46770 Integrated energy grids

Lecture 4 – Linearized Optimal Power Flow

Types of optimization problems and course structure

	One node	Network			
One time step	Economic dispatch or One-node dispatch optimization (Lecture 2)	Power		Gas flow (Lecture 6)	Heat flow (Lecture 7)
		Linearized AC power flow (Lecture 4)	AC power flow (Lecture 5)		
Multiple time steps	Multi-period optimization Join capacity and dispatch optimization in one node (Lecture 8)	Join capacity and dispatch optimization in a network (Lecture 10)			

Recap from previous lectures

Overview of relevant equations to model energy networks

Regardless of the type of network (power, gas, heat) we need:

- Equations that represent the balance of energy (or mass) in every node -> **Nodal balance equations**

- Equations that represent the transport of energy in every link -> **Link equations**

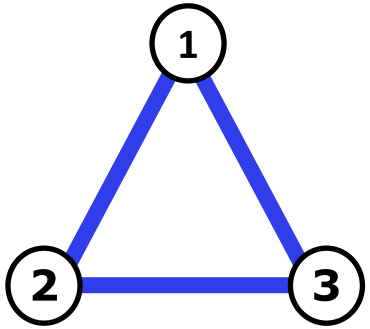
(these will depend on the physical applicable laws when transporting power, gas, heat)

- Information on how the nodes are connected by links -> **Matrices to capture the network topology**

  (we need a “map” of the network)

Laplacian matrix

Laplacian matrix $L_{ij} = D_{ij} - A_{ij}$



$$L_{ij} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} = \left[\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \right]$$

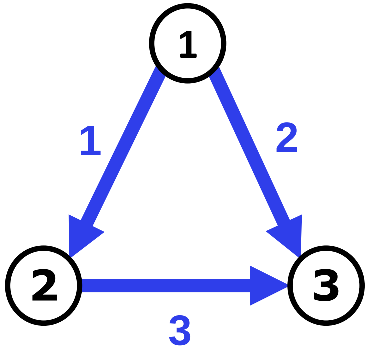
The Laplacian matrix is a "map" of the network, it contains information on how the nodes are connected.

If the links have different strength, we can add this information and we get the weighted Laplacian matrix.

Incidence matrix

Incidence matrix

$$K_{il} = \begin{cases} 1 & \text{if link } l \text{ starts at node } i \\ -1 & \text{if link } l \text{ ends at node } i \\ 0 & \text{otherwise} \end{cases}$$



$$\begin{array}{cc} \text{links:} \; \begin{matrix} 1 & \;\;2 & \;\;3 \end{matrix} & \text{nodes} \\ K_{il} = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{pmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{array}$$

The Incidence matrix contains information on the links connected to each node and allows us to impose energy conservation or nodal energy balances $I_i = \sum_l K_{il} I_l$

Dispatch optimization in a network

Assume we have a network of nodes i. In every node, we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity $G_{s,i}$ and a linear variable cost o_S. Determine the optimal economic dispatch (how much energy is being produced by each of them) to supply the demand d_i in a certain hour and the optimal AC power flow while minimizing the total system cost.

Economic dispatch in one node

$$\min_{g_s} \sum_s o_s g_s$$

subject to:

$$\sum_s g_s - d = 0$$

$$0 \le g_s \le G_s$$

How can we include power flows between nodes in our optimization problem? Or more specifically, what are our nodal energy balance equations and links equations?

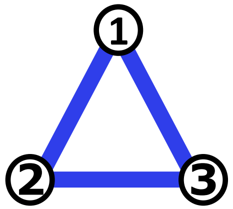
$$\min_{g_s} \sum_s o_s g_s$$

subject to:

$$\sum_s g_s - d = ?$$

$$0 \le g_s \le G_s$$

?



Economic dispatch with lines net transfer capacities

We can use the incidence matrix to impose nodal energy balance and limit the maximum power flowing through every line by its capacity. This is called transport model, trade model or net transfer capacities (NTC) model.

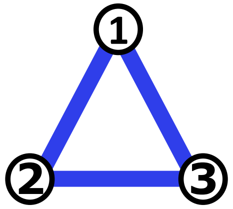
The solution is not unique, arbitrary flows can exist, and power losses are not represented.

Economic dispatch in one node

$$\begin{cases} \min_{g_s} \sum_s o_s g_s \\ \text{subject to:} \\ \sum_s g_s - d = 0 \\ 0 \leq g_s \leq G_s \end{cases}$$

Economic dispatch with AC power flow

$$\begin{cases} \min_{g_{s,i}} \sum_{s,i} o_s g_{s,i} \\ \text{subject to:} \\ \sum_{s,i} g_{s,i} - d_i = p_i = \sum_l K_{il} p_l \\ 0 \leq g_s \leq G_s \\ |p_l| \leq P_l \end{cases}$$



Nodal power balance

Lines Capacities

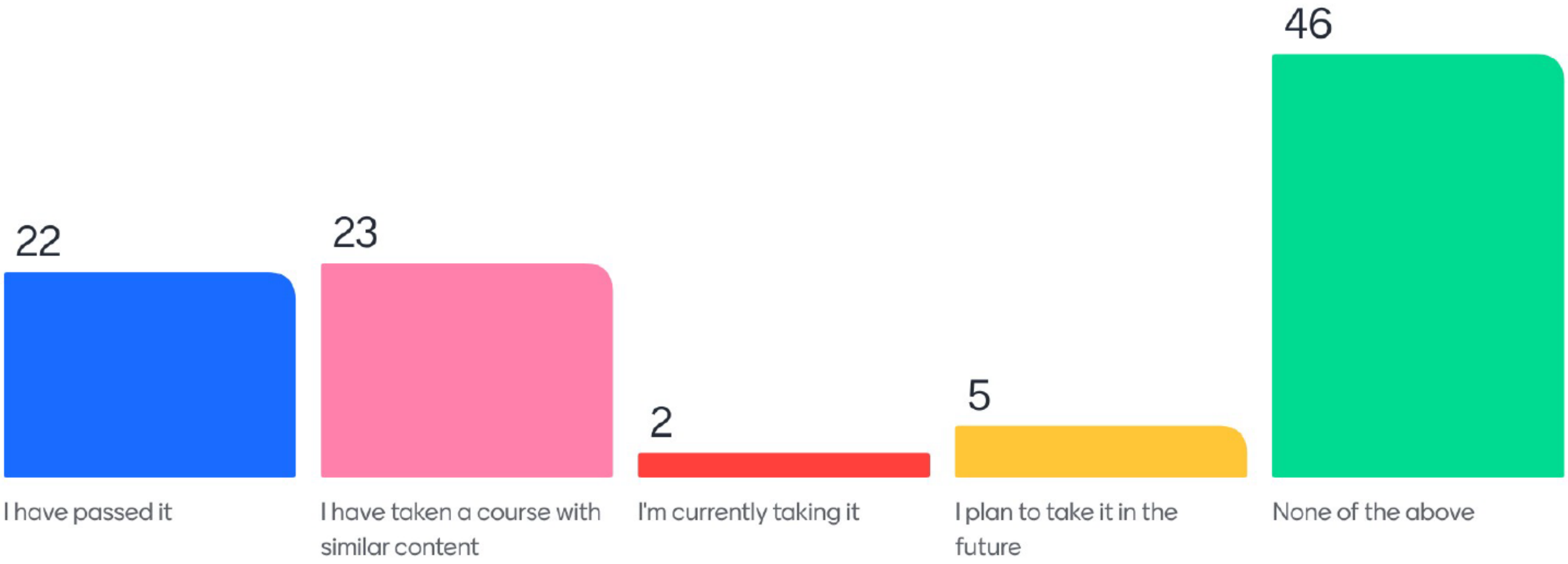
Modelling approaches for power flow in AC networks

Net Transfer Capacities	Linearized AC power flow (DC Power flow)	AC Power flow
$\sum_{s,i} g_{s,i} - d_i = \sum_{l} K_{il} p_l$ $\lvert p_l \rvert \leq P_l$		
× No unique solution × No representation of power losses		

Learning goals

- Review operating principles of AC power
- Write the equations describing the power flowing through an AC line and linearlize them
- Explain the analogy between DC circuits and linearized AC optimal power flow
- Calculate the linearlized power flows in an AC network by calculating the voltages using the inverse of the weighted Laplacian matrix
- Calculate the linearlized power flows in an AC network by using the Power Transfer Distribution Factors (PTDF) matrix
- Write the system cost minimization problem including AC power flows
- Formulate the optimal power flow problem on a computer

Regarding the course "Introduction to Electric Power Systems (46750)"



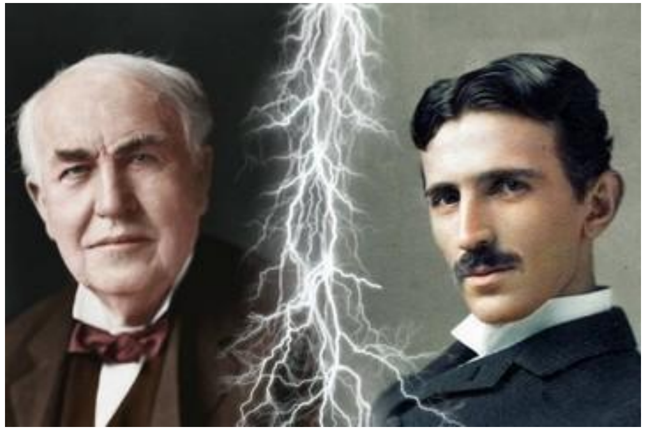
AC Power
(a short review)

Why do we use alternating current (AC) ?

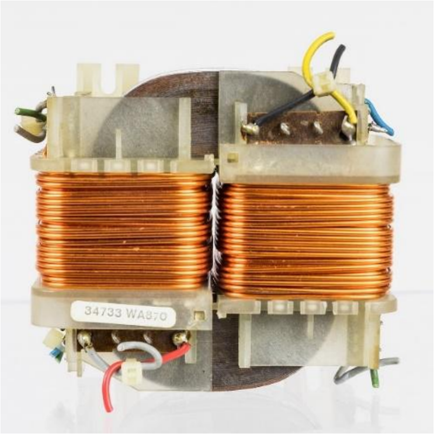
We need high voltage to transport power long-distances while keeping current (and power losses) low.

$$P = IV$$
$$losses \sim I^2 R$$

The war of currents (DC vs AC)



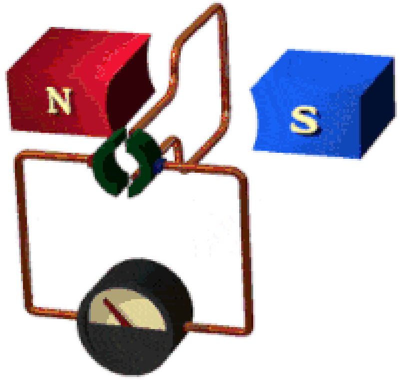
Transformers allow high-efficiency voltage conversion



$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

Ref: Wikipedia

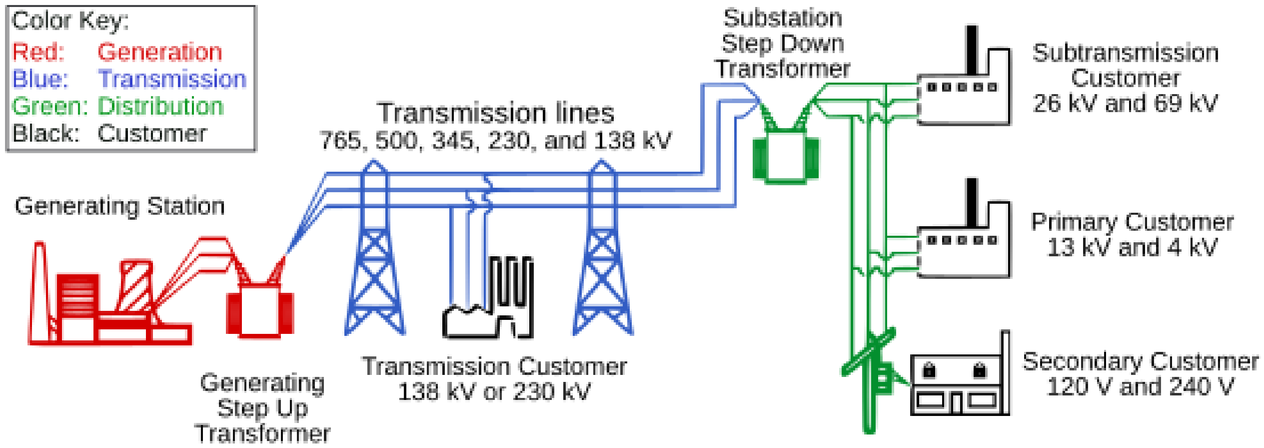
Generators convert mechanical rotational energy into AC electricity



Radial and meshed networks

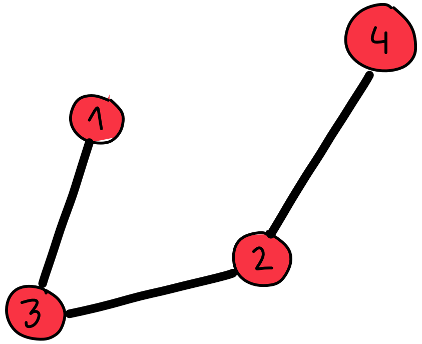
Power networks comprise transmission networks and distribution networks.

Transmission networks are typically meshed, while distribution networks are typically radial.



Ref: Wikipedia

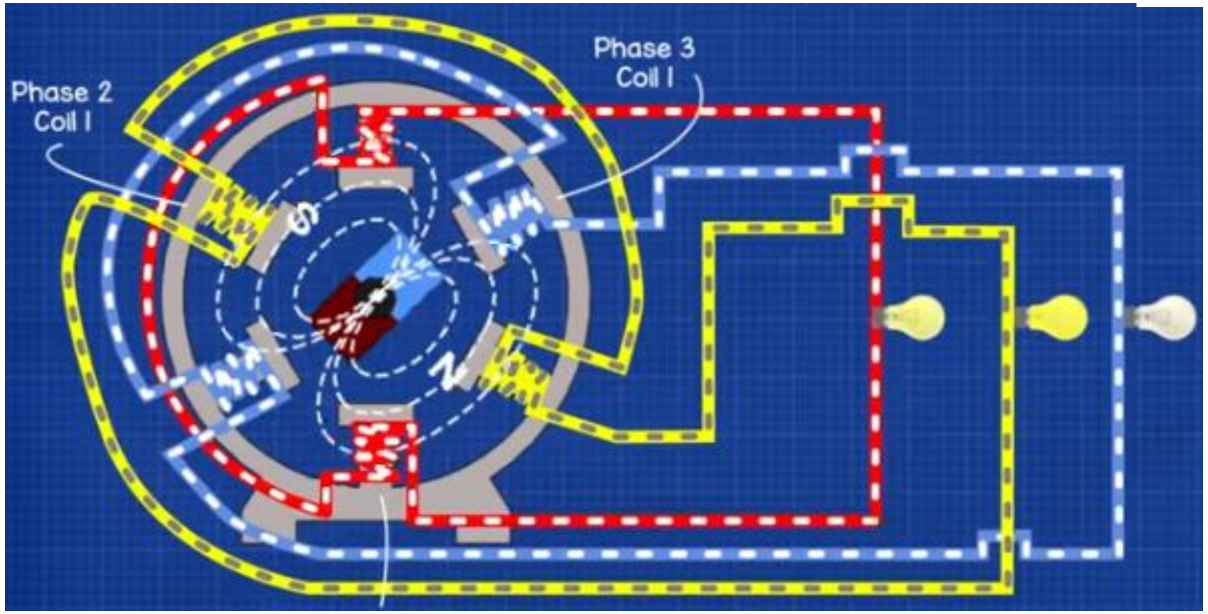
Radial Network: A network that was no cycles

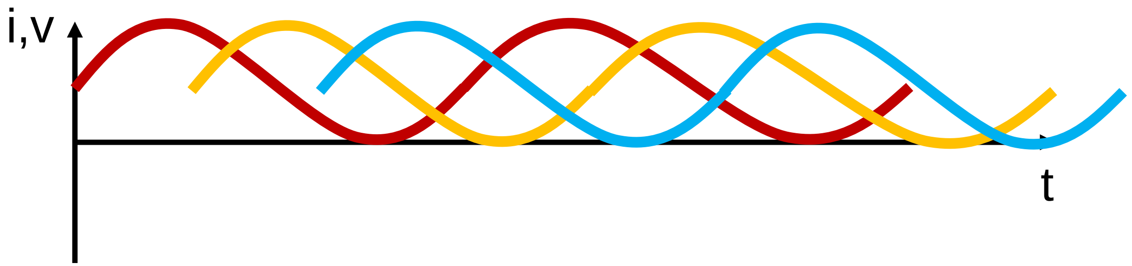


Alternating current

How does AC work?

Why 3 phases ?

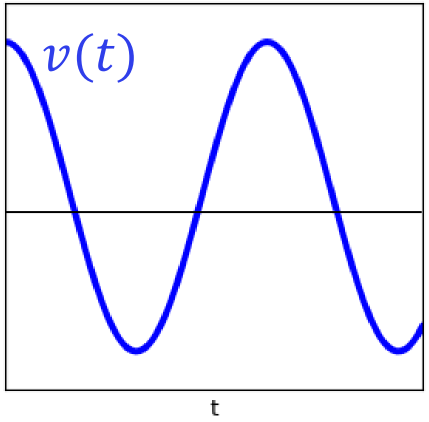


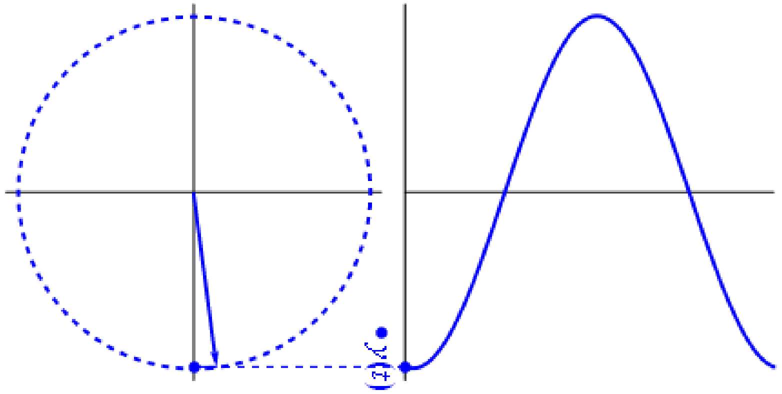


Under certain assumptions, three-phase network can be decomposed into three identical single-phase networks. This allows us to represent the entire system by a single phase.

AC voltage and current

$v(t) = V\cos(\omega t + \theta)$





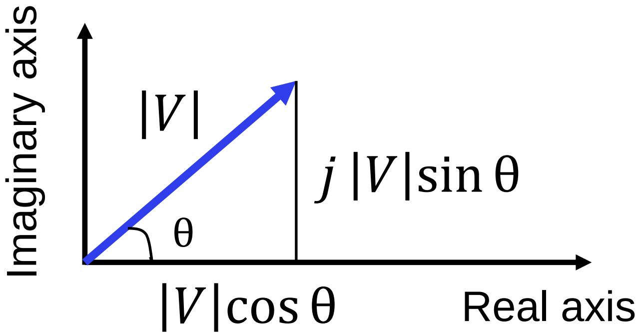
Phasor representation

Polar form: $\bar{X} = |X|\angle\theta$

Rectangular form: $\bar{X} = x + jy = |X|\cos(\theta) + j|X|\sin(\theta)$

Upper bar indicates complex number and $j = \sqrt{-1}$

Conjugate of an imaginary number $\bar{X}^* = |X|\angle - \theta \quad \bar{X}^* = x - jy$



$i(t) = I\cos(\omega t + \delta)$

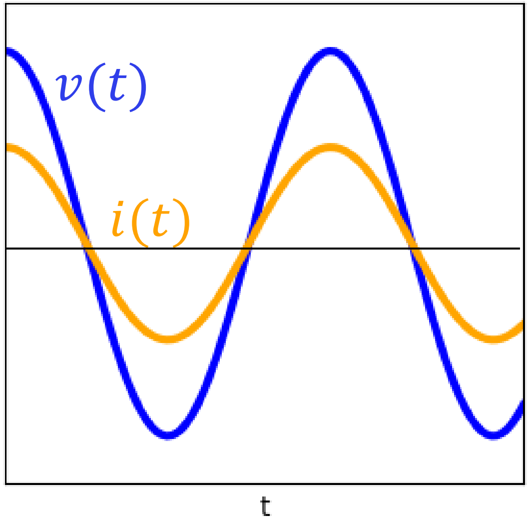
Current and Voltage are not necessarily in phase $\delta \neq \theta$

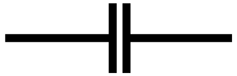
Resistive, capacity and inductive loads

Resistive loads

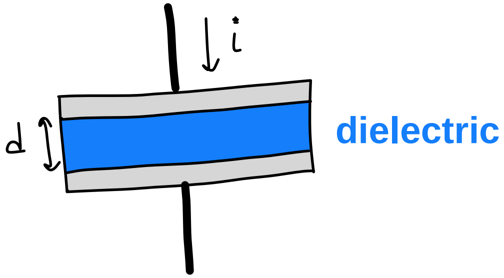
$$v(t) = V\cos(\omega t + \theta)$$

$$i(t) = \frac{v(t)}{R}$$

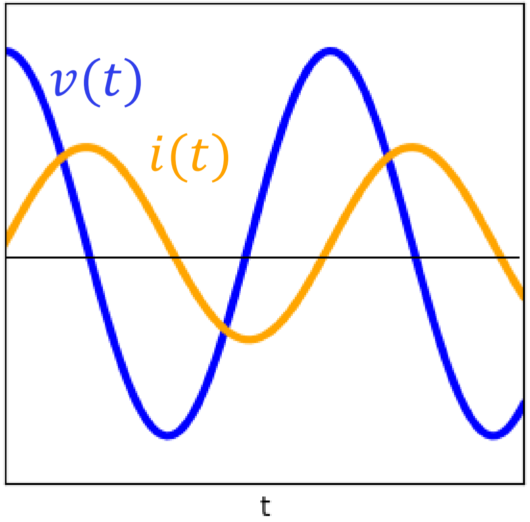




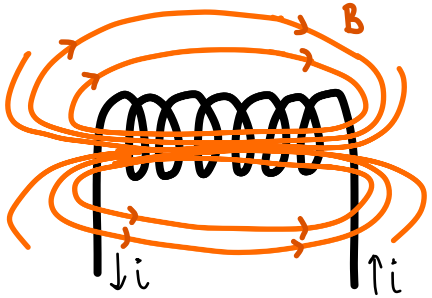
Capacity loads



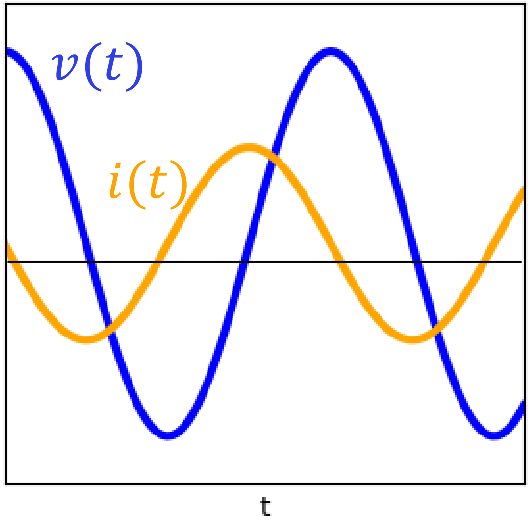
$$i(t) = C\frac{dv(t)}{dt}$$



Inductive loads



$$v(t) = L\frac{di(t)}{dt}$$



Complex Power

The **complex power** flowing is equal to the product of the voltage and the conjugate of the current (both complex numbers)

$$\underbrace{\bar{S}}_{\text{apparent power}} = \bar{V}\bar{I}^{*} = |V||I|\angle\theta-\delta = P + jQ = \underbrace{\mathrm{VI}\cos(\theta-\delta)}_{\text{active power}} + \mathrm{j}\,\underbrace{\mathrm{VI}\sin(\theta-\delta)}_{\text{reactive power}}$$

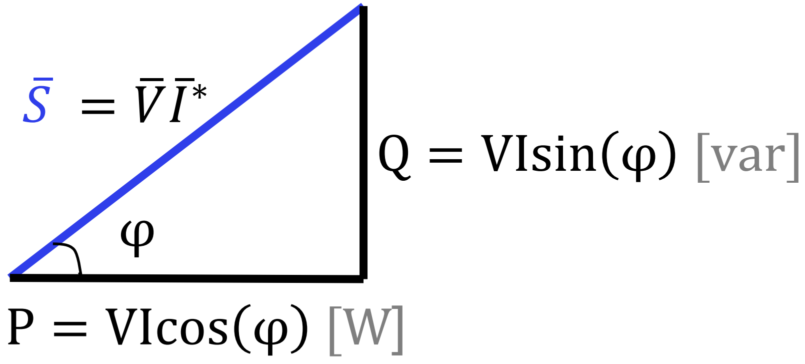
apparent power

active power

"useful" work
(from source to load)

reactive power

Not "useful" work
(flows back and forth)

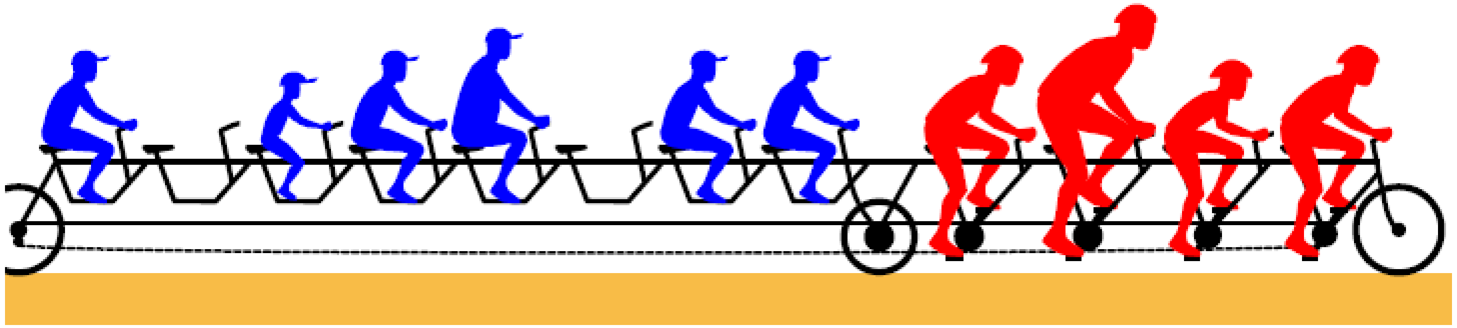


Power factor

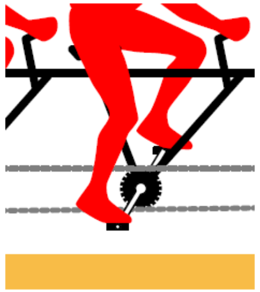
$$\underbrace{\cos(\theta-\delta)}_{\cos\varphi} = \frac{P}{S} = \frac{P}{\sqrt{P^2+Q^2}}$$

Alternating current

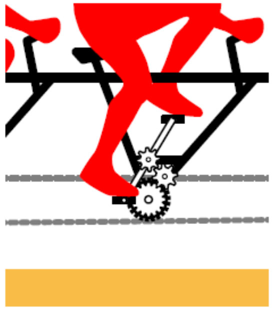
The tandem bicycle analogy



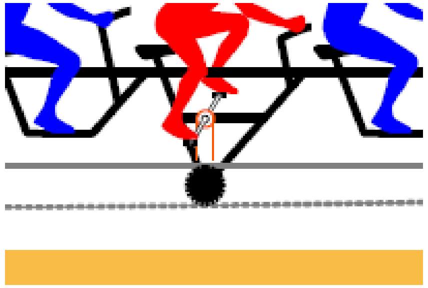
Synchronous generators using the grid frequency



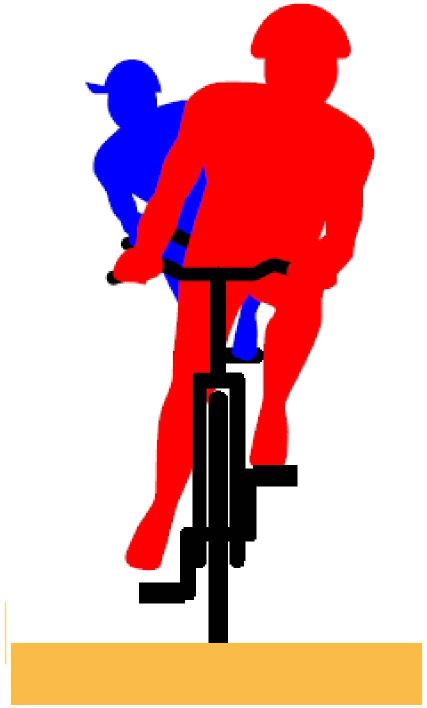
Synchronous generators using a different frequency



Other generators, e.g., solar PV



Reactive power



Further readings: Fassbinder 2005, [The electrical system as a tandem bicycle]

Generic loads

The **Impedance** represents the opposition to current in a line. It includes the resistance r associated to resistive loads and the reactance x associated the capacitive and inductive loads

$$\underbrace{z}_{\text{impedance}} = \underbrace{r}_{\text{resistance}} + j\underbrace{x}_{\text{reactance}}$$

The **Admittance** is the inverse impedance, and it represents how easily a line allows current to flow. It includes the conductance g and the susceptance b

$$\underbrace{y}_{\text{admittance}} = \underbrace{g}_{\text{conductance}} + j\underbrace{b}_{\text{susceptance}}$$

$$y = \frac{1}{z}$$

We can now write Ohm's law as

$$\bar{I}_{i\rightarrow j} = \bar{Y}_{ij}\left(\bar{V}_i - \bar{V}_j\right)$$

The bar above a variable denotes complex number

Per unit system

The per unit system is a method of normalizing voltage, current, power and impedance using their nominal values

In the following slides, we implicitly assume per-unit quantities.

For example:

$$V_{pu} = \frac{V}{V_{nom}}$$

For high-voltage transmission levels V_{nom}=765,000 V

$$y = Y_{pu} = \frac{Y}{Y_{nom}}$$

The per-unit system ensure consistent tolerance used in the optimization problem for optimality (objective function) and feasibility (constraints)

AC transmission lines

Overhead: naked metal and suspended on insulators, lower cost and easy maintenance, aluminum conductors (low cost, light weight)

Underground: need insulating cover

Transmission lines are a combination of resistive, capacitive, and inductive loads

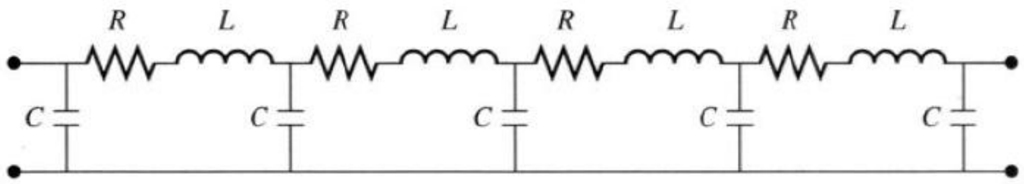
Resistance

$$R_{DC} = \frac{\rho l}{A}$$

AC resistance is higher than DC resistance due to skin effect which forces more current flow near the outer surface of the conductor.

Inductance A conductor carrying a current that varies in time produces a variable magnetic flux. The greater the spacing between the phases of a transmission line, the greater the inductance of the line.

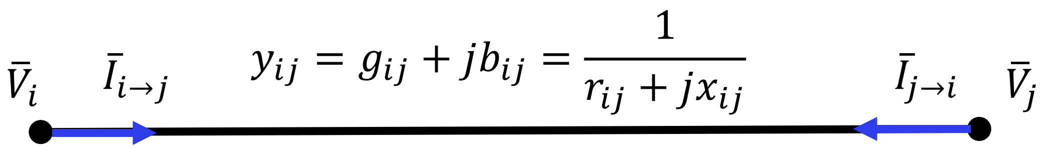
Capacitance Because we have a pair of conductors separated by a dielectric (air) The greater the spacing between the phases of a transmission line, the lower the capacitance of the line.



Modeling AC transmission lines

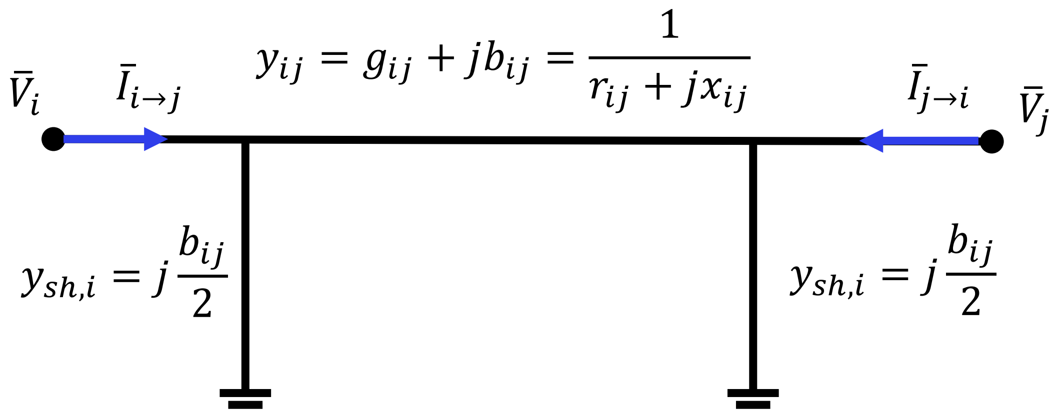
today

Simple series admittance model



next lecture

π model



Complex power through a line (I)

The complex power flowing through a line is equal to the product of the voltage difference and the conjugate of the current (both complex numbers)

$\bar{V}_i$ $\bar{I}_{i\to j}$ $y_{ij} = g_{ij} + jb_{ij} = \frac{1}{r_{ij} + jx_{ij}}$ $\bar{I}_{j\to i}$ $\bar{V}_j$

$$\bar{S}_{i\to j} = \bar{I}^*_{i\to j}\bar{V}_i = \bar{Y}^*_{ij}\left(\bar{V}^*_i - \bar{V}^*_j\right)\bar{V}_i = \bar{Y}^*_{ij}\bar{V}^*_i\bar{V}_i - \bar{Y}^*_{ij}\bar{V}^*_j\bar{V}_i$$

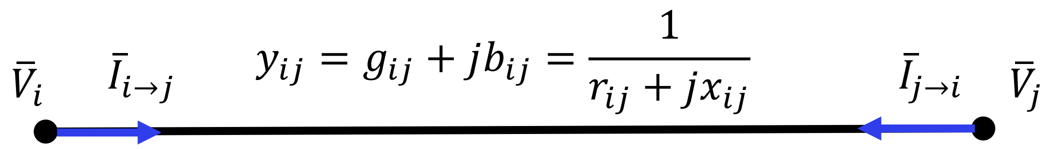
The power flowing in the opposite direction is not the same (due to power losses)

$$\bar{S}_{j\to i} = \bar{I}^*_{j\to i}\bar{V}_j \neq \bar{S}_{i\to j} = \bar{I}^*_{i\to j}\bar{V}_i$$

power losses

Complex power through a line (II)

The complex power flowing through a line is equal to the product of the voltage difference and the conjugate of the current (both complex numbers)



$$\bar{S}_{i\to j} = \bar{I}^*_{i\to j}\bar{V}_i = \bar{Y}^*_{ij}\bar{V}_i^*\bar{V}_i - \bar{Y}^*_{ij}\bar{V}_j^*\bar{V}_i$$

We can split the complex power into real (active power) and imaginary part (reactive power) and use rectangular form

$$\boxed{\bar{S}_{i\to j} = p_{i\to j} + \mathrm{j}q_{i\to j}}$$

$$\bar{V}_i^*\bar{V}_i = V_iV_i\angle 0 = |V_i|^2$$

$$\bar{Y}^*_{ij} = g_{ij} - \mathrm{j}b_{ij}$$

$$\bar{V}_j^*\bar{V}_i = V_iV_j\angle\theta_i - \theta_j = |V_i||V_j|\cos(\theta_i - \theta_j) + j|V_i||V_j|\sin(\theta_i - \theta_j)]$$

$$\boxed{\begin{aligned} p_{i\to j} &= g_{ij}|V_i|^2 - |V_i||V_j|\,[g_{ij}\cos(\theta_i - \theta_j) + b_{ij}\sin(\theta_i - \theta_j)] \\ q_{i\to j} &= -b_{ij}|V_i|^2 - |V_i||V_j|\,[g_{ij}\sin(\theta_i - \theta_j) - b_{ij}\cos(\theta_i - \theta_j)] \end{aligned}}$$

These equations are non-linear but we can simplify them under certain assumptions.

Linearized AC power flow (DC power flow)

Assumptions to linearize AC power flow (I)

Complex power flowing through a line $\bar{S}_{i\to j} = p_{i\to j} + \mathrm{j}q_{i\to j}$

$$p_{i\to j} = g_{ij}|\cancel{V_i}|^2 - |\cancel{V_i}||V_j|\,[g_{ij}\cos(\cancel{\theta_i - \theta_j}) + b_{ij}\underbrace{\sin(\theta_i - \theta_j)}_{\theta_i - \theta_j}]$$

$$q_{i\to j} = -b_{ij}|\cancel{V_i}|^2 - |\cancel{V_i}||V_j|\,[g_{ij}\underbrace{\sin(\theta_i - \theta_j)}_{\theta_i - \theta_j} - b_{ij}\cos(\cancel{\theta_i - \theta_j})]$$

$$p_{i\to j} = g_{ij} - [\,g_{ij} + b_{ij}(\theta_i - \theta_j)] = -b_{ij}(\theta_i - \theta_j)$$

$$q_{i\to j} = -b_{ij} - [g_{ij}(\theta_i - \theta_j) - b_{ij}] = -\cancel{g_{ij}}(\theta_i - \theta_j)$$

$$p_{i\to j} = -b_{ij}(\theta_i - \theta_j)$$

$$\boxed{p_{i\to j} = \frac{\theta_i - \theta_j}{x_{ij}}}$$

1º. Unitary voltage values close to 1

2º. Voltage angle differences are small

$$\sin(\theta_i - \theta_j) \approx (\theta_i - \theta_j)$$

$$cos(\theta_i - \theta_j) \approx 1$$

3º. Conductances g_{ij} are negligible relative to susceptances b_{ij} (resistance are much smaller than reactance)

$(r_{ij} \ll x_{ij} \quad g_{ij} \ll b_{ij} \quad b_{ij} = \frac{-1}{x_{ij}})$.

This implies that power losses are neglected. This also implies that reactive power flow $q_{i\to j}$ is neglected

Demonstration $b_{ij} = -1/x_{ij}$

$$y = g + jb = \frac{1}{r+jx} = \frac{r-jx}{(r+jx)(r-jx)} = \frac{r-jx}{r^2+x^2}$$

$$g = Re[\frac{r-jx}{r^2+x^2}] = \frac{r}{r^2+x^2}$$

$$b = Im[\frac{r-jx}{r^2+x^2}] = \frac{-x}{r^2+x^2}$$

For $r \ll x \quad b = \frac{-1}{x}$

Assumptions to linearize AC power flow (II)

Now we have obtained a linear relation between the active power p_l flowing through a line and the voltage angle differences $\theta_i - \theta_j$

This enables us to formulate linear optimization problems, that are convex and have a global minimum

$$p_{i\rightarrow j} = \frac{\theta_i - \theta_j}{x_{ij}}$$

θ_i is in radians and $p_{i\rightarrow j}$ is in per unit!

The per-unit power flows ensure consistent tolerance used in the optimization problem for optimality (objective function) and feasibility (constraints)

The same result is discussed in Optimization in Modern Power Systems (Lecture 3)

Analogy between DC and linearized AC power flow

The equation that relates the active power flow and the voltage angles is analogous to Ohm's law. This is the reason that justifies using the term "**DC approximation or DC power flow**" to name the linearized AC power flow.

$$I_l = \frac{V_i - V_j}{R_l} \qquad\qquad p_l = \frac{\theta_i - \theta_j}{x_l}$$

DC CIRCUITS	LINEARIZED AC POWER FLOW
Current flow I_l	Active power flow p_l
Voltage V_i	Voltage angle θ_i
Resistance R_l	Reactance x_l

Limitations of the Linearized AC Optimal Power flow

- The linearized AC approximation is only valid for stable situations, where voltage angle differences are small. It can not be used to analyze heavily-loaded systems, fast changes, restart from blackout, network splitting, etc.

- With the linear power flow approximation, we cannot estimate losses in the network because we have assumed resistance to be zero.

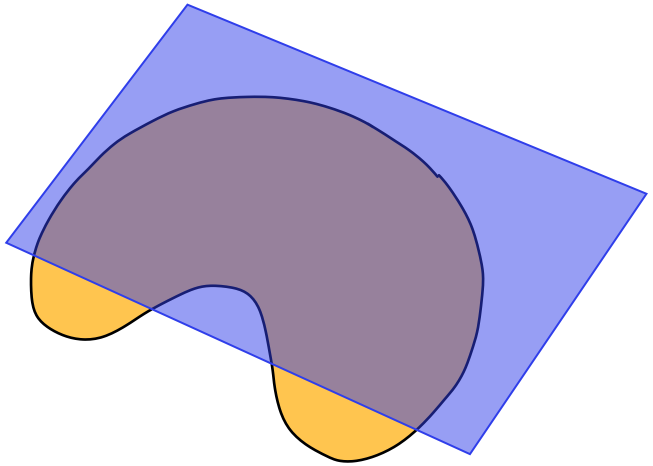
- Not guarantee to find a same solution that is feasible for the non-linearized AC optimal power flow

- We can use the linear power flow approach for long-term planning.

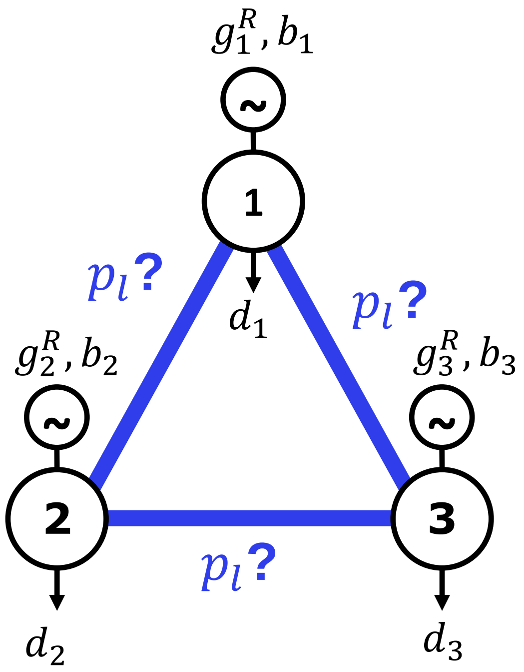
Limitations of the Linearized AC Optimal Power flow

Linearization of AC power flow is an **approximation** of the feasible space which might:

- may contain points that are not feasible
- may not contain some feasible points
- may not include original optimal solution



Optimal power flows. Stating the problem



In every node i, mismatch (i.e., renewable generation g_i^R minus demand d_i), is equal to local balance b_i plus injection p_i:

$$\Delta_i = g_i^R - d_i = b_i + p_i$$

The total sum of energy injection is zero (energy conservation):

$$\sum_i p_i = p_1 + p_2 + p_3 = 0$$

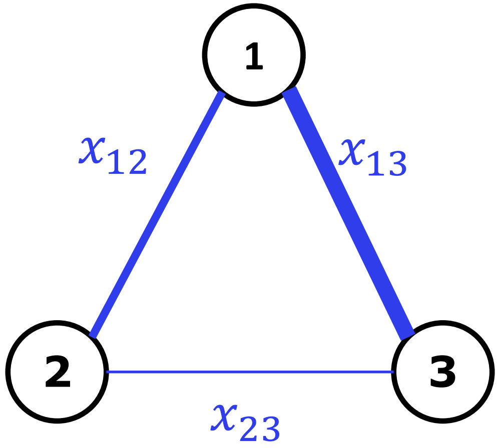
The **goal** of power flow analysis is to **find the power flows through the links of a network given the injection pattern for the nodes.**

We can use the incidence matrix to impose nodal energy balances $p_i = \sum_l K_{il} p_l$

However, to calculate the power flows, we need additional equations otherwise there are many possible solutions

Optimal power flows

We can use the **weighted Laplacian matrix**



The power flowing through every link $p_{i\to j}=p_l$ depends on the voltage angle difference across the nodes connected by the link

$$p_{i\to j}=p_l=\frac{\theta_i-\theta_j}{x_l}=\frac{1}{x_l}\sum_j K_{lj}\theta_j$$

And the power injected in every node p_i is equal to the sum of the power exported through the links

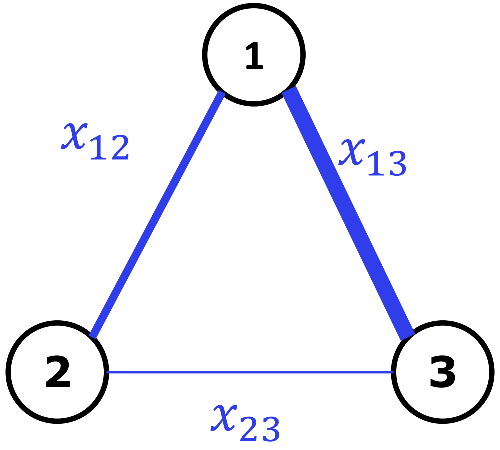
$$p_i=\sum_l K_{il}p_l=\underbrace{\sum_l K_{il}\frac{1}{x_l}\sum_j K_{lj}}_{L=KBK^T}\theta_j$$

$B_{ll}=\frac{1}{x_l}$ is the diagonal matrix of inverse of every link reactance

The **weighted Laplacian matrix (or Susceptance Matrix)** relates the power injections p_i and voltages angles θ_j in every node.

Calculating the power flows

The weighted Laplacian matrix relates the power injections p_i and voltage angles $\theta_j$in every node $p_i = \sum_j L_{ij}\,\theta_j$



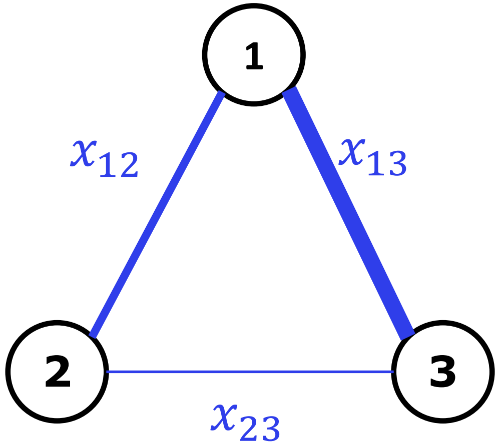
To obtain the power $p_{i\rightarrow j}$ flowing through the links, we follow two steps :

1º. Calculate the voltage angles θ_j using the inverse of the weighted Laplacian matrix $\theta_j = \sum_i (L^{-1})_{ji}\, p_i$ (the Laplacian is not invertible, and we have to use the strategies described in the previous lecture)

2º. Calculate the power flows $p_l = p_{i\rightarrow j}$ using the voltage angles and the transposed incidence matrix $p_l = \frac{1}{x_l}\sum_j K_{lj}\theta_j$

Power Transfer Distribution Factors (PTDF) matrix

An alternative method consists in using the Power Transfer Distribution Factors (PTDF) matrix



$$p_l = \frac{1}{x_l}\sum_j K_{lj}\theta_j = \frac{1}{x_l}\sum_{ji} K_{lj}(L^{-1})_{ji}p_i = \underbrace{\sum_i PTDF_{li}p_i}$$

The PTDF is calculated as the transpose incidence matrix times the inverse Laplacian matrix

The PTDF matrix measures the sensitivity of power flows in each link relative to incremental changes in nodal power injections throughout the network.

The linearized AC power flow based on PTDF formulation is used for market coupling of the European markets, each node corresponds to a biding zone and PTDFs are derived for the interconnections between countries

Economic dispatch with linearized AC optimal power flow

Assume we have a network of nodes i. In every node, we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity $G_{s,i}$ and a linear variable cost o_S. Determine the optimal economic dispatch (how much energy is being produced by each of them) to supply the demand d_i in a certain hour and the optimal AC power flow while minimizing the total system cost.

Economic dispatch in one node

$$\min_{g_s} \sum_s o_s g_s$$

subject to:

$$\sum_s g_s - d = 0$$

$$0 \leq g_s \leq G_s$$

Economic dispatch with linearized AC power flow

$$\min_{g_{s,i}} \sum_{s,i} o_s g_{s,i}$$

subject to:

$$\sum_{s,i} g_{s,i} - d_i = \sum_l K_{il} p_l$$

Nodal power balance

$$p_i = \sum_l K_{il} p_l = \sum_j L_{ij}\, \theta_j$$

$$0 \leq g_s \leq G_s$$

$$|p_l| \leq P_l$$

Lines capacities

$$p_l = \frac{\theta_l}{x_l}$$

Links equations

Kirchhoff's laws

Alternatively, we can solve the problem of calculating the power flows (given the power injection patterns), by imposing both Kirchhoff's laws

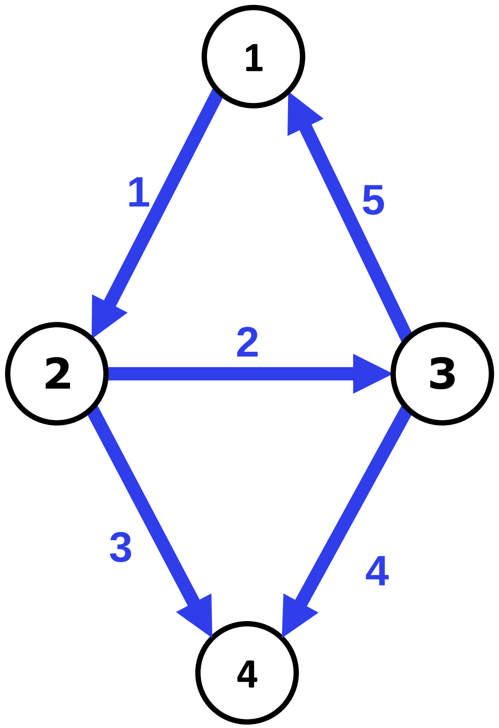
Kirchhoff's Currents Law can be imposed using the incidence matrix

$$p_i = \sum_l K_{il} p_l \qquad \forall i$$

Here, I_i is the current that node i wants to inject, while I_l is the current flowing throughout link l

Kirchhoff's Voltage Law can be imposed using the cycle matrix

$$\sum_l C_{lc}\, \theta_l = 0 \qquad \forall c$$



This method is computationally very efficient for large networks*.

Kirchhoff's voltage law can be written using power flows. This way, we avoid the need for using the variables θ in the problem.

$$p_{1\to 2} = \frac{\theta_1 - \theta_2}{x_{12}}$$

$$\sum_l C_{lc}\, x_l p_l = 0 \qquad \forall c$$

*See Hörsch et al, 2018. Linear optimal power flow using cycle flows

Economic dispatch with linearized AC optimal power flow

Assume we have a network of nodes i. In every node, we have a set of generators s (e.g., onshore wind, solar PV, gas power plant…) each of them has an installed capacity $G_{s,i}$ and a linear variable cost o_S. Determine the optimal economic dispatch (how much energy is being produced by each of them) to supply the demand d_i in a certain hour and the optimal AC power flow while minimizing the total system cost.

Economic dispatch in one node

$$\left\{\begin{array}{l} \min_{g_s} \sum_s o_s g_s \\ \text{subject to:} \\ \quad \sum_s g_s - d = 0 \\ \quad 0 \leq g_s \leq G_s \end{array}\right.$$

Economic dispatch with linearized AC power flow

$$\left\{\begin{array}{l} \min_{g_{s,i}} \sum_{s,i} o_s g_{s,i} \\ \text{subject to:} \\ \quad \sum_{s,i} g_{s,i} - d_i = \sum_l K_{il} p_l \\ \quad 0 \leq g_s \leq G_s \\ \quad |p_l| \leq P_l \\ \quad \sum_l C_{lc}\, x_l\, p_l = 0 \end{array}\right.$$

Nodal power balance

(Kirchoff's Current Law)

Lines Capacities

Links equations
(Kirchoff's Voltage Law)

Modelling approaches for power flow in AC networks

Net Transfer Capacities	Linearized AC power flow (DC Power flow)	AC Power flow
$\sum_{s,i} g_{s,i} - d_i = \sum_l K_{il} p_l$ $\lvert p_l \rvert \leq P_l$	$\sum_{s,i} g_{s,i} - d_i = \sum_l K_{il} p_l$ $\lvert p_l \rvert \leq P_l$ $\sum_l C_{lc}\, x_l\, p_l = 0$	
	✓ Linear ✓ Optimality guaranteed ✓ Computational tractable ✓ Good enough for long-term planning	
× No unique solution × No representation of power losses	× Not guarantee feasible power flows × No representation of power losses × Not good enough for heavily-loaded systems, fast changes, restart from blackout, network splitting, etc.	

* For radial networks, Net Transfer Capacities and Linearized AC power flow models are equivalent

Problems for this lecture

Problems 4.1, 4.2 (**Group 8**)

Problems 4.3, 4.4 (**Group 9**)