MAT 300 2-5 HW

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Problem 1. Indexing Sets

Let I_n where $n \in \mathbb{N}$ be the collection of intervals described in 2.3.11.

- a. Find $\cup_{n \in \mathbb{N}}$.
- b. Find $\cap_{n \in \mathbb{N}}$.
- c. How would the answer be different if the intervals were closed intervals instead of open intervals?

Let $C_t$where$t \in R$ be circles described in Example 2.3.11

- a. Find $\cup_{n \in \mathbb{R}}$.
- b. Find $\cap_{n \in \mathbb{R}}$.

Solution

Part (a)

- a. Since later intervals are always subsets of the original interval (because they are more restricted intervals), the resultant set is simply $[0, 1]$, because set elements are unique. If the intervals were closed, the answer would be $(0, 1)$
- b. Since later intervals are always subsets of the original interval (because they are more restricted intervals), the resultant intersection becomes smaller and smaller. Eventually, the set will become the interval $[0, 1/x]$ as x approaches infinity. This is the set 0. If the intervals were closed, the answer would be the empty set, $\varnothing$.
- c. (Addressed above)
- a. Since the subscript t is changing the x location of the circle, the union of these will be the 2D space between $y = 1$ and $y = -1$.
- b. Since for two elements in C_t with a difference in t of more than 1, no points will overlap, their intersection is the null set. The null set intersected with anything else is also the null set, so the total resultant set is the null set.

Problem 2. Subset of Intersection

Let A, B, and X be sets. Prove that $X \in A \cap B \Leftrightarrow X \subseteq A and X \subseteq B$.

Solution

Part (a)

In general, C is a subset of D if (and only if), for every element c of C, c is an element of D. C intersection D is defined as the set of all elements x such that both x is in C and x is in D. Combining these two definitions, for every element x in the set $X \subseteq A$, it is true that x is in A intersection B, and thus also true that x is in A, and that x is in B.

Problem 3. Set identities

Try proving the provided set identities.

Solution

Part (a)

In a sense, 1 and 2 are applying DeMorgan's laws.

(a) $C\ (A \cup B) = (C\ A) \cap (C\ B)$. $c \in Candc \notin A \cup Bmeans\ \not(c \in Aorc \in B)$, and by DeMorgan's law, this becomes $c \notin Aandc \notin Bbutc \in C$.

(b) $C\ (A \cap B) = (C\ A) \cup (C\ B)$. This uses the above method, but reverses "and" and "or".

(c) $B\ (B\ A) = A \cap B$. This means every element b in B that is not also in A, and then further every element b in B that is not in ($B\ A$, previously defined). Thus this means all elements in both A and B, thus the intersection of the two.

(d) $(A\ B) \cup (B\ A) = (A \cup B)\ (A \cap B)$. This is showing distributivity. Specifically, we are considering the set of all elements either where each is in A but not in B, or is in B but not in A. Thus, this is the union of both sets, except for the elements in both sets (the intersection set).

Problem 4. Counting and Indexing

What is the connection between questions 2.1 and 2.3?

Solution

Part (a)

Question 3 describes set-indexing, and and question 1 describes a theory of counting. It seems that set indexing certainly relies on counting, since an index is something which enumerates positions between two numbers (i.e. that counts). However, since both of these formalisms are describing nearly the same thing, it is very possible that if one was found first, it would more or less contain the other (i.e. if one found a theory of set indexing before counting, they would find that their indexing theory actually contained counting, even if it wasn't explicit).

Problem 5. Powerset Union

Let A and B be sets, and $P()$ denote the powerset.

(a) Provide a counterexample to show that it is not necessarily true that $P(AUB) = P(A)UP(B)$.

(b) Is it ever true that $P(AUB) = P(A)UP(B)$?

Solution

Part (a)

(a) Assume A and B are entirely separate sets, i.e. $a \in Aanda \notin Bandb \in B \notin A$. Then, the cardinality of A union B is the summation of the two, so $P(A)$ is $2^{(}|A| + |B|)$, but the cardinality of $P(A) \cup P(B)$ is $2^{|}A| + 2^{|}B| - 1$ because of the empty set produced by $P()$. Thus, these sets differ.

(b) For a trivial case, let $A = \varnothing$ and $B = \varnothing$. This is perhaps vacuously true.

Problem 6. Berry Paradox

Attempt to resolve the Berry paradox.

Solution

Part (a)

To the best of my knowedge, this can't be done! Suppose we found the least integer not nameable in fewer than 19 syllables. Then, this integer would be refered to by the provided sentence. This integer would now be named, so it would not satisfy the first property.