$$= \frac{e^{-x}}{24}\left(x - \frac{0-0-14}{24}\right)$$

$$= \frac{e^{-x}}{24}\left(x + \frac{7}{12}\right)$$

$$\therefore y = y_c + y_p = c_1 e^{-5x} + c_2 e^{x} + c_3 e^{2x} + \frac{e^{-x}}{24}\left(x + \frac{7}{12}\right)$$

Ans

8)

$$y''' + 4y'' + 3y' = x^2 \cos x - 3x$$

$$\Rightarrow (D^3 + 4D^2 + 3D)y = x^2 \cos x - 3x \quad \cdots (i)$$

A.E. $\Rightarrow$

$$m^3 + 4m^2 + 3m = 0$$

$$\Rightarrow m(m^2 + 4m + 3) = 0$$

$m = 0$	$m^2 + 4m + 3 = 0$ $m^2 + 3m + m + 3 = 0$ $m(m+3) + 1(m+3) = 0$ $(m+3)(m+1) = 0$	$\therefore m = 0, -1, -3$

$$\therefore y_c = c_1 + c_2 e^{-3x} + c_3 e^{-x}$$

$$(i) \Rightarrow y_p = \frac{1}{D^3 + 4D^2 + 3D} x^2 \cos x - \frac{1}{D^3 + 4D^2 + 3D} 3x$$

$$= \text{R.P. of } \frac{1}{D^3 + 4D^2 + 3D} x^2 e^{ix} - \frac{1}{3D}\left(1 + \frac{D^2}{3} + \frac{4D}{3}\right)^{-1} 3x$$

$$= \text{R.P of } e^{ix} \frac{1}{(D+i)^3+4(D+i)^2+3(D+i)} x^2 - \frac{1}{3D}\left(1-\frac{D^2}{3}+\frac{4D}{3}\right)^{-1} 3x$$

$$= \text{R.P. of } e^{ix} \frac{1}{D^3+3iD^2-3D-i+4D^2+8iD-4+3D+3i} x^2$$

$$- \frac{1}{3D}\left(3x - 0 - \frac{12}{3}\right)$$

$$= \text{R.P. of } e^{ix} \frac{1}{D^3+3iD^2+4D^2+8iD+2i-4} x^2 - \frac{1}{3}\left(\frac{3x^2}{2}-4x\right)$$

$$= \text{R.P. of } e^{ix} \frac{1}{D^3+D^2(3i+4)+8iD+(2i-4)} x^2 - \frac{1}{3}\left(\frac{3x^2}{2}-4x\right)$$

$$= \text{R.P. of } e^{ix} \frac{1}{2i-4}\left(1+\frac{D^3+D^2(3i+4)+8iD}{2i-4}\right)^{-1} x^2 - \frac{1}{3}\left(\frac{3x^2}{2}-4x\right)$$

$$= \text{R.P. of } \frac{e^{ix}}{2i-4}\left(1-\frac{D^3+D^2(3i+4)+8iD}{2i-4}+\left(\frac{D^3+D^2(3i+4)+8iD}{2i-4}\right)^2-\cdots\right)x^2$$

$$-\frac{1}{3}\left(\frac{3x^2}{2}-4x\right)$$

$$= \text{R.P. of } \frac{e^{ix}}{2i-4}\left(x^2-\frac{2(3i+4)+8i\cdot 2x}{2i-4}+\frac{-64\cdot 2x}{(2i-4)^2}\right)-\frac{1}{3}\left(\frac{3x^2}{2}-4x\right)$$

$$= \text{R.P. of } \frac{1}{2i-4}(\cos x+i\sin x)\left(x^2-\frac{6i+8+16xi}{2i-4}+\frac{-128x}{-4-16i+16}\right)-\frac{1}{3}\left(\frac{3x^2}{2}-4x\right)$$

$$= \;''$$

$$(D^2+1)y = x\cos x - \cos x$$

$$y_1 = \frac{1}{D^2+1}\, x\cos x$$

$$= x\,\frac{1}{D^2+1}\cos x - 2D\left[\frac{1}{D^2+1}\right]^2 \cos x$$

$$\cancel{= x\,\frac{x}{2}\left\{\cos x - \frac{2D}{D^4+2D^2+1}\cos x\right.}$$

$$\cancel{= \frac{x^2}{2}\sin x}$$

$$= \frac{x^2}{2}\sin x - \frac{2D}{D^2+1}\,\frac{1}{D^2+1}\cos x$$

$$= \frac{x^2}{2}\sin x - \frac{2D}{D^2+1}\,\frac{x}{2}\sin x$$

$$= \frac{x^2}{2}\sin x - \frac{1}{D^2+1}D(x\sin x)$$

$$= \frac{x^2}{2}\sin x - \frac{1}{D^2+1}(x\cos x + \sin x)$$

$$= \frac{x^2}{2}\sin x - \frac{1}{D^2+1}x\cos x - \frac{1}{D^2+1}\sin x$$

$$= \frac{x^2}{2}\sin x - \frac{1}{D^2+1}x\cos x + \frac{x}{2}\cos x$$

$$= \frac{x^2}{2}\sin x - y_1 + \frac{x}{2}\cos x$$

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$$y'' + y' + \frac{1}{4}y = e^{x}(\sin 3x - \cos 3x)$$

$$\Rightarrow \left(D^2 + D + \frac{1}{4}\right)y = e^{x}(\sin 3x - \cos 3x) \quad \ldots (i)$$

A.E. $\Rightarrow$

$$m^2 + m + \frac{1}{4} = 0$$

$$\Rightarrow \left(m + \frac{1}{2}\right)^2 = 0$$

$$m = -\frac{1}{2}, -\frac{1}{2}$$

$$\therefore y_c = c_1 e^{-\frac{1}{2}x} + c_2 x e^{-\frac{1}{2}x}$$

$$\therefore y_p = \frac{1}{D^2 + D + \frac{1}{4}} e^{x}(\sin 3x - \cos 3x)$$

$$= e^{x} \frac{1}{(D+1)^2 + D + 1 + \frac{1}{4}} (\sin 3x - \cos 3x)$$

$$= e^{x} \frac{1}{D^2 + 2D + 1 + D + 1 + \frac{1}{4}} (\sin 3x - \cos 3x)$$

$$= e^{x}\left[\frac{1}{D^2 + 3D + \frac{9}{4}} \sin 3x - \frac{1}{D^2 + 3D + \frac{9}{4}} \cos 3x\right]$$

$$= e^{x}\left[\frac{1}{-9 + 3D + \frac{9}{4}} \sin 3x - \frac{1}{-9 + 3D + \frac{9}{4}} \cos 3x\right]$$

$$= e^{x}\left[\frac{1}{3D - \frac{27}{4}} \sin 3x - \frac{1}{3D - \frac{27}{4}} \cos 3x\right]$$

$$= e^x \left[\frac{3D + \frac{27}{4}}{9D^2 - \frac{729}{16}} \sin 3x - \frac{3D + \frac{27}{4}}{9D^2 - \frac{729}{16}} \cos 3x \right]$$

$$= e^x \left[\frac{3D + \frac{27}{4}}{-81 - \frac{729}{16}} \sin 3x - \frac{3D + \frac{27}{4}}{-81 - \frac{729}{16}} \cos 3x \right]$$

$$= e^x \left[-\frac{16}{2025} \left(9\cos 3x + \frac{27}{4} \sin 3x \right) + \frac{16}{2025} \left(-9 \sin 3x + \frac{27}{4} \cos 3x \right) \right]$$

$$= e^x \left(-\frac{16}{225} \cos 3x - \frac{4}{75} \sin 3x - \frac{16}{225} \sin 3x + \frac{4}{75} \cos 3x \right)$$

$$= e^x \left(-\frac{4}{225} \cos 3x - \frac{28}{225} \sin 3x \right)$$

$\therefore$ G.S. $\Rightarrow$

$$y = y_c + y_p = c_1 e^{-\frac{1}{2}x} + c_2 x e^{-\frac{1}{2}x} + e^x \left(-\frac{4}{225} \cos 3x - \frac{28}{225} \sin 3x \right)$$

Ans

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$$y'' + y' + y = x \sin x$$

$$\Rightarrow (D^2 + D + 1) y = x \sin x \quad \cdots (i)$$

A.E. $\Rightarrow$ $m^2 + m + 1 = 0$

$$m = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3} i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$\therefore y_c = e^{-\frac{1}{2}x} \left[A \cos \frac{\sqrt{3}}{2} x + B \sin \frac{\sqrt{3}}{2} x \right]$$

$\therefore Y_p = \dfrac{1}{D^2+D+1} x \sin x$

$= x \dfrac{1}{D^2+D+1} \sin x - \dfrac{2D+1}{(D^2+D+1)^2} \sin x$

$= x \dfrac{1}{D} \sin x - \dfrac{2D+1}{D^2} \sin x$

$= -x\cos x + (2D+1) \sin x$

$= -x\cos x + 2\cos x + \sin x$

$\therefore$ G.S. $\Rightarrow$

$y = Y_c + Y_p = e^{-\frac{1}{2}x}\left[A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right] - x\cos x + 2\cos x + \sin x$

Ans

58|

$y'' + 4y = \cos^2 x$

$\Rightarrow (D^2+4)y = \frac{1}{2}(\cos 2x + 1)$

$\Rightarrow (D^2+4)y = \frac{1}{2}\cos 2x + \frac{1}{2}$

A.E. $\Rightarrow$

$m^2 + 4 = 0$

$m^2 = -4$

$\therefore m = \pm 2i$

$\therefore Y_c = A\cos 2x + B\sin 2x$

$$Y_p = \frac{1}{D^2+4}\left(\frac{1}{2}\cos 2x + \right) + \frac{1}{D^2+4}\frac{1}{2}$$

$$= x\frac{1}{2D}\frac{1}{2}\cos 2x + \frac{1}{4}\left(1+\frac{D^2}{4}\right)^{-1}\frac{1}{2}$$

$$= \frac{x}{2}\cdot\frac{1}{2}\cdot\sin 2x\cdot\frac{1}{2} + \frac{1}{4}\left(1-\dots\right)\frac{1}{2}$$

$$= \frac{x}{8}\sin 2x + \frac{1}{8}$$

$\therefore$ G.S. $\Rightarrow$

$$y = Y_c + Y_p = A\cos 2x + B\sin 2x + \frac{x}{8}\sin 2x + \frac{1}{8}$$

Ans

From MKC Books

14)

$$(D^3 - 5D^2 + 7D - 2)y = e^{2x}\cosh x$$

A.E. $\Rightarrow$

$$m^3 - 5m^2 + 7m - 2 = 0$$

$$m^3 - 2m^2 - 3m^2 + 6m + m - 2 = 0$$

$$m^2(m-2) - 3m(m-2) + 1(m-2) = 0$$

$$(m-2)(m^2 - 3m + 1) = 0$$

$m - 2 = 0$

$m = 2$

$$m^2 - 3m + 1 = 0$$

$$m = \frac{-9 \pm \sqrt{9-4}}{2} = \frac{-9 \pm \sqrt{5}}{2} = -\frac{9}{2} \pm \frac{\sqrt{5}}{2}$$

$$\therefore Y_c = c_1 e^{2x} + c_2 e^{-\left(\frac{9}{2}+\frac{\sqrt{5}}{2}\right)x} + c_3 e^{\left(-\frac{9}{2}+\frac{\sqrt{5}}{2}\right)x}$$

$$\therefore y_p = \frac{1}{D^3 - 5D^2 + 7D - 2} e^{2x} \cosh x$$

$$= e^{2x} \frac{1}{(D+2)^3 - 5(D+2)^2 + 7(D+2) - 2} \cosh x$$

$$= e^{2x} \frac{1}{D^3 + \cancel{3}6D^2 + 12D + 8 - 5D^2 - 20D - 20 + 7D + 14 - 2} \cosh x$$

$$= e^{2x} \frac{1}{D^3 + D^2 - D} \cosh x$$

$$= e^{2x} \frac{1}{-D - 1 - D} \cosh x$$

$$= e^{2x} \frac{1}{-2D - 1} \cosh x$$

$$= -e^{x} \frac{1}{2D+1} \cosh x$$

$$= -e^{x} \frac{2D-1}{4D^2-1} \cosh x$$

$$= -e^{x} \frac{2D-1}{-4-1} \cosh x$$

$$= \frac{e^{x}}{5} (2D-1) \cosh x$$

$$= \frac{e^{x}}{5} (2 \sinh x - \cosh x)$$

$\therefore$ G.S. $\Rightarrow$

$$y = y_c + y_p$$

$$= c_1 e^{2x} + c_2 e^{-(\frac{9}{2} + \frac{\sqrt{5}}{2})x} + c_3 e^{(-\frac{9}{2} + \frac{\sqrt{5}}{2})x}$$

$$+ \frac{e^{x}}{5} (2 \sinh x - \cosh x)$$

Ans

16)

$$(D^4+2D^2+1)y = x^2\cos^2 x$$

$$\Rightarrow (D^4+2D^2+1)y = x^2\cdot\frac{1}{2}(\cos 2x+1)$$

$$\Rightarrow (D^4+2D^2+1)y = \frac{1}{2}x^2\cos 2x + \frac{1}{2}x^2$$

A.E. $\Rightarrow$

$$m^4+2m^2+1=0$$

$$(m^2+1)^2=0$$

$$m=\pm i, \pm i$$

$$\therefore y_c = A\cos x + B\sin x + Cx\cos x + Dx\sin x$$

$$y_p = \frac{1}{D^4+2D^2+1}\frac{1}{2}x^2\cos 2x + \frac{1}{D^4+2D^2+1}\frac{1}{2}x^2$$

$$= \text{R.P. of } \frac{1}{D^4+2D^2+1}\frac{1}{2}x^2 e^{i2x} + \{1+(D^4+2D^2)\}^{-1}\frac{1}{2}x^2$$

$$= \text{R.P. of } e^{i2x}\frac{1}{(D+i2)^4+2(D+i2)^2+1}\frac{1}{2}x^2 + \left(1-(D^4+2D^2)+\cdots\right)\frac{1}{2}x^2$$

$$= \text{R.P. of } e^{i2x}\frac{1}{D^4+8D^3i-24D^2-32Di+16+2D^2+8Di-8+1}\frac{1}{2}x^2 + \frac{1}{2}x^2-2$$

$$= \text{R.P. of } e^{i2x}\frac{1}{D^4+8D^3i-22D^2-24Di+9}\frac{1}{2}x^2 + \left(\frac{1}{2}x^2-2\right)$$

$$= R.P. \text{ of } \frac{e^{i2x}}{9}\left(1+\frac{D^4+8D^3i-22D^2-24Di}{9}\right)^{-1}\frac{1}{2}x^2+\left(\frac{1}{2}x^2-2\right)$$

$$= R.P. \text{ of } \frac{e^{i2x}}{9}\left(1-\frac{D^4+8D^3i-22D^2-24Di}{9}+\left(\frac{D^4+8D^3i-22D^2-24Di}{9}\right)^2-\dots\right)\frac{1}{2}x^2$$

$$+\left(\frac{1}{2}x^2-2\right)$$

$$= \cancel{R.P. \text{ of } \frac{1}{9}(\cos 2x + i\sin 2x)}$$

$$= R.P. \text{ of } \frac{1}{9}(\cos 2x + i\sin 2x)\left(\frac{1}{2}x^2+\frac{-22+24xi}{9}-\frac{+576}{81}\right)+\left(\frac{1}{2}x^2-2\right)$$

$$= R.P. \text{ of } \frac{1}{9}\left(\frac{x^2}{2}\cos 2x+\frac{22}{9}\cos 2x+\frac{8}{3}xi\cos 2x-\frac{64}{9}\cos 2x+\frac{x^2}{2}i\sin 2x\right.$$

$$\left.+\frac{22}{9}i\sin 2x - \frac{8}{3}x\sin 2x-\frac{64}{9}i\sin 2x\right)$$

$$+\left(\frac{1}{2}x^2-2\right)$$

$$= \frac{1}{9}\left(\frac{x^2}{2}\cos 2x+\frac{22}{9}\cos 2x-\frac{64}{9}\cos 2x-\frac{8}{3}x\sin 2x\right)+\frac{x^2}{2}-2$$

$$= \frac{1}{9}\left(\frac{x^2}{2}\cos 2x-\frac{14}{3}\cos 2x-\frac{8}{3}x\sin 2x\right)+\frac{x^2}{2}-2$$

$\therefore$ G.S. $\Rightarrow$

$$y = y_c + y_p = A\cos x + B\sin x + Cx\cos x + Dx\sin x + \frac{x^2}{2} - 2$$

$$+\frac{1}{9}\left(\frac{x^2}{2}\cos 2x-\frac{14}{3}\cos 2x-\frac{8}{3}x\sin 2x\right)$$

Ans

Assignment Draft

Given,

$$y''' + Py'' + Qy' + Ry = f(x) \quad \ldots\ldots (i)$$

$$\therefore y_c = c_1y_1 + c_2y_2 + c_3y_3$$

Let,

$$y_p = u_1y_1 + u_2y_2 + u_3y_3 \quad \ldots\ldots (ii)$$

$$\Rightarrow y_p' = u_1y_1' + u_1'y_1 + u_2y_2' + u_2'y_2 + u_3y_3' + u_3'y_3$$

Let, $$\boxed{u_1'y_1 + u_2'y_2 + u_3'y_3 = 0} \quad \ldots (iii)$$

$$\Rightarrow y_p' = u_1y_1' + u_2y_2' + u_3y_3' \quad \ldots\ldots (iv)$$

$$\Rightarrow y_p'' = u_1y_1'' + u_1'y_1' + u_2y_2'' + u_2'y_2' + u_3y_3'' + u_3'y_3'$$

Let, $$\boxed{u_1'y_1' + u_2'y_2' + u_3'y_3' = 0} \quad \ldots\ldots (v)$$

$$\Rightarrow y_p'' = u_1y_1'' + u_2y_2'' + u_3y_3'' \quad \ldots (vi)$$

$$\Rightarrow y_p''' = u_1y_1''' + u_1'y_1'' + u_2y_2''' + u_2'y_2'' + u_3y_3''' + u_3'y_3'' \quad \ldots (vii)$$

Now substituting Y_p, Y_p', Y_p'', Y_p''' in (i)

$$\Rightarrow u_1 Y_1''' + u_1' Y_1'' + u_2 Y_2''' + u_2' Y_2'' + u_3 Y_3''' + u_3' Y_3'' + P\left(u_1 Y_1'' + u_2 Y_2'' + u_3 Y_3''\right)$$

$$+ Q\left(u_1 Y_1' + u_2 Y_2' + u_3 Y_3'\right) + R\left(u_1 Y_1 + u_2 Y_2 + u_3 Y_3\right) = f(x)$$

$$\Rightarrow u_1 \overbrace{\left(Y_1''' + PY_1'' + QY_1' + RY_1\right)}^{0} + u_2 \overbrace{\left(Y_2''' + PY_2'' + QY_2' + RY_2\right)}^{0} + u_3 \overbrace{\left(Y_3''' + PY_3'' + QY_3' + RY_3\right)}^{0}$$

$$+ u_1' Y_1'' + u_2' Y_2'' + u_3' Y_3'' = f(x)$$

$$\Rightarrow u_1' Y_1'' + u_2' Y_2'' + u_3' Y_3'' = f(x) \quad \text{---- (viii)}$$

Now,

$$\left.\begin{aligned}
\text{(iii)} &\Rightarrow u_1' Y_1 + u_2' Y_2 + u_3' Y_3 = 0 \\
\text{(v)} &\Rightarrow u_1' Y_1' + u_2' Y_2' + u_3' Y_3' = 0 \\
\text{(viii)} &\Rightarrow u_1' Y_1'' + u_2' Y_2'' + u_3' Y_3'' = f(x)
\end{aligned}\right\} \text{System of Lineen Equation}$$

$$\therefore D = \begin{vmatrix} Y_1 & Y_2 & Y_3 \\ Y_1' & Y_2' & Y_3' \\ Y_1'' & Y_2'' & Y_3'' \end{vmatrix} = W$$

$$\therefore D_1 = \begin{matrix} 0 & Y_2 & Y_3 \\ Y_1' & & Y \end{matrix}$$

$$D_1 = \begin{vmatrix} 0 & Y_2 & Y_3 \\ 0 & Y_2' & Y_3' \\ f(x) & Y_2'' & Y_3'' \end{vmatrix} = -Y_2\left(-Y_3' f(x)\right) + Y_3\left(-Y_2' f(x)\right)$$

$$= Y_2 Y_3' f(x) - Y_3 Y_2' f(x)$$

$$D_2 = \begin{vmatrix} Y_1 & 0 & Y_3 \\ Y_1' & 0 & Y_3' \\ Y_1'' & f(x) & Y_3'' \end{vmatrix} = Y_1\left(-Y_3' f(x)\right) + Y_3\left(Y_1' f(x)\right)$$

$$= Y_3 Y_1' f(x) - Y_1 Y_3' f(x)$$

$$D_3 = \begin{vmatrix} Y_1 & Y_2 & 0 \\ Y_1' & Y_2' & 0 \\ Y_1'' & Y_2'' & f(x) \end{vmatrix} = Y_1\left(Y_2' f(x)\right) - Y_2\left(Y_1' f(x)\right)$$

$$= Y_1 Y_2' f(x) - Y_2 Y_1' f(x)$$

$$\therefore u_1' = \frac{D_1}{W} = \frac{Y_2 Y_3' f(x) - Y_3 Y_2' f(x)}{W}$$

$$\Rightarrow u_1 = \int \frac{Y_2 Y_3' f(x) - Y_3 Y_2' f(x)}{W}\, dx$$

$$\therefore u_2' = \frac{D_2}{W} = \frac{Y_3 Y_1' f(x) - Y_1 Y_3' f(x)}{W}$$

$$\Rightarrow u_2 = \int \frac{Y_3 Y_1' f(x) - Y_1 Y_3' f(x)}{W}\, dx$$

$$\therefore u_3' = \frac{D_3}{W} = \frac{Y_1 Y_2' f(x) - Y_2 Y_1' f(x)}{W}$$

$$\Rightarrow u_3 = \int \frac{Y_1 Y_2' f(x) - Y_2 Y_1' f(x)}{W}\, dx$$

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$y''' + y' = \tan x$

A.E. $\Rightarrow$ $m^3 + m = 0$

$m(m^2+1) = 0$

$\therefore m = 0, \pm i$

$\therefore y_c = C_1 + C_2 \cos x + C_3 \sin x$

$\therefore y_1 = 1$

$\therefore y_2 = \cos x$

$\therefore y_3 = \sin x$

$$W = \begin{vmatrix} 1 & \cos x & \sin x \\ 0 & -\sin x & \cos x \\ 0 & -\cos x & -\sin x \end{vmatrix}$$

$= 1(\sin^2 x + \cos^2 x) - \cancel{\cos x}\ 0$

$= 1 \neq 0$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\, dx$

$\int u\, du$

$\frac{u^2}{2} = \frac{\sin^2 x}{2}$

$\cancel{\therefore u_1 = \int \frac{co}{}}$

$\therefore u_1 = \int \left(\cos x \cdot \cos x \tan x - \sin x(-\sin x) \tan x\right) dx$

$= \int \left(\cos x \sin x + \frac{\sin^3 x}{\cos x}\right) dx$

$= \frac{\sin^2 x}{2} + \int \sin x(1 - \cos^2 x)\sec x\, dx$

$= \frac{\sin^2 x}{2} + \int \sin x(\sec x - \cos x)\, dx$

$$= \frac{\cancel{x} \sin^2 x}{2} + \int \left(\sin x \sec x - \sin x \cos x \right) dx$$

$$= \frac{\cancel{x}\sin^2 x}{2} - \frac{\sin^2 x}{2} + \int \sin x \sec x \, dx$$

$$= \int \tan x \, dx$$

$$= \ln |\sec x|$$

$$u_2 = \int \cancel{\sin x \cdot 0} - 1 \cos x \tan x \, dx$$

$$= -\int \sin x \, dx$$

$$= \cos x$$

$$u_3 = \int 1 (-\sin x) \tan x \quad dx$$

$$= -\int \sin x \tan x \, dx$$

$$= -\int \frac{\sin^2 x}{\cos x} dx$$

$$= -\int \frac{1 - \cos^2 x}{\cos x} dx$$

$$= -\int \left(\sec x - \cos x \right) dx$$

$$= -\ln |\sec x + \tan x| + \sin x$$

$$\therefore Y_p = u_1 Y_1 + u_2 Y_2 + u_3 Y_3$$

$$= \ln|\sec x| + \underbrace{\cos^2 x + \sin^2 x}_{1} - \sin x \ln |\sec x + \tan x|$$

$\therefore$ G.S. $\Rightarrow$

$y = y_c + y_p = c_1 + c_2\cos x + c_3 \sin x + \ln(\sec x) + 1 - \sin x \ln(\sec x + \tan x)$ $\underline{Ans}$

$= c_4 + c_2 \cos x + c_3 \sin x + \ln(\sec x) - \sin x \ln(\sec x + \tan x)$ $\underline{Ans}$

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$y''' + 4y' = \sec 2x$

A.E. $\Rightarrow$

$m^3 + 4m = 0$

$m(m^2+4) = 0$

$m = 0, \pm 2i$

$\therefore y_c = c_1 + c_2 \cos 2x + c_3 \sin 2x$

$\therefore y_1 = 1$

$y_2 = \cos 2x$

$y_3 = \sin 2x$

$$W = \begin{vmatrix} 1 & \cos 2x & \sin 2x \\ 0 & -2\sin 2x & 2\cos 2x \\ 0 & -4\cos 2x & -4\sin 2x \end{vmatrix}$$

$= 8\sin^2 2x + 8\cos^2 2x$

$= 8 \neq 0$

$$\therefore u_1 = \int \frac{\left(\cos 2x\,(2\cos 2x)\sec 2x - \sin 2x\,(-2\sin 2x)\sec 2x\right)}{8}\,dx$$

$$= \int \frac{\left(2\cos 2x + 2\sin^2 2x \sec 2x\right)}{\underset{2}{\cancel{8}}}\,dx$$

$$= \frac{1}{8}\sin 2x + \int \frac{2(1-\cos^2 2x)\sec 2x}{8}\, dx$$

$$= \frac{1}{8}\sin 2x + \frac{2}{8}\int (\sec 2x - \cos 2x)\, dx$$

$$= \frac{1}{8}\sin 2x + \frac{2}{8}\cdot\frac{1}{2}\cdot \ln|\sec 2x + \tan 2x| - \frac{2}{8}\cdot\frac{1}{2}\sin 2x$$

$$= \frac{1}{8}\ln|\sec 2x + \tan 2x|$$

$$u_2 = \int -\frac{1(2\cos 2x)\sec 2x}{8}\, dx$$

$$= -\frac{2}{8}\int dx$$

$$= -\frac{2x}{8} = -\frac{1}{4}x$$

$$u_3 = \int \frac{1(-2\sin 2x)\sec 2x}{8}\, dx$$

$$= -\frac{2}{8}\int \sin 2x \sec 2x\, dx$$

$$= -\frac{2}{8}\int \tan 2x\, dx$$

$$= \frac{1}{8}\ln|\cos 2x| \cancel{dx}$$

$$= \cancel{\ln|\cos 2x|}$$

$$\therefore y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$$

$$= \frac{1}{8}\ln|\sec 2x + \tan 2x| - \frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x \ln|\cos 2x|$$

$\therefore$ G.S. $\Rightarrow$

$$y = c_1 + c_2\cos 2x + c_3\sin 2x + \frac{1}{8}\ln|\sec 2x + \tan 2x| - \frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x\ln|\cos 2x|$$

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$$y''' - 2y'' - y' + 2y = e^{4x}$$

A.E. $\Rightarrow$

$$m^3 - 2m^2 - m + 2 = 0$$

$$m = -1, 2, 1$$

$\therefore y_c = c_1 e^{-x} + c_2 e^{x} + c_3 e^{2x}$

$\therefore y_1 = e^{-x}$

$\therefore y_2 = e^{x}$

$\therefore y_3 = e^{2x}$

$$\therefore W = \begin{vmatrix} e^{-x} & e^{x} & e^{2x} \\ -e^{-x} & e^{x} & 2e^{2x} \\ e^{-x} & e^{x} & 4e^{2x} \end{vmatrix}$$

$$= e^{-x}(4e^{3x} - 2e^{3x}) - e^{x}(-4e^{x} - 2e^{x}) + e^{2x}(-e^{0} - e^{0})$$

$$= e^{-x}(2e^{3x}) - e^{x}(-6e^{x}) + e^{2x}(-1-1)$$

$$= 2e^{2x} + 6e^{2x} - 2e^{2x}$$

$$= 6e^{2x} \neq 0$$

$$u_1 = \int \frac{e^{x} \cdot 2e^{2x} \cdot e^{4x} - e^{2x} \cdot e^{x} \cdot e^{4x}}{6e^{2x}} \, dx$$

$$= \int \frac{2e^{7x} - e^{7x}}{6e^{2x}} \, dx$$

$$= \int \frac{e^{7x}}{6e^{2x}} \, dx = \frac{1}{6} \int e^{5x} \, dx = \frac{1}{30} e^{5x}$$

$$u_2 = \int \frac{e^{2x}(-e^{-x})\cdot e^{4x} - e^{-x}\cdot 2e^{2x}\cdot e^{4x}}{6e^{2x}}\,dx$$

$$= \int \frac{-e^{5x} - 2e^{5x}}{6e^{2x}}\,dx$$

$$= -\frac{1}{2}\int e^{3x}\,dx$$

$$= -\frac{1}{6}e^{3x}$$

$$u_3 = \int \frac{e^{-x}\cdot e^{x}\cdot e^{4x} - e^{x}\cdot(-e^{-x})\cdot e^{4x}}{6e^{2x}}\,dx$$

$$= \int \frac{e^{4x} + e^{4x}}{6e^{2x}}\,dx$$

$$= \frac{1}{3}\int e^{2x}\,dx$$

$$= \frac{1}{6}e^{2x}$$

$$\therefore Y_p = u_1 Y_1 + u_2 Y_2 + u_3 Y_3$$

$$= \frac{1}{30}e^{5x}\cdot e^{-x} - \frac{1}{6}e^{3x}\cdot e^{x} + \frac{1}{6}e^{2x}\cdot e^{2x}$$

$$= \frac{1}{30}e^{4x} - \frac{1}{6}e^{4x} + \frac{1}{6}e^{4x}$$

$$= \cancel{\frac{4}{45}e^{4x}}\ \frac{1}{30}e^{4x}$$

$\therefore$ G.S. $\Rightarrow$ $y = c_1e^{-x} + c_2e^{x} + c_3e^{2x} + \frac{1}{30}e^{4x}$

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$$y''' - 3y'' + 2y' = \frac{e^{2x}}{1+e^{x}}$$

A.E. $\Rightarrow$

$$m^3 - 3m^2 + 2m = 0$$

$$m(m^2 - 3m + 2) = 0$$

$m = 0$ | $m^2 - 3m + 2 = 0$

$m = 1, 2$

$\therefore y_c = c_1 + c_2 e^{x} + c_3 e^{2x}$

$\therefore y_1 = 1$

$y_2 = e^{x}$

$y_3 = e^{2x}$

$$W = \begin{vmatrix} 1 & e^{x} & e^{2x} \\ 0 & e^{x} & 2e^{2x} \\ 0 & e^{x} & 4e^{2x} \end{vmatrix}$$

$$= 4e^{3x} - 2e^{3x}$$

$$= 2e^{3x} \neq 0$$

$$u_1 = \int \frac{e^{x} \cdot 2e^{2x} \cdot \frac{e^{2x}}{1+e^{x}} - e^{2x} \cdot e^{x} \cdot \frac{e^{2x}}{1+e^{x}}}{2e^{3x}} \, dx$$

$$= \int \frac{\frac{2e^{5x}}{1+e^{x}} - \frac{e^{5x}}{1+e^{x}}}{2e^{3x}} \, dx$$

$$= \int \left(\frac{e^{5x}}{1+e^{x}} \cdot \frac{1}{2e^{3x}} \right) dx$$

$$= \int \frac{e^{2x}}{2+2e^x}\,dx$$

$$= \frac{1}{2}\int \frac{e^{2x}}{1+e^x}\,dx = \frac{1}{2}\int\left(e^x - \frac{e^x}{1+e^x}\right)dx$$

$$= \cancel{\frac{1+e^x}{2} - \frac{1}{2}\ln|1+e^x|}$$

$$= \frac{1}{2}e^x - \frac{1}{2}\ln(1+e^x)$$

$$u_2 = \int \frac{-1\cdot 2e^{2x}\cdot \frac{e^{2x}}{1+e^x}}{2e^{3x}}\,dx$$

$$= -\int\left(\frac{2e^{4x}}{1+e^x}\cdot\frac{1}{2e^{3x}}\right)dx$$

$$= -\int \frac{e^x}{1+e^x}\,dx$$

$$= -\ln(1+e^x)$$

$$u_3 = \int \frac{1\cdot e^x\cdot\frac{e^{2x}}{1+e^x}}{2e^{3x}}\,dx$$

$$= \int\left(\frac{e^{3x}}{1+e^x}\cdot\frac{1}{2e^{3x}}\right)dx$$

$$= \frac{1}{2}\int\frac{1}{1+e^x}\,dx = -\frac{1}{2}\int\frac{-e^{-x}}{1+e^{-x}}$$

$$= \cancel{\frac{1}{2}\left(\ln(1+e^x)+x\right) = \frac{1}{2}x + \frac{1}{2}\ln(1+e^x)}$$

$$= -\frac{1}{2}\ln(1+e^{-x})$$

$e^x = u-1$

$x = \ln(u-1)$

$u = 1+e^x$

$\frac{du}{dx} = e^x$

$du = e^x\cdot dx$

$u = 2x$

$du = 2\,dx$

$e^x = e^{\ln(u-1)}$

$= (u-1)$

$e^x\cdot e^x$

$\frac{e^{\ln(u-1)}}{u}\cdot du$

$\frac{u-1}{u}\,du$

$= \int\left(1-\frac{1}{u}\right)du$

$= u - \ln u$

$= (1+e^x) - \ln(1+e^x)$

$u = 1+e^x$

$\frac{du}{dx} = e^x$

$du = e^x\,dx$

$dx = \frac{1}{e^x}\,du$

$= \frac{1}{u-1}\,du$

$\int\frac{1}{u(u-1)}\,du$

$= \int\frac{1}{u^2-u}\,du$

$\therefore Y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$

$= \left\{ \frac{1+e^x}{2} - \frac{1}{2}\ln(1+e^x) \right\} - e^x \ln(1+e^x) + \frac{1}{2} x e^{2x} + \frac{1}{2} e^{2x} \ln(1+e^x)$

$\cancel{= \left(-\frac{1}{2} - e^x + \frac{1}{2} e^{2x}\right) \ln(1+e^x) + \frac{1+e^x}{2} + \frac{1}{2} x e^{2x}}$

$\therefore$ G.S. $\Rightarrow$

$$y = c_1 + c_2 e^x + c_3 e^{2x} + \frac{1+e^x}{2} - \frac{1}{2}\ln(1+e^x) - e^x \ln(1+e^x) + \frac{1}{2} x e^{2x} + \frac{1}{2} e^{2x} \ln(1+e^x)$$

$\therefore Y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$

$= 1 \cdot \left(\frac{1}{2} e^x - \frac{1}{2}\ln(1+e^x)\right) + e^x \cdot \left(-\ln(1+e^x)\right) + e^{2x} \cdot \left(-\frac{1}{2}\ln(1+e^{-x})\right)$

$= \frac{1}{2} e^x - \frac{1}{2}\ln(1+e^x) - e^x \ln(1+e^x) - \frac{1}{2} e^{2x} \ln(1+e^{-x})$

$\therefore$ G.S. $\Rightarrow$

$\therefore y = y_c + y_p$

$= c_1 + c_2 e^x + c_3 e^{2x} + \frac{1}{2} e^x - \frac{1}{2}\ln(1+e^x) - e^x \ln(1+e^x) - \frac{1}{2} e^{2x} \ln(1+e^{-x})$

$\boxed{\text{H.W}} \Rightarrow$ from Lecture-13

from Zillis Book - 4.6

1)

$y'' + y = \sec x$

A.E. $\Rightarrow$

$m^2 + 1 = 0$

$\therefore m = \pm i$

$\therefore y_c = A\cos x + B\sin x$

$\therefore y_1 = \cos x$

$y_2 = \sin x$

$\therefore W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$

$= \cos^2 x + \sin^2 x$

$= 1 \neq 0$

$\therefore u_1 = -\int \frac{\sin x \sec x}{1} dx$

$= -\int \tan x \, dx$

$\cancel{= -\ln|\sec x + \tan x|}$

$= \ln|\cos x|$

$\therefore u_2 = \int \cos x \sec x \, dx$

$= \int dx$

$= x$

$\therefore y_p = u_1 y_1 + u_2 y_2$

$= \cos x \ln|\cos x| + x\sin x$

$\therefore y = A\cos x + B\sin x + \cos x \ln|\cos x| + x\sin x$

4)

$$Y'' + Y = \sec\theta \tan\theta$$

A.E. $\Rightarrow$

$$m^2 + 1 = 0$$

$$\therefore m = \pm i$$

$$\therefore Y_c = A\cos\theta + B\sin\theta$$

$\therefore Y_1 = \cos\theta$

$Y_2 = \sin\theta$

$$\therefore W = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1 \neq 0$$

$$\therefore u_1 = -\int \sin\theta \sec\theta \cdot \tan\theta \, d\theta \qquad \therefore u_2 = \int \cos\theta \sec\theta \tan\theta \, d\theta$$

$$= -\int \tan^2\theta \, d\theta = \int (1 - \sec^2\theta) d\theta \qquad = \int \tan\theta \, d\theta$$

$$= x - \tan x$$

$$= \cancel{\ln|\cos^2\theta|} \qquad = -\ln|\cos\theta|$$

$$\therefore Y_p = u_1 Y_1 + u_2 Y_2$$

$$= \cancel{\cos\theta \ln|\cos^2\theta|} - \sin\theta \ln|\cos\theta|$$

$$x\cos\theta - \cancel{x\tan x}\sin\theta$$

$$\cancel{\therefore Y = A\cos\theta + B\sin\theta + \cos\theta \ln|\cos^2\theta| - \sin\theta \ln|\cos^2\theta|}$$

$$\therefore Y = A\cos\theta + B\sin\theta + x\cos\theta - \sin\theta - \sin\theta \ln|\cos\theta|$$

6)

$$y'' + y = \sec^2 x$$

A.E. $\Rightarrow$

$$m^2 + 1 = 0$$

$$\therefore m = \pm i$$

$$\therefore y_c = A\cos x + B\sin x$$

$$\therefore y_1 = \cos x$$

$$y_2 = \sin x$$

$$\therefore W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$= \cos^2 x + \sin^2 x$$

$$= 1 \neq 0$$

$u = \cos x$

$du = -\sin x\, dx$

$\sin x\, dx = -du$

$\therefore \int \frac{1}{u^2} du$

u^2

$\frac{u^{-1}}{-1} = -\frac{1}{u} = -\frac{1}{\cos x}$

$\frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}$

$$\therefore u_1 = -\int \sin x \sec^2 x\, dx$$

$$= -\int \frac{\sin x}{\cos^2 x} dx$$

$$= -\frac{1}{\cos x}$$

$$\therefore u_2 = \int \cos x \sec^2 x\, dx$$

$$= \int \sec x\, dx$$

$$= \ln|\sec x + \tan x|$$

$$\therefore y_p = u_1 y_1 + u_2 y_2$$

$$= -\cos x \sec x = \cancel{1} + \sin x \ln|\sec x + \tan x|$$

$$\therefore y = A\cos x + B\sin x \cancel{1} + \sin x \ln|\sec x + \tan x| + \cos x \sec x$$

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$$y'' + 3y' + 2y = \frac{1}{1+e^x}$$

A.E. $\Rightarrow$

$$m^2 + 3m + 2 = 0$$

$$\Rightarrow (m+2)(m+1) = 0$$

$$\therefore m = -1, -2$$

$$\therefore y_c = c_1 e^{-x} + c_2 e^{-2x}$$

$\therefore y_1 = e^{-x}$

$y_2 = e^{-2x}$

$$\therefore W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix}$$

$$= -2e^{-3x} + e^{-3x}$$

$$= -e^{-3x}$$

$$\therefore u_1 = -\int \frac{e^{-2x} \cdot \frac{1}{1+e^x}}{-e^{-3x}} dx \qquad \therefore u_2 = \int \frac{e^{-x} \cdot \frac{1}{1+e^x}}{-e^{-3x}} dx$$

$$= \int \frac{e^{-2x}}{1+e^x} \cdot \frac{1}{e^{-3x}} dx \qquad = -\int \frac{e^{-x}}{1+e^x} \cdot \frac{1}{e^{-3x}} dx$$

$$= \int \frac{e^x}{1+e^x} dx \qquad = -\int \frac{e^{2x}}{1+e^x} dx$$

$$= \ln|1+e^x|$$

$$= -\int \left(e^x - \frac{e^x}{1+e^x}\right) dx$$

$$= -e^x + \ln|1+e^x|$$

$\therefore y_p = u_1 y_1 + u_2 y_2 = \cancel{e^{-x} \ln|1+e^x|}$

$$= e^{-x} \ln|1+e^x| - e^{-2x} \cdot e^x + e^{-2x} \ln|1+e^x|$$

$$= e^{-x} \ln|1+e^x| - e^{-x} + e^{-2x} \ln|1+e^x|$$

$\therefore y = c_1 e^{-x} + c_2 e^{-2x} + e^{-x} \ln|1+e^x| - e^{-x} + e^{-2x} \ln|1+e^x|$

12)

$$y'' - 2y' + y = \frac{e^x}{1+x^2}$$

A.E. $\Rightarrow$

$$m^2 - 2m + 1 = 0$$

$$(m-1)^2 = 0$$

$$\therefore m = 1, 1$$

$\therefore y_c = c_1 e^x + c_2 x e^x$

$\therefore y_1 = e^x$

$y_2 = x e^x$

$$\therefore W = \begin{vmatrix} e^x & x e^x \\ e^x & x e^x + e^x \end{vmatrix}$$

$$= x e^{2x} + e^{2x} - x e^{2x}$$

$$= e^{2x} \neq 0$$

$$\therefore u_1 = -\int \frac{x e^x \cdot \frac{e^x}{1+x^2}}{e^{2x}} dx$$

$$= -\int \frac{x e^{2x}}{1+x^2} \cdot \frac{1}{e^{2x}} dx$$

$$= -\frac{1}{2} \int \frac{2x}{1+x^2} dx$$

$$= -\frac{1}{2} \ln|1+x^2|$$

$$\therefore u_2 = \int \frac{e^x \cdot \frac{e^x}{1+x^2}}{e^{2x}} dx$$

$$= \int \frac{e^{2x}}{1+x^2} \cdot \frac{1}{e^{2x}} dx$$

$$= \int \frac{1}{1+x^2} dx$$

$$= \tan^{-1} x$$

$$\therefore y_p = -\frac{1}{2} e^x \ln|1+x^2| + x e^x \tan^{-1} x$$

$$\therefore y = c_1 e^x + c_2 x e^x - \frac{1}{2} e^x \ln|1+x^2| + x e^x \tan^{-1} x$$

13

$y'' + 3y' + 2y = \sin e^x$

A.E. $\Rightarrow$

$m^2 + 3m + 2 = 0$

$\Rightarrow (m+2)(m+1) = 0$

$\therefore m = -1, -2$

$\therefore y_c = c_1 e^{-x} + c_2 e^{-2x}$

$\therefore y_1 = e^{-x}$

$y_2 = e^{-2x}$

$\therefore W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix}$

$= -2e^{-3x} + e^{-3x}$

$= -e^{-3x}$

$u = e^x$

$du = e^x dx$

$\int \sin u \, du$

$-\cos u$

$-\cos e^x$

$\cancel{u = e^{2x}}$

$\cancel{du = 2e^{2x} dx}$

$\int u \cdot \sin u \, du$

$u \quad \sin u$

$1 \quad -\cos u$

$0 \quad -\sin u$

$= -u\cos u + \sin u$

$\therefore u_1 = -\int \frac{e^{-2x} \cdot \sin e^x}{-e^{-3x}} dx$

$= \int e^x \sin e^x \, dx$

$= -\cos e^x$

$\therefore u_2 = \int \frac{e^{-x} \cdot \sin e^x}{-e^{-3x}} dx$

$= -\int e^{2x} \cdot \sin e^x \, dx$

$= -\left[-e^x \cos e^x + \sin e^x\right]$

$= e^x \cos e^x - \sin e^x$

$\therefore y_p = u_1 y_1 + u_2 y_2$

$= -e^{-x}\cos e^x + e^{-x}\cos e^x - e^{-2x}\sin e^x$

$= -e^{-2x}\sin e^x$

$\therefore y = c_1 e^{-x} + c_2 e^{-2x} - e^{-2x}\sin e^x$

Ans

|4|

$y'' - 2y' + y = e^t \text{ arctan} t$

$\Rightarrow y'' - 2y' + y = e^t \tan^{-1} t$

A.E. $\Rightarrow$

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$\therefore m = 1, 1$

$\therefore \cancel{y_c = c_1 e^x + c_2 x e^x}$

$\therefore y_c = c_1 e^t + c_2 t e^t$

$\therefore y_1 = e^t$

$y_2 = t e^t$

$\therefore W = \begin{vmatrix} e^t & t e^t \\ e^t & t e^t + e^t \end{vmatrix}$

$= t e^{2t} + e^{2t} - t e^{2t}$

$= e^{2t} \neq 0$

$\therefore u_1 = -\int \frac{t e^t \cdot e^t \tan^{-1} t}{e^{2t}} \, dt$

$= -\int t \tan^{-1} t \, dt$

$= \cancel{[t\int} = -\left[\tan^{-1} t \int t \, dt - \int\left\{\frac{d}{dx}(\tan^{-1} t)\int t \, dt\right\} dt\right]$

$= -\left[\frac{1}{2} t^2 \tan^{-1} t - \int \frac{1}{1+t^2} \cdot \frac{1}{2} t^2 \, dt\right]$

$= -\left[\frac{1}{2} t^2 \tan^{-1} t - \frac{1}{2}\int \frac{t^2}{1+t^2} \, dt\right]$

$= \cancel{-\left[\frac{1}{2} t^2 \tan^{-1} t - \frac{1}{2}\int\left(t - \frac{t}{1+t^2}\right) dt\right]}$

$$= -\left[\frac{1}{2}x^2\tan^{-1}x - \cancel{\frac{1}{2}\cdot\frac{1}{2}\cdot x^2 + \frac{1}{2}\cdot\frac{1}{2}\ln|1+x^2|}\right]$$

$$\cancel{= -\frac{1}{2}x^2\tan^{-1}x - \frac{1}{4}x^2 + \frac{1}{4}\ln|1+x^2|}$$

$$= -\left[\frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2}\,dx\right]$$

$$= -\left[\frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx\right]$$

$$= -\left[\frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x\right]$$

$$= -\frac{1}{2}x^2\tan^{-1}x + \frac{1}{2}x - \frac{1}{2}\tan^{-1}x$$

$$= -\frac{x^2+1}{2}\tan^{-1}x + \frac{1}{2}x$$

$$\therefore u_2 = \int \frac{e^x\cdot e^x\tan^{-1}x}{e^{2x}}\,dx$$

$$= \int \tan^{-1}x\,dx$$

$$= \tan^{-1}x\int 1\,dx - \int\left\{\frac{d}{dx}(\tan^{-1}x)\int 1\,dx\right\}dx$$

$$= x\tan^{-1}x - \int \frac{1}{1+x^2}\cdot x\,dx$$

$$= x\tan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$$

$$= x\tan^{-1}x - \frac{1}{2}\ln|1+x^2|$$

$\therefore Y_p = u_1 Y_1 + u_2 Y_2$

$= e^t\left(-\frac{t^2+1}{2}\tan^{-1}t + \frac{t}{2}\right) + te^t\left(t\tan^{-1}t - \frac{1}{2}\ln|1+t^2|\right)$

$= -\frac{1+t^2}{2}e^t\tan^{-1}t + \frac{1}{2}te^t + t^2e^t\tan^{-1}t - \frac{1}{2}te^t\ln|1+t^2|$

$\therefore Y = Y_c + Y_p$

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$y'' + 2y' + y = e^{-t}\cdot\ln t$

A.E. $\Rightarrow$ $m^2 + 2m + 1 = 0$

$\Rightarrow (m+1)^2 = 0$

$\therefore m = -1, -1$

$\therefore Y_c = c_1 e^{-x} + c_2 x e^{-x}$

$\therefore Y_1 = e^{-x}$

$Y_2 = xe^{-x}$

$W = \begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & -xe^{-x} + e^{-x} \end{vmatrix}$

$= -xe^{-2x} + e^{-2x} + xe^{-2x}$

$= e^{-2x} \neq 0$

$\therefore u_1 = -\int \frac{xe^{-x}}{}$

15

$$y'' + 2y' + y = e^{-t} \cdot \ln t$$

A.E. $\Rightarrow$

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0$$

$$\therefore m = -1, -1$$

$$\therefore y_c = c_1 e^{-t} + c_2 t e^{-t}$$

$$\therefore y_1 = e^{-t}$$

$$y_2 = t e^{-t}$$

$$W = \begin{vmatrix} e^{-t} & te^{-t} \\ -e^{-t} & -te^{-t} + e^{-t} \end{vmatrix}$$

$$= -te^{-2t} + e^{-2t} + te^{-2t}$$

$$= e^{-2t} \neq 0$$

$$\therefore u_1 = -\int \frac{te^{-t} \cdot e^{-t} \cdot \ln t}{e^{-2t}} \, dt$$

$$= -\int t \cdot \ln t \; dt$$

$$= -\left[\ln t \int t \, dt - \int \left\{ \frac{d}{dx}(\ln t) \int t \, dt \right\} dt \right]$$

$\cancel{=}$ ~~ln t~~

$$= -\left[\frac{1}{2} t^2 \cdot \ln t - \int \frac{1}{t} \cdot \frac{1}{2} t^2 \, dt \right]$$

$$= -\left[\frac{t^2}{2} \ln t - \frac{1}{2} \int t \, dt \right]$$

$$= -\left[\frac{t^2}{2} \ln t - \frac{t^2}{4} \right] = \frac{t^2}{4} - \frac{t^2}{2} \ln t$$

$$\therefore u_2 = \int \frac{e^{-t} \cdot e^{-t} \cdot \ln t}{e^{-2t}}\, dt$$

$$= \int \ln t \; dt$$

$$= \ln t \int dt - \int \left\{ \frac{d}{dx}(\ln t) \int dt \right\} dt$$

$$= t \ln t - \int \frac{t}{t}\, dt$$

$$= t \ln t - t$$

$$\therefore Y_p = u_1 Y_1 + u_2 Y_2$$

$$= e^{-t}\left(\frac{t^2}{4} - \frac{t^2}{2}\ln t\right) + te^{-t}\left(t \ln t - t\right)$$

$$= \frac{1}{4} e^{-t} t^2 - \frac{1}{2} e^{-t} t^2 \ln t + t^2 e^{-t} \ln t - t^2 e^{-t}$$

$$= \frac{1}{2} t^2 e^{-t} \ln t - \frac{3}{4} t^2 e^{-t}$$

$$\therefore Y = Y_c + Y_p$$

$$= c_1 e^{-t} + c_2 t e^{-t} + \frac{1}{2} t^2 e^{-t} \ln t - \frac{3}{4} t^2 e^{-t}$$

Ans

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$$2y'' + 2y' + y = 4\sqrt{x}$$

$$A.E. \Rightarrow \; 2m^2 + 2m + 1 = 0$$

$$\therefore m = \frac{-1 \pm i}{2} = -\frac{1}{2} \pm \frac{1}{2} i$$

$$\therefore Y_c = e^{-\frac{1}{2}x}\left[A \cos\frac{x}{2} + B \sin\frac{x}{2}\right]$$

$$\therefore Y_1 = e^{-\frac{1}{2}x} \cos\frac{x}{2}$$

$$\therefore Y_2 = e^{-\frac{1}{2}x} \sin\frac{x}{2}$$

$$\therefore W = \begin{vmatrix} e^{-\frac{1}{2}x}\cos\frac{x}{2} & \cancel{e^{-\frac{1}{2}x}\sin\frac{x}{2}}\ e^{-\frac{1}{2}x}\sin\frac{x}{2} \\ -\frac{1}{2}e^{-\frac{1}{2}x}\sin\frac{x}{2} - \frac{1}{2}e^{-\frac{1}{2}x}\cos\frac{x}{2} & \frac{1}{2}e^{-\frac{1}{2}x}\cos\frac{x}{2} - \frac{1}{2}e^{-\frac{1}{2}x}\sin\frac{x}{2} \end{vmatrix}$$

$$= \frac{1}{2}e^{-x}\cos^2\frac{x}{2} - \frac{1}{2}e^{-x}\cos\frac{x}{2}\sin\frac{x}{2}$$

$$- \left(-\frac{1}{2}e^{-x}\sin^2\frac{x}{2} - \frac{1}{2}e^{-x}\cos\frac{x}{2}\sin\frac{x}{2}\right)$$

$$= \frac{1}{2}e^{-x}\cos^2\frac{x}{2} + \frac{1}{2}e^{-x}\sin^2\frac{x}{2} - \frac{1}{2}e^{-x}\cos\frac{x}{2}\sin\frac{x}{2} + \frac{1}{2}e^{-x}\cos\frac{x}{2}\sin\frac{x}{2}$$

$$= \frac{1}{2}e^{-x} \neq 0$$

$$= \cancel{e^{-x} \neq 0}$$

$$\therefore u_1 = -\int \frac{e^{-\frac{1}{2}x}\sin\frac{x}{2}\cdot 4\sqrt{x}}{\frac{1}{2}e^{-x}}\,dx$$

$$\cancel{= -4\int e^{\frac{1}{2}x}\sin\frac{x}{2}\cdot x^{\frac{1}{2}}\,dx}$$

$$= -8\int e^{\frac{1}{2}x}\sin\frac{x}{2}\cdot\sqrt{x}\,dx$$

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$3y'' - 6y' + 6y = e^x \sec x$

A.E. $\Rightarrow$

$3m^2 - 6m + 6 = 0$

$m^2 - 2m + 2 = 0$

$m = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2}$

$= 1 \pm i$

$\therefore y_c = e^x [A\cos x + B\sin x]$

$\therefore y_1 = e^x \cos x$

$y_2 = e^x \sin x$

$$W = \begin{vmatrix} e^x \cos x & e^x \sin x \\ -e^x \sin x + e^x \cos x & e^x \cos x + e^x \sin x \end{vmatrix}$$

$= e^{2x}\cos^2 x + e^{2x}\cos x \sin x + e^{2x}\sin^2 x - e^{2x}\sin x \cos x$

$= e^{2x} \neq 0$

$\therefore u_1 = -\int \frac{e^x \sin x \cdot e^x \sec x}{e^{2x}} dx$

$= -\int \sin x \sec x \, dx$

$= -\int \tan x \, dx$

$= \ln|\cos x|$

$\therefore u_2 = \int \frac{e^x \cos x \cdot e^x \sec x}{e^{2x}} dx$

$= \int dx$

$= x$

$\therefore y_p = u_1 y_1 + u_2 y_2$

$= e^x \cos x \ln|\cos x| + x e^x \sin x$