# Notes Week 1 : Propositional Logic

# Propositional Logic

Statement T, F

propositional variable : $p, q, r, \ldots$

6 logical connectives : $\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \oplus$

difference $\vee$ and $\oplus$ : $T \vee T$ is true, $T \oplus T$ is false

implication : $p \rightarrow q$ is false iff. p is T and q is F

"1+1=3" $\rightarrow$ "the world is flat"

p is sufficient for q , q is necessary for p

p iff q , $p \leftrightarrow q$ , p is nec. & suff. for q

⚠ "if you register for the course, then you can take the exam"

precendence : $p \wedge q \rightarrow r$ $\qquad (\neg p) \wedge q$

$(p \wedge q) \rightarrow r \qquad p \wedge (q \rightarrow r)$

tautology, contradiction, contingency, satisfiability

logical equivalence : $p \equiv q$ iff. $p \leftrightarrow q$ is a tautology

used in proofs !

Important equivalences to remember:

$$p \to q \equiv \neg q \to \neg p \quad \text{(contrapositive)}$$

$$p \to q \equiv \neg p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q \quad \text{(De Morgan)}$$

associativity, commutativity $(p \wedge q) \wedge r$

distributivity

$$p \to q \not\equiv q \to p \quad \text{(converse)}$$

1. equivalence proofs
2. truth tables

Interesting facts:

- 3 connectives are enough: $\wedge, \vee, \neg$
- every prop. stat. can be converted to DNF or CNF

DNF $\underline{(p_1 \wedge \dots \wedge \neg p_n)} \vee (p_1 \wedge \dots p_n) \dots$

CNF $\vee \quad \vee \quad \wedge$

- DNF can get from truth tables
- DNF and CNF can have size $2^n$, if $n$ is size of original expr.

is ⊕ distributive?

$$p \land (q \oplus r) \equiv (p \land q) \oplus (p \land r)$$

$$\equiv \quad \text{def } \oplus$$

$$p \land ((q \land \neg r) \lor (\neg q \land r)) \qquad\qquad ((p \land q) \land \neg(p \land r)) \lor (\neg(p \land q) \land (p \land r))$$

$$\equiv \quad \text{De Morgan}$$

$$1 \qquad 2$$

$$(p \land q \land \neg r) \lor (p \land \neg q \land r) \qquad\qquad ((p \land q) \land (\neg p \lor \neg r)) \lor ((\neg p \lor q) \land (p \land r))$$

$$\equiv \quad \text{dist. } 1$$

$$\left( (\overset{F}{\cancel{p \land q \land \neg p}}) \lor (p \land q \land \neg r) \right) \lor$$

$$2$$

$$\left( (\cancel{\neg p \land p \land r}) \lor (q \land p \land r) \right)$$

$$p \vee (q \oplus r) \overset{?}{\not\equiv} (p \vee q) \oplus (p \vee r)$$

counter-ex : $T \vee (T \oplus T) \equiv T$

$$(T \vee T) \oplus (T \vee T) \equiv F$$

$$p \oplus (q \wedge r) \equiv (p \oplus q) \wedge (p \oplus r)$$

$$T \oplus (T \wedge F) \equiv T$$

$$(T \oplus T) \wedge (T \oplus F) \equiv F$$

Exploding CNF:

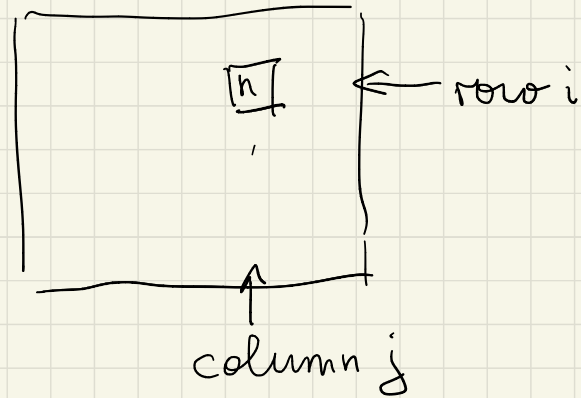
$$(p_1 \wedge q_1) \vee (p_2 \wedge q_2) \vee \cdots \vee (p_n \wedge q_n)$$

$$(p_1 \wedge q_1) \vee (p_2 \wedge q_2) \equiv$$

$$(p_1 \vee (p_2 \wedge q_2)) \wedge (q_1 \vee (p_2 \wedge q_2)) \equiv$$

$$(p_1 \vee p_2) \wedge (p_1 \vee q_2) \wedge (q_1 \vee p_2) \wedge (q_1 \vee q_2)$$

# Sudoku




$p(i,j,n)$

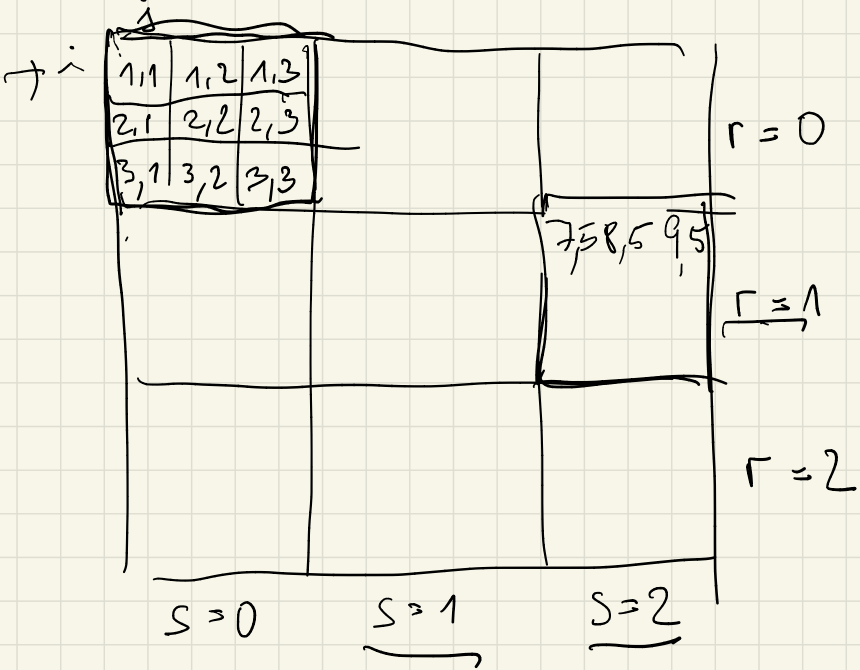
Every row condains every number: choosen $i,n$

$$p(i,1,n) \vee p(i,2,n) \vee \dots \vee p(i,9,n) = \bigvee_{j=1}^{9} p(i,j,n)$$

For row $i$

$$\bigvee_{j=1}^{9} p(i,j,1) \wedge \bigvee_{j=1}^{9} p(i,j,2) \wedge \dots = \bigwedge_{n=1}^{9} \bigvee_{j=1}^{9} p(i,j,n)$$

$$\bigwedge_{i=1}^{9} \bigwedge_{n=1}^{9} \bigvee_{j=1}^{9} p(i,j,n)$$




$p(1,1,n)$

$p(1,2,n)$

$p(1,3,n)$

$\vdots$

$p(3,3,n)$

$$\bigvee_{i=1}^{3} \bigvee_{j=1}^{3} p(i,j,n)$$

$$\bigwedge_{n=1}^{9} \bigwedge_{s=0}^{2} \bigwedge_{r=0}^{2} \bigvee_{i=1}^{3} \bigvee_{j=1}^{3} p(r \times 3 + i,\ s * 3 + j,\ n)$$

A "confusing" log. equivalence:

$$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$$

$$\equiv \neg (p \vee q) \vee r$$

$$\equiv (\neg p \wedge \neg q) \vee r$$

$$\equiv (\neg p \vee r) \wedge (\neg q \vee r)$$

$$\equiv (p \to r) \wedge (q \to r)$$

# Using truth tables to derive DNF

| p | q | r | s ... | $p \wedge q \to r \to s$ ... | S |
|---|---|---|---|---|---|
| ⋮ | | | | | |
| T | F | F | T | | T |
| ⋮ | | | | | |
| T | T | F | F | | T |

$$(p \wedge \neg q \wedge \neg r \wedge s) \vee (p \wedge q \wedge \neg r \wedge \neg s) \vee \ldots$$