# MAT 300 4-09 HW

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# Problem 1. 5.5.21

Let $A$ be a set, and $(s_i)$ be a sequence in $A$.

(a) If $(s_i)$ is constant, every subsequence of $(s_i)$ is constant.

(b) If $(s_i$ has distinct terms, then every subsequence of $(s_i)$ has distinct terms.

### Solution

### Part (a)

**Proof** Suppose there existed a non-constant subsequence of $s_i$, call it $s_{a_k}$. Then, by definition, $s_{a_i} \neq s_{a_j}$ for distinct natural numbers $i$ and $j$. Evaluate $a_i$, and $a_j$, which will be natural numbers. Call them $x$ and $y$ respectively. This means that $s_x s_y$ for distinct $x$ and $y$ ($x$ and $y$ must be distinct because, if they were not, then $s_{a_i}$ would equal $s_{a_j}$). This leads to a contradiction, namely that $s_x \neq s_y$ for distinct $x$ and $y$, or the original subsequence would be constant. By contradiction, every subsequence of a constant sequence must also be constant. ■

**Proof** Suppose that there existed a non-distinct subsequence of $s_i$. Call this subsequence $s_{a_k}$. By negating the definition of distinct, $s_{a_i} = s_{a_j}$ for some distinct $i$ and $j$. Evaluate $a_i$ and $a_j$ to natural numbers $x$ and $y$, so that $s_x = s_y$ for distinct $x$ and $y$. This is a contradiction with the fact that $s_i$ is distinct, so it is not possible to have a constant subsequence of a distinct sequence. ■

# Problem 2. 6.1.4

Use the well-ordering of $\mathcal{N}$ (Every non-empty set of natural numbers contains a least element) to prove the following:

(a) Every nonempty subset of $\mathcal{Z}$ that has a lower bound has a least element.

(b) Every nonempty subset of $\mathcal{Z}$ that has a lower bound has a greatest element.

### Solution

In each of these solutions, the subset on $\mathcal{Z}$ must be converted to a subset of $\mathcal{N}$.

### Part (a)

### Part (b)

# Problem 3. 6.2.10

Prove the following about the partial ordering on $\mathcal{N}$.

(a) Prove that $\mathcal{N}$ is partially ordered under the relation $|$.

(b) Is $|$ a *total* order on $\mathcal{N}$? Explain.

(c) Draw a lattice diagram the depicts the order $|$ on the set $\{1, 2..15\}$.

(d) Does $\{2, 3, 4, 5..\}$ have any minimal or maximal elements with respect to the order $|$?

**Solution**

**Part (a)**

**Part (b)**

**Part (c)**

**Part (d)**

# Problem 4. 6.2.3

Let $a$, $b$ be natural numbers. Suppose $a = qb + r$ with $0 \leq r < b$. What does the division algorithm yield when $-a$ is divided by $b$? Justify your answer. Those natural numbers $n$ that, when divided into $m$ using the division algorithm, yield a remainder of zero have a special status with respect to $m$. We say that $n$ divides $m$ evenly or simply that $n$ divides $m$.

**Solution**

**Part (a)**

# Problem 5. 6.2.8

Let $a, b, c$ be integers.

(a) If $a|b$ and $b|a$, then $a = +/ - b$.

(b) If $a|b$ and $b|c$, then $a|c$.

### Solution

### Part (a)