MAT 300 1-15 HW

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Problem 1. Comic Logic

Consider the following assertions:

(a) All cats have four legs

(b) I have four legs

(c) I am a cat

Are the assertions statements or predicates? Construct a truth table for "If A and B, then C" Now consider the actual truth values of the assertions made by the dog. Cross out the lines of the truth table that don't apply *in this particular instance.*

Solution

Part (a)

Line $(A \wedge B) \Rightarrow C$	A	B	C
1 T	T	T	T
2 F	T	T	F
3 T	T	F	T
4 T	T	F	F
5 T	F	T	T
6 T	F	T	F
7 T	F	F	T
8 T	F	F	F

In reality, both A and B are true, but C is clearly not. Therefore, only the first line of the truth table is "applicable" to the scenario in a technical sence.

Problem 2. Negation

Consider the statement “Marlene has brown hair”. When asked to negate this statement, some students are apt to say “Marlene has blonde hair” (Really, which students?). Explain why this is incorrect (Why does this problem have a hint. . . seriously?)

Solution

Part (a)

Obviously, Marlene could have any color of hair *besides* brown. For instance, it could be red, not blonde.

Problem 3. Line creation proof

Suppose that (a, b) and (c, d) are two distinct points in R^2.$Provethatthereexistsauniquelinepassingthroug$

Solution

Part (a)

A line is defined as a point and a slope, or $y = mx + e$ (e is used to distinguish from b, which is already defined). For a line to pass through (a, b) and (c, d), it must have a certain fixed slope and intercept, i.e. $b = ma + e$ and $c = md + e$. These two equations give unique values for the line...

Problem 4. Statement Negation

Negate the following statements and attempt to write the result as a positive statement.

(a) $x + y$ is even and $y + z$ is even.

(b) $x > 0$ and x is rational.

(c) Either l is parallel to m, or l and m are the same line (for fixed lines in 2D space).

(d) The roots of this polynomial are either all real or all complex. (Assume that real numbers are different from complex numbers with imaginary part 0, even though this is not true generally).

Solution

Part (a)

(a) $x + y$ is odd or $y + z$ is odd.

(b) $x < 0$ or x is not rational.

(c) l is not parallel to m and l and m are the not same line (for fixed lines in 2D space.

(d) The roots of this polynomial are not all real and not all complex. (Assume that real numbers are different from complex numbers with imaginary part 0, even though this is not true generally).

Problem 5. More Statement Negation

Negate the following statements, attempting to write the result positively.

(a) There exists a line in the plane passing through the points $(-1, 1), (2, -1), (3, 0)$

(b) There exists an odd prime number

(c) For all reals, $x, x^3 = x$

(d) Every positive integer is the sum of distinct powers of three

(e) For all positive real numbers x there exists a real y such that $y^2 = x$

(f) There exists a positive real number y such that for all reals x, $y^2 = x$

Solution

Part (a)

(a) No line exists in the plane passing through the points $(-1, 1), (2, -1), (3, 0)$.

(b) There does not exist an odd prime number.

(c) For all reals, $x, x^3 \neq x$

(d) Every positive integer is not the sum of distinct powers of three

(e) For all positive real numbers x there exists a real y such that $y^2 \neq x$

(f) There exists a positive real number y such that for all reals x, $y^2 = x$