North South University

Department of Mathematics and Physics

Assignment - 1

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7.2

10) $\int \sqrt{x} \ln x \, dx$

$= \ln x \int \sqrt{x}\, dx - \int \left\{ \frac{d}{dx} (\ln x) \int \sqrt{x}\, dx \right\} dx$

$= \ln x \cdot \frac{x^{3/2}}{3/2} - \int \frac{1}{x} \cdot \frac{x^{3/2}}{3/2} \, dx$

$= \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \int x^{1/2} \, dx$

$= \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \cdot \frac{x^{3/2}}{3/2} + C$

$= \frac{2}{3} x^{3/2} \ln x - \frac{4}{9} \cdot x^{3/2} + C$

Ans.

26)

$\int e^{3x} \cos 2x \, dx$

$= \cos 2x \int e^{3x} dx - \int \left\{ \frac{d}{dx} (\cos 2x) \int e^{3x} dx \right\} dx$

$= \cos 2x \cdot \frac{1}{3} e^{3x} - \int -2 \sin 2x \cdot \frac{1}{3} e^{3x} \, dx$

$= \frac{1}{3} e^{3x} \cos 2x + \frac{2}{3} \int e^{3x} \sin 2x \, dx$

$= \frac{1}{3} e^{3x} \cdot \cos 2x + \frac{2}{3} \left[\sin 2x \int e^{3x} dx - \int \left\{ \frac{d}{dx} (\sin 2x) \int e^{3x} dx \right\} dx \right.$

$= \frac{1}{3} e^{3x} \cos 2x + \frac{2}{3} \left[\sin 2x \cdot \frac{1}{3} e^{3x} - \int 2 \cos 2x \cdot \frac{1}{3} e^{3x} dx \right]$

$$= \frac{1}{3} e^{3x} \cos 2x + \frac{2}{9} e^{3x} \cdot \sin 2x - \frac{4}{9} \int e^{3x} \cos 2x \, dx$$

$$\Rightarrow \int e^{3x} \cos 2x \, dx + \frac{4}{9} \int e^{3x} \cos 2x \, dx = \frac{e^{3x} \cos 2x}{3} + \frac{2 \cdot e^{3x} \cdot \sin 2x}{9}$$

$$\Rightarrow \frac{13}{9} \int e^{3x} \cos 2x \, dx = \frac{e^{3x} \cos 2x}{3} + \frac{2 \cdot e^{3x} \cdot \sin 2x}{9}$$

$$\therefore \int e^{3x} \cos 2x \, dx = \frac{3 e^{3x} \cos 2x}{13} + \frac{2 e^{3x} \sin 2x}{13}$$

$$= \frac{1}{13} (3 \cos 2x + 2 \sin 2x) e^{3x} + C$$

Ans.

26) $\int \frac{x e^x}{(x+1)^2} dx$

Let,

$u = x e^x$

$du = (x e^x + e^x) dx$

$du = e^x (x+1) dx$

$dv = \frac{1}{(x+1)^2}$

$\int dv = \int \frac{1}{(x+1)^2} dx$

$v = \frac{(x+1)^{-1}}{-1}$

$\therefore v = -\frac{1}{(x+1)}$

$$\therefore \int \frac{xe^x}{(x+1)^2}\,dx$$

$$= xe^x \cdot \frac{-1}{(x+1)} - \int \frac{-1}{x+1}\,dx$$

$$= -\frac{xe^x}{x+1} + \int \frac{1}{x+1}\cdot e^x(x+1)\,dx$$

$$= -\frac{xe^x}{x+1} + \int e^x\,dx$$

$$= -\frac{xe^x}{x+1} + e^x + c$$

Ans.

32) $$\int_0^{\sqrt{3}/2} \sin^{-1}x\,dx$$

$$= \left[\sin^{-1}\int 1\,dx\right]_0^{\sqrt{3}/2} - \int_0^{\sqrt{3}/2} \left\{\frac{d}{dx}(\sin^{-1}x)\int 1\,dx\right\}dx$$

$$= \left[x\sin^{-1}x\right]_0^{\sqrt{3}/2} - \int_0^{\sqrt{3}/2} \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= \left[x\sin^{-1}x\right]_0^{\sqrt{3}/2} + \left[\sqrt{1-x^2}\right]_0^{\sqrt{3}/2}$$

$$= \frac{\sqrt{3}}{2}\sin^{-1}\frac{\sqrt{3}}{2} + \left[\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2} - \sqrt{1}\right]$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\pi}{3} + \frac{1}{2} - 1$$

$$= \frac{\pi\sqrt{3}}{6} - \frac{1}{2}$$

Ans

38) $\int_0^2 \ln(x^2+1)\,dx$

$= \left[\ln(x^2+1)\int 1\,dx\right]_0^2 - \int_0^2 \left\{\frac{d}{dx}\left(\ln(x^2+1)\right)\int 1\,dx\right\} dx$

$= x\ln(x^2+1)\Big]_0^2 - \int_0^2 \frac{2x^2}{x^2+1}\,dx$

$= x\ln(x^2+1)\Big]_0^2 - 2\int_0^2 \left(\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\right) dx$

$= 2\ln 5 - 2\int \left(1-\frac{1}{x^2+1}\right) dx$

$= 2\ln 5 - 2\left[x - \tan^{-1}x\right]_0^2$

$= 2\ln 5 - 2\left[2-\tan^{-1}2\right]$

$= 2\ln 5 - 4 + 2\tan^{-1}2$

Ans.

7.3

10) $\int \sin^3 x \cos^2 x \, dx$

$= \int \sin^2 x \cos^2 x \sin x \, dx$

$= \int (1-\cos^2 x) \cos^2 x \sin x \, dx$

Let,

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$\therefore = -\int (1-u^2) u^2 \, du$

$= -\int (u^2 - u^4) \, du$

$= -\frac{u^3}{3} + \frac{u^5}{5} + C$

$= \frac{-1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + C$

Ans

30) $\int \tan^5 x \sec^4 x \, dx$

$= \int \tan^5 x \sec^2 x \sec^2 x \, dx$

$= \int \tan^5 x (\tan^2 x + 1) \sec^2 x \, dx$

Let,

$u = \tan x$

$du = \sec^2 x \, dx$

$\therefore \int u^5 (u^2+1) \, du$

$= \int (u^7 + u^5) \, du$

$= \frac{u^8}{8} + \frac{u^6}{6} + c$

$= \frac{1}{8} \tan^8 x + \frac{1}{6} \tan^6 x + c$

Ans

42) $\int \tan^4 x$

$= \frac{\tan^3 x}{3} - \int \tan^2 x \, dx$

$= \frac{1}{3} \tan^3 x - \left[\frac{\tan x}{1} - \int \tan^0 x \, dx \right]$

$= \frac{1}{3} \tan^3 x - \tan x + x + c$

Ans

46) $\int_0^{\pi/6} \sec^3 2\theta \tan 2\theta \, d\theta$

$= \int_0^{\pi/6} \sec^2 2\theta \sec 2\theta \cdot \tan 2\theta \, d\theta$

Let,

$u = \sec 2\theta$

$du = \sec 2\theta \tan 2\theta \cdot 2 \, d\theta$

$\frac{1}{2} du = \sec 2\theta \tan 2\theta \, d\theta$

if, $\theta = \pi/6$

$u = \sec \frac{\pi}{3} = 2$

$\theta = 0$

$u = \sec 0 = 1$

$\therefore = \int_1^2 u^2 \cdot \frac{1}{2} du$

$= \frac{1}{2} \int_1^2 u^2 du$

$= \frac{1}{2} \left[\frac{u^3}{3} \right]_1^2$

$= \frac{1}{2} \left[\frac{8}{3} - \frac{1}{3} \right]$

$= \frac{7}{6}$ Ans

48) $\int_0^{1/4} \sec \pi x \tan \pi x \, dx$

Let,

$u = \pi x$

$du = \pi \, dx$

$dx = \frac{1}{\pi} du$

if, $\cancel{u=0}$

$x = 0,$

$u = 0$

if, $x = 1/4$

$u = \frac{\pi}{4}$

$\therefore \; = \frac{1}{\pi} \int_0^{\pi/4} \sec u \tan u \, du$

$= \frac{1}{\pi} \left[\sec u \right]_0^{\pi/4}$

$= \frac{1}{\pi} \left[\sec \frac{\pi}{4} - 1 \right]$

$= \frac{\sqrt{2}-1}{\pi}$ Ans

7.4

10) $\int x^3 \sqrt{5-x^2}\, dx$

Let,

$u = 5 - x^2$ | $x^2 = 5 - u$

$du = -2x\, dx$

$x\,dx = \dfrac{du}{-2}$

$\therefore = \int (5-u)\sqrt{u}\, \dfrac{du}{-2}$

$= -\dfrac{1}{2} \int (5u^{\frac{1}{2}} - u^{3/2})\, du$

$= -\dfrac{1}{2}\left[5 \cdot \dfrac{u^{3/2}}{3/2} - \dfrac{u^{5/2}}{5/2}\right] + c$

$= -\dfrac{1}{2} \cdot 5 \cdot \dfrac{2}{3} (5-x^2)^{3/2} + \dfrac{1}{2} \cdot \dfrac{2}{5} \cdot (5-x^2)^{5/2} + c$

$= -\dfrac{5}{3}(5-x^2)^{3/2} + \dfrac{1}{5}(5-x^2)^{5/2} + c$

Ans.

20) $\int \frac{\cos\theta}{\sqrt{2-\sin^2\theta}}\, d\theta$

Let,

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$= \int \frac{du}{\sqrt{2-u^2}}$

$= \sin^{-1}\frac{u}{\sqrt{2}} + c$

$= \sin^{-1}\left(\frac{\sin\theta}{\sqrt{2}}\right) + c$

Ans

40) $\int \frac{dx}{16x^2+16x+5}$

$= \int \frac{1}{(4x)^2 + 2\cdot 4x\cdot 2 + 2^2 + 1}\, dx$

$= \int \frac{1}{(4x+2)^2+1}\, dx$

Let,

$u = 4x+2$

$du = 4\, dx$

$dx = \frac{1}{4} du$

$\therefore = \frac{1}{4}\int \frac{1}{u^2+1}\, du$

$= \frac{1}{4}\tan^{-1}u + c$

$= \frac{1}{4}\tan^{-1}(4x+2) + c$

Ans.

46) $\int \frac{2x+3}{4x^2+4x+5}\,dx$

$= \int \frac{2x+3}{(2x)^2+2\cdot 2x\cdot 1+1^2+4}\,dx$

$= \int \frac{(2x+1)+2}{(2x+1)^2+4}\,dx$

Let,

$u = 2x+1$

$du = 2\,dx$

$\therefore = \frac{1}{2}\int \frac{u+2}{u^2+4}\,du$

$= \frac{1}{2}\int \left(\frac{u}{u^2+4}+\frac{2}{u^2+4}\right)du$

$= \frac{1}{2}\cdot\frac{1}{2}\int \frac{2u}{u^2+4}\,du + \frac{1}{2}\cdot 2\int \frac{1}{u^2+2^2}\,du$

$= \frac{1}{4}\ln(u^2+4) + \frac{1}{2}\tan^{-1}\frac{u}{2} + c$

$= \frac{1}{4}\ln\left((2x+1)^2+4\right) + \frac{1}{2}\tan^{-1}\left(\frac{2x+1}{2}\right) + c$

$= \frac{1}{4}\ln(4x^2+4x+5) + \frac{1}{2}\tan^{-1}\left(x+\frac{1}{2}\right) + c$

Ans.

48) $\int_0^4 \sqrt{x(4-x)}\,dx$

$= \int_0^4 \sqrt{4x-x^2}\,dx$

$= \int_0^4 \sqrt{-x^2+4x-4+4}\,dx$

$= \int_0^4 \sqrt{4-(x-2)^2}\,dx$

Let,

$x-2 = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

if, $x=0$,

$-2 = 2\sin\theta$

$\theta = \sin^{-1}(-1) = -\frac{\pi}{2}$

if, $x=4$,

$2 = 2\sin\theta$

$\theta = \sin^{-1}(1) = \frac{\pi}{2}$

$\therefore\ = \int_{-\pi/2}^{\pi/2} 2\cos\theta \cdot 2\cos\theta\,d\theta$

$= 4\int_{-\pi/2}^{\pi/2} \cos^2\theta\,d\theta$

$= 4 \cdot \frac{1}{2} \int_{-\pi/2}^{+\pi/2} (1+\cos 2\theta)\,d\theta$

$= 2\left[\theta + \frac{\sin 2\theta}{2}\right]_{-\pi/2}^{\pi/2} = 2\left(\pi/2 + \frac{\pi}{2}\right)$

$= 2\pi$ Ans