Session 83: Estimating Deviations

- Markov's Inequality
- Chebyshev's Inequality
- Examples

Markov's Inequality

Let X be a non-zero and non-negative random variable:

$$\exists s X(s) > 0 \land \forall s X(s) \geq 0$$

Let $p(X \geq a)$ denote the probability that the variable attains a value larger than a

Then $\forall M > 0 \; p(X \geq M \cdot E(X)) \leq \dfrac{1}{M}$

Example

Rolling a dice, where rolling a 6 is considered as success (Bernoulli trial)

When performing 6 trials the expectation value is to obtain 1 success.

Estimate the probability of having at least 3 successes:

$$p(X \geq 3) = p(X \geq 3E(X)) \leq \frac{1}{3}$$

Chebyshev's Inequality

Let X be a random variable on a sample space S with probability function p. If r is a positive real number, then

$$p(|X(s) - E(X)| \geq r) \leq V(X)/r^2$$

Example

Rolling a dice, where rolling a 6 is considered as success (Bernoulli trial)

When performing 6 trials the expectation value is to obtain 1 success.
The variance is *npq* = 6*1/6*5/6 = 5/6

Estimate the probability of having at least 3 successes:

$$p(|X(s) - E(X)| \geq 2) \leq V(X)/2^2 = \frac{5}{6 \cdot 2^2} = \frac{5}{24} \approx \frac{1}{5}$$

Note: this is a smaller probability than ⅓ obtained with the Markov inequality

Example

We can also compute the exact probability of having 3 or more successes using the binomial distribution

$$b(3:6,1/6) + b(4:6,1/6) + b(5:6,1/6) = \frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3}\frac{5^3}{6^6} + \frac{6 \cdot 5}{1 \cdot 2}\frac{5^2}{6^6} + \frac{6}{1}\frac{5}{6^6} \approx 0.06$$

Thus also the estimate using Chebyshev's Inequality was not very sharp

Summary

- Markov's Inequality
  - based on expectation value
- Chebyshev's Inequality
  - based on variance