# Session 73: Discrete Probability

- Finite probability
- Examples

# Short History of Probability

Girolamo Cardano
1501 - 1576

Pierre-Simon Laplace
1749 - 1827

- Cardano: first treatment of probability (notoriously short of money)
- Bayes: Bayes' Theorem, first concept of statistical data analysis
- Laplace: classical theory of probability was introduced in the 18th century, when he analysed games of chance.
- Kolmogorov: modern axiomatic probability theory

Thomas Bayes
1701 - 1761

Andrey Kolmogorov
1902 - 1987

# Probability of an Event

**Definitions**:

- An **experiment** is a procedure that yields one of a given set of possible **outcomes**: rolling the dice
- The **sample space S** of the experiment is the set of possible outcomes: faces of the dice S = {1,2,3,4,5,6}
- An **event E** is a subset of the sample space: e.g., E = {5,6}
- An event **occurs**, if the outcome belongs to the event: E occurs when rolling a 6

# Probability of an Event

**Definition**: If *S* is a finite sample space of equally likely outcomes, and *E* is an event, that is, a subset of *S*, then the **probability** of *E* is

$$p(E) = \frac{|E|}{|S|}$$

For every event *E*, we have $0 \leq p(E) \leq 1$.

Example, Dice : two fair dice, $6^2 = 36$ poss. outcomes

Probability to roll a sum of 7:

1+6, 2+5, 3+4, 4+3, 5+2, 6+1 6 outcomes

$\frac{6}{36} = \frac{1}{6}$ probability

Probability to roll at least one 6:

1+6, 2+6, 3+6, 4+6, 5+6, 6+6, 6+1, 6+2, 6+3, 6+4, 6+5, 11 outcomes

$\frac{11}{36}$ probability $\approx 30\%$

# Example

For Euromillions choose a set of 5 numbers out of 50 and 2 numbers out of 12.

What is the probability that a person picks the correct 5+2 numbers?

The number of ways to choose 5 numbers out of 50 is

$$C(50,5) = 50!/(45!5!) = 2'118'760$$

The number of ways to choose 2 numbers out of 12 is

$$C(12,2) = 66$$

Using the product rule, the probability of picking a winning combination is

$$1/(66*2'118'760) = 1/139'838'160 \approx 0.0000000072.$$

# Example

What is the probability that the numbers 11, 4, 17, 39, and 23 are drawn in that order from a bin with 50 balls labeled with the numbers 1,2, …, 50 if

1. The ball selected is not returned to the bin.
2. The ball selected is returned to the bin before the next ball is selected.

Use the product rule in each case.

1. *Sampling without replacement*: The probability is 1/254,251,200 since there are $50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 = 254{,}251{,}200$ ways to choose the five balls.
2. *Sampling with replacement*: The probability is $1/50^5 = 1/312{,}500{,}000$ since $50^5 = 312{,}500{,}000$.

# Summary

- Finite probability
  - Sample Space, Event, Probability
- Examples
  - Card Games, Lotteries