North South University

Department of Mathematics and Physics

Assignment - 2

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Course No : MAT-130

Course Title : Calculus and Analytical Geometry II

Section : 8

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7.5

10)

$$\int \frac{dx}{x^2-6x-7}$$

$$= \int \frac{1}{(x-7)(x+1)} dx$$

$$= \int \left(\frac{\frac{1}{8}}{x-7} + \frac{-\frac{1}{8}}{x+1} \right) dx$$

$$= \frac{1}{8}\int \frac{1}{x-7} dx - \frac{1}{8}\int \frac{1}{x+1} dx$$

$$= \frac{1}{8} \ln|x-7| - \frac{1}{8}\ln|x+1| + c$$

$$= \frac{1}{8} \ln\left|\frac{x-7}{x+1}\right| + c$$

Ans.

Here,

$$x^2-6x-7 = x^2-7x+x-7$$

$$= x(x-7)+1(x-7)$$

$$= (x-7)(x+1)$$

$$\frac{1}{(x-7)(x+1)} = \frac{A}{x-7} + \frac{B}{x+1}$$

$$\therefore 1 = A(x+1)+B(x-7)$$

$$= Ax + A + Bx - 7B$$

$$1 = (A+B)x + (A-7B)$$

$$\therefore A+B=0$$

$$A-7B=1$$

$$\Rightarrow A = 1+7B$$

$$\therefore 1+7B+B=0$$

$$8B=-1$$

$$\therefore B = -\frac{1}{8}$$

$$\therefore A = \frac{1}{8}$$

20) $\int \frac{3x+1}{3x^2+2x-1} dx$

$= \frac{1}{2} \int \frac{6x+2}{3x^2+2x-1} dx$

$= \frac{1}{2} \ln |3x^2+2x-1| + c$

Ans.

26) $\int \frac{2x^2-2x-1}{x^3-x^2} dx$

Here,

$x^3-x^2 = x^2(x-1)$

$\therefore \frac{2x^2-2x-1}{x^3-x^2} = \frac{2x^2-2x-1}{x^2(x-1)}$

$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$

$\therefore 2x^2-2x-1 = Ax(x-1) + B(x-1) + Cx^2$

$= Ax^2 - Ax + Bx - B + Cx^2$

$= (A+C)x^2 + (B-A)x - B$

$\therefore A+C = 2$

$B-A = -2$

$B = 1$

$\therefore A = 1+2 = 3$

$C = 2-3 = -1$

$= \int \left(\frac{3}{x} + \frac{1}{x^2} + \frac{-1}{x-1} \right) dx$

$= 3\int \frac{1}{x} dx + \int \frac{1}{x^2} dx - \int \frac{1}{x-1} dx$

$= 3 \ln|x| - \frac{1}{x} - \ln|x-1| + c$

Ans.

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$$\int \frac{dx}{x^3+2x}$$

Here,

$x^3+2x = x(x^2+2)$

$\therefore \frac{1}{x^3+2x} = \frac{1}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}$

$\therefore 1 = A(x^2+2) + (Bx+C)x$

$= Ax^2 + 2A + Bx^2 + Cx$

$= (A+B)x^2 + Cx + 2A$

$\therefore A+B = 0$

$C = 0$

$2A = 1 \Rightarrow A = \frac{1}{2}$

$\therefore B = -\frac{1}{2}$

$$= \int \left(\frac{\frac{1}{2}}{x} + \frac{-\frac{1}{2}x+0}{x^2+2} \right) dx$$

$$= \frac{1}{2}\int \frac{1}{x}\,dx - \frac{1}{2}\int \frac{x}{x^2+2}\,dx$$

$$= \frac{1}{2}\ln|x| - \frac{1}{4}\int \frac{2x}{x^2+2}\,dx$$

$$= \frac{1}{2}\ln|x| - \frac{1}{4}\ln|x^2+2| + c$$

$$= \frac{1}{4}\left(2\ln|x| - \ln|x^2+2|\right) + c$$

$$= \frac{1}{4}\ln\left|\frac{x^2}{x^2+2}\right| + c$$

Ans.

32) $\int \frac{x^3+x^2+x+2}{(x^2+1)(x^2+2)}\,dx$

Here,

$$\frac{x^3+x^2+x+2}{(x^2+1)(x^2+2)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2}$$

$\therefore x^3+x^2+x+2 = (Ax+B)(x^2+2)+(Cx+D)(x^2+1)$

$= Ax^3+2Ax+Bx^2+2B + Cx^3+Cx+Dx^2+D$

$= (A+C)x^3+(B+D)x^2+(2A+C)x + (2B+D)$

$\therefore A+C = 1 \Rightarrow C = 1-A$

$B+D = 1 \Rightarrow D = 1-B$

$2A+C = 1 \Rightarrow 2A+1-A = 1$

$\therefore A = 0$

$2B+D = 2$

$\therefore C = 1$

$\Rightarrow 2B+1-B = 2$

$\therefore B = 1$

$\therefore D = 0$

$= \int \left(\frac{1}{x^2+1} + \frac{x}{x^2+2}\right) dx$

$= \int \frac{1}{x^2+1}\,dx + \int \frac{x}{x^2+2}\,dx$

$= \tan^{-1}x + \frac{1}{2}\int \frac{2x}{x^2+2}\,dx$

$= \tan^{-1}x + \frac{1}{2}\ln|x^2+2| + C$

Ans.

6.9

20)

$$y = \sinh^{-1}\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+\frac{1}{x^2}}} \cdot \frac{d}{dx}\left(\frac{1}{x}\right)$$

$$= \frac{1}{\sqrt{1+\frac{1}{x^2}}} \cdot \left(-\frac{1}{x^2}\right)$$

$$= -\frac{1}{x^2\sqrt{1+\frac{1}{x^2}}}$$ Ans.

26)

$$y = \sinh^{-1}(\tanh x)$$

$$= \frac{1}{\sqrt{1+\tanh^2 x}} \cdot \frac{d}{dx}(\tanh x)$$

$$= \frac{1}{\sqrt{1+\tanh^2 x}} \cdot \operatorname{sech}^2 x$$

$$= \frac{\operatorname{sech}^2 x}{\sqrt{1+\tanh^2 x}}$$ Ans.

32/ $\int \operatorname{cosech}^2(3x)\, dx$

Let,

$u = 3x$

$\frac{du}{dx} = 3$

$\therefore dx = \frac{1}{3} du$

$\therefore \int \operatorname{cosech}^2(u) \cdot \frac{1}{3} du$

$= \frac{1}{3} \int \operatorname{cosech}^2 u \, du$

$= -\frac{1}{3} \coth u + c$

$= -\frac{1}{3} \coth(3x) + c$

Ans

38/ $\int \frac{dx}{\sqrt{x^2-2}} \quad (x > \sqrt{2})$

Let,

$x = \sqrt{2} u$

$dx = \sqrt{2}\, du$

$\therefore \int \frac{\sqrt{2}}{\sqrt{2u^2-2}} du$

$= \int \frac{\sqrt{2}}{\sqrt{2} \cdot \sqrt{u^2-1}} du$

$= \int \frac{1}{\sqrt{u^2-1}} du$

$= \cosh^{-1} u + c$

$= \cosh^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$ Ans

42) $\int \frac{dx}{\sqrt{9x^2-25}}$ $(x > 5/3)$

$= \int \frac{dx}{\sqrt{9(x^2-\frac{25}{9})}}$

$= \frac{1}{3} \int \frac{dx}{\sqrt{x^2-(\frac{5}{3})^2}}$

$= \frac{1}{3} \cosh^{-1}(\frac{x}{5/3}) + c$

$= \frac{1}{3} \cosh^{-1}(\frac{3x}{5}) + c$

Ans

6.1

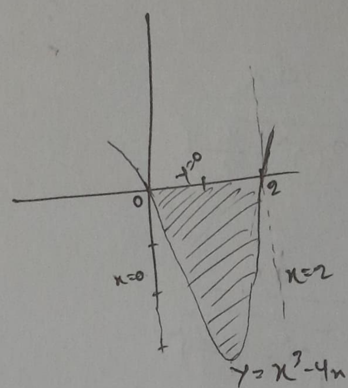
8|

$y = x^3 - 4x$

$y = 0$

$x = 0$

$x = 2$



Area, $A = \int_0^2 [0 - (x^3 - 4x)]\, dx$

$= \int_0^2 (4x - x^3)\, dx$

$= \left[4\frac{x^2}{2} - \frac{x^4}{4}\right]_0^2$

$= \left[2x^2 - \frac{x^4}{4}\right]_0^2$

$= 2 \cdot 4 - 16$

$= -4$

$\therefore$ Area is 4 ~~unit~~ unit.

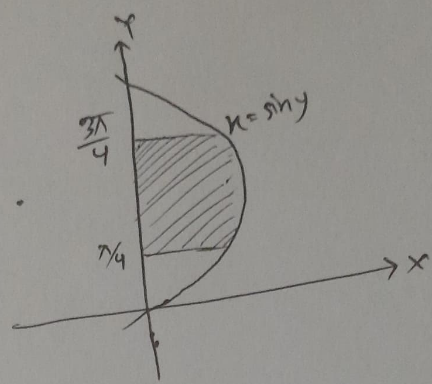
11)

$x = \sin y$

$x = 0$

$y = \frac{\pi}{4}$

$y = \frac{3\pi}{4}$



$$\therefore \text{Area, } A = \int_{\pi/4}^{3\pi/4} \sin y \, dy$$

$$= -\cos y \Big]_{\pi/4}^{3\pi/4}$$

$$= -\cos \frac{3\pi}{4} + \cos \frac{\pi}{4}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$

$$= \sqrt{2}$$

Therefore, area is $\sqrt{2}$

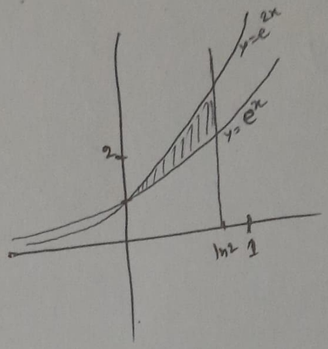
13)

$y = e^x$

$y = e^{2x}$

$x = 0$

$x = \ln 2$



$$\text{Area } A = \int_0^{\ln 2} (e^{2x} - e^x)\, dx$$

$$= \left[\frac{e^{2x}}{2} - e^x \right]_0^{\ln 2}$$

$$= \frac{e^{2\ln 2}}{2} - e^{\ln 2} - \frac{e^0}{2} + e^0$$

$$= \frac{4}{2} - 2 - \frac{1}{2} + 1$$

$$= \frac{1}{2}$$

Therefore, area is $\frac{1}{2}$.

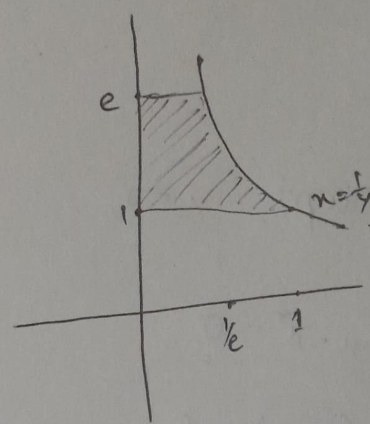
14]

$x = \frac{1}{y}$

$x = 0$

$y = 1$

$y = e$



Therefore,

$$\text{Area, } A = \int_1^e \frac{1}{y}\, dy$$

$$= \ln y \Big]_1^e$$

$$= \ln e - \ln 1$$

$$= 1 - 0$$

$$= 1.$$

Therefore, area is. 1.

37)

a) $y = 2x - x^2$

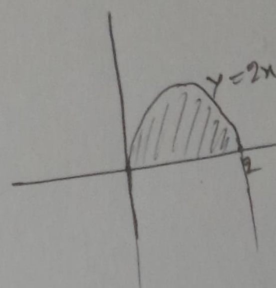
for x-intercept, $y = 0$;

$2x - x^2 = 0$

$x(2-x) = 0$

$x = 0$; $2 - x = 0$

$x = 2$



$\therefore$ Area, $A = \int_0^2 (2x - x^2)\, dx$

$= \left[\frac{2x^2}{2} - \frac{x^3}{3} \right]_0^2$

$= 2^2 - \frac{2^3}{3}$

$= 4 - \frac{8}{3}$

$= \frac{12-8}{3}$

$= 4/3$

Therefore, area 4/3

b)

$y = mx$ intercects $y = 2x - x^2$, where,

$$mx = 2x - x^2$$

$$\Rightarrow x^2 + mx - 2x = 0$$

$$\Rightarrow x^2 + (m-2)x = 0$$

$$\Rightarrow x(x + m - 2) = 0$$

$$\Rightarrow x = 0 \quad \text{and} \quad x = 2-m$$

Then area below the curve and above the line is

$$\int_0^{2-m} (2x - x^2 - mx)\, dx$$

$$= \int_0^{2-m} \left[(2-m)x - x^2\right] dx$$

$$= \left[(2-m)\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{2-m}$$

$$= \frac{(2-m)(2-m)^2}{2} - \frac{(2-m)^3}{3}$$

$$= \frac{3(2-m)^3 - 2(2-m)^3}{6}$$

$$= \frac{1}{6}(2-m)^3$$

So,

$$\frac{1}{6}(2-m)^3 = \frac{4}{3\cdot 2}$$

$$(2-m)^3 = \frac{2}{3}\times 6$$

$$(2-m)^3 = 4$$

$$2-m = \sqrt[3]{4}$$

$$m = 2-\sqrt[3]{4}$$

Therefores the value of m is, $(2-\sqrt[3]{4})$

and the equation of the line,

$$y = (2-\sqrt[3]{4})x$$

$$\Rightarrow y = 2x - \sqrt[3]{4}x$$

Ans