# Session 38: Advanced Big-O Facts

- Big-O for more functions
- Big-O for combined functions

# More big-$O$ facts

$\forall n > m$, $n, m$ constant:

$x^m$ is $O(x^n)$ but $x^n$ is not $O(x^m)$

$\forall a > 0, b > 0, n > m$, $a, b, n, m$ constant:

$\log_b(x^m)$ is $O(\log_a(x^n))$

$\log_a(x^n)$ is $O(\log_b(x^m))$

**and they are all $O(\log(x))$**

# Big-*O* Estimates for the Factorial Function

Factorial function

$$f(n) = n! = 1 \times 2 \times \cdots \times n\ .$$

$$n! = 1 \times 2 \times \cdots \times n \ \le \ n \times n \times \cdots \times n \ = \ n^n$$

$n!$ is $O(n^n)$ taking $C = 1$ and $k = 1.$

Logarithm of factorial function: log *n*!

Given that $n! \le n^n$ then $\log(n!) \le n\ log(n)$.

Hence, $\log(n!)$ is $O(n\ log(n))$ taking $C = 1$ and $k = 1$.

# Combinations of Functions

If $f(x)$ is $O(g(x))$ and $g(x)$ is $O(h(x))$ then $f(x)$ is $O(h(x))$

If $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$ then $(f_1 * f_2)(x)$ is $O(g_1(x) * g_2(x))$

If $f_1(x)$ and $f_2(x)$ are both $O(g(x))$ then $(f_1 + f_2)(x)$ is $O(g(x))$

If $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$ then $(f_1 + f_2)(x)$ is $O(\max(|g_1(x)|, |g_2(x)|))$

# Combinations of Functions

Proof for

If $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$ then $(f_1 + f_2)(x)$ is $O(\max(|g_1(x)|, |g_2(x)|))$

# Summary

- Big-O for powers, logarithms and factorials
- Big-O for sum and product of functions