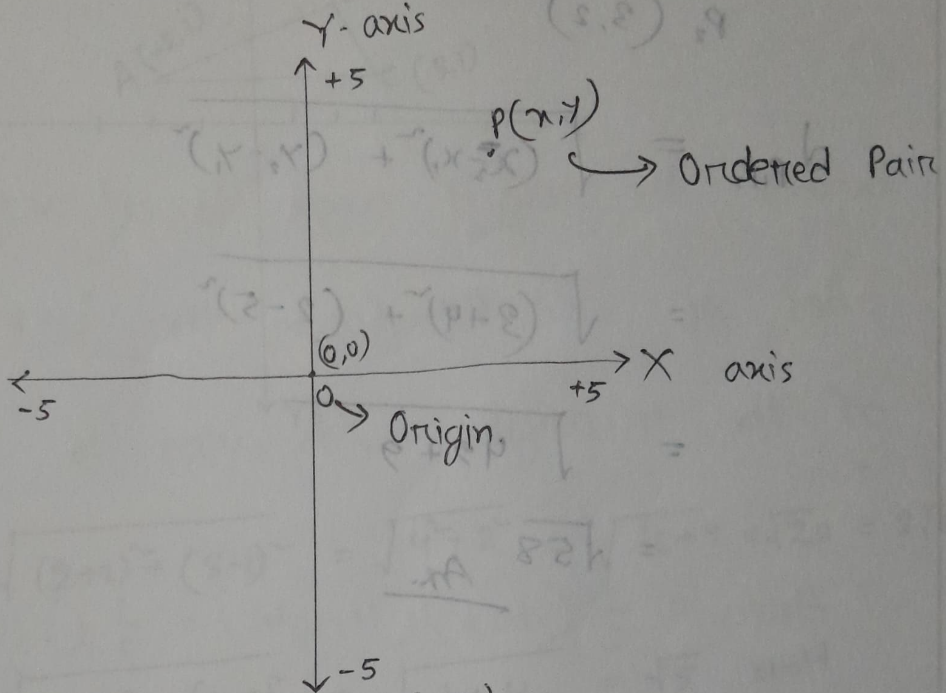
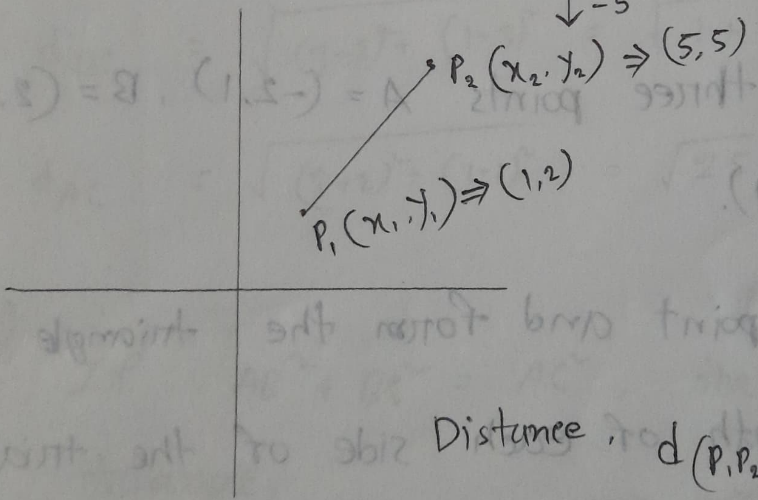
Chapter - One

Graphing





Distance, $d_{(P_1P_2)} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(5-1)^2 + (5-2)^2}$

$= \sqrt{4^2+3^2}$

$= \sqrt{25} = 5$ Ans

⊛ Figure 13.

$P_1 \ (-4, 5)$

$P_2 \ (3, 2)$

$$d_{P_1P_2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3+4)^2 + (2-5)^2}$$

$$= \sqrt{49 + 9}$$

$$= \sqrt{58}$$

Ans.

⊛ Consider the three points $A = (-2, 1)$, $B = (2, 3)$ and $C = (3, 1)$.

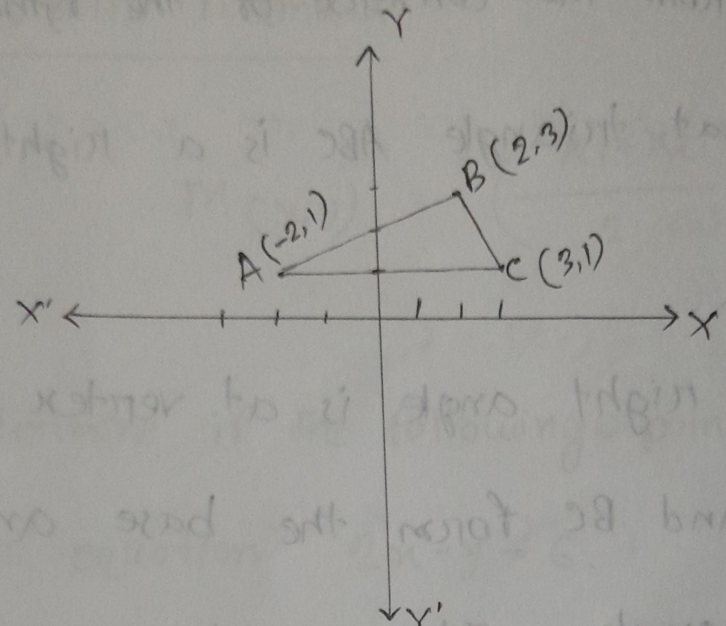
a) Plot each point and form the triangle ABC.

b) Find the length of each side of the triangle.

c) Verify that the triangle is a right triangle.

d) Find the area of the triangle.

a)



b)

$$d_{AB} = \sqrt{(2+2)^2 + (3-1)^2} = \sqrt{4^2 + 2^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} \text{ unit}$$

$$d_{BC} = \sqrt{(3-2)^2 + (1-3)^2} = \sqrt{1+4} = \sqrt{5} \text{ unit}$$

$$d_{AC} = \sqrt{(3+2)^2 + (1-1)^2} = \sqrt{25} = 5 \text{ unit}$$

c)

If $AB^2 + BC^2 = AC^2$, then it will be a right triangle.

$$L.H.S. = AB^2 + BC^2$$
$$= (d_{AB})^2 + (d_{BC})^2$$
$$= (2\sqrt{5})^2 + (\sqrt{5})^2$$
$$= 20 + 5$$
$$= 25$$
$$= 5^2$$
$$= (d_{AC})^2$$
$$= AC^2$$
$$= R.H.S.$$

It follows from the converse of the Pythagorean Theorem that triangle ABC is a right triangle.

d)

Because the right angle is at vertex B, the sides AB and BC form the base and height of the triangle. Its area is

$$\text{Area} = \frac{1}{2}(\text{Base})(\text{Height}) = \frac{1}{2}(2\sqrt{5})(\sqrt{5})$$

$$= 5 \text{ square units.}$$

* Midpoint Formula:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

* Determine if the following points are on the graph of the equation $2x - y = 6$.

a) $(2,3)$ b) $(2,-2)$

Solution

a) Let us substitute the given point in the above equation.

$$2 \cdot 2 - 3 = 6$$

$$\Rightarrow 4 - 3 = 6$$

$$\Rightarrow 1 = 6$$; this is not true.

Hence, the point $(2,3)$ is not on the graph of the above equation.

b) $(2,-2)$

Let us substitute the given point in the above equation.

$$2\cdot 2-(-2)=6$$

$$\Rightarrow 4+2=6$$

$$\Rightarrow 6=6$$: this is true.

Hence, the given point $(2,-2)$ is on the graph of the above equation.

⊛ Find the x-intercepts and y-intercepts of the graph of $y=x^2-4$.

$\Rightarrow$

In order to find the x-intercept(s), let us put $y=0$ in the above equation and solve it for x.

$$0=x^2-4$$

$$\Rightarrow x^2=4$$

$$\therefore x=\pm 2$$

Hence, the x-intercepts are -2 and 2.

In order to find the y-intercept(s), let us put $x=0$ in the above equation and solve it for y.

$$y = 0^2 - 4$$

$$\Rightarrow y = -4$$

Hence, the y-intercept is -4.

AYU - 1.1 (9th Edition)

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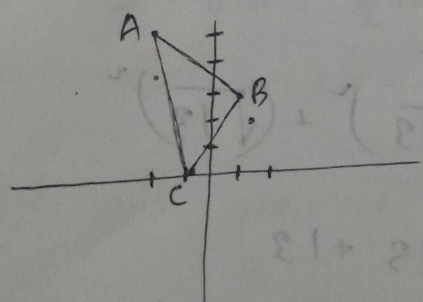
$A(-2,5)$, $B(1,3)$, $C(-1,0)$



To show that the triangle is a right triangle, we ~~we~~ need to show that the sum of the squares of the lengths of two of the sides equals the square of the length of the third side. Looking at figure, it seems reasonable to conjecture that the right angle is at vertex B. We shall check to see whether,

$$d_{AB} = \sqrt{(1+2)^2+(3-5)^2} = \sqrt{9+4} = \sqrt{13}$$

$$d_{BC} = \sqrt{(-1-1)^2+(0-3)^2} = \sqrt{4+9} = \sqrt{13}$$

$$d_{AC} = \sqrt{(-1+2)^2+(0-5)^2} = \sqrt{1+25} = \sqrt{26}$$

Now,

$$\therefore (d_{AB})^2 + (d_{BC})^2 = (\sqrt{13})^2 + (\sqrt{13})^2$$

$$= 13 + 13$$

$$= 26$$

And, $(d_{AC})^2 = (\sqrt{26})^2 = 26$

$$\therefore (d_{AB})^2 + (d_{BC})^2 = (d_{AC})^2$$

It follows from the converse of the Pythagorean Theorem that triangle ABC is a right triangle.

Because the right angle is at vertex B, the sides AB and BC form the base and height of the triangle. Its area is,

$$\text{Area} = \frac{1}{2}(\text{Base})(\text{Height})$$

$$= \frac{1}{2}(\sqrt{13})(\sqrt{13})$$

$$= \frac{1}{2} \cdot 13 = \frac{13}{2}$$ ~~square unit~~.

$$= 6.5$$ square unit.

Lecture - 3

Test an Equation for Symmetry

A graph is said to be symmetric with respect to the x-axis if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.

A graph is said to be symmetric with respect to the y-axis if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.

A graph is said to be symmetric with respect to the origin if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.

For the equation $y = \frac{x^2-4}{x^2+1}$:

a) find the intercepts

b) test for symmetry

Solution:

a)

In order to find the x-intercepts. Let us put $y=0$ in the given equation and solve it for x.

$$0 = \frac{x^2-4}{x^2+1}$$

$$\Rightarrow x^2-4 = 0$$

$$\Rightarrow x^2 = 4$$

$$\therefore x = \pm 2$$

Thus the x-intercepts are $+2$ and -2.

The points are $(2,0)$ and $(-2,0)$

2nd Part

In order to find the y-intercepts. Let us put $x=0$ in the given equation and solve it for y.

$$y = \frac{0^2-4}{0^2+1}$$

$$\therefore y = -4$$

Thus the y-intercept is -4.

The point is $(0,-4)$.

b)

In order to test the symmetry of the given equation let us replace y by $-y$.

$$-y = \frac{x^2-4}{x^2+1}$$

$\Rightarrow y = \frac{-x^2+4}{x^2+1}$; which is not equivalent to the given equation.

Thus the given equation is not symmetric with respect to the x-axis.

In order to test the symmetry of the given equation, let us replace x by $-x$.

$$y = \frac{(-x)^2-4}{(-x)^2+1}$$

$\Rightarrow y = \frac{x^2-4}{x^2+1}$; which is equivalent to the given equation.

Thus the given equation is symmetric with respect to the y-axis.

In order to test the symmetry of the given equation let us replace x by $-x$ and y by $-y$.

$$-y = \frac{(-x)^2-4}{(-x)^2+1}$$

$$\Rightarrow y = \frac{-x^2+4}{x^2+1}$$; which is not equivalent to the given equation.

Thus the given equation is not symmetric with respect to the origin.

Home Work (9th Edt)

1/

$$2(x+3)-1 = -7$$

$$\Rightarrow 2(x+3) = -6$$

$$\Rightarrow 2x+6 = -6$$

$$\Rightarrow 2x = -12$$

$$\therefore x = -6$$

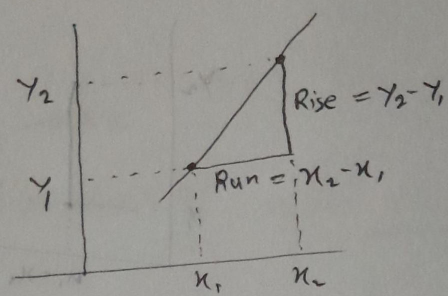
2/ $x^2-9=0$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = \pm\sqrt{9}$$

$$\therefore x = \pm 3$$

Slope

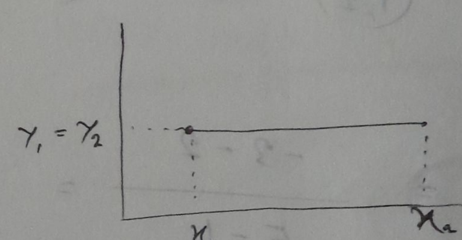


$$\text{Slope } m = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\Delta y}{\Delta x}$$

$$= \frac{\text{change in } y}{\text{chang in } x}$$

⊛ Slope of Horizontal Line.

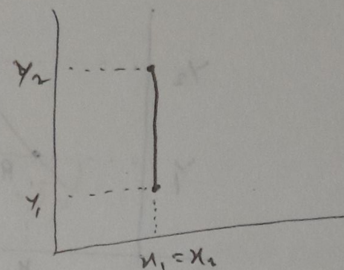


$$\text{Slope}, \ m = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{y_1 - y_1}{\Delta x}$$

$$= \frac{0}{\Delta x} = 0$$

Slope of Verrtical Line,



$$\text{Slope, } m = \frac{\text{Rise}}{\text{Run}}$$

$$= \frac{\Delta y}{0}$$

$= \text{undefined.}$

⊛ Find the slope m of the line containing the points (1,2) and (5,-3)

$\Rightarrow$

$$m = \frac{-3-2}{5-1} = \frac{-5}{4} \text{ Ans.}$$

⊛ Draw a graph of the line that contains the point (3,2) and has a slope of

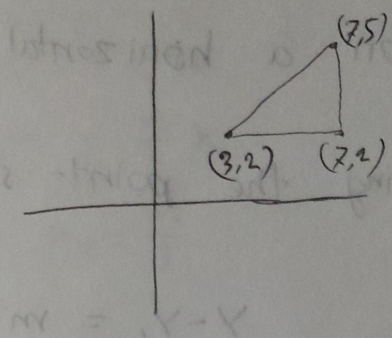
a) $\frac{3}{4}$ b) $-\frac{4}{5}$

⇒

a)

Given that

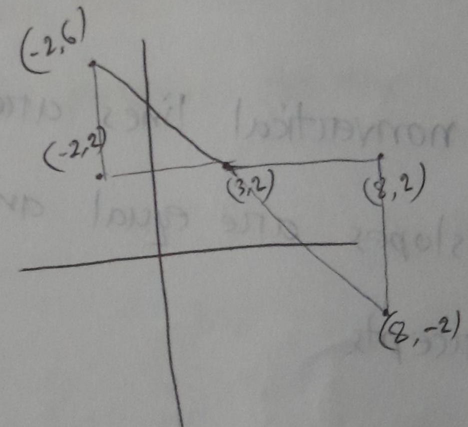
Slope, $m = \frac{3}{4}$ → Rise / → Run



b)

Given that

Slope, $m = -\frac{4}{5} = \frac{-4}{5} = \frac{4}{-5}$



✻ $m = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y}{x_2 - x_1}$

$\boxed{(y - y_1) = m(x - x_1)}$ → Point-slope form of an equation.

(*) Point $(3,2)$

for a horizontal line, $m=0$.

Using the point-slope form of an equation we have

$$y-y_1 = m(x-x_1)$$

$$\Rightarrow y-2 = 0(x-x_1)$$

$$\Rightarrow y=2$$

Ans

(*) Two nonvertical lines are parallel if and only if their slopes are equal and they have different y-intercepts.

(*) Show that the lines given by the following equations are ~~paralla~~ parallel:

$L_1: 2x+3y=5$

$L_2: 4x+6y=0$

$\Rightarrow$

and,

$2x + 3y = 6$

$3y = -2x + 6$

$y = \frac{-2}{3}x + 2$

$\therefore m_1 = \frac{-2}{3}$

$4x + 6y = 0$

$6y = -4x$

$y = \frac{-2}{3}x$

$\therefore m_2 = \frac{-2}{3}$

$\therefore m_1 = m_2 = \frac{-2}{3}$

Thus L_1 and L_2 are parallel to each other.

Find an equation for the line that contains the point $(2,-3)$ and is parallel to the line $2x+y=6$

$\Rightarrow$

$2x + y = 6$

$y = -2x + 6$

$\therefore m = -2$

By using ~~slope~~ point-slope form,

$y + 3 = -2(x-2)$

$y + 3 = -2x + 4$

$2x + y = 1$ Ans

* Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

$$\therefore m_1 m_2 = -1.$$

* Find an equation of the line that contains the point $(1,-2)$ and is perpendicular to the line $x+3y=6$.

$\Rightarrow$

$$x + 3y = 6$$

$$\Rightarrow 3y = -x + 6$$

$$\therefore y = \frac{-1}{3}x + 6$$

Hence, the slope $m_1 = \frac{-1}{3}$.

For perpendicular lines, we know that,

$$m_1 m_2 = -1$$

$$-\frac{1}{3} \times m_2 = -1$$

$$\therefore m_2 = 3$$

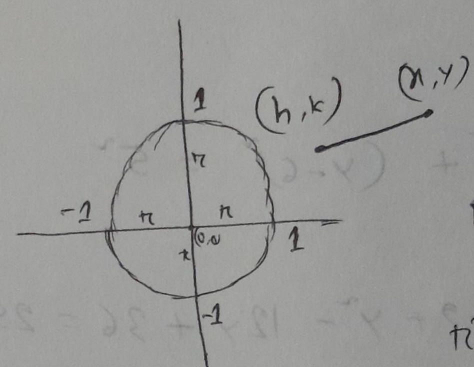
By using the point-slope form of an equation,

$$y + 2 = 3(x - 1)$$

$$\Rightarrow y + 2 = 3x - 3$$

$$\therefore y = 3x - 5 \quad \underline{\text{Ans}}$$

$$x^2 + y^2 = 1$$



$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

$$r^2 = (x-h)^2 + (y-k)^2$$

$r \Rightarrow$ Radious of the circle, fixed.

$(h,k) \Rightarrow$ Center of the circle, fixed.

$$(x-h)^2 + (y-k)^2 = r^2 \; \cdots \text{(i)}$$

If the center is brought at the origin

$$(x-0)^2 + (y-0)^2 = r^2$$

$$x^2 + y^2 = r^2 \; \cdots \text{(ii)}$$

If
$r = 1$;

$\Rightarrow x^2 + y^2 = 1$ ---(iii)

this is the equation of a unit circle.

* Write the standard form of the equation of the circle with radius 5 and center (-3, 6).

$\Rightarrow$

$(x+3)^2 + (y-6)^2 = 5^2$

$x^2 + 6x + 9 + y^2 - 12y + 36 = 25$

$x^2 + y^2 + 6x - 12y = -20$

$x^2 + y^2 + 6x - 12y + 20 = 0$

Ans

* For the circle $(x+3)^2 + (y-2)^2 = 16$, find the intercepts, if any, of its graph.

⇒

The given circle is $(x+3)^2 + (y-2)^2 = 16 \quad \cdots (i)$

To find the x-intercepts let us put $y=0$ in equation (i) and solve it for x.

$(x+3)^2 + (0-2)^2 = 16$

$(x+3)^2 + 4 = 16$

$(x+3)^2 = 16-4$

$(x+3)^2 = 12$

$x+3 = \sqrt{12} = \pm 2\sqrt{3}$

$\therefore x = \pm 2\sqrt{3} - 3$

Hence the x-intercepts of the circle are $2\sqrt{3}-3$ and $-2\sqrt{3}-3$.

To find the y-intercepts let us put $x=0$ in the equation (i) and solve it for y.

$\Rightarrow (0+3)^2 + (y-2)^2 = 16$

$\Rightarrow (y-2)^2 = 16-9$

$\Rightarrow y-2 = \pm\sqrt{7}$

$\therefore y = \pm\sqrt{7} + 2$

Hence, the y-intercepts of the circle are $\sqrt{7}+2$ and $-\sqrt{7}+2$.

⊛ General equation of a circle is,

$$x^2+y^2+ax+by+c=0$$

⊛ Find the general equation of the circle whose center is $(1,-2)$ and whose graph contains the point $(4,-2)$

$\Rightarrow$

By using distance formula,

$$r=\sqrt{(1-4)^2+(-2+2)^2}=3$$

Hence the equation of its circle is

$$(x-1)^2+(y+2)^2=3^2$$

$$\Rightarrow x^2+y^2-2x+4y-4=0$$

A

Chapter-2

Function and Their Graphs

* For the function f defined by $f(x) = 2x^2 - 3x$, evalute

a) $f(3) = 2\cdot 3^2 - 3\cdot 3$

$= 2\cdot 9 - 9$

$= 18 - 9$

$= 9$ Ans

b) $f(x) + f(3) = 2x^2 - 3x + 9$ Ans

c) $3f(x) = 3(\cancel{x}2x^2 - 3x)$

$= 6x^2 - 9x$ Ans

d) $f(-x) = 2\cdot(-x)^2 - 3\cdot(-x)$

$= 2x^2 + 3x$ Ans

e) $-f(x) = -(2x^2 - 3x)$

$= -2x^2 + 3x$ Ans

f) $f(3x) = 2\cdot(3x)^2 - 3\cdot(3x)$

$= 2\cdot 9x^2 - 9x$

$= 18x^2 - 9x$ Ans

g) $f(x+3) = 2\cdot(x+3)^2 - 3(x+3)$

$= 2\cdot(x^2+6x+9) - 3x - 9$

$= 2x^2 + 12x + 18 - 3x - 9$

$= 2x^2 + 9x + 9$ Ans

h)

$$\frac{f(x+h)-f(x)}{h} \; ; \; h \neq 0$$

$$= \frac{2(x+h)^2 - 3(x+h) - 2x^2 + 3x}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \frac{4xh + 2h^2 - 3h}{h}$$

$$= \frac{h(4x + 2h - 3)}{h}$$

$$= 4x + 2h - 3$$ Ans

b) $g(x) = \frac{3x}{x^2-4}$

Domain of $g(x)$ is

$\{x \mid x \neq -2, x \neq 2\}$

c) $h(t) = \sqrt{4-3t}$

$4-3t \geq 0$

$-3t \geq -4$

$t \leq \frac{4}{3}$

Domain of $h(t)$ is

$\{t \mid t \leq \frac{4}{3}\}$

$\Rightarrow (-\infty, \frac{4}{3}]$

d) $f(x) = \frac{\sqrt{3x+12}}{x-5}$

$3x+12 \geq 0$

$3x \geq -12$

$x \geq -4$

$x-5 \neq 0$

$x \neq 5$

$\therefore \{x \mid x \geq -4, x \neq 5\}$

$f(x) = x^2 + 9$ and $g(x) = 3x + 5$, then,

$$f(x) + g(x) = (x^2+9) + (3x+5)$$
$$= x^2 + 3x + 14$$

The new function $y = x^2 + 3x + 14$ is called the sum function $f+g$. Similarly,

$$f(x) \cdot g(x) = (x^2+9)(3x+5)$$
$$= 3x^3 + 5x^2 + 27x + 45$$

the new function $y = 3x^2 + 5x^2 + 27x + 45$ is called the product function $f \cdot g$.

If f and g are functions:

The sum $f+g$ is the function defined by

$$(f+g)(x) = f(x) + g(x)$$

the ~~d~~ domain of $f+g$ consists of the ~~o~~ numbers x that are in the domains of both f and g.

That is, domain of $f+g$ = domain of $f \cap$ domain of g.

The difference $f-g$ is the function defined by

$$(f-g)(x) = f(x) - g(x)$$

The domain of $f-g$ consists of the numbers x that are in the domains of both f and g. That is, domain of $f-g$ = domain of $f \cap$ domain of g.

The product $f \cdot g$ is the function defined by

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

The domain of $f \cdot g$ consists of the numbers x that are in the domains of both f and g. That is, domain of $f \cdot g$ = domain of $f \cap$ domain of g.

* The quotient $\frac{f}{g}$ is the function defined by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad ; g(x) \neq 0$$

The domain of $\frac{f}{g}$ consists of the numbers x for which $g(x) \neq 0$ and that are in the domains of both f and g. That is,

domain of $\frac{f}{g} = \{x \mid g(x) \neq 0\} \cap$ domain of f $\cap$ domain of g.

*

$f(x) = \frac{1}{x+2}$ and $g(x) = \frac{x}{x-1}$

a) $f(x) + g(x) = (f+g)(x)$

$$= \frac{1}{x+2} + \frac{x}{x-1}$$

$$= \frac{x-1+x^2+2x}{(x+2)(x-1)}$$

$$= \frac{x^2+3x-1}{(x+2)(x-1)}$$

$= \{x \mid x \neq -2, x \neq 1\}$

Even and Odd Function

* A function f is even, if and only if, whenever the point (x, y) is on the graph of f then the point $(-x, y)$ is also on the graph.

$$f(-x) = f(x)$$

* A function f is odd, if and only if, whenever the point (x, y) is on the graph of f then the point $(-x, -y)$ is also on the graph.

$$f(-x) = -f(x)$$

Theorem

A function is even if and only if its graph is symmetric with respect to the y-axis. A function is odd if and only if its graph is symmetric with respect to the origin.

⊛ Example,

$$g(x) = x^3 - 1$$

In order to test even and odd function. lets replace x by $-x$.

$$g(-x) = (-x)^3 - 1$$

$$= -x^3 - 1$$

$$= -(x^3+1) \neq g(x)$$

$$\neq -g(x)$$

So, the function is neither even nor odd function.

Local Maximum and Local Minimum

Suppose f is a function defined on an open interval containing c. If the value of f at c is greater than or equal to the values of f on I, then f has a local maximum at c.

If the value of f at c is less than or equal to the values of f on I, then f has a local minimum at c.

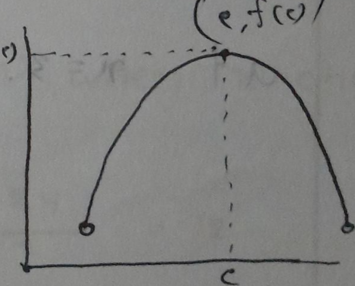
$\Rightarrow$

A function f has a local maximum at c if there is an open interval I containing c so that for all x in Z, $f(x) \leq f(c)$. We call $f(c)$ a local maximum value of f.

A function f has a local minimum at c if there is an open interval I containing c so that, for all x in Z, $f(x) \geq f(c)$. We call $f(c)$ a local minimum value of f.

<u>Local Maximum</u>

Open Interval



At c,

$f(x) = f(c)$

At all other points, except c,

$f(x) < f(c)$

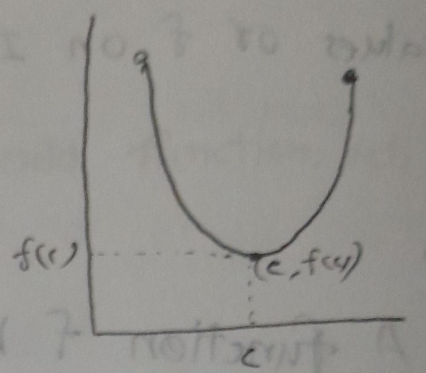
$f(x) \leq f(c) \Rightarrow$ Local maximum.

Local Minimum

Open Interval

At c,

$f(x) = f(c)$



At all other points, except c,

$f(x) > f(c)$

$f(x) \geq f(c) \Rightarrow$ Local minimum.

The absolute minimum of the function occurs at $x = 0$. The value of the absolute minimum

$f(0) = 1$.

The absolute ~~ma~~ maximum of the function occurs at $x = 3$. The value of the absolute maximum

$f(3) = 6$.

Average Rate of Change of a Function.

If a ~~nd~~ and b, $a \neq b$, are ~~in~~ in the domain of a function $y = f(x)$, the average rate of change of f from a to b is defined as,

$$\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} \; ; a \neq b$$

⊛ Find the average rate of change of $f(x) = 3x^2$.

a) From 1 to 3

b) From 1 to 5

c) From 1 to 7.

$\Rightarrow$

a) The average rate of change of $f(x)$ when x changes from 1 to 3 is,

$$\Delta f_1 = \frac{f(3) - f(1)}{3-1} = \frac{3 \cdot 3^2 - 3 \cdot 1^2}{3-1} = \frac{24}{2} = 12$$

b) from 1 to 5 is,

$$\Delta f_2 = \frac{f(5) - f(1)}{5-1} = \frac{3 \cdot 5^2 - 3 \cdot 1^2}{5-1} = \frac{72}{4} = 18$$

c)
from 1 to 7,

$$\Delta f_3 = \frac{f(7)-f(1)}{7-1} = \frac{3\cdot 7^2 - 3\cdot 1^2}{7-1} = \frac{144}{6} = 24$$

Suppose that $g(x) = 3x^2 - 2x + 3$.

a) Find the average rate of change of g from -2 to 1.

b) Find an equation of the secant line containing $(-2, g(-2))$ and $(1, g(1))$.

$\Rightarrow$

a)

The average rate of change of g from -2 to 1.

$$\Delta g = \frac{g(1) - g(-2)}{1-(-2)} = \frac{[3\cdot 1^2 - 2\cdot 1 + 3] - [3\cdot(-2)^2 - 2(-2) + 3]}{1+2}$$

$$= \frac{4-19}{3}$$

$$= -5$$

$g(-2) = 19 = y_1$

$g(1) = 4 = y_2$

b) The average rate of change of g from -2 to 1 is equal to m_{sec}.

Thus, the slope of the secant line is

$$m_{sec} = -5$$

Using the point-slope form of a line, we get the equation of the secant line,

$$y - y_1 = m_{sec}(x - x_1)$$

$$y - 19 = -5(x - (-2))$$

$$y - 19 = -5(x + 2)$$

$$y - 19 = -5x - 10$$

$$y = -5x + 9$$

Ans

(*) The function f is defined as

$$f(x) = \begin{cases} -2x+1 & \text{if } -3 \le x < 1 \\ 2 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

a) Find $f(-2)$, $f(1)$ and $f(2)$.

b) Determine the domain of f.

c) ~~Use~~ Locate any intercepts.

d) Graph f.

e) Use the graph to find the range of f.

f) Is f continuous on its domain?

~~Soltt~~ Solution

a) To find $f(-2)$, observe that when $x = -2$ the equation for f is given by $f(x) = -2x + 1$, So,

$$f(-2) = -2(-2) + 1$$

$$= 5$$

when, $x=1$, the equation for f is $f(x) = ~~2~~2$ so,

$$\cancel{f(2) = 2^2 = 4}$$

$$\therefore f(1) = 2$$

when $x=2$ the equation for f is $f(x) = x^2$, so.

$$f(2) = 2^2$$
$$= 4$$

b)

To find the domain of f, look at its definition. Since f is defined for all x greater than or equal to -3. the domain of f is $\{x \mid x \geq -3\}$, on the interval $[-3, \infty)$.

c) Let us put $x=0$, and ~~sto~~ solve the equation for y.

$$f(0) = -2 \cdot 0 + 1$$
$$= 1$$

$\therefore$ y-intercept is $(0,1)$

Let us put $y=0$ and solve the equation for x.

$f(x)=0=-2x+1$	$f(x)=0=2$	$f(x)=0=x^2$
$2x=1$	$\therefore 0=2$	$\therefore x=0$; $x>1$
$\therefore x=\frac{1}{2}$	No solution	Not acceptable.

$\therefore$ x-intercepts is ~~are~~ $(\frac{1}{2}, 0)$.

P.NO.: 99

Example - 4

a)

Charge $= \$5.50 + \$(0.064471 \times 300)$

$= \$24.84$

b) Charge $= \$5.50 + \$(0.064471 \times 1000) + \$(0.078391 \times 500)$

$= \$109.17$

e)

$$C(x) = \$5.50 + \$(0.064471 \times 1000) + \$(0.078391 \times (x-1000))$$

$$= 6.9971 + (0.078391(x-1000))$$

$$= 0.07831x - 8.42 \; ; \text{ if } x > 1000$$

if x is between 0 to 1000.

$$f(x) = \$5.50 + \$0.064471x \quad \text{if } 0 \le x \le 1000$$

$$\rightarrow C(x) = \begin{cases} 0.064471x + 5.50 & ; \text{ if } 0 \le x \le 1000 \\ 0.078391x - 8.42 & ; \text{ if } x > 1000 \end{cases}$$

Chapter - 3 : Linear and Quadratic Function

Linear function.

~~$f(x) y = mx + b$~~

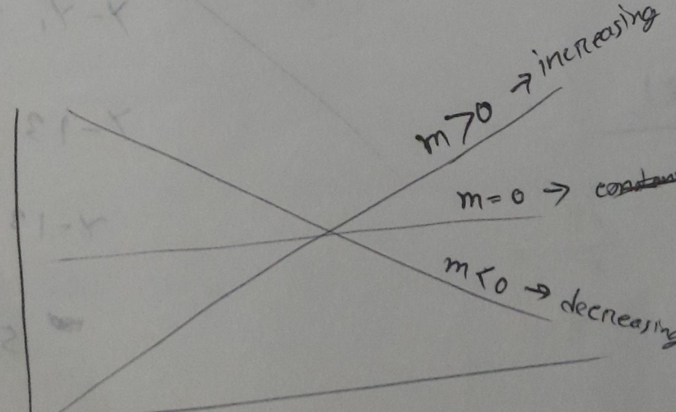
$$f(x) = y = mx + b$$

⊛ Average rate of change in ~~lea~~ linear function is constant.

⊛ Linear function have a constant average rate of change. That is, the average rate of change of a linear function $f(x) = mx + b$ is,

$$\frac{\Delta y}{\Delta x} = m$$

⊛



Theorem:

A linear function $f(x) = mx + b$ is increasing over ~~the~~ its domain if its slope, m, is positive. It is decreasing over its domain if its slope, m, is negative. It is constant over its domain if its slope, m, is zero.

Quadratic Function

$F(x) = 3x^2 - 5x + 1$ $\quad g(x) = -6x^2 + 1$ $\quad H(x) = \frac{1}{2}x^2 + \frac{2}{3}x$

A quadratic function is a function of the form

$$f(x) = ax^2 + bx + c \; ; \; a \neq 0$$

where, a, b and c are real numbers and $a \neq 0$. The domain of a quadratic function is the set of all real numbers.

*

$f(x) = y = 2x^2 + 8x + 5$

$y = 2(x^2 + 4x + 4) - 3$

$= 2x^2 + 8x + 8 - 3$

$= 2x^2 + 8x + 5$

$y = x^2$

$\downarrow$

$y = 2x^2$

$\downarrow$

$y = 2(x+2)^2$

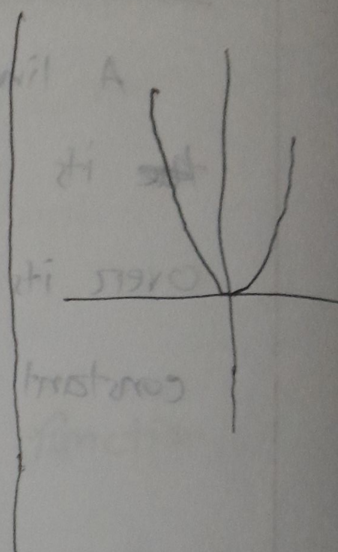
$\downarrow$

$y = 2(x+2)^2 - 3$

$\updownarrow$

$y = 2(x-h)^2 + k$

$= a(x-h)^2 + k$



$h = -2$

$k = -3$

; $(h, k) = (-2, -3)$ Vertex of the function.

The axis of symmetry is

$x = h = -2.$

$$f(x) = y = ax^2 + bx + c$$

$$= a\left[x^2 + \frac{b}{a}x\right] + c$$

$$= a\left[x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right] + c - \frac{b^2}{4a}$$

$$= a\left[x + \frac{b}{2a}\right]^2 + \left[\frac{4ac - b^2}{4a}\right]$$

$$\updownarrow$$

$$= a(x-h)^2 + k$$

Here, $h = \frac{-b}{2a}$

$k = \frac{4ac - b^2}{4a}$

$$\boxed{f(x) = ax^2 + bx + c = a(x-h)^2 + k}$$

$$\text{vertex} = (h, k)$$

$$= (x, y)$$

$$= \left\{\frac{-b}{2a}, f\left(-\frac{b}{2a}\right)\right\}$$

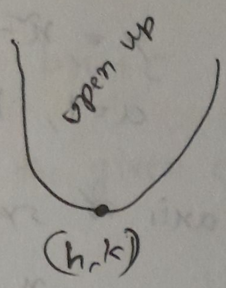
$f(x) = 2x^2 + 8x + 5$ | $a = 2$, $b = 8$, $c = 5$

$h = \frac{-b}{2a} = \frac{-8}{2 \cdot 2} = -2$

$k = f\left(-\frac{b}{2a}\right) = f(-2) = 2 \cdot (-2)^2 + 8(-2) + 5$
$= 2 \cdot 4 - 16 + 5$
$= 8 - 16 + 5$
$= -3$

vertex, $(h, k) = (-2, -3)$.

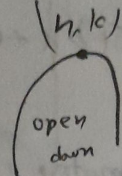
The graph of $f(x) = a(x-h)^2 + k$ is the parabola $y = ax^2$ shifted horizontally h units (replace x by $x-h$) and vertically k units (add k). As a result, the vertex is at (h, k), and the graph opens up if $a > 0$ and down if $a < 0$. The axis of symmetry is the vertical line $x = h$.



is the ventex

It is the minimum point

$a > 0$



is the ventex which is the maximum point

$a < 0$

* It is easier to obtain the venten of a quadratic function f by remembering that its x-coordinate is $h = -\frac{b}{2a}$. The y-coordinate k can then be found by ~~every~~ evaluating f at $-\frac{b}{2a}$. That is, $k = f\left(-\frac{b}{2a}\right)$.

* The graph of the parabola represented by the quandratic function $y = ax^2 + bx + c$ has an axis of symmetry represented by the equation of the vertical line $x = \frac{-b}{2a}$.

$y = ax^2 + bx + c$

axis of symmetry: $x = \frac{-b}{2a}$

$y = x^2 - 4x + 3$

$a = 1,\ b = -4,\ c = 3$

axis of symmetry: $x = 2$

$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)}$

$= 2$

$y = a(x - r_1)(x - r_2)$

axis of symmetry: $x = \frac{r_1 + r_2}{2}$

$y = a(x-1)(x-3)$

$a = 1,\ r_1 = 1,\ r_2 = 3$

$x = \frac{r_1 + r_2}{2}$

$= \frac{1+3}{2} = 2$

axis of symmetry: $x = 2$.

Without graphing, locate the vertex and axis of symmetry of the parabola defined by

$f(x) = -3x^2 + 6x + 1$. Does it open up or down.

$a = -3,\ b = 6,\ c = 1$

$a < 0 \Rightarrow$ open down

$x = \frac{-b}{2a} = \frac{-6}{2 \cdot (-3)} = \frac{-6}{-6} = 1$

vertex, $(1, f(1)) = (1, 4)$

$\rightarrow$ highest point

⊛

$f(x) = ax^2 + bx + c$

Set, $x=0$ in the given equation and solve it for y.

$f(0) = a\cdot 0^2 + b\cdot 0 + c$

$f(0) = c$

Set. $y=0$ in the given equation and solve it for x.

$f(x) = 0 = ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

here, $b^2-4ac = \Delta$; is called discriminant.

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

If, $\Delta > 0$; if Δ is positive then,

$$x = \frac{-b+\sqrt{\Delta}}{2a} \quad ; \quad x = \frac{-b-\sqrt{\Delta}}{2a}$$

Two real solutions

~~Two~~ Two x-intercepts.

Here,

$a > 0$, ~~hence~~ hence the parabola opens up.

vertex (h,k)

$h = -\frac{b}{2a}$

$k = f\left(\frac{-b}{2a}\right)$

The axis of symmetry

$x = h = -\frac{b}{2a}$

The y-intercept is c.

If $\Delta = 0$, then

$$x = \frac{-b}{2a}$$

If $\Delta < 0$; if Δ is negative then,

$$x = \frac{-b \pm \sqrt{-\Delta}}{2a}$$

$$x = -\frac{b}{2a} + i\frac{\sqrt{\Delta}}{2a} \quad ; \quad x = -\frac{b}{2a} - i\frac{\sqrt{\Delta}}{2a}$$

Two complex roots. No real solutions.

⊛ Graph $f(x) = -3x^2 + 6x + 1$ using its properties.

Determine the domain and the range of f.

Determine where f is increasing and where it is decreasing.

$\Rightarrow$

$f(x) = -3x^2 + 6x + 1$

Here,

$a = -3$

$b = 6$

$c = 1$

$h = -\frac{6}{2(-3)} = 1$

$k = f(h) = f(-\frac{b}{2a}) = f(1) = 4$

Set, $x = 0$ in $f(x)$

$f(0) = y = 1$

Set, $y = 0$ in $f(x)$ and solve for x.

$f(x) = -3x^2 + 6x - 1 = 0$

$\Delta = b^2 - 4ac = 48 > 0$

Here, $a < 0$, hence the parabola opens down.

Vertex $(h, k) = \left(-\frac{b}{2a}, f(-\frac{b}{2a})\right)$
$= 1, 4$

$h = -\frac{b}{2a}$

$k = f(-\frac{b}{2a})$

The axis of symmetry

$x = h = -\frac{b}{2a} = 1$

The y-intercept is 1

The x-intercept are

-0.15 and 2.15

Domain $= (-\infty, \infty)$

Range $= (-\infty, 4]$

Increasing $= (-\infty, 1)$

Decreasing $= (1, \infty)$

The descriminant is positive, Hence the function has two real solution and two x-intercepts.

$$x = \frac{-6 \pm \sqrt{48}}{-6}$$

$\therefore x = -0.15$ and $x = 2.15$.

(8)

a) Graph $f(x) = x^2 - 6x + 9$ by determining whether the graph opens up or down and by finding its vertex, axis of symmetry, y-intercept, and x-intercept, if any.

b) Determine the domain and the range of f.

c) Determine where f is increasing and where it is decreasing.

⇒

a)

$f(x) = x^2 - 6x + 9$

Here, $a = 1$
$b = -6$
$c = 9$

$\therefore b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 9$
$= 36 - 36$
$= 0$

$h = -b/2a = -\frac{-6}{2 \cdot 1} = \frac{6}{2} = 3$

Here, $a > 0$, hence the parabola opens up.

vertex. $(h, k) = \{-\frac{b}{2a}, f(-\frac{b}{2a})\}$
$= (3, 0)$

$h = -b/2a$

$k = f(-b/2a)$

The axis of symmetry

$x = h = \frac{-b}{2a} = 3$

y-intercept is 9

x-intercept is 3

Thus descriminant is equal to zero, hence it has one real solution.

Set, $x=0$ in $f(x)$

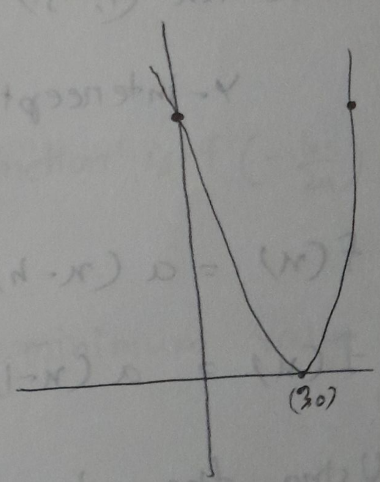
$$f(0) = 0^2 - 6\cdot 0 + 9$$
$$= 0 - 0 + 9$$
$$= 9$$

Set, $y=0$ in $f(x)$ and solve for x.

$$f(x) = x^2 - 6x + 9 = 0$$

$\cancel{x^2 - 3x - 3x + 9}$

$$x^2 - 2\cdot x\cdot 3 + 3^2 = 0$$
$$(x-3)^2 = 0$$
$$x - 3 = 0$$
$$x = 3$$



$$f(3) = 3^2 - 6\cdot 3 + 9$$
$$= 9 - 18 + 9$$
$$= 0$$

b) Domain $= (-\infty, \infty)$

Range $= (0, \infty)$

c)

increasing = ~~(-∞, 3)~~ $(3, \infty)$

Decreasing = $(-\infty, 3)$

⊛ Determine the quadratic function whose vertex is $(1, -5)$ and whose y-intercept is -3.

$\Rightarrow$

vertex $(1, -5) = (h, k)$

y-intercept $= -3$

$h = 1$

$k = -5$

$f(0) = -3$

$f(x) = a(x-h)^2 + k$

$f(x) = a(x-1)^2 - 5 \quad \cdots (i)$

Using the value of y-intercept we get

$f(0) = -3.$

$\Rightarrow a(0-1)^2 - 5 = -3$

$\Rightarrow a - 5 = -3$

$\Rightarrow a = -3+5$

$\therefore a = 2$

Substitute the value of 'a' in equation (1).

$$f(x) = 2(x-1)^2 - 5$$

$$= 2(x^2-2x+1) - 5$$

$$= 2x^2 - 4x + 2 - 5$$

$$\therefore f(x) = 2x^2 - 4x - 3$$

* Theorem.

1. If $a < 0$, the function has a maximum value (highest point)

   The maximum value of the function is $f\left(-\frac{b}{2a}\right)$

2. If $a > 0$, the function has a minimum value (lowest point)

   The minimum value of the function is $f\left(-\frac{b}{2a}\right)$

* Determine whether the quadratic function

$$f(x) = x^2 - 4x - 5$$

has a maximum or minimum value. Then find the maximum or minimum value.

$\Rightarrow$

$$f(x) = x^2 - 4x - 5$$

$$a = 1 > 0$$

Since $a > 0$, the function has the minimum value.

$$\frac{-b}{2a} = \frac{4}{2 \cdot 1} = 2$$

$$f(2) = 2^2 - 4 \cdot 2 - 5$$

$$= 4 - 8 - 5$$

$$= -9$$

So, the minimum value of the function is -9.

$\boxed{\text{H.W}}$ $\boxed{\text{9th Ed}}$

$\boxed{3.1}$

$R = xp$ ---- (i) $\Rightarrow$ Demand Equation

$x = 21000 - 150p$ ---- (ii)

$R = (21000 - 150p)p$

$R = 21000p - 150p^2$ ---- (iii)

Here, x is the total number of calculators sold at a price P.

P is price per calculator.

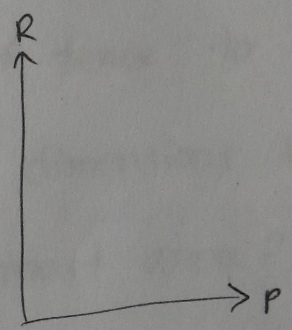
R is revenue earn by the company.

Here,

$a = 150$

$b = 21000$

$c = R$



b)

$P > 0$

and $x \geq 0$

$\Rightarrow 21000 - 150p \geq 0$

$P \leq 140$

Combining the above condition, we get the domain of R is

$\{P \mid 0 < P \leq 140\}$

c) Since, $a<0$, the vertex is the highest point, hence the parabola opens down.

$$h = -\frac{b}{2a} = \frac{-21000}{2\times(-150)}$$

$$= 70$$

d) So, the ~~b~~ maximum will be ~~R = 21000 × 70~~ - 150

$$R = 21000 \times 70 - 150 \times (70)^2$$

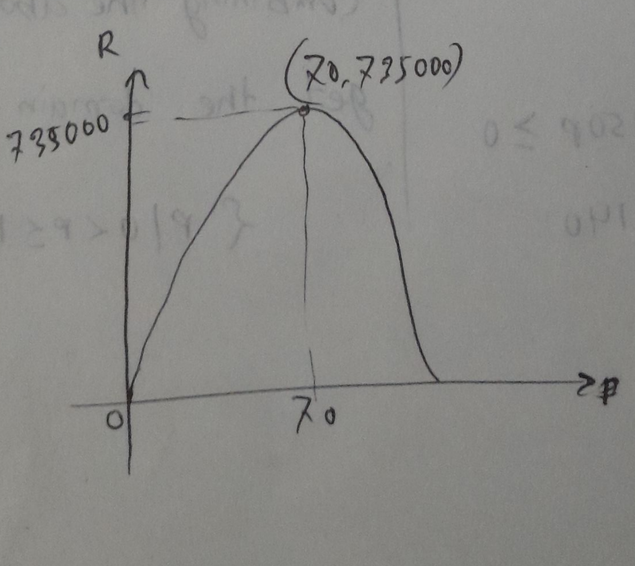
$$= 735000$$

e)

$$x = 21000 - 150\,(70)$$

$$x = 10\,500$$

f)



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g)

$R = 675000 = 21000p - 150p^2$

$\Rightarrow p^2 - 140p + 4500 = 0$

$(p-50)(p-90) = 0$

$p = 50$ and $p = 90$.

(*) A farmer has 2000 yards of fence to enclose a rectangular field. What are the dimensions of the rectangular that encloses the most area?

$\Rightarrow$

Perimeter $= 2x + 2w = 2000$

$\Rightarrow x + w = 1000$

$w = 1000 - x$ --(i)

Area, $A = xw$

$= x(1000-x)$

$A = 1000x - x^2$ ---(ii)

Here,

$a = -1$

$b = 1000$

$x = h = \frac{-b}{2a} = -\frac{1000}{2(-1)} = 500$

Area, $A = 1000x - x^2$

$= (1000 \times 500) - (500)^2$

$= 500000 - 250000$

$= 250000$ sq ~~un~~ yards.

So dimension is (500×500).

(20) A projectile is fired from a cliff 500 feet above the water at an inclination of 45° to the horizontal, with a muzzle velocity of 400 feet per second. In physics, it is established that height h of the projectile above the water can be modeled by

$$h(x) = \frac{-32x^2}{(400)^2} + x + 500$$

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Where

a) Find the maximum height of the projectile.

$\Rightarrow$

The height of the projectile is given by a quadratic function.

$$h(x) = \frac{-32x^2}{(400)^2} + x + 500$$

$$= \frac{-1}{5000}x^2 + x + 500$$

We, are looking for the maximum value of h. Since $a < 0$, the maximum value is obtained at the vertex, whose x-coordinate is.

$$x = -\frac{b}{2a} = -\frac{1}{2\left(-\frac{1}{5000}\right)} = \frac{5000}{2} = 2500$$

The maximum height of the projectile is

$$h(2500) = \frac{-1}{5000}(2500)^2 + 2500 + 500$$

$$= -1250 + 2500 + 500$$

$$= 1750 \text{ ft}$$

b) How far from the base of the cliff will the projectile strike the water?

$\Rightarrow$

The projectile will strike the water when the height is zero. To find the distance x traveled, solve the equation.

$$h(x) = \frac{-1}{5000}x^2 + x + 500 = 0$$

The discriminant of this quadratic equation is

$$b^2 - 4ac = 1^2 - 4\left(\frac{-1}{5000}\right)(500) = 1.4$$

Then

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1.4}}{2\left(-\frac{1}{5000}\right)}$$

$$= \begin{cases} -458 \\ 5458 \end{cases}$$

Discard the negative solution. The projectile will strike the water at a distance of about 5458 feet from the base of the cliff.

Chapter - 4

Date : ..../......./.......

Polynomial and Rational Functions.

General Equation of a Polymomial function:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Power Function

A power function of degree n is a monomial function of the form.

$$f(x) = ax^n$$

Where, a is a real number, $a \neq 0$, and $n > 0$ is an integer.

* If f is a function and r is a real number for which $f(r) = 0$, then r is called a real zero of f.

As a consequence of this definition, the following statements are equivalent.

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1. r is a real zero of a polynomial function f.

2. r is an x-intercept of the graph of f.

3. $x-r$ is a factor of f.

4. r is a solution to the equation $f(x)=0$.

* Find a polynomial ~~f~~ of degree 3 whose zeros are -3, 2 and 5.

$\Rightarrow$

$$f(x) = a(x+3)(x-2)(x-5)$$

* If $(x-r)^m$ is a factor of a polynomial f and $(x-r)^{m+1}$ is not a factor of f, then r is called a zero of multiplicity m of f.

* Identifying Zeros and Their Multiplicities.

For the polynomial function.

$$f(x) = 5(x-2)(x+3)^2\left(x-\frac{1}{2}\right)^4$$

⇒ There are three real zeros. These are, $2, -3, \frac{1}{2}$.

The multiplicity of 2 is 1

The multiplicity of -3 is 2

The multiplicity of $\frac{1}{2}$ is 4

⊛ If r is a zero of Even multiplicity:

⇒ Sign of $f(x)$ does not change from one side of r to the other side of r.

Graph touches x-axis at r.

⊛ If r is a zero of odd multiplicity

⇒ Sign of $f(x)$ changes from one side of r to the other side of r.

Graph crosses x-axis at r.

⊛ Turning Points.

⇒ Points on the graph where the graph changes from an increasing function to a decreasing function, or vice versa, are called turning points.

⊛ Theorem:

If f is a polynomial function of degree n, then f has at most $n-1$ turning points.

If the graph of a polynomial function f has $n-1$ turning points, the degree of f is at least n.

⇒ Based on the first part of the theorem, a polynomial function of degree 5 will have at most $5-1=4$ turning points. Based on the second part of the theorem, if a polynomial function has 3 turning points, then its degree must

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be at least 4.

⊛ End Behaviour:

⇒ For very large values of x, either positive or negative, the graph of $f(x) = (x+1)^2 (x-2)$ looks lik the graph of $y = x^3$.

⊛ Theorem:

For large values of x, either positive or negative, the graph of the polynomial function,

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

resembles the graph of the power function

$$y = a_n x^n.$$

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(*) Analysing the graph of a polynomial function.

⇒

Step 1: Determine the end behavior of the graph of the function.

Step 2: Find the x- and y-intercepts of the graph of the function.

Step 3: Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

Step 4: Use a graphing utility to graph the function

Step 5: Approximate the turning points of the graph.

Step 6: Use the information in steps 1 through 5 to draw a complete graph of the function by hand.

Step 7: Find the domain and the range of the function.

Step 8: Use the graph to determine where the function is increasing and where it is decreasing.

⊛ Remainder Theorem:

Let f be a polynomial function. If $f(x)$ is divided by $x-c$, then the remainder is $f(c)$.

⊛ Factor Theorem:

⇒ An important and useful conequence of the remainder theorem is called the factor theorem.

Let f be a polynomial function. Then $x-c$ is a factor of $f(x)$ if and only if $f(c)=0$.

The factor theorem actually consists of two separate statements.

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1. If $f(c)=0$, then $x-c$ is a factor of $f(x)$.

2. If $x-c$ is a factor of $f(x)$, then $f(c)=0$.

(*) Theorem:

A polynomial function cannot have more real zeros than its degree.

Proof: The proof is based on the Factor Theorem. If r is a real zero of a polynomial function f, then $f(r)=0$ and, hence, $x-r$ is a factor of $f(x)$. Each real zero corresponds to a factor of degree 1. Because f cannot have more first-degree factors than its degree, the result follows.

Rational Zeros Theorem

General equation,

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots a_1 x + a_0$$

Here, $a_n \neq 0$, $a_0 \neq 0$ }

where each coefficient is an integer. If $\frac{p}{q}$, in lowest terms, is a rational zeros of f, then p must be a factor of a_0 and q must be a factor of a_n.

List the

$$f(x) = 2x^3 + 11x^2 - 7x - 6$$

Here,

$a_0 = -6$

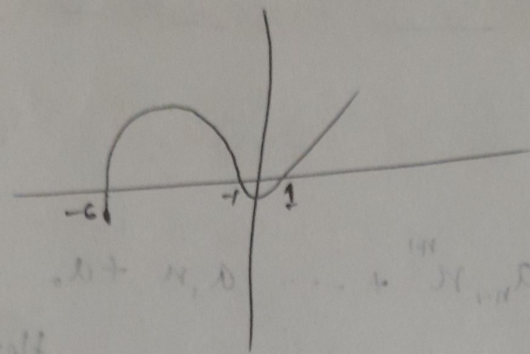
$a_3 = 2$

Factors of a_0 ; $p = \pm 1, \pm 2, \pm 3, \pm 6$

Factors of a_n ; $q = \pm 1, \pm 2$

$p/q = \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \pm 2, \pm 3, \pm 6$

Graph:



From the graph, -6 is real zeros

$(x+6)$ is a factor of the function.

$\therefore f(x) = 2x^3 + 11x^2 - 7x - 6$

$= (x+6)(2x^2 - x - 1)$

Now,

$2x^2 - x - 1 = 0$	$ax^2 + bx + c = 0$
$2x^2 - 2x + x - 1 = 0$	
$2x(x-1) + 1(x-1) = 0$	
$(x-1)(2x+1) = 0$	

$\therefore x - 1 = 0$ and $2x + 1 = 0$

$x = 1$ $\quad$ $x = -\frac{1}{2}$

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$\rightarrow f(x) = 2x^3 + 11x^2 - 7x - 6$

$= (x+6)(x-1)(2x+1)$

$f(-6) = 0$

$f(-\frac{1}{2}) = 0$

$f(1) = 0$

$\left.\begin{array}{l} x = -6 \\ x = -\frac{1}{2} \\ x = 1 \end{array}\right\}$ Real zeros of the given function.

Steps for Finding the real zeros of a Polynomial function.

Step 1: Use the degree of the polynomial function to determine the maximum number of zeros.

Step 2: If the polynomial function has integer coefficients, use the rational zeros theorem to identify those rational numbers that potentially can be zeros.

Step 3: Graph the polynomial function.

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Step 4: Use the Factor theorem to determine if the potential rational zero is a zero. If it is, we synthetic division or long divison to factor the polynomial function. Repeat step 4 until all the zeros of the polynomial function have been identified and the polynomial function is completely factored.

Find the zeros of $f(x) = x^5 - x^4 - 4x^3 + 8x^2 - 32x + 48$ write f in factored form.

Here, $a_0 = 48$

$a_5 = 1$

factor of a_0; $p = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48$

factor of a_5; $q = \pm 1$

$\therefore p/q = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48$

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Since -3 appears to be a zero and -3 is a potential rational zero, evalute $f(-3)$ and find that $f(-3)=0$.

By the Factor Theorem, $x+3$ is a factor of f. We use synthetic division to factor f.

Factor f as

$$f(x) = x^5 - x^4 - 4x^3 + 8x^2 - 32x + 48 = (x+3)(x^4 - 4x^3 + 8x^2 - 16x + 16)$$

Now work with the first depressed equation:

$$q_1(x) = x^4 - 4x^3 + 8x^2 - 16x + 16 = 0$$

It appears that 2 might be zero of even multiplicity. Check the potential rational zero 2 using synthetic division.

Since, $f(2) = 0$, then $x-2$ is a factor and

$$f(x) = (x+3)(x-2)(x^3 - 2x^2 + 4x - 8)$$

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The depressed equation $q_2(x) = x^3 - 2x^2 + 4x - 8 = 0$

can be factored by grouping.

$$x^3 - 2x^2 + 4x - 8 = (x^3 - 2x^2) + (4x - 8)$$

$$= x^2(x-2) + 4(x-2)$$

$$= (x-2)(x^2+4) = 0$$

$(x-2) = 0$ or $x^2+4=0$

$x = 2$

Since, $x^2+4=0$ has no real solutions. the real zeros of f are -3 and 2, with 2 being a zero or multiplicity 2. The factored form of f is.

$$f(x) = x^5 - x^4 - 4x^3 + 8x^2 - 32x + 48$$

$$= (x+3)(x-2)^2(x^2+4)$$

$(x-2)^2 = 0$	$x^2+4=0$	$x+3=0$
$x=2 \mid x=2$	$x^2 = -4$	$x=-3$
	$x = 2i \mid -2i$	

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complex zero is r

$f(r) = 0$

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_1 x + a_0$

If $r_1, r_2, r_3 \dots r_n$ are the complex zeros of the polynomial function, then $f(x) = a_n (x-r_1)(x-r_2)(x-r_3) \dots (x-r_n)$

H.W, 9th Ed

4.1

4.1

Given that,

Zeros $= -1, 1, 3$

degree $= 3$

if r is the real zero of a polynomial function then $(x-r)$ is a factor of f.

zeros

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Chapter - 5

Exponential and Logarithmic Function

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) = (g \cdot f)(x) = gof$$

$$x \xrightarrow{g} g(x) \xrightarrow{f} f(g(x)) = (f \cdot g)(x) = fog$$

$$x \xrightarrow{g} g(x) \xrightarrow{g} g(g(x)) = (g \cdot g)(x) = gog$$

$$x \xrightarrow{f} f(x) \xrightarrow{f} f(f(x)) = (f \cdot f)(x) = \cancel{fog}\ fof$$

Composit Function

(*)

$$f(x) = 2x^2 - 3$$

$$g(x) = 4x$$

a)

$$\begin{aligned} (fog)(1) &= f(g(1)) \\ &= f(4 \cdot 1) \\ &= f(4) \\ &= 2 \cdot 4^2 - 3 \\ &= 32 - 3 \\ &= 29 \end{aligned}$$

b)

$$\begin{aligned} (gof)(1) &= g(f(1)) \\ &= g(2 \cdot 1 - 3) \\ &= g(2 - 3) \\ &= g(-1) \\ &= 4 \cdot (-1) \\ &= -4 \end{aligned}$$

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c) $(f \circ f)(-2)$

$= f(f(-2))$

$= f(2(-2)^2 - 3)$

$= f(8-3)$

$= f(5)$

$= 2 \cdot 5^2 - 3$

$= 50 - 3$

$= 47$

Ans

d) $(g \circ g)(-1)$

$= g(g(-1))$

$= g(4(-1))$

$= g(-4)$

$= 4 \cdot (-4)$

$= -16$

Ans

(*) $f(x) = x^2 + 3x - 1$

$g(x) = 2x + 3$

a) $f \circ g$

$= f(g(x))$

$= f(2x+3)$

$= (2x+3)^2 + 3(2x+3) - 1$

$= 4x^2 + 12x + 9 + 6x + 9 - 1$

$= 4x^2 + 18x + 17$

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b) $g \cdot f$

$= g(f(x))$

$= g(x^2 + 3x - 1)$

$= 2(x^2 + 3x - 1) + 3$

$= 2x^2 + 6x - 2 + 3$

$= 2x^2 + 6x + 1$

A

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⊛ Find the domain of $f \cdot g$ if $f(x) = \frac{1}{x+2}$ and

$g(x) = \frac{4}{x-1}$.

$\therefore f \cdot g = f(g(x))$

$= f\left(\frac{4}{x-1}\right)$

$= \frac{1}{\frac{4}{x-1} + 2}$

$= \frac{1}{\frac{4+2x-2}{x-1}}$

$= \frac{x-1}{2x+2}$

Now, $2x+2 \neq 0$

$2x \neq -2$

$x \neq -1$

again, $x-1 \neq 0$

$x \neq 1$

domain of $f \cdot g = \{\mathbb{R} - \pm 1\}$

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⊛ If $f(x) = 3x-4$ and $g(x) = \frac{1}{3}(x+4)$, show that,

$$(f\circ g)(x) = (g\circ f)(x) = x$$

$\Rightarrow$

$$(f\circ g)(x) = f(g(x))$$
$$= f\left(\frac{1}{3}(x+4)\right)$$
$$= 3\cdot\frac{1}{3}\cdot(x+4) - 4$$
$$= x+4-4$$
$$= x$$

$$(g\circ f)(x) = g(f(x))$$
$$= g(3x-4)$$
$$= \frac{1}{3}(3x-4+4)$$
$$= \frac{1}{3}\cdot 3x$$
$$= x$$

$\therefore (f\circ g)(x) = (g\circ f)(x) = x$

Showed.

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$f(x) = \frac{1}{x+2}$

$(f \circ f)(x) = f(f(x))$

$= f\left(\frac{1}{x+2}\right)$

$= \frac{1}{\frac{1}{x+2} + 2}$

$= \frac{1}{\frac{1+2x+4}{x+2}}$

$= \frac{x+2}{2x+5}$

Now,

$2x+5 \neq 0$

$2x \neq -5$

$x \neq \frac{-5}{2}$

again,

$x+2 \neq 0$

$x \neq -2$

So, domain of $(f \circ f) = \{\mathbb{R} - (-\frac{5}{2}), (-2)\}$

~~Hor~~

⊛ Horizontal Line - Test Theorem,

If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

⊛ Theorem:

A function that is increasing on an interval I is a one-to-one function on I.

A function that is decreasing on an interval I is a one-to-one function on I.

⊛ Remember, if f is a one-to-one function, it has an inverse function, f^{-1}.

Domain of f = Range of f^{-1}

Range of f = Domain of f^{-1}

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⊛ $f^{-1}(f(x)) = x$, where x is in the domain of f

⊛ $f(f^{-1}(x)) = x$, where x is in the domain of f^{-1}

⊛ Verify that the inverse of $g(x) = x^3$ is $g^{-1}(x) = \sqrt[3]{x}$

$\Rightarrow$

$$g(g^{-1}(x)) = g(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$$

$$g^{-1}(g(x)) = g^{-1}(x^3) = \sqrt[3]{x^3} = x^{3 \cdot \frac{1}{3}} = x$$

$\therefore$ $g(g^{-1}(x)) = g^{-1}(g(x)) = x$

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* The function

$$f(x) = \frac{2x+1}{x-1} \quad ; \quad x \neq 1$$

is one-to-one. Find its inverse and check the result.

$\Rightarrow$

Replacing x by y and y by x.

$$x = \frac{2y+1}{y-1} \quad \cdots \text{(i)}$$

This is the implicit form of the inverse function.

$$2y + 1 = xy - x$$

$$xy - 2y = x + 1$$

$$y(x-2) = x+1$$

$$y = \frac{x+1}{x-2} \quad \cdots \text{(ii)}$$

This is the explicit form of the inverse function

is $f^{-1}(x) = \frac{x+1}{x-2}$.

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Check!

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2x+1}{x-1}\right)$$

$$= \frac{\frac{2x+1}{x-1}+1}{\frac{2x+1}{x-1}-2}$$

$$= \frac{2x+1+x-1}{x-1} \cdot \frac{x-1}{2x+1-2x+2}$$

$$= \frac{3x}{3}$$

$$= x$$

$$f(f^{-1}(x)) = f\left(\frac{x+1}{x-2}\right)$$

$$= \frac{2\cdot\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-1}$$

$$= \frac{2x+2+x-2}{x-2} \cdot \frac{x-2}{x+1-x+2}$$

$$= \frac{3x}{x}$$

$$= x$$

$$\therefore f(f^{-1}(x)) = f^{-1}(f(x)) = x.$$

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Laws of exponents

$f(0) = 5$

$f(x) = 2f(x-1)$

$\dfrac{f(x)}{f(x-1)} = 2$

If $x=1$

$f(1) = 2f(0) = 2 \cdot 5 = 5 \cdot 2^1$

If $x=2$

$f(2) = 2f(1) = 2 \cdot 5 \cdot 2^1 = 5 \cdot 2^2$

If $x=3$

$f(3) = 2f(2) = 2 \cdot 5 \cdot 2^2 = 5 \cdot 2^3$

If $x=x$;

$f(x) = 5 \cdot 2^x$

If $x = x+1$

$f(x+1) = 5 \cdot 2^{x+1}$

$$\frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = \cdots = \frac{f(x+1)}{f(x)} = \cdots = 2$$

$f(x) = Ca^x$

$f(x+1) = Ca^{x+1}$

$$\therefore \frac{f(x+1)}{f(x)} = \frac{Ca^{x+1}}{Ca^x} = a$$

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$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

$$\boxed{a^u = a^v \quad \therefore u = v}$$

Properties of the Exponential Function $f(x) = a^x$. $a > 1$.

1. The domain is the set of all real numbers or $(-\infty, \infty)$ using interval notation; the range is the set of positive real numbers or $(0, \infty)$ using interval notation

2. There are no x-intercepts; the y-intercept is 1.
3. The x-axis $(y = 0)$ is a horizontal asymptote as $x \to -\infty$ $\left[\lim_{x \to -\infty} a^x = 0\right]$

4. $f(x) = a^x$, where $a > 1$, is an increasing function and is one-to-one.

(*) Solve: $4^{2x-1} = 8^x$

$\Rightarrow 2^{2(2x-1)} = 2^{3 \cdot x}$

$\Rightarrow 2^{4x-2} = 2^{3x}$

$\Rightarrow 4x-2 = 3x$

$\Rightarrow x = 2$

Ans

(*) $e^{-x^2} = (e^x)^2 \cdot \frac{1}{e^3}$

$\Rightarrow e^{-x^2} = e^{2x} \cdot e^{-3}$

$e^{-x^2} = e^{2x-3}$

$-x^2 = 2x-3$

$x^2 + 2x = 3$

$x^2 + 2x - 3 = 0$

$x^2 + 3x - x - 3 = 0$

$x(x+3) - 1(x+3) = 0$

$(x+3)(x-1) = 0$

$x = \{-3, 1\}$

(*) $5^{x^2+8} = 125^{2x}$

$5^{x^2+8} = 5^{3 \cdot 2x}$

$x^2 + 8 = 6x$

$x^2 - 6x + 8 = 0$

$x^2 - 4x - 2x + 8 = 0$

$x(x-4) - 2(x-4) = 0$

$(x-4)(x-2) = 0$

$x = \{4, 2\}$

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$y = f(x) = a^x$

inverse implicit form. $x = a^y$

↳ logarithmic function.

$y = \log_a x \cong x = a^y$

$\boxed{\begin{matrix} \log_a x = y \\ x = a^y \end{matrix}}$

⊛ $y = \log_3 x$

$x = 3^y$

⊛ $4 = \log_3 81$

$\Rightarrow 81 = 3^4$

⊛

a) $1{\cdot}2^3 = m$

$3 = \log_{1\cdot 2} m$

b) $e^b = 9$

$b = \log_e 9$

c) $a^4 = 24$

$4 = \log_a 24$

⊛

a) $\log_a 4 = 5$

$4 = a^5$

b) $\log_e b = -3$

$b = e^{-3}$

c) $\log_3 5 = c$

$5 = 3^c$

⊛

a) $\log_2 16$

$= \log_2 2^4$

$= 4$

b) $\log_3 \frac{1}{27}$

$= \log_3 3^{-3}$

$= -3$

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properties of logarithmic functions.

1. The domain is the set of positive real numbers or $(0, \infty)$ using interval notation; the range is the set of all real numbers or $(-\infty, \infty)$ using ~~in~~ interval notation.

2. The x-intercept of the graph is 1. There is no y-intercept.

3. The y-axis $(x=0)$ is a vertical asymptote of the graph.

4. A logarithmic function is decreasing if $0<a<1$ and increasing if $a>1$.

5. The graph of f contains the points $(1,0)$, $(a,1)$ and $(\frac{1}{a}, -1)$.

6. The graph is smooth and continuous, with no corners or gaps.

a) $\log_3 (4x-7) = 2$

$4x - 7 = 3^2$

$4x - 7 = 9$

$4x = 16$

$x = 4$

b) $\log_x 64 = 2$

$64 = x^2$

$x = \pm\sqrt{64}$

$x = \pm 8$

$x \neq -8$

cannot be negative.

$\therefore x = 8$ Ans

$e^{2x} = 5$

$\ln e^{2x} = \ln 5$

$\ln 5 = 2x$

$x = \frac{\ln 5}{2}$

$= 0.8047$ B

$\boxed{\ln = \log_e}$

$\log_a 1 = 0$

$y = \log_a 1$

$a^y = 1$

$a^y = a^0$

$\therefore y = 0$

$\therefore \log_a 1 = 0$

$\log_a a = 1$

$y = \log_a a$

$a^y = a$

$\therefore y = 1$

$\therefore \log_a a = 1.$

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⊛ $\log_a (x\sqrt{x^2+1})$; $x>0$ Express all powers as factor.

$\Rightarrow \log_a x + \log_a(\sqrt{x^2+1})$

$= \log_a x + \log_a (x^2+1)^{\frac{1}{2}}$

$= \log_a x + \frac{1}{2} \log_a (x^2+1)$ Ans

⊛ $\ln \frac{x^2}{(x-1)^3}$; $x>1$

$= \ln x^2 - \ln (x-1)^3$

$= 2\ln x - 3\ln(x-1)$ Ans

⊛ $\log_a \frac{\sqrt{x^2+1}}{x^3(x+1)^4}$

$= \log_a(\sqrt{x^2+1}) - \log_a (x^3)(x+1)^4$

$= \frac{1}{2}\log_a(x^2+1) - \log_a x^3 + \log_a (x+1)^4$

$= \frac{1}{2} \log_a(x^2+1) - 3\log_a x + 4\log_a(x+1)$

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(*)

c) $\log_a x + \log_a 9 + \log_a (x^2+1) - \log_a 5$

$= \log_a (9x) + \log_a (x^2+1) - \log_a 5$

$= \log_a [9x \cdot (x^2+1)] - \log_a 5$

$= \log_a \left[\frac{9x \cdot (x^2+1)}{5} \right]$ Ans

(*)

$2 \log_5 x = \log_5 9$

$\log_5 x^2 = \log_5 9$

$x^2 = 9$

$x = \pm 3$

$\therefore x = 3$

(*) $\log_5 (x+6) + \log_5 (x+2) = 1$

$\log_5 (x+6) + \log_5 (x+2) = 1$

$(x+6)(x+2) = 5^1 = 5$

$\therefore x = -7 \mid x = -1$

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$\circledast$ $\ln x + \ln(x-4) = \ln(x+6)$

$\ln[x\cdot(x-4)] = x+6$

$\therefore x = 6 \mid x = -1$

$\circledast$

$5^{x-2} = 3^{3x+2}$

$\Rightarrow \ln 5^{x-2} = \ln 3^{3x+2}$

$\Rightarrow (x-2)\ln 5 = (3x+2)\ln 3.$

$\Rightarrow (\ln 5)x - 2\ln 5 = (3\ln 3)x + 2\ln 3$

$x[\ln 5 * 3\ln 3] = 2\ln 3 + 2\ln 5$

$\therefore x = \dfrac{2\ln 3 + 2\ln 5}{\ln 5 * 3\ln 3}$

$= -3.21$

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$$4^x - 2^x - 12 = 0$$

$$(2^x)^2 - 2^x - 12 = 0$$

$$\cancel{\therefore (2^x - 4)(2^x + 3) = 0}$$

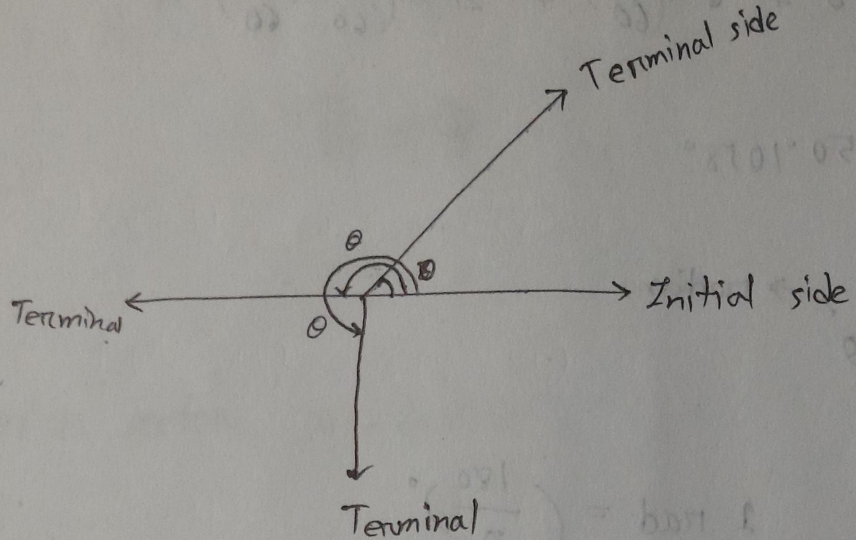
$$(2^x)^2 - 4 \cdot 2^x + 3 \cdot 2^x - 12 = 0$$

$$2^x(2^x - 4) + 3(2^x - 4) = 0$$

$$(2^x - 4)(2^x + 3) = 0$$

Chapter 6

Trigonometric Function



$$1^\circ = 60'$$

$$1' = 60''$$

$$32.25^\circ \rightarrow 0.25^\circ = \frac{\overset{5}{\cancel{25}}}{\underset{20}{\cancel{100}}} \times \overset{3}{\cancel{60}}$$

$$= 15'$$

$$32.25^\circ = 32^\circ\ 15'$$

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⊛

$$50^\circ 6' 21'' = 50^\circ + 6 \times 1' + 21 \times 1''$$

$$= 50^\circ + 6 \times \left(\frac{1}{60}\right)^\circ + 21 \times \left(\frac{1}{60} \times \frac{1}{60}\right)^\circ$$

$$= 50.1058^\circ$$

⊛ $s = r\theta$ → radians

$$1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ$$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radians.}$$

⊛

$$60^\circ = 60 \times 1^\circ$$

$$= 60 \times \frac{\pi}{180} \text{ radians}$$

$$= \frac{\pi}{3} \text{ radians}$$

⊛

$$\frac{\pi}{6} = \frac{\pi}{6} \times 1 \text{ radians}$$

$$= \frac{\pi}{6} \times \frac{180}{\pi} \text{ degrees}$$

$$= 30^\circ$$

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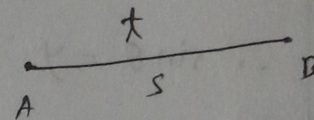
$$\frac{\theta}{\theta_1} = \frac{A}{A_1}$$

$$A = A_1 \times \frac{\theta}{\theta_1}$$

$$= \pi r^2 \times \frac{\theta}{2\pi}$$

$$= \frac{1}{2} r^2 \theta .$$



Area of a sector, $A = \frac{1}{2} r^2 \theta$

$$v = \frac{s}{t}$$

$$\omega = \frac{\theta}{t}$$

linear speed, $v = \frac{s}{t} = \frac{r\theta}{t}$

$$= r\left(\frac{\theta}{t}\right) = r \cdot \omega$$

$$\therefore v = r\omega$$

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Equation of a circle,

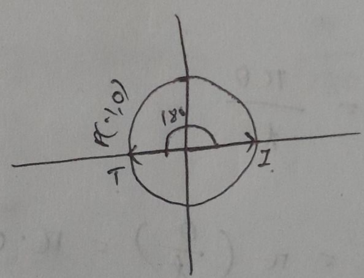
$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 = 1 \longrightarrow \text{unit circle}$$

$\therefore \sin\theta = y$ $\cos\theta = x$ $\tan\theta = \dfrac{y}{x}$

$\operatorname{cosec}\theta = \dfrac{1}{y}$ $\sec\theta = \dfrac{1}{x}$ $\cot\theta = \dfrac{x}{y}$

(*)



$\theta = 180° = \pi$

$\sin\theta = 0$ $\cos\theta = -1$ $\tan\theta = 0$

$\operatorname{cosec}\theta$ = undefined $\sec\theta = -1$ $\cot\theta$ = undefined

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$\sin(3\pi) = y = 0$

$\cos(-270°) = x = 0$

$\theta = \frac{\pi}{6}$

$\sin\left(\frac{\pi}{6}\right) = y = \frac{1}{2}$

$\cos\left(\frac{\pi}{6}\right) = x = \frac{\sqrt{3}}{2}$

$\tan\left(\frac{\pi}{6}\right) = \frac{y}{x} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

$\operatorname{cosec}\left(\frac{\pi}{6}\right) = \frac{1}{y} = \frac{1}{1/2} = 2$

$\sec\left(\frac{\pi}{6}\right) = \frac{1}{x} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$

$\cot\left(\frac{\pi}{6}\right) = \frac{x}{y} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

$\left.\begin{array}{l}\operatorname{cosec}\theta = \frac{1}{y} \\ \cot\theta = \frac{x}{y}\end{array}\right\} y \neq 0$

$\left.\begin{array}{l}\sec\theta = \frac{1}{x} \\ \tan\theta = \frac{y}{x}\end{array}\right\} x \neq 0$

$\pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi$

$\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$

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(*)

$\sin(\theta + 2\pi) = \sin\theta$ $\quad$ $\cos(\theta + 2\pi) = \cos\theta$

$\text{cosec}(\theta + 2\pi) = \text{cosec}\,\theta$ $\quad$ $\sec(\theta + 2\pi) = \sec\theta$

$\tan(\theta + \pi) = \tan\theta$

$\cot(\theta + \pi) = \cot\theta$

a) $\sin\frac{17\pi}{4} = \sin\left(\frac{\pi}{4} + 4\pi\right)$

$= \sin\frac{\pi}{4}$

$= y = 1$

b) $\cos(5\pi)$

$= \cos(\pi + 4\pi)$

$= \cos\pi$

$= x = -1$

c) $\tan\frac{5\pi}{4}$

$= \tan\left(\frac{\pi}{4} + \pi\right)$

$= \tan\frac{\pi}{4}$

$= \frac{y}{x}$

$= \frac{1/2}{1/2} = 1$

(*)

$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = \cos\theta$ $\quad$ $\tan(-\theta) = -\tan\theta$

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$ $\quad$ $\sec(-\theta) = \sec\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

d)

$$\tan\left(-\frac{37\pi}{4}\right) = -\tan\left(\frac{37\pi}{4}\right)$$

$$= -\tan\left(\frac{\pi}{4} + 9\pi\right)$$

$$= -\tan\left(\frac{\pi}{4}\right)$$

$$= -\frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}}$$

$$= -\frac{1/\sqrt{2}}{1/\sqrt{2}}$$

$$= -1$$

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Chapter 7

$y = \sin x \longrightarrow$ Domain $= (-\infty, \infty)$

Range $= [-1, 1]$

$x = \sin y$ ... implicit form

$y = \sin^{-1} x$ ... Explicit form

$\hookrightarrow$ Domain $[-1, 1]$

Range $(-\infty, \infty)$

(*) $\sin^{-1} 1 = ?$

Let, $\theta = \sin^{-1} 1$

$\sin\theta = 1 = \sin\frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{2}$

Thus, $\sin^{-1} = \frac{\pi}{2}$.

(*) $\sin^{-1}(\sin\frac{5\pi}{8})$

$= \sin^{-1}(\sin\frac{3\pi}{8})$

$= \frac{3\pi}{8}$

(*) $\cos^{-1}(\cos(-\frac{2\pi}{3}))$

$= \cos^{-1}(\cos\frac{2\pi}{3})$

$= \frac{2\pi}{3}$

(*) $\tan^{-1}(-\sqrt{3})$

$= -\tan^{-1}(\sqrt{3})$

$= -\frac{\pi}{3}$

(*)

Given that,

$$f(x) = y = 2\sin x - 1$$

Let replace x by y and y by x

$$x = 2\sin y - 1$$

$$2\sin y = x+1$$

$$\sin y = \frac{x+1}{2}$$

$$y = \sin^{-1}\left(\frac{x+1}{2}\right)$$

Thus the inverse function of $f(x) = 2\sin x - 1$ is

$$f^{-1}(x) = \sin^{-1}\left(\frac{x+1}{2}\right)$$

For $f(x)$,

Domain $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range $= [-3, 1]$

For, $f^{-1}(x)$,

Domain $= [-3, 1]$

Range $= \left[-\pi/2, \frac{\pi}{2}\right]$

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(*) Write $\sin(\tan^{-1}u)$ as an algebraic expression containing u.

Let,

$\tan^{-1} u = \theta$

So, $\sin[\tan^{-1} u] = \sin\theta = \sin\theta \dfrac{\cos\theta}{\cos\theta}$

$= \tan\theta \cdot \cos\theta$

$= \dfrac{\tan\theta}{\sec\theta}$

$= \dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

$= \dfrac{u}{\sqrt{1+u^2}}$ Ans

P.T.O.

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* Establish the identity :

$$\frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \cos\theta - \sin\theta$$

$$L.H.S. = \frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)}$$

$$= \frac{[\sin(-\theta)]^2 - [\cos(-\theta)]^2}{-\sin\theta - \cos\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{-\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{-(\sin\theta + \cos\theta)}$$

$$= -\sin\theta + \cos\theta$$

$$= \cos\theta - \sin\theta$$

$= R.H.S.$

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⊗

$$L.H.S = \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{(1-\sin\theta)(1+\sin\theta)}{\cos\theta\,(1+\sin\theta)}$$

$$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos\theta}{1+\sin\theta}$$

$= R.H.S.$

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Important for Final

1. $$\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$$

2. $$\frac{1-2\cos^2\theta}{\sin\theta\cos\theta} = \tan\theta - \cot\theta$$

3. $$\frac{1-\cos\theta}{1+\cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$$

4. $$\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$$

5. $$\frac{\tan\theta - \cot\theta}{\tan\theta + \cot\theta} + 2\cos^2\theta = 1$$

6. $$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$$