Quiz Questions: Advanced Counting

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1. The following relation gives the number a_n of bitstrings of length $2n$:
   A. $a_0 = 0, a_n = 4a_{n-1}$
   B. $a_0 = 1, a_n = 2a_{n-1}$
   C. $a_0 = 1, a_n = 4a_{n-1}$
   D. $a_0 = 0, a_n = 2a_{n-1}$

2. The solution to the recurrence relation $a_n = a_{n-1} + 2n$ with initial term $a_0 = 2$ is:
   A. $n(n+1) + 2$
   B. $3n^2$
   C. $2(1+n)$
   D. $4n + 7$

3. The recurrence relation $a_n = a_{n-1} + n\, a_{n-2} + 1$ is:
   A. linear, but not homogeneous
   B. homogeneous, but not with constant coefficients
   C. of degree 1 and linear
   D. with constant coefficients, but not linear

4. The generating function of a sequence a_n
   A. has roots that are used to construct a closed form solution of the sequence
   B. is a closed form solution for the sequence
   C. has the elements a_n as coefficients of x^n in its power series expansion
   D. recursively computes the elements of the sequence

5. The solution to the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$ has the following form
   A. $c_0 2^n + c_1 n$
   B. $c_0 + c_1 n$
   C. $2c_0 - n$
   D. $c_0 2^n + n c_1 2^n$

6. The characteristic equation of the recurrence relation $a_n = a_{n-1} - 2a_{n-2} + 3a_{n-3}$ is
   A. $r^3 - r^2 + 2r - 3 = 0$
   B. $r^2 + 2r - 3 = 0$
   C. $r^3 + r^2 - 2r + 3 = 0$
   D. $r^2 - 2r + 3 = 0$

7. The generating function for the sequence $a_n = 2^n$ is:
   A. $\frac{1}{1-x^2}$
   B. $\frac{1}{1+2x}$
   C. $(1+2x)^n$
   D. $\frac{1}{1-2x}$

8. Number of elements in $A_1 \cup A_2 \cup A_3$ if A_1 is subset of A_2 and $|A_2 \cap A_3| = 1$:
   A. $|A_1| + |A_2| + |A_3| - 1$
   B. $|A_1| + |A_3| - 1$
   C. $|A_2| + |A_3| - 1$
   D. $|A_3| + 1$

Solutions:

1. C
2. A
3. A
4. C
5. B
6. A
7. D
8. C