Advanced information, computation, communication I
EPFL - Fall semester 2020-2021

Week 1

September 22, 2020

Exercise 0 (Warm-up; unlike the multiple choice questions at the final exam, more than a single answer may be correct.). The law regarding alcohol drinking in Switzerland states the following: "If you are drinking strong alcohol, then you must be at least 18 years old." (for all x, $A(x) \to 18(x)$). Assume that you are hired as a police officer at Satellite and need to enforce the above law. Which student(s) are you going to arrest and why:

- ○ The 17-year old student drinking soda.
- ○ The 17-year old student drinking vodka.
- ○ The 18-year old student drinking water.
- ○ The 18-year old student drinking tequila.

Next Tuesday, Satellite organizes a cabaret night open to all students under the following conditions: "Everyone must be at least 18 years old and drink strong alcohol" (for all x, $A(x) \wedge 18(x)$). As previously, which student(s) are you going to arrest and why:

- ○ The 17-year old student drinking soda.
- ○ The 17-year old student drinking vodka.
- ○ The 18-year old student drinking water.
- ○ The 18-year old student drinking tequila.

Exercise 1. Determine the truth values (i.e., T or F) of the following propositions:

- ☐ $19 - 4 = 12$ if and only if 3 is a prime number.
- ☐ If $1 + 1 = 5$, then $1 + 1 = 3$.
- ☐ If the moon is a star, then so is the sun.
- ☐ If 5 is a prime number, then the earth is flat.
- ☐ $0 > 1$ if and only if $2 > 1$.
- ☐ Either Toronto is the capital of Canada or Hamburg is the capital of Germany.

Exercise 2. Construct a truth table for each of these compound propositions:

1. $p \oplus (p \vee q)$
2. $p \wedge (q \oplus u)$
3. $(p \wedge q) \oplus (p \wedge u)$
4. $(p \leftrightarrow q) \oplus (p \to q)$

Exercise 3. Without using truth tables, show the following logical equivalences:

1. $\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$
2. $p \oplus (q \wedge u) \not\equiv (p \oplus q) \wedge (p \oplus u)$.
3. $p \oplus q \equiv (p \vee q) \wedge (\neg p \vee \neg q)$.
4. $\neg(p \oplus q) \equiv (\neg p) \oplus q$.
5. $p \leftrightarrow q \equiv \neg(p \oplus q)$.

Exercise 4. Find a compound proposition with three variables p, q, u that is

1. True if and only if p is true, q is false, and u is false;
2. True if and only if exactly one of the variables is true;
3. True if and only if at least two of the variables are true.

Exercise 5. The negation of the statement "if it rains, the ground is wet" is

○ if the ground is not wet, it does not rain.

○ it rains and the ground is not wet.

○ if the ground is wet, it does not rain.

○ if it rains, the ground is not wet.

Exercise 6. Let C and D be two sets. The statement $\neg((D \subseteq C) \wedge (C \subset D))$

○ is a tautology.

○ is a contingency.

○ is a contradiction.

○ is not a compound proposition.

Exercise 7. (Rosen, exercise 38 p. 25) Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is available:

- Either Kevin or Heather, or both, are chatting.
- Either Randy or Vijay, but not both, are chatting.
- If Abbey is chatting, so is Randy.
- Vijay and Kevin are either both chatting or neither is.
- If Heather is chatting, then so are Abbey and Kevin.

Explain your reasoning.

Exercise 8. The negation of the statement "If I think, then I am" is given by:

○ I am not, and I think.

○ If I am not, then I do not think.

○ I am, and I do not think.

○ I do not think, or I am not.

Exercise 9. Tick the equivalent sentence of the following newspaper headline:

"UK minister refuses to rule out ignoring law preventing no-deal Brexit"

○ UK minister does not accept not to rule in not acknowledging law not approving no-deal Brexit.

○ UK minister accepts to rule in acknowledging law approving no-deal Brexit.

○ UK minister does not accept to rule out ignoring law tolerating Brexit with deal.

○ UK minister refuses to rule in acknowledging law approving no-deal Brexit.

Exercise 10. (From 2016 midterm exam)

(*français*) Soit p et q deux propositions. Considérons les deux propositions composées ci-dessous.

(*English*) Let p and q be two propositions. Consider the two compound propositions below.

$$((q \to p) \land \neg q) \to \neg p \qquad\qquad \big(((\neg q) \to (\neg p)) \land p\big) \to q$$

○ $\left\{\begin{array}{l}\text{Une seule des propositions composées est une tautologie, l'autre est une contingence.}\\ \text{One of the compound propositions is a tautology, the other is a contingency.}\end{array}\right.$

○ $\left\{\begin{array}{l}\text{Les deux propositions composées sont des contingences.}\\ \text{Both compound propositions are contingencies.}\end{array}\right.$

○ $\left\{\begin{array}{l}\text{Une seule des propositions composées est une contradiction, l'autre est une contingence.}\\ \text{One of the compound propositions is a contradiction, the other is a contingency.}\end{array}\right.$

○ $\left\{\begin{array}{l}\text{Une seule des propositions composées est une tautologie, l'autre est une contradiction.}\\ \text{One of the compound propositions is a tautology, the other is a contradiction.}\end{array}\right.$

Exercise 11. (From 2017 midterm exam)

(*français*) Pour $x, y \in \mathbf{Z}$, la proposition composée suivante est une tautologie

(*English*) For $x, y \in \mathbf{Z}$, the following compound proposition is a tautology

$$\neg(x > 1) \lor \neg(y \leq 0) \;\leftrightarrow\; \neg((x \leq 1) \land (y > 0))$$

○ $\left\{\begin{array}{l}\text{si la partie à gauche de “}\leftrightarrow\text{” est remplacée pas sa négation, et “}\leftrightarrow\text{” est remplacé par “}\to\text{”.}\\ \text{if the left hand side of “}\leftrightarrow\text{” is negated and “}\leftrightarrow\text{” is replaced by “}\to\text{”.}\end{array}\right.$

○ $\left\{\begin{array}{l}\text{et ne requiert aucun changement.}\\ \text{and does not require any changes.}\end{array}\right.$

○ $\left\{\begin{array}{l}\text{si “}\leftrightarrow\text{” est replacé par “}\leftarrow\text{”.}\\ \text{if “}\leftrightarrow\text{” is replaced by “}\leftarrow\text{”.}\end{array}\right.$

- ◯ si la partie à droite de "$\leftrightarrow$" est remplacée pas sa négation, et "$\leftrightarrow$" est remplacé par "$\rightarrow$".
  if the right hand side of "$\leftrightarrow$" is negated and "$\leftrightarrow$" is replaced by "$\rightarrow$".

Exercise 12. (From 2016 mock final exam)
The compound proposition $((\neg p \wedge q) \rightarrow (r \oplus q)) \vee (\neg s \leftrightarrow p)$ is

- ◯ a tautology.
- ◯ a contingency.
- ◯ a contradiction.
- ◯ incorrectly formatted.