Advanced information, computation, communication I
EPFL - Fall semester 2021-2022

Week 6

October 29, 2021

1 Open Questions

Exercise 1. (**)

```
function f1() {
  x=0
  i=1
  while (i ≤ n) {
    x=x+1
    i=x+x
  }
  a=x
}
```

```
function f2() {
  y=0
  j=1
  while (j ≤ n) {
    y=y+1
    j=y*y
  }
  b=y
}
```

After execution of the two program fragments `f1` and `f2`, it is the case that

- ◯ $a \approx \frac{n}{2}$, $b \approx \sqrt{n}$.
- ◯ $a \approx n$, $b \approx \log_2(n)$.
- ◯ $a \approx \frac{n}{2}$, $b \approx \log_2(n)$.
- ◯ $a \approx n$, $b \approx \sqrt{n}$.

Exercise 2. (**) The three algorithms below sort the input sequence $a_1, \ldots, a_n$ in ascending order.

Algorithm 1 Bubble Sort

```
for i ← 1 to n − 1 do
  for j ← 1 to n − i do
    if a_j > a_{j+1} then
      swap a_j and a_{j+1}
    end if
  end for
end for
```

Algorithm 2 Selection Sort

```
for i ← 1 to n − 1 do
  min ← i
  for j ← i + 1 to n do
    if a_min > a_j then
      min ← j
    end if
  end for
  if a_i > a_min then
    swap a_i and a_min
  end if
end for
```

Algorithm 3 Insertion Sort

```
for j ← 2 to n do
  i ← 1
  while a_j > a_i and i < j do
    i ← i + 1
  end while
  m ← a_j
  for k ← 0 to j − i − 1 do
    a_{j−k} ← a_{j−k−1}
  end for
  a_i ← m
end for
```

Use Bubble Sort, Selection Sort and Insertion Sort to sort the following sequence:

```
9, 12, -43, 20, -2, 3, 7, 28, 19
```

Exercise 3. (*) Recall the stable maximum matching problem/algorithm introduced in class (Session 34). Is a stable maximum matching unique? Either prove that every stable maximum matching is unique, or disprove it with a counterexample.

Exercise 4. (**) Let $\{A, B, C, D\}$ be a set of men, and $\{\alpha, \beta, \gamma, \delta\}$ a set of women. We want to match up men and women using the Gale-Shapley algorithm in two different ways. The preferences of men and women are given in the following lists, going from most preferable on the left to least preferable on the right.

Men	1st	2nd	3rd	4th
A	γ	δ	β	α
B	δ	γ	α	β
C	α	γ	β	δ
D	β	δ	α	γ

Women	1st	2nd	3rd	4th
α	D	A	B	C
β	C	B	A	D
γ	C	B	A	D
δ	D	A	B	C

1. If the men propose, and women accept/reject, what is the matching after the algorithm terminates?
2. If the women propose, and men accept/reject, what is the matching after the algorithm terminates?
3. Who is the best possible (stable) valid partner for "α"?

Exercise 5. (*) (Hint. The algorithmic steps are fairly similar for all four. Only showing steps for the first will suffice.) Use the cashier's algorithm to make change using quarters, dimes, nickels, and pennies for:

1. 87 cents.
2. 49 cents.
3. 99 cents.
4. 33 cents.

Exercise 6. (*) Describe an algorithm that determines whether a function f from a finite set $\{a_1, a_2, ..., a_n\}$ to its image $\{f(a_1), f(a_2), ..., f(a_n)\}$ is one-to-one.

Exercise 7. (***) Adapt the bubble sort algorithm so that it stops when no more swaps is required. Express this more efficient version of the algorithm in pseudocode.

Exercise 8. (*) Two strings are anagrams if each can be formed from the other by rearranging its characters. Devise an algorithm to determine whether two strings are anagrams.

Exercise 9. (*) (Book Chapter 3.1 Exercise 5) Describe an algorithm that takes as input a list of n integers in non-decreasing order, $a_1 \leq a_2 \leq ... \leq a_n$, and produces the list of all values that occur more than once.

2 Exam Questions

Exercise 10. (**)

$$L_{x_1} = (y_3, y_1, y_2) \qquad L_{y_1} = (x_2, x_1, x_3)$$
$$L_{x_2} = (y_2, y_3, y_1) \qquad L_{y_2} = (x_1, x_3, x_2)$$
$$L_{x_3} = (y_1, y_2, y_3) \qquad L_{y_3} = (x_3, x_2, x_1)$$

(*français*) Soit L_x pour $x \in X = \{x_1, x_2, x_3\}$ la liste de préférence de x donnée ci-dessus et soit L_y pour $y \in Y = \{y_1, y_2, y_3\}$ la liste de préférence de y donnée ci-dessus. Le couplage $\{(x_1, y_1), (x_2, y_3), (x_3, y_2)\}$ est

(*English*) Let L_x for $x \in X = \{x_1, x_2, x_3\}$ be the preference list of x as given above and let L_y for $y \in Y = \{y_1, y_2, y_3\}$ be the preference list of y as given above. The matching $\{(x_1, y_1), (x_2, y_3), (x_3, y_2)\}$ is

○ { instable. / unstable.

○ { stable et optimal pour Y. / stable and Y-optimal.

○ { stable et optimal pour X. / stable and X-optimal.

○ { stable, mais n'est pas un couplage stable optimal pour X ou pour Y. / stable but not a stable matching that is X- or Y-optimal.

Exercise 11. (***) Charlotte, Giulia, Kevin and Patrick are starting university next year. They have applied to EPFL, ETHZ, USI and HSG, and their preferences are listed as follows:

Student	most preferred	$\longrightarrow$	$\longrightarrow$	least preferred
Patrick	ETHZ	EPFL	USI	HSG
Giulia	EPFL	USI	ETHZ	HSG
Charlotte	USI	ETHZ	EPFL	HSG
Kevin	HSG	ETHZ	EPFL	USI

The universities, on the other hand, have their own lists of preferred students

University	most preferred	$\longrightarrow$	$\longrightarrow$	least preferred
EPFL	Giulia	Charlotte	Patrick	Kevin
ETHZ	Giulia	Patrick	Charlotte	Kevin
USI	Patrick	Charlotte	Giulia	Kevin
HSG	Patrick	Giulia	Charlotte	Kevin

Which of the matchings below is not stable?

○ (Kevin, EPFL) (Charlotte, USI)

○ (Kevin, ETHZ) (Patrick, HSG)

○ (Kevin, HSG) (Giulia, EPFL)

○ (Kevin, USI) (Patrick, ETHZ)

$*$ = *easy exercise, everyone should solve it rapidly*
$**$ = *moderately difficult exercise, can be solved with standard approaches*
$***$ = *difficult exercise, requires some idea or intuition or complex reasoning*