# Lecture 4: DC Optimal Power Flow

DTU Course 46770: Integrated Energy Grids

February 2025

**Problem 4.1.** This is a continuation of Problem 3.2 from Lecture 3.

Let us assume now that we are in an hour with an excess of wind generation in Denmark and a deficit in other countries so that the power injection $p_i$ of the different countries is as follows:

Germany= - 2 MW, DK1=5 MW, DK2=6 MW, Norway = - 8 MW, Sweden = -1 MW,

Determine the voltage angles $\theta_i$ and the flows $p_l$ in the lines of the network. Assume that $\theta_0$=0; i.e. the reference bus is at node 0 (Germany); and the reactance in all links is $x_l$=1.

*Hint 1: Remember the relations between power injection $p_i$ in every node and power flows $p_l$ presented in Lecture 4.*

$p_i = \sum_j L_{ij}\theta_j$

$p_l = \frac{1}{x_l}\sum_j K_{lj}\theta_j$

$L_{ij} = \sum_l K_{il}\frac{1}{x_l}K_{lj}$

Hint 2: you can use numpy.linalg.solve to solve the linear equation system.

**Problem 4.2.** This is a continuation of Problem 3.2 from Lecture 3.

a) Assuming that the reactance in the links is $x_l$=1, calculate the Power Transfer Distribution Factor (PTDF) matrix.

b) Assuming the power injection pattern described in Problem 4.1 determine the flows in the lines of the network.

**Problem 4.3.** This is a continuation of Problem 3.3 from Lecture 3. Using the Python package networkX.

a) Assuming that the reactance in the links is $x_l$=1, calculate the Power Transfer Distribution Factor (PTDF) matrix.

b) Assuming the power injection pattern described in Problem 4.1 determine the power flows in the lines of the network and plot them.

c) Assume now that the links unitary susceptance is $x_l$=[1, 0.5, 0.5, 0.5, 1], calculate the weighted Laplacian (or susceptance matrix), the PTDF matrix, the power flows and plot them.

**Problem 4.4.** This is a continuation of Problem 3.4 from Lecture 3. For the synchronous zone corresponding to Scandinavia:

a) Add to the network object the information on the links susceptance, calculate the weighted Laplacian (or susceptance matrix), and the Power Transfer Distribution Factor (PTDF) matrix.

b) Assuming that power injection in the nodes increases linearly from -1 in the first node to +1 in the last node, calculate and plot the power flows in the network.