CSE 273 / L-12 / 04.03.2025 /

Midterm Exam

No class

L-13 / 09.03.2025 /

⊛ Arden's Theorem Proof:

⇒ Theorem:

$R = Q + RP$; $P \notin \epsilon$

$P, Q \Rightarrow \in RE$

$\therefore R = QP^*$

Here,

$R = Q + RP$

$= Q + QP^*P$

$= Q(\epsilon + P^*P)$ [I₉]

$= QP^*$

$\therefore R = QP^* \Rightarrow$ one of the solution, not the only one.

Now,

$R = Q + RP$

$= Q + (Q + RP)P$

$= Q + QP + RP^2$

$= Q + QP + (Q + RP)P^2$

$= Q + QP + QP^2 + RP^3$

$= Q + QP + QP^2 + \cdots + QP^i + RP^{i+1}$

$= Q(\epsilon + P + P^2 + \cdots + P^i) + RP^{i+1}$

$= QP^* + RP^{i+1}$

$= QP^* + RP^{i+1}$

$= QP^* + QP^*P^{i+1}$ ← from first part

$= QP^* + QP^* = QP^*$

Pumping Lemma

- identify whether the expression is a RE or not.

⇒

P = Pumping length

$s \in L$

↳ at least P length

$|s| \geq P$

Then $s = xyz$

where,

(i) $|xy| \leq P$

(ii) $|y| > 0$

(iii) $i \geq 0$; $xy^iz \in L$

- if any one condition violate, then the expression is not a RE.
- if all condition satisfied still it not mandatory, it will be a RE. It may not be a RE.

Selection of P is critical.

- we need to take P such as, it could violate the condition.
- as well as choosing string is also important.

✳ $L = \{a^n b^n \mid n \geq 0\}$

$\Rightarrow$

$P = 5$

$s = a^5 b^5 = aaaaabbbbb$

Here,

$x = \in$

$y = a$

$z = aaaabbbbb$

For, $i = 3$,

$xy^3z = \in aaa\ aaaabbbbb = a^7 b^5 \notin L$

$\therefore L = \{a^n b^n \mid n \geq 0\}$ is not a valid ~~the~~ RE.

- we can't make the machine for this RE.
- Because here state is not finite.

✳ if we need to memorize past input for the future input, then it is not a RE.

L-14 / 16.03.2025

✳ Content free grammar & Language: (CFG)

- Regular grammar, ~~based on~~ right syntax and a valid context need to form a sentence.
- But in CFG, we just need right syntax to form a sentence. Identify syntactical errors.

$G = \{V, T, P, S\}$

⇒ V = set of variables

T = set of terminals

P = set of production Rule

S = start symbol

Let's imagine you are on a trip, DHAKA to chittagong. And you take a break in CUMZLLA. Here DHAKA & CUMZLLA are variables and chittagong is your final destination/terminal.

⇒ Terminals are written wing small letter.

⇒ Variables are written wing capital letter.

Production Rule:

- way or path

⇒ $A \longrightarrow \alpha$

left side,
- only one variable

Right side
- variable / terminal / combined.

$A \rightarrow aAb$

⇒ $T = \{a, b\}$

$V = \{A\}$

⇒ At the end you must use terminal to reach your destination. That means the last production rule must have a a option of terminals only.

CFG for palindrome:

$P \rightarrow 0S0 \mid 1S1$

$S \rightarrow 0 \mid 1 \mid \epsilon$

Derivative Process:

- Back tracking process
- two type
  - (i) Leftmost: Left side variable will be derivate first.
  - (ii) Rightmost: Right side variable will be derivate first.

⇒ Another version is Parse tree / Derivation tree.

Whatever you use, leftmost or rightmost, parse tree will be the same. If you find two parse tree for a grammar and a string, then the grammar is ambiguious. Not a valid grammar.

$S \Rightarrow^{*} 0110011$?

* means, any number of production rule can be use.

Quiz-2

08.04.2025

syllabus upto 25.03.2025

L-15 / 18.03.2025 /

Ambiguity test in CFG:

- if there are multiple way to derive at least one string, then the grammar is ambiguous.
- more than one parse tree.

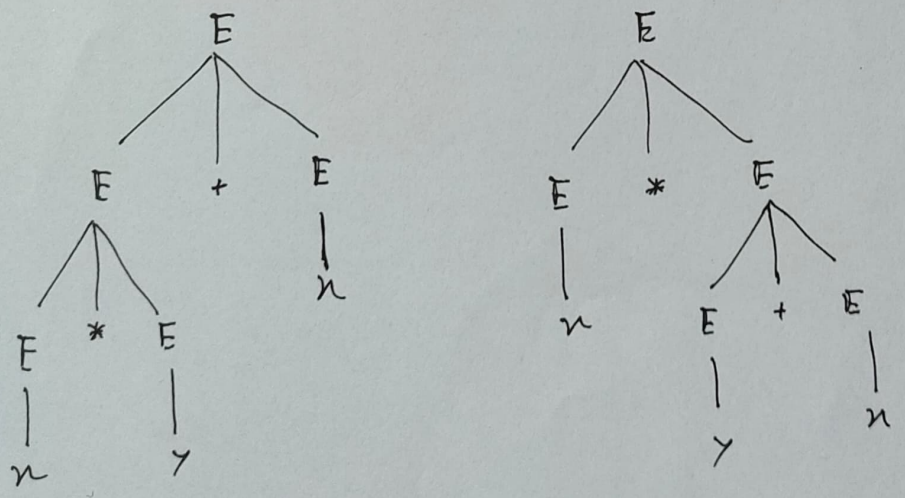
L: $E \longrightarrow E+E \mid E*E \mid (E) \mid x \mid y \mid z$

Here,

$V = \{E\}$

$T = \{x, y, z, +, *, (,)\}$

⇒ String: $x*y+x$



∴ two parse tree for a single string.

∴ grammar is ambiguous.

CFG Practice!

(i) exact string is given,

a
ab
010
ϵ

} no pattern, direct string given

∴ $S \rightarrow a | ab | 010 | \epsilon$

(ii) any number of a's

$S \rightarrow \epsilon | aS$

(iii) at least one length of a's

$S \rightarrow a | aS$

(iv) any string at least one length.

$S \rightarrow aS | bS | a | b$

(v) $0^n 1^n$; $n \geq 1$

$S \rightarrow \underbrace{01}_{\text{minimum length string}} | 0S1$

(vi) $0^n 1^n$; $n \geq 0$

$S \rightarrow \epsilon | 0S1$

(vii) $2^m 0^n 1^{n+1} 22$; $n \geq 0, m \geq 1$

divide in three part

$S \rightarrow ABC$

$A \rightarrow 2A | 2$

$B \rightarrow 0B1 | 1$

$C \rightarrow 22$

(viii) at most two 1's

$S \rightarrow \epsilon | 1 | 11$

Assignment-3:

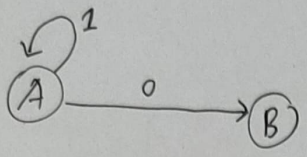
⇒ minimum 5x CFG
Relatively complex

06.04.2025

RE ⇒ RG

$\langle V, T, P, S \rangle$

all same except Production Rule.



~~A~~ For incoming only

$A \rightarrow A1$

$B \rightarrow A0$

For outgoing

$A \rightarrow 1A \mid 0B$

∴ RG:

$S \rightarrow \in \mid 0 \mid 1 \mid 1V \mid V1$

Quiz-2
Date changed
13.04.2025

L-16 / 23.03.2025 /

Construction of Reduced Grammar

Rules:

(i) Remove Useless symbols.

- Remove non-generating variable
- Non-reachable symbol remove

(ii) Remove $\in$-productions

(iii) Remove unit production

Remove non-generating variables:

- variables that do not produce any string.
- ⇒ that means there is no production rule for that variable.

$S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow b \mid D$
$E \rightarrow c$

→ Variable. but no production Rule exist for this.

W_1 = list of variable, that can produce string directly
$= \{A, B, E\}$

W_2 = list of generating variable may produce through other variable.
$= \{S, A, B, E\}$

W_3 = no more variable, ~~one~~ that can generate string.
$= \{S, A, B, E\} = W_2$ // stopping of algorithm.

Remove non-reachable symbols:

$S \rightarrow AB$ ⇒ starting production Rule.
$A \rightarrow a$
$B \rightarrow b$
$E \rightarrow c$ ⇒ no way to came here

$W_1 = \{S\}$
$W_2 = \{S, A, B\}$

CSE 273 / L-17 / 25.03.2025 /

✳ Remove Null Production:

$S \rightarrow aS \mid AB$

$A \rightarrow e$

$B \rightarrow e$

$D \rightarrow b$

Here, nullable variables

W_1 = Directly produce null
= $\{A, B\}$

W_2 = Second step null
= $\{S, A, B\}$ // no more variable that can produce null.

Here,

$S \rightarrow aS$ → S - sometime can produce null

$\rightarrow aS \mid a$ → without S possibility, S null

Again,

$S \rightarrow AB$ → any one, or both, or none can be null

$\rightarrow AB \mid A \mid B$ (B) → A null

$\rightarrow AB \mid A \mid B$ → B null

Therefore,

$S \rightarrow aS \mid AB \mid a \mid A \mid B$

$D \rightarrow b$

✳ Unit production Removal:

- both side have variable only. (Single variable)
- we must check the impact before remove.

⇒ $S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow C \mid b$
$C \rightarrow D$

$D \rightarrow E$
$E \rightarrow ab \mid aBa$

Here, Unit productions are
$B \rightarrow C$
$C \rightarrow D$
$D \rightarrow E$

⇒ Find out chain:

$W(S) = \{S\}$ ⇒ only S possible

$W(A) = \{A\}$ ⇒ only A possible

$W(B) = \{B, C, D, E\}$ ⇒ $B \to C \to D \to E$ chain

$W(C) = \{C, D, E\}$

$W(D) = \{D, E\}$

$W(E) = \{E\}$

Therefore,

If no chain exist, keep the previous production rule.

Start from lower set,

$E \to ab \mid aBa$

$A \to a$

$S \to AB$

$D \to ab \mid aBa$

$C \to ab \mid aBa$

$B \to b \mid ab \mid aBa$

Steps to reduce the grammar:

(i) Remove $\in$-production

(ii) Remove unit-production

(iii) Remove useless symbols

(a) Remove non-generating variables

(b) Remove non-reachable variable and terminals.

Practice - more from Slide