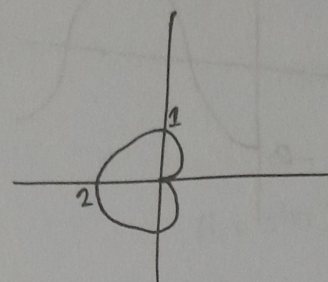
$r = a \pm b \cos\theta$

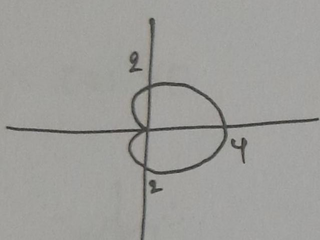
$r = a \pm b \sin\theta$

if $a = b$, then it will be heart shape.

$r = 1 - 1 \cos\theta$

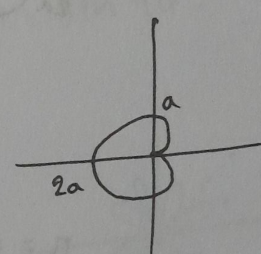


$r = 2 + 2 \cos\theta$

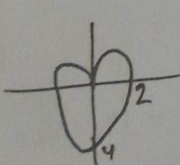


$r = a(1 - \cos\theta)$

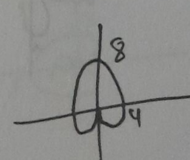
$= a - a\cos\theta$



$r = 2 - 2 \sin\theta$

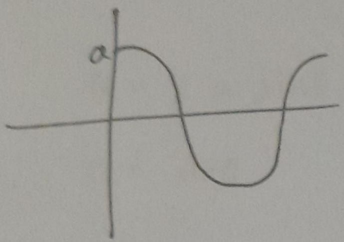


$r = 4 + 4 \sin\theta$

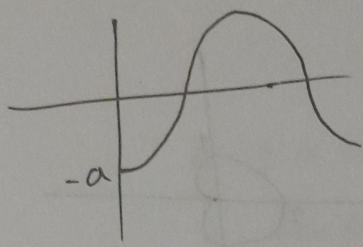


$r = \cos\theta$

$r = a\cos\theta$

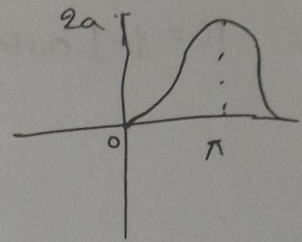


$r = -a\cos\theta$



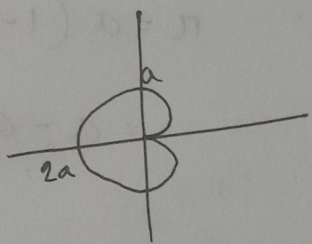
$r = -a\cos\theta + a$

$= a - a\cos\theta$

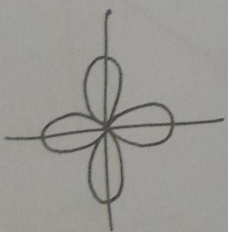


up to π its increasing

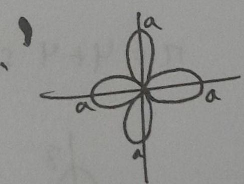
$r = a - a\cos\theta$



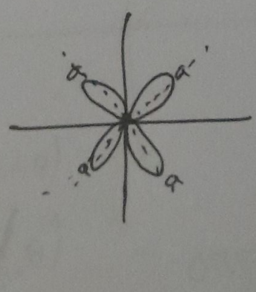
$r = \cos 2\theta$



$r = a\cos 2\theta$



$r = \sin 2\theta$



size

$r = a \sin n\theta$

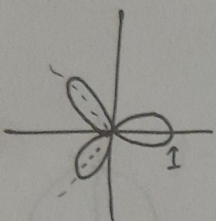
size

$r = a \cos n\theta$

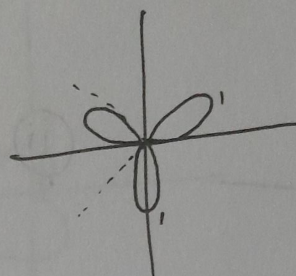
n → odd ⇒ n pattern

n → even ⇒ $2n$ pattern

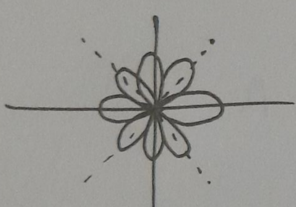
$r = \cos 3\theta$



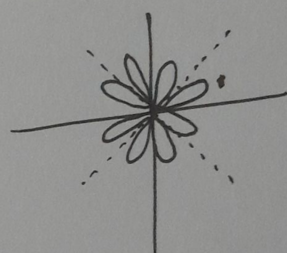
$r = \sin 3\theta$



$r = \cos 4\theta$



$r = \sin 4\theta$

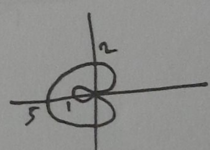


(i) $r = 2 - 3\cos\theta$

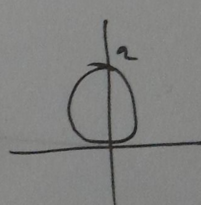
(ii) $r = 2\sin\theta$

(iii) $r = 3\cos 3\theta$

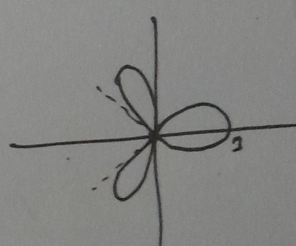
(i)



(ii)



(iii)



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⊛ Draw the graph:

i. $r = 2\sin\theta$

ii. $r = 2\cos 2\theta$

(iii) $r = 2$

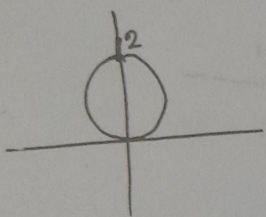
(iv) $r = 2 - 2\sin\theta$

(v) $r = 2\sin 3\theta$

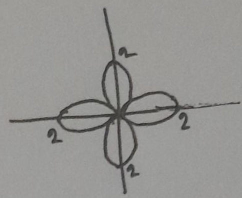
(vi) $r = 5 + 6\cos\theta$

⇒

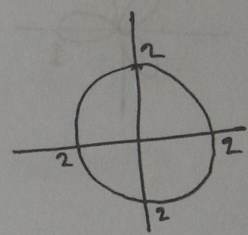
(i)



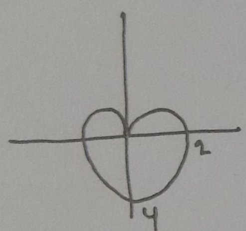
(ii)



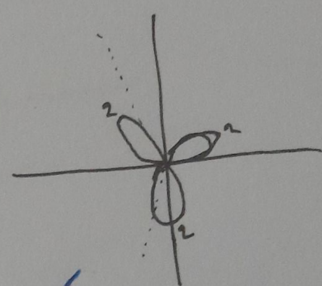
(iii)



(iv)



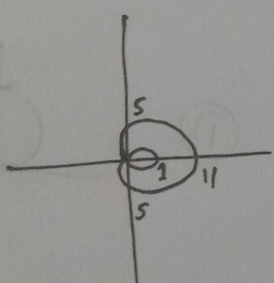
(v)



3 → Down   7 = Down

5 → Up   9 = Up

(vi)

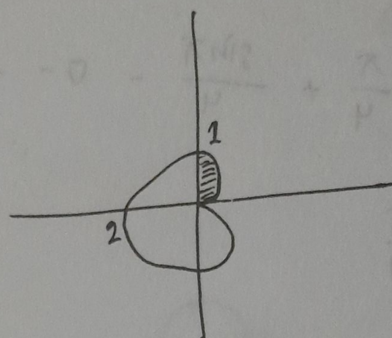


10.3

Area under the polar graph

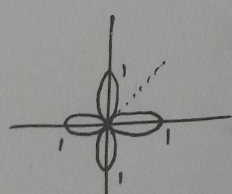
$$\text{area}, = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$$

⊛ $r = 1 - \cos\theta$



$$A = \frac{1}{2} \int_{0}^{\frac{\pi}{2}} (1-\cos\theta)^2 \, d\theta$$

⊛ $r = \cos 2\theta$



$$A = 8 \cdot \frac{1}{2} \int_{0}^{\pi/4} (\cos 2\theta)^2 \, d\theta$$

$$= 4 \int_{0}^{\pi/4} \cos^2 2\theta \, d\theta$$

$$= 4 \int_0^{\pi/4} \frac{1}{2} (1+\cos 4\theta)\, d\theta$$

$$= 2 \int_0^{\pi/4} (1+\cos 4\theta)\, d\theta$$

$$= 2 \left[\theta + \frac{\sin 4\theta}{4} \right]_0^{\pi/4}$$

$$= 2 \left[\frac{\pi}{4} + \frac{\sin \pi}{4} - 0 - \frac{\sin 0}{4} \right]$$

$$= 2 \cdot \frac{\pi}{4}$$

$$= \frac{\pi}{2} \quad \underline{\text{Ans}}$$

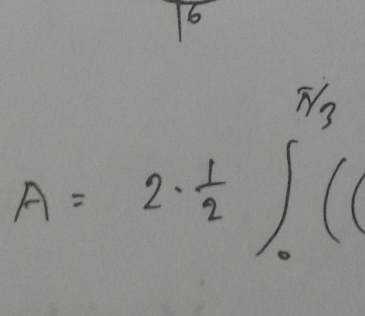
⊛ $r = 6$ $\qquad r = 4 + 4\cos\theta$

$$4 + 4\cos\theta = 6$$

$$4\cos\theta = 2$$

$$\cos\theta = \frac{1}{2}$$

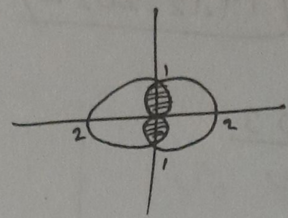
$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$



$$A = 2 \cdot \frac{1}{2} \int_0^{\pi/3} \left((4+4\cos\theta)^2 - 6^2 \right) d\theta$$

$\circledast$ $r = 1 - \cos\theta$

$r = 1 + \cos\theta$



$1 - \cos\theta = 1 + \cos\theta$

$2\cos\theta = 0$

$\cos\theta = 0$

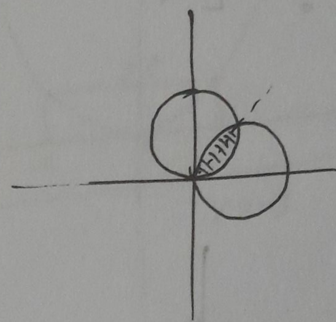
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$$A = 4 \cdot \frac{1}{2} \int_0^{\pi/2} (1-\cos\theta)^2 \, d\theta$$

$\circledast$

$r = 9\cos\theta$

$r = 9\sin\theta$



$9\cos\theta = 9\sin\theta$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$=$

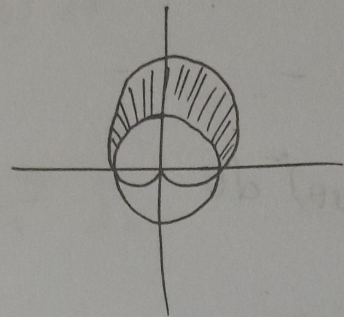
$$A = 2 \cdot \frac{1}{2} \int_0^{\pi/4} 81 \sin^2\theta \, d\theta$$

(L-21 / 14.12.2022 /

10·3

⊛ $r = 1 + \sin\theta$

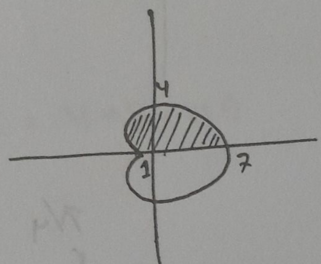
$r = 1$



$\therefore$

$$A = 2 \cdot \frac{1}{2} \int_{0}^{\pi/2} \left[(1+\sin\theta)^2 - 1^2 \right] d\theta$$

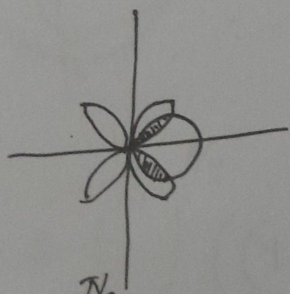
⊛ $r = 4 + 3\cos\theta$



$$A = \frac{1}{2} \int_{0}^{\pi} (4 + 3\cos\theta)^2 \, d\theta$$

⊛ $r = \sin 2\theta$

$r = \cos\theta$



$$\therefore A = 2 \left[\frac{1}{2} \int_{0}^{\pi/6} (\sin 2\theta)^2 d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} (\cos\theta)^2 d\theta \right.$$

$\sin 2\theta = \cos\theta$

$2\sin\theta\cos\theta = \cos\theta$

$2\sin\theta = 1$

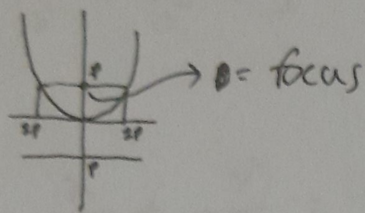
$\sin\theta = \frac{1}{2}$

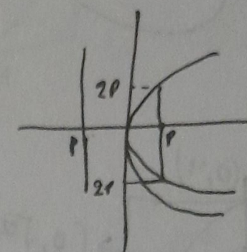
$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{6}, \frac{\pi}{2}$

10.4

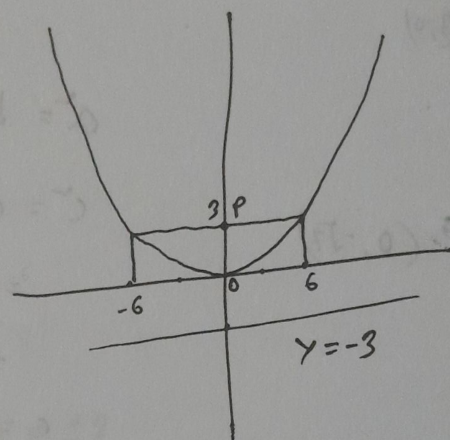
$x^2 = 4py$ | $y^2 = 4px$





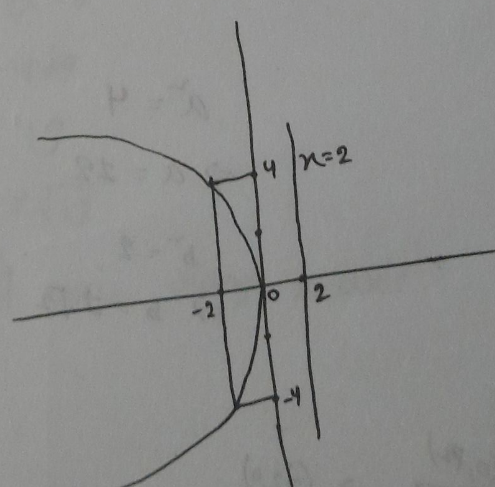
⊛ $x^2 = 12y$

$= 4 \cdot 3y$



⊛ $y^2 = -8x$

$= 4(-2)x$



⊛ Symetric about y-axis
passes through (5,2)
→ Find the equation:

$x^2 = 4py$

$4p = \frac{x^2}{y} = \frac{25}{2}$

$\boxed{\therefore x^2 = \frac{25}{2}y}$

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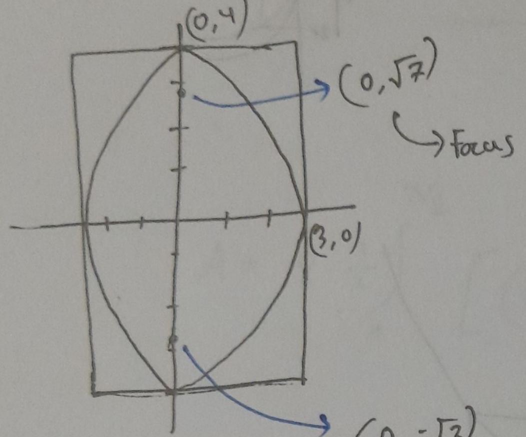
$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$ → major axis along the y-axis.

$a^2 = 16$

$\Rightarrow a = \pm 4$

$b^2 = 9$

$\Rightarrow b = \pm 3$



$a^2 = b^2 + c^2$

$c^2 = a^2 - b^2$

$= 16 - 9$

$= 7$

$c = \pm\sqrt{7}$

$$x^2 + 2y^2 = 4$$

$x^2 + 2y^2 = 4$

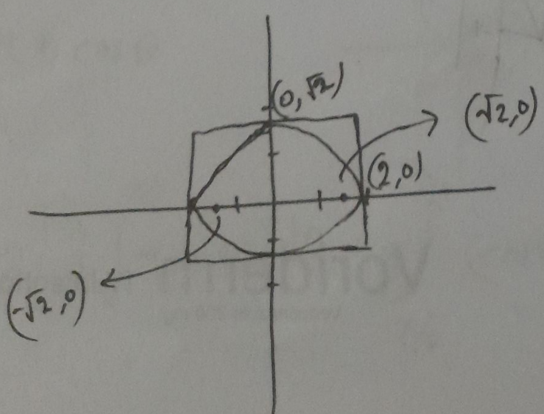
$\frac{x^2}{4} + \frac{y^2}{2} = 1$

$a^2 = 4$

$\Rightarrow a = \pm 2$

$b^2 = 2$

$\Rightarrow b = \pm\sqrt{2}$



$a^2 = b^2 + c^2$

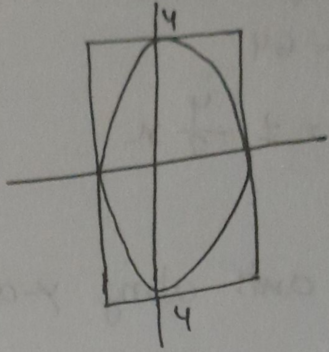
$c^2 = a^2 - b^2$

$= 4 - 2$

$= 2$

$c = \pm\sqrt{2}$

⊛ Focus $(0, \pm 2)$



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{12} + \frac{y^2}{16} = 1$$

$a = \pm 4$

$a^2 = 16$

$c = \pm 2$

$c^2 = 4$

$4 = a^2 - b^2$

$= 16 - b^2$

$b^2 = 16 - 4$

$b^2 = 12$

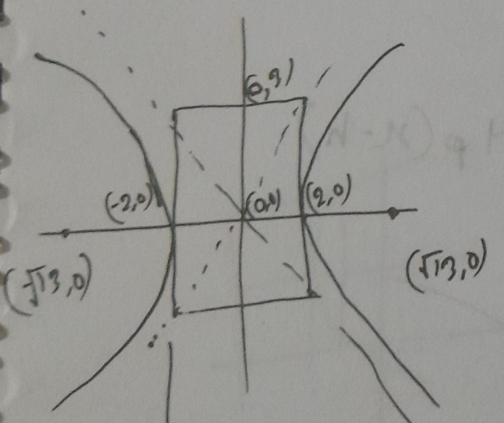
b

⊛

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

$a = \pm 2$

$b = \pm 3$



$c^2 = a^2 + b^2$

$= 4 + 9$

$= 13$

$c = \pm\sqrt{13}$

focal axis along x-axis

asymtote

$$y = \pm \frac{3}{2}x$$

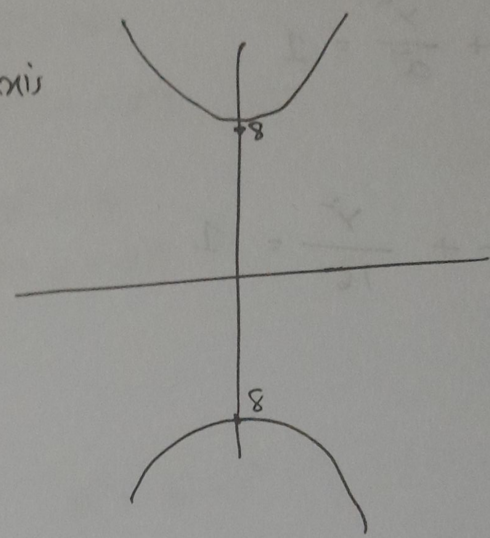
$(0, \pm 8)$

$a = 8$

$a^2 = 64$

$y = \pm \frac{4}{3} x$

→ focal axis along y-axis



$$\frac{y^2}{64} - \frac{x^2}{36} = 1$$

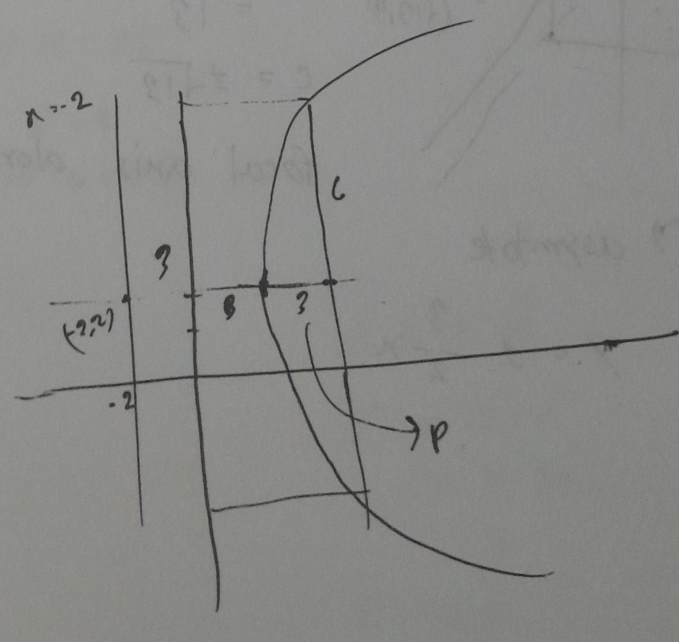
$x^2 = 4py$ $(0,0)$ $y^2 = 4px$

(h,k)

$(x-h)^2 = 4p(y+k)$ $(y-k)^2 = 4p(x-h)$

$(h,k) = (1,2)$

focus $= (4,2)$



$(y-k)^2 = 4p(x-h)$

$(y-2)^2 = 12(x-1)$

$y^2 - 8x - 6y - 23 = 0$

$y^2 - 6y = 8x + 23$

$y^2 - 2\cdot y\cdot 3 + 3^2 = 8x + 23 + 3^2 = 8x + 32$

$(y-3)^2 = 8(x+4)$

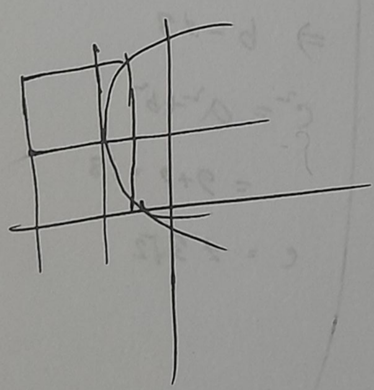
$4p = 8$

$\therefore p = 2$

$(h,k) = (-4, 3)$

focus $= (-4+2, 3)$

$= (-2, 3)$



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⊛ Describe the Graph

⊛ $x^2-y^2-4x+8y-21=0$

$1\ (x^2-4x) - 1\ (y^2-8y) = 21$

$1\ (x^2-2x\cdot 2+2^2) - 1\ (y^2-2y\cdot 4+4^2) = 21 + 1\cdot 4 - 1\cdot 16$

$1\ (x-2)^2 - 1\ (y-4)^2 = 9$

$$\frac{(x-2)^2}{9} - \frac{(y-4)^2}{9} = 1$$

$\Rightarrow$ 1. Conic: Hyperbola

2. $(h,k) = (2,4)$

3. Focal axis: along - x-axis

4. $a = \pm 3$

$b = \pm 3$

$c = \pm 3\sqrt{2}$

5. focus:

6. Asymptote

$a^2 = 9$

$\Rightarrow a = \pm 3$

$b^2 = 9$

$\Rightarrow b = \pm 3$

$c^2 = a^2 + b^2$

$= 9+9 = 18$

$c = \pm 3\sqrt{2}$

⊛

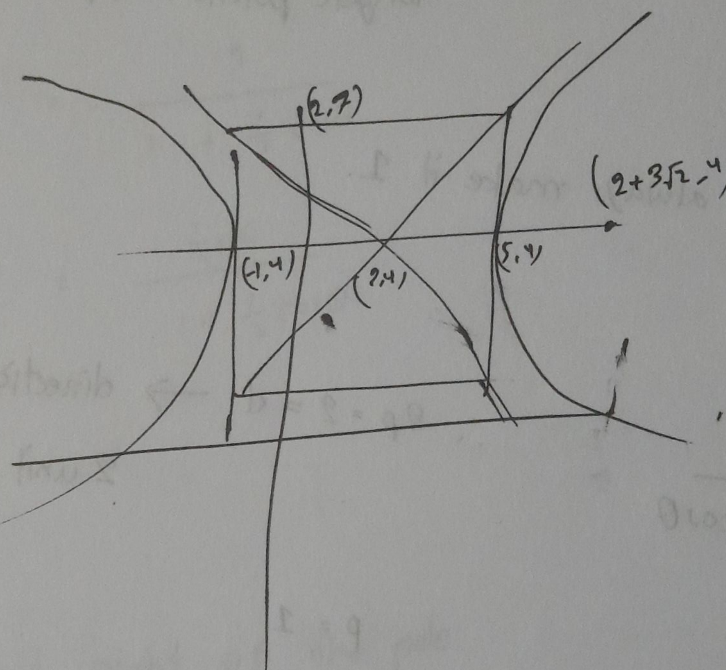
⊛ Assymptote:

$$\frac{(y-4)^2}{9} = \frac{(x-2)^2}{9}$$

$$(y-4) = \pm(x-2)$$

$$y = \pm(x-2)+4$$

Graph:



10.6

$$r = \frac{ed}{1 \pm e\cos\theta}$$

$$= \frac{ed}{1 \pm e\sin\theta}$$

$d \rightarrow$ 2p (Directics)

Equation of polar Condinates

focus in poll

longer portion of condoic = directics

$$r = \frac{2}{1-\cos\theta}$$

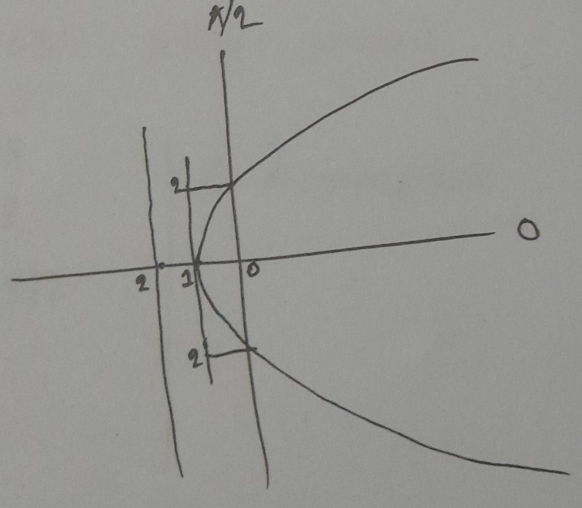
$1 \rightarrow$ always make it 1.

$$= \frac{1 \cdot 2}{1 - 1\cos\theta}$$

$\therefore 2p = 2 = d \rightarrow$ directics

2 unit Left of the polar

$p = 1$



⊛ Find a, b, c, d, e ?

Equation : $r = \dfrac{6}{2 + \cos\theta}$

$= \dfrac{6}{2(1 + \frac{1}{2}\cos\theta)}$

$= \dfrac{3}{1 + \frac{1}{2}\cos\theta}$

$= \dfrac{\frac{1}{2} \cdot 6}{1 + \frac{1}{2}\cos\theta}$

$\therefore e = \frac{1}{2}$ = elipse

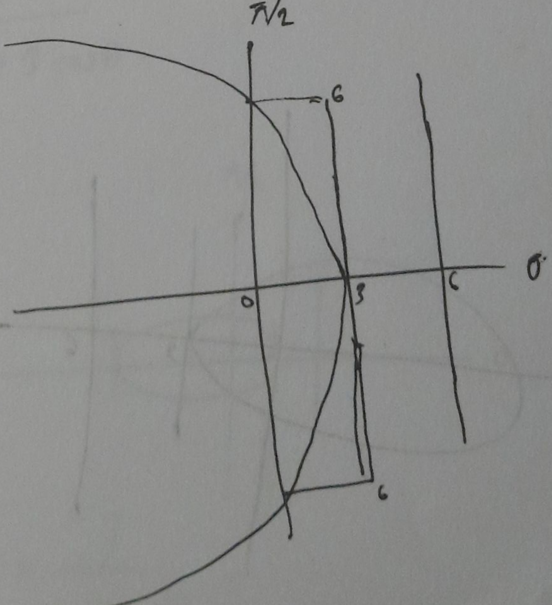
$d = 6$

directies : 6 unit right of the pole

$d = 6$

$2p = 6$

$p = 3$



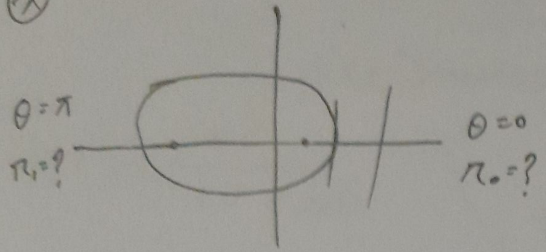
$\theta = \pi$

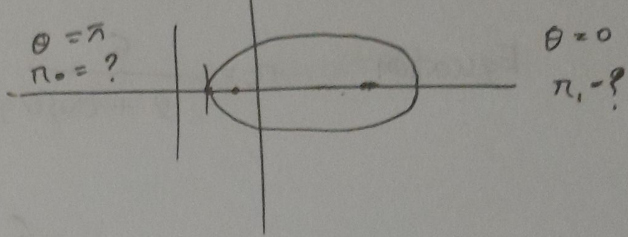
$r_1 = ?$

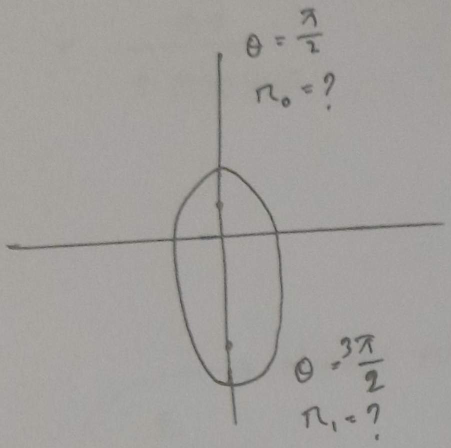
$\theta = 0$

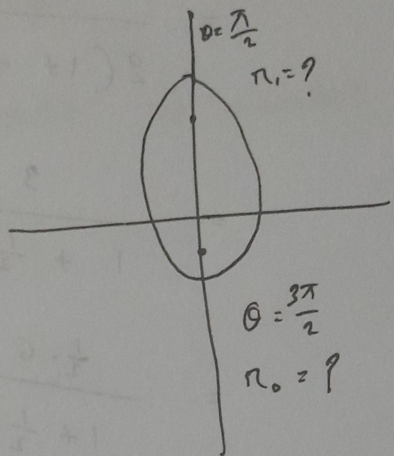
$r_0 = ?$

$a = \frac{1}{2}(r_1 + r_0) = \frac{1}{2}(6+2) = \frac{1}{2} \cdot 8 = 4$



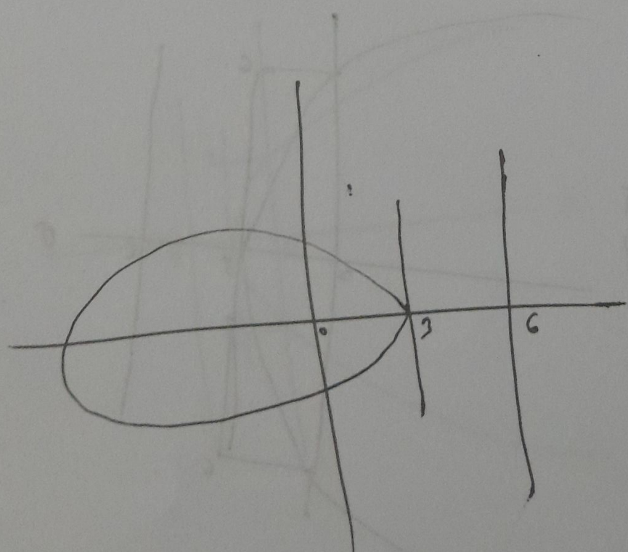






$$b = \sqrt{r_0 r_1} = \sqrt{2 \cdot 6} = \sqrt{12} = 2\sqrt{3}$$

$$c = \frac{1}{2}\left(r_1 - r_0\right) = \frac{1}{2}\left(6 - 2\right) = \frac{1}{2} \cdot 4 = 2$$



$$r = \frac{8}{1 - \sin\theta}$$

$e = \frac{3}{4}$; directrix, $x = 4$

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{16 + 0}$$

$$= \pm 4$$

$\therefore d = 4$

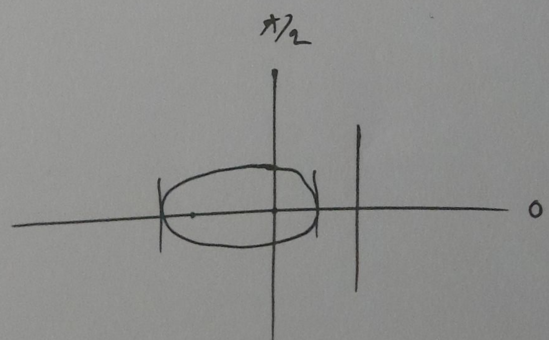
$$\therefore \frac{\frac{3}{4} \cdot 4}{1 + \frac{3}{4}\cos\theta} = \frac{3}{1 + \frac{3}{4}\cos\theta}$$

$$= \frac{12}{4 + 3\cos\theta}$$

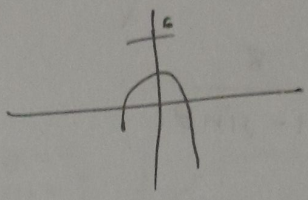
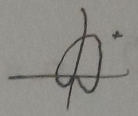
$x = 1$

$d = 1$

$e = 1$



$$\frac{1 \cdot 1}{1 + \cos\theta} = \frac{1}{1 + \cos\theta}$$

$e = 2$

$y = 6$

$d = 6$

$$\frac{2\cdot 6}{1+2\sin\theta} = \frac{12}{1+2\sin\theta}$$