North South University

Department of Mathematics and Physics

Assignment - 2

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Course Title : Introduction to Linear Algebra

Section : 10

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1.3

16/ Find all values of k, if any, that satisfy the equation.

$$\begin{bmatrix} 2 & 2 & k \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2+4+0 & 4+0+3k & 0+6+k \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 6 & 4+3k & 6+k \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 12+2(4+3k)+k(6+k) \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 12+8+6k+6k+k^2 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} k^2+12k+20 \end{bmatrix} = 0$$

Now,

$$k^2 + 12k + 20 = 0$$

$$\Rightarrow k^2 + 10k + 2k + 20 = 0$$

$$\Rightarrow k(k+10) + 2(k+10) = 0$$

$$\Rightarrow (k+10)(k+2) = 0$$

$$\therefore k = -10, -2$$

Therefore, the equation will satisfy if $k = -10, -2$.

24| Find the 4×4 matrix $A = [a_{ij}]$ whose entries satisfy the stated condition.

a) $a_{ij} = i + j$

$$\Rightarrow A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

b) $a_{ij} = i^{j-1}$

$\Rightarrow$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix}$$

c) $a_{ij} = \begin{cases} 1 & \text{if } |i-j| > 1 \\ -1 & \text{if } |i-j| \leq 1 \end{cases}$

$\Rightarrow$

$$A = \begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$$

1.4

18) Let A be the matrix

$$\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

In each part, compute the given quantity.

e) $p(A)$, where $p(x) = 2x^2 - x + 1$

$\Rightarrow$

$$p(x) = 2x^2 - x + 1$$

$$\therefore \; p(A) = 2(A)^2 - A + I$$

Now,

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 0+0 \\ 8+4 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix}$$

$$\therefore p(A) = 2 \cdot A^2 - A + I$$

$$= 2\begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ 24 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8-2+1 & 0-0+0 \\ 24-4+0 & 2-1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 0 \\ 20 & 2 \end{bmatrix}$$

Therefore,

$$p(A) = \begin{bmatrix} 7 & 0 \\ 20 & 2 \end{bmatrix}$$

f) $p(A)$, where $p(x) = x^3 - 2x + 4$

$\Rightarrow$

$$p(x) = x^3 - 2x + 4$$

$$\therefore p(A) = A^3 - 2A + 4I$$

Now,

From 'e' we get that,

$$A^2 = \begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix}$$

$$\therefore A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8+0 & 0+0 \\ 24+4 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ 28 & 1 \end{bmatrix}$$

$$\therefore p(A) = A^3 - 2A + 4I$$

$$= \begin{bmatrix} 8 & 0 \\ 28 & 1 \end{bmatrix} - 2\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ 28 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8-4+4 & 0-0+0 \\ 28-8+0 & 1-2+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ 20 & 3 \end{bmatrix}$$

Therefore,

$$p(A) = \begin{bmatrix} 8 & 0 \\ 20 & 3 \end{bmatrix}$$

1.5

15|

Use the inversion algorithm to find the inverse of the given matrix, if the inverse exists.

$$\begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$$

Let,

$$[A|I] = \left[\begin{array}{ccc|ccc} -1 & 3 & -4 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{r_1' = -r_1} \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[r_3' = r_3 + 4r_1]{r_2' = r_2 - 2r_1} \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 10 & -7 & 2 & 1 & 0 \\ 0 & -10 & 7 & -4 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{r_2' = \frac{1}{10} r_2} \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & \frac{-7}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & -10 & 7 & -4 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{r_3' = r_3 + 10r_2} \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & \frac{-7}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right]$$

Hence, this matrix is not inverseable.

1.6

6) Solve the system by inverting the coefficient matrix.

$$-x - 2y - 3z = 0$$
$$w + x + 4y + 4z = 7$$
$$w + 3x + 7y + 9z = 4$$
$$-w - 2x - 4y - 6z = 6$$

$\Rightarrow$

Rearranging the system,

$$w + 3x + 7y + 9z = 4$$
$$w + x + 4y + 4z = 7$$
$$-x - 2y - 3z = 0$$
$$-w - 2x - 4y - 6z = 6$$

matrix of co-efficient,

$$A = \begin{bmatrix} 1 & 3 & 7 & 9 \\ 1 & 1 & 4 & 4 \\ 0 & -1 & -2 & -3 \\ -1 & -2 & -4 & -6 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 \\ 7 \\ 0 \\ 6 \end{bmatrix} \quad \& \quad X = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$$

Now,

$$[A|I] = \left[\begin{array}{cccc|cccc} 1 & 3 & 7 & 9 & 1 & 0 & 0 & 0 \\ 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 0 & -1 & -2 & -3 & 0 & 0 & 1 & 0 \\ -1 & -2 & -4 & -6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_4' = R_4 + R_1]{R_2' = R_2 + R_3} \left[\begin{array}{cccc|cccc} 1 & 3 & 7 & 9 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -2 & 0 & 1 & 0 & 1 \\ 0 & -1 & -2 & -3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2' = -R_2} \left[\begin{array}{cccc|cccc} 1 & 3 & 7 & 9 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & -1 & -2 & -3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_4' = R_4 - R_2]{R_3' = R_3 + R_2} \left[\begin{array}{cccc|cccc} 1 & 3 & 7 & 9 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & 0 & -2 & -1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 3 & 1 & 1 & 1 & 0 & 2 \end{array}\right]$$

$$\xrightarrow{r_3' = r_3 + r_4} \left[\begin{array}{cccc|cccc} 1 & 3 & 7 & 9 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 & 0 & 2 \end{array}\right]$$

$$\xrightarrow{r_4' = r_4 - 3r_3} \left[\begin{array}{cccc|cccc} 1 & 3 & 7 & 9 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -2 & 1 & -3 & -1 \end{array}\right]$$

$$\xrightarrow[r_2' = r_2 - 2r_2]{r_1' = r_1 - 9r_4} \left[\begin{array}{cccc|cccc} 1 & 3 & 7 & 0 & 19 & -9 & 27 & 9 \\ 0 & 1 & 0 & 0 & 4 & -3 & 6 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -2 & 1 & -3 & -1 \end{array}\right]$$

$$\xrightarrow{r_1' = r_1 - 7r_3} \left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 0 & 12 & -9 & 20 & 2 \\ 0 & 1 & 0 & 0 & 4 & -3 & 6 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -2 & 1 & -3 & -1 \end{array}\right]$$

$$\xrightarrow{r_1' = r_1 - 3r_4} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 1 & 0 & 0 & 4 & -3 & 6 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -2 & 1 & -3 & -1 \end{array}\right]$$

This is the form of $[I \mid A^{-1}]$

$$\rightarrow A^{-1} = \begin{bmatrix} 0 & 0 & 2 & -1 \\ 4 & -3 & 6 & 1 \\ 1 & 0 & 1 & 1 \\ -2 & 1 & -3 & -1 \end{bmatrix}$$

We know,

$$X = A^{-1} B$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 & -1 \\ 4 & -3 & 6 & 1 \\ 1 & 0 & 1 & 1 \\ -2 & 1 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 0 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0+0+0-6 \\ 16-21+0+6 \\ 4+0+0+6 \\ -8+7+0-6 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 1 \\ 10 \\ -7 \end{bmatrix}$$

Therefore, the solution ~~is~~, are,

$$w = -6$$

$$x = 1$$

$$y = 10$$

$$z = -7$$

10) Solve the linear systems together by reducing the appropriate augmented matrix.

$$-x_1 + 4x_2 + x_3 = b_1$$

$$x_1 + 9x_2 - 2x_3 = b_2$$

$$6x_1 + 4x_2 - 8x_3 = b_3$$

i) $b_1 = 0,\ b_2 = 1,\ b_3 = 0$

ii) $b_1 = -3,\ b_2 = 4,\ b_3 = -5$

$\Rightarrow$

Augmented matrix is,

$$\left[\begin{array}{ccc|c|c} -1 & 4 & 1 & 0 & -3 \\ 1 & 9 & -2 & 1 & 4 \\ 6 & 4 & -8 & 0 & -5 \end{array}\right]$$

$$\xrightarrow{r_1' = -r_1} \left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 1 & 9 & -2 & 1 & 4 \\ 6 & 4 & -8 & 0 & -5 \end{array}\right]$$

$$\xrightarrow[r_3' = r_3 - 6r_1]{r_2' = r_2 - r_1} \left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 0 & 13 & -1 & 1 & 1 \\ 0 & 28 & -2 & 0 & -23 \end{array}\right]$$

$$\xrightarrow{r_2' = \frac{1}{13} r_2} \left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 0 & 1 & -\frac{1}{13} & \frac{1}{13} & \frac{1}{13} \\ 0 & 28 & -2 & 0 & -23 \end{array}\right]$$

$$\xrightarrow{r_3' = r_3 - 28 r_2} \left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 0 & 1 & -\frac{1}{13} & \frac{1}{13} & \frac{1}{13} \\ 0 & 0 & \frac{2}{13} & \frac{-28}{13} & \frac{-327}{13} \end{array}\right]$$

$$\xrightarrow{r_3' = \frac{13}{2} r_3} \left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 0 & 1 & \frac{-1}{13} & \frac{1}{13} & \frac{1}{13} \\ 0 & 0 & 1 & -14 & -\frac{327}{2} \end{array}\right]$$

$$\xrightarrow[r_2' = r_2 + \frac{1}{13} r_3]{r_1' = r_1 + r_3} \left[\begin{array}{ccc|c|c} 1 & -4 & 0 & -14 & \frac{-321}{2} \\ 0 & 1 & 0 & -1 & \frac{-25}{2} \\ 0 & 0 & 1 & -14 & \frac{-327}{2} \end{array}\right]$$

$$\xrightarrow{r_1' = r_1 + 4r_2} \left[\begin{array}{ccc|c|c} 1 & 0 & 0 & -18 & \frac{-427}{2} \\ 0 & 1 & 0 & -1 & \frac{-25}{2} \\ 0 & 0 & 1 & -14 & \frac{-327}{2} \end{array}\right]$$

Therefore, the solutions are,

i) $x_1 = -18$
$x_2 = -1$
$x_3 = -14$

ii) $x_1 = \frac{-427}{2}$
$x_2 = \frac{-25}{2}$
$x_3 = \frac{-327}{2}$

15| Determine conditions on the b_i's, if any, in order to guarantee that the linear system is consistent.

$$x_1 - 2x_2 + 5x_3 = b_1$$
$$4x_1 - 5x_2 + 8x_3 = b_2$$
$$-3x_1 + 3x_2 - 3x_3 = b_3$$

$\Rightarrow$

Augmented matrix:

$$\begin{bmatrix} 1 & -2 & 5 & b_1 \\ 4 & -5 & 8 & b_2 \\ -3 & 3 & -3 & b_3 \end{bmatrix}$$

$$\xrightarrow[r_3' = r_3 + 3r_1]{r_2' = r_2 - 4r_1} \begin{bmatrix} 1 & -2 & 5 & b_1 \\ 0 & 3 & -12 & b_2 - 4b_1 \\ 0 & -3 & 12 & b_3 + 3b_1 \end{bmatrix}$$

$$\xrightarrow{r_2' = \frac{1}{3} r_2} \begin{bmatrix} 1 & -2 & 5 & b_1 \\ 0 & 1 & -4 & \frac{b_2 - 4b_1}{3} \\ 0 & -3 & 12 & b_3 + 3b_1 \end{bmatrix}$$

$$\xrightarrow{r_3' = r_3 + 3r_2} \begin{bmatrix} 1 & -2 & 5 & b_1 \\ 0 & 1 & -4 & \frac{b_2 - 4b_1}{3} \\ 0 & 0 & 0 & b_3 + b_2 - b_1 \end{bmatrix}$$

It is now evident that third row in the matrix that the system has a solution if and only if b_1, b_2, and b_3 satisfy the condition.

$$b_3 + b_2 - b_1 = 0$$

$$\therefore \quad b_3 = b_1 - b_2$$

16| Determine conditions on the b_i's, if any, in order to guarantee that the linear system is consistent.

$$x_1 - 2x_2 - x_3 = b_1$$

$$-4x_1 + 5x_2 + 2x_3 = b_2$$

$$-4x_1 + 7x_2 + 4x_3 = b_3$$

$\Rightarrow$

Augmented matrix is,

$$\begin{bmatrix} 1 & -2 & -1 & b_1 \\ -4 & 5 & 2 & b_2 \\ -4 & 7 & 4 & b_3 \end{bmatrix}$$

$$\xrightarrow[r_3' = r_3 + 4r_1]{r_2' = r_2 + 4r_1} \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & -3 & -2 & b_2 + 4b_1 \\ 0 & -1 & 0 & b_3 + 4b_1 \end{bmatrix}$$

$$\xrightarrow{r_2' = -\frac{1}{3} r_2} \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & 1 & \frac{2}{3} & \frac{-b_2 - 4b_1}{3} \\ 0 & -1 & 0 & b_3 + 4b_1 \end{bmatrix}$$

$$\xrightarrow{r_3' = r_3 + r_2} \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & 1 & \frac{2}{3} & \frac{-b_2 - 4b_1}{3} \\ 0 & 0 & \frac{2}{3} & \frac{8b_1 - b_2 + 3b_3}{3} \end{bmatrix}$$

$$\xrightarrow{r_3' = \frac{3}{2} r_3} \begin{bmatrix} 1 & -2 & -1 & b_1 \\ 0 & 1 & \frac{2}{3} & \frac{-b_2 - 4b_1}{3} \\ 0 & 0 & 1 & \frac{8b_1 - b_2 + 3b_3}{2} \end{bmatrix}$$

$$\xrightarrow[r_2' = r_2 - \frac{2}{3} r_3]{r_1' = r_1 + r_3} \begin{bmatrix} 1 & -2 & 0 & \frac{10b_1 - b_2 + 3b_3}{2} \\ 0 & 1 & 0 & -4b_1 - b_3 \\ 0 & 0 & 1 & \frac{8b_1 - b_2 + 3b_3}{2} \end{bmatrix}$$

$$\xrightarrow{r_1' = r_1 + 2r_2} \begin{bmatrix} 1 & 0 & 0 & \frac{-6b_1 - b_2 - b_3}{2} \\ 0 & 1 & 0 & -4b_1 - b_3 \\ 0 & 0 & 1 & \frac{8b_1 - b_2 + 3b_3}{2} \end{bmatrix}$$

Hence,

$$x_1 = \frac{-6b_1 - b_2 - b_3}{2} = -3b_1 - \frac{1}{2} b_2 - \frac{1}{2} b_3$$

$$x_2 = -4b_1 - b_3$$

$$x_3 = \frac{8b_1 - b_2 + 3b_3}{2} = 4b_1 - \frac{b_2}{2} + \frac{3}{2} b_3$$

Therefore, the system is consistent for all b_1, b_2 and b_3.

1.7

11)

Find A^2, A^{-2} and A^{-k} (where k is any integer) by inspection.

$$A = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

$\Rightarrow$

A is a diagonal matrix. So, we can use diagonal matrix properties.

Therefore,

$$A^2 = \begin{bmatrix} (\frac{1}{2})^2 & 0 & 0 \\ 0 & (\frac{1}{3})^2 & 0 \\ 0 & 0 & (\frac{1}{4})^2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{9} & 0 \\ 0 & 0 & \frac{1}{16} \end{bmatrix}$$

Now,

$$A^{-1} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^{-2} = (A^{-1})^2$$

$$= \begin{bmatrix} 2^2 & 0 & 0 \\ 0 & 3^2 & 0 \\ 0 & 0 & 4^2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{bmatrix}$$

$$\therefore A^{-k} = (A^{-1})^k$$

$$= \begin{bmatrix} 2^k & 0 & 0 \\ 0 & 3^k & 0 \\ 0 & 0 & 4^k \end{bmatrix}$$

12)

Find A^2, A^{-2}, and A^{-k} (where k is any integer) by inspection

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$\Rightarrow$

'A' is a diagonal matrix, so we can use its properties.

$$A^2 = \begin{bmatrix} (-2)^2 & 0 & 0 & 0 \\ 0 & (-4)^2 & 0 & 0 \\ 0 & 0 & (-3)^2 & 0 \\ 0 & 0 & 0 & (2)^2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{-1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$A^{-2} = (A^{-1})^2 = \begin{bmatrix} (-\frac{1}{2})^2 & 0 & 0 & 0 \\ 0 & (\frac{-1}{4})^2 & 0 & 0 \\ 0 & 0 & (\frac{-1}{3})^2 & 0 \\ 0 & 0 & 0 & (\frac{1}{2})^2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{16} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

$$\therefore A^{-k} = (A^{-1})^k = \begin{bmatrix} (-\frac{1}{2})^k & 0 & 0 & 0 \\ 0 & (\frac{-1}{4})^k & 0 & 0 \\ 0 & 0 & (-\frac{1}{3})^k & 0 \\ 0 & 0 & 0 & (\frac{1}{2})^k \end{bmatrix}$$

25/

Find all values of x in order for A to be invertible

$$A = \begin{bmatrix} x-1 & x^2 & x^4 \\ 0 & x+2 & x^3 \\ 0 & 0 & x-4 \end{bmatrix}$$

$\Rightarrow$

'A' is a upper triangle matrix. We know that determinant of a upper triangle matrix is the product of the entries of the main diagonal.

$$\det(A) = (x-1)(x+2)(x-4)$$

a matrix can be ~~inl~~ invertible if its determinant is not equals to zero.

$$\therefore (x-1)(x+2)(x-4) \neq 0$$

$$\therefore x \neq 1,\ x \neq -2,\ x \neq 4$$

Therefore 'A' is invertible for all real numbers ~~expce~~ except 1, -2 and 4.

26) Find all values of x in order for A to be invertible.

$$A = \begin{bmatrix} x-\frac{1}{2} & 0 & 0 \\ x & x-\frac{1}{3} & 0 \\ x^2 & x^3 & x-\frac{1}{4} \end{bmatrix}$$

$\Rightarrow$

'A' is a lower triangle matrix. We know that determinant of a lower triangle matrix is the product of the entries of the main diagonal.

$\det(A) = (x-\frac{1}{2})(x-\frac{1}{3})(x-\frac{1}{4})$

a matrix can be invertible if its determinant is not equal to zero.

$\therefore (x-\frac{1}{2})(x-\frac{1}{3})(x-\frac{1}{4}) \neq 0$

$\therefore x \neq \frac{1}{2},\ x \neq \frac{1}{3},\ x \neq \frac{1}{4}$

Therefore, the value of x is all real number except $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.

35)

Let $A = [a_{ij}]$ be an $n \times n$ matrix. Determine whether

A is symmetric.

a) $a_{ij} = i^2 + j^2$

$\Rightarrow$

Lets find the 'entries a_{ji} of matrix A.

$$a_{ji} = j^2 + i^2$$
$$= i^2 + j^2$$

$$\therefore a_{ij} = a_{ji}$$

Therefore, matrix A is symmetric.

b) $a_{ij} = i^2 - j^2$

$\Rightarrow$

Lets find the entries a_{ji} of matrix A.

$$a_{ji} = j^2 - i^2$$

$$\therefore a_{ij} \neq a_{ji}$$

Therefore matrix A is not symmetric unless $n = 1$.

c) $a_{ij} = 2i + 2j$

$\Rightarrow$

Lets find the entries a_{ji} of matrix A.

$$a_{ji} = 2j + 2i$$
$$= 2i + 2j$$

$$\therefore a_{ij} = a_{ji}$$

Therefore, matrix A is symmetric.

d) $a_{ij} = 2i^2 + 2j^3$

$\Rightarrow$

Lets find the entries a_{ji} of matrix A.

$$a_{ji} = 2j^2 + 2i^3$$

$$\therefore a_{ij} \neq a_{ji}$$

Therefore, matrix A is not symmetric unless $n = 1$.
