MAT 300 2-26 HW

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Problem 1. 4.1.10

Indicate whether the following relations on the given sets are reflexive, symmetric, anti-symmetric, or transitive.

Solution

Part (a)

(a) $A = \{p : p \text{ is a person in alaska}\}$. $x\ y$ if x is at least as tall as y.
Reflexive, not symmetric, anti-symmetric, and transitive.

(b) $A = \mathcal{N}$. $x\ y$ if $x + y$ is even.
Reflexive, symmetric, not anti-symmetric, and transitive (though somewhat vacuously).

(c) $A = \mathcal{N}$. $x\ y$ if $x + y$ is odd.
Reflexive, symmetric, not anti-symmetric, and transitive, as above.

(d) $A = \mathcal{P}(\mathcal{N})$. $x\ y$ if $x \subseteq y$.
Reflexive, not symmetric, anti-symmetric, and transitive.

(e) $A = \mathcal{R}$. $x\ y$ if $x = 2y$.
Not reflexive, not symmetric, not anti-symmetric, and not transitive.

(f) $A = \mathcal{R}$. $x\ y$ if $x - y$ is irrational.
Not reflexive, symmetric, not anti-symmetric, and not transitive.

(g) $A = \{l : l \text{is a line in the Cartesian plane}\}$. $x\ y$ if x and y are parallel lines or if x and y are the same line.
Reflexive, symmetric, not anti-symmetric, and transitive.

Problem 2. 4.2.15

Let A be a partially ordered set. Prove that if A has a greatest element, then the greatest element is unique. (Assume two greatest elements and show they are the same).

Solution

Part (a)

Solution

Problem 3. 4.2.4

Let A be a set. Show that $\mathcal{P}(A)$ need not be totally ordered under the relation $\subseteq$.

Solution

Part (a)

Recall that a set A is said to be totally ordered if it has a relation which is anti-symmetric, transitive, and satisfies the "connex property": $a\ b$ or $b\ a$ for any a, b in the set A. While $\subseteq$ satisfies anti-symmetry and transitivity for $\mathcal{P}(A)$, it does not satisfy the connex property. For instance, the powerset of $\{0, 1\}$ contains the elements $\{0\}$ and $\{1\}$. Let these be the variables a and b in the connex property, and it is clear that it is not satisfied (because $\{0\} \nsubseteq \{1\}$ and $\{1\} \nsubseteq \{0\}$).

Problem 4. Title

Problem

Solution

Part (a)

Solution