North South University

Department of Mathematics and Physics

Assignment - 2

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Course No : MAT-120

Course Title : Calculus and Analytical Geometry I

Section : 13

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3.4

10)

Given that,

$$x = 1$$

$$y = 2$$

$$\frac{dx}{dt} = -2$$

$$\frac{dy}{dt} = 3$$

and,

$$z = x^3 y^2$$

differentiate w.r.t. t,

$$\frac{dz}{dt} = \frac{d}{dt}(x^3 y^2)$$

$$= x^3 \cdot \frac{d}{dt} y^2 + y^2 \cdot \frac{d}{dt} x^3$$

$$= x^3 \cdot 2y \cdot \frac{dy}{dt} + y^2 \cdot 3x^2 \cdot \frac{dx}{dt}$$

$$= 1^3 \cdot 2 \cdot 2 \cdot 3 - 2^2 \cdot 3 \cdot 1^2 \cdot 2$$

$$= 12 - 24$$

$$= -12 \text{ unit/second}$$

Therefore, z is decreasing at the rate of 12 units/second

11/

Given that,

$$r = 4$$

and $$\frac{d\theta}{dt} = \frac{2\pi}{60} = \frac{\pi}{30} \text{ rad/min}$$

We know that,

$$\text{Area}, \ A = \frac{1}{2} r^2 \theta$$

$$\Rightarrow \ A = \frac{1}{2} \cdot 4^2 \cdot \theta$$

$$\Rightarrow \ A = 8\theta$$

Now, differentiate w.r.t. t,

$$\Rightarrow \ \frac{dA}{dt} = 8 \frac{d\theta}{dt}$$

$$\therefore \ \frac{dA}{dt} = 8 \cdot \frac{\pi}{30} \text{ in}^2\text{/min}$$

$$= \frac{4\pi}{15} \text{ in}^2\text{/min}$$

Therefore,

area swept out rate $\frac{4\pi}{15}$ in²/min.

12|

Let,

radius $= r$

area $= A$

Given that,

$$\frac{dr}{dt} = 3 \text{ ft/sec}$$

after 10 second radius, $r = 3 \cdot 10$ ft
$= 30$ ft

$$\left.\frac{dA}{dt}\right|_{t=10} = ?$$

We know that,

$$A = \pi r^2$$

differentiate both side w.r.t. t

$$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2)$$

$$= \pi \cdot 2r \cdot \frac{dr}{dt} \quad [\pi \text{ is constant}]$$

$$= 2\pi \cdot 30 \cdot 3$$

$$\therefore \frac{dA}{dt} = 180\pi \text{ ft}^2\text{/sec}$$

Therefore area is increasing at the rate of 180π ft²/sec.

13)

Given that,

$$\frac{dA}{dt} = 6$$

and $A = 9$

$$\Rightarrow \pi r^2 = 9$$

$$\Rightarrow r^2 = \frac{9}{\pi}$$

$$\therefore r = \frac{3}{\sqrt{\pi}}$$

We know that,

$$A = \pi r^2$$

$$r^2 = \frac{A}{\pi}$$

differentiate both side w.r.t. t,

$$\frac{d}{dt} r^2 = \frac{1}{\pi} \cdot \frac{dA}{dt}$$

$$\Rightarrow 2r \cdot \frac{dr}{dt} = \frac{1}{\pi} \cdot \frac{dA}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{1}{2\pi r} \cdot \frac{dA}{dt}$$

$$= \frac{1}{2 \cdot \pi \cdot \frac{3}{\sqrt{\pi}}} \cdot 6$$

$$\therefore \frac{dr}{dt} = \frac{1}{\sqrt{\pi}} \text{ mi/h}$$

Therefore, radius increasing at the rate of $\frac{1}{\sqrt{\pi}}$ mi/h

14)

Given that,

radius, $r = 1$

Diameter, $D = 2$

$$\frac{dv}{dt} = 3$$

We know that,

$$v = \frac{4}{3}\pi r^3$$

$$\Rightarrow v = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

$$\Rightarrow v = \frac{\pi}{6}D^3$$

$$\Rightarrow D^3 = \frac{6}{\pi}v$$

differentiate both side w.r.t. t,

$$3 \cdot D^2 \cdot \frac{dD}{dt} = \frac{6}{\pi}\frac{dv}{dt}$$

$$\frac{dD}{dt} = \frac{2}{\pi D^2}\frac{dv}{dt}$$

$$= \frac{2}{\pi \cdot 4} \cdot 3$$

$$\therefore \frac{dD}{dt} = \frac{3}{2\pi} \text{ ft/min}$$

Therefore, diameter increasing at the rate of $\frac{3}{2\pi}$ ft/min

15/

Given that,

radius, $r = 9$

$$\frac{dr}{dt} = -15 \text{ cm/min}$$

We know that,

$$V = \frac{4}{3}\pi r^3$$

differentiate both side with r.t. t,

$$\frac{dV}{dt} = \frac{4\pi}{3} \cdot 3r^2 \cdot \frac{dr}{dt}$$

$$= 4 \cdot 9^2 \cdot (-15) \cdot \pi$$

$$= -4860\pi \text{ cm}^3/\text{min}$$

Therefore

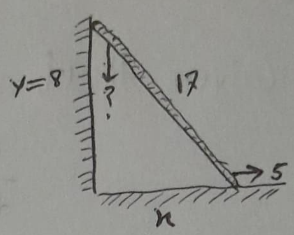
Air must be removed at the rate of 4860π cm³/min

16

Given that,

$$\frac{dx}{dt} = 5 \text{ ft/sec}$$

$$y = 8$$



From Pythagorean theorem,

$$x^2 + y^2 = 17^2$$

differentiate both side w.r.t. t,

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Hence,
$x^2 + 8^2 = 17^2$
$x = 15$

$$\Rightarrow 2 \cdot 15 \cdot 5 + 2 \cdot 8 \cdot \frac{dy}{dt} = 0$$

$$\Rightarrow \frac{dy}{dt} = \frac{-150}{16}$$

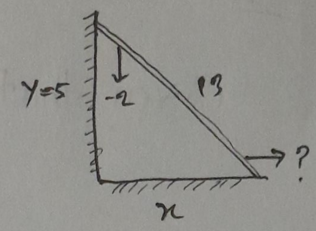
$$= -\frac{75}{8} \text{ ft/s}$$

Therefore,

the top of the ladder is moving down the wall at the rate of $\frac{75}{8}$ ft/sec.

17|

Given that,



$$\frac{dy}{dt} = -2 \text{ ft/s}$$

$$y = 5 \text{ ft}$$

From Pythagorean theorem,

$$x^2 + y^2 = 13^2$$

differentiate both side w.r.t. t,

$$2x\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} = 0$$

Here,

$$x^2 + 5^2 = 13^2$$

$$x = 12$$

$$\Rightarrow 2\cdot 12\cdot\frac{dx}{dt} - 2\cdot 5\cdot 2 = 0$$

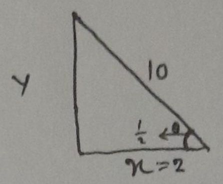
$$\Rightarrow \frac{dx}{dt} = \frac{20}{24}$$

$$= \frac{5}{6} \text{ ft/s}$$

Therefore ladder is moving away from the wall at the rate of $\frac{5}{6}$ ft/sec.

18/

Given that,



distance of the bottom of the plank from wall, $x = 2$ ft

acute angle $= \theta$

$$\frac{dx}{dt} = -6 \text{ in/s} = -\frac{1}{2} \text{ ft/sec}$$

We know,

$$\cos\theta = \frac{x}{10}$$

differentiate both side w.r.t. t

$$-\sin\theta \cdot \frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$$

$$\Rightarrow \frac{d\theta}{dt} = -\frac{1}{10\sin\theta} \cdot \frac{dx}{dt}$$

$$= -\frac{1}{10 \cdot \frac{\sqrt{96}}{10}} \cdot \left(-\frac{1}{2}\right)$$

$$= \frac{1}{2\sqrt{96}} \text{ rad/sec}$$

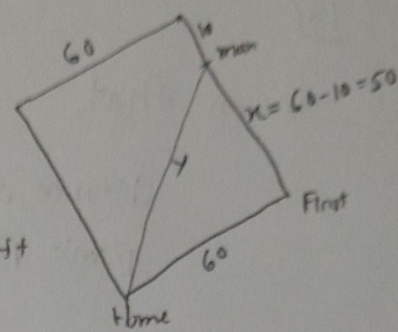
Here,

$$\sin\theta = \frac{y}{10} = \frac{\sqrt{10^2-2^2}}{10} = \frac{\sqrt{96}}{10}$$

Therefore,

angle is increasing at the rate of $\frac{1}{2\sqrt{96}}$ rad/sec.

19)



Let,

distance from first base, $x = 60-10$
$= 50$ ft

distance from home plate, $y = \sqrt{60^2+50^2}$
$= 10\sqrt{61}$

Given that,

$$\frac{dx}{dt} = 25 \text{ ft/sec}$$

From Pythagorean theorem,

$$x^2 + 60^2 = y^2$$

differentiate both side w.r.t. t,

$$2x \cdot \frac{dx}{dt} = 2y \cdot \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{2x}{2y} \cdot \frac{dx}{dt}$$

$$= \frac{50}{10\sqrt{61}} \cdot 25$$

$$= \frac{125}{\sqrt{61}} \text{ ft/sec}$$

Therefore, distance from home plate is increasing at the rate of $\frac{125}{\sqrt{61}}$ ft/sec.

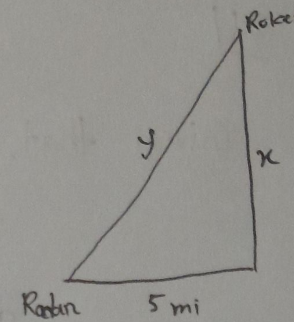
20)

Given that,

$$x = 4 \text{ mi}$$

$$\frac{dy}{dt} = 2000 \text{ mi/h}$$

$$\frac{dx}{dt} = ?$$



From Pythagorean theorem,

$$x^2 + 5^2 = y^2$$

differentiate w.r.t. t,

$$2x \cdot \frac{dx}{dt} = 2y \cdot \frac{dy}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{2y}{2x} \cdot \frac{dy}{dt}$$

$$= \frac{\sqrt{41}}{4} \cdot 2000$$

$$= 500\sqrt{41} \text{ mi/h}$$

Here,

$$x^2 + 25 = y^2$$

$$y = \sqrt{16+25}$$

$$= \sqrt{41}$$

Therefore,

Roket rising at the rate of $500\sqrt{41}$ mi/h

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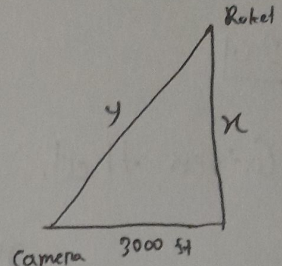


Given that,

$$x = 4000 \text{ ft}$$

$$\frac{dx}{dt} = 880 \text{ ft/sec}$$

From Pythagorean theorem,

$$x^2 + 3000^2 = y^2$$

differentiate both side w.r.t. t,

$$2x \cdot \frac{dx}{dt} = 2y \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{2x}{2y} \cdot \frac{dx}{dt}$$

$$= \frac{4000}{5000} \cdot 880$$

$$= 704 \text{ ft/sec}$$

Here.

$$x^2 + 3000^2 = y^2$$

$$y = \sqrt{4000^2 + 3000^2}$$

$$= 5000$$

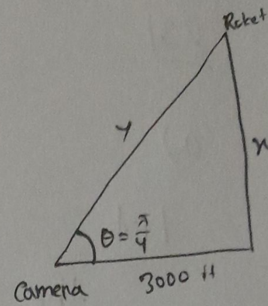
Therefore,

Camera to Roket distance increasing at the rate of 704 ft/sec.

22|

Given that,

$$\theta = \frac{\pi}{4}$$



$$\frac{d\theta}{dt} = 0.2 \text{ rad/sec}$$

We know,

$$\tan\theta = \frac{x}{3000}$$

$$\Rightarrow x = 3000 \tan\theta$$

differentiate both side w.r.t. t,

$$\frac{dx}{dt} = 3000.(\sec^2\theta).\frac{d\theta}{dt}$$

$$= 3000 \cdot (\sec^2\frac{\pi}{4}).(0.2)$$

$$= 1200 \text{ ft/sec}$$

Therefore,

Roket rising at the rate of 1200 ft/sec.

23)

a)

Let,

the altitude, x

then, $r - x = 3960$

at perigee, $\theta = 0$

So, $r = \dfrac{4995}{1+0.12\cos 0} = 4460$

therefore,

$$\text{altitude}, x = r - 3960$$
$$= 4460 - 3960$$
$$= 500 \text{ miles}$$

again,

at apogee,

$\theta = \pi$

So, $r = \dfrac{4995}{1+0.12\cos\pi} = 5676$

therefore,

$$\text{altitude}, x = r - 3960$$
$$= 5676 - 3960$$
$$= 1716 \text{ miles}$$

Therefore

altitude at ~~pre~~ perigee is 500 miles and at apogee is 1716 miles.

b)

Given that,

$\theta = 120°$

$\therefore r = \dfrac{4995}{1+0.12\cos 120} = 5314$

$\therefore$ altitude is, $x = 5314 - 3960 = 1354$ miles

We know,

the rate of change of altitude $\frac{dx}{dt}$ is same as the rate of change of r, $\frac{dr}{dt}$.

Given that,

$$r = \frac{4995}{1+0.12\cos\theta}$$

differentiate both side w.r.t. t,

$$\frac{dr}{dt} = \frac{4995\cdot(0.12)\cdot\sin\theta\cdot\frac{d\theta}{dt}}{(1+0.12\cos\theta)^2}$$

Here, $\frac{d\theta}{dt} = 2.7°/m$

$= \frac{2.7\pi}{180}$ r

$$= \frac{4995\cdot(0.12)\cdot(\sin 120)\cdot 2.7\pi}{(1+0.12\cos 120)^2\cdot 180}$$

$= 27.68$ mile/min

Therefore, altitude is increasing at the rate of 27.68 mile/mi

241

a)

Let,

horizontal distance is, x

Given that,

$$\frac{dx}{dt} = 300 \text{ mile/h}$$
$$= 440 \text{ ft/sec}$$

We know, $\theta = 30°$

$$\cot\theta = \frac{x}{4000}$$

differentiate both side w.r.t. t,

$$-\operatorname{cosec}^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{4000} \cdot \frac{dx}{dt}$$

$$\Rightarrow \frac{d\theta}{dt} = -\frac{1}{4000} \cdot 440 \cdot \sin^2\theta$$

$$= -0.0275 \text{ rad/sec}$$

$$= -1.6°/\text{sec}$$

therefore,

θ is decreasing at the rate of $1.6°$/sec.

b)

Let,

y be the distance between the observation point and the aircraft.

Given that,

$\theta = 30°$

$\frac{d\theta}{dt} = -0.0275$ rad/sec [from a]

We know,

$\operatorname{cosec}\theta = \frac{y}{4000}$

$\Rightarrow y = 4000 \operatorname{cosec}\theta$

differentiate both side w.r.t. t,

$\frac{dy}{dt} = -4000 (\operatorname{cosec}\theta \cdot \cot\theta) \cdot \frac{d\theta}{dt}$

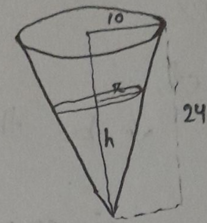
$= -4000 (\operatorname{cosec} 30 \cdot \cot 30) \cdot (-0.0275)$

$= 381$ ft/sec

Therefore,

distance increasing at the rate of 381 ft/sec.

251



Given that,

$$\frac{dV}{dt} = 20$$

Using similan triangle,

$$\frac{r}{h} = \frac{10}{24}$$

$$\therefore r = \frac{5}{12} h$$

The volume of waden in the tank at a depth h is

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{5}{12} h\right)^2 h$$

$$V = \frac{25}{432} \pi h^3$$

deffrenentiate both side w.r.t. t,

$$\frac{dV}{dt} = \frac{25}{144} \pi h^2 . \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{144}{25 . \pi . h^2} . \frac{dV}{dt}$$

$$= \frac{144}{25 . \pi . (16)^2} . 20$$

$$= \frac{9}{20\pi} \text{ ft/min}$$

Thenefone,
depth inneasing at the nate of $\frac{9}{20}\pi$ ft/min

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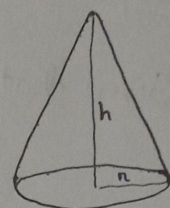
Given that,

$$\frac{dv}{dt} = 8$$

$$h = 6$$

$$r = \frac{1}{2}h$$



Now,

$$v = \frac{1}{3}\pi r^2 h$$

$$v = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$$

$$v = \frac{1}{12}\pi h^3$$

differentiate both side w.r.t. t,

$$\frac{dv}{dt} = \frac{1}{12}\pi \cdot 3 \cdot h^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{4}{\pi h^2} \cdot \frac{dv}{dt}$$

$$= \frac{4}{\pi \cdot 6^2} \cdot 8$$

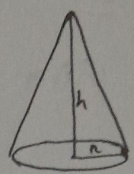
$$= \frac{8}{9\pi} \text{ ft/min}$$

Therefore,

height is increasing at the rate of $\frac{8}{9\pi}$ ft/min.

27/

Given that,



$$\frac{dh}{dt} = 5$$

$$h = 10$$

$$r = \frac{1}{2}h$$

Now,

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{12}\pi h^3$$

differentiate both side w.r.t. t,

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \cdot \frac{dh}{dt}$$

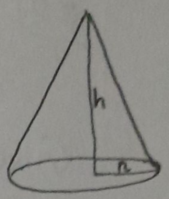
$$= \frac{1}{4} \cdot \pi \cdot 10^2 \cdot 5$$

$$= 125\pi \ ft^3/min.$$

Therefore,

sand puring at the rate of 125π $ft^3/min.$

28/



Let,

circumference is C

Given that,

$$\frac{dV}{dt} = 10$$

$$h = 8$$

$$r = \frac{1}{2}h$$

Now,

$$C = 2\pi r$$

$$\Rightarrow C = 2\pi \frac{1}{2} h$$

$$C = \pi h$$

differentiate both side w.r.t. t,

$$\frac{dC}{dt} = \pi \frac{dh}{dt}$$

$$= \pi \cdot \frac{4}{\pi h^2} \cdot \frac{dV}{dt}$$

$$= \frac{4}{8^2} \cdot 10$$

$$= \frac{5}{8} \text{ ft/min}$$

Here,

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \cancel{\frac{1}{12}} \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{\pi h^2} \cdot \frac{dV}{dt}$$

Therefore,

circumference increasing at the rate of $\frac{5}{8}$ ft/min.

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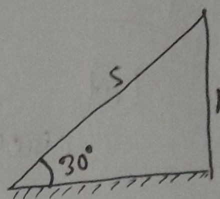


Given that,

$$\frac{ds}{dt} = 500 \text{ mi/h}$$

Now,

$$\sin\theta = \frac{h}{s}$$

$$\Rightarrow h = s\sin\theta$$

$$\Rightarrow h = s\sin 30 = \frac{1}{2}s$$

differentiate both side w.r.t. t,

$$\frac{dh}{dt} = \frac{1}{2}\frac{ds}{dt}$$

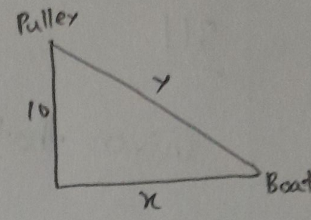
$$= \frac{1}{2}\cdot 500$$

$$= 250 \text{ mi/h}.$$

Therefore,

altitude increasing at the rate of 250 mi/h.

30|



Given that,

$$\frac{dy}{dt} = -20$$

$$y = 125$$

From Pythagorean theorem,

$$x^2 + 10^2 = y^2$$

differentiate both side w.r.t. t

$$2x \cdot \frac{dx}{dt} = 2y \cdot \frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{2y}{2x} \cdot \frac{dy}{dt}$$

$$= \frac{125}{15\sqrt{69}} \cdot (-20)$$

$$= -\frac{500}{3\sqrt{69}} \text{ ft/min}$$

Here,

$$x^2 + 10^2 = (125)^2$$

$$x = \sqrt{(125)^2 - 10^2}$$

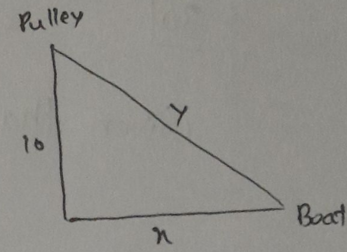
$$= \sqrt{15525}$$

$$= 15\sqrt{69}$$

Therefore,

The boat is approaching the dock at the rate of $\frac{500}{3\sqrt{69}}$ ft/min.

31)



Given that,

$$\frac{dx}{dt} = -12$$

$$y = 125$$

From Pythagorean theorem,

$$x^2 + 10^2 = y^2$$

differentiate both side w.r.t. t,

$$2x \cdot \frac{dx}{dt} = 2y \cdot \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{2x}{2y} \cdot \frac{dx}{dt}$$

$$= \frac{15\sqrt{69}}{125} \cdot (-12)$$

$$= -\frac{36\sqrt{69}}{25} \text{ ft/min}$$

Here,

$$x^2 + 10^2 = y^2$$

$$x = \sqrt{(125)^2 - 10^2}$$

$$= \sqrt{15525}$$

$$= 15\sqrt{69}$$

Therefore,

The rope must be pulled at the rate of

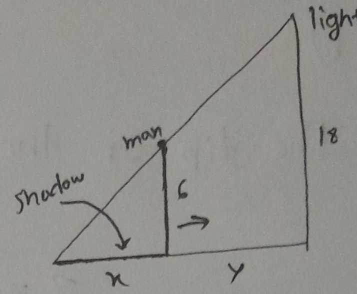
$$\frac{36\sqrt{69}}{25} \text{ ft/min.}$$

32/

a)

Given that,

$$\frac{dy}{dt} = -3$$



by similar triangle,

$$\frac{x}{6} = \frac{x+y}{18}$$

$$\Rightarrow 18x = 6x + 6y$$

$$\Rightarrow 12x = 6y$$

$$\Rightarrow x = \frac{1}{2}y$$

differentiate both side w.r.t. t,

$$\frac{dx}{dt} = \frac{1}{2}\frac{dy}{dt}$$

$$= \frac{1}{2}(-3)$$

$$= -\frac{3}{2} \text{ ft/sec}$$

Therefore,

Shadow length is decreasing at the rate of $\frac{3}{2}$ ft/sec.

b)

The tip of the shadow is, z

$$z = x + y$$

differentiate both side w.r.t. t,

$$\frac{dz}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$$

$$= -\frac{3}{2} + (-3)$$

$$= -\frac{9}{2} \text{ ft/sec}$$

Therefore,

the tip of the shadow is moving at the rate of $\frac{9}{2}$ ft/sec toward the street light.

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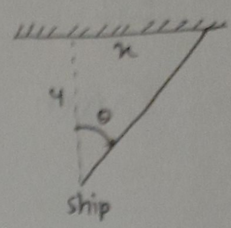

Given that,

$$\theta = \frac{\pi}{4}$$

$$\frac{d\theta}{dt} = \frac{2\pi}{10} = \frac{\pi}{5} \text{ rad/sec.}$$

Now,

$$\tan\theta = \frac{x}{4}$$

$$\Rightarrow x = 4\tan\theta$$

differentiate both side w.r.t. t,

$$\frac{dx}{dt} = 4 \cdot \sec^2\theta \cdot \frac{d\theta}{dt}$$

$$= 4 \cdot \left(\sec^2\frac{\pi}{4}\right) \cdot \frac{\pi}{5}$$

$$= \frac{8\pi}{5} \text{ km/sec}$$

Therefore,

beam moving at the rate of $\frac{8\pi}{5}$ km/sec.

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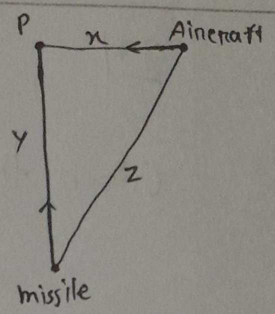

Given that,

$x = 2$

$y = 4$

$\frac{dx}{dt} = -600$

$\frac{dy}{dt} = -1200$

$\frac{dz}{dt} = ?$

Now,

$z^2 = x^2 + y^2$

differentiate both side w.r.t. t,

$$2z \cdot \frac{dz}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{2 \cdot 2 \cdot (-600) + 2 \cdot 4 \cdot (-1200)}{2 \cdot 2\sqrt{5}}$$

Here,

$x^2 + y^2 = z^2$

$z = \sqrt{2^2 + 4^2}$

$= 2\sqrt{5}$

$$= -600\sqrt{5} \text{ mi/h}$$

Therefore, the distance between missile and aircraft is decreasing at the rate of $600\sqrt{5}$ mi/h.

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7|

$$\lim_{x\to 0} \frac{e^x-1}{\sin x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x\to 0} \frac{e^x}{\cos x}$$

$$= \frac{e^0}{\cos 0}$$

$$= 1$$

Therefore, $\lim_{x\to 0} \frac{e^x-1}{\sin x} = 1$

8|

$$\lim_{x\to 0} \frac{\sin 2x}{\sin 5x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, usin L' Hospital Rule,

$$= \lim_{x\to 0} \frac{2\cos 2x}{5\cos 5x}$$

$$= \frac{2\cos 0}{5\cos 0}$$

$$= \frac{2}{5}$$

Therefore $\lim_{x\to 0} \frac{\sin 2x}{\sin 5x} = \frac{2}{5}$.

9)

$$\lim_{\theta \to 0} \frac{\tan\theta}{\theta} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{\theta \to 0} \frac{\sec^2\theta}{1}$$

$$= \frac{\sec^2 0}{1}$$

$$= 1$$

Therefore,

$$\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1$$

10)

$$\lim_{x \to 0} \frac{x e^x}{1 - e^x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x \to 0} \frac{x e^x + e^x}{-e^x}$$

$$= \frac{0+1}{-1}$$

$$= -1$$

Therefore,

$$\lim_{x \to 0} \frac{x e^x}{1 - e^x} = -1$$

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$$\lim_{x \to \pi^+} \frac{\sin x}{x - \pi} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L'Hospital Rule;

$$= \lim_{x \to \pi^+} \frac{\cos x}{1}$$

$$= \cos \pi$$

$$= -1$$

Therefore,

$$\lim_{x \to \pi^+} \frac{\sin x}{x - \pi} = -1$$

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$$\lim_{x \to 0^+} \frac{\sin x}{x^2} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L'Hospital Rule,

$$= \lim_{x \to 0^+} \frac{\cos x}{2x}$$

$$= +\infty$$

Therefore,

$$\lim_{x \to 0^+} \frac{\sin x}{x^2} = +\infty$$

13)

$$\lim_{x \to +\infty} \frac{\ln x}{x} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x \to +\infty} \frac{1/x}{1}$$

$$= \lim_{x \to +\infty} \frac{1}{x}$$

$$= 0$$

Therefore,

$$\lim_{x \to +\infty} \frac{\ln x}{x} = 0$$

14)

$$\lim_{x \to +\infty} \frac{e^{3x}}{x^2} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x \to +\infty} \frac{3e^{3x}}{2x} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

$$= \lim_{x \to +\infty} \frac{9e^{3x}}{2}$$

$$= +\infty$$

Therefore,

$$\lim_{x \to +\infty} \frac{e^{3x}}{x^2} = +\infty$$

15/

$$\lim_{x\to 0^+} \frac{\cot x}{\ln x} \quad \left[\text{form } \frac{\infty}{-\infty}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x\to 0^+} \frac{-\operatorname{cosec}^2 x}{\frac{1}{x}}$$

$$= \lim_{x\to 0^+} \frac{-x}{\operatorname{cosec}^{-2} x} \quad \left[\text{form } \frac{0}{0}\right]$$

$$= \lim_{x\to 0^+} \frac{-1}{\sin 2x}$$

$$= \lim_{x\to 0^+} -\operatorname{cosec} 2x$$

$$= -\infty$$

Therefore,

$$\lim_{x\to 0^+} \frac{\cot x}{\ln x} = -\infty$$

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$$\lim_{x\to 0^+} \frac{1-\ln x}{e^{1/x}} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to 0^+} \frac{-\frac{1}{x}}{\left(-\frac{1}{x^2}\right)\cdot e^{1/x}}$$

$$= \lim_{x \to 0^+} \left(-\frac{1}{x}\right)\left(-\frac{x^2}{e^{1/x}}\right)$$

$$= \lim_{x \to 0^+} \frac{x}{e^{1/x}}$$

$$= 0$$

Therefore,

$$\lim_{x \to 0^+} \frac{1 - \ln x}{e^{1/x}} = 0$$

[7]

$$\lim_{x \to +\infty} \frac{x^{100}}{e^x} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x \to \infty} \frac{100 \cdot x^{99}}{e^x} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

$$= \lim_{x \to +\infty} \frac{100 \cdot 99 \cdot x^{98}}{e^x} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

. . . .

. . . .

$$= \lim_{x \to +\infty} \frac{100 \cdot 99 \cdot \cdots \cdot 1}{e^x}$$

$$= 0$$

Therefore,

$$\lim_{x \to +\infty} \frac{x^{100}}{e^x} = 0$$

18)

$$\lim_{x\to 0^+} \frac{\ln(\sin x)}{\ln(\tan x)} \quad \left[\text{form } \frac{-\infty}{-\infty}\right]$$

Now, using L' Hospital Rule

$$= \lim_{x\to 0^+} \frac{\frac{\cos x}{\sin x}}{\frac{\sec^2 x}{\tan x}}$$

$$= \lim_{x\to 0^+} \frac{\cos x}{\sin x} \cdot \frac{\tan x}{\sec^2 x}$$

$$= \lim_{x\to 0^+} \frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x} \cdot \cos^2 x$$

$$= \lim_{x\to 0^+} \cos^2 x$$

$$= 1$$

Therefore,

$$\lim_{x\to 0^+} \frac{\ln(\sin x)}{\ln(\tan x)} = 1$$

19)

$$\lim_{x\to 0} \frac{\sin^{-1} 2x}{x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-(2x)^2}} \cdot 2}{1}$$

$$= \lim_{x \to 0} \frac{2}{\sqrt{1-4x^2}}$$

$$= 2$$

Therefore,

$$\lim_{x \to 0} \frac{\sin^{-1} 2x}{x} = 2$$

20)

$$\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x \to 0} \frac{1 - \frac{1}{1+x^2}}{3x^2}$$

$$= \lim_{x \to 0} \frac{1+x^2-1}{(1+x^2) \cdot 3x^2}$$

$$= \lim_{x \to 0} \frac{1}{3(1+x^2)}$$

$$= \frac{1}{3}$$

Therefore,

$$\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3} = \frac{1}{3}$$

21)

$$\lim_{x\to\infty} xe^{-x} = \lim_{x\to+\infty} \frac{x}{e^x} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to+\infty} \frac{1}{e^x}$$

$$= 0$$

Therefore,

$$\lim_{x\to+\infty} x e^{-x} = 0$$

22)

$$\lim_{x\to\pi^-} (x-\pi) \tan\tfrac{1}{2}x = \lim_{x\to\pi^-} \frac{x-\pi}{\cot(\frac{x}{2})} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to\pi^-} \frac{1}{-\frac{1}{2}\cdot \operatorname{cosec}^2 \frac{x}{2}}$$

$$= \frac{1}{-\frac{1}{2}\cdot \operatorname{cosec}^2 \frac{\pi}{2}}$$

$$= -2$$

Therefore,

$$\lim_{x\to\pi^-} (x-\pi) \tan\tfrac{1}{2}x = -2$$

23)

$$\lim_{x\to+\infty} x\sin\frac{\pi}{x} = \lim_{x\to+\infty} \frac{\sin\frac{\pi}{x}}{\frac{1}{x}} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to+\infty} \frac{\frac{-\pi}{x^2}\cdot\cos\frac{\pi}{x}}{\frac{-1}{x^2}}$$

$$= \lim_{x\to+\infty} \frac{-\pi}{x^2}\cdot\cos\frac{\pi}{x}\cdot\frac{-x^2}{1}$$

$$= \lim_{x\to+\infty} \pi\cdot\cos\frac{\pi}{x}$$

$$= \pi$$

Therefore,

$$\lim_{x\to+\infty} x\sin\frac{\pi}{x} = \pi$$

24)

$$\lim_{x\to0^+} \tan x \ln x = \lim_{x\to0^+} \frac{\ln x}{\cot x} \quad \left[\text{form } \frac{-\infty}{\infty}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to0^+} \frac{\frac{1}{x}}{-\operatorname{cosec}^2 x}$$

$$= \lim_{x\to0^+} \frac{1}{x}\cdot\frac{1}{-\operatorname{cosec}^2 x}$$

$$= \lim_{x\to0^+} \frac{1}{x}\cdot(-\sin^2 x)$$

$$= \lim_{x \to 0^+} \frac{-\sin^2 x}{x} \quad \left[\text{form } \frac{-0}{0}\right]$$

$$= \lim_{x \to 0^+} \frac{-2\sin x \cos x}{1}$$

$$= 0$$

Therefore,

$$\lim_{x \to 0^+} \tan x \ln x = 0$$

25)

$$\lim_{x \to \frac{\pi}{2}^-} \sec 3x \cos 5x = \lim_{x \to \frac{\pi}{2}^-} \frac{\cos 5x}{\cos 3x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{-5\sin 5x}{-3\sin 3x}$$

$$= \frac{-5(1)}{-3(-1)}$$

$$= -\frac{5}{3}$$

Therefore,

$$\lim_{x \to \frac{\pi}{2}^-} \sec 3x \cos 5x = -\frac{5}{3}$$

26)

$$\lim_{x\to\pi} (x-\pi)\cot x = \lim_{x\to\pi} \frac{x-\pi}{\tan x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x\to\pi} \frac{1}{\sec^2 x}$$

$$= \frac{1}{\sec^2 \pi}$$

$$= 1$$

Therefore,

$$\lim_{x\to\pi} (x-\pi)\cot x = 1$$

27)

$$\lim_{x\to+\infty} (1-3/x)^x \quad [\text{form } 1^\infty]$$

Let,

$$y = \left(1-\frac{3}{x}\right)^x$$

taking ln both side,

$$\ln y = \ln\left(1-\frac{3}{x}\right)^x$$

$$\ln y = x \cdot \ln\left(1-\frac{3}{x}\right)$$

$$\ln y = \frac{\ln\left(1-\frac{3}{x}\right)}{\frac{1}{x}}$$

Taking Limit both side,

$$\lim_{x \to +\infty} \ln y = \lim_{x \to +\infty} \frac{\ln\left(1-\frac{3}{x}\right)}{\frac{1}{x}} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x \to +\infty} \frac{\frac{1}{1-\frac{3}{x}} \cdot \frac{3}{x^2}}{\frac{-1}{x^2}}$$

$$= \lim_{x \to +\infty} \frac{-3}{1-\frac{3}{x}}$$

$$= -3$$

Now,

$$\lim_{x \to +\infty} \ln y = -3$$

$$\Rightarrow \ln \lim_{x \to +\infty} y = -3$$

$$\Rightarrow \lim_{x \to +\infty} \left(1-\frac{3}{x}\right)^x = e^{-3}$$

Therefore,

$$\lim_{x \to +\infty} \left(1-\frac{3}{x}\right)^x = e^{-3}$$

28)

$$\lim_{x\to 0} (1+2x)^{-\frac{3}{x}}$$

Let,

$$y = (1+2x)^{-\frac{3}{x}}$$

taking ln both side,

$$\ln y = \ln (1+2x)^{-\frac{3}{x}}$$

$$\Rightarrow \ln y = -\frac{3}{x} \ln (1+2x)$$

$$\Rightarrow \ln y = \frac{-3 \ln (1+2x)}{x}$$

taking limit both side,

$$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{-3 \ln (1+2x)}{x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x\to 0} \frac{-3 \cdot \frac{1}{1+2x} \cdot 2}{1}$$

$$= \lim_{x\to 0} \frac{-6}{1+2x}$$

$$= -6$$

Now,

$$\lim_{x \to 0} \ln y = -6$$

$$\Rightarrow \ln \lim_{x \to 0} y = -6$$

$$\therefore \lim_{x \to 0} (1+2x)^{-\frac{3}{x}} = e^{-6}$$

Therefore,

$$\lim_{x \to 0} (1+2x)^{-\frac{3}{x}} = e^{-6}$$

29)

$$\lim_{x \to 0} (e^x + x)^{\frac{1}{x}}$$

Let,

$$y = (e^x + x)^{\frac{1}{x}}$$

taking ln both side,

$$\ln y = \ln (e^x + x)^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x} \cdot \ln (e^x + x)$$

$$\ln y = \frac{\ln (e^x + x)}{x}$$

taking limit both side,

$$\lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{\ln (e^x + x)}{x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital rule,

$$= \lim_{x \to 0} \frac{e^x + 1}{e^x + x}$$

$$= \frac{1+1}{1+0}$$

$$= 2$$

Now,

$$\lim_{x \to 0} \ln y = 2$$

$$\Rightarrow \ln \lim_{x \to 0} y = 2$$

$$\therefore \lim_{x \to 0} (e^x + x)^{\frac{1}{x}} = e^2$$

Therefore,

$$\lim_{x \to 0} (e^x + x)^{\frac{1}{x}} = e^2$$

30)

$$\lim_{x \to +\infty} \left(1 + \frac{a}{x}\right)^{bx}$$

Let,

$$y = \left(1 + \frac{a}{x}\right)^{bx}$$

taking ln both side,

$$\ln y = \ln \left(1 + \frac{a}{x}\right)^{bx}$$

$$\Rightarrow \ln y = bx \ln \left(1 + \frac{a}{x}\right)$$

$$\Rightarrow \ln y = \frac{b \ln\left(1+\frac{a}{x}\right)}{\frac{1}{x}}$$

taking limit both side,

$$\lim_{x \to +\infty} \ln y = \lim_{x \to +\infty} \frac{b \ln\left(1+\frac{a}{x}\right)}{\frac{1}{x}} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x \to +\infty} \frac{b \cdot \frac{1}{1+\frac{a}{x}} \cdot a \cdot \frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x \to +\infty} \frac{ab}{1+\frac{a}{x}}$$

$$= ab$$

Now,

$$\lim_{x \to +\infty} \ln y = ab$$

$$\Rightarrow \ln\left(\lim_{x \to \infty} y\right) = ab$$

$$\therefore \lim_{x \to +\infty} \left(1+\frac{a}{x}\right)^{bx} = e^{ab}$$

Therefore,

$$\lim_{x \to +\infty} \left(1+\frac{a}{x}\right)^{bx} = e^{ab}$$

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$$\lim_{x\to 1} (2-x)^{\tan\left[\frac{\pi}{2}\cdot x\right]}$$

Let,

$$y = (2-x)^{\tan\left(\frac{\pi x}{2}\right)}$$

taking ln both side,

$$\ln y = \ln (2-x)^{\tan\left(\frac{\pi x}{2}\right)}$$

$$\Rightarrow \ln y = \tan\frac{\pi x}{2} \cdot \ln(2-x)$$

$$\Rightarrow \ln y = \frac{\ln(2-x)}{\cot\frac{\pi x}{2}}$$

taking limit both side,

$$\lim_{x\to 1} \ln y = \lim_{x\to 1} \frac{\ln(2-x)}{\cot\frac{\pi x}{2}} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to 1} \frac{\frac{1}{2-x}\cdot(-1)}{-\operatorname{cosec}^2\frac{\pi x}{2}\cdot\frac{\pi}{2}}$$

$$= \lim_{x\to 1} \frac{2\sin^2\frac{\pi x}{2}}{\pi(2-x)}$$

$$= \frac{2\sin^2\frac{\pi}{2}}{\pi(2-1)} = \frac{2}{\pi}$$

Now,

$$\lim_{x\to 1} \ln y = \frac{2}{\pi}$$

$$\therefore \lim_{x\to 1} (2-x)^{\tan\frac{\pi}{2}x} = e^{2/\pi}$$

Therefore,

$$\lim_{x\to 1} (2-x)^{\tan\frac{\pi}{2}x} = e^{2/\pi}$$

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$$\lim_{x\to +\infty} \left(\cos\frac{2}{x}\right)^{x^2}$$

Let,

$$y = \left(\cos\frac{2}{x}\right)^{x^2}$$

taking ln both side,

$$\ln y = \ln\left(\cos\frac{2}{x}\right)^{x^2}$$

$$\Rightarrow \ln y = x^2 \ln\left(\cos\frac{2}{x}\right)$$

$$\Rightarrow \ln y = \frac{\ln\left(\cos\frac{2}{x}\right)}{\frac{1}{x^2}}$$

taking limit both side,

$$\lim_{x\to+\infty} \ln y = \lim_{x\to+\infty} \frac{\ln\left(\cos\frac{2}{x}\right)}{\frac{1}{x^2}} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to+\infty} \frac{\frac{1}{\cos\frac{2}{x}} \cdot -\sin\frac{2}{x} \cdot \frac{-2}{x^2}}{\frac{-2}{x^3}}$$

$$= -\lim_{x\to+\infty} \frac{\tan\frac{2}{x}}{\frac{1}{x}} \quad \left[\text{form } \frac{0}{0}\right]$$

$$= -\lim_{x\to+\infty} \frac{\sec^2\frac{2}{x} \cdot \frac{-2}{x^2}}{\frac{-1}{x^2}}$$

$$= -2 \lim_{x\to+\infty} \sec^2\left(\frac{2}{x}\right)$$

$$= -2$$

Now,

$$\lim_{x\to+\infty} \ln y = -2$$

$$\therefore \lim_{x\to+\infty} \left(\cos\frac{2}{x}\right)^{x^2} = e^{-2}$$

Therefore,

$$\lim_{x\to+\infty} \left(\cos\frac{2}{x}\right)^{x^2} = e^{-2}$$

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$$\lim_{x \to 0} \left(\operatorname{cosec} x - \frac{1}{x}\right) \qquad [\infty - \infty]$$

$$= \lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$$

$$= \lim_{x \to 0} \frac{x - \sin x}{x \sin x} \qquad [\text{form } \tfrac{0}{0}]$$

Now, using L' Hospital Rule,

$$= \lim_{x \to 0} \frac{1 - \cos x}{x \cos x + \sin x} \qquad [\text{form } \tfrac{0}{0}]$$

$$= \lim_{x \to 0} \frac{\sin x}{\cos x + \cos x - x \sin x}$$

$$= \lim_{x \to 0} \frac{\sin x}{2\cos x - x \sin x}$$

$$= \frac{0}{2 \cdot 1 - 0}$$

$$= 0$$

Therefore,

$$\lim_{x \to 0} \left(\operatorname{cosec} x - \frac{1}{x}\right) = 0$$

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$$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{\cos 3x}{x^2}\right)$$

$$= \lim_{x\to 0}\frac{1-\cos 3x}{x^2} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x\to 0}\frac{3\sin 3x}{2x} \quad \left[\text{form } \frac{0}{0}\right]$$

$$= \lim_{x\to 0}\frac{9\cos 3x}{2}$$

$$= \frac{9}{2}$$

Therefore,

$$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{\cos 3x}{x^2}\right) = \frac{9}{2}$$

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$$\lim_{x\to +\infty}\left(\sqrt{x^2+x}-x\right) \quad [\text{form } \infty-\infty]$$

$$= \lim_{x\to +\infty}\frac{\sqrt{x^2+x}-x}{1}$$

$$= \lim_{x\to +\infty}\frac{\left(\sqrt{x^2+x}-x\right)\left(\sqrt{x^2+x}+x\right)}{\left(\sqrt{x^2+x}+x\right)}$$

$$= \lim_{x \to +\infty} \frac{x^2+x-x^2}{\sqrt{x^2+x}+x}$$

$$= \lim_{x \to +\infty} \frac{x}{\sqrt{x^2+x}+x} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x \to +\infty} \frac{1}{\sqrt{1+\frac{1}{x}}+1}$$

$$= \frac{1}{2}$$

Therefore,

$$\lim_{x \to \infty} \left(\sqrt{x^2+x}-x\right) = \frac{1}{2}$$

36)

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x-1}\right)$$

$$= \lim_{x \to 0} \frac{e^x-1-x}{x(e^x-1)}$$

$$= \lim_{x \to 0} \frac{e^x-1-x}{xe^x-x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x \to 0} \frac{e^x-1}{xe^x+e^x-1} \quad \left[\text{form } \frac{0}{0}\right]$$

$$= \lim_{x \to 0} \frac{e^x}{xe^x + 2e^x}$$

$$= \frac{1}{2}$$

Therefore,

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right) = \frac{1}{2}.$$

37)

$$\lim_{x \to +\infty} [x - \ln(x^2+1)] \quad [\text{form } \infty - \infty]$$

Let,

$$y = x - \ln(x^2+1)$$

$$e^y = e^{x - \ln(x^2+1)}$$

$$= e^x \cdot e^{-\ln(x^2+1)}$$

$$= e^x \cdot e^{\ln\left(\frac{1}{x^2+1}\right)}$$

$$= e^x \cdot \frac{1}{x^2+1}$$

$$\lim_{x \to +\infty} e^y = \lim_{x \to +\infty} \frac{e^x}{x^2+1} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

$$= \lim_{x \to +\infty} \frac{e^x}{2x} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

$$= \lim_{x \to +\infty} \frac{e^x}{2}$$

$$= +\infty$$

Now,

$$\lim_{x \to +\infty} e^y = +\infty$$

$$\Rightarrow \lim_{x \to +\infty} y = \ln(+\infty)$$

$$\lim_{x \to +\infty} [x - \ln(x^2+1)] = \infty$$

Therefore,

$$\lim_{x \to +\infty} [x - \ln(x^2+1)] = +\infty$$

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$$\lim_{x \to +\infty} [\ln x - \ln(1+x)] \qquad [\text{form } \infty - \infty]$$

$$= \lim_{x \to +\infty} \left(\ln \frac{x}{1+x}\right)$$

$$= \lim_{x \to +\infty} \ln \frac{1}{\frac{1}{x}+1}$$

$$= \ln \frac{\lim_{x \to +\infty} 1}{\lim_{x \to +\infty} \left(\frac{1}{x}+1\right)}$$

$$= \ln(1)$$

$$= 0$$

Therefore,

$$\lim_{x \to +\infty} [\ln x - \ln(1+x)] = 0$$

39)

$$\lim_{x\to 0^+} x^{\sin x}$$

Let,

$$Y = x^{\sin x}$$

taking ln both side,

$$\ln y = \ln x^{\sin x}$$

$$\ln y = \sin x \ln x$$

$$\ln y = \frac{\ln x}{\operatorname{cosec} x}$$

taking limit both side,

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln x}{\operatorname{cosec} x} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\operatorname{cosec} x \cot x}$$

$$= \lim_{x\to 0^+} \frac{\sin x}{x}(-\tan x)$$

$$= 1\cdot(-0)$$

$$\lim_{x\to 0^+} \ln y = 0$$

$$\therefore \lim_{x\to 0^+} x^{\sin x} = e^0 = 1$$

Therefore, $\lim_{x\to 0^+} x^{\sin x} = 1$

40)

$$\lim_{x \to 0^+} (e^{2x} - 1)^x$$

Let,

$$y = (e^{2x} - 1)^x$$

taking ln both side,

$$\ln y = \ln (e^{2x} - 1)^x$$

$$\ln y = x \cdot \ln (e^{2x} - 1)$$

$$\ln y = \frac{\ln (e^{2x} - 1)}{\frac{1}{x}}$$

taking limit both side,

$$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \frac{\ln (e^{2x} - 1)}{\frac{1}{x}} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x \to 0^+} \frac{-2 e^{2x} \cdot x^2 \cancel{(\quad)}}{e^{2x} - 1} \quad \left[\text{form } \frac{0}{0}\right]$$

$$= \lim_{x \to 0^+} \frac{-2 \cdot e^{2x} \cdot 2 \cdot 2x}{e^{2x} \cdot 2}$$

$$= \lim_{x \to 0^+} \frac{-8xe^{2x}}{2e^{2x}}$$

$$= \frac{0}{2}$$

$$= 0$$

Therefore, $\lim_{x \to 0^+} (e^{2x} - 1)^x = e^0 = 1$

41)

$$\lim_{x\to 0^+}\left[-\frac{1}{\ln x}\right]^x$$

Let,

$$y=\left[-\frac{1}{\ln x}\right]^x$$

taking ln both side,

$$\ln y = \ln\left(-\frac{1}{\ln x}\right)^x$$

$$= x\ln\left(-\frac{1}{\ln x}\right)$$

taking limit,

$$\lim_{x\to 0^+}\ln y = \lim_{x\to 0^+}\frac{\ln\left(-\frac{1}{\ln x}\right)}{\frac{1}{x}} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

using L' Hospital Rule,

$$= \lim_{x\to 0^+}\frac{x^2}{x\ln x}$$

$$= \lim_{x\to 0^+}\frac{x}{\ln x}$$

$$\lim_{x\to 0^+}\ln y = 0$$

$$\therefore \lim_{x\to 0^+}\left(-\frac{1}{\ln x}\right)^x = e^0 = 1$$

Therefore

$$\lim_{x\to 0^+}\left(-\frac{1}{\ln x}\right)^x = 1$$

42/

$$\lim_{x \to +\infty} x^{\frac{1}{x}} \quad [\text{form } \infty^0]$$

Let,

$$y = x^{\frac{1}{x}}$$

taking ln both side,

$$\ln y = \ln x^{1/x}$$

$$\ln y = \frac{1}{x} \ln x$$

taking limit,

$$\lim_{x \to +\infty} \ln y = \lim_{x \to +\infty} \frac{\ln x}{x} \quad [\text{form } \frac{\infty}{\infty}]$$

Using L' Hospital Rule,

$$= \lim_{x \to +\infty} \frac{\frac{1}{x}}{1}$$

$$= \lim_{x \to \infty} \frac{1}{x}$$

$$\lim_{x \to +\infty} \ln y = 0$$

$$\therefore \lim_{x \to +\infty} x^{1/x} = e^0 = 1$$

Therefore,

$$\lim_{x \to +\infty} x^{1/x} = 1$$

43)

$$\lim_{x\to+\infty} (\ln x)^{1/x} \quad [\text{form } \infty^0]$$

Let

$$y = (\ln x)^{1/x}$$

taking ln both side

$$\ln y = \ln (\ln x)^{1/x}$$

$$\ln y = \frac{1}{x} \ln\ln x$$

taking limit,

$$\lim_{x\to+\infty} \ln y = \lim_{x\to+\infty} \frac{\ln\ln x}{x} \quad [\text{for } \tfrac{\infty}{\infty}]$$

using L' Hospital Rule,

$$= \lim_{x\to+\infty} \frac{\frac{1}{x\ln x}}{1}$$

$$= \lim_{x\to+\infty} \frac{1}{x\ln x}$$

$$\lim_{x\to+\infty} \ln y = 0$$

$$\therefore \lim_{x\to+\infty} y = e^0 = 1$$

Therefore,

$$\lim_{x\to+\infty} (\ln x)^{1/x} = 1$$

441

$$\lim_{x\to 0^+} (-\ln x)^x \quad [\text{form } \infty^0]$$

Let,

$$y = (-\ln x)^x$$

taking ln both side,

$$\ln y = \ln (-\ln x)^x$$

$$\ln y = x \ln(-\ln x)$$

taking limit,

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(-\ln x)}{\frac{1}{x}} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

using L' Hospital Rule,

$$= \lim_{x\to 0^+} \frac{1}{-\ln x} \cdot \frac{1}{-x} \cdot \frac{x^2}{-1}$$

$$= \lim_{x\to 0^+} \frac{-x}{\ln x}$$

$$\lim_{x\to 0^+} \ln y = 0$$

$$\therefore \lim_{x\to 0^+} y = e^0 = 1$$

Therefore,

$$\lim_{x\to 0^+} (-\ln x)^x = 1$$

451

$$\lim_{x \to \frac{\pi}{2}^-} (\tan x)^{\frac{\pi}{2}-x}$$

Let, $y = (\tan x)^{\frac{\pi}{2}-x}$

$\ln y = \ln (\tan x)^{\frac{\pi}{2}-x}$ [taking ln both side]

$\ln y = (\frac{\pi}{2}-x) \ln(\tan x)$

taking limit,

$$\lim_{x \to \frac{\pi}{2}^-} \ln y = \lim_{x \to \frac{\pi}{2}^-} \frac{\ln(\tan x)}{\frac{1}{\frac{\pi}{2}-x}} \quad [\text{form } \frac{\infty}{\infty}]$$

using L' Hospital Rule,

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{\frac{\sec^2 x}{\tan x}}{\frac{1}{(\frac{\pi}{2}-x)^2}}$$

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{\frac{\pi}{2}-x}{\cos x} \cdot \frac{\frac{\pi}{2}-x}{\sin x}$$

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{\frac{\pi}{2}-x}{\cos x} \cdot \lim_{x \to \frac{\pi}{2}^-} \frac{\frac{\pi}{2}-x}{\sin x}$$

$$\lim_{x \to \frac{\pi}{2}^-} \ln y = 1 \cdot 0 = 0$$

$$\therefore \lim_{x \to \frac{\pi}{2}^-} y = e^0 = 1$$

Therefore,

$$\lim_{x \to \frac{\pi}{2}^-} (\tan x)^{\frac{\pi}{2}-x} = 1$$