at,

$t = 0, \quad T = 85^\circ F$

$\therefore \; 85 = 70 + ce^{k \cdot 0}$

$\therefore c = 85 - 70 = 15$

(i) $\Rightarrow \quad T = 70 + 15e^{kt} \quad \cdots$ (ii)

at,

$t = 1$ hour, $\quad T = 80^\circ F$

$\therefore \; 80 = 70 + 15e^{k}$

$\Rightarrow \; e^{k} = \dfrac{80-70}{15} = \dfrac{2}{3}$

$\therefore k = \ln\left(\frac{2}{3}\right)$

(ii) $\Rightarrow \quad T = 70 + 15e^{\ln\left(\frac{2}{3}\right) \cdot t} \quad \cdots$ (iii)

Let,

t^* be the time when, $T = 98 \cdot 6^\circ F$

$\therefore \; 98 \cdot 6 = 70 + 15e^{\ln\left(\frac{2}{3}\right) t^*}$

$\Rightarrow \; e^{\ln\left(\frac{2}{3}\right) t^*} = \dfrac{98 \cdot 6 - 70}{15} = \dfrac{143}{75}$

$\therefore \; t^* = \dfrac{\ln\left(\frac{143}{75}\right)}{\ln\left(\frac{2}{3}\right)} \cong -1 \cdot 59$ hour

Therefore, body was found 1·57 hours after the death.

Total time elapsed $= 1\cdot 57 + 1 \approx 2\cdot 57$ hours.

Ans

$\boxed{\text{H.W}}$ ⇒ from Lecture-9

Higher Order ODE with constant coefficient

1|

$$(D^2 - 2D - 3)y = 0$$

⇒

$\therefore$ A.E. ⇒ $m^2 - 2m - 3 = 0$

$$\Rightarrow m^2 - 3m + m - 3 = 0$$

$$\Rightarrow m(m-3) + 1(m-3) = 0$$

$$\Rightarrow (m-3)(m+1) = 0$$

$$\therefore m = -1, 3$$

$\therefore$ G.S. ⇒ $y = c_1 e^{-x} + c_2 e^{3x}$

Ans

2)

$$(D^3 - D)y = 0$$

$\therefore$ A.E. $\Rightarrow$

$$m^3 - m = 0$$

$$\Rightarrow m(m^2 - 1) = 0$$

$$\Rightarrow m(m+1)(m-1) = 0$$

$$\therefore m = 0, -1, 1$$

$\therefore$ G.S. $\Rightarrow$

$$y = c_1 e^{0.x} + c_2 e^{-x} + c_3 e^{x}$$

$$= c_1 + c_2 e^{-x} + c_3 e^{x}$$

3)

$$(D^2 + 1)y = 0$$

$\therefore$ A.E. $\Rightarrow$

$$m^2 + 1 = 0$$

$$\therefore m = \frac{-0 \pm \sqrt{0-4}}{2} = \frac{\pm\sqrt{-4}}{2} = \pm \frac{2}{2} i$$

$$= \pm i$$

$\therefore$ G.S. $\Rightarrow$

$$y = e^{0.x} [A \cos x + B \sin x]$$

$$= A \cos x + B \sin x$$

4)

$$(D^4-1)y=0$$

$\therefore$ A.E. $\Rightarrow$

$$m^4-1=0$$

$$\cancel{m^4=1}$$

$$\Rightarrow (m^2)^2-1=0$$

$$\Rightarrow (m^2+1)(m^2-1)=0$$

$$\Rightarrow (m^2+1)(m+1)(m-1)=0$$

$$\therefore m=-1,1,\pm i$$

$\therefore$ G.S. $\Rightarrow$

$$y=c_1e^{-x}+c_2e^{x}+A\cos x+B\sin x$$

5)

$$(D^3-5D^2+7D-2)y=0$$

$\therefore$ A.E. $\Rightarrow$

$$m^3-5m^2+7m-2=0$$

$$\Rightarrow m^3-2m^2-3m^2+6m+m-2=0$$

$$\Rightarrow m^2(m-2)-3m(m-2)+(m-2)=0$$

$$\Rightarrow (m-2)(m^2-3m+1)=0$$

$m-2=0$

$m=2$

$m^2-3m+1=0$

$$m=\frac{+3\pm\sqrt{9-4}}{2}=\frac{3\pm\sqrt{5}}{2}$$

$$\therefore m=2,\ \frac{3+\sqrt{5}}{2},\ \frac{3-\sqrt{5}}{2}$$

$\therefore$ G.S. $\Rightarrow$

$$y = c_1 e^{2x} + c_2 e^{\frac{3+\sqrt{5}}{2}.x} + c_3 e^{\frac{3-\sqrt{5}}{2}.x}$$

Ans

6|

$$(D^4 + 2D^2 + 1)\, y = 0$$

$\therefore$ A.E. $\Rightarrow$

$$m^4 + 2m^2 + 1 = 0$$

$$\Rightarrow (m^2+1)^2 = 0$$

$$\Rightarrow (m^2+1)(m^2+1) = 0$$

$m^2+1 = 0$	$m^2+1=0$
$m = \pm i$	$m = \pm i$

$\therefore m = \pm i, \pm i$

$\therefore$ G.S. $\Rightarrow$ $A\cos x + B\sin x + Cx\cos x + Dx\sin x$

[Confused]

Solved correct according to MBC.

$\boxed{\text{H.W.}}$ ⇒ from Lecture - 10

⊛ from MKC Book (Page - 107, Example)

4·5/

$$(D^2 - 3D + 2)y = e^x$$

Already Solved in Class

4·6/

$$(D^2 - 7D + 12)y = 2^x \quad \cdots (i)$$

A.E ⇒ $m^2 - 7m + 12 = 0$

$\Rightarrow m^2 - 4m - 3m + 12 = 0$

$\Rightarrow m(m-4) - 3(m-4) = 0$

$\Rightarrow (m-4)(m-3) = 0$

$\therefore m = 3, 4$

$\therefore Y_c = c_1 e^{3x} + c_2 e^{4x}$

(i) ⇒ $Y_p = \frac{1}{(D^2 - 7D + 12)} \cdot 2^x = \frac{1}{D^2 - 7D + 12} \cdot e^{x \ln 2}$

$$= \cancel{\frac{1}{12\left(1 + \frac{D^2 - 7D}{12}\right)} e^{x \ln 2}}$$

$$= \cancel{\frac{1}{12}}$$

$$= \frac{1}{(\ln 2)^2 - 7\cdot\ln 2 + 12} \cdot e^{x\ln 2}$$

$\therefore$ G.S. $\Rightarrow$ $y = y_c + y_p$

$$= c_1 e^{3x} + c_2 e^{4x} + \frac{e^{x\ln 2}}{(\ln 2)^2 - 7\cdot\ln 2 + 12}$$

4.71

$$(D^2 + 2D + 1)y = 2x + x^2 \quad \cdots (i)$$

A.E. $\Rightarrow$

$$m^2 + 2m + 1 = 0$$

$$\Rightarrow (m+1)^2 = 0$$

$$\Rightarrow (m+1)(m+1) = 0$$

$$\therefore m = -1, -1$$

$$\therefore y_c = c_1 e^{-x} + c_2 x e^{-x}$$

(i) $\Rightarrow$ $y_p = \frac{1}{D^2 + 2D + 1} \cdot (x^2 + 2x)$

$$= \frac{1}{(1 + D^2 + 2D)} (x^2 + 2x)$$

$$= (1 + D^2 + 2D)^{-1} (x^2 + 2x)$$

$$= \left(1 - (D^2+2D) + (D^2+2D)^2 - \cdots\right)(x^2+2x)$$

$$= \left[1 - (D^2+2D) + (D^4+4D^3+4D^2) - \cdots\right](x^2+2x)$$

x^2+2x

$D = 2x+2$

$D^2 = 2$

$$= x^2+2x - (2+4x+4) + (0+0+8)$$

$$= x^2+2x-2-4x-4+8$$

$$= x^2-2x+2$$

$\therefore$ G.S. $\Rightarrow$ $y = y_c + y_p$

$$= c_1 e^{-x} + c_2 x e^{-x} + x^2 - 2x + 2$$

Ans

$$(D^2+4D-2)y = 2x^2-3x+6$$

A.E. $\Rightarrow$

$$m^2+4m-2=0$$

$$\Rightarrow m = \frac{-4 \pm \sqrt{16+8}}{2}$$

$$= \frac{-4 \pm \sqrt{24}}{2}$$

$$= \frac{-4 \pm 2\sqrt{6}}{2}$$

$$= -2 \pm \sqrt{6}$$

$\therefore m = -2+\sqrt{6},\ -(2+\sqrt{6})$

$$\therefore y_c = c_1 e^{(-2+\sqrt{6})x} + c_2 e^{-(2+\sqrt{6})x}$$

$$\therefore y_p = \frac{1}{D^2+4D-2}(2x^2-3x+6)$$

$$= \frac{1}{-2\left(1-\frac{D^2+4D}{2}\right)}(2x^2-3x+6)$$

$2x^2-3x+6$

$D = 4x-3$

$D^2 = 4$

$$= -\frac{1}{2}\left(1-\frac{D^2+4D}{2}\right)^{-1}(2x^2-3x+6)$$

$$= -\frac{1}{2}\left[1+\frac{D^2+4D}{2}+\left(\frac{D^2+4D}{2}\right)^2+\cdots\right](2x^2-3x+6)$$

$$= -\frac{1}{2}\left[1+\frac{D^2+4D}{2}+\frac{D^4+8D^3+16D^2}{4}+\cdots\right](2x^2-3x+6)$$

$$= -\frac{1}{2}\left[2x^2-3x+6+\frac{4+16x-12}{2}+\frac{0+0+64}{4}\right]$$

$$= -x^2+\frac{3}{2}x-3-4x+2-8$$

$$= -x^2-\frac{5}{2}x-9$$

G. S. $\Rightarrow$

$$y = y_c + y_p$$

$$= c_1 e^{(-2+\sqrt{6})x} + c_2 e^{-(2+\sqrt{6})x} - x^2 - \frac{5}{2}x - 9$$

Ans

(*) $(D^2-5D+4)y = 8e^x$

A.E. $\Rightarrow$

$$m^2-5m+4=0$$

$$\Rightarrow m^2-4m-m+4=0$$

$$\Rightarrow m(m-4)-1(m-4)=0$$

$$\Rightarrow (m-4)(m-1)=0$$

$$\therefore m = 1,4$$

$$\therefore y_c = c_1e^x + c_2e^{4x}$$

$$y_p = \frac{1}{D^2-5D+4} \cdot 8e^x$$

$$= e^x \frac{1}{(D+1)^2-5(D+1)+4} 8$$

$$= e^x \frac{1}{D^2+2D+1-5D-5+4} \cdot 8$$

$$= e^x \frac{1}{D^2-3D} \cdot 8$$

$$= e^x \frac{1}{-3D(1-\frac{D}{3})} 8$$

$$= e^x \left(-\frac{1}{3D}\right)\left(1-\frac{D}{3}\right)^{-1} \cdot 8$$

$$= e^x \left(-\frac{1}{3D}\right)[1+\cdots] 8$$

$$y_p = \frac{1}{D^2-5D+4} 8e^x$$

$$= \frac{x}{2D-5} 8e^x$$

$$= \frac{x}{-3} 8e^x$$

$$= -\frac{8}{3}xe^x$$

$$= e^x \left(-\frac{1}{3D}\right) \cdot 8$$

$$= e^x \left(-\frac{1}{3} \cdot 8x\right)$$

$$= -\frac{8x}{3} e^x$$

$\therefore$ G.S. $\Rightarrow$

$$y = y_c + y_p$$

$$= c_1 e^x + c_2 e^{4x} - \frac{8x}{3} e^x$$

Ans.

$\boxed{\text{H.W.}}$ $\Rightarrow$ from Lecture -11

from MKC Book

Page : 109

Example :

4·8

$$(D^2 + 5D + 6) y = \sin 2x \quad \ldots\ldots (i)$$

A.E $\Rightarrow$

$$m^2 + 5m + 6 = 0$$

$$\Rightarrow m^2 + 3m + 2m + 6 = 0$$

$$\Rightarrow m(m+3) + 2(m+3) = 0$$

$$\Rightarrow (m+3)(m+2) = 0$$

$$\therefore m = -3, -2$$

$$\therefore y_c = c_1 e^{-3x} + c_2 e^{-2x}$$

(i) $\Rightarrow$ $y_p = \dfrac{1}{D^2+5D+6} \sin 2x$

$= \dfrac{1}{-2^2+5D+6} \sin 2x$

$= \dfrac{1}{5D+2} \sin 2x$

$= \dfrac{5D-2}{(5D)^2-2^2} \sin 2x$

$= \dfrac{5D-2}{25D^2-4} \sin 2x$

$= \dfrac{5D-2}{25(-2^2)-4} \sin 2x$

$= \dfrac{5D-2}{-104} \sin 2x$

$= -\dfrac{1}{104}\left(5 \cdot \cos 2x \cdot 2 - 2\sin 2x\right)$

$= -\dfrac{1}{104}\left(10\cos 2x - 2\sin 2x\right)$

$\therefore$ G.S. $\Rightarrow$

$y = y_c + y_p$

$= c_1 e^{-3x} + c_2 e^{-2x} - \dfrac{1}{104}\left(10\cos 2x - 2\sin 2x\right)$

Ans

4.71

$(D^2-5D+6)y = \sin(3x+2) \cdots (i)$

A.E. $\Rightarrow$

$m^2-5m+6=0$

$\Rightarrow (m-3)(m-2)=0$

$\therefore m=2,3$

$\therefore y_c = c_1e^{2x}+c_2e^{3x}$

(i) $\Rightarrow y_p = \frac{1}{D^2-5D+6}\sin(3x+2)$

$= \frac{1}{-3^2-5D+6}\sin(3x+2)$

$= -\frac{1}{5D+3}\sin(3x+2)$

$= -\frac{5D-3}{25D^2-9}\sin(3x+2)$

$= \frac{3-5D}{25(-3^2)-9}\sin(3x+2)$

$= \frac{3-5D}{-234}\sin(3x+2)$

$= -\frac{1}{234}(3\sin(3x+2)-5\cdot\cos(3x+2)\cdot 3)$

$= -\frac{1}{234}(3\sin(3x+2)-15\cos(3x+2))$

$\therefore$ G.S. $\Rightarrow$

$y = y_c+y_p$

$= c_1e^{2x}+c_2e^{3x}-\frac{1}{234}(3\sin(3x+2)$
$-15\cos(3x+2))$

Ans

4.10/

$$\frac{d^2y}{dx^2} + 4y = \sin 2x$$

$$\Rightarrow (D^2+4)y = \sin 2x \quad \cdots (i)$$

A.E. $\Rightarrow$

$$m^2+4=0$$

$$m^2=-4$$

$$m = \pm\sqrt{-4}$$

$$= \pm 2i$$

$\therefore$ $y_c = A\cos 2x + B\sin 2x$

(i) $\Rightarrow$ $y_p = \dfrac{1}{D^2+4}\sin 2x$

$$\boxed{= \frac{1}{-2^2+4}\sin 2x} \text{ [failure case]}$$

$$= \frac{x}{2}\int \sin 2x\, dx$$

$$= -\frac{x}{4}\cos 2x$$

$\therefore$ G.S. $\Rightarrow$

$$y = y_c + y_p$$

$$= A\cos 2x + B\sin 2x - \frac{x}{4}\cos 2x$$

Ans

4.11/

$$(D^2+2D+4)y = e^x \cos 2x \quad \cdots (i)$$

A.E. $\Rightarrow$ $m^2 + 2\cancel{0}m + 4 = 0$

$$m = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2\sqrt{3}i}{2}$$

$$= -1 \pm \sqrt{3}i$$

$$\therefore y_c = e^{-x}\left[A\cos\sqrt{3}x + B\sin\sqrt{3}x\right]$$

(i) $\Rightarrow$ $$y_p = \frac{1}{D^2+2D+4} e^x \cos 2x$$

$$= e^x \frac{1}{(D+1)^2 + 2(D+1) + 4} \cos 2x$$

$$= e^x \frac{1}{D^2+2D+1+2D+2+4} \cos 2x$$

$$= e^x \frac{1}{D^2+4D+\cancel{6}7} \cos 2x$$

$$= e^x \frac{1}{-2^2+4D+\cancel{6}7} \cos 2x$$

$$= e^x \frac{1}{4D+3} \cos 2x$$

$$= e^x \frac{4D-3}{16D^2-\cancel{4}9} \cos 2x$$

$$= e^x \frac{4D-3}{16(-2^2)-\cancel{4}9} \cos 2x$$

$$= e^x \frac{4D-3}{-\cancel{68}73} \cos 2x$$

$$= -\frac{e^x}{73\cancel{68}}\left(-4.\sin 2x \cdot 2 - 3\cos 2x\right)$$

$$= \frac{e^x}{\cancel{68}\,73}\left(8\sin 2x + 3\cos 2x\right)$$

$\therefore$ G.S. $\Rightarrow$

$$y = y_c + y_p$$

$$= e^{-x}\left[A\cos\sqrt{3}x + B\sin\sqrt{3}x\right] + \frac{e^x}{\cancel{68}\,73}\left(8\sin 2x + 3\cos 2x\right)$$

Ans

4.15

$$(D^2-4D+13)y=0 \quad ; \quad y(0)=-1, \; Dy(0)=2$$

A.E. $\Rightarrow$

$$m^2-4m+13=0$$

$$m=\frac{+4\pm\sqrt{16-52}}{2}$$

$$=\frac{4\pm\sqrt{-36}}{2}$$

$$=\frac{4\pm 6i}{2}$$

$$=2\pm 3i$$

$\therefore$ G.S. $\Rightarrow$

$$y=e^{2x}\left[A\cos 3x+B\sin 3x\right] \quad \ldots (i)$$

Given,

$$y(0)=-1$$

$\therefore (i) \Rightarrow$ $$-1=e^{0}\left[A\cos 0+B\sin 0\right]$$

$$\Rightarrow -1=A$$

$$\therefore A=-1$$

$$\therefore y=e^{2x}\left[-\cos 3x+B\sin 3x\right] \quad \ldots (ii)$$

$$\therefore Dy=e^{2x}\left(3\sin 3x+3B\cos 3x\right)+2e^{2x}\left(-\cos 3x+B\sin 3x\right) \quad \ldots (iii)$$

Given,

$$Dy(0)=2$$

(iii) $\Rightarrow$

$$2 = e^0(3\sin 0 + 3B\cos 0) + 2e^0(-\cos 0 + B\sin 0)$$

$$\Rightarrow 2 = 3B - 2$$

$$\Rightarrow 3B = 4$$

$$\therefore B = \frac{4}{3}$$

$$\therefore y = e^{2x}\left(-\cos 3x + \frac{4}{3}\sin 3x\right)$$

4·16/

$$(D^2 - 6D + 25)y = 0 \quad ; \quad y(0) = -3, \quad y'(0) = -1$$

A.E. $\Rightarrow$

$$m^2 - 6m + 25 = 0$$

$$m = \frac{6 \pm \sqrt{36-100}}{2}$$

$$= \frac{6 \pm \sqrt{-64}}{2}$$

$$= \frac{6 \pm 8i}{2}$$

$$= 3 \pm 4i$$

$$\therefore y = e^{3x}\left[A\cos 4x + B\sin 4x\right] \quad \cdots \text{(i)}$$

Given, $y(0) = -3$

(i) $\Rightarrow$ $-3 = e^0\left[A\cos 0 + B\sin 0\right]$

$$\Rightarrow -3 = A$$

$\therefore$ (i) $\Rightarrow$ $y = e^{3x}[-3\cos 4x + B\sin 4x]$ ....(ii)

$y' = e^{3x}(12\sin 4x + 4B\cos 4x) + 3e^{3x}(-3\cos 4x + B\sin 4x)$

Given, $y'(0) = -1$

$\Rightarrow -1 = e^0(12\sin 0 + 4B\cos 0) + 3\cdot e^0(-3\cos 0 + B\sin 0)$

$\Rightarrow -1 = 4B - 9$

$\Rightarrow 4B = 8$

$\therefore B = 2$

$\therefore y = e^{3x}(-3\cos 4x + 2\sin 4x)$

Ans.

4.17/

$(D^3 - 6D^2 + 9D)y = 0$; $y(0) = 0$, $y'(0) = 2$; $y''(0) = -6$

A.E. $\Rightarrow$ $m^3 - 6m^2 + 9m = 0$

$\Rightarrow m(m^2 - 6m + 9) = 0$

$\Rightarrow m(m-3)^2 = 0$

$\therefore m = 0, 3, 3$

$\therefore y = c_1 e^0 + c_2 e^{3x} + c_3 x e^{3x}$ ....(i)

Given, $y(0) = 0$

$\therefore 0 = c_1 + c_2 e^0 + 0$

$\Rightarrow 0 = c_1 + c_2$

$\therefore C_2 = -C_1$

$\therefore$ (i) $\Rightarrow$

$$Y = C_1 - C_1 e^{3x} + C_3 x e^{3x} \quad \text{--- (ii)}$$

$$Y' = -3C_1 e^{3x} + 3C_3 x e^{3x} + C_3 e^{3x} \quad \text{--- (iii)}$$

Given

$$Y'(0) = 2$$

$$\therefore \quad 2 = -3C_1 e^0 + 3C_3 \cdot 0 \cdot e^0 + C_3 e^0$$

$$\Rightarrow \quad 2 = -3C_1 + C_3$$

$\cancel{\therefore C_1}$

$\cancel{\therefore C_3 = 2 + 3C_1}$ $\therefore C_3 = 2 + 3C_1$

(iii) $\Rightarrow$

$\cancel{Y' = C_1 - C_1 e^{3x}}$

$$Y' = -3C_1 e^{3x} + 3(2+3C_1) x e^{3x} + (2+3C_1) e^{3x}$$

$$Y'' = -9C_1 e^{3x} + (6+9C_1) x \cdot e^{3x} \cdot 3 + (6+9C_1) e^{3x} + 3(2+3C_1) e^{3x}$$

$$Y'' = -9C_1 e^{3x} + (18+27C_1) x e^{3x} + (6+9C_1) e^{3x} + (6+9C_1) e^{3x}$$

Given $Y''(0) = -6$

$$\Rightarrow -6 = -9C_1 e^0 + (18+27C_1) \cdot 0 + (6+9C_1) e^0 + (6+9C_1) e^0$$

$$-6 = -9c_1 + 6 + 9c_1 + 6 + 9c_1$$

$$\Rightarrow -6 = 9c_1 + 12$$

$$\Rightarrow 9c_1 = -18$$

$$\therefore c_1 = -2$$

$$\therefore c_2 = 2$$

$$\therefore c_3 = 2 + 3(-2) = 2 - 6 = -4$$

$$\therefore y = -2 + 2e^{3x} - 4xe^{3x}$$

$\boxed{\text{H.W.}} \Rightarrow$ from Lecture 12

$$(D^2+1)y = x\cos x - \cos x \quad \cdots (i)$$

A.E. $\Rightarrow$ $m^2+1=0$

$m = \pm j$

$\therefore y_c = A\cos x + B\sin x$

$\therefore y_1 = \cos x$

$\therefore y_2 = \sin x$

$\therefore W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$

$= \cos^2 x + \sin^2 x = 1 \neq 0$

$\therefore (i) \Rightarrow$

$$y_p = \frac{1}{D^2+1} x\cos x - \frac{1}{D^2+1}\cos x$$

$$= x\frac{1}{D^2+1}\cos x - \frac{2D}{(D^2+1)^2}\cos x - \frac{1}{D^2+1}\cos x$$

$= \cancel{x}$ $-1+1$

$$= x\cdot\frac{x}{2}\int \cos x\, dx - \frac{2D}{D^4+2D^2+1}\cos x - \frac{x}{2}\int \cos x\, dx$$

$$= \frac{x^2}{2}\sin x - \frac{2D}{D^4+2D^2+1}\cos x - \frac{x}{2}\sin x$$

$$= \frac{x^2}{2}\sin x - \frac{x}{2}\sin x - (2D)\frac{x}{2}\int \cos x\, dx$$

$$= \frac{x^2}{2}\sin x - \frac{x}{2}\sin x - (2D)\frac{x}{2}\sin x$$

$$= \frac{x^2}{2}\sin x - \frac{x}{2}\sin x - 2\left(\frac{x\cos x}{2} + \frac{\sin x}{2}\right)$$

$=$

$$= \cancel{\frac{x^2}{4}\sin x - \frac{x}{2}\sin x} -$$

$$= \frac{x^2}{2}\sin x - \frac{x}{2}\sin x - x\cos x - \sin x$$

$$(D^2+1)y = x\cos x - \cos x$$

A.E. $\Rightarrow$

$$m^2+1 = 0$$

$$m^2 = -1$$

$$m = \pm i$$

$$\therefore y_c = A\cos x + B\sin x$$

$$y_p = \frac{1}{D^2+1} x\cos x - \frac{1}{D^2+1}\cos x$$

$$= x\frac{1}{D^2+1}\cos x - \frac{2D}{(D^2+1)^2}\cos x - \frac{1}{D^2+1}\cos x$$

$$= x\cdot\frac{x}{2}\int \cos x\,dx - (2D)\left(\frac{x}{2}\int \cos x\,dx\right) - \frac{x}{2}\int \cos x\,dx$$

$$= \frac{x^2}{2}\sin x - (2D)\left(\frac{x}{2}\sin x\right) - \frac{x}{2}\sin x$$

$$= \frac{x^2}{2}\sin x - D(x\sin x) - \frac{x}{2}\sin x$$

$$= \frac{x^2}{2}\sin x - (x\cos x + \sin x) - \frac{x}{2}\sin x$$

$$y_p = \frac{x^2}{2}\sin x - x\cos x - \sin x - \frac{x}{2}\sin x$$

$\therefore$ G.S. $\Rightarrow$

$$y = y_c + y_p = A\cos x + B\sin x + \frac{x^2}{2}\sin x - x\cos x - \sin x - \frac{x}{2}\sin x$$

Ans.

(*) From MKC Books:

Example 4.13/ $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = x^2 e^{3x} \cos 2x$

$$\Rightarrow \left(D^2 - 6D + 9\right)y = x^2 e^{3x} \cos 2x \quad \cdots (i)$$

A.E. $\Rightarrow$

$$m^2 - 6m + 9 = 0$$

$$(m-3)^2 = 0$$

$$\therefore m = 3, 3$$

$$\therefore y_c = c_1 e^{3x} + c_2 x e^{3x}$$

(i) $\Rightarrow y_p = \frac{1}{D^2 - 6D + 9} x^2 e^{3x} \cos 2x$

$$= e^{3x} \frac{1}{(D+3)^2 - 6(D+3) + 9} x^2 \cos 2x$$

$$= e^{3x} \frac{1}{D^2 + 6D + 9 - 6D - 18 + 9} x^2 \cos 2x$$

$$= e^{3x} \frac{1}{D^2} x^2 \cos 2x$$

$$= e^{3x} \left(\text{R.P. of } \frac{1}{D^2} x^2 e^{i2x} \right)$$

$$= e^{3x} \left(\text{R.P. of } e^{2ix} \frac{1}{(D+2i)^2} x^2 \right)$$

$$= e^{3x}\left(\text{R.P. of } e^{2ix}\frac{1}{D^2+4iD+4(-1)}x^2\right)$$

$$= e^{3x}\left(\text{R.P. of } e^{2ix}\frac{1}{D^2+4iD-4}x^2\right)$$

$$= e^{3x}\left(\text{R.P. of } e^{i2x}\cdot\frac{1}{-4}\left(1-\frac{D^2+4iD}{4}\right)^{-1}x^2\right)$$

$$= e^{3x}\left[\text{R.P. of } e^{i2x}\cdot\frac{1}{-4}\left(1+\frac{D^2+4iD}{4}+\left(\frac{D^2+4iD}{4}\right)^2+\cdots\right)x^2\right]$$

$$= e^{3x}\left[\text{R.P. of } e^{i2x}\cdot\frac{1}{-4}\left(1+\frac{D^2+4iD}{4}+\frac{D^4+8iD^3+16(-1)D^2}{16}+\cdots\right)x^2\right]$$

$$= e^{3x}\left[\text{R.P. of } e^{i2x}\cdot\frac{1}{-4}\left(x^2+\frac{2+4i\cdot 2x}{4}+\frac{0+0-16\cdot 2}{16}\right)\right]$$

$$= e^{3x}\left[\text{R.P. of } \frac{1}{-4}(\cos 2x+i\sin 2x)\left(x^2+\frac{1}{2}+2ix-2\right)\right]$$

$$= e^{3x}\left[\text{R.P. of } \frac{1}{-4}(\cos 2x+i\sin 2x)\cancel{\left(-\frac{1}{2}+2ix\right)}\left(x^2+2ix-\frac{3}{2}\right)\right]$$

$$\cancel{= e^{3x}\left[\text{R.P. of } \frac{1}{-4}\left(-\frac{1}{2}\cos 2x+2ix\cos 2x-\frac{1}{2}i\sin 2x-2x\sin 2x\right)\right]}$$

$$\cancel{= \frac{e^{3x}}{-4}\left(-\frac{1}{2}\cos 2x-2x\sin 2x\right)}$$

$$\cancel{= \frac{e^{3x}}{4}\left(\frac{1}{2}\cos 2x+2x\sin 2x\right)}$$

$$= e^{3x}\left[\text{R.P. of } \frac{1}{-4}\left(x^2\cos 2x+ix^2\sin 2x+2ix\cos 2x-2x\sin 2x-\frac{3}{2}\cos 2x-\frac{3}{2}i\sin 2x\right)\right]$$

$$y_p = -\frac{e^{3x}}{4}\left(x^2\cos 2x-2x\sin 2x-\frac{3}{2}\cos 2x\right)$$

∴ G.S. ⇒

$$y = y_c + y_p$$
$$= c_1 e^{3x} + c_2 x e^{3x} - \frac{e^{3x}}{4}\left(x^2 \cos 2x - 2x \sin 2x - \frac{3}{2}\cos 2x\right)$$

Ans

From page 122!

7|

$$(D^4 - 1)y = \sin x \quad \cdots (i)$$

A.E. ⇒

$$m^4 - 1 = 0$$
$$(m^2+1)(m^2-1) = 0$$
$$m = \pm 1, \pm i$$

$$\therefore y_c = c_1 e^{x} + c_2 e^{-x} + A\cos x + B\sin x$$

(i) ⇒

$$y_p = \frac{1}{D^4 - 1}\sin x$$
$$= \frac{x}{2}\int \sin x \, dx$$
$$= -\frac{x}{2}\cos x$$

∴ G.S. ⇒

$$y = y_c + y_p = c_1 e^{x} + c_2 e^{-x} + A\cos x + B\sin x - \frac{x}{2}\cos x$$

Ans

8|

$$(D^2 - 4D + 4)y = x^3 e^{2x} \quad \cdots \text{(i)}$$

A.E. $\Rightarrow$

$$m^2 - 4m + 4 = 0$$

$$(m-4)^2 = 0$$

$$m = 4, 4$$

$$\therefore Y_c = C_1 e^{4x} + C_2 x e^{4x}$$

(i) $\Rightarrow$ $$Y_p = \frac{1}{D^2 - 4D + 4} x^3 e^{2x}$$

$$= e^{2x} \frac{1}{(D+2)^2 - 4(D+2) + 4} x^3$$

$$= e^{2x} \frac{1}{D^2 + 4D + 4 - 4D - 8 + 4} x^3$$

$$= e^{2x} \frac{1}{D^2} x^3$$

$$= e^{2x} \frac{1}{D} \frac{x^4}{4}$$

$$= e^{2x} \frac{x^5}{20}$$

$\therefore$ G.S. $\Rightarrow$

$$Y = Y_c + Y_p = C_1 e^{4x} + C_2 x e^{4x} + e^{2x} \frac{x^5}{20}$$

Ans

9]

$$(D^3 - D)y = e^x x \quad \cdots (i)$$

A.E. $\Rightarrow$

$$m^3 - m = 0$$

$$\Rightarrow m(m^2-1) = 0$$

$$m(m+1)(m-1) = 0$$

$$\therefore m = -1, 0, 1$$

$$\therefore y_c = c_1 + c_2 e^{-x} + c_3 e^{x}$$

(i) $\Rightarrow$ $y_p = \dfrac{1}{D^3 - D} e^x x$

$$= e^x \frac{1}{(D+1)^3 - (D+1)} x$$

$$= e^x \frac{1}{D^3 + 3D^2 + 3D + 1 - D - 1} x$$

$$= e^x \frac{1}{D^3 + 3D^2 + 2D} x$$

$$= e^x \cdot \frac{1}{2D}\left(1 + \frac{D^2}{2} + \frac{3D}{2}\right)^{-1} x$$

$$= e^x \cdot \frac{1}{2D}\left[1 - \left(\frac{D^2}{2} + \frac{3D}{2}\right) + \cdots\right] x$$

$$= \frac{e^x}{2} \cdot \frac{1}{D}\left(x - \frac{3}{2}\right)$$

$$= \frac{e^x}{2} \cdot \left(\frac{x^2}{2} - \frac{3}{2}x\right)$$

$\therefore$ G.S. $\Rightarrow$

$$y = y_c + y_p$$

$$= c_1 + c_2 e^{-x} + c_3 e^{x} + \frac{e^x}{2}\left(\frac{x^2}{2} - \frac{3}{2}x\right)$$

10

$$(D^2-6D+13)y = 8e^{3x}\sin 4x \quad \cdots (i)$$

A.F. ⇒

$$m^2-6m+13=0$$

$$m = \frac{6\pm\sqrt{36-52}}{2} = \frac{6\pm\sqrt{-16}}{2} = \frac{6\pm 4i}{2}$$

$$= 3\pm 2i$$

$$\therefore y_c = e^{3x}[A\cos 2x + B\sin 2x]$$

(i) ⇒

$$y_p = \frac{1}{D^2-6D+13}\cdot 8e^{3x}\sin 4x$$

$$= e^{3x}\frac{1}{(D+3)^2-6(D+3)+13}8\sin 4x$$

$$= e^{3x}\frac{1}{D^2+6D+9-6D-18+13}8\sin 4x$$

$$= e^{3x}\frac{1}{D^2+4}8\sin 4x$$

$$= e^{3x}\frac{1}{-4^2+4}8\sin 4x$$

$$= e^{3x}\frac{8\sin 4x}{-12} = -\frac{2e^{3x}}{3}\sin 4x$$

$$\therefore y = e^{3x}[A\cos x + B\sin x] - \frac{2e^{3x}}{3}\sin 4x$$

(4)

$$(D^2+1)y = x\cos x - \cos x$$

A.E. $\Rightarrow$ $m^2+1=0$

$m = \pm 1$ $\therefore Y_c = A\cos x + B\sin x$

$$Y_p = \frac{1}{D^2+1}x\cos x - \frac{1}{D^2+1}\cos x \qquad \frac{x}{2}\int \cos x$$

$$= x\frac{1}{D^2+1}\cos x - \frac{2D}{(D^2+1)^2}\cos x - x\frac{1}{2D}\cos x$$

$$= x^2\frac{1}{2D}\cos x - \frac{2D}{D^4+2D^2+1}\cos x - \frac{x}{2}\sin x$$

$$= \frac{x^2}{2}\sin x - 2Dx\frac{1}{4D^3+4D}\cos x - \frac{x}{2}\sin x$$

$$= \frac{x^2}{2}\sin x - 2Dx^2\frac{1}{12D^2+4}\cos x - \frac{x}{2}\sin x$$

$$= \frac{x^2}{2}\sin x - 2Dx^2\frac{1}{-12+4}\cos x - \frac{x}{2}\sin x$$

$$\cancel{= \frac{x^2}{2}\sin x - \frac{x^2}{4}\sin x} - \frac{x}{2}\sin x$$

$$= \frac{x^2}{2}\sin x + \frac{1}{4}(2x\cos x - x^2\sin x) - \frac{x}{2}\sin x$$

$$= \frac{x^2}{2}\sin x + \frac{1}{2}x\cos x - \frac{x^2}{4}\sin x - \frac{x}{2}\sin x$$

$$= \frac{1}{4}x^2\sin x + \frac{1}{2}x\cos x - \frac{1}{2}x\sin x$$

111

$$(D^2-2D+1)y = xe^x \sin x \quad \cdots (i)$$

A.E. $\Rightarrow$

$$m^2 - 2m + 1 = 0$$

$$(m-1)^2 = 0$$

$$\therefore m = 1, 1$$

$$\therefore y_c = c_1 e^x + c_2 x e^x$$

(i) $\Rightarrow$ $$y_p = \frac{1}{D^2-2D+1} x e^x \sin x$$

$$= e^x \frac{1}{(D+1)^2 - 2(D+1) + 1} x \sin x$$

$$= e^x \frac{1}{D^2+2D+1-2D-2+1} x \sin x$$

$$= e^x \frac{1}{D^2} x \sin x$$

$$= e^x \left[x \cdot \frac{1}{D^2} \sin x - \frac{2D}{(D^2)^2} \sin x \right]$$

$$= e^x \left[-x\sin x - 2D(\sin x) \right]$$

$$= e^x (-x\sin x - 2\cos x)$$

$\therefore$ G.S. $\Rightarrow$ $$y = y_c + y_p = c_1 e^x + c_2 x e^x + e^x(-x\sin x - 2\cos x)$$

Ans

121

$$(D^2+1)y = x^2 \sin 2x \quad \cdots (i)$$

A.E. $\Rightarrow$

$$m^2+1=0$$

$$m = \pm i$$

$$\therefore y_c = A\cos x + B\sin x$$

(i) $\Rightarrow$ $$y_p = \frac{1}{D^2+1} x^2 \sin 2x$$

$$= \text{I.P. of } \frac{1}{D^2+1} x^2 e^{i2x}$$

$$= \text{''} \quad e^{i2x} \frac{1}{(D+i2)^2+1} x^2$$

$$= \text{''} \quad e^{i2x} \frac{1}{D^2+4iD-4+1} x^2$$

$$= \text{''} \quad e^{i2x} \frac{1}{D^2+4iD-3} x^2$$

$$= \text{''} \quad e^{i2x} \cdot \frac{1}{-3}\left(1-\frac{D^2+4iD}{3}\right)^{-1} x^2$$

$$= \text{''} \quad -\frac{e^{i2x}}{3}\left[1+\frac{D^2+4iD}{3}+\left(\frac{D^2+4iD}{3}\right)^2+\cdots\right] x^2$$

$$= \text{''} \quad -\frac{e^{i2x}}{3}\left[1+\frac{D^2}{3}+\frac{4iD}{3}+\frac{D^4+8iD^3-16D^2}{3}+\cdots\right] x^2$$

$$= \text{''} \quad -\frac{e^{i2x}}{3}\left(x^2+\frac{2}{3}+\frac{8ix}{3}-\frac{32}{3}\right)$$

$$= " \frac{1}{2}(\cos 2x + i\sin 2x)\left(x^2 + \frac{8ix}{3} - 10\right)$$

$$= " \frac{1}{2}\left(x^2\cos 2x + \frac{8}{3}ix\cos 2x - 10\cos 2x + ix^2\sin 2x - \frac{8}{3}x\sin 2x - i\,10\sin 2x\right)$$

$$Y_p = \frac{1}{2}\left(\frac{8}{3}x\cos 2x + x^2\sin 2x - 10\sin 2x\right)$$

$\therefore$ G.S. $\Rightarrow$

$$y = Y_c + Y_p$$

$$= A\cos x + B\sin x + \frac{1}{2}\left(\frac{8}{3}x\cos 2x + x^2\sin 2x - 10\sin 2x\right)$$

Ans

⊛ from Zillis Books

Exercise 4.5:

5| $y''' + 10y'' + 25y' = e^x$

$$\Rightarrow (D^3 + 10D^2 + 25D)y = e^x \quad \cdots (i)$$

A.E. $\Rightarrow$

$$m^3 + 10m^2 + 25m = 0$$

$$\Rightarrow m(m^2 + 10m + 25) = 0$$

$$\Rightarrow m(m+5)^2 = 0$$

$$\therefore m = 0, -5, -5$$

$$\therefore Y_c = c_1 + c_2 e^{-5x} + c_3 x e^{-5x}$$

(i) $\Rightarrow$ $$y_p = \frac{1}{D^3+10D^2+25D} e^x$$

$$= \frac{1}{1+10+25} e^x$$

$$= \frac{1}{36} e^x$$

$$\therefore \; y = y_c + y_p = c_1 + c_2 e^{-5x} + c_3 x e^{-5x} + \frac{1}{36} e^x$$

Ans

6|

$$y''' + 4y' = e^x \cos 2x$$

$$\Rightarrow (D^3 + 4D)y = e^x \cos 2x \; \cdots \text{(i)}$$

A.E. $\Rightarrow$ $m^3 + 4m = 0$

$$\Rightarrow m(m^2+4) = 0$$

$m = 0$ | $m^2+4=0$

$m^2 = -4$

$m = \pm\sqrt{-4}$

$= \pm 2i$

$$\therefore y_c = c_1 + A\cos 2x + B\sin 2x$$

(i) $\Rightarrow$ $$y_p = \frac{1}{D^3+4D} e^x \cos 2x$$

$$= e^x \frac{1}{(D+1)^3 + 4(D+1)} \cos 2x$$

$$= e^{x} \frac{1}{D^3 + 3D^2 + 3D + 1 + 4D + 4} \cos 2x$$

$$= e^{x} \frac{1}{D^3 + 3D^2 + 7D + 5} \cos 2x$$

$$= e^{x} \frac{1}{D \cdot (-2^2) + 3(-2^2) + 7D + 5} \cos 2x$$

$$= e^{x} \frac{1}{-4D - 12 + 7D + 5} \cos 2x$$

$$= e^{x} \frac{1}{3D - 7} \cos 2x$$

$$= e^{x} \frac{3D + 7}{9D^2 - 49} \cos 2x$$

$$= e^{x} \frac{3D+7}{9(-2^2) - 49} \cos 2x$$

$$= \frac{e^{x}}{-85} (3D+7) \cos 2x$$

$$= -\frac{e^{x}}{85} \left(-\sin 2x \cdot 2 \cdot 3 + 7\cos 2x\right)$$

$$= -\frac{e^{x}}{85} \left(7\cos 2x - 6\sin 2x\right)$$

$$\therefore y = y_c + y_p = C_1 + A\cos 2x + B\sin 2x - \frac{e^{x}}{85}\left(7\cos 2x - 6\sin 2x\right)$$

Ans

Ex

$$y''' + 2y'' - 13y' + 10y = xe^{-x}$$

$$\Rightarrow \left(D^3 + 2D^2 - 13D + 10\right)y = xe^{-x} \quad \ldots \text{(i)}$$

A.E $\Rightarrow$

$$m^3 + 2m^2 - 13m + 10 = 0$$

$$\therefore m = -5,\ 2,\ 1$$

$$\therefore y_c = c_1 e^{-5x} + c_2 e^{x} + c_3 e^{2x}$$

(i) $\Rightarrow$

$$y_p = \frac{1}{D^3 + 2D^2 - 13D + 10}\, xe^{-x}$$

$$= e^{-x} \frac{1}{(D-1)^3 + 2(D-1)^2 - 13(D-1) + 10}\, x$$

$$= e^{-x} \frac{1}{D^3 - 3D^2 + 3D - 1 + 2D^2 - 4D + 2 - 13D + 13 + 10}\, x$$

$$= e^{-x} \frac{1}{D^3 - D^2 - 14D + 24}\, x$$

$$= \frac{e^{-x}}{24}\left(1 + \frac{D^3 - D^2 - 14D}{24}\right)^{-1} x$$

$$= \frac{e^{-x}}{24}\left(1 - \frac{D^3 - D^2 - 14D}{24} + \cdots\right) x$$