Session 20: Introduction to Functions

- Definition of a Function
- Injection, Surjection, Bijection

Functions

Definition: Let A and B be nonempty sets. A **function** f from A to B is an assignment of exactly one element of B to each element of A.

If f is a function from A to B, we write $f : A \to B$.

Functions

Definition: Let A and B be nonempty sets. A **function** f from A to B is an assignment of exactly one element of B to each element of A.

If f is a function from A to B, we write $f : A \to B$.

We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A.

Functions

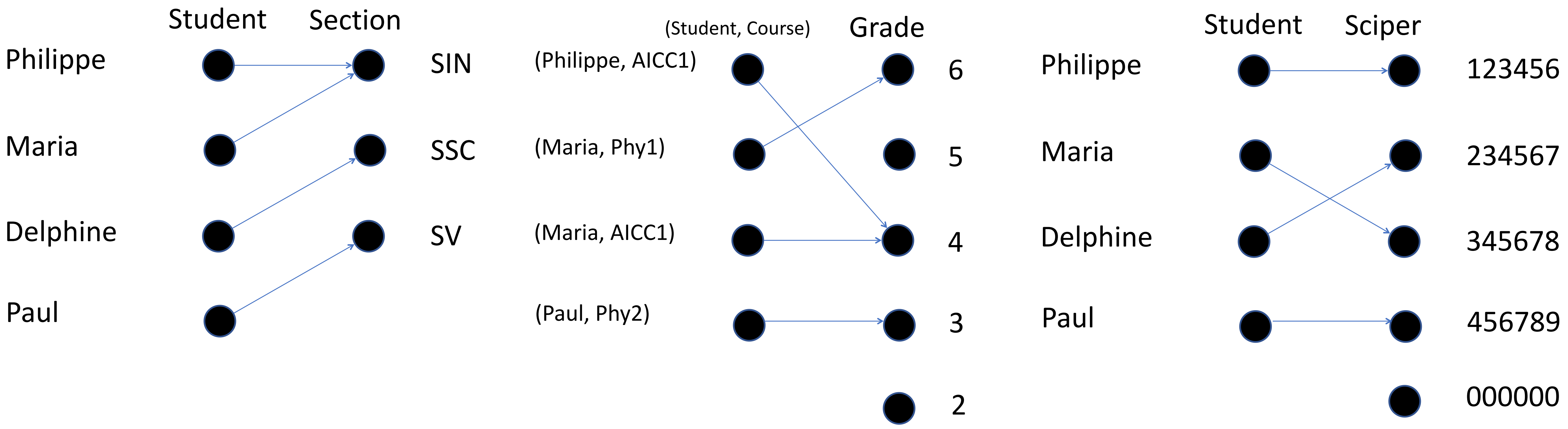
Definition: Let A and B be nonempty sets. A **function** f from A to B is an assignment of exactly one element of B to each element of A.

If f is a function from A to B, we write $f : A \rightarrow B$.

We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A.

- Functions are sometimes called **mappings** or **transformations**.

Example


Student
Section
Philippe
Maria
Delphine
Paul
SIN
SSC
SV
(Student, Course)
Grade
(Philippe, AICC1)
(Maria, Phy1)
(Maria, AICC1)
(Paul, Phy2)
6
5
4
3
2
Student
Sciper
Philippe
Maria
Delphine
Paul
123456
234567
345678
456789
000000

Functions - Terminology

Given a function $f: A \rightarrow B$**:**

Functions - Terminology

Given a function f: $A \to B$**:**

- We say f ***maps*** A to B *or* f is a ***mapping*** from A to B

Functions - Terminology

Given a function f: $A \rightarrow B$**:**

- We say f ***maps*** A to B *or* f is a ***mapping*** from A to B
- A is called the ***domain*** of f
- B is called the ***codomain*** of f

Functions - Terminology

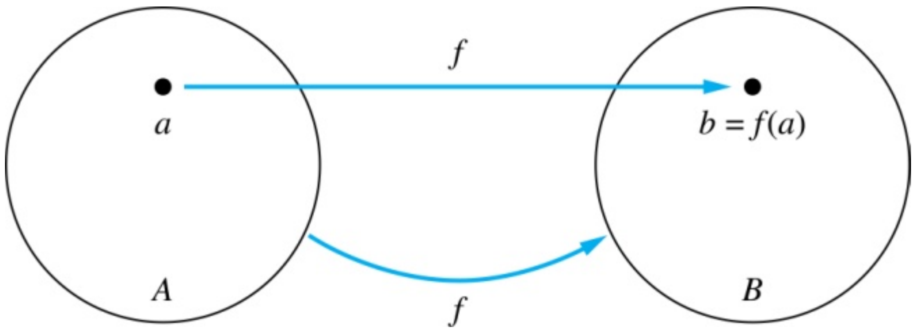
Given a function f: $A \rightarrow B$**:**

- We say f ***maps*** A to B *or* f is a ***mapping*** from A to B
- A is called the ***domain*** of f
- B is called the ***codomain*** of f
- If $f(a) = b$,
  - then b is called the ***image*** of a under f
  - a is called the ***preimage*** of b

Functions - Terminology

Given a function f: $A \rightarrow B$**:**

- We say f ***maps*** A to B *or* f is a ***mapping*** from A to B
- A is called the ***domain*** of f
- B is called the ***codomain*** of f
- If $f(a) = b$,
  - then b is called the ***image*** of a under f
  - a is called the ***preimage*** of b



Functions - Terminology

- The **range** of f is the set of all images of points in **A** under f. We denote it by $f(\boldsymbol{A})$.

Functions - Terminology

- The **range** of f is the set of all images of points in **A** under f. We denote it by $f(\boldsymbol{A})$.
- If $f: A \rightarrow B$ and $S \subseteq A$, then $f(S) = \{f(s) \mid s \in S\}$

Functions - Terminology

- The **range** of f is the set of all images of points in **A** under f. We denote it by $f(\boldsymbol{A})$.
- If f: $A \to B$ and $S \subseteq A$, then $f(S) = \{f(s) \mid s \in S\}$
- Two functions are ***equal*** when they have

Functions - Terminology

- The **range** of f is the set of all images of points in **A** under f. We denote it by $f(\boldsymbol{A})$.
- If f: $A \rightarrow B$ and $S \subseteq A$, then $f(S) = \{f(s) \mid s \in S\}$
- Two functions are ***equal*** when they have
    - the same domain

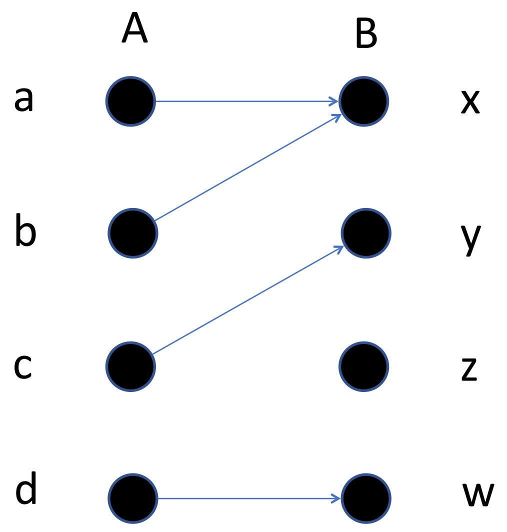
Functions - Terminology

- The **range** of f is the set of all images of points in **A** under f. We denote it by $f(\boldsymbol{A})$.
- If $f: A \rightarrow B$ and $S \subseteq A$, then $f(S) = \{f(s) \mid s \in S\}$
- Two functions are ***equal*** when they have
  - the same domain
  - the same codomain

Functions - Terminology

- The **range** of *f* is the set of all images of points in **A** under *f*. We denote it by *f*(***A***).
- If f: $A \rightarrow B$ and $S \subseteq A$, then $f(S) = \{f(s) \mid s \in S\}$
- Two functions are ***equal*** when they have
  - the same domain
  - the same codomain
  - and map each element of the domain to the same element of the codomain.

Example



f(a) = x

The image of d is w

The domain of f is ? {a, b, c, d}

The codomain of f is ? {x, y, z, w}

The preimage of y is ? c

The preimages of x are ? a, b

f(A) = {x, y, w)

f({a,b,c}) = {x, y}

Representing Functions

Functions may be specified in different ways

- An explicit statement of the assignment

  Table of students and their grades

- A formula

  $f(x) = x + 1$

- A computer program.

  A Python program that when given an integer n, produces the Number 2^n

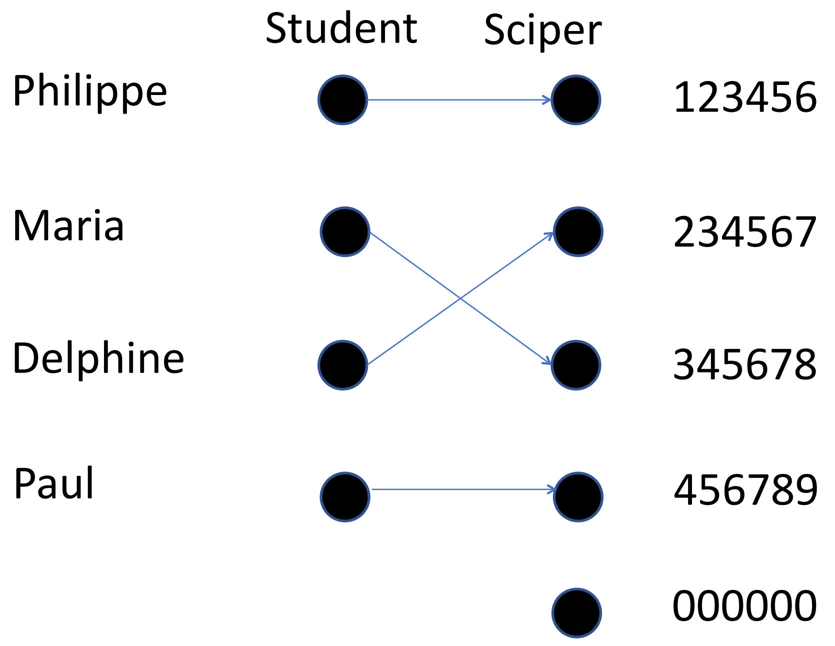
Injections

Definition: A function f is said to be **one-to-one**, or **injective**, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.

A function is said to be an **injection** if it is one-to-one.

Why important?

Every Sciper number can only be assigned to one student.



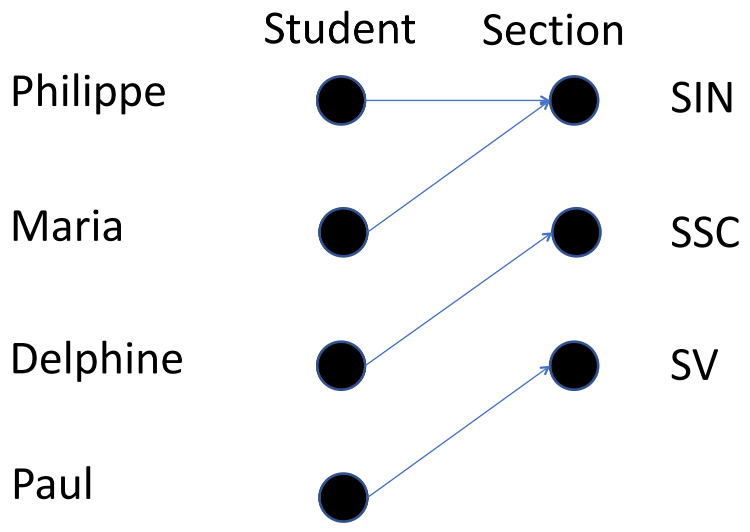
Surjections

Definition: A function *f* from *A* to *B* is called **onto** or **surjective**, if and only if for every element b ∈ *B* there is an element a ∈ *A* with *f*(a) = b.

A function *f* is called a **surjection** if it is **onto**.

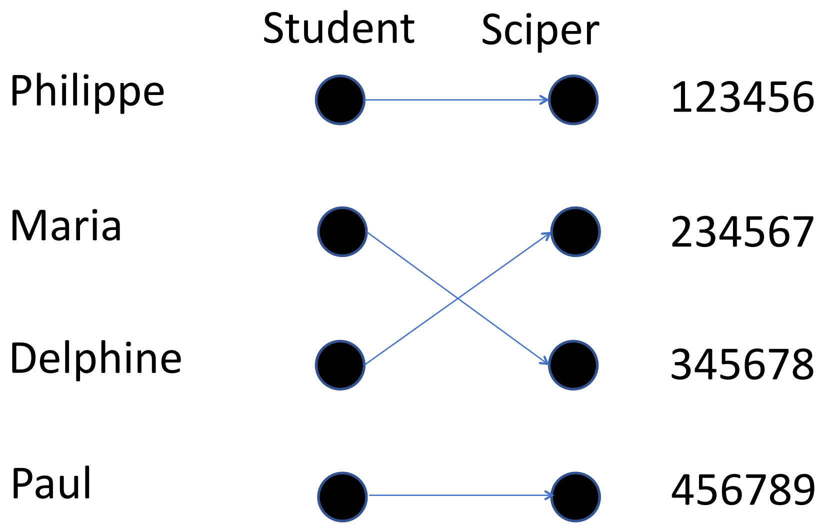
Why important?

Every Section has at least one student.

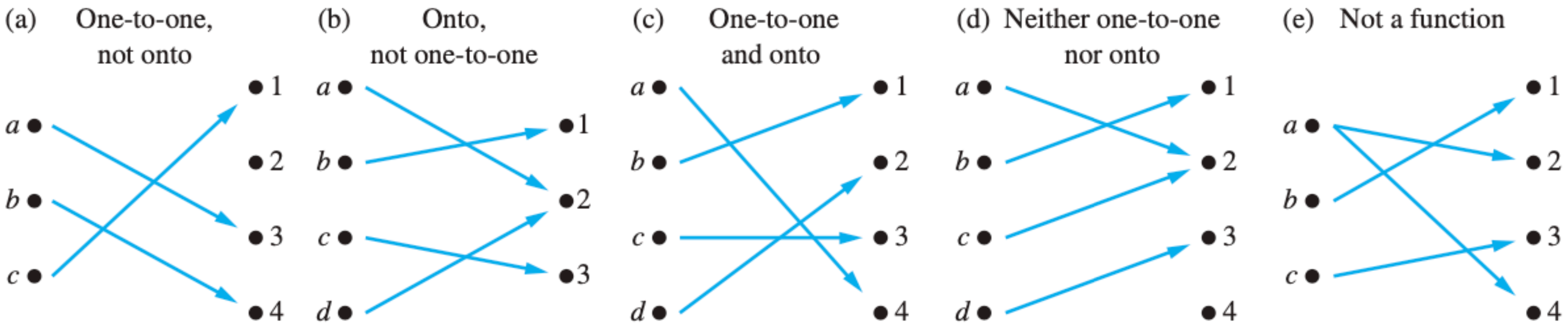


Bijections

Definition: A function f from A to B is a **one-to-one correspondence**, or a **bijection**, if it is both one-to-one and onto (surjective and injective).



Illustration



Showing that f is injective

Let f: $A \to B$ be a function

To show that f is injective:

Select arbitrary $x,y \in A$,

Show that if $f(x) = f(y)$, then $x = y$

Showing that f is injective

Let $f: A \to B$ be a function

To show that f is injective:

Select arbitrary $x, y \in A$,

Show that if $f(x) = f(y)$, then $x = y$

To show that f is not injective:

Find $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$

Showing that f is surjective

Let f: $A \rightarrow B$ be a function

To show that f is surjective:

Select arbitrary $y \in B$,

Find an element $x \in A$ such that $f(x) = y$

Showing that f is surjective

Let f: $A \to B$ be a function

To show that f is surjective:

Select arbitrary $y \in B$,

Find an element $x \in A$ such that $f(x) = y$

To show that f is not surjective :

Find $y \in B$ such that $f(x) \neq y$ for all $x \in A$

Example

N = **natural numbers** = {0, 1, 2, 3, ….}

Z = **integers** = {…, -3, -2, -1, 0, 1, 2, 3, …}

Is the function f: **Z** → **Z,** f(x) = x+1 surjective? yes

Is the function f: **N** → **N,** f(x) = x+1 surjective? no, 0 is not image of any element

Is the function f: **Z** → **Z,** f(x) = x+1 injective? yes

Is the function f: **N** → **N,** f(x) = x+1 injective? yes

Is the function f: **Z** → **Z,** f(x) = x^2 surjective? no, 3 is not image of any element

Is the function f: **Z** → **Z,** f(x) = x^2 injective? no, $-1^2 = 1^2$

Is the function f: **N** → **N,** f(x) = x^2 injective? yes

Summary

- Definition of a Function
  - domain, co-domain, image, pre-image, range, equality
- Injection, Surjection, Bijection
  - How to show these properties