

Session 73: Discrete Probability

- Finite probability
- Examples

Short History of Probability



Girolamo Cardano
1501 - 1576

- Cardano: first treatment of probability (notoriously short of money)
- Bayes: Bayes' Theorem, first concept of statistical data analysis
- Laplace: classical theory of probability was introduced in the 18th century, when he analysed games of chance.
- Kolmogorov: modern axiomatic probability theory



Pierre-Simon Laplace
1749 - 1827



Thomas Bayes
1701 - 1761



Andrey Kolmogorov
1902 - 1987

Probability of an Event

Definitions:

- An **experiment** is a procedure that yields one of a given set of possible **outcomes**: rolling the dice
- The **sample space S** of the experiment is the set of possible outcomes: faces of the dice $S = \{1,2,3,4,5,6\}$
- An **event E** is a subset of the sample space: e.g., $E = \{5,6\}$
- An event **occurs**, if the outcome belongs to the sample space: E occurs when rolling a 6

Probability of an Event

Definition: If S is a finite sample space of equally likely outcomes, and E is an event, that is, a subset of S , then the **probability** of E is

$$p(E) = \frac{|E|}{|S|}$$

For every event E , we have $0 \leq p(E) \leq 1$.

Example

What is the probability that when two dice are rolled, the sum of the numbers on the two dice is 7?

- By the product rule there are $6^2 = 36$ possible outcomes
- 6 of these sum up to 7: 1+6, 2+5, 3+4, 4+3, 5+2, 6+1
- Hence, the probability of obtaining a 7 is $6/36 = 1/6$

What is the probability to roll at least one 6?

- 11 of these contain a 6: 6+1, 6+2, 6+3, 6+5, 6+6, 5+6, 4+6, 3+6, 2+5, 1+6
- Hence the probability is $11/36$

Example

For Euromillions choose a set of 5 numbers out of 50 and 2 numbers out of 12.

What is the probability that a person picks the correct 5+2 numbers?

The number of ways to choose 5 numbers out of 50 is

$$C(50,5) = 50!/(45!5!) = 2'118'760$$

The number of ways to choose 2 numbers out of 12 is

$$C(12,2) = 66$$

Using the product rule, the probability of picking a winning combination is
 $1/66 * 2'118'760 = 1/139'838'160 \approx 0.0000000072$.

Example

What is the probability that the numbers 11, 4, 17, 39, and 23 are drawn in that order from a bin with 50 balls labeled with the numbers 1,2, ..., 50 if

1. The ball selected is not returned to the bin.
2. The ball selected is returned to the bin before the next ball is selected.

Use the product rule in each case.

1. *Sampling without replacement:* The probability is $1/254,251,200$ since there are $50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 = 254,251,200$ ways to choose the five balls.
2. *Sampling with replacement:* The probability is $1/50^5 = 1/312,500,000$ since $50^5 = 312,500,000$.

Summary

- Finite probability
 - Sample Space, Event, Probability
- Examples
 - Card Games, Lotteries