

Emulating SPS and how to use it

1. Neural emulation of SPS models (“Speculator”)
2. Hierarchical inference of $n(z)$ under SPS models

Justin Alsing, Oskar Klein Centre, Stockholm

Hiranya Peiris, Daniel Mortlock, Joel Leja, Boris Leistedt, George Efstathiou, ChangHoon Hahn and others

Why emulate SPS?

- Want to be able to analyse photometry under SPS models (for photo-z, and also galaxy evolution studies)
- SPS is too slow to scale up for LSST => need emulation

Emulating SPS spectra

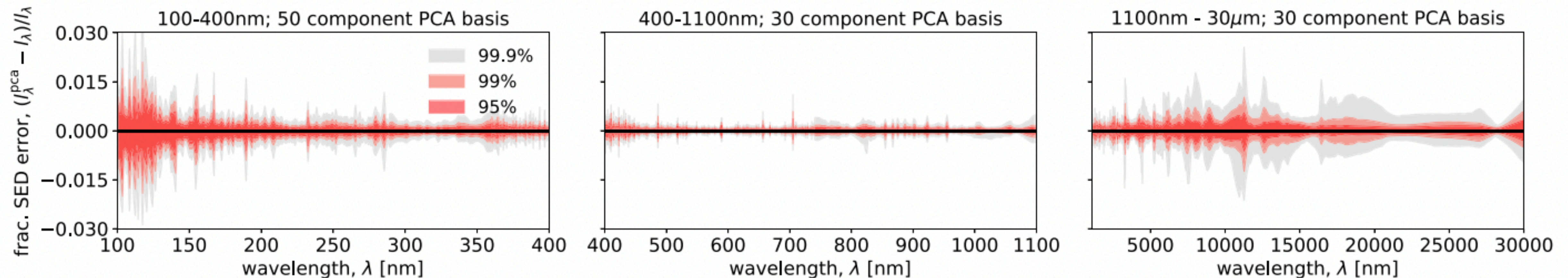
1. Generate training set of SPS parameters and spectra
2. PCA decompose the spectra
3. Train neural network to learn PCA coefficients, as a function of the SPS parameters

State-of-the-art 15-parameter Prospector-alpha model
Training set of a few million spectra (~ couple CPU days)

Parameter	Description	Prior
M	Total stellar mass formed	Log uniform $[10^7, 10^{12.5}]M_{\odot}$
$r_{\text{SFH}}^1, \dots, r_{\text{SFH}}^6$	Ratio of log SFR between adjacent bins	Clipped student's-t: $\sigma = 0.3, \nu = 2, r_{\text{SFH}}^i \leq 5$
t_{age}	Age of universe at the lookback time of galaxy	Uniform $[2.6, 13.7]$ Gyr, $(0 < z < 2.5)$
τ_2	Diffuse dust optical depth	Normal $\mu = 0.3, \sigma = 1, \text{min} = 0, \text{max} = 4$
τ_1	Birth cloud optical depth	Truncated normal in τ_1/τ_2 $\mu = 1, \sigma = 0.3, \text{min} = 0, \text{max} = 2$
n	Index of Calzetti et al. (2000) dust attn. curve	Uniform $[-1, 0.4]$
$\ln(Z_{\text{gas}}/Z_{\odot})$	Gas phase metallicity	Uniform $[-2, 0.5]$
f_{AGN}	Fraction of bolometric luminosity from AGN	Log uniform $[10^{-5}, 3]$
τ_{AGN}	Optical depth of AGN torus	Log uniform $[5, 150]$
$\ln(Z/Z_{\odot})$	Stellar metallicity	Truncated normal with μ and σ from Gallazzi et al. (2005) mass-metallicity relation (see Section 4), limits $\text{min} = -1.98, \text{max} = 0.19$
z	Redshift	Uniform $[0.5, 2.5]$

Emulating SPS spectra

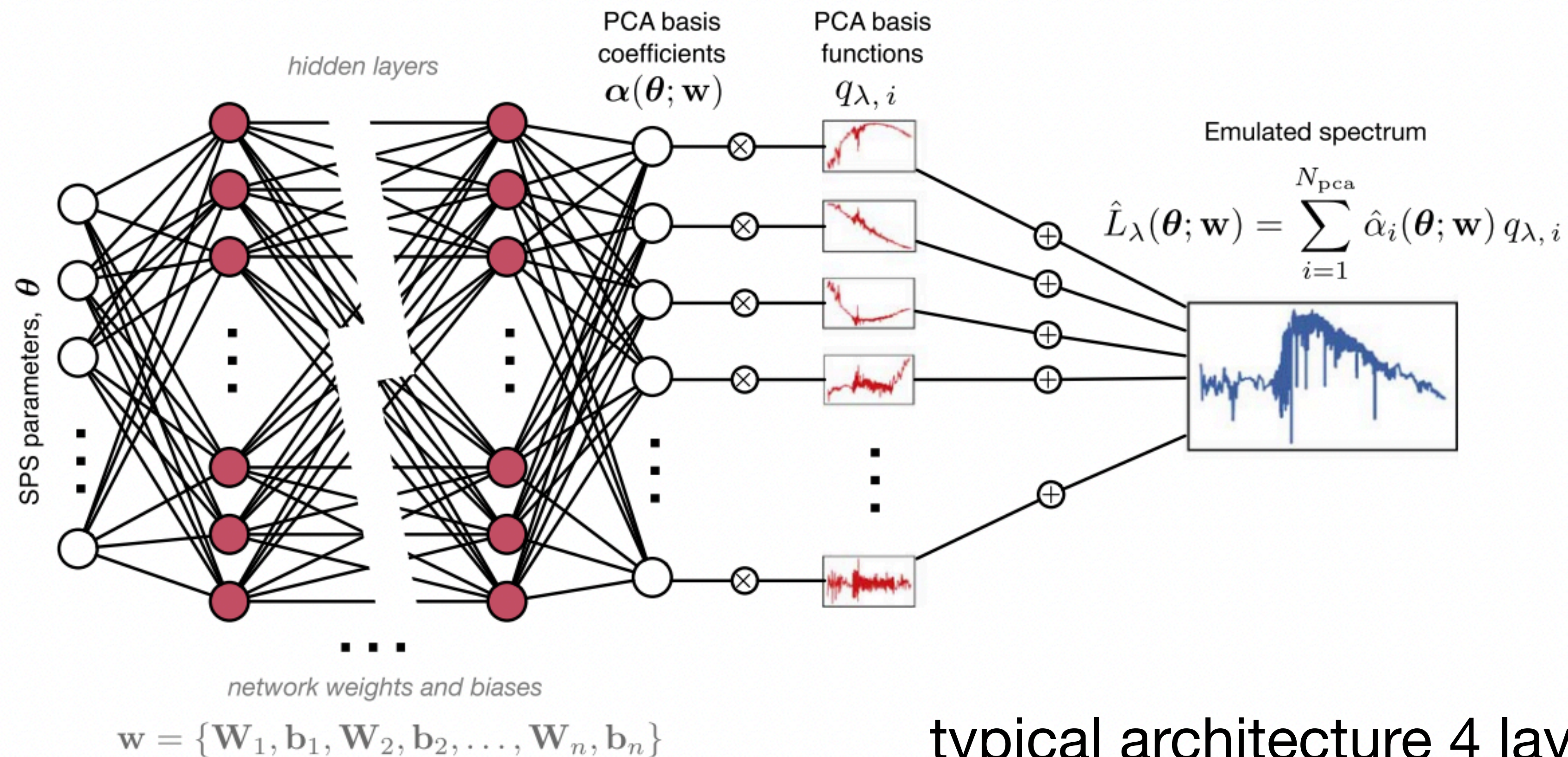
1. Generate training set of SPS parameters and spectra
2. PCA decompose the spectra
3. Train neural network to learn PCA coefficients, as a function of the SPS parameters



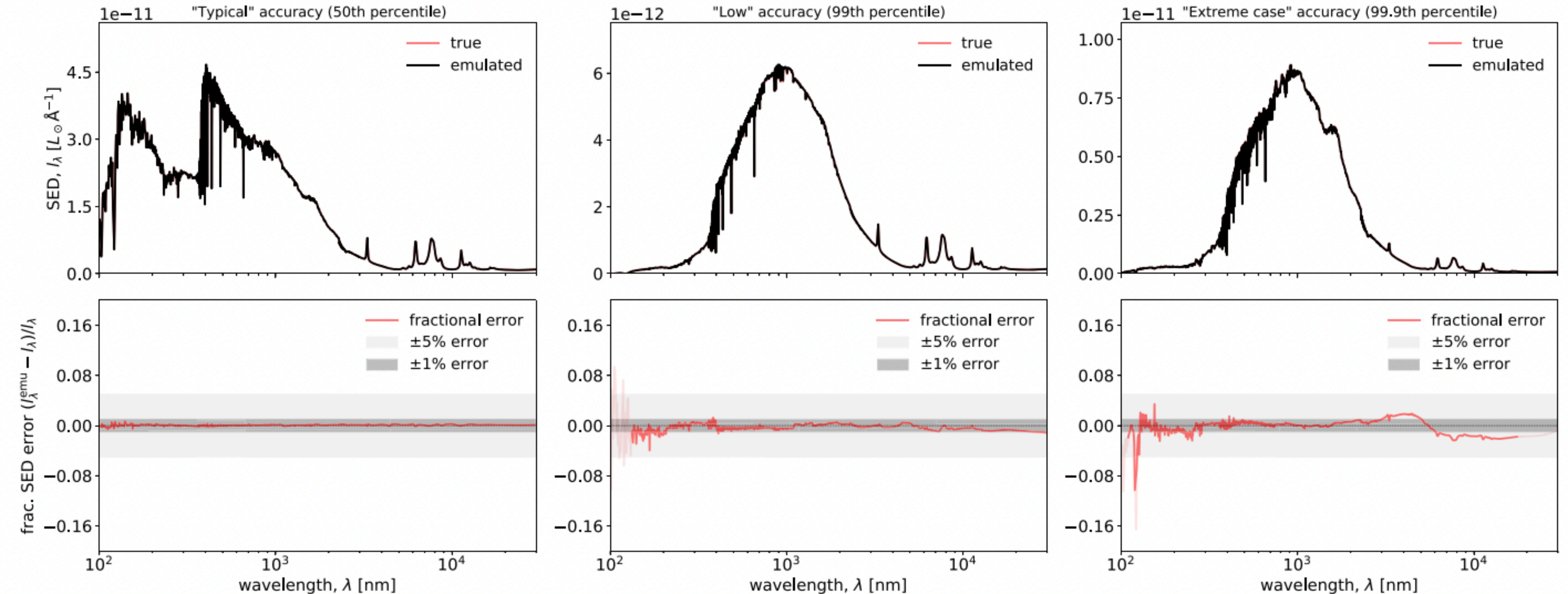
accuracy increases with more components if needed

Emulating SPS spectra

1. Generate training set of SPS parameters and spectra
2. PCA decompose the spectra
3. Train neural network to learn PCA coefficients



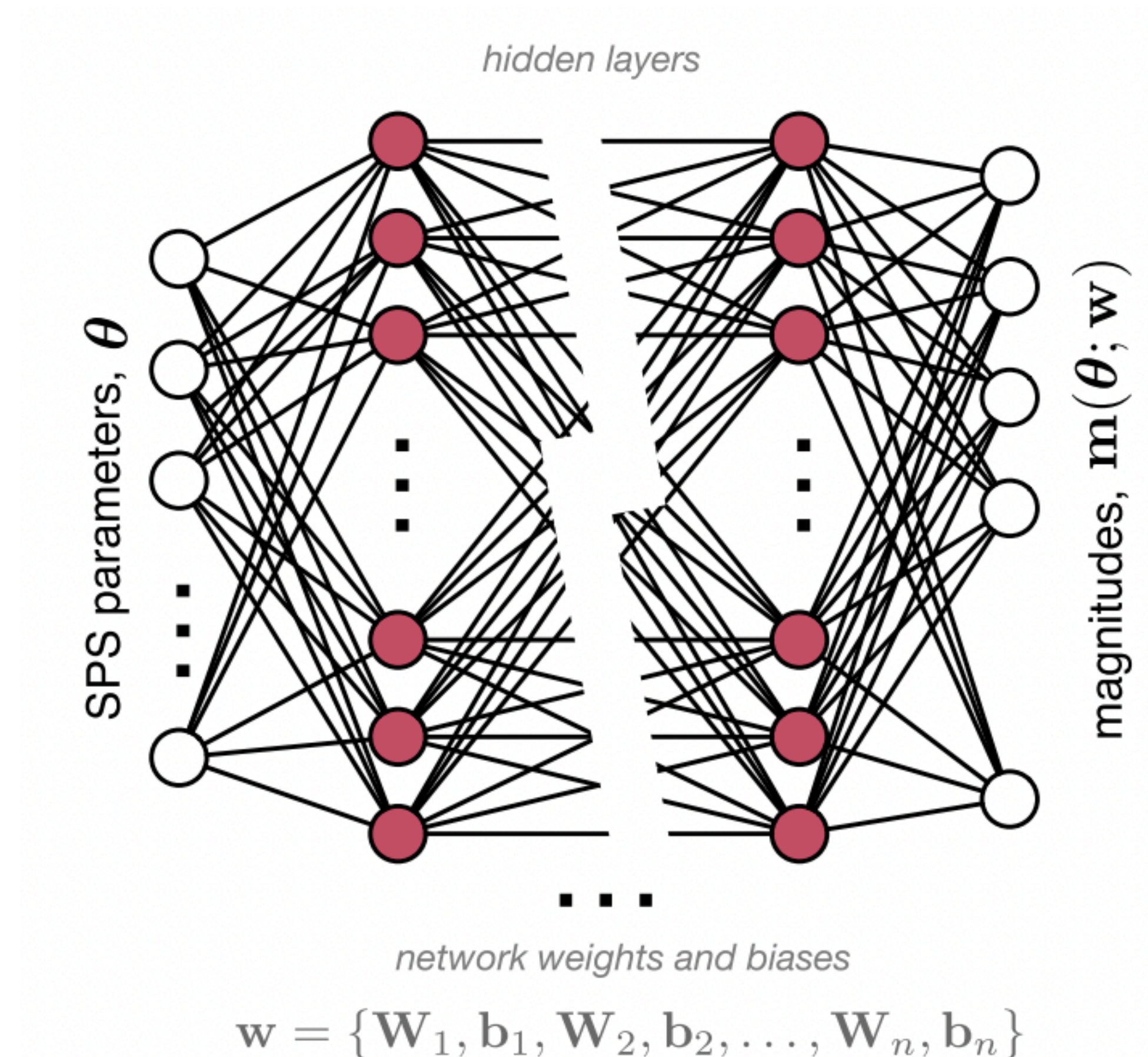
Emulating SPS spectra



10^4 x speed-up, %-level accuracy, differentiable

Emulating SPS photometry

1. Generate training set of SPS parameters and photometry
2. Train neural network to learn magnitudes directly

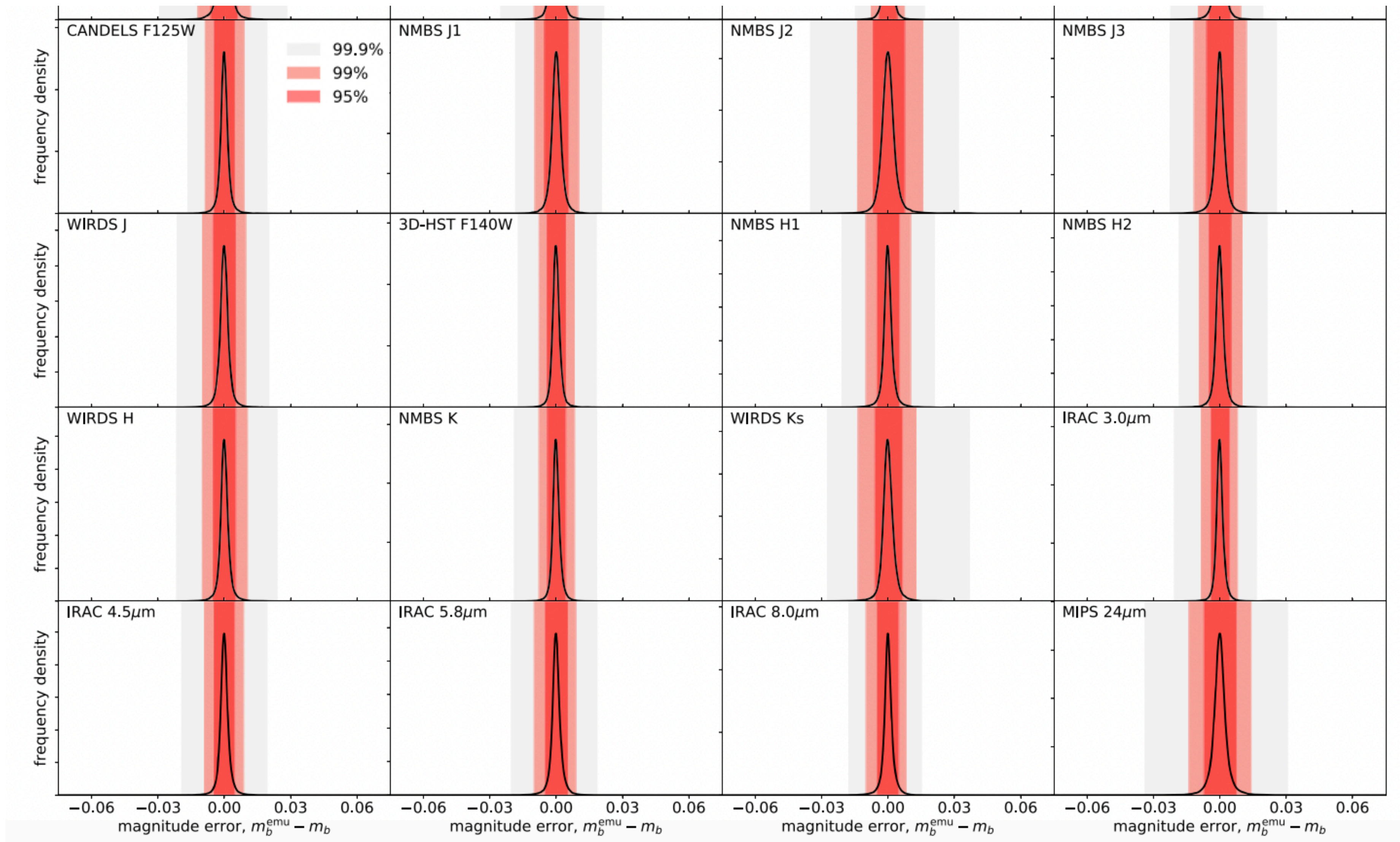


Emulating SPS photometry

10^4 x speed-up

< %-level
accuracy

Differentiable



What to do with SPS emulators

1. Fast SPS-parameter and redshift posteriors (<10 sec per galaxy)
2. Photometric likelihoods for embedding into LSS hierarchical models (eg BORG)
3. Forward modelling and hierarchical inference of cosmological redshift distributions $n(z)$

Forward modeling for $n(z)$

1. **Population model:** Galaxies properties drawn from population
2. **SPS model:** Generate photometry given physical properties
3. **Data model:** Add noise, do calibration
4. **Selection model:** Apply selection cuts

$$P(\theta_{1:N}, z_{1:N}, \psi, \eta | \hat{\mathbf{f}}_{1:N}) \propto P(\psi, \eta) \times \prod_{i=1}^{N_{\text{galaxies}}} P(\theta_i, z_i | \psi) \times P(\hat{\mathbf{f}}_i | \mathbf{f}(\theta_i, z_i), \eta) \times [P(S_i | \psi, \eta)]^{-1}$$

Diagram illustrating the forward modeling process for $n(z)$ using the joint posterior equation:

- joint posterior** points to the left side of the equation: $P(\theta_{1:N}, z_{1:N}, \psi, \eta | \hat{\mathbf{f}}_{1:N})$
- prior** points to the prior term: $P(\psi, \eta)$
- SPS parameters, redshift** points to the SPS model term: $P(\theta_i, z_i | \psi)$
- population model parameters** points to the population model term: $P(\theta_i, z_i | \psi)$
- observed fluxes** points to the observed fluxes term: $\hat{\mathbf{f}}_i$
- model fluxes** points to the model fluxes term: $\mathbf{f}(\theta_i, z_i)$
- data model parameters** points to the data model term: $P(\hat{\mathbf{f}}_i | \mathbf{f}(\theta_i, z_i), \eta)$
- selection integral** points to the selection term: $[P(S_i | \psi, \eta)]^{-1}$

Forward modeling for $n(z)$

Where does $n(z)$ fit into all this?

$$n(z) \equiv P(z|S) \xrightarrow{\text{chain rule, expand, etc}} \frac{1}{P(S)} \int \left[\iint P(S|\hat{\mathbf{f}}, \theta, z) P(\hat{\mathbf{f}}|\theta, z, \sigma) P(\sigma) d\hat{\mathbf{f}} d\sigma \right] P(\theta, z) d\theta$$

$n(z)$ is just an integral over selection x data model x population model

$$P(\theta_{1:N}, z_{1:N}, \psi, \eta | \hat{\mathbf{f}}_{1:N}) \propto P(\psi, \eta) \times \prod_{i=1}^{N_{\text{galaxies}}} P(\theta_i, z_i | \psi) \times P(\hat{\mathbf{f}}_i | \mathbf{f}(\theta_i, z_i), \eta) \times [P(S_i | \psi, \eta)]^{-1}$$

Diagram illustrating the components of the joint posterior probability:

- joint posterior** (points to the left side of the equation)
- prior** (points to $P(\psi, \eta)$)
- SPS parameters, redshift** (points to θ_i, z_i)
- population model parameters** (points to $P(\theta_i, z_i | \psi)$)
- observed fluxes** (points to $\hat{\mathbf{f}}_i$)
- model fluxes** (points to $\mathbf{f}(\theta_i, z_i)$)
- data model parameters** (points to $P(\hat{\mathbf{f}}_i | \mathbf{f}(\theta_i, z_i), \eta)$)
- selection integral** (points to $[P(S_i | \psi, \eta)]^{-1}$)

Forward modeling for $n(z)$

Advantages

- *Does not rely on spec-z calibration*
- *Auxiliary data (spec-z, extra surveys) can be included seamlessly (extended data vector or extra priors for objects with extra information)*
- *“Turns photo-z back into a physics problem”, synergies with galaxy-evo community*

$$P(\theta_{1:N}, z_{1:N}, \psi, \eta | \hat{\mathbf{f}}_{1:N}) \propto P(\psi, \eta) \times \prod_{i=1}^{N_{\text{galaxies}}} P(\theta_i, z_i | \psi) \times P(\hat{\mathbf{f}}_i | \mathbf{f}(\theta_i, z_i), \eta) \times [P(S_i | \psi, \eta)]^{-1}$$

The diagram illustrates the components of the forward modeling equation. Arrows point from the following labels to their corresponding parts in the equation:

- joint posterior** points to the entire equation.
- prior** points to $P(\psi, \eta)$.
- SPS parameters, redshift** points to $P(\theta_i, z_i | \psi)$.
- population model parameters** points to $P(\theta_i, z_i | \psi)$.
- observed fluxes** points to $\hat{\mathbf{f}}_i$.
- model fluxes** points to $\mathbf{f}(\theta_i, z_i)$.
- data model parameters** points to $P(\hat{\mathbf{f}}_i | \mathbf{f}(\theta_i, z_i), \eta)$.
- selection integral** points to $[P(S_i | \psi, \eta)]^{-1}$.

Forward modeling for $n(z)$

Research questions

1. *Can we forward model well enough to infer $n(z)$ with high-fidelity?*
2. *If so, can we actually “do” the inference under this model?*

The diagram shows the equation for the joint posterior probability, with arrows pointing from descriptive labels to specific parts of the formula. The labels and their corresponding parts are: 'prior' points to $P(\psi, \eta)$; 'population model parameters' points to $P(\theta_i, z_i | \psi)$; 'data model parameters' points to $P(\hat{\mathbf{f}}_i | \mathbf{f}(\theta_i, z_i), \eta)$; 'observed fluxes' points to $\hat{\mathbf{f}}_i$; 'model fluxes' points to $\mathbf{f}(\theta_i, z_i)$; 'selection integral' points to $[P(S_i | \psi, \eta)]^{-1}$; 'SPS parameters, redshift' points to θ_i, z_i ; and 'joint posterior' points to the entire left side of the equation.

$$P(\theta_{1:N}, z_{1:N}, \psi, \eta | \hat{\mathbf{f}}_{1:N}) \propto P(\psi, \eta) \times \prod_{i=1}^{N_{\text{galaxies}}} P(\theta_i, z_i | \psi) \times P(\hat{\mathbf{f}}_i | \mathbf{f}(\theta_i, z_i), \eta) \times [P(S_i | \psi, \eta)]^{-1}$$

Annotations:

- prior
- population model parameters
- data model parameters
- observed fluxes
- model fluxes
- selection integral
- SPS parameters, redshift
- joint posterior

Forward model

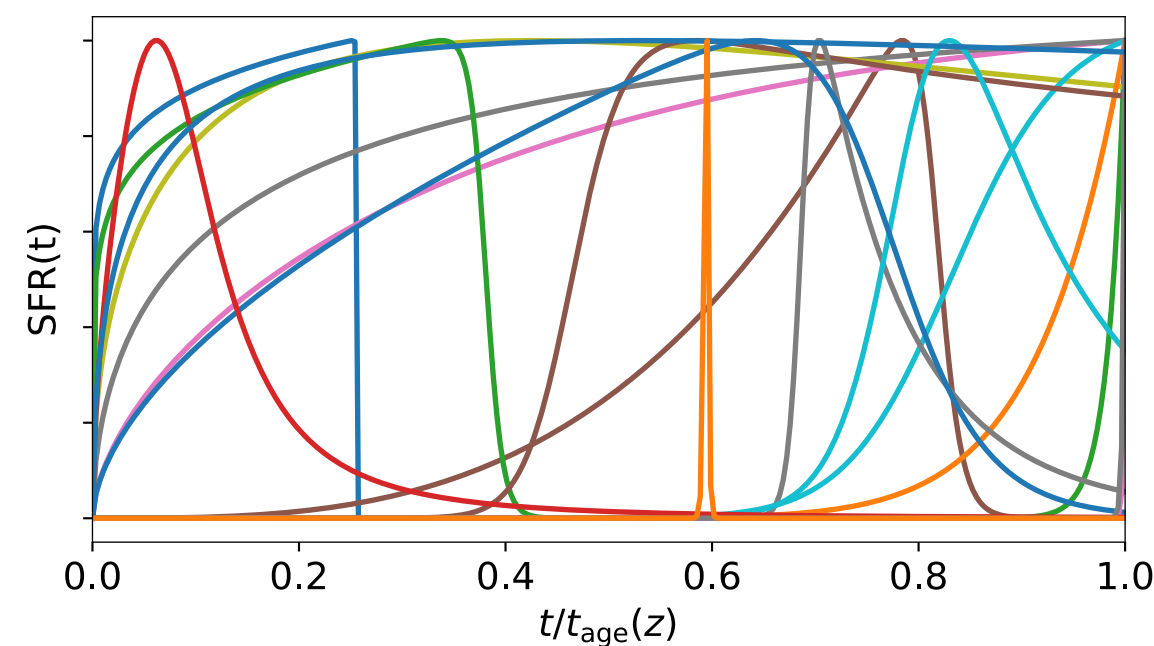
9-parameter SPS model

Mass, redshift

Gas phase metallicity (stellar metallicity history follows mass buildup)

$$\theta, z = (M, z, \log Z, \theta_{\text{SFH}}, \tau_1, \tau_2, n_{\text{dust}})$$

Double power-law SFH



Dust (ISM and birth cloud screens), attenuation law index

Forward model

Population model

Double Schechter function (Leja & Speagle 2021)
11 parameters, tight priors

Star-forming sequence (Leja & Speagle 2021)
Normalizing flow, 34 parameters, modest priors

$$P(\theta, z) = P(M, z)$$

Fundamental metallicity relation (Curti+ 2020)
5 parameters, modest priors

$$P(\text{SFR}(\boldsymbol{\theta}_{\text{SFH}}) | M, z)$$

$$P(\log Z | \text{SFR}(\boldsymbol{\theta}_{\text{SFH}}), M)$$

$$P(\tau_1 | \text{SFR}(\boldsymbol{\theta}_{\text{SFH}}), M)$$

Mass-dependent dust-SFR relation (similar to Tanaka 2015)
bilinear+scatter, 5 parameters, broad priors

Birth cloud ~ diffuse dust (Leja+ 2020)
fixed, broad

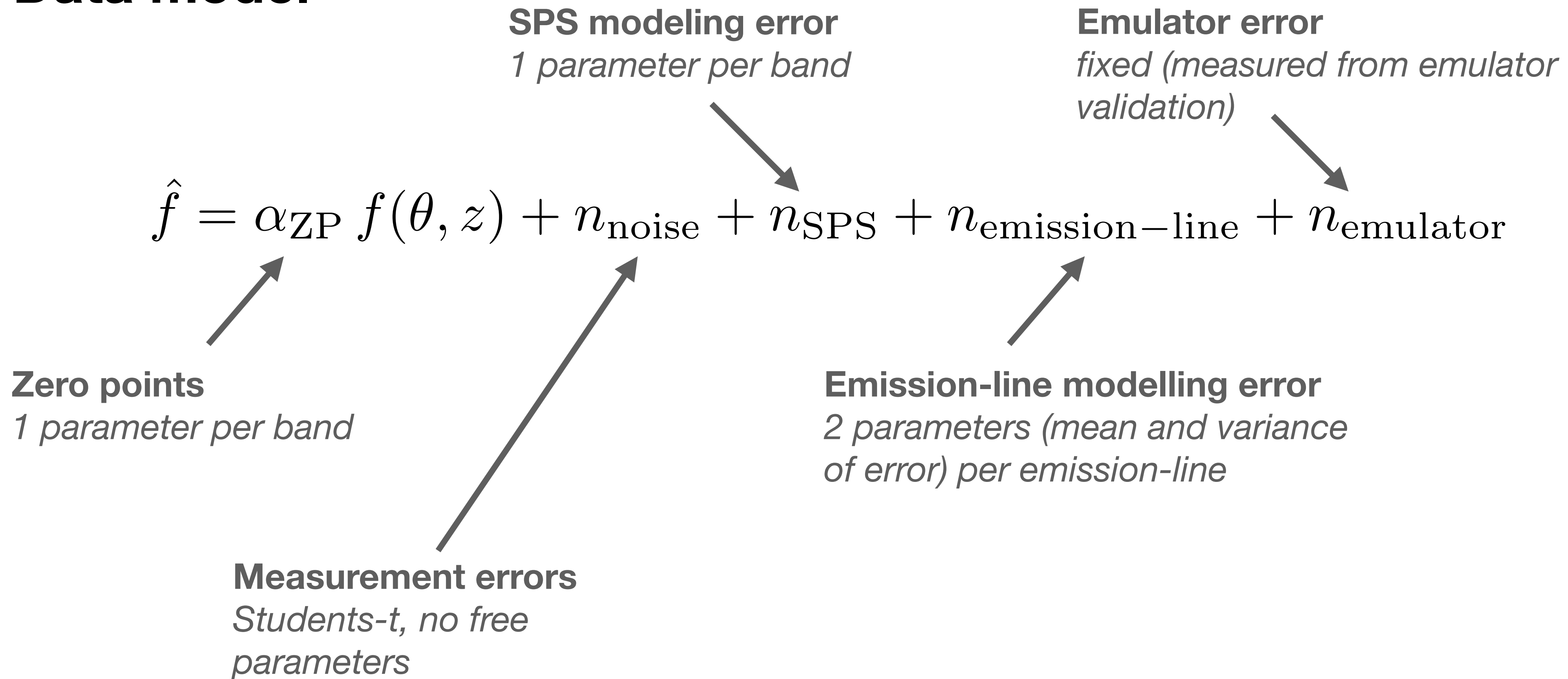
$$P(\tau_2 | \tau_1)$$

$$P(n_{\text{dust}} | \tau_1)$$

Dust index ~ optical depth
Quadratic relation + scatter: 4 parameters, modest priors

Forward model

Data model



Forward model

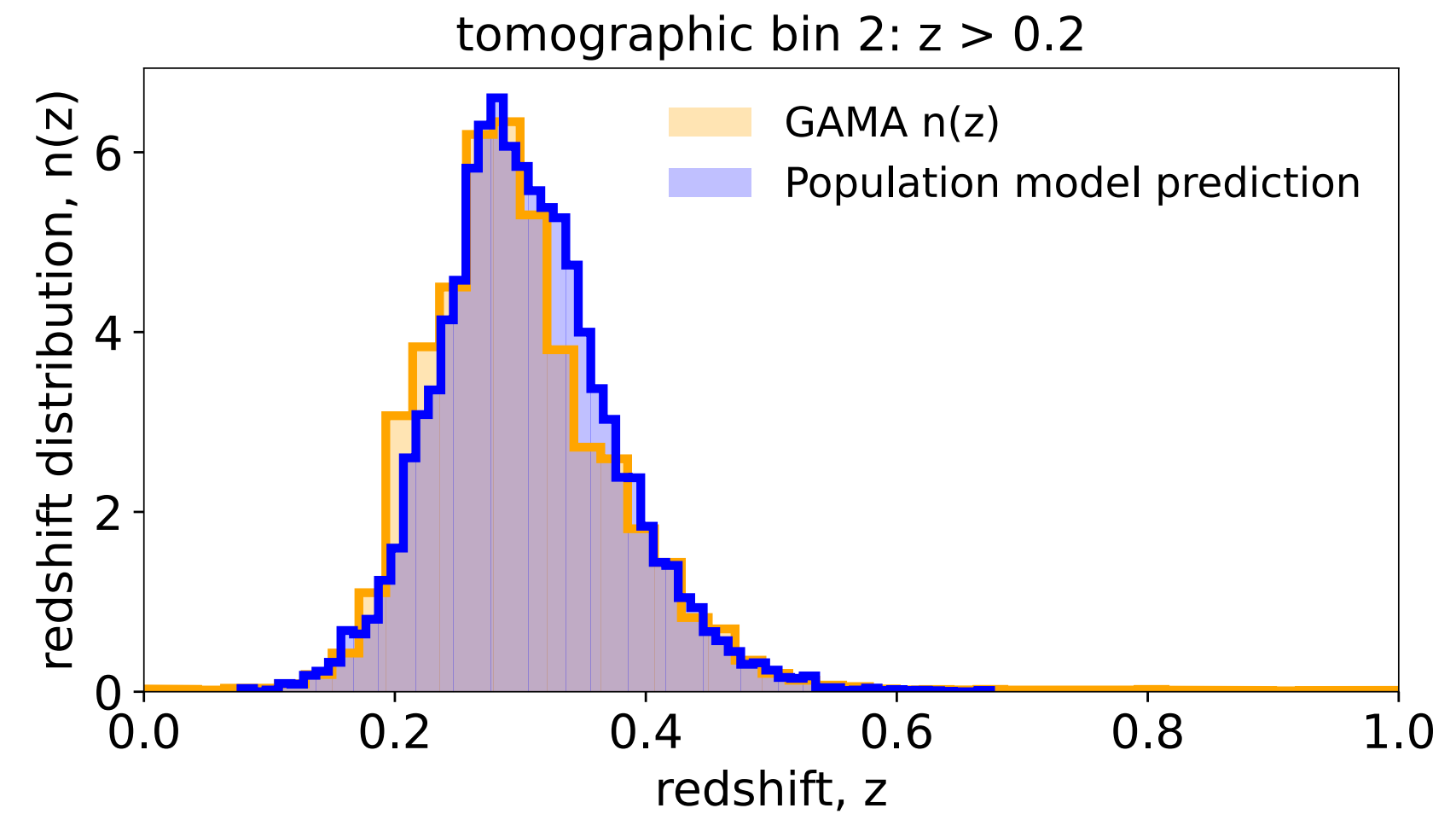
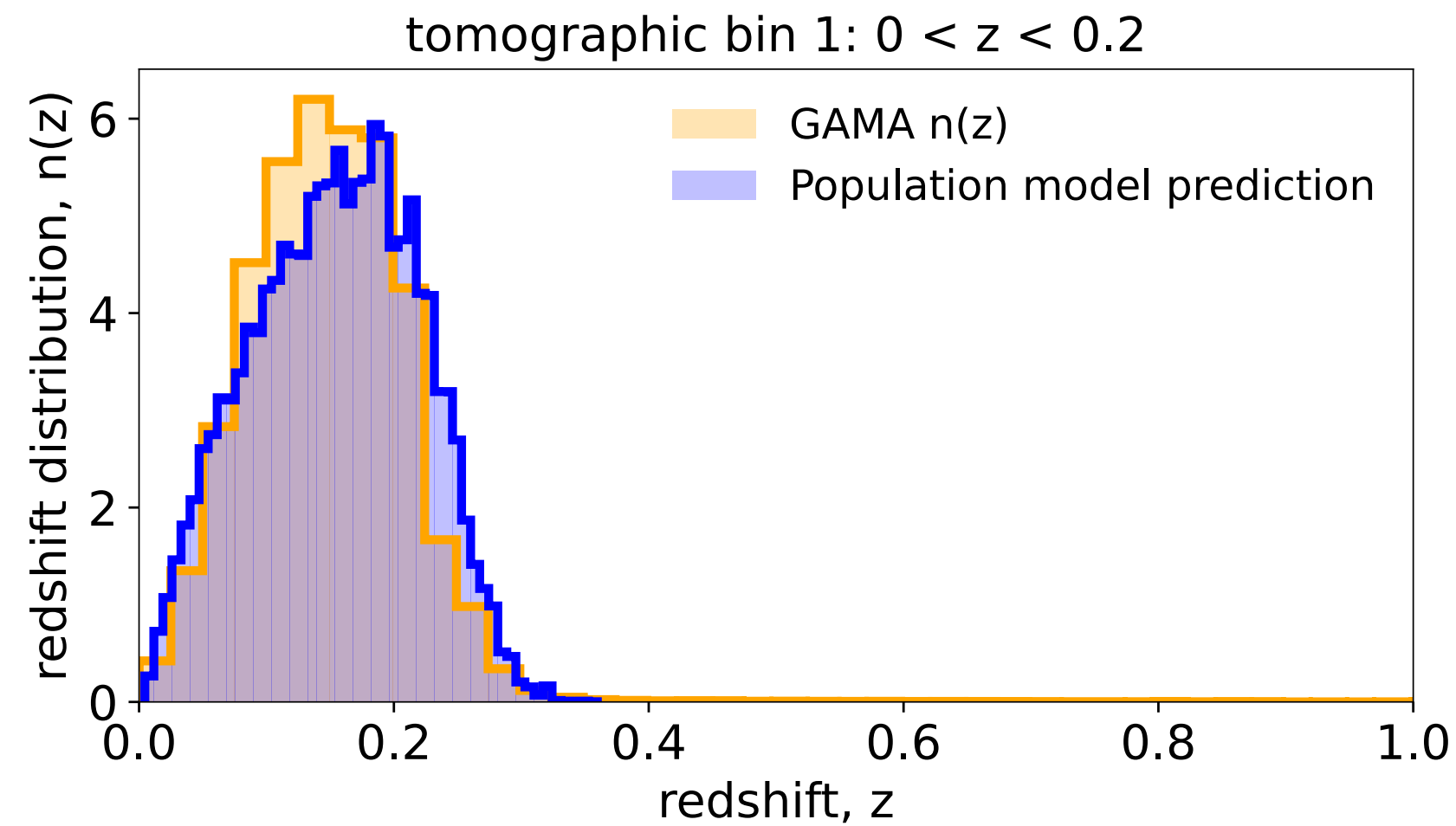
Selection model

Two spec-z surveys with straightforward selection cuts (great for validation)

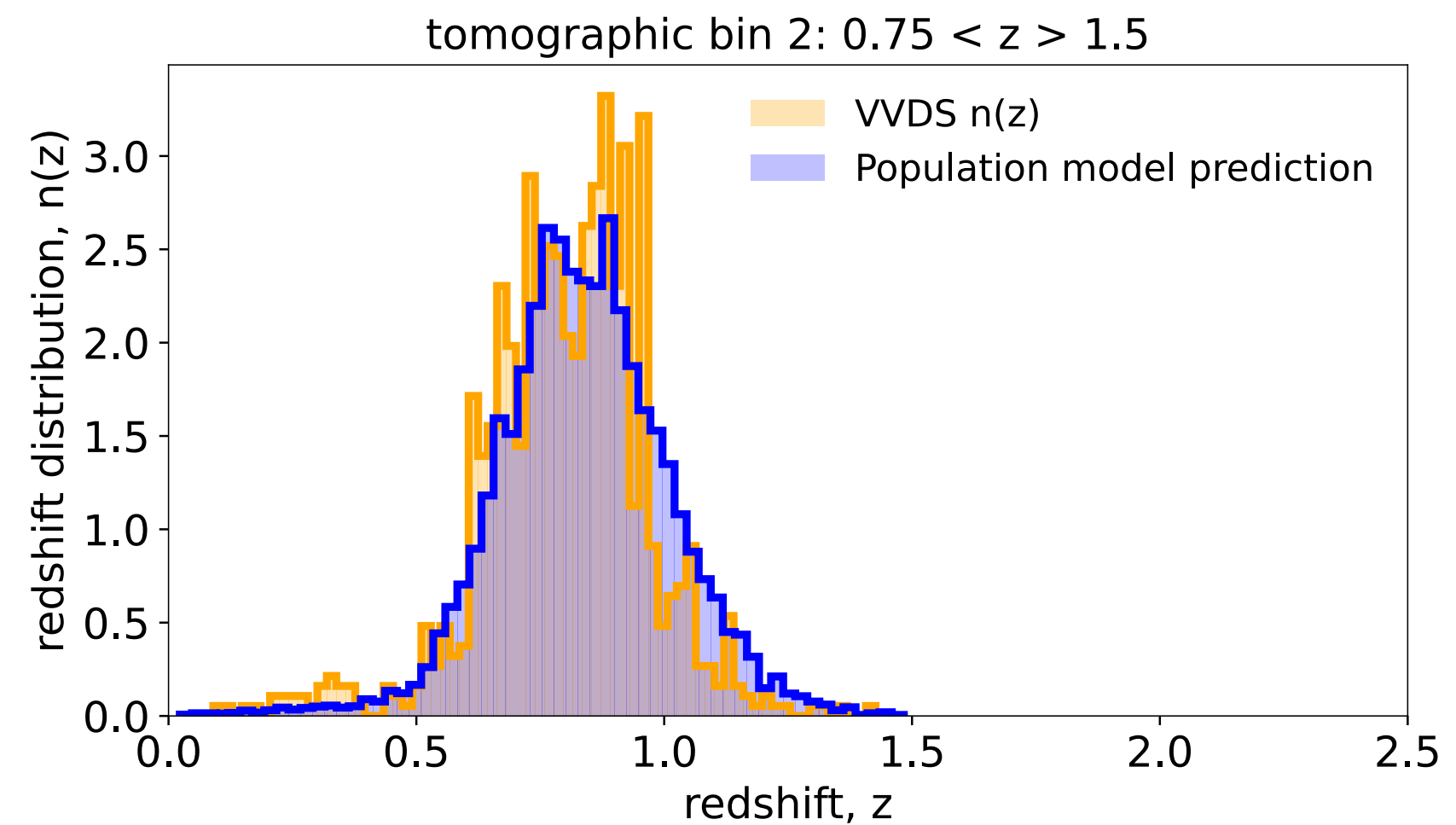
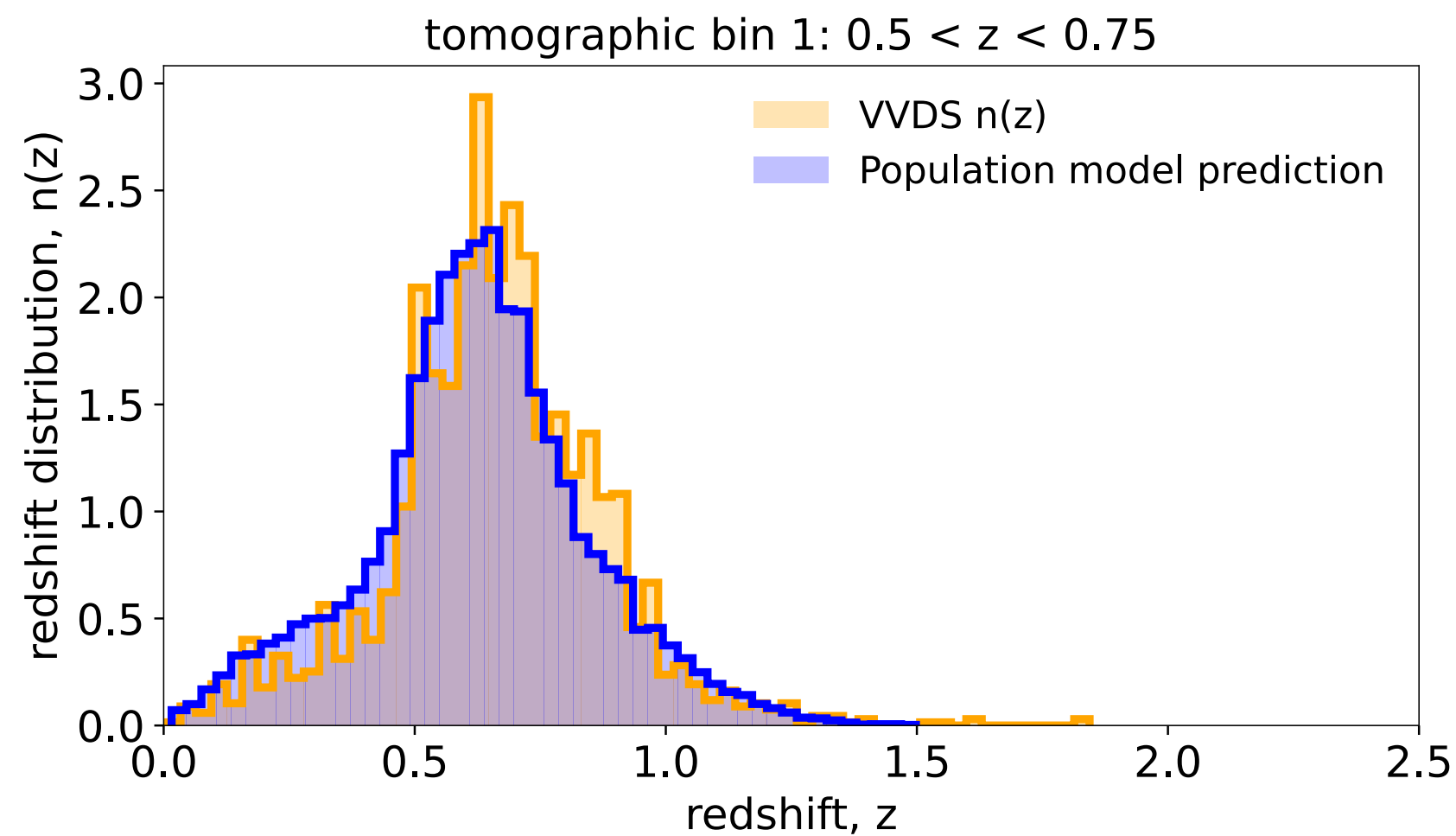
1. GAMA (ugriZYJHKs): $r < 19.65$, $(J-Ks) > 0.025$
2. VVDS (UBVRI): $I < 22.5$, *SG separation done at level of spectra*

How good is the baseline model?

GAMA $n(z)$
tomography
population model X
data model X
selection



VVDS $n(z)$
tomography
population model X
data model X
selection



Baseline model bias < 0.03 on $n(z)$ before parameter inference (no data!)

Doing the inference

Method 1: Hierarchical sampling

$$P(\theta_{1:N}, z_{1:N}, \psi, \eta | \hat{\mathbf{f}}_{1:N}) \propto P(\psi, \eta) \times \prod_{i=1}^{N_{\text{galaxies}}} P(\theta_i, z_i | \psi) \times P(\hat{\mathbf{f}}_i | \mathbf{f}(\theta_i, z_i), \eta) \times [P(S_i | \psi, \eta)]^{-1}$$

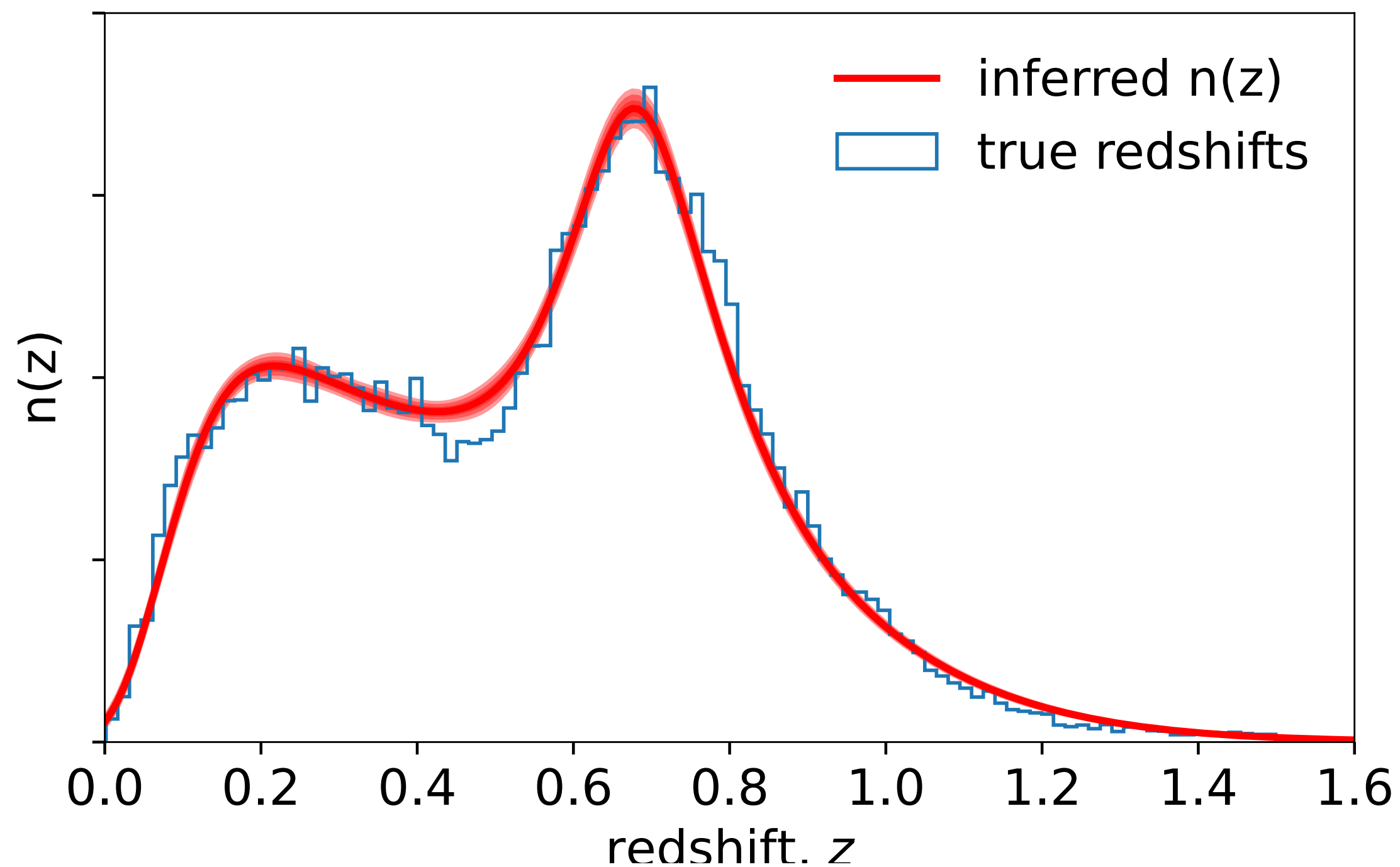
Emulating the selection term:

1. Generate mock galaxies (drawing hyper parameters from prior) to give training set {S, hyper-parameters}
2. Train a neural classifier to learn $P(S | \text{hyper-parameters})$
3. Embed into hierarchical model and sample

Doing the inference

Method 1: Hierarchical sampling

This works: validation test 200k galaxies with complex selection, small sub-set of hyper-parameters (5 hyper-parameters)



Scaling up to full hyper-parameter set may be challenging

Doing the inference

Method 2: Likelihood-free inference

Forward simulate mock catalogues (including selection)

Compress to handful of summary statistics

Use those summaries as basis for density-estimation LFI (Delfi) or similar

Advantages

- Avoids explicitly calculating the selection integrals
- Bypasses latent-parameter sampling (can do hyper-parameters only)
- Easy to increase model complexity (eg could include blending explicitly)

work-in-progress...