

How Bright Is That Object?

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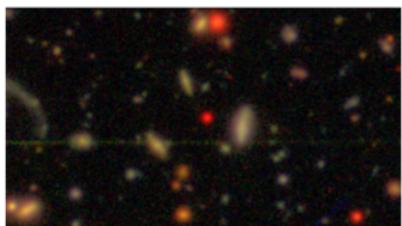
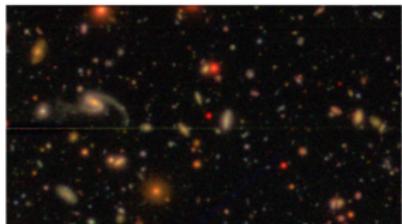
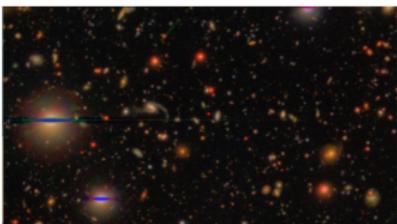
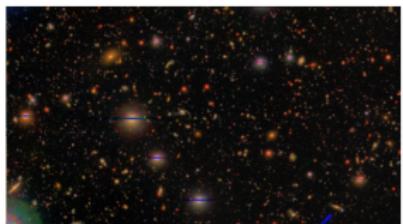
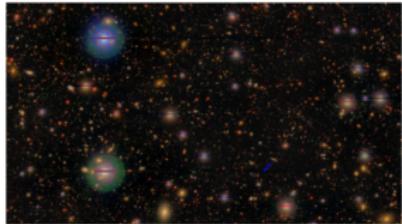
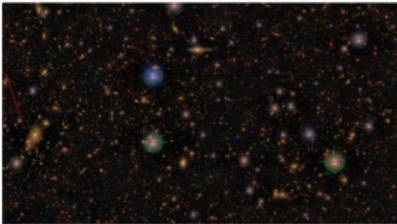
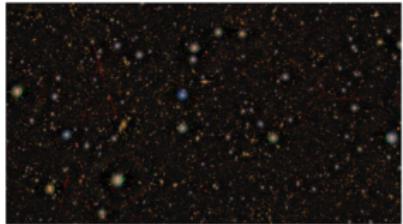


Introduction

What Will LSST Data Look Like?

The HSC project have have 1.5 hours per band in *gr*; c. 3 hours per band in *izy* in Cosmos (280/550 visits), and Michitaro Koike at NAOJ has written a nice tool 'hscMap' to visualise the true-colour images. I have an unofficial copy on my laptop, but [here](#)'s a link to NAOJ's page.

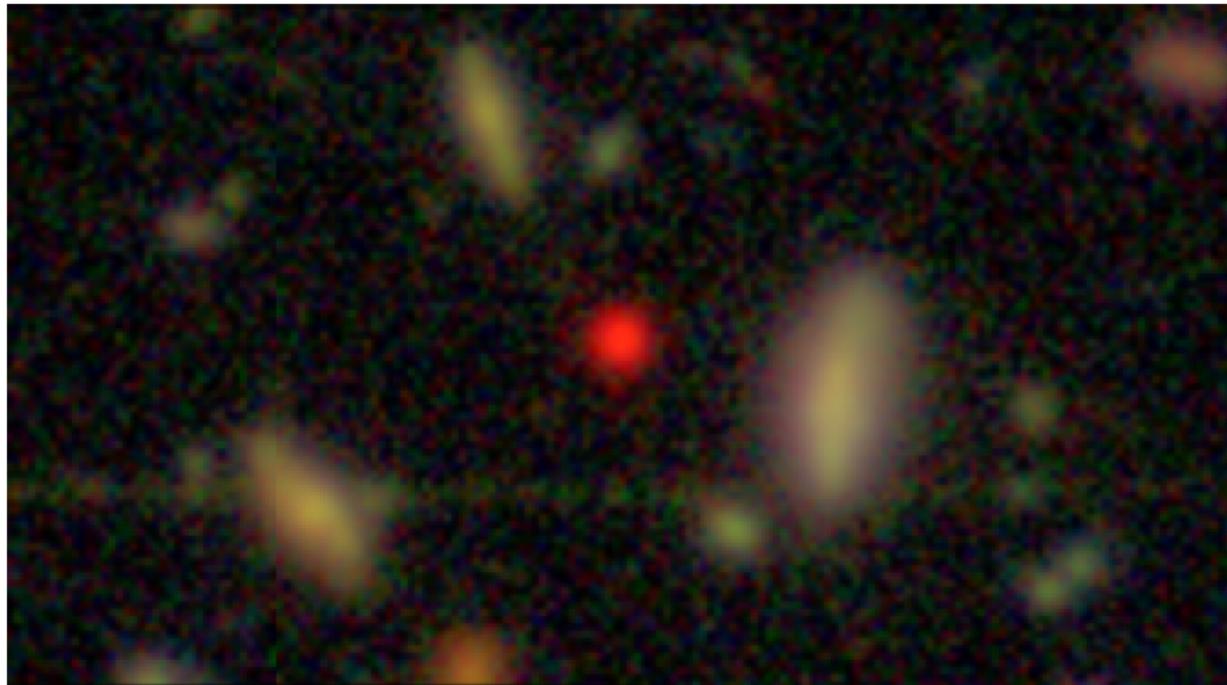
What Will LSST Data Look Like?



g-r-i true-colour composite of COSMOS

Background Levels

The Background Level



The background of the picture is black, but only because I've subtracted a pedestal from each image

Question I: Whence comes the pedestal ('sky') level?

List as many contributions as you can to the 'sky' level.

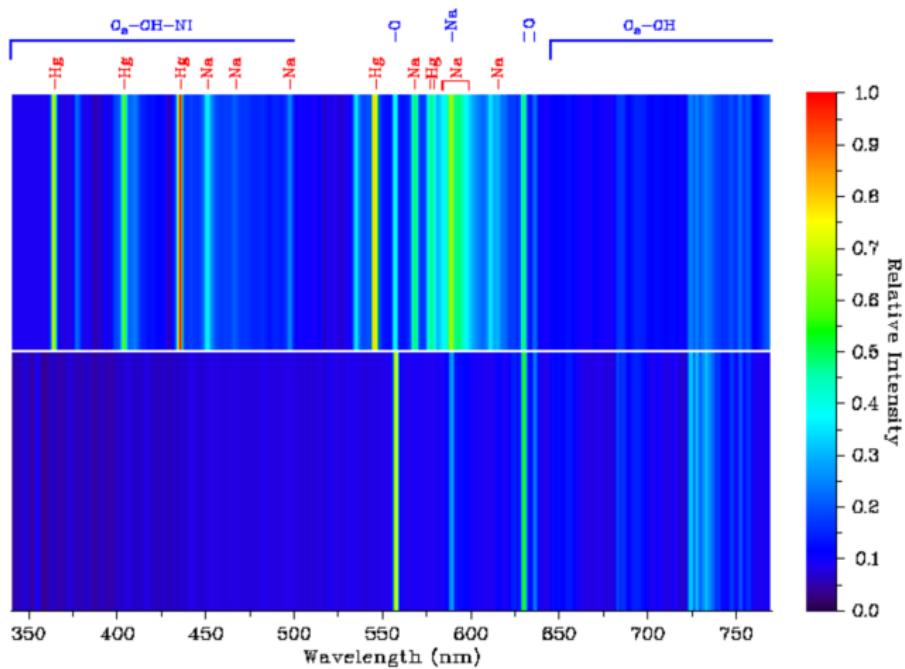
Answer I

Things I thought of were:

- Night sky emission (O_2 , OH)
- Zodiacal light/Gegenschein
- Starlight scattered from the atmosphere
- Moonlight scattered from the atmosphere
- Galactic cirrus
- Extra-Galactic background

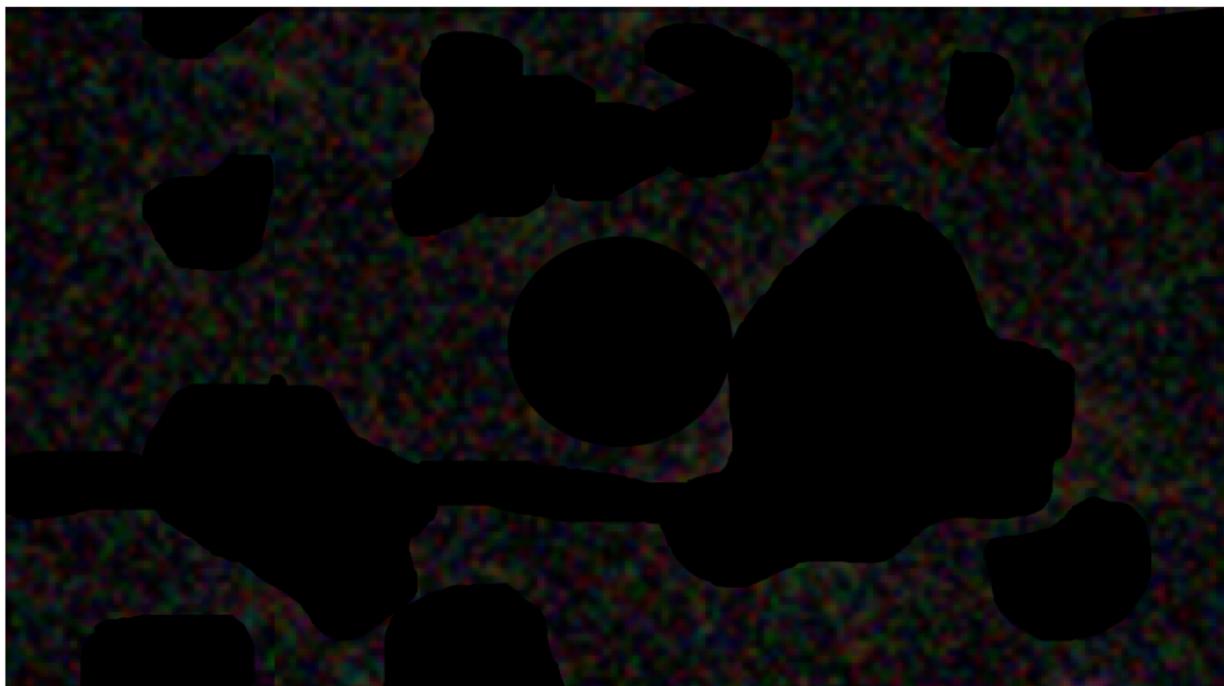
- Night sky emission (Na , Hg , ...)
- Scattered and Ghost light from the telescope
- Dark current in the CCDs
- Glow from the ion pumps

Answer I



The Background Level

Let's ignore spatial structure in the sky; this isn't really a good approximation but it would take us too far afield to address the problem. I've masked the obvious objects.



Question 2: What measurement on the unmasked pixels should I use to define the sky level?

Answer 2

Popular answers are:

- The Mean
- The Mode
- The Median
- A Clipped Mean

Question: Does the Central Limit Theorem guarantee that a Poisson distribution's median \rightarrow mean as $n \rightarrow \infty$?

Answer: No; median - mean $\rightarrow 1/6$.

The correct answer is, "The mean of everything that wouldn't have been detected if it'd been under your object". A clipped mean is safer, though.

We can subtract the background level, but we cannot (of course) subtract its noise.

Stars

Calibration

We really want to know how bright stars are in physical units such as Janskys ($10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$) or AB magnitudes (zero-point 3631 Jy);

$$S_\nu [\text{mJy}] \equiv 10^{-0.4(m_{\text{AB}} - 23.90)}$$

Measuring absolute fluxes is difficult, so in practice we measure only *relative* fluxes, and defer the conversion to Jy to a future discussion.

In other words, we use some algorithm to measure our target star's brightness, then apply the *same* algorithm to a standard star of known brightness, and knowing the ratio we deduce the desired flux.

These days we have standard stars over most of the sky (from SDSS and PanSTARRS, and soon DES). Even better, it seems likely that we'll be able to use GAIA to provide above-the-atmosphere standards over the entire sky.

Definitions

Let us assume that the star's profile, the PSF, is given by $\phi(\mathbf{x})$, normalised such that

$$\sum_i \phi(\mathbf{x}_i) = 1$$

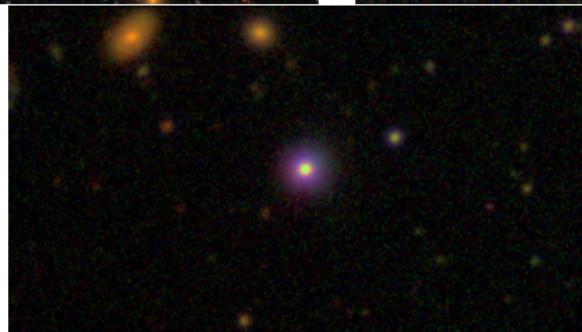
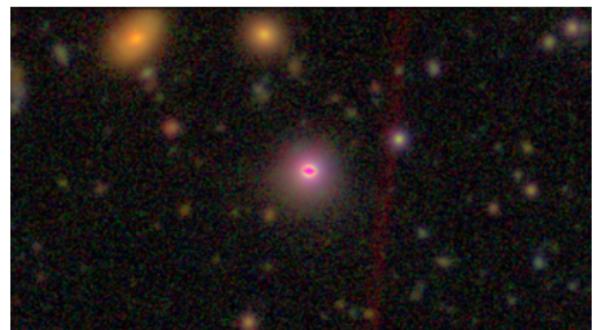
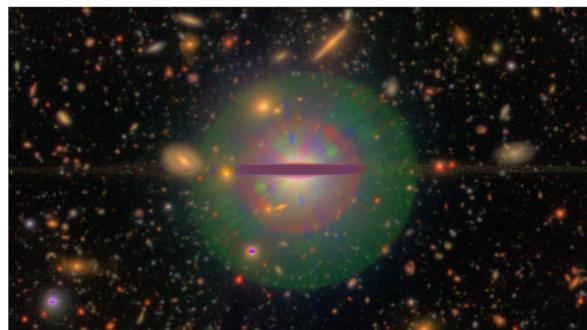
and that its amplitude (and flux) is A , measured in electrons. Let's take the noise in the background ϵ to be homoschedastic and Gaussian: $N(0, \sigma^2)$ (i.e. the per pixel variance is σ^2)

Let's agree to ignore uncertainties in the (subtracted) background level.

N.b. because we took A to be measured in electrons, each of which corresponds to a photon, A 's standard deviation is \sqrt{A} .

Bright Stars

It's easier to measure bright stars than faint as they are less sensitive to background subtleties and contamination from neighbours.



Question 3: What pixel measurements should I make to find the number of counts in a bright star?

By 'bright' I mean that the background noise is negligible relative to the photon noise in the star.

Answer 3

Choose a largish circular aperture of radius R centered on the star, and add up all the pixel values.

If you are worried about spatial structure in the background, choose a larger annulus with the same centre and estimate the sky level there.

Question 4: Does it matter if the inner radius of the background annulus is too small?

Note that we're asking about the choice of pixels, not the annulus's area (which affects the variance of the sky estimate).

Answer 4:

No, not if you are using an sky estimator (such as the mean) that is linear in the counts.

You'll remove some fraction of the light from the star, but because you did exactly the same thing to your standard star the ratio is still correct. This is really not very different from choosing a small value of R .

Question 5: What is the variance of your estimate of the star's flux?

You'll remember that the star's profile is $A\phi(\mathbf{x})$ and the background variance is σ^2 .

Answer 5:

Our model is that

$$l_i = A\phi(\mathbf{x}_i) + \epsilon_i$$

and we estimate the flux as

$$\hat{A} = \frac{1}{\sum_{|\mathbf{x}_i| \leq R} \phi(\mathbf{x}_i)} \sum_{|\mathbf{x}_i| \leq R} l_i \equiv f \sum_{|\mathbf{x}_i| \leq R} l_i$$

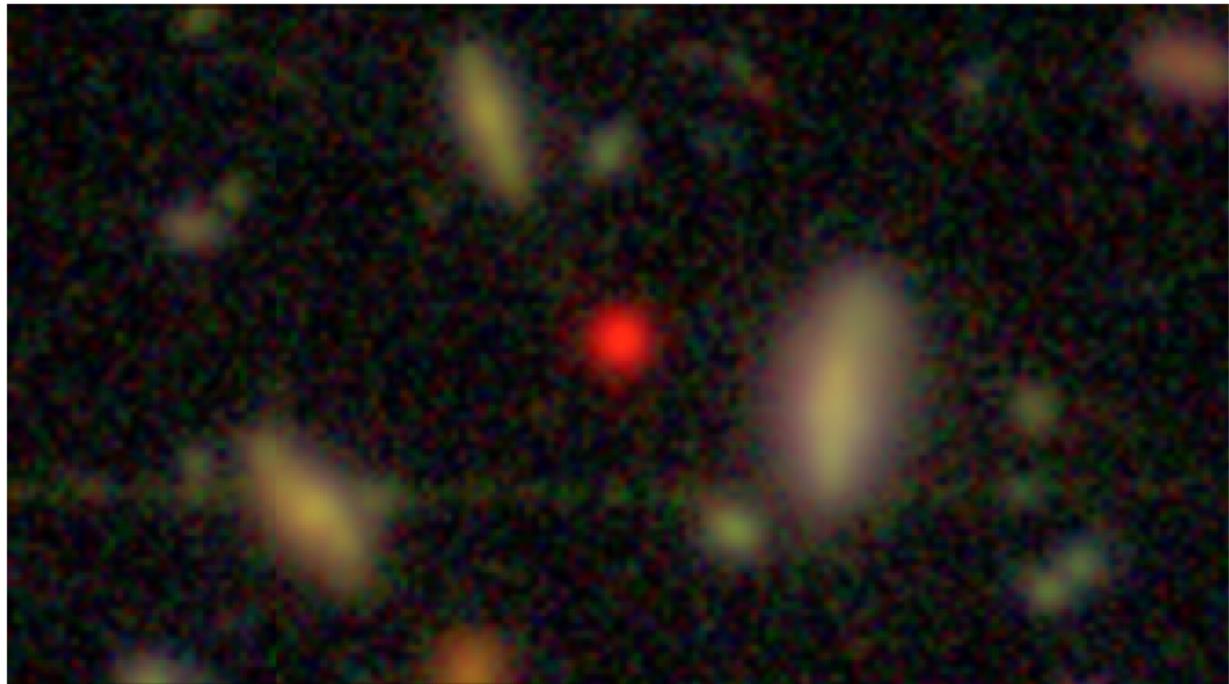
(N.b. $f > 1$).

\hat{A} 's variance is

$$\text{Var}(\hat{A}) = f^2 \sum_{|\mathbf{x}_i| \leq R} (A\phi(\mathbf{x}_i) + \sigma^2) = f^2 (A + \pi R^2 \sigma^2)$$

We said that the star was 'bright', so neglecting the background noise we recover the irreducible Poisson noise as $R \rightarrow \infty$ and $f \rightarrow 1$.

Faint Stars



That red spot is a faint star. How bright is it? Let's assume that it's isolated.

Question 6: What pixel measurements should I make to find the number of counts in a faint star?

By 'faint' I mean that the photon noise in the star is negligible relative to the background noise.

Assume that you know the star's PSF and centroid (maybe we are interested in measuring its variability, or the optical flux of a well-localised LIGO source).

Answer 6

We'll use a maximum likelihood estimator. Our model is that

$$l_i = A\phi(\mathbf{x}_i) + \epsilon_i$$

so

$$\ln \mathcal{L} = \sum_i \frac{(l_i - A\phi(\mathbf{x}_i))^2}{\sigma_i^2}$$

and differentiating with respect to A we find that

$$\hat{A} = \frac{\sum_i l_i \phi(\mathbf{x}_i) / \sigma_i^2}{\sum_i \phi^2(\mathbf{x}_i) / \sigma_i^2}$$

Because we assumed that the star's photon noise was negligible, $\sigma_i = \sigma$ and this reduces to

$$\hat{A} = \frac{\sum_i l_i \phi(\mathbf{x}_i)}{\sum_i \phi^2(\mathbf{x}_i)}$$

with variance

$$\text{Var}(A) = \frac{\sigma^2}{\sum_i \phi^2(\mathbf{x}_i)} \equiv n_{\text{eff}} \sigma^2$$

Question 7: What is \hat{A} 's variance if the PSF is a Gaussian?

i.e. what's the value of

$$n_{\text{eff}} \equiv \frac{1}{\sum_i \phi^2(\mathbf{x}_i)}$$

if $\phi \sim N(0, \alpha^2)$? What's n_{eff} if ϕ is constant over a disk of radius R ?

Bonus question: If you're quick with integrals and good at numerical optimisation, what's the optimal value of R for a circular aperture for faint stars with this seeing profile, and what's the lowest variance that you can obtain?

Answer 7

For the Gaussian case,

$$\text{Var}(A) = 4\pi\alpha^2 \sigma^2$$

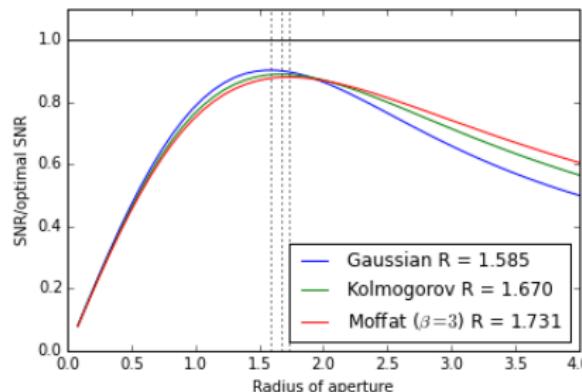
For the uniform disk, remembering that ϕ sums to unity and thus

$$\phi(r) = \begin{cases} 1/(\pi R^2) & r < R \\ 0 & r > R, \end{cases}$$

we have

$$\text{Var}(A) = \pi R^2 \sigma^2$$

Bonus answer:



Question 8: Should I have kept the per-pixel variance?

I replaced σ_i by σ (arguing that it was essentially the same for faint objects). Was this a good idea?

Answer 8

Yes.

Using constant weighting for every pixel is slightly suboptimal (it overweights the slightly-noisier core of the brighter stars), but more importantly if the PSF model isn't quite right the discrepancies between the data and the model are weighted *differently* for bright and faint stars.

This violates our promise that we will faithfully measure the ratio of fluxes for bright stars (e.g. our standards) and faint ones (whose properties we care about). We've traded noise for bias, and this is almost always a bad idea. If you're not convinced, remember that for bright sources the photon noise is likely to be sub-dominant to systematic errors anyway.

For a Gaussian PSF, using a constant weight increases the photon noise from the object by 33%.

Galaxies

Galaxies are harder

The image of a star is defined once we know its flux, centroid (assuming we've measured the PSF, ϕ). Galaxies are more complicated



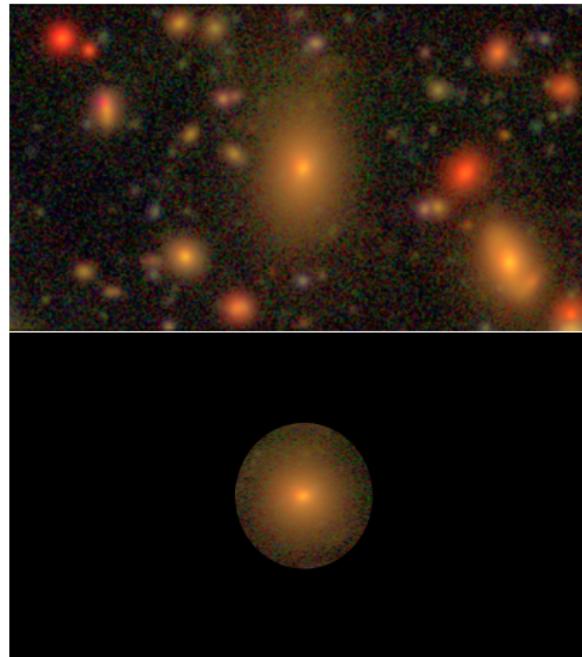
Galaxies are more complicated

They have:

- a variety of radial profiles
- non-circular isophotes
- complicated morphology
- colour gradients
- large dynamic range
- a tendency to cluster

Note that astronomers use the word 'colour' to mean the ratio of the flux measured through different filters. Because we traditionally use logarithmic units (magnitudes), the colour is expressed as the difference of two magnitudes, e.g. $g - r$.

Simple Galaxies



Let's start with the simple case; isolated circular galaxies where we're only interested in one band

Question 9: What radius should I use to measure a circular galaxy's aperture flux?

What pixel measurements should I make to find the number of counts in some band (e.g. g) in a bright circular galaxy with known centroid? We can use a circular aperture, but how should we choose the radius?

By 'bright' I again mean that the background noise is negligible relative to the photon noise in the galaxy.

Answer 9

- An isophotal radius (e.g. 25 mag arcsec $^{-2}$)
- A multiple (e.g. 2.5) of the Kron radius
- A multiple (e.g. 2) of the Petrosian radius

The Kron radius R_K is defined as

$$R_K \equiv \frac{\int_A r I(r) 2\pi r dr}{\int_A I(r) 2\pi r dr}$$

for some area A around the galaxy.

The Petrosian radius R_P is defined in terms of the Petrosian ratio \mathcal{R}_P ;

$$\mathcal{R}_P(r) \equiv \frac{I(r)}{\frac{1}{\pi r^2} \int_0^r I(s) 2\pi s ds};$$

$$\mathcal{R}_P(R_P) = f_P$$

SDSS used $f_P = 0.2$

Elliptical Galaxies

Galaxies with elliptical isophotes are not really much more complicated once we've chosen an axis ratio and position angle a/b and ψ (or equivalently e_1 and e_2). The radius r from the previous slide are interpreted as the size of the ellipse (conventionally \sqrt{ab}).

The problem comes in defining the shape of the ellipse. Ideally it would:

- be robustly measurable
- reduce to circular for marginally resolved objects

HSC uses 'adaptive Gaussian moments', but I'm not all that happy with the results.

We need to find a way to impose some sort of prior before LSST goes on the sky.

If there are isophotal twists (e.g. in S0s, or triaxial elliptical galaxies) the problem is, of course, ill-posed.

Question 10: What is 'Formularum'?

In the sentence

- The radius r formularum on the previous slide are interpreted as the size of the ellipse (conventionally \sqrt{ab}).

what is 'formularum'?

Answer 10

The genitive plural of "formula"

Question 11: What pixel measurements should I make to find the number of counts in a faint galaxy?

What pixel measurements should I make to find the number of counts in some band (e.g. g) in a faint circular galaxy?

By 'faint' I mean that the photon noise in the galaxy is negligible relative to the background noise.

Answer II

We'll use a maximum likelihood estimator; the problem is that the appropriate model is not obvious. A popular choice is a Sérsic profile

$$S(r; r_e, n) = A e^{-(r/r_e)^{1/n}}$$

with flux $A r_e^2 f(n)$, so our model becomes

$$I_i = A S(\mathbf{x}_i; r_e, n) \otimes \phi(\mathbf{x}_i) + \epsilon_i$$

so

$$\ln \mathcal{L} = \sum_i \frac{(I_i - A S(\mathbf{x}_i; r_e, n) \otimes \phi(\mathbf{x}_i))^2}{\sigma_i^2}$$

and we can estimate \hat{A} (and \hat{n} and \hat{r}_e) numerically; note that each step involves a convolution with the PSF (and this can be tricky if r_e is comparable to the size of the pixels).

As with stars, dividing each pixel by its variance can lead to biases as a function of magnitude; this is particularly a problem with \hat{A} but biases in the other parameters are also worrying.

Elliptical Galaxies

In reality we fit two more parameters (e.g. a/b and ψ or e_1 and e_2) to allow for the galaxy's ellipticity.

Question 12: Why are you not following the Bayesian Way?

I used a maximum-likelihood estimator, with implicit uninformative priors. What priors would have been better?

Answer 12

- Using a positivity prior on r_e seems pretty safe.
- Priors on n are reasonably non-controversial.
- priors on e_1 and e_2 are easier than using a/b (an uninformative prior for ψ is fine).
- Priors on A get us into well-known problems with faint counts (they show up in stellar photometry too).
- In general, informative priors lead to violating the condition that making an object a factor of two fainter changes our estimate of its flux by some other factor. Whether this is a problem is a scientific question that I'm not going to answer here; render unto DESC that which is DESC's.

Galaxy Colours

What should we do if we want to measure the colour of our galaxy (assumed to be free of colour gradients for now)?

The problem is that the seeing can be different in each band.



Question 13: How should I handle Variable Seeing?

This is only a problem if the galaxy's size is comparable to the seeing; so please ponder that case.

Answer 13

If we're using model-fit photometry, and the model is good, there's nothing to be done as the PSF is included in the model.

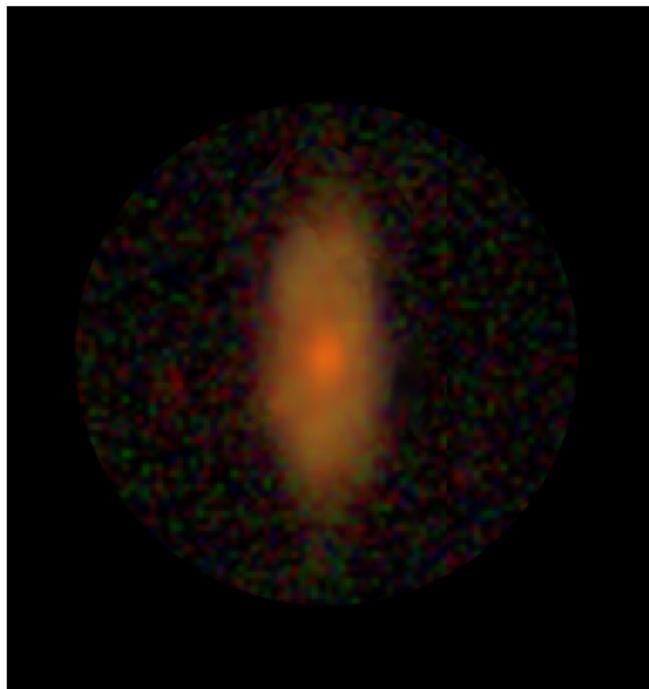
If you're wedded to aperture photometry you'll need to come up with some scheme for correcting the Kron/Petrosian/... radius and aperture for the seeing.

A popular alternative is to degrade all the images to the same seeing. This is the only totally safe thing to do, but comes with a noise penalty.

Well, as an alternative you *could* use an enormous aperture, but that carries an unacceptable noise penalty for most applications.

Colour Gradients

Real galaxies have colour gradients



Colour Gradients

How should we handle this? There are (at least) three cases:

- We care about the total flux
- We care about the colour of the entire galaxy
- We care about the colour of some well-defined stellar population

The latter is the case when we're trying to estimate a photometric redshift; we weight each kpc² differently --- so we deduce the wrong star formation history --- so it has the colour of stellar population at the proper redshift and the photo-z will be correct.

In many cases colour gradients are better thought of as multiple components with distinct colours (e.g. a bulge and a disk). This isn't always true (e.g. metallicity gradients in giant ellipticals).

The Curious Case of the Constant PSF

If the PSF in each band is the same (or has been made the same) and we don't require a total flux or a total colour (e.g. to guess a star formation history) we can use *any* 'aperture' to measure a colour.

A smart choice is a model of the galaxy (as it has good S/N properties), but you have to use the same model in each band, and worry about biases due to the model being presumably a better fit in the band in which it was originally fit. This bias is still present if you use a Kron or Petrosian aperture.

Question 14: How should I measure colours if the PSFs are different in each band?

Please consider both cases:

- We care about the colour of the entire galaxy
- We care about the colour of some well-defined stellar population

Answer 14

Tricky. If we fit a model that's flexible enough to recover the total flux then the PSF convolution allows for the PSF variation and all is well for both cases considered. The sort of models that people like are two component with fixed structural parameters and free amplitudes (think bulge+disk).
Or we can degrade the seeing to a constant and refer to the previous discussion.

Question 15: What should you do if the galaxies are not isolated?

Answer 15

Panic.



And/or invite me back to another DESC DE School to talk about deblenders.