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Step 4: Use the Factor theorem to determine if the potential rational zero is a zero. If it is, we synthetic division or long divison to factor the polynomial function. Repeat step 4 until all the zeros of the polynomial function have been identified and the polynomial function is completely factored.

✱ Find the zeros of $f(x) = x^5 - x^4 - 4x^3 + 8x^2 - 32x + 48$ write f in factored form.

Here, $a_0 = 48$

$a_5 = 1$

factor of a_0 ; $p = \pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm16, \pm24, \pm48$

factor of a_5 ; $q = \pm1$

$\therefore P/q = \pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm16, \pm24, \pm48$

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Since -3 appears to be a zero and -3 is a potential rational zero, evalute $f(-3)$ and find that $f(-3)=0$.

By the Factor Theorem, $x+3$ is a factor of f. We use synthetic division to factor f.

Factor f as

$f(x) = x^5 - x^4 - 4x^3 + 8x^2 - 32x + 48 = (x+3)(x^4 - 4x^3 + 8x^2 - 16x + 16)$

Now work with the first depressed equation:

$q_1(x) = x^4 - 4x^3 + 8x^2 - 16x + 16 = 0$

It appears that 2 might be zero of even multiplicity.

Check the potential rational zero 2 using synthetic division.

Since, $f(2) = 0$, then $x-2$ is a factor and

$f(x) = (x+3)(x-2)(x^3 - 2x^2 + 4x - 8)$

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The depressed equation $q_2(x) = x^3 - 2x^2 + 4x - 8 = 0$

can be factored by grouping.

$$x^3 - 2x^2 + 4x - 8 = (x^3 - 2x^2) + (4x - 8)$$

$$= x^2(x-2) + 4(x-2)$$

$$= (x-2)(x^2+4) = 0$$

$(x-2) = 0$ or $x^2 + 4 = 0$

$x = 2$

Since, $x^2 + 4 = 0$ has no real solutions. the real zeros of f are -3 and 2, with 2 being a zero of multiplicity 2. The factored form of f is.

$$f(x) = x^5 - x^4 - 4x^3 + 8x^2 - 32x + 48$$

$$= (x+3)(x-2)^2(x^2+4)$$

$(x-2)^2 = 0$	$x^2 + 4 = 0$	$x + 3 = 0$
$x = 2 \mid x = 2$	$x^2 = -4$	$x = -3$
	$x = 2i \mid -2i$	

complex zero is r

$f(r) = 0$

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_1 x + a_0$

If $r_1, r_2, r_3 \dots r_n$ are the complex zeros of the polynomial function, then $f(x) = a_n (x-r_1)(x-r_2)(x-r_3)\dots(x-r_n)$

H.W, 9th Ed

4.1

4.1

Given that,

Zeros $= -1, 1, 3$

degree $= 3$

if r is the real zero of a polynomial function then $(x-r)$ is a factor of f.

zeros

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Chapter - 5

Exponential and Logarithmic Function

$$\left.\begin{array}{l} x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) = (g \cdot f)(x) = g \circ f \\ x \xrightarrow{g} g(x) \xrightarrow{f} f(g(x)) = (f \cdot g)(x) = f \circ g \\ x \xrightarrow{g} g(x) \xrightarrow{g} g(g(x)) = (g \cdot g)(x) = g \circ g \\ x \xrightarrow{f} f(x) \xrightarrow{f} f(f(x)) = (f \cdot f)(x) = \cancel{f \circ g}\ f \circ f \end{array}\right\}$$ Composit Function

(*)

$f(x) = 2x^2 - 3$

$g(x) = 4x$

a) $(f \circ g)(1) = f(g(1))$

$= f(4 \cdot 1)$

$= f(4)$

$= 2 \cdot 4^2 - 3$

$= 32 - 3$

$= 29$

b) $(g \circ f)(1) = g(f(1))$

$= g(2 \cdot 1 - 3)$

$= g(2 - 3)$

$= g(-1)$

$= 4 \cdot (-1)$

$= -4$

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c) $(f \circ f)(-2)$

$= f(f(-2))$

$= f(2(-2)^2 - 3)$

$= f(8-3)$

$= f(5)$

$= 2 \cdot 5^2 - 3$

$= 50 - 3$

$= 47$

Ans

d) $(g \circ g)(-1)$

$= g(g(-1))$

$= g(4(-1))$

$= g(-4)$

$= 4 \cdot (-4)$

$= -16$

Ans

(*) $f(x) = x^2 + 3x - 1$

$g(x) = 2x + 3$

a) $f \circ g$

$= f(g(x))$

$= f(2x+3)$

$= (2x+3)^2 + 3(2x+3) - 1$

$= 4x^2 + 12x + 9 + 6x + 9 - 1$

$= 4x^2 + 18x + 17$

Ans

b) $g \cdot f$

$= g(f(x))$

$= g(x^2 + 3x - 1)$

$= 2(x^2 + 3x - 1) + 3$

$= 2x^2 + 6x - 2 + 3$

$= 2x^2 + 6x + 1$

Ans

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⊛ Find the domain of $f \cdot g$ if $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{4}{x-1}$.

$$\therefore f \cdot g = f(g(x))$$

$$= f\left(\frac{4}{x-1}\right)$$

$$= \frac{1}{\frac{4}{x-1} + 2}$$

$$= \frac{1}{\frac{4+2x-2}{x-1}}$$

$$= \frac{x-1}{2x+2}$$

Now, $2x+2 \neq 0$

$2x \neq -2$

$x \neq -1$

again, $x-1 \neq 0$

$x \neq 1$

domain of $f \cdot g = \{\mathbb{R} - \pm 1\}$

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⊛ If $f(x) = 3x-4$ and $g(x) = \frac{1}{3}(x+4)$, show that,

$$(f\circ g)(x) = (g\circ f)(x) = x$$

⇒

$$(f\circ g)(x) = f(g(x))$$

$$= f\left(\frac{1}{3}(x+4)\right)$$

$$= 3\cdot\frac{1}{3}\cdot(x+4) - 4$$

$$= x+4-4$$

$$= x$$

$$(g\circ f)(x) = g(f(x))$$

$$= g(3x-4)$$

$$= \frac{1}{3}(3x-4+4)$$

$$= \frac{1}{3}\cdot 3x$$

$$= x$$

$\therefore (f\circ g)(x) = (g\circ f)(x) = x$

Showed.

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⊛ $f(x) = \frac{1}{x+2}$

$$(f \circ f) = f(f(x))$$

$$= f\left(\frac{1}{x+2}\right)$$

$$= \frac{1}{\frac{1}{x+2}+2}$$

$$= \frac{1}{\frac{1+2x+4}{x+2}}$$

$$= \frac{x+2}{2x+5}$$

Now,

$$2x+5 \neq 0$$

$$2x \neq -5$$

$$x \neq \frac{-5}{2}$$

again,

$$x+2 \neq 0$$

$$x \neq -2$$

So, domain of $(f \cdot f) = \{\mathbb{R} - (-\frac{5}{2}), (-2)\}$

⊛ ~~Hor~~

Horizontal Line-Test Theorem.

If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

Theorem:

A function that is increasing on an interval I is a one-to-one function on I.

A function that is decreasing on an interval I is a one-to-one function on I.

Remember, if f is a one-to-one function, it has an inverse function, f^{-1}.

Domain of f = Range of f^{-1}

Range of f = Domain of f^{-1}

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⊛ $f^{-1}(f(x)) = x$, where x is in the domain of f

⊛ $f(f^{-1}(x)) = x$, where x is in the domain of f^{-1}

⊛ Verify that the inverse of $g(x) = x^3$ is $g^{-1}(x) = \sqrt[3]{x}$

$\Rightarrow$

$$g(g^{-1}(x)) = g(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$$

$$g^{-1}(g(x)) = g^{-1}(x^3) = \sqrt[3]{x^3} = x^{3 \cdot \frac{1}{3}} = x$$

$\therefore g(g^{-1}(x)) = g^{-1}(g(x)) = x$

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* The function

$$f(x) = \frac{2x+1}{x-1} \; ; \; x \neq 1$$

is one-to-one. find its inverse and check the result.

$\Rightarrow$

Replacing x by y and y by x.

$$x = \frac{2y+1}{y-1} \quad \cdots \text{(i)}$$

This is the implicit form of the inverse function.

$$2y+1 = xy - x$$

$$xy - 2y = x+1$$

$$y(x-2) = x+1$$

$$y = \frac{x+1}{x-2} \quad \cdots \text{(ii)}$$

This is the explicit form of the inverse function

is $f^{-1}(x) = \dfrac{x+1}{x-2}$.

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Cheek!

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2x+1}{x-1}\right)$$

$$= \frac{\frac{2x+1}{x-1}+1}{\frac{2x+1}{x-1}-2}$$

$$= \frac{2x+1+x-1}{x-1} \cdot \frac{x-1}{2x+1-2x+2}$$

$$= \frac{3x}{3}$$

$$= x$$

$$f(f^{-1}(x)) = f\left(\frac{x+1}{x-2}\right)$$

$$= \frac{2\cdot\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-1}$$

$$= \frac{2x+2+x-2}{x-2} \cdot \frac{x-2}{x+1-x+2}$$

$$= \frac{3x}{x}$$

$$= x$$

$$\therefore f(f^{-1}(x)) = f^{-1}(f(x)) = x.$$

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Laws of exponents

$f(0) = 5$

$f(x) = 2f(x-1)$

$\frac{f(x)}{f(x-1)} = 2$

If $x=1$

$f(1) = 2f(0) = 2 \cdot 5 = 5 \cdot 2^1$

If $x=2$

$f(2) = 2f(1) = 2 \cdot 5 \cdot 2^1 = 5 \cdot 2^2$

If $x=3$

$f(3) = 2f(2) = 2 \cdot 5 \cdot 2^2 = 5 \cdot 2^3$

If $x=x$;

$f(x) = 5 \cdot 2^x$

If $x = x+1$

$f(x+1) = 5 \cdot 2^{x+1}$

$$\frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = \cdots = \frac{f(x+1)}{f(x)} = \cdots = 2$$

$f(x) = Ca^x$

$f(x+1) = Ca^{x+1}$

$$\therefore \frac{f(x+1)}{f(x)} = \frac{Ca^{x+1}}{Ca^x} = a$$

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$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

$$\boxed{\begin{array}{c} a^u = a^v \\ \therefore u = v \end{array}}$$

⊛

Properties of the Exponential Function $f(x) = a^x$. $a > 1$.

1. The domain is the set of all real numbers or $(-\infty, \infty)$ using interval notation; the range is the set of positive real numbers or $(0, \infty)$ using interval notation

2. There are no x-intercepts; the y-intercept is 1.
3. The x-axis $(y = 0)$ is a horizontal asymptote as $x \to -\infty$ $\left[\lim_{x \to -\infty} a^x = 0\right]$

4. $f(x) = a^x$, where $a > 1$, is an increasing function and is one-to-one.

* Solve: $4^{2x-1} = 8^x$

$\Rightarrow 2^{2(2x-1)} = 2^{3 \cdot x}$

$\Rightarrow 2^{4x-2} = 2^{3x}$

$\Rightarrow 4x-2 = 3x$

$\Rightarrow x = 2$

Ans

* $e^{-x^2} = (e^x)^2 \cdot \frac{1}{e^3}$

$\Rightarrow e^{-x^2} = e^{2x} \cdot e^{-3}$

$e^{-x^2} = e^{2x-3}$

$-x^2 = 2x-3$

$x^2 + 2x = 3$

$x^2 + 2x - 3 = 0$

$x^2 + 3x - x - 3 = 0$

$x(x+3) - 1(x+3) = 0$

$(x+3)(x-1) = 0$

$x = \{-3, 1\}$.

* $5^{x^2+8} = 125^{2x}$

$5^{x^2+8} = 5^{3 \cdot 2x}$

$x^2 + 8 = 6x$

$x^2 - 6x + 8 = 0$

$x^2 - 4x - 2x + 8 = 0$

$x(x-4) - 2(x-4) = 0$

$(x-4)(x-2) = 0$

$x = \{4, 2\}$.

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$y = f(x) = a^x$

inverse implicit form. $x = a^y$

↳ logarithmic function.

$y = \log_a x \cong x = a^y$

$$\boxed{\begin{array}{l} \log_a x = y \\ x = a^y \end{array}}$$

✻ $y = \log_3 x$

$x = 3^y$

✻ $4 = \log_3 81$

$\Rightarrow 81 = 3^4$

✻

a) $1.2^3 = m$

$3 = \log_{1.2} m$

b) $e^b = 9$

$b = \log_e 9$

c) $a^4 = 24$

$4 = \log_a 24$

✻

a) $\log_a 4 = 5$

$4 = a^5$

b) $\log_e b = -3$

$b = e^{-3}$

c) $\log_3 5 = c$

$5 = 3^c$

✻

a) $\log_2 16$

$= \log_2 2^4$

$= 4$

b) $\log_3 \frac{1}{27}$

$= \log_3 3^{-3}$

$= -3$

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※

properties of logarithmic functions.

1. The domain is the set of positive real numbers or $(0, \infty)$ using interval notation; the range is the set of all real numbers or $(-\infty, \infty)$ using ~~in~~ interval notation.

2. The x-intercept of the graph is 1. There is no y-intercept.

3. The y-axis $(x=0)$ is a vertical asymptote of the graph.

4. A logarithmic function is decreasing if $0<a<1$ and increasing if $a>1$.

5. The graph of f contains the points $(1,0)$, $(a,1)$ and $(\frac{1}{a}, -1)$.

6. The graph is smooth and continuous, with no corners or gaps.

a) $\log_3 (4x-7) = 2$

$4x - 7 = 3^2$

$4x - 7 = 9$

$4x = 16$

$x = 4$

b) $\log_x 64 = 2$

$64 = x^2$

$x = \pm\sqrt{64}$

$x = \pm 8$

$x \neq -8$

cannot be negative.

$\therefore x = 8$ Ans

$e^{2x} = 5$

$\ln e^{2x} = \ln 5$

$\ln 5 = 2x$

$x = \frac{\ln 5}{2}$

$= 0.8047$ Ans

$\boxed{\ln = \log_e}$

$\log_a 1 = 0$

$y = \log_a 1$

$a^y = 1$

$a^y = a^0$

$\therefore y = 0$

$\therefore \log_a 1 = 0$

$\log_a a = 1$

$y = \log_a a$

$a^y = a$

$\therefore y = 1$

$\therefore \log_a a = 1.$

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* $\log_a (x\sqrt{x^2+1})$; $x > 0$ Express all powers as factor.

$\Rightarrow \log_a x + \log_a (\sqrt{x^2+1})$

$= \log_a x + \log_a (x^2+1)^{\frac{1}{2}}$

$= \log_a x + \frac{1}{2} \log_a (x^2+1)$ Ans

* $\ln \frac{x^2}{(x-1)^3}$; $x > 1$

$= \ln x^2 - \ln (x-1)^3$

$= 2 \ln x - 3 \ln (x-1)$ Ans

* $\log_a \frac{\sqrt{x^2+1}}{x^3 (x+1)^4}$

$= \log_a (\sqrt{x^2+1}) - \log_a (x^3)(x+1)^4$

$= \frac{1}{2} \log_a (x^2+1) - \log_a x^3 + \log_a (x+1)^4$

$= \frac{1}{2} \log_a (x^2+1) - 3 \log_a x + 4 \log_a (x+1)$

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c) $\log_a x + \log_a 9 + \log_a (x^2+1) - \log_a 5$

$= \log_a (9x) + \log_a (x^2+1) - \log_a 5$

$= \log_a [9x \cdot (x^2+1)] - \log_a 5$

$= \log_a \left[\frac{9x \cdot (x^2+1)}{5} \right]$ Ans

$2 \log_5 x = \log_5 9$

$\log_5 x^2 = \log_5 9$

$x^2 = 9$

$x = \pm 3$

$\therefore x = 3$

$\log_5 (x+6) + \log_5 (x+2) = 1$

$\log_5 (x+6) + \log_5 (x+2) = 1$

$(x+6)(x+2) = 5^1 = 5$

$\therefore x = -7 \mid x = -1$

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⊛ $\ln x + \ln(x-4) = \ln(x+6)$

$\ln[x \cdot (x-4)] = x+6$

$\therefore x = 6 \mid x = -1$

⊛

$5^{x-2} = 3^{3x+2}$

$\Rightarrow \ln 5^{x-2} = \ln 3^{3x+2}$

$\Rightarrow (x-2)\ln 5 = (3x+2)\ln 3.$

$\Rightarrow (\ln 5)x - 2\ln 5 = (3\ln 3)x + 2\ln 3$

$x[\ln 5 * 3\ln 3] = 2\ln 3 + 2\ln 5$

$$\therefore x = \frac{2\ln 3 + 2\ln 5}{\ln 5 * 3\ln 3}$$

$= -3.21$

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⊛ $4^x - 2^x - 12 = 0$

$(2^x)^2 - 2^x - 12 = 0$

~~$\therefore (2^x - 4)(2^x + 3) = 0$~~

$(2^x)^2 - 4 \cdot 2^x + 3 \cdot 2^x - 12 = 0$

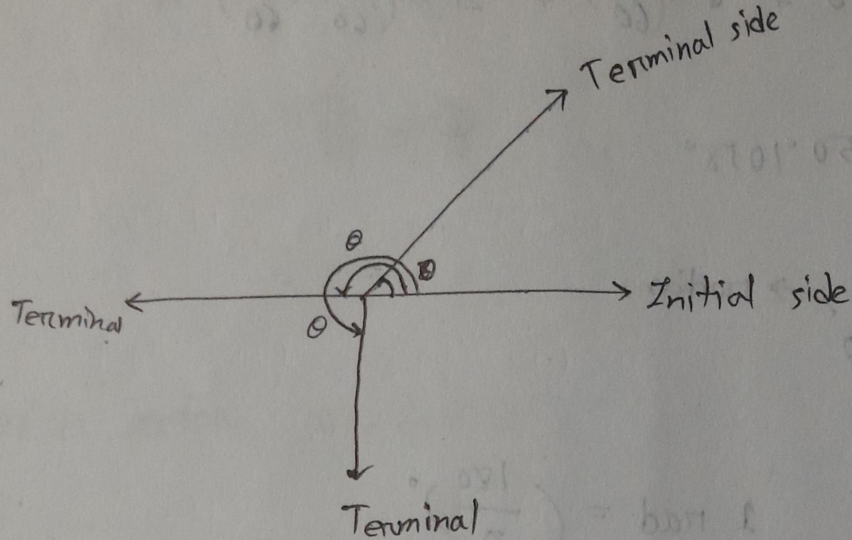
$2^x(2^x - 4) + 3(2^x - 4) = 0$

$(2^x - 4)(2^x + 3) = 0$

Anxux

Chapter 6

Trigonometric Function



$1^\circ = 60'$

$1' = 60''$

$32.25^\circ \rightarrow 0.25^\circ = \frac{\cancel{25}^{5}}{\cancel{100}_{20}} \times \cancel{60}^{3}$

$= 15'$

$32.25^\circ = 32^\circ\ 15'$

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⊛

$50^\circ\, 6'\, 21'' = 50^\circ + 6 \times 1' + 21 \times 1''$

$= 50^\circ + 6 \times \left(\frac{1}{60}\right)^\circ + 21 \times \left(\frac{1}{60} \times \frac{1}{60}\right)^\circ$

$= 50.1058^\circ$

⊛ $s = r\theta$ → radians

$1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ$

$1 \text{ degree} = \frac{\pi}{180}$ radians.

⊛ $60^\circ = 60 \times 1^\circ$

$= 60 \times \frac{\pi}{180}$ radians

$= \frac{\pi}{3}$ radians

⊛ $\frac{\pi}{6} = \frac{\pi}{6} \times 1$ radians

$= \frac{\pi}{6} \times \frac{180}{\pi}$ degrees

$= 30^\circ$

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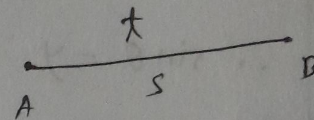
$$\frac{\theta}{\theta_1} = \frac{A}{A_1}$$

$$A = A_1 \times \frac{\theta}{\theta_1}$$

$$= \pi r^2 \times \frac{\theta}{2\pi}$$

$$= \frac{1}{2} r^2 \theta .$$



Area of a sector, $A = \frac{1}{2} r^2 \theta$

$$v = \frac{s}{t}$$

$$\omega = \frac{\theta}{t}$$

linear speed, $v = \frac{s}{t} = \frac{r\theta}{t}$

$$= r\left(\frac{\theta}{t}\right) = r \cdot \omega$$

$$\therefore v = r\omega$$

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Equation of a circle,

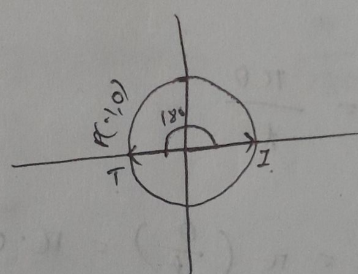
$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 = 1 \longrightarrow \text{unit circle}$$

$\therefore \sin\theta = y$ $\cos\theta = x$ $\tan\theta = \dfrac{y}{x}$

$\operatorname{cosec}\theta = \dfrac{1}{y}$ $\sec\theta = \dfrac{1}{x}$ $\cot\theta = \dfrac{x}{y}$

(*)



$\theta = 180° = \pi$

$\sin\theta = 0$ $\cos\theta = -1$ $\tan\theta = 0$

$\operatorname{cosec}\theta$ = undefined $\sec\theta = -1$ $\cot\theta$ = undefined

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(*) $\sin(3\pi) = y = 0$

$\cos(-270^\circ) = x = 0$

(*) $\theta = \frac{\pi}{6}$

$\sin\left(\frac{\pi}{6}\right) = y = \frac{1}{2}$

$\cos\left(\frac{\pi}{6}\right) = x = \frac{\sqrt{3}}{2}$

$\tan\left(\frac{\pi}{6}\right) = \frac{y}{x} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

$\operatorname{cosec}\left(\frac{\pi}{6}\right) = \frac{1}{y} = \frac{1}{1/2} = 2$

$\sec\left(\frac{\pi}{6}\right) = \frac{1}{x} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$

$\cot\left(\frac{\pi}{6}\right) = \frac{x}{y} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

(*) $\left.\begin{array}{l} \operatorname{cosec}\theta = \frac{1}{y} \\ \cot\theta = \frac{x}{y} \end{array}\right\} y \neq 0$ $\qquad$ $\left.\begin{array}{l} \sec\theta = \frac{1}{x} \\ \tan\theta = \frac{y}{x} \end{array}\right\} x \neq 0$

$\pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi$ $\qquad$ $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$

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$$\sin(\theta + 2\pi) = \sin\theta \qquad \cos(\theta+2\pi) = \cos\theta$$

$$\operatorname{cosec}(\theta + 2\pi) = \operatorname{cosec}\theta \qquad \sec(\theta+2\pi) = \sec\theta$$

$$\tan(\theta + \pi) = \tan\theta$$

$$\cot(\theta + \pi) = \cot\theta$$

a) $\sin\frac{17\pi}{4} = \sin\left(\frac{\pi}{4} + 4\pi\right)$

$= \sin\frac{\pi}{4}$

$= y = 1$

b) $\cos(5\pi)$

$= \cos(\pi + 4\pi)$

$= \cos\pi$

$= x = -1$

c) $\tan\frac{5\pi}{4}$

$= \tan\left(\frac{\pi}{4} + \pi\right)$

$= \tan\frac{\pi}{4}$

$= \frac{y}{x}$

$= \frac{1/2}{1/2} = 1$

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(*)

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$ $\sec(-\theta) = \sec\theta$ $\cot(-\theta) = -\cot\theta$

d) $\tan\left(-\frac{37\pi}{4}\right) = -\tan\left(\frac{37\pi}{4}\right)$

$= -\tan\left(\frac{\pi}{4} + 9\pi\right)$

$= -\tan\left(\frac{\pi}{4}\right)$

$= -\frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}}$

$= -\frac{1/\sqrt{2}}{1/\sqrt{2}}$

$= -1$

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Chapter 7

$y = \sin x \longrightarrow$ Domain $= (\infty, \infty)$

Range $= [-1, 1]$

$x = \sin y$ ... implicit form

$y = \sin^{-1} x$ ... Explicit form

Domain $[-1, 1]$

Range $(-\infty, \infty)$

* $\sin^{-1} 1 = ?$

Let, $\theta = \sin^{-1} 1$

$\sin\theta = 1 = \sin\frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{2}$

Thus, $\sin^{-1} = \frac{\pi}{2}$.

* $\sin^{-1}(\sin\frac{5\pi}{8})$

$= \sin^{-1}(\sin\frac{3\pi}{8})$

$= \frac{3\pi}{8}$

* $\cos^{-1}(\cos(-\frac{2\pi}{3}))$

$= \cos^{-1}(\cos\frac{2\pi}{3})$

$= \frac{2\pi}{3}$

* $\tan^{-1}(-\sqrt{3})$

$= -\tan^{-1}(\sqrt{3})$

$= -\frac{\pi}{3}$

⊛

Given that,

$$f(x) = y = 2\sin x - 1$$

Let replace x by y and y by x

$$x = 2\sin y - 1$$

$$2\sin y = x+1$$

$$\sin y = \frac{x+1}{2}$$

$$y = \sin^{-1}\left(\frac{x+1}{2}\right)$$

Thus the inverse function of $f(x) = 2\sin x - 1$ is

$$f^{-1}(x) = \sin^{-1}\left(\frac{x+1}{2}\right)$$

For $f(x)$,

Domain $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range $= [-3, 1]$

For, $f^{-1}(x)$,

Domain $= [-3, 1]$

Range $= [-\pi/2, \frac{\pi}{2}]$

Date : ..../......../........

(Q) Write $\sin(\tan^{-1}u)$ as an algebraic expression containing u.

Let,

$\tan^{-1} u = \theta$

So, $\sin[\tan^{-1} u] = \sin\theta = \sin\theta \dfrac{\cos\theta}{\cos\theta}$

$= \tan\theta \cdot \cos\theta$

$= \dfrac{\tan\theta}{\sec\theta}$

$= \dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

$= \dfrac{u}{\sqrt{1+u^2}}$ Ans

P.T.O.

Date : ..../......../........

Establish the identity :

$$\frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \cos\theta - \sin\theta$$

$$L.H.S. = \frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)}$$

$$= \frac{[\sin(-\theta)]^2 - [\cos(-\theta)]^2}{-\sin\theta - \cos\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{-\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{-(\sin\theta + \cos\theta)}$$

$$= -\sin\theta + \cos\theta$$

$$= \cos\theta - \sin\theta$$

$$= R.H.S.$$

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$$L.H.S = \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{(1-\sin\theta)(1+\sin\theta)}{\cos\theta\,(1+\sin\theta)}$$

$$= \frac{1-\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos\theta}{1+\sin\theta}$$

$= R.H.S.$

Date : ..../........../..........

Important for Final

1. $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$

2. $\dfrac{1-2\cos^2\theta}{\sin\theta\cos\theta} = \tan\theta - \cot\theta$

3. $\dfrac{1-\cos\theta}{1+\cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$

4. $\dfrac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

5. $\dfrac{\tan\theta - \cot\theta}{\tan\theta + \cot\theta} + 2\cos^2\theta = 1$

6. $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$