Cross-correlations of dark energy probes
or
More than the sum of the parts

Jo Dunkley
DESC School

- What is a cross-correlation?
- Why is it useful?
- How do I measure it?
- How do I calculate it?

Dark Energy probes from LSST and non-LSST

Probe	What property does it measure?

LSS	clustering of galaxies / DM.
BAO	$D_A(z)$, $H(z)$
SN	$H(z)$ expansion rate.
WL	clustering of lenses, distances
Clusters	$N(M, z)$
CMB lensing	clustering of ~~DM~~ lenses, distances.
21cm	clustering of ~~H~~ neutral H. $D_A(z)$
CIB	clustering of high-z galaxies

This lesson: focus on clustering probes

$$\delta(x) \equiv \frac{\delta\rho(x)}{\rho} \longrightarrow \delta(k)$$

$$\langle \delta(k)\,\delta(k')\rangle = (2\pi)^3\,\hat{\delta}(k-k')\,P(k)$$

$$\delta(k,z) \propto k^2\,\phi_i(k)\,D(z)$$

$$\propto \phi_i(k)\,D(z)$$

Matter

$$\langle \delta^2 \rangle \propto k^4 \langle \phi_i^2 \rangle D^2(z) \qquad \propto k\,D^2(z)$$

$$\propto \langle \phi_i^2 \rangle D^2(z) \qquad \propto k^{-3} D^2(z)$$

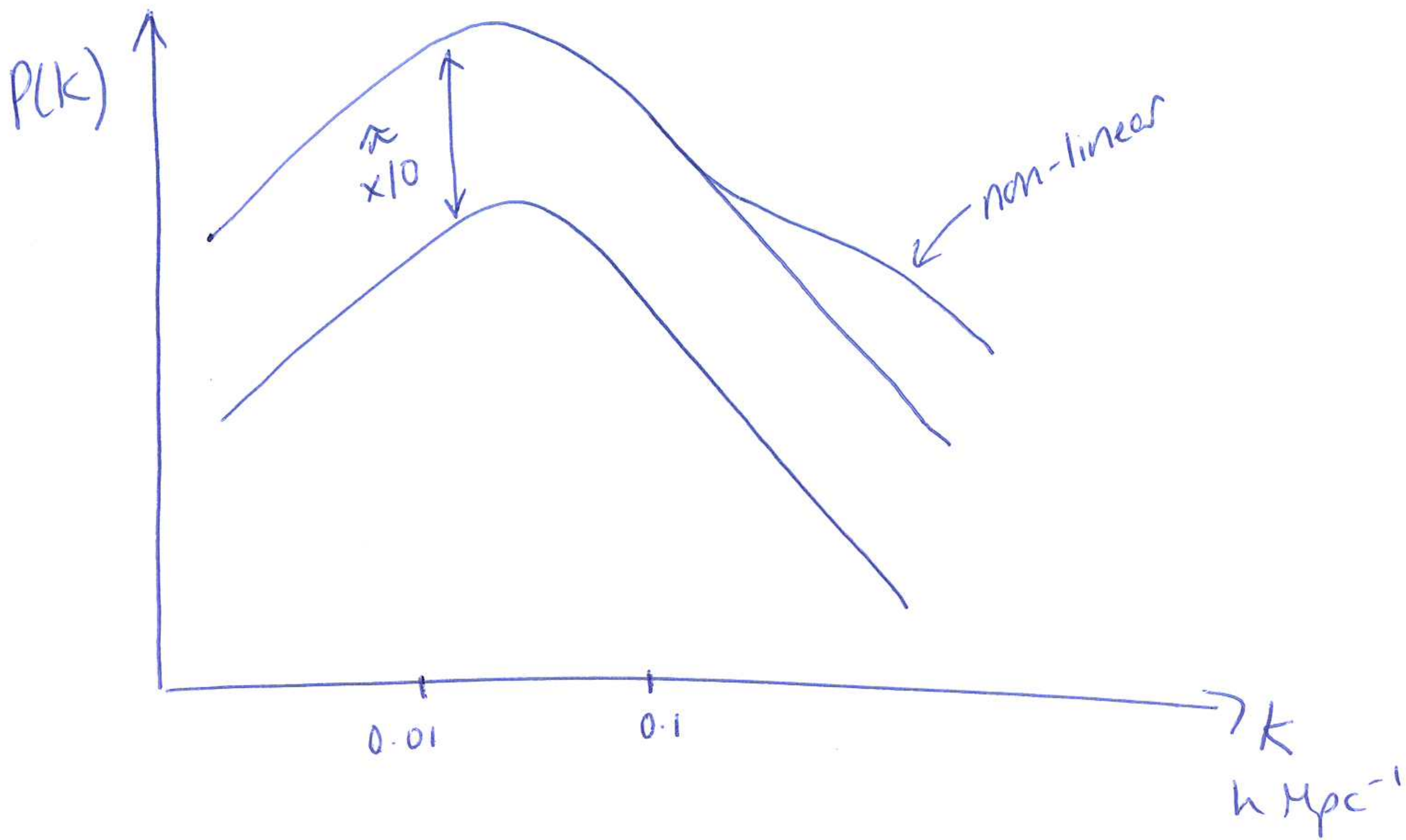
radiation.

$$\left[\, k^3 \langle \phi_i^2 \rangle \equiv P_i(k) \,\right]$$

What does P(k,z) look like?

at z=0 and z=2

What does $P(k,z)$ look like at $z=0,2$?



What does P(k,z) look like?

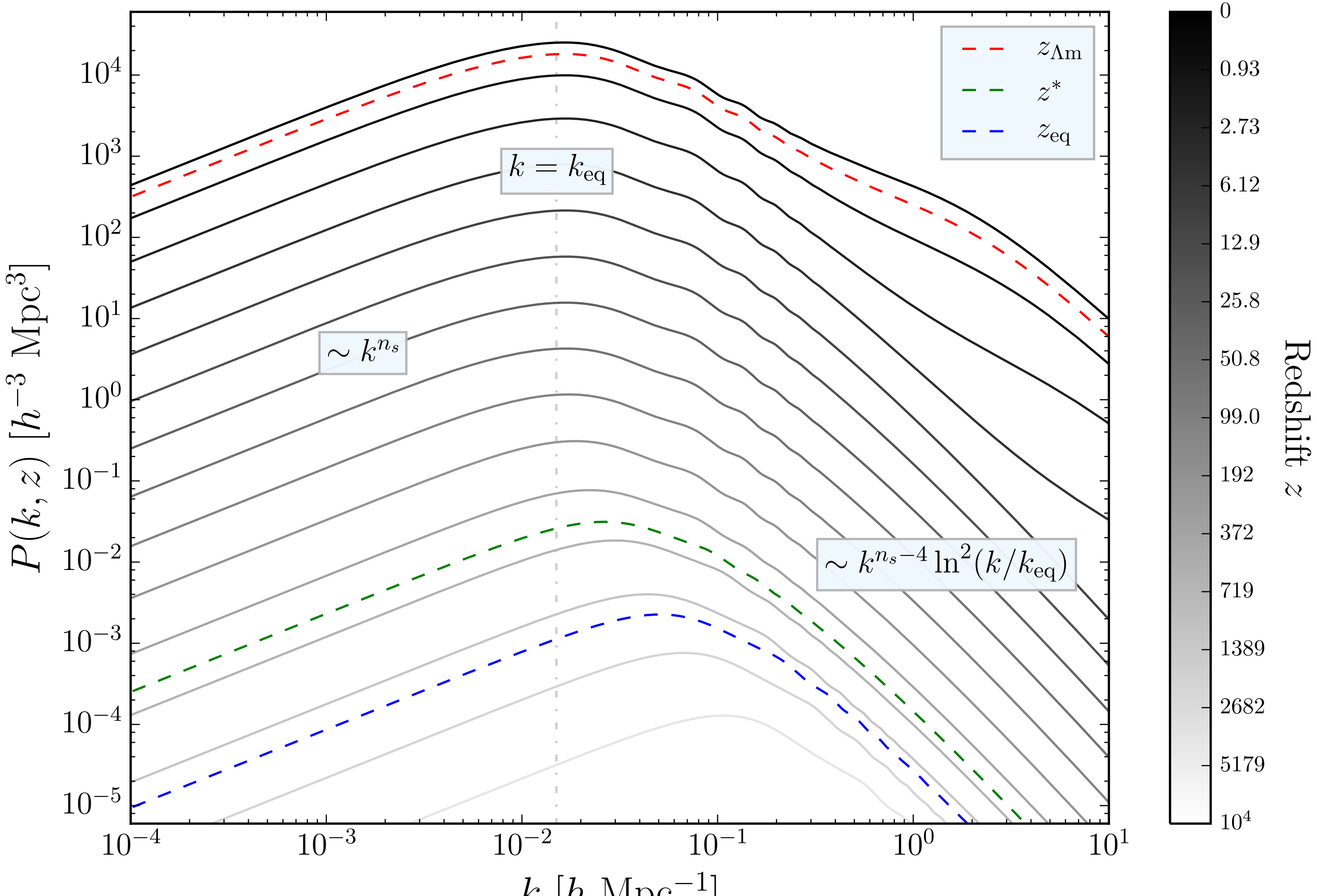


Fig: Rupert Allison

How does D(a) depend on Omega_L?

e.g. for 0.8,0.6 in flat universe

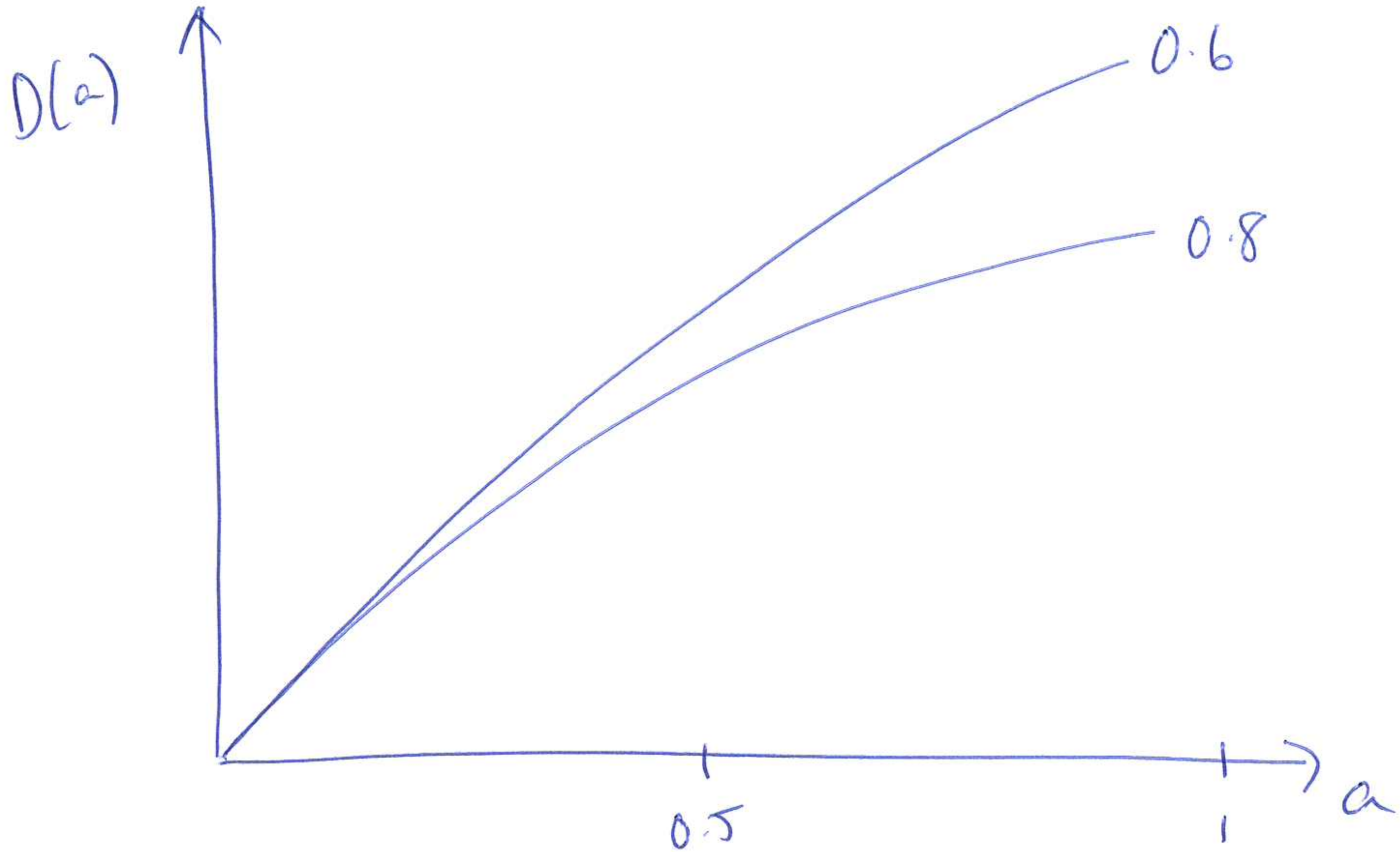
D(a) ~ a in matter-dominated

D(a) ~ const in lambda-dominated

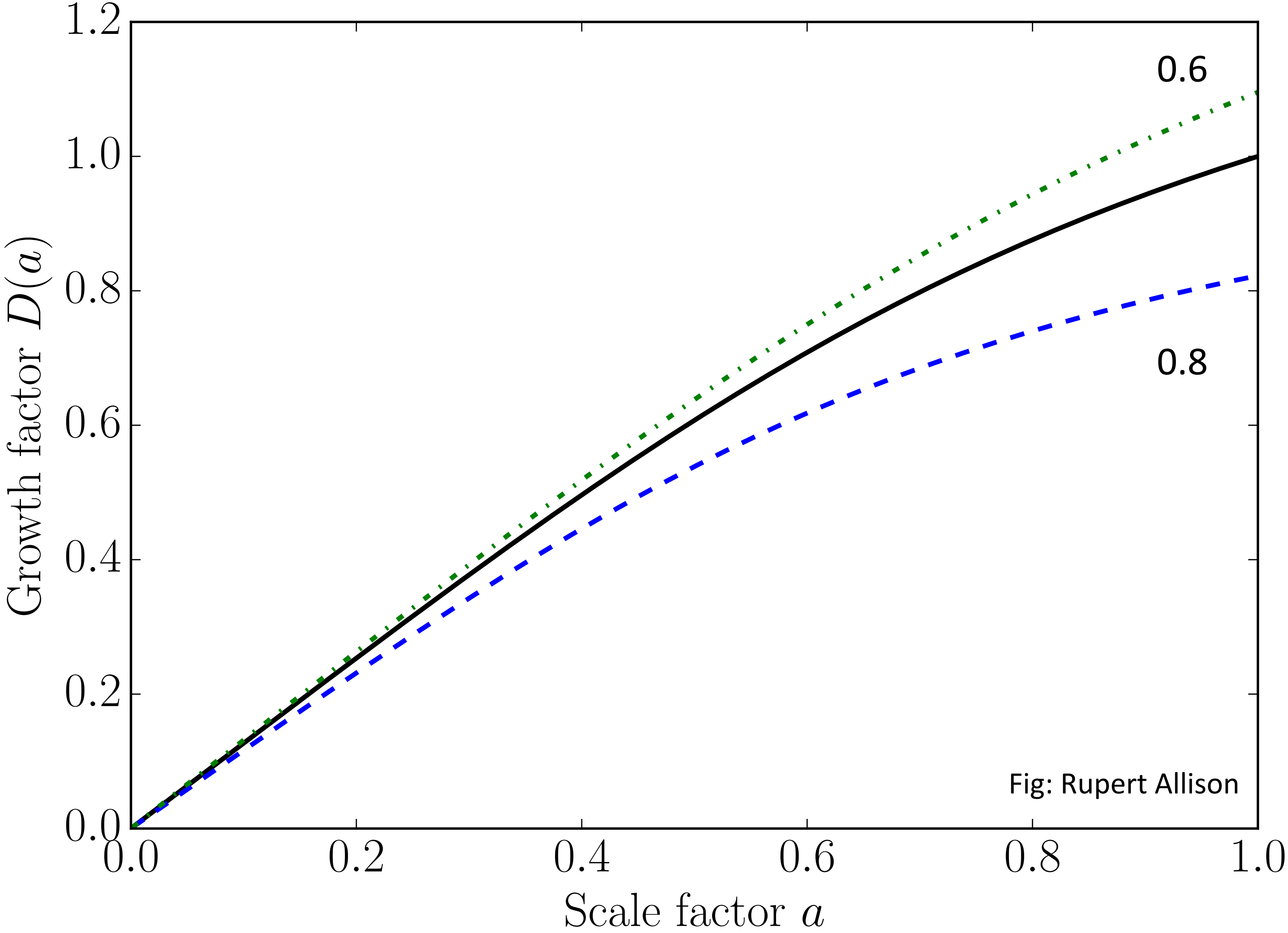
$D(a) \propto a$ matter dominated

$D(a) \propto$ const Λ-dominated

What does $D(a)$ look like for $\Omega_\Lambda = 0.6$
$= 0.8$

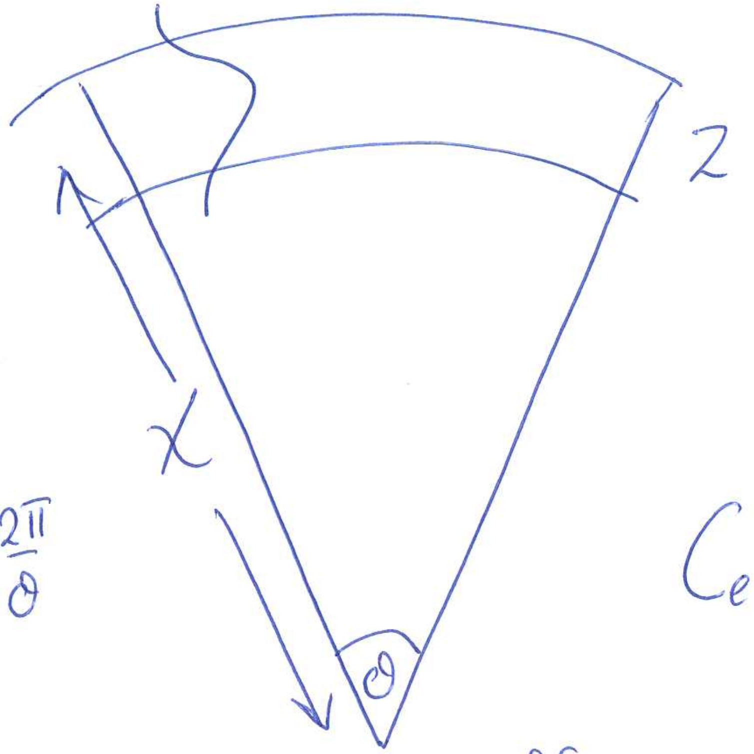


Growth function



How do we measure $P(k,z)$ in practice?

3



$\ell \approx \frac{2\pi}{\theta}$

$$\delta(\hat{n}, \chi)\, q(\chi) = \sum a_{\ell m} Y_{\ell m}$$

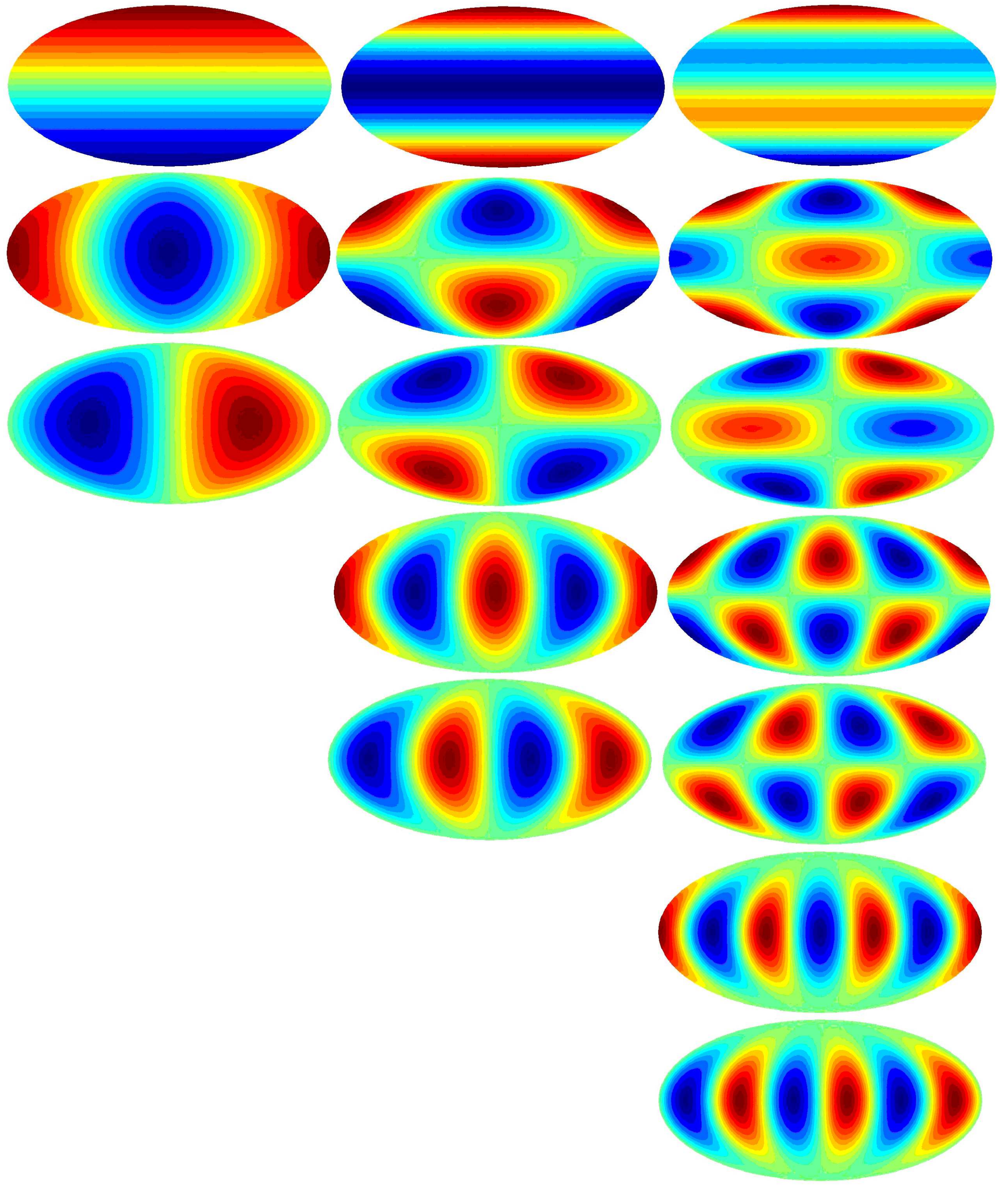
$$C_\ell^{\delta\delta} = \frac{1}{2\ell+1} \sum a_{\ell m}^{\delta} a_{\ell m}^{\delta\, T}$$

$$C_\ell^{\delta\delta} = \int \frac{d\chi}{\chi^2}\, q^2(\chi)\, P_m(k, \chi)$$

$k = \frac{\ell}{\chi}$

$$C_\ell^{\delta_1\delta_2} = \frac{1}{2\ell+1} \sum a_{\ell m}^{\delta_1} a_{\ell m}^{\delta_2\, T}$$

$$C_\ell^{\delta_1\delta_2} = \int \frac{d\chi}{\chi^2}\, q_1(\chi)\, q_2(\chi)\, P_m(k, \chi)$$



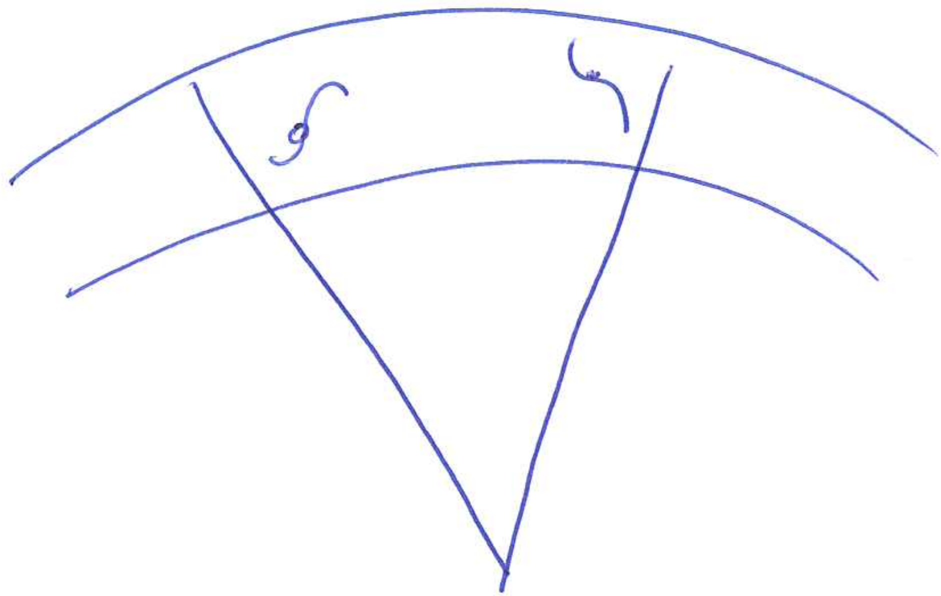
In practice: different measurements of P(k,z)

- Galaxy density
- Galaxy lensing
- CMB lensing
- …
- Cross-correlations!

In practice

Galaxy density

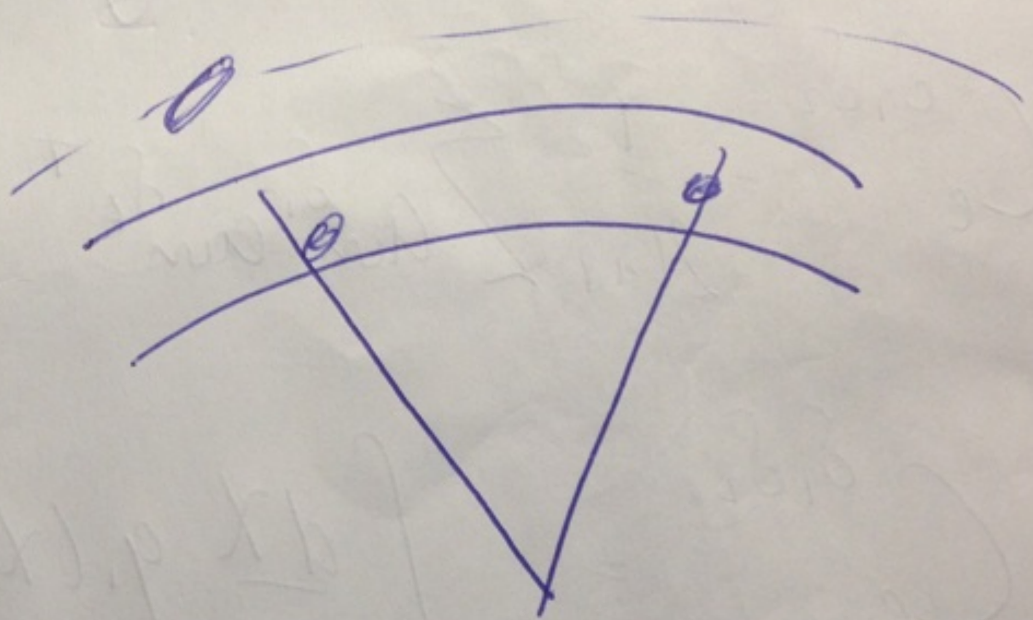
$$\delta_g = b\,\delta$$



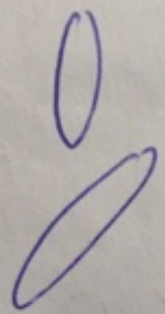
$$C_\ell^{gg} = b^2(z) \int_0^\infty \frac{d\chi}{\chi^2} \left(\frac{dn}{d\chi}\right)^2 P(k, z)$$

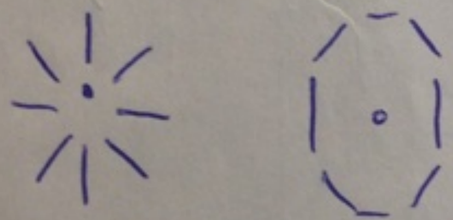
e.g 4 bins for LSST

Galaxy lensing



Sheer γ_1 γ_2





$$C_\ell^{\gamma\gamma} = (1+m)^2 \int_0^\infty \frac{d\chi}{\chi^2}\left[\frac{3}{2}\,\frac{\Omega_m H_0^2\,\chi}{c^2 a(\chi)}\cdot\underbrace{\frac{\chi_s-\chi}{\chi_s}}_{\downarrow}\right]^2 P(k,z)$$

$$\int dz_s\,\frac{dn_s}{dz}\,\frac{\chi_s-\chi}{\chi_s}$$

$$\equiv$$

$$C_\ell^{\kappa\kappa}.$$

General cross-correlation

$$C_\ell^{AB} = \int \frac{d\chi}{\chi^2}\, W_A(\chi) W_B(\chi)\; P_m\!\left(k = \frac{\ell + 1/2}{\chi}, \chi\right),$$

What redshifts does LSST see for galaxies and lenses?

sketch dN/dz

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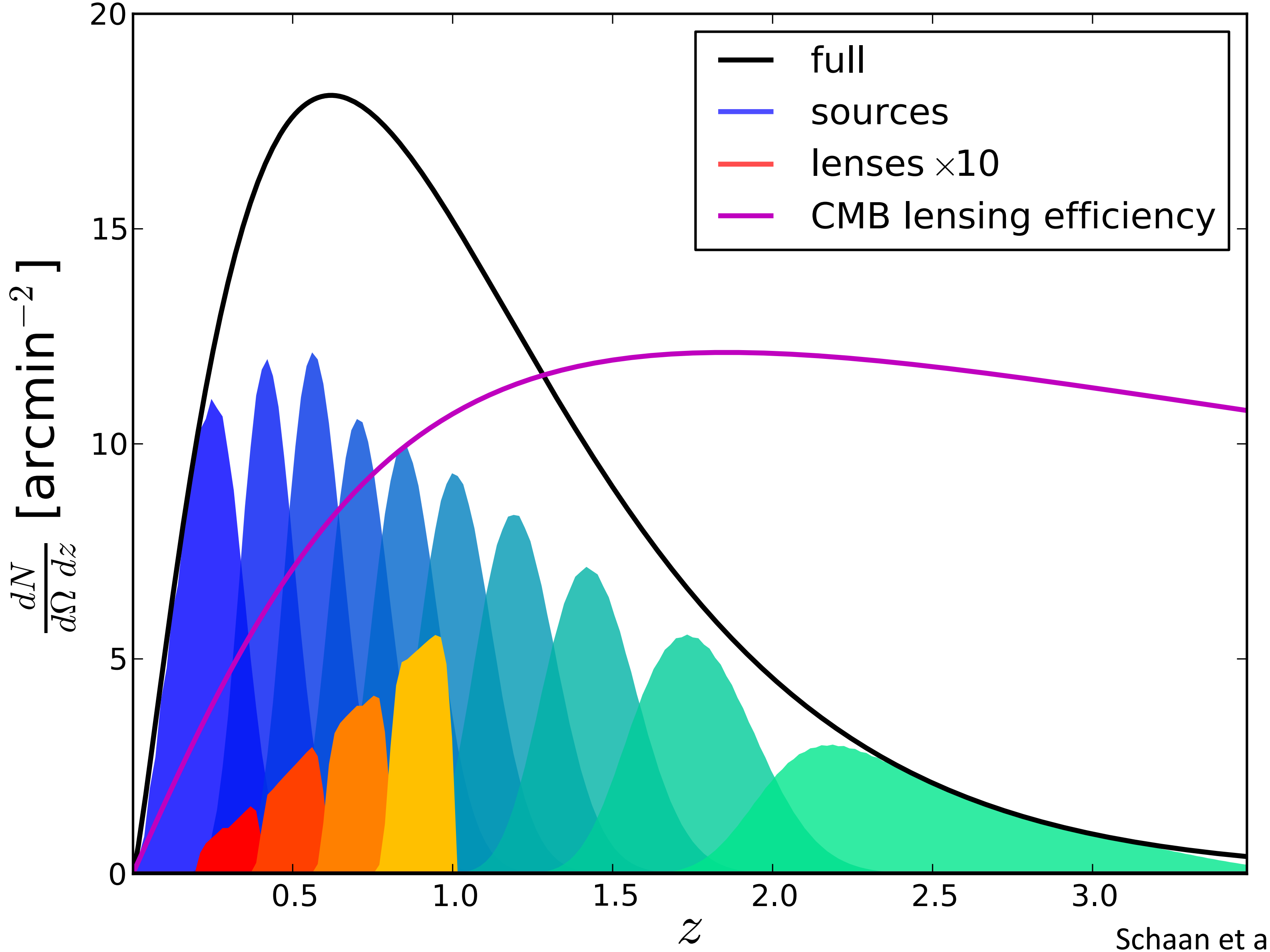
sketch dN/dz



Schaan et al 2016

Why are they so useful?
or, Systematics, systematics, systematics

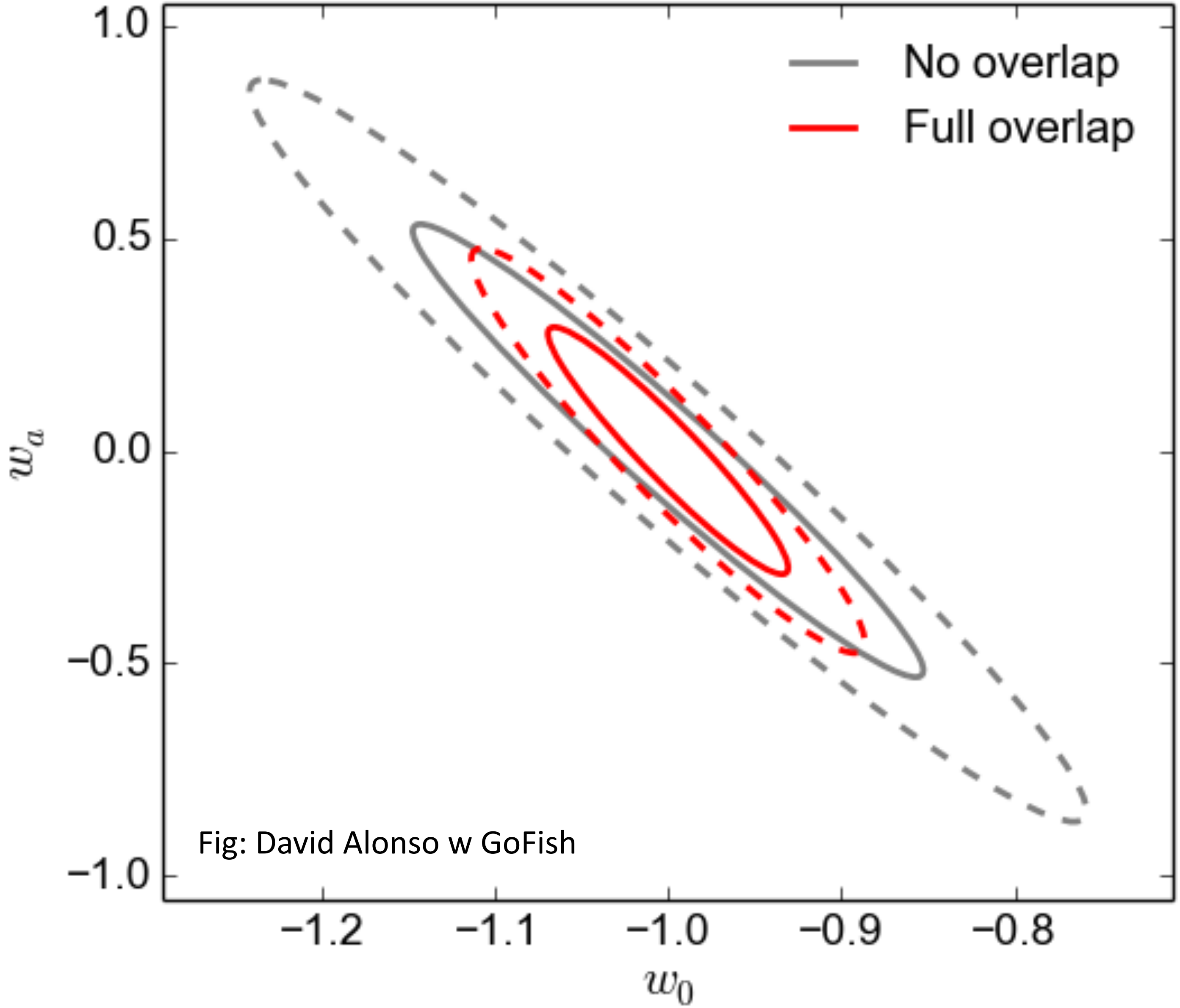
- Different tracers see same fluctuations

- Small change in growth looks just like nuisance parameters

- Use different tracers to constrain nuisance parameters AND growth (galaxy bias, multiplicative bias, photometric redshift uncertainty etc) - *known effects*

- Use different tracers to mitigate unexpected systematic residuals - *unknown effects*

Example 1: galaxy bias



Assume have LSS and WL, but don't know galaxy biases. Use overlap to constrain bias.

$$C_\ell^{gg} = b^2 \int \frac{d\chi}{\chi^2} W_g(\chi)^2 P(\ell/\chi, \chi)$$

$$C_\ell^{g\gamma} = b \int \frac{d\chi}{\chi^2} W_g(\chi) W_\gamma(\chi) P(\frac{\ell}{\chi}, \chi)$$

$$C_\ell^{\gamma\gamma} = \int \frac{d\chi}{\chi^2} W_\gamma(\chi)^2 P(\ell/\chi, \chi)$$

Example 2: residual systematic

Example 2

$$\gamma = \gamma_c + \gamma_s$$

$$g = g_c + g_s$$

$$C_e^{\gamma\gamma} = C_{e\,c}^{\gamma\gamma} + \underline{C_{e\,s}^{\gamma\gamma}}$$

$$C_e^{\gamma g} = C_{e\,c}^{\gamma g}.$$

How do I calculate these theory predictions?

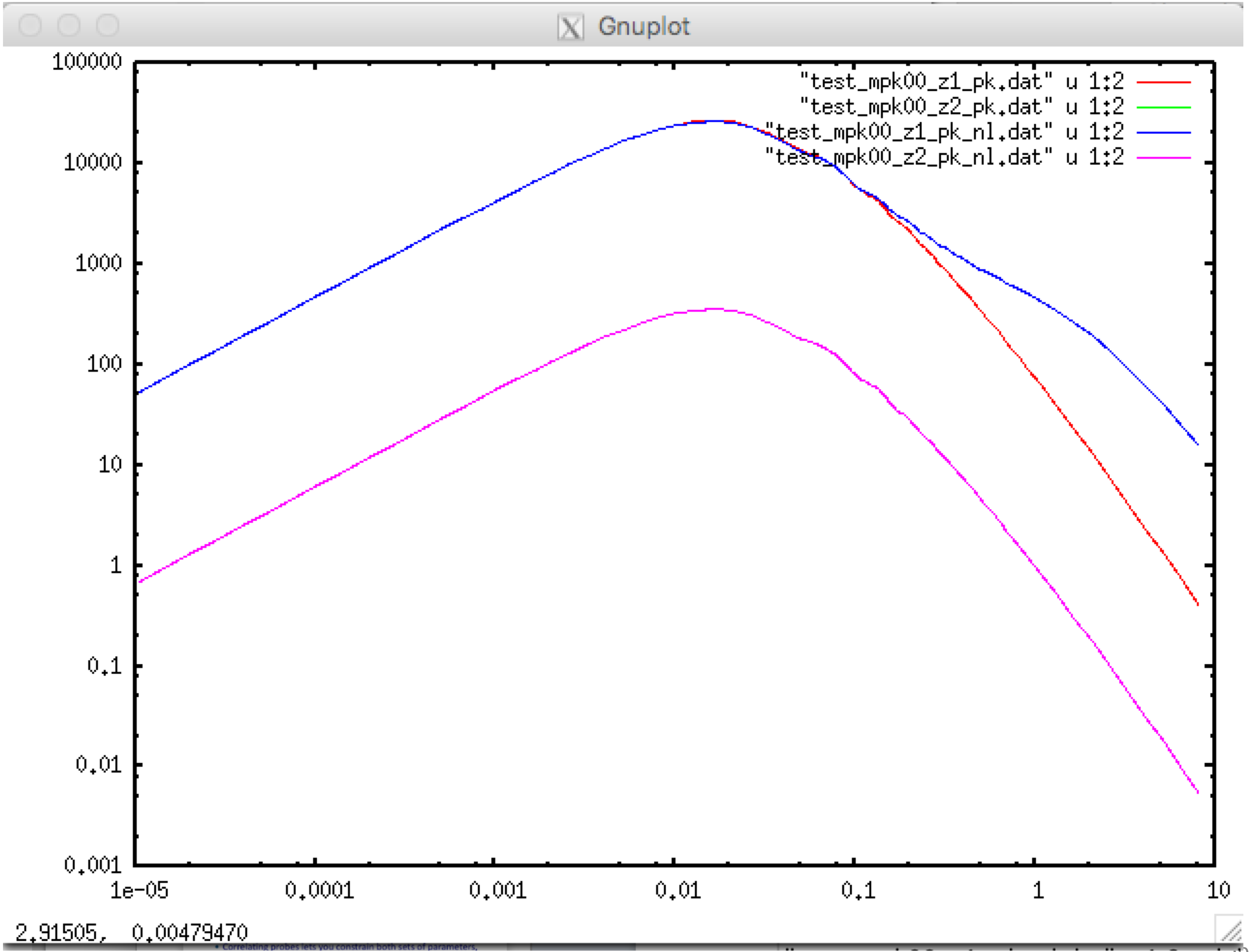
- Baseline = CLASS or CAMB for P(k, z)

- On top of that: integration and interpolation codes
  - CosmoSIS (Joe Zuntz et al)
  - CosmoLike (Elisabeth Krause, Tim Eifler)
  - GoFish (David Alonso, Elisa Chisari)
  - Many others!

- This is part of current TJP work - getting standard integrals into core cosmology library

Example: use CLASS to compute P(k, z=0) and P(k, z=10)

```
>emacs -nw test_mpk.ini
[search for z_pk, and change z_pk to 0,10]
>./class test_mpk.ini
>cd output
>gnuplot
>set logscale
>plot "test_mpk00_z1_pk.dat" u 1:2 w l, "test_mpk00_z2_pk.dat" u 1:2 w l
>plot "test_mpk00_z1_pk.dat" u 1:2 w l, "test_mpk00_z2_pk.dat" u 1:2 w l,
"test_mpk00_z1_pk_nl.dat" u 1:2 w l, "test_mpk00_z2_pk_nl.dat" u 1:2 w l
```


Gnuplot
"test_mpk00_z1_pk.dat" u 1:2
"test_mpk00_z2_pk.dat" u 1:2
"test_mpk00_z1_pk_nl.dat" u 1:2
"test_mpk00_z2_pk_nl.dat" u 1:2
100000
10000
1000
100
10
1
0.1
0.01
0.001
1e-05
0.0001
0.001
0.01
0.1
1
10
2.91505, 0.00479470

Summary

- Many of the DE probes measure clustering of dark matter
- A key distinguishing measurement is growth function

- With one probe this can be confused with nuisance parameters like galaxy bias, multiplicative bias, photo-z uncertainty
- Correlating probes lets you constrain both sets of parameters, and be more robust to unknown systematics

- Key parts of TJP work include:

  Cross-correlation theory be part of core cosmology library

  Building full covariance matrix for joint likelihood