Mistakes in Proofs

- Examples for direct and indirect proofs
- Other proof methods
- Mistakes in proofs

Example 1: where is the problem?

Proof that 1 = -1

$$
\begin{aligned}
-1 &= (-1)^1 && (step\ 1)\\
&= (-1)^{\frac{2}{2}} && (step\ 2)\\
&= ((-1)^2)^{\frac{1}{2}} && (step\ 3)\\
&= 1^{\frac{1}{2}} && (step\ 4)\\
&= 1 && (step\ 5)
\end{aligned}
$$

Example 2: where is the problem?

Find a solution of $\sqrt{2x^2-1} = x$

$$\begin{aligned}
\sqrt{2x^2-1} &= x && (step\ 1)\\
2x^2-1 &= x^2 && (step\ 2)\\
x^2-1 &= 0 && (step\ 3)\\
(x+1)(x-1) &= 0 && (step\ 4)\\
x = 1 \vee x &= -1 && (step\ 5)
\end{aligned}$$

Example 3: where is the problem?

$(p \rightarrow q) \vee (q \rightarrow p)$ is a tautology

Let p := “n is odd” and q := “n is prime”

However, neither $(p \rightarrow q)$ nor $(q \rightarrow p)$ is true.

So $(p \rightarrow q) \vee (q \rightarrow p)$ is not a tautology?