Session 80: Expected Value

- Expected Value
- Examples

Expected Value

Definition: The **expected value** (or **expectation** or **mean**) of the random variable *X* on the sample space *S* is equal to

$$E(X) = \sum_{s \in S} p(s)X(s)$$

Example

Expected Value of a Dice: Let X be the number that comes up when a fair dice is rolled. What is the expected value of X?

Expected Value

Theorem 1: If X is a random variable and $p(X = r)$ is the probability distribution with $p(X = r) = \sum_{s \in S, X(s)=r} p(s)$ then $E(X) = \sum_{r \in X(S)} p(X = r)r$

Example

What is the expected value of the sum of the numbers that appear when a pair of fair dice is rolled?

Let X be the random variable equal to the sum of the numbers that appear when a pair of fair dice is rolled.
The range of X is {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

$$
\begin{aligned}
&p(X = 2) = p(X = 12) = 1/36,\\
&p(X = 3) = p(X = 11) = 2/36 = 1/18,\\
&p(X = 4) = p(X = 10) = 3/36 = 1/12,\\
&p(X = 5) = p(X = 9) = 4/36 = 1/9,\\
&p(X = 6) = p(X = 8) = 5/36,\\
&p(X = 7) = 6/36 = 1/6.
\end{aligned}
$$

therefore

$$
\begin{aligned}
E(X) &= 2 \cdot \frac{1}{36} + 3 \cdot \frac{1}{18} + 4 \cdot \frac{1}{12} + 5 \cdot \frac{1}{9} + 6 \cdot \frac{5}{36} + 7 \cdot \frac{1}{6}\\
&\quad + 8 \cdot \frac{5}{36} + 9 \cdot \frac{1}{9} + 10 \cdot \frac{1}{12} + 11 \cdot \frac{1}{18} + 12 \cdot \frac{1}{36}\\
&= 7.
\end{aligned}
$$

Expected Value of Bernoulli trials

Theorem 2: The expected number of successes when n mutually independent Bernoulli trials are performed, where p is the probability of each trial, is np.

Summary

- Expected Value
- Examples
    - Expected Value of Bernoulli trials