50	10	169	166	170
P9	50	50	175	178	178
P10	50	80	181	173	175
P11	50	100	180	180	180
P12	50	200	175	177	174
P13	50	300	174	180	176
P14	50	500	168	175	172
P15	50	1000	174	174	172
P16	80	10	172	166	171
P17	80	50	178	172	173
P18	80	80	178	174	175
P19	80	100	172	177	174
P20	80	200	177	176	177
P21	80	300	179	177	178
P22	80	500	173	175	171
P23	80	1000	174	170	173
P24	100	10	169	171	172
P25	100	50	179	175	177
P26	100	80	175	171	180
P27	100	100	177	176	178
P28	100	200	180	175	176
P29	100	300	173	175	182
P30	100	500	169	166	174
P31	100	1000	175	175	170
P32	200	10	173	172	172
P33	200	50	181	176	173
P34	200	80	176	179	180
P35	200	100	173	178	175
P36	200	200	179	173	175
P37	200	300	178	172	173
P38	200	500	173	167	173
P39	200	1000	176	170	177
P40	300	10	167	175	169
P41	300	50	179	178	174
P42	300	80	173	176	182
P43	300	100	179	178	179
P44	300	200	179	174	179
P45	300	300	177	177	176
P46	300	500	176	170	176
P47	300	1000	168	168	177
P48	500	10	170	175	168
P49	500	50	177	176	175
P50	500	80	169	177	176
P51	500	100	172	176	175
P52	500	200	175	169	171
P53	500	300	174	177	178
P54	500	500	171	174	169
P55	500	1000	167	169	168
P56	1000	10	171	170	172
P57	1000	50	171	173	174
P58	1000	80	170	167	175
P59	1000	100	172	175	174
P60	1000	200	169	167	175
P61	1000	300	175	171	175
P62	1000	500	171	167	176
P63	1000	1000	172	175	175

Benchmark	Category	#inst	Gurobi_exact		Gurobi_heur		SCIP		Cplex		Knitro		LS-IQCQP	
			#feas	#win	#feas	#win	#feas	#win	#feas	#win	#feas	#win	#feas	#win
Time Limit 10 Seconds														
QP	QUBO	23	23	8	23	14	23	1	23	3	23	0	23	16
	LCQP	99	99	56	99	73	87	23	99	30	80	10	99	73
	QCLP	10	10	1	10	2	10	1	9	2	10	2	10	7
	QCQP	5	5	0	5	1	5	0	4	0	5	0	5	5
	QUBO	19	19	6	19	13	19	2	19	2	19	0	19	9
	LCQP	52	52	46	52	51	46	45	52	45	45	21	52	51
	QCLP	2	2	2	2	2	2	2	2	2	2	2	2	2
	QCQP	11	11	11	11	11	11	11	11	11	11	11	11	11
Total		221	221	130	221	167	203	85	220	95	195	46	221	175
Time Limit 60 Seconds														
QP	QUBO	23	23	11	23	10	23	2	23	5	23	0	23	18
	LCQP	99	99	64	99	74	98	26	99	37	87	12	99	74
	QCLP	10	10	1	10	5	10	1	10	3	10	3	10	4
	QCQP	5	5	1	5	1	5	0	5	0	5	0	5	5
	QUBO	19	19	10	19	14	19	2	19	3	19	0	19	10
	LCQP	52	52	46	52	52	52	45	52	45	45	21	52	51
	QCLP	2	2	2	2	2	2	2	2	2	2	2	2	2
	QCQP	11	11	11	11	11	11	11	11	11	11	11	11	11
Total		221	221	146	221	169	220	89	221	106	202	49	221	175
Time Limit 300 Seconds														
QP	QUBO	23	23	14	23	10	23	5	23	9	23	0	23	17
	LCQP	99	99	73	99	77	99	30	99	42	88	15	99	76
	QCLP	10	10	2	10	6	10	1	10	3	10	5	10	6
	QCQP	5	5	1	5	3	5	0	5	0	5	0	5	5
	QUBO	19	19	12	19	16	19	5	19	8	19	0	19	10
	LCQP	52	52	46	52	52	52	45	52	45	46	21	52	50
	QCLP	2	2	2	2	2	2	2	2	2	2	2	2	2
	QCQP	11	11	11	11	11	11	11	11	11	11	11	11	11
Total		221	221	161	221	177	221	99	221	120	203	54	221	177

Table 4: Comparison of the number of #win and #feas instances with state-of-the-art IQP solvers using the average result over 20 under different seed settings.

Instance name	variable count	constraint count
QP_9048.lp	202	1
QP_9030.lp	10000	5000
QP_7164.lp	420	225
QP_7159.lp	364	196
QP_7154.lp	312	169
QP_7149.lp	264	144
QP_7144.lp	220	121
QP_7139.lp	180	100
QP_7127.lp	1000	50
QP_6941.lp	2203	315
QP_6799.lp	2075	297
QP_6764.lp	2071	297
QP_6757.lp	2046	297
QP_6647.lp	627	33
QP_6597.lp	600	60
QP_6487.lp	618	309
QP_6324.lp	640	16
QP_5980.lp	150	8381
QP_5971.lp	150	5587
QP_5962.lp	150	2793
QP_5944.lp	100	2475
QP_5935.lp	100	1237
QP_5922.lp	500	0
QP_5909.lp	250	0
QP_5882.lp	150	0
QP_5881.lp	120	0
QP_5875.lp	200	0
QP_5755.lp	400	0
QP_5725.lp	343	0
QP_5721.lp	300	0
QP_3980.lp	235	48
QP_3931.lp	316	80
QP_3923.lp	395	80
QP_3913.lp	300	61
QP_3883.lp	182	1456
QP_3877.lp	630	0
QP_3865.lp	525	50
QP_3860.lp	435	8120
QP_3852.lp	231	0
QP_3850.lp	1225	0
QP_3841.lp	300	4600
QP_3838.lp	780	0
QP_3834.lp	50	1
QP_3832.lp	561	0
QP_3822.lp	861	0
QP_3815.lp	192	64
QP_3803.lp	190	2280
QP_3780.lp	168	72
QP_3775.lp	180	60
QP_3772.lp	380	4560
QP_3762.lp	90	480
QP_3757.lp	552	8096
QP_3752.lp	462	6160
QP_3751.lp	150	50
QP_3750.lp	210	70
QP_3745.lp	325	0
QP_3738.lp	435	0
QP_3714.lp	120	40
QP_3709.lp	600	50

Table 5: Instances detail

Instance name	variable count	constraint count
QP_3706.lp	703	0
QP_3705.lp	378	0
QP_3703.lp	225	30
QP_3693.lp	1128	0
QP_3650.lp	946	0
QP_3642.lp	1035	0
QP_3614.lp	210	44
QP_3587.lp	240	46
QP_3584.lp	528	10912
QP_3565.lp	276	0
QP_3562.lp	63	42
QP_3506.lp	496	0
QP_3413.lp	400	40
QP_3402.lp	144	24
QP_3380.lp	8904	823
QP_3361.lp	1024	64
QP_3347.lp	676	52
QP_3307.lp	256	32
QP_2957.lp	484	44
QP_2880.lp	625	50
QP_2733.lp	324	36
QP_2512.lp	100	20
QP_2492.lp	196	28
QP_2359.lp	306	3264
QP_2357.lp	240	2240
QP_2315.lp	595	13090
QP_2096.lp	300	6925
QP_2087.lp	276	6096
QP_2085.lp	253	5336
QP_2077.lp	231	4642
QP_2073.lp	210	4011
QP_2067.lp	190	3440
QP_2060.lp	171	2926
QP_2055.lp	153	2466
QP_2047.lp	136	2057
QP_2036.lp	324	324
QP_2029.lp	299	299
QP_2022.lp	275	275
QP_2017.lp	252	252
QP_1976.lp	152	152
QP_10074.lp	75	10
QP_10073.lp	75	6
QP_10072.lp	75	10
QP_10071.lp	200	11
QP_10070.lp	200	11
QP_10069.lp	200	10
QP_10068.lp	200	11
QP_10067.lp	200	5
QP_10066.lp	200	11
QP_10065.lp	200	11
QP_10064.lp	200	11
QP_10063.lp	200	5
QP_10062.lp	200	10
QP_10061.lp	200	5
QP_10060.lp	200	10
QP_10059.lp	200	10
QP_10058.lp	200	11
QP_10057.lp	200	11
QP_10056.lp	175	5

Table 6: Instances detail

Instance name	variable count	constraint count
QP_10055.lp	175	5
QP_10054.lp	175	11
QP_10053.lp	150	10
QP_10052.lp	150	6
QP_10051.lp	150	10
QP_10050.lp	150	5
QP_10049.lp	150	10
QP_10048.lp	150	5
QP_10047.lp	150	10
QP_10046.lp	150	6
QP_10045.lp	150	10
QP_10044.lp	150	6
QP_10043.lp	150	10
QP_10042.lp	125	5
QP_10041.lp	125	6
QP_10040.lp	125	6
QP_0752.lp	250	1
QP_0633.lp	75	1
QP_0067.lp	80	1
_st_testph4.lp	3	10
_st_testgr3.lp	20	20
_st_testgr1.lp	10	5
_st_test8.lp	24	20
_st_test6.lp	10	5
_st_test5.lp	10	11
_st_test4.lp	6	5
_st_test3.lp	13	10
_st_test2.lp	6	2
_st_test1.lp	5	1
_st_mi qp3.lp	2	1
_st_mi qp2.lp	4	3
_st_mi qp1.lp	5	1
_prob03.lp	2	1
_prob02.lp	6	8
_pb351575.lp	525	50
_pb351555.lp	525	50
_pb351535.lp	525	50
_pb302075.lp	600	50
_pb302055.lp	600	50
_pb302035.lp	600	50
_nvs24.lp	10	10
_nvs23.lp	9	9
_nvs19.lp	8	8
_nvs18.lp	6	6
_nvs17.lp	7	7
_nvs15.lp	4	1
_nvs13.lp	5	5
_nvs12.lp	4	4
_nvs11.lp	3	3
_nvs10.lp	2	2
_nvs03.lp	3	2

Table 7: Instances detail

Instance name	variable count	constraint count
_Graphpart_3pm-0334-0334.lp	108	36
_Graphpart_3pm-0333-0333.lp	81	27
_Graphpart_3pm-0244-0244.lp	96	32
_Graphpart_3pm-0234-0234.lp	72	24
_Graphpart_3g-0444-0444.lp	192	64
_Graphpart_3g-0344-0344.lp	144	48
_Graphpart_3g-0334-0334.lp	108	36
_Graphpart_3g-0333-0333.lp	81	27
_Graphpart_3g-0244-0244.lp	96	32
_Graphpart_3g-0234-0234.lp	72	24
_Graphpart_2pm-0099-0999.lp	243	81
_Graphpart_2pm-0088-0888.lp	192	64
_Graphpart_2pm-0077-0777.lp	147	49
_Graphpart_2pm-0066-0066.lp	108	36
_Graphpart_2pm-0055-0055.lp	75	25
_Graphpart_2pm-0044-0044.lp	48	16
_Graphpart_2g-1010-0824.lp	300	100
_Graphpart_2g-0099-9211.lp	243	81
_Graphpart_2g-0088-0088.lp	192	64
_Graphpart_2g-0077-0077.lp	147	49
_Graphpart_2g-0066-0066.lp	108	36
_Graphpart_2g-0055-0062.lp	75	25
_Graphpart_2g-0044-1601.lp	48	16
_crossdock_15x8.lp	240	46
_chimera_selby-c8-0nc8-02.lp	507	0
_chimera_selby-c8-0nc8-01.lp	507	0
_chimera_selby-c16-02.lp	2031	0
_chimera_selby-c16-01.lp	2031	0
_chimera_rfr-02.lp	2032	0
_chimera_rfr-01.lp	2032	0
_chimera_mis-02.lp	2032	0
_chimera_mis-01.lp	2032	0
_chimera_mgw-c8-507-0nc8-02.lp	508	0
_chimera_mgw-c8-507-0nc8-01.lp	508	0
_chimera_mgw-c8-439-0nc8-002.lp	440	0
_chimera_mgw-c8-439-0nc8-001.lp	440	0
_chimera_mgw-c16-2031-02.lp	2032	0
_chimera_mgw-c16-2031-01.lp	2032	0
_chimera_lga-02.lp	964	0
_chimera_lga-01.lp	1120	0
_chimera_k64maxcut-02.lp	1145	0
_chimera_k64maxcut-01.lp	1101	0
_chimera_k64ising-01.lp	1192	0
_ball_mk4_15.lp	30	1
_ball_mk4_10.lp	20	1
_ball_mk4_05.lp	10	1
_ball_mk3_30.lp	30	1
_ball_mk3_20.lp	20	1
_ball_mk3_10.lp	10	1
_ball_mk2_30.lp	30	1
_ball_mk2_10.lp	10	1

Table 8: Instances detail