Topological Origin of the Proton-to-Electron Mass Ratio in a Chiral Quantum Substrate

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Abstract

In the Standard Model of particle physics, the masses of hadrons are empirical parameters determined experimentally. This paper presents an alternative hydrodynamic model of mass generation, in which stable particles are treated as topological defects (solitons) within a physical, superfluid vacuum medium (hereafter referred to as the **Substrate**). Based on classical vorticity theorems and the Faddeev-Niemi model, we identify the electron with the fundamental vortex loop (unknot) and the proton with the trefoil knot (3_1). We analytically derive the ratio of their rest energies, demonstrating that it results from the relative volume of the configuration space. Specifically, by reducing the 7-dimensional moduli space of the knot by the 2 orientational degrees of freedom common to the unknot, we identify exactly 5 effective dimensions of knotting, yielding a geometric factor of π^5. The obtained theoretical result, $\mu = 6\pi^5 \approx 1836.118$, agrees with the CODATA experimental value within a margin of error of 0.002%, suggesting a geometric mechanism for baryonic mass quantization.

1 Introduction

The origin of the hierarchical mass spectrum of elementary particles remains one of the deepest open questions in theoretical physics. While the Higgs mechanism accounts for the masses of leptons and gauge bosons, the mass of baryons (like the proton) arises predominantly from the strong interaction energy [1]. Calculating this mass *ab initio* requires complex Lattice QCD simulations and does not offer a simple analytical relationship based on geometry.

Historical concepts treating matter as vortex structures in a continuum (Lord Kelvin [2]) have been revitalized by modern topological field theories. Faddeev and Niemi [3] proved that stable, knotted soliton solutions exist in non-linear sigma models. Parallel to this, in 1951, F. Lenz noted a striking correlation: the proton-to-electron mass ratio is almost exactly $6\pi^5$ [4]. For decades, this has been regarded as a numerological coincidence lacking a physical mechanism.

In this paper, we propose a physical derivation of the Lenz relation based on topological hydrodynamics. We posit that rest mass is a measure of the excitation energy of a vortex in a superfluid Substrate. We show that the transition from a trivial topology (electron) to a knotted topology (proton) involves a specific scaling of the phase space volume, dictated by the symmetry group S_3 and the dimensionality of the moduli space.

2 Theoretical Framework: Dynamics of the Chiral Substrate

We postulate that the physical vacuum possesses the properties of a **quantum fluid (Substrate)**, described by a macroscopic order parameter Ψ. The dynamics are governed by an extended Gross-Pitaevskii equation (GPE) with a parity-breaking term:

$$i\hbar\frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m^*}\nabla^2 + g|\Psi|^2 - i\chi(\mathbf{n}\cdot\nabla\times)\right]\Psi \tag{1}$$

Here, χ is the chirality parameter. The inclusion of the $-i\chi(\nabla\times)$ term, analogous to the Dzyaloshinskii-Moriya interaction in magnetic systems [6, 7], is a necessary condition for the stability of 3D knot structures, preventing the collapse of the vortex filament.

3 Topological Particle Model

The model assumes the existence of a fundamental quantum of circulation (a "vortex filament"), which serves as the building block of matter.

3.1 The Electron: Fundamental Loop ($N = 1$)

According to Helmholtz's second theorem [5], a vortex line in a fluid cannot end within the medium; it must form a closed loop. We identify the electron with the simplest stable structure: a single torus (unknot, 0_1).

- **Unitary Mass:** We define the mass of this ground state, m_e, as the natural energy unit. It represents the total energy cost of creating a single, unknotted vortex loop, including its intrinsic toroidal geometry and circulation.

3.2 The Proton: Knotted State ($N = 3$)

We model the proton as a composite structure where the same fundamental vortex filament is tied into a **trefoil knot** (3_1).

- This topology corresponds to the baryonic structure (3 loops corresponding to 3 valence quarks).
- It is the simplest non-trivial knot, stable in a chiral medium.

4 Analytical Derivation of the Mass Ratio

We seek the dimensionless ratio $\mu = m_p/m_e$. We postulate that this ratio is the product of the topological multiplicity and the relative volume of the configuration space (moduli space) required to form a knot from an unknot.

$$\mu = \Omega_{topo} \cdot \Gamma_{geom} \tag{2}$$

A. Topological Factor (Ω): Symmetry Group S_3

The proton consists of 3 loop components. In a chiral medium, where loop orientation is coupled to the flow, the system is described by the **Permutation Group** S_3. The number of distinct configurational microstates corresponds to the order of this group:

$$\Omega = |S_3| = 3! = 6 \tag{3}$$

This represents the 6-fold degeneracy of the knotted state compared to the single state of the electron (S_1, $1! = 1$).

B. Geometric Factor (Γ): Relative Moduli Space

This factor represents the geometric cost of knotting. According to soliton theory (e.g., Faddeev-Niemi models), the full moduli space for a trefoil knot in 3D is **7-dimensional** (typically associated with 3 translations, 3 rotations, and 1 scale/phase parameter).

However, we are calculating the mass of the proton *relative* to the electron. The electron (torus), existing in the same 3D space, already possesses **2 rotational degrees of freedom** (defining the orientation of its symmetry axis on the sphere S^2). These orientational modes are common to both structures and do not contribute to the *excess* energy of knotting.

To find the effective degrees of freedom required specifically for the **knotting process**, we must subtract the dimensions of the reference state (electron) from the dimensions of the knotted state (proton):

$$D_{eff} = D_{knot}(7) - D_{unknot}(2) = 5 \tag{4}$$

These **5 effective dimensions** correspond to the internal constraints required to stabilize the knot (spatial curvature modes and internal phase locking) that are absent in the trivial loop. In the statistical mechanics of coherent states, each independent cyclic degree of freedom contributes a factor of π to the partition function. Therefore, the geometric volume of the excited state (proton) scales relative to the ground state (electron) by a factor of:

$$\Gamma = \prod_{i=1}^{5} \pi = \pi^5 \tag{5}$$

C. Final Result

Combining the symmetry factor with the geometric scaling, we obtain:

$$\mu_{LSV} = 6\pi^5 \tag{6}$$

5 Results and Discussion

Comparison with experimental data (CODATA 2018):

- **Theoretical Value (LSV):** $\mu \approx 1836.1181$
- **Experimental Value:** $\mu_{exp} \approx 1836.1526$
- **Difference:** $\Delta\mu \approx 0.034\, m_e$ (0.002%)

Interpretation of the Deviation: The model assumes an ideal topological knot ($6\pi^5$). However, the physical proton carries an electric charge $+1e$, which introduces a self-energy correction (electromagnetic mass). The observed deviation of $0.034\, m_e$ is consistent with the magnitude of radiative corrections $\mathcal{O}(\alpha)$ expected in quantum electrodynamics. This suggests that the primary mass contribution ($> 99.99\%$) is topological (hydrodynamic/strong), while the remainder is electromagnetic.

6 Conclusions

This paper provides a geometric derivation for the proton-to-electron mass ratio. By treating the electron as a fundamental vortex loop and the proton as a trefoil knot in a chiral Substrate, we show that the mass ratio $6\pi^5$ emerges naturally from the reduction of the soliton moduli space (7D - 2D = 5D) and the permutation symmetry of the loop components (S_3). This result supports the hypothesis that fundamental particles are topological excitations of a unified quantum medium.

References

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