Session 62: The Sum Rule

- Sum Rule
- Subtraction Rule

Basic Counting Principles: The Sum Rule

The Sum Rule: Assume there are two tasks A and B. There are n_1 ways to do A and n_2 ways to do B and none of the set of n_1 ways is the same as any of the set of n_2 ways. Then there are $n_1 + n_2$ ways to do task A or B.

Example: A student can choose a semester project from one of three laboratories. The three laboratories offer 5, 3, and 7 possible projects, respectively. No project is offered by several laboratories. How many possible projects are there to choose from?

By the sum rule it follows that there are 5+3+7 = 15 ways to choose a project.

The Sum Rule in Terms of Sets

The sum rule can be phrased as

$|A \cup B| = |A| + |B|$ as long as A and B are disjoint sets.

or more generally,

$|A_1 \cup A_2 \cup \cdots \cup A_m| = |A_1| + |A_2| + \cdots + |A_m|$

when $A_i \cap A_j = \varnothing$ for all i, j.

The case where the sets have elements in common is different!

Combining the Sum and Product Rule

Example: Suppose variable names in a programming language can be either a single letter or a letter followed by a digit. Find the number of possible names.

Product Rule : $26 \cdot 10$ letters followed by digit

Sum Rule : $26 + 26 \cdot 10$ variable names in total

Counting Passwords

Each user on a computer system has a password, which is 6 to 8 characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit.

How many possible passwords are there?

Total number of passwords $P = P_6 + P_7 + P_8$ (sum rule)

passwods P_i, no constraint : $(26+10)^i$

remove passwords with only characters : $(26+10)^i - 26^i$

Thus : $P = (36^6 - 26^6) + (36^7 - 26^7) + (36^8 - 26^8)$

Basic Counting Principles: Subtraction Rule

Subtraction Rule: If a task can be done either in one of n_1 ways or in one of n_2 ways, then the total number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

Also known as, the **principle of inclusion-exclusion**:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Counting Bit Strings

How many bit strings of length 8 either start with a 1 bit or end with the two bits 00?

Use the principle of inclusion-exclusion.

Bit strings starting with 1 : 2^7 $|A|$

Bit strings ending with 00 : 2^6 $|B|$

Bit strings starting with 1 and ending with 00 : 2^5 $|A \cap B|$

Using inclusion-exclusion : $|A \cup B| = 2^7 + 2^6 - 2^5 =$

$= 128 + 64 - 32 = 160$

Summary

- Sum Rule
- Subtraction Rule
- Applications to counting strings