e)

Given integral,

$$\int \frac{dy}{y\sqrt{5y^2-3}}$$

$$= \int \frac{dy}{y\sqrt{5\left(y^2-\frac{3}{5}\right)}}$$

$$= \frac{1}{\sqrt{5}} \int \frac{dy}{y\sqrt{y^2-\left(\frac{\sqrt{3}}{\sqrt{5}}\right)^2}}$$

$$= \frac{1}{\sqrt{5}} \cdot \frac{1}{\frac{\sqrt{3}}{\sqrt{5}}} \cdot \sec^{-1} \frac{y}{\sqrt{3}/\sqrt{5}} + c$$

$$= \frac{1}{\sqrt{3}} \sec^{-1} \frac{\sqrt{5}y}{\sqrt{3}} + c$$

Therefore the value of the given integral is,

$$\frac{1}{\sqrt{3}} \sec^{-1} \frac{\sqrt{5}y}{\sqrt{3}} + c.$$

5.4

35)

We need to make $n$ subintervals.

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$$

Now,
point, $x_k = a + k\Delta x$

$$= 1 + k\cdot\frac{3}{n}$$

$$= 1 + \frac{3k}{n}$$

Now,
Area, $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\cdot\Delta x$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(1+\frac{3k}{n}\right)\cdot\frac{3}{n}$$

Here, $f(x) = \frac{x}{2}$

$$\therefore f\left(1+\frac{3k}{n}\right) = \frac{1+\frac{3k}{n}}{2}$$

$$\therefore \lim_{n\to\infty} \sum_{k=1}^{n} f\left(1+\frac{3k}{n}\right)\cdot\frac{3}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1}{2}\left(1+\frac{3k}{n}\right)\frac{3}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{3}{2}\left(\frac{1}{n}+\frac{3}{n^2}k\right)$$

$$= \lim_{n\to\infty} \frac{3}{2}\left[\sum_{k=1}^{n}\frac{1}{n} + \sum_{k=1}^{n}\frac{3}{n^2}k\right]$$

$$= \lim_{n\to\infty} \frac{3}{2}\left[1+\frac{3}{n^2}\cdot\frac{1}{2}n(n+1)\right]$$

$$= \lim_{n\to\infty} \frac{3}{2}\left[1+\frac{3}{2}\frac{n+1}{n}\right]$$

$$= \lim_{n\to\infty} \frac{3}{2}\left[1+\frac{3}{2}\cdot\left(1+\frac{1}{n}\right)\right]$$

$$= \frac{3}{2}\left(1+\frac{3}{2}\right)$$

$$= \frac{15}{4}$$

Therefore area is $\frac{15}{4}$.

36)

We need to make $n$ sub intervals.

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{5-0}{n} = \frac{5}{n}$$

Now, point, $x_k = a + k\cdot\Delta x$

$$= 0 + k\cdot\frac{5}{n}$$

$$= \frac{5k}{n}$$

Now,

$$\text{Area, } A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\cdot\Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{5k}{n}\right)\cdot\frac{5}{n}$$

Here $f(x) = 5-x$

$$\therefore f\left(\frac{5k}{n}\right) = 5-\frac{5k}{n}$$

$$\therefore \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{5k}{n}\right)\cdot\frac{5}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(5-\frac{5k}{n}\right)\frac{5}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{25}{n}-\frac{25}{n^2}k\right)$$

$$= \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{25}{n} - \sum_{k=1}^{n} \frac{25}{n^2}k\right)$$

$$= \lim_{n\to\infty} \left[25 - \frac{25}{n^2}\cdot\frac{1}{2}n(n+1)\right]$$

$$= \lim_{n\to\infty} \left[25-\frac{25}{2}\left(\frac{n+1}{n}\right)\right]$$

$$= \lim_{n\to\infty} \left[25-\frac{25}{2}\left(1+\frac{1}{n}\right)\right]$$

$$= 25-\frac{25}{2}$$

$$= \frac{25}{2}$$

Therefore, area is $\frac{25}{2}$.

371

We need to make n sub intervals.

$\therefore$ each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

Now, point, $x_k = a + k \cdot \Delta x$

$$= 0 + k \cdot \frac{3}{n}$$

$$= \frac{3k}{n}$$

Now, Area, $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{3k}{n}\right) \cdot \frac{3}{n}$$

Here,

$$f(x) = 9 - x^2$$

$$\therefore f\left(\frac{3k}{n}\right) = 9 - \left(\frac{3k}{n}\right)^2 = 9 - \frac{9}{n^2} k^2$$

$$\therefore \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{3k}{n}\right) \cdot \frac{3}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(9 - 9\frac{k^2}{n^2}\right) \frac{3}{n}$$

$$= \lim_{n\to\infty} \left[\frac{27}{n} \sum_{k=1}^{n} \left(1 - \frac{k^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty} \left[27 - \frac{27}{n^3} \sum_{k=1}^{n} k^2\right]$$

$$= 27 - 27\left(\frac{1}{3}\right)$$

$$= 18$$

Therefore
Area = 18.

38)

We need to make n intervals.

$\therefore$ Each sub Interval length,

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

Now point, $x_k = a + k \cdot \Delta x$

$$= 0 + k \cdot \frac{3}{n}$$

$$= \frac{3k}{n}$$

Now, Area, $A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} f\left(\frac{3k}{n}\right) \cdot \frac{3}{n}$$

Here,

$$f(x) = 4 - \frac{1}{4} x^2$$

$$\therefore f\left(\frac{3k}{n}\right) = 4 - \frac{1}{4}\left(\frac{3k}{n}\right)^2$$

$$= 4 - \frac{1}{4} \cdot \frac{9k^2}{n^2}$$

$$= 4 - \frac{9}{4} \cdot \frac{k^2}{n^2}$$

$$\therefore \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{3k}{n}\right)\frac{3}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(4 - \frac{9}{4}\frac{k^2}{n^2}\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \left[\sum_{k=1}^{n} \left(\frac{12}{n} - \frac{27k^2}{4n^3}\right)\right]$$

$$= \lim_{n\to\infty} \left[\sum_{k=1}^{n} \frac{12}{n} - \frac{27}{4n^3}\sum_{k=1}^{n} k^2\right]$$

$$= \lim_{n\to\infty} \left[12 - \frac{27}{4n^3} \cdot \frac{1}{6} n(n+1)(2n+1)\right]$$

$$= \lim_{n\to\infty} \left(12 - \frac{9}{8}\frac{(n+1)(2n+1)}{n^2}\right)$$

$$= \lim_{n\to\infty} \left[12 - \frac{9}{8}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right]$$

$$= 12 - \frac{9}{8}\cdot 1 \cdot 2$$

$$= \frac{39}{4}$$

Therefore Area $= \frac{39}{4}$.

39)

We need to make $n$ intervals,

Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{6-2}{n} = \frac{4}{n}$$

Now, point, $x_k = a + k\cdot\Delta x$

$$= 2 + k\cdot\frac{4}{n}$$

$$= 2 + \frac{4k}{n}$$

Now, Area, $A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$

$= \lim_{n \to \infty} \sum_{k=1}^{n} f\left(2 + \frac{4k}{n}\right) \frac{4}{n}$

Here, $f(x) = x^3$

$\therefore f\left(2 + \frac{4k}{n}\right) = \left(2 + \frac{4k}{n}\right)^3$

$$\therefore \lim_{n \to \infty} \sum_{k=1}^{n} f\left(2 + \frac{4k}{n}\right) \frac{4}{n} = \lim_{n \to \infty} \sum_{k=1}^{n} \left(2 + \frac{4k}{n}\right)^3 \cdot \frac{4}{n}$$

$$= \lim_{n \to \infty} \left[\frac{32}{n} \sum_{k=1}^{n} \left(1 + \frac{2}{n}k\right)^3\right]$$

$$= \lim_{n \to \infty} \left[\frac{32}{n} \sum_{k=1}^{n} \left(1 + \frac{6}{n}k + \frac{12}{n^2}k^2 + \frac{8}{n^3}k^3\right)\right]$$

$$= \lim_{n \to \infty} \left[\frac{32}{n} \left(\sum_{k=1}^{n} 1 + \frac{6}{n}\sum_{k=1}^{n} k + \frac{12}{n^2}\sum_{k=1}^{n} k^2 + \frac{8}{n^3}\sum_{k=1}^{n} k^3\right)\right]$$

$$= \lim_{n \to \infty} \left[\frac{32}{n} \left(n + \frac{6}{n} \cdot \frac{1}{2} n(n+1) + \frac{12}{n^2} \cdot \frac{1}{6} n(n+1)(2n+1) + \frac{8}{n^3} \cdot \frac{1}{4} n^2 (n+1)^2\right)\right]$$

$$= \lim_{n \to \infty} \left[32 \left(1 + 3\frac{n+1}{n} + 2\frac{(n+1)(2n+1)}{n^2} + 2\frac{(n+1)^2}{n^2}\right)\right]$$

$$= \lim_{n \to \infty} \left[32 \left(1 + 3\left(1 + \frac{1}{n}\right) + 2\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) + 2\left(1 + \frac{1}{n}\right)^2\right)\right]$$

$$= 32\left(1 + 3 \cdot 1 + 2 \cdot 1 \cdot 2 + 2 \cdot 1^2\right)$$

$$= 320$$

Therefore Area, $A = 320$.

4o)

We need to make $n$ sub intervals.

∴ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{-1+3}{n} = \frac{2}{n}$$

Now, point, $x_k = a + k.\Delta x$

$$= -3 + k.\frac{2}{n}$$

$$= \frac{2k}{n} - 3$$

Now, Area, $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k).\Delta x$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{2k}{n}-3\right).\frac{2}{n}$$

Here, $f(x) = 1 - x^3$

$$\therefore f\left(\frac{2k}{n}-3\right) = 1 - \left(\frac{2k}{n}-3\right)^3$$

$$\therefore \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{2k}{n}-3\right).\frac{2}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left\{1-\left(\frac{2k}{n}-3\right)^3\right\}\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n}\left[28 - \frac{54}{n}k + \frac{36}{n^2}k^2 - \frac{8}{n^3}k^3\right]$$

$$= \lim_{n\to\infty} \left[\frac{2}{n}\left(28n - 27(n+1) + 6\frac{(n+1)(2n+1)}{n} - 2\frac{(n+1)^2}{n}\right]\right.$$

$$= \lim_{n\to\infty} \left[2\left(28 - 27\left(1+\frac{1}{n}\right) + 6\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - 2\left(1+\frac{1}{n}\right)^2\right)\right]$$

$$= 2(28 - 27 + 12 - 2) = 22$$

Therefore, Area, $A = 22$.

4.1)

We need to make $n$ sub intervals,

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$$

points, $x_k = a + (k-1)\Delta x$

$$= 1 + (k-1)\frac{3}{n}$$

Now, Area,

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\cdot \Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(1+(k-1)\frac{3}{n}\right)\frac{3}{n}$$

Here,

$$f(x) = \frac{x}{2}$$

$$\therefore f\left(1+(k-1)\frac{3}{n}\right) = \frac{1+(k-1)\frac{3}{n}}{2}$$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{2}\left(1+(k-1)\frac{3}{n}\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{2}\cdot\frac{3}{n} + \frac{1}{2}(k-1)\frac{9}{n^2}$$

$$= \lim_{n\to\infty} \frac{3}{2n}\cdot\frac{1}{n}\cdot\sum_{k=1}^{n}(n+3k-3)$$

$$= \lim_{n\to\infty} \frac{3}{2n^2}\left[\sum_{k=1}^{n} n + \sum_{k=1}^{n} 3k - 3\sum_{k=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} \frac{3}{2n^2}\left[n\sum_{k=1}^{n} 1 + 3\sum_{k=1}^{n} k - 3\sum_{k=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} \frac{3}{2n^2}\left[n^2 + \frac{3}{2}\cdot\frac{n(n+1)}{2} - 3n\right]$$

$$= \lim_{n\to\infty} \frac{3}{2}\left[1 + \frac{3}{2}\left(1+\frac{1}{n}\right) - 3\frac{1}{n}\right]$$

$$= \frac{3}{2}\left[1 + \frac{3}{2}(1+0) - 0\right]$$

$$= \frac{3}{2}\left[1+\frac{3}{2}\right]$$

$$= \frac{3}{2}\cdot\frac{5}{2} = \frac{15}{4}$$

Therefore, area is $\frac{15}{4}$.

42|

We need to make $n$ sub intervals.

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{5-0}{n} = \frac{5}{n}$$

points, $x_k = a + (k-1)\Delta x$

$= 0 + (k-1)\cdot\frac{5}{n}$

$= (k-1)\frac{5}{n}$

Now, Area,

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left((k-1)\frac{5}{n}\right) \frac{5}{n}$$

Here,

$$f(x) = 5 - x$$

$$\therefore f\left((k-1)\frac{5}{n}\right) = 5 - (k-1)\frac{5}{n}$$

$$\therefore A = \lim_{n\to\infty} \sum_{k=1}^{n} \left(5 - (k-1)\frac{5}{n}\right)\frac{5}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{25}{n} - \frac{25}{n^2}(k-1)\right]$$

$$= \lim_{n\to\infty} \left[\frac{25}{n}\sum_{k=1}^{n} 1 - \frac{25}{n^2}\sum_{k=1}^{n}(k-1)\right]$$

$$= \lim_{n\to\infty} \left(25 - \frac{25}{2}\frac{n-1}{n}\right)$$

$$= \lim_{n\to\infty} \left[25 - \frac{25}{2}\left(1 - \frac{1}{n}\right)\right]$$

$$= 25 - \frac{25}{2} \cdot 1$$

$$= \frac{25}{2}$$

Therefore area is $\frac{25}{2}$.

43)

We need to make n sub intervals,

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

points, $x_k = a + (k-1)\Delta x$

$$= 0 + (k-1)\frac{3}{n}$$

$$= (k-1)\frac{3}{n}$$

Now, Area,

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\cdot\Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left((k-1)\frac{3}{n}\right)\cdot\frac{3}{n}$$

Here,

$$f(x) = 9 - x^2$$

$$\therefore f\left((k-1)\frac{3}{n}\right) = 9 - \left((k-1)\frac{3}{n}\right)^2$$

$$= 9 - \frac{9}{n^2}(k-1)^2$$

$$\therefore A = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(9 - \frac{9}{n^2}(k-1)^2\right)\frac{3}{n}\right]$$

$$= \lim_{n\to\infty} \left[\frac{27}{n} \sum_{k=1}^{n} \left(1 - \frac{(k-1)^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty} \left[27 - \frac{27}{n^3} \sum_{k=1}^{n} k^2 + \frac{54}{n^3} \sum_{k=1}^{n} k - \frac{27}{n^2}\right]$$

$= \lim_{n \to \infty}$

$= 27 - 27\left(\frac{1}{3}\right) + 0 + 0$

$= 18$

Therefore the value or area is 18.

44/

We need to make n sub intervals,

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

points, $x_k = a + (k-1)\Delta x$

$= 0 + (k-1)\frac{3}{n}$

$= (k-1)\frac{3}{n}$

Now, Area,

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} f\left((k-1)\frac{3}{n}\right) \cdot \frac{3}{n}$$

Here, $f(x) = 4 - \frac{1}{4}x^2$

$\therefore f\left((k-1)\frac{3}{n}\right) = 4 - \frac{1}{4} \cdot \left((k-1)\frac{3}{n}\right)^2$

$= 4 - \frac{1}{4} \cdot \frac{9}{n^2}(k-1)^2 = 4 - \frac{9}{4n^2}(k-1)^2$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(4 - \frac{9}{4n^2}(k-1)^2\right)\frac{3}{n}\right]$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{12}{n} - \frac{27k^2}{4n^3} + \frac{27k}{2n^3} - \frac{27}{4n^3}$$

$$= \lim_{n\to\infty} 12 - \frac{27}{4n^3}\cdot\frac{1}{6}n(n+1)(2n+1) + \frac{27}{2n^3}\frac{n(n+1)}{2} - \frac{27}{4n^2}$$

$$= \lim_{n\to\infty} \left[12 - \frac{9}{8}\frac{(n+1)(2n+1)}{n^2} + \frac{27}{4n} + \frac{27}{4n^2} - \frac{27}{4n^2}\right]$$

$$= \lim_{n\to\infty} 12 - \frac{9}{8}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + 0 + 0 - 0$$

$$= 12 - \frac{9}{8}1\cdot 2$$

$$= \frac{39}{4}$$

Therefore, area is $\frac{39}{4}$.

45|

We need to make $n$ sub intervals.

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{n} = \frac{4}{n}$$

points, $x_k = a + (k-\frac{1}{2})\Delta x$

$$= 0 + (k-\tfrac{1}{2})\frac{4}{n}$$

$$= (k-\tfrac{1}{2})\frac{4}{n}$$

Now, area,

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} f\left(\left(k-\frac{1}{2}\right)\frac{4}{n}\right)\frac{4}{n}$$

Here

$f(x) = 2x$

$$\therefore f\left(\left(k-\frac{1}{2}\right)\frac{4}{n}\right) = 2 \cdot \frac{4}{n}\left(k-\frac{1}{2}\right)$$

$$= \frac{8}{n}\left(k-\frac{1}{2}\right)$$

$$\therefore A = \lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{8}{n}\left(k-\frac{1}{2}\right)\right)\frac{4}{n}$$

$$= \lim_{n \to \infty} \left[\sum_{k=1}^{n} \left(\frac{32}{n^2}k - \frac{16}{n^2}\right)\right]$$

$$= \lim_{n \to \infty} \left[\frac{32}{n^2} \cdot \frac{n(n+1)}{2} - \frac{16}{n^2} \cdot n\right]$$

$$= \lim_{n \to \infty} \left(16 \cdot \frac{n+1}{n} - \frac{16}{n}\right)$$

$$= \lim_{n \to \infty} \left[16 \cdot \left(1+\frac{1}{n}\right) - 16 \cdot \frac{1}{n}\right]$$

$$= 16 \cdot (1+0) - 16 \cdot 0$$

$$= 16$$

Therefore, area is 16.

46)

We need to make $n$ sub intervals,

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n}$$

points, $x_k = a + (k - \frac{1}{2}) \Delta x$

$$= 1 + (k - \tfrac{1}{2}) \frac{4}{n}$$

Now, area

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \Delta x$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} f\left(1 + (k - \tfrac{1}{2})\tfrac{4}{n}\right) \cdot \frac{4}{n}$$

Here,

$$f(x) = 6 - x$$

$$\therefore f\left(1 + (k - \tfrac{1}{2}) \tfrac{4}{n}\right) = 6 - 1 + (k - \tfrac{1}{2}) \tfrac{4}{n}$$

$$= 5 - (k - \tfrac{1}{2}) \frac{4}{n}$$

$$\therefore A = \lim_{n \to \infty} \sum_{k=1}^{n} \left(5 - (k - \tfrac{1}{2}) \tfrac{4}{n}\right) \frac{4}{n}$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} \left[\frac{20}{n} - \frac{16k}{n^2} + \frac{8}{n^2}\right]$$

$$= \lim_{n \to \infty} \left[\frac{20}{n} \cdot n - \frac{16}{n^2} \cdot \frac{n(n+1)}{2} + \frac{8}{n^2} \cdot n\right]$$

$$= \lim_{n \to \infty} 20 - 8 \cdot \left(1 + \tfrac{1}{n}\right) + 8 \frac{1}{n}$$

$= 20 - 8.1 + 0$

$= \cancel{k} 12$

Therefore area is 12.

47)

We need to make n sub intervals.

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$$

$\therefore$ points,

$$x_k = a + (k - \tfrac{1}{2})\Delta x$$

$$= 0 + (k - \tfrac{1}{2})\frac{1}{n}$$

$$= (k - \tfrac{1}{2})\frac{1}{n}$$

Now,

$$\text{Area, } A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \cdot \Delta x$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} f\left((k - \tfrac{1}{2})\frac{1}{n}\right) \cdot \frac{1}{n}$$

Here, $f(x) = x^2$

$$\therefore f\left((k - \tfrac{1}{2})\frac{1}{n}\right) = \left((k - \tfrac{1}{2})\frac{1}{n}\right)^2$$

$$= \frac{1}{n^2}(k - \tfrac{1}{2})^2$$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \left( \frac{1}{n^2} (k - \frac{1}{2})^2 \right) \frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n^3} (k^2 - k + \frac{1}{4})$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n^3} k^2 - \frac{1}{n^3} k + \frac{1}{4n^3}$$

$$= \frac{1}{3} + 0 + 0$$

$$= \frac{1}{3}$$

Therefore the area is $\frac{1}{3}$.

48)

We need to make $n$ sub intervals.

$\therefore$ Each sub interval length,

$$\Delta x = \frac{b-a}{n} = \frac{1+1}{n} = \frac{2}{n}$$

points,

$$x_k = a + (k - \tfrac{1}{2}) \Delta x$$

$$= -1 + (k - \tfrac{1}{2}) \cdot \frac{2}{n}$$

Now, area,

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k) \Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(-1 + (k-\tfrac{1}{2}) \tfrac{2}{n}\right) \cancel{\Delta x}\, \frac{2}{n}$$

Here, $f(x) = x^2$

$\therefore f\left(-1+(k-\frac{1}{2})\frac{2}{n}\right) = \left(-1+(k-\frac{1}{2})\frac{2}{n}\right)^2$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \left(-1+(k-\tfrac{1}{2})\tfrac{2}{n}\right)^2 \frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{8k^2}{n^3} - \frac{8k}{n^3} + \frac{2}{n^3} - \frac{2}{n}$$

$$= \frac{8}{3} + 0 + 0 - 2$$

$$= \frac{2}{3}$$

Therefore,

Area is $\frac{2}{3}$.