# Session 15: Introduction to Sets

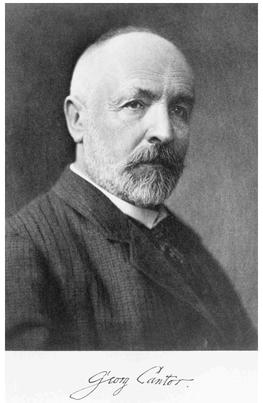
- Sets
- Specification of sets
- Sets of Numbers
- Special sets

# Sets, Functions and Relations

- Basic abstractions in mathematics and computer science
- Data structures are constructed using these abstractions
  - Unordered collections ~ sets
  - Order collections ~ sequences, which are functions
  - Networks, graphs ~ relations
  - Databases ~ relations
  - Objects ~ functions
- Computing is modelled using these abstractions
  - Finite state machines ~ relations
  - Programs are decomposed into functions
  - Cost of programs is expressed as functions

# Some History

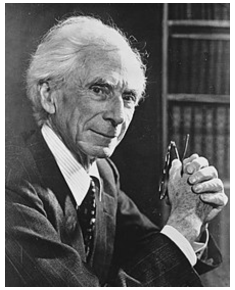
- Cantor, Founder of set theory, discovered uncountable sets
- Peano, Introduced notations ( $\in$ , $\cup$ , $\cap$ ) and axioms for natural numbers based on set theory
- ZF developed the currently widely accepted axioms for set theory
- Wrote Pincipia Mathematica, deriving all mathematics from primitive axioms, know for his antimony on set theory



Georg Cantor, 1845 - 1918

Guiseppe Peano, 1858 - 1932

Zermelo Fraenkel



Bertrand Russell, 1872 - 1970

# Introduction

- Sets are one of the basic building blocks in discrete mathematics.
  - Basis for counting, functions, relations
  - Programming languages have set operations
  - Databases are sets of discrete objects

- Set theory is an important branch of mathematics.
  - Many different systems of axioms have been used to develop set theory.
  - Here we are not concerned with a formal set of axioms for set theory.
  - Instead, we will use what is called **naïve set theory**.

# Sets

- A **set** is an unordered collection of objects.
  - the students in this class
  - the chairs in this room

- The objects in a set are called the **elements** of the set.
- A set is said to **contain** its elements.

- The notation $\boldsymbol{a} \in \boldsymbol{A}$ denotes that $a$ is an element of the set $A$.
- If $a$ is not an element of $A$, write $\boldsymbol{a} \notin \boldsymbol{A}$ $\equiv \neg a \in A$

# Describing a Set: Roster Method

Listing all elements of a set

$S = \{a, b, c, d\}$

$\forall x \, (x \in S \leftrightarrow x = a \lor x = b \lor x = c \lor x = d)$

# Describing a Set: Roster Method

Listing all elements of a set

$$S = \{a, b, c, d\}$$

- Order not important: $S = \{a, b, c, d\} = \{b, c, a, d\}$
- Multiple occurrences not important: $S = \{a, b, c, d\} = \{a, b, c, b, c, d\}$

Elipses (…) may be used to describe a set without listing all of the members when the pattern is clear

$$S = \{a, b, c, d, \ldots\ldots, z\}$$

# Examples

Set of all vowels in the English alphabet: $\{a, e, i, o, u\}$

Set of all  odd positive integers less than 10: $\{1, 3, 5, 7, 9\}$

Set of all positive integers less than 100: $\{1, 2, \ldots, 99\}$

Set of all integers less than 0: $\{-1, -2, \ldots\}$

# Sets of Numbers

**N** = **natural numbers** = {0, 1, 2, 3, ….}

**Z** = **integers** = {…, -3, -2, -1, 0, 1, 2, 3, …}

**Z**$^{+}$ = **positive integers** = {1, 2, 3, …..}

**R** = set of **real numbers**

**R**$^{+}$ = set of **positive real numbers**

**C** = set of **complex numbers**

**Q** = set of ***rational numbers***

# Set-Builder Notation

Specify the property or properties that all members must satisfy:

$$S = \{x \mid P(x)\}$$

- P(x) may be expressed in natural language or predicate logic

$$\forall x\,(x \in S \leftrightarrow P(x))$$

# Examples

$S = \{x \mid x \text{ is a positive integer less than } 100\}$

$O_1 = \{x \mid x \text{ is an odd positive integer less than } 10\}$ $\{1, 3, 5, 7, 9\}$

$O_2 = \{x \in \mathbf{Z}^+ \mid x \text{ is odd and } x < 10\}$ $O_1 = O_2$

$P = \{x \mid \text{Prime}(x)\}$

$\mathbf{Q}^+ = \{x \in \mathbf{R} \mid x = p/q, \text{ for some positive integers } p, q\}$

# Interval Notation

For sets of numbers

$[a,b] = \{x \mid a \le x \le b\}$

$[a,b) = \{x \mid a \le x < b\}$

$(a,b] = \{x \mid a < x \le b\}$

$(a,b) = \{x \mid a < x < b\}$

**closed interval** [a,b]

**open interval** (a,b)

# Universal Set and Empty Set

The **universal set U** is the set containing everything currently under consideration.

- Sometimes implicit
- Sometimes explicitly stated
- Contents depend on the context

The **empty set** is the set with no elements.

$\forall x \ (x \notin \emptyset)$

- Denoted as ∅ or {}

# Some things to remember

Sets can be elements of sets.

$$\{\{1, 2, 3\}, a, \{b, c\}\}$$

$$\{\mathbf{N}, \mathbf{Z}, \mathbf{Q}, \mathbf{R}\}$$

The empty set is different from a set containing the empty set.

$$\varnothing \neq \{\varnothing\}$$

# Summary

- Set definition
  - Roster method
  - Set Builder Notation
- Sets of Numbers
- Interval Notation
- Empty and Universal Set