Long-Lived Distributed Relative Localization of Robot Swarms

Alejandro Cornejo
School of Engineering and Applied Sciences
Harvard University
Cambridge MA 02138

Radhika Nagpal
School of Engineering and Applied Sciences
Harvard University
Cambridge MA 02138

***Abstract*—This paper studies the problem of having mobile robots in a multi-robot system maintain an estimate of the relative position and relative orientation of near-by robots in the environment. This problem is studied in the context of large swarms of simple robots which are capable of measuring only the distance to near-by robots.**

We present two distributed localization algorithms with different trade-offs between their computational complexity and their coordination requirements. The first algorithm does not require the robots to coordinate their motion. It relies on a non-linear least squares based strategy to allow robots to compute the relative pose of near-by robots. The second algorithm borrows tools from distributed computing theory to coordinate which robots must remain stationary and which robots are allowed to move. This coordination allows the robots to use standard trilateration techniques to compute the relative pose of near-by robots. Both algorithms are analyzed theoretically and validated through simulations.

I. Introduction

Most tasks which can be performed effectively by a group of robots require the robots to have some information about the relative positions and orientations of other nearby robots. For example in flocking [**?**] robots use the relative orientation of each of its neighbors to control their own heading, in formation control [**?**] robots control their own position as a function of the relative position of their neighbors, and in mapping [**?**] robots use the relative position and relative orientation of their neighbors to interpret the information collected by neighboring robots. However, most of the existing work on localization addresses localization of a single robot, requires landmarks with known positions on the environment, or relies on complex and expensive sensors. Many environments of interest prevent the use of landmarks, and complex and/or costly sensors are not available in swarm platforms, which are composed of large numbers of low-cost robots.

We study the problem of having each robot in a multi-robot system compute the relative pose (position and orientation) of close-by robots relying only on distance estimates to close-by robots. The algorithms described in this paper are fully distributed, and the computations performed at each robot rely only on information available in its local neighborhood. This problem is long-lived, since for any mobile robot, the set of close-by robots and their relative pose changes during the execution. We consider a general problem formulation which does not require explicit control over the motions performed by the robots. This allows composing solutions to this problem with motion-control algorithms to implement different higher-level behaviors. Furthermore, we study this problem in a robot swarm setting, which imposes sensor and computational restrictions on the solutions. The table below summarizes the communication and computational complexity requirements of the two distributed algorithms proposed in this paper.

	Communication	Computation
Algorithm 1	$O(1)$	$O(\varepsilon^{-2})$
Algorithm 2	$O(\Delta)$	$O(1)$

Fig. 1: Communication and computational requirements of the algorithms proposed in this paper. Communication costs are measured per round, and computational costs are per round per robot localized. Δ denotes the maximum degree of the graph, and ε represents the error.

In a typical swarm platform, the communication, computation and sensing capabilities of individual robots are fairly limited. The communication limitations of the individual robots in a swarm platform rule out any strategy that requires collecting large amounts of data at hub locations, and yet, the simplicity of the individual robots demand some form of cooperation. Moreover, the computational constraints of individual robots exclude the possibility of storing and updating complex models of the world or other robots.

Therefore, to fully exploit the potential of a robot swarm platform, it is paramount to use decentralized strategies that allow individual robots to coordinate locally to complete global tasks. This is akin to the behavior observed in swarms of insects, which collectively perform a number of complex tasks which are unsurmountable by a single individual, all while relying on fairly primitive forms of local communication.

A. Related work

The problem of localization using distance sensors has received a lot of attention, most of it focusing on landmark-based localization*. Using only connectivity information to stationary landmarks with known positions [**?**], it is possible to approximate the position of mobile nodes. When distance measurements to the landmarks are available, systems such as the Cricket Location-Support System (which uses ultrasound sensors) can localize mobile nodes within a predefined region, and it has been shown how to obtain finer grained position information using a similar setup [**?**].

The more general case of fixed stationary landmarks with unknown initial positions has also been considered in the literature [**? ?**]. The case where the set of robots that play the role of stationary landmarks changes through an execution has also been considered [**?**], but in contrast to the present work, it requires knowledge of the initial landmark positions and provides no explicit mechanism for coordinating which robots play the role of landmarks.

One of the few landmark-free localization methods is the robust quadrilaterals work [**?**], which is based on rigidity theory. However, in contrast to the present work it is designed primarily for static sensor networks and does not recover the relative orientation of nodes.

B. Road map.

Section II describes the formal system model and problem formulation. Sections III and IV present and analyze two different algorithms for the localization problem. Finally Section V evaluates the performance of these algorithms through simulations.

II. System Model

Let V be a collection of robots deployed in a planar environment. The *pose* (aka kinematic state) of robot $u \in V$ at time $t \in \mathbb{R}^+$ is described by a tuple $pose_{u_t} = \langle p_{v_t}, \phi_{u_t} \rangle$ where $p_{u_t} \in \mathbb{R}^2$ represents the *position* of robot u at time t, and $\phi_{v_t} \in [0, 2\pi)$ represents the *orientation* of robot u at time t. Robots do *not* know their position or orientation.

Each robot has its own local coordinate system which changes as a function of its pose. Specifically, at time t the local coordinate system of robot u has the origin at its own position p_{u_t} and has the x-axis aligned with its own orientation ϕ_{u_t}. All sensing at a robot is recorded in its local coordinate system.

For $\theta \in [0, 2\pi)$ let R_θ and $\psi(\theta)$ denote rotation matrix of θ and a unit vector of angle θ. The position of robot w at time t' in the local coordinate system of robot u at time t is defined as $p_{w_{t'}}|_{u_t} = R_{-\phi_{u_t}}(p_{w_{t'}} - p_{u_t}) = \left\|p_{w_{t'}} - p_{u_t}\right\| \psi(\theta_{w'_t}|_{u_t})$, and the orientation of robot w at time t' in the local coordinate system of robot u at time t is defined as $\phi_{w_{t'}}|_{u_t} = \phi_{w_{t'}} - \phi_{u_t}$. Hence the pose of robot w at time t' in the local coordinate system of robot u at time t is described by the tuple $pose_{w_{t'}}|_{u_t} = \left\langle p_{w_{t'}}|_{u_t}, \phi_{w_{t'}}|_{u_t} \right\rangle$.

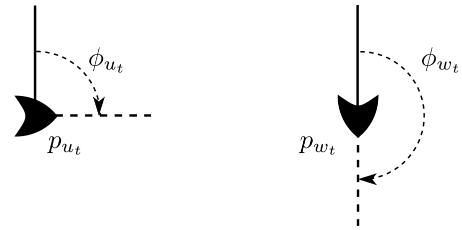


Fig. 2: In the global coordinate system robot u is pointing right and robot w is pointing down. In robot u's local coordinate system robot p_{w_t} is in front of robot p_{u_t}, and in robot w's local coordinate system robot p_{u_t} is to the right of robot p_{w_t}.

The communication graph at time t is a directed graph $G_t = (V, E_t)$, where $E_t \subseteq V \times V$ as a set of of directed edges such that $(u, v) \in E_t$ if and only if a message sent by robot u at time t is received by robot v. The neighbors of robot u at time t are the set of robots from which u can receive a message at time t, denoted by $N_{u_t} = \{v \mid (v, u) \in E_t\}$.

For simplicity and ease of exposition, it is assumed that computation, communication and sensing proceeds in synchronous lock-step rounds $\{1, 2, \ldots\}$. In practice synchronizers [**?**] can be used to simulate perfect synchrony in any partially synchronous system. If robot u receives a message from robot w at round i then robot u can identify the message originated from w, and estimate the distance $\|p_{v_i} - p_{w_i}\| = d_i(u, w)$†.

Robots are capable of using odometry to estimate their pose change between rounds in their own local coordinate system. Specifically at round j a robot $u \in V$ can estimate its translation change $p_{u_i}|_{u_j}$ with respect to round $i < j$ and its orientation change $\phi_{u_i}|_{u_j}$ with respect to round $i < j$. It is assumed that odometry estimates are reliable over intervals of two or three rounds (i.e. $i >= j - 3$), but suffer from drift over longer time intervals.

A. Problem Formulation

Formally, the problem statement requires that at every round i, each robot u computes the relative pose $pose_{w_i}|_{u_i}$ of every neighboring robot $w \in N_{u_i}$. Robots can only perceive each other through distance sensing. For a robot u to compute the pose of a neighboring robot w at a particular round, it must rely on the distance measurements and communication graph in the previous rounds, as well as the odometry estimates of u and w in previous rounds.

* Landmark-based localization assumes the environment contains a set of landmarks with known positions, and to which the robots can measure either the distance or the angle. GPS is an example of this type of localization, where satellites on known orbits play the role of landmarks. It is also common for a subset of robots or sensors with known positions to be the landmarks.

† Many swarm of platforms, including the Kilobots[**?**], use the same hardware (i.e., infrared transceivers) as a cost-effective way to implement both communication and sensing.

The algorithms considered do *not* require controlling the motion performed by each robot, which allows these algorithms to be run concurrently with any motion control algorithm. Moreover, the algorithms are tailored for large swarms of simple robots, and as such the size of the messages or the computation requirements do not depend on global parameters such as the size or diameter of the network.

III. Localization without Coordination

This section describes a distributed localization algorithm that requires no motion coordination between robots and uses minimal communication. Each robot localizes its neighbors by finding the solution to a system of non-linear equations. For simplicity, this section assumes that distance sensing and odometry estimation is perfect (e.g. noiseless). Section V describes how the results presented here can be easily extended to handle noisy measurements.

Consider any pair of robots a and b for a contiguous interval of rounds $I \subset \mathbb{N}$. To simplify notation let $p_{a_j \to b_k} = p_{b_k} - p_{a_j}$ denote the vector, in the global coordinate system, that starts at p_{a_j} and ends at p_{b_k}.

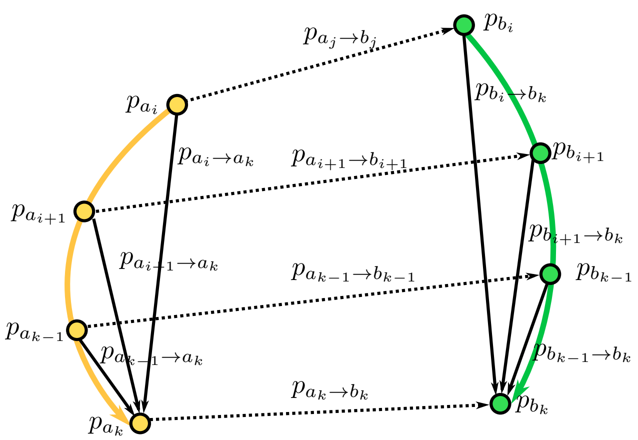


Fig. 3: Robot a and b in rounds $I = \{i, \ldots, k\}$.

Observing Fig. 3 it is easy to see that starting at p_{a_i} (and in general starting at any p_{a_j} for some $j < k$) there are at least two ways to arrive to p_{b_k}. For instance, by first traversing a dotted line and then a solid line or vice versa. Indeed, this holds since by definition for all $j \leq k$ we have:

$$p_{a_j \to a_k} + p_{a_k \to b_k} = p_{a_j \to b_k} = p_{a_j \to b_j} + p_{b_j \to b_k}. \tag{1}$$

For $j = k$ the equation 1 is vacuously true, and for $j < k$ this equation can be massaged to express a constraint on the relative pose of robots a and b in terms of quantities that individual robots can either sense or compute.

$$\begin{aligned}
p_{a_j \to a_k} - p_{b_j \to b_k} + p_{a_k \to b_k} &= p_{a_j \to b_j} \\
-R_{\phi_{a_k}} p_{a_j}|_{a_k} + R_{\phi_{b_k}} p_{b_j}|_{b_k} + R_{\phi_{a_k}} p_{b_k}|_{a_k} &= R_{\phi_{a_j}} p_{b_j}|_{a_j} \\
p_{a_j}|_{a_k} + R_{\phi_{b_k}|_{a_k}} p_{b_j}|_{b_k} + p_{b_k}|_{a_k} &= R_{\phi_{a_j} - \phi_{a_k}} p_{b_j}|_{a_j} \\
\left\| p_{a_j}|_{a_k} + R_{\phi_{b_k}|_{a_k}} p_{b_j}|_{b_k} + p_{b_k}|_{a_k} \right\| &= \left\| p_{b_j}|_{a_j} \right\| \\
\left\| -p_{a_j}|_{a_k} + R_{\phi_{b_k}|_{a_k}} p_{b_j}|_{b_k} + d_i(a,b)\psi(\theta_{b_k}|_{a_k}) \right\| &= d_j(a,b)
\end{aligned} \tag{2}$$

Dissecting equation 2; $d_j(a,b)$ and $d_k(a,b)$ are known and correspond to the estimated distance between robot a and b at round j and k respectively; $p_{a_j}|_{a_k}$ and $p_{b_j}|_{b_k}$, are also known, and correspond to the odometry estimates from round j to round k taken by robot a and b respectively; finally $\phi_{b_k}|_{a_k}$ and $\theta_{b_k}|_{a_k}$ are both unknown and correspond to the relative position and orientation of robot b at round k in the local coordinate system of robot a at round k.

Considering equation 2 over a series of rounds yields a non-linear system that, if well-behaved, allows a robot to estimate the relative pose of another. The following distributed algorithm leverages the constraints captured by this system to allow every robot to compute the relative pose of its neighbors.

Algorithm 1 Localization without Coordination

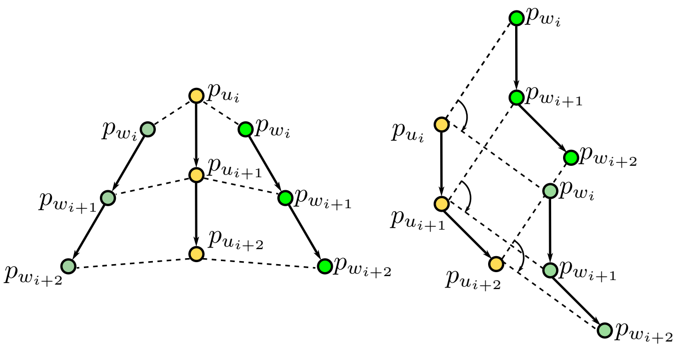
1: **for each** robot $u \in V$ and every round $k \in \{1, \ldots\}$ **do**
2: **broadcast** $\langle p_{u_{k-1}}|_{u_k}, \phi_{u_{k-1}}|_{u_k} \rangle$
3: **receive** $\langle p_{w_{k-1}}|_{w_k}, \phi_{w_{k-1}}|_{w_k} \rangle$ for $w \in N_{u_k}$
4: $I = \{k - \delta, k\}$
5: **for each** $w \in \bigcap_{j \in I} N_{u_j}$ **do**
6: integrate odometry $p_{u_j}|_{u_k}, \phi_{u_j}|_{u_k}$ for $j \in I$
7: find $\hat{\theta}_{w_k}|_{u_k}, \hat{\phi}_{w_k}|_{u_k}$ such that (2) holds $\forall j \in I$
8: $pose_{w_k}|_{u_k} \leftarrow \left\langle d_k(u,w)\psi(\hat{\theta}_{w_k}|_{u_k}), \hat{\phi}_{w_k}|_{u_k} \right\rangle$

At each round of Algorithm 1 every robot sends a constant amount of information (its odometry measurements for that round) and therefore its message complexity is $O(1)$. The computational complexity of Algorithm 1 is dominated by solving the system of non-linear equations (line 7), which can be done by numerical methods [?] in $O(\varepsilon^{-2})$ where ε is the desired accuracy.

The parameter δ in (line 4) of Algorithm 1 corresponds to the number of rounds over which equation 2 is considered. Since there are two unknowns then to avoid an undetermined system it must be required that $\delta \geq 2$, and it will be shown that in practice $\delta = 2$ suffices.

Regardless of the choice of δ there are motion patterns for which any algorithm that does not enforce a very strict motion coordination (which includes Algorithm 1, which enforces no motion coordination) cannot recover the relative pose of neighboring robots. These motions are referred to as *degenerate*, and are described next (see

Fig. 4). First, if during δ rounds two robots follow a linear trajectory, then the relative pose between these robots can only be recovered up to a *flip ambiguity*. Second, if during δ rounds one robot follows a displaced version of the trajectory followed by another robot, then it is possible to infer the relative orientation of the robots, but a *rotation ambiguity* prevents the recovery of the relative position. A degenerate motion can be a flip ambiguity, a rotation ambiguity, or a combination of both.



(a) Flip Ambiguity (b) Rotation Ambiguity

Fig. 4: Yellow robot cannot fully resolve the relative position of green robot using the available distance measurements due to degenerate motions.

Fortunately degenerate motions are rare. More precisely degenerate motions are a set of measure zero (for example, this implies that if the motions are random, then with probability 1 they are not degenerate). This can be shown to be a consequence of the generic rigidity of a triangular prism in Euclidean 2-space, see [**?**] for a thorough treatment of rigidity. We conclude this section with the following theorem, which formalizes the properties of Algorithm 1.

Theorem 1. *If at round i, robots u and w have been neighbors for a contiguous interval of δ or more rounds, and perform non-degenerate motions, then at round i Algorithm 1 computes $pose_{w_i}|_{u_i}$ at u and $pose_{u_i}|_{w_i}$ at w.*

IV. Localization with Coordination

This section describes a distributed localization algorithm that uses a simple stop/move motion coordinate scheme, and requires communication proportional to the number of neighbors. Using the aforementioned motion coordination scheme allows robots to compute the relative pose of neighboring robots through trilateration with no sensing errors. Section V generalizes this to consider noise.

By collecting multiple distance estimates a moving robot can use trilateration to compute the relative position of a stationary robot. Two such distance estimates already suffice to allow the moving robot to compute the relative position of a stationary robot up to a flip ambiguity (i.e., a reflection along the line that passes through the coordinates at which the measurements were taken).

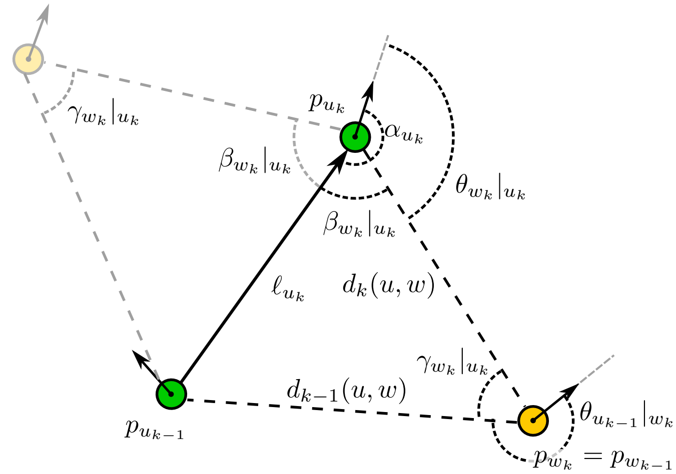


Fig. 5: Moving robot (green) uses trilateration to compute the relative position of stationary robot (yellow) up to a flip ambiguity.

Consider two neighboring robots u and w where from round $k-1$ to round k robot u moves while robot w remains stationary (see Fig. 5). Robot u can compute the relative position $p_{w_k}|_{u_k}$ of robot w at round k up to a flip ambiguity, relying only on the distance measurements to robot w at round $k-1$ and round k, and its own odometry for round k. Specifically the cosine yields the following.

$$\ell_{u_k} = \left\|p_{u_{k-1}}|_{u_k}\right\| \qquad \alpha_{u_k} = \measuredangle(p_{u_{k-1}}|_{u_k})$$

$$\beta_{w_k}|_{u_k} = \cos^{-1}\left(\frac{\ell_{u_k}^2 + d_k^2(u,w) - d_{k-1}^2(u,w)}{2\ell_{u_k} d_k(u,w)}\right) \tag{3}$$

$$\gamma_{w_k}|_{u_k} = \cos^{-1}\left(\frac{d_k^2(u,w) + d_{k-1}^2(u,w) - \ell_{u_k}^2}{2d_k(u,w)d_{k-1}(u,w)}\right) \tag{4}$$

$$\theta_{w_k}|_{u_k} = \alpha_{u_k} \mp \beta_{w_k}|_{u_k} \tag{5}$$

$$\theta_{u_k}|_{w_k} = \theta_{u_{k-1}}|_{w_k} \pm \gamma_{w_k}|_{u_k} \tag{6}$$

In order for robot u to fully determine the relative pose of robot w at round k (ignoring the flip ambiguity) it remains only to compute $\phi_{w_k}|_{u_k}$. Observe that given knowledge of $\theta_{u_{k-1}}|_{w_k}$, robot u can leverage Eq. 6 to compute $\theta_{u_k}|_{w_k}$ using the correction term $\gamma_{w_k}|_{u_k}$ computed through the cosine law. The following identity can be leveraged to easily recover $\phi_{w_k}|_{u_k}$ using $\theta_{u_k}|_{w_k}$ and $\theta_{w_k}|_{u_k}$.

$$\phi_{u_k}|_{w_k} = \theta_{w_k}|_{u_k} - \theta_{u_k}|_{w_k} + \pi \pmod{2\pi} \tag{7}$$

Summing up, if robot u moves from round $k-1$ to round k while robot w remains stationary, then using $d_{k-1}(u,w)$, $d_k(u,w)$ and $p_{u_{k-1}}|_{u_k}$ robot u can compute the relative position of robot w at time k. Additionally, if knowledge of $\theta_{u_{k-1}}|_{w_k}$ is available robot u can also compute the relative orientation of robot w at time k. Both the position and orientation are correct up to a flip ambiguity.

A robot can resolve the flip ambiguity in position and orientation by repeating the above procedure and checking for consistency of the predicted position and orientation. We refer to motions which preserve symmetry and therefore prevent the flip ambiguity from being resolved (for instance, collinear motions) as degenerate.

(a) Flip Ambiguity (b) Unambiguous

Fig. 6: Moving robot (yellow) localizing a stationary robot (green) using distance measurements (dashed lines) and odometry (solid arrows).

To bootstrap the previous trilateration procedure and allow robot u to recover the orientation of robot w, robot w —which remains stationary from round $k-1$ to round k— must somehow compute $\theta_{u_{k-1}}|_{w_{k-1}} = \theta_{u_{k-1}}|_{w_k}$ and communicate it to robot u by round k.

Note that the distance measurements between a stationary robot and a moving robot are invariant to rotations of the moving robot around the stationary robot. This prevents a stationary robot from recovering the relative position of a moving neighbor using any number of distance estimates.

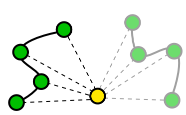


Fig. 7: Stationary robot (yellow) cannot compute the relative position of the moving robot (green), since all distance measurements (dashed lines) are invariant to rotations around the stationary robot.

Therefore to successfully use the aforementioned trilateration procedure requires coordinating the motion of the robots in a manner that gives every robot a chance to move and ensures that when a robot is moving its neighbors remain stationary. Formally, a *motion-schedule* is an algorithm that at each round classifies every robots as being either mobile or stationary. A motion-schedule is *well-formed* if at every round i the set of robots classified as mobile define an independent set of the communication graph G_i (i.e. no two mobile robots are neighbors). The *length* of a motion-schedule is the maximum number of rounds that any robot must wait before it is classified as mobile. A motion-schedule is *valid* if it is well-formed and has finite length.

The validity of a motion-schedule ensures that mobile robots can use trilateration to compute the relative positions of all its neighbors, and having a motion-schedule of finite length guarantees every robot gets a chance to move. The next subsection provides a description of a distributed algorithm that produces a valid motion-schedule. Algorithm 2 describes a distributed localization algorithm that leverages a valid motion-schedule and trilateration.

Algorithm 2 Localization with Coordination

$\Theta_{u_0} \leftarrow \emptyset \; \forall u \in V$
for each robot $u \in V$ and every round $k \in \{1, \ldots\}$ **do**
  broadcast $\langle p_{u_{k-1}}|_{u_k}, \phi_{u_{k-1}}|_{u_k}, \Theta_{u_{k-1}} \rangle$
  receive $\langle p_{w_{k-1}}|_{w_k}, \phi_{w_{k-1}}|_{w_k}, \Theta_{u_{k-1}} \rangle$ for $w \in N_{u_k}$
  if state = mobile **then**
    $\Theta_{u_k} \leftarrow \left\{ \hat{\theta}_{w_k}|_{u_k} \text{ through Eq. (4-5)} \right\}$
    $\hat{\phi}_{w_k}|_{u_k} \leftarrow$ use Eq. (6-7) $\forall w \in N_{u_k}$
    use previous state resolve *flip* in Θ_{u_k}
  else
    update Θ_{u_k} through $\phi_{w_k - 1}|_{w_k}, p_{w_{k-1}}|_{w_k}$
  $\hat{pose}_{w_k}|_{u_k} \leftarrow \langle d_k(u,w)\psi(\hat{\theta}_{w_k}|_{u_k}), \hat{\phi}_{w_k}|_{u_k} \rangle \; \forall w \in N_{u_k}$
  state $\leftarrow$ MOTION-SCHEDULER
  if state = mobile **then**
    move according to MOTION-CONTROLLER
  else
    remain stationary

At each round of Algorithm 2 every robot sends a message containing its own odometry estimates and $\Theta_{u_{k-1}}$, which is the set of previous position estimates (one for each of its neighbor), and therefore its message complexity is $O(\Delta)$. Mobile robots use trilateration to compute the relative position and relative orientation of its neighbors, and when possible stationary robots update the relative position and orientation of mobile robots using the received odometry estimates. In either case, the amount of computation spent by Algorithm 2 to localize each robot is constant.

Theorem 2. *(Assuming a valid motion-schedule.) If at round i, robots u and w have been neighbors for a contiguous set of rounds during which robot u performed a non-degenerate motion, then at round i Algorithm 2 computes $pose_{w_i}|_{u_i}$ at u.*

A. Motion Scheduling

As a straw-man distributed algorithm that requires no communication and outputs a valid motion-schedule, consider an algorithm that assigns a single mobile robot to each round, in a round robin fashion (i.e. at round i let robot $k = i \mod n$ be mobile and let the remaining $n-1$ robots be stationary). Although the motion-schedule produced by such an algorithm is valid, it is not suited for a swarm setting, since it exhibits no parallelism and the time required for a robot to move is linear on the number of robots.

Finding a motion-schedule that maximizes the number of mobile robots at any particular round is tantamount to finding a maximum independent set (aka MaxIS) of the communication graph, which is NP-hard. Similarly, finding a motion-schedule with minimal length implies finding a vertex-coloring with fewest colors of the communication graph, which is also NP-hard.

Algorithm 3 describes a motion-schedule with the more modest property of having the set of moving robots at each round define a maximal independent set (aka MIS) of the communication graph. Once a robot is classified as being mobile, it does not participate on subsequent MIS computations, until each of its neighbors has also been classified as mobile. Given these properties, it is not hard to show that for any robot u and a round k, the number of rounds until robot u is classified as mobile is bounded by the number of neighbors of robot u at round k.

Algorithm 3 Motion-Scheduler

1: **if** $\forall w \in N_u$ state$_w$ = inactive **then**
2:     state$_u \leftarrow$ compete
3: **if** state$_u$ = compete **then**
4:     **if** u is selected in distributed MIS **then**
5:         state$_u \leftarrow$ inactive
6:         **output** mobile
7: **output** stationary

Theorem 3. *Algorithm 3 defines a valid motion-schedule with length* $\Delta + 1$.

The description of Algorithm 3 utilizes a distributed MIS algorithm as a subroutine (line 4 in the pseudo-code). However, it should be noted that the problem of finding an MIS with a distributed algorithm is a fundamental symmetry breaking problem and is far from trivial. Fortunately, the MIS problem has been studied extensively by the distributed computing community, and extremely efficient solutions have been proposed under a variety of communication models [**?** **?** **?**]. The classic solution [**?**] requires $O(\log n)$ communication rounds and every node uses a total of $O(\log n)$ [**?**] bits of communication. For a wireless network settings, it is known [**?**] how to find an MIS exchanging at most $O(\log^* n)$[‡] bits. Due to lack of space, for the purposes of this paper it should suffice to know that it is possible to implement a distributed MIS protocol in the lower communication layers without significant overhead.

V. Algorithm Evaluation

To evaluate the performance of the proposed localization algorithms, this section considers a generalization of the system model described in Section II where the distance estimates and the odometry estimates are subject to noise from an independent zero-mean distribution. In particular we assume multi-variate Gaussian noise with zero-mean and covariance matrix $\Sigma = diag(\sigma_d, \sigma_\phi, \sigma_x, \sigma_y)$.

Algorithm 1 relied on finding a zero in a non-linear system of equations constructed using the distance estimates and odometry estimates pertinent to that robot. When these estimates are subject to noise, the corresponding non-linear system is no longer guaranteed to have a zero. To cope with noisy measurements it suffices to instead look for the point that minimizes the mean-squared error. This incurs in no additional computational overhead, since it can be accomplished using the same numerical methods used in the noiseless case.

To understand the sensitivity of Algorithm 1 to the various sources of error (distance estimates, orientation odometry and translation odometry), we carried out extensive simulations. For each simulation trial robots are deployed randomly in a region of 10m x 10m, and at each round each robot is allowed to perform a motion with a random orientation change between $[-\pi/4, \pi/4]$ and a translation change which is normally distributed with a mean of 3m and a variance of 0.5m. The length of each trial is 20 rounds. The plots below show the mean squared error (MSE) in the computed position (blue) and orientation (red) over 50 random trials for various different noise parameters. Since to initialize the position and orientation estimates Algorithm 2 requires at least three rounds, the first three rounds of every trial were discarded.

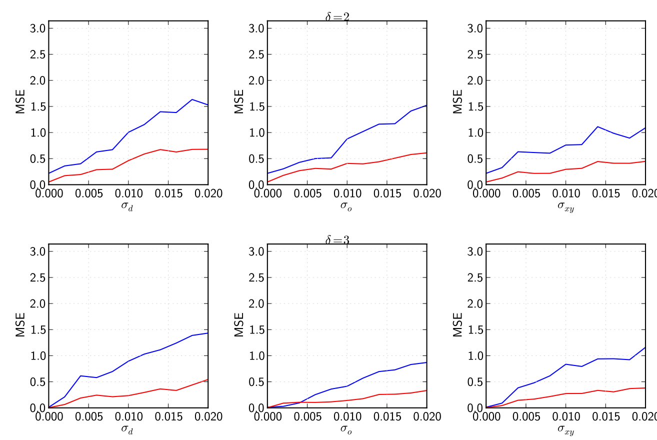


Fig. 8: Each plot shows MSE of the position (blue) and orientation (red) as a function of one component of the variance Σ. From left to right, each column shows the MSE as a function of σ_d, σ_o and $\sigma_{x,y}$. The top row shows the results with $\delta = 2$ and the bottom row for $\delta = 3$.

Not surprisingly the results produced by Algorithm 1 are sensitive to errors in all axis, although it is slightly more robust to errors in the translation odometry than in the distance sensing. Furthermore, the relative orientation estimate was consistently more tolerant to noise than the position estimate. As it would be expected, for all the different noise settings, increasing the parameter δ from

[‡]The iterated logarithm function counts the number of times the logarithm is applied to the argument before the result is less or equal to 1. It is an extremely slowly growing function, for instance the iterated logarithm of the number of atoms in the universe is less than 5.

2 to 3 consistently reduced the MSE in both position and orientation produced by Algorithm 1. However, increasing δ also increases the computational costs of the algorithm and only gives diminishing returns.

In the case of Algorithm 2, to perform trilateration using estimates subject to zero-mean noise corresponds we instead perform trilateration using the expected value of the estimates conditioned on the information available.

To understand the sensitivity of Algorithm 2 to the different sources of error, we used the same simulation environment and parameters as with Algorithm 1, with one exception. Namely, to keep the number of motions per trial for Algorithm 1 and Algorithm 2 roughly the same, the length of the trial was doubled, since at each round, for every pair of nodes, only one of them will be mobile and the other will remain stationary.

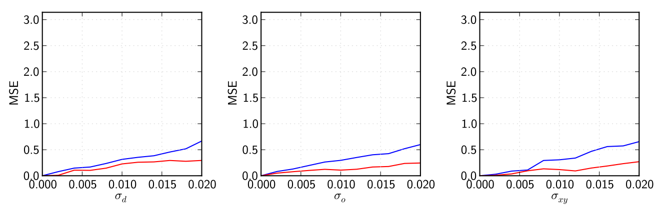


Fig. 9: Each plot shows MSE of the position (blue) and orientation (red) as a function of one component of the variance Σ. From left to right, each column shows the MSE as a function of σ_d, σ_o and $\sigma_{x,y}$.

The pose estimates produced by Algorithm 2 are for the most part equally affected by noise in either of the dimension. As it was the case with Algorithm 1, the relative orientation estimate was consistently more tolerant to noise than the position estimate. Overall compared to Algorithm 1, the results show that Algorithm 2 is in all respects less sensitive to noise.

A. Motion Control and Localization

To conclude we empirically explore the feasibility of composing existing motion control algorithms with the proposed localization algorithms. For its simplicity we consider the canonical problem of flocking [**?**]. Informally, flocking describes an emergent behavior of a collection of agents with no central coordination that move cohesively despite having no common a priori sense of direction.

Flocking behavior has received a lot of attention in the scientific community. Vicsek et al. [**?**] studied flocking from a physics perspective through simulations. The work of Vicsek et al. focused on the emergence of alignment in self-driven particle systems. Flocking has also been studied from a control theoretic perspective, for example in the work of Olfati-Saber [**?**] and Jadbabaie et al. [**?**], where the emphasis is on the robustness of the eventual alignment process despite the local and unpredictable nature of the communication.

For the purposes of this section we consider the standard and most simplistic flocking behavior, where each robot aligns its heading with its neighbors. Namely, at each round every robot steers its own orientation to the average orientation of its neighbors. It has been shown [**? ?**] that under very mild assumptions of the connectivity of the communication graph, the following procedure converges to a state where all robots share the same orientation.

Fig. 10 (on the following page) shows the results of the described average-based flocking algorithm when combined with Algorithm 1 to provide relative orientation estimates. Initially the first rounds the robots move erratically while the position and orientation estimates are initialized, and soon after the orientations of all the robots converge. Increasing the error in the distance sensing and odometry measurements is translated in greater inaccuracy in the resulting relative orientation estimates, which affects the resulting flocking state.

VI. Conclusions

We presented two distributed algorithms to solve the relative localization problem tailored for swarms of simple robots. The algorithms have different communication and computational requirements, as well as different robustness to sensing errors. Specifically, having greater communication and coordination allows us to reduce the required computational complexity and increase the robustness to sensing errors. In future work, we hope to further whether this trade-off is inherent to the problem or not.

Fig. 10: Final configuration of 6 robots after four 40 round runs of a flocking algorithm composed with Algorithm 1 to provide relative position and relative orientation estimates. All runs have the same initial random configuration. From left to right the variance of all noise parameters was increased.