North South University

Department of Mathematics and Physics

Assignment-3

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5.2

5)

Given that,

$$\frac{d}{dx}\left(\sqrt{x^3+5}\right)$$

$$= \frac{d}{dx}(x^3+5)^{\frac{1}{2}}$$

$$= \frac{1}{2} \cdot (x^3+5)^{-\frac{1}{2}} \cdot 3x^2$$

$$= \frac{3x^2}{2\sqrt{x^3+5}}$$

Therefore,

Corresponding integration formula

$$\int \frac{3x^2}{2\sqrt{x^3+5}}\, dx = \sqrt{x^3+5} + C$$

6)

Given that,

$$\frac{d}{dx}\left(\frac{x}{x^2+3}\right)$$

$$= \frac{(x^2+3)\cdot 1 - x \cdot 2x}{(x^2+3)^2}$$

$$= \frac{x^2+3-2x^2}{(x^2+3)^2}$$

$$= \frac{-x^2+3}{(x^2+3)^2}$$

Therefore,

Corresponding integration formula,

$$\int \frac{-x^2+3}{(x^2+3)^2}\,dx = \frac{x}{x^2+3} + C$$

7)

Given that,

$$\frac{d}{dx}\left[\sin(2\sqrt{x})\right]$$

$$= \cos(2\sqrt{x}) \cdot \frac{d}{dx}(2\sqrt{x})$$

$$= \cos(2\sqrt{x}) \cdot 2 \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{\cos(2\sqrt{x})}{\sqrt{x}}$$

Therefore,

Corresponding integration formula,

$$\int \frac{\cos(2\sqrt{x})}{\sqrt{x}}\,dx = \sin(2\sqrt{x}) + C$$

8)

Given that,

$$\frac{d}{dx}[\sin x - x\cos x]$$

$$= \frac{d}{dx}(\sin x) - \frac{d}{dx}(x\cos x)$$

$$= \cos x - (x \cdot (-\sin x) + \cos x \cdot 1)$$

$$= \cos x + x\sin x - \cos x$$

$$= x\sin x$$

Therefore,

Corresponding integration formula,

$$\int x\sin x\, dx = \sin x - x\cos x + c$$

15/

Given that,

$$\int x(1+x^3)\,dx$$

$$= \int (x+x^4)\,dx$$

$$= \int x\,dx + \int x^4\,dx$$

$$= \frac{x^2}{2} + \frac{x^5}{5} + c$$

Now checking answere

$$\frac{d}{dx}\left(\frac{x^2}{2} + \frac{x^5}{5}\right)$$

$$= \frac{d}{dx}\cdot\frac{x^2}{2} + \frac{d}{dx}\cdot\frac{x^5}{5}$$

$$= \frac{1}{2}\cdot 2x + \frac{1}{5}\cdot 5x^4$$

$$= x+x^4$$

$$= x(1+x^3)$$

Therefore the value of the given integral is

$$\frac{x^2}{2} + \frac{x^5}{5} + c$$

16)

Given that,

$$\int (2+y^2)^2 dy$$

$$= \int (4 + 4y^2 + y^4)\, dy$$

$$= \int 4\, dy + \int 4y^2\, dy + \int y^4\, dy$$

$$= 4y + 4 \cdot \frac{y^3}{3} + \frac{y^5}{5} + c$$

$$= 4y + \frac{4}{3}y^3 + \frac{1}{5}y^5 + c$$

Now checking the answer,

$$\frac{d}{dy}\left(4y + \frac{4}{3}y^3 + \frac{1}{5}y^5 + c\right)$$

$$= \frac{d}{dy}4y + \frac{d}{dy}\frac{4}{3}y^3 + \frac{d}{dy}\frac{1}{5}y^5 + \frac{d}{dy}c$$

$$= 4 + \frac{4}{3}\cdot 3y^2 + \frac{1}{5}\cdot 5y^4 + 0$$

$$= 4 + 4y^2 + y^4$$

$$= (2+y^2)^2$$

Therefore, the value of the given integral is,

$$4y + \frac{4}{3}y^3 + \frac{1}{5}y^5 + c$$

17)

Given that,

$$\int x^{1/3}(2-x)^2\,dx$$

$$= \int x^{1/3}(4-4x+x^2)\,dx$$

$$= \int (4x^{1/3} - 4x^{4/3} + x^{7/3})\,dx$$

$$= \int 4x^{1/3}\,dx - \int 4x^{4/3}\,dx + \int x^{7/3}\,dx$$

$$= 4\cdot\frac{x^{4/3}}{4/3} - 4\cdot\frac{x^{7/3}}{7/3} + \frac{x^{10/3}}{10/3} + C$$

$$= 3x^{4/3} - \frac{12}{7}x^{7/3} + \frac{3}{10}x^{10/3} + C$$

Now, checking answer,

$$\frac{d}{dx}\left[3x^{4/3} - \frac{12}{7}x^{7/3} + \frac{3}{10}x^{10/3} + C\right]$$

$$= \frac{d}{dx}\cdot 3\cdot x^{4/3} - \frac{d}{dx}\cdot\frac{12}{7}x^{7/3} + \frac{d}{dx}\frac{3}{10}x^{10/3} + \frac{d}{dx}C$$

$$= 3\cdot\frac{4}{3}\cdot x^{1/3} - \frac{12}{7}\cdot\frac{7}{3}\cdot x^{4/3} + \frac{3}{10}\cdot\frac{10}{3}\cdot x^{7/3} + 0$$

$$= 4x^{1/3} - 4x^{4/3} + x^{7/3}$$

$$= x^{1/3}(4-4x+x^2)$$

$$= x^{1/3}(2-x)^2$$

Therefore the value of the given integral is,

$$3x^{4/3} - \frac{12}{7}x^{7/3} + \frac{3}{10}x^{10/3} + c$$

18/

Given that,

$$\int (1+x^2)(2-x)\,dx$$

$$= \int (2 - x + 2x^2 - x^3)\,dx$$

$$= \int 2\,dx - \int x\,dx + \int 2x^2\,dx - \int x^3\,dx$$

$$= 2x - \frac{x^2}{2} + 2\cdot\frac{x^3}{3} - \frac{x^4}{4} + c$$

$$= 2x - \frac{1}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4 + c$$

Now checking answer,

$$\frac{d}{dx}\left[2x - \frac{1}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4 + c\right]$$

$$= \frac{d}{dx}2x - \frac{d}{dx}\cdot\frac{1}{2}x^2 + \frac{d}{dx}\frac{2}{3}x^3 - \frac{d}{dx}\frac{1}{4}x^4 + \frac{d}{dx}c$$

$$= 2 - \frac{1}{2}\cdot 2\cdot x + \frac{2}{3}\cdot 3\cdot x^2 - \frac{1}{4}\cdot 4\cdot x^3 + 0$$

$$= 2 - x + 2x^2 - x^3$$

$$= (1+x^2)(2-x)$$

Therefore, the value of the given integral is,

$$2x - \frac{1}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4 + c$$

19/

Given that,

$$\int \frac{x^5 + 2x^2 - 1}{x^4}\, dx$$

$$= \int (x + 2x^{-2} - x^{-4})\, dx$$

$$= \int x\, dx + \int 2x^{-2}\, dx - \int x^{-4}\, dx$$

$$= \frac{x^2}{2} + 2\frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + c$$

$$= \frac{1}{2}x^2 - \frac{2}{x} + \frac{1}{3x^3} + c$$

Now checking answer,

$$\frac{d}{dx}\left[\frac{1}{2}x^2 - \frac{2}{x} + \frac{1}{3x^3} + c\right]$$

$$= \frac{d}{dx}\frac{1}{2}x^2 - \frac{d}{dx}2\cdot x^{-1} + \frac{d}{dx}\frac{1}{3}\cdot x^{-3} + \frac{d}{dx}c$$

$$= \frac{1}{2}\cdot 2x - 2(-1)\cdot x^{-2} + \frac{1}{3}\cdot(-3)\cdot x^{-4} + 0$$

$$= x + 2x^{-2} - x^{-4}$$

$$= \frac{x^4(x + 2x^{-2} - x^{-4})}{x^4}$$

$$= \frac{x^5 + 2x^2 - 1}{x^4}$$

Therefore, the value of the given integral is,

$$\frac{1}{2}x^2 - \frac{2}{x} + \frac{1}{3x^3} + C$$

20/

Given that,

$$\int \frac{1 - 2t^3}{t^3} \, dt$$

$$= \int (t^{-3} - 2) \, dt$$

$$= \int t^{-3} \, dt - \int 2 \, dt$$

$$= \frac{t^{-2}}{-2} - 2t + C$$

$$= -\frac{1}{2} t^{-2} - 2t + C$$

Now checking answer,

$$\frac{d}{dt}\left[-\frac{1}{2}t^{-2} - 2t + C\right]$$

$$= \frac{d}{dt}\left(-\frac{1}{2} \cdot t^{-2}\right) - \frac{d}{dt} 2t + \frac{d}{dt} C$$

$$= -\frac{1}{2} \cdot (-2) \cdot t^{-3} - 2 + 0$$

$$= t^{-3} - 2$$

$$= \frac{t^3(t^{-3}-2)}{t^2}$$

$$= \frac{1-2t^3}{t^2}$$

Therefore, the value of the given integral is,

$$-\frac{1}{2}t^{-2} - 2t + C$$

21|

Given that,

$$\int \left[\frac{2}{x} + 3e^x\right] dx$$

$$= \int 2 \cdot \frac{1}{x}\, dx + \int 3e^x\, dx$$

$$= 2\ln x + 3e^x + c$$

Now checking answer,

$$\frac{d}{dx}\left[2\ln x + 3e^x + c\right]$$

$$= \frac{d}{dx} 2\ln x + \frac{d}{dx} 3e^x + \frac{d}{dx} c$$

$$= 2 \cdot \frac{1}{x} + 3e^x + 0$$

$$= \frac{2}{x} + 3e^x$$

Therefore, the value of the given integral is,

$$2\ln x + 3e^x + c.$$

22)

Given that,

$$\int \left[\frac{1}{2t} - \sqrt{2}e^{t} \right] dt$$

$$= \int \frac{1}{2t}\, dt - \int \sqrt{2}\, e^{t}\, dt$$

$$= \frac{1}{2} \int \frac{1}{t}\, dt - \sqrt{2} \int e^{t}\, dt$$

$$= \frac{1}{2} \ln t - \sqrt{2}\, e^{t} + C$$

Now checking answer,

$$\frac{d}{dt}\left[\frac{1}{2} \ln t - \sqrt{2}\, e^{t} + C \right]$$

$$= \frac{d}{dt} \cdot \frac{1}{2} \cdot \ln t - \frac{d}{dt} \cdot \sqrt{2} \cdot e^{t} + \frac{d}{dt} C$$

$$= \frac{1}{2} \cdot \frac{1}{t} - \cdot \sqrt{2}\, e^{t} + 0$$

$$= \frac{1}{2t} - \sqrt{2}\, e^{t}$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \ln t - \sqrt{2}\, e^{t} + C$$

23)

Given that,

$$\int [3 \sin x - 2 \sec^2 x] \, dx$$

$$= \int 3 \sin x \, dx - \int 2 \sec^2 x \, dx$$

$$= -3 \cos x - 2 \tan x + c$$

Now checking answer,

$$\frac{d}{dx} [-3 \cos x - 2 \tan x + c]$$

$$= \frac{d}{dx} (-3 \cos x) - \frac{d}{dx} 2 \tan x + \frac{d}{dx} c$$

$$= 3 \sin x - 2 \sec^2 x + 0$$

$$= 3 \sin x - 2 \sec^2 x$$

Therefore, the value of the given integral is,

$$-3 \cos x - 2 \tan x + c$$

24)

Given that,

$$\int [\operatorname{cosec}^2 x - \sec x \tan x]\, dx$$

$$= \int \operatorname{cosec}^2 x\, dx - \int \sec x \tan x\, dx$$

$$= -\cot x - \sec x + c$$

Now checking answer

$$\frac{d}{dx}[-\cot x - \sec x + c]$$

$$= \frac{d}{dx}(-\cot x) - \frac{d}{dx}\sec x + \frac{d}{dx}c$$

$$= \operatorname{cosec}^2 x - \sec x \tan x + 0$$

$$= \operatorname{cosec}^2 x - \sec x \tan x$$

Therefore, the value of the given integral is,

$$-\cot x - \sec x + c$$

25)

Given that,

$$\int \sec x(\sec x + \tan x)\, dx$$

$$= \int (\sec^2 x + \sec x \tan x)\, dx$$

$$= \int \sec^2 x\, dx + \int \sec x \tan x\, dx$$

$$= \tan x + \sec x + C$$

Now checking answer,

$$\frac{d}{dx}[\tan x + \sec x + C]$$

$$= \frac{d}{dx}\tan x + \frac{d}{dx}\sec x + \frac{d}{dx}C$$

$$= \sec^2 x + \sec x \tan x + 0$$

$$= \sec x(\sec x + \tan x)$$

Therefore the value of the given integral is,

$$\tan x + \sec x + C$$

26)

Given that,

$$\int \operatorname{cosec} x(\sin x + \cot x)\,dx$$

$$= \int (\operatorname{cosec} x \sin x + \operatorname{cosec} x \cot x)\,dx$$

$$= \int \left(\frac{1}{\sin x} \sin x + \operatorname{cosec} x \cot x\right) dx$$

$$= \int (1 + \operatorname{cosec} x \cot x)\,dx$$

$$= \int 1\,dx + \int \operatorname{cosec} x \cot x\,dx$$

$$= x - \operatorname{cosec} x + c$$

Now checking answer,

$$\frac{d}{dx}[x - \operatorname{cosec} x + c]$$

$$= \frac{d}{dx} x - \frac{d}{dx} \operatorname{cosec} x + \frac{d}{dx} c$$

$$= 1 + \operatorname{cosec} x \cancel{\tan} \cot x + 0$$

$$= \operatorname{cosec} x \left(\frac{1}{\operatorname{cosec} x} + \cot x\right)$$

$$= \operatorname{cosec} x (\sin x + \cot x)$$

Therefore, the value of the given integral is,

$$x - \operatorname{cosec} x + c$$

27)

Given that,

$$\int \frac{\sec\theta}{\cos\theta} d\theta$$

$$= \int \frac{\sec\theta}{\frac{1}{\sec\theta}} d\theta$$

$$= \int \sec^2\theta \, d\theta$$

$$= \tan\theta + C$$

Now checking answer;

$$\frac{d}{d\theta}[\tan\theta + C]$$

$$= \frac{d}{d\theta}\tan\theta + \frac{d}{d\theta}C$$

$$= \sec^2\theta + 0$$

$$= \sec\theta \cdot \sec\theta$$

$$= \sec\theta \cdot \frac{1}{\cos\theta}$$

$$= \frac{\sec\theta}{\cos\theta}$$

Therefore, the value of the given integral is

$$\tan\theta + C.$$

28)

Given that,

$$\int \frac{dy}{\operatorname{cosec} y}$$

$$= \int \frac{1}{\operatorname{cosec} y} dy$$

$$= \int \sin y \, dy$$

$$= -\cos y + C$$

Now checking answer,

$$\frac{d}{dy}[-\cos y + c]$$

$$= \frac{d}{dy}(-\cos y) + \frac{d}{dy} c$$

$$= \sin y + 0$$

$$= \frac{1}{\operatorname{cosec} y}$$

Therefore, the value of the given integrals is,

$$-\cos y + C.$$

29|

Given that,

$$\int \frac{\sin x}{\cos^2 x}\, dx$$

$$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\, dx$$

$$= \int \tan x \sec x\, dx$$

$$= \sec x + c$$

Now checking answer,

$$\frac{d}{dx}\left[\sec x + c\right]$$

$$= \frac{d}{dx} \sec x + \frac{d}{dx} c$$

$$= \sec x \tan x + 0$$

$$= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \frac{\sin x}{\cos^2 x}$$

Therefore, the value of the given integral is,

$\sec x + c$.

30|

Given that,

$$\int \left[\phi + \frac{2}{\sin^2\phi} \right] d\phi$$

$$= \int \phi \, d\phi + \int \frac{2}{\sin^2\phi} \, d\phi$$

$$= \int \phi \, d\phi + 2\int \operatorname{cosec}^2\phi \, d\phi$$

$$= \frac{\phi^2}{2} - 2\cot\phi + C$$

Now checking answer,

$$\frac{d}{d\phi}\left[\frac{\phi^2}{2} - 2\cot\phi + C \right]$$

$$= \frac{d}{d\phi}\frac{\phi^2}{2} - \frac{d}{d\phi} 2\cot\phi + \frac{d}{d\phi} C$$

$$= \frac{1}{2} \cdot 2\phi^1 + 2\operatorname{cosec}^2\phi + 0$$

$$= \phi + 2\operatorname{cosec}^2\phi$$

$$= \phi + \frac{2}{\sin^2\phi}$$

Therefore, the value of the given integral is,

$$\frac{\phi^2}{2} - 2\cot\phi + C$$

31|

Given that,

$$\int [1+\sin^2\theta \operatorname{cosec}\theta]\, d\theta$$

$$= \int \left[1 + \sin^2\theta \frac{1}{\sin\theta}\right] d\theta$$

$$= \int [1+\sin\theta]\, d\theta$$

$$= \int 1\, d\theta + \int \sin\theta\, d\theta$$

$$= \theta - \cos\theta + c$$

Now checking answer,

$$\frac{d}{d\theta}[\theta - \cos\theta + c]$$

$$= \frac{d}{d\theta}\theta - \frac{d}{d\theta}\cos\theta + \frac{d}{d\theta}c$$

$$= 1 + \sin\theta + 0$$

$$= 1 + \frac{\sin\theta \cdot \sin\theta}{\sin\theta}$$

$$= 1 + \sin^2\theta \cdot \frac{1}{\sin\theta}$$

$$= 1 + \sin^2\theta \operatorname{cosec}\theta$$

Therefore the value of the given integral is

$\theta - \cos\theta + c$.

32/

Given that,

$$\int \frac{\sec x + \cos x}{2\cos x}\,dx$$

$$= \int \left[\frac{\sec x}{2\cos x} + \frac{\cos x}{2\cos x}\right] dx$$

$$= \int \left[\frac{1}{2}\sec^2 x + \frac{1}{2}\right] dx$$

$$= \int \frac{1}{2}\sec^2 x\,dx + \int \frac{1}{2}\,dx$$

$$= \frac{1}{2}\tan x + \frac{1}{2}x + c$$

Now checking answer,

$$\frac{d}{dx}\left[\frac{1}{2}\tan x + \frac{1}{2}x + c\right]$$

$$= \frac{d}{dx}\frac{1}{2}\cdot\tan x + \frac{d}{dx}\frac{1}{2}\cdot x + \frac{d}{dx}c$$

$$= \frac{1}{2}\sec^2 x + \frac{1}{2} + 0$$

$$= \frac{\sec^2 x + 1}{2}$$

$$= \frac{\cos x(\sec^2 x + 1)}{\cos x \cdot 2}$$

$$= \frac{\sec x + \cos x}{2\cos x}$$

Therefore, the value of the given integral is, $\frac{1}{2}\tan x + \frac{1}{2}x + c$

33)

Given that,

$$\int \left[\frac{1}{2\sqrt{1-x^2}} - \frac{3}{1+x^2} \right] dx$$

$$= \int \frac{1}{2\sqrt{1-x^2}} dx - \int \frac{3}{1+x^2} dx$$

$$= \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx - 3 \int \frac{1}{1+x^2} dx$$

$$= \frac{1}{2} \sin^{-1} x - 3 \tan^{-1} x + C$$

Now checking answer,

$$\frac{d}{dx} \left[\frac{1}{2} \sin^{-1} x - 3 \tan^{-1} x + c \right]$$

$$= \frac{d}{dx} \frac{1}{2} . \sin^{-1} x - \frac{d}{dx} . 3 . \tan^{-1} x + \frac{d}{dx} c$$

$$= \frac{1}{2} . \frac{1}{\sqrt{1-x^2}} - 3 . \frac{1}{1+x^2} + 0$$

$$= \frac{1}{2\sqrt{1-x^2}} - \frac{3}{1+x^2}$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \sin^{-1} x - 3 \tan^{-1} x + C$$

34/

Given that,

$$\int \left[\frac{4}{x\sqrt{x^2-1}} + \frac{1+x+x^3}{1+x^2} \right] dx$$

$$= \int \left[\frac{4}{x\sqrt{x^2-1}} + \frac{1}{1+x^2} + \frac{x+x^3}{1+x^2} \right] dx$$

$$= \int \left[\frac{4}{x\sqrt{x^2-1}} + \frac{1}{1+x^2} + x \right] dx$$

$$= \int \frac{4}{x\sqrt{x^2-1}} dx + \int \frac{1}{1+x^2} dx + \int x \, dx$$

$$= 4\sec^{-1}x + \tan^{-1}x + \frac{x^2}{2} + c$$

Now checking answer,

$$\frac{d}{dx}\left[4\sec^{-1}x + \tan^{-1}x + \frac{x^2}{2} + c\right]$$

$$= \frac{d}{dx} 4.\sec^{-1}x + \frac{d}{dx}\tan^{-1}x + \frac{d}{dx}\frac{x^2}{2} + \frac{d}{dx}c$$

$$= 4\frac{1}{x^2\sqrt{x^2-1}} + \frac{1}{1+x^2} + \frac{1}{2}.2x + 0$$

$$= \frac{4}{x^2\sqrt{x^2-1}} + \frac{1}{1+x^2} + x$$

$$= \frac{4}{x^2\sqrt{x^2-1}} + \frac{1+x+x^3}{1+x^2}$$

Therefore, the value of the given integral is,

$$4\sec^{-1}x + \tan^{-1}x + \frac{x^2}{2} + c$$

43)

a)

Given that,

$$\frac{dy}{dx} = \sqrt[3]{x}$$

$$\Rightarrow dy = \sqrt[3]{x}\, dx$$

$$\Rightarrow \int dy = \int x^{\frac{1}{3}}\, dx$$

$$\Rightarrow y = \frac{x^{4/3}}{4/3} + c$$

$$\Rightarrow y = \frac{3x^{4/3}}{4} + c$$

Now,

if, $x = 1$

$y = 2$

$$\therefore 2 = \frac{3\cdot 1^{4/3}}{4} + c$$

$$\Rightarrow c = 2 - \frac{3}{4} = \frac{5}{4}$$

Hence, the particular solutions,

$$y = \frac{3x^{4/3}}{4} + \frac{5}{4}$$

b)

Given that,

$$\frac{dy}{dt} = \sin t + 1$$

$$\Rightarrow dy = \sin t + 1 \quad dt$$

$$\Rightarrow \int dy = \int \sin t + 1 \quad dt$$

$$\Rightarrow y = -\cos t + t + C$$

Now,

if

$$t = \frac{\pi}{3}$$

then, $y = \frac{1}{2}$

$$\therefore \frac{1}{2} = -\cos\frac{\pi}{3} + \frac{\pi}{3} + C$$

$$\Rightarrow \frac{1}{2} = -\frac{1}{2} + \frac{\pi}{3} + C$$

$$\Rightarrow C = \frac{1}{2} + \frac{1}{2} - \frac{\pi}{3}$$

$$\therefore C = 1 - \frac{\pi}{3}$$

Hence, the particular solutions,

$$y = -\cot t + t + 1 - \frac{\pi}{3}$$

c)

$$\frac{dy}{dx} = \frac{x+1}{\sqrt{x}}$$

$$\Rightarrow dy = \frac{x+1}{\sqrt{x}} dx$$

$$\Rightarrow \int dy = \int \left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} \right) dx$$

$$\Rightarrow y = \int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$$

$$= \int \sqrt{x}\, dx + \int \frac{1}{\sqrt{x}} dx$$

$$= \frac{x^{3/2}}{3/2} + 2\sqrt{x} + c$$

$$\therefore y = \frac{2x^{3/2}}{3} + 2\sqrt{x} + c$$

Now,

if, $x = 1$

then, $y = 0$

$$\therefore \quad 0 = \frac{2\, 1^{3/2}}{3} + 2\sqrt{1} + c$$

$$\Rightarrow 0 = \frac{2}{3} + 2 + c$$

$$\therefore c = -\frac{8}{3}$$

Hence, the particular solutions,

$$y = \frac{2x^{3/2}}{3} + 2\sqrt{x} - \frac{8}{3}$$

44]

a)

Given that,

$$\frac{dy}{dx} = \frac{1}{(2x)^3}$$

$$\Rightarrow dy = \frac{1}{8x^3} dx$$

$$\Rightarrow \int dy = \int \frac{1}{8} \cdot x^{-3} dx$$

$$\Rightarrow y = \frac{1}{8} \cdot \frac{x^{-2}}{-2} + c$$

$$\therefore y = -\frac{1}{16} x^{-2} + c$$

Now,

if, $x = 1$

then, $y = 0$

$$\therefore 0 = -\frac{1}{16} (1)^{-2} + c$$

$$\therefore c = \frac{1}{16}$$

Hence, the particular solutions,

$$y = -\frac{1}{16} x^{-2} + \frac{1}{16}$$

b)

Given that,

$$\frac{dy}{dt} = \sec^2 t - \sin t$$

$$\Rightarrow dy = (\sec^2 t - \sin t)\, dt$$

$$\Rightarrow \int dy = \int (\sec^2 t - \sin t)\, dt$$

$$\Rightarrow y = \int \sec^2 t\, dt - \int \sin t\, dt$$

$$\therefore y = \tan t + \cos t + C$$

Now,

if $t = \frac{\pi}{4}$

then, $y = 1$

$$\therefore 1 = \tan\frac{\pi}{4} + \cos\frac{\pi}{4} + C$$

$$\Rightarrow 1 = 1 + \frac{\sqrt{2}}{2} + C$$

$$\therefore C = -\frac{\sqrt{2}}{2}$$

Hence, the particular solutions is,

$$y = \tan t + \cos t - \frac{\sqrt{2}}{2}$$

c)

Given that,

$$\frac{dy}{dx} = x^2\sqrt{x^3}$$

$$\Rightarrow dy = x^2\sqrt{x^3}\ dx$$

$$\Rightarrow \int dy = \int x^2 \cdot x^{3\frac{1}{2}}\ dx$$

$$\Rightarrow y = \int x^2 \cdot x^{3/2}\ dx$$

$$= \int x^{7/2}\ dx$$

$$= \frac{x^{9/2}}{9/2} + c$$

$$\therefore y = \frac{2x^{9/2}}{9} + c$$

Now,

if, $x = 0$

$y = 0$

$$\therefore 0 = \frac{2 \cdot 0^{9/2}}{9} + c$$

$$\therefore c = 0$$

Hence, the particular solution is,

$$y = \frac{2x^{9/2}}{9}$$

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a)

Given that,

$$\frac{dy}{dx} = 4e^x$$

$$\Rightarrow dy = 4e^x dx$$

$$\Rightarrow \int dy = \int 4e^x dx$$

$$\Rightarrow y = 4e^x + c$$

Now,

if, $x=0$

then, $y = 1$

$$\therefore 1 = 4e^0 + c$$

$$c = 1-4 = -3$$

Hence, the particulan solution is,

$$y = 4e^x + -3$$

b)

Given that,

$$\frac{dy}{dt} = \frac{1}{t}$$

$$\Rightarrow dy = \frac{1}{t}\, dt$$

$$\Rightarrow \int dy = \int \frac{1}{t}\, dt$$

$$\Rightarrow y = \ln|t| + c$$

Now,

if, $t = -1$

then. $y = 5$

$$\therefore \quad 5 = \ln|-1| + c$$

$$\Rightarrow 5 = 0 + c$$

$$\therefore c = 5$$

Hence, the particular solution is,

$$y = \ln|t| + 5.$$

46)

a)

Given that,

$$\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$$

$$\Rightarrow dy = \frac{3}{\sqrt{1-x^2}}\, dx$$

$$\Rightarrow \int dy = \int 3 \cdot \frac{1}{\sqrt{1-x^2}}\, dx$$

$$\therefore y = 3\sin^{-1}x + C$$

Now,

if, $x = \frac{\sqrt{3}}{2}$

then $y = 0$

$$\therefore 0 = 3\sin^{-1}\frac{\sqrt{3}}{2} + C$$

$$\Rightarrow 0 = 3 \cdot \frac{\pi}{3} + C$$

$$\therefore C = \cancel{\frac{\pi}{2}} -\pi$$

Hence, the particular solutions is,

$$y = 3\sin^{-1}x - \pi$$

b)

Given that,

$$\frac{dy}{dx} = \frac{x^2-1}{x^2+1}$$

$$\Rightarrow dy = \frac{x^2-1}{x^2+1}\,dx$$

$$\Rightarrow \int dy = \int \frac{x^2+1-2}{x^2+1}\,dx$$

$$\Rightarrow y = \int \left(\frac{x^2+1}{x^2+1} - \frac{2}{x^2+1}\right) dx$$

$$= \int \left(1 - \frac{2}{x^2+1}\right) dx$$

$$= \int 1\,dx - \int \frac{2}{x^2+1}\,dx$$

$$\therefore y = x - 2\tan^{-1}x + c$$

Now,

if, $x = 1$

then $y = \frac{\pi}{2}$

$$\therefore \frac{\pi}{2} = 1 - 2\tan^{-1}1 + c$$

$$\Rightarrow \frac{\pi}{2} = 1 - 2\frac{\pi}{4} + c$$

$$\therefore c = \frac{\pi}{2} + \frac{\pi}{2} - 1 = \pi - 1$$

Hence, the particular solutions is,

$$y = x - 2\tan^{-1}x + \pi - 1$$

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Given that,

$$s = s(t)$$

$$v(t) = s'(t)$$

Here,

$$v(t) = 32t$$

$$\Rightarrow \frac{ds}{dt} = 32t$$

$$\Rightarrow ds = 32t\, dt$$

$$\Rightarrow \int ds = \int 32t\, dt$$

$$\Rightarrow s(t) = 32 \cdot \frac{t^2}{2} + C$$

$$\therefore s(t) = 16t^2 + C$$

Now, $s(0) = 20$

Here, $t = 0$
$s = 20$

$$\therefore 20 = 16 \cdot 0^2 + C$$

$$\therefore C = 20 - 0 = 20$$

Therefore,

$$s(t) = 16t^2 + 20$$