Advanced information, computation, communication I
EPFL - Fall semester 2021-2022

Week 10

November 26, 2021

1 Open Questions

Exercise 1. (*) Each user on a computer system has a password, which is seven or eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least two digits. How many possible passwords are there?

Exercise 2. (*)

1. A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?
2. Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?

Exercise 3. (**) An exam has 12 questions, with 4 possible answers for each question. How many students should complete the exam to ensure that at least 3 students will submit the exact same answers?

Exercise 4. (*)

1. How many functions are there from $A = \{0, 1, 2, 3\}$ to $B = \{0, 1, 2\}$?
2. How many injective functions are there from $A = \{0, 1, 2, 3\}$ to $B = \{0, 1, 2, 3, 4, 5, 6\}$?

Exercise 5. (*) How many bit strings of length 10 contain

1. exactly four 1s?
2. at most four 1s?
3. at least four 1s?
4. an equal number of 0s and 1s?

Exercise 6. (**) How many distinct five-card poker hands contain:

1. *One pair* (poker hand containing two cards of the same kind and three cards of three other kinds).
2. *Two pairs* (poker hand containing two cards of the same kind, two cards of another kind and one card of a third kind).
3. *Three of a kind* (poker hand containing three cards of the same kind and two cards of two other kinds).

Exercise 7. (**) Prove the hockey-stick identity using a mathematical argument (as opposed to a combinatorial argument):
For any integers n and r with $0 \leq r \leq n$, we have

$$\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}.$$

2 Exam Questions

Exercise 8. (*) Suppose that in the future every telephone in the world is assigned a number that contains a country code that is 1 to 3 digits long that is of the form X, XX, XXX followed by a 10-digit telephone number of the form NXX-NXX-$XXXX$, where N can take any values from 2 through 9 and X any values from 0 to 9. How many unique phone numbers would be available worldwide according to this numbering plan?

- ◯ 12876000
- ◯ 7.104×10^{12}
- ◯ 6.4×10^{15}
- ◯ 3.058×10^{12}

Exercise 9.

1. (*)The number of distinct triples (x_1, x_2, x_3) of non-negative integers x_1, x_2, x_3 such that $x_1 + x_2 + x_3 = 8$ equals
   - ◯ 495.
   - ◯ 330.
   - ◯ 165.
   - ◯ 45.
2. (*)The number of distinct triples (x_1, x_2, x_3) of non-negative integers x_1, x_2, x_3 such that $x_1 + x_2 + x_3 \leq 8$ equals
   - ◯ 495.
   - ◯ 330.
   - ◯ 165.
   - ◯ 45.
3. (**)The number of distinct quadruples (x_1, x_2, x_3, x_4) of non-negative integers x_1, x_2, x_3, x_4 such that $x_1 + x_2 + x_3 + x_4 < 8$ equals
   - ◯ 495.
   - ◯ 330.
   - ◯ 165.
   - ◯ 45.
4. (**)The number of distinct quadruples (x_1, x_2, x_3, x_4) of non-negative integers x_1, x_2, x_3, x_4 such that $x_i \geq i$ and $x_1 + x_2 + x_3 + x_4 \leq 18$ equals
   - ◯ 495.
   - ◯ 330.
   - ◯ 165.
   - ◯ 45.
5. (***)The number of distinct quintuples $(x_1, x_2, x_3, x_4, x_5)$ of non-negative integers x_1, x_2, x_3, x_4, x_5 such that $x_1 \geq 3$, $x_2 \geq 3$, $x_3 \geq 0$, $x_4 \geq 8$ and $0 \leq x_5 \leq 3$, and $x_1 + x_2 + x_3 + x_4 + x_5 < 24$ equals
   - ◯ 2002.

- ◯ 1750.
- ◯ 715.
- ◯ 210.

$*$ = *easy exercise, everyone should solve it rapidly*
$**$ = *moderately difficult exercise, can be solved with standard approaches*
$***$ = *difficult exercise, requires some idea or intuition or complex reasoning*