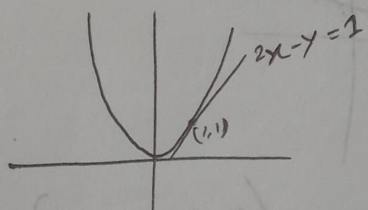
Slope of a tangent line.

i.e. Slope at a single point.

$$= \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

Find the equation of a tangent line to the parabola $y = x^2$ at a point $(1,1)$.

$\Rightarrow$



Now, we know that, slope of a tangent line,

$$m_{tan} = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

Here,

$x_0 = 1$

$$\therefore m_{tan} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(1+h)^2 - f(1)}{h}$$

Here,

$f(x) = x^2$

$f(1) = 1$

$f(1+h) = (1+h)^2$

$$= \lim_{h \to 0} \frac{1+2h+h^2-1}{h}$$

$$= \lim_{h \to 0} \frac{2h+h}{h}$$

$$= \lim_{h \to 0} (2+h)$$

$$= 2$$

$\therefore m_{tan} = 2$

and point $(1,1)$.

$\therefore$ equation of tangent line,

$$(y-y_1) = m(x-x_1)$$

$$y-1 = 2(x-1)$$

$$y-1 = 2x-2$$

$$2x-y = 1$$

(*) Find the equation of a tangent line to the parabola $f(x) = \frac{1}{x}$ at $x=2$.

$\Rightarrow$

Now, we know that, slope of a tangent line,

$$m_{tan} = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

Here,
$f(x) = \frac{1}{x}$
$f(2+h) = \frac{1}{2+h}$
$f(2) = \frac{1}{2}$

$$= \lim_{h \to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2-2-h}{2(2+h)}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{2(2+h) \cdot h}$$

$$= -\frac{1}{2(2+0)} = \frac{-1}{4}$$

$m_{tan} = -\frac{1}{4}$

point $(2, \frac{1}{2})$

equation,

$$y - \frac{1}{2} = -\frac{1}{4}(x-2)$$

$$y - \frac{1}{2} = -\frac{1}{4}x + \frac{1}{2}$$

$$y = -\frac{1}{4}x + 1$$

$$4y = -x + 4$$

$$\therefore x + 4y = 4$$ Ans

⊛ velocity $= \dfrac{\text{tatal distance}}{\text{tatal time}}$

$$s = f(t)$$

$$V_{ag} = \frac{S_2 - S_1}{t_2 - t_1}$$

$$= \frac{f(t_0+h) - f(t_0)}{t_0 + h - t_0}$$

$$= \frac{f(t_0+h) - f(t_0)}{h}$$

Velocity

Average Velocity

$$\frac{f(t_0+h) - f(t_0)}{h}$$

Instanteneous velocity

$$\lim_{h \to 0} \frac{f(t_0+h) - f(t_0)}{h}$$

⊛ A particle moves on a line away from its initial position so that after t hours it is

$$s = 3t^2 + t \text{ mile.}$$

a) Find the average velocity over $t=1$ to $t=3$ hour

b) Find the instantneous velocity at $t=1$ hour.

$\Rightarrow$

a)

at $t=1$, $s(1) = 3+1 = 4$

$t=3$, $s(3) = 3\times3^2+3 = 30$

$$\text{Average Velocity} = \frac{s(3)-s(1)}{3-1}$$

$$= \frac{30-4}{2} = 13 \text{ mile/hour.}$$

b)

Now, intanteneous velocity,

$$V_{is} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{3(1+h)^2 + (1+h) - 4}{h}$$

$$= \lim_{h \to 0} \frac{3(1+2h+h^2) + 1 + h - 4}{h}$$

$$= \lim_{h \to 0} \frac{6h + 3h^2 + h}{h}$$

$$= \lim_{h \to 0} 7 + 3h$$

$= 7$ mile/hour

Ans

H.W.

$s(t) = 1 + 5t - 2t^2$

a) $t = 1$ to $t = 3$

b) $t = 1$?

$t = 2$?

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2.2/

Average rate of change $= \dfrac{f(x_0+h) - f(x_0)}{h}$

$P(x_0, f(x_0))$

$Q(x_0+h, f(x_0+h))$

Instantaneous rate of change,

$$\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

⊛ $\lim_{h \to 0} \dfrac{f(x_0+h) - f(x_0)}{h}$

i) Instantaneous rate of change at $x = x_0$

ii) Slope at a single point at $x = x_0$

iii) Slope of tangent line at $x = x_0$

iv) Also this limit has special name, i.e. called derivative at $x = x_0$, and define by

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

⊛ Generally for any values of x, we will define,

Derivative of function $f(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

For $x = 2$,

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

For $x = -1$

$$f'(-1) = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$$

⊛ Find the derivative of $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x}+\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}} \quad \underline{\text{Ans}}$$

⊛ Find the tangent line to the curve $f(x) = \sqrt{x}$ ~~of~~ at $x = 4$.

$\Rightarrow$ at $x = 4$,

$f(x) = \sqrt{x}$

$f(4) = \sqrt{4} = 2$

$\therefore$ point $(4,2)$

$\therefore$ slope at $x = 4$,

from previous question

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2}$

$= \frac{1}{4}$

Therefore,

equation of a tangent line,

$$y-2 = \frac{1}{4}(x-4)$$

$$y-2 = \frac{1}{4}x-1$$

$$y = \frac{1}{4}x+1$$

Ans

Differentiability!

If this limit $f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$ exists, then we would say that the function $f(x)$ is differentiable at a point $x = x_0$.

Lef hand derivative (L.H.D.)

$$Lf'(x_0) = \lim_{h \to 0^-} \frac{f(x_0+h)-f(x_0)}{h}$$

Right hand derivative (R.H.D)

$$Rf'(x_0) = \lim_{h \to 0^+} \frac{f(x_0+h)-f(x_0)}{h}$$

If, L.H.D = R.H.D,

$$\lim_{h \to 0^-} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h \to 0^+} \frac{f(x_0+h)-f(x_0)}{h}$$, then

$\lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$ exist and then function is differentiable at $x = x_0$.

⊛ Show that, $f(x) = |x|$ is not differentiable at $x=0$.

$$f(x) = |x|$$

$$= \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

Now,

L.H.D, at $x=4$,

$$Lf'(0) = \lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0^-} \frac{-h-0}{h}$$

$$= \lim_{h \to 0^-} -1$$

$$= -1$$

Here,
$f(x) = -x$
$f(0+h) = -(0+h) = -h$
$f(0) = 0$

R.H.D at $x=0$,

$$Rf'(0) = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0^+} \frac{h-0}{h}$$

$$= \lim_{h \to 0^+} 1$$

$$= 1$$

Here,
$f(x) = x$
$f(0+h) = h$
$f(0) = 0$

$\therefore$ L.H.D $\neq$ R.H.D

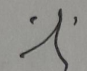
Hence, Limit doesn't exists.

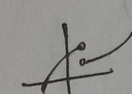
Therefore, the function is not differentiable at $x=0$.

(proved)

* When does a function not have derivative at a point.

i) A corner 'V' or 'Λ'

ii) A cusp 

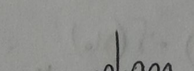
iii) A discontinuity 

iv) Vertical tangency.

(tangent line will be vertical)

$\therefore$ slope of tangent line = undefined.

i.e. Derivative = undefined

v) $\sin\frac{1}{x}$ 

⊛ Relation between differentiability and continuity:

<u>Theorem</u>: If a function differentiable, then the function is continuous at $x = x_0$.

For continuity, we have to show,

$$\lim_{x \to x_0} f(x) = f(x_0)$$

$$\lim_{x \to x_0} f(x) - f(x_0) = 0$$

let,

$h = x - x_0 \quad \therefore x = x_0 + h$

if $x \to x_0$

then $h \to 0$

$$\Rightarrow \lim_{h \to 0} [f(x_0+h) - f(x_0)] = 0$$

$$\Rightarrow \lim_{h \to 0} \left[\frac{f(x_0+h) - f(x_0)}{h} \times h \right]$$

$$\Rightarrow \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} \times \lim_{h \to 0} h$$

if a function differentiable, then $f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$ exists.

$$= f'(x_0) \times 0 = 0$$

if a function differentiable then,

$$\lim_{h \to 0} [f(x_0+h) - f(x_0)] = 0$$

$$\therefore \lim_{h \to 0} f(x_0+h) = f(x_0)$$

Show that $f(x) = \begin{cases} x & 0 \le x < \frac{1}{2} \\ 1-x & \frac{1}{2} \le x \le 2 \end{cases}$ is continuous but not differentiable at $x = \frac{1}{2}$. Also draw the graph.

$\Rightarrow$ Check the continuity at $x = \frac{1}{2}$.

1st check

$f(x) = 1-x$

$\therefore f\left(\frac{1}{2}\right) = 1 - \frac{1}{2} = \frac{1}{2}$

2nd check

$$\lim_{x \to \frac{1}{2}^-} f(x) = \lim_{x \to \frac{1}{2}^-} x$$

$$= \frac{1}{2}$$

$$\lim_{x \to \frac{1}{2}^+} f(x) = \lim_{x \to \frac{1}{2}^+} 1-x$$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

$\therefore$ L.H.L = R.H.L.

$\therefore$ Limit exist

3rd check

$$\lim_{x \to \frac{1}{2}} f(x) = f(x)$$

Therefore, this function is continuous at $x = \frac{1}{2}$.

2nd Part

Now for derivative :

L.H.D at $x=\frac{1}{2}$,

$$Lf'(\tfrac{1}{2}) = \lim_{h\to 0^-} \frac{f(\frac{1}{2}+h)-f(\frac{1}{2})}{h}$$

Here,
$f(x)=x$
$f(\frac{1}{2}+h)=\frac{1}{2}+h$
$f(\frac{1}{2})=\frac{1}{2}$

$$= \lim_{h\to 0^-} \frac{\frac{1}{2}+h-\frac{1}{2}}{h}$$

$$= 1$$

R.H.D at $x=\frac{1}{2}$

$$Rf'(\tfrac{1}{2}) = \lim_{h\to 0^+} \frac{f(\frac{1}{2}+h)-f(\frac{1}{2})}{h}$$

Here,
$f(x)=1-x$
$f(1/2+h)=1-\frac{1}{2}-h$
$=\frac{1}{2}-h$
$f(\frac{1}{2})=1-\frac{1}{2}=\frac{1}{2}$

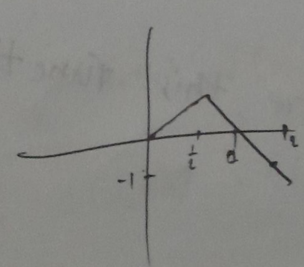
$$= \lim_{h\to 0^+} \frac{\frac{1}{2}-h-\frac{1}{2}}{h}$$

$$= -1$$

$\therefore$ L.H.D $\neq$ R.H.D

Hence function is not differentiable at $x=\frac{1}{2}$.

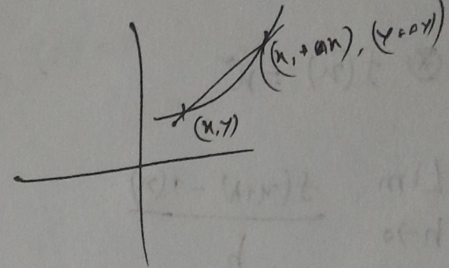
Graph :



(*) Show that $f(x) = \begin{cases} x^2+2 & x \le 1 \\ -x+4 & x > 1 \end{cases}$ check at $x=1$

(*) $f(x) = \begin{cases} x^2+1 & x \le 1 \\ 2x & x > 1 \end{cases}$ check at $x=1$

(*) Derivative of $f(x)$,



$f'(x)$

$\frac{dy}{dx}$

y_1

y'

$$\lim_{\Delta x \to 0} \frac{y+\Delta y - y}{x+\Delta x - x}$$

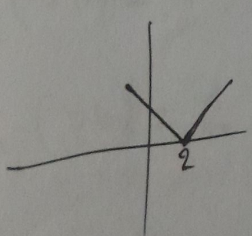
$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$f'(x) = \frac{dy}{dx}$$

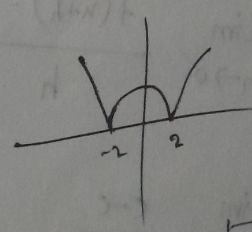
(*) Find all points where the following function fails to differentiable.

a) $f(x) = |x-2|$

b) $f(x) = |x^2-4|$



$\boxed{x=2}$



$\boxed{x=-2,2}$

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2.3/

⊛ Technique of Derivative:

⊛ $f(x) = x^2$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2h + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h}$$

$$= 2x + 0 = \cancel{2h}\ 2x$$

⊛ if $f(x) = x^2$

$$\frac{d}{dx}(x^2) = 2x.$$

⊛ $f(x) = c$, $f(x+h) = c$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{c-c}{h}$$

$$= 0$$

i) Derivative of constant function:

$f(x) = c,$

i.e. $f'(x) = \frac{d}{dx}(c) = 0$

$\frac{d}{dx}(1) = 0$ | $\frac{d}{dx}(5) = 0$

$\frac{d}{dx}(-\sqrt{2}) = 0$

ii) Derivative of power function

$f(x) = x^n$

$f'(x) = \frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(x^2) = 2x^{2-1} = 2x$

$\frac{d}{dx}(x^4) = 4x^3$

$\frac{d}{dx}(x^{\frac{1}{2}}) = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}(x^{-8}) = -8x^{-8-1}$

$= -8x^{-9}$

※ $\frac{d}{dx}(f(x) \pm g(x))$

$= \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$

※ $\frac{d}{dx}(c \cdot f(x))$

$= c \cdot \frac{d}{dx}(f(x))$

If $f(x) = 8x^5 + 7x^3 + 3x - 1$

$$f'(x) = \frac{d}{dx}(8x^5 + 7x^3 + 3x - 1)$$

$$= 5.8x^4 + 7.3.x^2 + 3.1.x^0 - 0$$

$$= 40x^4 + 21x^2 + 3$$

If $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$

$$f'(x) = \frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}-1}$$

$$= \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$$ Ans

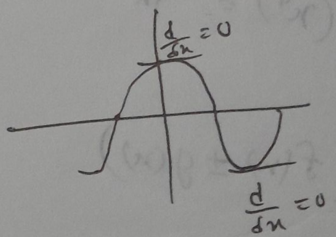
Horizontal tangent line:

$\Rightarrow$ tangent line is horizontal

$\Rightarrow$ slope $= 0$

$\Rightarrow \frac{dy}{dx} = 0$

$\Rightarrow f'(x) = 0$



At what points the function $f(x) = x^3 - 3x + 4$ have horizontal tangent line.

$\Rightarrow$

Here, $f(x) = x^3 - 3x + 4$

$\therefore f'(x) = 3x^2 - 3$

For horizontal tangent line, $f'(x) = 0$

$$3x^2 - 3 = 0$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

$x = 1$, $f(1) = 1 - 3 + 4 = 2$; point $(1, 2)$

$x = -1$, $f(-1) = -1 + 3 + 4 = 6$; point $(-1, 6)$

⊛ Higher derivative,

$$y = f(x)$$

$$\frac{dy}{dx} = f'(x)$$

$$\frac{d^2y}{dx^2} = f''(x)$$

$$\frac{d^3y}{dx^3} = f'''(x)$$

$\vdots$

$$\frac{d^ny}{dx^n} = f^n(x)$$

$$y = x^3 + 2x^2 - 1$$

$$\frac{dy}{dx} = 3x^2 + 4x$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(3x^2 + 4x)$$

$$\frac{d^2y}{dx^2} = 6x + 4$$

$$\frac{d^3y}{dx^3} = 6x$$

$$\frac{d^4y}{dx^4} = 0$$

if $y = 4x^4 + 2x^3 + 3$

find $y_3(0)$ / $\left.\frac{d^3y}{dx^3}\right|_{x=0}$ / $f'''(0)$ / $y'''(0)$

$$y = 4x^4 + 2x^3 + 3$$

$$\frac{dy}{dx} = 16x^3 + 6x^2$$

$$\frac{d^2y}{dx^2} = 48x^2 + 12x$$

$$\frac{d^3y}{dx^3} = 96x + 12$$

$$y_3|_{x=0} = 96 \cdot 0 + 12 = 12$$ Ans

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}\left(f(x)\right) \pm \frac{d}{dx}\left(g(x)\right)$$

2.4

Product and Quotient Rule

Product Rule

$$\frac{d}{dx}\left(f(x) \cdot g(x)\right)$$

$$= f(x) \cdot \frac{d}{dx}\left(g(x)\right) + g(x) \cdot \frac{d}{dx}\left(f(x)\right)$$

$$\frac{d}{dx}(uv) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$$

$$= \frac{g(x) \cdot \frac{d}{dx}\left(f(x)\right) - f(x) \cdot \frac{d}{dx}\left(g(x)\right)}{\left(g(x)\right)^2}$$

$$\boxed{\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \frac{dv}{dx}}{v^2}}$$

$$\boxed{\frac{L.D.H - H.D.L}{L^2}}$$

* $f(x) = (x^2+1)(x^2-1)$

$f'(x) = \frac{d}{dx}\{(x^2+1)(x^2-1)\}$

$= (x^2+1)(2x) + (x^2-1)(2x)$

$= 2x^3 + 2x + 2x^3 - 2x$

$= 4x^3$

Ans

* If $y = \frac{2x-1}{2x+3}$, find $\left.\frac{dy}{dx}\right|_{x=1}$

$\Rightarrow$

$y = \frac{2x-1}{2x+3}$

$\frac{dy}{dx} = \frac{dy}{dx}\frac{2x-1}{2x+3}$

$= \frac{(2x+3)\cdot\frac{dy}{dx}(2x-1) - (2x-1)\cdot\frac{dy}{dx}(2x+3)}{(2x+3)^2}$

$= \frac{(2x+3)\cdot 2 - (2x-1)\cdot 2}{(2x+3)^2}$

$= \frac{4x+6-4x+2}{(2x+3)^2}$

$= \frac{8}{(2x+3)^2}$

$\therefore \frac{dy}{dx} = \frac{8}{(2x+3)^2}$

$\left.\frac{dy}{dx}\right|_{x=1} = \frac{8}{(2+3)^2}$

$= \frac{8}{25}$ Ans

Find all values of x at which the tangent line to the curve is horizontal, $y = \frac{x^2-1}{x+2}$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2-1}{x+2}\right)$$

$$= \frac{(x+2)\frac{d}{dx}(x^2-1) - (x^2-1)\frac{d}{dx}(x+2)}{(x+2)^2}$$

$$= \frac{(x+2)\cdot 2x - (x^2-1)\cdot 1}{(x+2)^2}$$

$$= \frac{2x^2+4x-x^2+1}{(x+2)^2}$$

$$= \frac{x^2+4x+1}{(x+2)^2}$$

Now, for horizontal tangent line,

$$\frac{dy}{dx} = 0$$

$$\frac{x^2+4x+1}{(x+2)^2} = 0$$

$$x^2+4x+1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16-4}}{2\cdot 1}$$

$$= \frac{-4 \pm \sqrt{12}}{2}$$

$$= \frac{-4 \pm 2\sqrt{3}}{2}$$

$$= -2 \pm \sqrt{3}$$

$\therefore x = -2+\sqrt{3}, -2-\sqrt{3}$

Ans

* Find all values of x at which the tangent line to the curve $y = \frac{x^2+1}{x-1}$ is Parallel to the line $y = x$

$\Rightarrow$

Here, $y = \frac{x^2+1}{x-1}$

we know that slope of tangent line is

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x-1}\right) = \frac{(x-1)\cdot 2x - (x^2+1)\cdot 1}{(x-1)^2} = \frac{2x^2-2x-x^2-1}{(x-1)^2}$$

$$= \frac{x^2-2x-1}{(x-1)^2}$$

$\therefore$ slope of tangent line,

$$m_1 = \frac{dy}{dx} = \frac{x^2-2x-1}{(x-1)^2}$$

and slope of the line $y = x$ is

$$m_2 = 1$$

Now, if two lines are parallel, then,

$$m_1 = m_2$$

$$\frac{x^2-2x-1}{(x-1)^2} = 1$$

$$x^2-2x-1 = (x-1)^2$$

$$x^2-2x-1 = x^2-2x+1$$

$$-1 = 1 \text{ (Not posible)}$$

$\therefore$ points are none.

(*) $y = \frac{12}{x}$; parallel to the line $3x + y = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{12}{x}\right)$$

$$= \frac{x \cdot \frac{d}{dx}(12) - 12 \cdot \frac{d}{dx} \cdot x}{x^2}$$

$$= \frac{0 - 12}{x^2}$$

$$= \frac{-12}{x^2}$$

$$\therefore m_1 = \frac{-12}{x^2}$$

slope of the line $3x + y = 0$

$$\Rightarrow y = -3x$$

$$m_2 = -3$$

Now,

$$m_1 = m_2$$

$$\frac{-12}{x^2} = -3$$

$$-12 = -3x^2$$

$$x^2 = 4$$

$$x = \pm 2$$

$x = 2$, $y = \frac{12}{2} = 6 \Rightarrow (2, 6)$

$x = 2$, $y = \frac{12}{-2} = -6 \Rightarrow (-2, -6)$

⊛ $y = 2x^2 - 4x$; perpendicular to the line $y = \frac{1}{4}x + 5$

$\Rightarrow$

$y = 2x^2 - 4x$

$\frac{dy}{dx} = 4x - 4$

$m_1 = 4x - 4$ / $m_2 = \frac{1}{4}$

$\therefore (4x-4)\frac{1}{4} = -1$

$4x - 4 = -4$

$4x = 0$

$x = 0$

$\therefore x = 0$

$y = 2 \cdot 0 - 4 \cdot 0 = 0$

$\therefore$ point $(0,0)$.

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Derivative of Trig. Fn.

$y = \sin x$

$\frac{dy}{dx} = \frac{d}{dx}(\sin x)$

$= \cos x$

$y = \cos x$

$\frac{dy}{dx} = \frac{d}{dx}(\cos x)$

$= -\sin x$

$y = \tan x$

$\frac{dy}{dx} = \frac{d}{dx}(\tan x)$

$= \sec^2 x$

$y = \sec x$

$\frac{dy}{dx} = \frac{d}{dx}(\sec x)$

$= \sec x \tan x$

$y = \operatorname{cosec} x$

$\frac{dy}{dx} = \frac{d}{dx}(\operatorname{cosec} x)$

$= -\operatorname{cosec} x \cot x$

$y = \cot x$

$\frac{dy}{dx} = \frac{d}{dx}(\cot x)$

$= -\operatorname{cosec}^2 x$

⊛ $\frac{d}{dx}(x^2 + x\cos x)$

$= 2x + \frac{d}{dx}(x\cos x)$

$= 2x + x(-\sin x) + \cos x \cdot 1$

$= 2x - x\sin x + \cos x$ Ans

⊛ $f(x) = \sec x - \sqrt{2}\tan x$

$f'(x) = \frac{d}{dx}(\sec x - \sqrt{2}\tan x)$

$= \sec x \tan x - \sqrt{2}\sec^2 x$

⊛ $f(x) = \dfrac{\sin x \sec x}{1+ s \tan x}$; find $f'(x) = ?$

$\Rightarrow$

$$f(x) = \frac{\sin x \sec x}{1+ x\tan x}$$

$$= \frac{\sin x \cdot \frac{1}{\cos x}}{1+ x \cdot \frac{\sin x}{\cos x}}$$

$$= \frac{\sin x}{\cos x + x \sin x}$$

$$f'(x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x + x\sin x}\right)$$

$$= \frac{(\cos x + x\sin x)\cdot \frac{d}{dx}(\sin x) - \sin x \cdot \frac{d}{dx}(\cos x + x\sin x)}{(\cos x + x\sin x)^2}$$

$$= \frac{(\cos x + x\sin x)\cdot \cos x - \sin x\,(-\sin x + x\cos x + \sin x)}{(\cos x + x\sin x)^2}$$

$$= \frac{\cos^2 x}{(\cos x + x\sin x)^2}$$

⊛ Find the all values of x in the interval $[-2\pi, 2\pi]$ at which the graph of f has horizontal tangent line.

i) $f(x) = \sin x$ ii) $f(x) = x + \cos x$ iii) $f(x) = \tan x$

$\Rightarrow$

i) $f(x) = \sin x$

$f'(x) = \cos x$

For H. tangent line

$f'(x) = 0$

$\therefore \cos x = 0$

$x = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}$

ii) $f(x) = x + \cos x$

$f'(x) = 1 - \sin x$

for, H. tangent line,

$f'(x) = 0$

$1 - \sin x = 0$

$\therefore \sin x = 1$

$\therefore x = -\frac{3\pi}{2}, \frac{\pi}{2}$

iii) $f(x) = \tan x$

$f'(x) = \sec^2 x$

for H. tangent line,

$f'(x) = 0$

$\sec^2 x = 0$

$\frac{1}{\cos^2 x} = 0$

$1 = 0$ [Not possible]

$\therefore$ No points

* $y = x \sin x - x \cos x$

Find $\frac{d^2y}{dx^2} = ?$

* Find the tangent line for, $f(x) = \tan x$ at, $x=0$ and $x = \frac{\pi}{4}$

$\Rightarrow$

Here,

$f(x) = \tan x$

$f'(x) = \sec^2 x$

$\therefore$ slope at $x=0$

$f'(0) = \sec^2 0 = 1$

if $x=0$, $y = f(0) = \tan 0 = 0$

point $(0,0)$

tangent line,

$y - 0 = 1(x-0)$

$y = x$

slope at $x = \frac{\pi}{4}$

$f'(\frac{\pi}{4}) = \sec^2 \frac{\pi}{4} = 2$

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The chain Rule

$\frac{d}{dx}(x\cos x) = x\frac{d}{dx}(\cos x) + \cos x \cdot \frac{d}{dx}x$

$= -x\sin x + \cos x$

$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1})$

$= -1x^{-2} = -x^{-2}$

$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{x \cdot \frac{d}{dx}(1) - 1 \cdot \frac{d}{dx}(x)}{x^2}$

$= \frac{-1}{x^2} = -x^{-2}$

Derivative of composite function

$y = f(g(x))$

$\frac{dy}{dx} = \frac{d}{dx}(f(g(x)))$

$= f'(g(x)) \cdot \frac{d}{dx}(g(x))$

⊛ $y = x^4$

$$\frac{dy}{dx} = 4x^3$$

⊛ $y = (3x+1)^7$

$$\frac{dy}{dx} = 7(3x+1)^6 \cdot \frac{d}{dx}(3x+1)$$

$$= 7(3x+1)^6 \cdot 3$$

$$= 21(3x+1)^6$$

⊛ $y = 7x^6 + 2$

$$\frac{dy}{dx} = 7 \cdot 6x^5 + 0$$

$$= 42x^5$$

⊛ $y = \sqrt{7x^6+2} = (7x^6+2)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(7x^6+2)^{\frac{1}{2}-1} \cdot \frac{d}{dx}(7x^6+2)$$

$$= \frac{1}{2}(7x^6+2)^{-\frac{1}{2}} \cdot 42x^5$$

$$= 21x^5(7x^6+2)^{-\frac{1}{2}}$$

(*) If $y = \cos^2 x$

$y = (\cos x)^2$

$$\frac{dy}{dx} = 2(\cos x)^{2-1} \cdot \frac{d}{dx}(\cos x)$$

$$= 2\cos x \cdot (-\sin x)$$

$$= -2\cos x \sin x$$

(*) $y = \sin^5 x$

$y = (\sin x)^5$

$$\frac{dy}{dx} = 5(\sin x)^4 \cdot \frac{d}{dx}(\sin x)$$

$$= 5\sin^4 x \cdot \cos x$$

(*) $f(x) = \sqrt{x^3 - 2x + 5}$

$f(x) = (x^3 - 2x + 5)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(x^3 - 2x + 5)^{-\frac{1}{2}} \cdot \frac{d}{dx}(x^3 - 2x + 5)$$

$$= \frac{1}{2}(x^3 - 2x + 5)^{-\frac{1}{2}} \cdot (3x^2 - 2)$$

$$y = \sin 5x$$

$$\frac{dy}{dx} = \cos 5x \cdot \frac{d}{dx}(5x)$$

$$= 5 \cos 5x$$

$$y = (x + \tan 7x)^3$$

$$\frac{dy}{dx} = 3(x + \tan 7x)^2 \cdot \frac{d}{dx}(x + \tan 7x)$$

$$= 3(x + \tan 7x)^2 \cdot \left(1 + \sec^2 7x \cdot \frac{d}{dx}(7x)\right)$$

$$= 3(x + \tan 7x)^2 \cdot (1 + 7\sec^2 7x)$$

Find $\frac{dy}{dx}$ from the followings.

i) $y = \sin(3x^2)$ ii) $y = \cos(\cos x)$ iii) $y = \sqrt{x^3 + \operatorname{cosec} x}$

Solve /

i) $y = \sin(3x^2)$

$$\frac{dy}{dx} = \cos 3x^2 \cdot \frac{d}{dx}(3x^2)$$

$$= \cos 3x^2 \cdot 6x$$

$$= 6x \cdot \cos 3x^2$$