Chapter - 2

Determinants

2.1

$$A = \begin{bmatrix} 2 & -1 \\ 3 & -4 \end{bmatrix}$$ is a matrix

$$\det(A) = |A| = \begin{vmatrix} 2 & -1 \\ 3 & -4 \end{vmatrix} = -8+3 = -5$$

⊛

$$A = \begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 6 \\ 1 & 4 & 8 \end{bmatrix}$$

Minor of 3 : $M_{11} = \begin{vmatrix} 5 & 6 \\ 4 & 8 \end{vmatrix} = 40-24 = 16$

$$M_{23} = \begin{vmatrix} 3 & 1 \\ 1 & 4 \end{vmatrix} = 12-1 = 11$$

$$M_{21} = \begin{vmatrix} 1 & -4 \\ 4 & 8 \end{vmatrix} = 8+16 = 24$$

$$M_{22} = \begin{vmatrix} 3 & -4 \\ 1 & 8 \end{vmatrix} = 24+4 = 28$$

Cofacton

$$1+1=2 = \text{even} = (+)$$

$$C_{11} = (-1)^{(+)} M_{11}$$

$$2+1=3 = \text{odd} = (-1)$$

$$C_{21} = (-1) M_{21}$$

$$2+2=4 = \text{even} = (+1)$$

$$C_{22} = (+1) M_{22}$$

$$A = \begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 6 \\ 1 & 4 & 8 \end{bmatrix}$$

$$C_{21} = -M_{21} = -24$$

$$C_{22} = M_{22} = 28$$

$$C_{23} = -M_{23} = -11$$

$$\det(A) = 2 \cdot C_{21} + 5 \cdot C_{22} + 6 \cdot C_{23}$$

$$= 2(-24) + 5(28) + 6(-11)$$

$$= 26$$

Ans

- Cofacton expansion method to find det (A)

⊛ We can find det (A) by any row or collumn.

row × row cofactor

or

Collumn × Column cofactor

⊛

Cofactor of row-2 × entry of row 1 or 3 = 0.

L-9/29.06.2022/

2.2/

* Find determinant by row reduction:

$$A = \begin{bmatrix} 2 & 5 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 6 \end{bmatrix}$$

$\det(A) = 2 \cdot 1 \cdot 6 = 12$

*

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 1 \\ 3 & -4 & 2 \end{vmatrix} = - \begin{vmatrix} 5 & 6 & 1 \\ 2 & 3 & 4 \\ 3 & -4 & 2 \end{vmatrix}$$

*

$$\begin{vmatrix} 2 & 6 & 4 \\ 5 & 6 & 1 \\ 3 & -4 & 2 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & 2 \\ 5 & 6 & 1 \\ 3 & -4 & 2 \end{vmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}$$

$$\det(A) = \ ?$$

$$\underline{2 \cdot 3 \ /}$$

$$\det(A+B) \neq \det(A) + \det(B)$$

$$\det(AB) = \det(A) \cdot \det(B)$$

$$A = \begin{bmatrix} 1 & 3 \\ 5 & -4 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{19} \begin{bmatrix} -4 & -3 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{19} & \frac{3}{19} \\ \frac{5}{19} & -\frac{1}{19} \end{bmatrix}$$

$$\det(A^{-1}) =$$

* $AA^{-1} = I$

$\Rightarrow \det(AA^{-1}) = \det(I) = 1$

$\Rightarrow \det(A) \cdot \det(A^{-1}) = 1$

* $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

matrix of cofactors $= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

Transpose of the matrix $= \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} = \operatorname{adj}(A).$

Now,

$A \cdot \operatorname{adj}(A) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$

$= \begin{bmatrix} \det(A) & 0 & 0 \\ 0 & \det(A) & 0 \\ 0 & 0 & \det(A) \end{bmatrix}$

$$= \det(A) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \det(A) \cdot I$$

Now,

$$A \cdot \text{adj}(A) = \det(A) \cdot I$$

$$\Rightarrow A \cdot \frac{\text{adj}(A)}{\det(A)} = I$$

$$\therefore A^{-1} = \frac{\text{adj}(A)}{\det(A)}$$

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$$

$$\text{Number of cofactors} = \begin{bmatrix} 12 & 6 & -16 \\ 4 & 2 & 16 \\ 12 & -10 & 16 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} 12 & 4 & 12 \\ 6 & 2 & -10 \\ -16 & 16 & 16 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{12}{64} & \frac{4}{64} & \frac{12}{64} \\ \frac{6}{64} & \frac{2}{64} & \frac{-10}{64} \\ \frac{-16}{64} & \frac{16}{64} & \frac{16}{64} \end{bmatrix}$$

$$\det(A) = 3 \cdot 12 + 2 \cdot 6 + (-1)(-16)$$

$$= 36 + 12 + 16$$

$$= 64$$

251)

$$A = \begin{bmatrix} 4 & 5 & 0 \\ 4 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 2 & 5 & 0 \\ 3 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}$$

Replace with constant

$$A_2 = \begin{bmatrix} 4 & 2 & 0 \\ 4 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 4 & 5 & 2 \\ 4 & 1 & 3 \\ 1 & 5 & 1 \end{bmatrix}$$

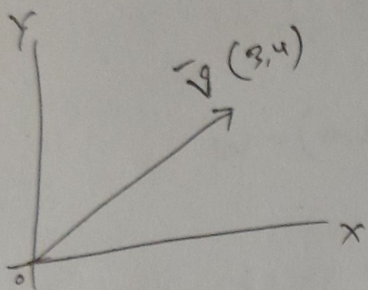
Cramer's rule:

$$x = \frac{\det(A_1)}{\det(A)} \qquad y = \frac{\det(A_2)}{\det(A)}$$

$$z = \frac{\det(A_3)}{\det(A)}$$

L-10 / 04.07.2024

3.1 / Chapter-3 / Vector /

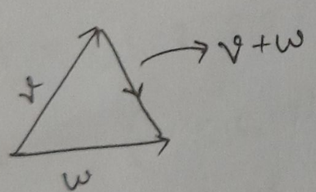


Magnitude / length / norm of the vector v : $\|v\|$

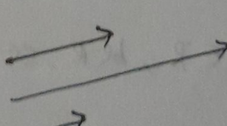
$\bar{v} = (1, -3, 4, -2)$

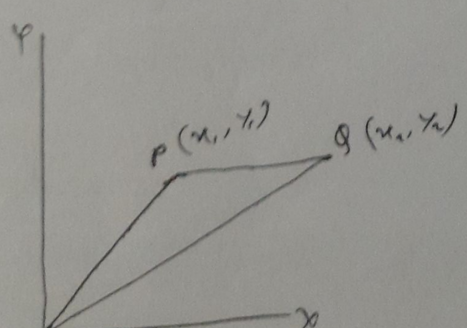
$\|v\| = \sqrt{1^2 + (-3)^2 + 4^2 + (-2)^2}$

$= \sqrt{1+9+16+4} = \sqrt{30}$



$v, 2v, \frac{1}{2}v \Rightarrow$ Parallel vector





$\bar{OP_1} = (x_1, y_1)$

$\bar{OP_2} = (x_2, y_2)$

$\bar{P_1P_2} = (x_2 - x_1, y_2 - y_1)$

$P_1\ (2,-1,4)$

$P_2\ (7,5,-8)$

$\vec{v} = \overrightarrow{P_1P_2}$

$v = (7-2,\ 5+1,\ -8-4)$

$= (5,6,-12)$

$\mathbb{R}$: set of all real numbers.

$X = \{a,b\}$

$Y = \{1,2,3\}$

$X \times Y = \{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3)\}$

$R \times R = R^2\ \{(a,b) : a \in \mathbb{R},\ b \in R\}$

Euclidn 2 space.

$R^3 = \{(a,b,c) : a \in R,\ b \in R,\ c \in R\}$

Eucliden -3 space

. . . –

. . . .

. . . .

. . . .

$v = (1, 2, -3) \in R^3$

$v = (1, -2, 3, 5) \in R^4$

$w = (a, b, c, d) \in R^4$

$v = w$

$(a, b, c, d) = (1, -2, 3, 5)$

$a = 1$

$b = -2$

$c = 3$

$d = 5$

$v = (1, -2, 3, 5)$

$w = (-2, 5, 6, -2)$

$v + w = (-1, 3, 9, 3)$

$3v = (3, -6, 9, 15)$

3.2/

Unit vector

$v = (5, -1, 3)$

$\|v\| = \sqrt{5^2 + (-1)^2 + 3^2} = \sqrt{25+1+9} = \sqrt{35}$

unit vector

$\hat{v} = \frac{v}{\|v\|} = \frac{5, -1, 3}{\sqrt{35}} = \left(\frac{5}{\sqrt{35}}, \frac{-1}{\sqrt{35}}, \frac{3}{\sqrt{35}}\right)$

Standard unit vector

In $\mathbb{R}^3$, $i = (1,0,0)$, $j = (0,1,0)$, $k = (0,0,1)$

are standard unit vector.

In $\mathbb{R}^4$, $e_1 = (1,0,0,0)$, $e_2 = (0,1,0,0)$, $e_3 = (0,0,1,0)$

$e_4 = (0,0,0,1)$

are standard unit vector

u, v

$\|u - v\| = d(u, v)$

$u = (1, 3, -2, 7)$

$v = (0, 7, 2, 2)$

$\|u - v\| = \|(1, -4, -4, 5)\|$

$= \sqrt{1^2 + (-4)^2 + (-4)^2 + 5}$

$= \sqrt{1 + 16 + 16 + 25}$

Dot product / Eudiden inner product

$u \cdot v = \|u\| \|v\| \cos\theta$

$= u_1 v_1 + u_2 v_2$

$u = (u, u_2)$ $\quad$ $u = \cancel{(2,-3,1)}\ (2, -3, 1)$ $\quad$ $\cos\theta = ?$

$v = (v, v_2)$ $\quad$ $v = (1, 4, -4)$

$u \cdot v = (2 \cdot -12 \cdot -4)$

$= 2 - 16 = -4$

$$\|u\| = \sqrt{4+9+1} = \sqrt{14}$$

$$\|v\| = \sqrt{1^2+(-4)^2+4^2} = \sqrt{33}$$

$$\cos\theta = \frac{u \cdot v}{\|u\| \|v\|}$$

$u \cdot v = 0 \Rightarrow u \perp v$

u orthogonal to v : $u \perp v$

L-11 / 06.07.2022 /

3.3 /

Orthogonality

$u \cdot v = 0 \Rightarrow$ Dot product

Eucliden innen product

⊛

$u = (-2, 3, 1, 4)$

$v = (1, 2, 0, -1)$

$u \cdot v = (-2 + 6 + 0 - 4)$

$= 0$

$\therefore u \perp v$

⊛ In $\mathbb{R}^3$

i, j, k are.

$i = (1,0,0)$

$j = (0,1,0)$

$k = (0,0,1)$

$i \cdot j = (0+0+0) = 0$

$j \cdot k = 0$

$k \cdot i = 0$

$i \cdot i = 1$

$j \cdot j = 1$

$k \cdot k = 1$

⊛ vector component of u along a

$$\text{Proj}_a u = \frac{u \cdot a}{\|a\|^2} a$$

vector component of u orthogonal to u

$u - \text{Proj}_a u$

Example - 5 /

$u = (2,-1,3)$, $a = (4,-1,2)$

$$\text{Proj}_a u = \frac{u \cdot a}{\|a\|^2} a$$

$$= \frac{8+1+6}{4^2+(-1)^2+2^2} a$$

$$= \frac{15}{21} (4,-1,2)$$

$$= \frac{5}{7} (4,-1,2)$$

$$= \left(\frac{20}{7}, \frac{-5}{7}, \frac{10}{7}\right)$$

vector component of u orthogonal to a

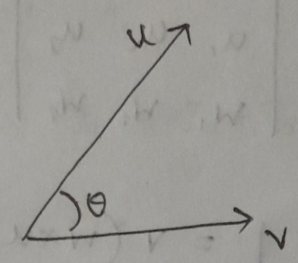
$$u - \text{Proj}_a u = (2,-1,3) - \left(\frac{20}{7}, \frac{-5}{7}, \frac{10}{7}\right)$$

$$= \left(2-\frac{20}{7}, -1+\frac{5}{7}, 3-\frac{10}{7}\right)$$

$$= \left(\frac{-6}{7}, \frac{-2}{7}, \frac{11}{7}\right)$$

3.5

Cross Product



$u \times v$ is a vector

$= \|u\| \, \|v\| \sin\theta \, \hat{n}$

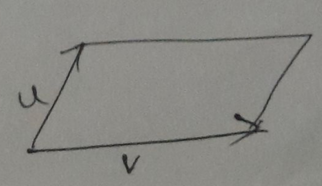
$\boxed{\begin{array}{l} u \perp (u \times v) \\ v \perp (u \times v) \end{array}}$ $\boxed{\begin{array}{l} u \cdot (u \times v) = 0 \\ v \cdot (u \times v) = 0 \end{array}}$

$u = (u_1, u_2, u_3) = u_1 i + u_2 j + u_3 k$

$v = (v_1, v_2, v_3) = v_1 i + v_2 j + v_3 k$

$$\bar{u} \times \bar{v} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = i \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - j \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + k \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

(*) $\|u \times v\|$ = area of the parallelogram determined by u and v



✳ u, v, w

$$u\cdot(v\times w) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = -\begin{vmatrix} v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$= \begin{vmatrix} v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = v\,(w\times u)$$

✳

$$\begin{tikzcd} & u \arrow[dl] & \\ v \arrow[rr] & & w \arrow[ul] \end{tikzcd}$$

$$u\cdot(v\times w) = v\cdot(w\times u) = w\cdot(u\times v)$$

$|u\cdot(v\times w)|$ = volume of the parallegram determined by u, v, w.

L-12/ 18.07.2022/

Electricity

For the point A,

$$I_1 = I_2 + I_3$$

$$7I_1 + 3I_3 - 30 = 0$$

$$11I_2 - 50 - 3I_3 = 0$$

1/

$\therefore I_1 = I_2 + I_3$

$\Rightarrow I_1 - I_2 - I_3 = 0$

$\therefore 13I_2 + 5I_1 + 8 = 0$

$\Rightarrow 5I_1 + 13I_2 = -8$

$\therefore -3 + 14I_3 - 13I_2 = 0$

$\Rightarrow -13I_2 + 14I_3 = 3$

$\therefore$ Augmented matrix:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 5 & 13 & 0 & -8 \\ 0 & -13 & 14 & 3 \end{bmatrix}$$

2)

$\therefore I_3 = I_2 + I_1$

$\Rightarrow I_1 + I_2 - I_3 = 0$

$\therefore 6 - 2I_1 + 2I_2 = 0$

$\Rightarrow -2I_1 + 2I_2 = -6$

$\therefore 8 - 4I_3 - 2I_2 = 0$

$\Rightarrow -2I_2 - 4I_3 = -8$

$\therefore \begin{bmatrix} 1 & 1 & -1 & 0 \\ -2 & 2 & 0 & -6 \\ 0 & -2 & -4 & -8 \end{bmatrix}$

4.1

Vector Space

Let V be a non-empty set of objects.

Two operations, addition and scaler multiplication, are

define in V.

$V = \mathbb{R}$ is a vector space

$V = \mathbb{R}^n$ is a vector space; $n \in \mathbb{N}$

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}$$

$= M_{22} \longrightarrow$ Notation.

Exercise:

1) a) $u = (-1, 2)$

$v = (3, 4)$

$k = 3$

$u + v = (-1+3, \ 2+4) = (2, 6)$

$ku = (0, ku_2) = (0, 3 \cdot 2) = (0, 6)$

e)

$$1u = (1 \cdot u_1 , 1 \cdot u_2) = (0, u_2) \neq u$$

Axiom 10 is fails and hence that v is not a vector space.

$M_{22} \Rightarrow$ 2 by 2 matrix of real numbers

$P_2 \Rightarrow$ all polynomial equation ; degree ≤ 2.

⊛ V, vector space,

$W \subset V$ is a subspace of V,

i) $u, v \in W \Rightarrow u+v \in W$

ii) k, u scalar, $u \in W \Rightarrow ku \in W$

⊛ x-axis is a subspace of W

⊛ $W = \{a + bx + cx^2 : a, b, c \in \mathbb{Z}\}$

$u = 2 + 3x + 7x^2 \in W$

rational

$\frac{1}{2}u = 1 + \left(\frac{3}{2}\right)x + \left(\frac{7}{2}\right)x^2 \notin W$

W is not subspace of P_2

⊛ V is vector in V and

$$W = k_1 u_1 + k_2 u_2 + \cdots + k_n u$$

where,

$u, u_1, \ldots u_n \in V$ and $k u_2 \cdots k_n$ are scalar

⊛ $V = \mathbb{R}^2$

$u = (1,-1)$, $v = (2,1) \in \mathbb{R}^2$

$\Rightarrow 2u + 3v$

$= 2(1,-1) + 3(2,1)$

$= (2,-2) + (6,3)$

$= (8,1) \in \mathbb{R}^3$

⊛

$S = \{w_1, w_2, w_3\}$, $w_1, w_2, w_3 \in V$

W = set of all possible linear combination if the vector is S.

= spanned by S.

W = span (S)

✲ $V = \mathbb{R}^3$

standard unit vector in $\mathbb{R}^3$:

$$S = \{ e_1 = (1,0,0),\ e_2 = (0,1,0),\ e_3 = (0,0,1) \}$$

$$k_1 e_1 + k_2 e_2 + k_3 e_3$$

Let

$$2e_1 + 3e_2 + 4e_3 = (2,3,4) \in \mathbb{R}^3$$

✲ P_2:

standard unit vector:

$$P_2 = \text{span}\{1, x, x^2\}$$

$$P_3 = \text{span}\{1, x, x^2, x^3\}$$

$$P_n = \text{span}\{1, x, x^2, x^3, \dots, x^n\}$$

✲ Standard unit matrix

$$M_{22} = \text{span}\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

|L-14 / 27.07.2022|

4.2

$v_1 = (1,1,2)$, $v_2 = (1,0,1)$, $v_3 = (2,1,3)$ span $\mathbb{R}^3$?

Take any vector $(a,b,c) \in \mathbb{R}^3$

Set,

$$(a,b,c) = k_1 v_1 + k_2 v_2 + k_3 v_3$$

$$= k_1(1,1,2) + k_2(1,0,1) + k_3(2,1,3)$$

$$= (k_1, k_1, 2k_1) + (k_2, 0, k_2) + (2k_3, k_3, 3k_3)$$

$$= (k_1 + k_2 + 2k_3, k_1 + 0 + k_3, 2k_1 + k_2 + 3k_3)$$

$\Rightarrow$

$$\left.\begin{aligned} k_1 + k_2 + 2k_3 &= a \\ k_1 \quad\quad + k_3 &= b \\ 2k_1 + k_2 + 3k_3 &= c \end{aligned}\right\} \text{System of linear equation}$$

Co-effecient matrix,

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 1(0-1) - 1(3-2) + 2(1-0)$$

$$= -1 - 1 + 2$$

$$= 0$$

$\det(A)$ is zero. So, system has no solution.

So, k_1, k_2, k_3 do not exist.

So, v_1, v_2, v_3 do not span $\mathbb{R}^3$.

4.3

Linear Independence

$v_1 = (2,3)$, $v_2 = (4,6)$

Relation:

$$v_2 = 2v_1$$

$$2v_1 - v_2 = (0,0)$$

Here, scalers,

$$2 \cdot 1 = 1 \neq 0$$

So, vectors v_1, v_2 are dependent.

⊛ $w_1 = (2,3)$, $w_2 = (5,-1)$

$0 \cdot w_1 + 0 \cdot w_2 = (0,0)$

No, other scaler can't make it zero.

So, vector w_1, w_2 are independent.

⊛ In $\mathbb{R}^3$

$e_1 = (1,0,0)$ $\quad e_2 = (0,1,0)$ $\quad e_3 = (0,0,1)$

Take $\quad k_1 e_1 + k_2 e_2 + k_3 e_3 = 0$

$k_1 (1,0,0) + k_2 (0,1,0) + k_3 (0,0,1) = (0,0,0)$

$\Rightarrow (k_1, k_2, k_3) = (0,0,0)$

$\therefore \left.\begin{array}{l} k_1 = 0 \\ k_2 = 0 \\ k_3 = 0 \end{array}\right\}$ only one solution is zero. So, they are linearly independent.

$\therefore \{e_1, e_2, e_3\}$ is a linear independent set.

⊛ In P_2

$\{1, x, x^2\}$ is linearly independent.

$$k_1 + k_2 x + k_3 x^2 = 0$$

$$\Rightarrow k_1 = 0, \ k_2 = 0, \ k_3 = 0$$

⊛ In homogenus system, if determinate is 0 then they are dependent, else independent.

⊛ Linear homogenus system having equation more than variable, then system has only zero solution. So, they are linearly independent.

L-15/ 08.08.2022/

$\boxed{4.4}$

S Basis for V

$\Rightarrow$ S is linearly independent

$\Rightarrow$ S spans V

* R^3

$S = \{v_1, v_2, v_3\}$ is independent

then it is spannis as well

and vice-versa

* R^3

standard unit basis for $R^3 = \{e_1, e_2, e_3\}$

standard basis for P_2 ! $\{1, x, x^2\}$

M_{22} ! $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

Example-9/

$S = \{ v_1 = (1,2,1),\ v_2 = (2,9,0),\ v_3 = (3,3,4) \}$

is a basis for $\mathbb{R}^3$

$v = (5,-1,9)$ $\quad (v)_S = ?$

Take, $\quad v = c_1 v_1 + c_2 v_2 + c_3 v_3$

$$(5,-1,9) = c_1(1,2,1) + c_2(2,9,0) + c_3(3,3,4)$$

$$= (c_1 + 2c_2 + 3c_3,\ 2c_1 + 9c_2 + 3c_3,\ c_1 + 4c_3)$$

$\therefore$

$$c_1 + 2c_2 + 3c_3 = 5$$

$$2c_1 + 9c_2 + 3c_3 = -1$$

$$c_1 \qquad + 4c_3 = 9$$

Augmented matrix.

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 9 & 3 & -1 \\ 1 & 0 & 4 & 9 \end{bmatrix} \longrightarrow \text{Solve it.}$$

by solving,

$c_1 = 1$

$c_2 = -1$

$c_3 = 2$

$\therefore (v)_s = (1, -1, 2)$

* In R^3

$v = (5, 2, -3)$

$S = \{(1,0,0), (0,1,0), (0,0,1)\}$ basis for R^3

$(v)_s = ?$

$(5, 2, -3) = c_1(1,0,0) + c_2(0,1,0) + c_3(0,0,1)$

$= (c_1, c_2, c_3)$

$\therefore c_1 = 5$

$c_2 = 2$

$c_3 = -3$

$\therefore (v)_s = (5, 2, -3)$

$\therefore (v)_s = v$ (Theorem)

(proved)

L-16 / 10.08.2022 /

4.5

Dimension of a vector space

$\dim(\mathbb{R}^3) = 3$

dimension = Positive integers

$P_2 = \{1, x, x^2\}$

$\dim(P_2) = 3$

$\dim(M_{22}) = 4 = 2\times 2$

$\dim(P_n) = n+1$

$= \{1, x, x^2, \dots, x^n\}$

Theorem 4.5.2

Let V be a finite-dimensional vector space, and let $\{v_1, v_2, \dots, v_n\}$ be any basis.

a) If a set has more than n ~~ve~~ vectors, then it is linearly dependent.

b) If a set has fewer than n vectors, then it does not span V.

Theorem 4.5.3

Let S be a nonempty set of vectors in a vector space V.

a) If S is a linearly independent set, and if v is a vector in V that is outside of span(S), then the set $S \cup \{v\}$ that results by inserting v into S is still linearly independent.

b) If v is a vector in S that is expressible as a linear combination of other vectors in S, and if $S - \{v\}$ denotes the set obtained by removing v from S, then $S - \{v\}$ span the same space; that is,

$$\text{span}(S) = \text{span}(S - \{v\})$$

⊛ Set of non zero solution of a homogenow system then the set is a subspace of this R^n space.

Example - 3

$x_1 = -s-t$

$x_2 = s$

$x_3 = -t$

$x_4 = 0$

$x_5 = t$

$$(x_1, x_2, x_3, x_4, x_5) = (-s-t, s, -t, 0, t)$$

$$= (-s, s, 0, 0, 0) + (-t, 0, -t, 0, t)$$

$$= s(-1, 1, 0, 0, 0) + t(-1, 0, -1, 0, 1)$$

$$(x_1, x_2, x_3, x_4, x_5) = s v_1 + t v_2$$

$\therefore$ v_1, v_2 span $\mathbb{R}^5$

$\{v_1, v_2\}$ is a basis for the solution space and dimension for the solution space 2.