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Preliminary Study on A Lightning Location Algorithm with Voronoi Diagram as the Constraint for Nonlinear Optimization

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Abstract—**In this paper, we propose a lightning location algorithm with the Voronoi diagram as the constraint for nonlinear optimization. From the open literature, this is the first time the Voronoi diagram concept is introduced to address the lightning location problem. The Voronoi diagram is employed to identify the station closest to the source and determine a small region (Voronoi cell) that contained the source. Through such a step, the location of the source is dramatically narrowed down to a subregion within which the constraint nonlinear optimization method is then used to estimate the accurate location of the source. A simple simulation, assuming a distributed network and a single lightning source, is conducted to assess the algorithm. A comparison with the conventional algorithms suggests promising integration of the Voronoi diagram into the lightning location technique.**

Keywords- Voronoi diagram, lightning location, TOA, least square, grid search

I. INTRODUCTION

Lightning location algorithm (LLA) is a critical part of the lightning location system (LLS), determining the operation and the performance of the entire system. At present, two categories of lightning location techniques are employed for the operational LLSs deployed on the ground by detecting electromagnetic signals, namely Time of Arrival (TOA) [1] and Magnetic Direction Finding (MDF) [2]. Other techniques in use are the variations or combinations of the two [3]. In particular, in the case of ultra-short baseline LLS, the TOA technique evolves to interferometric direction-finding [4].

For the TOA technique, there are many algorithms to implement it, depending on the coverage of the LLS and the desired location accuracy. If the curvature of the earth can be ignored within the coverage of the LLS, the TOA technique is usually implemented in the cartesian coordinate system model, and the algorithms adopted generally include Koshak & Solakiewicz algorithm [5], Chan algorithm [6] and least square algorithm family [7], as well as their combination [8]. However, when LLS covers a wide range, such as the nationwide VLF/LF LLS, the curvature of the earth cannot be ignored. In this case, the TOA technique needs to be implemented in spherical or oblate spheroidal coordinate model, and the algorithms adopted include Koshak algorithm [9], Zhao algorithm [10], etc. In recent years, with the development of GPU-based parallel computing technology, grid search has been applied to the implementation of TOA technique, and is considered to have great potential for the LLS with different coverage [11].

The above algorithms have good performance when the TOA measurements are accurate. However, if the time measurements are seriously interfered, bad solution is likely to be obtained. In this paper, an LLA with Voronoi diagram as the constraint for nonlinear optimization was proposed. This algorithm narrows the search range by providing a rough but reliable estimate of the solution, thus ensuring faster convergence and higher location accuracy. The following parts of this paper presents the principle of the algorithm, a simulation considering the scenario of 2D planar location, and finally a brief discussion.

II. Principle of the Algorithm

A. Voronoi diagram

Assume there are n points on the plane, $P = \{p_1, p_2, \cdots, p_n\}$, $2 \le n \le \infty$. Then on the same plane, the Euclidean distance of any point $X(x, y)$ and $p_i(x_i, y_i)$ is:

$$d(p_i, X) = \sqrt{(x_i - x)^2 + (y_i - y)^2} \tag{1}$$

The Voronoi cell or Voronoi region, $V(p_i)$, associated with the point, p_i, is the set of all points on the plane whose distance, $d(p_i, X)$ is not greater than their distance to the other points p_j, where $j \neq i$, as described in formula (2) [12].

$$V(p_i, X) = \{X | d(p_i, X) \le d(p_j, X)\}, \forall i \neq j, j = 1,2, \cdots, n \tag{2}$$

Figure 1 showed the Voronoi diagram (also called Voronoi partition) with 7 randomly generated points on the plane. The Delaunay triangulation of the 7 points is also shown in red dashed lines. From the plot we can see that the Voronoi cell associated with the points is defined by the perpendicular bisectors of the Delaunay edges attached to the points. Also, the vertices of the Voronoi edges are located at the circumcenters of the Delaunay triangles.

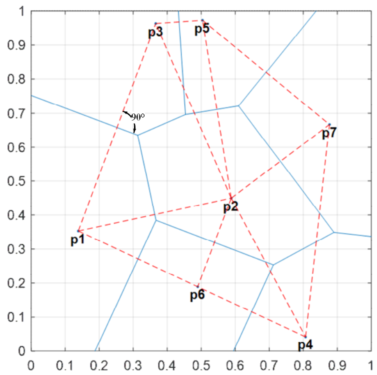

Figure 1. Voronoi diagram and the corresponding Delaunay triangulation with 7 randomly generated points on the plane. The blue lines indicate the Voronoi diagram and the red dashed lines indicate the Delaunay triangulation.

B. The algorithm description

If the above points are regarded as lightning detection stations, we can find that the lightning occurred in the Voronoi cells should be the first to arrive at the corresponding detection stations. Taking $V(p_2)$ as an example, according to the definition of Voronoi diagram, the lightning occurred in $V(p_2)$ should be the first to be detected by p_2 station. Based on this idea, we consider that the arriving order of the lightning emitted signals at the distributed stations can be used to estimate the rough location of the lightning, that is, the Voronoi polygon cell of the corresponding detection station.

Mathematically, the problem can be described in formula (3), where the parameter vector $X = (t, x, y)^T$ that indicates the occurrence time and the position of the lightning is the finding to minimize the sum-of-squares function $\mathrm{F}(X)$. The position of the ith detection station is indicated as $S(i)$ and the corresponding time of arrival observation is indicated as $TOA(i)$. The convex Voronoi polygon is represented with linear inequities, which is set as the constraint for the iterative nonlinear optimization. Note that in the Cartesian coordinate, the edges of the Voronoi polygons can always be represented by linear equalities, so linear inequities can be used to indicate the inner side of the Voronoi polygons. c indicates the propagation speed of the lightning signals.

$$\mathrm{F}(X) = \min \sum_{i=1}^{m} \left(\frac{\sqrt{\left((X(2) - S(i,1))^2 + (X(3) - S(i,2))^2\right)}}{c} + X(1) - TOA(i) \right)^2 \tag{3}$$
$$\text{subject to}$$
$$AX \le B$$

The procedure of the algorithm includes, firstly, the determination of the coarse estimate of the lightning location based on the topology of the distributed stations and the arriving order of the lightning signal, and successively, the constrained nonlinear optimization within the Voronoi cell. For the second step, the centroid of the Voronoi cell can be input to the iterative optimization process as an initial guess. Otherwise, the solution of Chan algorithm or Koshak & Solakiewicz algorithm are taken as the initial guess, and then the iteration is conducted with the Voronoi cell as a boundary.

III. Simulation Study

A. Configuration

12 detection stations are assumed on a planar cartesian coordinate model with a unit of kilometer, and their locations are (0 0), (-20 -10), (0 -30), (20 0), (10 20), (-10 20), (-30 15), (20 -20), (30 25), (35 -15), (-25 -35), (5,35), as indicated by the triangles in Fig.2. The edges of the Voronoi polygons that determine by the 12 stations are shown in blue lines. To further elaborate the proposed algorithm, a lightning source is assumed at (5,5) indicated by the star. From the figure, we can see that the lightning source is located within the Voronoi cell corresponding to the station (0, 0), so the station (0, 0) will be the first to detect the lightning source, if it does not miss it for some reasons. Fig. 2 also shows the process of locating the lightning source with the time difference of arrival (TDOA) measured by the detection stations. The hyperbolas determined by the two pairs of detection stations intersecte exactly at where lightning occurred.

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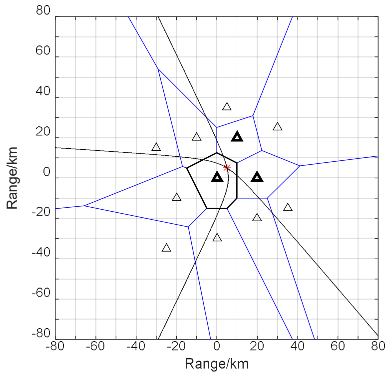
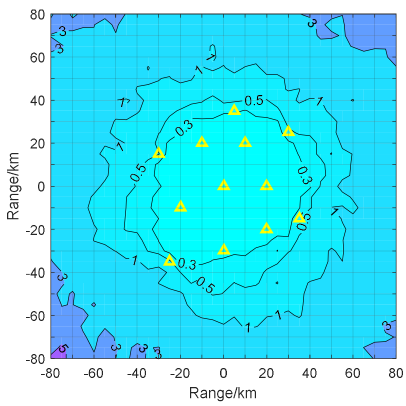


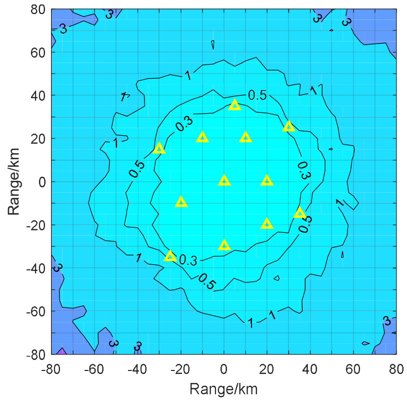
Figure 2. Voronoi diagram for 12 assumed lightning detection stations on a planar cartesian coordinate model. The triangles indicate the detection stations. The star indicates the lightning source. The hyperbolas are determined by stations represented by bold triangles and intersect exactly at the star.

The 80 km×80 km computation domain is divided into grids with 5 km spacing. The Monte Carlo simulation is performed at each grid assuming that 30 lightning sources with occurrence time $t = 0$ occurred at the grid. An error followed the normal distribution is added to the arrival time at each station. The processed arrival times are input into Chan algorithm [6], Chan and unconstraint least square combined algorithm [8], as well as the proposed Chan and Voronoi diagram constraint least square combined algorithm. The derived position was compared with the known position for each grid. Arithmetic mean error was obtained when 30 lightning sources were located for each grid.

B. Results

Figure 3(a), 3(b) and 3(c) show the error distribution for Chan algorithm, Chan and unconstraint least square algorithm (Chan+LS), as well as Chan and Voronoi constraint least square algorithm (Chan+Voronoi LS) when the standard deviation of the timing error δ = 1 us. From the figure we can see that while the error distribution changes little after the unconstraint least square optimization, the error around the boundary of the computation domain is significantly reduced after the Voronoi constraint least square optimization. Note that in Figure 3(c) the error range of 1km-3km is expanded.





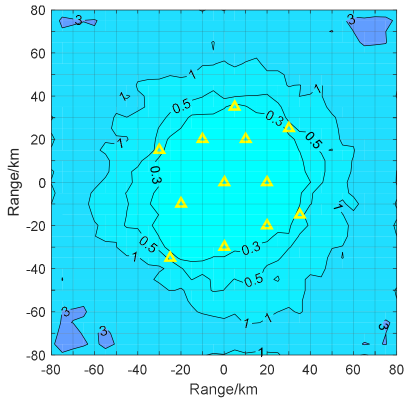
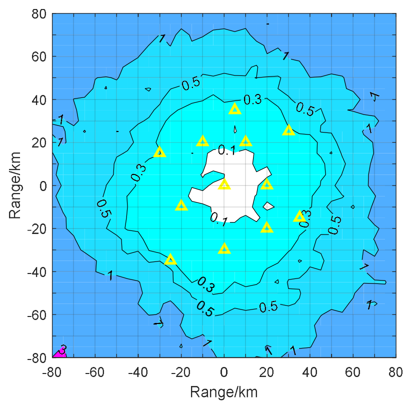


Figure 3. Error distribution for (a)Chan algorithm, (b)Chan+LS algorithm, (c)Chan+Voronoi constraint LS alrorithm, when assuming the timing error followed normal distribution with a mean of 0 μs and a standard deviation of 1 us.

When smaller timing error with the standard deviation δ = 0.6 us is input to the three algorithms, we obtain the error distributions as shown in Figure 4(a), 4(b) and 4(c). As expected, the errors decrease in the domain. Compared with the unconstraint least square algorithm, the Voronoi constraint least square algorithm expands the high accuracy area in the center of the network and improves the location accuracy around the boundary of the domain.



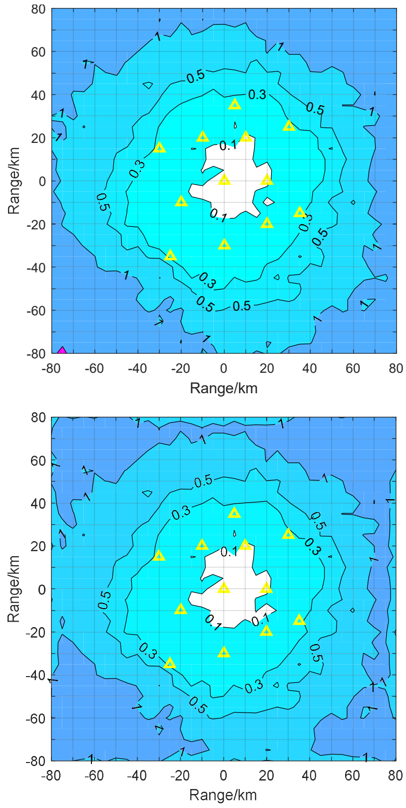
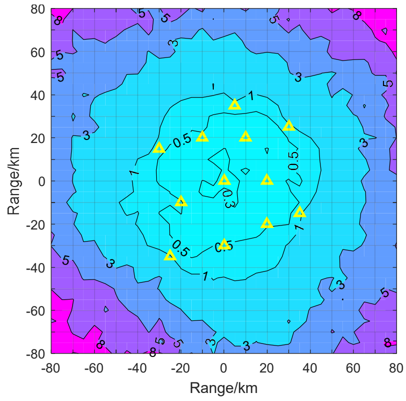


Figure 4. Error distribution for (a)Chan algorithm, (b)Chan+LS algorithm, (c)Chan+Voronoi constraint LS alrorithm, when assuming the timing error followed normal distribution with a mean of 0 μs and a standard deviation of 0.6 us.

When greater timing error with the standard deviation $\delta = 2$ us is applied to the three algorithms, the error distributions are obtained, as shown in Figure 5(a), 5(b) and 5(c). As expected, the errors increase in the domain. The location accuracy is improved by the unconstraint least square algorithm, compared with the Chan algorithm, and is further improved by the Voronoi constraint least square algorithm.



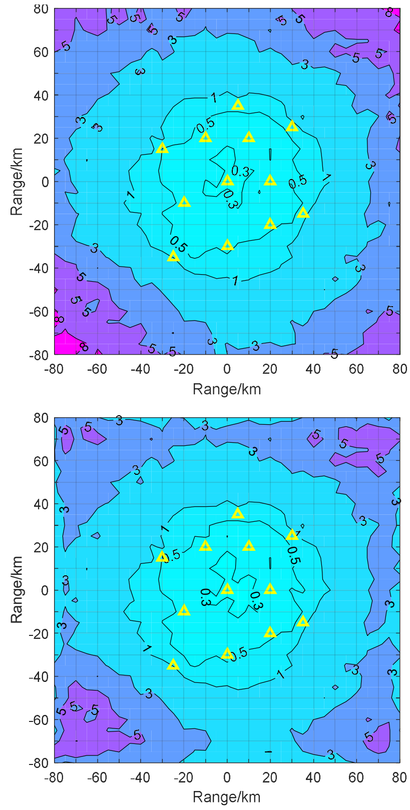


Figure 5. Error distribution for (a)Chan algorithm, (b)Chan+LS algorithm, (c)Chan+Voronoi constraint LS algorithm, when assuming the timing error followed normal distribution with a mean of 0 μs and a standard deviation of 2 us.

IV. DISCUSSION

In this paper, Voronoi diagram is used as the boundary for the least square optimization to locate the lightning sources. From the simulation, we speculate that the greater the timing error, the more the proposed algorithm reducing the error. It should be noted that the Voronoi diagram is open outside of the network and the detection range is taken as the boundary to close the Voronoi cell. If a box constraint with $-80\ \text{km} \le x \le 80\ \text{km}$ and $-80\ \text{km} \le y \le 80\ \text{km}$ is added to the least square optimization, the location error could also be reduced outside of the network (not shown).

In this simulation, the network is relatively dense, and it is assumed that all stations are involved in the calculation, so the error distribution of the three algorithms is not very different, particularly when the timing error is small. It is speculated that when the detection stations are not very dense, the improvement could be more obvious through the Voronoi diagram constraint optimization.

In this paper, the detection station closest to the lightning source is assumed to receive the corresponding electromagnetic signal. In practice, it is necessary to extend the constraint to the surrounding Voronoi cells if in the nearest Voronoi cell the location result is not the desired one.

It should be mentioned that if the grid search method is used for lightning location, the search range will be greatly reduced when Voronoi diagram is added as the constraint.

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