Session 41: Complexity Analyses

- Linear Search
- Binary Search
- Bubble Sort
- Insertion Sort

How much detail is needed?

procedure $max(a_1, a_2, \ldots., a_n$: integers)

$max := a_1$

for $i := 2$ to n

if $max < a_i$ then $max := a_i$

return max

Worst Case Complexity of Linear Search

procedure *linearsearch*(x: integer,
$a_1, a_2, \ldots, a_n$: distinct integers)

$i := 1$

$\boxed{\begin{array}{l}\textbf{while } (i \le n \text{ and } x \ne a_i) \\ \qquad i := i + 1\end{array}}$

if $i \le n$ **then** *location* $:= i$

else *location* $:= 0$

return *location*

Worst Case Complexity of Binary Search

procedure binary search(x: integer, $a_1, a_2, \ldots, a_n$: increasing integers)

$i := 1$

$j := n$

while $i < j$

$m := \lfloor (i + j)/2 \rfloor$

if $x > a_m$ **then** $i :=$ m + 1

else $j :=$ m

if $x = a_i$ **then** *location* $:= i$

else *location* := 0

return *location*

Worst Case Complexity of Bubble Sort

procedure *bubblesort*($a_1,\ldots,a_n$: real numbers with $n \geq 2$)

  for i := 1 to $n - 1$

    for j := 1 to $n - i$

      if $a_j > a_{j+1}$

      then interchange a_j and a_{j+1}

Worst Case Complexity of Insertion Sort

procedure *insertion sort*($a_1,\ldots,a_n$:
real numbers with $n \geq 2$)

for j := 2 to n

$i := 1$

while $a_j > a_i$ **and** $i < j$

$i := i + 1$

$m := a_j$

for k := 0 to $j - i - 1$

$a_{j\text{-}k} := a_{j\text{-}k\text{-}1}$

$a_i := m$

Summary

Worst case complexities

- Linear Search: $\Theta(n)$
- Binary Search: $\Theta(\log(n))$
- Bubble Sort: $\Theta(n^2)$
- Insertion Sort: $\Theta(n^2)$