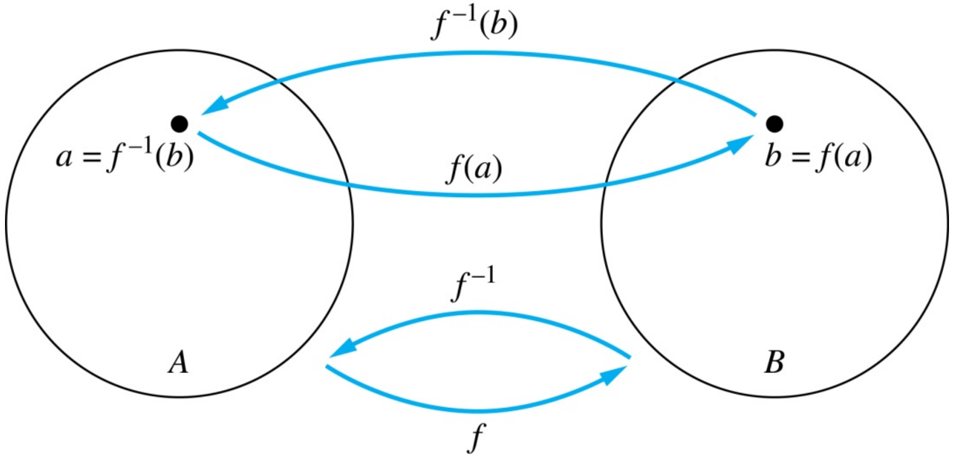
Session 21: More on Functions

- Inverse Function
- Function Composition
- Partial Functions
- Graphs of Functions

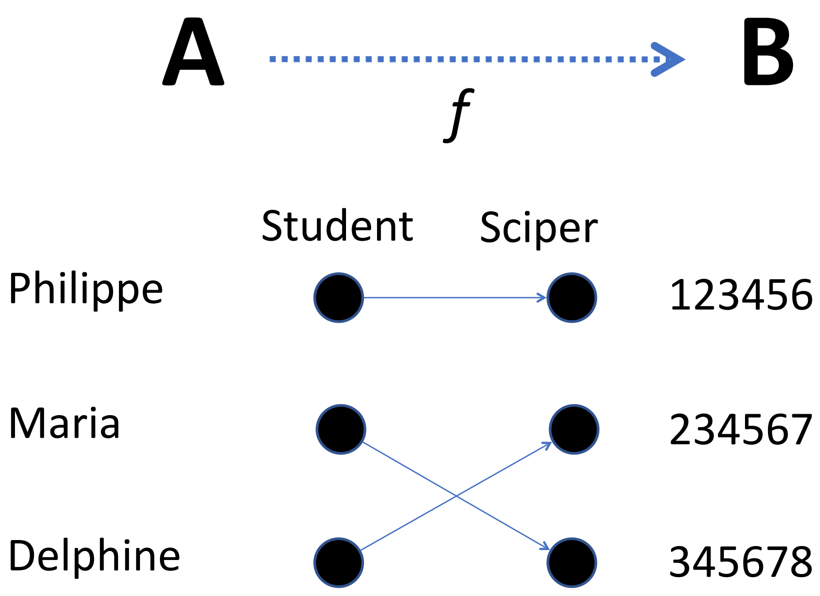
Inverse Functions

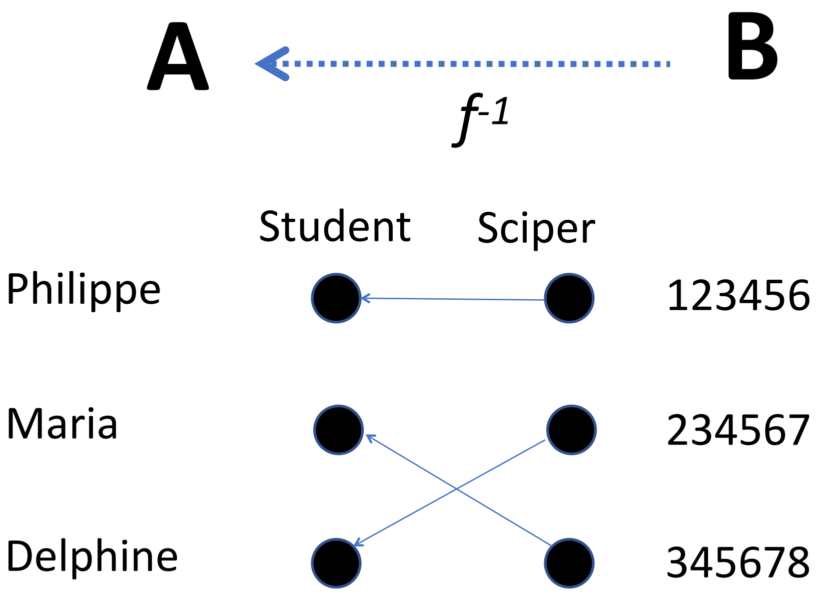
Definition: Let f be a bijection from A to B. Then the *inverse* of f, denoted f^{-1}, is the function from B to A defined as

$$f^{-1}(y) = x \text{ iff } f(x) = y$$



Example





No inverse exists unless *f* is a bijection. Why?

Example

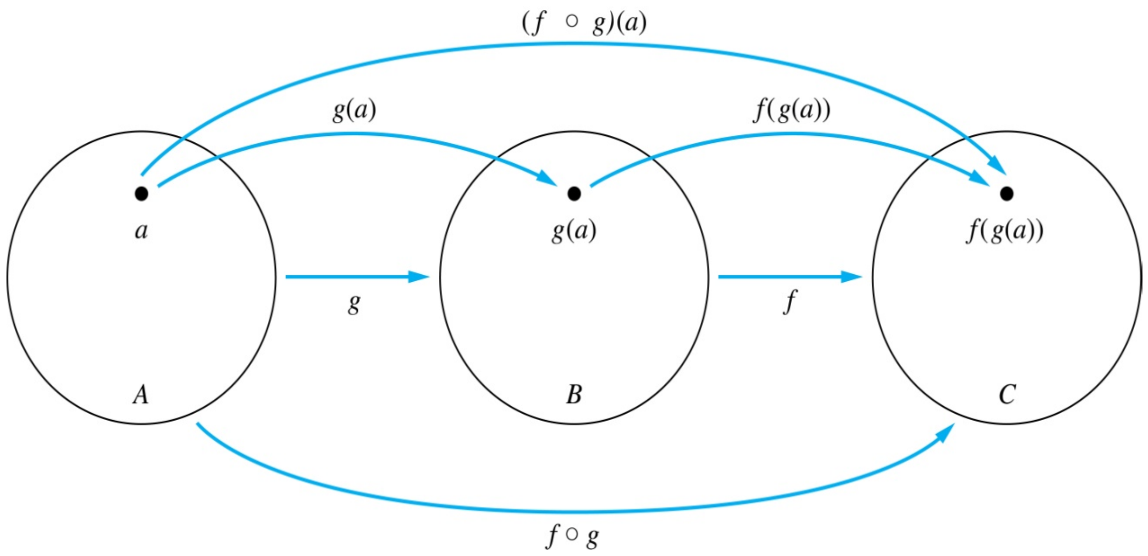
Is the function f: $\mathbf{Z} \to \mathbf{Z}$, $f(x) = x+1$ invertible?

Is the function f: $\mathbf{R} \to \mathbf{R}$, $f(x) = x^2$ invertible?

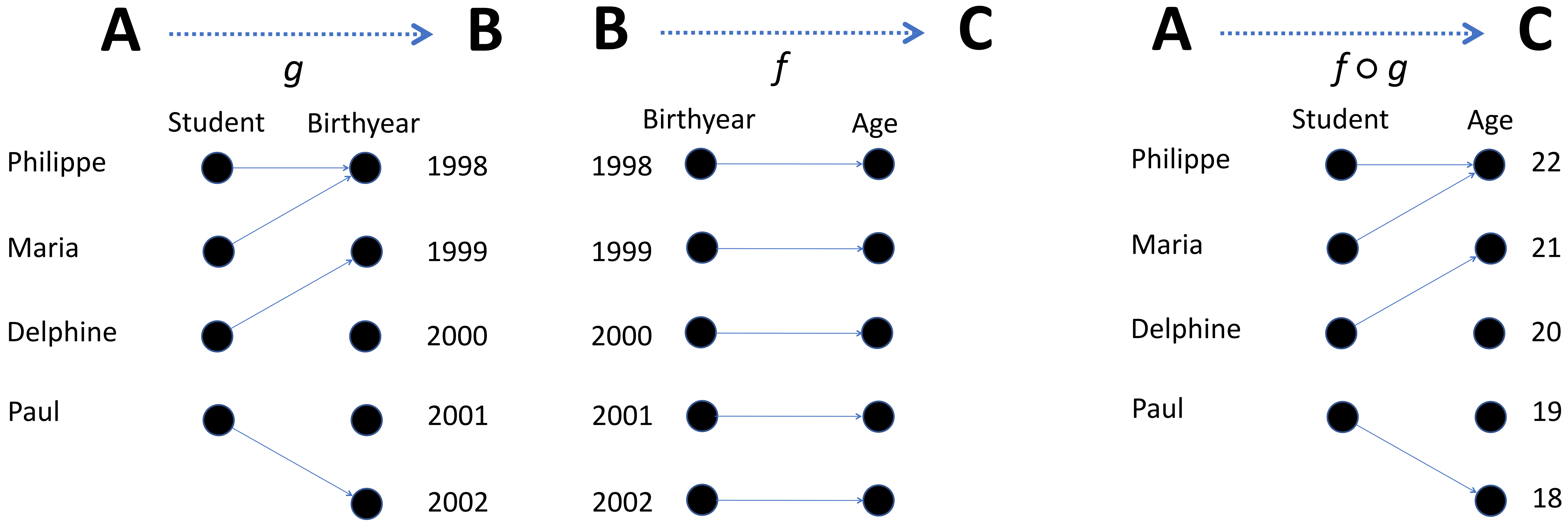
Composition

Definition: Let f: $B \rightarrow C$, g: $A \rightarrow B$. The **composition** of f with g, denoted $f \circ g$ is the function from A to C defined by

$$f \circ g(x) = f(g(x))$$



Example


A
g
B
Student
Birthyear
Philippe
Maria
Delphine
Paul
1998
1999
2000
2001
2002
B
f
C
Birthyear
Age
1998
1999
2000
2001
2002
A
f ∘ g
C
Student
Age
Philippe
Maria
Delphine
Paul
22
21
20
19
18

Example

If $f(x) = x^2$ and $g(x) = x+1$, then

$$f(g(x)) =$$

and

$$g(f(x)) =$$

Composition is not commutative!

Partial Functions

Definition: A **partial function** f from a set A to a set B is an assignment to each element a in a <u>subset</u> of A**,** called the *domain of definition* of f, of a unique element b in B.

- The sets A and B are called the **domain** and **codomain** of f, respectively.
- We say that f is **undefined** for elements in A that are not in the domain of definition of f.
- When the domain of definition of f equals A, we say that f is a ***total function***.

Example

f: **Z** $\rightarrow$ **R** where $f(n) = \sqrt{n}$ is a partial function from **Z** to **R** where the domain of definition is the set of nonnegative integers.

The domain of the function is **N.**

f is undefined for negative integers.

Summary

- Inverse Function
  - Only for bijections
- Function Composition
  - Not commutative
- Partial Functions