Mistakes in Proofs

- Examples for direct and indirect proofs
- Other proof methods
- Mistakes in proofs

Example 1: where is the problem?

Proof that 1 = -1

$$
\begin{aligned}
-1 &= & (-1)^1 && (step\ 1)\\
&= & (-1)^{\frac{2}{2}} && (step\ 2)\\
&\boxed{=} & \boxed{((-1)^2)^{\frac{1}{2}}} && \boxed{(step\ 3)}\\
&= & 1^{\frac{1}{2}} && (step\ 4)\\
&= & 1 && (step\ 5)
\end{aligned}
$$

Power Law

$\forall x \geq 0 \quad x^{a \cdot b} = (x^a)^b$

The condition on x is not satisfied

Example 2: where is the problem?

Find a solution of $\sqrt{2x^2-1} = x$

$$\begin{aligned} \sqrt{2x^2-1} &= x && (step\ 1) \\ \boxed{2x^2-1 = x^2 \quad (step\ 2)} \\ x^2-1 &= 0 && (step\ 3) \\ (x+1)(x-1) &= 0 && (step\ 4) \\ x = 1 \vee x = -1 & && (step\ 5) \end{aligned}$$

$\forall x \forall y\ (x = y \rightarrow x^2 = y^2)$

The inverse is not true:

counterexample:

$x = -1,\ y = 1$

$(-1)^2 = 1^2 \not\rightarrow -1 = 1$

We want to show: $\sqrt{2x^2-1} = x \leftrightarrow x = 1 \vee x = -1$

Example 3: where is the problem?

$(p \to q) \vee (q \to p)$ is a tautology

Let p := "n is odd" and q := "n is prime"

However, neither $(p \to q)$ nor $(q \to p)$ is true.

So $(p \to q) \vee (q \to p)$ is not a tautology?

Correct formulation: $p(n)$ = "n is odd" and $q(n)$ = "n is prime"

Then $\forall n \, (p(n) \to q(n))$ is not true

$\forall n \, (q(n) \to p(n))$ is not true

But $\forall n \, ((p(n) \to q(n)) \vee (q(n) \to p(n)))$ is true, but this is not equivalent to $\forall n \, (p(n) \to q(n)) \vee \forall n \, (q(n) \to p(n))$