

Chapten : 1

L-1

01.06.2022

Linear Equation:

$$y = mx + c$$

$$x + y = z$$

✳

$$x + 3y + 5z = 10$$

$$2x - y + 4z = -5$$

$$3x + 2y = z$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 10 \\ 2 & -1 & 4 & -5 \\ 3 & 2 & 0 & z \end{array} \right]$$

Coefficient Matrix Constant part
Augmented Matrix

Target Matrix:

$$\left[\begin{array}{cccc} 1 & 2 & -1 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & 5 \end{array} \right]$$

→ One Solution

$$\left[\begin{array}{cccc} 1 & 2 & -1 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 0 & 5 \end{array} \right]$$

→ No solution

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 1 \\ 0 & 1 & 5 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

→ Many Solution

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$$x + y + 2z = 9$$

$$2x + 4y - 3z = 1$$

mit dem

$$2x + 4y - 3z = 1$$

$$3x + 6y - 5z = 0$$

$$x = v + w$$

Augmented Matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{array} \right] \xrightarrow{\begin{array}{l} R_1 \rightarrow R_1 - R_2 \\ R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{array}} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & -11 & -27 \end{array} \right] \xrightarrow{\begin{array}{l} R_2 \rightarrow R_2 - R_3 \\ R_3 \rightarrow R_3 - \frac{3}{2}R_2 \end{array}} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & \frac{-7}{2} & \frac{-17}{2} \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{aligned} R'_1 &= R_1 - 2R_2 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & \frac{-7}{2} & \frac{-17}{2} \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\text{pivot 1}} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & \frac{-7}{2} & \frac{-17}{2} \\ 0 & 0 & 0 & 0 \end{array} \right] \\ R'_2 &= R_2 - 3R_1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & \frac{-7}{2} & \frac{-17}{2} \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\text{pivot 2}} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & \frac{-7}{2} & \frac{-17}{2} \\ 0 & 0 & 0 & 0 \end{array} \right] \end{aligned}$$

$$R'_1 = \frac{1}{2} R_2 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & \frac{-7}{2} & \frac{-17}{2} \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\text{pivot 1}} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & \frac{-7}{2} & \frac{-17}{2} \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\pi_3' = \pi_3 - 3\pi_1$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & -\frac{7}{2} & \frac{-17}{2} \\ 0 & 0 & \frac{-1}{2} & \frac{-3}{2} \end{bmatrix}$$

\$= 3x + y + 10\$
\$+ 3x + y + 10\$
\$= 6x + 2y + 10\$
\$\perp = x\$

$$\pi_3' = \cancel{\pi_3} - 2\pi_2$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & -\frac{7}{2} & \frac{-17}{2} \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

in row-echelon form
\$(P.S.I) = (S.E.R)\$

and solving from bottom \$\leftarrow\$

$$\therefore z = 3$$

$$\therefore y + \frac{-7}{2}z = \frac{-17}{2}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow y + \frac{-7 \cdot 3}{2} = \frac{-17}{2}$$

$$\Rightarrow y = \frac{-17}{2} + \frac{21}{2}$$

$$\therefore y = \frac{-17 + 21}{2} = \frac{4}{2} = 2$$

$$\therefore x = 10 - y - z$$

$$\therefore x = 10 -$$

$$\therefore x = 10 -$$

$$\therefore x = 10 -$$

$$\therefore x + y + 2z = 9$$

$$\Rightarrow x + 2 + 2 \cdot 3 = 9$$

$$\Rightarrow x = 9 - 8$$

$$\therefore x = 1$$

$$\therefore (x, y, z) = (1, 2, 3)$$

Ans

→ Reduced row-echelon form

⊗

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 7 \end{array} \right]$$

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a)

$$2x = -1$$

b)

$$3x_2 - x_3 - x_4 = -1$$

$$-4x = -6$$

$$x = -1$$

$$3x = 0$$

$$5x_1 + 2x_2 - 3x_4 = -6$$

c)

$$\begin{array}{l} x + 2y + 3z = 4 \\ -4x - 3y - 2z = -1 \end{array}$$

$$\begin{array}{l} 5x - 6y + z = 1 \\ -8x \end{array}$$

$$d) \quad 3x_1 + x_2 - 4x_3 = 3$$

$$-4x_1 + 4x_2 + x_4 = -3$$

$$-x_1 + 3x_2 - 2x_4 = -9$$

$$-x_4 = -2$$

15]

$$y_1 = ax_1^2 + bx_1 + c$$

$$y_2 = ax_2^2 + bx_2 + c$$

$$y_3 = ax_3^2 + bx_3 + c$$

Augmented Matrix:

$$\left[\begin{array}{cccc|c} x_1^2 & x_1 & 1 & y_1 \\ x_2^2 & x_2 & 1 & y_2 \\ x_3^2 & x_3 & 1 & y_3 \end{array} \right]$$

Example - 5:

The augmented matrix for the system is,

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{array} \right]$$

$$\begin{array}{l} R_2' = R_2 - 2R_1 \\ R_3' = R_3 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{array} \right]$$

$$\xrightarrow{R_4' = -R_2} R_4' = -R_2$$

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{array} \right]$$

$$R'_3 = R_3 - 5R_2$$

$$R'_4 = R_4 - 4R_2$$

all bases selected except
row 2 & 3

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 \end{array} \right]$$

$$R_3 \leftrightarrow R_4$$

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$R'_3 = \frac{1}{6} R_3$$

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$R'_2 = R_2 - 3R_3$$

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$R_1' = R_1 + 2R_2$$

$$\rightarrow \left[\begin{array}{cccccc|c} 1 & 3 & 0 & 4 & 2 & 10 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \text{reduced row-echelon matrix in echelon}$$

$$\therefore x_1 + 3x_2 + 4x_3 + 2x_5 = 0$$

$$x_3 + 2x_4 = 0$$

$$x_3 = \frac{1}{3}$$

$\therefore x_1, x_2, x_3$ are leading variables.

$$\therefore x_1 = -3x_2 - 4x_4 - 2x_5$$

$$\therefore x_3 = -2x_4$$

$$\therefore x_3 = \frac{1}{3}$$

$\therefore x_2, x_4, x_5$ are free variables.

Take,

$$x_2 = r$$

$$x_4 = s$$

$$x_5 = t$$

$$\therefore x_1 = -3n - 4s - 2t$$

$x_2 = \pi$ (not zero and not multiple of powers of 5)

$$x_3 = -2s$$

$$x_4 = s \begin{bmatrix} 0 & 0 & 8 & 0 & 8- & 8+ & 1 \end{bmatrix}$$

$$x_5 = t \begin{bmatrix} 0 & 8- & 8+ & 2- & 2+ & 0 & 8 \end{bmatrix}$$

$$x_6 = \frac{1}{3} \begin{bmatrix} 0 & 21 & 0 & 01 & 2 & 0 & 0 \end{bmatrix}$$

where, π, s, t can be any real numbers.

$$\textcircled{*} \quad 2x + y + 5z = 0 \quad \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 01 & 2 & 0 & 0 \\ 0 & 01 & 0 & 8 & 8 & 0 & 0 \end{array} \right] \quad \text{linear homogenous equation.}$$

$$\left[\begin{array}{cccccc} 0 & 0 & 8 & 0 & 8- & 8+ & 1 \\ 0 & 8- & 0 & 8 & 1 & 0 & 0 \\ 0 & 21 & 0 & 01 & 2 & 0 & 0 \\ 0 & 21 & 0 & 0 & 8 & 0 & 0 \end{array} \right]$$

P.T.O.

Example - 6:

The augmented matrix for the system is

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & 0 \\ 2 & 6 & 6 & 8 & 4 & 18 & 0 \end{array} \right]$$

$$R_2' = R_2 - 2R_1$$

$$R_4' = R_4 - 2R_1$$

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & 0 \\ 0 & 0 & 4 & 8 & 0 & 18 & 0 \end{array} \right]$$

$$R_2' = -R_2$$

$$\longrightarrow$$

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & 0 \\ 0 & 0 & 4 & 8 & 0 & 18 & 0 \end{array} \right]$$

$$R_3' = R_3 - 5R_2$$

$$\xrightarrow{R_4' = R_4 - 4R_2}$$

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$R_3 \leftrightarrow R_4$$

$$\xrightarrow{R_3 \leftrightarrow R_4}$$

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$R_4' = \frac{1}{6}R_4$$

$$\xrightarrow{R_4' = \frac{1}{6}R_4}$$

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$r'_1 = r_2 - 3r_3$$

$$\rightarrow \left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$r'_1 = r_1 + 2r_2$$

$$\rightarrow \left[\begin{array}{ccccccc} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \leftarrow$$

$$\therefore x_1 + 3x_2 + 4x_4 + 2x_5 = 0$$

$$x_2 + 2x_4 = 0$$

$$x_6 = 0$$

$$\therefore x_1 = -3x_2 - 4x_4 - 2x_5$$

$$x_3 = -2x_4$$

$$x_6 = 0$$

Let, $x_2 = \pi$

$$x_4 = s \quad \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 2 & 3 \end{matrix} = 8 \quad \begin{matrix} 1 & 2 & 1 \\ 0 & 3 & 2 \end{matrix} = A$$

3×3 2×2 2×2

$$\therefore x_1 = -3\pi - 4s - 2t$$

$$x_2 = \pi \quad \text{Ans}$$

$$x_3 = -2s + t \quad \begin{matrix} 8s+8+t & 8+0+1 \\ 0+2+8 & 0+2+1 \\ 0+0+0 & 0+0+0 \end{matrix} = 9A$$

3×3 2×2 2×2

$$x_4 = s$$

$$x_5 = t \quad \begin{matrix} 81 & 08 & 88 & 09 \\ 01 & 28 & 11 & 8 \end{matrix}$$

4×4

Ans

$$x_6 = 0 \quad \begin{matrix} 1 & 2 & 0 \\ 1 & 0 & 2 \end{matrix} = A \quad \text{Ans}$$

$$x_7 = 0 \quad \begin{matrix} 2 & 0 \\ 0 & 1 \end{matrix} = A$$

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1.3

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix}_{2 \times 3} \quad \text{&} \quad B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 2 & 5 & 2 \end{bmatrix}_{3 \times 4}$$

$n_1 \times c_1$ $n_2 \times c_2$

$$c_1 = n_2$$

$$\therefore AB = \begin{bmatrix} 4+0+8 & 1-2+28 & 4+6+20 & 3+2+8 \\ 8+0+0 & 2-6+0 & 8+18+0 & 6+6+0 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 27 & 30 & 13 \\ 8 & -4 & 26 & 12 \end{bmatrix}_{2 \times 4}$$

$$\textcircled{*} \quad A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & -1 \end{bmatrix}_{2 \times 3}$$

$$A^T = \begin{bmatrix} 2 & 5 \\ 3 & 6 \\ 4 & -1 \end{bmatrix}_{3 \times 2}$$

23)

a) $A = \begin{bmatrix} a & 0 & 0 & 0 & 0 & 0 \\ 0 & b & 0 & 0 & 0 & 0 \\ 0 & 0 & c & 0 & 0 & 0 \\ 0 & 0 & 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & 0 & f \end{bmatrix}$

b) $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & 0 & a_{33} & a_{34} & a_{35} & a_{36} \\ 0 & 0 & 0 & a_{44} & a_{45} & a_{46} \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}$

d) $A = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & a_{65} & a_{66} \end{bmatrix}$

24)

a)

$$\left[\begin{array}{cccc|ccc} 2 & 3 & 4 & 5 & 0 & 0 & 0 \\ 3 & 4 & 5 & 6 & 0 & 1 & 0 \\ 4 & 5 & 6 & 7 & 0 & 0 & 0 \\ 5 & 6 & 7 & 8 & 0 & 0 & 0 \end{array} \right]$$

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= A (c)

b)

$$\left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 4 & 8 & 0 & 0 & 0 \\ 1 & 3 & 9 & 27 & 0 & 0 & 0 \\ 1 & 4 & 16 & 64 & 0 & 0 & 0 \end{array} \right]$$

(d)

= A

$$\left[\begin{array}{ccccc|cc} 0 & 0 & 0 & 0 & 11 & 11 \\ 0 & 0 & 0 & 22 & 22 & 11 \\ 0 & 0 & 22 & 22 & 22 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

(e)

= A

$$\left[\begin{array}{ccccc|cc} 0 & 0 & 0 & 22 & 22 & 11 \\ 0 & 0 & 22 & 22 & 22 & 0 \\ 0 & 0 & 22 & 22 & 22 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccccc|cc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccccc|cc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccccc|cc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

(Theorem) 1.4.1 Properties of Matrix Arithmetic

Theorem: 1.4.1 Properties of Matrix Arithmetic

Properties of $A + B = B + A$ with respect to addition

b) $A + (B+C) = (A+B)+C$

c) $A(BC) = (AB)C$

d) $A(B+C) = AB+AC$

e) $(B+C)A = BA+CA$

f) $A(B-C) = AB-AC$

g) $(B-C)A = BA-CA$

h) $a(B+C) = aB + aC$

i) $a(B-C) = aB - aC$

j) ~~$(a+b)c = ac + bc$~~

k) $(a-b)c = ac - bc$

l) $a(bc) = (ab)c$

m) $a(bc) = (ab)c = b(ac)$

* Identity Matrix / Unit Matrix (Product)

* In general math: If product of two non-zero numbers is 1, then they are inverse of each other.

* Square matrix whose determinant is zero is singular matrix

* Rules of inverse,

i. square matrix

ii. same size

iii. Non-singular

$$AB = I = BA$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det(A) = ad - bc = 1(d-a) \quad (1)$$

$$\det(A) = (ad) \quad (1)$$

$$(ad)^{-1} = (d-a)^{-1} \cdot (ad)^{-1} \quad (1)$$

$$\textcircled{*} \quad A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

\Rightarrow

$$|A| = \begin{vmatrix} 2 & -5 \\ -1 & 3 \end{vmatrix} = 6 - 5 = 1$$

$$|B| = \begin{vmatrix} 3 & 5 \\ 1 & 2 \end{vmatrix} = 6 - 5 = 1$$

$$\textcircled{*} \quad A = \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix} \quad |A| = \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 2 & 1 \end{bmatrix} = 2 - 8 = -6$$

$$\det(A) = -3 - 8 = -11$$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} -3 & -4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{11} & \frac{4}{11} \\ \frac{2}{11} & -\frac{1}{11} \end{bmatrix}$$

$$\textcircled{*} \quad B = \begin{bmatrix} 5 & 2 \\ 3 & -1 \end{bmatrix}$$

$$\det(B) = -5 - 6 = -11$$

$$\therefore B^{-1} = \frac{1}{\det(B)} \begin{bmatrix} -1 & -2 \\ -3 & 5 \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} -1 & -2 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{3}{11} & \frac{-5}{11} \end{bmatrix}$$

$$\textcircled{*} \quad A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \quad \left| \begin{array}{l} 1 = 2+(-1) = 1 \\ 2 = (-1) \cdot 3 + 2 = -3+2 = -1 \end{array} \right. \quad \begin{bmatrix} 2-(-1) & -5 \\ -1 & 1 \end{bmatrix} = A$$

$$\det(A) = 6 - 5 = 1$$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \left| \begin{array}{l} 1 = 2 \cdot 3 - 1 \cdot 1 = 6 - 1 = 5 \\ 2 = 1 \cdot 5 - 3 \cdot 1 = 5 - 3 = 2 \end{array} \right. = B \quad \left| \begin{array}{l} 1 = 2 \cdot 3 - 1 \cdot 1 = 6 - 1 = 5 \\ 2 = 1 \cdot 5 - 3 \cdot 1 = 5 - 3 = 2 \end{array} \right. = |A|$$

$$B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \quad \left| \begin{array}{l} 1 = 2 \cdot 3 - 1 \cdot 1 = 6 - 1 = 5 \\ 2 = 1 \cdot 5 - 3 \cdot 1 = 5 - 3 = 2 \end{array} \right. = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = |B|$$

$$\det(B) = 6 - 5 = 1$$

$$\therefore B^{-1} = \frac{1}{1} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = A \quad \left| \begin{array}{l} 1 = 2 \cdot 1 - (-1) \cdot 3 = 2 + 3 = 5 \\ 2 = (-1) \cdot (-5) - 2 \cdot 3 = 5 - 6 = -1 \end{array} \right. = A$$

$\left| \begin{array}{l} 1 = 2 \cdot 1 - (-1) \cdot 3 = 2 + 3 = 5 \\ 2 = (-1) \cdot (-5) - 2 \cdot 3 = 5 - 6 = -1 \end{array} \right. = (A)^{-1} b$

$$\textcircled{*} \quad \begin{bmatrix} 2x + 3y & 1 \\ x - 5y & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad \left| \begin{array}{l} 1 = 2 \cdot 1 - 3 \cdot (-5) = 2 + 15 = 17 \\ 2 = 2 \cdot 2 - 1 \cdot (-5) = 4 + 5 = 9 \end{array} \right. = (AB)^{-1} b$$

$$\begin{bmatrix} 2 & -5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$Ax = B$$

$$A^{-1} Ax = A^{-1} B$$

$$x = A^{-1} B$$

$$A^{-1} = \frac{1}{-13} \begin{bmatrix} -5 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{13} & \frac{3}{13} \\ \frac{1}{13} & -\frac{2}{13} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} \frac{5}{13} & \frac{3}{13} \\ \frac{1}{13} & -\frac{2}{13} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{13} + \frac{21}{13} \\ \frac{1}{13} - \frac{14}{13} \end{bmatrix} = \begin{bmatrix} \frac{26}{13} \\ \frac{-13}{13} \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\textcircled{*} (AB)^{-1} = B^{-1}A^{-1}$$

$$\Rightarrow (AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1}$$

$$= AIA^{-1}$$

$$= AA^{-1}$$

$$= I$$

$$(B^{-1}A^{-1})AB = B^{-1}(A^{-1}A)B$$

$$\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} = A$$

$$= B^{-1}IB$$

$$= B^{-1}B$$

$$= I \quad \text{Durchaus das mit holds} \Rightarrow (A) \text{ ist}$$

$$\textcircled{4} \quad \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1A$$

$$\textcircled{5} \quad (AB)^T = B^T A^T$$

$$(A \pm B)^T = A^T \pm B^T$$

$$\textcircled{6} \quad (AB)^{-1} = B^{-1} A^{-1}$$

$$A^{-2} = (A^{-1})^{-1} = (A^{-1})^2$$

$$A^2 \cdot A^3 = A^5$$

$$A^{-n} = (A^{-1})^n = (A^n)^{-1}$$

$$(A^{-1})^{-1} = A$$

$$A(AB)A = (AA)(BA) \Leftarrow$$

Matrix Polynomial

$$A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$$

$$P(x) = 2x^2 - 5x + 4$$

$P(A)$ = matrix polynomial

$$= 2A^2 - 5A + 4I$$

\Rightarrow

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \cdot 2 + 1 \cdot 4 & -6 - 3 \\ 8 + 4 & -12 + 1 \end{bmatrix} = \begin{bmatrix} -8 & -9 \\ 12 & -11 \end{bmatrix}$$

$$P(A) = 2A^2 - 5A + 4I$$

$$= 2 \begin{bmatrix} -8 & -9 \\ 12 & -11 \end{bmatrix} - 5 \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} + 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & -18 \\ 24 & -22 \end{bmatrix} - \begin{bmatrix} 10 & -15 \\ 20 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -16 - 10 + 4 & -18 + 15 + 0 \\ 24 + 20 + 0 & -22 - 5 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} -22 & -3 \\ 44 & -23 \end{bmatrix}$$

A
 $A^2 - A - I =$

$$(A - I) =$$

$$\textcircled{*} \quad (A^T)^{-1} = (A^{-1})^T$$

$$(A^T)(A^{-1})^T = I$$

$$\Rightarrow (A^{-1}A)^T = I$$

$$\Rightarrow I^T = I$$

$$\therefore I = I \quad \text{CP+AB} \rightarrow A\beta = (A) ?$$

Sample Question: $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$

verify that $(A^{-1})^T = (A^T)^{-1}$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 21 & 01 \\ 02 & 03 \end{bmatrix} = \begin{bmatrix} 21 & 01 \\ 02 & 03 \end{bmatrix}$$

\textcircled{*} A square matrix A is said to be idempotent if

$$A^2 = A.$$

$$\begin{bmatrix} 0+21+21 & 1+0+01 \\ 0+2-22 & 0+0+02 \end{bmatrix} =$$

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$$\begin{aligned} a) \quad (I-A)^2 &= (I-A)(I-A) \\ &= I^2 - IA - AI + A^2 \\ &= I - A - A + A \\ &= (I-A) \end{aligned}$$

$\therefore (I-A)$ is idempotent.

b)

$$\begin{aligned}
 (2A - I)(2A - I) &= 4A^2 - 2AI - 2IA + I^2 \\
 &= 4A^2 - 2A - 2A + I \\
 &= 4A - 4A + I \\
 &= I
 \end{aligned}$$

so, $(2A - I)^{-1} = (2A - I)$

(Proved)

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

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Elementary Matrix

Defination: An $n \times n$ matrix is called an elementary matrix if it can be obtained from the $n \times n$ identity matrix I_n by performing a single row operation.

$$\begin{bmatrix} 0 & 0 & 1 & | & 0 & 0 & 1 \\ 0 & 1 & 0 & | & 0 & 1 & 0 \\ 1 & 0 & 0 & | & 0 & 0 & 1 \end{bmatrix}$$

Q) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \text{an elementary matrix}$

$$\left\{ \begin{array}{l} R'_3 = R_2 + 2R_1 \\ I = \end{array} \right.$$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \xrightarrow{\text{row } 3 \rightarrow R'_3 = R_2 + 2R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{an elementary matrix.}$

Sample Question:

Q) Find out the operation or an elementary

$[A | I] \rightarrow [I | A^{-1}]$



$$\left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 5 & 7 & 2 & 0 & 1 & 0 \\ 8 & 1 & 2 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 5 & 0 & 2 & 1 \\ 2 & 2 & 2 & 0 & 1 & 0 \\ 1 & 0 & 3 & 1 & 0 & 0 \end{array} \right]$$

④ Find A^{-1}

$$A = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 0 & 1 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 1 & 0 & 0 \end{array} \right]$$

$$[A | I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array} \right]$$

$$\pi'_2 = \pi_2 - 2\pi_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array} \right]$$

$$\pi'_3 = \pi_3 + 2\pi_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & +2 & 1 \end{array} \right]$$

$$\pi'_1 = -\pi_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array} \right]$$

$$\begin{array}{l} R_1' = R_1 - 3R_3 \\ \hline R_2' = R_2 + 3R_3 \end{array} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 6 & 3 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array} \right]$$

$$R_1' = R_1 - 2R_2 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array} \right] = A$$

$$= \left[I \mid A^{-1} \right] \quad \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 8 & 8 & 1 \\ 0 & 1 & 0 & 8 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right] = [E \mid A]$$

$$A^{-1} = \left[\begin{array}{ccc} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 8 & 8 & 1 \\ 0 & 1 & 0 & 8 & 0 & 0 \\ 1 & 0 & -2 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 8 & 8 & 1 \\ 0 & 1 & 0 & 8 & 0 & 0 \\ 1 & 0 & -2 & 0 & 0 & 0 \end{array} \right]$$

Example - 5/

$$A = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{bmatrix}$$

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 2 & 4 & -1 & 0 & 1 & 0 \\ -1 & 2 & 5 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{aligned} R_2' &= R_2 - 2R_1 \\ R_3' &= R_3 + R_1 \end{aligned} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & -8 & -9 & -2 & 1 & 0 \\ 0 & 8 & 9 & 1 & 0 & 1 \end{array} \right] = A$$

$$R_2' = -\frac{1}{8}R_2 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & 1 & \frac{9}{8} & \frac{1}{4} & -\frac{1}{8} & 0 \\ 0 & 8 & 9 & 1 & 0 & 1 \end{array} \right]$$

$$R_3' = R_3 - 8R_2 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & 1 & \frac{9}{8} & \frac{1}{4} & -\frac{1}{8} & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{array} \right]$$

A is not ~~inter~~ invertible.

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(*)

$$x_1 + 2x_2 + 3x_3 = 5$$

$$2x_1 + 5x_2 + 3x_3 = 3$$

$$x_1 + 8x_2 = 17$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 2 & 5 & 3 & 3 \\ 1 & 8 & 0 & 17 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & -3 & -7 \\ 0 & 6 & -3 & 12 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & -3 & -7 \\ 0 & 0 & 15 & 30 \end{array} \right] = [I | A]$$

Hence,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 8 & 0 \end{bmatrix} \rightarrow \text{Coefficient matrix.}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix}$$

Logic!

$$AX = B$$

$$A^{-1}(AX) = A^{-1}B$$

$$X = A^{-1}B$$

Q

$$\therefore A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 8 \\ 0 & 0 & 15 \\ 0 & 0 & 30 \end{bmatrix}$$

$$= \begin{bmatrix} -200 + 48 + 153 \\ 65 - 15 - 51 \\ 25 - 6 - 17 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 8 \\ 0 & 0 & 15 \\ 0 & 0 & 30 \end{bmatrix}$$

Ans

1.7

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = {}^T A$$

Diagonal Matrix

$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is a diagonal matrix.

(*) $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$\det(A) = 2 \cdot (-3) \cdot (-1) = 6 \neq 0$

$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$

$$\textcircled{1} \quad D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$D^2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{bmatrix} = A$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

$$\textcircled{2} \quad A^4 = \begin{bmatrix} 16 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 81 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

Triangular Matrix

$$A = \begin{bmatrix} 16 & 0 & 0 \\ 5 & 1 & 0 \\ 2 & -3 & 81 \end{bmatrix}$$

$$\det(A) = 16 \cdot 1 \cdot 81 = 1296$$

Symmetric Matrix

A square matrix A is symmetric if,

$$A = A^T$$

$$\begin{bmatrix} 2 & -3 & 4 \\ -3 & 7 & -2 \\ 4 & -2 & -1 \end{bmatrix}$$

Here,

$$a_{12} = a_{21}$$

$$a_{ij} = a_{ji}$$

$$a_{13} = a_{31}$$

$$a_{23} = a_{32}$$

⑧

$$\begin{bmatrix} -5 & 6 & 3 \\ 6 & 1 & a \\ 3 & a & -3 \end{bmatrix}$$

$a = \text{any real number}$

$$(A^{-1})^T = (A^T)^{-1}$$

