# NORTH SOUTH UNIVERSITY

Department of Mathematics & Physics

Assignment – 01

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| Name | : Joy Kumar Ghosh |
| Student ID | : 2211424 6 42 |
| Course No. | : MAT 250 |
| Course Title | : Calculus and Analytic Geometry IV |
| Section | : 16 |
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13.2

1|

$$\lim_{(x,y)\to(1,3)} (4xy^2 - x)$$

$$= 4\cdot1\cdot3^2 - 1$$

$$= \cancel{48-1}\ 36-1$$

$$= 35$$

Ans

3|

$$\lim_{(x,y)\to(-1,2)} \frac{xy^3}{x+y}$$

$$= \frac{(-1)\cdot 2^3}{-1+2}$$

$$= -8$$

Ans.

5|

$$\lim_{(x,y)\to(0,0)} \ln(1+x^2y^3)$$

$$= \ln(1+0\cdot0)$$

$$= \ln(1)$$

$$= 0$$

Ans.

7)

$$\lim_{(x,y)\to(0,0)} \frac{3}{x^2+2y^2}$$

Along $x=0$:

$$\lim_{(x,y)\to(0,0)} \frac{3}{x^2+2y^2}\Bigg|_{x=0}$$

$$= \lim_{(x,y)\to(0,0)} \frac{3}{2y^2}$$ ; Does not exist.

Along $y=0$:

$$\lim_{(x,y)\to(0,0)} \frac{3}{x^2+2y^2}\Bigg|_{y=0}$$

$$= \lim_{(x,y)\to(0,0)} \frac{3}{x^2}$$ ; Does not exist

$\therefore \lim_{(x,y)\to(0,0)} \frac{3}{x^2+2y^2}$ does not exist.

9)

~~Given~~

Let, $z = x^2+y^2$

$$\therefore \lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = \lim_{z\to 0^+} \frac{\sin z}{z}$$

$$= 1$$

Ans

10)

Let,

$z = x^2+y^2$

$$\therefore \lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2} = \lim_{z\to 0^+} \frac{1-\cos z}{z}$$

$$= \lim_{z\to 0^+} \frac{(1-\cos z)(1+\cos z)}{z(1+\cos z)}$$

$$= \lim_{z\to 0^+} \frac{1-\cos^2 z}{z(1+\cos z)}$$

$$= \lim_{z\to 0^+} \frac{\sin^2 z}{z(1+\cos z)}$$

$$= \lim_{z\to 0^+} \frac{\sin z}{z} \cdot \lim_{z\to 0^+} \frac{\sin z}{1+\cos z}$$

$$= 1 \cdot \frac{0}{1+1}$$

$$= 0$$

Ans

11)

Let,

$$z = x^2+y^2$$

$$\therefore \lim_{(x,y)\to(0,0)} e^{-\frac{1}{x^2+y^2}} = \lim_{z\to 0^+} e^{-\frac{1}{z}}$$

$$= 0$$

Ans.

12)

Let,

$$z = x^2+y^2$$

$$\therefore \lim_{(x,y)\to(0,0)} \frac{e^{-\frac{1}{\sqrt{x^2+y^2}}}}{\sqrt{x^2+y^2}} = \lim_{z\to 0^+} \frac{e^{-\frac{1}{\sqrt{z}}}}{\sqrt{z}}$$

$$= \lim_{z\to 0^+} \frac{1}{\sqrt{z}\cdot e^{\frac{1}{\sqrt{z}}}}$$

Let,

$$\frac{1}{\sqrt{z}} = w \quad \left| \begin{array}{l} z\to 0^+ \\ w\to \infty \end{array} \right.$$

$$\therefore \lim_{z\to 0^+} \frac{1}{\sqrt{z}}\cdot\frac{1}{e^{\frac{1}{\sqrt{z}}}} = \lim_{w\to\infty} \frac{w}{e^w} \quad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \lim_{w\to\infty} \frac{1}{e^w} \quad [\text{L.H.}]$$

$$= 0$$

A

13|

$$\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)(x^2-y^2)}{(x^2+y^2)}$$

$$= \lim_{(x,y)\to(0,0)} (x^2-y^2)$$

$$= 0-0$$

$$= 0$$

Ans.

15|

$$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$$

Along $x=0$:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}\Bigg|_{x=0}$$

$$= \lim_{(x,y)\to(0,0)} \frac{0}{2y^2}$$

$$= 0$$

Along $y=0$:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}\Bigg|_{y=0}$$

$$= \lim_{(x,y)\to(0,0)} \frac{0}{3x^2} = 0$$

Along $y = x$:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}\Bigg|_{y=x}$$

$$= \lim_{(x,y)\to(0,0)} \frac{x^2}{3x^2+2x^2}$$

$$= \lim_{x\to 0} \frac{x^2}{5x^2}$$

$$= \frac{1}{5}$$

$\therefore$ So, Limit does not exist.

23)

Let,

$(r,\theta)$ be polar coordinates of the point $(x,y)$. Then,

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r^2 = x^2+y^2$$

Then,

$$\lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2}\,\ln(x^2+y^2) = \lim_{r\to 0^+} r\ln r^2$$

$$= \lim_{r\to 0^+} \frac{2\ln r}{\frac{1}{r}} \quad \left[\frac{\infty}{\infty} \text{ Form}\right]$$

$$= \lim_{r\to 0^+} \frac{2\cdot\frac{1}{r}}{-\frac{1}{r^2}}$$

$$= \lim_{r \to 0^+} \frac{2}{r} \cdot (-r^2)$$

$$= \lim_{r \to 0^+} (-2r)$$

$$= 0$$

Ans

24/

Let,

$(r, \theta)$ be polar coordinates of the point $(x, y)$. Then,

$$x = r \cos\theta$$

$$y = r \sin\theta$$

$$r^2 = x^2 + y^2$$

$$\therefore \lim_{(x,y) \to (0,0)} y \ln(x^2+y^2) = \lim_{r \to 0^+} r \sin\theta \ln r^2$$

$$= \lim_{r \to 0^+} r \sin\theta \cdot 2 \ln r$$

$$= \lim_{r \to 0^+} 2r(\ln r) \sin\theta$$

$$= 0$$

Ans

25|

Let,

$(r, \theta)$ be polar coordinates of $(x, y)$. Then,

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r^2 = x^2 + y^2$$

$$\therefore \lim_{(x,y)\to(0,0)} \frac{x^2y^2}{\sqrt{x^2+y^2}} = \lim_{r\to 0^+} \frac{r^2\cos^2\theta \cdot r^2\sin^2\theta}{r}$$

$$= \lim_{r\to 0^+} \frac{r^4 \cdot \cos^2\theta \cdot \sin^2\theta}{r}$$

$$= \lim_{r\to 0^+} r^3 \cdot \cos^2\theta \cdot \sin^2\theta$$

$$= 0$$

Ans

26|

Let,

$(r, \theta)$ be polar coordinates of the point $(x, y)$. Then,

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r^2 = x^2 + y^2$$

$$\therefore \lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+2y^2}} = \lim_{r\to 0^+} \frac{r\cos\theta \cdot r\sin\theta}{\sqrt{(r\cos\theta)^2 + 2(r\sin\theta)^2}}$$

$$= \lim_{r\to 0^+} \frac{r^2\cos\theta \cdot \sin\theta}{\sqrt{r^2\cos^2\theta + 2r^2\sin^2\theta}}$$

$$= \lim_{r\to 0^+} \frac{r^2\cos\theta \cdot \sin\theta}{\sqrt{r^2(\cos^2\theta + 2\sin^2\theta)}}$$

$$= \lim_{r\to 0^+} \frac{r^2\cos\theta \sin\theta}{\sqrt{r^2(\cos^2\theta + \sin^2\theta + \sin^2\theta)}}$$

$$= \lim_{r\to 0^+} \frac{r^2\cos\theta \cdot \sin\theta}{\sqrt{r^2(1+\sin^2\theta)}} \le \frac{r^2}{\sqrt{r^2}}$$

$$= \lim_{r\to 0^+} r$$

$$= 0$$

Ans