Video 11: Inference Rules in Propositional Logic

- Important inference rules
- Examples
- Fallacies

Inference Rules

1. Propositional Logic: Inference Rules
2. Predicate Logic: Inference rules for propositional logic plus additional inference rules to handle variables and quantifiers

Conjunction and Modus Ponens

$$\begin{array}{l} p \\ q \\ \hline \therefore p \land q \end{array}$$

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Conjunction and Modus Ponens

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Corresponding Tautology:

(p ∧ q) → (p ∧ q)

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Corresponding Tautology:

(p ∧ (p →q)) → q

Inference Rule: Modus Tollens

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Corresponding Tautology:

$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$

Inference Rule: Modus Tollens

$$
\begin{array}{l}
p \to q \\
\neg q \\
\hline
\therefore \neg p
\end{array}
$$

Corresponding Tautology:

$(\neg q \land (p \to q)) \to \neg p$

Example:

p := "I have passed AICC"

q := "I can advance to year 2 of the studies"

Inference Rule: Modus Tollens

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Corresponding Tautology:

$(\neg q \wedge (p \to q)) \to \neg p$

Example:
p := “I have passed AICC”
q := “I can advance to year 2 of the studies”

Premises
“If I have passed AICC, I can advance to year 2 of the studies”
“I cannot advance to year 2 of the studies.”

Inference Rule: Modus Tollens

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Corresponding Tautology:

$(\neg q \wedge (p \to q)) \to \neg p$

Example:
p := “I have passed AICC”
q := “I can advance to year 2 of the studies”

Premises
“If I have passed AICC, I can advance to year 2 of the studies”
“I cannot advance to year 2 of the studies.”

Conclusion
“I did not pass AICC.”

Hypothetical Syllogism

$$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

Corresponding Tautology:

((p →q) ∧ (q→r)) → (p→ r)

Hypothetical Syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

Corresponding Tautology:
$((p \to q) \wedge (q \to r)) \to (p \to r)$

Example:
r := “I can take Analysis 4”

Hypothetical Syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

Corresponding Tautology:
$((p \to q) \wedge (q \to r)) \to (p \to r)$

Example:
r := "I can take Analysis 4"

Premises
"If I have passed AICC, I can advance to year 2 of the studies"
"If I can advance to year 2 of the studies, I can take Analysis 4"

Hypothetical Syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

Corresponding Tautology:

$((p \to q) \land (q \to r)) \to (p \to r)$

Example:

r := “I can take Analysis 4”

Premises

“If I have passed AICC, I can advance to year 2 of the studies”

“If I can advance to year 2 of the studies, I can take Analysis 4”

Conclusion

“If I have passed AICC, I can take Analysis 4”

Resolution

$$\begin{array}{l} \neg p \lor r \\ p \lor q \\ \hline \therefore q \lor r \end{array}$$

Corresponding Tautology:

((¬p ∨ r) ∧ (p ∨ q)) → (q ∨ r)

Resolution

$$\begin{array}{l} \neg p \vee r \\ p \vee q \\ \hline \therefore q \vee r \end{array}$$

Corresponding Tautology:

$((\neg p \vee r) \wedge (p \vee q)) \rightarrow (q \vee r)$

Example:
p := “The weather is nice
q := “I am at home”
r := “I am at the beach”

Resolution

$$\begin{array}{l} \neg p \vee r \\ p \vee q \\ \hline \therefore q \vee r \end{array}$$

Corresponding Tautology:

$((\neg p \vee r) \wedge (p \vee q)) \rightarrow (q \vee r)$

Example:

p := “The weather is nice

q := “I am at home”

r := “I am at the beach”

Premises:

“The weather is bad or I am at the beach”

“The weather is nice or I am at home”

Resolution

$$\begin{array}{l} \neg p \lor r \\ p \lor q \\ \hline \therefore q \lor r \end{array}$$

Corresponding Tautology:

$((\neg p \lor r) \land (p \lor q)) \rightarrow (q \lor r)$

Example:
p := “The weather is nice
q := “I am at home”
r := “I am at the beach”

Premises:
“The weather is bad or I am at the beach”
“The weather is nice or I am at home”

Conclusion:
“I am at home or at the beach”

Resolution

$$\begin{array}{l} \neg p \vee r \\ p \vee q \\ \hline \therefore q \vee r \end{array}$$

Corresponding Tautology:

$((\neg p \vee r) \wedge (p \vee q)) \rightarrow (q \vee r)$

Example:
p := “The weather is nice
q := “I am at home”
r := “I am at the beach”

Premises:
“The weather is bad or I am at the beach”
“The weather is nice or I am at home”

Conclusion:
“I am at home or at the beach”

Resolution plays an important role in automated theorem proofing and AI

It allows to eliminate propositional variables from the premises

Other Inference Rules

$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification

Simpler form of resolution

Dual to Conjunction

Simpler form of Modus Ponens

Valid Arguments

Attention: even seemingly “obvious” conclusions imply an argument

Example: From $p \wedge (p \rightarrow q)$ conclude q

$p \wedge (p \rightarrow q)$	
p	simplification
$p \rightarrow q$	simplification
q	modus ponens

Fallacies!

$((p \rightarrow q) \wedge q) \rightarrow p$ *is not a tautology*

- **fallacy of affirming the conclusion**

Fallacies!

$((p \rightarrow q) \land q) \rightarrow p$ *is not a tautology*

- **fallacy of affirming the conclusion**

Example:

- If you do every problem in this book, then you will learn discrete mathematics. You learned discrete mathematics.
- Therefore, you did every problem in this book?

Fallacies!

$((p \rightarrow q) \land \neg p) \rightarrow \neg q$ is not a tautology

- **fallacy of denying the hypothesis**

Fallacies!

$((p \rightarrow q) \wedge \neg p) \rightarrow \neg q$ is not a tautology

- **fallacy of denying the hypothesis**

Example:

- If you do every problem in this book, then you will learn discrete mathematics. You did not do every problem in this book.
- Therefore, you did not learn discrete mathematics?

Summary

- Modus Ponens, Modus Tollens
- Hypothetical Syllogism
- Resolution
- How to build valid arguments
- Fallacies
  - affirming the conclusion
  - denying the hypothesis