Advanced information, computation, communication I
EPFL - Fall semester 2021-2022

Week 9

November 19, 2021

1 Open questions

Exercise 1. (*) Prove that if a and b are nonzero integers, a divides b, and $a + b$ is odd, then a is odd.

Exercise 2. (*) Let m be a positive integer. Show that $a \equiv b \pmod{m}$ if a **mod** $m = b$ **mod** m.

Exercise 3. (**) Suppose that a and b are integers, $a \equiv 11 \pmod{19}$, and $b \equiv 3 \pmod{19}$. Find the integer c with $0 \leq c \leq 18$ such that:

1. $c \equiv 13a \pmod{19}$.
2. $c \equiv 7a + 3b \pmod{19}$.
3. $c \equiv a^3 + 4b^3 \pmod{19}$.

Exercise 4. (**) Show that the hexadecimal expansion of a positive integer can be obtained from its binary expansion by grouping together blocks of four binary digits, adding initial zeros if necessary, and translating each block of four binary digits into a single hexadecimal digit.

Exercise 5. (*) Find the sum and product of each of these pairs of numbers. Express your answers as a hexadecimal expansion.

1. $(1AE)_{16}, (BBC)_{16}$
2. $(ABCDE)_{16}, (1111)_{16}$

Exercise 6. (**) Suppose that n and b are positive integers with $b \geq 2$ and the base b expansion of n is $n = (a_m a_{m-1} ... a_1 a_0)_b$. Find the base b expansion of:

1. b^n
2. $\lfloor n/b \rfloor$

Exercise 7. (**) Find the decimal expansion of the number with the $n-$digit base seven expansion $(111...111)_7$ (with n 1's). [Hint: Use the formula for the sum of the terms of a geometric progression.]

Exercise 8. (**) Express in pseudocode the trial division algorithm for determining whether an integer is prime.

Exercise 9. (***) Express in pseudocode an algorithm for finding the prime factorisation of an integer.

Exercise 10. Show that a positive integer is divisible by 3 if and only if the sum of its decimal digits is divisible by 3.

Exercise 11. (*) Find $\gcd(92928, 123552)$ and $\mathrm{lcm}(92928, 123552)$, and verify that $\gcd(92928, 123552) \cdot \mathrm{lcm}(92928, 123552) = 92928 \cdot 123552$.

2 Exam questions

Exercise 12. (***) The sum $((AAAAAA)_{16}^{(AAAAAA)_{16}} + (BBBBBB)_{16}^{(BBBBBB)_{16}}) \mod 8$ is:

- ◯ 1,
- ◯ 3,
- ◯ 5,
- ◯ 7.

Exercise 13. (***) The maximum number of divisions performed when executing the Euclidean algorithm to compute the $\gcd(n_1, n_2)$ for two integers n_1, n_2 with $25 \geq n_1 \geq n_2 \geq 0$ is:

- ◯ 4,
- ◯ 5,
- ◯ 6,
- ◯ 7.

Exercise 14. (*) Let R be a relation on the set of integers such that $(x, y) \in R$ if $\gcd(x, y)$ is a prime number. Then:

- ◯ R is an equivalence relation,
- ◯ R is reflexive and symmetric but not transitive,
- ◯ R is symmetric but not reflexive and not transitive,
- ◯ R is symmetric and transitive, but not reflexive.

$*$ = *easy exercise, everyone should solve it rapidly*
$**$ = *moderately difficult exercise, can be solved with standard approaches*
$***$ = *difficult exercise, requires some idea or intuition or complex reasoning*