Mission to Mars!
Sutton Trust 2021: Introduction to Aerospace Engineering

9th July, 2021

In this exercise we're going to design a mission to Mars, looking at the engineering principles behind rocket launches and interplanetary travel. The aim of this exercise is to serve as an introduction to some of the topics studied by aerospace engineering students and to be an exercise in problem solving. The questions might be quite challenging, depending on how much maths and physics you've done before, so if you're finding them difficult, don't worry! The point of this is to see how maths can be applied to real world problems by engineers, and get an idea of the kind of stuff you'll learn as an engineering student. After having a go at the questions individually, we'll talk through the solutions in the zoom session and then move on to an online coding exercise based on the topics covered in the sheet, which will allow us to visualise the problems.

Q1 **Lift off.**

To get into orbit a spacecraft needs to gain both altitude and speed, to get above the atmosphere and then stay in orbit once its there. Before heading to Mars, we want to launch into a Low Earth Orbit (LEO, an orbit between 200 and 2000 km altitude above the Earth). In this question we will work out how much fuel we need to get there.

a) The speed required to stay in (a circular) orbit depends on the orbital altitude, and is given by:

$$v = \sqrt{\frac{GM_E}{r}}$$

where $G = 6.67 \times 10^{-11}$ m^3 kg^{-1}s^{-2} is the universal gravitational constant, $M_E = 5.972 \times 10^{24}$ kg is the mass of the Earth, and r is the "orbital radius", the distance from the centre of the Earth to the spacecraft (in metres). How fast does the rocket need to go to be in a circular orbit with altitude 800 km? (HINT: the radius of the Earth is 6370 km).

b) When launching, we need to take into account the rotation of the Earth.

i) Is it better to launch from near the Equator or from Scotland ($\sim 57^\circ$ North), and why?

ii) What is the angular speed of a point on the Equator, in deg/s? What is the angular speed in rad/s? What is the linear speed, in m/s? (HINT: how long does it take the Earth to rotate 360 degrees, how long does it take to rotate 1 degree?)

c) In spaceflight engineering, we often talk about ***delta-v*** (Δv). This is the *change* (or delta) in velocity required to perform launches or manoeuvres in space, and is a measure of the "effort" required to move around in space.

i) What is the minimum Δv required to get into a circular, equatorial orbit of 800 km altitude, when launching from the Equator? Is it better to launch towards the East or the West?

ii) We only want to put enough fuel in our rocket to reach orbit, (where someone is going to come and refuel us). We need to make sure we don't put to much fuel in, because the more fuel we put in our rocket, the more fuel we need, because now our rocket's heavier, so then we need more fuel... etc. This is all described by Tsiolkovsky's Rocket Equation, which is given by:

$$\Delta v = I_{sp} g \ln \frac{m_0}{m_f}$$

I_{sp} is the *specific impulse* of the rocket engine, a measure of how efficiently the engine produces thrust. m_0 is the initial mass of our rocket + fuel, (the wet-mass), and m_f is the final mass (the dry-mass) after all the fuel has been burnt. We have borrowed a Rocketdyne F-1 engine from NASA for our rocket, which gives us a specific impulse of 263 s. If $m_f = 5000$ kg, how much fuel do we need to get into orbit?

iii) Can you think of any ways we can reduce the amount of fuel needed to get into orbit? (How is our launch different to real rockets?)

Q2 **Heading to Mars**

Now that we're in LEO, we want to send our spacecraft onto an interplanetary trajectory and rendezvous with Mars. Although we are in Earth orbit, at the same time we are orbiting the Sun *with* the Earth. This means that rather than just flying in a straight line towards Mars, we need to follow a curved path, due to the Sun's gravity. Timing is everything! Since Earth and Mars are both orbiting the Sun at different rates, we need to work out when to leave Earth so that our spacecraft and Mars both arrive at the same point in space at the same time.

Figure one shows the path we will follow between Earth and Mars. This type of trajectory is called a "Hohmann Transfer". We begin in a circular orbit around the Sun (the same orbit as the Earth). We then perform a manoeuvre which places us on an *elliptical* orbit (Kepler's laws tell us that the orbit of a planet/spacecraft is an ellipse with the Sun at one focus). With this orbit, one edge of our ellipse just touches the Earth's orbit, and the other just touches Mars' orbit. Once we reach Mars, we then perform another manoeuvre to change our orbit to match Mars', and will then be captured by the planet.

a) i) It takes Earth one year to orbit the Sun, how many degrees does Earth move in a single day?

ii) The Martian year is 687 days, how many degrees does Mars move in a single (Earth) day?

iii) If it takes us 259 days to get to Mars following the path in Figure 1, what is the angle θ shown in the diagram? (HINT: how far does Mars travel in 259 days).

iv) Where is Earth in its orbit when we arrive at Mars?

v) When Mars is on the far side of the Sun from the Earth, it's called a "solar conjunction". The next time this will happen will be the 8th October, 2021. How many days after this date do we want to launch?

b) We're now going to calculate our Δv budget, the Δv required for the first manoeuvre that puts us into the elliptical orbit, and the Δv required at Mars rendezvous for capture.

i) The "orbital radius" of Earth (distance from the Sun to the Earth) is 1 "Astronomical Unit" (AU), which is around 1.5×10^{11} m. Mars' orbital radius is 1.5 AU. What is the "semi-major axis" of the ellipse which is our trajectory? (HINT: the semi-major axis of an ellipse is half of its longest diameter, look this up for a diagram.)

ii) What is the semi-major axis of the Earth's orbit? (HINT: what is the longest diameter of a circle? What is the semi-major axis of a circle?)

iii) The speed of a spacecraft following an elliptical orbit changes over time, depending on where in the orbit the spacecraft is. Orbital speed is given by:

$$v^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$$

where $\mu = 1.327 \times 10^{20}$ m^3 s^{-2} for an orbit around the Sun, r is the distance from the Sun to the point of the orbit we are interested in, and a is the semi-major axis of the orbit. What is the orbital speed of our spacecraft before we begin our interplanetary trajectory? (HINT: a is the semi-major axis of the Earth's orbit here, what is r?)

iv) What orbital speed does our spacecraft need to begin travelling to Mars?

v) What is the Δv of our first manoeuvre?

vi) What Δv is required to be captured by Mars on arrival? (HINT: we are changing from the elliptical orbit of our interplanetary trajectory, to a circular orbit the same as Mars.)

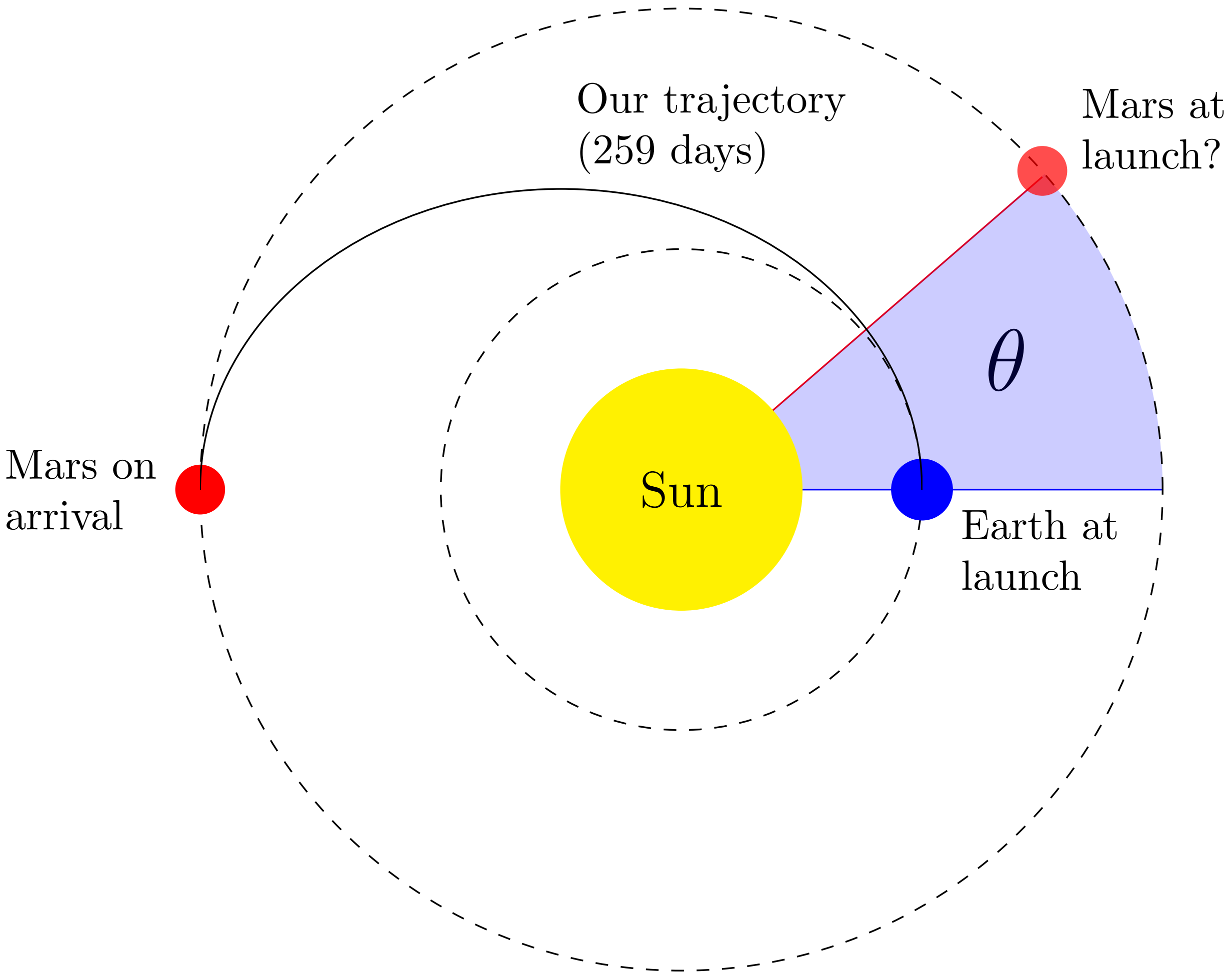

Figure 1: Our trajectory and the relative position of Earth and Mars.