46770 Integrated energy grids

Lecture 5 – Optimal Power Flow

Types of optimization problems and course structure

	One node	Network			
		Power			
One time step	Economic dispatch or One-node dispatch optimization (Lecture 2)	Linearized AC power flow (Lecture 4)	AC power flow (Lecture 5)	Gas flow (Lecture 6)	Heat flow (Lecture 7)
Multiple time steps	Multi-period optimization Join capacity and dispatch optimization in one node (Lecture 8)	Join capacity and dispatch optimization in a network (Lecture 10)			

Recap from previous lectures

Generic loads

The **Impedance** represents the opposition to current in a line. It includes the resistance r associated to resistive loads and the reactance x associated the capacitive and inductive loads

$$z = r + jx$$

impedance **resistance** **reactance**

The **Admittance** is the inverse impedance, and it represents how easily a line allows current to flow. It includes the conductance g and the susceptance b

$$y = g + jb$$

admittance **conductance** **susceptance**

$$y = \frac{1}{z}$$

We can now write Ohm’s law as

$$\bar{I}_{i\to j} = \bar{Y}_{ij}\left(\bar{V}_i - \bar{V}_j\right)$$

The bar above a variable denotes complex number

AC transmission lines

Transmission lines are a combination of resistive, capacitive, and inductive loads

Simple series admittance model

$\bar{V}_i$ $\quad \bar{I}_{i\to j}$ $\quad y_{ij} = g_{ij} + jb_{ij} = \dfrac{1}{r_{ij} + jx_{ij}}$ $\quad \bar{I}_{j\to i}$ $\quad \bar{V}_j$

$$\bar{S}_{i\to j} = \bar{I}^*_{i\to j}\bar{V}_i = \bar{Y}^*_{ij}\left(\bar{V}_i^* - \bar{V}_j^*\right)\bar{V}_i = \bar{Y}^*_{ij}\bar{V}_i^*\bar{V}_i - \bar{Y}^*_{ij}\bar{V}_j^*\bar{V}_i$$

Assumptions to linearize AC power flow (I)

Complex power flowing through a line $\bar{S}_{i\to j} = p_{i\to j} + \mathrm{j}q_{i\to j}$

$$p_{i\to j} = g_{ij}|\cancelto{1}{V_i}|^2 - |\cancelto{1}{V_i}||V_j|\,[g_{ij}\cos(\cancelto{1}{\theta_i - \theta_j}) + b_{ij}\underbrace{\sin(\theta_i - \theta_j)}_{\theta_i - \theta_j}]$$

$$q_{i\to j} = -b_{ij}|\cancelto{1}{V_i}|^2 - |\cancelto{1}{V_i}||V_j|\,[g_{ij}\underbrace{\sin(\theta_i - \theta_j)}_{\theta_i - \theta_j} - b_{ij}\cos(\cancelto{1}{\theta_i - \theta_j})]$$

$$p_{i\to j} = g_{ij} - [\,g_{ij} + b_{ij}(\theta_i - \theta_j)] = -b_{ij}(\theta_i - \theta_j)$$

$$q_{i\to j} = -b_{ij} - [g_{ij}(\theta_i - \theta_j) - b_{ij}] = -\cancelto{0}{g_{ij}}(\theta_i - \theta_j)$$

$$p_{i\to j} = -b_{ij}(\theta_i - \theta_j)$$

$$\boxed{p_{i\to j} = \frac{\theta_i - \theta_j}{x_{ij}}}$$

1º. Unitary voltage values close to 1

2º. Voltage angle differences are small

$$\sin(\theta_i - \theta_j) \approx (\theta_i - \theta_j)$$

$$cos(\theta_i - \theta_j) \approx 1$$

3º. Conductances g_{ij} are negligible relative to susceptances b_{ij} (resistance are much smaller than reactance)
$(r_{ij} \ll x_{ij} \quad g_{ij} \ll b_{ij} \quad b_{ij} = \frac{-1}{x_{ij}})$.

This implies that power losses are neglected. This also implies that reactive power flow $q_{i\to j}$ is neglected

Analogy between DC and linearized AC power flow

The equation that relates the active power flow and the voltage angles is analogous to Ohm's law. This is the reason that justifies using the term "**DC approximation or DC power flow**" to name the linearized AC power flow.

$$I_l = \frac{V_i - V_j}{R_l} \qquad\qquad p_l = \frac{\theta_i - \theta_j}{x_l}$$

DC CIRCUITS	LINEARIZED AC POWER FLOW
Current flow I_l	Active power flow p_l
Voltage V_i	Voltage angle θ_i
Resistance R_l	Reactance x_l

Economic dispatch with linearized AC optimal power flow

Assume we have a network of nodes i. In every node, we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity $G_{s,i}$ and a linear variable cost o_S. Determine the optimal economic dispatch (how much energy is being produced by each of them) to supply the demand d_i in a certain hour and the optimal AC power flow while minimizing the total system cost.

Economic dispatch with linearized AC power flow

$$\min_{g_{s,i}} \sum_{s,i} o_s g_{s,i}$$

subject to:

$$\sum_{s,i} g_{s,i} - d_i = p_i = \sum_{l} K_{il} p_l$$

Nodal power balance

$$0 \leq g_{s,i} \leq G_{s,i}$$

$$|p_l| \leq P_l$$

Lines capacities

$$p_l = \frac{\theta_{i-}\theta_j}{x_l}$$

Physical relations in the links

Modelling approaches for power flow in AC networks

Net Transfer Capacities	Linearized AC power flow (DC Power flow)	AC Power flow
$\sum_s g_{s,i} - d_i = \sum_l K_{il} p_l$ $\lvert p_l \rvert \leq P_l$	$\sum_s g_{s,i} - d_i = p_i = \sum_l K_{il} p_l$ $\lvert p_l \rvert \leq P_l$ $p_l = \frac{\theta_{i-}\theta_j}{x_l}$ (alternative to last constraint $\sum_l C_{lc}\, x_l\, p_l = 0$)	
	✓ Linear ✓ Optimality guaranteed ✓ Computational tractable ✓ Good enough for long-term planning	
× No unique solution × No representation of power losses	× Not guarantee feasible power flows × No representation of power losses × Not good enough for fast changes, restart from blackout, network splitting, etc.	

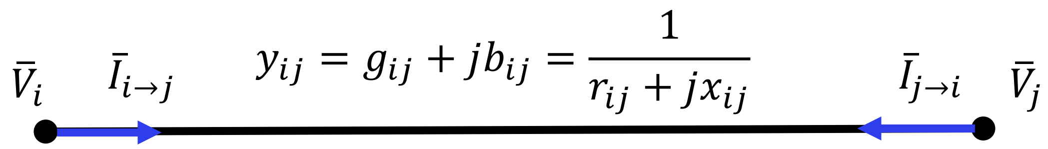
Learning goals

- Write the system cost minimization problem including AC optimal power flow
- Describe and apply the Newton-Raphson method to solve nonlinear AC power flows
- Formulate the optimal power flow problem on a computer.
- Describe approaches to convexify the AC optimal power flow
- Explain the differences between linear approximations and convex relaxations
- Define contingency and explain the N-1 security criterion

For a comprehensive discussion on the setting the equations for AC power flows, calculating the Jacobian matrix, and Newton-Raphson technique for solving AC power flows, check Lecture 7 in DTU course 46700 Introduction to Electric Power Systems

Complex power through a line (I)

The complex power flowing through a line is equal to the product of the voltage difference and the conjugate of the current (both complex numbers)



$$\bar{S}_{i\to j} = \bar{I}^*_{i\to j}\bar{V}_i = \bar{Y}^*_{ij}\left(\bar{V}_i^* - \bar{V}_j^*\right)\bar{V}_i = \bar{Y}^*_{ij}\bar{V}_i^*\bar{V}_i - \bar{Y}^*_{ij}\bar{V}_j^*\bar{V}_i$$

$$\boxed{\bar{S}_{i\to j} = p_{i\to j} + \mathrm{j}q_{i\to j}}$$

$$p_{i\to j} = g_{ij}|V_i|^2 - |V_i||V_j|\,[g_{ij}\cos(\theta_i - \theta_j) + b_{ij}\sin(\theta_i - \theta_j)]$$

$$q_{i\to j} = -b_{ij}|V_i|^2 - |V_i||V_j|\,[g_{ij}\sin(\theta_i - \theta_j) - b_{ij}\cos(\theta_i - \theta_j)]$$

Under certain assumptions we can linearized these equations (what we did in the lecture last week)

We can also use them directly.

Economic dispatch with AC optimal power flow

Economic dispatch with AC power flow

$$\min_{g_{s,i}} \sum_{s,i} o_s g_{s,i}$$

subject to:

$$\sum_{s,i} \bar{g}_{s,i} - \bar{d}_i = \bar{S}_i = \sum_{l} K_{il} \bar{S}_{l(i \to j)}$$

$$0 \le g_s \le G_s$$

$$\left|\bar{S}_{i \to j}\right|^2 \le \bar{S}^2_{i \to j,max}$$

$$\bar{S}_{i \to j} = \bar{Y}^*_{ij} \bar{V}^*_i \bar{V}_i - \bar{Y}^*_{ij} \bar{V}^*_j \bar{V}_i$$

Nodal balance equations

Capacities limits

These equations include complex numbers which entail constrains for both the real and imaginary parts

The bar above a variable denotes complex number

Economic dispatch with AC optimal power flow

Economic dispatch with AC power flow

$$\min_{g_{s,i}} \sum_{s,i} o_s g_{s,i}$$

subject to:

$$\sum_{s,i} g_{s,i}^{p} - d_i^{p} = p_i = \sum_{l} K_{il} p_{l(i\to j)}$$

$$\sum_{s,i} g_{s,i}^{q} - d_i^{q} = q_i = \sum_{l} K_{il} q_{l(i\to j)}$$

$$0 \leq g_s \leq G_s$$

$$\left|p_{i\to j}^2 + q_{i\to j}^2\right|^2 \leq \bar{S}_{i\to j,max}^2$$

$$0 \leq p_s \leq P_s$$

$$-Q_s \leq q_s \leq Q_s$$

The optimization problem is not linear, and the unknown variables are: V_i, θ_i, $p_{i\to j}$, $q_{i\to j}$, $g_{s,i}$

Complex power flowing through a line $\bar{S}_{i\to j} = p_{i\to j} + \mathrm{j}q_{i\to j}$

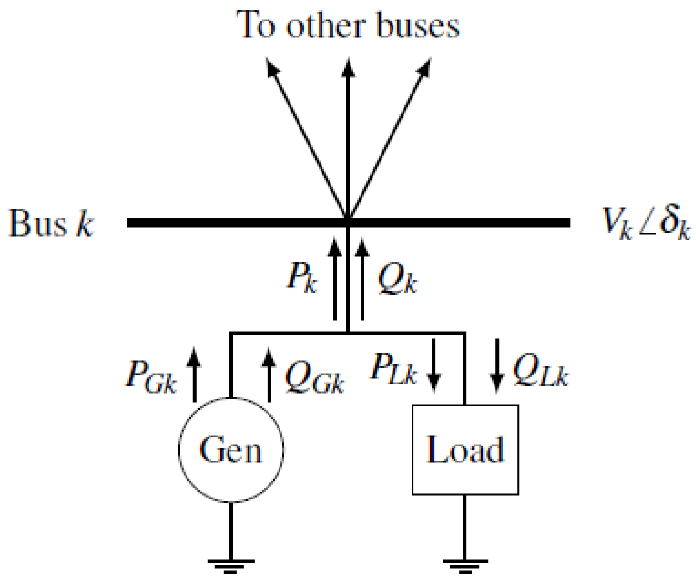
$$p_{i\to j} = g_{ij}|V_i|^2 - |V_i||V_j|\,[g_{ij}\cos(\theta_i - \theta_j) + b_{ij}\sin(\theta_i - \theta_j)]$$

$$q_{i\to j} = -b_{ij}|V_i|^2 - |V_i||V_j|\,[g_{ij}\sin(\theta_i - \theta_j) - b_{ij}\cos(\theta_i - \theta_j)]$$

(Side note: this was already presented in previous courses)

This is the same formulation that you learnt in Lecture 7 of DTU course 46700 Introduction to Power Systems

We have a set of nonlinear equations that relate the power flow and the injection in the nodes



Every bus has for variables: net real power $P_{Gi} - P_{Li}$, net reactive power $Q_{Gi} - Q_{Li}$, voltage magnitude V_i, and phase angle θ_i

- **Load (PQ) bus**: $P_{Gi} - P_{Li}$ and $Q_{Gi} - Q_{Li}$ given
- **Voltage controlled (PV) bus**: : $P_{Gi} - P_{Li}$ and V_i
- **Slack bus**: V_i and θ_i given

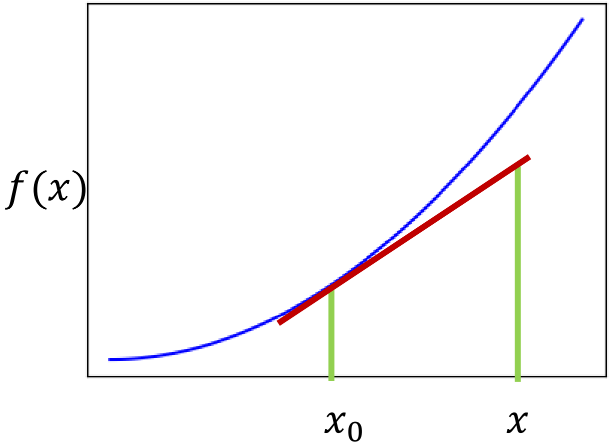
Modelling approaches for power flow in AC networks

Net Transfer Capacities	Linearized AC power flow (DC Power flow)	AC Power flow
$\sum_s g_{s,i} - d_i = \sum_l K_{il} p_l$ $\lvert p_l \rvert \leq P_l$	$\sum_s g_{s,i} - d_i = p_i = \sum_l K_{il} p_l$ $\lvert p_l \rvert \leq P_l$ $p_l = \frac{\theta_i - \theta_j}{x_l}$ (alternative to last constraint $\sum_l C_{lc}\, x_l\, p_l = 0$)	$\sum_{s,i} g_{s,i} - d_i = \bar{S}_i = \sum_l K_{il} \bar{S}_{l(i\to j)}$ $\lvert \bar{S}_{i\to j} \rvert^2 \leq \bar{S}^2_{i\to j,max}$ $\bar{S}_{i\to j} = \bar{Y}^*_{ij} \bar{V}^*_i \bar{V}_i - \bar{Y}^*_{ij} \bar{V}^*_j \bar{V}_i$
	✓ Linear ✓ Optimality guaranteed ✓ Computational tractable ✓ Good enough for long-term planning	✓ Feasible AC power flows
× No unique solution × No representation of power losses	× Not guarantee feasible power flows × No representation of power losses × Not good enough for fast changes, restart from blackout, network splitting, etc.	× Non-linear × No optimality guaranteed × High computational complexity

Newton-Raphson method

Newton-Raphson method (I)

Assume we have a set of nonlinear equations, each of them $y = f(x)$



We do a Taylor series expansion

$$y = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0} (x - x_0) + (higher\ order\ terms)$$

0

We neglect higher order terms and solve for x:

$$x = x_0 + \left[\left.\frac{\partial f}{\partial x}\right|_{x=x_0}\right]^{-1} [y - f(x_0)]$$

The Newton-Raphson method replaces x by old values $x(i)$ and x by new values $x(i+1)$:

$$x(i+1) = x(i) + \left[\left.\frac{\partial f}{\partial x}\right|_{x=x(i)}\right]^{-1} [y - f(x(i))]$$

These slides are reproduced from Lecture 7 in DTU course 46700 Introduction to Power Systems, prepared by Tilman Weckesser

Newton-Raphson method (II)

The Jacobian matrix collects all the partial derivatives

$$J(i) = \left.\frac{\partial f}{\partial x}\right|_{x=x(i)} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots \\ \dots & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}_{x=x(i)}$$

$$x(i+1) = x(i) + \underbrace{\left[\left.\frac{\partial f}{\partial x}\right|_{x=x(i)}\right]^{-1}}_{J^{-1}(i)} [y - f(x(i))]$$

Newton-Raphson method (III)

Steps to compute solution

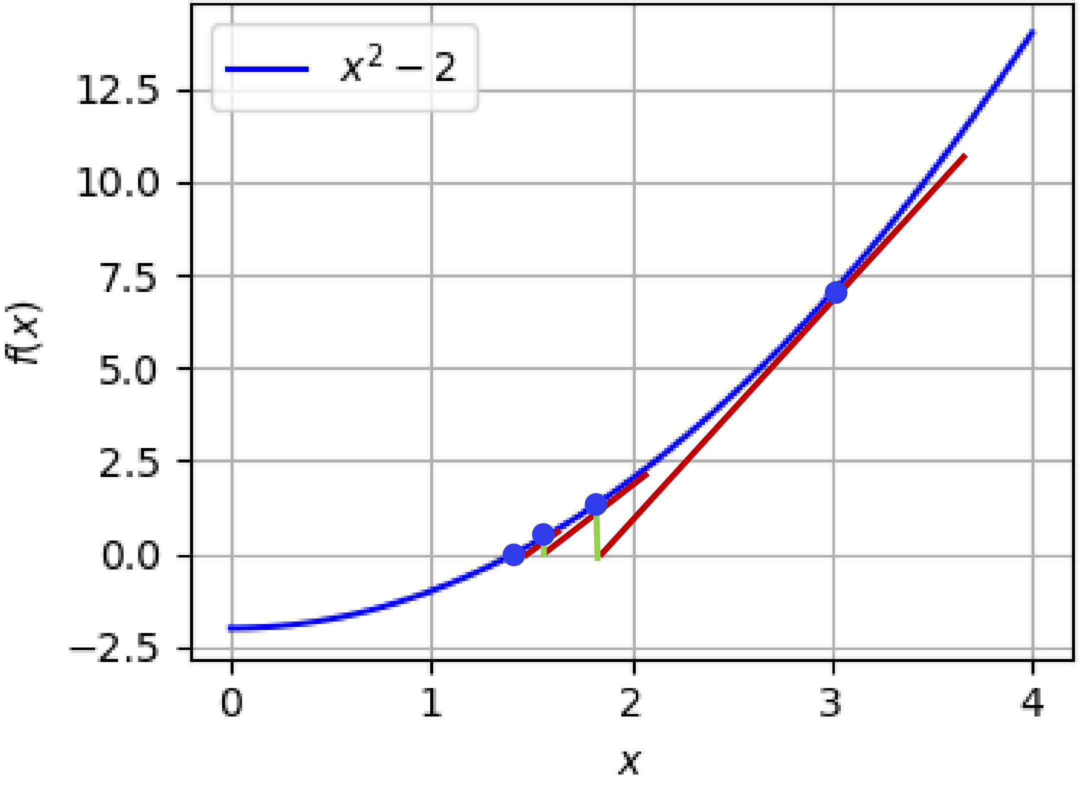
1. Select an initial guess for $x(i)$
2. Compute $y - f(x(i))$ and $J(i)$
3. Solve $J(i)[x(i+1) - x(i)] = [y - f(x(i))]$
4. Compute $x(i+1)$
5. Check for convergence $\left|\frac{x(i+1)-x(i)}{x(i)}\right| < \varepsilon$

Example: $y = f(x) = x^2 - 2 = 0$

$$x(i+1) = x(i) + \left[\left.\frac{\partial f}{\partial x}\right|_{x=x(i)}\right]^{-1} [y - f(x(i))]$$

$$x(i+1) = x(i) + \frac{1}{2x(i)} [0 - (x(i)^2 - 2)]$$

$\boldsymbol{i}$	$\boldsymbol{x(i)}$	$\boldsymbol{J(i)}$	$\boldsymbol{y - f(x(i))}$	$\boldsymbol{\Delta x(i)}$
0	3	0.167	-7	-1.167
1	1.833	0.272	-1.361	-0.371
2	1.462	0.342	-0.138	-0.047
3	1.415	0.353	-0.002	-0.001



Newton-Raphson method (IV)

When we apply the Newton-Raphson method to the AC power flow equations, the y values are the active p_i and reactive power q_i, the x values are the angles θ_i and voltage magnitudes V_i, and the functions $f(x)$ are the AC power flow equations

$$y = \begin{bmatrix} p_{i\to j} \\ q_{i\to j} \end{bmatrix} \qquad x = \begin{bmatrix} \theta_i \\ V_i \end{bmatrix} \qquad f(x) = \begin{bmatrix} p_{i\to j}(\theta_i, V_i) \\ q_{i\to j}(\theta_i, V_i) \end{bmatrix}$$

$$\sum_{s,i} g_{s,i}^p - d_i^p = p_i = \sum_l K_{il} p_{l(i\to j)} \qquad p_{i\to j} = g_{ij}|V_i|^2 - |V_i||V_j| \left[g_{ij}\cos(\theta_i - \theta_j) + b_{ij}\sin(\theta_i - \theta_j)\right]$$

$$\sum_{s,i} g_{s,i}^q - d_i^q = q_i = \sum_l K_{il} q_{l(i\to j)} \qquad q_{i\to j} = -b_{ij}|V_i|^2 - |V_i||V_j| \left[g_{ij}\sin(\theta_i - \theta_j) - b_{ij}\cos(\theta_i - \theta_j)\right]$$

Convex relaxation of AC power flows

Convex relaxation of AC optimal power flow

Economic dispatch with AC power flow

$$\begin{cases} \min_{g_{s,i}} \sum_{s,i} o_s g_{s,i} \\ \\ \text{subject to:} \\ \sum_{s,i} \bar{g}_{s,i} - \bar{d}_i = \bar{S}_i = \sum_{l} K_{il} \bar{S}_{l(i \to j)} \\ \bar{S}_{i \to j} = \bar{Y}_{ij}^* \bar{V}_i^* \bar{V}_i - \bar{Y}_{ij}^* \boxed{\bar{V}_j^* \bar{V}_i} \\ 0 \le g_s \le G_s \\ \left| \bar{S}_{i \to j} \right|^2 \le \bar{S}_{i \to j,max}^2 \end{cases}$$

The intuition to relax the optimal power flow comes from the realization that the non-linearities comes from the term $\bar{V}_j^* \bar{V}_i$

This example is based on the presentation by Coffrin and Roald (2018)

Convex relaxation of AC optimal power flow

We define the auxiliary variable $\bar{W}_{ij} = \bar{V}_j^* \bar{V}_i$ and relax the equality constraint into an inequality constraint to transform the non-convex constraint into a convex constraint.

$$\bar{W}_{ij} = \bar{V}_j^* \bar{V}_i$$

$$(\bar{V}_j^* \bar{V}_i)(\bar{V}_i^* \bar{V}_j) = (\bar{V}_j^* \bar{V}_i)(\bar{V}_i^* \bar{V}_j)$$

$$\left|\bar{V}_j^* \bar{V}_i\right|^2 = (\bar{V}_i^* \bar{V}_i)(\bar{V}_j^* \bar{V}_j)$$

$$\left|\bar{W}_{ij}\right|^2 = (\bar{W}_{ii})\left(\bar{W}_{jj}\right)$$

$$\left|\bar{W}_{ij}\right|^2 \leq (\bar{W}_{ii})(\bar{W}_{jj})$$

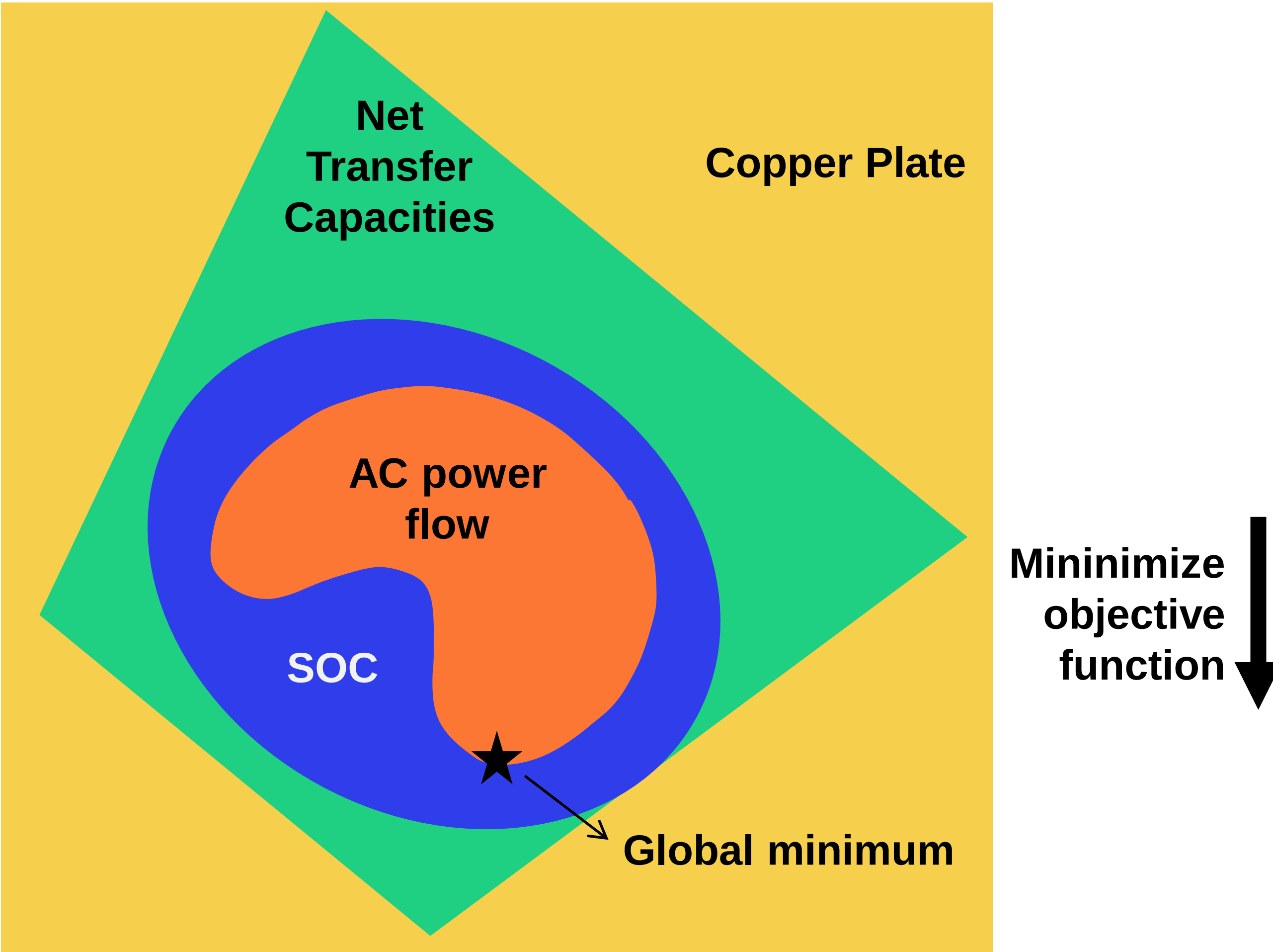
This approach is known as Second Order Program (SOC)

This example is based on the presentation by Coffrin and Roald (2018)

Convex relaxation of AC Optimal Power flow

Convex **Relaxation** of AC power flow is an **extension** of the feasible space which contains all feasible points and therefore also the true optimal solution.

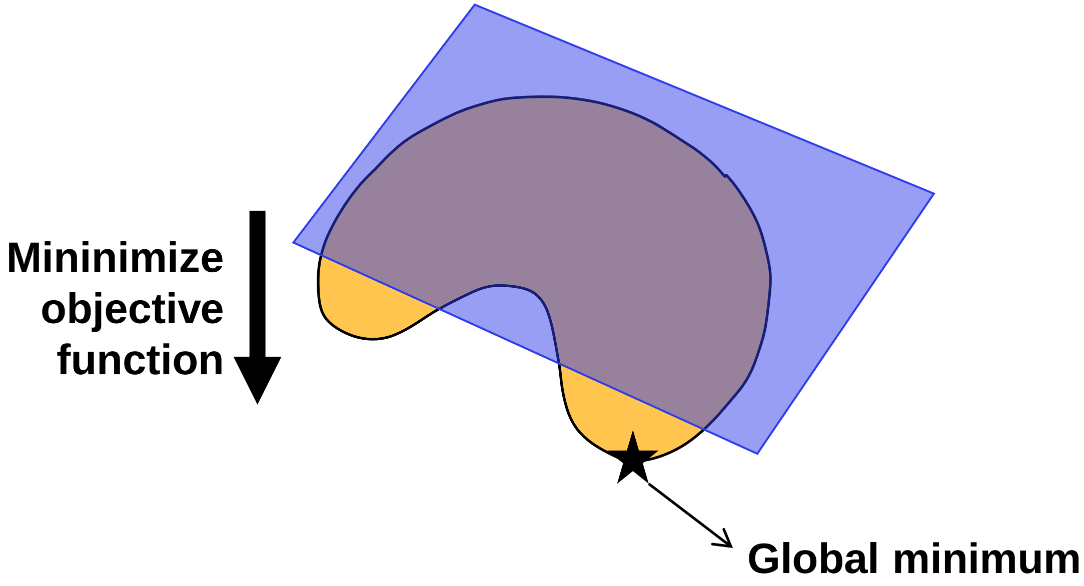
A relaxation can be convex or non-convex, linear or non-linear, relaxations are not unique



Limitations of the Linearized AC Optimal Power flow

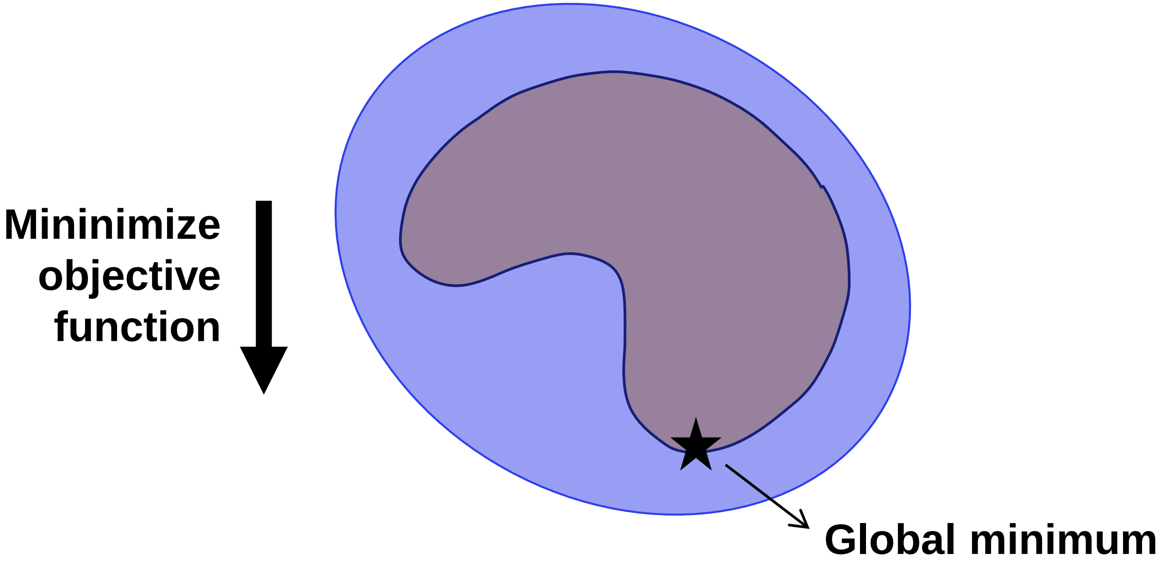
Linearization of AC power flow is an **approximation** of the feasible space which:

- may contain points that are not feasible
- may not contain some feasible points
- may not include original optimal solution



Convex **Relaxation** of AC power flow is an **extension** of the feasible space which:

- contains the true optimal solution
- may contain points that are not feasible
- contains all feasible points and



Convex relaxation to include losses in linearized AC power flows

Losses in AC power flow (I)

We have written the active power flow in a line as

$$p_{i\to j} = g_{ij}|V_i|^2 - |V_i||V_j|\,[g_{ij}\cos(\theta_i-\theta_j) + b_{ij}\sin(\theta_i-\theta_j)]$$

Following the convention that departing flows are positive and arriving flows are negative, the losses are the difference between power sent from i to j and power received from j to i.

$$\Psi_l = p_{i\to j} + p_{j\to i} = g_{ij}|V_i|^2 - |V_i||V_j|\,[g_{ij}\cos(\theta_i-\theta_j) + b_{ij}\sin(\theta_i-\theta_j)]$$
$$+g_{ij}|V_j|^2 - |V_j||V_i|\,[g_{ij}\cos(\theta_j-\theta_i) + b_{ij}\sin(\theta_j-\theta_i)]$$

$$\cos(-a) = \cos(a)$$
$$\sin(-a) = -\sin(a)$$

$$\Psi_l = p_{i\to j} + p_{j\to i} = g_{ij}(|V_i|^2 + |V_j|^2) - 2|V_i||V_j|\,g_{ij}\cos(\theta_i-\theta_j)$$

This approach is based on Neuman et al., Applied Energy (2022)

Losses in AC power flow (II)

$$\Psi_l = p_{i\to j} + p_{j\to i} = g_{ij}(|V_i|^2 + \left|V_j\right|^2) - 2|V_i|\left|V_j\right| g_{ij}\cos\left(\theta_i - \theta_j\right)$$

Assuming close to nominal per-unit voltage magnitudes $|V_i| \sim 1$

$$\Psi_l = 2g_{ij}(1 - 2\cos\left(\theta_i - \theta_j\right))$$

$$p_l = \frac{\theta_i - \theta_j}{x_l} \qquad \Psi_l = 2g_{ij}(1 - \cos(p_l x_l))$$

Losses in AC power flow (III)

Approximating $\cos(a) \approx 1 - \frac{a^2}{2}$

$$\Psi_l = 2g_{ij}(1 - \cos(p_l x_l)) = 2g_{ij}\left(1 - 1 + \frac{p_l^2 x_l^2}{2}\right) = g_{ij} p_l^2 x_l^2$$

By inserting $g_{ij} \approx \frac{r_{ij}}{x_{ij}^2}$

$$\boxed{\Psi_l = r_{ij} p_l^2}$$

This is still a quadratic equality constraint that leads to a quadratically constrained quadratic program (QCQP)

We can make it convex by building a convex envelope around the constraint.

This way we keep the problem linear while including transmission losses.

Demonstration $g \approx \frac{r}{x^2}$

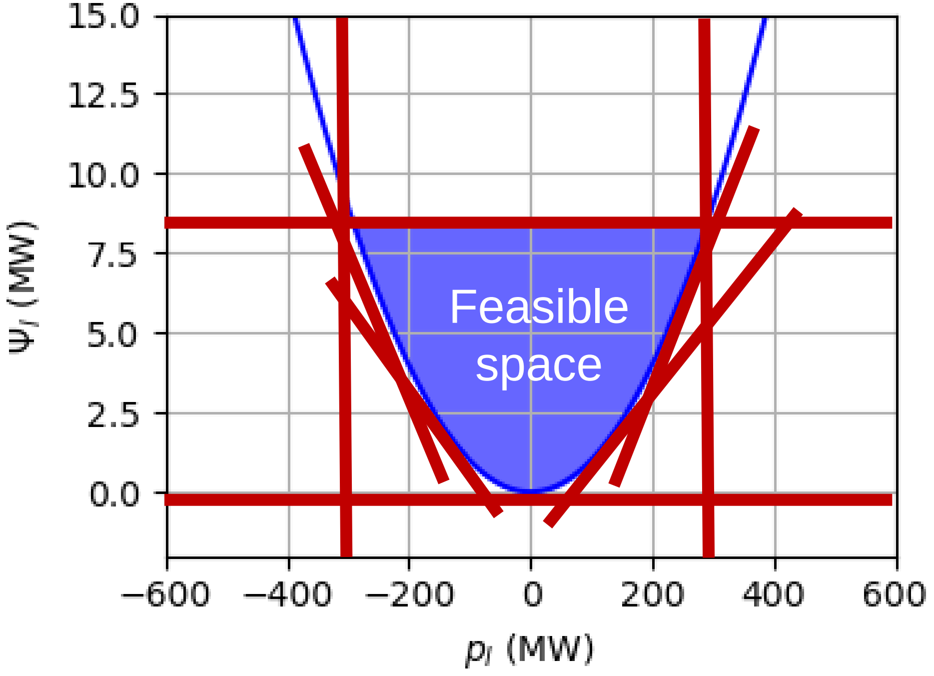
$$y = g + jb = \frac{1}{r+jx} = \frac{r-jx}{(r+jx)(r-jx)} = \frac{r-jx}{r^2+x^2}$$

$$g = Re[\frac{r-jx}{r^2+x^2}] = \frac{r}{r^2+x^2}$$

$$b = Im[\frac{r-jx}{r^2+x^2}] = \frac{-x}{r^2+x^2}$$

For $r \ll x$ $\quad r = \frac{1}{x^2}$

Losses in AC power flow (IV)



We can make it convex by building a convex envelope around the constraint.

$$\boxed{\Psi_l = r_{ij} p_l^2}$$

substituted by

$$\Psi_l \geq 0$$

$$p_l \leq P_l \quad \rightarrow \quad \Psi_l \leq r_{ij} P_l^2$$

$$\Psi_l \geq m_k \cdot p_l + a_k \qquad \forall k = 1 \ldots . n$$
$$\Psi_l \leq -m_k \cdot p_l + a_k \qquad \forall k = 1 \ldots . n$$

Set of linear equations that represent a convex envelope of the nonlinear constraint

Economic dispatch with linearized AC optimal power flow and losses

Economic dispatch with linearized AC power flow

$$\min_{g_{s,i}} \sum_{s,i} o_s g_{s,i}$$

Nodal power balance

subject to:

$$\sum_{s,i} g_{s,i} - d_i = p_i = \sum_{l} K_{il} p_l + \frac{|K_{il}|}{2} \Psi_l$$

Losses must be accounted for in the nodal balance

$$0 \leq g_{s,i} \leq G_{s,i}$$

$$|p_l| \leq P_l - \Psi_l$$

$$p_l = \frac{\theta_{i-}\theta_j}{x_l}$$

Lines capacities

Links equations

Losses reduce the transmission capacity of a line

Power system resilience and N-1 criterion

Contingency and N-1 criterion

Power system resilience is the ability of the system to withstand major disruption (caused by uncontrolled events or unanticipated loss of system components) with acceptable degradation of power delivery to consumers and to recover within an acceptable time.

Contingency: an incident that deviate from the planned operation and can affect the security of the power system (e.g. line or loads outages, deviations of the power generation due to solar or wind forecast errors)

N-1 security criterion:

Power system must remain secure in the event of a component outage that leads to a system operation with N-1 components.

An **approximation for the N-1 security criterion** when using linearized AC power flow consists **in limiting the power that can flow through every line to 70% of its thermal rating**. This also allows room for reactive power.

Power flow vs optimal power flow

Power flow vs Optimal Power Flow

	Power flow analysis	Optimal power flow
Unknowns	$V_i,\ \theta_i,\ p_{i\to j},\ q_{i\to j}$	$V_i,\ \theta_i,\ p_{i\to j},\ q_{i\to j},\ g_{s,i}$
Objective	• Find the flows in the links of a network given the injection pattern for the nodes. • Determines voltage magnitudes and angles at each bus • Computes real and reactive power flows for lines connecting buses, as well as lines losses	• Find the optimal flows in the links of a network that minimizes the total system costs (while supplying demand in every node)
Applications	• Feasibility analysis: for dispatch (obtained in market clearing), identify grid overloading) • Security analysis: simulate outages to identify possible grid overloading	• Find optimal dispatch that minimizes system cost and ensures power flow feasibility • Find optimal capacities to avoid bottlenecks (long-term planning) • Find optimal redispatch after an outage to restore feasibility

Additional materials: Short videos

Carleton Coffrin, Line Roald, 2018 Convex Relaxations in Power System Optimization,

A series of 8 short videos

https://www.youtube.com/watch?v=gB43TmcoUpA

Problems for this lecture

Review tutorial on PyPSA

https://martavp.github.io/integrated-energy-grids/intro-pypsa.html

PyPSA uses Linopy and NetworkX

Problems 5.1, 5.2 (**Group 10**)

Problems 5.3, 5.4 (**Group 11**)

Not part of this course, but for those students who wants to learn more:

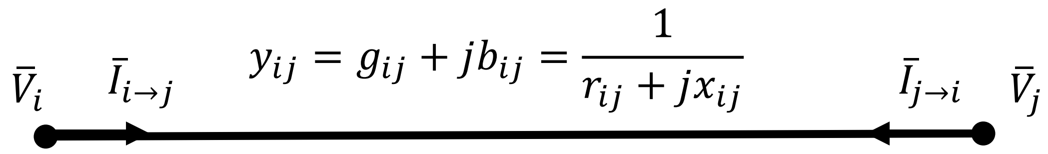
Dispatch problem with German network

EXTRA MATERIALS

π model of transmission lines

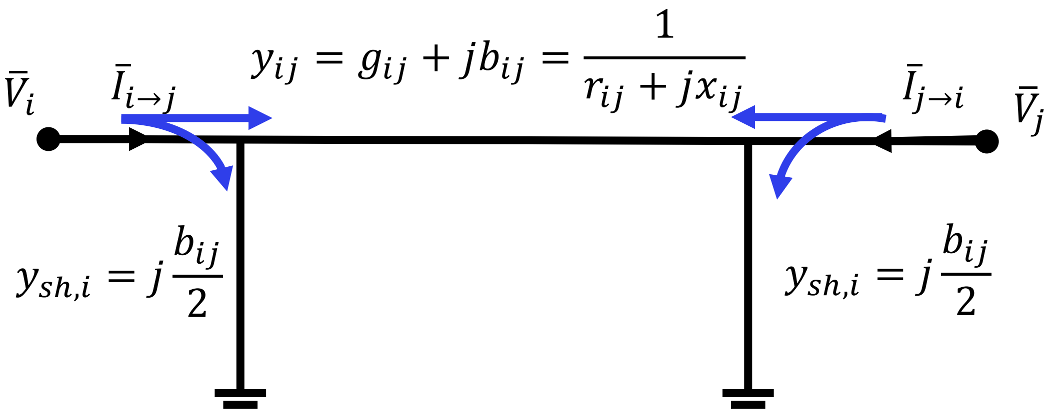
Modeling AC transmission lines

Simple series admittance model



$$\bar{S}_{i\to j} = \bar{I}^*_{i\to j}\bar{V}_i = \bar{Y}^*_{ij}\left(\bar{V}^*_i - \bar{V}^*_j\right)\bar{V}_i = \bar{Y}^*_{ij}\bar{V}^*_i\bar{V}_i - \bar{Y}^*_{ij}\bar{V}^*_j\bar{V}_i$$

π model



The current entering node i flows along the series admittance y_{ij} but some flows along the shunt susceptance $y_{sh,i}$

$$\bar{S}_{i\to j} = \bar{I}^*_{i\to j}\bar{V}_i + \bar{I}^*_{sh,i}\bar{V}_i = \bar{Y}^*_{ij}\left(\bar{V}^*_i - \bar{V}^*_j\right)\bar{V}_i + \bar{Y}^*_{sh,i}\bar{V}^*_i\bar{V}_i$$
$$= (\bar{Y}^*_{ij} + \bar{Y}^*_{sh,i})\bar{V}^*_i\,\bar{V}_i - \bar{Y}^*_{ij}\bar{V}^*_j\bar{V}_i$$