North South University

Department of Electrical and Computer Engineering

Homework - 3

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Course No : CSE 173

Course Title : Discrete Mathematics

Section : 1

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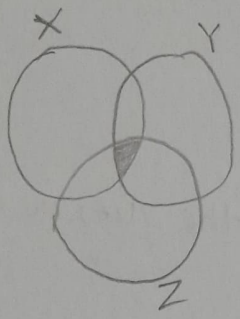
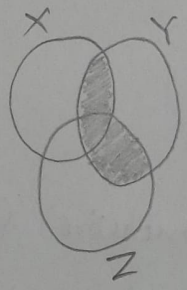
Ans. to the ques. no. 01

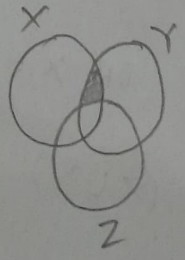
a) $X \cap (Y \cap Z)$



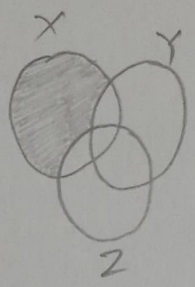
b) $(X \cap Y) \cup (Y \cap Z)$



c) $X \cap (Y - Z)$



d) $(X-Y-Z)$



Ans. to the ques.no. 02

a)

$$X \oplus Y = (X \cup Y) - (X \cap Y)$$

L.H.S. $= X \oplus Y$

Here, $\oplus$ means exclusive OR. That means X or Y but not both.

$\therefore$ L.H.S. $= \{x | x \in X \oplus Y\}$

$= \{x | x \in X$ OR $x \in Y$ AND $x \notin X$ AND $x \notin Y\}$

$= \{x | x \in (X \cup Y)$ AND $x \notin (X \cap Y)\}$

$= \{x | x \in (X \cup Y) - (X \cap Y)\}$

$= (X \cup Y) - (X \cap Y)$

$=$ R.H.S.

(Proved)

b)

$$(Y - X) \cup (Z - X) = (Y \cup Z) - X$$

L.H.S. $= \{x \mid x \in (Y - X) \cup (Z - X)\}$

$= \{x \mid x \in (Y - X) \text{ OR } x \in (Z - X)\}$

$= \{x \mid x \in Y \text{ AND } x \notin X \text{ OR } x \in Z \text{ AND } x \notin X\}$

$= \{x \mid x \in Y \text{ OR } x \in Z \text{ AND } x \notin X\}$

$= \{x \mid x \in (Y \cup Z) \text{ AND } x \notin X\}$

$= \{x \mid x \in (Y \cup Z) - X\}$

$= (Y \cup Z) - X$

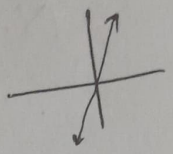
= R.H.S.

(Proved)

Ans. to the ques. no 03

Given that,

$$f(x) = 6x \quad ; \quad f : Z \to Z$$



As this a function of a straight line, its domain is $-\infty$ to ∞. Ans it's co-domain is also $-\infty$ to ∞.

As we know any function is one-to-one when every domain elements has an unique image. Here for every x value has its own unique $f(x)$. So, this function is one-to-one function.

An onto function is such that for every element in the codomain there exists an element in domain which maps to it, but here suppose for $f(x) = 5$, we don't have any x value for $f(x)$ gives output 5. Hence this function is not onto function.

Ans. to the ques. no 04

i)

Let the function $f: N \to N$, given by $f(x) = 2x$

Now,

Let us consider two elements x_1 and x_2 in the domain of f. So we get,

$$f(x_1) = 2x_1$$

$$f(x_2) = 2x_2$$

Now,

$$f(x_1) = f(x_2)$$

$$\Rightarrow 2x_1 = 2x_2$$

$$\therefore x_1 = x_2$$

Hence, the given function is one-to-one.

Now,

let $f(x) = y$, such that $y \in N$

we get,

$$2x = y$$

$$x = \frac{y}{2}$$

Put, $y = 1$

then, $x = \frac{1}{2} = 0.5$ which does not belongs to N

Hence, the given function is not onto.

ii)

Let the function $f: N \to N$, given by $f(1) = f(2) = 1$

Here,

$f(x) = f(1) = 1$ and

$f(x) = f(2) = 1$

Since, different elements 1, 2 have same image 1.

∴ f is not one-to-one

Let, $f(x) = y$, such that $y \in N$

Here, y is a natural number and for every 'y', there is a value of x which is natural number.

Hence, f is onto function.

iii)

Let the function $f(x) = x$; for $x : R \to R$

$f(x_1) = x_1$ and $f(x_2) = x_2$

$f(x_1) = f(x_2)$

$x_1 = x_2$

Hence, the function is one-to-one function.

Now, let $f(x) = y$, where $y \in R$, $x = y$

Now, for all $y \in R$, $x \in R$.

Hence, $f(x) = x$ is onto function as well.

Ans. to the ques. no. 05

Given,

$f(x) = x^2 + 1$

$g(x) = x + 2$

$fog(x) = f(g(x))$

$= f(x+2)$

$= (x+2)^2 + 1$

$= x^2 + 4x + 4 + 1$

$= x^2 + 4x + 5$

$$gof(x) = g(f(x))$$
$$= g(x^2+1)$$
$$= x^2+1+2$$
$$= x^2+3$$

Therefore, $fog(x) \neq gof(x)$ for the given $f(x)$ and $g(x)$.

Ans. to the ques. no. 06

Given that,

$$f(x) = \frac{x+4}{2x-5}$$

$$\therefore y = \frac{x+4}{2x-5}$$

Now, interchange the variables x and y.

$$x = \frac{y+4}{2y-5}$$

$$\Rightarrow x(2y-5) = y+4$$

$$\Rightarrow 2xy - 5x = y+4$$

$$\Rightarrow 2xy - y = 5x+4$$

$$\Rightarrow y(2x-1) = 5x+4$$

$$\therefore y = \frac{5x+4}{2x-1}$$

Therefore, inverse function is,

$$f^{-1}(x) = \frac{5x+4}{2x-1}$$

Ans to the quer. no. 07

Given that,

$$H(m) = 3m-2$$

$$\therefore y = 3m-2$$

Now interchange variables m and y.

$$m = 3y-2$$

$$\Rightarrow 3y = m+2$$

$$\therefore y = \frac{m+2}{3}$$

Therefore inverse function of $H(m) = 3m-2$ is

$$H^{-1}(m) = \frac{m+2}{3}$$

Now,

$$H \circ H^{-1}(m) = H\left(H^{-1}(m)\right)$$

$$= H\left(\frac{m+2}{3}\right)$$

$$= 3 \cdot \frac{m+2}{3} - 2$$

$$= m+2-2$$

$$= m$$

$$= R.H.S.$$

$\therefore$ L.H.S. = R.H.S. (showed).