



⊗

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

→ Vacuously True

⊗ Predicate!

⇒ Every student in CSE 425 has done CSE 173.

Predicate Definition {  $P(x)$ : Student  $x$  from CSE 425 has done CSE 173  
 $x \in \text{Domain}$   
 ↪ CSE 425

⇒ True/False?

- process to obtain is Quantification

Quantifier {  $\forall$  → Universal (every, none, nobody, all)  
 $\exists$  → Existential (some, a few, at least a few, exactly 3)

⊗  $\forall x P(x) \Rightarrow \text{True/False?}$

$$P(x=a) \wedge P(x=b) \wedge \dots \wedge P(x=z)$$

⊗  $\exists x P(x)$

$P(x)$ :  $x$  owns a GPU

⇒ there exist at least one  $x$  such that  $P(x)$

$$P(x=a) \vee P(x=b) \vee \dots \vee P(x=z)$$



L-27/02.12.2024/

## \* Theory:

- Logic Programming  $\rightarrow$  Propositional Algebra

- Predicate Calculus

- Inference

$\Rightarrow$  Rules

- resolution

- resolution principle

- used to resolve queries  
in logic programming } Backward  
Chaining

\*  $x \in \text{Domain}$

$\forall x P(x)$ : For all  $x$   $P(x)$

$\exists x P(x)$ : There exist at least one  $x$  such that  $P(x)$

$\Rightarrow$

$x \in \text{NSU}$

$P(x)$ :  $x$  is from SEC-1

$Q(x)$ :  $x$  is good

$\Rightarrow \forall x (P(x) \rightarrow Q(x))$

$\Rightarrow x \in \text{NSU}$

$P(x)$ :  $x$  from SEC-1

$S(x)$ :  $x$  owns a super computer

$\exists x P(x) \wedge S(x)$

## \* Inference!

- to infer
- to guess based on available information & logical reasonings / deductions.

- we have rules  
 → are in propositional form.

## \* Rules:

$$\textcircled{i} \quad \begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

$$\textcircled{ii} \quad \begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

$$\textcircled{iii} \quad \begin{array}{l} p \wedge q \\ \hline \therefore p, q \end{array}$$

$$\textcircled{iv} \quad \begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

$$\textcircled{v} \quad \begin{array}{l} p \dots \dots \dots \\ \hline \therefore p \vee q \vee r \vee s \vee \dots \end{array} \quad \text{True}$$

$$\textcircled{vi} \quad \begin{array}{l} \text{Hypothetical} \\ \text{Syllogism} \\ p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

$$\textcircled{vii} \quad \begin{array}{l} \text{Disjunction} \\ \text{Syllogism} \\ p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

## $\textcircled{viii}$ Resolution → Clausal Form

$$\begin{array}{l} \boxed{p \vee q} \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array} \quad \text{True}$$

## \* Clausal Form:

- disjunction of literals or the single literal itself.

- Extracted form

- Conjunctive Normal Form (CNF)  $\Rightarrow \underbrace{(\dots \vee \dots)}_{\text{clausal form}} \wedge (\dots \vee \dots)$



## Resolution Principle

\*  $\neg P \rightarrow (Q \wedge R)$

$\equiv \neg(\neg P) \vee (Q \wedge R)$  Definition of implication

$\equiv P(Q \wedge R)$

$\equiv \underbrace{(P \vee Q)}_{\text{Clause-1}} \wedge \underbrace{(P \vee R)}_{\text{Clause-2}}$

### \* Resolvent:

$\Rightarrow C_1 = P \vee Q \vee R : C'_1 = Q \vee R$   
 $\Rightarrow C_2 = \neg P \vee \neg S \vee T : C'_2 = \neg S \vee T$

$\vee \Rightarrow \underbrace{C'_1 \vee C'_2}_{\text{Resolvent}}$



1.  $T \rightarrow (M \vee E) \rightarrow \neg T \vee M \vee E$

2.  $S \rightarrow \neg E \rightarrow \neg S \vee \neg E$

3.  $T \wedge S \rightarrow \begin{matrix} T \\ S \end{matrix}$  clausal itself

$\hookrightarrow M$

$\hookrightarrow$  Desired Conclusion

$\neg M$

$\rightarrow$  adding negated conclusion as a clausal form



1.  $\neg T \vee M \vee E$

2.  $\neg S \vee \neg E$

3.  $T$

4.  $S$

5.  $\neg M$

6.  $\neg T \vee M \vee \neg S$

7.  $M \vee \neg S$

8.  $M$

9. □

$\rightarrow$  Now calculate the resolvent?

$\therefore$  Resolvent 1,2

$\therefore$  Resolvent 3,6

$\therefore$  Resolvent 7,6

$\therefore$  Resolvent 8,5

$\rightarrow$  By definition, it is a FALSE FLAG

∴ So conclusion M is True (Proved by Contradiction)

⊛ Example:

Sister(x, z) : x is a sister of z

Parent(z, y) : z is the parent of y

Rule:

Head of the Horn Clause

Aunt(x, y) :- Female(x), Sister(x, z), Parent(z, y)

Horn Clause

x is the aunt  
of y

⊛

$\forall x \forall y \forall z (Female(x) \wedge Sister(x, z) \wedge Parent(z, y))$

(transform it to proposition)

Instantiation Process

Definition of Aunt

$x \in \text{Domain}$

$\forall x P(x) \quad ] \text{ True}$

$\rightarrow P(c)$

specific  
subject



L-28/03.12.2024/

## ⊗ Prolog knowledge Base

- Facts (Predicate)
- Rules: Defined as Horn Clause

Proposition Topic  
from 173 must  
read

"to be define" :- "Predicate Calculus"

→ sequence of predicate

|||

"sequence of predicate" → "term to be defined."

## ⊗ Previous Example:

Female (Sita)

Sister (Sita, Geetha)

Parents (Geetha, Mohan)

→ x is sister of z

Horn clause with Head.

Rule: Aunt (x, y) :- Female (x), Sister (x, z), Parents (z, y)

→ x is the Aunt of y

→ z is the parent of y

"," ≡ AND ≡ Conjunction ≡  $\wedge$

;" ≡ OR ≡ Disjunction ≡  $\vee$

⊗  $\forall x \forall y \forall z \left( \left( \text{Female}(x), \text{Sister}(x, z), \text{Parent}(z, y) \right) \rightarrow \text{Aunt}(x, y) \right)$

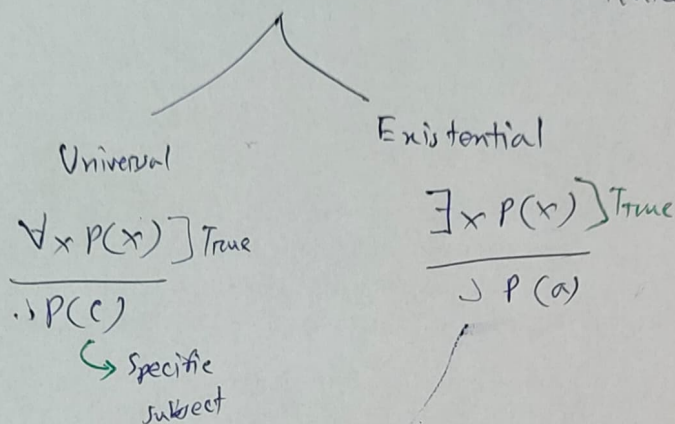
∴ Aunt (Sita, Mohan) = ?

⇒

1. Female (Sita)
2. Sister (Sita, Geetha)
3. Parent (Geetha, Mohan)

Clause.  
- each as propositional form: subject is sep specified.

\* Taking instantiation of the rules



$$\begin{aligned}
 4. & \left( \text{Female}(\text{Sita}) \wedge \text{Sister}(\text{Sita}, \text{Geetha}) \wedge \text{Parent}(\text{Geetha}, \text{Mohan}) \right) \rightarrow \text{Aunt}(\text{Sita}, \text{Mohan}) \\
 & \equiv \neg \left( \text{Female}(\text{Sita}) \wedge \text{Sister}(\text{Sita}, \text{Geetha}) \wedge \text{Parent}(\text{Geetha}, \text{Mohan}) \right) \vee \text{Aunt}(\text{Sita}, \text{Mohan}) \\
 & \equiv \neg \text{Female}(\text{Sita}) \vee \neg \text{Sister}(\text{Sita}, \text{Geetha}) \vee \neg \text{Parent}(\text{Geetha}, \text{Mohan}) \vee \text{Aunt}(\text{Sita}, \text{Mohan})
 \end{aligned}$$

$$5. \neg \text{Aunt}(\text{Sita}, \text{Mohan})$$

$$6. \square \Rightarrow \text{[Line 4 and 1, 2, 3, 5 all are eliminated as opposite form and both are True.]}$$

↪ Empty clause

↪ FALSE



## \* Practice on Swiss Prolog

→ always use trace to understand the step.

### \* Question Pattern:

- Graph will be given
  - define edge on connected lines, (visual graph)
  - ~~wright~~ ,
  - write the rule and proposition
  - need to write all edge predicate, undirect  $\Rightarrow$  add both way
- $\Rightarrow$  edge graph
- $\Rightarrow$  Mathematical function (area, square, factorial, Fibonacci)
- $\Rightarrow$  Relationship - family tree
- $\Rightarrow$  List Example (May not important for exam)

L-29/04.12.2024/

### \* lambda Calculus:

function  $\Rightarrow$  Prefix of a statement.

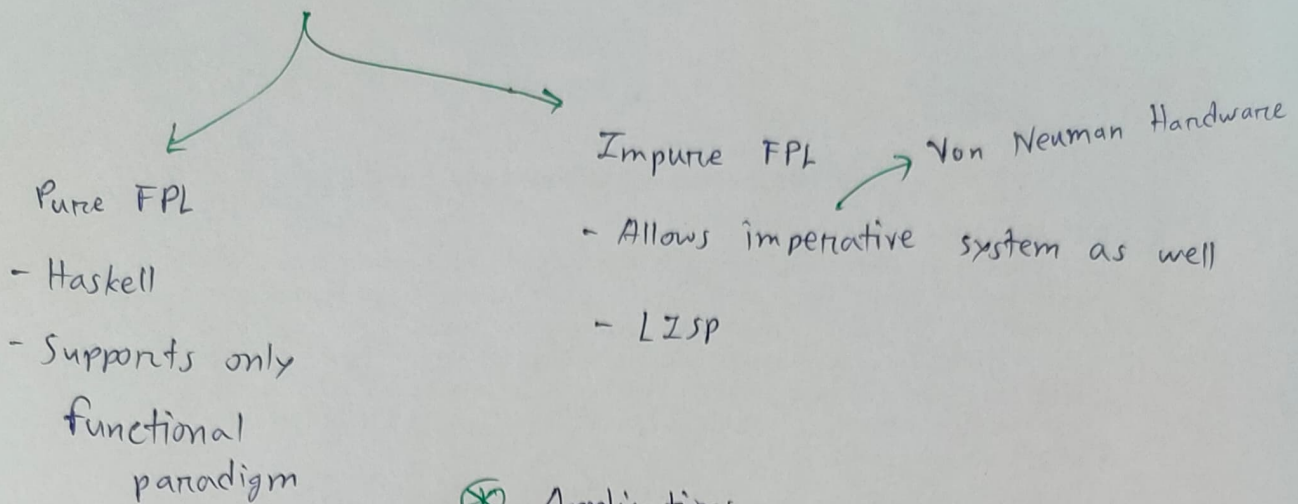
~~etc~~

### \* Functional Programming Language (FPL):

OCaml PL

- Handle symbolic computation
- List processing application
- flexible storage of values.
- we are concerned more on the functionality & less about "memory related" variable storage.

⇒ Two sub-class



### ⊗ Applications

- mainly AI area
- Natural Language Processing
- Speech and vision processing.

### ⊗ Example!

<u>C</u>	<u>FPL</u>
<pre>sum = 0; for (i = 1; i ≤ n; i++) {     sum = sum + i; }</pre>	<pre>sum [1, 2, 3, ..., n]</pre> <p>→ function does the computation.</p>
<pre>x = 7;</pre>	<pre>x = f(y)</pre>

### ⊗ Functional Programming Language ⇒ Elements

- List
- Function
- Composition ( $g \circ f, f \circ g$ )
- Recursive



## \* Lambda Calculus:

$\langle \text{expression} \rangle \rightarrow \langle \text{name} \rangle / \langle \text{function} \rangle / \langle \text{application} \rangle$

$\langle \text{function} \rangle \rightarrow \lambda \langle \text{name} \rangle. \langle \text{expression} \rangle$

$\langle \text{application} \rangle \rightarrow \frac{\langle \text{expression} \rangle}{E} \frac{\langle \text{expression} \rangle}{E}$   
 $\swarrow$   
 $E_1, E_2, E_3, \dots, E_n$

\*  $() \Rightarrow$  for separating expression.

$(+ 3 4) \Rightarrow 3 + 4 = 7$

$((+ 3) 4) \Rightarrow 3 + 4 = 7$   
 $\swarrow$   
 as input of

$\lambda x. \underline{x * x}$   
 $\nearrow$  bound variable  
 $\nearrow$  function of  $x$   
 $\searrow$  expression

\* Left association:

$((((E_1) E_2) E_3) \dots E_n)$

\* Free variable:

$(\lambda x. x)$   
 $\swarrow$  Body  
 - expression  
 -  $x$   
 - bound

$(\lambda y. y x)$   
 $\swarrow$  body  
 - expression  
 -  $y x$   
 $\swarrow$  Bound  $\searrow$  Free

\*  $(+ (\times 5 6) (\times 8 3))$

$\rightarrow (+ (30) (\times 8 3))$

$\rightarrow (+ 30 24)$

$\rightarrow 54$

$$\textcircled{*} (\lambda x (\lambda y. x+y) 5) ((\lambda y. y * y) 6) = ?$$

$$\rightarrow (\lambda x. x+5) ((\lambda y. y * y) 6)$$

$$\rightarrow (\lambda x. x+5) (6 * 6)$$

$$\rightarrow (6 * 6) + 5$$

$$\rightarrow 41.$$

One Question must from  
Lambda function

$\textcircled{*}$  Map function.

Final Exam

- 19.12.2024