Session 74: Complements and Unions of Events

- Complements
- Unions
- Examples

The Probability of Complements of Events

Theorem 1: Let *E* be an event in sample space *S*. The probability of the event $\bar{E} = S - E$, the complementary event of *E*, is given by

$$p(\overline{E}) = 1 - p(E).$$

Example

What is the probability that when two dice are rolled, to not roll a 6?

- Let *E* be the event to roll a 6.
- This probability is 11/36.
- Thus the probability of not rolling a 6 is 1 – 11/36 = 25/36

Example

A sequence of 10 bits is chosen randomly. What is the probability that at least one of these bits is 0?

The Probability of Unions of Events

Theorem 2: Let E_1 and E_2 be events in the sample space S. Then

$$p(E_1 \cup E_1) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$$

Proof: Given the inclusion-exclusion formula

$|A \cup B| = |A| + |B| - |A \cap B|$, it follows that

$$p(E_1 \cup E_2) = \frac{|E_1 \cup E_2|}{|S|} = \frac{|E_1| + |E_2| - |E_1 \cap E_2|}{|S|}$$

$$= \frac{|E_1|}{|S|} + \frac{|E_2|}{|S|} - \frac{|E_1 \cap E_2|}{|S|}$$

$$= p(E_1) + p(E_2) - p(E_1 \cap E_2). \qquad \blacktriangleleft$$

Example

What is the probability that a positive integer selected at random from the set of positive integers not exceeding 100 is divisible by either 2 or 5?

- Let E_1 be the event that the integer is divisible by 2 and E_2 be the event that it is divisible 5.
- Then the event that the integer is divisible by 2 **or** 5 is $E_1 \cup E_2$
- The event that the integer is divisible by 2 **and** 5 is $E_1 \cap E_2$
- It follows that:

$$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$$
$$= 50/100 + 20/100 - 10/100 = 3/5.$$

Example

When rolling two dice, what is the probability to roll a 6?

Summary

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