Session 22: Relations

- Introduction to Relations
- Operation on Relations

Binary Relations

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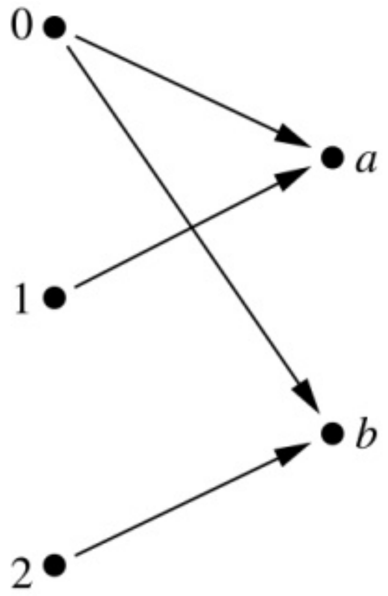
Example:

Let $A = \{0,1,2\}$ and $B = \{a,b\}$

$\{(0, a), (0, b), (1,a) , (2, b)\}$ is a relation from A to B.

Representation of Relations

Possible representation of relations from a set *A* to a set *B*



directed graph

R	a	b
0	×	×
1	×	
2		×

table

Functions and Relations

- A function $f : A \rightarrow B$ can also be defined as a subset of $A \times B$, i.e. as a relation.
- A function f from A to B contains one, and only one ordered pair (a, b) for every element $a \in A$.

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$$\forall x, y_1, y_2[[(x, y_1) \in f \wedge (x, y_2) \in f] \to y_1 = y_2]$$

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Relations are more general than functions!

Combining Relations

Given two relations R_1 and R_2, we can combine them using basic set operations to form new relations such as $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 - R_2$, and $R_2 - R_1$.

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Example:

Let $A = \{1,2,3\}$ and $B = \{1,2,3,4\}$.

Let $R_1 = \{(1,1),(2,2),(3,3)\}$ and $R_2 = \{(1,1),(1,2),(1,3),(1,4)\}$

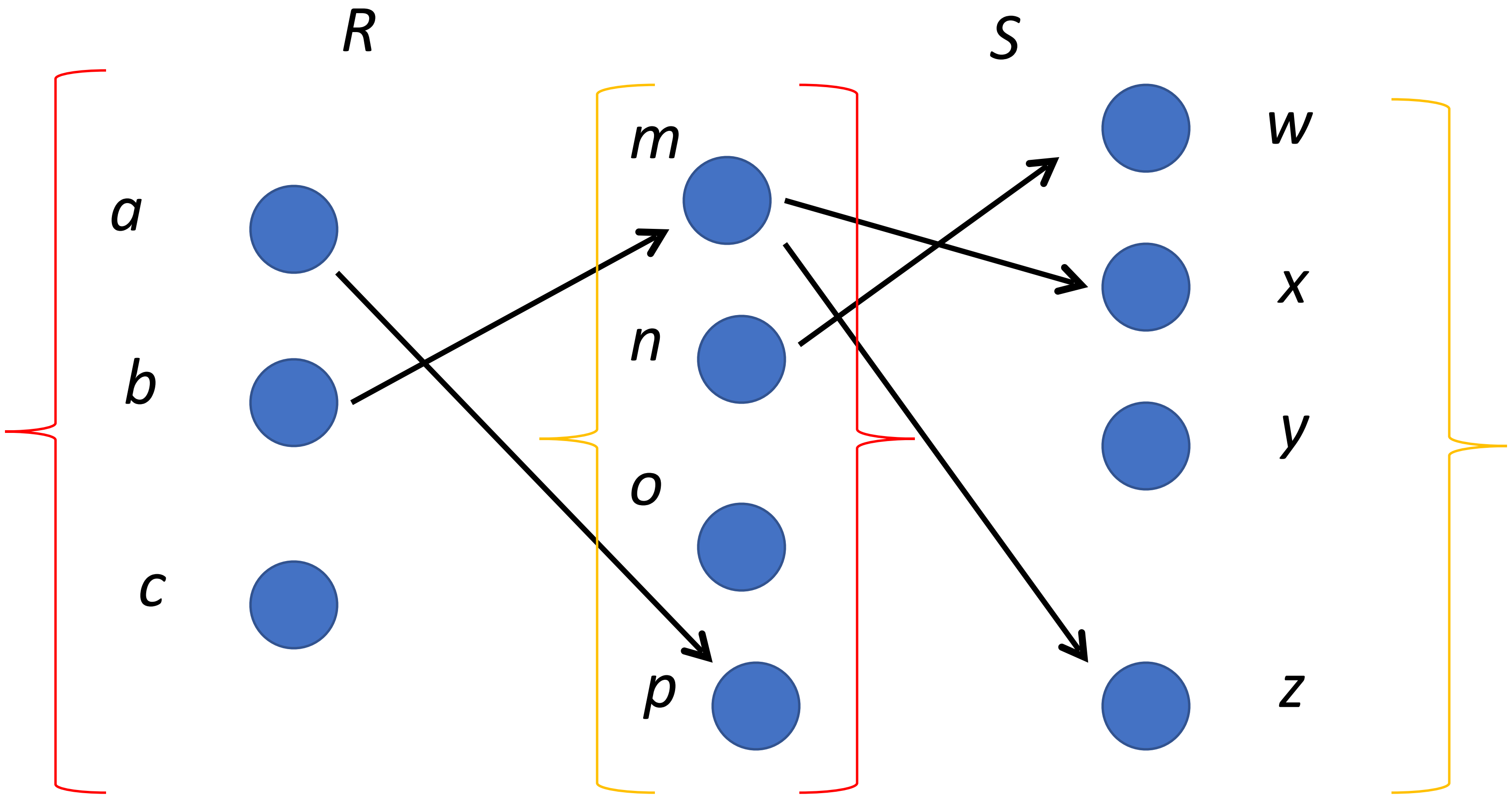
Composition of Relations

Definition: Let R be a relation from a set A to a set B. Let S be a relation from B to a set C.

The **composite** of R and S is the relation consisting of ordered pairs (a, c), where $a \in A$, $c \in C$, and for which there exists an element $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$.

We denote the composite of R and S by $S \circ R$.

Example



N-ary Relations

Definition: Let $A_1, A_2, \ldots, A_n$ be sets. An **n-ary relation** on these sets is a subset of $A_1 \times A_2 \times \cdots \times A_n$. The sets $A_1, A_2, \ldots, A_n$ are called the **domains** of the relation, and n is called its **degree**.

Example

Database tables are n-ary relations

TABLE 1 Students.

Student_name	*ID_number*	*Major*	*GPA*
Ackermann	231455	Computer Science	3.88
Adams	888323	Physics	3.45
Chou	102147	Computer Science	3.49
Goodfriend	453876	Mathematics	3.45
Rao	678543	Mathematics	3.90
Stevens	786576	Psychology	2.99

Summary

- Binary Relations
- Set-operations on Relations
- Composition of Relations
- N-ary Relations