Supplemental Notes Week 2

Precedence of Quantifiers

$\forall$, $\exists$ have higher precedence than all other logical connectives!

$\neg \forall x P(x)$ means $\neg (\forall x P(x))$

$\forall x P(x) \to Q(x)$ means $(\forall x P(x)) \to Q(x)$

$\underbrace{\forall x P(x) \wedge Q(x)}$ means $(\forall x P(x)) \wedge Q(x)$

Attention: this is often interpreted as

$\forall x (P(x) \wedge Q(x))$!

this is sloppy !

	precedence
$\forall$ $\exists$	0
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

Domains of Variables

- every variable has its domain
- frequently, e.g. in mathematical statements, the variables share the same domain

3 ways to specify the domain (cf. slide 23, week 2)

1. x is from the domain "students in the class" and $\forall x\, J(x)$
2. x is from the domain "students" and $\forall x\, (S(x) \to J(x))$
3. x is from the domain "students" and $\forall x \in S\; J(x)$

$S(x) :=$ "x is student in the class"

$S \;:=$ the set of all students in the class

Contrapositive of quantified statement

$$\forall x\ P(x) \to \exists y\ Q(y) \equiv \neg \exists y\ Q(y) \to \neg \forall x\ P(x)$$
$$\equiv \forall y\ \neg Q(y) \to \exists x\ \neg P(x)$$

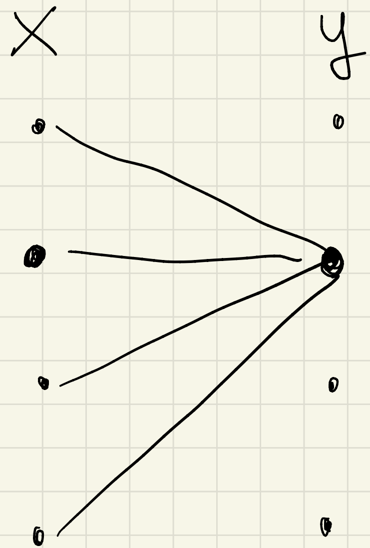
Expressing: " there exist exactly two "

$$\exists x \exists y\ \left(P(x) \land P(y) \land x \neq y \land \forall z\ \left(P(z) \to (z = x \lor z = y)\right)\right)$$

Negation of $\forall x\ \left(P(x) \to Q(x)\right)$

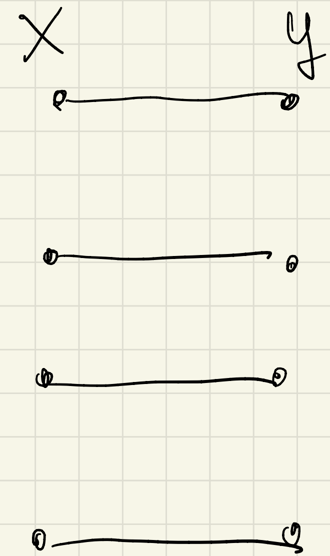
$$\neg \forall x\ \left(P(x) \to Q(x)\right) \equiv \exists x\ \neg\left(\neg P(x) \lor Q(x)\right)$$
$$\equiv \exists x\ \left(P(x) \land \neg Q(x)\right)$$

Why is $\forall x \exists y\, P(x,y) \not\equiv \exists y \forall x\, P(x,y)$?



Here both are true

line means $P(x, y)$ is True



Here $\forall x \exists y\, P(x,y)$ is True

but

$\exists y \forall x\, P(x,y)$ is not true

Solution to Question 2 of Quiz 1:

(a bit more complex than expected ☺)

$$\forall x \; (\neg x = Lee \rightarrow c(x)) \wedge \neg c(Lee)$$

$$\neg(\forall x \; (\neg x = Lee \rightarrow c(x)) \wedge \neg c(Lee))$$

$$\neg \forall x \; (\neg x = Lee \rightarrow c(x)) \vee c(Lee)$$

$$\exists x \; \neg (\neg x = Lee \rightarrow c(x)) \vee c(Lee)$$

$$\exists x \; \neg (x = Lee \vee c(x)) \vee c(Lee)$$

$$\exists x \; (\neg x = Lee \wedge \neg c(x)) \vee c(Lee)$$