Session 45: Proofs by Mathematical Induction

- Proofs of summation formulas
- Inequalities
- Divisibility Results
- Number of Subsets

Summation Formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Inequalities

Show that $2^n < n!$, for every integer $n \geq 4$.

Divisibility Results

Show that $n^3 - n$ is divisible by 3, for every positive integer n.

Number of Subsets of a Finite Set

Theorem: If *S* is a finite set with n elements, where *n* is a nonnegative integer, then *S* has 2^n subsets.

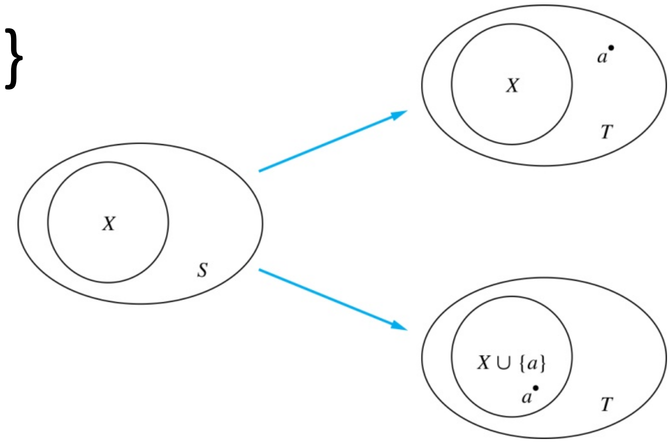
Proof: Let $P(n)$ be the proposition that a set with *n* elements has 2^n subsets.

- BASIS STEP : $P(0)$ is true, because the empty set has only itself as a subset and $2^0 = 1$.
- INDUCTIVE STEP: Assume $P(k)$ is true for an arbitrary nonnegative integer *k*.

Number of Subsets – Inductive Step

Inductive Hypothesis: For an arbitrary nonnegative integer k, every set with k elements has 2^k subsets.

- Let T be a set with $k + 1$ elements.
- Then, for some a, $T = S \cup \{a\}$, where $a \in T$ and $S = T - \{a\}$
- Hence $|S| = k$.
- For each subset X of S, there are exactly two subsets of T, i.e., X and $X \cup \{a\}$.
- By the inductive hypothesis S has 2^k subsets.
- Since there are two subsets of T for each subset of S, the number of subsets of T is $2 \cdot 2^k = 2^{k+1}$.



Summary

- Proofs of summation formulas
- Inequalities
- Divisibility Results
- Number of Subsets