

Quiz Questions: Propositional Logic

1. Which of the following is the negation of the statement "I drive to work if and only if it is rainy"?

- A. If I drive to work, then it is not rainy.
- B. I drive to work if it is not rainy.
- C. I drive to work if and only if it is not rainy.
- D. I do not drive to work if and only if it is not rainy.

Note:

$$\begin{aligned}\neg(X \leftrightarrow Y) &\Leftrightarrow \neg((X \rightarrow Y) \wedge (Y \rightarrow X)) \\ &\Leftrightarrow \neg(X \rightarrow Y) \vee \neg(Y \rightarrow X) \\ &\Leftrightarrow (X \wedge \neg Y) \vee (Y \wedge \neg X).\end{aligned}$$

So the negation of "X is true if and only if Y is true" is "Either X is true and Y is false, or X is false and Y is true." This is in turn equivalent to "X is true if and only if Y is false" (just compare the cases when they are each true). So you also get that

$$\neg(X \leftrightarrow Y) \Leftrightarrow X \leftrightarrow \neg Y \Leftrightarrow \neg X \leftrightarrow Y$$

2. Which each of these compound propositions is satisfiable?

- A. $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$
- B. $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$
- C. $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

3. Which of the propositions is logically equivalent to $(p \wedge q) \vee (\neg p \wedge \neg q)$?

- A. $p \leftrightarrow \neg q$
- B. $p \leftrightarrow q$
- C. $\neg q \leftrightarrow p$
- D. $q \leftrightarrow \neg p$

4. Which of the following propositions is correct?

- A. The inverse of the implication $p \rightarrow q$ is logically equivalent to $p \rightarrow q$.
- B. The converse of the implication $p \rightarrow q$ is logically equivalent to the inverse of $p \rightarrow q$.
- C. The contrapositive of the implication $p \rightarrow q$ is logically equivalent to the inverse of $p \rightarrow q$.
- D. The converse of the implication $p \rightarrow q$ is logically equivalent to the contrapositive of $p \rightarrow q$.

5. Select those statements that are true (multiple answers are possible)

- A. If $1 + 1 = 3$, then $2 + 2 = 4$
- B. If $1 + 1 = 2$, then $2 + 2 = 5$
- C. If monkeys can fly, then $1 + 1 = 3$
- D. If $1 + 1 = 2$ if and only if $2 + 3 = 4$

6. Suppose m and t are the propositions

m : “you are a member of the team”

t : “you take afternoon classes.”

Express in English the compound proposition $m \rightarrow \neg t$.

- A. You are a member of the team only if you take afternoon classes.
- B. You are a member of the team only if you don't take afternoon classes.
- C. If you don't take afternoon classes, then you are a member of the team.
- D. If you take afternoon classes, then you are a member of the team.

7. Suppose h and c are these propositions:

h : “I go hiking”

c : “it is a cold day.”

Express in symbols the compound proposition

“I don't go hiking when it is a cold day.”

- A. $h \rightarrow c$.
- B. $c \rightarrow \neg h$.
- C. $\neg c \rightarrow h$.
- D. $\neg h \rightarrow c$.

8. The negation of the statement “If I think, then I am” is given by:

- A. I am not, and I think.
- B. If I am not, then I do not think.
- C. I am, and I do think.
- D. I do not think, or I am not.