1) A number y is an additive inverse of x if x+y=0. Suppose you want to prove that all real numbers have additive inverses. What quantifiers on the variables are understood?

   a. ∀x∀y, where the universe for x and y is the set of all real numbers
   b. ∃x∀y, where the universe for x and y is the set of all real numbers
   c. ∀x∃y, where the universe for x and y is the set of all real numbers
   d. ∃x∃y, where the universe for x and y is the set of all real numbers

C) [CORRECT] For all **x** (in real numbers) there exists a **y** that is the additive inverse of x. Therefore by definition the ∀x∃y quantifier is understood.

2) Which of the following is the negation of the following statement: "Everyone in the class except Lee has a computer".

   a. Someone in the class other than Lee does not have a laptop computer or Lee has a laptop computer
   b. Lee and someone else in the class have a laptop computer
   c. Lee is the only student in the class with a laptop computer
   d. Someone in the class other than Lee does not have a laptop computer and Lee does not have a laptop computer

A) [CORRECT] The given statement is a conjunction: all students in the class (other than Lee) have laptop computers AND Lee does not have a laptop computer. This statement has the form $p \wedge q$. The negation has the form $\neg p \vee \neg q$. The negation $\neg q$ is "Lee has a laptop computer". However, the statement p has a universal quantifier; its negation states that "it is false that all students in the class (other than Lee) have a laptop computer," which is equivalent to "someone in the class (other than Lee) has a laptop computer." Therefore the negation of the given statement is "Someone in the class other than Lee does not have a laptop computer, or Lee has a laptop computer."

3) Which of the following is the negation of ∀x (P(x)→Q(x))?

   a. ∃x(P(x)→Q(x))
   b. ∃x(P(x) ∧ ¬Q(x))
   c. ∃x(¬P(x)→ ¬Q(x))
   d. ∃x(¬P(x) ∧ Q(x))

B) [CORRECT] Negating a statement with a universal quantifier yields a statement of the form $\exists x \neg(P(x) \rightarrow Q(x))$. To obtain the negation of $P(x) \rightarrow Q(x)$, rewrite $P(x) \rightarrow Q(x)$ as $\neg P(x) \vee Q(x)$ and use one of De Morgan's laws:

$$\neg(P(x) \rightarrow Q(x)) \equiv \neg(\neg P(x) \vee Q(x)) \equiv P(x) \wedge \neg Q(x).$$

4) The negation of ∀x∃y∀z Q(x,y,z) is:

   a. ¬ (∀x∃y∀z ¬Q(x,y,z))
   b. ∃x∀y∃z ¬Q(x,y,z)
   c. ∀x∃y∀z ¬Q(x,y,z)

B) [CORRECT] You need to use the rules for negating quantified statements three times:

$$\neg(\forall x \exists y \forall z\, Q(x,y,z)) \equiv \exists x \neg(\exists y \forall z\, Q(x,y,z)) \equiv \exists x \forall y \neg(\forall z\, Q(x,y,z)) \equiv \exists x \forall y \exists z \neg Q(x,y,z).$$

5) Express the following statement in symbols:

"Every Mathematics Major is taking a Computer Science course,"

using the following: M (x) is the statement "x is a Mathematics Major", C(y) is the statement "y is a Computer Science course", T (x, y) is the statement "x is taking y", the universe for x is the set of all students, and the universe for y is the set of all courses.

a. ∀x∃y[M (x) → (C(y) ∧ T(x, y))]
b. ∀x∃y[M (x) ∧ C(y) ∧ T(x, y)]
c. ∀x ∃y[M (x) → T(x,C(y))]
d. ∃y∀x[M(x) → (T(x, y) ∧ C(y))]
e. ∀y∃x(M (x) ∧ C(y) ∧ T(x, y))

A) [CORRECT] This response says that if x is any Mathematics Major, then there is a Computer Science course that x is taking. This is the given statement.

6) Consider the statement

∀x ∃y(M (x) —> C(x, y))

where M(x) means "x is a Mathematics Major", C(x, y) means "x completed y", the universe for x consists of all students and the universe for y consists of all computer projects. Which of these statements is the English translation of the statement?

a. There is a computer project that every Mathematics Major completed.
b. Every Mathematics Major completed every computer project.
c. Every Mathematics Major completed at least one computer project.
d. Some Mathematics Major failed to complete the entire set of computer projects.
e. Every computer project was completed by at least one Mathematics Major.

C) [CORRECT] The original statement says that no matter what Mathematics Major is selected, that person has completed at least one computer project.

7) Which of these statements says that "Every number has exactly one additive inverse."? Assume that the universe for all variables consists of all real numbers

a. ∀x ∃y ∀z [(x+ y= 0) ∧ ((x+ z= 0) → (y= z))]
b. ∀x∀y∃z(x+ y= x+ z= 0)
c. ∀x ∃y(x + y= 0)
d. ∀x∃y∃z[(x+ y= 0) ∧ (x+ z= 0)]

A) [CORRECT] The first part of the predicate says that every number x has an additive inverse y. The second part says that if it also happens that $x + z = 0$, then the number z must be the same as the number y. This says that x has exactly one additive inverse.

8) Which of these statements is the negation of the following statement

∀x∃y(P(x, y) ∧ (∃z R(x, y, z)))

a. $\forall x \exists y(\neg P(x, y) \lor \exists z(\neg R(x, y, z)))$
b. $\exists x \forall y(\neg P(x, y) \lor \forall z(\neg R(x, y, z)))$
c. $\exists x \forall y(P(x, y) \lor \forall z R(x, y, z))$
d. $\forall x \exists y(P(x, y) \land \forall z(\neg R(x, y, z)))$