## Chapter - 1

06.06.2022

L-2

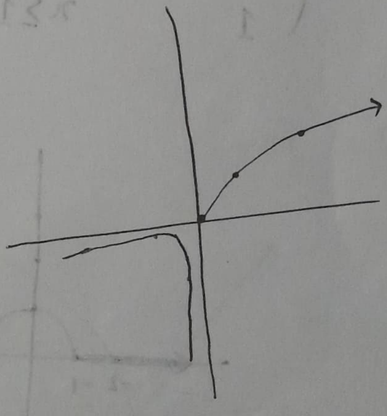
0.1

### Piecewise Defined Function

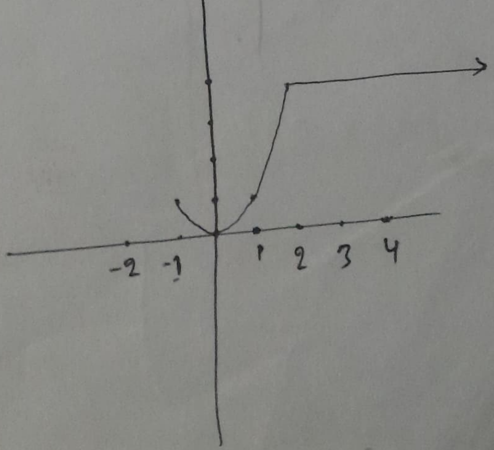
⊛ $f(x) = |x|$

$$f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

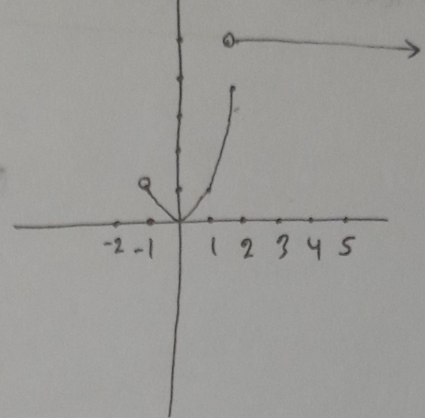
⊛ $$f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ \frac{1}{x} & x < 0 \end{cases}$$



⊛ $$f(x) = \begin{cases} x^2 & -1 \leq x \leq 2 \\ 4 & x > 2 \end{cases}$$



$$f(x) = \begin{cases} x^2 & -1 < x \le 2 \\ 5 & x > 2 \end{cases}$$

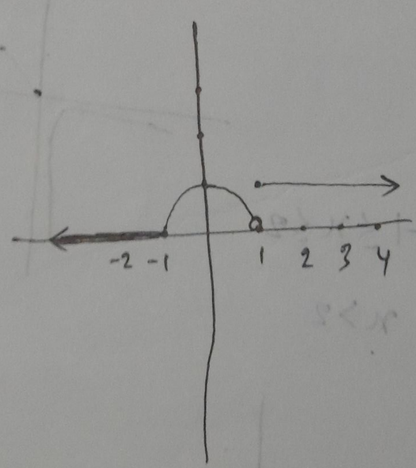



$y = +\sqrt{a^2-x^2}$ upper half

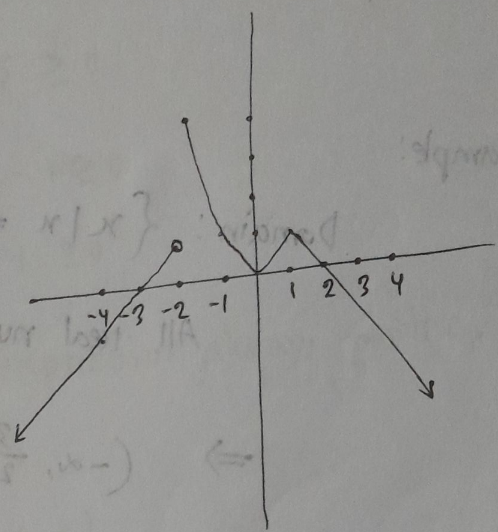
$y = -\sqrt{a^2-x^2}$ lower half

Circle

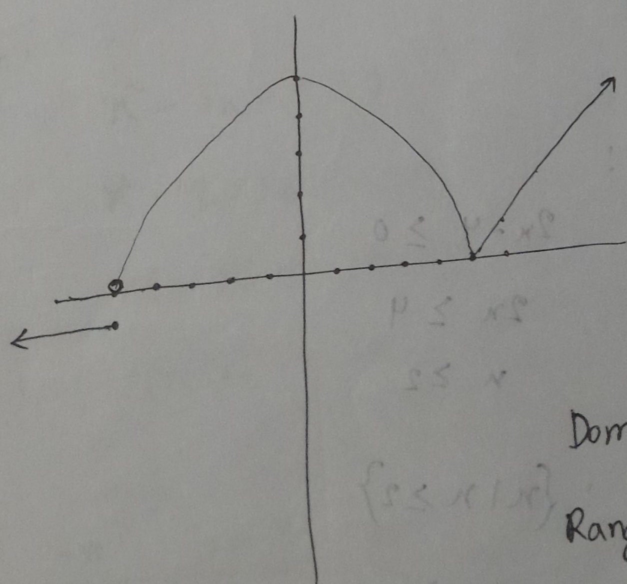
$$f(x) = \begin{cases} 0 & x \le -1 \\ \sqrt{1-x^2} & -1 < x < 1 \\ 1 & x \ge 1 \end{cases}$$



$$f(x) = \begin{cases} x+3 & x < -2 \\ x^2 & -2 \le x < 1 \\ -x+2 & x \ge 1 \end{cases}$$




$$f(x) = \begin{cases} -1 & x \le -5 \\ \sqrt{25-x^2} & -5 < x < 5 \\ x-5 & x \ge 5 \end{cases}$$



Domain : $(-\infty, \infty)$

Range : $[-1, \infty)$

⊛ If output is imaginary number or undefined then the input will not include in the domain.

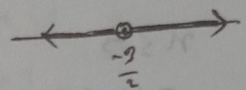
Sample:

Domain: $\{x \mid x \neq -\frac{3}{2}\}$

All real number except $-\frac{3}{2}$.

$\Rightarrow (-\infty, \frac{-3}{2}) \cup (-\frac{3}{2}, \infty)$

$\Rightarrow$



⊛ $G(x) = \sqrt{2x-4}$

$\Rightarrow$

For domain:

$2x - 4 \geq 0$

$2x \geq 4$

$x \geq 2$

$\therefore$ Domain: $\{x \mid x \geq 2\}$

$\Rightarrow [2, \infty)$

⊛ $H(x) = \sqrt{x^2-2x+5}$

For domain:

$$x^2-2x+5 \geq 0$$

$$x^2-2x+1-4 \geq 0$$

$$(x-1)^2+4 \geq 0$$

This expression always positive.

Therefore,

Domain: $(-\infty, \infty)$

⊛ $f(x) = \sqrt{x^2-5x+6}$

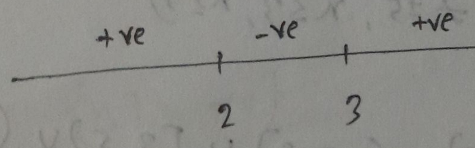
For domain:

$$x^2-5x+6 \geq 0$$

$$x^2-3x-2x+6 \geq 0$$

$$x(x-3)-2(x-3) \geq 0$$

$$(x-3)(x-2) \geq 0$$




∴ Domain: $(-\infty, 2] \cup [3, \infty)$

⊛ $f(x) = 1 - 2x + x^2$

Domain: $(-\infty, \infty)$

⊛ $f(x) = \sqrt{|x|}$

Domain: $(-\infty, \infty)$

⊛ $f(x) = \sqrt{5x + 10}$

For domain,

$$5x + 10 \geq 0$$

$$5x \geq -10$$

$$x \geq -2$$

Domain: $[-2, \infty)$

⊛ $f(x) = \dfrac{x+3}{4 - \sqrt{x^2 - 9}}$

For domain,

$$4 - \sqrt{x^2-9} \neq 0$$

$$4 \neq \sqrt{x^2-9}$$

$$x^2 - 9 \neq 16$$

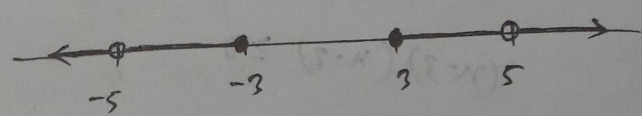
$$x^2 \neq 25$$

$$x \neq \pm 5$$

again,

$$x^2 - 9 \geq 0$$

$$(x+3)(x-3) \geq 0$$




Domain: $\{x \mid x \neq \pm 5,\ x \leq 3,\ x \geq 3\}$

$\Rightarrow (-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$

* $h(x) = \sin x$

Domain: $(-\infty, \infty)$

* $f(x) = \cos x$

Domain: $(-\infty, \infty)$

* $f(x) = \tan x = \dfrac{\sin x}{\cos x}$

Domain: $\{x \mid x \neq \frac{\pi}{2}, \frac{3\pi}{2} \dots\}$

* $f(x) = \sec x = \dfrac{1}{\cos x}$

Domain: $\{x \mid x \neq \frac{\pi}{2}, \frac{3\pi}{2} \dots\}$

* $f(x) = \cot x = \dfrac{\cos x}{\sin x}$

Domain: $\{x \mid x \neq 0, \pi, 2\pi, 3\pi \dots\}$

* $f(x) = \operatorname{cosec} x$

Domain: $\{x \mid x \neq 0, \pi, 2\pi \dots\}$

* $f(x) = \dfrac{1}{1-\sin x}$

For domain:

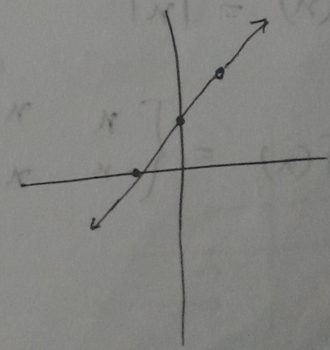
$1 - \sin x \neq 0$

$\sin x \neq 1$

$x \neq \frac{\pi}{2}, \frac{9\pi}{2} \dots$

* The effect of algebric operation on the domain.

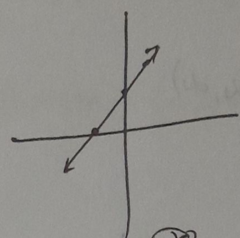
$\Rightarrow \quad f(x) = \dfrac{x^2-4}{x-2}$



Domain: $\{x \mid x \neq 2\}$

$$f(x) = \frac{(x+2)(x-2)}{(x-2)}$$

$$= x+2$$



Domain : $(-\infty, \infty)$

Domain : $\{x \mid x \neq 2\}$

$$f(x) = \frac{x\sqrt{x} + \sqrt{x}}{x+1}$$

$$= \frac{\sqrt{x}\,(x+1)}{x+1}$$

$$= \sqrt{x}$$

Domain: $[0, \infty)$

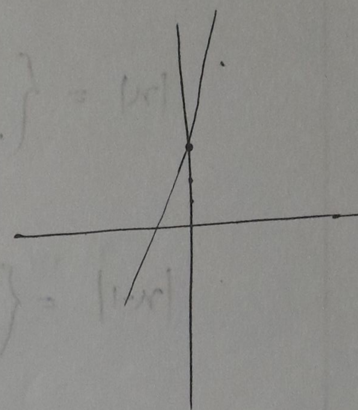
Express the absolute value function in piecewise form.

① $f(x) = |x|$

$$\therefore f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

(ii) $f(x) = |x| + 2x+3$

$\therefore f(x) = \begin{cases} x + 2x + 3 & x \geq 0 \\ -x + 2x + 3 & x < 0 \end{cases}$



$= \begin{cases} 3x+3 & x \geq 0 \\ x+3 & x < 0 \end{cases}$

(iii) $f(x) = |x-2|$

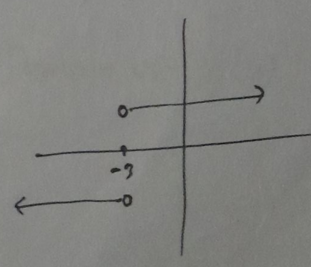
$\Rightarrow f(x) = \begin{cases} (x-2) & x-2 \geq 0 \\ -(x-2) & x-2 < 0 \end{cases}$

$= \begin{cases} x-2 & x \geq 2 \\ 2-x & x < 2 \end{cases}$

(iv) $f(x) = \dfrac{|x+3|}{x+3}$

$\Rightarrow f(x) = \begin{cases} \dfrac{x+3}{x+3} & x+3 \geq 0 \\ \dfrac{-(x+3)}{x+3} & x+3 < 0 \end{cases}$

$= \begin{cases} 1 & x \geq -3 \\ -1 & x < -3 \end{cases}$

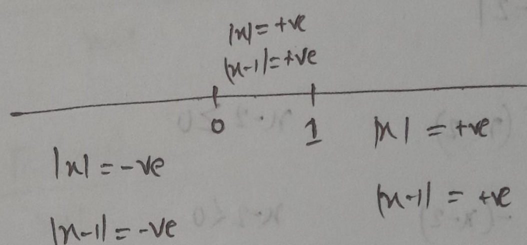



Domain: $\{x \mid x \neq -3\}$

(v) $g(x) = |x| + |x-1|$

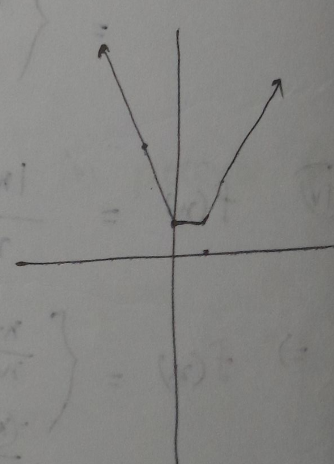
$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$|x-1| = \begin{cases} (x-1) & x \geq 1 \\ -(x-1) & x < 1 \end{cases}$$




$$\therefore f(x) = \begin{cases} -x-(x-1) & x \leq 0 \\ x-(x-1) & 0 < x < 1 \\ x+(x-1) & x \geq 1 \end{cases}$$

$$= \begin{cases} -2x+1 & x \leq 0 \\ 1 & 0 < x < 1 \\ 2x-1 & x \geq 1 \end{cases}$$



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## New function from the old.

1. Operation of function:

i. Addition
ii. Substraction
iii. Multiplication
iv. Division

⊛ If f(x) and g(x) are two function, then

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(f \cdot g)(x) = f(x) \cdot g(x)$$

Domain: Interceection of the domain f(x) and g(x)

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

" and $g(x) \neq 0$

⊛ $f(x) = \sqrt{x-1}$ and $g(x) = x^2+2$

$$\therefore (f+g)(x) = f(x) + g(x)$$
$$= \sqrt{x-1} + x^2+2$$

Domain of f(x): $[1,\infty)$

Domain of g(x): $(-\infty,\infty)$

∴ Domain: $[1,\infty)$

$\therefore \left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$

$= \frac{x^2+2}{\sqrt{x-1}}$

Here, $\sqrt{x-1} \neq 0$

$x-1 \neq 0$

$x \neq 1$

$\therefore$ Domain: $(1, \infty)$

❀ Composit of function:

$f(x)$ and $g(x)$ are two function, then

$(f \circ g)(x) = \underbrace{f}_{\text{outer function}}(\underbrace{g(x)}_{\text{inner function}})$

$(g \circ f)(x) = g(f(x))$

$(f \circ f)(x) = f(f(x))$

$(g \circ g)(x) = g(g(x))$

Domain of composit function:

calculation/resulting domain + inner function domain.

⊛ $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1}{x}$

$$\therefore (g \circ f)(x) = g(f(x))$$
$$= g\left(\frac{1}{x-1}\right)$$
$$= \frac{1}{\frac{1}{x-1}}$$
$$= x-1$$

Here, $x-1 \neq 0$

$x \neq 1$

$\therefore$ Domain: $\{x \mid x \neq 1\}$

$f(x) = 2 - x^2$ and $g(x) = \sqrt{x}$

$\therefore (f \circ g)(x) = f(g(x))$

$= f(\sqrt{x})$

$= 2 - (\sqrt{x})^2$

$= 2 - x$

Here,

$x \geq 0$

$\therefore$ Domain: $\{x \mid x \geq 0\}$

$\Rightarrow [0, \infty)$

3) Transformation / Translation

i. Shifting $\rightarrow$ Horizontal, Vertical

ii. Stretching / Compression $\rightarrow$ Horizontal, Vertical

iii. Reflection $\rightarrow$ Horizontal, Vertical

i. <u>Shifting:</u>

$f(x)+c \Rightarrow$ Vertical shifting by c units up.

$f(x)-c \Rightarrow$ Vertical shifting by c units down.

$f(x-c) \Rightarrow$ Horizontal shifting by c units right.

$f(x+c) \Rightarrow$ Horizontal shifting by c unit left.

ii. <u>Stretching/Compression</u>

Vertical:

$c \cdot f(x) \Rightarrow$ Vertical stretching/compression

if $c>1$, vertical streched

if $c<1$, vertical compressed

$f(c \cdot x) \Rightarrow$ Horizontal Stretching/Compression

if, $c>1$, Horizontally compressed

if, $c<1$, Horizontally stretched

iii. <u>Reflection</u>: Multiply by $-1$.

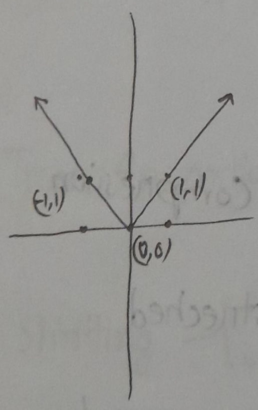
$-f(x) \Rightarrow$ Vertical reflection

$f(-x) \Rightarrow$ Horizontal reflection

⊛ <u>Graph the function:</u>

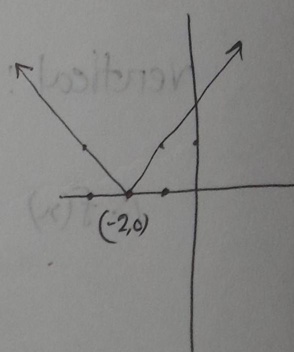
⊛ $f(x) = 2|x+2|+1$

$\Rightarrow \quad |x| \longrightarrow |x+2| \longrightarrow 2|x+2| \longrightarrow 2|x+2|+1$



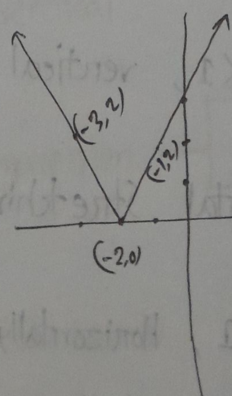

$x$ is replace by $x+2$

H. shifting by 2 units Left




multiply by 2

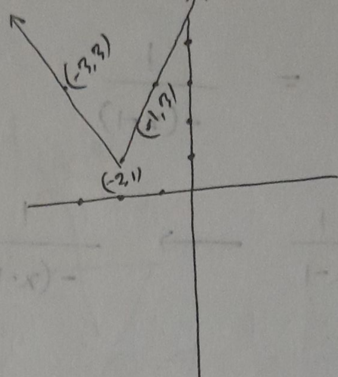
V. stretch



adding 1 $\longrightarrow$

V. shifting by 1

units up

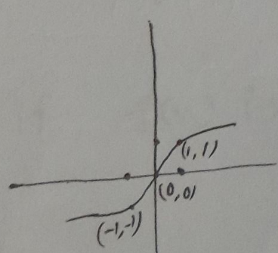



∴ Domain: $(-\infty, \infty)$

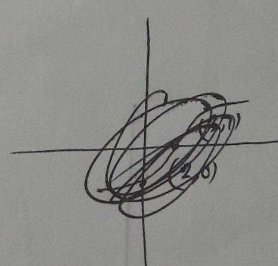
Range: $[1, \infty)$

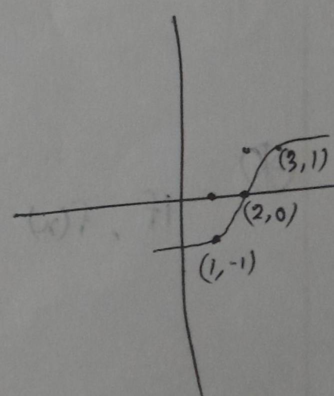
⊛ $y = \sqrt[3]{2-x} = \sqrt[3]{-x+2}$

∴ $y = \sqrt[3]{x} \longrightarrow \sqrt[3]{x-2} \longrightarrow \sqrt[3]{\cancel{-x+2}\ -(x-2)}$

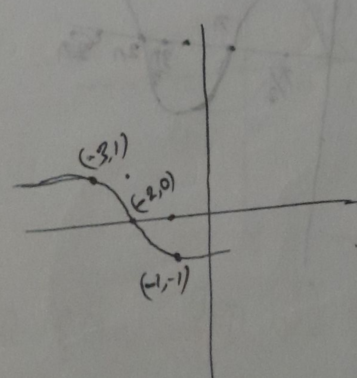



$\longrightarrow$



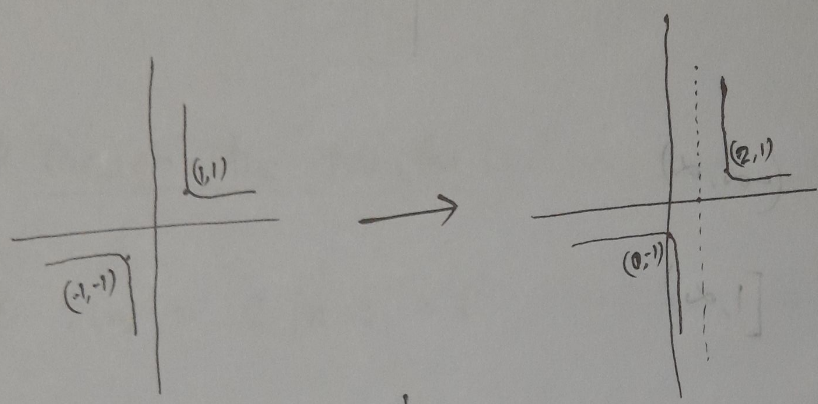


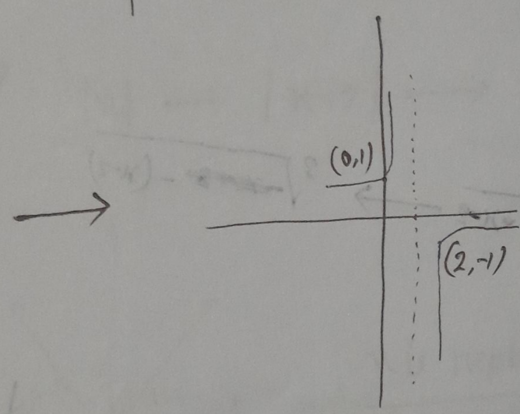

$\longrightarrow$



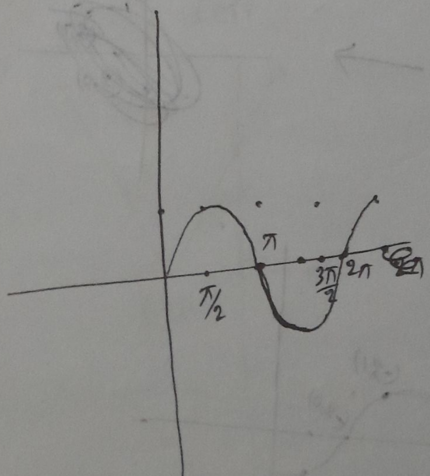
$f(x) = \frac{1}{1-x} = \frac{1}{-(x-1)}$

$\Rightarrow \frac{1}{x} \longrightarrow \frac{1}{x-1} \longrightarrow \frac{1}{-(x-1)}$



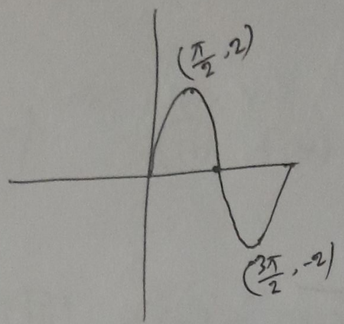




$\therefore$ if, $f(x)$

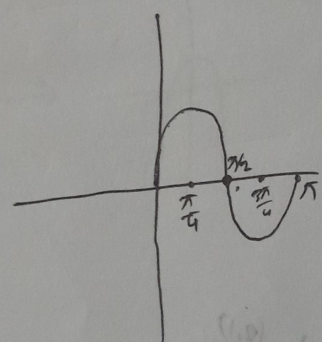



Draw, $2f(x)$, $f(2x)$

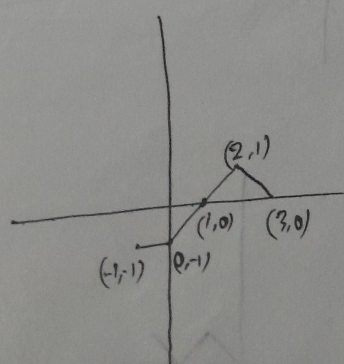
$\xrightarrow{2f(x)}$




$f(2x) \rightarrow$ Horizontal Compression, $x$-co-ordinate multiply by $\frac{1}{2}$.



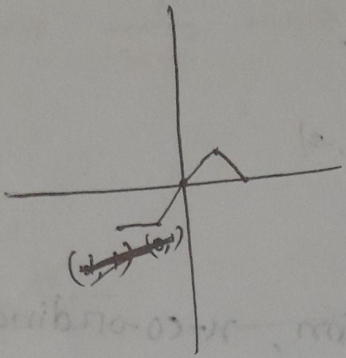

⊛ If, $f(x)$ is



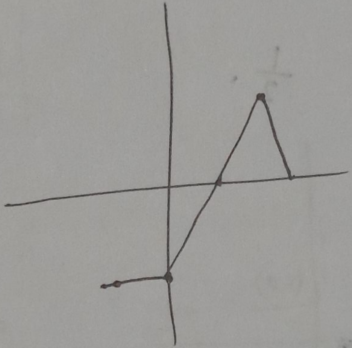

Draw (i) $f(x+1)$ (ii) $f(2x)$ (iii) $y = |f(x)|$ (iv) $y = 1 - |f(x)| = -|f(x)| + 1$

⇒

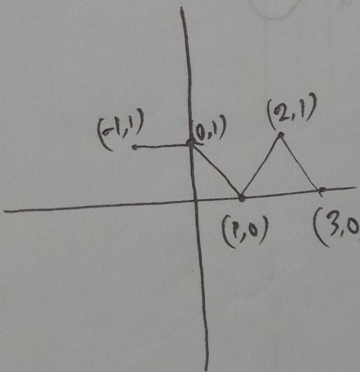
(i)



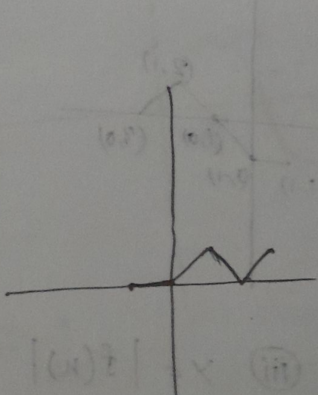
(ii)



(iii)




(iv)

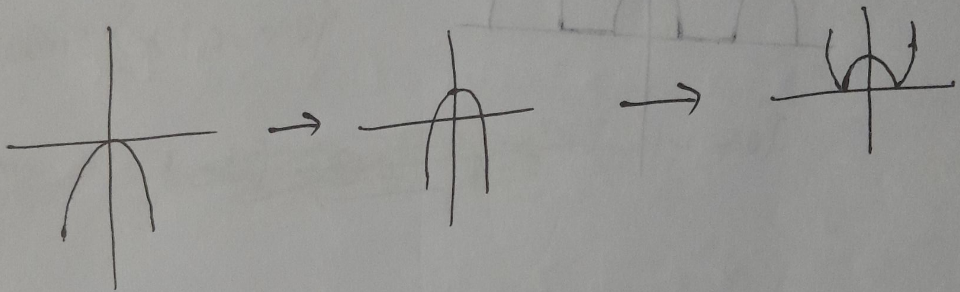


⊛ Draw the graph of the followings

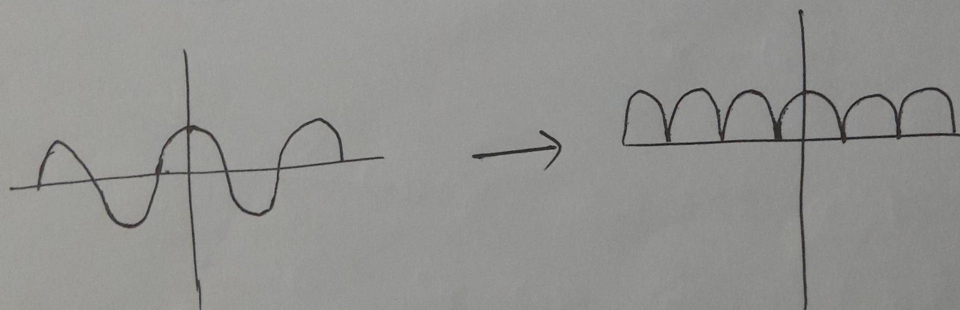
(i) $f(x) = |1-x^2|$ (ii) $f(x) = |\cos x|$

(iii) $f(x) = \cos x + |\cos x|$

(i) $f(x) = -x^2 \to 1-x^2 \longrightarrow |1-x^2|$

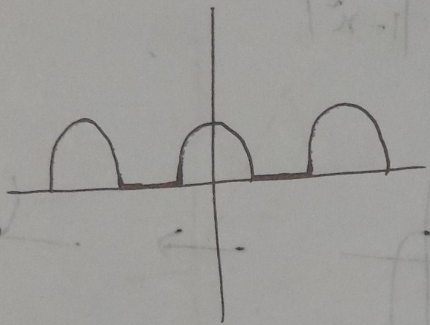


(ii)



(iii) $\cos x + |\cos x| = \begin{cases} \cos x + \cos x & \cos x \geq 0 \\ \cos x - \cos x & \cos x < 0 \end{cases}$

$= \begin{cases} 2\cos x & \cos x \geq 0 \\ 0 & \cos x < 0 \end{cases}$



H.W. 0.1

Q1

a) $f(x) = \dfrac{1}{x-3}$

For domain,

$$x - 3 \neq 0$$

$$x \neq 3$$

$\therefore$ Domain: $\{x \mid x \neq 3\}$

$\therefore$ Range: $\cancel{R-\{0\}}\ \cancel{(R-0)}\ \cancel{\{R-0\}}\ \{y \mid y \neq 0\}$

0.3

## Families of Curves

$y = b \longrightarrow$ Families of horizontal Line.

Linear Function $\longleftarrow y = mx + b \longrightarrow$ Families of Strait Line

Slope

y-intercept

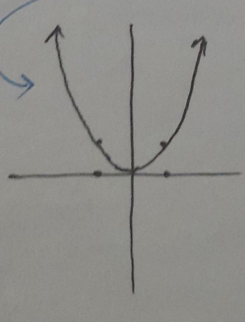
$x = a \longrightarrow$ Families of vertical Line.

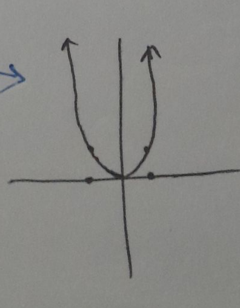
## Power Function

$f(x) = x^n$

If $n = 2, 4, 6, 8 \dots$ (even)

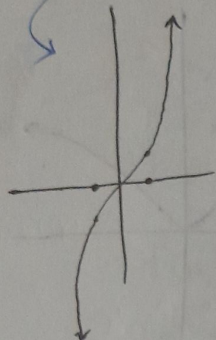
then, $f(x) = x^2$, $f(x) = x^4$, $f(x) = x^6 \dots$

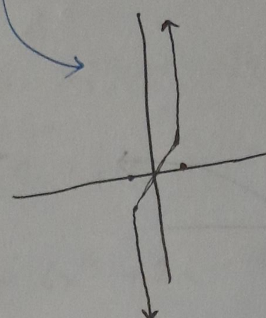


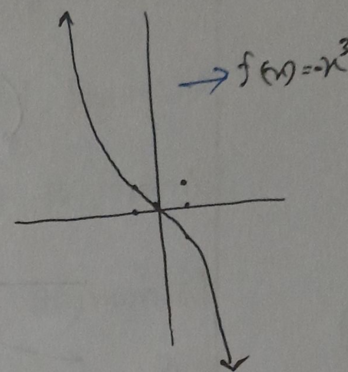


If, $n = 3, 5, 7, \ldots$ (odd)

then, $f(x) = x^3$, $f(x) = x^5$, $f(x) = x^7 \ldots$

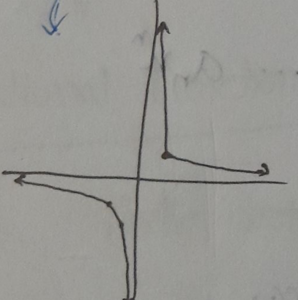




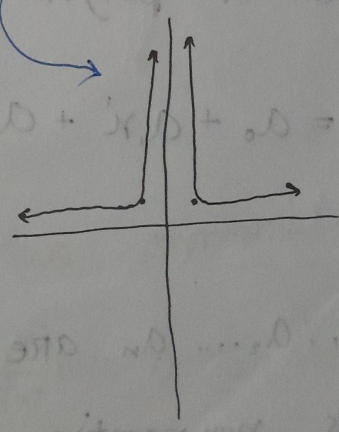



⊛ $f(x) = \frac{1}{x^n}$

$f(x) = \frac{1}{x}$, $f(x) = \frac{1}{x^2}$



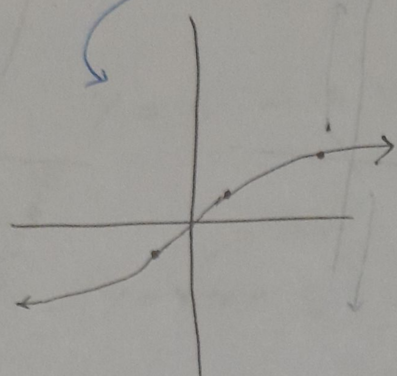
odd

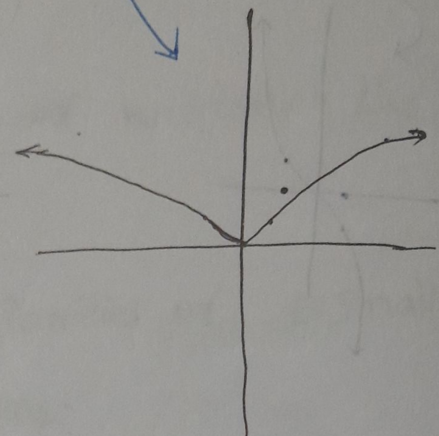


even

$f(x) = x^{2/3}$

$f(x) = (x^{1/3})^2$





## Polynomial Function

General form of polynomial function,

$$f(x) = a_0 + a_1x^1 + a_2x^2 + \dots a_nx^n$$

Here,

$a_0, a_1, a_2 \dots a_n$ are co-efficient

$n$ is non-negative integer

Highest power of $x$ is called its degree.

Domain of polynomial function $(-\infty, \infty)$

Constant function

$f(x) = -3 \rightarrow$ Polynomial, Degree $= 0$

Linear Function

$f(x) = 2x+4 \rightarrow$ Polynomial, Degree $= 1$

Quadratic Function

$f(x) = 2x^2 + 3x + \sqrt{3} \rightarrow$ Polynomial, Degree $= 2$

$h(x) = 6x^3 + 3x^{-2} + 4x + 1 \rightarrow$ Not Polynomial

$f(x) = 3x^2 + 6x^{1/3} + 2x \rightarrow$ Not Polynomial

## ⊛ Graph of Polynomial Function:

⊛ Graph of Polynomial function always smooth and continuous.

## ⊛ Rational Function:

Ratio between two polynomial function.

$$f(x) = \frac{P(x)}{Q(x)}$$

Domain : $\{x \mid Q(x) \neq 0\}$

* $f(x) = \dfrac{x^2+2x}{x-1}$

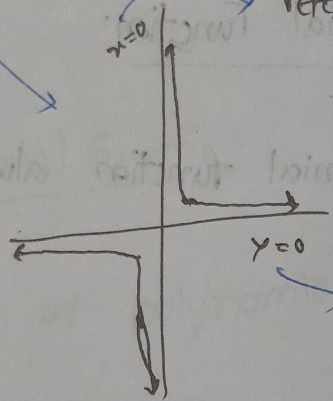
Domain : $\{x \mid x \neq 1\}$

* $G(x) = \dfrac{x^2+2\sqrt{x}+3}{x^2+1}$ → Not Rational

Not Polynomial

* $f(x) = \dfrac{1}{x}$ → Basic Rational Function

Vertical assymptote




Horizontal Asymptote

* <u>Asymptote:</u>

Asymptote drawn by doted Line.

There are three types of asymptote.

(i) Horizontal Asymptote (H.A.) → Form , $y=b$

(ii) Vertical Asymptote (V.A.) → Form $x=a$

(iii) Oblique Asymptote (O.A.) → Form , $y=mx+C$

⊛ Finding V.A.:

⇒ Solve the denominator

$f(x) = \frac{x+2}{x-3}$

⇒ Solve,

$x-3=0$

$x=3$

$\therefore x=3$ is V.A.

⊛ $f(x) = \frac{x^2+2}{(x+1)(x-2)}$

Solve, $(x+1)(x-2)=0$

$x=-1,\ x=2$

$\therefore$ V.A.: $x=-1,\ x=2$.

⊛ Finding H.A.:

⇒ If, degree of top polynomial < degree of bottom polynomial, then,

H.A.: $y=0$

⇒ If, degree of top polynomial = degree of bottom polynomial, then,

$$\text{H.A.: } y = \frac{\text{leading co-efficient of top polynomial}}{\text{leading co-efficient of bottom polynomial}}$$

⇒ If, degree of top polynomial > degree of bottom polynomial, then, No H.A.

Only Oblique Asymptote is here.

⊛ $f(x) = \dfrac{2x+1}{x^2-2x}$

$= \dfrac{2x+1}{x(x-2)}$

V.A. : $x = 0,\ x = 2$

H.A. : $y = 0$

⊛ $f(x) = \dfrac{2x^2+3x+1}{4x^2+6x+2}$

H.A. : $y = \dfrac{2}{4} = \dfrac{1}{2}$

⊛ $y = \dfrac{x^2+5x+1}{x^2+1}$

H.A. : $y = \dfrac{1}{1} = 1$

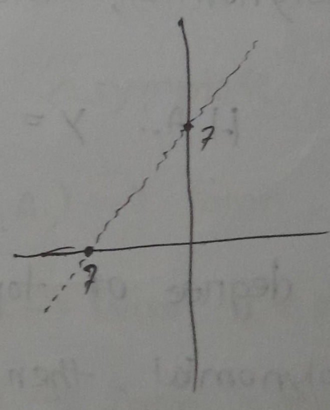
V.A. : No V.A. here.

because its imaginary number

⊛ $f(x) = \dfrac{x^2+5x+3}{x-2}$

Here,

$$x-2)\ x^2+5x+3\ (x+7$$
$$\underline{\overset{(-)}{x^2}\ \overset{(+)}{-\ 2x}}$$
$$7x+3$$
$$\underline{\overset{(-)}{7x}\ \overset{(+)}{-\ 14}}$$
$$17$$




$\therefore$ O.A. : $y = x + 7$

⊛ Families of

$$y = A \sin \omega x \quad \& \quad y = A \cos \omega x$$

Here,

Amplitude $= |A|$

Period, $P = \frac{2\pi}{\omega}$

⊛ $y = \sin x$

Comparing with $y = A \sin \omega x$

$A = 1$

$\omega = 1$

Period $= \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi$

⊛ $y = 2 \sin 4x$

Graph: $\sin x \rightarrow 2 \sin x \longrightarrow 2 \sin 4x$ (Long Process)

⊛ $y = 3 \sin 2\pi x$

Comparing with $y = A \sin \omega x$

$A = |3| = 3$

y - co-ordinate lie between -3 to 3.

$$\text{Period} = \frac{2\pi}{\underbrace{2\pi}_{\omega}} = 1$$

⊛ Graph : $y = -2\sin\left(\frac{\pi}{2}x\right)$ using key points.

Here,

$\text{Amplitute} = |-2| = 2$

$\text{Period} = \frac{2\pi}{\pi/2} = 4$

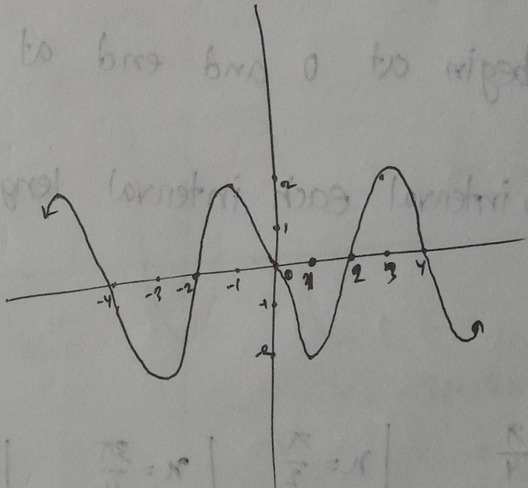
One circle begin at 0 and end at $x = 4$.

Devide by 4 sub interval, each interval length, $\frac{4-0}{4} = 1$

Now points,

| | | | | |
|---|---|---|---|---|
| $x = 0$ | $x = 1$ | $x = 2$ | $x = 3$ | $x = 4$ |
| $y = -2\sin(\frac{\pi}{2}\cdot 0)$ | $y = -2\sin\frac{\pi}{2}$ | $y = -2\sin(\frac{\pi}{2}\cdot 2)$ | $y = -2\sin(\frac{\pi}{2}\cdot 3)$ | $y = -2\sin(\frac{\pi}{2}\cdot 4)$ |
| $= 0$ | $= -2\cdot 1$ | $= 0$ | $= -2(-1)$ | $= 0$ |
| | $= -2$ | | $= 2$ | |
| $(0,0)$ | $(1,-2)$ | $(2,0)$ | $(3,2)$ | $(4,0)$ |

Graph:




Domain: $(-\infty, \infty)$

Range: $[-2, 2]$

* Graph, $y = -3\cos 2x + 1$ using key points

Now,

$y = -3\cos 2x$

Here,

Amplitude, $A = |-3| = 3$

y- co-ordinate line between $-3$ to $3$.

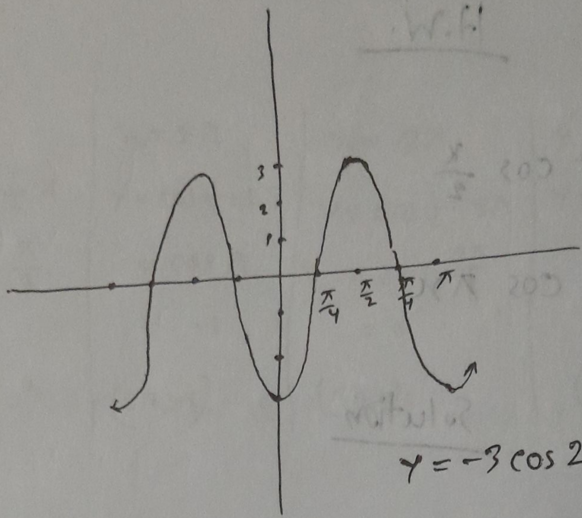
Period, $= \frac{2\pi}{2} = \pi$

One circle begin at $0$ and end at $\pi$.

Deride 4 sub interval each interval length $= \frac{\pi - 0}{4} = \frac{\pi}{4}$

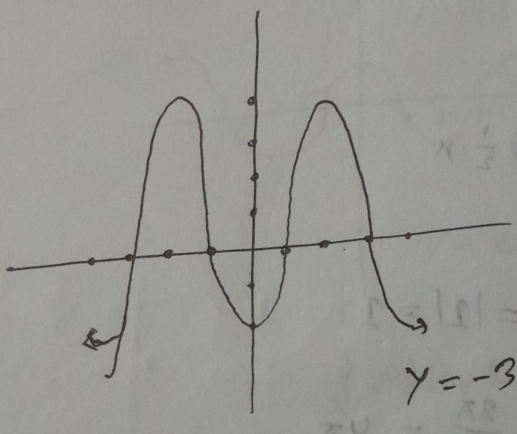
Now, points,

| | | | | |
|---|---|---|---|---|
| $x = 0$ | $x = \frac{\pi}{4}$ | $x = \frac{\pi}{2}$ | $x = \frac{3\pi}{4}$ | $x = \pi$ |
| $y = -3\cos 2\cdot 0$ | $y = -3\cos 2\cdot\frac{\pi}{4}$ | $y = -3\cos\pi$ | $y = -3\cos 2\cdot\frac{3\pi}{4}$ | $y = -3\cos 2\cdot\pi$ |
| $= -3\cos 0$ | $= -3\cos\frac{\pi}{2}$ | $= -3(-1)$ | $= 0$ | $= -3$ |
| $= -3$ | $= 0$ | $= 3$ | $(\frac{3\pi}{4}, 0)$ | $(\pi, -3)$ |
| $(0, -3)$ | $(\frac{\pi}{4}, 0)$ | $(\frac{\pi}{2}, 3)$ | | |




$y = -3\cos 2x$

$\downarrow$ adding 1



$y = -3\cos 2x + 1$

Domain : $(-\infty, \infty)$

Range : $[-2, 4]$

H.W.

1. $y = 2 + \cos \frac{x}{2}$

2. $y = -2 \cos \pi x$

Solution

1)

$y = 2 + \cos \frac{x}{2}$

$\Rightarrow y = \cos \frac{x}{2} + 2$

Now,

$y = \cos \cancel{x} \frac{1}{2} \cdot x$

Here,

Amplitute $= |1| = 1$

Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

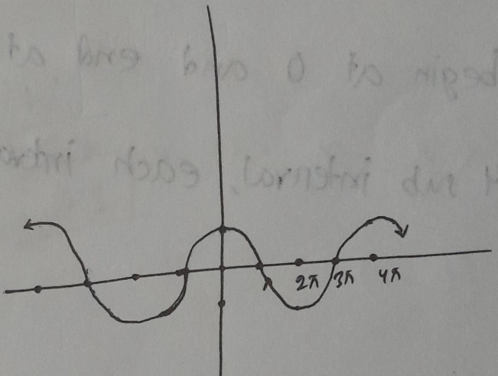
One circle begin at $0$ and end at $4\pi$.

Devided by 4 sub interval, each interval length $\frac{4\pi - 0}{4} = \pi$

Now points,

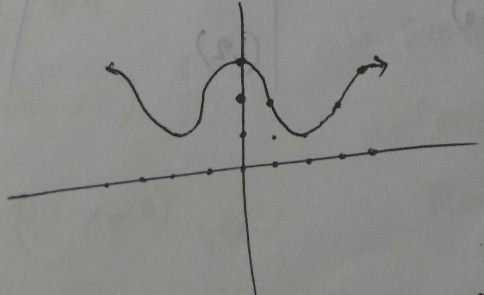
| | | | | |
|---|---|---|---|---|
| $x=0$ | $x=\pi$ | $x=2\pi$ | $x=3\pi$ | $x=4\pi$ |
| $y=\cos\frac{1}{2}\cdot 0$ | $y=\cos\frac{1}{2}\pi$ | $y=\cos\frac{1}{2}\cdot 2\pi$ | $y=\cos\frac{1}{2}\cdot 3\pi$ | $y=\cos\frac{1}{2}\cdot 4\pi$ |
| $=\cos 0$ | $=\cos\frac{\pi}{2}$ | $=\cos\pi$ | $=\cos\frac{3\pi}{2}$ | $=\cos 2\pi$ |
| $=1$ | $=0$ | $=-1$ | $=0$ | $=1$ |
| $(0,1)$ | $(\pi,0)$ | $(2\pi,-1)$ | $(3\pi,0)$ | $(4\pi,1)$ |

Graph:




$y=\cos\frac{1}{2}\cdot x$

$\downarrow$ adding 2



$y=\cos\frac{x}{2}+2$

2)

$Y = -2 \cos \pi x$

Here,

Amplitute $= |-2| = 2$

Y- co-ordinat line between $-2$, to $2$.

Period $= \frac{2\pi}{\pi} = 2$

One cincle begin at 0 and end at 2.

Derided by 4 sub interval, each interval length $\frac{2-0}{4} = \frac{1}{2}$

Now points,

$x = 0$

$y = -2 \cos \pi . 0$

$= -2 \cos 0$

$= -2$

$(0, -2)$

$x = \frac{1}{2}$

$y = -2 \cos \pi . \frac{1}{2}$

$= -2 \cos \frac{\pi}{2}$

$= 0$

$(\frac{1}{2}, 0)$

$x = 1$

$y = -2 \cos \pi . 1$

$= -2 \cos \pi$

$= 2$

$(1, 2)$

$x = \frac{3}{2}$

$y = -2 \cos \pi . \frac{3}{2}$

$= -2 \cos \frac{2\pi}{2}$

$= 0$

$(\frac{3}{2}, 0)$

$x = 2$

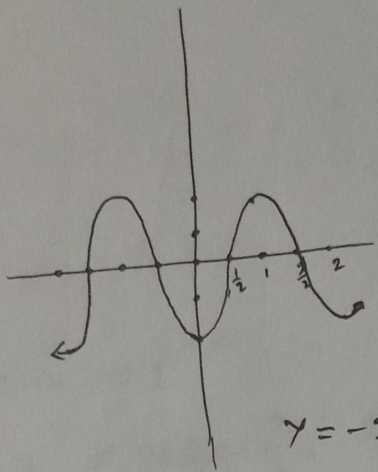
$y = -2 \cos \pi . 2$

$= -2 \cos 2\pi$

$= -2$

$(2, -2)$

Graph!




$y = -2\cos \pi x$

H.W

$\boxed{0.1}$

9)

a) $f(x) = \dfrac{1}{x-3}$

For, domain,

$x - 3 \neq 0$

$x \neq 3$

$\therefore$ Domain : $\{x : x \neq 3\}$

$\therefore$ Range : $\{y : y \neq 0\}$

b) $f(x) = \dfrac{x}{|x|}$

For domain,

$|x| \neq 0$

$x \neq 0$

$\therefore$ Domain : $\{x : x \neq 0\}$

~~Range : $y : y \neq 0$ $\{y : y \neq 0\}$~~

Range : $\{-1, 1\}$

c)

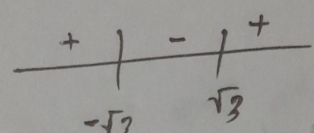
$g(x) = \sqrt{x^2-3}$

For domain,

$x^2 - 3 \geq 0$

$x^2 \geq 3$

$\cancel{x \geq \sqrt{3}}$




Domain: $\{x : x \leq -\sqrt{3} \text{ and } x \geq \sqrt{3}\}$

Range: $[0, \infty)$

d)

$G(x) = \sqrt{x^2-2x+5}$

For domain,

$x^2 - 2x + 5 \geq 0$

$x^2 - 2x + 1 + 4 \geq 0$

$(x-1)^2 + 4 \geq 0$

This expression is always positive

so, domain: $(-\infty, \infty)$

Range: ~~[illegible]~~ $[2, \infty)$

e)

$$h(x) = \frac{1}{1-\sin x}$$

For domain,

$1 - \sin x \neq 0$

$\sin x \neq 1$

$x \neq \frac{\pi}{2}, \frac{5\pi}{2}$

$\therefore x \neq (2n + \frac{1}{2})\pi \;;\; n = 0, \pm 1, \pm 2 \ldots$

Range: $-1 \leq \sin x \leq 1$ $\quad y \geq \frac{1}{2}$

$0 \leq 1 - \sin x \leq 2$

f) $H(x) = \sqrt{\frac{x^2-4}{x-2}} = \sqrt{x+2}$

For domain:

$x - 2 \neq 0$

$x \neq 2$

$\frac{x^2-4}{x-2} \geq 0$

$\frac{(x+2)(x-2)}{x-2} \geq 0$

$x + 2 \geq 0$

$x \geq -2$

$\therefore$ domain : $\{x : x \geq -2 \text{ and } x \neq 2\}$

Range: $[0, 2) \cup (2, +\infty)$

10)

a)

$f(x) = \sqrt{3-x}$

$3 - x \geq 0$

$-x \geq 0 - 3$

$x \leq 3$

Domain: $\{x : x \leq 3\}$

b) $F(x) = \sqrt{4 - x^2}$

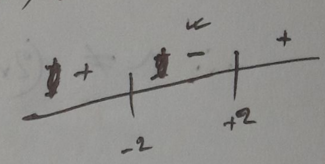
$4 - x^2 \geq 0$

$-x^2 \geq -4$

$x^2 \leq 4$

$x^2 - 4 \leq 0$

$(x+2)(x-2) \leq 0$




~~Domain: $(-\infty, -2] \cup [2, +\infty)$~~

Domain: $[-2, 2]$

c)

$g(x) = 3 + \sqrt{x}$

Fon domain,

$x \geq 0$

D: $\{x : x \geq 0\}$

d) $G(x) = x^3 + 2$

D: $\mathbb{R}$

e) $h(x) = 3 \sin x$

D: $\mathbb{R}$

f)

f) $H(x) = (\sin\sqrt{x})^{-2} = \dfrac{1}{(\sin\sqrt{x})^2}$

For domain,

$x \geq 0$

And,

$\cancel{\sin\sqrt{x} \neq 0}$

$\sqrt{x} \neq 0$

$\sqrt{x} \neq n\pi \quad ; n = 0, \pm 1, \pm 2, \pm 3 \ldots$

Range: $y \geq 1$.

## |0.2|

$y = x^2$ / $y = \sqrt{x}$ / $y = \frac{1}{x}$ / $y = |x|$ / $y = \sqrt[3]{x}$

5]

$y = -2(x+1)^2 - 3$

$y = x^2 \rightarrow (x+1)^2 \rightarrow 2(x+1)^2 \rightarrow -2(x+1)^2 \rightarrow -2(x+1)^2 - 3$

6]

$y = \frac{1}{2}(x-3)^2 + 2$

$y = x^2 \rightarrow (x-3)^2 \rightarrow \frac{1}{2}(x-3)^2 \rightarrow \frac{1}{2}(x-3)^2 + 2$

7]

$y = x^2 + 6x$ $\cancel{= x(x+6)}$

$y = x^2 + 2 \cdot 3x + 3^2 - 9$

$y = (x+3)^2 - 9$

$y = x^2 \rightarrow (x+3)^2 \rightarrow (x+3) - 9$

8)

$y = \frac{1}{2}(x^2 - 2x + 3)$

$= \frac{1}{2}(x^2 - 2x + 1 + 2)$

$= \frac{1}{2}((x-1)^2 + 2)$

$y = x^2 \rightarrow (x-1)^2 \rightarrow (x-1)^2 + 2 \rightarrow \frac{1}{2}((x-1)^2 + 2)$

9)

$y = 3 - \sqrt{x+1}$

$y = \sqrt{x} \rightarrow \sqrt{x+1} \rightarrow -\sqrt{x+1} \rightarrow -\sqrt{x+1} + 3$

10)

$y = 1 + \sqrt{x-4}$

$y = \sqrt{x} \rightarrow \sqrt{x-4} \rightarrow \sqrt{x-4} + 1$

11)

$y = \frac{1}{2}\sqrt{x} + 1$

$y = \sqrt{x} \rightarrow \frac{1}{2}\sqrt{x} \rightarrow \frac{1}{2}\sqrt{x} + 1$

12)

$y = -\sqrt{3x}$

$y = \sqrt{x} \rightarrow \sqrt{3x} \rightarrow -\sqrt{3x}$

13)

$y = \frac{1}{x-3}$

$y = \frac{1}{x} \rightarrow \frac{1}{(x-3)}$

14)

$y = \frac{1}{1-x} = \frac{1}{-(x-1)}$

$y = \frac{1}{x} \rightarrow \frac{1}{x-1} \rightarrow \frac{1}{-(x-1)}$

15)

$y = 2 - \frac{1}{x+1}$

$y = \frac{1}{x} \rightarrow \frac{1}{x+1} \rightarrow -\frac{1}{x+1} \rightarrow -\frac{1}{x+1} + 2$

16)

$y = \frac{x-1}{x} = \frac{x}{x} - \frac{1}{x} = 1 - \frac{1}{x}$

$y = \frac{1}{x} \rightarrow -\frac{1}{x} \rightarrow -\frac{1}{x} + 1$

17)

$y = |x+2| - 2$

$y = |x| \rightarrow |x+2| \rightarrow |x+2| - 2$

18)

$y = 1 - |x-3|$

$y = |x| \rightarrow |x-3| \rightarrow -|x-3| \rightarrow -|x-3|+1$

19)

$y = |2x-1| + 1$

$y = |x| \rightarrow |2x| \rightarrow |2x-1| \rightarrow |2x-1|+1$

20) $y = \sqrt{x^2-4x+4} = \sqrt{(x - 2\cdot x\cdot 2 + 2^2)} = \sqrt{(x-2)^2} = |x-2|$

$y = |x| \rightarrow |x-2|$

21)

$y = 1 - 2\sqrt[3]{x}$

$y = \sqrt[3]{x} \rightarrow 2\sqrt[3]{x} \rightarrow -2\sqrt[3]{x} \rightarrow -2\sqrt[3]{x}+1$

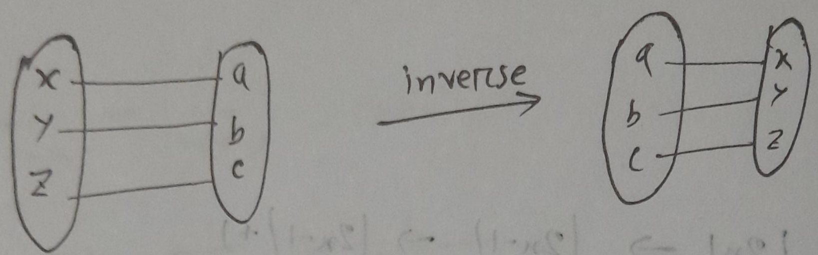
22) $y = \sqrt[3]{x-2} - 3$

$\Rightarrow y = \sqrt[3]{x} \rightarrow \sqrt[3]{x-2} \rightarrow \sqrt[3]{x-2} - 3$

L-5/ 15.06.2022

## 0.4

## Inverse function:




One to one function

If a function one to one, then interchanging range and domain, then we will get the new function, that is called inverse function.

Domain of $f^{-1}$ = Range of f

Range of $f^{-1}$ = Domain of f

Getting Inverse function from the equation:

Example: If $f(x) = x^3 - 1$, find inverse of $f(x)$.

$\Rightarrow \quad f(x) = x^3 - 1$

$y = x^3 - 1$

Interchange $x$ and $y$,

$x = y^3 - 1$

solve for $y$,

$y^3 = x + 1$

$\therefore y = \sqrt[3]{x+1}$

$\therefore f^{-1}(x) = \sqrt[3]{x+1}$

<u>Checking:</u>

$f(f^{-1}(x)) = f(\sqrt[3]{x+1})$

$= (\sqrt[3]{x+1})^3 - 1$

$= x + 1 - 1$

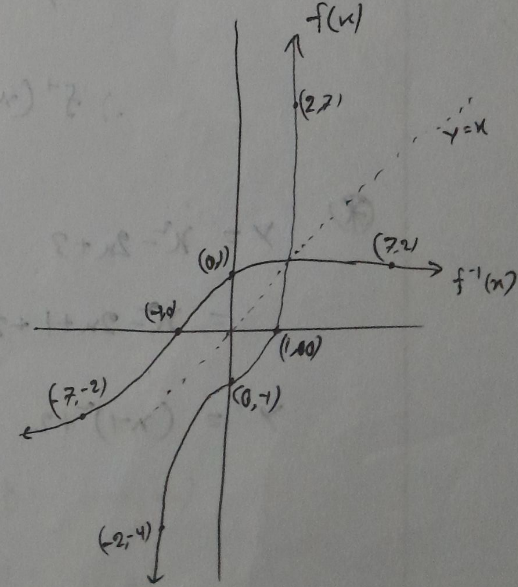
$= x$

$f^{-1}(f(x)) = f^{-1}(x^3 - 1)$

$= \sqrt[3]{x^3 - 1 + 1}$

$= \sqrt[3]{x^3}$

$= x$




$\therefore f(f^{-1}(x)) = f^{-1}(f(x)) = x$ (Proved)

(*) If $f(x) = (x-1)^2$ are one to one?

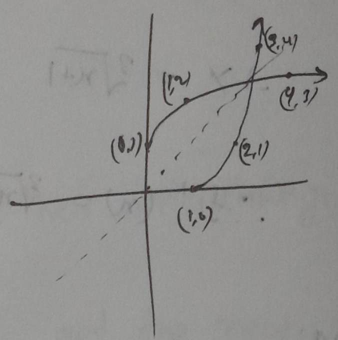
restricted domain $x \geq 1$

$$f(x) = (x-1)^2$$

$$y = (x-1)^2$$

interchange, $x$ and $y$,

$$x = (y-1)^2$$




solve for $y$,

$$x = (y-1)^2$$

$$y-1 = \sqrt{x}$$

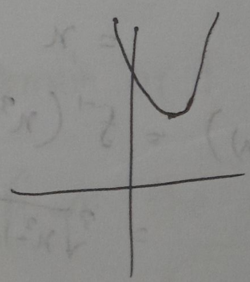
$$y = \sqrt{x}+1$$

$$\therefore f^{-1}(x) = \sqrt{x}+1$$

(*) $y = x^2 - 2x + 3$

$$= x^2 - 2x + 1 + 2$$

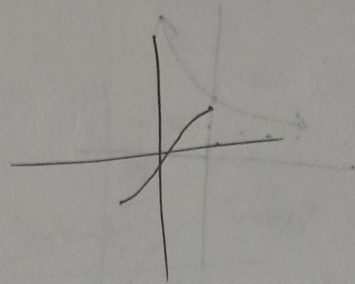
$$y = (x-1)^2 + 2$$



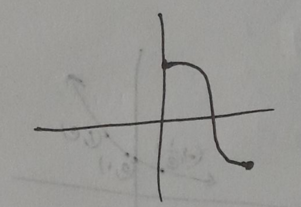
D: $[1, \infty)$

R: $[2, \infty)$

* $f(x) = \sin x$ ; $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$



* $f(x) = \cos x$ ; $0 \le x \le \pi$

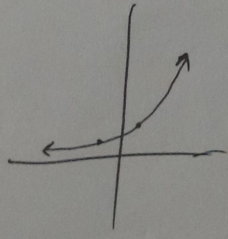


0.5 /

Exponential Function:

$f(x) = a^x$ ; $a > 0$, $a \neq 1$

$f(x) = 2^x$



D: $(-\infty, \infty)$

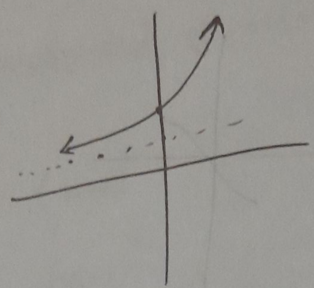
R: $(0, \infty)$

H.A: $y = 0$

Increasing

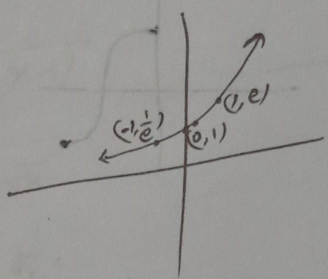
$(0,1)$, $(1,a)$, $(-1,\frac{1}{a})$

$f(x) = 2^x + 1$



$e = 2.7182 \ldots$ natural no.

$f(x) = e^x$




(*)

$f(x) = a^x$

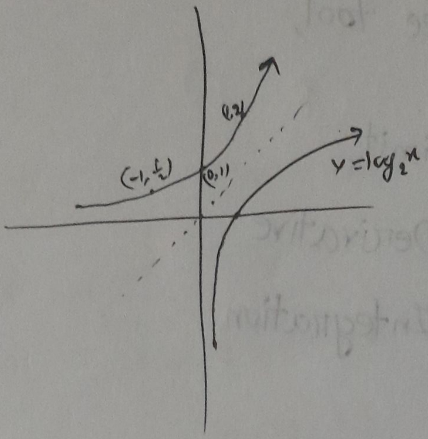
$Y = a^x$

$\Rightarrow \quad x = a^y$

$y = \log_a x$

$f^{-1}(x) = \log_a x$

⊛ $y = \log_a x$ , if and only if $x = a^y$




$\longrightarrow$ D: $(0, \infty)$

R: $(-\infty, \infty)$

V.A.: $x = 0$

L-6/22.06.2022/

# Chapter - 1

There are three tool,

i. Limit
ii. Derivative
iii. Integration

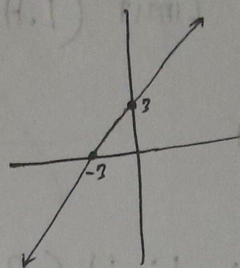
## 1.1 / Limit

### Limit of a function:

If $f(x)$ has a limit $L$ at an input $a$, this means that, $f(x)$ gets closer and closer to $L$ as $x$ moves closer and closer to $a$, Then we can write:

$$\lim_{x \to a} f(x) = L$$

⊛ $f(x) = x+3$




$\lim_{x \to 2} x+3 = 5$

$\lim_{x \to 1} f(x) = 4$

$\lim_{x \to -1} f(x) = 2$

$\lim_{x \to -2} f(x) = 1$

$f(x) = x+3$

$f(1) = 4$

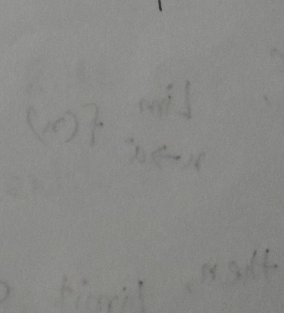
$f(-1) = 2$

$f(-2) = 1$

$f(2) = 5$

⊛ $f(x) = \dfrac{x^2-4}{x-2}$

$f(x) = x+2 \longrightarrow D: x \neq 2$




$f(2) = \frac{0}{0}$ = undefined

$\lim_{x \to} x+2 = 4$

$\lim_{x \to -1} x+2 = 1$

$\lim_{x \to 4} x+2 = 6$

⊛ Left Hand Limit (L.H.L.)

$$\lim_{x\to a^-} f(x)$$

⊛ Right Hand Limit (R.H.L.)

$$\lim_{x\to a^+} f(x)$$

⊛

If, L.H.L = R.H.L

Then we will say

Limit $\lim_{x\to a} f(x)$ exist.

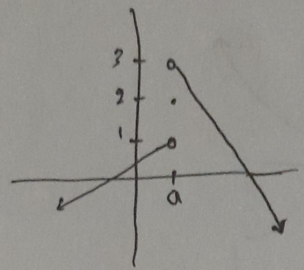
$$\Rightarrow \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$$

$$\therefore \lim_{x\to a} f(x) = L$$

⊛ If, $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$

then, Limit does not exist. (DNE).

(*) $f(x)$




Here,

$\lim_{x \to a^-} f(x) = 1$

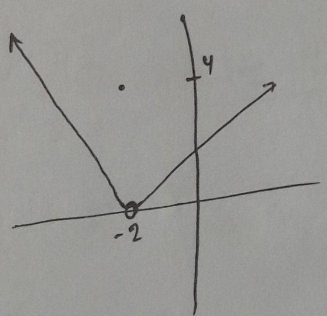
$\lim_{x \to a^+} f(x) = 3$

$f(a) = 2$

$\therefore$ L.H.L $\neq$ R.H.L.

$\therefore$ Limit does not exist.

(*)




Here,

$\lim_{x \to -2^+} f(x) = 0$

$\lim_{x \to -2^-} f(x) = 0$

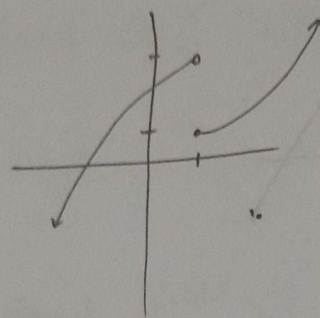
$f(-2) = 4$

$\therefore$ L.H.L = R.H.L

$\therefore$ Limit exist.

$\therefore \lim_{x \to -2} f(x) = 0$

⊛



Here,

$\lim_{x \to 3^-} f(x) = 5$

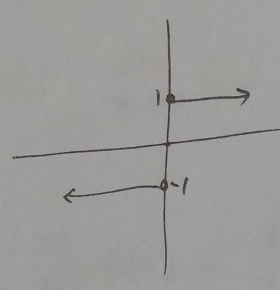
$\lim_{x \to 3^+} f(x) = 2$

$f(x) = 2$

$\therefore$ L.H.L $\neq$ R.H.L.

$\therefore$ Limit does not exist.

⊛




Here,

$\lim_{x \to 0^-} f(x) = -1$

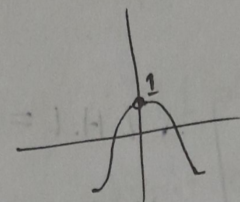
$\lim_{x \to 0^+} f(x) = 1$

$\therefore$ L.H.L $\neq$ R.H.L.

$\therefore$ Limit does not exist.

⊛

$$\lim_{x \to 0} \frac{\sin x}{x} = ?$$




$$\lim_{x \to 0^-} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0^+} \frac{\sin x}{x} = 1$$

$f(0)$ = undefined

$\therefore$ L.H.L = R.H.L

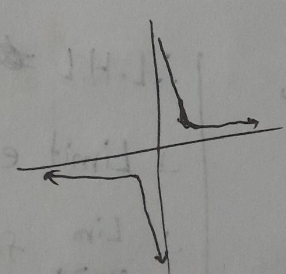
$\therefore$ Limit exist.

$$\therefore \lim_{x \to a} \frac{\sin x}{x} = 1$$

## ⊛ Infinite Limit

⊛

$$f(x) = \frac{1}{x}$$



Here,

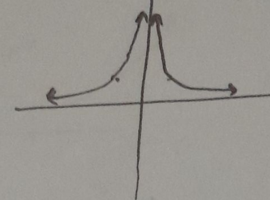
$$\lim_{x \to 0^+} f(x) = \alpha$$

$$\lim_{x \to 0^-} f(x) = -\alpha$$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ Limit does not exist.

$f(x) = \frac{1}{x^2}$



Here,

$\lim_{x \to 0^-} f(x) = \infty$

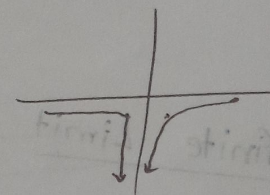
$\lim_{x \to 0^+} f(x) = \infty$

$\therefore$ L.H.L = R.H.L

$\therefore$ Limit exist.

$\therefore \lim_{x \to 0} f(x) = \infty$

$f(x) = -\frac{1}{x^2}$



Here,

$\lim_{x \to 0^-} f(x) = -\infty$

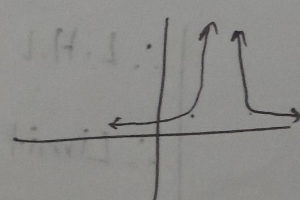
$\lim_{x \to 0^+} f(x) = -\infty$

$\therefore$ L.H.L $\cancel{\neq}$ = R.H.L.

$\therefore$ Limit exist.

$\therefore \lim_{x \to 0} f(x) = -\infty$

$f(x) = \frac{1}{(x-2)^2}$



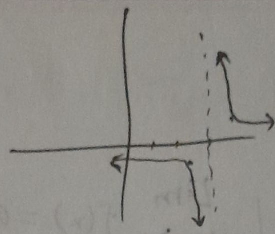
$\lim_{x \to 2^-} f(x) = \infty$

$\lim_{x \to 2^+} f(x) = \infty$

Limit exist.

$\lim_{x \to 2} f(x) = \infty$

⊛ $f(x) = \frac{1}{x-3}$



Here,

$\lim_{x \to 3^-} f(x) = -\infty$

$\lim_{x \to 3^+} f(x) = +\infty$

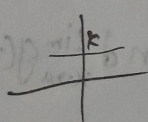
$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ Limit does not exist.

## 1.2

## Computing Limit

⊛ $f(x) = k$



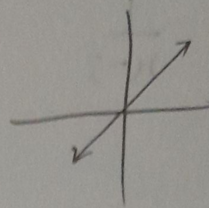

$\therefore \lim_{x \to a} f(x) = k$

$\lim_{x \to -1} f(x) = k$

$\lim_{x \to -2} f(x) = k$

$\lim_{x \to -3} 3 = 3$

$$f(x) = x$$



$f(0) = 0$

$f(1) = 1$

$f(-2) = -2$

$$\lim_{x \to a} f(x) = a$$

$$\lim_{x \to 2} f(x) = 2.$$

$$\lim_{x \to 3} f(x) = 3$$

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

## Limit Laws:

If, $\lim_{x \to a} f(x) = L_1$ and $\lim_{x \to a} g(x) = L_2$

(i) $$\lim_{x \to a} \left(f(x) \pm g(x)\right) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$

$$= L_1 \pm L_2$$

(ii) $$\lim_{x \to a} \left(f(x) \cdot g(x)\right) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

$$= L_1 \cdot L_2$$

(iii) $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} = \dfrac{L_1}{L_2}$ ; $L_2 \neq 0$

(iv) $\lim\limits_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x\to a} f(x)} = \sqrt[n]{L_1}$

(v) $\lim\limits_{x\to a} k.f(x) = k.\lim\limits_{x\to a} f(x) = k.L_1$

(vi) $\lim\limits_{x\to a} (f(x))^n = \left(\lim\limits_{x\to a} f(x)\right)^n = (L_1)^n$

(vii) $\lim\limits_{x\to 1} (x^2+x+1)$

$= \lim\limits_{x\to 1} x^2 + \lim\limits_{x\to 1} x + \lim\limits_{x\to 1} 1$

$= 1 + 1 + 1$

$= 3$

Ans

## ⊛ Limit of Polynomial Function:

$$f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

$$\lim_{x \to 1} f(x) = f(1)$$

⊛

$$\lim_{x \to -2} (x^2 + 2x + 3) = (-2)^2 + 2(-2) + 3$$

$$= 4 - 4 + 3$$

$$= 3$$

Ans

⊛

$$\lim_{x \to 3} (3x^3 + 6x - 3) = 3 \cdot 3^3 + 6 \cdot 3 - 3$$

$$= 3 \cdot 27 + 18 - 3$$

$$= 81 + 18 - 3$$

$$= 96$$

Ans.

## ⊛ Limit of Rational Function:

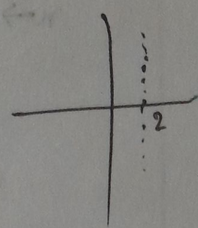
$$f(x) = \frac{P(x)}{Q(x)}$$

$$\lim_{x \to a} f(x) = \lim_{x \to a} \frac{P(x)}{Q(x)} = \frac{\lim_{x \to a} P(x)}{\lim_{x \to a} Q(x)} = \frac{P(a)}{Q(a)} ; \; Q(a) \neq 0$$

* $$\lim_{x\to 2} \frac{x^2+3}{x^3+6} = \frac{\lim_{x\to 2} x^2+3}{\lim_{x\to 2} x^3+6} = \frac{2^2+3}{2^3+6} = \frac{7}{14} = \frac{1}{2}$$

Ans

* $$\lim_{x\to 2} \frac{x-3}{x-2}$$




Now,

$$\lim_{x\to 2^-} \frac{x-3}{x-2} = \infty$$

$$\lim_{x\to 2^+} \frac{x-3}{x-2} = -\infty$$

L.H.L $\neq$ R.H.L.

$\therefore$ Limit does not exist.

L-7 / 24.06.2022 /

## Finding Limit for Polynomial Function

$$f(x) = x^3 + 2x^2 + 3$$

$$\lim_{x \to 1} f(x) = f(1)$$

$$= 1^3 + 2 \cdot 1^2 + 3$$

$$= 1 + 2 + 3$$

$$= 6$$

Ans

## Finding Limit for Rational Function.

$$f(x) = \frac{P(x)}{Q(x)}$$

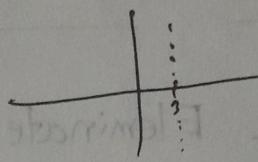
$$\lim_{x \to a} \frac{P(x)}{Q(x)} = \frac{P(a)}{Q(a)} \quad ; \quad Q(a) \neq 0$$

$$f(x) = \frac{x^2+6}{x-2}$$

$$\lim_{x \to 1} f(x) = \frac{1^2+6}{1-2} = \frac{7}{-1} = -7$$

$$f(x) = \frac{x+2}{x-3}$$

$$\lim_{x \to 3} f(x) = ?$$




$f(2.99) = \frac{4.99}{-0.01} = -499$

$f(2.999) = -4999$

$f(3.001) = \frac{5.001}{0.001} = 5001$

$f(3.00001) = 5000001$

Here,

$$\lim_{x \to 3^-} f(x) = -\infty$$

$$\lim_{x \to 3^+} f(x) = \infty$$

$\therefore$ L.H.L. $\neq$ R.H.L.

$\therefore$ Limit does not exist.

$$\lim_{x \to 3} \frac{1}{|x-3|} = ?$$

$f(2.99) = \frac{1}{0.001} = 1000$

$f(2.9999) = 100000$

$f(3.001) = \frac{1}{0.0001} = 1000$

$f(3.00001) = 100000$

Here,

$$\lim_{x \to 3^-} \frac{1}{|x-3|} = \infty$$

$$\lim_{x \to 3^+} \frac{1}{|x-3|} = \infty$$

$\therefore$ L.H.L = R.H.L

$$\therefore \lim_{x \to 3} \frac{1}{|x-3|} = \infty$$

## ⊛ Finding Limit involving Radicals:

Rules: Eleminate the zero demominactor.

⊛ $\lim_{x\to\sqrt{3}} \frac{x^2-3}{x-\sqrt{3}}$

$= \lim_{x\to\sqrt{3}} \frac{x^2-(\sqrt{3})^2}{(x-\sqrt{3})}$

$= \lim_{x\to\sqrt{3}} \frac{(x+\sqrt{3})(x-\sqrt{3})}{(x-\sqrt{3})}$

$= \lim_{x\to\sqrt{3}} (x+\sqrt{3})$

$= \sqrt{3}+\sqrt{3}$

$= 2\sqrt{3}$

Ans

⊛ $\lim_{y\to 4} \frac{4-y}{2-\sqrt{y}}$

$= \lim_{y\to 4} \frac{(2-\sqrt{y})(2+\sqrt{y})}{2-\sqrt{y}}$

$= \lim_{y\to 4} (2+\sqrt{y})$

$= 2+\sqrt{4}$

$= 4$

Ans

⊛

$\lim_{y\to 6^-} \frac{y+6}{y^2-36}$

$= \lim_{y\to 6^-} \frac{y+6}{(y+6)(y-6)}$

$= \lim_{y\to 6^-} \frac{1}{y-6}$

$= -\infty$

$\lim_{y\to 6^+} \frac{1}{y-6} = \infty$

⊛ Limit of Piece-wise defined function.

⊛ If, $f(x) = \begin{cases} x-1 & x \le 3 \\ 3x-7 & x > 3 \end{cases}$

i) $\lim_{x \to 3^-} f(x)$ ii) $\lim_{x \to 3^+} f(x)$ iii) $\lim_{x \to 3} f(x)$ iv) $\lim_{x \to 5} f(x)$

v) $\lim_{x \to 1} f(x)$

$\Rightarrow$

i) $\lim_{x \to 3^-} f(x)$

$= \lim_{x \to 3} x-1$

$= 3-1$

$= 2$

ii) $\lim_{x \to 3^+} f(x)$

$= \lim_{x \to 3^+} 3x-7$

$= 3 \cdot 3 - 7$

$= 9-7$

$= 2$

iii) L.H.L = R.H.L

$\therefore \lim_{x \to 3} f(x) = 2$

iv) $\lim_{x \to 5} f(x)$

$= \lim_{x \to 5} 3x-7$

$= 3 \cdot 5 - 7$

$= 15-7$

$= 8$

v) $\lim_{x \to 1} f(x)$

$= \lim_{x \to 1} x-1$

$= 1-1$

$= 0$

Q. If,

$$f(x) = \begin{cases} x-2 & x<0 \\ x^2 & 0 \le x \le 2 \\ 2x & x>2 \end{cases}$$

i) $\lim_{x\to 0} f(x)$ ii) $\lim_{x\to 1} f(x)$ iii) $\lim_{x\to 2} f(x)$

$\Rightarrow$

i) $\lim_{x\to 0} f(x) = ?$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x-2$

$= 0-2$

$= -2$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2$

$= 0^2$

$= 0$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ Limit does not exist.

ii) $\lim_{x\to 1} f(x)$

$= \lim_{x\to 1} x^2$

$= 1^2$

$= 1$

iii) $\lim_{x\to 2} f(x) = ?$

$\lim_{x\to 2^-} f(x)$

$= \lim_{x\to 2^-} x^2$

$= 2^2$

$= 4$

$\lim_{x\to 2^+} f(x)$

$= \lim_{x\to 2^+} 2x$

$= 2\cdot 2$

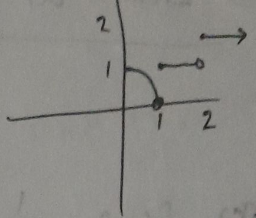
$= 4$

$\therefore$ L.H.L = R.H.L

$\therefore \lim_{x\to 2} f(x) = 4.$

⊛ If,

$$f(x) = \begin{cases} \sqrt{1-x^2} & 0 \le x < 1 \\ 1 & 1 \le x < 2 \\ 2 & x \ge 2 \end{cases}$$




i) $\lim_{x \to 1} f(x)$ ii) $\lim_{x \to 2} f(x)$

⇒

i) $\lim_{x \to 1} f(x)$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \sqrt{1-x^2} = \sqrt{1-1^2} = 0$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 1 = 1$$

∴ L.H.L ≠ R.H.L.

∴ Limit does not exist.

ii) $\lim_{x \to 2} f(x) = ?$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 1 = 1$$

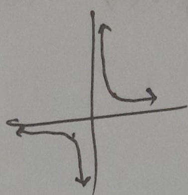
$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 2 = 2$$

∴ L.H.L ≠ R.H.L.

∴ Limit does not exist.

## ⊛ Limit at infinity (End behavior of function / Horizontal asymptote):

⊛ $f(x) = \frac{1}{x}$



$\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to -\infty} \frac{1}{x} = 0.$

H.A.: $y = 0$

## ⊛ Limit Laws for Limit at infinity.

(i) $\lim_{x \to \pm\infty} (f(x))^n = \left(\lim_{x \to \pm\infty} f(x)\right)^n$

(ii) $\lim_{x \to \pm\infty} k \cdot f(x) = k \cdot \left(\lim_{x \to \pm\infty} f(x)\right)$

(iii) $\lim_{x \to \pm\infty} k = k$

⊛ $\lim_{x \to +\infty} \left(\frac{1}{x^n}\right)$

$= \left(\lim_{x \to +\infty} \frac{1}{x}\right)^n$

$= (0)^n = 0$

## Limit of Polynomial function as $x \to \pm\infty$.

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

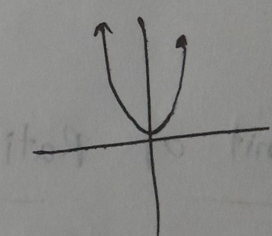
End behaviour,

$$\lim_{x \to +\infty} f(x) = \lim_{x \to \infty} (a_n x^n + \cdots + a_0)$$

$$= \lim_{x \to +\infty} a_n x^n \left(1 + \left(\frac{a_{n-1}}{a_n} \cdot \frac{1}{x}\right) + (\;)\cdot\frac{1}{x^2} + \cdots \frac{a_0}{a_n x^n}\right)$$

$$= a_n x^n$$

## $f(x) = 3x^6 + 7x^3 + 1$



End behaviour:

$$f(x) = 3x^6$$

$$\lim_{x \to \infty} 3x^6 + 7x^3 + 1$$

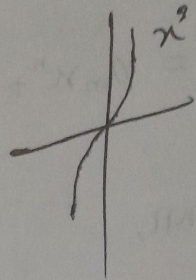
$$= \lim_{x \to \infty} 3x^6$$

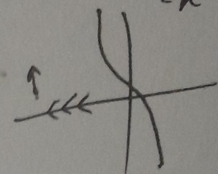
$$= +\infty$$

$$\lim_{x \to -\infty} -3x^5 + 6x^4 + 7x^2 + 2$$

$$= \lim_{x \to -\infty} -3x^5$$

$$= +\infty$$

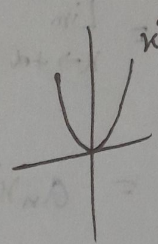


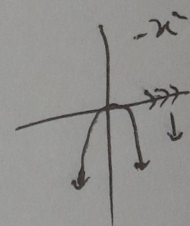




$$\lim_{x \to +\infty} -x^8 + 7x^6 + 7x + 1$$

$$= \lim_{x \to +\infty} -x^8$$

$$= -\infty$$




## Limit of Rational Function as $x \to \pm\infty$.

$$\lim_{x \to \pm\infty} \frac{P(x)}{Q(x)}$$

Rules:

⇒ Find the end behaviour of Top and bottom polynomial.

⇒ Cancel the common factor

⇒ Apply the limit.

$$\lim_{x\to +\infty} \frac{3x^3+6x^2+2}{4x^4+7x+1}$$

$$= \lim_{x\to \infty} \frac{3x^3}{4x^4}$$

$$= \lim_{x\to \infty} \frac{3}{4x}$$

$$= \frac{3}{4} \cdot \lim_{x\to \infty} \frac{1}{x}$$

$$= \frac{3}{4} \cdot 0 = 0$$

Ans.

$$\lim_{x\to \infty} \frac{5x^3-2x^2+1}{1-3x}$$

$$= \lim_{x\to \infty} \frac{5x^3}{-3x}$$

$$= \lim_{x\to \infty} \frac{-5x^2}{3}$$

$$= \frac{5}{3} \cdot \lim_{x\to \infty} -x^2$$

$$= \frac{5}{3} \cdot (-\infty)$$

$$= -\infty$$

Ans.

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## ⊛ Limit at infinity for rational function:

⊛ $f(x) = \dfrac{2x^4 + 7x^2 + 3x - 1}{5x^5 + 6x^4 + 3x + 2}$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{2x^4 + 7x^2 + 3x - 1}{5x^5 + 6x^4 + 3x + 2}$$

$$\boxed{\begin{array}{l} \lim_{x \to \infty} \frac{1}{x} = 0 \\ \lim_{x \to -\infty} \frac{1}{n} = 0 \end{array}}$$

$$= \lim_{x \to \infty} \frac{2x^4}{5x^5}$$

$$= \frac{2}{5} \lim_{x \to \infty} \frac{1}{x}$$

$$= 0$$

⊛ Find the H.A. for

$$f(x) = \frac{3x^3 + 2x + 1}{6x^4 + 7x + 3}$$

⇒

We know that, for H.A.

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{3x^3 + 2x + 1}{6x^4 + 7x + 3}$$

$$\therefore \lim_{x \to \infty} \frac{3x^3 + 2x + 1}{6x^4 + 7x + 3} = \lim_{x \to \infty} \frac{3x^3}{6x^4}$$

$$= \frac{1}{2} \lim_{x \to \infty} \frac{1}{x}$$

$$= 0$$

$\therefore \lim_{x \to -\infty} \dfrac{3x^3+2x+1}{6x^4+7x+3} = \lim_{x \to -\infty} \dfrac{3x^3}{6x^4}$

$= 0$

$\therefore$ H.A. is $y = 0$.

Find the H.A. of

$$f(x) = \frac{x^3-2}{|x|^3+1}$$

$\Rightarrow$

We know that, for H.A.

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x^3-2}{|x|^3+1}$$

$\therefore \lim_{x \to \infty} \dfrac{x^3-2}{|x|^3+1} = \lim_{x \to \infty} \dfrac{x^3}{x^3} = 1$

$\therefore \lim_{x \to -\infty} \dfrac{x^3-2}{|x|^3+1} = \lim_{x \to -\infty} \dfrac{x^3-2}{-x^3+1} = \lim_{x \to -\infty} \dfrac{x^3}{-x^3} = -1$

$\therefore$ H.A. are,

$y = 1$

$y = -1$

## Limit at infinity for involving radicals.

$$\lim_{x \to -\infty} \frac{\sqrt{x^2+2}}{3x-6}$$

$$= \lim_{x \to -\infty} \frac{\frac{\sqrt{x^2+2}}{|x|}}{\frac{3x-6}{|x|}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{x^2+2}{x^2}}}{\frac{3x-6}{-x}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{1+\frac{2}{x^2}}}{-3+\frac{6}{x}}$$

$$= \frac{\lim_{x \to -\infty} \sqrt{1+\frac{2}{x^2}}}{\lim_{x \to -\infty} \left(-3+\frac{6}{x}\right)}$$

$$= \frac{\sqrt{\lim_{x \to -\infty} \left(1+\frac{2}{x^2}\right)}}{-3+\lim_{x \to -\infty} \frac{6}{x}}$$

$$= \frac{\sqrt{1+\lim_{x \to \infty} \frac{2}{x^2}}}{-3+0}$$

$$= \frac{\sqrt{1+0}}{-3}$$

$$= \frac{\sqrt{1}}{-3} = -\frac{1}{3} \text{ Ans}$$

$$\lim_{x \to -\infty} \frac{\sqrt{3x^4+x}}{x^2-8}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{3x^4+x}{x^2}}}{\frac{x^2-8}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{3x^4+x}{x^4}}}{1-\frac{8}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{3+\frac{1}{x^3}}}{1-\frac{8}{x^2}}$$

$$= \frac{\sqrt{\lim_{x \to -\infty} 3 + \lim_{x \to -\infty} \frac{1}{x^3}}}{\lim_{x \to -\infty} 1 - \lim_{x \to -\infty} \frac{8}{x^2}}$$

$$= \frac{\sqrt{3+0}}{1-0}$$

$$= \frac{\sqrt{3}}{1} = \sqrt{3}$$ Ans

$$\lim_{x\to\infty} \sqrt{x^2+3} - x$$

$$= \lim_{x\to\infty} \frac{(\sqrt{x^2+3} - x)(\sqrt{x^2+3} + x)}{(\sqrt{x^2+3} + x)}$$

$$= \lim_{x\to\infty} \frac{(\sqrt{x^2+3})^2 - x^2}{\sqrt{x^2+3} + x}$$

$$= \lim_{x\to\infty} \frac{x^2+3-x^2}{\sqrt{x^2+3}+x} = \lim_{x\to\infty} \frac{3}{\sqrt{x^2+3}+x}$$

$$= \lim_{x\to\infty} \frac{\frac{3}{|x|}}{\frac{\sqrt{x^2+3}+x}{|x|}}$$

$$= \lim_{x\to\infty} \frac{\frac{3}{x}}{\sqrt{\frac{x^2+3}{x^2}}+1}$$

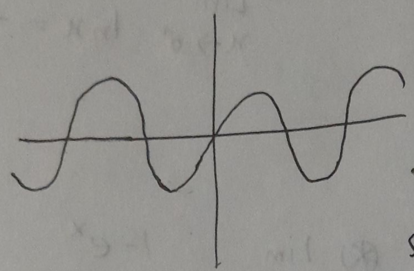
$$= \frac{\lim_{x\to\infty} \frac{3}{x}}{\sqrt{\lim_{x\to\infty}\left(1+\frac{3}{x^2}\right)} + \lim_{x\to\infty} 1}$$

$$= \frac{0}{1+1} = \frac{0}{2}$$

$$= 0$$ Ans

## ⊛ Limit at infinity for Trigonometry / exponential / Logarithmic.

⊛ Trigonometry:

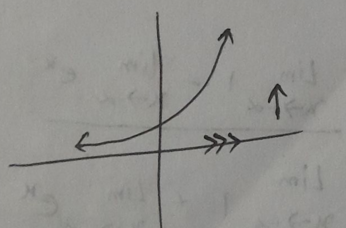


$x \to \infty$

$\sin x / \cos x \to$ Repeating

→ No limit for $x \to \pm\infty$ for trigonometry function.

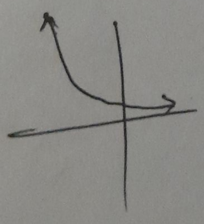
⊛ $y = e^x$



$$\lim_{x \to \infty} e^x = +\infty$$

$$\lim_{x \to -\infty} e^x = 0$$

$y = e^{-x}$

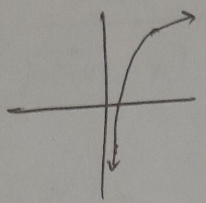


$$\lim_{x \to \infty} e^{-x} = 0$$

$$\lim_{x \to -\infty} e^{-x} = +\infty$$

## Logarithmic function:

$y = \ln x$



$\lim_{x \to \infty} \ln x = +\infty$

$\lim_{x \to 0^+} \ln x = -\infty$

$\lim_{x \to +\infty} \ln\left(\frac{2}{x^2}\right)$

$= \ln\left(\lim_{x \to +\infty} \frac{2}{x^2}\right)$

$= \ln 0$

$= -\infty$

$\lim_{x \to -\infty} \frac{1-e^x}{1+e^x}$

$= \frac{\lim_{x \to -\infty} (1-e^x)}{\lim_{x \to -\infty} (1+e^x)}$

$= \frac{\lim_{x \to -\infty} 1 - \lim_{x \to -\infty} e^x}{\lim_{x \to -\infty} 1 + \lim_{x \to -\infty} e^x}$

$= \frac{1-0}{1+0} = 1$

$$\lim_{x \to -\infty} \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$$

$$= \lim_{x \to -\infty} \frac{e^{2x}+1}{e^{2x}-1} = \frac{\lim_{x \to -\infty} e^{2x} + \lim_{x \to -\infty} 1}{\lim_{x \to -\infty} e^{2x} - \lim_{x \to -\infty} 1}$$

$$= \frac{0+1}{0-1} = \frac{1}{-1} = -1$$

Ans

$$\lim_{x \to \infty} \left(1+\frac{1}{x}\right)^{x} = e$$

$$\lim_{x \to -\infty} \left(1+\frac{1}{x}\right)^{x} = e$$

$$\lim_{x \to \infty} \left(1+\frac{1}{x}\right)^{2x}$$

$$= \lim_{x \to \infty} \left(1+\frac{1}{x}\right)^{x \cdot 2}$$

$$= e^{2}$$

Ans

## Continuity :

* There are three conditions for a function $f(x)$ to be continuous at a point $x = a$,

(i) Function defined at $x = a$, i.e. $f(a)$ exists.

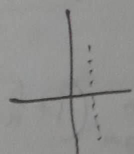
(ii) Limit exists, i.e. $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

then, $\lim_{x \to a} f(x)$ exists.

(iii) $\lim_{x \to a} f(x) = f(a)$.

* Types of discontinuous :

* If, function not defined,



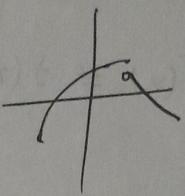
$$f(x) = \frac{1}{x-2}$$

$$f(2) = \text{undefined}$$

infinity discontinuous.

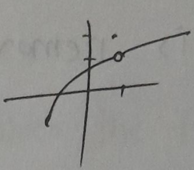
⊛ if,

$$\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x)$$



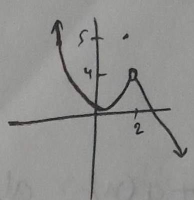
Jump discontinuous.

⊛ $\lim_{x\to a} f(x) \neq f(a)$



Removable discontinuity

⊛ $f(x) = \begin{cases} x^2 & x<2 \\ 5 & x=2 \\ -x+6 & x>2 \end{cases}$




Check the continuity at $x=2$.

$\Rightarrow$

1st Check

$f(2)=5$ ; defined

2nd Check

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2$$

$$= 4$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} -x+6$$

$$= -2+6$$

$$= 4$$

$\therefore$ L.H.L. = R.H.L.

$\therefore \lim_{x\to 2} f(x) = 4$ ; Limit exists

3rd Check

$$\lim_{x\to 2} f(x) \neq f(2)$$

Hence, the function is not continuous at $x=2$.

This is removable discontinuity.

If $f(x)$ and $g(x)$ are two continuous function.

then,

① $f(x) \pm g(x)$ also continuous at $x=a$

(ii) $f(x) \cdot g(x)$ also continuous at $x=a$

(iii) $\frac{f(x)}{g(x)}$ also continuous at $x=a$; $g(a) \neq 0$

if $g(a)=0$, then discontinuous at $x=a$.

Polynomial function continuous everywhere.

(*) Rational function $f(x) = \frac{P(x)}{Q(x)}$ also continuous everywhere.

; $Q(x) \neq 0$

if $Q(x) = 0$, on that point will be discontinuous.

(*) Find the value of $x$ at which the following function not continuous.

(i) $f(x) = x^4 - 7x^3 + 3x^2 + 2$

$\Rightarrow$ Polynomial function. so, there is no discontinuous point.

(ii) $f(x) = \frac{x+2}{x^2+4}$

$\Rightarrow$ $x^2+4$ is always positive and greater than 0.

So, there is no discontinous point.

(iii) $f(x) = \sqrt[3]{x-8}$

$\Rightarrow$ There is no discontinuous point.

(iv) $g(x) = \frac{x+2}{x^2-4}$

$\Rightarrow$ $x^2-4 \neq 0$

$x^2 \neq 4$

$x \neq \pm 2$

This function is discontinuous at $x = \pm 2$.

(v) $f(x) = \frac{x-3}{2x^2+x}$

$\Rightarrow$

$x(2x+1) \neq 0$

$x \neq 0$, $2x+1 \neq 0$

$x \neq -\frac{1}{2}$

$\therefore$ this function is discontinuous at $x = 0, -\frac{1}{2}$.

(vi) $f(x) = \frac{2x+1}{4x^2+4x+5}$

$= \frac{2x+1}{(2x+1)^2+4}$

There is no discontinuous point.

(*) Show that $f(x) = |x|$ continuous everywhere.

$\Rightarrow$ $$f(x) = |x|$$

$$= \begin{cases} x & \geq 0 \\ -x & < 0 \end{cases}$$

if $x \geq 0$, $f(x) = x$, polynomial. $\therefore$ continuous $(0, \infty)$

If $x < 0$, $f(x) = -x$, polynomial. $\therefore$ continuous $(-\infty, 0)$

Only possible discontinuity at $x=0$.

Now, check for $x=0$,

1st condition

$$f(x) = x$$
$$f(0) = 0 \text{ ; defined}$$

2nd Check

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} -x = 0$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x = 0$$

$\therefore$ L.H.L = R.H.L.

$\therefore \lim_{x \to 0} f(x) = 0$

Limit exist.

3rd Check

$$\lim_{x \to 0} f(x) = f(0)$$

Hence, this function is continuous at $x=0$. also

therefore,

the function is continuous everywhere.

(*) $f(x) = |3x-2|$

$\Rightarrow$

$$f(x) = \begin{cases} 3x-2 & 3x-2 \geq 0 \\ -(3x-2) & 3x-2 < 0 \end{cases}$$

$$= \begin{cases} 3x-2 & x \geq \frac{2}{3} \\ -3x+2 & x < \frac{2}{3} \end{cases}$$

if $x > \frac{2}{3}$, $f(x) = 3x-2$, polynomial; continuous $(\frac{2}{3}, \infty)$

if $x < \frac{2}{3}$, $f(x) = -3x+2$, Polynomial; continuous $(-\infty, \frac{2}{3})$

Only possible discontinuity at $x = \frac{2}{3}$

Now check for $x = \frac{2}{3}$

1st Check

$f(x) = 3x - 2$

$f(\frac{2}{3}) = 0$

2nd check

$\lim_{x \to \frac{2}{3}^-} f(x) = \lim_{x \to \frac{2}{3}^-} -3x+2$

$= 0$

$\lim_{x \to \frac{2}{3}^+} f(x) = \lim_{x \to \frac{2}{3}^+} 3x-2$

$= 0$

$\therefore$ L.H.L = R.H.L.

$\therefore \lim_{x \to \frac{2}{3}} f(x) = 0$

3rd Check

$\lim_{x \to 0} f(x) = f(\frac{2}{3})$

Hence the function is continuous at $x = \frac{2}{3}$ also.

Therefore,

the function is continuous everywhere.

Extra:

* $f(x) = |x-2|$
* $f(x) = x + |x-2|$

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* If $f(x) = \begin{cases} 2x-7 & x \geq 3 \\ kx^2 & x < 3 \end{cases}$

For what value of k, the function continuous everywhere.

$\Rightarrow$

1st Check

$f(3) = 2 \cdot 3 - 7 = -1$

2nd check

$\lim\limits_{x \to 3^+} 2x - 7 = -1$

$\lim\limits_{x \to 3^-} kx^2 = 9k$

if Limit exist, then L.H.L = R.H.L

$9k = -1$

$k = -\frac{1}{9}$

if $k = -\frac{1}{9}$, then limit exist.

$\therefore \lim\limits_{x \to 3} f(x) = -1 = f(3)$

$\therefore$ for $k = -\frac{1}{9}$ the function is continuous.

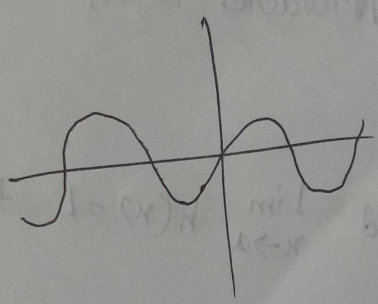
1.6 /

## Continuity for Trigonometry function

$f(x) = \sin x$ $\quad f(x) = \tan x$ $\quad f(x) = \sec x$

$f(x) = \cos x$ $\quad f(x) = \cot x$ $\quad f(x) = \operatorname{cosec} x$

⊛ For $\sin x$ and $\cos x$



for $\sin x$ and $\cos x$ function continuous everywhere.

$$\lim_{x \to c} \sin x = \sin c$$

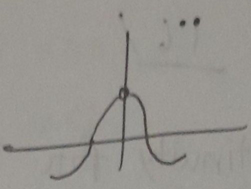
$$\lim_{x \to c} \cos x = \cos c$$

⊛ $\lim_{x \to c} \tan x = \tan c = \frac{\sin c}{\cos c}$ ; if $\cos c \neq 0$

$f(x) = \sec x = \frac{1}{\cos x}$

⊛ $\lim_{x \to c} \cot x = \cot c = \frac{\cos c}{\sin c}$ ; if $\sin c \neq 0$

* $\lim_{x \to 0} \frac{\sin x}{x} = 1$



## Squeezing Theorem

If, $f(x)$, $g(x)$ and $h(x)$ are function satisfying

$g(x) \leq f(x) \leq h(x)$, if $g(x)$ and $h(x)$ has some

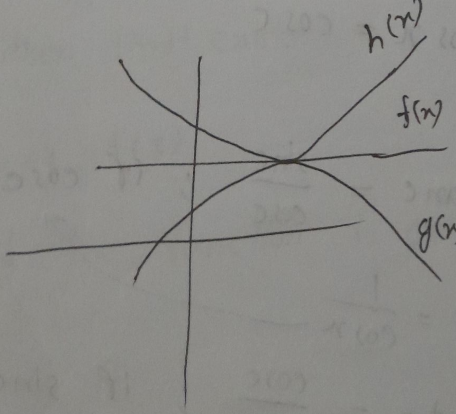
limit $L$ as $x$ approaches to $a$,

Say,

$\lim_{x \to a} g(x) = L$ and $\lim_{x \to a} h(x) = L$ then,

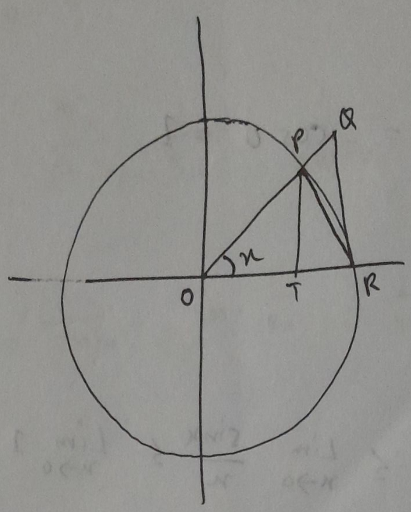
- Squeezing Theorem state that,

$$\lim_{x \to a} f(x) = L$$



Proov, $\lim_{x \to 0} \frac{\sin x}{x} = 1$

Let a unit circle, whose center at (0,0) and radiow 1.




area of the triange OPR $\leq$ are of sector OPR $\leq$ are of triange OQR

$$\frac{1}{2} \times OR \times PT \leq \frac{1}{2} r^2 \theta \leq \frac{1}{2} \times OR \times QR$$

$$\frac{1}{2} \times 1 \times \sin x \leq \frac{1}{2} \cdot 1 \cdot x \leq \frac{1}{2} \times 1 \times \tan x$$

$\frac{PT}{1} = \sin x$

$PT = \sin x$

$\frac{QR}{OR} = \tan x$

$QR = OR \tan x$

$= 1 \cdot \tan x$

$= \tan x$

$$\frac{\sin x}{2} \leq \frac{x}{2} \leq \frac{\tan x}{2}$$

$$\sin x \leq x \leq \tan x$$

$$1 \leq \frac{x}{\sin x} < \frac{1}{\cos x}$$

$$1 \geq \frac{\sin x}{x} \geq \cos x$$

$$\therefore \cos x \le \frac{\sin x}{x} \le 1$$

$$\therefore g(x) \le f(x) \le h(x)$$

Now,

$$\lim_{x \to 0} \cos x = \cos 0 = 1$$

$$\lim_{x \to 0} 1 = 1$$

$$\therefore \lim_{x \to 0} \cos x \le \lim_{x \to 0} \frac{\sin x}{x} \le \lim_{x \to 0} 1$$

Hence, by squeezing theorem,

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

---

(*)

$$\lim_{x \to 0} \frac{\sin 5x}{5x} = 1$$

$$\lim_{x \to 0} \frac{x}{\sin x} = 1$$

$$\lim_{x \to 0} \frac{\sin 2t}{2t} = 1$$

$$\lim_{\theta \to 0} \frac{\sin 7\theta}{7\theta} = 1$$

$$\lim_{t \to 0} \frac{\tan t}{t} = \lim_{t \to 0} \frac{\sin t}{t} \cdot \cos t$$

$$= 1 \cdot \cos t$$

$$= \cos 0 = 1$$

(*)

$$\lim_{x\to 0} \frac{\tan 7x}{\sin 3x}$$

$$= \lim_{x\to 0} \frac{\sin 7x}{\cos 7x} \cdot \frac{1}{\sin 3x}$$

$$= \lim_{x\to 0} \frac{\sin 7x}{7x} \cdot 7x \cdot \frac{3x}{\sin 3x} \cdot \frac{1}{3x} \cdot \frac{1}{\cos 7x}$$

$$= \lim_{x\to 0} \frac{\sin 7x}{7x} \cdot \lim_{x\to 0} \frac{3x}{\sin 3x} \cdot 7x \cdot \frac{1}{3x} \cdot \lim_{x\to 0} \frac{1}{\cos 7x}$$

$$= \frac{7}{3} \cdot \left[1 \cdot 1 \cdot \frac{1}{\cos 0}\right]$$

$$= \frac{7}{3} [1 \cdot 1 \cdot 1]$$

$$= \frac{7}{3}$$ Ans

(*)

$$\lim_{x\to 0} \frac{1-\cos x}{x}$$

$$= \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x} \cdot \frac{1}{1+\cos x}$$

$$= \left(\lim_{x\to 0} \frac{\sin x}{x}\right)^2 \cdot \lim_{x\to 0} \frac{x}{1+\cos x}$$

$$= 1^2 \cdot \frac{0}{1+1} = 0$$ Ans

(*) $\lim_{x\to 0} \frac{\tan 3x^2 + \sin^2 5x}{x^2}$

$= \lim_{x\to 0} \frac{\tan 3x^2}{x^2} + \lim_{x\to 0} \frac{\sin^2 5x}{x^2}$

$= \lim_{x\to 0} \frac{\tan 3x^2}{3x^2} \cdot 3 + \lim_{x\to 0} \frac{\sin^2 5x}{(5x)^2} \cdot 25$

$= 1 \cdot 3 + \lim_{x\to 0} \left(\frac{\sin 5x}{5x}\right)^2 \cdot 25$

$= 3 + 1 \cdot 25$

$= 28$ Ans

(*) Discuss the Limit:

a) $\lim_{x\to 0} \sin \frac{1}{x}$  b) $\lim_{x\to 0} x \sin \frac{1}{x}$

$\Rightarrow$

a) $\lim_{x\to 0^+} \frac{1}{x} = +\infty$

$\lim_{x\to 0^-} \frac{1}{x} = -\infty$

$\lim_{x\to 0} \sin \frac{1}{x} = \text{DNE}$

b)

$$\lim_{x \to 0} x \sin\frac{1}{x}$$

we know that,

$$-1 \leq \sin\frac{1}{x} \leq 1$$

$$-x \leq x \sin\frac{1}{x} \leq x$$

$$\lim_{x \to 0^-} x = 0$$

$$\lim_{x \to 0^+} x = 0$$

Hence, by squeezing theorem

$$\lim_{x \to 0} x \sin\frac{1}{x} = 0.$$

c) $x^2 \cos\frac{1}{x}$

$$-1 \leq \cos\frac{1}{x} < 1$$

$$x^2 \leq x^2 \cos\frac{1}{x} \leq x^2$$

$$\lim_{x \to 0^-} x^2 = 0$$

$$\lim_{x \to 0^+} x^2 = 0$$

$$\therefore \lim_{x \to 0} x^2 \cos\frac{1}{x} = 0$$

$$\lim_{x \to \pi} \frac{\pi - x}{\sin x}$$

Let,

$$t = \pi - x$$

$$\therefore x = \pi - t$$

as $x \to \pi$ then $t \to 0$

$$\lim_{t \to 0} \frac{t}{\sin(\pi - t)} = \lim_{t \to 0} \frac{t}{\sin t} = 1$$

$$\lim_{x \to 0} e^{\sin x} = e^{\lim_{x \to 0} \sin x} = e^0 = 1$$

$$\lim_{x \to \infty} \cos(2 \tan^{-1} x) = \cos\left(\lim_{x \to \infty} 2 \tan^{-1} x\right)$$

$$= \cos\left(2 \times \frac{\pi}{2}\right)$$

$$= \cos \pi$$

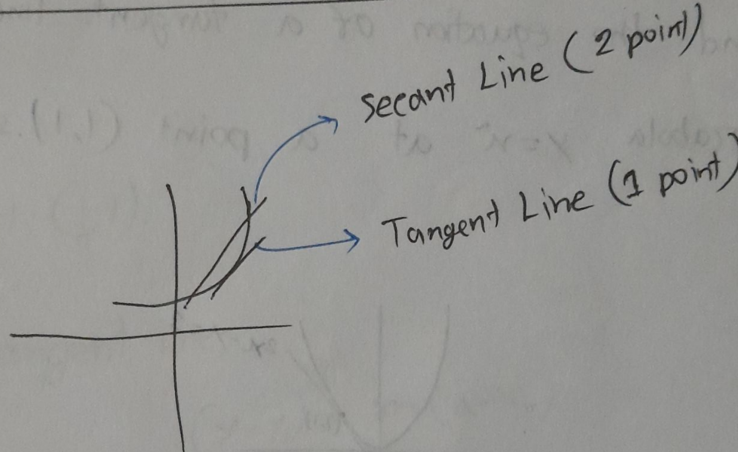
$$= -1$$

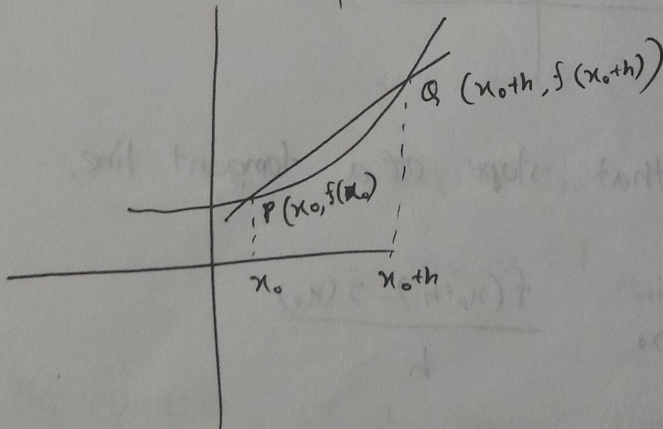
# Chapter - 2

## Derivative

### 2.1

### Tangent line and rate of change




Slope of PQ (secant line) on average rate of change.

$$m_{PQ} \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(x_0+h) - f(x_0)}{(x_0+h) - x_0}$$

$$= \frac{f(x_0+h) - f(x_0)}{h}$$