MAT 300 1-15 HW

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Problem 1. Quantification and Ordering

Consider the predicate about integers "x = 2y", which contains two free variables. There are six distinct ways to use quantification to turn this predicate into a statement (Why six?). Find all six statements and determine the truth or falsehood of each.

Solution

Part (a)

Given "there exists" and "for all", there are six distinct quantifications because identical quantifiers cannot be swapped to make a new quantification, but otherwise order matters when quantifying. Assume x and y are taken from the real numbers.

(a) There exists an x and there exists a y such that $x = 2y$. This is true. It is saying that given some arbitrary number, its double exists.

(b) There exists an x such that for all y, y $x = 2y$. This is false. It is saying that given some arbitrary number, all of the real numbers are it doubled.

(c) There exists a y such that for all x, y $x = 2y$. This is false for the same reason as 2, except halved instead of doubled.

(d) For all y and for all x, $x = 2y$. This is false. It is saying that all real numbers are doubles/halves of each other.

(e) For all y, there exists an x such that $x = 2y$. This is true, since x is assigned after y.

(f) For all x, there exists a y such that $x = 2y$. This is true, since y is assigned after x.

Problem 2. Verifying Tautology by Truth Table

Verify that $(A \Rightarrow (B \vee C)) \Leftrightarrow ((A \wedge \neg B) \Rightarrow C)$

Solution

Part (a)

A	B	C	B $\vee C$	$\neg B$	A $\wedge \neg B$	A $\Rightarrow (B \vee C)$	(A $\wedge \neg B) \Rightarrow C$
T	T	T	T	F	F	T	T
T	T	F	T	F	F	T	T
T	F	T	T	T	T	T	T
T	F	F	F	T	T	F	F
F	T	T	T	F	F	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	F	T	T
F	F	F	F	T	F	T	T

As the last two columns are identical, $(A \Rightarrow (B \vee C)) \Leftrightarrow ((A \wedge \neg B) \Rightarrow C)$ is a tautology.

Problem 3. Free Variables and Predicates

Suppose we understand the free variable z to refer to (a) books, (b) automobiles, and (c) pencils. For each context, give an examples where the following:

- For all z, A(z) is true.
- For all z, B(z) is false, but there exists at least one z that makes B(z) true.

Solution

Part (a)

Assuming books means literary books (which excludes books that are empty or consist of only pictures):

- "A(z) = z contains words."
- "B(z) = z is about russian history"

Part (b)

Assuming automobiles doesn't include tanks or robots:

- "A(z) = z has wheels"
- "A(z) = z is painted red"

Part (c)

Pencils seem to be well defined.

- "A(z) = z can write"
- "A(z) = z is mechanical"

Problem 4. Quantification

Is it possible to have a predicate $T(x)$ such that for all x, $T(x)$ is true, but there exists some x such that $T(x)$ is false?

Solution

Part (a)

No. If the second statement were true, this would be a contradiction to the first statement.

Problem 5. If-then

Consider the statements:

(a) P: All dogs eat meat

(b) Q: Rome is in Italy

(c) R: Chocolate prevents cavities

(d) S: The moon is made of green cheese

Determine which of the following are true:

(a) If P, then Q

(b) If P, then R

(c) If R, then S

(d) If S, then Q.

(e) If Q, then S.

Solution

Part (a)

Assume P and Q are true, but R and S are false.

(a) If P, then Q: True (Vacuously)

(b) If P, then R: False

(c) If R, then S: True (Vacuously)

(d) If S, then Q: True (Vacuously)

(e) If Q, then S: False