# Gauge groups in PyR@TE 3

## Contents



This ancillary file provides information about the first few representations of some special unitary, special orthogonal, symplectic and exceptional simple Lie algebras that can be implemented in PyR@TE 3. In particular, each representation is assigned a set of Dynkin labels, which constitutes a unique identifier in the case where several representations of a given gauge group have the same dimension. We also provide the Dynkin index $T(R)$ of each representation since it is used in PyR@TE to normalize the generators according to

$$\mathrm{Tr}\left(t^a t^b\right) = T(R)\,\delta^{ab}\,,$$

hence impacting the actual expression of the $\beta$-functions.

# 1 $SU$ family

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SU2 | A1 | 3 | 1 | **2** | [1] | 1/2 | Pseudo-real |
| | | | | **3** | [2] | 2 | Real (adjoint) |
| | | | | **4** | [3] | 5 | Pseudo-real |
| | | | | **5** | [4] | 10 | Real |
| | | | | **6** | [5] | 35/2 | Pseudo-real |
| | | | | **7** | [6] | 28 | Real |
| | | | | **8** | [7] | 42 | Pseudo-real |
| | | | | **9** | [8] | 60 | Real |
| | | | | **10** | [9] | 165/2 | Pseudo-real |
| | | | | **11** | [10] | 110 | Real |
| SU3 | A2 | 8 | 2 | **3** | [1, 0] | 1/2 | Complex |
| | | | | $\overline{\mathbf{3}}$ | [0, 1] | 1/2 | Complex |
| | | | | **6** | [0, 2] | 5/2 | Complex |
| | | | | $\overline{\mathbf{6}}$ | [2, 0] | 5/2 | Complex |
| | | | | **8** | [1, 1] | 3 | Real (adjoint) |
| | | | | **10** | [0, 3] | 15/2 | Complex |
| | | | | $\overline{\mathbf{10}}$ | [3, 0] | 15/2 | Complex |
| | | | | **15** | [2, 1] | 10 | Complex |
| | | | | $\overline{\mathbf{15}}$ | [1, 2] | 10 | Complex |
| | | | | $\mathbf{15}^{'}$ | [4, 0] | 35/2 | Complex |
| | | | | $\overline{\mathbf{15}^{'}}$ | [0, 4] | 35/2 | Complex |
| | | | | **21** | [0, 5] | 35 | Complex |
| | | | | $\overline{\mathbf{21}}$ | [5, 0] | 35 | Complex |
| | | | | **24** | [1, 3] | 25 | Complex |
| | | | | $\overline{\mathbf{24}}$ | [3, 1] | 25 | Complex |
| | | | | **27** | [2, 2] | 27 | Real |
| | | | | **28** | [0, 6] | 63 | Complex |
| | | | | $\overline{\mathbf{28}}$ | [6, 0] | 63 | Complex |
| | | | | **35** | [1, 4] | 105/2 | Complex |
| | | | | $\overline{\mathbf{35}}$ | [4, 1] | 105/2 | Complex |

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SU4 | A3 | 15 | 3 | $\mathbf{4}$ | [1, 0, 0] | 1/2 | Complex |
| | | | | $\overline{\mathbf{4}}$ | [0, 0, 1] | 1/2 | Complex |
| | | | | $\mathbf{6}$ | [0, 1, 0] | 1 | Real |
| | | | | $\mathbf{10}$ | [0, 0, 2] | 3 | Complex |
| | | | | $\overline{\mathbf{10}}$ | [2, 0, 0] | 3 | Complex |
| | | | | $\mathbf{15}$ | [1, 0, 1] | 4 | Real (adjoint) |
| | | | | $\mathbf{20}$ | [0, 1, 1] | 13/2 | Complex |
| | | | | $\overline{\mathbf{20}}$ | [1, 1, 0] | 13/2 | Complex |
| | | | | $\mathbf{20}'$ | [0, 2, 0] | 8 | Real |
| | | | | $\mathbf{20}''$ | [0, 0, 3] | 21/2 | Complex |
| | | | | $\overline{\mathbf{20}''}$ | [3, 0, 0] | 21/2 | Complex |
| | | | | $\mathbf{35}$ | [0, 0, 4] | 28 | Complex |
| | | | | $\overline{\mathbf{35}}$ | [4, 0, 0] | 28 | Complex |
| | | | | $\mathbf{36}$ | [2, 0, 1] | 33/2 | Complex |
| | | | | $\overline{\mathbf{36}}$ | [1, 0, 2] | 33/2 | Complex |
| | | | | $\mathbf{45}$ | [0, 1, 2] | 24 | Complex |
| | | | | $\overline{\mathbf{45}}$ | [2, 1, 0] | 24 | Complex |
| | | | | $\mathbf{50}$ | [0, 3, 0] | 35 | Real |
| | | | | $\mathbf{56}$ | [5, 0, 0] | 63 | Complex |
| | | | | $\overline{\mathbf{56}}$ | [0, 0, 5] | 63 | Complex |
| SU5 | A4 | 24 | 4 | $\mathbf{5}$ | [1, 0, 0, 0] | 1/2 | Complex |
| | | | | $\overline{\mathbf{5}}$ | [0, 0, 0, 1] | 1/2 | Complex |
| | | | | $\mathbf{10}$ | [0, 1, 0, 0] | 3/2 | Complex |
| | | | | $\overline{\mathbf{10}}$ | [0, 0, 1, 0] | 3/2 | Complex |
| | | | | $\mathbf{15}$ | [2, 0, 0, 0] | 7/2 | Complex |
| | | | | $\overline{\mathbf{15}}$ | [0, 0, 0, 2] | 7/2 | Complex |
| | | | | $\mathbf{24}$ | [1, 0, 0, 1] | 5 | Real (adjoint) |
| | | | | $\mathbf{35}$ | [0, 0, 0, 3] | 14 | Complex |
| | | | | $\overline{\mathbf{35}}$ | [3, 0, 0, 0] | 14 | Complex |
| | | | | $\mathbf{40}$ | [0, 0, 1, 1] | 11 | Complex |
| | | | | $\overline{\mathbf{40}}$ | [1, 1, 0, 0] | 11 | Complex |
| | | | | $\mathbf{45}$ | [0, 1, 0, 1] | 12 | Complex |
| | | | | $\overline{\mathbf{45}}$ | [1, 0, 1, 0] | 12 | Complex |
| | | | | $\mathbf{50}$ | [0, 0, 2, 0] | 35/2 | Complex |
| | | | | $\overline{\mathbf{50}}$ | [0, 2, 0, 0] | 35/2 | Complex |
| | | | | $\mathbf{70}$ | [2, 0, 0, 1] | 49/2 | Complex |
| | | | | $\overline{\mathbf{70}}$ | [1, 0, 0, 2] | 49/2 | Complex |
| | | | | $\mathbf{70}'$ | [0, 0, 0, 4] | 42 | Complex |
| | | | | $\overline{\mathbf{70}'}$ | [4, 0, 0, 0] | 42 | Complex |
| | | | | $\mathbf{75}$ | [0, 1, 1, 0] | 25 | Real |

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SU6 | A5 | 35 | 5 | $\mathbf{6}$ | [1, 0, 0, 0, 0] | 1/2 | Complex |
| | | | | $\overline{\mathbf{6}}$ | [0, 0, 0, 0, 1] | 1/2 | Complex |
| | | | | $\mathbf{15}$ | [0, 1, 0, 0, 0] | 2 | Complex |
| | | | | $\overline{\mathbf{15}}$ | [0, 0, 0, 1, 0] | 2 | Complex |
| | | | | $\mathbf{20}$ | [0, 0, 1, 0, 0] | 3 | Pseudo-real |
| | | | | $\mathbf{21}$ | [2, 0, 0, 0, 0] | 4 | Complex |
| | | | | $\overline{\mathbf{21}}$ | [0, 0, 0, 0, 2] | 4 | Complex |
| | | | | $\mathbf{35}$ | [1, 0, 0, 0, 1] | 6 | Real (adjoint) |
| | | | | $\mathbf{56}$ | [0, 0, 0, 0, 3] | 18 | Complex |
| | | | | $\overline{\mathbf{56}}$ | [3, 0, 0, 0, 0] | 18 | Complex |
| | | | | $\mathbf{70}$ | [0, 0, 0, 1, 1] | 33/2 | Complex |
| | | | | $\overline{\mathbf{70}}$ | [1, 1, 0, 0, 0] | 33/2 | Complex |
| | | | | $\mathbf{84}$ | [0, 1, 0, 0, 1] | 19 | Complex |
| | | | | $\overline{\mathbf{84}}$ | [1, 0, 0, 1, 0] | 19 | Complex |
| | | | | $\mathbf{105}$ | [0, 0, 1, 0, 1] | 26 | Complex |
| | | | | $\overline{\mathbf{105}}$ | [1, 0, 1, 0, 0] | 26 | Complex |
| | | | | $\mathbf{105}'$ | [0, 0, 0, 2, 0] | 32 | Complex |
| | | | | $\overline{\mathbf{105}'}$ | [0, 2, 0, 0, 0] | 32 | Complex |
| | | | | $\mathbf{120}$ | [2, 0, 0, 0, 1] | 34 | Complex |
| | | | | $\overline{\mathbf{120}}$ | [1, 0, 0, 0, 2] | 34 | Complex |
| SU7 | A6 | 48 | 6 | $\mathbf{7}$ | [1, 0, 0, 0, 0, 0] | 1/2 | Complex |
| | | | | $\overline{\mathbf{7}}$ | [0, 0, 0, 0, 0, 1] | 1/2 | Complex |
| | | | | $\mathbf{21}$ | [0, 1, 0, 0, 0, 0] | 5/2 | Complex |
| | | | | $\overline{\mathbf{21}}$ | [0, 0, 0, 0, 1, 0] | 5/2 | Complex |
| | | | | $\mathbf{28}$ | [2, 0, 0, 0, 0, 0] | 9/2 | Complex |
| | | | | $\overline{\mathbf{28}}$ | [0, 0, 0, 0, 0, 2] | 9/2 | Complex |
| | | | | $\mathbf{35}$ | [0, 0, 1, 0, 0, 0] | 5 | Complex |
| | | | | $\overline{\mathbf{35}}$ | [0, 0, 0, 1, 0, 0] | 5 | Complex |
| | | | | $\mathbf{48}$ | [1, 0, 0, 0, 0, 1] | 7 | Real (adjoint) |
| | | | | $\mathbf{84}$ | [3, 0, 0, 0, 0, 0] | 45/2 | Complex |
| | | | | $\overline{\mathbf{84}}$ | [0, 0, 0, 0, 0, 3] | 45/2 | Complex |
| | | | | $\mathbf{112}$ | [1, 1, 0, 0, 0, 0] | 23 | Complex |
| | | | | $\overline{\mathbf{112}}$ | [0, 0, 0, 0, 1, 1] | 23 | Complex |
| | | | | $\mathbf{140}$ | [0, 1, 0, 0, 0, 1] | 55/2 | Complex |
| | | | | $\overline{\mathbf{140}}$ | [1, 0, 0, 0, 1, 0] | 55/2 | Complex |
| | | | | $\mathbf{189}$ | [2, 0, 0, 0, 0, 1] | 45 | Complex |
| | | | | $\overline{\mathbf{189}}$ | [1, 0, 0, 0, 0, 2] | 45 | Complex |
| | | | | $\mathbf{196}$ | [0, 0, 0, 0, 2, 0] | 105/2 | Complex |
| | | | | $\overline{\mathbf{196}}$ | [0, 2, 0, 0, 0, 0] | 105/2 | Complex |

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SU8 | A7 | 63 | 7 | **8** | [1, 0, 0, 0, 0, 0, 0] | 1/2 | Complex |
| | | | | $\overline{\mathbf{8}}$ | [0, 0, 0, 0, 0, 0, 1] | 1/2 | Complex |
| | | | | **28** | [0, 1, 0, 0, 0, 0, 0] | 3 | Complex |
| | | | | $\overline{\mathbf{28}}$ | [0, 0, 0, 0, 0, 1, 0] | 3 | Complex |
| | | | | **36** | [2, 0, 0, 0, 0, 0, 0] | 5 | Complex |
| | | | | $\overline{\mathbf{36}}$ | [0, 0, 0, 0, 0, 0, 2] | 5 | Complex |
| | | | | **56** | [0, 0, 1, 0, 0, 0, 0] | 15/2 | Complex |
| | | | | $\overline{\mathbf{56}}$ | [0, 0, 0, 0, 1, 0, 0] | 15/2 | Complex |
| | | | | **63** | [1, 0, 0, 0, 0, 0, 1] | 8 | Real (adjoint) |
| | | | | **70** | [0, 0, 0, 1, 0, 0, 0] | 10 | Real |
| | | | | **120** | [3, 0, 0, 0, 0, 0, 0] | 55/2 | Complex |
| | | | | $\overline{\mathbf{120}}$ | [0, 0, 0, 0, 0, 0, 3] | 55/2 | Complex |
| | | | | **168** | [1, 1, 0, 0, 0, 0, 0] | 61/2 | Complex |
| | | | | $\overline{\mathbf{168}}$ | [0, 0, 0, 0, 0, 1, 1] | 61/2 | Complex |
| | | | | **216** | [0, 1, 0, 0, 0, 0, 1] | 75/2 | Complex |
| | | | | $\overline{\mathbf{216}}$ | [1, 0, 0, 0, 0, 1, 0] | 75/2 | Complex |
| | | | | **280** | [2, 0, 0, 0, 0, 0, 1] | 115/2 | Complex |
| | | | | $\overline{\mathbf{280}}$ | [1, 0, 0, 0, 0, 0, 2] | 115/2 | Complex |
| SU9 | A8 | 80 | 8 | **9** | [1, 0, 0, 0, 0, 0, 0, 0] | 1/2 | Complex |
| | | | | $\overline{\mathbf{9}}$ | [0, 0, 0, 0, 0, 0, 0, 1] | 1/2 | Complex |
| | | | | **36** | [0, 1, 0, 0, 0, 0, 0, 0] | 7/2 | Complex |
| | | | | $\overline{\mathbf{36}}$ | [0, 0, 0, 0, 0, 0, 1, 0] | 7/2 | Complex |
| | | | | **45** | [2, 0, 0, 0, 0, 0, 0, 0] | 11/2 | Complex |
| | | | | $\overline{\mathbf{45}}$ | [0, 0, 0, 0, 0, 0, 0, 2] | 11/2 | Complex |
| | | | | **80** | [1, 0, 0, 0, 0, 0, 0, 1] | 9 | Real (adjoint) |
| | | | | **84** | [0, 0, 1, 0, 0, 0, 0, 0] | 21/2 | Complex |
| | | | | $\overline{\mathbf{84}}$ | [0, 0, 0, 0, 0, 1, 0, 0] | 21/2 | Complex |
| | | | | **126** | [0, 0, 0, 1, 0, 0, 0, 0] | 35/2 | Complex |
| | | | | $\overline{\mathbf{126}}$ | [0, 0, 0, 0, 1, 0, 0, 0] | 35/2 | Complex |
| | | | | **165** | [3, 0, 0, 0, 0, 0, 0, 0] | 33 | Complex |
| | | | | $\overline{\mathbf{165}}$ | [0, 0, 0, 0, 0, 0, 0, 3] | 33 | Complex |
| | | | | **240** | [1, 1, 0, 0, 0, 0, 0, 0] | 39 | Complex |
| | | | | $\overline{\mathbf{240}}$ | [0, 0, 0, 0, 0, 0, 1, 1] | 39 | Complex |
| | | | | **315** | [0, 1, 0, 0, 0, 0, 0, 1] | 49 | Complex |
| | | | | $\overline{\mathbf{315}}$ | [1, 0, 0, 0, 0, 0, 1, 0] | 49 | Complex |
| | | | | **396** | [2, 0, 0, 0, 0, 0, 0, 1] | 143/2 | Complex |
| | | | | $\overline{\mathbf{396}}$ | [1, 0, 0, 0, 0, 0, 0, 2] | 143/2 | Complex |

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SU10 | A9 | 99 | 9 | **10** | [1, 0, 0, 0, 0, 0, 0, 0, 0] | 1/2 | Complex |
| | | | | $\overline{\mathbf{10}}$ | [0, 0, 0, 0, 0, 0, 0, 0, 1] | 1/2 | Complex |
| | | | | **45** | [0, 1, 0, 0, 0, 0, 0, 0, 0] | 4 | Complex |
| | | | | $\overline{\mathbf{45}}$ | [0, 0, 0, 0, 0, 0, 0, 1, 0] | 4 | Complex |
| | | | | **55** | [2, 0, 0, 0, 0, 0, 0, 0, 0] | 6 | Complex |
| | | | | $\overline{\mathbf{55}}$ | [0, 0, 0, 0, 0, 0, 0, 0, 2] | 6 | Complex |
| | | | | **99** | [1, 0, 0, 0, 0, 0, 0, 0, 1] | 10 | Real (adjoint) |
| | | | | **120** | [0, 0, 1, 0, 0, 0, 0, 0, 0] | 14 | Complex |
| | | | | $\overline{\mathbf{120}}$ | [0, 0, 0, 0, 0, 0, 1, 0, 0] | 14 | Complex |
| | | | | **210** | [0, 0, 0, 1, 0, 0, 0, 0, 0] | 28 | Complex |
| | | | | $\overline{\mathbf{210}}$ | [0, 0, 0, 0, 0, 1, 0, 0, 0] | 28 | Complex |
| | | | | **220** | [3, 0, 0, 0, 0, 0, 0, 0, 0] | 39 | Complex |
| | | | | $\overline{\mathbf{220}}$ | [0, 0, 0, 0, 0, 0, 0, 0, 3] | 39 | Complex |
| | | | | **252** | [0, 0, 0, 0, 1, 0, 0, 0, 0] | 35 | Pseudo-real |
| | | | | **330** | [1, 1, 0, 0, 0, 0, 0, 0, 0] | 97/2 | Complex |
| | | | | $\overline{\mathbf{330}}$ | [0, 0, 0, 0, 0, 0, 0, 1, 1] | 97/2 | Complex |
| | | | | **440** | [0, 1, 0, 0, 0, 0, 0, 0, 1] | 62 | Complex |
| | | | | $\overline{\mathbf{440}}$ | [1, 0, 0, 0, 0, 0, 0, 1, 0] | 62 | Complex |

# 2 $SO$ family

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SO3 | B1 | 3 | 1 | **2** | [1] | 1/2 | Pseudo-real |
| | | | | **3** | [2] | 2 | Real (adjoint) |
| | | | | **4** | [3] | 5 | Pseudo-real |
| | | | | **5** | [4] | 10 | Real |
| | | | | **6** | [5] | 35/2 | Pseudo-real |
| | | | | **7** | [6] | 28 | Real |
| | | | | **8** | [7] | 42 | Pseudo-real |
| | | | | **9** | [8] | 60 | Real |
| | | | | **10** | [9] | 165/2 | Pseudo-real |
| | | | | **11** | [10] | 110 | Real |
| SO5 | B2 | 10 | 2 | **4** | [0, 1] | 1/2 | Pseudo-real |
| | | | | **5** | [1, 0] | 1 | Real |
| | | | | **10** | [0, 2] | 3 | Real (adjoint) |
| | | | | **14** | [2, 0] | 7 | Real |
| | | | | **16** | [1, 1] | 6 | Pseudo-real |
| | | | | **20** | [0, 3] | 21/2 | Pseudo-real |
| | | | | **30** | [3, 0] | 27 | Real |
| | | | | **35** | [1, 2] | 21 | Real |
| | | | | **35′** | [0, 4] | 28 | Real |
| | | | | **40** | [2, 1] | 29 | Pseudo-real |
| | | | | **55** | [4, 0] | 77 | Real |

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SO6 | D3 | 15 | 3 | $\mathbf{4}$ | [0, 1, 0] | 1/2 | Complex |
| | | | | $\overline{\mathbf{4}}$ | [0, 0, 1] | 1/2 | Complex |
| | | | | $\mathbf{6}$ | [1, 0, 0] | 1 | Real |
| | | | | $\mathbf{10}$ | [0, 0, 2] | 3 | Complex |
| | | | | $\overline{\mathbf{10}}$ | [0, 2, 0] | 3 | Complex |
| | | | | $\mathbf{15}$ | [0, 1, 1] | 4 | Real (adjoint) |
| | | | | $\mathbf{20}$ | [1, 0, 1] | 13/2 | Complex |
| | | | | $\overline{\mathbf{20}}$ | [1, 1, 0] | 13/2 | Complex |
| | | | | $\mathbf{20}'$ | [2, 0, 0] | 8 | Real |
| | | | | $\mathbf{20}''$ | [0, 0, 3] | 21/2 | Complex |
| | | | | $\overline{\mathbf{20}''}$ | [0, 3, 0] | 21/2 | Complex |
| | | | | $\mathbf{35}$ | [0, 0, 4] | 28 | Complex |
| | | | | $\overline{\mathbf{35}}$ | [0, 4, 0] | 28 | Complex |
| | | | | $\mathbf{36}$ | [0, 2, 1] | 33/2 | Complex |
| | | | | $\overline{\mathbf{36}}$ | [0, 1, 2] | 33/2 | Complex |
| | | | | $\mathbf{45}$ | [1, 0, 2] | 24 | Complex |
| | | | | $\overline{\mathbf{45}}$ | [1, 2, 0] | 24 | Complex |
| | | | | $\mathbf{50}$ | [3, 0, 0] | 35 | Real |
| | | | | $\mathbf{56}$ | [0, 5, 0] | 63 | Complex |
| | | | | $\overline{\mathbf{56}}$ | [0, 0, 5] | 63 | Complex |
| SO7 | B3 | 21 | 3 | $\mathbf{7}$ | [1, 0, 0] | 1 | Real |
| | | | | $\mathbf{8}$ | [0, 0, 1] | 1 | Real |
| | | | | $\mathbf{21}$ | [0, 1, 0] | 5 | Real (adjoint) |
| | | | | $\mathbf{27}$ | [2, 0, 0] | 9 | Real |
| | | | | $\mathbf{35}$ | [0, 0, 2] | 10 | Real |
| | | | | $\mathbf{48}$ | [1, 0, 1] | 14 | Real |
| | | | | $\mathbf{77}$ | [3, 0, 0] | 44 | Real |
| | | | | $\mathbf{105}$ | [1, 1, 0] | 45 | Real |
| | | | | $\mathbf{112}$ | [0, 1, 1] | 46 | Real |
| | | | | $\mathbf{112}'$ | [0, 0, 3] | 54 | Real |
| | | | | $\mathbf{168}$ | [2, 0, 1] | 85 | Real |
| | | | | $\mathbf{168}'$ | [0, 2, 0] | 96 | Real |

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SO8 | D4 | 28 | 4 | **8** | [1, 0, 0, 0] | 1 | Real |
| | | | | **8**$'$ | [0, 0, 0, 1] | 1 | Real |
| | | | | **8**$''$ | [0, 0, 1, 0] | 1 | Real |
| | | | | **28** | [0, 1, 0, 0] | 6 | Real (adjoint) |
| | | | | **35** | [0, 0, 0, 2] | 10 | Real |
| | | | | **35**$'$ | [0, 0, 2, 0] | 10 | Real |
| | | | | **35**$''$ | [2, 0, 0, 0] | 10 | Real |
| | | | | **56** | [0, 0, 1, 1] | 15 | Real |
| | | | | **56**$'$ | [1, 0, 1, 0] | 15 | Real |
| | | | | **56**$''$ | [1, 0, 0, 1] | 15 | Real |
| | | | | **112** | [3, 0, 0, 0] | 54 | Real |
| | | | | **112**$'$ | [0, 0, 0, 3] | 54 | Real |
| | | | | **112**$''$ | [0, 0, 3, 0] | 54 | Real |
| | | | | **160** | [1, 1, 0, 0] | 60 | Real |
| | | | | **160**$'$ | [0, 1, 0, 1] | 60 | Real |
| | | | | **160**$''$ | [0, 1, 1, 0] | 60 | Real |
| | | | | **224** | [1, 0, 0, 2] | 100 | Real |
| | | | | **224**$'$ | [1, 0, 2, 0] | 100 | Real |
| | | | | **224**$''$ | [0, 0, 2, 1] | 100 | Real |
| | | | | **224**$'''$ | [2, 0, 0, 1] | 100 | Real |
| | | | | **224**$''''$ | [0, 0, 1, 2] | 100 | Real |
| | | | | **224**$'''''$ | [2, 0, 1, 0] | 100 | Real |
| | | | | **294** | [0, 0, 0, 4] | 210 | Real |
| | | | | **294**$'$ | [0, 0, 4, 0] | 210 | Real |
| | | | | **294**$''$ | [4, 0, 0, 0] | 210 | Real |
| | | | | **300** | [0, 2, 0, 0] | 150 | Real |
| | | | | **350** | [1, 0, 1, 1] | 150 | Real |
| SO9 | B4 | 36 | 4 | **9** | [1, 0, 0, 0] | 1 | Real |
| | | | | **16** | [0, 0, 0, 1] | 2 | Real |
| | | | | **36** | [0, 1, 0, 0] | 7 | Real (adjoint) |
| | | | | **44** | [2, 0, 0, 0] | 11 | Real |
| | | | | **84** | [0, 0, 1, 0] | 21 | Real |
| | | | | **126** | [0, 0, 0, 2] | 35 | Real |
| | | | | **128** | [1, 0, 0, 1] | 32 | Real |
| | | | | **156** | [3, 0, 0, 0] | 65 | Real |
| | | | | **231** | [1, 1, 0, 0] | 77 | Real |
| | | | | **432** | [0, 1, 0, 1] | 150 | Real |

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SO10 | D5 | 45 | 5 | **10** | [1, 0, 0, 0, 0] | 1 | Real |
| | | | | **16** | [0, 0, 0, 0, 1] | 2 | Complex |
| | | | | $\overline{\mathbf{16}}$ | [0, 0, 0, 1, 0] | 2 | Complex |
| | | | | **45** | [0, 1, 0, 0, 0] | 8 | Real (adjoint) |
| | | | | **54** | [2, 0, 0, 0, 0] | 12 | Real |
| | | | | **120** | [0, 0, 1, 0, 0] | 28 | Real |
| | | | | **126** | [0, 0, 0, 0, 2] | 35 | Complex |
| | | | | $\overline{\mathbf{126}}$ | [0, 0, 0, 2, 0] | 35 | Complex |
| | | | | **144** | [1, 0, 0, 1, 0] | 34 | Complex |
| | | | | $\overline{\mathbf{144}}$ | [1, 0, 0, 0, 1] | 34 | Complex |
| | | | | **210** | [0, 0, 0, 1, 1] | 56 | Real |
| | | | | **210**$'$ | [3, 0, 0, 0, 0] | 77 | Real |
| | | | | **320** | [1, 1, 0, 0, 0] | 96 | Real |
| | | | | **560** | [0, 1, 0, 0, 1] | 182 | Complex |
| | | | | $\overline{\mathbf{560}}$ | [0, 1, 0, 1, 0] | 182 | Complex |
| SO11 | B5 | 55 | 5 | **11** | [1, 0, 0, 0, 0] | 1 | Real |
| | | | | **32** | [0, 0, 0, 0, 1] | 4 | Pseudo-real |
| | | | | **55** | [0, 1, 0, 0, 0] | 9 | Real (adjoint) |
| | | | | **65** | [2, 0, 0, 0, 0] | 13 | Real |
| | | | | **165** | [0, 0, 1, 0, 0] | 36 | Real |
| | | | | **275** | [3, 0, 0, 0, 0] | 90 | Real |
| | | | | **320** | [1, 0, 0, 0, 1] | 72 | Pseudo-real |
| | | | | **330** | [0, 0, 0, 1, 0] | 84 | Real |
| | | | | **429** | [1, 1, 0, 0, 0] | 117 | Real |
| | | | | **462** | [0, 0, 0, 0, 2] | 126 | Real |
| SO12 | D6 | 66 | 6 | **12** | [1, 0, 0, 0, 0, 0] | 1 | Real |
| | | | | **32** | [0, 0, 0, 0, 1, 0] | 4 | Pseudo-real |
| | | | | **32**$'$ | [0, 0, 0, 0, 0, 1] | 4 | Pseudo-real |
| | | | | **66** | [0, 1, 0, 0, 0, 0] | 10 | Real (adjoint) |
| | | | | **77** | [2, 0, 0, 0, 0, 0] | 14 | Real |
| | | | | **220** | [0, 0, 1, 0, 0, 0] | 45 | Real |
| | | | | **352** | [1, 0, 0, 0, 0, 1] | 76 | Pseudo-real |
| | | | | **352**$'$ | [1, 0, 0, 0, 1, 0] | 76 | Pseudo-real |
| | | | | **352**$''$ | [3, 0, 0, 0, 0, 0] | 104 | Real |
| | | | | **462** | [0, 0, 0, 0, 0, 2] | 126 | Real |
| | | | | **462**$'$ | [0, 0, 0, 0, 2, 0] | 126 | Real |
| | | | | **495** | [0, 0, 0, 1, 0, 0] | 120 | Real |
| | | | | **560** | [1, 1, 0, 0, 0, 0] | 140 | Real |
| | | | | **792** | [0, 0, 0, 0, 1, 1] | 210 | Real |

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SO13 | B6 | 78 | 6 | **13** | [1, 0, 0, 0, 0, 0] | 1 | Real |
| | | | | **64** | [0, 0, 0, 0, 0, 1] | 8 | Pseudo-real |
| | | | | **78** | [0, 1, 0, 0, 0, 0] | 11 | Real (adjoint) |
| | | | | **90** | [2, 0, 0, 0, 0, 0] | 15 | Real |
| | | | | **286** | [0, 0, 1, 0, 0, 0] | 55 | Real |
| | | | | **442** | [3, 0, 0, 0, 0, 0] | 119 | Real |
| | | | | **715** | [0, 0, 0, 1, 0, 0] | 165 | Real |
| | | | | **715′** | [1, 1, 0, 0, 0, 0] | 165 | Real |
| | | | | **768** | [1, 0, 0, 0, 0, 1] | 160 | Pseudo-real |
| | | | | **1287** | [0, 0, 0, 0, 1, 0] | 330 | Real |
| | | | | **1716** | [0, 0, 0, 0, 0, 2] | 462 | Real |
| SO14 | D7 | 91 | 7 | **14** | [1, 0, 0, 0, 0, 0, 0] | 1 | Real |
| | | | | **64** | [0, 0, 0, 0, 0, 1, 0] | 8 | Complex |
| | | | | $\overline{\mathbf{64}}$ | [0, 0, 0, 0, 0, 0, 1] | 8 | Complex |
| | | | | **91** | [0, 1, 0, 0, 0, 0, 0] | 12 | Real (adjoint) |
| | | | | **104** | [2, 0, 0, 0, 0, 0, 0] | 16 | Real |
| | | | | **364** | [0, 0, 1, 0, 0, 0, 0] | 66 | Real |
| | | | | **546** | [3, 0, 0, 0, 0, 0, 0] | 135 | Real |
| | | | | **832** | [1, 0, 0, 0, 0, 0, 1] | 168 | Complex |
| | | | | $\overline{\mathbf{832}}$ | [1, 0, 0, 0, 0, 1, 0] | 168 | Complex |
| | | | | **896** | [1, 1, 0, 0, 0, 0, 0] | 192 | Real |
| | | | | **1001** | [0, 0, 0, 1, 0, 0, 0] | 220 | Real |
| | | | | **1716** | [0, 0, 0, 0, 0, 0, 2] | 462 | Complex |
| | | | | $\overline{\mathbf{1716}}$ | [0, 0, 0, 0, 0, 2, 0] | 462 | Complex |
| SO15 | B7 | 105 | 7 | **15** | [1, 0, 0, 0, 0, 0, 0] | 1 | Real |
| | | | | **105** | [0, 1, 0, 0, 0, 0, 0] | 13 | Real (adjoint) |
| | | | | **119** | [2, 0, 0, 0, 0, 0, 0] | 17 | Real |
| | | | | **128** | [0, 0, 0, 0, 0, 0, 1] | 16 | Real |
| | | | | **455** | [0, 0, 1, 0, 0, 0, 0] | 78 | Real |
| | | | | **665** | [3, 0, 0, 0, 0, 0, 0] | 152 | Real |
| | | | | **1105** | [1, 1, 0, 0, 0, 0, 0] | 221 | Real |
| | | | | **1365** | [0, 0, 0, 1, 0, 0, 0] | 286 | Real |
| | | | | **1792** | [1, 0, 0, 0, 0, 0, 1] | 352 | Real |
| | | | | **2940** | [4, 0, 0, 0, 0, 0, 0] | 952 | Real |

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SO16 | D8 | 120 | 8 | **16** | [1, 0, 0, 0, 0, 0, 0, 0] | 1 | Real |
| | | | | **120** | [0, 1, 0, 0, 0, 0, 0, 0] | 14 | Real (adjoint) |
| | | | | **128** | [0, 0, 0, 0, 0, 0, 0, 1] | 16 | Real |
| | | | | **128′** | [0, 0, 0, 0, 0, 0, 1, 0] | 16 | Real |
| | | | | **135** | [2, 0, 0, 0, 0, 0, 0, 0] | 18 | Real |
| | | | | **560** | [0, 0, 1, 0, 0, 0, 0, 0] | 91 | Real |
| | | | | **800** | [3, 0, 0, 0, 0, 0, 0, 0] | 170 | Real |
| | | | | **1344** | [1, 1, 0, 0, 0, 0, 0, 0] | 252 | Real |
| | | | | **1820** | [0, 0, 0, 1, 0, 0, 0, 0] | 364 | Real |
| | | | | **1920** | [1, 0, 0, 0, 0, 0, 1, 0] | 368 | Real |
| | | | | **1920′** | [1, 0, 0, 0, 0, 0, 0, 1] | 368 | Real |
| | | | | **3740** | [4, 0, 0, 0, 0, 0, 0, 0] | 1122 | Real |

# 3 *Sp* family

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SP2 | C1 | 3 | 1 | **2** | [1] | 1/2 | Pseudo-real |
| | | | | **3** | [2] | 2 | Real (adjoint) |
| | | | | **4** | [3] | 5 | Pseudo-real |
| | | | | **5** | [4] | 10 | Real |
| | | | | **6** | [5] | 35/2 | Pseudo-real |
| | | | | **7** | [6] | 28 | Real |
| | | | | **8** | [7] | 42 | Pseudo-real |
| | | | | **9** | [8] | 60 | Real |
| | | | | **10** | [9] | 165/2 | Pseudo-real |
| | | | | **11** | [10] | 110 | Real |
| SP4 | C2 | 10 | 2 | **4** | [1, 0] | 1/2 | Pseudo-real |
| | | | | **5** | [0, 1] | 1 | Real |
| | | | | **10** | [2, 0] | 3 | Real (adjoint) |
| | | | | **14** | [0, 2] | 7 | Real |
| | | | | **16** | [1, 1] | 6 | Pseudo-real |
| | | | | **20** | [3, 0] | 21/2 | Pseudo-real |
| | | | | **30** | [0, 3] | 27 | Real |
| | | | | **35** | [2, 1] | 21 | Real |
| | | | | **35′** | [4, 0] | 28 | Real |
| | | | | **40** | [1, 2] | 29 | Pseudo-real |
| | | | | **55** | [0, 4] | 77 | Real |
| SP6 | C3 | 21 | 3 | **6** | [1, 0, 0] | 1/2 | Pseudo-real |
| | | | | **14** | [0, 1, 0] | 2 | Real |
| | | | | **14′** | [0, 0, 1] | 5/2 | Pseudo-real |
| | | | | **21** | [2, 0, 0] | 4 | Real (adjoint) |
| | | | | **56** | [3, 0, 0] | 18 | Pseudo-real |
| | | | | **64** | [1, 1, 0] | 16 | Pseudo-real |
| | | | | **70** | [1, 0, 1] | 20 | Real |
| | | | | **84** | [0, 0, 2] | 36 | Real |
| | | | | **90** | [0, 2, 0] | 30 | Real |
| | | | | **126** | [0, 1, 1] | 93/2 | Pseudo-real |
| | | | | **126′** | [4, 0, 0] | 60 | Real |
| | | | | **189** | [2, 1, 0] | 72 | Real |

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| SP8 | C4 | 36 | 4 | **8** | [1, 0, 0, 0] | 1/2 | Pseudo-real |
| | | | | **27** | [0, 1, 0, 0] | 3 | Real |
| | | | | **36** | [2, 0, 0, 0] | 5 | Real (adjoint) |
| | | | | **42** | [0, 0, 0, 1] | 7 | Real |
| | | | | **48** | [0, 0, 1, 0] | 7 | Pseudo-real |
| | | | | **120** | [3, 0, 0, 0] | 55/2 | Pseudo-real |
| | | | | **160** | [1, 1, 0, 0] | 30 | Pseudo-real |
| | | | | **288** | [1, 0, 0, 1] | 70 | Pseudo-real |
| | | | | **308** | [0, 2, 0, 0] | 77 | Real |
| | | | | **315** | [1, 0, 1, 0] | 70 | Real |
| SP10 | C5 | 55 | 5 | **10** | [1, 0, 0, 0, 0] | 1/2 | Pseudo-real |
| | | | | **44** | [0, 1, 0, 0, 0] | 4 | Real |
| | | | | **55** | [2, 0, 0, 0, 0] | 6 | Real (adjoint) |
| | | | | **110** | [0, 0, 1, 0, 0] | 27/2 | Pseudo-real |
| | | | | **132** | [0, 0, 0, 0, 1] | 21 | Pseudo-real |
| | | | | **165** | [0, 0, 0, 1, 0] | 24 | Real |
| | | | | **220** | [3, 0, 0, 0, 0] | 39 | Pseudo-real |
| | | | | **320** | [1, 1, 0, 0, 0] | 48 | Pseudo-real |
| | | | | **715** | [4, 0, 0, 0, 0] | 182 | Real |
| | | | | **780** | [0, 2, 0, 0, 0] | 156 | Real |
| SP12 | C6 | 78 | 6 | **12** | [1, 0, 0, 0, 0, 0] | 1/2 | Pseudo-real |
| | | | | **65** | [0, 1, 0, 0, 0, 0] | 5 | Real |
| | | | | **78** | [2, 0, 0, 0, 0, 0] | 7 | Real (adjoint) |
| | | | | **208** | [0, 0, 1, 0, 0, 0] | 22 | Pseudo-real |
| | | | | **364** | [3, 0, 0, 0, 0, 0] | 105/2 | Pseudo-real |
| | | | | **429** | [0, 0, 0, 1, 0, 0] | 55 | Real |
| | | | | **429′** | [0, 0, 0, 0, 0, 1] | 66 | Real |
| | | | | **560** | [1, 1, 0, 0, 0, 0] | 70 | Pseudo-real |
| | | | | **572** | [0, 0, 0, 0, 1, 0] | 165/2 | Pseudo-real |
| | | | | **1365** | [4, 0, 0, 0, 0, 0] | 280 | Real |
| | | | | **1650** | [0, 2, 0, 0, 0, 0] | 275 | Real |

# 4 Exceptional algebras

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| G2 | G2 | 14 | 2 | **7** | [0, 1] | 1 | Real |
| | | | | **14** | [1, 0] | 4 | Real (adjoint) |
| | | | | **27** | [0, 2] | 9 | Real |
| | | | | **64** | [1, 1] | 32 | Real |
| | | | | **77** | [0, 3] | 44 | Real |
| | | | | **77′** | [2, 0] | 55 | Real |
| | | | | **182** | [0, 4] | 156 | Real |
| | | | | **189** | [1, 2] | 144 | Real |
| | | | | **273** | [3, 0] | 351 | Real |
| | | | | **286** | [2, 1] | 286 | Real |
| | | | | **378** | [0, 5] | 450 | Real |
| F4 | F4 | 52 | 4 | **26** | [0, 0, 0, 1] | 3 | Real |
| | | | | **52** | [1, 0, 0, 0] | 9 | Real (adjoint) |
| | | | | **273** | [0, 0, 1, 0] | 63 | Real |
| | | | | **324** | [0, 0, 0, 2] | 81 | Real |
| | | | | **1053** | [1, 0, 0, 1] | 324 | Real |
| | | | | **1053′** | [2, 0, 0, 0] | 405 | Real |
| | | | | **1274** | [0, 1, 0, 0] | 441 | Real |
| | | | | **2652** | [0, 0, 0, 3] | 1071 | Real |
| | | | | **4096** | [0, 0, 1, 1] | 1536 | Real |
| | | | | **8424** | [1, 0, 1, 0] | 3726 | Real |
| | | | | **10829** | [1, 0, 0, 2] | 4998 | Real |
| E6 | E6 | 78 | 6 | **27** | [1, 0, 0, 0, 0, 0] | 3 | Complex |
| | | | | $\overline{\mathbf{27}}$ | [0, 0, 0, 0, 1, 0] | 3 | Complex |
| | | | | **78** | [0, 0, 0, 0, 0, 1] | 12 | Real (adjoint) |
| | | | | **351** | [0, 0, 0, 1, 0, 0] | 75 | Complex |
| | | | | $\overline{\mathbf{351}}$ | [0, 1, 0, 0, 0, 0] | 75 | Complex |
| | | | | **351′** | [0, 0, 0, 0, 2, 0] | 84 | Complex |
| | | | | $\overline{\mathbf{351'}}$ | [2, 0, 0, 0, 0, 0] | 84 | Complex |
| | | | | **650** | [1, 0, 0, 0, 1, 0] | 150 | Real |
| | | | | **1728** | [1, 0, 0, 0, 0, 1] | 480 | Complex |
| | | | | $\overline{\mathbf{1728}}$ | [0, 0, 0, 0, 1, 1] | 480 | Complex |
| | | | | **2430** | [0, 0, 0, 0, 0, 2] | 810 | Real |
| | | | | **2925** | [0, 0, 1, 0, 0, 0] | 900 | Real |
| | | | | **3003** | [0, 0, 0, 0, 3, 0] | 1155 | Complex |
| | | | | $\overline{\mathbf{3003}}$ | [3, 0, 0, 0, 0, 0] | 1155 | Complex |
| | | | | **5824** | [0, 0, 0, 1, 1, 0] | 2016 | Complex |
| | | | | $\overline{\mathbf{5824}}$ | [1, 1, 0, 0, 0, 0] | 2016 | Complex |
| | | | | **7371** | [0, 1, 0, 0, 1, 0] | 2520 | Complex |
| | | | | $\overline{\mathbf{7371}}$ | [1, 0, 0, 1, 0, 0] | 2520 | Complex |

| Group | Lie algebra | Dim. | Rank | Representations | | | |
|---|---|---|---|---|---|---|---|
| | | | | Name / Dim. | Dynkin labels | Index | Reality |
| E7 | E7 | 133 | 7 | **56** | [0, 0, 0, 0, 0, 1, 0] | 6 | Pseudo-real |
| | | | | **133** | [1, 0, 0, 0, 0, 0, 0] | 18 | Real (adjoint) |
| | | | | **912** | [0, 0, 0, 0, 0, 0, 1] | 180 | Pseudo-real |
| | | | | **1463** | [0, 0, 0, 0, 0, 2, 0] | 330 | Real |
| | | | | **1539** | [0, 0, 0, 0, 1, 0, 0] | 324 | Real |
| | | | | **6480** | [1, 0, 0, 0, 0, 1, 0] | 1620 | Pseudo-real |
| | | | | **7371** | [2, 0, 0, 0, 0, 0, 0] | 2106 | Real |
| | | | | **8645** | [0, 1, 0, 0, 0, 0, 0] | 2340 | Real |
| | | | | **24320** | [0, 0, 0, 0, 0, 3, 0] | 8640 | Pseudo-real |
| | | | | **27664** | [0, 0, 0, 1, 0, 0, 0] | 8580 | Pseudo-real |
| E8 | E8 | 248 | 8 | **248** | [0, 0, 0, 0, 0, 0, 1, 0] | 30 | Real (adjoint) |
| | | | | **3875** | [1, 0, 0, 0, 0, 0, 0, 0] | 750 | Real |
| | | | | **27000** | [0, 0, 0, 0, 0, 0, 2, 0] | 6750 | Real |
| | | | | **30380** | [0, 0, 0, 0, 0, 1, 0, 0] | 7350 | Real |
| | | | | **147250** | [0, 0, 0, 0, 0, 0, 0, 1] | 42750 | Real |
| | | | | **779247** | [1, 0, 0, 0, 0, 0, 1, 0] | 251370 | Real |
| | | | | **1763125** | [0, 0, 0, 0, 0, 0, 3, 0] | 682500 | Real |
| | | | | **2450240** | [0, 0, 0, 0, 1, 0, 0, 0] | 889200 | Real |
| | | | | **4096000** | [0, 0, 0, 0, 0, 1, 1, 0] | 1536000 | Real |
| | | | | **4881384** | [2, 0, 0, 0, 0, 0, 0, 0] | 1968300 | Real |