Advanced information, computation, communication I
EPFL - Fall semester 2021-2022

Week 2

October 1, 2021

1 Open Questions

Exercise 1. (*) (Rosen, exercise 8, sec. 1.4 in 8^{th} edition) Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.

1. $\forall x(R(x) \to H(x))$
2. $\exists x(R(x) \to H(x))$
3. $\forall x(R(x) \land H(x))$
4. $\exists x(R(x) \land H(x))$

Exercise 2. (**) (Rosen, exercise 9, sec. 1.5 in 8^{th} edition) Let $L(x, y)$ be the statement "x loves y", where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements:

1. Everybody loves Sharon.
2. Everybody loves somebody.
3. There is somebody whom everybody loves.
4. Nobody loves everybody.
5. There is somebody whom Daisy does not love.
6. There is somebody whom no one loves.
7. There is exactly one person whom everybody loves.
8. There are exactly two people whom Marsellus loves.
9. Everyone loves himself or herself.
10. There is someone who loves no one besides himself or herself.

Exercise 3. (*) (Rosen, exercise 16, sec. 1.4 in 8^{th} edition) Determine the truth value of each of these statements if the domain consists of all integers.

1. $\exists x(x^2 = 2)$
2. $\exists x(x^2 = -1)$
3. $\forall x(x^2 + 2 \geq 1)$
4. $\forall x(x^2 \neq x)$

Exercise 4. (*) (Rosen, exercise 26, sec. 1.5 in 8^{th} edition) Let $Q(x, y)$ be the statement "$x + y = x - y$." If the domain for both variables consists of all integers, what are the truth values?

1. $Q(1, 1)$
2. $Q(2, 0)$
3. $\forall y Q(1, y)$
4. $\exists x Q(x, 2)$
5. $\exists x \exists y Q(x, y)$
6. $\forall x \exists y Q(x, y)$
7. $\exists y \forall x Q(x, y)$
8. $\forall y \exists x Q(x, y)$
9. $\forall x \forall y Q(x, y)$

Exercise 5. (*) (Rosen, exercise 37, sec. 1.4 in 8^{th} edition) Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

1. $\forall x (x^2 \geq x)$
2. $\forall x (x > 0 \vee x < 0)$
3. $\forall x (x = 1)$

2 Exam Questions

Exercise 6. (**) Given the two statements below, where the domain of discourse is $\mathbf{R}$ for both x and y,

$$\exists y \forall x (x \neq 0 \rightarrow xy = 1) \qquad\qquad \exists x \forall y (xy < 0 \rightarrow xy > 0)$$

- ◯ They are both false.
- ◯ Only the first is true.
- ◯ Only the second is true.
- ◯ They are both true.

Exercise 7. (**) Consider the two statements below, where $P(x, y)$ is a propositional function and the domain of discourse is $\mathbf{Z}_{\geq 0}$ for x, y and z:

$$(\exists y \forall x \, P(x, y)) \rightarrow (\forall x \exists y \, P(x, y)) \qquad\qquad (\neg \exists x \ x^x = x!) \rightarrow \forall y, z \ y \neq z.$$

- ◯ They are both false.
- ◯ Only the first is true.
- ◯ Only the second is true.
- ◯ They are both true.

Exercise 8. (**) Let E be a set of endpoints on a network, let P be a set of paths connecting those endpoints, and let $C(p, x, y)$ be the proposition that path $p \in P$ connects endpoints x and y with $x, y \in E$. The statement "there are at least two paths connecting every two distinct endpoints on the network" can be expressed by

- ○ $\forall x, y \in E \left(x \neq y \to \exists p, q \in P \left(p \neq q \wedge (C(p, x, y) \vee C(q, x, y))\right)\right)$.
- ○ $\forall x, y \in E \left(x \neq y \wedge \exists p, q \in P \left(p \neq q \wedge C(p, x, y) \wedge C(q, x, y)\right)\right)$.
- ○ $\neg\left(\exists x, y \in E \left(x \neq y \ \wedge\ \forall p, q \in P \left(p = q \vee \neg C(p, x, y) \vee \neg C(q, x, y)\right)\right)\right)$.
- ○ $\neg\left(\exists x, y \in E \left(x \neq y \ \wedge\ \forall p, q \in P \left(p = q \wedge \neg C(p, x, y) \wedge \neg C(q, x, y)\right)\right)\right)$.

Exercise 9. (**) Given the propositional function $T(x)$, the statement $\exists! x\, T(x)$ is logically equivalent to

- ○ $\neg(\forall x\, [T(x) \to \exists y \neq x\, T(y)])$.
- ○ $\exists x \forall y ((\neg T(y)) \vee (y = x))$.
- ○ $\exists x (T(x) \vee \forall y [(\neg T(y)) \vee (y = x)])$.
- ○ $\exists x (T(x) \wedge \forall y [T(y) \wedge (y = x)])$.

Exercise 10. (**) Given the propositional functions $G(x)$: "x is a boy", $F(y)$: "y is a girl", and $A(z)$: "z likes computers", the statement "all boys like computers and there is a girl that does not like computers" can be expressed by

- ○ $\neg[(\exists y\, (G(y) \wedge \neg A(y))) \vee (\forall x\, (F(x) \to A(x)))]$.
- ○ $\neg[(\forall x\, (F(x) \to A(x))) \vee (\exists y\, (G(y) \to \neg A(y)))]$.
- ○ $(\exists x\, (F(x) \to \neg A(x)) \wedge (\forall y\, ((\neg G(y)) \vee A(y)))$.
- ○ $(\forall y\, (G(y) \wedge A(y))) \wedge (\exists x\, (F(x) \wedge \neg A(x)))$.

Exercise 11. (*) Which expressions below are equivalent to $\neg(\forall x \exists y\, P(x, y))$. Explain.

- ○ $\exists x \forall y\ \neg P(x, y)$;
- ○ $\exists x \exists y\ \neg P(x, y)$.

$*$ = *easy exercise, everyone should solve it rapidly*
$**$ = *moderately difficult exercise, can be solved with standard approaches*
$***$ = *difficult exercise, requires some idea or intuition or complex reasoning*