North South University

Department of Electrical and Computer Engineering

Homework 2

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Course No : CSE 173

Course Title : Discrete Mathematics

Section : 1

Date : 26 July, 2022

Homework 2

Ans. to the ques.no.01

a)

Given statement:

All cats have parasites.

Let,

x : cat

Predicate:

c(x) : x has parasites.

Expression : $\forall x \; c(x)$

Negation of this expression:

$$\neg(\forall x \, c(x)) \equiv \exists x \, \neg c(x)$$

In English test :

There exists a cat which does not have parasites.

b)

There is a cow that can add two numbers.

Let,

x : cow

Predicate:

$c(x)$: x can add two numbers.

Expression:

$\exists x \; c(x)$

Negation of this expression:

$$\neg(\exists x \; c(x)) \equiv \forall x \; \neg c(x)$$

In English text:

No cow can add two numbers.

c)

Given statement:

Every monkey you encounter can climb.

Let,

x: monkey

Predicate:

$m(x)$: x can climb

Expression:

$\forall x \; m(x)$

Negation of this expression:

$$\neg(\forall x \; m(x)) \equiv \exists x \; \neg m(x)$$

In English text:

There is a monkey you encounter that cannot climb.

d)

Given statement:

There is a fish that can speak Bengali.

Let,

x: fish

Predicate:

$f(x)$: x can speak Bengali.

Expression:

$\exists x\ f(x)$

Negation of this expression:

$\neg(\exists x\ f(x)) \equiv \forall x\ \neg f(x)$

In English text:

No fish can speak Bengali.

e)

Given statement:

There exists a horse that can fly and catch bird as needed.

Let,

x: horse

Predicate:

$f(x)$: x can fly.

$c(x)$: x can catch bird.

Expression:

$$\exists x \, (f(x) \wedge c(x))$$

Negation of this expression:

$$\neg(\exists x \, (f(x) \wedge c(x)) \equiv \forall x \, \neg(f(x) \wedge c(x))$$

In English text:

There is no horse that can fly and catch bird as needed.

Ans. to the ques. no. 02

Given that,

$Q(x,y)$

x: All students in CSE173

y: All TV reality shows

a)

Given statement:

There is a student at CSE 173 who is a contestant on a TV reality show.

Expression:

$\exists x \exists y \, Q(x,y)$

b)

Given statement:

No student at CSE 173 has ever been a contestant on a TV reality show.

Expression:

$\forall x \forall y \, \neg Q(x,y)$

c)

Given statement:

There is a student at CSE 173 who is a contestant on Close-up and Bangladeshi Idol.

Let,

c: Close-up.

b: Bangladeshi Idol.

Expression:

$$\exists x\,(Q(x,c) \land Q(x,b))$$

d)

Given statement:

Every TV reality show aired so far had a student from CSE 173 as a contestant.

Expression:

$$\forall y\, \exists x\, Q(x,y)$$

e)

Given statement:

At least two students from CSE 173 are the contestants on Bangladeshi Idol.

Let,

b: Bangladeshi Idol

Expression:

$$\exists x\, Q(x, b)$$

Ans. to the ques. no. 03

a)

Given that,

$$\forall x\, [P(x) \vee Q(x)]$$

Negation of this expression:

$$\neg(\forall x\, [P(x) \vee Q(x)]$$

$$\equiv \exists x\, [\neg(P(x) \vee Q(x))]$$

$$\equiv \exists x\, [\neg P(x) \wedge \neg Q(x)]$$

b)

Given that,

$$\exists y\,[\,P(y) \vee (Q(y) \vee R(y))]$$

Negation of this expression:

$$\neg(\exists y\,[P(y) \vee (Q(y) \vee R(y))]$$

$$\equiv \forall y\,[\neg(P(y) \vee (Q(y) \vee R(y)))]$$

$$\equiv \forall y\,[\,\neg P(y) \wedge \neg(Q(y) \vee R(y))]$$

$$\equiv \forall y\,[\,\neg P(y) \wedge (\neg Q(y) \wedge \neg R(y))]$$

c)

Given that,

$$\exists x\,[(P(x) \wedge Q(x)) \vee (Q(x) \wedge \neg P(x))]$$

Negation of this expression:

$$\neg(\exists x\,[(P(x) \wedge Q(x)) \vee (Q(x) \wedge \neg P(x))]$$

$$\equiv \forall x\,[\neg((P(x) \wedge Q(x)) \vee (Q(x) \wedge \neg P(x)))]$$

$$\equiv \forall x\,[\,\neg(P(x) \wedge Q(x)) \wedge \neg(Q(x) \wedge \neg P(x))]$$

$$\equiv \forall x [(\neg P(x) \lor \neg Q(x)) \land (\neg Q(x) \lor P(x))]$$

Ans. to the ques. no. 04

a)

Given statement:

If m and n are both negative, their product is always positive.

Here,

For both negative numbers m and n: $\forall m \forall n (m<0 \land n<0)$

Product for them is always positive: $mn>0$

Therefore,

Complete statement:

$$\forall m \forall n [(m<0 \land n<0) \to mn>0]$$

b)

Given statement:

Assume m and n are positive, then average of m and n positive.

Here,

For both positive numbers m and n:

$$\forall m \forall n \ (m>0 \wedge n>0)$$

Average for them is positive:

$$\frac{m+n}{2} > 0$$

Complete statement:

$$\forall m \forall n \left[(m>0 \wedge n>0) \rightarrow \frac{m+n}{2} > 0 \right]$$

c)

Given statement:

If m and n are negative, m-n is not necessarily negative.

Here,

For both negative numbers m and n:

$$\exists m \exists n\ (m<0 \land n<0)$$

Difference of them is not necessarily negative:

$$\neg(m-n<0)$$

complete statement:

$$\exists m \exists n\ [(m<0) \land (n<0) \land \neg(m-n<0)]$$

P.T.O.

Homework 3

Ans. to the ques. no. 01

We can quickly reduce a proof to checking just a few simple cases because $2x^2 > 14$ when $|x| \geq 3$ and $5y^2 > 14$ when $|y| \geq 2$. This leaves the cases when x equals $-2, -1, 0, 1,$ or 2 and y equals $-1, 0$ or 1. We can finish using an exhaustive proof. To dispense with the remaining cases, we note that possible values for $2x^2$ are $0, 2,$ and 8 and possible values for $5y^2$ are 0 and 5, and the largest sum of possible values for $2x^2$ and $5y^2$ is 13. Consequently, it is impossible for $2x^2 + 5y^2 = 14$ to hold when x and y are integers.

Ans to the ques. no. 02

Proof by contraposition:

if $\underbrace{x^2-2a+7 \text{ is even}}_{p}$, then $\underbrace{x \text{ is odd.}}_{q}$

$\therefore p \rightarrow q \equiv \neg q \rightarrow \neg p$

$\neg q$: x is not odd. (that's means even)

$\neg p$: x^2-2a+7 is not even. (that means odd)

$\therefore x = 2k$; k is an integer

Now,

$$x^2-2a+7 = (2k)^2-2a+7$$
$$= 4k^2-2a+7$$
$$= 4k^2-2a+6+1$$
$$= 2(2k^2-a+3)+1$$
$$= 2 \cdot \text{integer} + 1$$
$$= \text{odd}$$

(Proved)

Ans. to the ques. no. 03

Given statement:

If $\underbrace{3n^2+4n+1 \text{ is even}}_{p}$, then $\underbrace{3n+1 \text{ is even}}_{q}$ or $\underbrace{n+1 \text{ is even}}_{r}$

$$p \rightarrow (q \vee r)$$

Proof by contrapositive:

$$p \rightarrow (q \vee r) \equiv \neg(q \vee r) \rightarrow \neg p$$

Now,

$$\neg(q \vee r) \equiv \neg q \wedge \neg r$$

$\equiv$ $(3n+1)$ is not even AND $(n+1)$ is not even

$\equiv$ $(3n+1)$ is odd AND $(n+1)$ is odd

$\equiv$ $3n$ is even AND n is even

$\equiv$ n is even

if n is even, then n^2 is even and therefore $3n^2$ is also even.

if n is even, the $4n$ is also even.

Therefore,

$$3n^2 + 4n + 1 \equiv \text{even} + \text{even} + 1$$

$$\equiv \text{odd}$$

Therefore,

"$3n^2 + 4n + 1$ is not even" is true.

(proved)

Ans. to the ques. no. 04

Proof by contradiction:

Lets assume that, there exists a natural number n that is both even and odd.

Since, n is even: $n = 2k$; k is an integer

Since, n is odd: $n = \cancel{2k+} 2i + 1$; i is an integer

Therefore,

$$2k = 2i + 1$$

$$\Rightarrow 2k - 2i = 1$$

$$\Rightarrow 2(k - i) = 1$$

$$\therefore k - i = \frac{1}{2}$$

Since ~~x~~ k and i are integers (k-i) must also be an integer. But we got (k-i) as $\frac{1}{2}$, which is not an integer. So, this is a contradiction, hence our assumption that "there exists a natural number n that is both even and odd" is false.

Hence, we can conclude that no natural number n is both even and odd.

Ans. to the ques. no. 05

a)

Given that,

$$1+3+5+\cdots\cdots+(2n-1)=n^2$$

Proof by mathematical induction:

Basic step:

put $n=1$

$L.H.S = 2\cdot1-1 = 1$

$R.H.S = 1^2 = 1$

Inductive step:

Lets assume that the given statement is true for $n=k$. That is $p(k)$ is true.

$\Rightarrow 1+3+5+\dots+(2k-1) = k^2$

Now,

We have to show that the statement is true for $n=k+1$. That is we have to show that,

$1+3+5+\dots+(2k-1)+(2k+1) = (k+1)^2$

$L.H.S. = 1+3+5+\dots+(2k-1)+(2k+1)$

$= k^2+2k+1$

$= (k+1)^2$

$= R.H.S.$

(proved)

b)

Given statement:

4^n-1 is a multiple of 3

Basic step:

$n=1$

$$4^1 - 1 = 4 - 1$$
$$= 3$$; which is multiple of 3.

Inductive step:

Lets assume that this statement is true for $n = k$.

That means that $(4^k - 1)$ is multiple of 3.

Now, we have to show that, the statement is true for $(k+1)$. That means that $(4^{k+1} - 1)$ is multiple of 3.

Now,

$$4^{k+1} - 1 = 4^k \cdot 4 - 1$$
$$= 4^k (3+1) - 1$$
$$= 3 \cdot 4^k + 4^k - 1$$
$$= 3 \cdot 4^k + 3m$$ [m is a positive integer]
$$= 3(4^k + m)$$
$$= 3 \cdot \text{integer}$$; which is multiple of 3

(proved)

c)

Given statement:

$$2+4+6+\cdots+2n = n(n+1)$$

Basic step:

$n = 1$

L.H.S. $= 2\cdot 1 = 2$

R.H.S $= 1(1+1)$

$= 1\cdot 2 = 2$

Inductive step:

Lets assume that the given statement is true for $n=k$. That is $p(k)$ is true.

$$\Rightarrow 2+4+6+\cdots+2k = k(k+1)$$

Now,

We have to show that the statement is true for $n=k+1$. That is we have to show that,

$$2+4+6+\cdots+2k+(2k+2) = (k+1)(k+2)$$

L.H.S. $= 2+4+6+\cdots+2k+(2k+2)$

$= k(k+1)+2k+2$

$= k(k+1)+2(k+1)$

$= (k+1)(k+2)$

$= R.H.S.$

(proved)

d)

Given statement :

$$-1 + 2 + 5 + 8 + \dots + (3n-4) = \frac{n}{2}(3n-5)$$

Basic step :

$n = 1$

$L.H.S. = 3 \cdot 1 - 4 = -1$

$R.H.S. = \frac{1}{2}(3 \cdot 1 - 5)$

$= \frac{1}{2} \cdot (-2)$

$= -1$

Inductive step :

Lets assume that the given statement is true for $n=k$.

That is $p(k)$ is true.

$$\Rightarrow -1 + 2 + 5 + 8 + \dots + (3k-4) = \frac{k}{2}(3k-5)$$

Now,

We have to show that the statement is true for $n = k+1$.

That is we have to show that,

$$-1+2+5+8+\cdots+(3k-4)+(3k-1) = \frac{k+1}{2}\left(3(k+1)-5\right) = \frac{3k^2+k-2}{2}$$

$$\text{L.H.S.} = -1+2+5+8+\cdots+(3k-4)+(3k-1)$$

$$= \frac{k}{2}(3k-5)+(3k-1)$$

$$= \frac{k(3k-5)+2(3k-1)}{2}$$

$$= \frac{3k^2-5k+6k-2}{2}$$

$$= \frac{3k^2+k-2}{2}$$

$$= \text{R.H.S.}$$

(proved)