Session 18: Set Operations

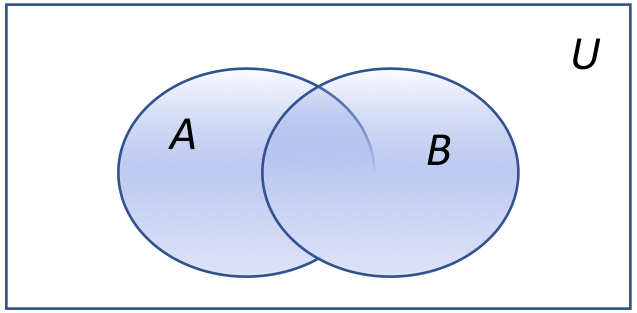
- Set Operations
  - Union
  - Intersection
  - Complement
  - Difference
  - Symmetric Difference

Union

Definition: Let *A* and *B* be sets. The **union** of the sets *A* and *B*, denoted by $A \cup B$, is the set:

$$\{x | x \in A \vee x \in B\}$$

Venn Diagram for $A \cup B$



Example: $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$

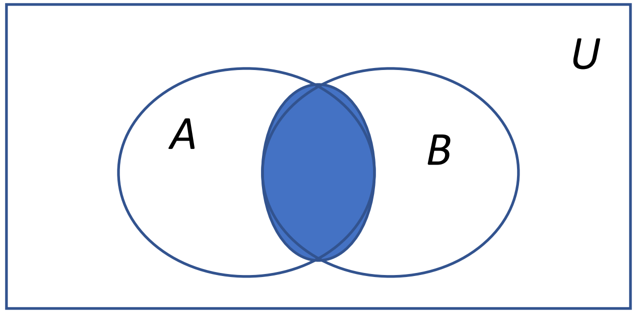
Intersection

Definition: The **intersection** of sets *A* and *B*, denoted by $A \cap B$, is

$$\{x | x \in A \wedge x \in B\}$$

If the intersection is empty, then *A* and *B* are said to be **disjoint**.

Venn Diagram for $A \cap B$



Example: $\{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$

$\{1, 2, 3\} \cap \{4, 5, 6\} = \varnothing$

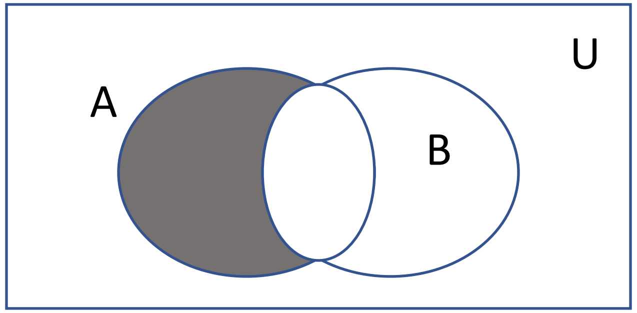
Difference

Definition: The **difference** of sets A and B, denoted by $A - B$, is the set containing the elements of A that are not in B.

$$A - B = \{x \mid x \in A \wedge x \notin B\} = A \cap \overline{B}$$

The difference of A and B is also called the **complement** of B with respect to A.

Venn Diagram for $A - B$



Example: $\{1, 2, 3\} - \{3, 4, 5\} = \{1, 2\}$

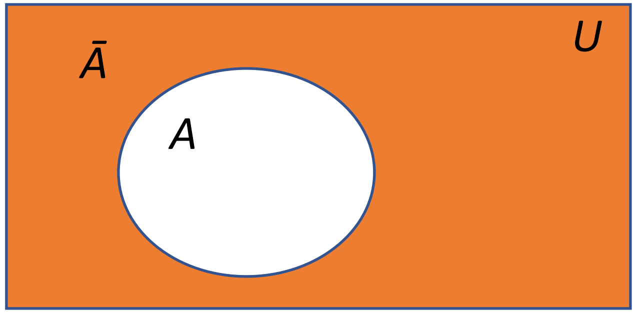
Complement

Definition: If A is a set, then the complement of the A with respect to the universe U, denoted by $\bar{A}$ is the set

$$\bar{A} = U - A = \{x \in U \mid x \notin A\}$$

The complement of A is also denoted by A^c.

Venn Diagram for Complement



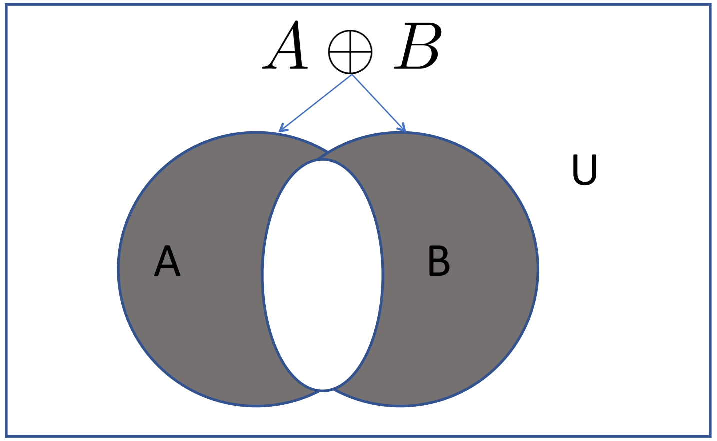
Example: If U is the positive integers, $\{x \mid x > 70\}^c = \{x \mid x \leq 70\}$

Symmetric Difference

Definition: The **symmetric difference** of sets *A* and *B*, denoted by A⊕B is the set

$$(A - B) \cup (B - A)$$

Venn Diagram



Example: *A* = {1, 2, 3, 4, 5} , *B* ={4, 5, 6, 7, 8}, A⊕B = {1, 2, 3, 6, 7, 8}

Analogy Set Operations – Propositional Calculus Connectives

$\cup$ corresponds to $\vee$ $\qquad$ $A \cup B = \{x / x \in A \vee x \in B\}$

$\cap$ corresponds to $\wedge$ $\qquad$ $A \cap B = \{x / x \in A \wedge x \in B\}$

$\bar{A}$ corresponds to $\neg$ $\qquad$ $\bar{A} = \{x \in U \;/\; \neg\, x \in A\} = \{x \in U \;/\; x \notin A\}$

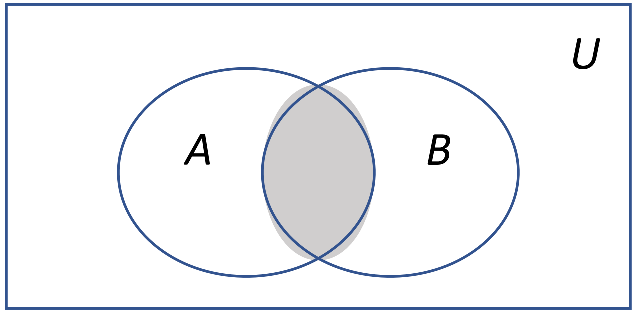
$\oplus$ corresponds to $\oplus$ $\qquad$ $A \oplus B = \{x / x \in A \oplus x \in B\}$

Cardinality of Set Union

Inclusion-Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Venn Diagram for *A*, *B*, $A \cap B$, $A \cup B$



Summary

- Set Operations
- Analogy to Propositional Logic
- Inclusion-Exclusion