$$= \frac{\sqrt{5-2\cdot 0}}{-1-3\cdot 0}$$

$$= \frac{\sqrt{5}}{-1}$$

$$= -\sqrt{5}$$

Therefore,

$$\lim_{x\to-\infty} \frac{\sqrt{5x^2-2}}{x+3} = -\sqrt{5}$$

26)

Given that,

$$\lim_{x\to+\infty} \frac{\sqrt{5x^2-2}}{x+3}$$

$$= \lim_{x\to+\infty} \frac{\frac{\sqrt{5x^2-2}}{|x|}}{\frac{x+3}{|x|}}$$

$$= \lim_{x\to+\infty} \frac{\sqrt{\frac{5x^2-2}{x^2}}}{\frac{x+3}{x}}$$

$$= \lim_{x\to+\infty} \frac{\sqrt{5-\frac{2}{x^2}}}{1+\frac{3}{x}}$$

$$= \frac{\sqrt{\lim_{x\to+\infty} 5 - 2\cdot\lim_{x\to+\infty}\frac{1}{x^2}}}{\lim_{x\to+\infty} 1 + 3\cdot\lim_{x\to+\infty}\frac{1}{x}}$$

$$= \frac{\sqrt{5-2\cdot 0}}{1+3\cdot 0}$$

$$= \sqrt{5}$$

Therefore,

$$\lim_{x \to +\infty} \frac{\sqrt{5x^2-2}}{x+3} = \sqrt{5}$$

27)

Given that,

$$\lim_{y \to -\infty} \frac{2-y}{\sqrt{7+6y^2}}$$

$$= \lim_{y \to -\infty} \frac{\frac{2-y}{|y|}}{\frac{\sqrt{7+6y^2}}{|y|}}$$

$$= \lim_{y \to -\infty} \frac{\frac{2-y}{-y}}{\sqrt{\frac{7+6y^2}{y^2}}}$$

$$= \lim_{y \to -\infty} \frac{-\frac{2}{y}+1}{\sqrt{\frac{7}{y^2}+6}}$$

$$= \frac{-2\cdot \lim_{y \to -\infty} \frac{1}{y} + \lim_{y \to -\infty} 1}{\sqrt{7\cdot \lim_{y \to -\infty} \frac{1}{y^2} + \lim_{y \to -\infty} 6}}$$

$$= \frac{-2\cdot 0+1}{\sqrt{7\cdot 0+6}} = \frac{1}{\sqrt{6}}$$

Therefore,

$$\lim_{y \to -\infty} \frac{2-y}{\sqrt{7+6y^2}} = \frac{1}{\sqrt{6}}$$

28)

Given that,

$$\lim_{y \to +\infty} \frac{2-y}{\sqrt{7+6y^2}}$$

$$= \lim_{y \to +\infty} \frac{\frac{2-y}{|y|}}{\frac{\sqrt{7+6y^2}}{|y|}}$$

$$= \lim_{y \to +\infty} \frac{\frac{2-y}{y}}{\sqrt{\frac{7+6y^2}{y^2}}}$$

$$= \lim_{y \to +\infty} \frac{\frac{2}{y} - 1}{\sqrt{\frac{7}{y^2} + 6}}$$

$$= \frac{2 \cdot \lim_{y \to +\infty} \frac{1}{y} - \lim_{y \to +\infty} 1}{\sqrt{7 \cdot \lim_{y \to +\infty} \frac{1}{y^2} + \lim_{y \to +\infty} 6}}$$

$$= \frac{2 \cdot 0 - 1}{\sqrt{7 \cdot 0 + 6}}$$

$$= -\frac{1}{\sqrt{6}}$$

Therefore,

$$\lim_{y \to +\infty} \frac{2-y}{\sqrt{7+6y^2}} = -\frac{1}{\sqrt{6}}$$

29|

Given that,

$$\lim_{x \to -\infty} \frac{\sqrt{3x^4+x}}{x^2-8}$$

$$= \lim_{x \to -\infty} \frac{\frac{\sqrt{3x^4+x}}{|x^2|}}{\frac{x^2-8}{|x^2|}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{3x^4+x}{x^4}}}{\frac{x^2-8}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{3+\frac{1}{x^3}}}{1-\frac{8}{x^2}}$$

$$= \frac{\sqrt{\lim_{x \to -\infty} 3 + \lim_{x \to -\infty} \frac{1}{x^3}}}{\lim_{x \to -\infty} 1 - 8 \cdot \lim_{x \to -\infty} \frac{1}{x^2}}$$

$$= \frac{\sqrt{3+0}}{1-8 \cdot 0}$$

$$= \sqrt{3}$$

Therefore, $\lim_{x \to -\infty} \frac{\sqrt{3x^4+x}}{x^2-8} = \sqrt{3}$

30)

Given that,

$$\lim_{x \to +\infty} \frac{\sqrt{3x^4+x}}{x^2-8}$$

$$= \lim_{x \to +\infty} \frac{\frac{\sqrt{3x^4+x}}{x^2}}{\frac{x^2-8}{x^2}}$$

$$= \lim_{x \to +\infty} \frac{\sqrt{\frac{3x^4+x}{x^4}}}{1-\frac{8}{x^2}}$$

$$= \lim_{x \to +\infty} \frac{\sqrt{3+\frac{1}{x^3}}}{1-\frac{8}{x^2}}$$

$$= \frac{\sqrt{\lim_{x \to +\infty} 3 + \lim_{x \to +\infty} \frac{1}{x^3}}}{\lim_{x \to +\infty} 1 - \lim_{x \to +\infty} \frac{8}{x^2}}$$

$$= \frac{\sqrt{3+0}}{1-0}$$

$$= \sqrt{3}$$

Therefore,

$$\lim_{x \to +\infty} \frac{\sqrt{3x^4+x}}{x^2-8} = \sqrt{3}$$

311

Given that,

$$\lim_{x\to+\infty}\left(\sqrt{x^2+3}-x\right)$$

$$=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2+3}-x\right)\left(\sqrt{x^2+3}+x\right)}{\left(\sqrt{x^2+3}+x\right)}$$

$$=\lim_{x\to+\infty}\frac{x^2+3-x^2}{\sqrt{x^2+3}+x}$$

$$=\lim_{x\to+\infty}\frac{3}{\sqrt{x^2+3}+x}$$

$$=\lim_{x\to+\infty}\frac{\frac{3}{|x|}}{\frac{\sqrt{x^2+3}+x}{|x|}}$$

$$=\lim_{x\to+\infty}\frac{\frac{3}{x}}{\frac{\sqrt{x^2+3}}{x}+1}$$

$$=\lim_{x\to+\infty}\frac{\frac{3}{x}}{\sqrt{\frac{x^2+3}{x^2}}+1}$$

$$=\lim_{x\to+\infty}\frac{\frac{3}{x}}{\sqrt{1+\frac{3}{x^2}}+1}$$

$$=\frac{3.\lim_{x\to+\infty}\frac{1}{x}}{\lim_{x\to+\infty}\sqrt{1+\frac{3}{x^2}}+\lim_{x\to+\infty}1}=\frac{3.0}{1+1}=0$$

Therefore,

$$\lim_{x\to+\infty}\left(\sqrt{x^2+3}-x\right)=0$$

32|

Given that,

$$\lim_{x\to+\infty}\left(\sqrt{x^2-3x}-x\right)$$

$$=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2-3x}-x\right)\left(\sqrt{x^2-3x}+x\right)}{\left(\sqrt{x^2-3x}+x\right)}$$

$$=\lim_{x\to+\infty}\frac{x^2-3x-x^2}{\sqrt{x^2-3x}+x}$$

$$=\lim_{x\to+\infty}\frac{-3x}{\sqrt{x^2-3x}+x}$$

$$=\lim_{x\to+\infty}\frac{\frac{-3x}{x}}{\frac{\sqrt{x^2-3x}}{x}+1}$$

$$=\lim_{x\to+\infty}\frac{-3}{\sqrt{1-\frac{3}{x}}+1}$$

$$=\frac{\lim_{x\to+\infty}-3}{\sqrt{\lim_{x\to+\infty}1-3.\lim_{x\to+\infty}\frac{1}{x}}+\lim_{x\to+\infty}1}$$

$$=\frac{-3}{\sqrt{1-3\cdot0}+1}=\frac{-3}{1+1}=\frac{-3}{2}$$

Therefore,

$$\lim_{x \to +\infty} \left(\sqrt{x^2-3x} - x\right) = \frac{-3}{2}$$

33)

Given that,

$$\lim_{x \to -\infty} \frac{1-e^x}{1+e^x}$$

$$= \frac{\lim_{x \to -\infty} (1-e^x)}{\lim_{x \to -\infty} (1+e^x)}$$

$$= \frac{\lim_{x \to -\infty} 1 - \lim_{x \to -\infty} e^x}{\lim_{x \to -\infty} 1 + \lim_{e \to -\infty} e^x}$$

$$= \frac{1-0}{1+0}$$

$$= 1$$

Therefore,

$$\lim_{x \to -\infty} \frac{1-e^x}{1+e^x} = 1$$

34)

Given that,

$$\lim_{x\to+\infty} \frac{1-e^{x}}{1+e^{x}}$$

$$= \lim_{x\to+\infty} \frac{e^{-x}-1}{e^{-x}+1}$$

$$= \frac{\lim_{x\to+\infty} e^{-x} - \lim_{x\to+\infty} 1}{\lim_{x\to+\infty} e^{-x} + \lim_{x\to+\infty} 1}$$

$$= \frac{0-1}{0+1}$$

$$= -1$$

Therefore,

$$\lim_{x\to+\infty} \frac{1-e^{x}}{1+e^{x}} = -1$$

35)

Given that,

$$\lim_{x\to+\infty} \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$$

$$= \lim_{x\to+\infty} \frac{1+e^{-2x}}{1-e^{-2x}} \quad [\text{devide by } e^{x}]$$

$$= \frac{\lim_{x\to+\infty} 1 + \lim_{x\to+\infty} e^{-2x}}{\lim_{x\to+\infty} 1 - \lim_{x\to+\infty} e^{-2x}}$$

$$= \frac{1+0}{1-0}$$

$$= 1$$

Therefore,

$$\lim_{x \to +\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} = 1$$

36|

Given that,

$$\lim_{x \to -\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$= \lim_{x \to -\infty} \frac{\frac{e^x + e^{-x}}{e^{-x}}}{\frac{e^x - e^{-x}}{e^{-x}}}$$

$$= \lim_{x \to -\infty} \frac{e^{2x} + 1}{e^{2x} - 1}$$

$$= \frac{\lim_{x \to -\infty} e^{2x} + \lim_{x \to -\infty} 1}{\lim_{x \to -\infty} e^{2x} - \lim_{x \to -\infty} 1}$$

$$= \frac{0+1}{0-1}$$

$$= -1$$

Therefore, $\lim_{x \to -\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} = -1$

37/

Given that,

$$\lim_{x \to +\infty} \ln\left(\frac{2}{x^2}\right)$$

$$= \ln\left(\lim_{x \to +\infty} \frac{2}{x^2}\right)$$

$$= \ln 0$$

$$= -\infty$$

Therefore,

$$\lim_{x \to +\infty} \ln\left(\frac{2}{x^2}\right) = -\infty$$

38/

Given that,

$$\lim_{x \to 0^+} \ln\left(\frac{2}{x^2}\right)$$

$$= \ln\left(\lim_{x \to 0^+} \frac{2}{x^2}\right)$$

$$= \ln +\infty$$

$$= +\infty$$

Therefore,

$$\lim_{x \to 0^+} \ln\left(\frac{2}{x^2}\right) = +\infty.$$

39)

Given that

$$\lim_{x \to +\infty} \frac{(x+1)^x}{x^x}$$

$$= \lim_{x \to +\infty} \left(\frac{x+1}{x}\right)^x$$

$$= \lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x$$

$$= e$$

Therefore,

$$\lim_{x \to +\infty} \frac{(x+1)^x}{x^x} = e$$

40)

Given that,

$$\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^{-x}$$

$$= \lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^{x \cdot (-1)}$$

$$= e^{-1}$$

Therefore,

$$\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^{-x} = e^{-1}$$

1.5

1)

Given that,

$$f(x) = 5x^4 - 3x + 7$$

Here, $f(x)$ is a polynomial function. Polynomial functions are always continuous.

Therefore,

There is no discontinuous point.

2)

Given that,

$$f(x) = \sqrt[3]{x-8}$$

Here, cubic root is alway defineable. So this function has a continuous curve.

Therefore,

There is no value of x for which the function $f(x)$ is not continuous.

13|

Given that,

$$f(x) = \frac{x+2}{x^2+4}$$

Here, (x^2+4) is ~~aly~~ always positive and greater than zero.

Therefore
there is no discontinuaw point.

14|

Given that,

$$f(x) = \frac{x+2}{x^2-4}$$

Here,

$$x^2-4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

Therefore
The function $f(x)$ is not continuaw at $x=2$ and $x=-2$.

15)

Given that,

$$f(x) = \frac{x}{2x^2+x}$$

Here,

$$2x^2 + x = 0$$

$$\Rightarrow \cancel{x(2x+1) = 0}$$

$$\Rightarrow x(2x+1) = 0$$

$\therefore x = 0$ and $2x+1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

Therefore,

The function is not continuow at $x = -\frac{1}{2}$ and $x = 0$.

16)

Given that,

$$f(x) = \frac{2x+1}{4x^2+4x+5}$$

$$= \frac{2x+1}{(2x+1)^2+4}$$

Here, $(2x+1)^2+4$ is alway positive and greater than zero

Therefore, the function is continuow everywhere.

17)

Given that,

$$f(x) = \frac{3}{x} + \frac{x-1}{x^2-1}$$

$$= \frac{3x^2-3+x^2-x}{x(x^2-1)}$$

$$= \frac{4x^2-x-3}{x(x^2-1)}$$

Here,

$$x(x^2-1) = 0$$

$\therefore x = 0$ and $x^2-1=0$

$$x^2 = 1$$

$$x = \pm 1$$

Therefore, the function is not continuous at $x=0$, $x=1$ and $x=-1$.

18)

Given that,

$$f(x) = \frac{5}{x} + \frac{2x}{x+4}$$

$$= \frac{5x+20+2x^2}{x(x+4)}$$

$$= \frac{2x^2+5x+20}{x(x+4)}$$

Here,

$$x(x+4) = 0$$

$\therefore x = 0$ and $x + 4 = 0$

$x = -4$

Therefore,

the function is not continuous at $x = 0$ and $x = -4$.

19)

Given that,

$$f(x) = \frac{x^2+6x+9}{|x|+3}$$

Here, $|x|+3$ is always positive and greater than zero.

Hence, there is no discontinuous point.

20)

Given that,

$$f(x) = \left| 4 - \frac{8}{x^4+x} \right|$$

$$= \left| \frac{4x^4+4x-8}{x^4+x} \right|$$

Here,

$$x^4+x=0$$

$$x(x^3+1)=0$$

$\therefore x=0$ and $x^3+1=0$

$$x^3=-1$$

$$x=-1$$

Therefore the function is not continuous at $x=0$ and $x=-1$.

<u>21)</u>

Given that,

$$f(x)=\begin{cases} 2x+3, & x\le 4 \\ 7+\frac{16}{x}, & x>4 \end{cases}$$

For $x<4$, $f(x)=2x+3$ is always continuous at $(-\infty,4)$

For $x>4$, $f(x)=7+\frac{16}{x}$ is always continuous at $(4,\infty)$

For $x=4$,

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} 2x+3$$

$$= 2\cdot 4+3$$

$$= 11$$

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} 7 + \frac{16}{x}$$

$$= \lim_{x\to 4^+} 7 + \frac{\lim_{x\to 4^+} 16}{\lim_{x\to 4^+} x}$$

$$= 7 + \frac{16}{4}$$

$$= 7+4$$

$$= 11$$

$$f(4) = 2\cdot 4+3$$

$$= 11$$

$$\therefore \lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = f(4) = 11$$

Therefore,

the function is continuous every where.

22|

Given that,

$$f(x) = \begin{cases} \frac{3}{x-1} & , x \neq 1 \\ 3 & , x = 1 \end{cases}$$

For $x=1$,

$$f(1) = 3$$

So, function is defined.

Now, checking limit at $x=1$ for ~~fo~~ the function $f(x)$.

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{3}{x-1}$$

$$= -\infty$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{3}{x-1}$$

$$= +\infty$$

$\therefore$ L.H.L $\neq$ R.H.L.

Hence, ~~function~~ ~~d~~ limit doesn't exist.

Therefore,

the function is not continuous at $x=1$.

29/

a)

Given that,

$$f(x) = \begin{cases} 7x-2, & x \le 1 \\ kx^2, & x > 1 \end{cases}$$

For $x = 1$,

$$f(1) = 7.1 - 2$$
$$= 5$$

So, function is defined.

Now, checking limit,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 7x-2$$
$$= 7.1-2$$
$$= 5$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} kx^2$$
$$= k.1^2$$
$$= k$$

if limit exist, then, L.H.L. = R.H.L.

$$\Rightarrow 5 = k$$

Therefore, value of k is 5.

b)

Given that,

$$f(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x+k & x > 2 \end{cases}$$

For $x=2$,

$f(2) = k.2^2 = 4k$

Now, $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} kx^2$

$= k.2^2$

$= 4k$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 2x+k$

$= 2.2+k$

$= 4+k$

if limit exist then,

$L.H.L = R.H.L$

$\Rightarrow 4k = 4+k$

$\Rightarrow 3k = 4$

$\therefore k = \frac{4}{3}$

Therefore, value of k is $\frac{4}{3}$.

30)

a)

Given that,

$$f(x) = \begin{cases} 9-x^2, & x \geq -3 \\ \frac{k}{x^2}, & x < -3 \end{cases}$$

For $x = -3$,

$$f(-3) = 9-(-3)^2 = 0$$

Now,

$$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \frac{k}{x^2}$$

$$= \frac{\lim_{x \to -3^-} k}{\lim_{x \to -3^-} x^2}$$

$$= \frac{k}{9}$$

$$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} 9-x^2$$

$$= 9-(-3)^2$$

$$= 0$$

if limit exist then,

$$\text{L.H.L} = \text{R.H.L.}$$

$$\Rightarrow \frac{k}{9} = 0$$

$$\therefore k = 0$$

Therefore, value of k is 0.

b)

Given that,

$$f(x) = \begin{cases} 9-x^2, & x \geq 0 \\ \frac{k}{x^2}, & x < 0 \end{cases}$$

For $x=0$,

$$f(0) = 9-0^2$$
$$= 9$$

Now,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{k}{x^2}$$
$$= +\infty$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 9-x^2$$
$$= 9-0$$
$$= 9$$

$\therefore$ L.H.L $\neq$ R.H.L

So, limit doesn't exist at $x=0$.

Therefore, the function is not continuous at $x=0$.

So, there is no k value which makes the function continuous every where.