31.05.2022
Tuesday
Lecture - 1

Chapter - 1

The Foundations : Logic and Proofs

Propositional Logic

Argument

i. Man is mortal.
ii. Socrates is a human.
⇒ So, Socrates will die

→ Propositions
→ Special Statement

⊛ Propositions : Statements that have definitive truth values.

⊛

× 1. Where is my book ?
2. $2+7=33$
3. CSE 173 is a bad course.
× 4. $x+25=1$ → Here, we don't know is it true or false because x is unknown.

Transformation of Propositions.

Process :

Logical Operator

i. Negation ($\neg$, $\sim$

It is not the case ~~of~~ that p.

Variable

p : Man is mortal

$\neg p$: Man is not mortal

Negation of p.

Truth table :

p	$\neg p$
T	F
F	T

⊛ Compound Propositions:

Conjunction

"Today is Tuesday. $\rightarrow p$

AND $\Rightarrow \wedge \rightarrow$ Conjuction

I eat rice." $\rightarrow q$

$\Rightarrow$ $p \wedge q \Rightarrow$ p conjuction q.

Truth Table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

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Disjunction:

Given that, p.q are propositions.

Then their disjunction denoted as $p \vee q$. (or)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

By default disjunction is inclusive or.

Exclusive or:

p or q, but not both

⇒ In $p \oplus q$, one of p, and q must be true, but not both.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statement

Implication ⇒ "$\longrightarrow$"

If p and q are prepositions, then $p \rightarrow q$ is a conditional statement or implication which is read as "if p, then q".

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Different ways of expressing "$p \rightarrow q$"

⇒ if p, then q

⇒ if p, q

⇒ q unless $\neg p$

⇒ q if p

⇒ q whenever p

⇒ q follows from p

⇒ p implies q

⇒ p only if q

⇒ q when p

⇒ p is sufficient for q

⇒ q is necessary for p

⇒ a necessary condition for p is q

⇒ a sufficient condition for q is p.

Converse of $p \to q \Rightarrow q \to p$

Contrapositive of $p \to q \Rightarrow \neg q \to \neg p$

inverse of $p \to q \Rightarrow \neg p \to \neg q$

~~Bi~~ Biconditional Statement

"$\leftrightarrow$" bi-implication

If p and q are propositions. ~~there~~ then we can form the biconditional proposition $p \leftrightarrow q$, read as "p if and only if q."

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

when two values are same

$(p \leftrightarrow q) \equiv (p \to q) \wedge (q \to p)$

Logically equivalent

$\neg (p \leftrightarrow q) \equiv p \oplus q$

⊛

p	q	$\neg p$	$p \to q$	$\neg p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$p \to q \equiv \neg p \vee q$

⊛ some alternative ways "p if and only if q". is expressed in English:

$\Rightarrow$ p is necessary and sufficient for q

$\Rightarrow$ if p then q, and conversely

$\Rightarrow$ p iff q.

⊛

If today is Friday, then 2+3 = 6.

$\underbrace{\text{today is Friday}}_{p}$, $\underbrace{2+3=6}_{q}$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

F F T $\longrightarrow$ Vacuously true.

⊛ If I stay at home, then I am sick

I stay at home only if I am sick.

⊛ $p \oplus q = (p \land \neg q) \overset{\vee}{+} (q \land \neg p)$

⊛

Demorgans Law:

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

$$\neg(p \lor q) \equiv \neg p \land \neg q$$

⊛ $\neg(P_1 \vee P_2 \dots \vee \dots \vee P_n) \equiv \neg P_1 \wedge \neg P_2 \wedge \dots \wedge \neg P_n$

⊛ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ ⟷ Associative Law

$\neg$	High
$\wedge$	
$\vee$	
$\rightarrow$	
$\leftrightarrow$	Low

⊛ Distributive Law:

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$(p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r)$

$(p \wedge q) \vee (r \wedge s) \equiv R \vee (r \wedge s) \equiv (R \vee r) \wedge (R \vee s)$

Types of Compound Propositions

⊛ Tautology : When all the outputs/truth values are "true."

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

⊛ Contradiction : When all the outputs/truth values are "false."

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

⊛ Contingency : Outputs True/False mixed.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Transformation of $p \rightarrow q$ into other conditional forms.

(i) Converse of $p \rightarrow q$: $q \rightarrow p$

p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

$$\therefore p \rightarrow q \not\equiv q \rightarrow p$$

(ii) inverse of $p \rightarrow q$: $\neg p \rightarrow \neg q$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

$$\therefore p \rightarrow q \not\equiv \neg p \rightarrow \neg q$$

(iii) contropositive of $p \to q$:- $\neg q \to \neg p$

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \to \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$\therefore p \to q \equiv \neg q \to \neg p$

proof technique.

L-4/12.06.2022/

(*) Show that below expression is a Tautology:

$(P \wedge q) \rightarrow (P \vee q)$

P	q	$P \wedge q$	$P \vee q$	$(P \wedge q) \rightarrow (P \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

$$\underbrace{(P \wedge q)}_{R} \rightarrow \underbrace{(P \vee q)}_{S}$$

$\equiv R \rightarrow S$

$\equiv \neg R \vee S$ Defn of implication

$\equiv \neg(P \wedge q) \vee (P \vee q)$ $\because R = P \wedge q, \because S = P \vee q$

$\equiv (\neg P \vee \neg q) \vee (P \vee q)$ Demorgan's Low

$\equiv (\neg P \vee P) \vee (\neg q \vee q)$ Associative Law

$\equiv T \vee T$ Negation Law

$\equiv T$

$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$: check if tautology

Predicate

All students from CSE173, SEC-1 are good.

$P(x)$: $\underbrace{x}_{\text{Subject}}$ $\underbrace{\text{from CSE 173, SEC-1 is good}}_{\text{Predicate}}$

→ Propositional function

Predicated statement
$P(x)$:
$x \in$ SEC-1 → Domain

For all → Universal

There's at least one → Existential

Quantification process:

→ Evaluates if predicated statement is True or False

Quantifier

→ Existential
There's atleast one
some, a few, some of them

→ Universal
All, each, everybody

Universal Quantifier:

For All $\xrightarrow{\text{Notation}}$ $\forall x \; P(x)$

$x \in$ CSE173, SEC-1

* There exists at least $\overset{\to x}{\boxed{\text{one student}}}$ in CSE173, SEC-1 who ~~is from~~ plays football.

$P(x)$: x in CSE173 ... plays football

Existential:

$\exists x \; P(x)$

p	q	r	$\neg p$	$(p \vee q)$	$(\neg p \vee r)$	$(q \vee r)$	$((p \vee q) \wedge (\neg p \vee r))$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	T	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	F	T	T	F	T
F	F	F	T	F	T	F	F	T

$\therefore ((p \vee q) \wedge (\neg p \vee r)) \longrightarrow (q \vee r) \equiv T$

L-5 / 14.06.2022

* Quantification

→ Quantifier

For all

→ Universal → $\forall x$

→ Existential → $\exists x$

There exist

$\forall x \; P(x)$ $\boxed{T/F?}$

- Predicate
- Propositional function

$x \in$ Domain

$$P(x_1) \wedge P(x_2) \wedge P(x_3) \dots \wedge (Px_n) \equiv T$$

$\exists x \; P(x)$ $\boxed{T/F}$?

$$P(x_1) \vee P(x_2) \dots \vee P(x_n)$$

↓

$T \equiv T$

⊛

$\forall x P(x)$, $P(x) = x^{100} \geq x^{20}$

$x \in N \rightarrow T$

$x \in R^{+} \rightarrow F$ ⟶ $x = 0.5$ → Counter example

$0.5^{100} < 0.5^{20}$

$\exists x P(x)$, $P(x) = \frac{1}{x} > x$

$x \in N \rightarrow F$

$x \in Z \rightarrow T$

⟶ $x = -2$ → Counter Example

$\frac{1}{-2} = -0.5 > -2$

⊛

$\neg(\forall x P(x)) \equiv ? \equiv \exists x \neg P(x)$

$\neg(\exists x P(x)) \equiv ? \equiv \forall x \neg P(x)$

⊛ $\underbrace{\text{Every students}}_{\forall x}$ $\underbrace{\text{from SEC-1 has done CSE 115.}}_{P(x)}$

↓

Negation

It is not the case that every student from SEC-1 has done CSE 115.

So, there exists at least one student who has not done CSE115.

$\exists x \neg P(x)$.

⊛ $\neg(P(x_1) \wedge P(x_2) \ldots \wedge P(x_n))$

$\equiv \neg P(x_1) \vee \neg P(x_2) \vee \ldots \neg P(x_n)$

$\equiv \exists x \neg P(x)$.

⊛ All Birds can fly.

$P(x)$: x can fly

$x \in$ All Birds $\longrightarrow \forall x P(x)$

$x \in$ Animal kingdom $\rightarrow$

$P(x)$: x is Bird

$q(x)$: x can fly

if x is a bird, it can fly

$\forall x \, (P(x) \rightarrow q(x))$

⇒ All student from SEC-1 have taken CSE115

$x \in NSU$

$P(x)$: Student x is from SEC-1

$q(x)$: Student x has taken CSE 115

$$\forall x \, (P(x) \rightarrow q(x))$$

⇒ There exists at least one student in SEC-1 who has done CSE 2115

$\exists x \, P(x) \Rightarrow x \in$ SEC-1

$P(x)$: x is from SEC-1

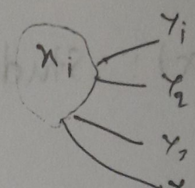
$q(x)$: x has done CSE 215

$$\exists x \, (P(x) \wedge q(x))$$

L-6 / 19.06.2022

Nested Quantifier

$\forall x \, \forall y$



$\rightarrow x, y \in \mathbb{Z}$

$\forall x \, \exists y \quad x+y=0 \rightarrow T$

$\downarrow \quad \downarrow$

2 ; y: −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5

$2 \rightarrow -2 \rightarrow 2-2=0 \rightarrow T$

⊛ $\forall x \, \forall y \quad x+y=0 \longrightarrow F$

$x, y \in \mathbb{Z}$

⊛

$\forall x \, \exists y \quad x+y=0 \longrightarrow F$

$x \in \mathbb{Z}, \; y \in \mathbb{R}^+$

⊛ $\forall x \, \exists y \quad x \cdot y = 0 \longrightarrow F$

$x \in \mathbb{Z}, \; y \in \mathbb{R}^+$

⊛ $\forall x \, \exists y \quad x \cdot y = 0 \longrightarrow T$

$x \in \mathbb{Z}, \; y \in \mathbb{R}^+ \cup \{0\}$

⊛ $\forall x \; \exists y \quad x \cdot y = 1 \longrightarrow F$

$x \in \mathbb{Z},\ y \in \mathbb{R}^+ \cup \{0\}$

⊛ $x \in \mathbb{Z},\ y \in \mathbb{R}^+ \cup \{0\}$

$\forall x \; \exists y \quad x \cdot y = 0 \longrightarrow T$

⊛ $\exists x \; \forall y \quad x \cdot y = 0 \longrightarrow T$

⊛ $\exists x \; \forall y \quad x \cdot y = 0 \longrightarrow F$

$x \in \mathbb{Z} - \{0\},\ y \in \mathbb{R}^+ \cup \{0\}$

⊛ Every real number except zero has a multiplicative inverse.

$$\forall x \left(x \neq 0 \longrightarrow \exists y \left(xy = 1 \right) \right)$$

$$\neg \forall x \, \exists y \, \underbrace{(xy = 1)}_{Q(xy)} \quad x \in \mathbb{Z}$$

$$\neg \forall x \, Q(x,y) \equiv \exists x \, \neg Q(x,y) \quad x \in \mathbb{Z}$$

$$\equiv \exists x \, \neg \exists y \, (x,y = 1) \quad x \in \mathbb{Z}$$

$$\equiv \exists x \, \forall y \, \neg (xy = 1) \quad x \in \mathbb{Z}$$

$$\equiv \exists x \, \forall y \quad xy \neq 1 \quad x \in \mathbb{Z}$$

⊛ There does ~~not~~ not exists a $\underbrace{\text{person}}_{w}$ who has taken a ~~fly~~ $\underbrace{\text{flight}}_{f}$ on every $\underbrace{\text{airline}}_{a}$ in the world.

a → $\exists$ every → $\forall$

$\Rightarrow$

$p(w,f)$: Person "w" has taken a flight "f".

$q(f,a)$: flight "f" is in airline "a".

$$\neg\left(\exists w \, \forall a \, \exists f \, \left(p(w,f) \wedge q(f,a)\right)\right)$$

(*) $\neg(\exists w \underbrace{\forall a \exists f (p(w,f) \wedge q(f,a))}_{Q})$

$$\neg(\exists w\, Q) \equiv \forall w\, \neg Q$$

$$= \forall w\, \neg(\forall a\, \underbrace{\exists f\, (p(w,f) \wedge q(f,a))}_{M}) =$$

$$= \forall w\, (\neg \forall a\, M) =$$

$$= \forall w\, \exists a\, \neg M =$$

$$= \forall w\, \exists a\, \neg(\exists f\, (p(w,f) \wedge q(f,a))) =$$

$$= \forall w\, \exists a\, \forall f\, \neg(p(w,f) \wedge q(f,a)) =$$

$$= \forall w\, \exists a\, \forall f\, (\neg p(w,f) \vee \neg q(f,a))$$

Inference (Process)

→ Argument

↓

Sequence of statement that with conclusions.

Valid

Invalid

Argument

Man is mortal

Socrates is a human

So, socrates will die.

→ Conclusion

If you have an RFID, then you get attendance.

You have a RFID

therefore

You get attendance.

Argument form

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Premises

→ Conclusion

L-7 / 21.06.2022

⊛ Inference

→ to infer

→ Process to deduce the conclusion from collected information

⊛ If you $\underbrace{\text{have an RFID}}_{p}$, you $\underbrace{\text{get attendance}}_{q}$.

⇒ You $\underbrace{\text{have an RFID}}_{p}$

therefore / so / thus

You $\underbrace{\text{get attendance.}}_{q}$

i.e. ⇒ Premises $\begin{cases} p \to q \\ p \end{cases}$

conclusion → $\therefore q$

$$\left[\left((p \to q) \land p\right) \to q\right]$$

should be tautology

Argument

- Valid
- Invalid

Valid → When all the premises are true, the conclusion must be true

→ How do we check it?

- Tautology Approach
- Critical row approach

Tautology Approach ↓

p	q	$p \to q$	$(p \to q) \wedge p$	$((p \to q) \wedge p) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

↓ Tautology

⊛ Critical row approach:

1. $p \vee q$

2. $p \rightarrow r$

3. $q \rightarrow r$

$\therefore r$

$$\left[\left((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \right) \rightarrow r \right]$$

should be true.

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	F	T	T
F	F	F	F	T	T

$\therefore \Rightarrow$ Tautology / True

Rule

Modus Ponens (Mode that affirms)

1. $p \rightarrow q$
2. p

$\therefore q$

Modus Tollens (Mode that denies)

1. $p \rightarrow q$
2. $\neg q$

$\therefore \neg p$

Conjunction Rule:

p
q

$p \wedge q$

1. $p \wedge q \rightarrow r$
2. p
3. q
4. $? = p \wedge q$ (Conjunction Rule)
5. $? = r$ (Modus Ponens)

⊛ Simplification:

$$\frac{1.\ p \wedge q}{p, q}$$

⊛ Addition Rule:

$$\frac{p}{\therefore p \vee \text{"anything"}}$$

⊛ $(p \vee s) \rightarrow q$

$$\frac{\begin{array}{c} p \\ r \end{array}}{p \vee s}$$

⊛ Disjunctive Syllogism:

$$\frac{\begin{array}{c} p \vee q \\ \neg p \end{array}}{\therefore q}$$

⊛ Resolution:

$$\frac{\begin{array}{c} p \vee q \\ \neg p \vee r \end{array}}{\therefore q \vee r}$$

⊛ Hypothetical Syllogism:

$$\frac{\begin{array}{c} p \rightarrow q \\ q \rightarrow r \end{array}}{\therefore p \rightarrow r}$$

(*)

$\underbrace{\text{It is not sunny this afternoon}}_{\neg p}$, and $\underbrace{\text{it is colder than yesterday}}_{q}$. $\underbrace{\text{We will go for swimming}}_{r}$, only if $\underbrace{\text{it is sunny}}_{p}$. If $\underbrace{\text{we do not go for swimming}}_{\neg r}$, then $\underbrace{\text{we will take a canoe trip}}_{s}$. If $\underbrace{\text{we take a canoe trip}}_{s}$, then $\underbrace{\text{we will be ~~to~~ home by sunset}}_{t}$.

<u>Conclusion:</u> We will reach home by sunset.

$\Rightarrow$

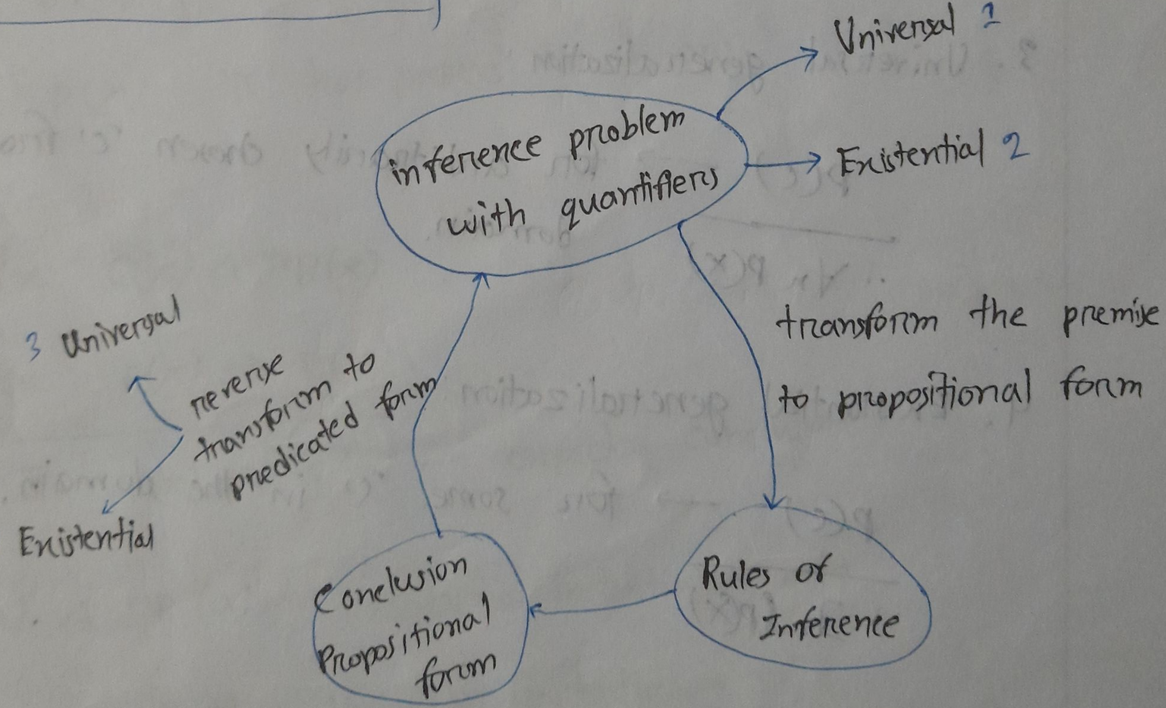
1. $\neg p \land q$ Premise 1
2. $r \rightarrow p$ Premise 2
3. $\neg r \rightarrow s$ Premise 3
4. $s \rightarrow t$ Premise 4
5. $\neg p$ Simplification of 1
6. $\neg r$ Modus Tollens 2,5

7. s Modus Ponens 3,6

8. t Modus Ponens 4,7

Quantifiers/ $\rightarrow$ Rules of Inference

$$\begin{array}{c} p \rightarrow q \\ p \\ \hline \therefore q \end{array} \qquad \begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array} \quad \rightarrow \text{Propositional form}$$



Rules

1. Universal instantiation:

$$\frac{\forall x\, P(x) \quad T}{\therefore P(c)}$$ → for any randomly chosen specific 'c'.

2. Existential instantiation:

$$\frac{\exists x\, P(x) \quad T}{\therefore P(c)}$$ → specific 'c'

3. Universal generalization:

$$\frac{P(c)}{\therefore \forall x\, P(x)}$$ → for arbitrarily chosen 'c' from the domain.

4. Existential generalization:

$$\frac{P(c)}{\therefore \exists x\, P(x)}$$ → for some 'c' in the domain.

⊛ A student in this class has not read the book.

Everyone from this class passed the first exam.

Therefore,

Someone, who passed the first exam has not read the book.

$\Rightarrow$

$C(x)$: 'x' is in this class.

$B(x)$: 'x' has read the book.

$P(x)$: 'x' has passed the exam.

1. $\exists x \quad C(x) \land \neg B(x)$

2. $\forall x \quad C(x) \rightarrow P(x)$

$\exists x \quad P(x) \land \neg B(x)$ → expected conclusion.

3. $C(a) \land \neg B(a)$ Existential instantiation of 1

4. $C(a) \rightarrow P(a)$ Universal instantiation of 2

5. $C(a)$ Simplification of 3