SM

PyR@TE 3.0

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Contents

1 Model

1.1 Gauge groups

Name	Type	Abelian	Coupling constant
U1Y	$U(1)$	True	g_1
SU2L	$SU(2)$	False	g_2
SU3c	$SU(3)$	False	g_3

1.2 Fermions

Name	Generations	U1Y × SU2L × SU3c
Q	3	$(+\frac{1}{6}, \mathbf{2}, \mathbf{3})$
L	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
u_R	3	$(+\frac{2}{3}, \mathbf{1}, \mathbf{3})$
d_R	3	$(-\frac{1}{3}, \mathbf{1}, \mathbf{3})$
e_R	3	$(-1, \mathbf{1}, \mathbf{1})$

1.3 Scalars

Name	Complex	Expression	Generations	U1Y × SU2L × SU3c
H	True	$\frac{1}{\sqrt{2}}\,(\Pi + i\,\Sigma)$	1	$(+\frac{1}{2}, \mathbf{2}, \mathbf{1})$

2 Lagrangian

2.1 Definitions

$$\tilde{H}_i = \epsilon_{i,j} H^\dagger_j$$

2.2 Yukawa couplings

$$-\mathcal{L}_Y = \; + Y_{u\,f_1,f_2} \tilde{H}_i \overline{Q}_{f_1,i,a} u_{R\,f_2,a} + Y_{d\,f_1,f_2} \overline{Q}_{f_1,i,a} H_i d_{R\,f_2,a} + Y_{e\,f_1,f_2} \overline{L}_{f_1,i} H_i e_{R\,f_2} + \text{h.c.}$$

2.3 Quartic couplings

$$-\mathcal{L}_Q = +\lambda H_i^\dagger H_i H_{i_1}^\dagger H_{i_1}$$

2.4 Scalar mass couplings

$$-\mathcal{L}_{sm} = +\mu H_i^\dagger H_i$$

3 Renormalization Group Equations

3.1 Convention

$$\beta\left(X\right) \equiv \mu \frac{dX}{d\mu} \equiv \frac{1}{\left(4\pi\right)^2}\beta^{(1)}(X) + \frac{1}{\left(4\pi\right)^4}\beta^{(2)}(X)$$

3.2 Definitions and substitutions

$$Y_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix} \quad , \quad Y_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix} \quad , \quad Y_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

3.3 Gauge couplings

$$\beta^{(1)}(g_1) = \frac{41}{10} g_1^3$$

$$\beta^{(2)}(g_1) = +\frac{199}{50} g_1^5 + \frac{27}{10} g_1^3 g_2^2 + \frac{44}{5} g_1^3 g_3^2 - \frac{17}{10} g_1^3 \left|y_t\right|^2 - \frac{1}{2} g_1^3 \left|y_b\right|^2 - \frac{3}{2} g_1^3 \left|y_\tau\right|^2$$

$$\beta^{(1)}(g_2) = -\frac{19}{6} g_2^3$$

$$\beta^{(2)}(g_2) = +\frac{9}{10} g_1^2 g_2^3 + \frac{35}{6} g_2^5 + 12 g_2^3 g_3^2 - \frac{3}{2} g_2^3 \left|y_t\right|^2 - \frac{3}{2} g_2^3 \left|y_b\right|^2 - \frac{1}{2} g_2^3 \left|y_\tau\right|^2$$

$$\beta^{(1)}(g_3) = -7 g_3^3$$

$$\beta^{(2)}(g_3) = +\frac{11}{10} g_1^2 g_3^3 + \frac{9}{2} g_2^2 g_3^3 - 26 g_3^5 - 2 g_3^3 \left|y_t\right|^2 - 2 g_3^3 \left|y_b\right|^2$$

3.4 Yukawa couplings

$$\beta^{(1)}(y_t) = +\frac{9}{2}y_t\left|y_t\right|^2 + y_t\left|y_\tau\right|^2 + \frac{3}{2}y_t\left|y_b\right|^2 - \frac{17}{20}g_1^2 y_t - \frac{9}{4}g_2^2 y_t - 8g_3^2 y_t$$

$$\begin{aligned}\beta^{(2)}(y_t) = &-12y_t\left|y_t\right|^4 - \frac{1}{4}y_t\left|y_b\right|^4 - \frac{9}{4}y_t\left|y_\tau\right|^4 - \frac{11}{4}y_t\left|y_b\right|^2\left|y_t\right|^2 + \frac{5}{4}y_t\left|y_b\right|^2\left|y_\tau\right|^2 - \frac{9}{4}y_t\left|y_t\right|^2\left|y_\tau\right|^2\\
&-12\lambda y_t\left|y_t\right|^2 + 6\lambda^2 y_t + \frac{393}{80}g_1^2 y_t\left|y_t\right|^2 + \frac{225}{16}g_2^2 y_t\left|y_t\right|^2 + 36g_3^2 y_t\left|y_t\right|^2 + \frac{15}{8}g_1^2 y_t\left|y_\tau\right|^2\\
&+\frac{15}{8}g_2^2 y_t\left|y_\tau\right|^2 + \frac{7}{80}g_1^2 y_t\left|y_b\right|^2 + \frac{99}{16}g_2^2 y_t\left|y_b\right|^2 + 4g_3^2 y_t\left|y_b\right|^2 + \frac{1187}{600}g_1^4 y_t - \frac{9}{20}g_1^2 g_2^2 y_t\\
&+\frac{19}{15}g_1^2 g_3^2 y_t - \frac{23}{4}g_2^4 y_t + 9g_2^2 g_3^2 y_t - 108g_3^4 y_t\end{aligned}$$

$$\beta^{(1)}(y_b) = +\frac{3}{2}y_b\left|y_t\right|^2 + \frac{9}{2}y_b\left|y_b\right|^2 + y_b\left|y_\tau\right|^2 - \frac{1}{4}g_1^2 y_b - \frac{9}{4}g_2^2 y_b - 8g_3^2 y_b$$

$$\begin{aligned}\beta^{(2)}(y_b) = &-\frac{11}{4}y_b\left|y_b\right|^2\left|y_t\right|^2 - \frac{1}{4}y_b\left|y_t\right|^4 + \frac{5}{4}y_b\left|y_t\right|^2\left|y_\tau\right|^2 - 12y_b\left|y_b\right|^4 - \frac{9}{4}y_b\left|y_\tau\right|^4 - \frac{9}{4}y_b\left|y_b\right|^2\left|y_\tau\right|^2\\
&-12\lambda y_b\left|y_b\right|^2 + 6\lambda^2 y_b + \frac{91}{80}g_1^2 y_b\left|y_t\right|^2 + \frac{99}{16}g_2^2 y_b\left|y_t\right|^2 + 4g_3^2 y_b\left|y_t\right|^2 + \frac{237}{80}g_1^2 y_b\left|y_b\right|^2\\
&+\frac{225}{16}g_2^2 y_b\left|y_b\right|^2 + 36g_3^2 y_b\left|y_b\right|^2 + \frac{15}{8}g_1^2 y_b\left|y_\tau\right|^2 + \frac{15}{8}g_2^2 y_b\left|y_\tau\right|^2 - \frac{127}{600}g_1^4 y_b - \frac{27}{20}g_1^2 g_2^2 y_b\\
&+\frac{31}{15}g_1^2 g_3^2 y_b - \frac{23}{4}g_2^4 y_b + 9g_2^2 g_3^2 y_b - 108g_3^4 y_b\end{aligned}$$

$$\beta^{(1)}(y_\tau) = +3y_\tau\left|y_t\right|^2 + 3y_\tau\left|y_b\right|^2 + \frac{5}{2}y_\tau\left|y_\tau\right|^2 - \frac{9}{4}g_1^2 y_\tau - \frac{9}{4}g_2^2 y_\tau$$

$$\begin{aligned}\beta^{(2)}(y_\tau) = &-\frac{27}{4}y_\tau\left|y_t\right|^2\left|y_\tau\right|^2 + \frac{3}{2}y_\tau\left|y_b\right|^2\left|y_t\right|^2 - \frac{27}{4}y_\tau\left|y_b\right|^2\left|y_\tau\right|^2 - \frac{27}{4}y_\tau\left|y_t\right|^4 - \frac{27}{4}y_\tau\left|y_b\right|^4\\
&-3y_\tau\left|y_\tau\right|^4 - 12\lambda y_\tau\left|y_\tau\right|^2 + 6\lambda^2 y_\tau + \frac{17}{8}g_1^2 y_\tau\left|y_t\right|^2 + \frac{45}{8}g_2^2 y_\tau\left|y_t\right|^2 + 20g_3^2 y_\tau\left|y_t\right|^2\\
&+\frac{5}{8}g_1^2 y_\tau\left|y_b\right|^2 + \frac{45}{8}g_2^2 y_\tau\left|y_b\right|^2 + 20g_3^2 y_\tau\left|y_b\right|^2 + \frac{537}{80}g_1^2 y_\tau\left|y_\tau\right|^2 + \frac{165}{16}g_2^2 y_\tau\left|y_\tau\right|^2\\
&+\frac{1371}{200}g_1^4 y_\tau + \frac{27}{20}g_1^2 g_2^2 y_\tau - \frac{23}{4}g_2^4 y_\tau\end{aligned}$$

3.5 Quartic couplings

$$\beta^{(1)}(\lambda) = +24\lambda^2 - \frac{9}{5}g_1^2\lambda - 9g_2^2\lambda + \frac{27}{200}g_1^4 + \frac{9}{20}g_1^2 g_2^2 + \frac{9}{8}g_2^4 + 12\lambda\left|y_t\right|^2 + 12\lambda\left|y_b\right|^2 + 4\lambda\left|y_\tau\right|^2$$

$$- 6\left|y_t\right|^4 - 6\left|y_b\right|^4 - 2\left|y_\tau\right|^4$$

$$\begin{aligned}\beta^{(2)}(\lambda) = \; & -312\lambda^3 + \frac{108}{5}g_1^2\lambda^2 + 108g_2^2\lambda^2 + \frac{1887}{200}g_1^4\lambda + \frac{117}{20}g_1^2g_2^2\lambda - \frac{73}{8}g_2^4\lambda - \frac{3411}{2000}g_1^6 \\ & - \frac{1677}{400}g_1^4g_2^2 - \frac{289}{80}g_1^2g_2^4 + \frac{305}{16}g_2^6 - 144\lambda^2\left|y_t\right|^2 - 144\lambda^2\left|y_b\right|^2 - 48\lambda^2\left|y_\tau\right|^2 \\ & + \frac{17}{2}g_1^2\lambda\left|y_t\right|^2 + \frac{5}{2}g_1^2\lambda\left|y_b\right|^2 + \frac{15}{2}g_1^2\lambda\left|y_\tau\right|^2 + \frac{45}{2}g_2^2\lambda\left|y_t\right|^2 + \frac{45}{2}g_2^2\lambda\left|y_b\right|^2 + \frac{15}{2}g_2^2\lambda\left|y_\tau\right|^2 \\ & + 80g_3^2\lambda\left|y_t\right|^2 + 80g_3^2\lambda\left|y_b\right|^2 - \frac{171}{100}g_1^4\left|y_t\right|^2 + \frac{9}{20}g_1^4\left|y_b\right|^2 - \frac{9}{4}g_1^4\left|y_\tau\right|^2 + \frac{63}{10}g_1^2g_2^2\left|y_t\right|^2 \\ & + \frac{27}{10}g_1^2g_2^2\left|y_b\right|^2 + \frac{33}{10}g_1^2g_2^2\left|y_\tau\right|^2 - \frac{9}{4}g_2^4\left|y_t\right|^2 - \frac{9}{4}g_2^4\left|y_b\right|^2 - \frac{3}{4}g_2^4\left|y_\tau\right|^2 - 3\lambda\left|y_t\right|^4 \\ & - 42\lambda\left|y_b\right|^2\left|y_t\right|^2 - 3\lambda\left|y_b\right|^4 - \lambda\left|y_\tau\right|^4 - \frac{8}{5}g_1^2\left|y_t\right|^4 + \frac{4}{5}g_1^2\left|y_b\right|^4 - \frac{12}{5}g_1^2\left|y_\tau\right|^4 - 32g_3^2\left|y_t\right|^4 \\ & - 32g_3^2\left|y_b\right|^4 + 30\left|y_t\right|^6 - 6\left|y_b\right|^4\left|y_t\right|^2 - 6\left|y_b\right|^2\left|y_t\right|^4 + 30\left|y_b\right|^6 + 10\left|y_\tau\right|^6\end{aligned}$$

3.6 Scalar mass couplings

$$\beta^{(1)}(\mu) = -\frac{9}{10}g_1^2\mu - \frac{9}{2}g_2^2\mu + 12\lambda\mu + 6\mu\left|y_t\right|^2 + 6\mu\left|y_b\right|^2 + 2\mu\left|y_\tau\right|^2$$

$$\begin{aligned}\beta^{(2)}(\mu) = \; & +\frac{1671}{400}g_1^4\mu + \frac{9}{8}g_1^2g_2^2\mu - \frac{145}{16}g_2^4\mu + \frac{72}{5}g_1^2\lambda\mu + 72g_2^2\lambda\mu - 60\lambda^2\mu + \frac{17}{4}g_1^2\mu\left|y_t\right|^2 \\ & + \frac{5}{4}g_1^2\mu\left|y_b\right|^2 + \frac{15}{4}g_1^2\mu\left|y_\tau\right|^2 + \frac{45}{4}g_2^2\mu\left|y_t\right|^2 + \frac{45}{4}g_2^2\mu\left|y_b\right|^2 + \frac{15}{4}g_2^2\mu\left|y_\tau\right|^2 + 40g_3^2\mu\left|y_t\right|^2 \\ & + 40g_3^2\mu\left|y_b\right|^2 - 72\lambda\mu\left|y_t\right|^2 - 72\lambda\mu\left|y_b\right|^2 - 24\lambda\mu\left|y_\tau\right|^2 - \frac{27}{2}\mu\left|y_t\right|^4 - 21\mu\left|y_b\right|^2\left|y_t\right|^2 \\ & - \frac{27}{2}\mu\left|y_b\right|^4 - \frac{9}{2}\mu\left|y_\tau\right|^4\end{aligned}$$

3.7 Vacuum-expectation values

Definitions:

$$H \;:\; \frac{1}{\sqrt{2}}\Pi_2 \to \frac{1}{\sqrt{2}}\left(\Pi_2 + v\right)$$

Gauge fixing:

$$\xi \to 1$$

RGEs:

$$\beta^{(1)}(v) = +\frac{3}{5}g_1^2 v + 3g_2^2 v - 3v\left|y_t\right|^2 - 3v\left|y_b\right|^2 - v\left|y_\tau\right|^2$$

$$\begin{aligned}\beta^{(2)}(v) = & -\frac{1221}{800}g_1^4 v + \frac{9}{16}g_1^2 g_2^2 v + \frac{199}{32}g_2^4 v - \frac{103}{40}g_1^2 v\left|y_t\right|^2 - \frac{43}{40}g_1^2 v\left|y_b\right|^2 - \frac{81}{40}g_1^2 v\left|y_\tau\right|^2 \\ & - \frac{63}{8}g_2^2 v\left|y_t\right|^2 - \frac{63}{8}g_2^2 v\left|y_b\right|^2 - \frac{21}{8}g_2^2 v\left|y_\tau\right|^2 - 20g_3^2 v\left|y_t\right|^2 - 20g_3^2 v\left|y_b\right|^2 + \frac{27}{4}v\left|y_t\right|^4 \\ & - \frac{3}{2}v\left|y_b\right|^2\left|y_t\right|^2 + \frac{27}{4}v\left|y_b\right|^4 + \frac{9}{4}v\left|y_\tau\right|^4 - 6\lambda^2 v\end{aligned}$$

A Group theoretical information

A.1 Gauge groups

Group	Lie algebra	Dim.	Rank	Representations		
				Name / Dim.	Dynkin labels	Type
SU2	A1	3	1	$\mathbf{2}$	[1]	Pseudo-real
				$\overline{\mathbf{2}}$	[1, True]	Pseudo-real
SU3	A2	8	2	$\mathbf{3}$	[1, 0]	Complex
				$\overline{\mathbf{3}}$	[0, 1]	Complex