Advanced information, computation, communication I
EPFL - Fall semester 2021-2022

# Week 7

November 5, 2021

## 1 Open Questions

**Exercise 1.** (**) Recall the exercise from last week where we used Bubble Sort, Selection Sort and Insertion Sort to sort the following sequence:

`9, 12, -43, 20, -2, 3, 7, 28, 19`

The pseudocode for all three is provided below

**Algorithm 1** Bubble Sort

**for** $i \leftarrow 1$ **to** $n-1$ **do**
  **for** $j \leftarrow 1$ **to** $n-i$ **do**
    **if** $a_j > a_{j+1}$ **then**
      swap $a_j$ and $a_{j+1}$

**Algorithm 2** Selection Sort

**for** $i \leftarrow 1$ **to** $n-1$ **do**
  $\min \leftarrow i+1$
  **for** $j \leftarrow i+1$ **to** $n$ **do**
    **if** $a_{\min} > a_j$ **then**
      $\min \leftarrow j$
  **if** $a_i > a_{\min}$ **then**
    swap $a_i$ and $a_{\min}$

**Algorithm 3** Insertion Sort

**for** $j \leftarrow 2$ **to** $n$ **do**
  $i \leftarrow 1$
  **while** $a_j > a_i$ and $i < j$ **do**
    $i \leftarrow i+1$
  $m \leftarrow a_j$
  **for** $k \leftarrow 0$ **to** $j-i-1$ **do**
    $a_{j-k} \leftarrow a_{j-k-1}$
  $a_i \leftarrow m$

1. How many comparisons are done in each of the algorithms?
2. How many swaps are done in each of the algorithms?
3. What is the approximate overall cost of the two algorithms for an input sequence of length $n+1$?

**Exercise 2.** (*)

1. Show that $5x$ is $o(x^2)$.
2. Show that $2x^2$ is not $o(x^2)$.
3. Show that $1/x$ is $o(x)$.

**Exercise 3.** (**) Let $f$ be arbitrary functions from $\mathbf{N}$ to $\mathbf{R}_{>0}$.

Let $g_1, g_2$ be two functions from $\mathbf{N}$ to $\mathbf{R}_{>0}$ such that $g_1$ and $g_2$ are both $\Theta(f)$.

1. Show that the function $g_1 + g_2$ is $\Theta(f)$ or provide a counterexample.
2. Show that the function $g_1 g_2$ is $\Theta(f^2)$ or provide a counterexample.

Let $g_3, g_4$ be two functions from $\mathbf{N}$ to $\mathbf{R}$ such that $g_3$ and $g_4$ are both $\Theta(f)$.

3. Show that the function $g_3 + g_4$ is $\Theta(f)$ or provide a counterexample.
4. Show that the function $g_3 g_4$ is $\Theta(f^2)$ or provide a counterexample.

Let $g$ be a function from $\mathbf{N}$ to $\mathbf{R}_{>0}$ such that $g$ is $O(f)$.

5. Show that $2^g$ is $O(2^f)$, or provide a counterexample.

**Exercise 4.** (*) What is the largest $n$ for which one can solve within a minute using an algorithm that requires $f(n)$ bit operations, where each bit operation is carried out in $10^{-12}$ seconds, with these functions $f(n)$?

a. $\log n$

b. $1,000,000n$

c. $n^2$

## 2 Exam Questions

**Exercise 5.** (***) How many comparisons among list elements does insertion sort perform when sorting the following list of length $2n$, $n \geq 1$, in ascending order:

$$2n-1, 2n-3, \ldots, 3, 1, 2n, 2n-2, \ldots, 4, 2$$

○ $\frac{1}{2}(n^2+3n-2)$

○ $\frac{1}{2}(n^2+5n-4)$

○ $\frac{1}{2}(n^2+7n-6)$

○ $\frac{1}{2}(n^2+n)$

**Exercise 6.** (***) Which of the following functions has the fastest growth when $n$ goes to infinity?

○ $2^{(\log_2(\log_2 n))^2}$

○ $(\log_2 n)^{2(\log_2 n)^2}$

○ $(\log_2(n^2))^{\log_2(n^2)}$

○ $n^{\log_2(\log_2 n)}$

**Exercise 7.** (**) Which function below grows fastest when $n$ goes to infinity?

○ $(\log_3(33))^{n-3}$

○ $3^n$

○ $n^{3\log_3(n)}$

○ $n^3\log_3(n)$

**Exercise 8.** (**) Consider the two statements below, where $k$ and $\ell$ are constants with $k > \ell \geq 2$ and $m \to \infty$:

$$\log_m(k) \text{ is } \Theta(\log_m(\ell)) \qquad k^{\log_\ell(m)} \text{ is } O(\ell^{\log_k(m)}).$$

○ They are both false.

○ Only the first is true.

- ◯ Only the second is true.
- ◯ They are both true.

**Exercise 9.** (*) Consider the following two statements:

$$f \text{ is } o(f) \qquad\qquad f \text{ is } o(g) \text{ implies } f \text{ is } O(g).$$

- ◯ Only the second is true.
- ◯ They are both false.
- ◯ Only the first is true.
- ◯ They are both true.

**Exercise 10.** (**) Given the two statements below, where $d > 0$ is an integer constant and $a_i$ are strictly positive integers for all $i \in \mathbf{Z}$, with $\max_{i\in\mathbf{Z}} a_i = D$, $\min_{i\in\mathbf{Z}} a_i = M$ for $D > 0, M > 0$,

$$\sum_{i=0}^{n} a_i i^d \text{ is } \Theta(n^{d+1}) \qquad\qquad \sum_{i=0}^{d} a_i n^i \text{ is } \Theta(n^d)$$

- ◯ They are both true.
- ◯ Only the first is true.
- ◯ Only the second is true.
- ◯ They are both false.