Session 21: More on Functions

- Inverse Function
- Function Composition
- Partial Functions
- Graphs of Functions

Inverse Functions

Definition: Let *f* be a bijection from *A* to *B*. Then the *inverse* of *f*, denoted f^{-1}, is the function from *B* to *A* defined as

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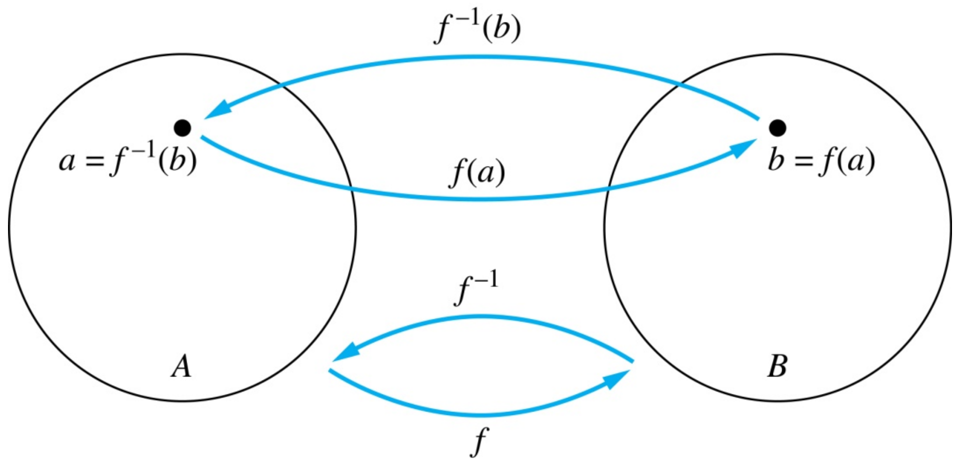
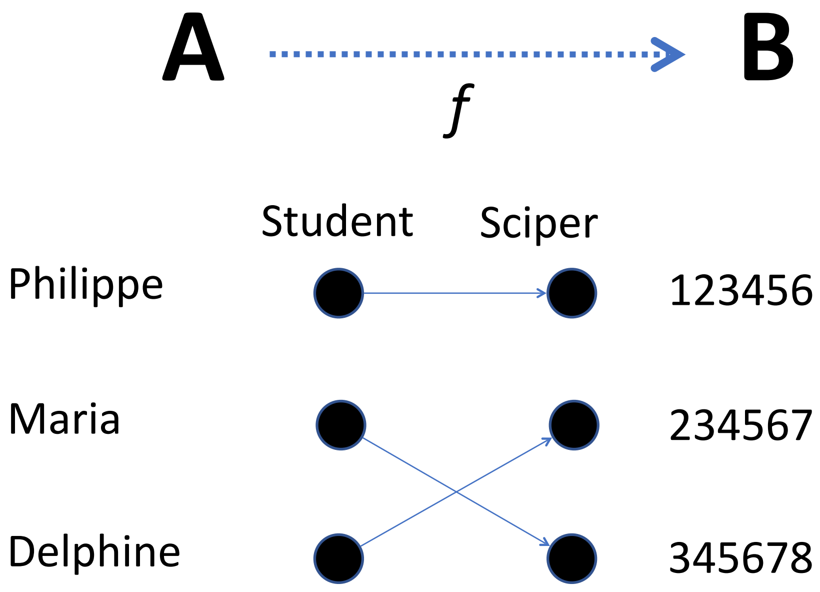
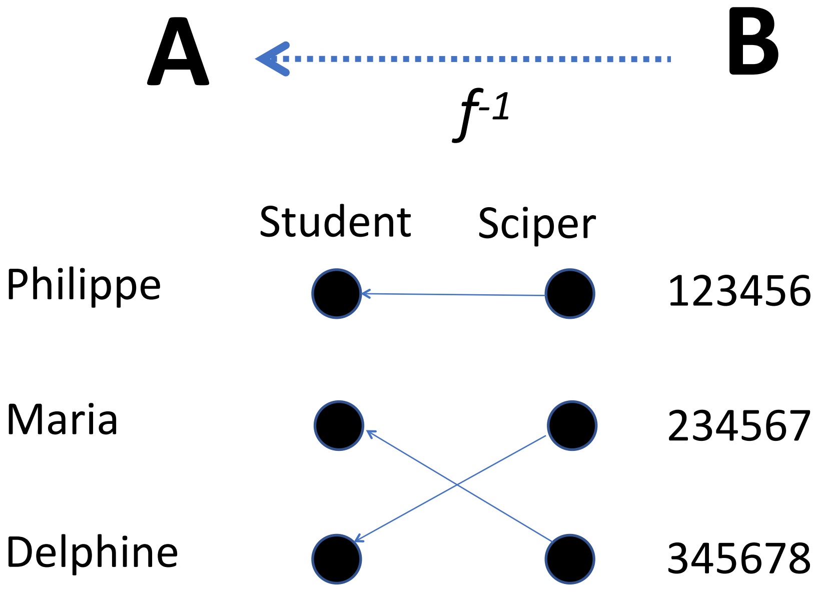


Example


A
B
f
Student
Sciper
Philippe
Maria
Delphine
123456
234567
345678


A
B
f-1
Student
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Philippe
Maria
Delphine
123456
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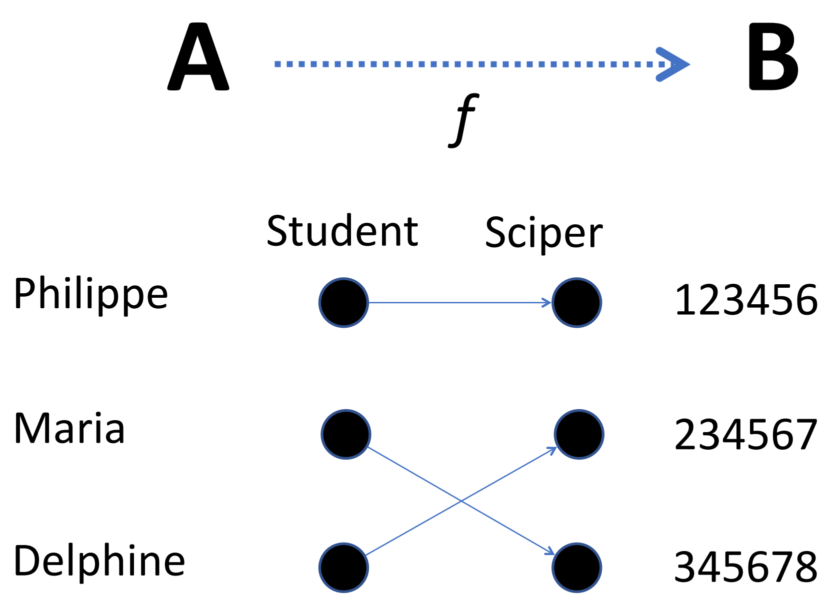

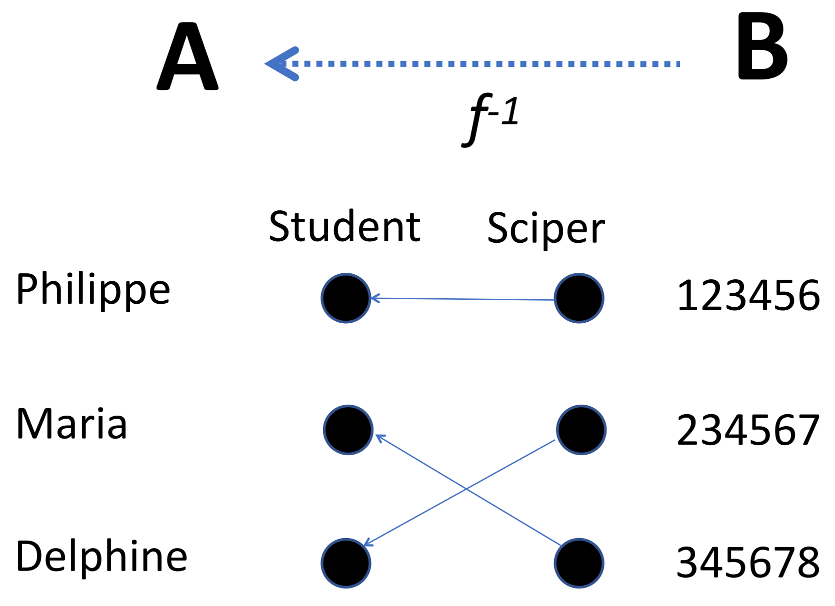

No inverse exists unless *f* is a bijection. Why?

Example

Is the function f: $\mathbf{Z} \to \mathbf{Z}$, $f(x) = x+1$ invertible?

Is the function f: $\mathbf{R} \to \mathbf{R}$, $f(x) = x^2$ invertible?

Composition

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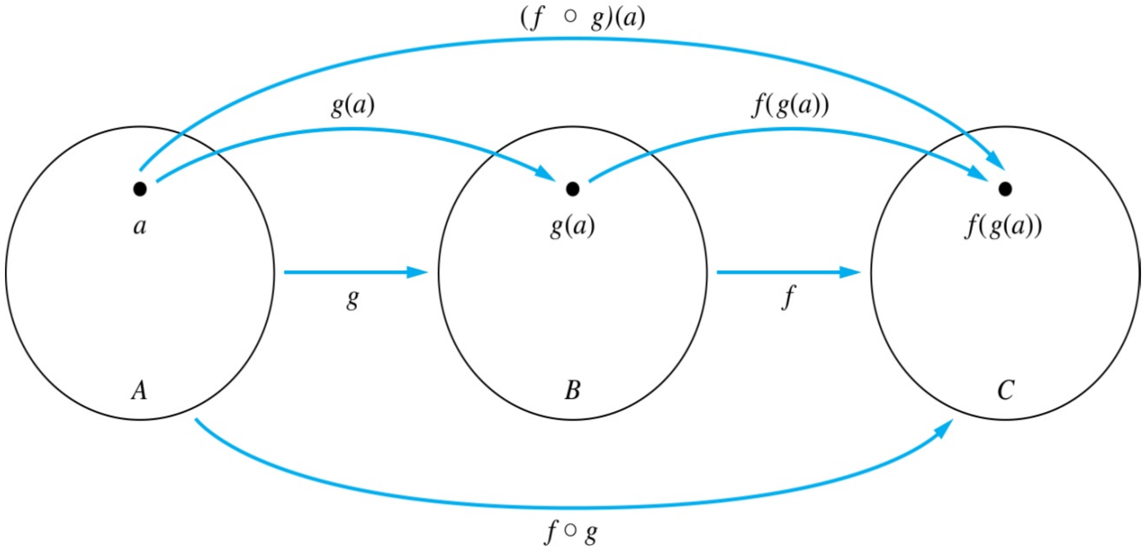
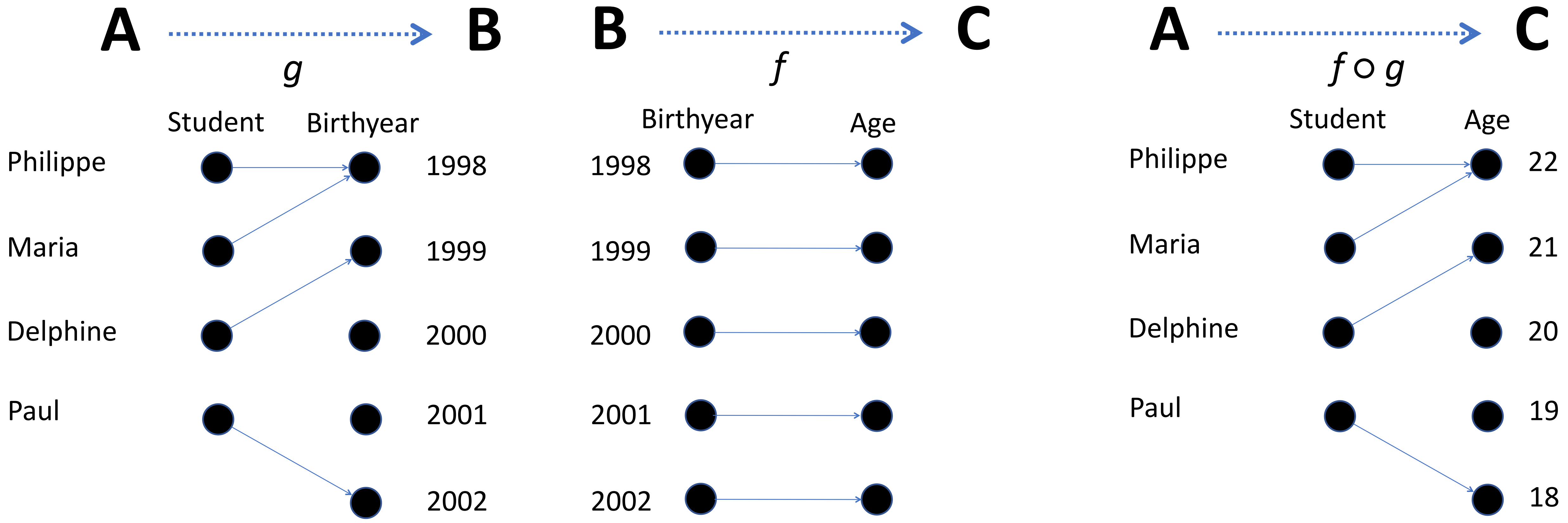


Example


A
g
B
Student
Birthyear
Philippe
Maria
Delphine
Paul
1998
1999
2000
2001
2002
B
f
C
Birthyear
Age
1998
1999
2000
2001
2002
A
f ∘ g
C
Student
Age
Philippe
Maria
Delphine
Paul
22
21
20
19
18

Example

If $f(x) = x^2$ and $g(x) = x+1$, then

$$f(g(x)) =$$

and

$$g(f(x)) =$$

Composition is not commutative!

Partial Functions

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- When the domain of definition of f equals A, we say that f is a ***total function***.

Example

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f is undefined for negative integers.

Summary

- Inverse Function
  - Only for bijections
- Function Composition
  - Not commutative
- Partial Functions