Session 17: Constructing Sets

- How to build new sets from existing sets
- Size of sets

Power Sets

Definition: The set of all subsets of a set *A*, denoted $\mathcal{P}(A)$, is called the *power set* of *A*.

Example: If *A* = {a, b} then $\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

Tuples

Definition: The **ordered n-tuple** $(a_1, a_2, \ldots.., a_n)$ is the ordered collection that has a_1 as its first element and a_2 as its second element and so on until a_n as its last element.

- Two n-tuples are equal if and only if their corresponding elements are equal.

  $(a_1, a_2, \ldots.., a_n) = (b_1, b_2, \ldots.., b_n)$ iff. $a_1 = b_1$ and … and $a_n = b_n$

- 2-tuples are called **ordered pairs**.

Cartesian Product

Definition: The **Cartesian Product** of two sets A and B, denoted by $A \times B$, is the set of ordered pairs (a, b) where $a \in A$ and $b \in B$

$$A \times B = \{(a, b) | a \in A \wedge b \in B\}$$

Definition: A subset R of the Cartesian product $A \times B$ is called a **relation** from the set A to the set B.

Example

$A = \{a, b\}$ $B = \{1, 2, 3\}$

Cartesian Product: $\{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

A relation: $\{(a,1), (b,2), (c,3)\}$

Note: In general $A \times B$ is not equal to $B \times A$

Cartesian Product

Definition: The **Cartesian Products** of the sets A_1, A_2, ……, A_n, denoted by $A_1 \times A_2 \times \ldots\ldots \times A_n$, is the set of ordered n-tuples $(a_1, a_2,\ldots\ldots, a_n)$ where a_i belongs to A_i for i = 1, … n.

$$A_1 \times A_2 \times \cdots \times A_n = \\ \{(a_1, a_2, \ldots, a_n) | a_i \in A_i \text{ for } i = 1, 2, \ldots n\}$$

Example

$A \times B \times C$ where $A = \{0, 1\}$, $B = \{1, 2\}$ and $C = \{0, 1, 2\}$

$A \times B \times C =$ $\{(0, 1, 0), (0, 1, 1), \ldots\}$

Note: $A \times B \times C \neq (A \times B) \times C$

$\neq A \times (B \times C)$

Truth Sets of Predicates

Definition: Given a predicate *P* and a domain *D*, we define the **truth set** of *P* to be the set of elements in *D* for which *P*(*x*) is true.

The truth set of *P*(x) is denoted by

$$\{x \in D | P(x)\}$$

Example: The truth set of *P*(*x*) where the domain is the integers and *P*(*x*) := |*x*| = 1 is the set {-1, 1}

Set Cardinality

Definition: If there are exactly n distinct elements in a set S where n is a nonnegative integer, we say that S is **finite**. Otherwise it is **infinite**.

Definition: The *cardinality* of a finite set S, denoted by $|S|$, is the number of (distinct) elements of S.

Examples

If a set has n elements, then the cardinality of the power set is 2^n.

If $|A| = n$ and $|B| = m$, then $|A \times B| = n*m$.

The set of integers is infinite.

Examples

$|\{1,2,3\}| =$ 3

$|\varnothing| =$ 0

$|\{\varnothing\}| =$ 1

Summary

- Power sets
- Tuples and Cartesian Product
- Cardinality of sets