

Session 20: Introduction to Functions

- Definition of a Function
- Injection, Surjection, Bijection

Functions

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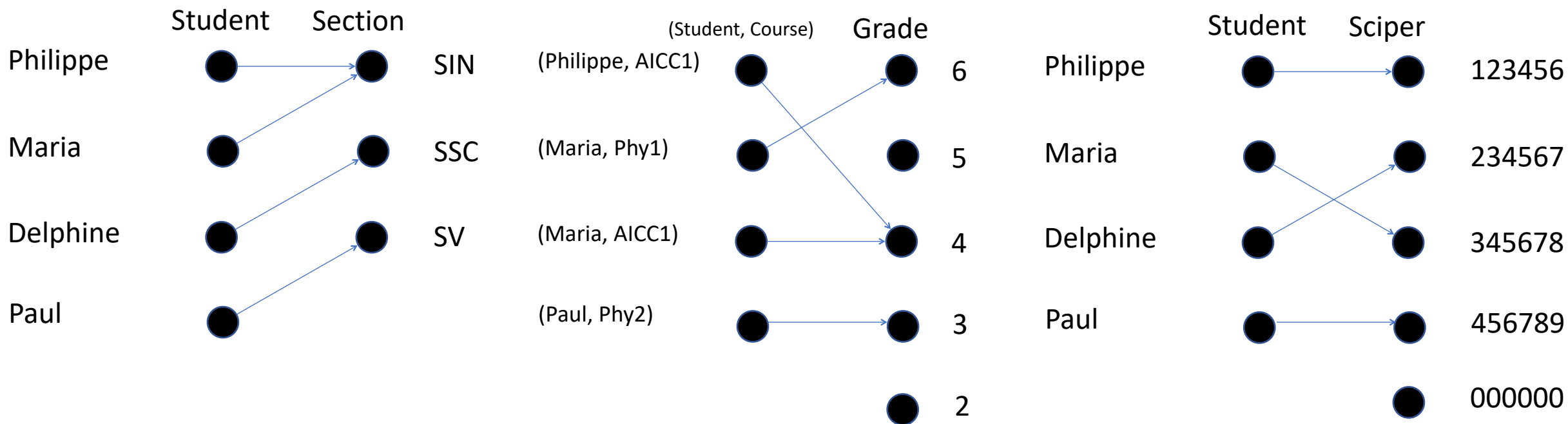
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- Functions are sometimes called **mappings** or **transformations**.

Example



Functions - Terminology

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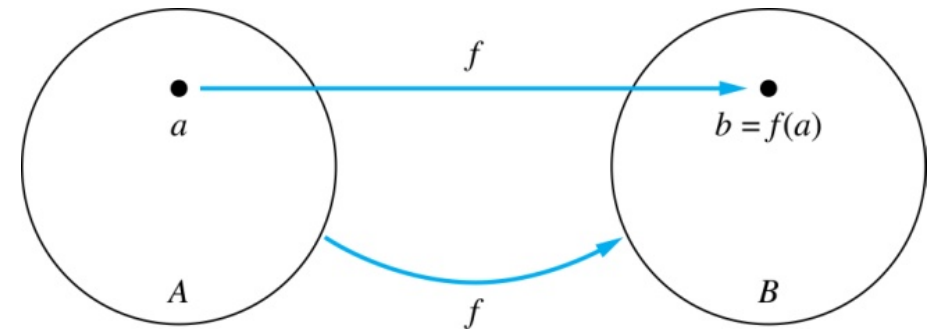
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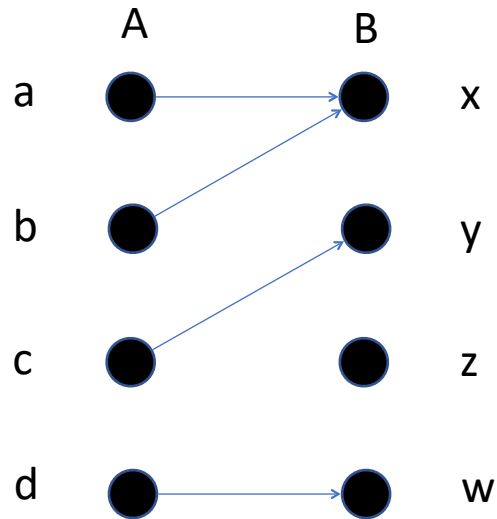
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- Two functions are ***equal*** when they have
 - the same domain
 - the same codomain
 - and map each element of the domain to the same element of the codomain.

Example



$$f(a) =$$

The image of d is

The domain of f is ?

The codomain of f is ?

The preimage of y is ?

The preimages of x are ?

$$f(A) =$$

$$f(\{a,b,c\}) =$$

Representing Functions

Functions may be specified in different ways

- An explicit statement of the assignment

Table of students and their grades

- A formula

$$f(x) = x + 1$$

- A computer program.

A Python program that when given an integer n , produces the Number 2^n

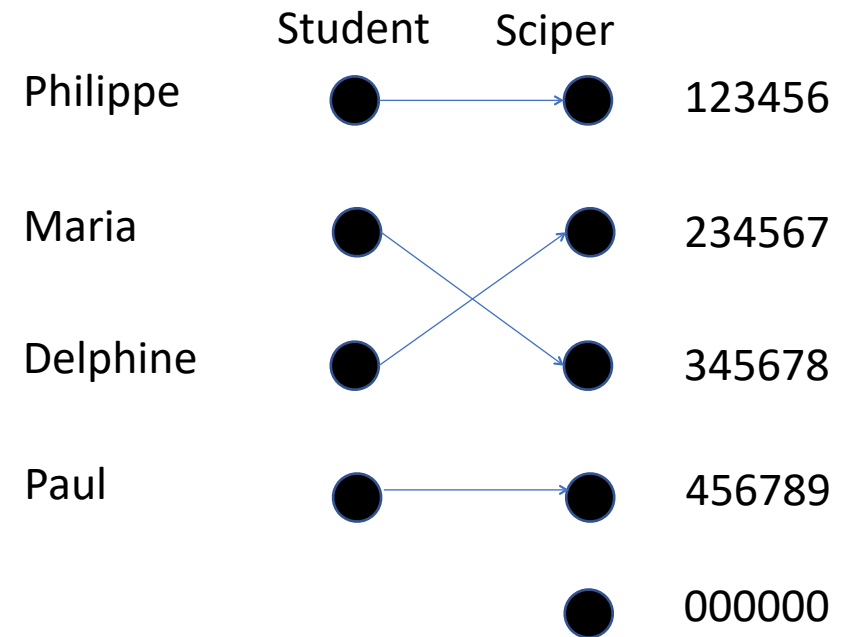
Injectons

Definition: A function f is said to be **one-to-one**, or **injective**, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f .

A function is said to be an **injection** if it is one-to-one.

Why important?

Every Sciper number can only be assigned to one student.



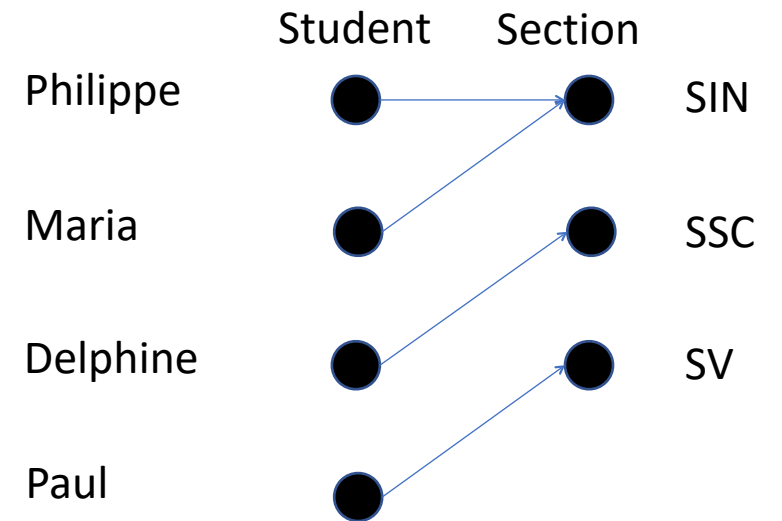
Surjections

Definition: A function f from A to B is called **onto** or **surjective**, if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$.

A function f is called a **surjection** if it is **onto**.

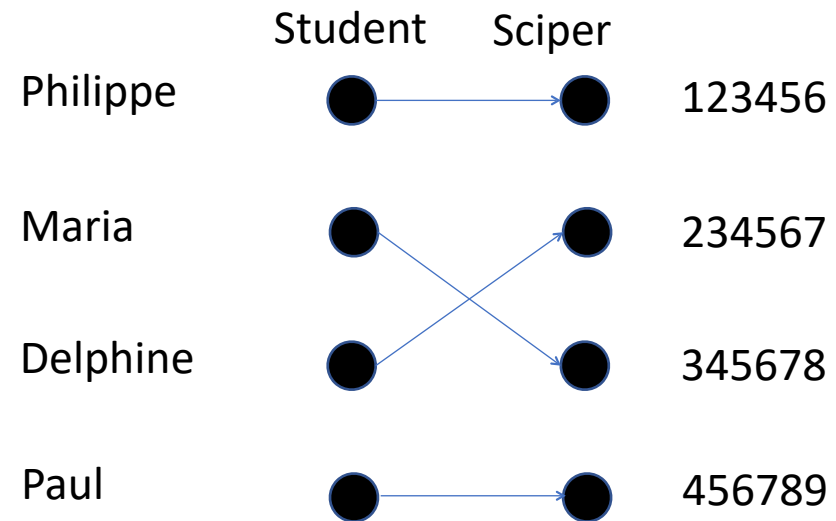
Why important?

Every Section has at least one student.



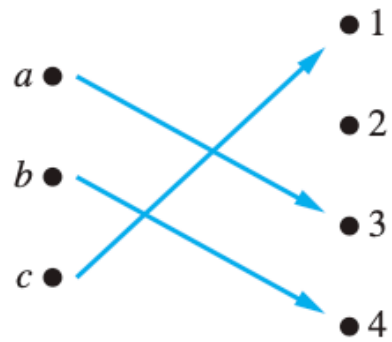
Bijections

Definition: A function f from A to B is a **one-to-one correspondence**, or a **bijection**, if it is both one-to-one and onto (surjective and injective).

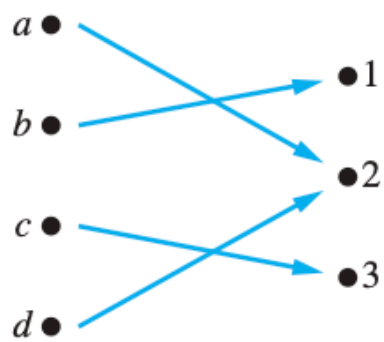


Illustration

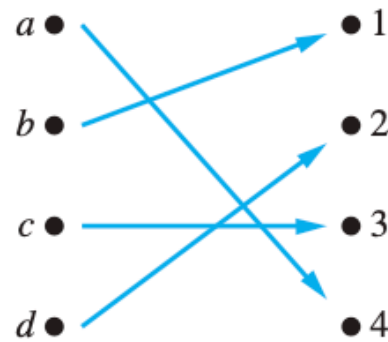
(a) One-to-one,
not onto



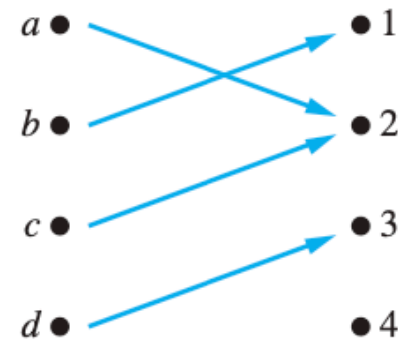
(b) Onto,
not one-to-one



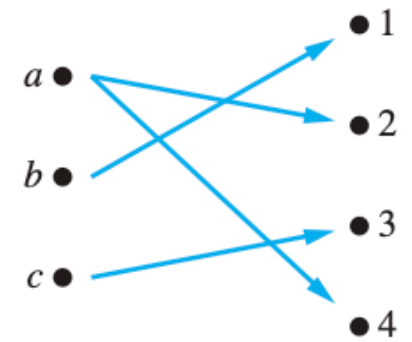
(c) One-to-one
and onto



(d) Neither one-to-one
nor onto



(e) Not a function



Showing that f is injective

Let $f: A \rightarrow B$ be a function

To show that f is injective:

Select arbitrary $x, y \in A$,

Show that if $f(x) = f(y)$, then $x = y$

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To show that f is not injective:

 Find $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$

Showing that f is surjective

Let $f: A \rightarrow B$ be a function

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To show that f is not surjective :

 Find $y \in B$ such that $f(x) \neq y$ for all $x \in A$

Example

N = natural numbers = $\{0, 1, 2, 3, \dots\}$

Z = integers = $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Is the function $f: \mathbf{Z} \rightarrow \mathbf{Z}, f(x) = x+1$ surjective?

Is the function $f: \mathbf{N} \rightarrow \mathbf{N}, f(x) = x+1$ surjective?

Is the function $f: \mathbf{Z} \rightarrow \mathbf{Z}, f(x) = x+1$ injective?

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Summary

- Definition of a Function
 - domain, co-domain, image, pre-image, range, equality
- Injection, Surjection, Bijection
 - How to show these properties