Session 46: Strong Induction

- Principle of Strong Induction
- Examples of Strong Induction

Strong Induction

To prove that $P(n)$ is true for all positive integers n, where $P(n)$ is a propositional function, complete two steps:

Basis Step: Show that P(1) is true

Inductive Step: Show that $\forall k\ ([P(1) \land P(2) \land \cdots \land P(k)] \rightarrow P(k+1))$ is true for all positive integers k.

- Strong Induction is sometimes called the *second principle of mathematical induction* or *complete induction*.

Properties of Strong Induction

- We can always use strong induction instead of mathematical induction.
- But there is no reason to use it if it is simpler to use mathematical induction.
- In fact, the principles of mathematical induction, strong induction, and the well-ordering property are all equivalent.
- Sometimes it is clear how to proceed using one of the three methods, but not the other two.

Example of Strong Induction

Theorem: Every positive integer n can be written as a sum of distinct powers of two, that is, there exists a set of integers S = $\{k_1,\ldots,k_m\}$ such that $n = \sum_{j=1}^{m} 2^{k_j}$.

Summary

- Principle of Strong Induction
- Proofs can be sometimes simpler with strong induction