NORTH SOUTH UNIVERSITY

Department of Mathematics & Physics

Assignment – 01

Name	: Joy Kumar Ghosh
Student ID	: 2211424 6 42
Course No.	: MAT 361
Course Title	: Probability and Statistics
Section	: 10
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2.1.1/

a)

Given,

the random variable is x.

$x: \{0, 1, 2, 3, 4\}$

$P(x=0) = 0.08$

$P(x=1) = 0.11$

$P(x=2) = 0.27$

$P(x=3) = 0.33$

$P(x=4) = ?$

We know,

$$P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) = 1$$

$$\therefore P(x=4) = 1 - 0.08 - 0.11 - 0.27 - 0.33$$

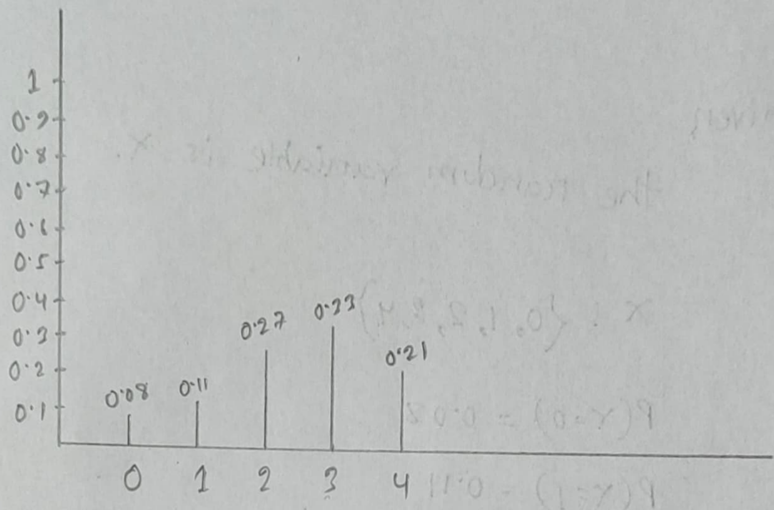
$$= 0.21$$

b)

Probability mass function,

x	0	1	2	3	4
$P(x)$	0.08	0.11	0.27	0.33	0.21

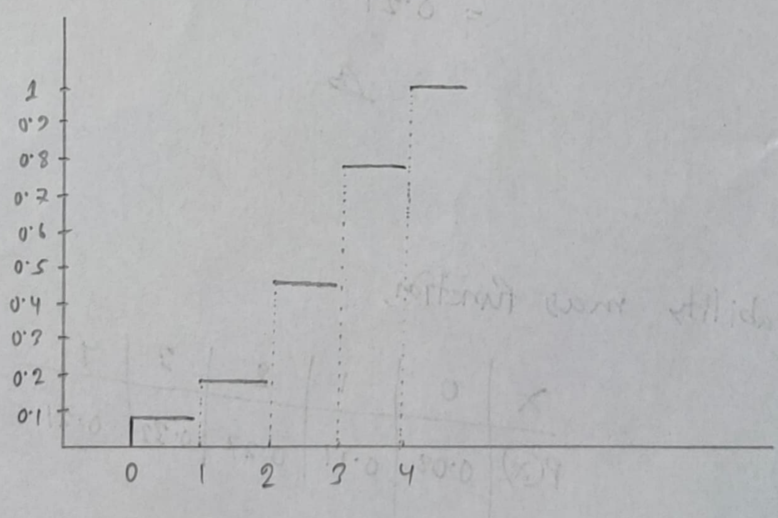
Graph:



c)

CDF:

X	0	1	2	3	4
P(x)	0·08	0·11	0·27	0·33	0·21
F(x)	0·08	0·19	0·46	0·79	1



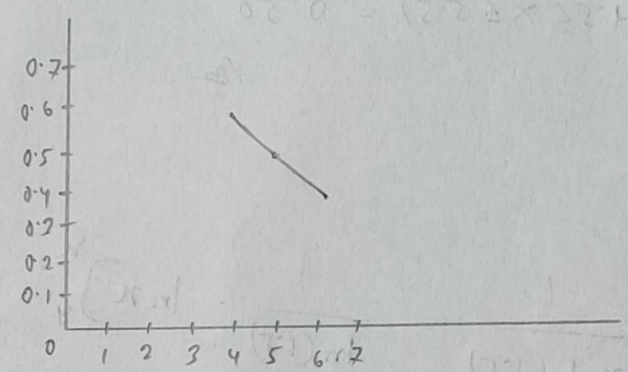
2.2.2/

a)

Given density function,

$$f(x) = \frac{1}{x \ln(1.5)} \quad ; \quad 4 \le x \le 6$$

sketch of this function,



b)

Here,

$$\int_4^6 \frac{1}{x \ln(1.5)} = \frac{1}{\ln(1.5)} \cdot \ln x \Big]_x^6$$

$$= \frac{\ln 6 - \ln 4}{\ln(1.5)}$$

$$= 1$$

Therefore, the total area under the density function is equal to 1.

c)

$$\int_{4\cdot5}^{5\cdot5} \frac{1}{x\ln(1\cdot5)} = \frac{1}{\ln(1\cdot5)} \cdot \ln x \Big]_{4\cdot5}^{5\cdot5}$$

$$= \frac{\ln(5\cdot5) - \ln(4\cdot5)}{\ln(1\cdot5)}$$

$$= 0\cdot50$$

$\therefore P(4\cdot5 \le X \le 5\cdot5) = 0\cdot50$

d)

$$\int_{4}^{x} \frac{1}{x\ln(1\cdot5)} = \frac{1}{\ln(1\cdot5)} \cdot \ln x \Big]_{4}^{x}$$

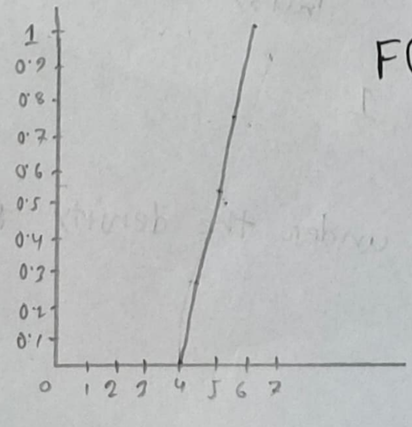
$$= \frac{\ln x - \ln 4}{\ln(1\cdot5)}$$

Therefore,

~~Cumulat~~

Cumulative distribution function is,

$$F(x) = \frac{\ln x - \ln 4}{\ln(1\cdot5)}$$



2.2.6/

a)

Given,

probability density function,

$$f(x) = A(0.5-(x-0.25)^2)$$

$$0.125 \le x \le 0.5$$

$$\therefore \int_{0.125}^{0.5} A(0.5-(x-0.25)^2)\,dx$$

$$= A\int_{0.125}^{0.5} (0.5 - x^2 + 0.5x - 0.0625)\,dx$$

$$= A\int_{0.125}^{0.5} (0.4375 - x^2 + 0.5x)\,dx$$

$$= A\left[0.4375x - \frac{x^3}{3} + 0.25x^2\right]_{0.125}^{0.5}$$

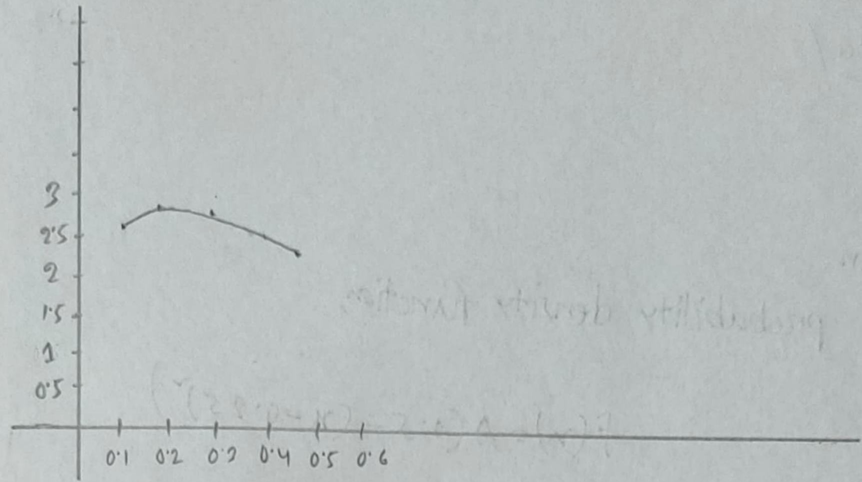
$$= A\left[0.21875 - \frac{1}{24} + 0.0625 - \frac{7}{128} + \frac{1}{1536} - \frac{1}{256}\right]$$

$$= A\,\frac{93}{512}$$

Here,

$$A\,\frac{93}{512} = 1$$

$$\therefore A = \frac{512}{93} \text{ Ans}$$



b)

Given,

density function,

$$f(x) = \frac{512}{93}\left(0.5-(x-0.25)^2\right) \quad ; \quad 0.125 \le x \le 0.5$$

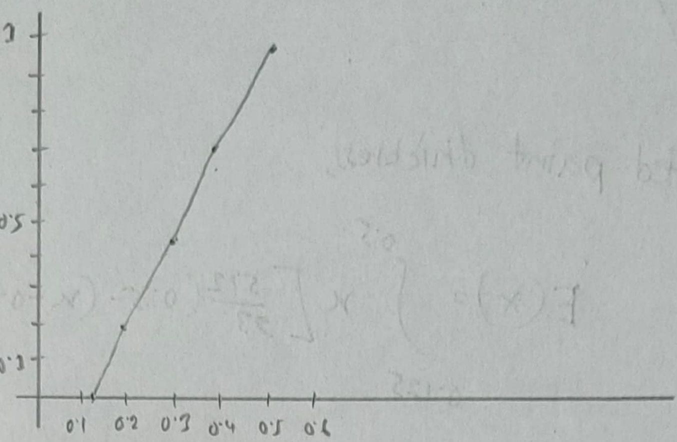
$$\therefore \int_{0.125}^{x} \frac{512}{93}\left(0.5-(x-0.25)^2\right) dx$$

$$= \frac{512}{93}\left[0.4375x - \frac{x^3}{3} + 0.25x^2\right]_{0.125}^{x}$$

$$= \frac{512}{93}\left[0.4375x - \frac{x^3}{3} + 0.25x^2 - \frac{89}{1536}\right]$$

$\therefore$ CDF,

$$F(x) = \frac{512}{93}\left[0.4375x - \frac{x^3}{3} + 0.25x^2 - \frac{89}{1536}\right]$$



c)

$$P(0.125 \le x \le 0.2) = \frac{512}{93}\left[0.4375x - \frac{x^3}{3} + 0.25x^2 - \frac{87}{1536}\right]$$

where, $x = 0.2$

$$= \frac{512}{93}\left[0.4375 \times 0.2 - \frac{(0.2)^3}{3} + 0.25 \times 0.2^2 - \frac{87}{1536}\right]$$

$$= 0.20$$

2.3.1/

Probability mass function,

x	0	1	2	3	4
P(x)	0.08	0.11	0.27	0.33	0.21

$$E(x) = \Sigma\, x \cdot P(x)$$

$$= (0 \times 0.08) + (1 \times 0.11) + (2 \times 0.27) + (3 \times 0.33) + (4 \times 0.21)$$

$$= 2.48$$

2.3.12/

Expected paint thickness,

$$E(x) = \int_{0.125}^{0.5} x\left[\frac{512}{93}\left(0.5-(x-0.25)^2\right)\right] dx$$

$$= \int_{0.125}^{0.5} \cancel{x}\ \frac{512}{93} x\left(0.4375 - x^2 + 0.5x\right) dx$$

$$= \int_{0.125}^{0.5} \left(\frac{224}{93}x - \frac{512}{93}x^3 + \frac{256}{93}x^2\right) dx$$

$$= \left[\frac{112}{93}x^2 - \frac{128}{93}x^4 + \frac{256}{279}x^3\right]_{0.125}^{0.5}$$

$$= \frac{92}{279} - \frac{181}{8928}$$

$$= \frac{307}{992}$$

$$= 0.31$$

Ans

Median paint thickness.

when,

$$CDF = 0.5$$

$$\Rightarrow \frac{512}{93}\left[0.4375x - \frac{x^3}{3} + 0.25x^2 - \frac{89}{1536}\right] = 0.5$$

$$\Rightarrow 0.4375x - \frac{x^3}{3} + 0.25x^2 - \frac{89}{1536} = \frac{93}{1024}$$

$$\Rightarrow -\frac{x^3}{3} + 0.25x^2 + 0.4375x - \frac{457}{3072} = 0$$

$$\therefore x = -1,\ 1.44,\ 0.31$$

Hence,

$$0.125 \le x \le 0.5$$

$\therefore x = 0.31$ is the median.

Ans

2.4.5/

Given, density function,

$$f(x) = \frac{1}{x\ln(1.5)} \quad ; \quad 4 \le x \le 6$$

a)

$$E(x) = \int_4^6 \frac{1}{\ln(1.5)} \, dx$$

$$= \frac{1}{\ln(1.5)} \left. x \right]_4^6$$

$$= \frac{2}{\ln(1.5)}$$

$$= 4.93$$

$$E(x^2) = \int_4^6 \frac{x}{\ln(1.5)} \, dx$$

$$= \frac{1}{\ln(1.5) \times 2} \left. x^2 \right]_4^6$$

$$= \frac{20}{\ln(1.5) \times 2}$$

$$= 24.66$$

$\therefore$ Variance, $V(x) = E(x^2) - [E(x)]^2 = 24.66 - 24.30$

$= 0.36$

b)

We know,

$$SD(x) = \sqrt{V(x)} = \sqrt{0.36}$$
$$= 0.6$$

Ans

c)

From 2.22,

$$CDF = \frac{\ln x - \ln 4}{\ln(1.5)}$$

for upper quantiles,

$$CDF = 0.75$$

$$\Rightarrow \frac{\ln x - \ln 4}{\ln(1.5)} = 0.75$$

$$\Rightarrow \ln x = 1.69$$

$$\therefore x = e^{1.69}$$
$$= 5.42$$

Ans

For lower quantiles,

$$CDF = 0.25$$

$$\Rightarrow \frac{\ln x - \ln 4}{\ln(1.5)} = 0.25$$

$$\Rightarrow \ln x = 1.49$$

$$\Rightarrow x = e^{1.49}$$
$$= 4.44$$

Ans

d)

We know,

inten quantile range $IQR = Q_3 - Q_1$

$= 5.42 - 4.44$

$= 0.98$

2.5.3/

a) $$\int_4^6 \int_{-2}^{3} A(x-3)y \, dx\, dy = A\int_4^6 y\left[\frac{x^2}{2} - 3x\right]_{-2}^{3} dy$$

$$= A\int_4^6 -\frac{25}{2} y \, dy$$

$$= -\frac{25A}{2}\left[\frac{y^2}{2}\right]_4^6$$

$$= -\frac{25A}{2} \cdot 10$$

$$= -125A$$

We know,

$-125A = 1$

$\therefore A = -\frac{1}{125}$

$\therefore f(x,y) = -\frac{1}{125}(x-3)y$

$= \frac{1}{125}(3-x)y$

b)

$$P(0 \le x \le 1,\ 4 \le y \le 5) = \frac{1}{125}\int_4^5\int_0^1 (3-x)y\,dx\,dy$$

$$= \frac{1}{125}\int_4^5 y\left[3x - \frac{x^2}{2}\right]_0^1 dy$$

$$= \frac{1}{50}\int_4^5 y\,dy$$

$$= \frac{1}{50}\cdot\frac{y^2}{2}\Big]_4^5$$

$$= \frac{1}{50}\cdot\frac{9}{2}$$

$$= \frac{9}{100}$$ Ans

c)

$$f_x(x) = \frac{1}{125}\int_4^6 (3-x)y\,dy$$

$$= \frac{3-x}{125}\cdot\frac{y^2}{2}\Big]_4^6$$

$$= \frac{3-x}{125}\cdot 10$$

$$= \frac{2}{25}(3-x)\ ;\ -2 \le x \le 3$$

$$f_y(y) = \frac{1}{125}\int_{-2}^{3} (3-x)y\,dx$$

$$= \frac{y}{125}\left[3x - \frac{x^2}{2}\right]_{-2}^{3}$$

$$= \frac{y}{125}\cdot\frac{25}{2} = \frac{y}{10}\ ;\ 4 \le y \le 6$$

d)

Here,

$$f_x(x)\cdot f_y(y) = \frac{2}{25}(3-x)\cdot\frac{y}{10}$$

$$= \frac{1}{125}(3-x)y$$

$$= f(x,y)$$

$\therefore$ X & Y are independent random variables.

e)

$$f_Y(y) = \frac{y}{10}$$

$$\therefore f_Y(5) = \frac{5}{10} = \frac{1}{2}$$

$$\therefore \cancel{f_{X|Y=5}(x) = \frac{f(x,y)}{f_Y(y)}}$$

$$\cancel{= \frac{\frac{1}{125}(3-x)y}{\frac{1}{2}}}$$

$$\cancel{= \frac{2}{125}(3-x)y}$$

$$\therefore f_{X|Y=5}(x) = \frac{f(x,5)}{f_Y(5)} = \frac{\frac{1}{125}(3-x)\cdot 5}{\frac{1}{2}}$$

$$= \frac{2}{25}(3-x)$$

Ans