Chapter - 1

06.06.2022

L-2

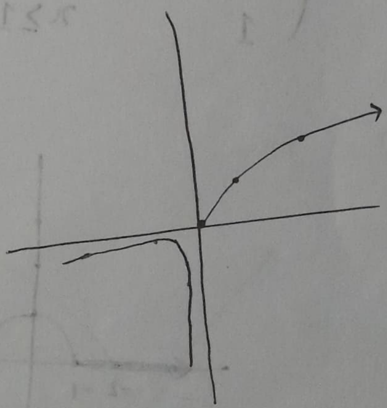
0.1

Piecewise Defined Function

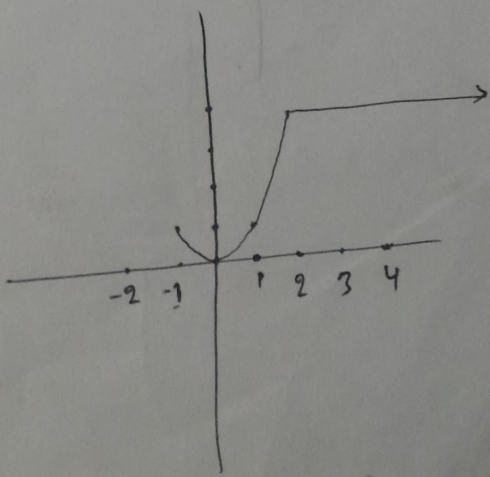
* $f(x) = |x|$

$$f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

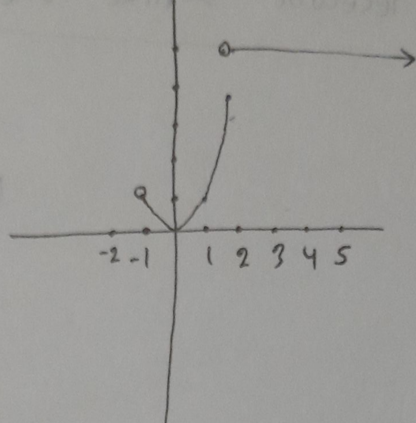
* $$f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ \frac{1}{x} & x < 0 \end{cases}$$



* $$f(x) = \begin{cases} x^2 & -1 \leq x \leq 2 \\ 4 & x > 2 \end{cases}$$



$$f(x) = \begin{cases} x^2 & -1 < x \le 2 \\ 5 & x > 2 \end{cases}$$

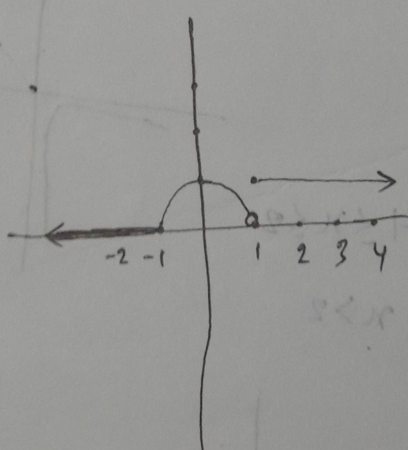


$y = +\sqrt{a^2 - x^2}$ upper half

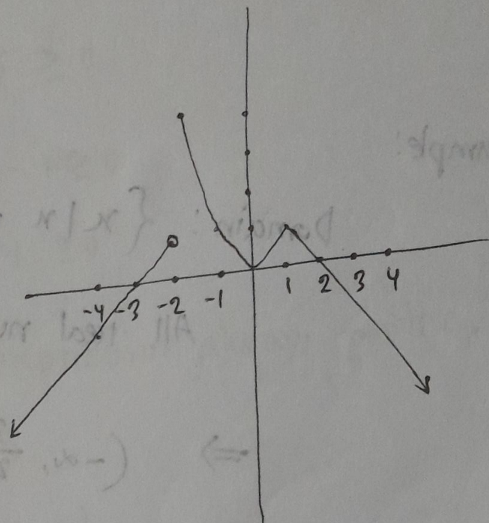
$y = -\sqrt{a^2 - x^2}$ lower half

Circle

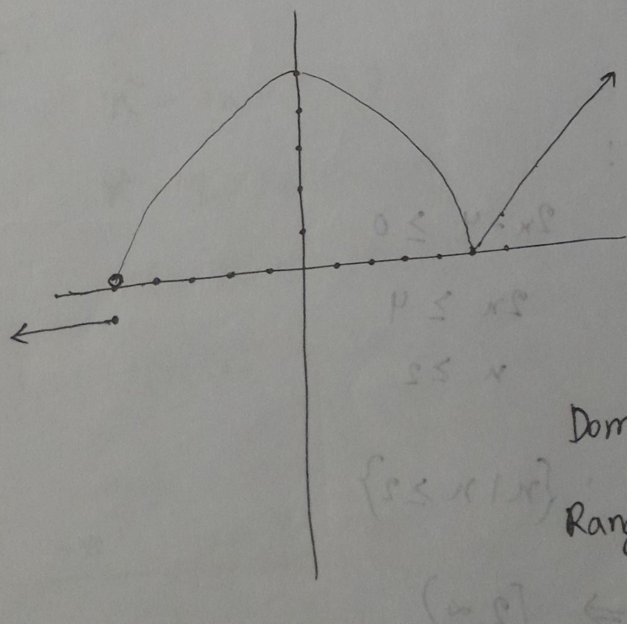
$$f(x) = \begin{cases} 0 & x \le -1 \\ \sqrt{1-x^2} & -1 < x < 1 \\ 1 & x \ge 1 \end{cases}$$



⊛ $f(x) = \begin{cases} x+3 & x < -2 \\ x^2 & -2 \leq x < 1 \\ -x+2 & x \geq 1 \end{cases}$



⊛ $f(x) = \begin{cases} -1 & x \leq -5 \\ \sqrt{25-x^2} & -5 < x < 5 \\ x-5 & x \geq 5 \end{cases}$



Domain : $(-\infty, \infty)$

Range : $[-1, \infty)$

⊛ If output is imaginary number or undefined then the input will not include in the domain.

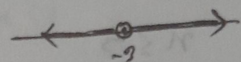
Sample:

Domain: $\{x \mid x \neq -\frac{3}{2}\}$

All real number except $-\frac{3}{2}$.

$\Rightarrow (-\infty, \frac{-3}{2}) \cup (-\frac{3}{2}, \infty)$

$\Rightarrow$



$-\frac{3}{2}$

⊛ $G(x) = \sqrt{2x-4}$

$\Rightarrow$

For domain:

$2x - 4 \geq 0$

$2x \geq 4$

$x \geq 2$

$\therefore$ Domain: $\{x \mid x \geq 2\}$

$\Rightarrow [2, \infty)$

⊛ $H(x) = \sqrt{x^2-2x+5}$

For domain:

$$x^2 - 2x + 5 \geq 0$$

$$x^2 - 2x + 1 - 4 \geq 0$$

$$(x-1)^2 + 4 \geq 0$$

This expression always positive.

Therefore,

Domain: $(-\infty, \infty)$

⊛ $f(x) = \sqrt{x^2-5x+6}$

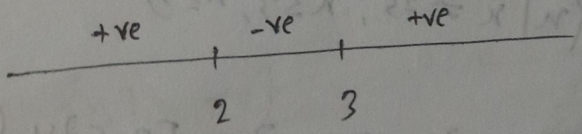
For domain:

$$x^2 - 5x + 6 \geq 0$$

$$x^2 - 3x - 2x + 6 \geq 0$$

$$x(x-3) - 2(x-3) \geq 0$$

$$(x-3)(x-2) \geq 0$$



∴ Domain: $(-\infty, 2] \cup [3, \infty)$

* $f(x) = 1 - 2x + x^2$

Domain : $(-\infty, \infty)$

* $f(x) = \sqrt{|x|}$

Domain: $(-\infty, \infty)$

* $f(x) = \sqrt{5x + 10}$

For domain,

$5x + 10 \geq 0$

$5x \geq -10$

$x \geq -2$

Domain : $[-2, \infty)$

* $f(x) = \dfrac{x+3}{4 - \sqrt{x^2 - 9}}$

For domain,

$4 - \sqrt{x^2-9} \neq 0$

$4 \neq \sqrt{x^2-9}$

$x^2 - 9 \neq 16$

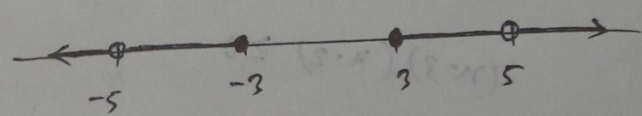
$x^2 \neq 25$

$x \neq \pm 5$

again,

$x^2 - 9 \geq 0$

$(x+3)(x-3) \geq 0$



Domain: $\{x \mid x \neq \pm 5,\ x \leq 3,\ x \geq 3\}$

$\Rightarrow (-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$

* $h(x) = \sin x$

Domain: $(-\infty, \infty)$

* $f(x) = \cos x$

Domain: $(-\infty, \infty)$

* $f(x) = \tan x = \frac{\sin x}{\cos x}$

Domain: $\{x \mid x \neq \frac{\pi}{2}, \frac{3\pi}{2} \dots\}$

* $f(x) = \sec x = \frac{1}{\cos x}$

Domain: $\{x \mid x \neq \frac{\pi}{2}, \frac{3\pi}{2} \dots\}$

* $f(x) = \cot x = \frac{\cos x}{\sin x}$

Domain: $\{x \mid x \neq 0, \pi, 2\pi, 3\pi \dots\}$

* $f(x) = \operatorname{cosec} x$

Domain: $\{x \mid x \neq 0, \pi, 2\pi \dots\}$

* $f(x) = \frac{1}{1 - \sin x}$

For domain:

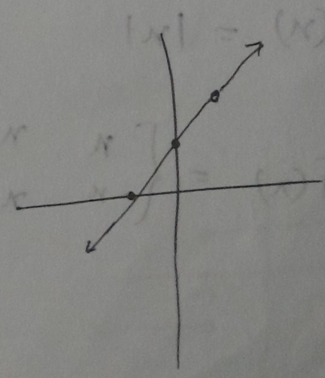
$$1 - \sin x \neq 0$$

$$\sin x \neq 1$$

$$x \neq \frac{\pi}{2}, \frac{9\pi}{2} \dots$$

* The effect of algebric operation on the domain.

$\Rightarrow f(x) = \frac{x^2 - 4}{x - 2}$

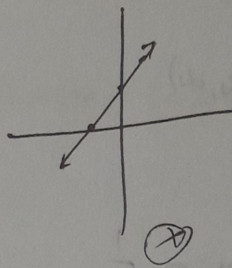


Domain: $\{x \mid x \neq 2\}$

$$f(x) = \frac{(x+2)(x-2)}{(x-2)}$$

$$= x+2$$

Domain : $(-\infty, \infty)$ ✗



Domain : $\{x \mid x \neq 2\}$ ✓

✱ $$f(x) = \frac{x\sqrt{x} + \sqrt{x}}{x+1}$$

$$= \frac{\sqrt{x}(x+1)}{x+1}$$

$$= \sqrt{x}$$

Domain : $[0, \infty)$

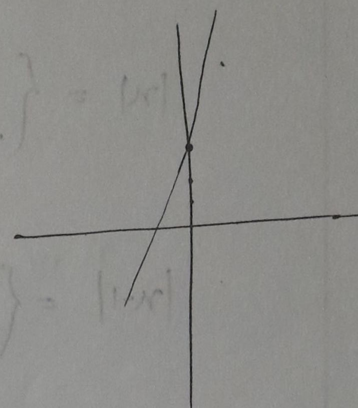
✱ Express the absolute value function in piecewise form.

① $f(x) = |x|$

$$\therefore f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

(ii) $f(x) = |x| + 2x + 3$

$$\therefore f(x) = \begin{cases} x + 2x + 3 & x \geq 0 \\ -x + 2x + 3 & x < 0 \end{cases}$$



$$= \begin{cases} 3x + 3 & x \geq 0 \\ x + 3 & x < 0 \end{cases}$$

(iii) $f(x) = |x-2|$

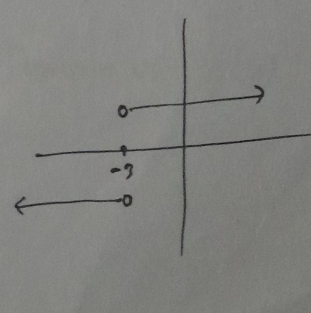
$$\Rightarrow f(x) = \begin{cases} (x-2) & x-2 \geq 0 \\ -(x-2) & x-2 < 0 \end{cases}$$

$$= \begin{cases} x-2 & x \geq 2 \\ 2-x & x < 2 \end{cases}$$

(iv) $f(x) = \dfrac{|x+3|}{x+3}$

$$\Rightarrow f(x) = \begin{cases} \dfrac{x+3}{x+3} & x+3 \geq 0 \\ \dfrac{-(x+3)}{x+3} & x+3 < 0 \end{cases}$$

$$= \begin{cases} 1 & x \geq -3 \\ -1 & x < -3 \end{cases}$$

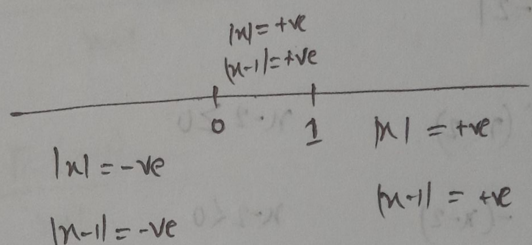


Domain: $\{x \mid x \neq -3\}$

(v) $g(x) = |x| + |x-1|$

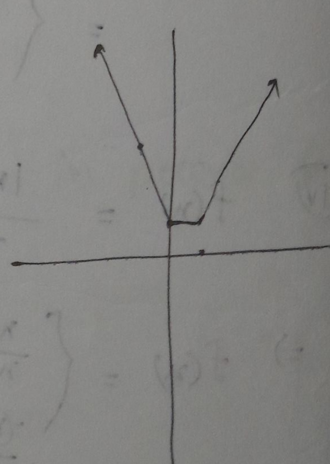
$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$|x-1| = \begin{cases} (x-1) & x \geq 1 \\ -(x-1) & x < 1 \end{cases}$$



$$\therefore f(x) = \begin{cases} -x-(x-1) & x \leq 0 \\ x-(x-1) & 0 < x < 1 \\ x+(x-1) & x \geq 1 \end{cases}$$

$$= \begin{cases} -2x+1 & x \leq 0 \\ 1 & 0 < x < 1 \\ 2x-1 & x \geq 1 \end{cases}$$



L-3 / 08.06.2022

0.2

New function from the old.

1. Operation of function:

i. Addition

ii. Substraction

iii. Multiplication

iv. Division

If $f(x)$ and $g(x)$ are two function, then

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(f.g)(x) = f(x) \cdot g(x)$

Domain: Interceection of the domain $f(x)$ and $g(x)$

$(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$ " and $g(x) \neq 0$

$f(x) = \sqrt{x-1}$ and $g(x) = x^2+2$

$\therefore (f+g)(x) = f(x) + g(x)$

$= \sqrt{x-1} + x^2+2$

Domain of $f(x)$: $[1, \infty)$

Domain of $g(x)$: $(-\infty, \infty)$

$\therefore$ Domain: $[1, \infty)$

$\therefore \left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$

$= \frac{x^2+2}{\sqrt{x-1}}$

Here, $\sqrt{x-1} \neq 0$

$x-1 \neq 0$

$x \neq 1$

$\therefore$ Domain: $(1, \infty)$

❀ Composit of function:

$f(x)$ and $g(x)$ are two function, then

$(f \circ g)(x) = f(g(x))$

outer function: f

inner function: $g(x)$

$(g \circ f)(x) = g(f(x))$

$(f \circ f)(x) = f(f(x))$

$(g \circ g)(x) = g(g(x))$

Domain of composit function:

calculation/resulting domain + inner function domain.

⊛ $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1}{x}$

$$\therefore (g \circ f)(x) = g(f(x))$$

$$= g\left(\frac{1}{x-1}\right)$$

$$= \frac{1}{\frac{1}{x-1}}$$

$$= x-1$$

Here, $x-1 \neq 0$

$x \neq \cancel{0}1$

$\therefore$ Domain: $\{x \mid x \neq 1\}$

(*) $f(x) = 2-x^2$ and $g(x) = \sqrt{x}$

$\therefore (f \circ g)(x) = f(g(x))$

$= f(\sqrt{x})$

$= 2-(\sqrt{x})^2$

$= 2-x$

Here,

$x \geq 0$

$\therefore$ Domain : $\{x \mid x \geq 0\}$

$\Rightarrow [0, \infty)$

3) Transformation / Translation

i. Shifting $\begin{cases} \to \text{Horizontal} \\ \to \text{Vertical} \end{cases}$

ii. Stretching / Compression $\begin{cases} \to \text{Horizontal} \\ \to \text{Vertical} \end{cases}$

iii. Reflection $\begin{cases} \to \text{Horizontal} \\ \to \text{Vertical} \end{cases}$

i. Shifting:

$f(x)+c \Rightarrow$ Vertical shifting by c units up.

$f(x)-c \Rightarrow$ Vertical shifting by c units down.

$f(x-c) \Rightarrow$ Horizontal shifting by c units right.

$f(x+c) \Rightarrow$ Horizontal shifting by c units left.

ii. Stretching / Compression

Vertical:

$c \cdot f(x) \Rightarrow$ Vertical stretching / Compression

if $c>1$, vertical streched

if $c<1$, vertical compressed

$f(c \cdot x) \Rightarrow$ Horizontal Stretching / Compression

if, $c>1$, Horizontally compressed

if, $c<1$, Horizontally stretched

iii. Reflection: Multiply by -1.

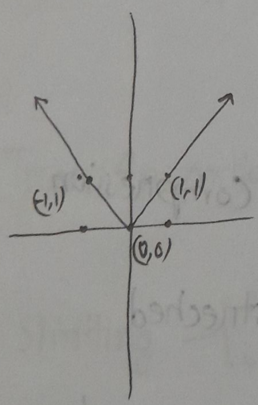
$-f(x) \Rightarrow$ Vertical reflection

$f(-x) \Rightarrow$ Horizontal reflection

⊛ Graph the function:

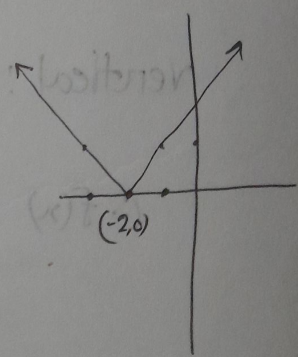
⊛ $f(x) = 2|x+2| + 1$

$\Rightarrow \quad |x| \longrightarrow |x+2| \longrightarrow 2|x+2| \longrightarrow 2|x+2| + 1$



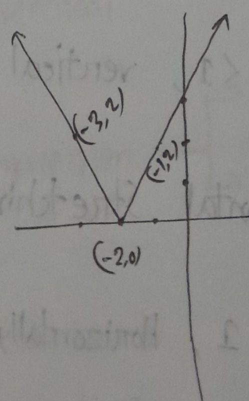
x is replace by x+2

H. shifting by 2 units Left



multiply by 2

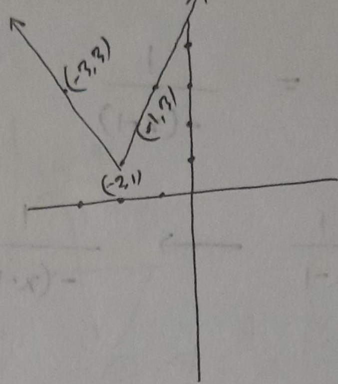
V. stretch



adding 1 $\longrightarrow$

V. shifting by 1

units up

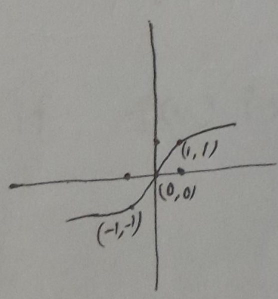


$\therefore$ Domain: $(-\infty, \infty)$

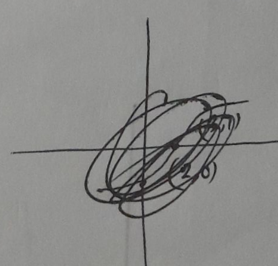
Range: $[1, \infty)$

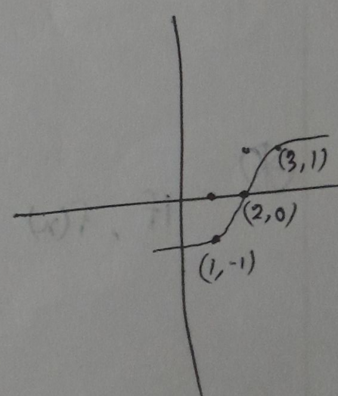
$$y = \sqrt[3]{2-x} = \sqrt[3]{-x+2}$$

$$\therefore\ y = \sqrt[3]{x} \longrightarrow \sqrt[3]{x-2} \longrightarrow \sqrt[3]{\cancel{-x+2}\ -(x-2)}$$

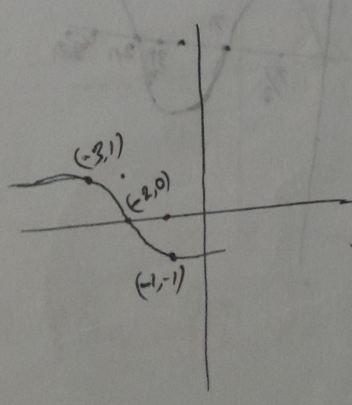


$\longrightarrow$



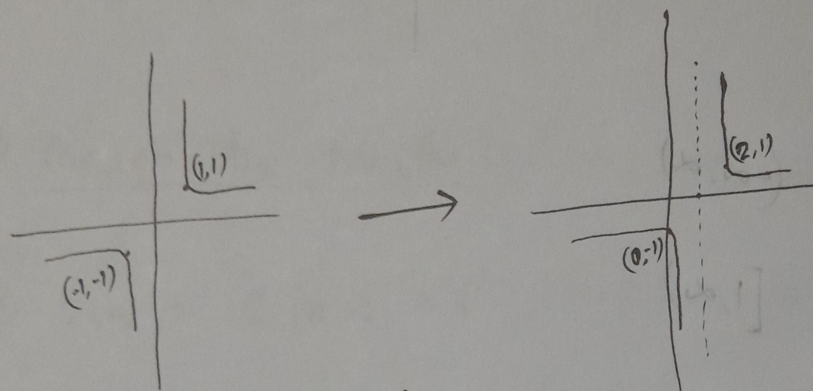


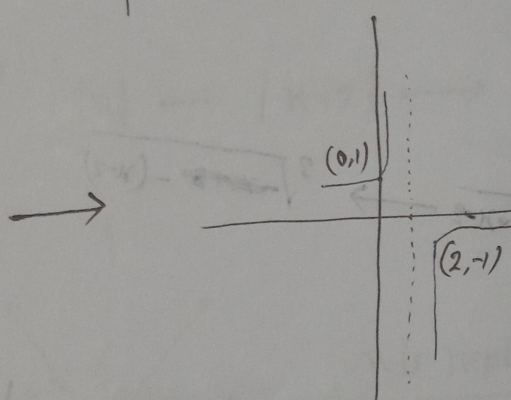
$\longrightarrow$



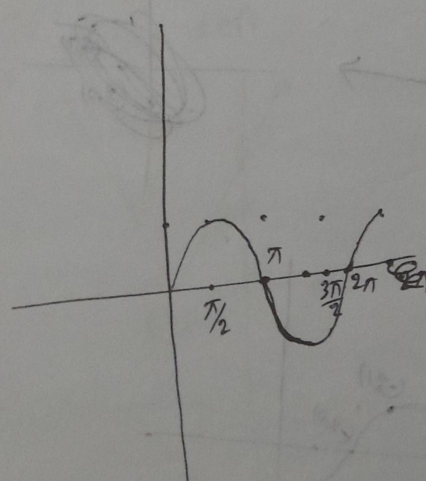
* $f(x) = \dfrac{1}{1-x} = \dfrac{1}{-(x-1)}$

$\Rightarrow \dfrac{1}{x} \longrightarrow \dfrac{1}{x-1} \longrightarrow \dfrac{1}{-(x-1)}$



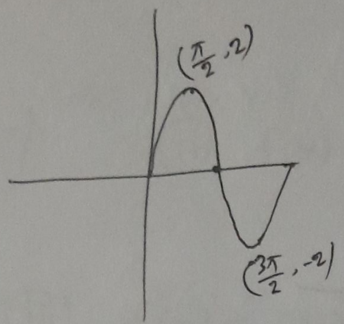


* ∴ if, $f(x)$

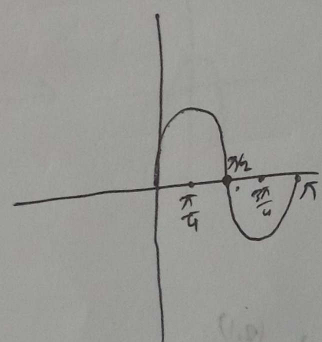


Draw, $2f(x)$, $f(2x)$

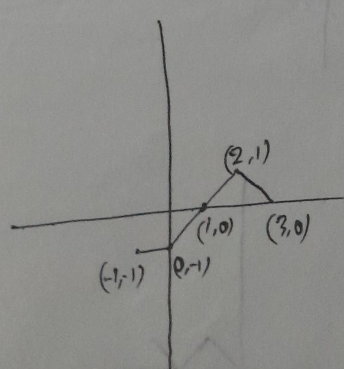
$2f(x)$ →



$f(2x)$ → Horizontal Compression, x-co-ordinate multiply by $\frac{1}{2}$.



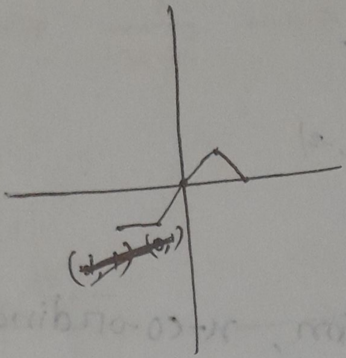
⊛ If, $f(x)$ is



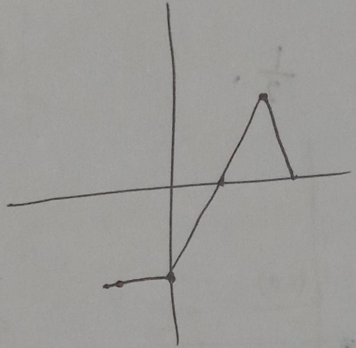
Draw (i) $f(x+1)$ (ii) $f(2x)$ (iii) $y = |f(x)|$ (iv) $y = 1 - |f(x)| = -|f(x)| + 1$

⇒

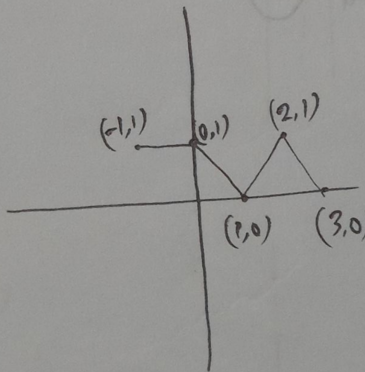
(i)



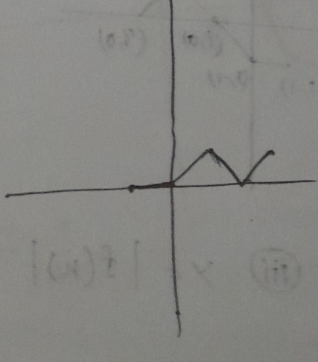
(ii)



(iii)



(iv)

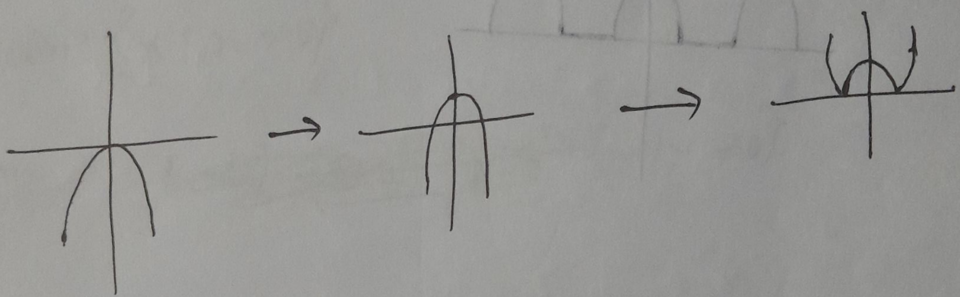


⊛ Draw the graph of the followings

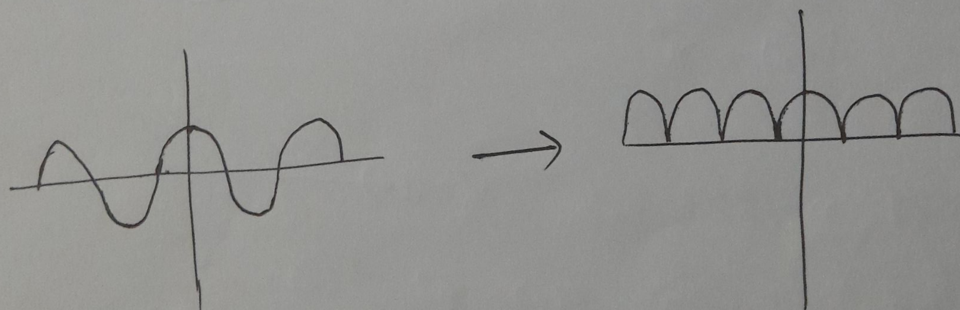
(i) $f(x) = |1-x^2|$ (ii) $f(x) = |\cos x|$

(iii) $f(x) = \cos x + |\cos x|$

(i) $f(x) = -x^2 \rightarrow 1-x^2 \longrightarrow |1-x^2|$

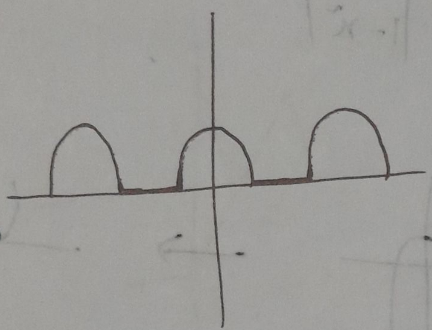


(ii)



(iii) $\cos x + |\cos x| = \begin{cases} \cos x + \cos x & \cos x \geq 0 \\ \cos x - \cos x & \cos x < 0 \end{cases}$

$= \begin{cases} 2\cos x & \cos x \geq 0 \\ 0 & \cos x < 0 \end{cases}$



$\boxed{\text{H.W. } \mid 0.1}$

9/

a) $f(x) = \dfrac{1}{x-3}$

For domain,

$$x-3 \neq 0$$
$$x \neq 3$$

$\therefore$ Domain : $\{x \mid x \neq 3\}$

$\therefore$ Range : $\cancel{R-\{0\}}$ $\cancel{(R-0)}$ $\cancel{\{R-0\}}$ $\{y \mid y \neq 0\}$

L-4 / 13.06.2022

0.3

Families of Curves

$y = b$ → Families of horizontal Line.

Linear Function ← $y = \overset{\text{slope}}{m}x + \underset{\text{y-intercept}}{b}$ → Families of Strait Line

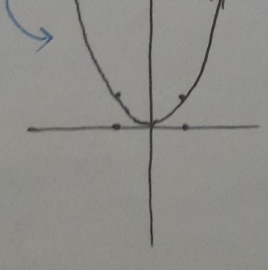
$x = a$ → Families of vertical Line.

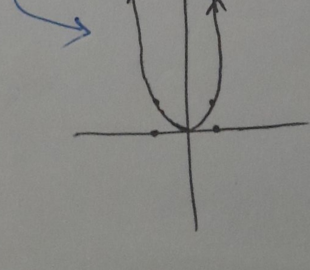
Power Function

$f(x) = x^n$

If $n = 2, 4, 6, 8 \dots$ (even)

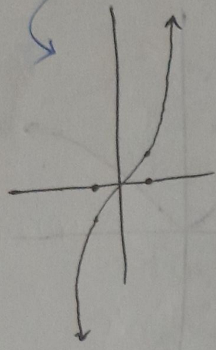
then, $f(x) = x^2$, $f(x) = x^4$, $f(x) = x^6 \dots$

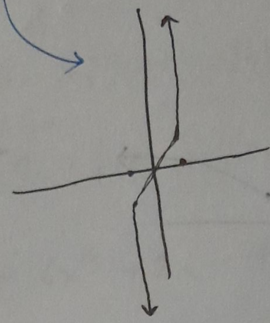


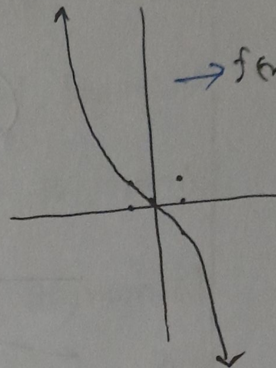


If, $n = 3, 5, 7, \ldots$ (odd)

then, $f(x) = x^3$, $f(x) = x^5$, $f(x) = x^7 \ldots$

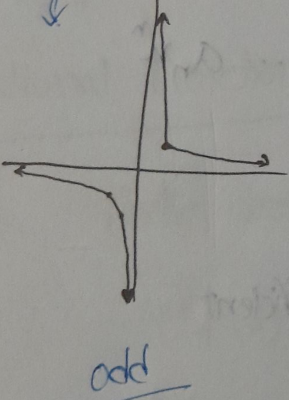




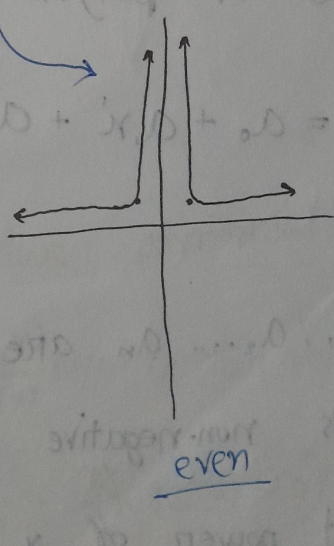


⊛ $f(x) = \dfrac{1}{x^n}$

$f(x) = \dfrac{1}{x}$, $f(x) = \dfrac{1}{x^2}$



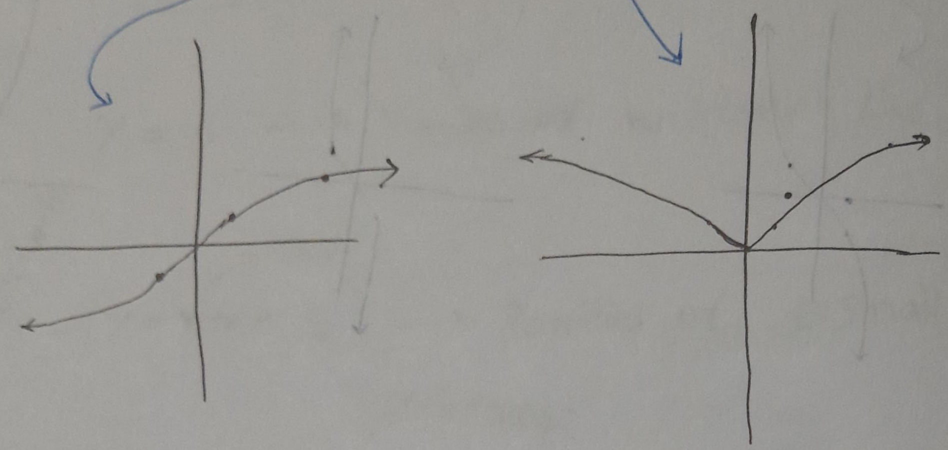
odd



even

* $f(x) = x^{2/3}$

$f(x) = (x^{1/3})^2$



* Polynomial Function

General form of polynomial function,

$$f(x) = a_0 + a_1x^1 + a_2x^2 + \cdots a_nx^n$$

Here,

$a_0, a_1, a_2 \ldots a_n$ are co-efficient

n is non-negative integer

Highest power of x is called its degree.

Domain of polynomial function $(-\infty, \infty)$

Constant function

$f(x) = -3 \rightarrow$ Polynomial, Degree = 0

Linear Function

$f(x) = 2x+4 \rightarrow$ Polynomial, Degree = 1

Quadratic Function

$f(x) = 2x^2 + 3x + \sqrt{3} \rightarrow$ Polynomial, Degree = 2

$h(x) = 6x^3 + 3x^{-2} + 4x + 1 \rightarrow$ Not Polynomial

$f(x) = 3x^2 + 6x^{1/3} + 2x \rightarrow$ Not Polynomial

⊛ Graph of Polynomial Function:

⊛ Graph of Polynomial function always smooth and continuous.

⊛ Rational Function:

Ratio between two polynomial function.

$$f(x) = \frac{P(x)}{Q(x)}$$

Domain : $\{x \mid Q(x) \neq 0\}$

⊛ $f(x) = \dfrac{x^2+2x}{x-1}$

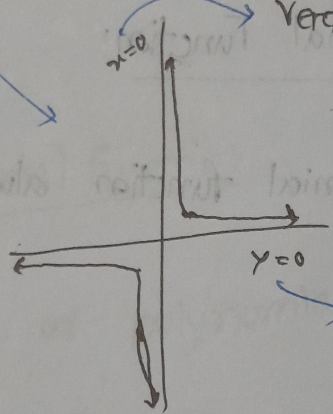
Domain : $\{x \mid x \neq 1\}$

Not Polynomial

⊛ $G(x) = \dfrac{x^2+2\sqrt{x}+3}{x^2+1}$ → Not Rational

⊛ $f(x) = \dfrac{1}{x}$ → Basic Rational Function

Vertical assymptote



Horizontal Asymptote

⊛ **Asymptote:**

Asymptote drawn by doted Line.

There are three types of asymptote.

(i) Horizontal Asymptote (H.A.) → Form, $y=b$

(ii) Vertical Asymptote (V.A.) → Form $x=a$

(iii) Oblique Asymptote (O.A.) → Form, $y=mx+c$

⊛ Finding V.A.:

⇒ Solve the denominator

$f(x) = \dfrac{x+2}{x-3}$

⇒ Solve,

$x-3=0$

$x=3$

$\therefore x=3$ is V.A.

⊛ $f(x) = \dfrac{x^2+2}{(x+1)(x-2)}$

Solve, $(x+1)(x-2)=0$

$x=-1,\ x=2$

$\therefore$ V.A.: $x=-1,\ x=2$.

⊛ Finding H.A.:

⇒ If, degree of top polynomial < degree of bottom polynomial, then,

H.A.: $y=0$

⇒ If, degree of top polynomial = degree of bottom polynomial, then,

$$\text{H.A.: } y = \frac{\text{leading co-efficient of top polynomial}}{\text{leading co-efficient of bottom polynomial}}$$

⇒ If, degree of top polynomial > degree of bottom polynomial, then, No H.A.

Only Oblique Asymptote is here.

⊛ $f(x) = \dfrac{2x+1}{x^2-2x}$

$= \dfrac{2x+1}{x(x-2)}$

V.A. : $x=0, x=2$

H.A. : $y=0$

⊛ $f(x) = \dfrac{2x^2+3x+1}{4x^2+6x+2}$

H.A. : $y = \dfrac{2}{4} = \dfrac{1}{2}$

⊛ $y = \dfrac{x^2+5x+1}{x^2+1}$

H.A. : $y = \dfrac{1}{1} = 1$

V.A. : No V.A. here.

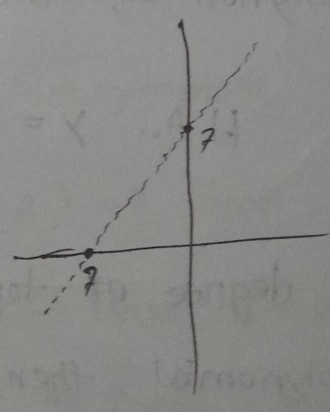
because its imaginary number

⊛ $f(x) = \dfrac{x^2+5x+3}{x-2}$

Here,

$x-2)\ x^2+5x+3\ (x+7$

$\quad x^2 - 2x$
$\quad (-)\ \ (+)$
$\quad\overline{\qquad 7x+3}$
$\quad\quad 7x - 14$
$\quad\quad (-)\ \ (+)$
$\quad\quad\overline{\qquad 17}$



$\therefore$ O.A. : $y = x+7$

⊛ Families of

$$y = A \sin \omega x \quad \& \quad y = A \cos \omega x$$

Here,

$$\text{Amplitute} = |A|$$

$$\text{Period, } P = \frac{2\pi}{\omega}$$

⊛ $y = \sin x$

Comparing with $y = A \sin \omega x$

$A = 1$

$\omega = 1$

$$\text{Period} = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi$$

⊛ $y = 2 \sin 4x$

Graph: $\sin x \rightarrow 2 \sin x \longrightarrow 2 \sin 4x$ (Long Process)

$y = 3 \sin 2\pi x$

Comparing with $y = A \sin \omega x$

$A = |3| = 3$

y - co-ordinate line between -3 to 3.

$$\text{Period} = \frac{2\pi}{\underbrace{2\pi}_{\omega}} = 1$$

Graph : $y = -2\sin\left(\frac{\pi}{2}x\right)$ using key points.

Here,

$\text{Amplitute} = |-2| = 2$

$$\text{Period} = \frac{2\pi}{\pi/2} = 4$$

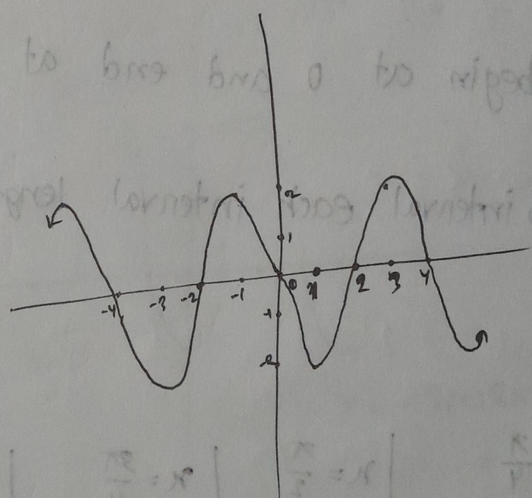
One circle begin at 0 and end at $x = 4$.

Devide by 4 sub interval, each interval length, $\frac{4-0}{4} = 1$

Now points,

$x = 0$	$x = 1$	$x = 2$	$x = 3$	$x = 4$
$y = -2\sin\left(\frac{\pi}{2}\cdot 0\right)$	$y = -2\sin\frac{\pi}{2}$	$y = -2\sin\left(\frac{\pi}{2}\cdot 2\right)$	$y = -2\sin\left(\frac{\pi}{2}\cdot 3\right)$	$y = -2\sin\left(\frac{\pi}{2}\cdot 4\right)$
$= 0$	$= -2\cdot 1$	$= 0$	$= -2(-1)$	$= 0$
	$= -2$		$= 2$	
$(0,0)$	$(1,-2)$	$(2,0)$	$(3,2)$	$(4,0)$

Graph:



Domain: $(-\infty, \infty)$

Range: $[-2, 2]$

* Graph, $y = -3\cos 2x + 1$ using key points

Now,

$y = -3\cos 2x$

Here,

Amplitude, $A = |-3| = 3$

y- co-ordinate line between -3 to 3.

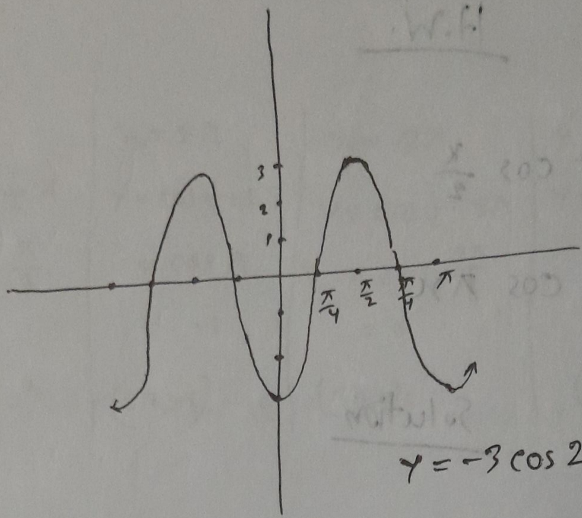
Period, $= \frac{2\pi}{2} = \pi$

One circle begin at 0 and end at π.

Deride 4 sub interval each interval length $= \frac{\pi - 0}{4} = \frac{\pi}{4}$

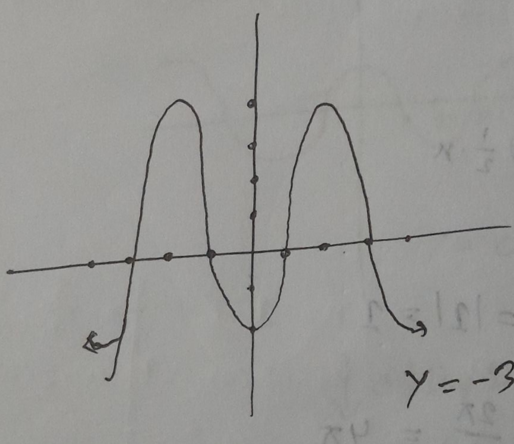
Now, points,

$x = 0$	$x = \frac{\pi}{4}$	$x = \frac{\pi}{2}$	$x = \frac{3\pi}{4}$	$x = \pi$
$y = -3\cos 2\cdot 0$	$y = -3\cos 2\cdot\frac{\pi}{4}$	$y = -3\cos\pi$	$y = -3\cos 2\cdot\frac{3\pi}{4}$	$y = -3\cos 2\cdot\pi$
$= -3\cos 0$	$= -3\cos\frac{\pi}{2}$	$= -3(-1)$	$= 0$	$= -3$
$= -3$	$= 0$	$= 3$		
$(0, -3)$	$(\frac{\pi}{4}, 0)$	$(\frac{\pi}{2}, 3)$	$(\frac{3\pi}{4}, 0)$	$(\pi, -3)$



$y = -3\cos 2x$

$\Big\downarrow$ adding 1



$y = -3\cos 2x + 1$

Domain : $(-\alpha, \alpha)$

Range : $[-2, 4]$