MAT 300 3-19 HW

ID: 1213399809
Name: Lucas Saldyt (lsaldyt@asu.edu)
Collaborators: $\varnothing$

Problem 1. 4.12

Let A be a partially ordered set. Suppose $X \subseteq Y \subseteq A$.

(a) Assuming that all the least upper bounds and greatest upper bounds exist, prove that $glb(Y) \leq glb(X) \leq lub(X) \leq lub(Y)$

(b) Find two subets X and Y of $\mathcal{R}$ for which X is a proper subset of Y and $glb(Y) = glb(X)$ and $lub(X) = lub(Y)$.

Solution

Part (a)

Theorem 0.0.1. *Since X is a subset of Y, its greatest and least values are, at most/least the greatest and least values of Y*

Proof Consider the *glb* of X. Since X is a subset of Y, this *glb* necessarily exists in Y. However, since Y possibly has more elements than X, there could exist an arbitrary element in Y which makes a *glb* which is possibly smaller than the *glb* in X. However, no greater *glb* could exist, because it would not include X's *glb*. Consider the *lub* of X. Since X is a subset of Y, this *lub* necessarily exists in Y. However, since Y possibly has more elements than X, there could exist an arbitrary element in Y which makes a *lub* which is possibly greater than the *lub* in X. However, no smaller *lub* could exist, because it would not include X's *lub*. ■

Part (b)

Simply create an interval, Y, and a second interval, X, which is equal to Y, except that middle portions of the interval are missing. For instance, Let $Y = [0.0, 1.0]$ and $X = [0.0, 0.5) \cup (0.5, 1.0]$.

Problem 2. 5.1.12

Give an example of a function $f : \mathcal{R} \to \mathcal{R}$ in which:

(a) f is one-to-one but not onto.

(b) f is onto but not one-to-one.

(c) f is both one-to-one and onto.

(d) f is neither one-to-one nor onto.

Solution

Part (a)

(a) $f(x) = \sqrt{x}$

(b) $f(x) = x^3$

(c) $f(x) = x + \pi$

(d) $f(x) = sin(x)$

Problem 3. 5.1.14

For each function $f\colon \mathcal{R} \to \mathcal{R}$, either show that f is one-to-one or prove that it is not.

(a) $f(x) = \frac{x}{2} + 6$

(b) $f(x) = sin(x)$

(c) $f(x) = x^3 - x$

Solution

Part (a)

This function is one-to-one. Any given $y = f(x)$ is mapped to by $x = 2y - 12$. Suppose two values of x, call them a and b, mapped onto the same y. This would imply $y = \frac{a}{2} + 6$ and $y = \frac{b}{2} + 6$, which implies that $a = b$, and thus f is one-to-one, by contradiction.

Part (b)

This function is not one-to-one. For instance, 0 is mapped to by multiples of π, so, for instance, 0 and π map onto the same value, and the function is not one-to-one.

Part (c)

This function is not one-to-one. Consider $y = 0$. This is true both when $x = 1$, or $x = 0$, and so the function is not one-to-one.

Problem 4. 5.1.15

For each function $f\colon \mathcal{R} \to \mathcal{R}$, either show that f is onto or prove that it is not.

(a) $f(x) = \frac{x}{2} + 6$

(b) $f(x) = sin(x)$

(c) $f(x) = x^3 - x$

Solution

Part (a)

This function is onto. $x = 2y - 12$ shows that any y in Y (where Y is the co-domain) is reachable by arbitrary x.

Part (b)

Consider $y = 1.1$. There is no x for which $sin(x) = 1.1$.

Part (c)

This function is onto. Since every odd-degree polynomial with real coefficients has at least one real root, this root is usable to reach a given y by arbitrary x.

Problem 5. 5.2.4

Suppose that $f\colon A \to B$ and $g\colon B \to C$ are functions. Give proofs/counterexamples.

(a) If $g \circ f$ is one-to-one, must f be one-to-one?

(b) If $g \circ f$ is one-to-one, must g be one-to-one?

(c) If $g \circ f$ is onto, must f onto?

(d) If $g \circ f$ is onto, must g onto?

Solution

Part (a)

If $g \circ f$ is one-to-one, f must be one-to-one. Suppose f is not one-to-one, i.e. there is some b in the codomain of f which is mapped to by two values, x and y. Then, g may map b to any other value, call it c. This implies that $f \circ g$ maps to c by both x and y, and thus $g \circ f$ is not one-to-one. By contradiction, f must be one-to-one.

Part (b)

If $g \circ f$ is one-to-one, g may be one-to-one, but does not have to be. Suppose g is not one-to-one, i.e. there is some b in the codomain of g which is mapped to by two values, x and y. If both x and y are in B, then there is a problem and $g \circ f$ is not one-to-one. If they are not, then everything is nominal. Thus, in some cases there is a contradiction, and it is implied that g must be one-to-one.

Part (c)

If $g \circ f$ is onto, f must be onto. Suppose f is not onto, i.e. there is some value b in the codomain which it cannot reach from some x. Since b is not reachable, g cannot use it to reach an arbitrary value $c \in C$, and thus $g \circ f$ would not be onto. By contradiction, f must be onto if $g \circ f$ is onto.

Part (d)

If $g \circ f$ is onto, g must be onto. Suppose g is not onto, i.e. there is some value c in the codomain which it cannot reach from some x. Since c is not reachable from g from any starting value, $g \circ f$ is not onto. By contradiction, g must be onto if $g \circ f$ is onto.