Session 75: Probability Distributions

- Limitations of Laplace’s Probability
- Probability Distribution
- Laws on Probability Distributions

Limitation of Laplace’s Definition

Laplace’s definition from the previous section, assumes that all outcomes are equally likely. This is not always the case.

Example: The sum of rolling two dice.

Set of possible outcomes: S = {2,3,4,5,6,7,8,9,10,11,12}

Probability of sum 2: need to roll (1,1), thus 1/36

Probability of sum 3: need to roll (1,2) or (2,1), thus 1/18

Probability of sum 4: need to roll (1,3), (2,2) or (3,1), thus 1/12

Probability Distribution

Definition: Let S be a sample space of an experiment with a finite or countable number of outcomes. We assign a probability $p(s)$ to each outcome s, so that:

1. $0 \leq p(s) \leq 1$ for each $s \in S$
2. $\sum_{s \in S} p(s) = 1$

The function p from the set of all outcomes of the sample space S is called a **probability distribution**.

Example

The sum of rolling two dice.

Set of possible outcomes: S = {2,3,4,5,6,7,8,9,10,11,12}

The Probability Distribution is given by:

$$p(7-i) = \frac{6-|i|}{36} \quad \text{for } -5 \le i \le 5$$

Example

What probabilities should we assign to the outcomes H (heads) and T (tails) when the coin is biased so that heads come up twice as often as tails?

Uniform Distribution

Definition: Suppose that S is a set with n elements. The **uniform distribution** assigns the probability $1/n$ to each element of S.

Example: Consider again the coin flipping example, but with a fair coin.

Then $p(H) = p(T) = 1/2$.

Probability of an Event

Definition: The **probability** of the event *E* is the sum of the probabilities of the outcomes in *E*.

$$p(E) = \sum_{s \in E} p(s)$$

Example

Suppose that a die is biased so that 3 appears twice as often as each other number, but that the other five outcomes are equally likely.

What is the probability that an odd number appears when we roll this die?

Probabilities of Complements and Unions

Complements: $p\left(\bar{E}\right) = 1 - p(E)$ still holds.

Proof:

$$p\left(\bar{E}\right) = \sum_{s \in \bar{E}} p(s) = \sum_{s \in S - E} p(s) = \sum_{s \in S} p(s) - \sum_{s \in E} p(s) = 1 - p(E)$$

Union: $p\left(E_1 \cup E_2\right) = p\left(E_1\right) + p\left(E_2\right) - p\left(E_1 \cap E_2\right)$ still holds.

Combinations of Events

Theorem 3: If E_1, E_2, … is a sequence of pairwise disjoint events in a sample space S, then

$$p\left(\bigcup_i E_i\right) = \sum_i p(E_i)$$

Summary

- Limitations of Laplace's Probability
- Probability Distribution
  - Uniform distribution
  - Probability of events
- Laws on Probability Distributions
  - Complement and Union
  - Disjoint events