Session 10: Valid Arguments

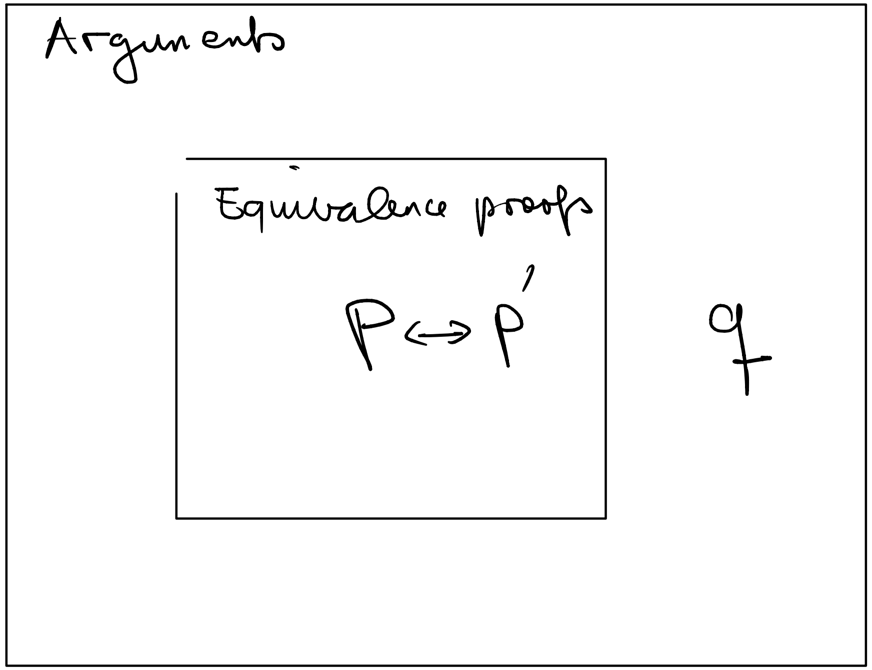
- Arguments
- Argument Forms
- Inference Rules
- Valid Arguments

Deriving Knowledge

Assume you no something, a proposition p

If $p \Leftrightarrow p'$, then p and p' represent the same knowledge

If $p \rightarrow q$, then we also know q, by knowing p
but q is inferred knowledge, and not necessarily allows to "reproduce" p



Example

Assume the following rule holds

> "If I have passed AICC, I can advance to year 2 of the studies"

Example

what I know

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"If I have passed AICC, I can advance to year 2 of the studies"

And assume that you know

"I have passed AICC"

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Assume the following rule holds

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Then you would conclude

what I derive

“I can advance to year 2 of the studies”

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Then you would conclude

“I can advance to year 2 of the studies”

This is a **valid argument**

arguments allow
to infer knowledge

Example

Assume the following rule holds

“If I have passed AICC, I can advance to year 2 of the studies”

And assume that you know

p := “I have passed AICC”

Then you would conclude

q : = “I can advance to year 2 of the studies”

Example

Assume the following rule holds

$$p \rightarrow q$$

And assume that you know

$$p$$

Then you would conclude

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This is a **valid argument form**

- It is true for any p and q

It is written as

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

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$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

It is called **Modus Ponens**

Why is the Argument Form Valid?

We know that $(p \land (p \to q)) \to q$ is a tautology

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So if we know that p and $p \to q$ are True, then the premise $(p \wedge (p \to q))$ is True, and therefore also q is True

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So if we know that p and $p \rightarrow q$ are True, then the premise $(p \wedge (p \rightarrow q))$ is True, and therefore also q is True

This holds for any tautology of the form $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$

Arguments in Propositional Logic

- A **argument** in propositional logic is a sequence of propositions.
  - All but the final proposition are called **premises**.
  - The last statement is the **conclusion**.
  - The argument is valid if the premises imply the conclusion.

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- An **argument form** is an argument that is valid no matter what propositions are substituted into its propositional variables

- **Inference rules** are simple argument forms that will be used to construct more complex argument forms

Using the Inference Rules to Build Valid Arguments

- A **valid argument** is a sequence of statements.
  - Each statement is either a premise or follows from previous statements by inference rules.
  - The last statement is called conclusion.
- A valid argument takes the following form:

$$
\begin{array}{ll}
 & \text{Step}_1 \\
 & \text{Step}_2 \\
 & \quad \vdots \\
 & \text{Step}_n \\
\hline
\therefore & \text{Conclusion}
\end{array}
$$

Example:

Inference Rule:

$$\begin{array}{l} p \\ p \rightarrow q \\ \hline q \end{array}$$

Argument Form:

$$\begin{array}{l} r \wedge s \\ (r \wedge s) \rightarrow q \\ \hline q \end{array}$$

Argument:

$$\begin{array}{ll} \left.\begin{array}{l} p \\ p \rightarrow (r \wedge s) \\ (r \wedge s) \rightarrow q \\ (r \wedge s) \end{array}\right\} & \text{premises} \\ \hline q & \text{conclusion} \end{array}$$

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> “If I have passed AICC and if I have passed Analysis 1 and if I have passed Linear Algebra and …. (list all your courses here), I can advance to year 2 of the studies”

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> p_1 := "I have passed AICC"

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And assume that you know

> p_1 := “I have passed AICC”
>
> p_2 := “I have passed Analysis 1”

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Assume the following rule holds

“If I have passed AICC and if I have passed Analysis 1 and if I have passed Linear Algebra and …. (list all your courses here), I can advance to year 2 of the studies”

And assume that you know

p_1 := “I have passed AICC”

p_2 := “I have passed Analysis 1”

p_3 := “I have passed Linear Algebra”

…

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q : = “I can advance to year 2 of the studies”

Using a Truth Table

Now build the truth table for $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$

to show that the argument holds

where n = 20 is the number of courses

The table will have 2^{20} = 1’048’576 rows, which is not very practical

Using Inference Rules

We have another inference rule: **Conjunction** Inference Rule
($p \wedge q \rightarrow p \wedge q$ is a tautology)

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Now we can provide the argument in a much simpler way!

Building the Argument

Write down what we know (the premises)

p_1

p_2

…

p_n

$(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$

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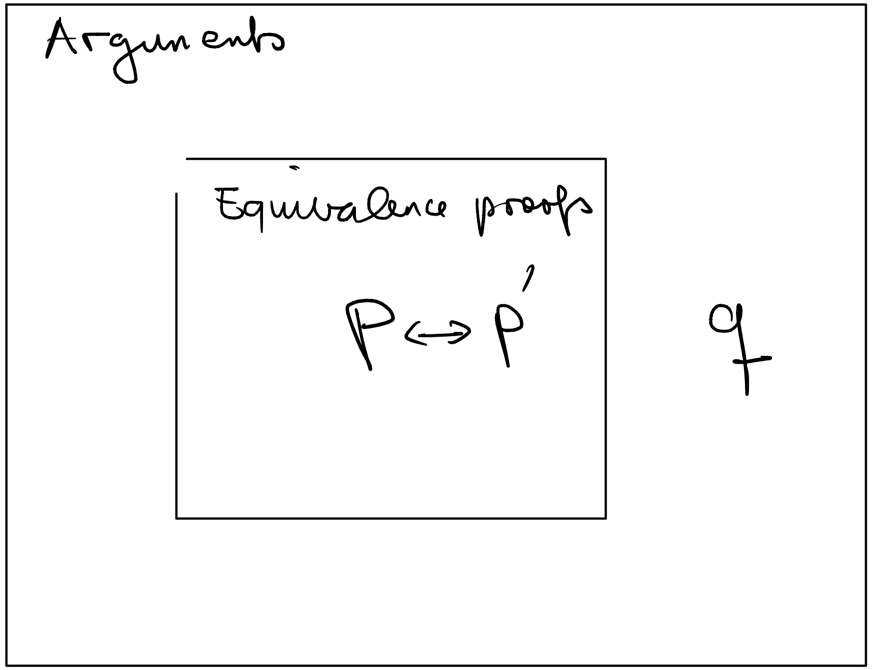

Summary

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Logic Programming (Example)

Predicates : lecturer(L, C), student(S, C) (prop. functions)

Facts : lecturer(karl, C2100). (propositions which
student(franck, CS100). etc are true)

Rules : teaches(L, S) :- lecturer(L, C), student(S, C)

equivalent to : $\forall x y z\, (\text{lecturer}(x,z) \wedge \text{student}(y,z) \rightarrow \text{teaches}(x,y))$

Queries : ? teaches(karl, franck) PROLOG

TRUE

? teaches(karl, X)

X = franck