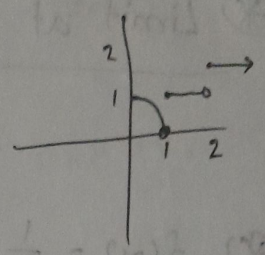
⊛ If,

$$f(x) = \begin{cases} \sqrt{1-x^2} & 0 \le x < 1 \\ 1 & 1 \le x < 2 \\ 2 & x \ge 2 \end{cases}$$



i) $\lim_{x \to 1} f(x)$ ii) $\lim_{x \to 2} f(x)$

$\Rightarrow$

i) $\lim_{x \to 1} f(x)$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \sqrt{1-x^2}$

$= \sqrt{1-1^2}$

$= 0$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 1$

$= 1$

$\therefore$ L.H.L $\neq$ R.H.L.

$\therefore$ Limit does not exist.

ii) $\lim_{x \to 2} f(x) = ?$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 1$

$= 1$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 2$

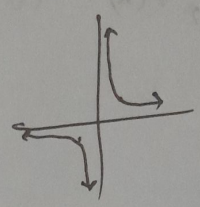
$= 2$

$\therefore$ L.H.L $\neq$ R.H.L.

$\therefore$ Limit does not exist.

⊛ Limit at infinity (End behaviour of function / Horizontal asymtote):

⊛ $f(x) = \frac{1}{x}$



$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to -\infty} \frac{1}{x} = 0$$

H.A.: $y = 0$

⊛ Limit Laws for Limit at infinity.

(i) $\lim_{x \to \pm\infty} (f(x))^n = \left(\lim_{x \to \pm\infty} f(x)\right)^n$

(ii) $\lim_{x \to \pm\infty} k \cdot f(x) = k \cdot \left(\lim_{x \to \pm\infty} f(x)\right)$

(iii) $\lim_{x \to \pm\infty} k = k$

⊛ $\lim_{x \to +\infty} \left(\frac{1}{x^n}\right)$

$= \left(\lim_{x \to +\infty} \frac{1}{x}\right)^n$

$= (0)^n = 0$

⊛ Limit of Polynomial function as $x \to \pm\infty$.

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_n x + a_0$$

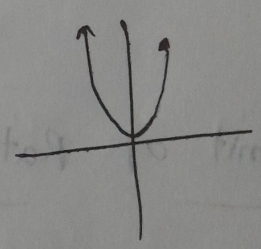
End behaviour,

$$\lim_{x \to +\infty} f(x) = \lim_{x \to \infty} \left(a_n x^n + \cdots + a_0 \right)$$

$$= \lim_{x \to +\infty} a_n x^n \left(1 + \left(\frac{a_{n-1}}{a_n} \cdot \frac{1}{x} \right) + \left(\cdot \frac{1}{x^2} + \cdots \frac{a_0}{a_n x^n} \right) \right)$$

$$= a_n x^n$$

⊛ $f(x) = 3x^6 + 7x^3 + 1$



End behaviour:

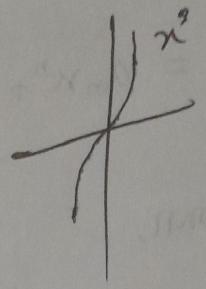
$$f(x) = 3x^6$$

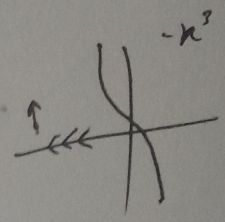
$$\lim_{x \to \infty} 3x^6 + 7x^3 + 1$$

$$= \lim_{x \to \infty} 3x^6$$

$$= +\infty$$

$$\lim_{x \to -\infty} -3x^5 + 6x^4 + 7x^2 + 2$$

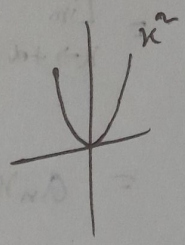


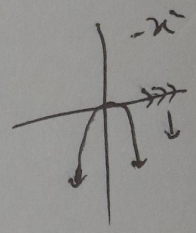


$$= \lim_{x \to -\infty} -3x^5$$

$$= +\infty$$

$$\lim_{x \to +\infty} -x^8 + 7x^6 + 7x + 1$$





$$= \lim_{x \to +\infty} -x^8$$

$$= -\infty$$

Limit of Rational Function as $x \to \pm\infty$.

$$\lim_{x \to \pm\infty} \frac{P(x)}{Q(x)}$$

Rules:

⇒ Find the end behaviour of Top and bottom polynomial.

⇒ Cancel the common factor

⇒ Apply the limit.

(*) $\lim_{x \to +\infty} \dfrac{3x^3 + 6x^2 + 2}{4x^4 + 7x + 1}$

$= \lim_{x \to \infty} \dfrac{3x^3}{4x^4}$

$= \lim_{x \to \infty} \dfrac{3}{4x}$

$= \dfrac{3}{4} \cdot \lim_{x \to \infty} \dfrac{1}{x}$

$= \dfrac{3}{4} \cdot 0 = 0$

Ans.

(*) $\lim_{x \to \infty} \dfrac{5x^3 - 2x^2 + 1}{1 - 3x}$

$= \lim_{x \to \infty} \dfrac{5x^3}{-3x}$

$= \lim_{x \to \infty} \dfrac{-5x^2}{3}$

$= \dfrac{5}{3} \cdot \lim_{x \to \infty} -x^2$

$= \dfrac{5}{3} \cdot (-\infty)$

$= -\infty$

Ans

L-8/27.06.2022

⊛ Limit at infinity for rational function:

⊛ $f(x) = \dfrac{2x^4+7x^2+3x-1}{5x^5+6x^4+3x+2}$

$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to-\infty} \frac{1}{x} = 0$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{2x^4+7x^2+3x-1}{5x^5+6x^4+3x+2}$$

$$= \lim_{x\to\infty} \frac{2x^4}{5x^5}$$

$$= \frac{2}{5} \lim_{x\to\infty} \frac{1}{x}$$

$$= 0$$

⊛ Find the H.A. for

$$f(x) = \frac{3x^3+2x+1}{6x^4+7x+3}$$

$\Rightarrow$

We know that, for H.A.

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{3x^3+2x+1}{6x^4+7x+3}$$

$$\therefore \lim_{x\to\infty} \frac{3x^3+2x+1}{6x^4+7x+3} = \lim_{x\to\infty} \frac{3x^3}{6x^4}$$

$$= \frac{1}{2} \lim_{x\to\infty} \frac{1}{x}$$

$$= 0$$

$$\therefore \lim_{x \to -\infty} \frac{3x^3+2x+1}{6x^4+7x+3} = \lim_{x \to -\infty} \frac{3x^3}{6x^4}$$

$$= 0$$

$\therefore$ H.A. is $y=0$.

⊛ Find the H.A. of

$$f(x) = \frac{x^3-2}{|x|^3+1}$$

$\Rightarrow$

We know that, for H.A.

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x^3-2}{|x|^3+1}$$

$$\therefore \lim_{x \to \infty} \frac{x^3-2}{|x|^3+1} = \lim_{x \to \infty} \frac{x^3}{x^3} = 1$$

$$\therefore \lim_{x \to -\infty} \frac{x^3-2}{|x|^3+1} = \lim_{x \to -\infty} \frac{x^3-2}{-x^3+1} = \lim_{x \to -\infty} \frac{x^3}{-x^3} = -1$$

$\therefore$ H.A. are,

$y = 1$

$y = -1$

Limit at infinity for involving radicals.

$$\lim_{x \to -\infty} \frac{\sqrt{x^2+2}}{3x-6}$$

$$= \lim_{x \to -\infty} \frac{\frac{\sqrt{x^2+2}}{|x|}}{\frac{3x-6}{|x|}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{x^2+2}{x^2}}}{\frac{3x-6}{-x}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{1+\frac{2}{x^2}}}{-3+\frac{6}{x}}$$

$$= \frac{\lim_{x \to -\infty} \sqrt{1+\frac{2}{x^2}}}{\lim_{x \to -\infty} \left(-3+\frac{6}{x}\right)}$$

$$= \frac{\sqrt{\lim_{x \to -\infty} \left(1+\frac{2}{x^2}\right)}}{-3+\lim_{x \to -\infty} \frac{6}{x}}$$

$$= \frac{\sqrt{1+\lim_{x \to \infty} \frac{2}{x^2}}}{-3+0}$$

$$= \frac{\sqrt{1+0}}{-3}$$

$$= \frac{\sqrt{1}}{-3} = -\frac{1}{3}$$ Ans

(*) $$\lim_{x \to -\infty} \frac{\sqrt{3x^4+x}}{x^2-8}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{3x^4+x}{x^2}}}{\frac{x^2-8}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{3x^4+x}{x^4}}}{1-\frac{8}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{3+\frac{1}{x^3}}}{1-\frac{8}{x^2}}$$

$$= \frac{\sqrt{\lim_{x \to -\infty} 3 + \lim_{x \to -\infty} \frac{1}{x^3}}}{\lim_{x \to -\infty} 1 - \lim_{x \to -\infty} \frac{8}{x^2}}$$

$$= \frac{\sqrt{3+0}}{1-0}$$

$$= \frac{\sqrt{3}}{1} = \sqrt{3}$$ Ans

$$\lim_{x\to\infty} \sqrt{x^2+3} - x$$

$$= \lim_{x\to\infty} \frac{(\sqrt{x^2+3} - x)(\sqrt{x^2+3} + x)}{(\sqrt{x^2+3} + x)}$$

$$= \lim_{x\to\infty} \frac{(\sqrt{x^2+3})^2 - x^2}{\sqrt{x^2+3} + x}$$

$$= \lim_{x\to\infty} \frac{x^2+3-x^2}{\sqrt{x^2+3}+x} = \lim_{x\to\infty} \frac{3}{\sqrt{x^2+3} + x}$$

$$= \lim_{x\to\infty} \frac{\frac{3}{|x|}}{\frac{\sqrt{x^2+3} + x}{|x|}}$$

$$= \lim_{x\to\infty} \frac{\frac{3}{x}}{\sqrt{\frac{x^2+3}{x^2}} + 1}$$

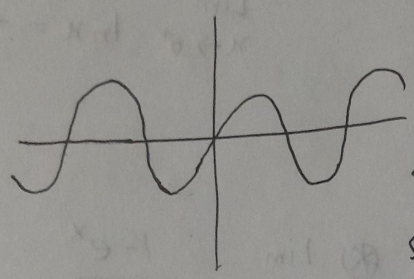
$$= \frac{\lim_{x\to\infty} \frac{3}{x}}{\sqrt{\lim_{x\to\infty} \left(1 + \frac{3}{x^2}\right)} + \lim_{x\to\infty} 1}$$

$$= \frac{0}{1+1} = \frac{0}{2}$$

$$= 0$$ Ans

⊛ Limit at infinity for Trigonometry / exponential / Logarithmic.

⊛ Trigonometry:

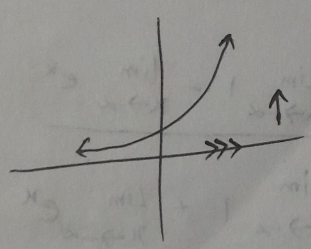


$x \to \alpha$

$\sin x / \cos x \to$ Repeating

∴ No limit for $x \to \pm\alpha$ for trigonometry function.

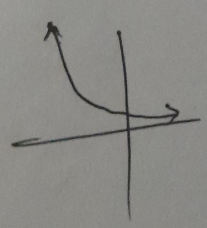
⊛ $y = e^x$



$$\lim_{x \to \alpha} e^x = +\alpha$$

$$\lim_{x \to -\alpha} e^x = 0$$

$y = e^{-x}$

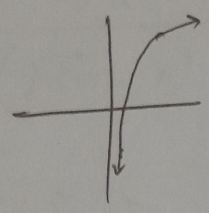


$$\lim_{x \to \alpha} e^{-x} = 0$$

$$\lim_{x \to -\alpha} e^{-x} = +\alpha$$

⊛ Logarithmic function:

$Y = \ln x$



$$\lim_{x \to \alpha} \ln x = +\alpha$$

$$\lim_{x \to 0^+} \ln x = -\alpha$$

⊛ $\lim_{x \to +\alpha} \ln\left(\frac{2}{x^2}\right)$

$= \ln\left(\lim_{x \to +\alpha} \frac{2}{x^2}\right)$

$= \ln 0$

$= -\alpha$

⊛ $\lim_{x \to -\alpha} \frac{1 - e^x}{1 + e^x}$

$$= \frac{\lim_{x \to -\alpha} (1 - e^x)}{\lim_{x \to -\alpha} (1 + e^x)}$$

$$= \frac{\lim_{x \to -\alpha} 1 - \lim_{x \to -\alpha} e^x}{\lim_{x \to -\alpha} 1 + \lim_{x \to -\alpha} e^x}$$

$$= \frac{1 - 0}{1 + 0} = 1$$

$$\lim_{x \to -\infty} \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

$$= \lim_{x \to -\infty} \frac{e^{2x} + 1}{e^{2x} - 1} = \frac{\lim_{x \to -\infty} e^{2x} + \lim_{x \to -\infty} 1}{\lim_{x \to -\infty} e^{2x} - \lim_{x \to -\infty} 1}$$

$$= \frac{0+1}{0-1} = \frac{1}{-1} = -1$$ Ans

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = e$$

$$\lim_{x \to -\infty} \left(1 + \frac{1}{x}\right)^{x} = e$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{2x}$$

$$= \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x \cdot 2}$$

$$= e^{2}$$ Ans

Continuity :

There are three conditions for a function $f(x)$ to be continuous at a point $x=a$,

(i) Function defined at $x=a$, i.e. $f(a)$ exists.

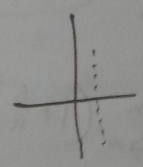
(ii) Limit exists, i.e. $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

then, $\lim_{x \to a} f(x)$ exists.

(iii) $\lim_{x \to a} f(x) = f(a)$.

Types of discontinuous :

If, function not defined,



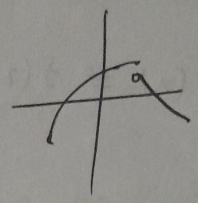
$$f(x) = \frac{1}{x-2}$$

$f(2)$ = undefined

infinity discontinuous.

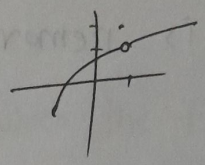
⊛ if,

$$\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x)$$



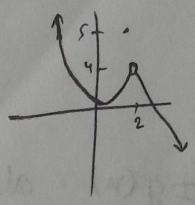
Jump discontinuous.

⊛ $\lim_{x \to a} f(x) \neq f(a)$



Removable discontinuity

⊛ $f(x) = \begin{cases} x^2 & x < 2 \\ 5 & x = 2 \\ -x+6 & x > 2 \end{cases}$



Check the continuity at $x=2$.

$\Rightarrow$

1st Check

$f(2) = 5$; defined

2nd Check

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2$$

$$= 4$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} -x+6$$

$$= -2+6$$

$$= 4$$

$\therefore$ L.H.L. = R.H.L.

$\therefore \lim_{x \to 2} f(x) = 4$; Limit exists

3rd Check

$$\lim_{x \to 2} f(x) \neq f(2)$$

Hence, the function is not continuous at $x=2$.

This is removable discontinuity.

* If $f(x)$ and $g(x)$ are two continuous function.

then,

(i) $f(x) \pm g(x)$ also continuous at $x=a$

(ii) $f(x) \cdot g(x)$ also continuous at $x=a$

(iii) $\frac{f(x)}{g(x)}$ also continuous at $x=a$; $g(a) \neq 0$

if $g(a) = 0$, then discontinuous at $x=a$.

* Polynomial function continuous everywhere.

⊛ Rational function $f(x) = \frac{P(x)}{Q(x)}$ also continuous everywhere.

; $Q(x) \neq 0$

if $Q(x) = 0$, on that point will be discontinuous.

⊛ Find the value of x at which the following function not continuous.

(i) $f(x) = x^4 - 7x^3 + 3x^2 + 2$

⇒ Polynomial function. so, there is no discontinuous point.

(ii) $f(x) = \frac{x+2}{x^2+4}$

⇒ x^2+4 is always positive and greater than 0.

So, there is no discontinous point.

(iii) $f(x) = \sqrt[3]{x-8}$

⇒ There is no discontinuous point.

(iv) $g(x) = \frac{x+2}{x^2-4}$

$\Rightarrow$

$x^2-4 \neq 0$

$x^2 \neq 4$

$x \neq \pm 2$

This function is discontinuous at $x = \pm 2$.

(v) $f(x) = \frac{x-3}{2x^2+x}$

$\Rightarrow$

$x(2x+1) \neq 0$

$x \neq 0$, $2x+1 \neq 0$

$x \neq -\frac{1}{2}$

$\therefore$ this function is discontinuous at $x = 0, -\frac{1}{2}$.

(vi) $f(x) = \frac{2x+1}{4x^2+4x+5}$

$= \frac{2x+1}{(2x+1)^2+4}$

There is no discontinuous point.

⊛ Show that $f(x) = |x|$ continuous everywhere.

$\Rightarrow$ $f(x) = |x|$

$$= \begin{cases} x & \geq 0 \\ -x & < 0 \end{cases}$$

if $x \geq 0$, $f(x) = x$, polynomial. $\therefore$ continuous $(0, \infty)$

If $x < 0$, $f(x) = -x$, polynomial. $\therefore$ continuous $(-\infty, 0)$

Only possible discontinuity at $x = 0$.

Now, check for $x = 0$,

1st condition

$f(x) = x$

$f(0) = 0$; defined

2nd Check

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} -x = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x = 0$

$\therefore$ L.H.L = R.H.L.

$\therefore \lim_{x \to 0} f(x) = 0$

Limit exist.

3rd check

$$\lim_{x \to 0} f(x) = f(0)$$

Hence, this function is continuow at $x=0$. also

therefor,

the function is continuow everywhen.

(*) $f(x) = |3x-2|$

$\Rightarrow$

$$f(x) = \begin{cases} 3x-2 & 3x-2 \geq 0 \\ -(3x-2) & 3x-2 < 0 \end{cases}$$

$$= \begin{cases} 3x-2 & x \geq \frac{2}{3} \\ -3x+2 & x < \frac{2}{3} \end{cases}$$

if $x > \frac{2}{3}$, $f(x) = 3x-2$, polynomial; continuow, $(\frac{2}{3}, \infty)$

if $x < \frac{2}{3}$, $f(x) = -3x+2$, Polynomial; continuow $(-\infty, \frac{2}{3})$

Only possible discontinuity at $x = \frac{2}{3}$

Now check for $x = \frac{2}{3}$

1st check

$$f(x) = 3x - 2$$

$$f\left(\frac{2}{3}\right) = 0$$

2nd check

$$\lim_{x \to \frac{2}{3}^-} f(x) = \lim_{x \to \frac{2}{3}^-} -3x+2$$

$$= 0$$

$$\lim_{x \to \frac{2}{3}^+} f(x) = \lim_{x \to \frac{2}{3}^+} 3x-2$$

$$= 0$$

$\therefore$ L.H.L = R.H.L.

$$\therefore \lim_{x \to \frac{2}{3}} f(x) = 0$$

Extra:

- $f(x) = |x-2|$
- $f(x) = x + |x-2|$

3rd check

$$\lim_{x \to 0} f(x) = f\left(\frac{2}{3}\right)$$

Hence the function is continuous at $x = \frac{2}{3}$ also.

Therefore,

the function is continuous everywhere.

L-9 / 29.06.2022 /

⊛ If $f(x) = \begin{cases} 2x-7 & x \geq 3 \\ kx^2 & x < 3 \end{cases}$

For what value of k, the function continuous everywhere.

$\Rightarrow$

1st Check

$f(3) = 2 \cdot 3 - 7 = -1$

2nd check

$\lim_{x \to 3^+} 2x - 7 = -1$

$\lim_{x \to 3^-} kx^2 = 9k$

if Limit exist, then L.H.L = R.H.L

$9k = -1$

$k = -\frac{1}{9}$

if $k = -\frac{1}{9}$, then limit exist.

$\therefore \lim_{x \to 3} f(x) = -1 = f(3)$

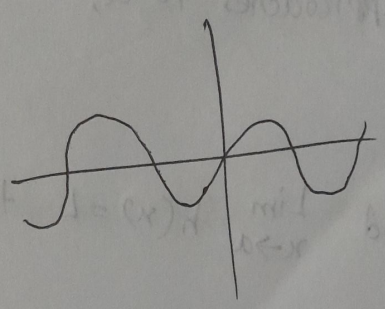
$\therefore$ for $k = -\frac{1}{9}$ the function is continuous.

Continuity for Trigonmetry function

$f(x) = \sin x$ $\quad$ $f(x) = \tan x$ $\quad$ $f(x) = \sec x$

$f(x) = \cos x$ $\quad$ $f(x) = \cot x$ $\quad$ $f(x) = \operatorname{cosec} x$

⊛ For $\sin x$ and $\cos x$



for $\sin x$ and $\cos x$ function continuous everywhere.

$$\lim_{x \to c} \sin x = \sin c$$

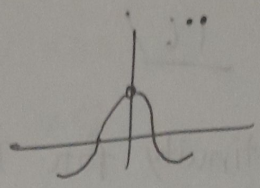
$$\lim_{x \to c} \cos x = \cos c$$

⊛ $\lim_{x \to c} \tan x = \tan c = \dfrac{\sin c}{\cos c}$; if $\cos c \neq 0$

$f(x) = \sec x = \dfrac{1}{\cos x}$

⊛ $\lim_{x \to c} \cot x = \cot c = \dfrac{\cos c}{\sin c}$; if $\sin c \neq 0$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$



Squeezing Theorem

If, $f(x)$, $g(x)$ and $h(x)$ are function satisfying

$g(x) \leq f(x) \leq h(x)$, if $g(x)$ and $h(x)$ has some

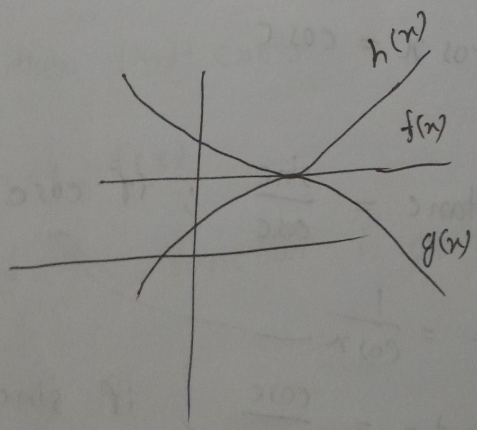
limit L as x approaches to a,

Say,

$\lim_{x \to a} g(x) = L$ and $\lim_{x \to a} h(x) = L$ then,

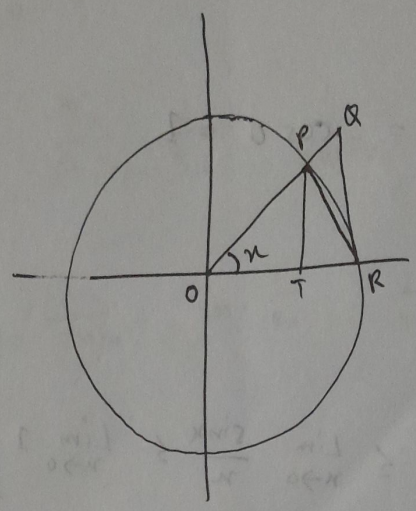
• Squeezing Theorem state that,

$$\lim_{x \to a} f(x) = L$$



⊛ Proov, $\lim_{x \to 0} \frac{\sin x}{x} = 1$

Let a unit circle, whose center at (0,0) and radiow 1.



area of the triange OPR $\leq$ are of sector OPR $\leq$ are of triange OQR

$$\frac{1}{2} \times OR \times PT \leq \frac{1}{2} r^2 \theta \leq \frac{1}{2} \times OR \times QR$$

$$\frac{1}{2} \times 1 \times \sin x \leq \frac{1}{2} \cdot 1 \cdot x \leq \frac{1}{2} \times 1 \times \tan x$$

$\frac{PT}{1} = \sin x$

$PT = \sin x$

$\frac{QR}{OR} = \tan x$

$QR = OR \tan x$

$= 1 \cdot \tan x$

$= \tan x$

$$\frac{\sin x}{2} \leq \frac{x}{2} \leq \frac{\tan x}{2}$$

$$\sin x \leq x \leq \tan x$$

$$1 \leq \frac{x}{\sin x} < \frac{1}{\cos x}$$

$$1 \geq \frac{\sin x}{x} \geq \cos x$$

$$\therefore \cos x \leq \frac{\sin x}{x} \leq 1$$

$$\therefore g(x) \leq f(x) \leq h(x)$$

Now,

$$\lim_{x \to 0} \cos x = \cos 0 = 1$$

$$\lim_{x \to 0} 1 = 1$$

$$\therefore \lim_{x \to 0} \cos x \leq \lim_{x \to 0} \frac{\sin x}{x} \leq \lim_{x \to 0} 1$$

Hence, by squeezing theorem,

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$

(*)

$$\lim_{x \to 0} \frac{\sin 5x}{5x} = 1$$

$$\lim_{x \to 0} \frac{x}{\sin x} = 1$$

$$\lim_{x \to 0} \frac{\sin 2t}{2t} = 1$$

$$\lim_{\theta \to 0} \frac{\sin 7\theta}{7\theta} = 1$$

$$\lim_{t \to 0} \frac{\tan t}{t} = \lim_{t \to 0} \frac{\sin t}{t} \cdot \cos t$$

$$= 1 \cdot \cos t$$

$$= \cos 0 = 1$$

(*)

$$\lim_{x\to 0} \frac{\tan 7x}{\sin 3x}$$

$$= \lim_{x\to 0} \frac{\sin 7x}{\cos 7x} \cdot \frac{1}{\sin 3x}$$

$$= \lim_{x\to 0} \frac{\sin 7x}{7x} \cdot 7x \cdot \frac{3x}{\sin 3x} \cdot \frac{1}{3x} \cdot \frac{1}{\cos 7x}$$

$$= \lim_{x\to 0} \frac{\sin 7x}{7x} \cdot \lim_{x\to 0} \frac{3x}{\sin 3x} \cdot 7x \cdot \frac{1}{3x} \cdot \lim_{x\to 0} \frac{1}{\cos 7x}$$

$$= \frac{7}{3} \cdot \left[1 \cdot 1 \cdot \frac{1}{\cos 0}\right]$$

$$= \frac{7}{3} [1 \cdot 1 \cdot 1]$$

$$= \frac{7}{3} \text{ Ans}$$

(*) $$\lim_{x\to 0} \frac{1-\cos x}{x}$$

$$= \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x} \cdot \frac{1}{1+\cos x}$$

$$= \left(\lim_{x\to 0} \frac{\sin x}{x}\right)^2 \cdot \lim_{x\to 0} \frac{x}{1+\cos x}$$

$$= 1^2 \cdot \frac{0}{1+1} = 0 \text{ Ans}$$

$$\lim_{x\to 0} \frac{\tan 3x^2 + \sin^2 5x}{x^2}$$

$$= \lim_{x\to 0} \frac{\tan 3x^2}{x^2} + \lim_{x\to 0} \frac{\sin^2 5x}{x^2}$$

$$= \lim_{x\to 0} \frac{\tan 3x^2}{3x^2} \cdot 3 + \lim_{x\to 0} \frac{\sin^2 5x}{(5x)^2} \cdot 25$$

$$= 1 \cdot 3 + \lim_{x\to 0} \left(\frac{\sin 5x}{5x}\right)^2 \cdot 25$$

$$= 3 + 1 \cdot 25$$

$$= 28 \text{ Ans}$$

Discuss the Limit:

a) $\lim_{x\to 0} \sin \frac{1}{x}$ b) $\lim_{x\to 0} x \sin \frac{1}{x}$

$\Rightarrow$

a) $\lim_{x\to 0^+} \frac{1}{x} = +\infty$

$\lim_{x\to 0^-} \frac{1}{x} = -\infty$

$\lim_{x\to 0} \sin \frac{1}{x} = \text{DNE}$

b) $\lim_{x\to 0} x \sin\frac{1}{x}$

we know that,

$$-1 \le \sin\frac{1}{x} \le 1$$

$$-x \le x\sin\frac{1}{x} \le x$$

$$\lim_{x\to 0^-} x = 0$$

$$\lim_{x\to 0^+} x = 0$$

Hence, by squeezing theorem

$$\lim_{x\to 0} x\sin\frac{1}{x} = 0.$$

c) $x^2\cos\frac{1}{x}$

$$-1 \le \cos\frac{1}{x} < 1$$

$$x^2 \le x^2\cos\frac{1}{x} \le x^2$$

$$\lim_{x\to 0^-} x^2 = 0$$

$$\lim_{x\to 0^+} x^2 = 0$$

$$\therefore \lim_{x\to 0} x^2\cos\frac{1}{x} = 0$$

(*) $\lim_{x\to \pi} \frac{\pi - x}{\sin x}$

Let,

$$t = \pi - x$$

$$\therefore x = \pi - t$$

as $x \to \pi$ then $t \to 0$

$$\lim_{t\to 0} \frac{t}{\sin(\pi - t)} = \lim_{t\to 0} \frac{t}{\sin t} = 1$$

(*) $\lim_{x\to 0} e^{\sin x} = e^{\lim_{x\to 0}\sin x} = e^0 = 1$

(*) $\lim_{x\to\infty} \cos(2\tan^{-1}x) = \cos\left(\lim_{x\to\infty} 2\tan^{-1}x\right)$

$$= \cos\left(2\times\frac{\pi}{2}\right)$$

$$= \cos\pi$$

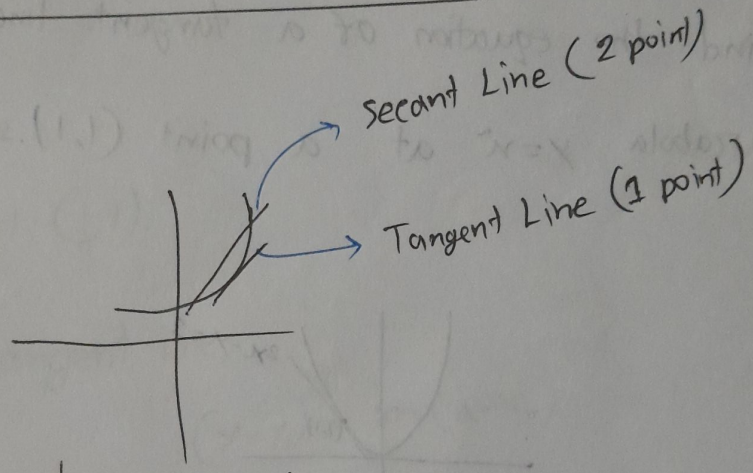
$$= -1$$

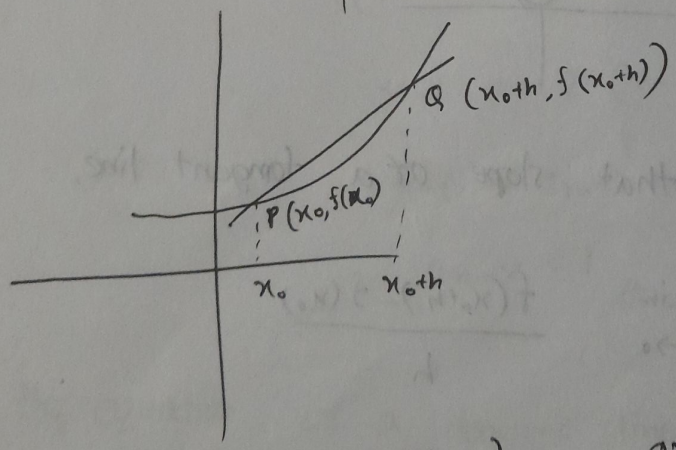
Chapter - 2

Derivative

2.1

Tangent line and rate of change





Slope of PQ (secant line) or average rate of change.

$$m_{PQ} \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(x_0+h) - f(x_0)}{(x_0+h) - x_0}$$

$$= \frac{f(x_0+h) - f(x_0)}{h}$$