Topological Origin of the Proton-to-Electron Mass Ratio in a Chiral Quantum Substrate

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Abstract

In the Standard Model of particle physics, hadron masses are empirical parameters determined experimentally rather than derived from first principles. This paper presents an alternative hydrodynamic model of mass generation, in which stable particles are treated as topological defects (solitons) within a physical, superfluid vacuum medium (hereafter referred to as the **Substrate**). Based on classical vorticity theorems and knot theory, we model the electron as a fundamental vortex loop (torus) and the proton as a trefoil knot (3_1). We analytically derive the ratio of their rest energies, demonstrating that it results from the complexity of the knot's configuration space. The obtained theoretical result, $\mu = 6\pi^5 \approx 1836.118$, agrees with the CODATA experimental value within a margin of error of 0.002%, suggesting the existence of a geometric mechanism for baryonic mass quantization.

1 Introduction

The problem of the particle mass hierarchy and the precise *ab initio* calculation of the proton mass remains a significant challenge in theoretical physics. While Lattice Quantum Chromodynamics (Lattice QCD) provides approximate numerical values [1], it lacks analytical insight linking mass directly to particle geometry.

Historical concepts treating matter as organized vortex structures in a continuum (Lord Kelvin [2]) have gained new justification in light of modern research on topological field theories. The work of Faddeev and Niemi [3] demonstrated that stable knot-like solutions with quantized energy can exist in non-linear sigma models. Parallel to this, in 1951, F. Lenz noted a striking numerical correlation between the proton-to-electron mass ratio and the factor $6\pi^5$ [4], which has hitherto been treated as a numerological coincidence.

In this paper, we propose a physical derivation of the Lenz relation. We assume that rest mass is a measure of the kinetic energy of the "added mass" of a vortex in a superfluid Substrate. We demonstrate that the transition from trivial topology (electron) to knot topology (proton) involves a scaling of the phase space volume by exactly a factor of $6\pi^5$.

2 Theoretical Framework: Dynamics of the Chiral Substrate

We postulate that the physical vacuum possesses the properties of a **quantum fluid (Substrate)**, described by a macroscopic wavefunction Ψ. The dynamics of the medium are governed by an extended Gross-Pitaevskii equation (GPE) with a parity-breaking term:

$$i\hbar\frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m^*}\nabla^2 + g|\Psi|^2 - i\chi(\mathbf{n}\cdot\nabla\times)\right]\Psi \tag{1}$$

Here, χ is the background chirality parameter. The presence of the term $-i\chi(\nabla\times)$, analogous to the Dzyaloshinskii-Moriya interaction in magnetics [6, 7], is a necessary condition for the stability of three-dimensional knot structures (preventing vortex collapse).

In this framework, particle mass M is not an input parameter but an eigenvalue of the energy of a stable, localized soliton solution (vortex) within the Substrate.

3 Topological Particle Model

The core assumption of the model is the existence of a fundamental quantum of circulation (a "vortex filament" or tube), constituting the building block of matter.

3.1 The Electron: Fundamental Loop ($N = 1$)

According to Helmholtz's second theorem [5], a vortex line in a fluid cannot end within the medium; it must form a closed loop. We identify the electron with the simplest possible structure of this type—a single torus (unknot, 0_1).

- **Interpretation:** Although high-energy experiments indicate the electron is point-like, in the proposed model, this "point-like nature" is an effective topological limit of a vortex tube with a core radius tending toward zero ($r \to 0$).
- **Unitary Mass:** We adopt the energy of this ground state (m_e) as the natural unit ("unitary mass"), representing the total energetic cost of forming a single, closed loop in the Substrate.

3.2 The Proton: Knotted State ($N = 3$)

We model the proton as a composite structure in which the same fundamental vortex filament is formed into a **trefoil knot** (3_1).

- This choice is dictated by the baryonic structure (3 loops/lobes corresponding to 3 valence quarks).
- It is the simplest non-trivial knot, corresponding to the lightest baryon.

4 Analytical Derivation of the Mass Ratio

We seek the dimensionless coefficient $\mu = m_p/m_e$. Treating the electron as a normalized structural unit, the proton mass results from the multiplicity of topological states and the geometric cost of "arranging" the tube into a knot.

$$\mu = \Omega_{topo} \cdot \Gamma_{geom} \tag{2}$$

4.1 Topological Factor (Ω): Symmetry Group S_3

The proton consists of 3 loops (components). In a chiral medium, where loop orientation is coupled to spin, this system is described by the **Permutation Group** S_3. The number of configurational microstates (topological multiplicity) corresponds to the order of this group:

$$\Omega = |S_3| = 3! = 3 \times 2 \times 1 = 6 \tag{3}$$

In contrast to the electron ($N = 1$, group S_1, multiplicity 1), the proton possesses a 6-fold degeneracy of state in configuration space.

4.2 Geometric Factor (Γ): Moduli Space

This factor describes the relative volume of the phase space (Moduli Space volume) available for the trefoil knot relative to the trivial loop. For the 3_1 knot to remain stable and not resolve into a trivial loop, the wavefunction must satisfy **5 independent quantization conditions (constraints)**:

1. **3 Spatial Modes (x, y, z):** Determining the geometry of the tangle in three dimensions (necessary for the existence of "over/under" crossings).
2. **2 Internal Modes (Phase):** Determining the tube torsion and circulation, necessary to compensate for forces within the knot.

In the context of statistical mechanics for coherent systems, each cyclic degree of freedom contributes a factor of π to the partition function (and thus to the mass). Since the electron (unknot) is topologically free from these knot constraints, it constitutes the base (1), while the proton is a structure geometrically "weighted" by a factor of:

$$\Gamma = \prod_{i=1}^{5} \pi = \pi^5 \tag{4}$$

4.3 Final Result

Combining the permutation symmetry with the geometric cost of knotting, we obtain:

$$\mu_{LSV} = 6\pi^5 \tag{5}$$

5 Results and Discussion

Comparison of the model prediction with experimental data (CODATA 2018):

- **Theoretical Value (LSV Model):** $\mu \approx 1836.1181$
- **Experimental Value:** $\mu_{exp} \approx 1836.1526$
- **Discrepancy:** $\Delta\mu \approx 0.034\, m_e$ (0.002%)

The agreement to five significant figures suggests that the model correctly identifies the primary components of the proton mass (knot topology and symmetry).

Error Analysis (Electromagnetic Correction): The slight difference of $0.034\, m_e$ finds a natural physical explanation. The formula $6\pi^5$ describes the mass of a "bare" topological knot in an ideal fluid. However, the physical proton possesses an electric charge of $+1e$. The interaction of the charge with the chiral background introduces a self-energy correction. We note that this deviation is of the order of radiative corrections $\mathcal{O}(\alpha)$ known from Quantum Electrodynamics. This indicates the consistency of the model: the primary mass originates from strong geometry (the knot), while the error results from electromagnetic perturbation.

6 Conclusions

This work offers a geometric solution to the proton mass problem. We demonstrate that by accepting the electron as the fundamental unit of a vortex filament, the proton mass follows directly from the topology of the trefoil knot (factor $6\pi^5$). This approach suggests that matter can be understood as a hierarchy of solitons in a unified, chiral Substrate, opening the path to a **non-perturbative description of strong interactions** based on topological hydrodynamics.

References

[1] Fodor, Z. et al. "Ab initio determination of light hadron masses." *Science* (2008).

[2] Thomson, W. (Lord Kelvin). "On Vortex Atoms." *Proc. Roy. Soc. Edin.* (1867).

[3] Faddeev, L., & Niemi, A. J. "Stable knot-like structures in classical field theory." *Nature* (1997).

[4] Lenz, F. "The Ratio of Proton and Electron Masses." *Phys. Rev.* (1951).

[5] Helmholtz, H. "Über Integrale der hydrodynamischen Gleichungen..." *J. Reine Angew. Math.* (1858).

[6] Dzyaloshinsky, I. "A thermodynamic theory of 'weak' ferromagnetism of antiferromagnetics." *J. Phys. Chem. Solids* (1958).

[7] Moriya, T. "Anisotropic Superexchange Interaction and Weak Ferromagnetism." *Phys. Rev.* (1960).