Video 11: Inference Rules in Propositional Logic

- Important inference rules
- Examples
- Fallacies

Inference Rules

1. Propositional Logic: Inference Rules
2. Predicate Logic: Inference rules for propositional logic plus additional inference rules to handle variables and quantifiers

Conjunction and Modus Ponens

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Corresponding Tautology:
(p ∧ q) → (p ∧ q)

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Corresponding Tautology:
(p ∧ (p →q)) → q

Inference Rule: Modus Tollens

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Corresponding Tautology:

$(\neg q \land (p \to q)) \to \neg p$

Example:
p := “I have passed AICC”
q := “I can advance to year 2 of the studies”

Premises
“If I have passed AICC, I can advance to year 2 of the studies”
“I cannot advance to year 2 of the studies.”

Conclusion
“I did not pass AICC.”

A trivial inference rule: if $p \equiv q$ then

$$\begin{array}{l} p \\ \hline \therefore q \end{array}$$

is a valid argument.

Note: $p \to q$ is a tautology

<u>Example</u>:

$$\left.\begin{array}{l} p \to q \\ \neg q \end{array}\right\} \text{premises}$$

$\neg q \to \neg p$ contraposition

$\neg p$ modus ponens

Hypothetical Syllogism

Also called:
- chain rule
- transitivity of implication

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

Corresponding Tautology:
((p →q) ∧ (q→r)) → (p→ r)

Example:
r := "I can take Analysis 4"

Premises
"If I have passed AICC, I can advance to year 2 of the studies"
"If I can advance to year 2 of the studies, I can take Analysis 4"

Conclusion
"If I have passed AICC, I can take Analysis 4"

Using truth tables for showing an argument:

The "dumb" argument prover: (DAB)

1. build the complete truth table for all statements in the premises and conclusion
2. remove all rows, where not <u>all</u> premises are T
3. check whether the column of the conclusion contains only T

Example :

p_1

p_2

$\vdots$

p_n

$(p_1 \land \dots \land p_n) \to q$

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q

variables			premises			conclusion
p_1	$\dots$	p_n	$p_1 \land p_2$	$\dots$	$(p_1 \land \dots p_n) \to q$	q
T	$\dots$	T		T		$\boxed{T}$
~~T~~	~~$\dots$~~	~~F~~		~~F~~		~~F~~

Here DAB performs a lot of "unnecessary work" that can with additional intelligence be avoided (we just need the rows where $p_1 \dots p_n$ are all true)

Example

$p_1 \to p_2$

$p_2 \to p_3$

$\vdots$

$p_{n-1} \to p_n$

$p_1 \to p_n$

which rows do eliminate?

p_1	$\dots$	p_n	$p_1 \to p_2$	$\dots$	$p_{n-1} \to p_n$	$p_1 \to p_n$
T	$\dots$	T				
T	$\dots$	F				

Here it is less evident how to safe work!

Resolution

$$\begin{array}{l} \neg p \vee r \\ p \vee q \\ \hline \therefore q \vee r \end{array}$$

Corresponding Tautology:

((¬p ∨ r) ∧ (p ∨ q)) → (q ∨ r)

Example:
p := “The weather is nice
q := “I am at home”
r := “I am at the beach”

Premises:
“The weather is bad or I am at the beach”
“The weather is nice or I am at home”

Conclusion:
“I am at home or at the beach”

Resolution plays an important role in automated theorem proofing and AI

It allows to eliminate propositional variables from the premises

Resolution rules them all

Chain Rule is a "hidden" resolution

$p \to q$

$q \to r$

$\neg p \lor q$ equivalenc

$\neg q \lor r$ equivalence

$\neg p \lor r$ resolution

$p \to r$

Other Inference Rules

$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism	Simpler form of resolution
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition	Dual to Conjunction
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification	Simpler form of Modus Ponens

Valid Arguments

Attention: even seemingly "obvious" conclusions imply an argument

Example: From $p \wedge (p \rightarrow q)$ conclude q

$p \wedge (p \rightarrow q)$	
p	simplification
$p \rightarrow q$	simplification
q	modus ponens

Fallacies!

$((p \to q) \land q) \to p$ *is not a tautology*

- **fallacy of affirming the conclusion**

Example:

- If you do every problem in this book, then you will learn discrete mathematics. You learned discrete mathematics.
- Therefore, you did every problem in this book?

Fallacies!

$((p \to q) \wedge \neg p) \to \neg q$ is not a tautology

- **fallacy of denying the hypothesis**

Example:

- If you do every problem in this book, then you will learn discrete mathematics. You did not do every problem in this book.
- Therefore, you did not learn discrete mathematics?

Summary

- Modus Ponens, Modus Tollens
- Hypothetical Syllogism
- Resolution
- How to build valid arguments
- Fallacies
  - affirming the conclusion
  - denying the hypothesis