Video 12: Arguments in Predicate Logic

- Inference Rules for Quantifiers
- Building valid arguments

Handling Quantified Statements

- Valid arguments for quantified statements are a sequence of statements.
- Each statement is either a premise or follows from previous statements by rules of inference which include:
  - Rules of Inference for Propositional Logic
  - Rules of Inference for Quantified Statements

Universal Instantiation (UI)

$$\frac{\forall x P(x)}{\therefore P(c)}$$

Example: The domain consists of all Men and Socrates is a Man

Premise:
"All men are mortal."

Conclusion:
"Therefore, Socrates is mortal."

$$\frac{\forall x \, \mathrm{Mortal}(x)}{\mathrm{Mortal}(\mathrm{Socrates})}$$

Solution for Socrates Example

If we choose a more general domain, e.g. all beings, including gods and spirits, we need a more elaborate proof to build a valid argument

- Both rules for propositional logic and quantifiers

Step	**Reason**
1. $\forall x(Man(x) \rightarrow Mortal(x))$	Premise
2. $Man(Socrates) \rightarrow Mortal(Socrates)$	UI from (1)
3. $Man(Socrates)$	Premise
4. $Mortal(Socrates)$	MP from (2) and (3)

Universal Generalization (UG)

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$$

Used often implicitly in Mathematical Proofs.

Attention: you must not make any assumptions about c

Existential Instantiation (EI)

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$$

Example:

"There is someone who knows Java in the class."
"Let's call her *a* and say that *a* knows Java"

Note: we do not know who is "a"!

Existential Generalization (EG)

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$$

Example:

“Michelle knows Java in the class.”
“Therefore, someone knows Java in the class.”

Universal Modus Ponens

$$\begin{array}{l} \forall x(P(x) \to Q(x)) \\ P(a), \text{where } a \text{ is a particular} \\ \quad\text{element in the domain} \\ \hline \therefore Q(a) \end{array}$$

Universal Modus Ponens combines universal instantiation and modus ponens into one rule.

This rule could be used in the Socrates example.

Summary

- Inference Rules for Quantifiers
  - Universal Instantiation
  - Universal Generalization
  - Existential Instantiation
  - Existential Generalization
  - Universal Modus Ponens

Use the rules of inference to construct a valid argument showing that the conclusion

"Someone who passed the first exam has not read the book."

follows from the premises

"A student in this class has not read the book."

"Everyone in this class passed the first exam."

Define predicates:

$C(x) :=$ x is a student in the class

$B(x) :=$ x read the book

$E(x) :=$ x passed the exam

Premises: ① $\exists x\,(C(x) \wedge \neg B(x))$

② $\forall x\,(C(x) \rightarrow E(x))$

Conclusion $\exists x\,(E(x) \wedge \neg B(x))$

③ $C(a) \rightarrow E(a)$, all a UI (from 2)

④ $C(\hat{a}) \wedge \neg B(\hat{a})$, some $\hat{a}$ EI (from 1)

⑤ $C(\hat{a})$ simplification (from 3, since true for all a in particular also for $\hat{a}$)

⑥ $E(\hat{a})$ modus ponens (from 4, 5)

⑦ $\neg B(\hat{a})$ simplif (from 4)

⑧ $E(\hat{a}) \wedge \neg B(\hat{a})$ Conjunction (from 6, 7)

⑨ $\exists x\,(E(x) \wedge \neg B(x))$ EG (from 8)