NORTH SOUTH UNIVERSITY

Department of Electrical and Computer Engineering

Assignment – 05

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Course Title	: Data Structures and Algorithm
Section	: 06
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General Case of a Recursion

```
{
    if (base case)
        solve it;
    else
        reduce the problem using recursion;
}
```

Now we need to find the base case and the recursive call order.

$^{n}C_{r}$ = n choose r, is a formula to represent the number of ways to choose r items from a set of n items.

$$= \frac{n!}{(n-r)!\, r!}$$

Now, let's consider the selection process in two cases:

1. The first item is selected: In this case, we choose one item from the set of n items, leaving us with (r-1) items to choose from the remaining (n-1) items. We can select these (r-1) items in $^{(n-1)}C_{(r-1)}$ ways.

2. The first item is not selected: In this case, we do not choose the first item, leaving us with r items to choose from the remaining $(n-1)$ items. We can select these r items in ${}^{(n-1)}C_r$ ways.

By considering these two cases, we can express ${}^nC_r$ recursively as:

$$ {}^nC_r = {}^{(n-1)}C_{(r-1)} + {}^{(n-1)}C_r $$

Let's prove it,

$$ {}^{(n-1)}C_{(r-1)} + {}^{(n-1)}C_r = \frac{(n-1)!}{(r-1)!\,(n-1-r+1)!} + \frac{(n-1)!}{r!\,(n-1-r)!} $$

$$ = \frac{(n-1)!}{(r-1)!\,(n-r)!} + \frac{(n-1)!}{r!\,(n-r-1)!} $$

$$ = \frac{(n-1)!}{(r-1)!\,(n-r)(n-r-1)!} + \frac{(n-1)!}{r(r-1)!(n-r-1)!} $$

$$ = \frac{r(n-1)! + (n-r)(n-1)!}{r(r-1)!\,(n-r)\,(n-r-1)!} $$

$$= \frac{(n-1)!\ (r+n-r)}{r!\ (n-r)!}$$

$$= \frac{n\,(n-1)!}{r!\ (n-r)!}$$

$$= \frac{n!}{r!\ (n-r)!}$$

$$= {}^{n}C_{r}$$

(Proved)

Now, let's find the base case:

We know that,

$${}^{n}C_{0} = 1$$

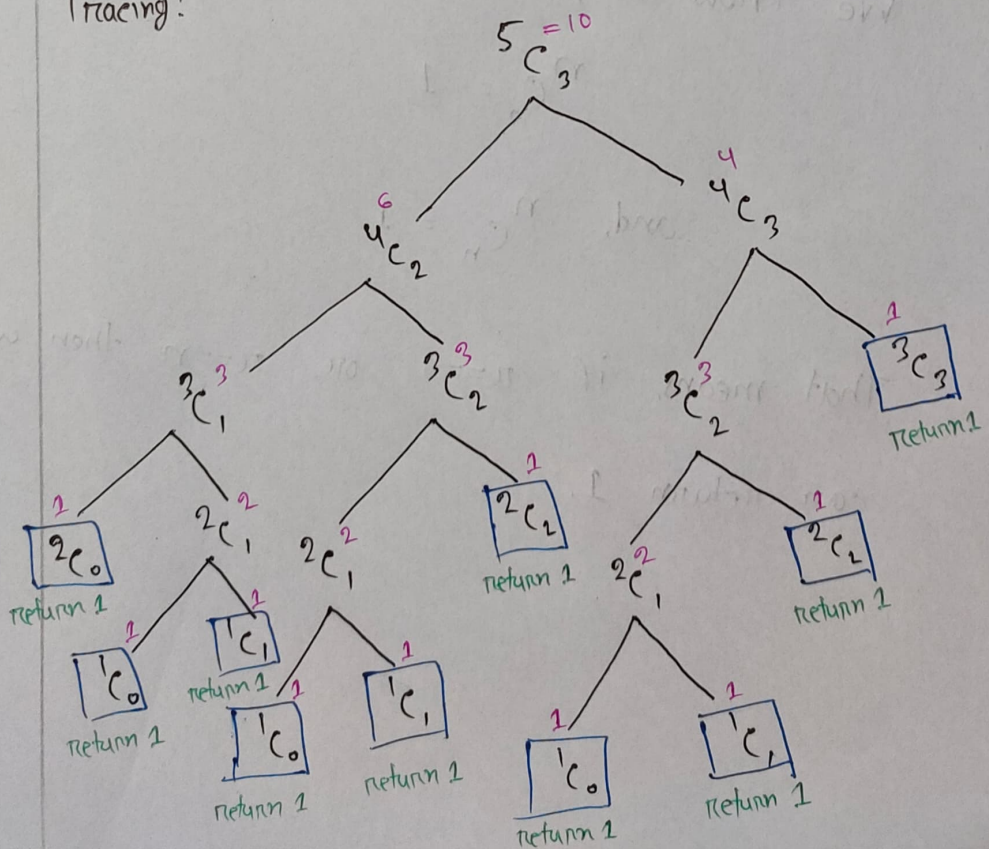
and, $${}^{n}C_{n} = 1.$$

That means, if $r == 0$ or, $r == n$ then we can return 1.

Hence, the recursive function is,

```
int nCr(int n, int r){
    if(r==0 || r==n){
        return 1;
    }
    else{
        return nCr(n-1, r-1) + nCr(n-1, r);
    }
}
```

Tracing:



Sources: Deep search on Google, Long time conversation with chatGPT, Finally I learn a bit. Then I rewrite in my own word.