Session 15: Introduction to Sets

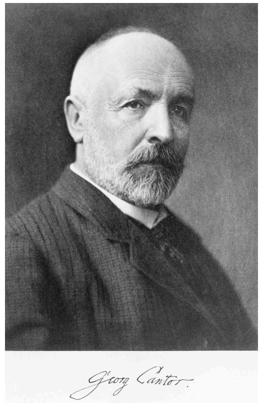
- Sets
- Specification of sets
- Sets of Numbers
- Special sets

Sets, Functions and Relations

- Basic abstractions in mathematics and computer science
- Data structures are constructed using these abstractions
  - Unordered collections ~ sets
  - Order collections ~ sequences, which are functions
  - Networks, graphs ~ relations
  - Databases ~ relations
  - Objects ~ functions
- Computing is modelled using these abstractions
  - Finite state machines ~ relations
  - Programs are decomposed into functions
  - Cost of programs is expressed as functions

Some History

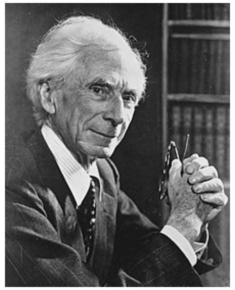
- Cantor, Founder of set theory, discovered uncountable sets
- Peano, Introduced notations (∈ , ∪ , ∩) and axioms for natural numbers based on set theory
- ZF developed the currently widely accepted axioms for set theory
- Wrote Pincipia Mathematica, deriving all mathematics from primitive axioms, know for his antimony on set theory



Georg Cantor, 1845 - 1918

Guiseppe Peano, 1858 - 1932

Zermelo Fraenkel



Bertrand Russell, 1872 - 1970

Introduction

- Sets are one of the basic building blocks in discrete mathematics.
  - Basis for counting, functions, relations
  - Programming languages have set operations
  - Databases are sets of discrete objects

- Set theory is an important branch of mathematics.
  - Many different systems of axioms have been used to develop set theory.
  - Here we are not concerned with a formal set of axioms for set theory.
  - Instead, we will use what is called **naïve set theory**.

Sets

- A **set** is an unordered collection of objects.
  - the students in this class
  - the chairs in this room

- The objects in a set are called the **elements** of the set.
- A set is said to **contain** its elements.

- The notation $\boldsymbol{a} \in \boldsymbol{A}$ denotes that a is an element of the set A.
- If a is not an element of A, write $\boldsymbol{a} \notin \boldsymbol{A}$ $\equiv \neg a \in A$

Describing a Set: Roster Method

Listing all elements of a set

$$S = \{a, b, c, d\}$$

- Order not important: $S = \{a, b, c, d\} = \{b, c, a, d\}$

Examples

Set of all vowels in the English alphabet:

Set of all odd positive integers less than 10:

Set of all positive integers less than 100:

Set of all integers less than 0:

Sets of Numbers

N = **natural numbers** = {0, 1, 2, 3, ….}

Z = **integers** = {…, -3, -2, -1, 0, 1, 2, 3, …}

$\mathbf{Z}^+$ = **positive integers** = {1, 2, 3, …..}

R = set of **real numbers**

$\mathbf{R}^+$ = set of **positive real numbers**

C = set of **complex numbers**

Q = set of ***rational numbers***

Set-Builder Notation

Specify the property or properties that all members must satisfy:

$$S = \{x \mid P(x)\}$$

- P(x) may be expressed in natural language or predicate logic

Examples

$S = \{x \mid x$ is a positive integer less than 100$\}$

$O_1 = \{x \mid x$ is an odd positive integer less than 10$\}$

$O_2 = \{x \in \mathbf{Z}^+ \mid x$ is odd and $x < 10\}$

$P = \{x \mid \text{Prime}(x)\}$

$\mathbf{Q}^+ = \{x \in \mathbf{R} \mid x = p/q$, for some positive integers $p, q\}$

Interval Notation

For sets of numbers

$[a,b] = \{x \mid a \le x \le b\}$

$[a,b) = \{x \mid a \le x < b\}$

$(a,b] = \{x \mid a < x \le b\}$

$(a,b) = \{x \mid a < x < b\}$

closed interval [a,b]

open interval (a,b)

Universal Set and Empty Set

The **universal set U** is the set containing everything currently under consideration.

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The **empty set** is the set with no elements.

- Denoted as $\emptyset$ or $\{\}$

Some things to remember

Sets can be elements of sets.

$\{\{1, 2, 3\}, a, \{b, c\}\}$

$\{\mathbf{N}, \mathbf{Z}, \mathbf{Q}, \mathbf{R}\}$

The empty set is different from a set containing the empty set.

$\varnothing \neq \{\varnothing\}$

Summary

- Set definition
  - Roster method
  - Set Builder Notation
- Sets of Numbers
- Interval Notation
- Empty and Universal Set