North South University

Department of Mathematics and Physics

Assignment-3

Name : Joy kumar Ghosh

Student ID : 2211424 6 42

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5.2

5)

Given that,

$$\frac{d}{dx}\left(\sqrt{x^3+5}\right)$$

$$= \frac{d}{dx}(x^3+5)^{\frac{1}{2}}$$

$$= \frac{1}{2}\cdot(x^3+5)^{-\frac{1}{2}}\cdot 3x^2$$

$$= \frac{3x^2}{2\sqrt{x^3+5}}$$

Therefore,

Corresponding integration formula

$$\int \frac{3x^2}{2\sqrt{x^3+5}}\,dx = \sqrt{x^3+5}+c$$

6)

Given that,

$$\frac{d}{dx}\left(\frac{x}{x^2+3}\right)$$

$$= \frac{(x^2+3)\cdot 1 - x\cdot 2x}{(x^2+3)^2}$$

$$= \frac{x^2+3-2x^2}{(x^2+3)^2}$$

$$= \frac{-x^2+3}{(x^2+3)^2}$$

Therefore,

Corresponding integration formula,

$$\int \frac{-x^2+3}{(x^2+3)^2}\,dx = \frac{x}{x^2+3} + c$$

7)

Given that,

$$\frac{d}{dx}\left[\sin(2\sqrt{x})\right]$$

$$= \cos(2\sqrt{x}) \cdot \frac{d}{dx}(2\sqrt{x})$$

$$= \cos(2\sqrt{x}) \cdot 2 \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{\cos(2\sqrt{x})}{\sqrt{x}}$$

Therefore,

Corresponding integration formula,

$$\int \frac{\cos(2\sqrt{x})}{\sqrt{x}}\,dx = \sin(2\sqrt{x}) + c$$

8)

Given that,

$$\frac{d}{dx}[\sin x - x\cos x]$$

$$= \frac{d}{dx}(\sin x) - \frac{d}{dx}(x\cos x)$$

$$= \cos x \quad - \quad (x\cdot(-\sin x) + \cos x\cdot 1)$$

$$= \cos x + x\sin x - \cos x$$

$$= x\sin x$$

Therefore,

Corresponding integration formula,

$$\int x\sin x\, dx = \sin x - x\cos x + c$$

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Given that,

$$\int x(1+x^3)\,dx$$

$$= \int (x+x^4)\,dx$$

$$= \int x\,dx + \int x^4\,dx$$

$$= \frac{x^2}{2} + \frac{x^5}{5} + c$$

Now checking answere

$$\frac{d}{dx}\left(\frac{x^2}{2} + \frac{x^5}{5}\right)$$

$$= \frac{d}{dx}\cdot\frac{x^2}{2} + \frac{d}{dx}\cdot\frac{x^5}{5}$$

$$= \frac{1}{2}\cdot 2x + \frac{1}{5}\cdot 5x^4$$

$$= x+x^4$$

$$= x(1+x^3)$$

Therefore the value of the given integral is

$$\frac{x^2}{2} + \frac{x^5}{5} + c$$

16)

Given that,

$$\int (2+y^2)^2\, dy$$

$$= \int (4 + 4y^2 + y^4)\, dy$$

$$= \int 4\, dy + \int 4y^2\, dy + \int y^4\, dy$$

$$= 4y + 4 \cdot \frac{y^3}{3} + \frac{y^5}{5} + c$$

$$= 4y + \frac{4}{3}y^3 + \frac{1}{5}y^5 + c$$

Now checking the answer,

$$\frac{d}{dy}\left(4y + \frac{4}{3}y^3 + \frac{1}{5}y^5 + c\right)$$

$$= \frac{d}{dy}4y + \frac{d}{dy}\frac{4}{3}y^3 + \frac{d}{dy}\frac{1}{5}y^5 + \frac{d}{dy}c$$

$$= 4 + \frac{4}{3} \cdot 3y^2 + \frac{1}{5} \cdot 5y^4 + 0$$

$$= 4 + 4y^2 + y^4$$

$$= (2+y^2)^2$$

Therefore, the value of the given integral is,

$$4y + \frac{4}{3}y^3 + \frac{1}{5}y^5 + c$$

17)

Given that,

$$\int x^{1/3}(2-x)^2\,dx$$

$$= \int x^{1/3}(4-4x+x^2)\,dx$$

$$= \int (4x^{1/3} - 4x^{4/3} + x^{7/3})\,dx$$

$$= \int 4x^{1/3}\,dx - \int 4x^{4/3}\,dx + \int x^{7/3}\,dx$$

$$= 4\cdot\frac{x^{4/3}}{4/3} - 4\cdot\frac{x^{7/3}}{7/3} + \frac{x^{10/3}}{10/3} + c$$

$$= 3x^{4/3} - \frac{12}{7}x^{7/3} + \frac{3}{10}x^{10/3} + c$$

Now, checking answer,

$$\frac{d}{dx}\left[3x^{4/3} - \frac{12}{7}x^{7/3} + \frac{3}{10}x^{10/3} + c\right]$$

$$= \frac{d}{dx}\cdot 3\cdot x^{4/3} - \frac{d}{dx}\cdot\frac{12}{7}x^{7/3} + \frac{d}{dx}\frac{3}{10}x^{10/3} + \frac{d}{dx}c$$

$$= 3\cdot\frac{4}{3}\cdot x^{1/3} - \frac{12}{7}\cdot\frac{7}{3}\cdot x^{4/3} + \frac{3}{10}\cdot\frac{10}{3}\cdot x^{7/3} + 0$$

$$= 4x^{1/3} - 4x^{4/3} + x^{7/3}$$

$$= x^{1/3}(4-4x+x^2)$$

$$= x^{1/3}(2-x)^2$$

Therefore the value of the given integral is,

$$3x^{4/3} - \frac{12}{7} x^{7/3} + \frac{3}{10} x^{10/3} + c$$

18|

Given that,

$$\int (1+x^2)(2-x)\,dx$$

$$= \int (2 - x + 2x^2 - x^3)\,dx$$

$$= \int 2\,dx - \int x\,dx + \int 2x^2\,dx - \int x^3\,dx$$

$$= 2x - \frac{x^2}{2} + 2\cdot\frac{x^3}{3} - \frac{x^4}{4} + c$$

$$= 2x - \frac{1}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4 + c$$

Now checking answer,

$$\frac{d}{dx}\left[2x - \frac{1}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4 + c\right]$$

$$= \frac{d}{dx} 2x - \frac{d}{dx}\cdot\frac{1}{2}x^2 + \frac{d}{dx}\frac{2}{3}x^3 - \frac{d}{dx}\frac{1}{4}x^4 + \frac{d}{dx} c$$

$$= 2 - \frac{1}{2}\cdot 2\cdot x + \frac{2}{3}\cdot 3\cdot x^2 - \frac{1}{4}\cdot 4\cdot x^3 + 0$$

$$= 2 - x + 2x^2 - x^3$$

$$= (1+x^2)(2-x)$$

Therefore, the value of the given integral is,

$$2x - \frac{1}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4 + c$$

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Given that,

$$\int \frac{x^5 + 2x^2 - 1}{x^4}\, dx$$

$$= \int (x + 2x^{-2} - x^{-4})\, dx$$

$$= \int x\, dx + \int 2x^{-2}\, dx - \int x^{-4}\, dx$$

$$= \frac{x^2}{2} + 2\frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + c$$

$$= \frac{1}{2}x^2 - \frac{2}{x} + \frac{1}{3x^3} + c$$

Now checking answer,

$$\frac{d}{dx}\left[\frac{1}{2}x^2 - \frac{2}{x} + \frac{1}{3x^3} + c\right]$$

$$= \frac{d}{dx}\frac{1}{2}x^2 - \frac{d}{dx}2 \cdot x^{-1} + \frac{d}{dx}\frac{1}{3} \cdot x^{-3} + \frac{d}{dx}c$$

$$= \frac{1}{2} \cdot 2x - 2(-1) \cdot x^{-2} + \frac{1}{3} \cdot (-3) \cdot x^{-4} + 0$$

$$= x + 2x^{-2} - x^{-4}$$

$$= \frac{x^4(x + 2x^{-2} - x^{-4})}{x^4}$$

$$= \frac{x^5+2x^2-1}{x^4}$$

Therefore, the value of the given integral is,

$$\frac{1}{2}x^2 - \frac{2}{x} + \frac{1}{3x^3} + C$$

20|

Given that,

$$\int \frac{1-2t^3}{t^3}\,dt$$

$$= \int (t^{-3} - 2)\,dt$$

$$= \int t^{-3}\,dt - \int 2\,dt$$

$$= \frac{t^{-2}}{-2} - 2t + C$$

$$= -\frac{1}{2}t^{-2} - 2t + C$$

Now checking answer,

$$\frac{d}{dt}\left[-\frac{1}{2}t^{-2} - 2t + C\right]$$

$$= \frac{d}{dt}\left(-\frac{1}{2}\cdot t^{-2}\right) - \frac{d}{dt}2t + \frac{d}{dt}C$$

$$= -\frac{1}{2}\cdot(-2)\cdot t^{-3} - 2 + 0$$

$$= t^{-3} - 2$$

$$= \frac{t^3(t^{-3}-2)}{t^3}$$

$$= \frac{1-2t^3}{t^3}$$

Therefore, the value of the given integral is,

$$-\frac{1}{2}t^{-2} - 2t + C$$

21|

Given that,

$$\int \left[\frac{2}{x} + 3e^x\right] dx$$

$$= \int 2 \cdot \frac{1}{x}\, dx + \int 3e^x\, dx$$

$$= 2\ln x + 3e^x + C$$

Now checking answer,

$$\frac{d}{dx}\left[2\ln x + 3e^x + c\right]$$

$$= \frac{d}{dx} 2\ln x + \frac{d}{dx} 3e^x + \frac{d}{dx} c$$

$$= 2 \cdot \frac{1}{x} + 3e^x + 0$$

$$= \frac{2}{x} + 3e^x$$

Therefore, the value of the given integral is,

$$2\ln x + 3e^x + C.$$

22)

Given that,

$$\int \left[\frac{1}{2t} - \sqrt{2}e^{t}\right] dt$$

$$= \int \frac{1}{2t}\,dt - \int \sqrt{2}\,e^{t}\,dt$$

$$= \frac{1}{2}\int \frac{1}{t}\,dt - \sqrt{2}\int e^{t}\,dt$$

$$= \frac{1}{2}\ln t - \sqrt{2}\,e^{t} + C$$

Now checking answer,

$$\frac{d}{dt}\left[\frac{1}{2}\ln t - \sqrt{2}\,e^{t} + C\right]$$

$$= \frac{d}{dt}\cdot\frac{1}{2}\cdot\ln t - \frac{d}{dt}\cdot\sqrt{2}\cdot e^{t} + \frac{d}{dt}C$$

$$= \frac{1}{2}\cdot\frac{1}{t} - \cdot\sqrt{2}\,e^{t} + 0$$

$$= \frac{1}{2t} - \sqrt{2}\,e^{t}$$

Therefore, the value of the given integral is,

$$\frac{1}{2}\ln t - \sqrt{2}\,e^{t} + C$$

23)

Given that,

$$\int [3\sin x - 2\sec^2 x]\, dx$$

$$= \int 3\sin x\, dx - \int 2\sec^2 x\, dx$$

$$= -3\cos x - 2\tan x + c$$

Now checking answer,

$$\frac{d}{dx}[-3\cos x - 2\tan x + c]$$

$$= \frac{d}{dx}(-3\cos x) - \frac{d}{dx} 2\tan x + \frac{d}{dx} c$$

$$= 3\sin x - 2\sec^2 x + 0$$

$$= 3\sin x - 2\sec^2 x$$

Therefore, the value of the given integral is,

$$-3\cos x - 2\tan x + c$$

24)

Given that,

$$\int [\operatorname{cosec}^2 x - \sec x \tan x]\, dx$$

$$= \int \operatorname{cosec}^2 x\, dx - \int \sec x \tan x\, dx$$

$$= -\cot x - \sec x + c$$

Now checking answer

$$\frac{d}{dx}[-\cot x - \sec x + c]$$

$$= \frac{d}{dx}(-\cot x) - \frac{d}{dx}\sec x + \frac{d}{dx}c$$

$$= \operatorname{cosec}^2 x - \sec x \tan x + 0$$

$$= \operatorname{cosec}^2 x - \sec x \tan x$$

Therefore, the value of the given integral is,

$$-\cot x - \sec x + c$$

25)

Given that,

$$\int \sec x(\sec x + \tan x)\, dx$$

$$= \int (\sec^2 x + \sec x \tan x)\, dx$$

$$= \int \sec^2 x\, dx + \int \sec x \tan x\, dx$$

$$= \tan x + \sec x + c$$

Now checking answer,

$$\frac{d}{dx}[\tan x + \sec x + c]$$

$$= \frac{d}{dx}\tan x + \frac{d}{dx}\sec x + \frac{d}{dx}c$$

$$= \sec^2 x + \sec x \tan x + 0$$

$$= \sec x(\sec x + \tan x)$$

Therefore the value of the given integral is,

$$\tan x + \sec x + c$$

26)

Given that,

$$\int \operatorname{cosec} x(\sin x + \cot x)\, dx$$

$$= \int (\operatorname{cosec} x \sin x + \operatorname{cosec} x \cot x)\, dx$$

$$= \int \left(\frac{1}{\sin x} \sin x + \operatorname{cosec} x \cot x\right) dx$$

$$= \int (1 + \operatorname{cosec} x \cot x)\, dx$$

$$= \int 1\, dx + \int \operatorname{cosec} x \cot x\, dx$$

$$= x - \operatorname{cosec} x + c$$

Now checking answer,

$$\frac{d}{dx}[x - \operatorname{cosec} x + c]$$

$$= \frac{d}{dx} x - \frac{d}{dx} \operatorname{cosec} x + \frac{d}{dx} c$$

$$= 1 + \operatorname{cosec} x \cancel{\tan} \cot x + 0$$

$$= \operatorname{cosec} x \left(\frac{1}{\operatorname{cosec} x} + \cot x\right)$$

$$= \operatorname{cosec} x (\sin x + \cot x)$$

Therefore, the value of the given integral is,

$$x - \operatorname{cosec} x + c$$

27)

Given that,

$$\int \frac{\sec\theta}{\cos\theta}\, d\theta$$

$$= \int \frac{\sec\theta}{\frac{1}{\sec\theta}}\, d\theta$$

$$= \int \sec^2\theta\, d\theta$$

$$= \tan\theta + C$$

Now checking answer;

$$\frac{d}{d\theta}\left[\tan\theta + C\right]$$

$$= \frac{d}{d\theta}\tan\theta + \frac{d}{d\theta}C$$

$$= \sec^2\theta + 0$$

$$= \sec\theta \cdot \sec\theta$$

$$= \sec\theta \cdot \frac{1}{\cos\theta}$$

$$= \frac{\sec\theta}{\cos\theta}$$

Therefore, the value of the given integral is

$$\tan\theta + C.$$

28)

Given that,

$$\int \frac{dy}{\operatorname{cosec} y}$$

$$= \int \frac{1}{\operatorname{cosec} y}\, dy$$

$$= \int \sin y \, dy$$

$$= -\cos y + C$$

Now checking answer,

$$\frac{d}{dy}\left[-\cos y + c\right]$$

$$= \frac{d}{dy}(-\cos y) + \frac{d}{dy} c$$

$$= \sin y + 0$$

$$= \frac{1}{\operatorname{cosec} y}$$

Therefore, the value of the given integrals is,

$$-\cos y + C.$$

29|

Given that,

$$\int \frac{\sin x}{\cos^2 x}\, dx$$

$$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\, dx$$

$$= \int \tan x \sec x\, dx$$

$$= \sec x + c$$

Now checking answer,

$$\frac{d}{dx}\left[\sec x + c\right]$$

$$= \frac{d}{dx}\sec x + \frac{d}{dx}c$$

$$= \sec x \tan x + 0$$

$$= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \frac{\sin x}{\cos^2 x}$$

Therefore, the value of the given integral is,

$\sec x + c.$

30|

Given that,

$$\int \left[\phi + \frac{2}{\sin^2\phi}\right] d\phi$$

$$= \int \phi \, d\phi + \int \frac{2}{\sin^2\phi} \, d\phi$$

$$= \int \phi \, d\phi + 2\int \csc^2\phi \, d\phi$$

$$= \frac{\phi^2}{2} - 2\cot\phi + c$$

Now checking answer,

$$\frac{d}{d\phi}\left[\frac{\phi^2}{2} - 2\cot\phi + c\right]$$

$$= \frac{d}{d\phi}\frac{\phi^2}{2} - \frac{d}{d\phi} 2\cot\phi + \frac{d}{d\phi} c$$

$$= \frac{1}{2} \cdot 2\phi^1 + 2\csc^2\phi + 0$$

$$= \phi + 2\csc^2\phi$$

$$= \phi + \frac{2}{\sin^2\phi}$$

Therefore, the value of the given integral is,

$$\frac{\phi^2}{2} - 2\cot\phi + c$$

31)

Given that,

$$\int [1+\sin^2\theta \operatorname{cosec}\theta]\, d\theta$$

$$= \int \left[1 + \sin^2\theta \frac{1}{\sin\theta}\right] d\theta$$

$$= \int [1+\sin\theta]\, d\theta$$

$$= \int 1\, d\theta + \int \sin\theta\, d\theta$$

$$= \theta - \cos\theta + c$$

Now checking answer,

$$\frac{d}{d\theta}[\theta - \cos\theta + c]$$

$$= \frac{d}{d\theta}\theta - \frac{d}{d\theta}\cos\theta + \frac{d}{d\theta}c$$

$$= 1 + \sin\theta + 0$$

$$= 1 + \frac{\sin\theta \cdot \sin\theta}{\sin\theta}$$

$$= 1 + \sin^2\theta \cdot \frac{1}{\sin\theta}$$

$$= 1 + \sin^2\theta \operatorname{cosec}\theta$$

Therefore the value of the given integral is

$\theta - \cos\theta + c$.

32/

Given that,

$$\int \frac{\sec x + \cos x}{2\cos x} dx$$

$$= \int \left[\frac{\sec x}{2\cos x} + \frac{\cos x}{2\cos x}\right] dx$$

$$= \int \left[\frac{1}{2}\sec^2 x + \frac{1}{2}\right] dx$$

$$= \int \frac{1}{2}\sec^2 x\, dx + \int \frac{1}{2} dx$$

$$= \frac{1}{2}\tan x + \frac{1}{2}x + c$$

Now checking answer,

$$\frac{d}{dx}\left[\frac{1}{2}\tan x + \frac{1}{2}x + c\right]$$

$$= \frac{d}{dx}\frac{1}{2}.\tan x + \frac{d}{dx}\frac{1}{2}.x + \frac{d}{dx}c$$

$$= \frac{1}{2}\sec^2 x + \frac{1}{2} + 0$$

$$= \frac{\sec^2 x + 1}{2}$$

$$= \frac{\cos x(\sec^2 x + 1)}{\cos x \cdot 2}$$

$$= \frac{\sec x + \cos x}{2\cos x}$$

Therefore, the value of the given integral is, $\frac{1}{2}\tan x + \frac{1}{2}x + c$

33)

Given that,

$$\int \left[\frac{1}{2\sqrt{1-x^2}} - \frac{3}{1+x^2} \right] dx$$

$$= \int \frac{1}{2\sqrt{1-x^2}} dx - \int \frac{3}{1+x^2} dx$$

$$= \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx - 3 \int \frac{1}{1+x^2} dx$$

$$= \frac{1}{2} \sin^{-1} x - 3 \tan^{-1} x + C$$

Now checking answer,

$$\frac{d}{dx} \left[\frac{1}{2} \sin^{-1} x - 3 \tan^{-1} x + c \right]$$

$$= \frac{d}{dx} \frac{1}{2} \cdot \sin^{-1} x - \frac{d}{dx} \cdot 3 \cdot \tan^{-1} x + \frac{d}{dx} c$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{1-x^2}} - 3 \cdot \frac{1}{1+x^2} + 0$$

$$= \frac{1}{2\sqrt{1-x^2}} - \frac{3}{1+x^2}$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \sin^{-1} x - 3 \tan^{-1} x + C$$

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Given that,

$$\int \left[\frac{4}{x\sqrt{x^2-1}} + \frac{1+x+x^3}{1+x^2}\right] dx$$

$$= \int \left[\frac{4}{x\sqrt{x^2-1}} + \frac{1}{1+x^2} + \frac{x+x^3}{1+x^2}\right] dx$$

$$= \int \left[\frac{4}{x\sqrt{x^2-1}} + \frac{1}{1+x^2} + x\right] dx$$

$$= \int \frac{4}{x\sqrt{x^2-1}} dx + \int \frac{1}{1+x^2} dx + \int x \, dx$$

$$= 4\sec^{-1}x + \tan^{-1}x + \frac{x^2}{2} + c$$

Now checking answer,

$$\frac{d}{dx}\left[4\sec^{-1}x + \tan^{-1}x + \frac{x^2}{2} + c\right]$$

$$= \frac{d}{dx} 4.\sec^{-1}x + \frac{d}{dx}\tan^{-1}x + \frac{d}{dx}\frac{x^2}{2} + \frac{d}{dx}c$$

$$= 4\frac{1}{x^2\sqrt{x^2-1}} + \frac{1}{1+x^2} + \frac{1}{2}.2x + 0$$

$$= \frac{4}{x^2\sqrt{x^2-1}} + \frac{1}{1+x^2} + x$$

$$= \frac{4}{x^2\sqrt{x^2-1}} + \frac{1+x+x^3}{1+x^2}$$

Therefore, the value of the given integral is,

$$4\sec^{-1}x + \tan^{-1}x + \frac{x^2}{2} + c$$

43)

a)

Given that,

$$\frac{dy}{dx} = \sqrt[3]{x}$$

$$\Rightarrow dy = \sqrt[3]{x}\ dx$$

$$\Rightarrow \int dy = \int x^{\frac{1}{3}}\ dx$$

$$\Rightarrow y = \frac{x^{4/3}}{4/3} + c$$

$$\Rightarrow y = \frac{3x^{4/3}}{4} + c$$

Now,

if, $x = 1$

$y = 2$

$$\therefore 2 = \frac{3 \cdot 1^{4/3}}{4} + c$$

$$\Rightarrow c = 2 - \frac{3}{4} = \frac{5}{4}$$

Hence, the particular solutions,

$$y = \frac{3x^{4/3}}{4} + \frac{5}{4}$$

b)

Given that,

$$\frac{dy}{dt} = \sin t + 1$$

$$\Rightarrow dy = \sin t + 1 \ dt$$

$$\Rightarrow \int dy = \int \sin t + 1 \ dt$$

$$\Rightarrow y = -\cos t + t + c$$

Now,

if

$$t = \frac{\pi}{3}$$

then, $y = \frac{1}{2}$

$$\therefore \frac{1}{2} = -\cos\frac{\pi}{3} + \frac{\pi}{3} + c$$

$$\Rightarrow \frac{1}{2} = -\frac{1}{2} + \frac{\pi}{3} + c$$

$$\Rightarrow c = \frac{1}{2} + \frac{1}{2} - \frac{\pi}{3}$$

$$\therefore c = 1 - \frac{\pi}{3}$$

Hence, the particular solutions,

$$y = -\cot t + t + 1 - \frac{\pi}{3}$$

c)

$$\frac{dy}{dx} = \frac{x+1}{\sqrt{x}}$$

$$\Rightarrow dy = \frac{x+1}{\sqrt{x}}\, dx$$

$$\Rightarrow \int dy = \int \left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right) dx$$

$$\Rightarrow y = \int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$$

$$= \int \sqrt{x}\, dx + \int \frac{1}{\sqrt{x}}\, dx$$

$$= \frac{x^{3/2}}{3/2} + 2\sqrt{x} + c$$

$$\therefore y = \frac{2x^{3/2}}{3} + 2\sqrt{x} + c$$

Now,

if, $x = 1$

then, $y = 0$

$$\therefore \quad 0 = \frac{2\, 1^{3/2}}{3} + 2\sqrt{1} + c$$

$$\Rightarrow 0 = \frac{2}{3} + 2 + c$$

$$\therefore c = -\frac{8}{3}$$

Hence, the particular solutions,

$$y = \frac{2x^{3/2}}{3} + 2\sqrt{x} - \frac{8}{3}$$

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a)

Given that,

$$\frac{dy}{dx} = \frac{1}{(2x)^3}$$

$$\Rightarrow dy = \frac{1}{8x^3} dx$$

$$\Rightarrow \int dy = \int \frac{1}{8} \cdot x^{-3} dx$$

$$\Rightarrow y = \frac{1}{8} \cdot \frac{x^{-2}}{-2} + c$$

$$\therefore y = -\frac{1}{16} x^{-2} + c$$

Now,

if, $x = 1$

then, $y = 0$

$$\therefore 0 = -\frac{1}{16} (1)^{-2} + c$$

$$\therefore c = \frac{1}{16}$$

Hence, the particular solutions,

$$y = -\frac{1}{16} x^{-2} + \frac{1}{16}$$

b)

Given that,

$$\frac{dy}{dt} = \sec^2 t - \sin t$$

$$\Rightarrow dy = (\sec^2 t - \sin t)\, dt$$

$$\Rightarrow \int dy = \int (\sec^2 t - \sin t)\, dt$$

$$\Rightarrow y = \int \sec^2 t\, dt - \int \sin t\, dt$$

$$\therefore y = \tan t + \cos t + C$$

Now,

if $t = \frac{\pi}{4}$

then, $y = 1$

$$\therefore 1 = \tan\frac{\pi}{4} + \cos\frac{\pi}{4} + C$$

$$\Rightarrow 1 = 1 + \frac{\sqrt{2}}{2} + C$$

$$\therefore C = -\frac{\sqrt{2}}{2}$$

Hence, the particular solutions is,

$$y = \tan t + \cos t - \frac{\sqrt{2}}{2}$$

c)

Given that,

$$\frac{dy}{dx} = x^2\sqrt{x^3}$$

$$\Rightarrow dy = x^2\sqrt{x^3}\ dx$$

$$\Rightarrow \int dy = \int x^2 \cdot x^{3\frac{1}{2}}\ dx$$

$$\Rightarrow y = \int x^2 \cdot x^{3/2}\ dx$$

$$= \int x^{7/2}\ dx$$

$$= \frac{x^{9/2}}{9/2} + c$$

$$\therefore y = \frac{2x^{9/2}}{9} + c$$

Now,

if, $x = 0$

$y = 0$

$$\therefore 0 = \frac{2 \cdot 0^{9/2}}{9} + c$$

$$\therefore c = 0$$

Hence, the particular solition is,

$$y = \frac{2x^{9/2}}{9}$$

45|

a)

Given that,

$$\frac{dy}{dx} = 4e^{x}$$

$$\Rightarrow dy = 4e^{x}\,dx$$

$$\Rightarrow \int dy = \int 4e^{x}\,dx$$

$$\Rightarrow y = 4e^{x} + c$$

Now,

if, $x=0$

then, $y = 1$

$\therefore$ $1 = 4e^{0} + c$

$c = 1-4 = -3$

Hence, the particular solution is,

$$y = 4e^{x} - 3$$

b)

Given that,

$$\frac{dy}{dt} = \frac{1}{t}$$

$$\Rightarrow dy = \frac{1}{t} dt$$

$$\Rightarrow \int dy = \int \frac{1}{t} dt$$

$$\Rightarrow y = \ln|t| + c$$

Now,

if, $t = -1$

then. $y = 5$

$$\therefore 5 = \ln|-1| + c$$

$$\Rightarrow 5 = 0 + c$$

$$\therefore c = 5$$

Hence, the particular solution is,

$$y = \ln|t| + 5.$$

46)

a)

Given that,

$$\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$$

$$\Rightarrow dy = \frac{3}{\sqrt{1-x^2}}\,dx$$

$$\Rightarrow \int dy = \int 3 \cdot \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\therefore y = 3\sin^{-1}x + c$$

Now,

if, $x = \frac{\sqrt{3}}{2}$

then $y = 0$

$$\therefore 0 = 3\sin^{-1}\frac{\sqrt{3}}{2} + c$$

$$\Rightarrow 0 = 3 \cdot \frac{\pi}{3} + c$$

$$\therefore c = \cancel{-\frac{\pi}{2}} -\pi$$

Hence, the particular solutions is,

$$y = 3\sin^{-1}x - \pi$$

b)

Given that,

$$\frac{dy}{dx} = \frac{x^2-1}{x^2+1}$$

$$\Rightarrow dy = \frac{x^2-1}{x^2+1} dx$$

$$\Rightarrow \int dy = \int \frac{x^2+1-2}{x^2+1} dx$$

$$\Rightarrow y = \int \left(\frac{x^2+1}{x^2+1} - \frac{2}{x^2+1}\right) dx$$

$$= \int \left(1 - \frac{2}{x^2+1}\right) dx$$

$$= \int 1\, dx - \int \frac{2}{x^2+1} dx$$

$$\therefore y = x - 2\tan^{-1}x + c$$

Now,

if, $x = 1$

then $y = \frac{\pi}{2}$

$$\therefore \frac{\pi}{2} = 1 - 2\tan^{-1}1 + c$$

$$\Rightarrow \frac{\pi}{2} = 1 - 2\frac{\pi}{4} + c$$

$$\therefore c = \frac{\pi}{2} + \frac{\pi}{2} - 1 = \pi - 1$$

Hence, the particular solutions is,

$$y = x - 2\tan^{-1}x + \pi - 1$$

47)

Given that,

$$s = s(t)$$

$$v(t) = s'(t)$$

Here,

$$v(t) = 32t$$

$$\Rightarrow \frac{ds}{dt} = 32t$$

$$\Rightarrow ds = 32t\,dt$$

$$\Rightarrow \int ds = \int 32t\,dt$$

$$\Rightarrow s(t) = 32 \cdot \frac{t^2}{2} + c$$

$$\therefore s(t) = 16t^2 + c$$

Now, $s(0) = 20$

Here, $t = 0$

$s = 20$

$$\therefore 20 = 16 \cdot 0^2 + c$$

$$\therefore c = 20 - 0 = 20$$

Therefore,

$$s(t) = 16t^2 + 20$$

48)

Given that,

$$s = s(t)$$

$$v(t) = s'(t)$$

Here,

$$v(t) = \cos t$$

$$\Rightarrow \frac{ds}{dt} = \cos t$$

$$\Rightarrow ds = \cos t \, dt$$

$$\Rightarrow \int ds = \int \cos t \, dt$$

$$\Rightarrow s(t) = \sin t + C$$

Now,

$$s(0) = 2$$

Here, $t = 0$

$s = 2$

$$\therefore 2 = \sin 0 + C$$

$$\Rightarrow C = 2 - 0 = 2$$

Therefore,

$$s(t) = \sin t + 2$$

49)

Given that,

$$s = s(t)$$

$$v(t) = s'(t)$$

Here,

$$v(t) = 3\sqrt{t}$$

$$\Rightarrow \frac{ds}{dt} = 3\sqrt{t}$$

$$\Rightarrow ds = 3\sqrt{t}\, dt$$

$$\Rightarrow \int ds = \int 3t^{\frac{1}{2}}\, dt$$

$$\Rightarrow s(t) = 3 \cdot \frac{t^{3/2}}{3/2} + c$$

$$= 2t^{3/2} + c$$

Now,

$$s(4) = 1$$

Here, $t = 4$

$s = 1$

$$\therefore 1 = 2 \cdot 4^{3/2} + c$$

$$\therefore c = 1 - 16 = -15$$

Therefore,

$$s(t) = 2t^{3/2} - 15$$

Sol

Given that,

$$s = s(t)$$

$$v(t) = s'(t)$$

Here,

$$v(t) = 3e^t$$

$$\Rightarrow \frac{ds}{dt} = 3e^t$$

$$\Rightarrow ds = 3e^t \, dt$$

$$\Rightarrow \int ds = \int 3e^t \, dt$$

$$\Rightarrow s(t) = 3e^t + c$$

Now,

$$s(1) = 0$$

Here, $t = 1$

$s = 0$

$$\therefore 0 = 3e^1 + c$$

$$\therefore c = -3e$$

Therefore,

$$s(t) = 3e^t - 3e$$

53/

Given that,

$$\frac{dy}{dx} = 2x+1$$

$$\Rightarrow dy = (2x+1)\,dx$$

$$\Rightarrow \int dy = \int (2x+1)\,dx$$

$$\Rightarrow y = 2\cdot\frac{x^2}{2} + x + c$$

$$\therefore y = x^2+x+c$$

From point $(-3,0)$ we get that,

$$x=-3$$
$$y=0$$

$$\therefore 0 = (-3)^2 - 3 + c$$

$$\therefore c = -9+3 = -6$$

Therefore the equation of the curve is,

$$y = x^2+x-6$$

54/

Given that,

$$\frac{dy}{dx} = (x+1)^2$$

$$\Rightarrow dy = (x^2+2x+1)\,dx$$

$$\Rightarrow \int dy = \int (x^2+2x+1)\,dx$$

$$\Rightarrow y = \frac{x^3}{3} + 2\cdot\frac{x^2}{2} + x + c$$

$$\therefore y = \frac{1}{3}x^3 + x^2 + x + C$$

From point $(-2, 8)$, we get that,

$$x = -2$$
$$y = 8$$

$$\therefore 8 = \frac{1}{3}(-2)^3 + (-2)^2 + (-2) + C$$

$$\Rightarrow 8 = \frac{-8}{3} + 4 - 2 + c$$

$$\therefore c = \frac{26}{3}$$

Therefore, the equation of the curve is,

$$y = \frac{1}{3}x^3 + x^2 + x + \frac{26}{3}$$

55)

Given that,

$$\frac{dy}{dx} = -\sin x$$

$$\Rightarrow dy = -\sin x \, dx$$

$$\Rightarrow \int dy = \int -\sin x \, dx$$

$$\therefore y = \cos x + c$$

From point (0,2) we get that,

$$x = 0$$
$$y = 2$$

$$\therefore 2 = \cos 0 + c$$

$$\Rightarrow 2 = 1 + c$$

$$\therefore c = 1$$

Therefore,

the equation of the curve is,

$$y = \cos x + 1$$

56)

Given that,

$$\frac{dy}{dx} = x^2$$

$$\Rightarrow dy = x^2 dx$$

$$\Rightarrow \int dy = \int x^2 dx$$

$$\Rightarrow y = \frac{x^3}{3} + c$$

$$\therefore y = \frac{1}{3}x^3 + c$$

From the point $(-1,2)$ we get that,

$x = -1$
$y = 2$

$$\therefore 2 = \frac{1}{3}(-1)^3 + c$$

$$\Rightarrow 2 = -\frac{1}{3} + c$$

$$\therefore c = 2 + \frac{1}{3} = \frac{7}{3}$$

Therefore,
the equation of the curve is,

$$y = \frac{1}{3}x^3 + \frac{7}{3}$$

57)

Given that,

$$\frac{d^2y}{dx^2} = 6x$$

$$\Rightarrow \frac{dy}{dx} = \int 6x\,dx$$

$$\therefore \frac{dy}{dx} = 3x^2 + C_1 \quad \cdots (i)$$

Now, the slope of the tangent line, -3

$$\therefore \frac{dy}{dx} = -3 \quad (\text{when } x = 1)$$

From (i),

$$-3 = 3(1)^2 + C_1$$

$$\therefore C_1 = -3 - 3 = -6$$

$$\therefore \frac{dy}{dx} = 3x^2 - 6$$

$$\Rightarrow dy = (3x^2 - 6)\,dx$$

$$\Rightarrow \int dy = \int (3x^2 - 6)\,dx$$

$$\Rightarrow y = 3 \cdot \frac{x^3}{3} - 6x + C_2$$

$$\therefore y = x^3 - 6x + C_2 \quad \cdots (ii)$$

Now,

if, $x = 1$

then

$y = 5 - 3 \cdot 1 = 2$

$\therefore$ from (ii),

$2 = 1^3 - 6 \cdot 1 + c_2$

$\therefore c_2 = 2 - 1 + 6 = 7$

Therefore,

the equation of the curve is,

$y = x^3 - 6x + 7$.

5.3

1)

a)

Given integral,

$$\int 2x(x^2+1)^{23}\,dx$$

Let,

$u = x^2+1$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x^2+1)$

$\Rightarrow \frac{du}{dx} = 2x$

$\therefore du = 2x\,dx$

Therefore,

$$\int 2x(x^2+1)^{23}\,dx$$

$$= \int u^{23}\,du$$

$$= \frac{u^{24}}{24} + C$$

$$= \frac{(x^2+1)^{24}}{24} + C$$

Therefore,

the value of the integral is,

$$\frac{1}{24}(x^2+1)^{24} + C.$$

b)

Given integral,

$$\int \cos^3 x \sin x \, dx$$

Let,

$$u = \cos x$$

$$\Rightarrow \frac{du}{dx} = -\sin x$$

$$\therefore -du = \sin x \, dx$$

Therefore,

$$\int \cos^3 x \sin x \, dx$$

$$= \int u^3 (-du)$$

$$= -\int u^3 \, du$$

$$= -\frac{u^4}{4} + c$$

$$= -\frac{\cos^4 x}{4} + c$$

Therefore the value of the given integral is

$$-\frac{\cos^4 x}{4} + c$$

2)

a)

Given integral,

$$\int \frac{1}{\sqrt{x}} \sin\sqrt{x}\, dx$$

Let,

$$u = \sqrt{x}$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$\therefore 2\,du = \frac{1}{\sqrt{x}}\,dx$$

Therefore,

$$\int \frac{1}{\sqrt{x}} \sin\sqrt{x}\, dx$$

$$= \int \sin u \cdot 2\,du$$

$$= 2\int \sin u\, du$$

$$= -2\cos u + C$$

$$= -2\cos\sqrt{x} + C$$

Therefore the value of the given integral is,

$$-2\cos\sqrt{x} + C.$$

b)

Given integral,

$$\int \frac{3x\,dx}{\sqrt{4x^2+5}}$$

Let,

$$u = 4x^2+5$$

$$\Rightarrow \frac{du}{dx} = 8x$$

$$\Rightarrow du = 8x\,dx$$

$$\therefore x\,dx = \frac{1}{8}\,du$$

Therefore,

$$\int \frac{3x\,dx}{\sqrt{4x^2+5}}$$

$$= \int \frac{3\cdot\frac{1}{8}\,du}{\sqrt{u}}$$

$$= \frac{3}{8}\int \frac{1}{\sqrt{u}}\,du$$

$$= \frac{3}{8}\cdot 2\cdot\sqrt{u} + C$$

$$= \frac{3}{4}\sqrt{4x^2+5} + C$$

Therefore the value of the given integral is,

$$\frac{3}{4}\sqrt{4x^2+5} + C$$

3)

a)

Given integral,

$$\int \sec^2(4x+1)\,dx$$

Let,

$$u = 4x+1$$

$$\Rightarrow \frac{du}{dx} = 4$$

$$\therefore dx = \frac{1}{4}du$$

Therefore,

$$\int \sec^2(4x+1)\,dx$$

$$= \int \sec^2 u \cdot \frac{1}{4}du$$

$$= \frac{1}{4}\int \sec^2 u\,du$$

$$= \frac{1}{4}\tan u + C$$

$$= \frac{1}{4}\tan(4x+1) + C$$

Therefore the value of the given integral is

$$\frac{1}{4}\tan(4x+1) + C$$

b)

Given integral,

$$\int y\sqrt{1+2y^2}\,dy$$

Let,

$u = 1+2y^2$

$\Rightarrow \dfrac{du}{dy} = 4y$

$\Rightarrow du = 4y\,dy$

$\therefore y\,dy = \frac{1}{4}du$

Therefore,

$$\int y\sqrt{1+2y^2}\,dy$$

$$= \int \sqrt{u}\cdot\frac{1}{4}du$$

$$= \frac{1}{4}\int \sqrt{u}\,du$$

$$= \frac{1}{4}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{4}\cdot\frac{u^{3/2}}{3/2} + c$$

$$= \frac{1}{4}\cdot\frac{2}{3}\cdot u^{3/2} + c$$

$$= \frac{1}{6}\cdot(1+2y^2)^{3/2} + c$$

Therefore the value of the given integral is,

$$\frac{1}{6}(1+2y^2)^{3/2} + c$$

4)

a)

Given integral,

$$\int \sqrt{\sin \pi\theta} \, \cos \pi\theta \, d\theta$$

Let,

$$u = \sin \pi\theta$$

$$\Rightarrow \frac{du}{d\theta} = \cos \pi\theta \cdot \pi$$

$$\therefore \frac{1}{\pi} du = \cos \pi\theta \, d\theta$$

Therefore

$$\int \sqrt{\sin \pi\theta} \, \cos \pi\theta \, d\theta$$

$$= \int \sqrt{u} \cdot \frac{1}{\pi} du$$

$$= \frac{1}{\pi} \int \sqrt{u} \, du$$

$$= \frac{1}{\pi} \cdot \frac{u^{3/2}}{3/2} + c$$

$$= \frac{1}{\pi} \cdot \frac{2}{3} \cdot u^{3/2} + c$$

$$= \frac{2}{3\pi} \cdot (\sin \pi\theta)^{3/2} + c$$

Therefore the value of the given integral is,

$$\frac{2}{3\pi} \sin^{3/2}(\pi\theta) + c$$

b)

Given integral,

$$\int (2x+7)(x^2+7x+3)^{4/5}\,dx$$

Let,

$$u = x^2+7x+3$$

$$\Rightarrow \frac{du}{dx} = 2x+7$$

$$\therefore du = (2x+7)\,dx$$

Therefore,

$$\int (2x+7)(x^2+7x+3)^{4/5}\,dx$$

$$= \int u^{4/5}.\,du$$

$$= \frac{u^{9/5}}{9/5} + c$$

$$= \frac{5}{9}\cdot u^{9/5} + c$$

$$= \frac{5}{9}(x^2+7x+3)^{9/5} + c$$

Therefore the value of the given integral is,

$$\frac{5}{9}(x^2+7x+3)^{9/5} + c$$

5]

a)

Given integral,

$$\int \cot x \operatorname{cosec}^2 x \, dx$$

Let,

$$u = \cot x$$

$$\Rightarrow \frac{du}{dx} = -\operatorname{cosec}^2 x$$

$$\therefore -du = \operatorname{cosec}^2 x \, dx$$

Therefore,

$$\int \cot x \operatorname{cosec}^2 x \, dx$$

$$= \int u \cdot (-du)$$

$$= -\int u \, du$$

$$= -\frac{u^2}{2} + c$$

$$= -\frac{1}{2} \cot^2 x + c$$

Therefore, the value of the given integral is,

$$-\frac{1}{2} \cot^2 x + c$$

b)

Given integral,

$$\int (1+\sin t)^9 \cos t \, dt$$

Let,

$$u = \cancel{\cot}\ 1+\sin t$$

$$\Rightarrow \frac{du}{dt} = \cos t$$

$$\therefore du = \cos t \, dt$$

Therefore,

$$\int (1+\sin t)^9 \cos t \, dt$$

$$= \int u^9 \, du$$

$$= \frac{u^{10}}{10} + C$$

$$= \frac{1}{10}(1+\sin t)^{10} + C$$

Therefore, the value of the given integral is,

$$\frac{1}{10}(1+\sin t)^{10} + C.$$

6)

a)

Given integral,

$$\int \cos 2x \, dx$$

Let,

$u = 2x$

$\Rightarrow \frac{du}{dx} = 2$

$\therefore dx = \frac{1}{2} du$

Therefore,

$$\int \cos 2x \, dx$$

$$= \int \cos u \, \frac{1}{2} du$$

$$= \frac{1}{2} \int \cos u \, du$$

$$= \frac{1}{2} \sin u + c$$

$$= \frac{1}{2} \sin 2x + c$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \sin 2x + c.$$

b)

Given integral,

$$\int x \sec^2 x^2 \, dx$$

Let,

$$u = x^2$$

$$\Rightarrow \frac{du}{dx} = 2x$$

$$\therefore \frac{1}{2} du = x \, dx$$

Therefore,

$$\int x \sec^2 x^2 \, dx$$

$$= \int \sec^2 u \, \frac{1}{2} du$$

$$= \frac{1}{2} \int \sec^2 u \, du$$

$$= \frac{1}{2} \tan u + c$$

$$= \frac{1}{2} \tan x^2 + c$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \tan x^2 + c.$$

7)

a)

Given integral,

$$\int x^2 \sqrt{1+x}\, dx$$

Let,

$u = 1+x$

$\Rightarrow \frac{du}{dx} = 1$

$\therefore dx = du$

again,

$u = 1+x$

$x = u-1$

Therefore,

$$\int x^2 \sqrt{1+x}\, dx$$

$$= \int (u-1)^2 \sqrt{u}\, du$$

$$= \int (u^2 - 2u + 1)\, u^{1/2}\, du$$

$$= \int (u^{5/2} - 2u^{3/2} + u^{1/2})\, du$$

$$= \frac{u^{7/2}}{7/2} - 2 \cdot \frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + c$$

$$= \frac{2}{7} u^{7/2} - \frac{4}{5} u^{5/2} + \frac{2}{3} u^{3/2} + c$$

$$= \frac{2}{7} (1+x)^{7/2} - \frac{4}{5} (1+x)^{5/2} + \frac{2}{3} (1+x)^{3/2} + c$$

Ans

b)

Given integral,

$$\int [\operatorname{cosec}(\sin x)]^2 \cos x \, dx$$

Let,

$u = \sin x$

$\Rightarrow \frac{du}{dx} = \cos x$

$\therefore du = \cos x \, dx$

Therefore,

$$\int [\operatorname{cosec}(\sin x)]^2 \cos x \, dx$$

$$= \int [\operatorname{cosec} u]^2 \, du$$

$$= \int \operatorname{cosec}^2 u \, du$$

$$= -\cot u + c$$

$$= -\cot(\sin x) + c$$

Therefore, the value of the given integral is,

$$-\cot(\sin x) + c.$$

8)

a)

Given integral,

$$\int \sin(x-\pi)\,dx$$

Let, $u = x-\pi$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

Therefore,

$$\int \sin(x-\pi)\,dx$$

$$= \int \sin u \, du$$

$$= -\cos u + c$$

$$= -\cos(x-\pi) + c$$

Therefore the value of the given integral is,

$$-\cos(x-\pi) + c.$$

b)

Given integral,

$$\int \frac{5x^4}{(x^5+1)^2}\,dx$$

Let,

$u = x^5 + 1$

$\Rightarrow \frac{du}{dx} = 5x^4$

$\therefore du = 5x^4\,dx$

Therefore,

$$\int \frac{5x^4\,dx}{(x^5+1)^2}$$

$$= \int \frac{1}{u^2}\,du$$

$$= \cancel{\ln u + c}$$

$$= \cancel{\ln(x^5+1) + c}$$

$$= \int u^{-2}\,du$$

$$= \frac{u^{-1}}{-1} + c$$

$$= -\frac{1}{u} + c$$

$$= -\frac{1}{x^5+1} + c$$

Ans

9)

a)

Given integral,

$$\int \frac{dx}{x \ln x}$$

Let,

$$u = \ln x$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{x}$$

$$\therefore du = \frac{1}{x} dx$$

Therefore,

$$\int \frac{dx}{x \ln x}$$

$$= \int \frac{1}{u} du$$

$$= \ln u + c$$

$$= \ln(\ln x) + c$$

Therefore the value of the given integral is,

$$\ln(\ln x) + c$$

b)

Given integral,

$$\int e^{-5x}\,dx$$

Let,

$$u = -5x$$

$$\Rightarrow \frac{du}{dx} = -5$$

$$\therefore dx = -\frac{1}{5}\,du$$

Therefore,

$$\int e^{-5x}\,dx$$

$$= \int e^{u} \cdot \frac{-1}{5}\,du$$

$$= -\frac{1}{5}\int e^{u}\,du$$

$$= -\frac{1}{5} \cdot e^{u} + C$$

$$= -\frac{1}{5} e^{-5x} + C$$

Therefore, the value of the given integral is,

$$-\frac{1}{5} e^{-5x} + C$$

10)

a)

Given integral,

$$\int \frac{\sin 3\theta}{1+\cos 3\theta} d\theta$$

Let, $u = 1+\cos 3\theta$

$\Rightarrow \cancel{\frac{du}{d\theta} = -\sin 3\theta \cdot 0}$

$\Rightarrow \frac{du}{d\theta} = -\sin 3\theta \cdot 3$

$\Rightarrow du = -3 \sin 3\theta \, d\theta$

$\therefore \sin 3\theta \, d\theta = -\frac{1}{3} du$

Therefore,

$$\int \frac{\sin 3\theta}{1+\cos 3\theta} d\theta$$

$$= \int \frac{1}{u} \left(-\frac{1}{3}\right) du$$

$$= -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln u + c$$

$$= -\frac{1}{3} \ln(1+\cos 3\theta) + c$$

Therefore, the value of the given integral is,

$$-\frac{1}{3} \ln(1+\cos 3\theta) + c.$$

b)

Given integral,

$$\int \frac{e^x}{1+e^x}\,dx$$

let, $u = \cancel{1+\cos 3\theta}\ 1+e^x$

$$\Rightarrow \frac{du}{dx} = e^x$$

$$\therefore du = e^x\,dx$$

Therefore,

$$\int \frac{e^x}{1+e^x}\,dx$$

$$= \int \frac{1}{u}\,du$$

$$= \ln u + c$$

$$= \ln(1+e^x) + c$$

Therefore the value of the given integral is,

$$\ln(1+e^x) + c$$

15)

Given integral,

$$\int (4x-3)^9 \, dx$$

Let,

$$u = 4x-3$$

$$\Rightarrow \frac{du}{dx} = 4$$

$$\therefore dx = \frac{1}{4} du$$

Therefore,

$$\int (4x-3)^9 \, dx$$

$$= \int u^9 \tfrac{1}{4} du$$

$$= \frac{1}{4} \int u^9 du$$

$$= \frac{1}{4} \cdot \frac{u^{10}}{10} + c$$

$$= \frac{1}{40} (4x-3)^{10} + c$$

Therefore, the value of the given integral is,

$$\frac{1}{40} (4x-3)^{10} + c.$$

16/

Given integral,

$$\int x^3\sqrt{5+x^4}\, dx$$

Let,

$$u = 5+x^4$$

$$\Rightarrow \frac{du}{dx} = 4x^3$$

$$\therefore x^3\, dx = \frac{1}{4}\, du$$

Therefore,

$$\int x^3\sqrt{5+x^4}\, dx$$

$$= \int \sqrt{u} \cdot \frac{1}{4}\, du$$

$$= \frac{1}{4}\int u^{\frac{1}{2}}\, du$$

$$= \frac{1}{4} \cdot \frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{4} \cdot \frac{2}{3} \cdot u^{3/2} + C$$

$$= \frac{1}{6} \cdot (5+x^4)^{3/2} + C$$

Therefore, the value of the given integral is,

$$\frac{1}{6} \cdot (5+x^4)^{3/2} + C.$$

17/

Given integral,

$$\int \sin 7x \, dx$$

Let,

$$u = 7x$$

$$\Rightarrow \frac{du}{dx} = 7$$

$$\therefore dx = \frac{1}{7} du$$

Therefore, $\int \sin 7x \, dx$

$$= \int \sin u \cdot \frac{1}{7} du$$

$$= \frac{1}{7} \int \sin u \, du$$

$$= \frac{1}{7} \cdot (-\cos u) + c$$

$$= -\frac{1}{7} \cos 7x + c$$

Therefore, the value of the given integral is,

$$-\frac{1}{7} \cos 7x + C$$

18/

Given integral,

$$\int \cos \frac{x}{3} \, dx$$

Let, $u = \frac{x}{3}$

$$\Rightarrow \frac{du}{dx} = \frac{1}{3}$$

$\therefore \; dx = 3\,du$

Therefore,

$$\int \cos\frac{x}{3}\,dx$$

$$= \int \cos u \; 3\,du$$

$$= 3\int \cos u\,du$$

$$= 3\sin u + c$$

$$= 3\sin\frac{x}{3} + c$$

Therefore, the value of the given integral is,

$$3\sin\frac{x}{3} + c$$

19/

Given integral,

$$\int \sec 4x \tan 4x\,dx$$

Let,

$$u = 4x$$

$$\Rightarrow \frac{du}{dx} = 4$$

$$\therefore dx = \frac{1}{4}du$$

Therefore, $\int \sec 4x \tan 4x\,dx$

$$= \int \sec u \tan u \cdot \frac{1}{4}du$$

$$= \frac{1}{4}\int \sec u \tan u\,du$$

$$= \frac{1}{4}\sec u + c$$

$= \frac{1}{4} \sec 4x + c$

Therefore the value of the given integral is,

$\frac{1}{4} \sec 4x + C.$

20)

Given integral,

$\int \sec^2 5x \, dx$

Let,

$u = 5x$

$\Rightarrow \frac{du}{dx} = 5$

$\therefore dx = \frac{1}{5} du$

Therefore, $\int \sec^2 5x \, dx$

$= \int \sec^2 u . \frac{1}{5} du$

$= \frac{1}{5} \int \sec^2 u \, du$

$= \frac{1}{5} \tan u + c$

$= \frac{1}{5} \tan 5x + C$

Therefore, the value of the given integral is,

$\frac{1}{5} \tan 5x + C.$

21)

Given integral,

$$\int e^{2x}\,dx$$

Let,

$u = 2x$

$\Rightarrow \frac{du}{dx} = 2$

$\therefore dx = \frac{1}{2}du$

Therefore,

$$\int e^{2x}\,dx$$

$$= \int e^{u}\cdot\frac{1}{2}du$$

$$= \frac{1}{2}\int e^{u}du$$

$$= \frac{1}{2}e^{u}\,\cancel{du} + c$$

$$= \frac{1}{2}e^{2x} + c$$

Therefore the value of the given integral, $\frac{1}{2}e^{2x}+c$

22)

Given integral,

$$\int \frac{dx}{2x}$$

Let, $u = 2x$

$\Rightarrow \frac{du}{dx} = 2$

$\therefore dx = \frac{1}{2}du$

Therefore,

$$\int \frac{dx}{2x}$$

$$= \int \frac{1}{u} \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln u \cancel{d} + c$$

$$= \frac{1}{2} \ln(2x) + c$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \ln(2x) + c$$

23)

Given integral,

$$\int \frac{dx}{\sqrt{1-4x^2}}$$

$$= \int \frac{dx}{\sqrt{1-(2x)^2}}$$

Let,

$$u = 2x$$

$$\Rightarrow \frac{du}{dx} = 2$$

$$\therefore dx = \frac{1}{2} du$$

Therefore,

$$\int \frac{dx}{\sqrt{1-(2x)^2}}$$

$$= \int \frac{1}{\sqrt{1-u^2}} \cdot \frac{1}{2}\, du$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\, du$$

$$= \frac{1}{2} \sin^{-1} u + c$$

$$= \frac{1}{2} \sin^{-1} 2x + c$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \sin^{-1} 2x + c$$

24/

Given integral,

$$\int \frac{dx}{1+16x^2}$$

$$= \int \frac{dx}{1+(4x)^2}$$

Let, $u = 4x$

$$\Rightarrow \frac{du}{dx} = 4$$

$$\therefore dx = \frac{1}{4}\, du$$

Therefore,

$$\int \frac{dx}{1+(4x)^2}$$

$$= \int \frac{1}{1+u^2} \cdot \frac{1}{4} du$$

$$= \frac{1}{4} \int \frac{1}{1+u^2} du$$

$$= \frac{1}{4} \tan^{-1} u + c$$

$$= \frac{1}{4} \tan^{-1} 4x + c$$

Therefore the value of the given integral is,

$$\frac{1}{4} \tan^{-1} 4x + c.$$

25)

Given integral, $\int t\sqrt{7t^2+12}\, dt$

Let,

$$u = 7t^2+12$$

$$\Rightarrow \frac{du}{dt} = 14t$$

$$\Rightarrow du = 14t\, dt$$

$$\therefore t\, dt = \frac{1}{14} du$$

Therefore,

$$\int t\sqrt{7t^2+12}\,dt$$

$$= \int \sqrt{u} \cdot \frac{1}{14}\,du$$

$$= \frac{1}{14}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{14} \cdot \frac{u^{3/2}}{3/2} + c$$

$$= \frac{1}{14} \cdot \frac{2}{3} \cdot u^{3/2} + c$$

$$= \frac{1}{21} \cdot (7t^2+12)^{3/2} + c$$

Therefore, the value of the given integral is,

$$\frac{1}{21} \cdot (7t^2+12)^{3/2} + c$$

2(c)

Given integral,

$$\int \frac{x}{\sqrt{4-5x^2}}\,dx$$

Let,

$$u = 4-5x^2$$

$$\Rightarrow \frac{du}{dx} = -10x$$

$$\Rightarrow du = -10x\,dx$$

$$\therefore x\,dx = -\frac{1}{10}\,du$$

Therefore,

$$\int \frac{x}{\sqrt{4-5x^2}}\,dx$$

$$= \int \frac{1}{\sqrt{u}} \cdot \left(-\frac{1}{10}\right) \cdot du$$

$$= -\frac{1}{10} \int \frac{1}{\sqrt{u}}\,du$$

$$\cancel{= -\frac{1}{10} \ln u + c}$$

$$\cancel{= -\frac{1}{10} \ln (4-5x^2) + c}$$

$$= -\frac{1}{10} \int u^{-\frac{1}{2}}\,du$$

$$= -\frac{1}{10} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= -\frac{1}{5}\sqrt{(4-5x^2)} + c$$

Therefore, the value of the given integral is,

$$-\frac{1}{5}\sqrt{(4-5x^2)} + c.$$

27/

Given that,

$$\int \frac{6}{(1-2x)^3}\,dx$$

Let, $u = 1-2x$

$$\Rightarrow \frac{du}{dx} = -2$$

$$\therefore dx = -\frac{1}{2}\,du$$

Therefore,

$$\int \frac{6}{(1-2x)^3}\,dx$$

$$= \int \frac{6}{u^3}\cdot\left(-\frac{1}{2}\right)du$$

$$= -\frac{6}{2}\int u^{-3}\,du$$

$$= -\frac{6}{2}\cdot\frac{u^{-2}}{-2} + c$$

$$= \frac{6}{4}\cdot(1-2x)^{-2} + c$$

$$= \frac{3}{2}\cdot(1-2x)^{-2} + c$$

Therefore, the value of the given integral is,

$$\frac{3}{2}(1-2x)^{-2} + c.$$

28|

Given integral,

$$\int \frac{x^2+1}{\sqrt{x^3+3x}}\,dx$$

Let, $u = x^3+3x$

$$\Rightarrow \frac{du}{dx} = 3x^2+3$$

$$\Rightarrow \frac{du}{dx} = 3(x^2+1)$$

$$\therefore \frac{1}{3}du = (x^2+1)\,dx$$

Therefore,

$$\int \frac{x^2+1}{\sqrt{x^3+3x}}\,dx$$

$$= \int \frac{1}{\sqrt{u}} \cdot \frac{1}{3}du$$

$$= \frac{1}{3}\int u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{3} \cdot 2 \cdot \sqrt{u} + c$$

$$= \frac{2}{3}\sqrt{x^3+3x} + c$$

Therefore, the value of the given integral is,

$$\frac{2}{3}\sqrt{x^3+3x} + c$$

29)

Given integral,

$$\int \frac{x^3}{(5x^4+2)^3}\,dx$$

Let,

$$u = 5x^4 + 2$$

$$\Rightarrow \frac{du}{dx} = 20x^3$$

$$\therefore x^3\,dx = \frac{1}{20}\,du$$

Therefore,

$$\int \frac{x^3}{(5x^4+2)^3}\,dx$$

$$= \int \frac{1}{u^3}\cdot\frac{1}{20}\,du$$

$$= \frac{1}{20}\int u^{-3}\,du$$

$$= \frac{1}{20}\cdot\frac{u^{-2}}{-2} + c$$

$$= -\frac{1}{40}\cdot(5x^4+2)^{-2} + c$$

$$= -\frac{1}{40}\cdot(5x^4+2)^{-2} + c$$

Therefore, the value of the given integral is,

$$-\frac{1}{40}(5x^4+2)^{-2} + c.$$

30|

Given integeral,

$$\int \frac{\sin(1/x)}{3x^2}\,dx$$

Let,

$$u = \frac{1}{x}$$

$$\Rightarrow \frac{du}{dx} = -\frac{1}{x^2}$$

$$\therefore \frac{1}{x^2}\,dx = -du$$

Therefore,

$$\int \frac{\sin(1/x)}{3x^2}\,dx$$

$$= \int \frac{\sin u}{3}(-du)$$

$$= -\frac{1}{3}\int \sin u\,du$$

$$= -\frac{1}{3}(-\cos u) + c$$

$$= \frac{1}{3}\cos\frac{1}{x} + c$$

Therefore, the value of the given integral is

$$\frac{1}{3}\cos\frac{1}{x} + c$$

31)

Given integral,

$$\int e^{\sin x} \cos x \, dx$$

Let,

$$u = \sin x$$

$$\Rightarrow \frac{du}{dx} = \cos x$$

$$\therefore du = \cos x \, dx$$

Therefore,

$$\int e^{\sin x} \cos x \, dx$$

$$= \int e^{u} \, du$$

$$= e^{u} + c$$

$$= e^{\sin x} + c$$

Therefore, the value of the given integral is,

$$e^{\sin x} + c.$$

32)

Given integral,

$$\int x^3 e^{x^4} \, dx$$

Let, $u = x^4$

$$\Rightarrow \frac{du}{dx} = 4x^3$$

$$\therefore \frac{1}{4} du = x^3 \, dx$$

Therefore,

$$\int x^3 e^{x^4}\,dx$$

$$= \int e^u \cdot \frac{1}{4}\,du$$

$$= \frac{1}{4}\int e^u\,du$$

$$= \frac{1}{4} e^u + c$$

$$= \frac{1}{4} e^{x^4} + c$$

Therefore the value of the given integral is,

$$\frac{1}{4} e^{x^4} + c.$$

33)

Given integral,

$$\int x^2 e^{-2x^3}\,dx$$

Let, $u = -2x^3$

$$\Rightarrow \frac{du}{dx} = -6x^2$$

$$\Rightarrow du = -6x^2\,dx$$

$$\Rightarrow -\frac{1}{6}\,du = x^2\,dx$$

Therefore, $\int x^2 e^{-2x^3}\,dx$

$$= \int e^u \cdot \left(-\frac{1}{6}\right) \cdot du$$

$$= -\frac{1}{6}\int e^{u}\, du$$

$$= -\frac{1}{6}e^{u} + C$$

$$= -\frac{1}{6}e^{-2x^3} + C$$

Therefore, the value of the given integral is,

$$-\frac{1}{6}e^{-2x^3} + C$$

34/

Given integral,

$$\int \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\, dx$$

Let, $u = e^{x} - e^{-x}$

$$\Rightarrow \frac{du}{dx} = e^{x} - e^{-x}\cdot(-1)$$

$$\Rightarrow \frac{du}{dx} = e^{x} + e^{-x}$$

$$\therefore du = (e^{x} + e^{-x})\, dx$$

Therefore, $\displaystyle\int \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\, dx$

$$= \int \frac{1}{u}\, du$$

$$= \ln u + C$$

$$= \ln(e^{x} - e^{-x}) + C \quad \text{Ans.}$$

35|

Given integral,

$$\int \frac{e^x}{1+e^{2x}} dx$$

Let,

$$u = e^x$$

$$\Rightarrow \frac{du}{dx} = e^x$$

$$\therefore du = e^x dx$$

Therefore,

$$\int \frac{e^x}{1+e^{2x}} dx$$

$$= \int \frac{1}{1+u^2} du$$

$$= \tan^{-1} u + c$$

$$= \tan^{-1}(e^x) + c$$

Therefore, the value of the given integral is,

$$\tan^{-1}(e^x) + c$$

36|

Given integral,

$$\int \frac{t}{t^4+1} dt$$

$$= \int \frac{t}{(t^2)^2+1} dt$$

Let,

$u = t^2$

$\Rightarrow \frac{du}{dt} = 2t$

$\Rightarrow t\,dt = \frac{1}{2}du$

Therefore,

$$\int \frac{t}{(t^2)^2+1}\,dt$$

$$= \int \frac{1}{u^2+1} \cdot \frac{1}{2}du$$

$$= \frac{1}{2}\int \frac{1}{u^2+1}\,du$$

$$= \frac{1}{2}\tan^{-1}u + c$$

$$= \frac{1}{2}\tan^{-1}(t^2) + c$$

Therefore, the value of the given integral is,

$$\frac{1}{2}\tan^{-1}(t^2) + c$$

37|

Given integral,

$$\int \frac{\sin(5/x)}{x^2}\,dx$$

Let, $u = \frac{5}{x}$

$\Rightarrow \frac{du}{dx} = -\frac{5}{x^2}$

$\Rightarrow -\frac{1}{5}du = \frac{1}{x^2}dx$

Therefore,

$$\int \frac{\sin(5/x)}{x^2}\,dx$$

$$= \int \sin u \left(-\frac{1}{5}\right) du$$

$$= -\frac{1}{5} \int \sin u\, du$$

$$= -\frac{1}{5} (-\cos u) + C$$

$$= \frac{1}{5} \cos \frac{5}{x} + C$$

Therefore the value of the given integral is,

$$\frac{1}{5} \cos \frac{5}{x} + C$$

38)

Given integral,

$$\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}\,dx$$

Let,

$$u = \sqrt{x}$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$\therefore 2\,du = \frac{1}{\sqrt{x}}\,dx$$

Therefore $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}}\,dx$

$$= \int \sec^2 u \cdot 2\,du$$

$= 2\int \sec^2 u \, du$

$= 2 \tan u + c$

$= 2 \tan \sqrt{x} + c$

Therefore the value of the given integral is,

$2 \tan \sqrt{x} + c$

39/

Given integral,

$$\int \cos^4 3t \sin 3t \, dt$$

Let,

$u = \cos 3t$

$\Rightarrow \dfrac{du}{dx} = -\sin 3t \cdot 3$

$\Rightarrow \dfrac{du}{dx} = -3 \sin 3t$

$\therefore -\frac{1}{3} du = \sin 3t \, dx$

Therefore,

$\int \cos^4 3t \sin 3t \, dt$

$= \int u^4 \cdot (-\frac{1}{3}) du$

$= -\frac{1}{3} \int u^4 \, du$

$= -\frac{1}{3} \cdot \dfrac{u^5}{5} + c$

$= -\frac{1}{3} \cdot \dfrac{\cos^5(3t)}{5} + c$ Ans

40|

Given integral,

$$\int \cos 2x \sin^5 2x \, dx$$

Let,

$$u = \sin 2x$$

$$\Rightarrow \frac{du}{dx} = \cos 2x \cdot 2$$

$$\Rightarrow \frac{1}{2} du = \cos 2x \, dx$$

Therefore,

$$\int \cos 2x \sin^5 2x \, dx$$

$$= \int u^5 \cdot \frac{1}{2} \cdot du$$

$$= \frac{1}{2} \int u^5 \, du$$

$$= \frac{1}{2} \cdot \frac{u^6}{6} + C$$

$$= \frac{1}{12} \cdot \sin^6(2x) + C$$

Therefore, the value of the given integral is,

$$\frac{1}{12} \cdot \sin^6(2x) + C.$$

41)

Given integral,

$$\int x \sec^2(x^2)\, dx$$

Let,
$$u = x^2$$

$$\Rightarrow \frac{du}{dx} = 2x$$

$$\therefore \frac{1}{2} du = x\, dx$$

Therefore

$$\int x \sec^2(x^2)\, dx$$

$$= \int \sec^2 u \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \int \sec^2 u\, du$$

$$= \frac{1}{2} \tan u + c$$

$$= \frac{1}{2} \tan x^2 + c$$

Therefore the value of the given integral is,

$$\frac{1}{2} \tan x^2 + c$$

42)

Given integral,

$$\int \frac{\cos 4\theta}{(1+2\sin 4\theta)^4}\, d\theta$$

Let,
$$u = 1 + 2\sin 4\theta$$

$$\Rightarrow \frac{du}{d\theta} = 2\cos 4\theta \cdot 4 = 8\cos 4\theta$$

$\therefore \frac{1}{8} du = \cos 4\theta \, d\theta$

Therefore,

$$\int \frac{\cos 4\theta}{(1+2\sin 4\theta)^4} d\theta$$

$$= \int \frac{1}{u^4} \cdot \frac{1}{8} \cdot du$$

$$= \frac{1}{8} \int u^{-4} du$$

$$= \frac{1}{8} \cdot \frac{u^{-3}}{-3} + c$$

$$= -\frac{1}{24} \cdot (1+2\sin 4\theta)^{-3} + c$$

Therefore, the value of the given integral is,

$$-\frac{1}{24} \cdot (1+2\sin 4\theta)^{-3} + c$$

43)

Given integral,

$$\int \cos 4\theta \sqrt{2-\sin 4\theta} \, d\theta$$

Let, $u = 2 - \sin 4\theta$

$$\Rightarrow \frac{du}{d\theta} = -\cos 4\theta \cdot 4$$

$$\Rightarrow \frac{du}{d\theta} = -4\cos 4\theta$$

$\therefore -\frac{1}{4} du = \cos 4\theta \, d\theta$

Therefore,

$$\int \cos 4\theta \sqrt{2-\sin 4\theta}\, d\theta$$

$$= \int \sqrt{u} \cdot \left(-\frac{1}{4}\right) du$$

$$= -\frac{1}{4} \int u^{\frac{1}{2}}\, du$$

$$= -\frac{1}{4} \cdot \frac{u^{3/2}}{3/2} + c$$

$$= -\frac{1}{4} \cdot \frac{2}{3} \cdot u^{3/2} + c$$

$$= -\frac{1}{6} (2-\sin 4\theta)^{3/2} + c$$

Therefore the value of the given integral is,

$$-\frac{1}{6} (2-\sin 4\theta)^{3/2} + c$$

44)

Given integral,

$$\int \tan^3 5x \sec^2 5x\, dx$$

Let,

$$u = \tan 5x$$

$$\Rightarrow \frac{du}{dx} = \sec^2 5x \cdot 5$$

$$\therefore \frac{1}{5} du = \sec^2 5x\, dx$$

Therefore,

$$\int \tan^3 5x \sec^2 5x \, dx$$

$$= \int u^3 \cdot \frac{1}{5} du$$

$$= \frac{1}{5} \int u^3 du$$

$$= \frac{1}{5} \cdot \frac{u^4}{4} + c$$

$$= \frac{1}{20} \tan^4 5x + c$$

Therefore the value of the given integeral,

$$\frac{1}{20} \tan^4 5x + c$$

45|

Given integral,

$$\int \frac{\sec^2 x \, dx}{\sqrt{1-\tan^2 x}}$$

Let, $u = \tan x$

$$\Rightarrow \frac{du}{dx} = \sec^2 x$$

$$\therefore du = \sec^2 x \, dx$$

Therefore, $\displaystyle\int \frac{\sec^2 x \, dx}{\sqrt{1-\tan^2 x}}$

$$= \int \frac{1}{\sqrt{1-u^2}} du$$

$= \sin^{-1} u + c$

$= \sin^{-1}(\tan x) + c$

Therefore the value of the given integral is,

$$\sin^{-1}(\tan x) + c.$$

46)

Given integral,

$$\int \frac{\sin\theta}{\cos^2\theta + 1}\, d\theta$$

Let, $u = \cos\theta$

$\Rightarrow \dfrac{du}{d\theta} = -\sin\theta$

$\therefore -du = \sin\theta\, d\theta$

Therefore, $\displaystyle\int \frac{\sin\theta}{\cos^2\theta+1}\, d\theta$

$= \displaystyle\int \frac{1}{u^2+1}(-du)$

$= -\displaystyle\int \frac{1}{u^2+1}\, du$

$= -\tan^{-1} u + c$

$= -\tan^{-1}(\cos\theta) + c$

Ans

47)

Given integral,

$$\int \sec^3 2x \tan 2x \, dx$$

Let, $u = \sec 2x$

$$\Rightarrow \frac{du}{dx} = \sec 2x \tan 2x \cdot 2$$

$$\therefore \frac{1}{2} du = \sec 2x \tan 2x \, dx$$

Therefore,

$$\int \sec^3 2x \tan 2x \, dx$$

$$= \int u^2 \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \int u^2 du$$

$$= \frac{1}{2} \cdot \frac{u^3}{3} + c$$

$$= \frac{1}{6} \cdot \sec^3 2x + c$$

Therefore the value of the given integral,

$$\frac{1}{6} \cdot \sec^3 2x + c.$$

48/

Given Integral,

$$\int [\sin(\sin\theta)] \cos\theta \, d\theta$$

Let,

$u = \sin\theta$

$\Rightarrow \frac{du}{d\theta} = \cos\theta$

$\therefore du = \cos\theta \, d\theta$

Therefore,

$$\int [\sin(\sin\theta)] \cos\theta \, d\theta$$

$$= \int \sin u \, du$$

$$= -\cos u + c$$

$$= -\cos(\sin\theta) + c$$

Therefore the value of the given integral is, $-\cos(\sin\theta) + c$

49/

Given integral,

$$\int \frac{dx}{e^x}$$

$$= \int e^{-x} \, dx$$

Let, $u = -x$

$\Rightarrow \frac{du}{dx} = -1$

$\therefore dx = -du$

Therefore,
$$\int e^{-x}\,dx$$
$$= \int e^{u}(-du)$$
$$= -\int e^{u}\,du$$
$$= -e^{u} + c$$
$$= -e^{-x} + c$$

Therefore the value of the given integral is,
$$-e^{-x} + c.$$

50|

Given integral,
$$\int \sqrt{e^{x}}\,dx$$
$$= \int e^{x/2}\,dx$$

Let,
$$u = \frac{x}{2}$$
$$\Rightarrow \frac{du}{dx} = \frac{1}{2}$$
$$\therefore 2du = dx$$

Therefore,
$$\int e^{x/2}\,dx$$
$$= \int e^{u} \cdot 2\,du$$
$$= 2\int e^{u}\,du$$
$$= 2e^{u} + c$$
$$= 2e^{x/2} + c$$

Therefore the value of the given integral is,
$$2e^{x/2} + c.$$

51)

Given integral,
$$\int \frac{dx}{\sqrt{x}\, e^{(2\sqrt{x})}}$$

Let, $u = 2\sqrt{x}$

$$\Rightarrow \frac{du}{dx} = \frac{1}{\sqrt{x}}$$
$$\therefore du = \frac{1}{\sqrt{x}}\,dx$$

Therefore, $$\int \frac{dx}{\sqrt{x}\, e^{2\sqrt{x}}}$$
$$= \int \frac{1}{e^{u}}\,du$$

$$= -e^{-u} + c$$

$$= -e^{-2\sqrt{x}} + c$$

Therefore, the value of the given integral,

$$-e^{-2\sqrt{x}} + c.$$

52)

Given integral,

$$\int \frac{e^{\sqrt{2y+1}}}{\sqrt{2y+1}}\,dy$$

Let, $u = \sqrt{2y+1}$

$$\Rightarrow \frac{du}{dy} = \frac{1}{2\sqrt{2y+1}} \cdot 2$$

$$\therefore du = \frac{1}{\sqrt{2y+1}}\,dy$$

Therefore, $\int \frac{e^{\sqrt{2y+1}}}{\sqrt{2y+1}}\,dy$

$$= \int e^{u}\,du$$

$$= e^{u} + c$$

$$= e^{\sqrt{2y+1}} + c$$

Ans.

53)

Given integral,

$$\int \frac{y}{\sqrt{2y+1}}\,dy$$

Let,

$u = 2y+1$ again,

$u = 2y+1$

$\Rightarrow \frac{du}{dy} = 2$

$y = \frac{1}{2}(u-1)$

$\therefore \frac{1}{2}du = dy$

Therefore,

$$\int \frac{y}{\sqrt{2y+1}}\,dy$$

$$= \int \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\,dy$$

$$= \frac{1}{2}\int (u-1)\,u^{-\frac{1}{2}}\ \cancel{dy}\ \frac{1}{2}du$$

$$= \frac{1}{4}\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du$$

$$= \frac{1}{4}\left(\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right) + c$$

$$= \frac{1}{6}u^{3/2} - \frac{1}{2}u^{1/2} + c$$

$$= \frac{1}{6}(2y+1)^{3/2} - \frac{1}{2}(2y+1)^{1/2} + c$$

Ans.

54)

Given integral,

$$\int x\sqrt{4-x}\, dx$$

Let,

$u = 4-x$ again, $u = 4-x$

$\Rightarrow \frac{du}{dx} = -1$ $x = 4-u$

$\therefore dx = -du$

Therefore,

$$\int x\sqrt{4-x}\, dx$$

$$= \int (4-u)\sqrt{u}\,(-du)$$

$$= -\int (4 \cdot u^{1/2} - u^{3/2})\, du$$

$$= -4 \cdot \frac{u^{3/2}}{3/2} + \frac{u^{5/2}}{5/2} + c$$

$$= -\frac{8}{3} u^{3/2} + \frac{2}{5} u^{5/2} + c$$

$$= -\frac{8}{3}(4-x)^{3/2} + \frac{2}{5}(4-x)^{5/2} + c$$

Therefore, the value of the given integral,

$$-\frac{8}{3}(4-x)^{3/2} + \frac{2}{5}(4-x)^{5/2} + c$$

55|

Given integral,

$$\int \sin^3 2\theta \, d\theta$$

$$= \int \sin^2 2\theta \sin 2\theta \, d\theta$$

$$= \int (1-\cos^2 2\theta) \sin 2\theta \, d\theta$$

Let,

$$u = \cos 2\theta$$

$$\Rightarrow \frac{du}{d\theta} = -\sin 2\theta \cdot 2$$

$$\Rightarrow \frac{du}{d\theta} = -2\sin 2\theta$$

$$\therefore -\frac{1}{2} du = \sin 2\theta \, d\theta$$

Therefore,

$$\int (1-\cos^2 2\theta) \sin 2\theta \, d\theta$$

$$= \int (1-u^2) \cdot \left(-\frac{1}{2}\right) du$$

$$= -\frac{1}{2} \int (1-u^2) \, du$$

$$= -\frac{1}{2}\left(u - \frac{u^3}{3}\right) + C$$

$$= -\frac{1}{2}\cos 2\theta + \frac{1}{6}\cos^3 2\theta + C$$

Ans

57|

Given integral,

$$\int \frac{t+1}{t}\,dt$$

$$= \int \left(\frac{t}{t} + \frac{1}{t}\right) dt$$

$$= \int \left(1 + \frac{1}{t}\right) dt$$

$$= \int 1\,dt + \int \frac{1}{t}\,dt$$

$$= t + \ln t + c$$

Therefore the value of the given integral is,

$$t + \ln t + c.$$

58|

Given integral,

$$\int e^{2\ln x}\,dx$$

$$= \int e^{\ln x^2}\,dx$$

$$= \int x^2\,dx$$

$$= \frac{x^3}{3} \cancel{dx} + c$$

$$= \frac{1}{3}x^3 + c$$

Therefore the value of the given integral,

$$\frac{1}{3}x^3 + c.$$

59)

Given integral,

$$\int [\ln(e^x) + \ln(e^{-x})]\,dx$$

Here,

$$\ln e^x + \ln e^{-x} = \ln(e^x e^{-x})$$

$$= \ln 1$$

$$= 0$$

$$\therefore \int (\ln e^x + \ln e^{-x})\,dx = c$$

Therefore, the value of the given integral, is c.

60)

Given integral,

$$\int \cot x\,dx$$

$$= \int \frac{\cos x}{\sin x}\,dx$$

Let,

$$u = \sin x$$

$$\Rightarrow \frac{du}{dx} = \cos x$$

$$\therefore du = \cos x\,dx$$

Therefore,

$$\int \frac{\cos x}{\sin x}\,dx = \int \frac{1}{u}\,du$$

$$= \ln u + c$$

$$= \ln(\sin x) + c$$

Therefore, the value of the given integral,

$$\ln(\sin x) + c.$$

6 //

a) Given integral,

$$\int \frac{dx}{\sqrt{9-x^2}}$$

$$= \int \frac{dx}{\sqrt{3^2-x^2}}$$

$$= \sin^{-1}\frac{x}{3} + c$$

Therefore the value of the given integral is, $\sin^{-1}\frac{x}{3} + c$

b)
Given integral,

$$\int \frac{dx}{5+x^2}$$

$$= \int \frac{dx}{(\sqrt{5})^2 + x^2}$$

$$= \frac{1}{\sqrt{5}} \tan^{-1}\frac{x}{\sqrt{5}} + c$$

Therefore the value of the given integral is,

$$\frac{1}{\sqrt{5}} \tan^{-1}\frac{x}{\sqrt{5}} + c.$$

e)

Given integral,

$$\int \frac{dx}{x\sqrt{x^2-\pi}}$$

$$= \int \frac{dx}{x\sqrt{x^2-(\sqrt{\pi})^2}}$$

$$= \frac{1}{\sqrt{\pi}} \sec^{-1} \frac{x}{\sqrt{\pi}} + c$$

Therefore the value of the given integral is,

$$\frac{1}{\sqrt{\pi}} \sec^{-1} \frac{x}{\sqrt{\pi}} + c$$

62)

a) Given integral.

$$\int \frac{e^x}{4+e^{2x}}\, dx$$

Let,

$$u = e^x$$

$$\frac{du}{dx} = e^x$$

$$\therefore du = e^x dx$$

Therefore $\int \frac{e^x}{4+e^{2x}}\, dx$

$$= \int \frac{du}{2^2+u^2}$$

$$= \frac{1}{2} \tan^{-1} \frac{u}{2} + c$$

$$= \frac{1}{2} \tan^{-1} \frac{e^x}{2} + c$$

The refore, the value of the given integral sis,

$$\frac{1}{2} \tan^{-1} \frac{e^x}{2} + c$$

b)

Given integral;

$$\int \frac{dx}{\sqrt{9-4x^2}}$$

$$= \int \frac{dx}{2\sqrt{\frac{9}{4}-x^2}}$$

$$= \frac{1}{2} \int \frac{dx}{\sqrt{\left(\frac{3}{2}\right)^2-x^2}}$$

$$= \frac{1}{2} \sin^{-1} \frac{x}{3/2} + c$$

$$= \frac{1}{2} \sin^{-1} \frac{2x}{3} + c$$

The refore, the value of the given integral is,

$$\frac{1}{2} \sin^{-1} \frac{2x}{3} + c.$$

e)

Given integral,

$$\int \frac{dy}{y\sqrt{5y^2-3}}$$

$$= \int \frac{dy}{y\sqrt{5\left(y^2-\frac{3}{5}\right)}}$$

$$= \frac{1}{\sqrt{5}} \int \frac{dy}{y\sqrt{y^2-\left(\frac{\sqrt{3}}{\sqrt{5}}\right)^2}}$$

$$= \frac{1}{\sqrt{5}} \cdot \frac{1}{\frac{\sqrt{3}}{\sqrt{5}}} \cdot \sec^{-1} \frac{y}{\sqrt{3}/\sqrt{5}} + c$$

$$= \frac{1}{\sqrt{3}} \sec^{-1} \frac{\sqrt{5}y}{\sqrt{3}} + c$$

Therefore the value of the given integral is,

$$\frac{1}{\sqrt{3}} \sec^{-1} \frac{\sqrt{5}y}{\sqrt{3}} + c.$$

5.4

35/

We need to make n subintervals.

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$$

Now,

point, $x_k = a + k\Delta x$

$= 1 + k \cdot \frac{3}{n}$

$= 1 + \frac{3k}{n}$

Now,

Area, $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$

$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(1+\frac{3k}{n}\right) \cdot \frac{3}{n}$

Here, $f(x) = \frac{x}{2}$

$\therefore f\left(1+\frac{3k}{n}\right) = \frac{1+\frac{3k}{n}}{2}$

$\therefore \lim_{n\to\infty} \sum_{k=1}^{n} f\left(1+\frac{3k}{n}\right) \cdot \frac{3}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1}{2}\left(1+\frac{3k}{n}\right)\frac{3}{n}\right)$

$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{3}{2}\left(\frac{1}{n} + \frac{3}{n^2}k\right)$

$$= \lim_{n\to\infty} \frac{3}{2}\left[\sum_{k=1}^{n}\frac{1}{n} + \sum_{k=1}^{n}\frac{3}{n^2}k\right]$$

$$= \lim_{n\to\infty} \frac{3}{2}\left[1+\frac{3}{n^2}\cdot\frac{1}{2}n(n+1)\right]$$

$$= \lim_{n\to\infty} \frac{3}{2}\left[1+\frac{3}{2}\frac{n+1}{n}\right]$$

$$= \lim_{n\to\infty} \frac{3}{2}\left[1+\frac{3}{2}\cdot\left(1+\frac{1}{n}\right)\right]$$

$$= \frac{3}{2}\left(1+\frac{3}{2}\right)$$

$$= \frac{15}{4}$$

Therefore area is $\frac{15}{4}$.

36)

We need to make n sub intervals.

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{5-0}{n} = \frac{5}{n}$$

Now, point, $x_k = a + k\cdot\Delta x$

$$= 0 + k\cdot\frac{5}{n}$$

$$= \frac{5k}{n}$$

Now,

$$\text{Area, } A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\cdot \Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{5k}{n}\right)\cdot \frac{5}{n}$$

Here $f(x) = 5 - x$

$$\therefore f\left(\frac{5k}{n}\right) = 5 - \frac{5k}{n}$$

$$\therefore \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{5k}{n}\right)\cdot \frac{5}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(5 - \frac{5k}{n}\right)\frac{5}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{25}{n} - \frac{25}{n^2}k\right)$$

$$= \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{25}{n} - \sum_{k=1}^{n} \frac{25}{n^2}k\right)$$

$$= \lim_{n\to\infty} \left[25 - \frac{25}{n^2}\cdot\frac{1}{2}n(n+1)\right]$$

$$= \lim_{n\to\infty} \left[25 - \frac{25}{2}\left(\frac{n+1}{n}\right)\right]$$

$$= \lim_{n\to\infty} \left[25 - \frac{25}{2}\left(1 + \frac{1}{n}\right)\right]$$

$$= 25 - \frac{25}{2}$$

$$= \frac{25}{2}$$

Therefore, area is $\frac{25}{2}$.

37/

We need to make n sub intervals.

$\therefore$ each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

Now, point, $x_k = a + k \cdot \Delta x$

$$= 0 + k \cdot \frac{3}{n}$$

$$= \frac{3k}{n}$$

Now, Area, $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{3k}{n}\right) \cdot \frac{3}{n}$$

Here,

$$f(x) = 9 - x^2$$

$$\therefore f\left(\frac{3k}{n}\right) = 9 - \left(\frac{3k}{n}\right)^2 = 9 - \frac{9}{n^2} k^2$$

$$\therefore \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{3k}{n}\right) \cdot \frac{3}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(9 - 9\frac{k^2}{n^2}\right) \frac{3}{n}$$

$$= \lim_{n\to\infty} \left[\frac{27}{n} \sum_{k=1}^{n} \left(1 - \frac{k^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty} \left[27 - \frac{27}{n^3} \sum_{k=1}^{n} k^2\right]$$

$$= 27 - 27\left(\frac{1}{3}\right)$$

$$= 18$$

Therefore

Area = 18.

38|

We need to make n intervals.

$\therefore$ Each sub Interval length,

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

Now point, $x_k = a + k \cdot \Delta x$

$$= 0 + k \cdot \frac{3}{n}$$

$$= \frac{3k}{n}$$

Now, Area, $A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} f\left(\frac{3k}{n}\right) \cdot \frac{3}{n}$$

Here,

$$f(x) = 4 - \frac{1}{4}x^2$$

$$\therefore f\left(\frac{3k}{n}\right) = 4 - \frac{1}{4}\left(\frac{3k}{n}\right)^2$$

$$= 4 - \frac{1}{4} \cdot \frac{9k^2}{n^2}$$

$$= 4 - \frac{9}{4} \cdot \frac{k^2}{n^2}$$

$$\therefore \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{3k}{n}\right)\frac{3}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(4-\frac{9}{4}\frac{k^2}{n^2}\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \left[\sum_{k=1}^{n} \left(\frac{12}{n} - \frac{27k^2}{4n^3}\right)\right]$$

$$= \lim_{n\to\infty} \left[\sum_{k=1}^{n} \frac{12}{n} - \frac{27}{4n^3}\sum_{k=1}^{n} k^2\right]$$

$$= \lim_{n\to\infty} \left[12 - \frac{27}{4n^3} \cdot \frac{1}{6}n(n+1)(2n+1)\right]$$

$$= \lim_{n\to\infty} \left(12 - \frac{9}{8}\frac{(n+1)(2n+1)}{n^2}\right)$$

$$= \lim_{n\to\infty} \left[12 - \frac{9}{8}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right]$$

$$= 12 - \frac{9}{8}\cdot 1 \cdot 2$$

$$= \frac{39}{4}$$

Therefore Area $= \frac{39}{4}$.

39|

We need to make n intervals,

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{6-2}{n} = \frac{4}{n}$$

Now, point, $x_k = a + k\cdot\Delta x$

$$= 2 + k\cdot\frac{4}{n}$$

$$= 2 + \frac{4k}{n}$$

Now, Area, $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\cdot \Delta x$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(2+\frac{4k}{n}\right)\frac{4}{n}$$

Here, $f(x) = x^3$

$$\therefore f\left(2+\frac{4k}{n}\right) = \left(2+\frac{4k}{n}\right)^3$$

$$\therefore \lim_{n\to\infty} \sum_{k=1}^{n} f\left(2+\frac{4k}{n}\right)\frac{4}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(2+\frac{4k}{n}\right)^3 \cdot \frac{4}{n}$$

$$= \lim_{n\to\infty} \left[\frac{32}{n} \sum_{k=1}^{n} \left(1+\frac{2}{n}k\right)^3\right]$$

$$= \lim_{n\to\infty} \left[\frac{32}{n} \sum_{k=1}^{n} \left(1+\frac{6}{n}k+\frac{12}{n^2}k^2+\frac{8}{n^3}k^3\right)\right]$$

$$= \lim_{n\to\infty} \left[\frac{32}{n}\left(\sum_{k=1}^{n} 1 + \frac{6}{n}\sum_{k=1}^{n} k + \frac{12}{n^2}\sum_{k=1}^{n} k^2 + \frac{8}{n^3}\sum_{k=1}^{n} k^3\right)\right]$$

$$= \lim_{n\to\infty} \left[\frac{32}{n}\left(n + \frac{6}{n}\cdot\frac{1}{2}n(n+1) + \frac{12}{n^2}\cdot\frac{1}{6}n(n+1)(2n+1) + \frac{8}{n^3}\cdot\frac{1}{4}n^2(n+1)^2\right)\right]$$

$$= \lim_{n\to\infty} \left[32\left(1+3\frac{n+1}{n}+2\frac{(n+1)(2n+1)}{n^2}+2\frac{(n+1)^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty} \left[32\left(1+3\left(1+\frac{1}{n}\right)+2\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)+2\left(1+\frac{1}{n}\right)^2\right)\right]$$

$$= 32\left(1+3\cdot1+2\cdot1\cdot2+2\cdot1^2\right)$$

$$= 320$$

Therefore Area, $A = 320$.

4o)

We need to make n sub intervals.

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{-1+3}{n} = \frac{2}{n}$$

Now, point, $x_k = a + k \cdot \Delta x$

$$= -3 + k \cdot \frac{2}{n}$$

$$= \frac{2k}{n} - 3$$

Now, Area, $A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} f\left(\frac{2k}{n} - 3\right) \cdot \frac{2}{n}$$

Here, $f(x) = 1 - x^3$

$$\therefore f\left(\frac{2k}{n} - 3\right) = 1 - \left(\frac{2k}{n} - 3\right)^3$$

$$\therefore \lim_{n \to \infty} \sum_{k=1}^{n} f\left(\frac{2k}{n} - 3\right) \cdot \frac{2}{n} = \lim_{n \to \infty} \sum_{k=1}^{n} \left\{1 - \left(\frac{2k}{n} - 3\right)^3\right\} \frac{2}{n}$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} \frac{2}{n}\left[28 - \frac{54}{n}k + \frac{36}{n^2}k^2 - \frac{8}{n^3}k^3\right]$$

$$= \lim_{n \to \infty} \left[\frac{2}{n}\left(28n - 27(n+1) + 6\frac{(n+1)(2n+1)}{n} - 2\frac{(n+1)^2}{n}\right]$$

$$= \lim_{n \to \infty} \left[2\left(28 - 27\left(1 + \frac{1}{n}\right) + 6\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) - 2\left(1 + \frac{1}{n}\right)^2\right)\right]$$

$$= 2(28 - 27 + 12 - 2) = 22$$

Therefore, Area, $A = 22$.

<u>4.1)</u>

We need to make n sub intervals,

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$$

points, $x_k = a + (k-1)\Delta x$

$$= 1 + (k-1)\frac{3}{n}$$

Now, Area,

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} f\left(1 + (k-1)\frac{3}{n}\right)\frac{3}{n}$$

Here,

$$f(x) = \frac{x}{2}$$

$$\therefore f\left(1 + (k-1)\frac{3}{n}\right) = \frac{1 + (k-1)\frac{3}{n}}{2}$$

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{2}\left(1 + (k-1)\frac{3}{n}\right)\frac{3}{n}$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{2} \cdot \frac{3}{n} + \frac{1}{2}(k-1)\frac{9}{n^2}$$

$$= \lim_{n \to \infty} \frac{3}{2n} \cdot \frac{1}{n} \cdot \sum_{k=1}^{n} (n + 3k - 3)$$

$$= \lim_{n\to\infty} \frac{3}{2n^2}\left[\sum_{k=1}^{n} n + \sum_{k=1}^{n} 3k - 3\sum_{k=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} \frac{3}{2n^2}\left[n\sum_{k=1}^{n} 1 + 3\sum_{k=1}^{n} k - 3\sum_{k=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} \frac{3}{2n^2}\left[n^2 + \frac{3}{2}\cdot\frac{n(n+1)}{2} - 3n\right]$$

$$= \lim_{n\to\infty} \frac{3}{2}\left[1 + \frac{3}{2}\left(1+\frac{1}{n}\right) - 3\frac{1}{n}\right]$$

$$= \frac{3}{2}\left[1 + \frac{3}{2}(1+0) - 0\right]$$

$$= \frac{3}{2}\left[1+\frac{3}{2}\right]$$

$$= \frac{3}{2}\cdot\frac{5}{2} = \frac{15}{4}$$

Therefore, area is $\frac{15}{4}$.

42|

We need to make n sub intervals,

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{5-0}{n} = \frac{5}{n}$$

points, $x_k = a + (k-1)\Delta x$

$$= 0 + (k-1)\cdot\frac{5}{n}$$

$$= (k-1)\frac{5}{n}$$

Now, Area,

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left((k-1)\frac{5}{n}\right)\frac{5}{n}$$

Here

$$f(x) = 5-x$$

$$\therefore f\left((k-1)\frac{5}{n}\right) = 5-(k-1)\frac{5}{n}$$

$$\therefore A = \lim_{n\to\infty} \sum_{k=1}^{n} \left(5-(k-1)\frac{5}{n}\right)\frac{5}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{25}{n} - \frac{25}{n^2}(k-1)\right]$$

$$= \lim_{n\to\infty} \left[\frac{25}{n}\sum_{k=1}^{n} 1 - \frac{25}{n^2}\sum_{k=1}^{n}(k-1)\right]$$

$$= \lim_{n\to\infty} \left(25 - \frac{25}{2}\frac{n-1}{n}\right)$$

$$= \lim_{n\to\infty} \left[25 - \frac{25}{2}\left(1-\frac{1}{n}\right)\right]$$

$$= 25 - \frac{25}{2}\cdot 1$$

$$= \frac{25}{2}$$

Therefore area is $\frac{25}{2}$.

43]

We need to make n sub intervals,

∴ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

points, $x_k = a + (k-1)\Delta x$

$$= 0 + (k-1)\frac{3}{n}$$

$$= (k-1)\frac{3}{n}$$

Now, Area,

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\cdot\Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left((k-1)\frac{3}{n}\right)\cdot\frac{3}{n}$$

Here,

$$f(x) = 9 - x^2$$

$$\therefore f\left((k-1)\frac{3}{n}\right) = 9 - \left((k-1)\frac{3}{n}\right)^2$$

$$= 9 - \frac{9}{n^2}(k-1)^2$$

$$\therefore A = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(9 - \frac{9}{n^2}(k-1)^2\right)\frac{3}{n}\right]$$

$$= \lim_{n\to\infty} \left[\frac{27}{n} \sum_{k=1}^{n} \left(1 - \frac{(k-1)^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty} \left[27 - \frac{27}{n^3} \sum_{k=1}^{n} k^2 + \frac{54}{n^3} \sum_{k=1}^{n} k - \frac{27}{n^2}\right]$$

$= \lim_{n \to \infty}$

$= 27 - 27\left(\frac{1}{3}\right) + 0 + 0$

$= 18$

Therefore the value or area is 18.

44)

We need to make n sub intervals,

$\therefore$ Each sub interval length,

$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$

point, $x_k = a + (k-1)\Delta x$

$= 0 + (k-1)\frac{3}{n}$

$= (k-1)\frac{3}{n}$

Now, Area,

$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$

$= \lim_{n \to \infty} \sum_{k=1}^{n} f\left((k-1)\frac{3}{n}\right) \cdot \frac{3}{n}$

Here, $f(x) = 4 - \frac{1}{4}x^2$

$\therefore f\left((k-1)\frac{3}{n}\right) = 4 - \frac{1}{4} \cdot \left((k-1)\frac{3}{n}\right)^2$

$= 4 - \frac{1}{4} \cdot \frac{9}{n^2}(k-1)^2 = 4 - \frac{9}{4n^2}(k-1)^2$

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} \left[\left(4 - \frac{9}{4n^2}(k-1)^2\right) \frac{3}{n} \right]$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} \frac{12}{n} - \frac{27k^2}{4n^3} + \frac{27k}{2n^3} - \frac{27}{4n^3}$$

$$= \lim_{n \to \infty} 12 - \frac{27}{4n^3} \cdot \frac{1}{6} n(n+1)(2n+1) + \frac{27}{2n^3} \frac{n(n+1)}{2} - \frac{27}{4n^2}$$

$$= \lim_{n \to \infty} \left[12 - \frac{9}{8} \frac{(n+1)(2n+1)}{n^2} + \frac{27}{4n} + \frac{27}{4n^2} - \frac{27}{4n^2} \right]$$

$$= \lim_{n \to \infty} 12 - \frac{9}{8}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) + 0 + 0 - 0$$

$$= 12 - \frac{9}{8} 1 \cdot 2$$

$$= \frac{39}{4}$$

Therefore, area is $\frac{39}{4}$.

45|

We need to make n sub intervals.

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{n} = \frac{4}{n}$$

points, $x_k = a + (k - \frac{1}{2})\Delta x$

$$= 0 + (k - \tfrac{1}{2}) \frac{4}{n}$$

$$= (k - \tfrac{1}{2}) \frac{4}{n}$$

Now, area,

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} f\left(\left(k - \tfrac{1}{2}\right)\tfrac{4}{n}\right) \tfrac{4}{n}$$

Here

$f(x) = 2x$

$\therefore f\left(\left(k - \tfrac{1}{2}\right)\tfrac{4}{n}\right) = 2 \cdot \tfrac{4}{n}\left(k - \tfrac{1}{2}\right)$

$= \tfrac{8}{n}\left(k - \tfrac{1}{2}\right)$

$$\therefore A = \lim_{n \to \infty} \sum_{k=1}^{n} \left(\tfrac{8}{n}\left(k - \tfrac{1}{2}\right)\right) \tfrac{4}{n}$$

$$= \lim_{n \to \infty} \left[\sum_{k=1}^{n} \left(\frac{32}{n^2}k - \frac{16}{n^2}\right)\right]$$

$$= \lim_{n \to \infty} \left[\frac{32}{n^2} \cdot \frac{n(n+1)}{2} - \frac{16}{n^2} \cdot n\right]$$

$$= \lim_{n \to \infty} \left(16 \cdot \frac{n+1}{n} - \frac{16}{n}\right)$$

$$= \lim_{n \to \infty} \left[16 \cdot \left(1 + \tfrac{1}{n}\right) - 16 \cdot \tfrac{1}{n}\right]$$

$= 16 \cdot (1+0) - 16 \cdot 0$

$= 16$

Therefore, area is 16.

46)

We need to make n sub intervals,

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n}$$

points, $x_k = a + (k - \frac{1}{2}) \Delta x$

$$= 1 + (k - \tfrac{1}{2}) \frac{4}{n}$$

Now, area

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k) \Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(1 + (k - \tfrac{1}{2})\tfrac{4}{n}\right) \cdot \frac{4}{n}$$

Here,

$f(x) = 6 - x$

$$\therefore f\left(1 + (k - \tfrac{1}{2})\tfrac{4}{n}\right) = 6 - 1 + (k - \tfrac{1}{2})\frac{4}{n}$$

$$= 5 - (k - \tfrac{1}{2})\frac{4}{n}$$

$$\therefore A = \lim_{n\to\infty} \sum_{k=1}^{n} \left(5 - (k - \tfrac{1}{2})\tfrac{4}{n}\right)\frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{20}{n} - \frac{16k}{n^2} + \frac{8}{n^2}\right]$$

$$= \lim_{n\to\infty} \left[\frac{20}{n} \cdot n - \frac{16}{n^2} \cdot \frac{n(n+1)}{2} + \frac{8}{n^2} \cdot n\right]$$

$$= \lim_{n\to\infty} 20 - 8 \cdot (1 + \tfrac{1}{n}) + 8 \frac{1}{n}$$

$= 20 - 8.1 + 0$

$= ~~12~~ \; 12$

Therefore area is 12.

<u>471</u>

We need to make n sub intervals.

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$$

$\therefore$ points,

$$x_k = a + (k - \tfrac{1}{2})\Delta x$$

$$= 0 + (k - \tfrac{1}{2})\frac{1}{n}$$

$$= (k - \tfrac{1}{2})\frac{1}{n}$$

Now,

$$\text{Area, } A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} f\left((k - \tfrac{1}{2})\frac{1}{n}\right) \cdot \frac{1}{n}$$

Here, $f(x) = x^2$

$$\therefore f\left((k - \tfrac{1}{2})\frac{1}{n}\right) = \left((k - \tfrac{1}{2})\frac{1}{n}\right)^2$$

$$= \frac{1}{n^2}(k - \tfrac{1}{2})^2$$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1}{n^2}\left(k-\frac{1}{2}\right)^2\right)\frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n^3}\left(k^2 - k + \frac{1}{4}\right)$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n^3}k^2 - \frac{1}{n^3}k + \frac{1}{4n^3}$$

$$= \frac{1}{3} + 0 + 0$$

$$= \frac{1}{3}$$

Therefore the area is $\frac{1}{3}$.

48)

We need to make n sub intervals.

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{1+1}{n} = \frac{2}{n}$$

points,

$$x_k = a + \left(k - \frac{1}{2}\right)\Delta x$$

$$= -1 + \left(k - \frac{1}{2}\right)\cdot\frac{2}{n}$$

Now, area,

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(-1 + \left(k-\frac{1}{2}\right)\frac{2}{n}\right) \cancel{\Delta x}\ \frac{2}{n}$$

Here, $f(x) = x^2$

$\therefore f\left(-1+(k-\frac{1}{2})\frac{2}{n}\right) = \left(-1+(k-\frac{1}{2})\frac{2}{n}\right)^2$

$$A = \lim_{n\to\alpha} \sum_{k=1}^{n} \left(-1+(k-\tfrac{1}{2})\tfrac{2}{n}\right)^2 \frac{2}{n}$$

$$= \lim_{n\to\alpha} \sum_{k=1}^{n} \frac{8k^2}{n^3} - \frac{8k}{n^3} + \frac{2}{n^3} - \frac{2}{n}$$

$$= \frac{8}{3} + 0 + 0 - 2$$

$$= \frac{2}{3}$$

Therefore,

Area is $\frac{2}{3}$.