North South University

Department of Mathematics and Physics

Assignment-4

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Section : 8

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6.4

4)

$$x = \frac{1}{3}(y^2+2)^{3/2} \quad ; \text{ from } y=0 \text{ to } y=1.$$

$$\frac{dx}{dy} = \frac{1}{3}\cdot\frac{3}{2}(y^2+2)^{\frac{1}{2}} \cdot \frac{d}{dy}(y^2+2)$$

$$= \frac{1}{2}(y^2+2)^{\frac{1}{2}} \cdot 2y$$

$$= y\sqrt{y^2+2}$$

$$\therefore \text{Arc length} = \int_0^1 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_0^1 \sqrt{1+y^2(y^2+2)}\, dy$$

$$= \int_0^1 \sqrt{1+y^4+2y^2}\, dy$$

$$= \int_0^1 \sqrt{(y^2+1)^2}\, dy$$

$$= \int_0^1 (y^2+1)\, dy$$

$$= \left.\frac{y^3}{3}+y\right]_0^1 = \frac{1}{3}+1 = \frac{4}{3} \text{ Ans}$$

6)

$$y = \frac{x^6+8}{16x^2} \quad \text{from } x=2 \text{ to } x=3$$

$$\frac{dy}{dx} = \frac{16x^2 \cdot 6x^5 - (x^6+8)\,32x}{256\,x^4}$$

$$= \frac{96x^7 - 32x^7 - 256x}{256x^4}$$

$$= \frac{64x^7 - 256x}{256x^4}$$

$$= \frac{64x\,(x^6-4)}{256\,x^4}$$

$$= \frac{x^6-4}{4x^3}$$

$$= \frac{1}{4}x^3 - x^{-3}$$

$$\therefore \left(\frac{dy}{dx}\right)^2 = \left(\frac{1}{4}x^3 - x^{-3}\right)^2$$

$$= \frac{1}{16}x^6 - 2\cdot\frac{1}{4}x^3\cdot x^{-3} + (x^{-3})^2$$

$$= \frac{1}{16}x^6 - \frac{1}{2} + x^{-6}$$

$\therefore$ Arc length $= \displaystyle\int_2^3 \sqrt{1 + \frac{1}{16}x^6 - \frac{1}{2} + x^{-6}}\ dx$

$= \displaystyle\int_2^3 \sqrt{\frac{1}{16}x^6 + \frac{1}{2} + x^{-6}}\ dx$

$= \displaystyle\int_2^3 \sqrt{\left(\frac{1}{4}x^3 + x^{-3}\right)^2}\ dx$

$= \displaystyle\int_2^3 \left(\frac{1}{4}x^3 + x^{-3}\right) dx$

$= \left.\frac{1}{4}\cdot\frac{x^4}{4} + \frac{x^{-2}}{-2}\right]_2^3$

$= \left.\frac{1}{16}x^4 - \frac{1}{2}\cdot\frac{1}{x^2}\right]_2^3$

$= \frac{3^4}{16} - \frac{1}{2\cdot 3^2} - \frac{2^4}{16} + \frac{1}{2\cdot 2^2}$

$= \frac{595}{144}$ Ans.

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$x = \frac{1}{8} y^4 + \frac{1}{4} y^{-2}$ from $y = 1$ to $y = 4$

$$\frac{dx}{dy} = \frac{1}{8} \cdot 4y^3 + \frac{1}{4} \cdot (-2) \cdot y^{-3}$$

$$= \frac{1}{2} y^3 - \frac{1}{2} y^{-3}$$

$$\left(\frac{dx}{dy}\right)^2 = \left(\frac{1}{2} y^3 - \frac{1}{2} y^{-3}\right)^2$$

$$= \frac{1}{4} y^6 - 2 \cdot \frac{1}{2} y^3 \cdot \frac{1}{2} y^{-3} + \frac{1}{4} y^{-6}$$

$$= \frac{1}{4} y^6 - \frac{1}{2} + \frac{1}{4} y^{-6}$$

$\therefore$ Arc length $= \int_1^4 \sqrt{1 + \frac{1}{4} y^6 - \frac{1}{2} + \frac{1}{4} y^{-6}} \, dy$

$$= \int_1^4 \sqrt{\frac{1}{4} y^6 + \frac{1}{2} + \frac{1}{4} y^{-6}} \, dy$$

$$= \int_1^4 \sqrt{\left(\frac{1}{2} y^3 + \frac{1}{2} y^{-3}\right)^2} \, dy$$

$$= \int_1^4 \left(\frac{1}{2} y^3 + \frac{1}{2} y^{-3}\right) dy$$

$$= \left. \frac{1}{2} \cdot \frac{y^4}{4} + \frac{1}{2} \cdot \frac{y^{-2}}{-2} \right]_1^4$$

$$= \left. \frac{1}{8} y^4 - \frac{1}{4} y^{-2} \right]_1^4$$

$$= \frac{4^4}{8} - \frac{1}{4 \cdot 4^2} - \frac{1^4}{8} + \frac{1}{4 \cdot 1^2}$$

$$= \frac{2055}{64} \text{ Ans.}$$

30)

$x = \cos t + t \sin t$

$y = \sin t - t \cos t$; $0 \leq t \leq \pi$

$$\frac{dx}{dt} = -\sin t + t \cos t + \sin t$$

$$= t \cos t$$

$$\left(\frac{dx}{dt}\right)^2 = t^2 \cos^2 t$$

$$\frac{dy}{dt} = \cos t + t \sin t - \cos t$$

$$= t \sin t$$

$$\left(\frac{dy}{dt}\right)^2 = t^2 \sin^2 t$$

$$\therefore L = \int_0^{\pi} \sqrt{t^2\cos^2 t + t^2\sin^2 t}\ dt$$

$$= \int_0^{\pi} \sqrt{t^2(\cos^2 t + \sin^2 t)}\ dt$$

$$= \int_0^{\pi} t\ dt$$

$$= \left.\frac{t^2}{2}\right]_0^{\pi}$$

$$= \frac{\pi^2}{2} \quad \underline{\text{Ans.}}$$

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$$x = e^t(\sin t + \cos t) \quad ; \quad 1 \le t \le 4$$

$$y = e^t(\cos t - \sin t)$$

$$\frac{dx}{dt} = \frac{d}{dt}(e^t \sin t + e^t \cos t)$$

$$= e^t \cos t + e^t \sin t + e^t \sin t + e^t \cos t$$

$$= 2e^t \cos t$$

$$\left(\frac{dx}{dt}\right)^2 = 4e^{2t}\cos^2 t$$

$$\frac{dy}{dt} = \frac{d}{dt}\left(e^t \cos t - e^t \sin t\right)$$

$$= -e^t \sin t + e^t \cos t - e^t \sin t - e^t \cos t$$

$$= -2\, e^t \sin t$$

$$\left(\frac{dy}{dt}\right)^2 = 4\, e^{2t} \sin^2 t$$

$$\therefore L = \int_1^4 \sqrt{4 e^{2t} \cos^2 t + 4 e^{2t} \sin^2 t}\; dt$$

$$= \int_1^4 \sqrt{4 e^{2t} (\cos^2 t + \sin^2 t)}\; dt$$

$$= \int_1^4 2 \sqrt{e^{2t}}\; dt$$

$$= \int_1^4 2e^t \; dt$$

$$= 2e^t \Big]_1^4$$

$$= 2e^4 - 2e^1$$

$$= 2\,(e^4 - e)$$

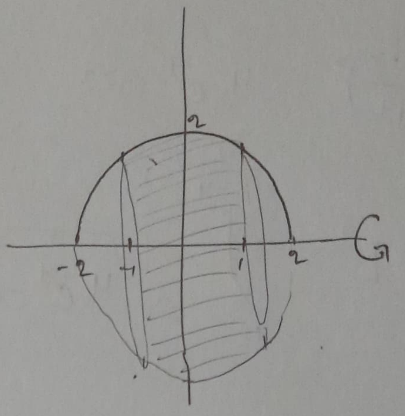
Ans.

6.5

3)

$y = \sqrt{4-x^2}$; $-1 \le x \le 1$

revolved about x-axis



$\therefore S.A. = 2\pi \int_{-1}^{1} \sqrt{4-x^2} \cdot \sqrt{1+\left(\frac{d}{dx}\left(\sqrt{4-x^2}\right)\right)^2}\, dx$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \cdot \frac{d}{dx}(4-x^2)$

$= \frac{1}{2\sqrt{4-x^2}} \cdot (-2x)$

$= -\frac{x}{\sqrt{4-x^2}}$

$\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{4-x^2}$

$$\therefore SA = 2\pi \int_{-1}^{1} \sqrt{4-x^2} \cdot \sqrt{1+\frac{x^2}{4-x^2}} \, dx$$

$$= 2\pi \int_{-1}^{1} \sqrt{4-x^2} \sqrt{\frac{4}{4-x^2}} \, dx$$

$$= 2\pi \int_{-1}^{1} \sqrt{4-x^2} \cdot \frac{2}{\sqrt{4-x^2}} \, dx$$

$$= 2\pi \int_{-1}^{1} 2 \, dx$$

$$= 4\pi \int_{-1}^{1} dx$$

$$= 4\pi [x]_{-1}^{1}$$

$$= 4\pi (1+1)$$

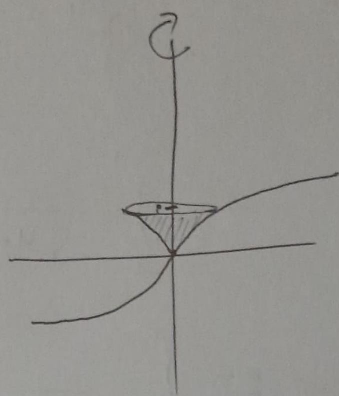
$$= 8\pi$$

Ans

6]

$x = y^3 \; ; \; 0 \le y \le 1$

revolved about y-axis



$$\frac{dx}{dy} = 3y^2$$

$$\left(\frac{dx}{dy}\right)^2 = 9y^4$$

$$\therefore S.A. = 2\pi \int_0^1 y^3 \cdot \sqrt{1+9y^4} \; dy$$

Let,

$u = 1+9y^4$

$du = 36y^3 \, dy$

$y^3 dy = \frac{1}{36} du$

y	u
0	1
1	10

$$= 2\pi \int_1^{10} \sqrt{u} \cdot \frac{1}{36} du$$

$$= \frac{\pi}{18} \int_1^{10} \sqrt{u} \; du$$

$$= \frac{\pi}{18} \int_1^{10} u^{\frac{1}{2}} \; du$$

$$= \frac{\pi}{18} \cdot \left[\frac{u^{3/2}}{3/2} \right]_1^{10}$$

$$= \frac{\pi}{18} \cdot \frac{2}{3} \left[u^{3/2} \right]_1^{10}$$

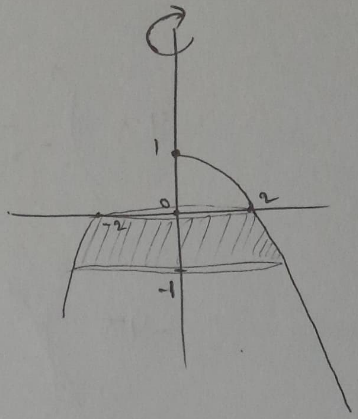
$$= \frac{\pi}{27} \left(10^{3/2} - 1\right)$$

Ans.

8)

$x = 2\sqrt{1-y}$; $-1 \le y \le 0$

revolved about y-axis



$$\frac{dx}{dy} = 2 \cdot \frac{1}{2\sqrt{1-y}} \cdot (-1) + 0$$

$$= -\frac{1}{\sqrt{1-y}}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{1}{1-y}$$

$$\therefore S.A. = 2\pi \int_{-1}^{0} 2\sqrt{1-y} \cdot \sqrt{1 + \frac{1}{1-y}} \; dy$$

$$= 2\pi \int_{-1}^{0} 2\sqrt{1-y} \sqrt{\frac{2-y}{1-y}} \; dy$$

$$= 2\pi \int_{-1}^{0} 2\sqrt{1-y} \cdot \frac{\sqrt{2-y}}{\sqrt{1-y}} \; dy$$

$$= 2\pi \int_{-1}^{0} 2\sqrt{2-y}\; dy$$

$$= 4\pi \int_{-1}^{0} \sqrt{2-y}\; dy$$

Let,

$u = 2-y$

$du = -dy$

$dy = -du$

y	u
-1	3
0	2

$$= 4\pi \int_{3}^{2} -\sqrt{u}\; du$$

$$= -4\pi \cancel{\int_{3}^{2}} \left[\frac{u^{3/2}}{3/2}\right]_3^2$$

$$= -4\pi \cdot \frac{2}{3}\left[u^{3/2}\right]_3^2$$

$$= -\frac{8}{3}\pi\left(2^{3/2} - 3^{3/2}\right)$$

$$= -\frac{8}{3}\pi \cdot 2\sqrt{2} + \frac{8}{3}\pi \cdot 3\sqrt{3}$$

$$= 8\pi\sqrt{3} - \frac{16}{3}\pi\sqrt{2}$$

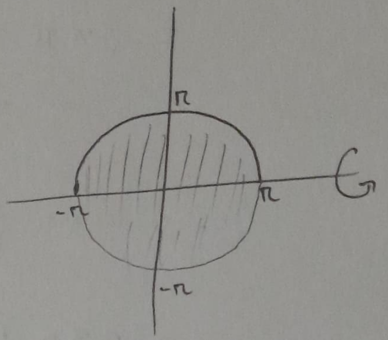
$$= 8\pi\left(\sqrt{3} - \frac{2}{3}\sqrt{2}\right)$$

An

25)

$y = \sqrt{r^2 - x^2}$

revolved about x-axis



$$\frac{dy}{dx} = \frac{1}{2\sqrt{r^2-x^2}} \cdot \frac{d}{dx}(r^2-x^2)$$

$$= \frac{1}{2\sqrt{r^2-x^2}} \cdot (-2x)$$

$$= -\frac{x}{\sqrt{r^2-x^2}}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{r^2-x^2}$$

$$\therefore S.A = 2\pi \int_{-r}^{r} \left(\sqrt{r^2-x^2} \cdot \sqrt{1 + \frac{x^2}{r^2-x^2}}\right) dx$$

$$= 2\pi \int_{-r}^{r} \left(\sqrt{r^2-x^2} \cdot \sqrt{\frac{r^2}{r^2-x^2}}\right) dx$$

$$= 2\pi \int_{-r}^{r} \left(\sqrt{r^2-x^2} \cdot \frac{r}{\sqrt{r^2-x^2}}\right) dx$$

$$= 2\pi \int_{-r}^{r} r \, dx$$

$$= 2\pi r \int_{-r}^{r} dx$$

$$= 2\pi r \left[x\right]_{-r}^{r}$$

$$= 2\pi r (r+r)$$

$$= 4\pi r^2$$

(Showed).

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$x = \cos^2 t$; $0 \le t \le \pi/2$
$y = \sin^2 t$

revolved about y-axis.

$$\frac{dx}{dt} = 2\cos t\,(-\sin t)$$

$$= -2\sin t \cos t$$

$$\left(\frac{dx}{dt}\right)^2 = 4\sin^2 t \cos^2 t$$

$$\frac{dy}{dt} = 2\sin t \cos t$$

$$\left(\frac{dy}{dt}\right)^2 = 4\sin^2 t \cos^2 t$$

$$\therefore S.A. = 2\pi \int_0^{\pi/2} \cos^2 t \sqrt{4\sin^2 t\cos^2 t + 4\sin^2 t \cos^2 t}\; dt$$

$$= 2\pi \int_0^{\pi/2} \cos^2 t \sqrt{8\sin^2 t\cos^2 t}\; dt$$

$$= 2\pi \cdot 2\sqrt{2} \int_0^{\pi/2} \cos^2 t \cdot \sin t \cdot \cos t \; dt$$

$$= 4\sqrt{2}\,\pi \int_0^{\pi/2} \cos^3 t \sin t \; dt$$

$$= 4\sqrt{2}\,\pi \int_0^{\pi/2} \cos^2 t \sin t \cos t \; dt$$

$$= 4\sqrt{2}\,\pi \int_0^{\pi/2} (1-\sin^2 t) \sin t \cos t \; dt$$

Let,
$u = \sin t$
$du = \cos t \, dt$

t	u
0	0
$\pi/2$	1

$$= 4\sqrt{2}\,\pi \int_0^{1} (1-u^2)\, u \; du$$

$$= 4\sqrt{2}\,\pi \int_0^{1} (u-u^3)\, du$$

$$= 4\sqrt{2}\,\pi \left[\frac{u^2}{2} - \frac{u^4}{4}\right]_0^1$$

$$= 4\sqrt{2}\,\pi \left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= \cancel{8\sqrt{2}}\ \sqrt{2}\,\pi$$

Ans.