MAGNETIC RECONNECTION IN SPACE AND LABORATORY PLASMAS AND ITS IMPLICATION IN TOKAMAK PLASMA CONFINEMENT

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Abstract

Magnetic reconnection is the topological restructuring of magnetic field lines in high temperature plasmas. The aim of this paper will be to describe the fundamental physics of magnetic reconnection and discuss the various MHD models to describe magnetic reconnection. Magnetic reconnection as observed in both space plasmas and laboratory plasmas results in significant changes in kinetic and thermal properties of the plasma. This inherently deteriorates the ability to confine plasma in the laboratory. The second part of this paper will describe the fundamentals of sawtooth instabilities triggering magnetic reconnection in tokamak plasma devices. Due to the undesirable effects of magnetic reconnection in laboratory plasmas, several techniques have been developed to control sawtooth instabilities. The efficacy of these techniques to include current drive schemes and ion population control will be discussed.

1 Reconnection Defined

Reconnection is the rearranging of the magnetic topology where magnetic field lines are broken and then recombine and is shown in Figure 1.[Priest,Yamada98,Yamada10] This is an important process to understand in astrophysics, laboratory plasma science, and others due to it's disruptive effects. The process of reconnection changes the macroscopic quantities of plasmas through:[Priest]

- conversion of magnetic energy into heat
- accelerate plasma by converting magnetic energy into kinetic energy
- create shockwaves, current filamentation, and turbulence
- affect fluxes of fast particles and heat due to changes in global magnetic field lines

The first two effects are especially important for stability of confined plasmas due to causing major and minor disruptions in tokomak discharges that will be discussed in this paper.[Yamada10]

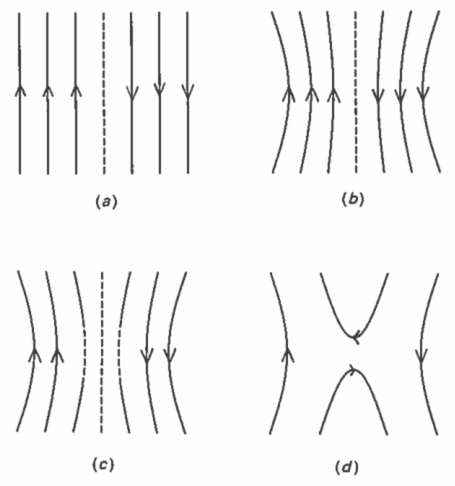

Figure 1: An intial perturbation generates magnetic pressure to counter. However, magnetic field diffusion occurs rapidly in locations of strong flux gradient. As magnetic field diffuses, perturbation is allowed to grow and alter topology.[Hood]

1.1 Reconnection in Space Plasmas

Magnetic reconnection is seen directly in many aspects of space plasmas. Solar flares and coronal mass ejections were one of the first events studied to understand magnetic reconnection.[Yamada10] Coronal mass ejections and solar flares result form converting magnetic energy into heat and particle energy during a magnetic reconnection event.[Priest] Additionally, a small scale reconnection model is thought to be responsible for the higher density slow solar wind originating from the Sun's equatorial region.[Priest] Reconnection between solar wind magnetic field lines and the Earth's magnetoshpere field lines is also thought to be responsible for the penetration of solar particles to penetrate the magnetosphere(Figure 2).[Frey,Priest]

Understanding reconnection in space plasma is vitally important to protecting space satellites, astronauts, and even terrestrial utilities from the damaging effects of highly energized charged particles from solar plasma.

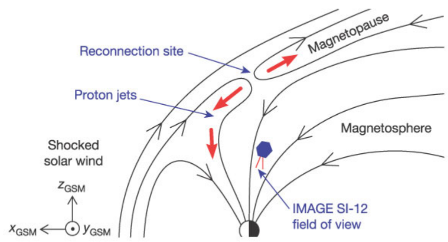

Figure 2: Reconnection at magnetopause[Frey]

1.2 Reconnection in Laboratory Plasmas

Reconnection is believed to be responsible for various anomalies in tokomak, reverse pinch, and spheromak devies. In tokomak devices, magnetic reconnection manifests in a periodic reorganization of core plasma density and temperature(Figure 3).

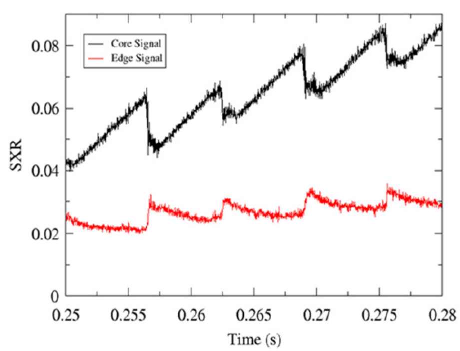

Figure 3: Soft X-ray emissions give core temperature indications. This shows the core plasma temperature periodically dropping associated with an increase in edge plasma temperature.[Chapman11]

In reverse pinch and spheromak device, magnetic reconnection plays a role in the magnetic relaxation. Magnetic relaxation is a self-organization of confined plasma that is thought to be a preferred state of minimal helicity.[Garcia,Yamada10] Magnetic reconnection events are believed to be a driving factor in changes of helicity and therefore a driving factor for confined plasma self-organization.[Garcia] Understanding these phenomena is important and is a driving factor in the development of new confinement techniques in the field of plasma fusion.[Champan11,Aymar,Wagner]

1.3 Fundamental Physics of Reconnection

Davidson, Priest, and Garcia all present very good introductions to the relevant magnetohydrodynamics of magnetic reconnection, in particular Chapter

6 of Eric Priests textbook. For this paper we will focus on the derivation of magnetic diffusion and how this plays a role in magnetic reconnection.
Beginning with the differential forms of Ampere's Circuit Law and Gauss' Magnetic Law:

$$\bigtriangledown \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \textbf{ or } \bigtriangledown \times \vec{B} = \mu \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

$$\bigtriangledown \cdot \vec{B} = 0$$

And combining them with Ohm's Law:

$$\vec{J} = \sigma\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

We arrive at the result,

$$\frac{\partial \vec{B}}{\partial t} = -\bigtriangledown \times \left(-\vec{v} \times \vec{B} + \frac{\vec{J}}{\sigma}\right)$$

or

$$\frac{\partial \vec{B}}{\partial t} = \bigtriangledown \times \left(\vec{v} \times \vec{B}\right) - \bigtriangledown \times \left(\eta \bigtriangledown \times \vec{B}\right) \textbf{ where } \eta \equiv \frac{1}{\mu\sigma}$$

To simplify further we can use $\bigtriangledown \times (\bigtriangledown \times \vec{a}) = \bigtriangledown(\bigtriangledown \cdot \vec{a}) - \bigtriangledown^2 \vec{a}$,

$$\frac{\partial \vec{B}}{\partial t} = \bigtriangledown \times \left(\vec{v} \times \vec{B}\right) - \eta \bigtriangledown^2 \vec{B}$$

We can introduce a new non-dimensional number called the magnetic Reynold's number, R_m

$$R_m = \frac{\left|\bigtriangledown \times \vec{v} \times \vec{B}\right|}{\left|\eta \bigtriangledown^2 \vec{B}\right|} \sim \frac{u_0 L}{\eta}$$

Typically velocity and length scales are much larger than η and this results in $R_m \gg 1$ and diffusivity can be neglected (this is considered idealized)[Garcia];

$$\frac{\partial \vec{B}}{\partial t} = \bigtriangledown \times \left(\vec{v} \times \vec{B}\right)$$

In this case, the magnetic field lines are similar to elastic bands frozen in the vector field.[Davidson,Garcia] This means that any flux line through a closed loop will be conserved and the magnetic flux lines provide a restoring force to a disturbance. If, however, the gradient of the magnetic field is very large, diffusion can no longer be neglected.

Now we can revisit Figure 1 and discuss the evolution of this process further. In Figure 1a, normal magnetic flux lines exist in anti-parallel directions. In Figure 1b, a perturbation pushes outward (not shown) and the ideal magnetic flux lines respond with a restoring force. As the perturbation grows and the restoring force grows, the magnetic flux lines become closer and closer. This increases the gradient of the magnetic field and diffusion sets in. In Figure 1d, magnetic flux lines diffuse and reconnect. In the Sun this perturbation is a manifestation of differential rotation rates that tend to twist the magnetic field lines. In tokomaks, this perturbation is caused from a kink in the magnetic field due to poloidal and toroidal magnetic interactions.[Yamada10,Hastie,Champan11,Yun,Yamda94] This will be discussed in the next section.

2 Sawtooth Instabilities in Tokomak Devices

Tokomak plasmas are generally susceptible to two types of instabilities, microscopic and macroscopic. Microscopic instabilities generally dictate transport properties of a confined plasma, but in general do not cause significant disruption of confinement. Macroscopic instabilities however, can and often do result in total loss of plasma confinement.[Hastie] Sawtooth instabilities are unique in that they are macroscopic instabilities but do not result in a termination of discharge.[Hastie] They can however trigger other instabilities. In this section, we will discuss the characteristics of sawtooth instabilities in tokomak devices to include; cycle characteristics, energy transport, and problems associated with sawtooth instabilities. We will then discuss the current theories on causes of sawtooth instabilities and the methods of controlling them.

2.1 Sawtooth Characteristics

Sawtooth characteristics were discovered in 1974 and have now been verified on all tokomak devices.[Hastie] Sawtooth instabilities are periodic redistributions of core plasma and exist in three phases(Figure 4);[Chapman11]

- an approximately linear increase in both plasma temperature and density
- an oscillatory precursor phase
- a final rapid drop in both temperature and plasma density.

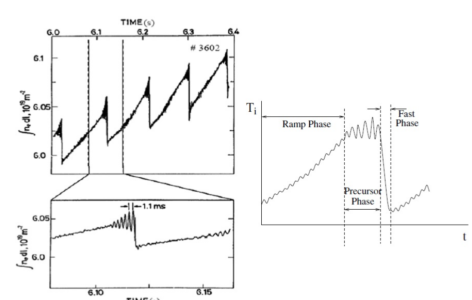

Figure 4: Sawtooth characteristic from JET showing all three phases[Chapman11]

When a sawtooth crash occurs, high energy electrons diffuse to cooler portions of the plasma resulting in a flattening T_e profile. This characteristic can be seen in both Figures 5 and 6.

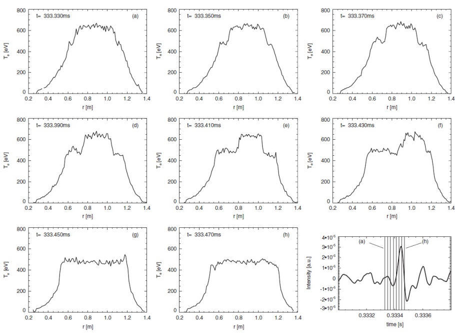

Figure 5: Electron temperature profile every $20\mu s$. In f, the temperature flattens out more on the interior where the higher magnetic field exists. The crash occurs between frames f and g.[Chapman10]

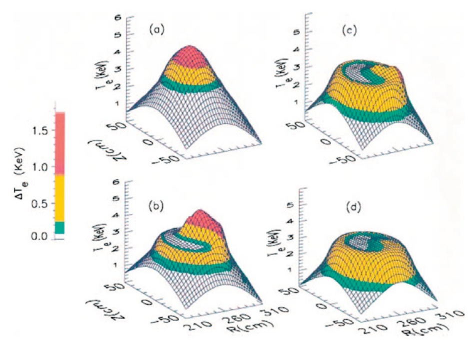

Figure 6: Two dimensional profile of electron temperature duing sawtooth crash. Similar flattening characteristics on the interior side can be seen.[Yamada94]

2.2 Tokamak Plasma Stability

Before we can discuss the theories for sawtooth instabilities, we need to qualitatively discuss confined plasma stability. To do this we will discuss the safety factor, $q = \frac{m}{n}$ where m is the poloidal number and n is the toroidal number. Essentially the safety factor is the ratio to the number of times a field line will go around toroidally for one poloidal circuit.[O'Brien] A more in depth review of MHD stability theory than is provided here will show that low rational numbers for q are least stable and tend to develop helical perturbations that can lead to kinks.[O'Brien] An interesting result of MHD stability analysis is that if an

$m = n = 1$ kink instability exists(Figure 7), the central safety factor, q_0, must be greater than one to diminish the perturbation.[Hopcraft] The large gradient of magnetic field can cause a transition from ideal to diffusion MHD where magnetic reconnection can occur.[Priest] This is where Kadomtsev comes in.

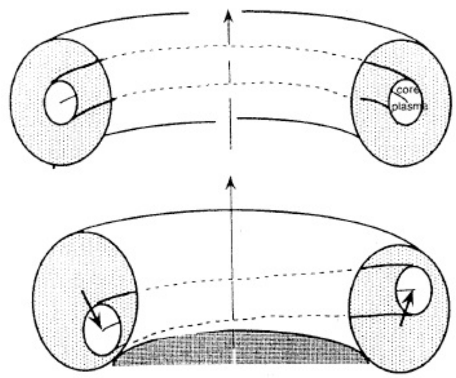

Figure 7: View of normal and $m = n = 1$ kink instability in a tokomak.[Hastie]

2.3 Sawtooth Theories

The initial theory for the cause of sawtooth instabilities was by Boris Kadomtsev who postulated that the cycle begins with an $m = n = 1$ kink instability where the central saftey factor $q_0 < 1$.[Yamada94,Hastie,Chapman11] Kadomtsev postulated that the kink instability would drive magnetic reconnection to restore a stable reconfiguration causing a transient the re-established a $q_0 > 1$ condition in the plasma. Ohmic heating was then thought to be responsible for driving T_e up and forcing $q_0 < 0$ to restart the cycle.[Hastie] Kadomtsev's model has been the standard driving much of the research in sawtooth oscillations.

Initial experiments indeed supported the Kadomtsev model[Sykes], but as devices got larger and data collection became more accurate, discrepancies were noted. One discrepancy is that expected central safety factors, q_0, should be restored to a value greater than one after a sawtooth crash. However increased precision yield experimental measurements showing q_0 never reaches 1.0 post crash(See Figures 8 and 9). This is indicative that full Kadomtsev reconnection is not occurring and has led to a theory that Kadomtsev reconnection is interrupted due to rapid heat diffusion through the reconnection region.[Yamada10]

Additional data has been collected that deviates from the Kadomtsev model. Reconnection times are significantly faster than those predicted by the Kadomtsev model, since the Kadomtsev model incorporates the Sweet-Parker model for reconnection. Reconnective theory predicts sawtooth crash times of $\tau_K \approx 340\mu s$ while actual sawtooth crashes occur on timescales $\leq 20\mu s$.[Yamada94,Chapman10,Chapman11]

As Chapman explains, this is not surprising since the Sweet-Parker model of reconnection is only applicable to collisional plasmas, while tokamak plasmas have a collisionless behavior.

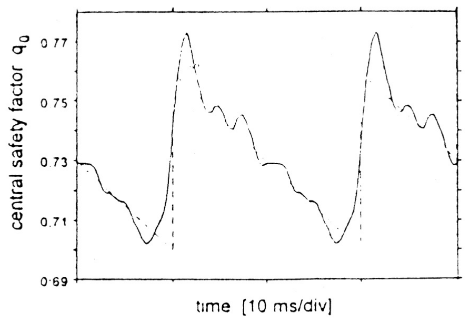

Figure 8: q_0 for TEXTOR.[Hastie]

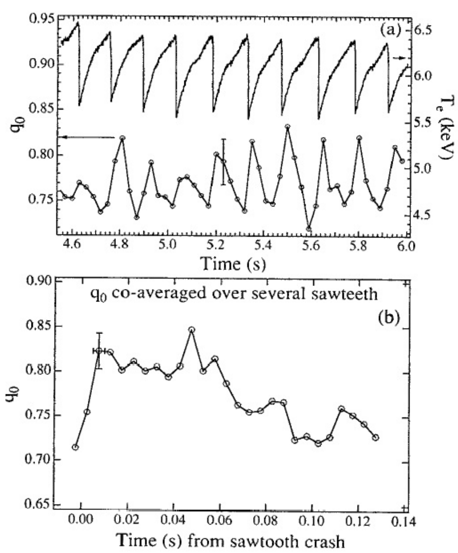

Figure 9: q_0 measurements on the TFTR.[Yamada94]

3 References

1. Aymar, R. *et al* "The ITER design" 2001 *Plasma Phys. Control Fusion* **44** 519.

2. Chapman, I. T. "Controlling sawtooth oscillations in tokamak plasmas" 2011 *Plasma Phys. Control Fusion* **53** 1.

3. Chapman, I. T. *et al* "Magnetic Reconnection Triggering Magnetohydrodynamic Instabilities during a Sawtooth Crash in a Tokamak Plasma" 2010 *Phys. Rev. Lett.* **105** 255002.

4. Chapman, I. T. *et al* "The physics of sawtooth stabilization" 2007 *Plasma Phys. Control Fusion* **49** 12B.

5. Davidson, P. A. An Introduction to Magnetohydrodynamics, Cambridge University Press, 2001.

6. Frey, H. U. *et al* "Continuous magnetic reconnection at Earth's magnetopause" 2003 *Letters to Nature* **426**.

7. Garcia-Martinez, Pablo L. "Dynamics of Magnetic Relaxation in Spheromaks" In Zheng, Linjin (Ed.), Topics in Magnetohydrodynamics (pp. 85-116). Cambridge University Press, 1993.

8. Haas, F. A. "Turulence in fluids and fusion plasmas" In Dendy, Richard (Ed.), Plasma Physics: An Introductory Course (pp. 103-128). Cambridge University Press, 1993.

9. Harry, John Earnest, Introduction to Plasma Technology, Wiley-VCH, 2010.

10. Hastie, R. J. "Sawtooth Instability in Tokamak Plasmas" 1998 *Astrophysics and Space Science* **256** 177-204.

11. Hood, A. W. "Solar plasmas" In Dendy, Richard (Ed.), Plasma Physics: An Introductory Course (pp. 267-289). Cambridge University Press, 1993.

12. Priest, Eric, Magnetohydrodynamics of the Sun, Cambridge University Press, 2014.

13. Sykes, A and J. A. Wesson. "Relaxation Instability in Tokamaks" 1976 *Phys. Rev. Lett.* **37** 140.

14. Wagner, F. "A quarter-century of H-mode studies" 2007 *Plasma Phys. Control. Fusion* **49** B12.

15. Wagner, F. "The Physics Basis of ITER Confinement" 2009 *AIP Conf. Proc.* **1095** 31.

16. Yamada, Masaaki *et al* "Investigation of magnetic reconnection during a sawtooth crash in a hightemperature tokamak plasma" 1994 *Phys. Plasmas* **1** 3269.

17. Yamada, Masaaki *et al* "Magnetic Reconnection" 2010 *Rev. Mod. Phys.* **82** 603.

18. Yun, G. S. *et al* "Appearance and Dynamics of Helical Flux Tubes under Electron Cyclotron Resonance Heating in the Core of KSTAR Plasmas" 2012 *Phys. Rev. Lett.* **109** 145003.