Session 47: Recursively Defined Functions

- Recursively Defined Functions
- Fibonacci Numbers

Recursively Defined Functions

Definition: A **recursive** or **inductive definition** of a function *f* nonegative integers as domain consists of two steps.

- BASIS STEP: Specify the value of the function at zero.
- RECURSIVE STEP: Give a rule for finding its value at an integer from its values at smaller integers.

A function $f(n)$ is a sequence a_0, a_1, … , where $f(i) = a_i$.

Example

Suppose f is defined by

$$f(0) = 3,$$

$$f(n + 1) = 2 f(n) + 3$$

Example

Recursive definition of the factorial function $n!$

$f(0) = 1$

$f(n + 1) = (n + 1) \cdot f(n)$

For a sequence a_k give a recursive definition of $f(n) = \sum_{k=0}^{n} a_k$

Fibonacci Numbers

The Fibonacci numbers are defined as follows:

$f_0 = 0$

$f_1 = 1$

$f_n = f_{n-1} + f_{n-2}$

Property of Fibonacci Numbers

Show that whenever $n \geq 3$, $f_n > \alpha^{n-2}$, where $\alpha = (1 + \sqrt{5})/2$.

Summary

- Recursively Defined Functions
- Fibonacci Numbers
- Proving properties of Recursively Defined Functions using induction