(*)

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\coth x) = \operatorname{cosech}^2 x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$$

$$\int \cosh x \, dx = \sinh x + c$$

$$\int \sinh x \, dx = \cosh x + c$$

$$\int \operatorname{sech}^2 x \, dx = \tanh x + c$$

$$\int \operatorname{cosech}^2 x \, dx = -\coth x + c$$

$$\int \operatorname{sech} x \tanh x = -\operatorname{sech} x + c$$

$$\int \operatorname{cosech} x \coth x = -\operatorname{cosech} x + c$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$

$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}} \quad ; x > 1$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2} \quad |x| < 1$$

$$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2} \quad |x| > 1$$

$$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}} \quad 0 < x < 1$$

$$\frac{d}{dx}(\operatorname{cosech}^{-1} x) = -\frac{1}{|x|\sqrt{1+x^2}} \quad x \neq 0$$

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right)+c \Rightarrow \ln\left(x+\sqrt{x^2+a^2}\right)+c$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\left(\frac{x}{a}\right)+c \Rightarrow \ln\left(x+\sqrt{x^2-a^2}\right)+c$$

$$\int \frac{1}{a^2-x^2}\,dx = \left\{\begin{array}{ll} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+c & ;|x|<a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right)+c & ;|x|>a \end{array}\right\} \Rightarrow \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+c \quad ;|x|\neq a$$

$$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\,\mathrm{sech}^{-1}\left|\frac{x}{a}\right|+c \Rightarrow -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{|x|}\right)+c$$

$$0<|x|<a$$

$$\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\,\mathrm{cosech}^{-1}\left|\frac{x}{a}\right|+c \Rightarrow -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2+x^2}}{|x|}\right)+c$$

$$x\neq 0$$

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L-7 / 26.10.2022 /

$$\int \frac{dx}{\sqrt{4x^2-9}}$$

$$= \int \frac{dx}{2\sqrt{x^2-\frac{9}{4}}}$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{x^2-\frac{9}{4}}}\,dx$$

$$= \frac{1}{2}\cosh^{-1}\left(\frac{x}{3/2}\right)+c$$

$$= \frac{1}{2}\cosh^{-1}\left(\frac{2x}{3}\right)+c$$

Ans.

$y = \cosh x$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\sinh^2 x} = \cosh x$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\cosh^2 x = 1+\sinh^2 x$$

$$\boxed{\sinh x = \frac{e^x - e^{-x}}{2}}$$

⊛ $\int_0^{\ln 2} \cosh x \, dx$

$= \sinh x \Big]_0^{\ln 2}$

$= \dfrac{e^x - e^{-x}}{2} \Big]_0^{\ln 2}$

$= \dfrac{e^{\ln 2} - e^{-\ln 2}}{2} - 0$

$= \dfrac{2 - e^{\ln \frac{1}{2}}}{2} = \dfrac{2 - \frac{1}{2}}{2} = \dfrac{\frac{4-1}{2}}{2} = \dfrac{3}{2} \times \dfrac{1}{2} = \dfrac{3}{4}$ Ans

⊛ Show that,

$$\int_{-a}^{a} e^{xt} \, dx = \frac{2}{t} \sinh(at)$$

$L.H.S. = \int_{-a}^{a} e^{xt} \, dx$

$= \dfrac{e^{xt}}{t} \Big]_{-a}^{a}$

$= \dfrac{e^{at}}{t} - \dfrac{e^{-at}}{t} = \dfrac{e^{at} - e^{-at}}{t}$

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$$= \frac{2\left(e^{at} - e^{-at}\right)}{2 \cdot t}$$

$$= \frac{2}{t} \cdot \frac{e^{at} - e^{-at}}{2}$$

$$= \frac{2}{t} \cdot \sinh(at)$$

(Showed)

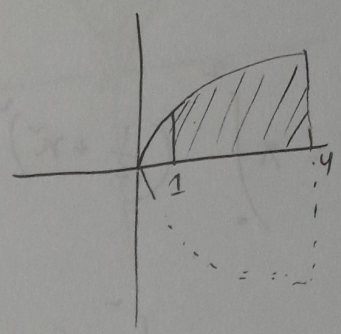
L-8 / 31.10.2022 /

L-9 / 02.11.2022 /

$$V = \int_a^b \pi r^2 \, dx$$

$$= \pi \int_a^b (f(x))^2 \, dx$$

⊛ $y = \sqrt{x}$; $[1, 4]$



$$V = \pi \int_1^4 (\sqrt{x})^2 \, dx$$

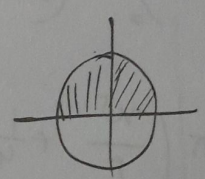
$$= \pi \int_1^4 x \, dx$$

$$= \pi \left[\frac{x^2}{2} \right]_1^4$$

$$= \pi \left(\frac{16}{2} - \frac{1}{2} \right)$$

$$= \frac{15}{2} \pi$$

⊛



$x^2 + y^2 = r^2$

$y^2 = r^2 - x^2$

$y = \pm \sqrt{r^2 - x^2}$

$+ \rightarrow$ upper part

$- \rightarrow$ down part

$$\therefore V = 2\pi \int_0^r \left(\sqrt{r^2 - x^2}\right)^2$$

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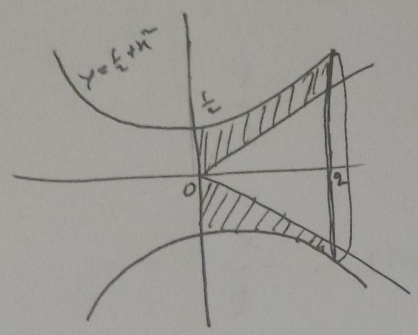
(*)



$$V = \pi \int_0^2 \left(\frac{1}{2} + x^2\right)^2 dx - \pi \int_0^2 x^2 \, dx$$

$$= \pi \int_0^2 \left(x^4 + x^2 + \frac{1}{4} - x^2\right) dx$$

$$= \pi \int_0^2 \left(x^4 + \frac{1}{4}\right) dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{1}{4} x\right]_0^2$$

$$= \ldots$$

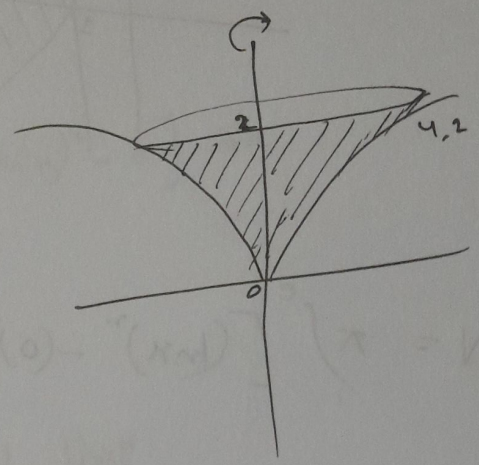
L-10 / 07.11.2022

6.2

⊛ $y = \sqrt{x} \Rightarrow y^2 = x$

$y = 2$

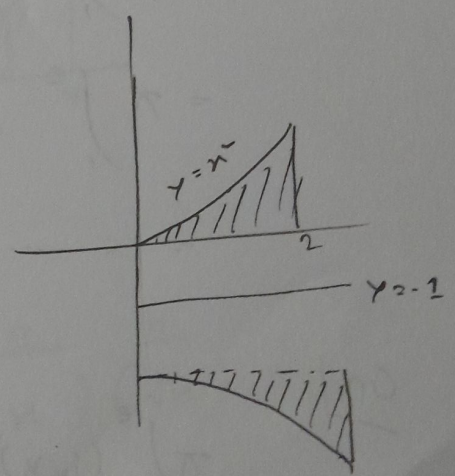
$x = 0$



$$V = \pi \int_0^2 \left\{ (y^2) - (0^2) \right\} dy$$

⊛ $y = x^2 \quad [0, 2]$

rotated about $y = -1$



$$V = \pi \int_0^2 \left[(x^2+1)^2 - (1)^2 \right] dx$$

$$= \pi \int_0^2 (x^4 + 2x^2)\, dx$$

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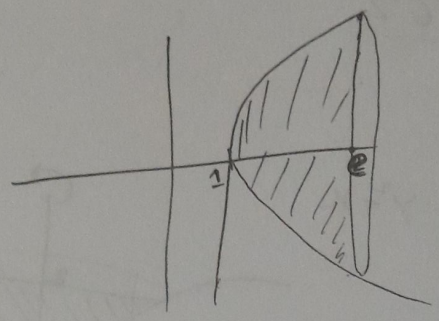
6.3

$y = \ln x$

$x = e$

x-axis



$$V = \pi \int_1^e \left[(\ln x)^2 - (0)^2 \right] dx$$

$$= \pi \int_1^e \left[(\ln x)^2 - 0^2 \right] dx$$

$$= \pi \int_1^e (\ln x)^2 \, dx$$

$$= \pi \int_1^e u^2 \cdot e^u \, du$$

$$= \ldots$$

let,

$u = \ln x \Rightarrow e^u = x$

$du = \frac{1}{x} dx$

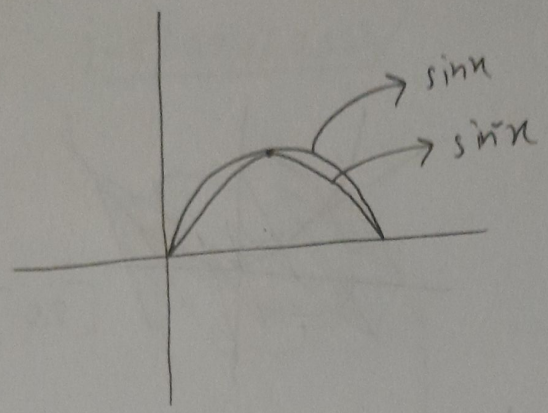
$x \, du = dx$

$e^u du = dx$

Or/

$$\pi \int_1^e (\ln x)^2 \cdot 1 \, dx$$

$$= \boxed{\int u v \, dx} \ldots\ldots$$



$$V = \pi \int_0^{\pi} \left[(\sin^2 x)^2 - 0 \right] dx$$

6.3

Cylindical Shell

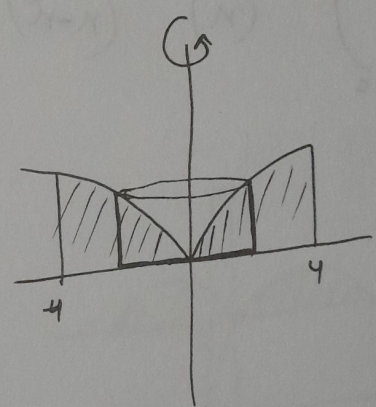
$y = \sqrt{x}$

$x = 1$

$x = 4$

x-axis

rotate about y-axis



$$V = 2\pi \int_1^4 \underbrace{(x)}_{r} \underbrace{(\sqrt{x})}_{h} \, dx$$

$$= 2\pi \int_1^4 x\sqrt{x} \, dx$$

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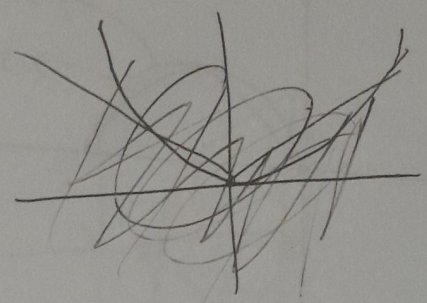
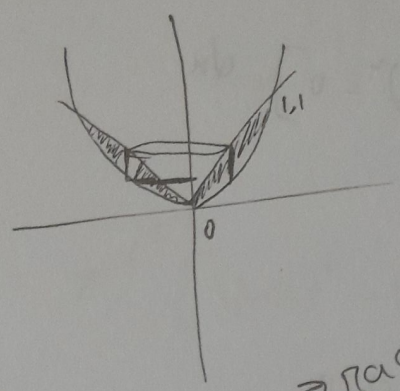
$y = x^2$

$y = x$





$$V = 2\pi \int_0^1 \underbrace{(x)}_{\text{radius}} \underbrace{(x - x^2)}_{\text{height}} \, dx$$

L-11 / 09.11.2022

$y = x^2$

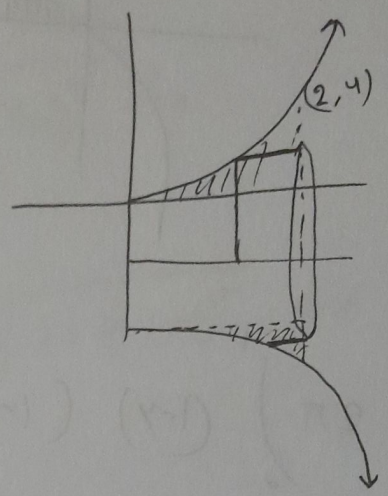
$x = \pm\sqrt{y}$

$x = \sqrt{y}$

(*) $y = x^2 \quad [0,2]$

$y = -1$

↳ rotate



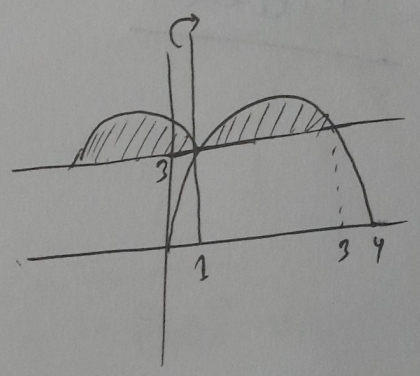
$$V = 2\pi \int_0^4 (y+1)\,(2-\sqrt{y})\,dy$$

(*) $y = 4x - x^2$

$y = 3$

$x = 1$

↳ rotate



$$V = 2\pi \int_1^3 (x-1)\,(4x - x^2 - 3)\,dx$$

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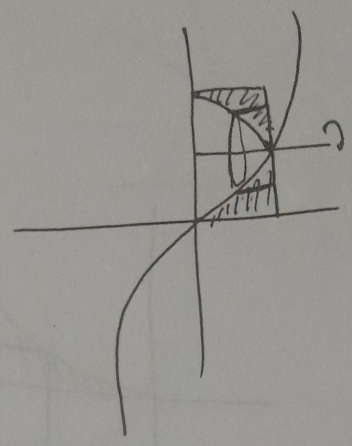
$y = x^3$

$y = 0$

$y = 1$

$\hookrightarrow$ rotate



$y = x^3$

$x = \sqrt[3]{y}$

$$V = 2\pi \int_0^1 (1-y)\,(1-\sqrt[3]{y})\,dy$$

Midterm

L-14 / 21.11.2022 /

6.5

$$S.A. = 2\pi \int_a^b f(x) \cdot \sqrt{1+(f'(x))^2}\, dx$$

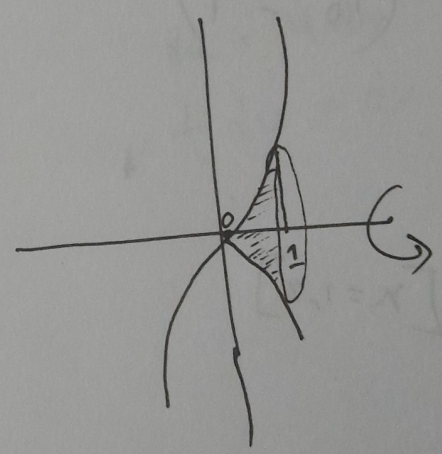
or

$$S.A = 2\pi \int_c^d f(y) \sqrt{1+(f'(y))^2}\, dy$$

(*)

$y = x^3 \quad [x = 0, 1]$

x- axis



$$SA = 2\pi \int_0^1 x^3 \sqrt{1+(3x^2)^2}\, dx$$

u	x
1	0
10	1

$$= 2\pi \int_0^1 x^3 \sqrt{1+9x^4}\, dx$$

Let,

$u = 1+9x^4$

$du = 36x^3\, dx$

$x^3 dx = \frac{1}{36} du$

$$= 2\pi \int_1^{10} \sqrt{u}\, \frac{1}{36}\, du$$

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$$= 2\pi \frac{1}{36} \int_1^{10} \sqrt{u}\, du$$

$$= 2\pi \frac{1}{36} \int_1^{10} u^{\frac{1}{2}}\, du$$

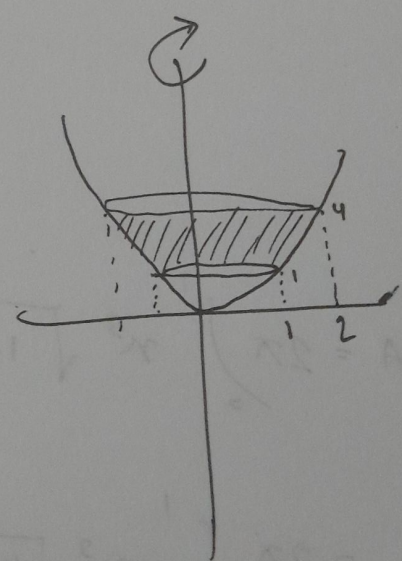
$$= \frac{\pi}{18} \left[\frac{u^{3/2}}{3/2} \right]_1^{10}$$

$$= \frac{\pi}{18} \cdot \frac{2}{3} \left[u^{3/2} \right]_1^{10}$$

$$= \frac{\pi}{27} \left(10^{3/2} - 1 \right)$$

(*) $y = x^2 \; [x = 1, 2]$

y-axis



$$SA = 2\pi \int_1^4 \sqrt{y}\sqrt{1+\frac{1}{4y}}\, dy$$

$$= 2\pi \int_1^4 \sqrt{y}\sqrt{\frac{1}{4y}(4y+1)}\, dy$$

$$= 2\pi \int_1^4 \sqrt{y}\cdot\frac{1}{2\sqrt{y}}\sqrt{(4y+1)}\, dy$$

$$= \pi \int_1^4 \sqrt{4y+1}\, dy$$

$x = \sqrt{y}$

$\frac{dx}{dy} = \frac{1}{2}y^{-\frac{1}{2}}$

$\left(\frac{dx}{dy}\right)^2 = \frac{1}{4}y^{-1}$

$= \frac{1}{4y}$

Let,

$u = 1+4y$

$du = 4\,dy$

$\frac{1}{4}du = dy$

y	u
1	5
4	17

$$= \pi \int_5^{17} \sqrt{u}\,\frac{du}{4}$$

$$= \frac{\pi}{4}\int_5^{17} u^{\frac{1}{2}}\, du$$

$$= \frac{\pi}{4}\left[\frac{u^{3/2}}{3/2}\right]_5^{17}$$

$$= \frac{\pi}{4}\cdot\frac{2}{3}\left[u^{3/2}\right]_5^{17}$$

$$= \frac{\pi}{6}\left[(17)^{3/2} - (5)^{3/2}\right]$$

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7.8

Improper Integral

$$\int_{-\infty}^{b} f(x)\,dx \qquad \int_{a}^{\infty} f(x)\,dx$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{\infty} f(x)\,dx$$

$$\int_{1}^{\infty} \frac{1}{x^2}\,dx \qquad \int_{1}^{\infty} \frac{1}{x^3}\,dx = \frac{1}{2}$$

$$= \lim_{b\to\infty} \int_{1}^{b} \frac{1}{x^2}\,dx \qquad \int_{1}^{\infty} \frac{1}{x}\,dx = \infty$$

$$= \lim_{b\to\infty} \left[-\frac{1}{x}\right]_{1}^{b}$$

$$= \lim_{b\to\infty} \left[-\frac{1}{b} + 1\right]$$

$$= \underline{1}$$

if, power > 1 $\Rightarrow$ converges

Limited area

finite area

power ≤ 1 $\Rightarrow$ diverges, ∞ area

$$\circledast \int_0^{\infty} (1-x)\, e^{-x}\, dx$$

$$= \lim_{b \to \infty} \int_0^{b} (1-x)\, e^{-x}\, dx$$

$$\begin{array}{ccc} 1-x & + & e^{-x} \\ & \searrow & -e^{-x} \\ -1 & - & \\ & \searrow & e^{-x} \\ 0 & & \end{array}$$

$$= \lim_{b \to \infty} \left[e^{-x} - (1-x)e^{-x} \right]_0^b$$

$$= \lim_{b \to \infty} \left[e^{-x} - e^{-x} + xe^{-x} \right]_0^b$$

$$= \lim_{b \to \infty} \left[xe^{-x} \right]_0^b$$

$$= \lim_{b \to \infty} \left[be^{-b} \right]$$

$$= \lim_{b \to \infty} \left(\frac{b}{e^b} \right) \left[\text{form } \frac{\infty}{\infty} \right]$$

$$= \lim_{b \to \infty} \left(\frac{1}{e^b} \right) \rightarrow \text{L'Hopital Rules.}$$

$$= 0$$

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L-15/23.11.2022/

(*) $$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \int_{-\infty}^{0} \frac{1}{1+x^2}\,dx + \int_{0}^{\infty} \frac{1}{1+x^2}\,dx$$

$$= \lim_{a\to-\infty} \int_{a}^{0} \frac{1}{1+x^2}\,dx + \lim_{b\to\infty} \int_{0}^{b} \frac{1}{1+x^2}\,dx$$

$$\therefore \lim_{a\to-\infty} \int_{a}^{0} \frac{1}{1+x^2}\,dx = \lim_{a\to-\infty} \tan^{-1}x \Big]_{a}^{0}$$

$$= \lim_{a\to-\infty} (-\tan^{-1}a)$$

$$= \frac{\pi}{2}$$

$$\therefore \lim_{b\to\infty} \int_{0}^{b} \frac{1}{1+x^2}\,dx = \lim_{b\to\infty} \tan^{-1}x \Big]_{0}^{b}$$

$$= \lim_{b\to\infty} (\tan^{-1}b)$$

$$= \frac{\pi}{2}$$

$$\therefore \int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

discontinue

$$\int_a^{b} f(x)\,dx = \lim_{k \to b^-} \int_a^{k} f(x)\,dx$$

$$\int_{a}^{b} f(x)\,dx = \lim_{k \to a^+} \int_k^{b} f(x)\,dx$$

Discontinue

discontinue

$$\int_a^{b} f(x)\,dx = \int_a^{c} f(x)\,dx + \int_c^{b} f(x)\,dx$$

$$= \lim_{k \to c^-} \int_a^{k} f(x)\,dx + \lim_{k \to c^+} \int_k^{b} f(x)\,dx$$

$$\int_0^1 \frac{1}{\sqrt{1-x}}\,dx = \lim_{k \to 1^-} \int_0^k \frac{1}{\sqrt{1-x}}\,dx$$

$$= \lim_{k \to 1^-} \left[-2\sqrt{1-x}\right]_0^k$$

$$= \lim_{k \to 1^-} \left[-2\sqrt{1-k} + 2\right]$$

$$= 2$$

Ans

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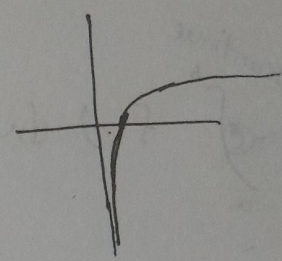
$$\circledast \int_1^2 \frac{1}{1-x}\,dx = \lim_{k\to 1^+} \int_k^2 \frac{1}{1-x}\,dx$$

$$= \lim_{k\to 1^+} \Big[-\ln|1-x|\Big]_k^2$$

$$= \lim_{k\to 1^+} \Big(-\ln(1) + \ln|1-k|\Big)$$

$$= \lim_{k\to 1^+} \ln|1-k|$$



$$= -\infty$$

$$\circledast \int_1^4 \frac{dx}{(x-2)^{2/3}} = \int_1^2 \frac{1}{(x-2)^{2/3}}\,dx + \int_2^4 \frac{1}{(x-2)^{2/3}}\,dx$$

$$= \lim_{k\to 2^-} \int_1^k \frac{1}{(x-2)^{2/3}}\,dx + \lim_{k\to 2^+} \int_k^4 \frac{1}{(x-2)^{2/3}}\,dx$$

$$\therefore \lim_{k\to 2^-} \int_1^k \frac{1}{(x-2)^{2/3}}\,dx = \lim_{k\to 2^-} \Big[3(x-2)^{1/3}\Big]_1^k$$

$$= \lim_{k\to 2^-} \Big(3(k-2)^{1/3} + 3\Big)$$

$$= 3$$

$$\therefore \lim_{k\to 2^+} \int_k^4 \frac{1}{(x-2)^{2/3}}\,dx = \lim_{k\to 2^+} 3(x-2)^{1/3}\Big]_k^4$$

$$= \lim_{k\to 2^+} \left(3\cdot 2^{1/3} - 3(k-2)^{1/3}\right)$$

$$= 3\cdot 2^{1/3}$$

$$\therefore \int_1^4 \frac{dx}{(x-2)^{2/3}} = 3 + 3\cdot\sqrt[3]{2}$$

Ans.

$$\circledast \int_0^2 \frac{dx}{(x-1)^2}$$

$$= -\frac{1}{x-1}\Big]_0^2$$

$= -1-1 = -2$ (Function is positive but area negative) ✗

$$\circledast \int_0^2 \frac{1}{(x-1)^2}\,dx = \int_0^1 \frac{1}{(x-1)^2}\,dx + \int_1^2 \frac{1}{(x-1)^2}\,dx$$

$$= \lim_{k\to 1^-} \int_0^k \frac{1}{(x-1)^2}\,dx + \lim_{k\to 1^+} \int_k^2 \frac{1}{(x-1)^2}\,dx$$

$\therefore \lim_{k\to 1^-} \left[-\frac{1}{k-1} - 1\right] = \infty$ (No need to check other part)

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Ans.: diverse.

L-16 / 28.11.2022

7.8

(*) $\int_{-\infty}^{0} e^{4x}\,dx$

$= \lim_{a\to-\infty} \int_{a}^{0} e^{4x}\,dx$

$= \lim_{a\to-\infty} \left[\frac{e^{4x}}{4}\right]_{a}^{0}$

$= \lim_{a\to-\infty} \left[\frac{1}{4} - \frac{e^{4a}}{4}\right]$

$= \frac{1}{4}$ Ans.

(*) $\int_{-\infty}^{\infty} \frac{x}{(x^2+5)^2}\,dx$

$= \int_{-\infty}^{0} \frac{x}{(x^2+5)^2}\,dx + \int_{0}^{\infty} \frac{x}{(x^2+5)^2}\,dx$

$= \lim_{a\to-\infty} \int_{a}^{0} \frac{x}{(x^2+5)^2}\,dx + \lim_{b\to\infty} \int_{0}^{b} \frac{x}{(x^2+5)^2}\,dx$

$$\therefore \lim_{a \to -\infty} \int_{a}^{0} \frac{x}{(x^2+5)^2}\, dx$$

Let,

$u = x^2+5$

$du = 2x \cdot dx$

$x\,dx = \frac{1}{2}\, du$

x	u
a	a^2+5
0	5

$$= \lim_{a \to -\infty} \int_{a^2+5}^{5} \frac{1}{u^2} \cdot \frac{1}{2}\, du$$

$$= \lim_{a \to -\infty} \left[\frac{1}{2} \cdot \frac{1}{-1} \cdot u^{-1} \right]_{a^2+5}^{5}$$

$$= \lim_{a \to -\infty} \left[\frac{1}{-2u} \right]_{a^2+5}^{5}$$

$$= \lim_{a \to -\infty} \left[\frac{1}{-10} + \frac{1}{2(a^2+5)} \right]$$

$$= -\frac{1}{10}$$

$$\therefore \lim_{b \to \infty} \int_{0}^{b} \frac{x}{(x^2+5)^2}\, dx$$

$$= \lim_{b \to \infty} \left[-\frac{1}{2u} \right]_{5}^{b^2+5}$$

$$= \lim_{b \to \infty} \left(-\frac{1}{2(b^2+5)} + \frac{1}{10} \right)$$

$$= \frac{1}{10}$$

$$\therefore \int_{-\infty}^{\infty} \frac{x}{(x^2+5)^2}\, dx = \frac{1}{10} - \frac{1}{10} = 0$$

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(*) $$\int_{\pi/3}^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}}\, dx$$

$$= \lim_{k \to \pi/3^+} \int_{k}^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}}\, dx$$

$$= \lim_{k \to \pi/3^+} \left[\sqrt{1-2\cos x} \right]_{k}^{\pi/2}$$

$$= \lim_{k \to \pi/3^+} \left(\sqrt{1-2\cos\frac{\pi}{2}} - \sqrt{1-2\cos k} \right)$$

$$= \lim_{k \to \frac{\pi}{3}^+} \left(1 - \sqrt{1-2\cos k} \right)$$

$$= 1 - \sqrt{1-2\cdot\tfrac{1}{2}}$$

$$= 1 - 0$$

$$= 1$$

Ans,

Let,

$$u = 1 - 2\cos x$$

$$du = 2\sin x\, dx$$

$$\therefore \int \frac{1}{\sqrt{u}} \cdot \frac{1}{2}\, du$$

$$= \frac{1}{2} \int u^{-\frac{1}{2}}\, du$$

$$= \frac{1}{2} \cdot \frac{u^{\frac{1}{2}}}{1/2}$$

$$= \sqrt{u}$$

$$= \sqrt{1-2\cos x}$$

$$\boxed{10.1}$$

We need to eliminate $\int$ by some alzebra operation.

L-17/30.11.2022/

$x = f(A)$

$y = g(t)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$\frac{dy}{dt} \to 0, *$

$\frac{dx}{dt} \to 0, *$

$y = f(x)$

$\frac{*}{*}$ ✓ $\quad \frac{0}{*} \quad \frac{*}{0} \quad \frac{0}{0}$

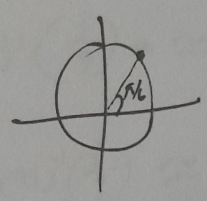
$\longleftrightarrow \quad \updownarrow$

single point

(*)

$x = \cos t$

$y = \sin t$



$\therefore x^2 + y^2 = 1$

$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{6}} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=\frac{\pi}{6}} = \left.\frac{\cos t}{-\sin t}\right|_{t=\frac{\pi}{6}}$$

$$= -\cot t \Big|_{t=\frac{\pi}{6}}$$

$$= -\sqrt{3}$$

$\therefore$ Slope $= -\sqrt{3}$

(*) If need to calculate equation, then use 't' to find the point co-ordinate then use point-slope formula.

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point - slope equation

$$\boxed{y - y_1 = m(x - x_1)}$$

⊛ Where the paper plane flying horizontally?

$x = t - 3\sin t$

$y = 4 - 3\cos t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3\sin t}{1 - 3\cos t}$$

$$\frac{dy}{dt} = 0$$

$$3\sin t = 0$$

$$\sin t = 0$$

$$t = \sin^{-1}(0) = 0, \pi, 2\pi, 3\pi$$

when

$t = 0, \quad \frac{dx}{dt} = 1 - 3\cos 0 = -2 \neq 0$

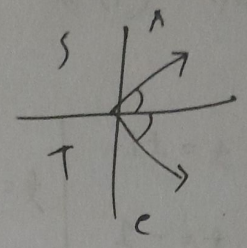
$t = \pi, \quad \frac{dx}{dt} = 1 - 3\cos\pi = 1 + 3 = 4 \neq 0$

$t = 2\pi, \quad \frac{dx}{dt} = 1 - 3\cos 2\pi = 1 - 3 = -2 \neq 0$

$t = 3\pi, \quad \frac{dx}{dt} = 1 - 3\cos 3\pi = 1 + 3 = 4 \neq 0$

$\therefore t = 0, 3.14, 6.28, 9.42$

(*) When flying verticaly:



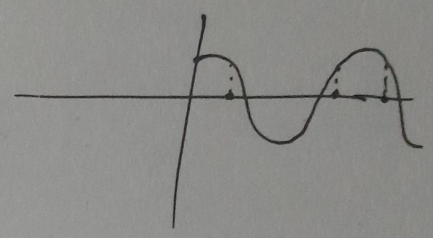
$$\frac{dx}{dt} = 1 - 3\cos t = 0$$

$$-3\cos t = -1$$

$$\cos t = \frac{1}{3}$$

$$t = \cos^{-1}\left(\frac{1}{3}\right)$$

$$t = ?$$



$$t = \cos^{-1}\left(\frac{1}{3}\right) \approx 1.23$$

$$t = 2\pi - \cos^{-1}\left(\frac{1}{3}\right) \approx 5.05$$

$$t = 2\pi + \cos^{-1}\left(\frac{1}{3}\right) \approx 7.51$$

(*) $x = t^2$

$y = t^3$

$$\frac{dy}{dx} = ? \ (1,1)$$

$\therefore t = \pm\sqrt{x}$

$\therefore y = (\pm\sqrt{x})^3 = \pm x^{3/2}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2}{2t} = \frac{3t}{2}$$

for (1,1)

$1 = t^2 \Rightarrow t = \pm 1$

$1 = t^3 \Rightarrow t = 1$

$$\therefore \left.\frac{dy}{dx}\right|_{t=1} = \frac{3 \cdot 1}{2} = \frac{3}{2}$$

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$\Rightarrow$ At point $(1,-1)$

$$\left.\begin{array}{l} 1 = t^2 \Rightarrow t = \pm 1 \\ -1 = t^3 \Rightarrow t = -1 \end{array}\right| \; t = -1$$

$$\therefore \left.\frac{dy}{dx}\right|_{t=-1} = \frac{3t^2}{2t} = \frac{3}{-2} = -\frac{3}{2}$$

$\Rightarrow$ at point $(0,0)$

$$\left.\begin{array}{l} 0 = t^2 \Rightarrow t = 0 \\ 0 = t^3 \Rightarrow t = 0 \end{array}\right| \; t = 0$$

$$\left.\frac{dy}{dx}\right|_{t=0} = \frac{3t^2}{2t} = \frac{0}{0}$$

$\circledast$ $\dfrac{d^2y}{dx^2} = ?$ $(1,1), (1,-1)$

$\to t = 1$ $\quad$ $\to t = -1$

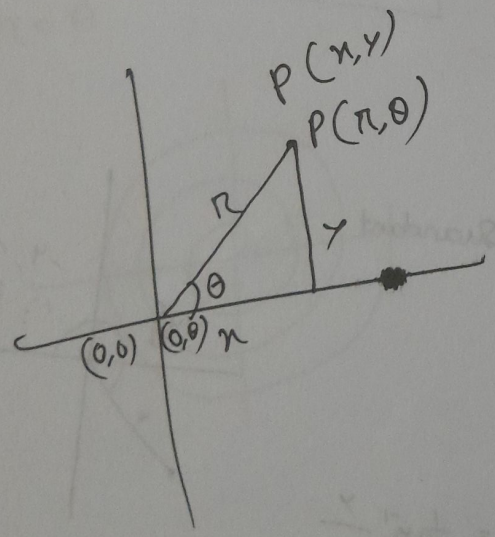
$$y' = \frac{dy}{dx} = \frac{3}{2}t$$

$$\therefore \frac{d^2y}{dx^2} = \frac{dy'}{dx} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}} = \frac{3/2}{2t} = \frac{3}{4t}$$

$$\therefore \left.\frac{d^2y}{dx^2}\right|_{t=1} = \frac{3}{4}$$

$$\therefore \left.\frac{d^2y}{dx^2}\right|_{t=-1} = -\frac{3}{4}$$

Polar Co-ordinate



Poll $(0,\theta)$

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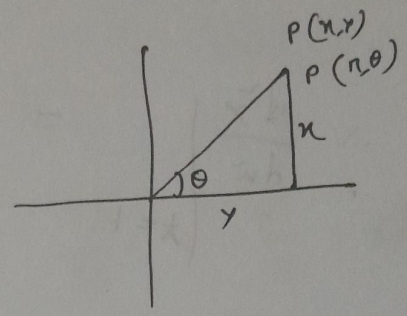
L-18 / 05.12.2022

10.2

* $(r,\theta) = (6, \frac{2\pi}{3})$

Find $(x,y) = (-3, 3\sqrt{3})$



$x = r\cos\theta$

$= 6\cos\frac{2\pi}{3}$

$= -3$

$y = r\sin\theta$

$= 6\sin\frac{2\pi}{3}$

$= 3\sqrt{3}$

$x^2+y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$\boxed{x^2+y^2 = r^2}$

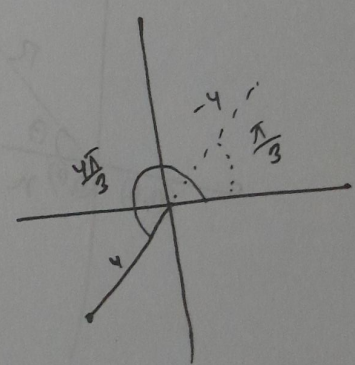
$\frac{y}{x} = \tan\theta$

$\boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$

*

$(x,y) = (-2, -2\sqrt{3}) \longrightarrow$ 3rd Quandant

$(r,\theta) = ? = (4, \frac{4\pi}{3}), (-4, \frac{\pi}{3})$



$r = \sqrt{x^2+y^2} = \sqrt{4+4\cdot 3}$

$= \sqrt{4+12}$

$= \pm 4$

$\theta = \tan^{-1}\frac{y}{x}$

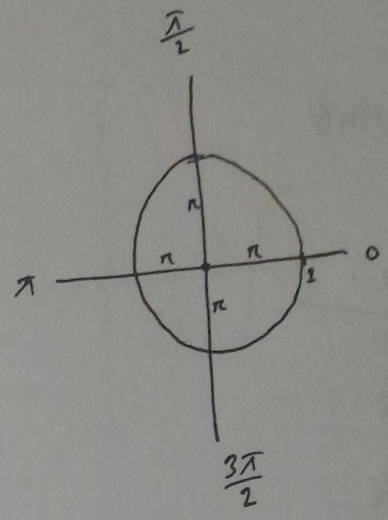
$= \tan^{-1}\frac{-2\sqrt{3}}{-2}$

$= \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}, \frac{4\pi}{3}$

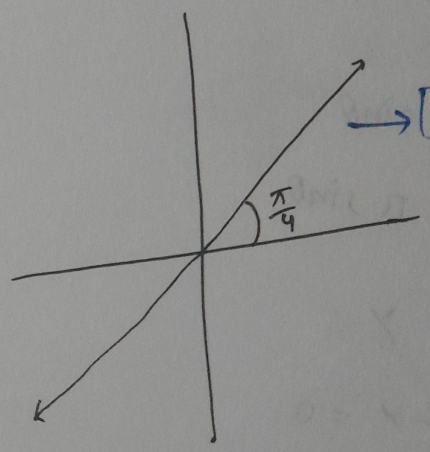
$r = 1$

$\boxed{r = a} \rightarrow$ families of circle



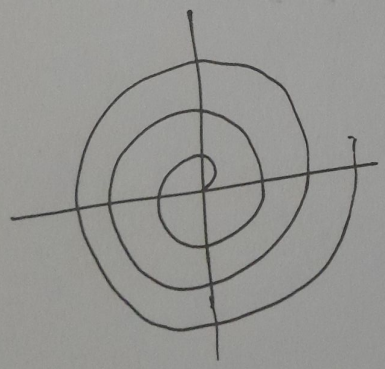
$\theta = \frac{\pi}{4}$

$\boxed{\theta = \theta_0} \rightarrow$ families of straight line.



$\boxed{r \geq 0} \rightarrow$ including zero (ray)

$r = \theta$



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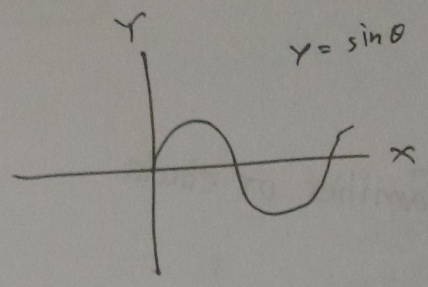
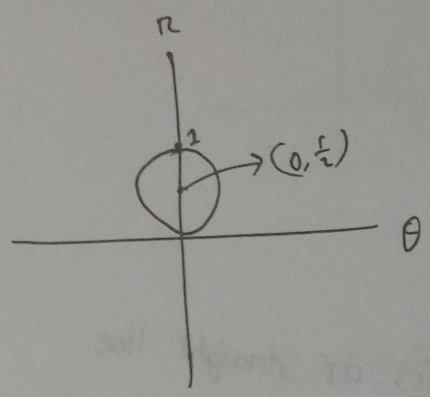
(*)

$r = \sin\theta$

θ	r
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
π	1





$$(x-0)^2 + (y-\tfrac{1}{2})^2 = (\tfrac{1}{2})^2$$

families of circle

$$\boxed{r = 2\cdot a\cdot \sin\theta}$$

$$r = 2\cdot\tfrac{1}{2}\cdot \sin\theta$$

$\Rightarrow$ $r = \sin\theta$

$r^2 = r\sin\theta$

$x^2+y^2 = y$

$x^2+y^2-y = 0$

$x^2+y^2-2\cdot y\cdot\frac{1}{2}+(\frac{1}{2})^2 = (\frac{1}{2})^2$

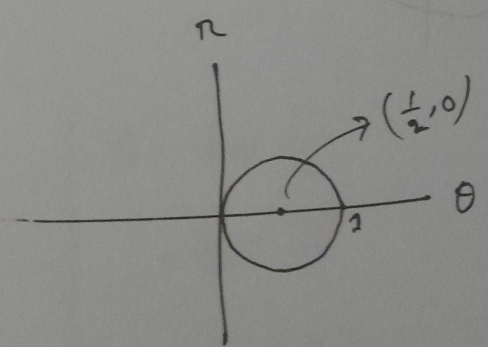
$(x-0)^2 + (y-\frac{1}{2})^2 = (\frac{1}{2})^2$

(*)

$r = \cos\theta$

$$\boxed{r = 2\cdot a\cdot \cos\theta}$$

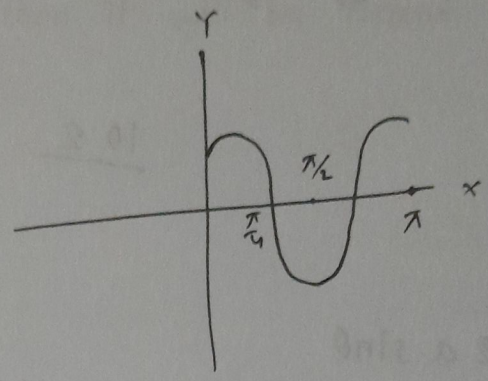
families of circle

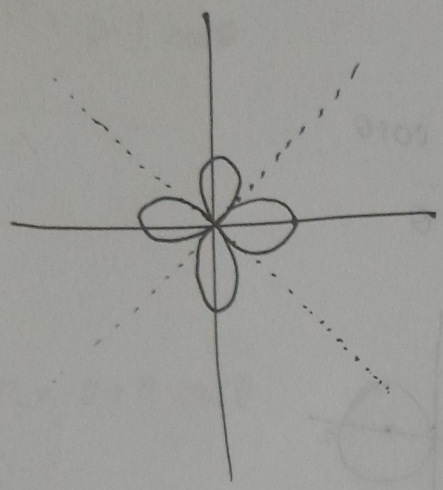


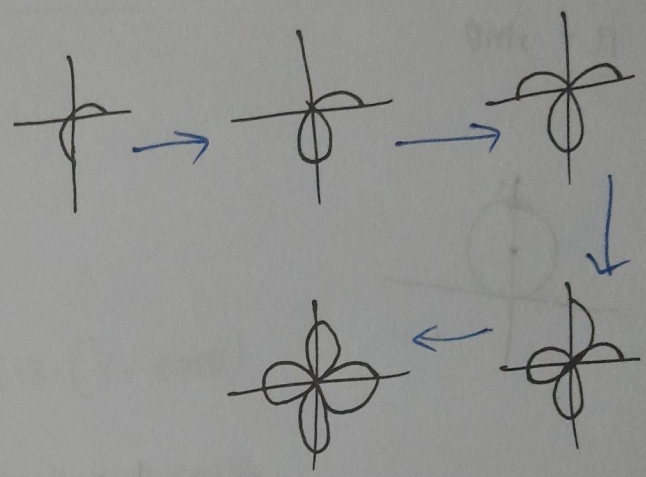
$$(x-\tfrac{1}{2})^2 + (y-0)^2 = (\tfrac{1}{2})^2$$

(*)

$r = \cos 2\theta$







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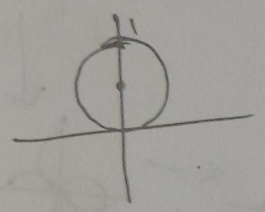
L-19 07.12.2022

10.2

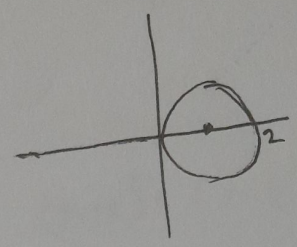
$r = 2a\sin\theta$

$r = \sin\theta$

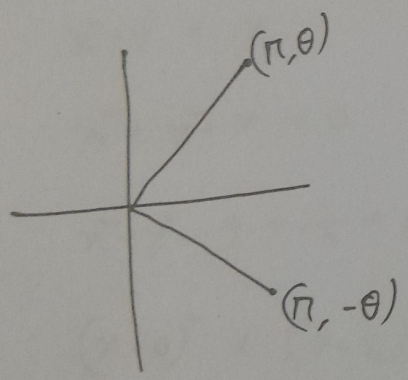


$r = 2a\cos\theta$

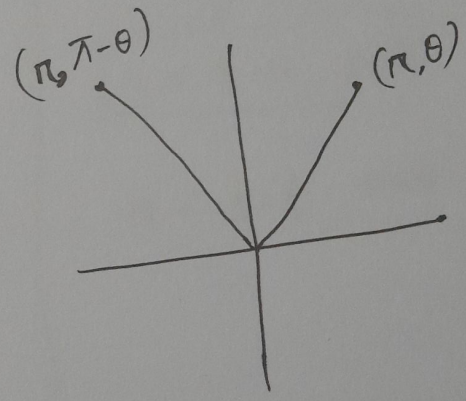
$r = \cos\theta$



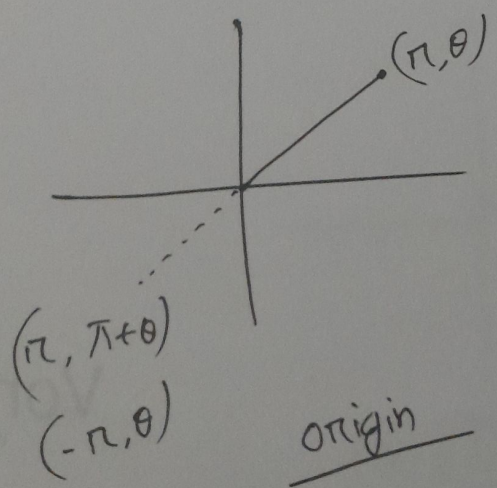
Symetrie



x-axis



y-axis



origin