North South University

Department of Mathematics and Physics

Assignment - 1

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Course No : MAT-125

Course Title : Introduction to Linear Algebra

Section : 10

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Section 1.1

14)

In each part, find the augmented matrix for the given system of linear equations.

b)

$$\begin{aligned} 2x_1 \quad\quad\quad + 2x_3 &= 1 \\ 3x_1 - x_2 + 4x_3 &= 7 \\ 6x_1 + x_2 - x_3 &= 0 \end{aligned}$$

Solution:

Augmented matrix:

$$\begin{bmatrix} 2 & 0 & 2 & 1 \\ 3 & -1 & 4 & 7 \\ 6 & 1 & -1 & 0 \end{bmatrix}$$

c)

$$
\begin{aligned}
x_1 + 2x_2 \qquad\qquad - x_4 + x_5 &= 1 \\
3x_2 + x_3 \qquad - x_5 &= 2 \\
x_3 + 7x_4 \qquad &= 1
\end{aligned}
$$

Solution:

Augmented matrix:

$$
\begin{bmatrix}
1 & 2 & 0 & -1 & 1 & 1 \\
0 & 3 & 1 & 0 & -1 & 2 \\
0 & 0 & 1 & 7 & 0 & 1
\end{bmatrix}
$$

Section - 1.2

4)

In each part, suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given reduced row echelon form. Solve the system.

c)

$$\begin{bmatrix} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution:

The corresponding system of equations is,

$$x_1 - 6x_2 + 3x_5 = -2$$

$$x_3 + 4x_5 = 7$$

$$x_4 + 5x_5 = 8$$

Here, x_1, x_3, x_4 are leading variables.

Solving for the leading variables we obtain,

$$x_1 = 6x_2 - 3x_5 - 2$$

$$x_3 = -4x_5 + 7$$

$$x_4 = -5x_5 + 8$$

Here, x_2 and x_5 are free variables.

Let,

$$x_2 = r$$

$$x_5 = s$$

Now,

$$x_1 = 6r - 3s - 2$$
$$x_2 = r$$
$$x_3 = -4s + 7$$
$$x_4 = -5s + 8$$
$$x_5 = s$$

where, r, s can be any real numbers.

6)

Solve the linear system by Gauss-Jordan elimination.

$$2x_1 + 2x_2 + 2x_3 = 0$$
$$-2x_1 + 5x_2 + 2x_3 = 1$$
$$8x_1 + x_2 + 4x_3 = -1$$

Solution:

The corresponding augmented matrix is,

$$\begin{bmatrix} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix}$$

$$\xrightarrow{r_1' = \frac{1}{2} r_1} \begin{bmatrix} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix}$$

$$\xrightarrow[r_3' = r_3 - 8r_1]{r_2' = r_2 + 2r_1} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{bmatrix}$$

$$\xrightarrow{r_2' = \frac{1}{7} r_2} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & -7 & -4 & -1 \end{bmatrix}$$

$$r_3' = r_3 + 7r_1 \quad \longrightarrow \quad \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$r_1' = r_1 - r_2 \quad \begin{bmatrix} 1 & 0 & \frac{3}{7} & -\frac{1}{7} \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The above matrix is in reduced row echelon form.

The corresponding system or equation is,

$$x_1 + \frac{3}{7}x_3 = -\frac{1}{7}$$

$$x_2 + \frac{4}{7}x_3 = \frac{1}{7}$$

Here, x_1 and x_2 are leading variables.

Solving for the leading variables we obtain,

$$x_1 = -\frac{3}{7}x_3 - \frac{1}{7}$$

$$x_2 = -\frac{4}{7}x_3 + \frac{1}{7}$$

Here, x_3 is free variables.

Let, $x_3 = r$

Now,

$$x_1 = -\frac{3}{7}r - \frac{1}{7}$$

$$x_2 = -\frac{4}{7}r + \frac{1}{7}$$

$$x_3 = r$$

where, r can be any real number.

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Solve the given homogeneous linear system by any method.

$$\begin{aligned} v + 3w - 2x &= 0 \\ 2u + v - 4w + 3x &= 0 \\ 2u + 3v + 2w - x &= 0 \\ -4u - 3v + 5w - 4x &= 0 \end{aligned}$$

Solution:

The corresponding augmented matrix is,

$$\begin{bmatrix} 0 & 1 & 3 & -2 & 0 \\ 2 & 1 & -4 & 3 & 0 \\ 2 & 3 & 2 & -1 & 0 \\ -4 & -3 & 5 & -4 & 0 \end{bmatrix}$$

$$\xrightarrow{r_1 \rightleftharpoons r_2} \begin{bmatrix} 2 & 1 & -4 & 3 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 2 & 3 & 2 & -1 & 0 \\ -4 & -3 & 5 & -4 & 0 \end{bmatrix}$$

$$\xrightarrow{r_1 = \frac{1}{2} r_1} \begin{bmatrix} 1 & \frac{1}{2} & -2 & \frac{3}{2} & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 2 & 3 & 2 & -1 & 0 \\ -4 & -3 & 5 & -4 & 0 \end{bmatrix}$$

$$\xrightarrow[r_4' = r_4 + 4r_1]{r_3' = r_3 - 2r_1} \begin{bmatrix} 1 & \frac{1}{2} & -2 & \frac{3}{2} & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 2 & 6 & -4 & 0 \\ 0 & -1 & -3 & 2 & 0 \end{bmatrix}$$

$$\xrightarrow[r_4' = r_4 + r_2]{r_3' = r_3 - 2r_2} \begin{bmatrix} 1 & \frac{1}{2} & -2 & \frac{3}{2} & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$r_1' = r_1 - \frac{1}{2} r_2 \begin{bmatrix} 1 & 0 & -\frac{7}{2} & \frac{5}{2} & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The above matrix is in reduced row echelon form.

The corresponding system of equation is,

$$u - \frac{7}{2}w + \frac{5}{2}x = 0$$

$$v + 3w - 2x = 0$$

Here, u and v are leading variables.

Solving for the leading variables we obtain,

$$u = \frac{7}{2}w - \frac{5}{2}x$$

$$v = -3w + 2x$$

Here, w and x are free variables.

Let,

$$w = s$$

$$x = t$$

Now,

$$u = \frac{7}{2}s - \frac{5}{2}t$$

$$v = -3s + 2t$$

$w = s$ and $x = t$

where, s and t can be any real numbers.

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Determine the values of a for which the system has no solutions, exactly one solution, or infinitely many solutions.

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2-14)z = a+2$$

Solution:

The corresponding augmented matrix is,

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{bmatrix}$$

$$\xrightarrow[r_3' = r_3 - 4r_1]{r_2' = r_2 - 3r_1} \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{bmatrix}$$

$$r_2' = -\frac{1}{7} r_2 \longrightarrow \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & -7 & a^2-2 & a-14 \end{bmatrix}$$

$$r_3' = r_3 + 7r_2 \quad \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & a^2-16 & a-4 \end{bmatrix}$$

The above matrix is in row echelon form.

The corresponding system of equations is,

$$x + 2y - 3z = 4$$
$$y - 2z = \frac{10}{7}$$
$$(a^2-16)z = a-4$$

Considering the last equations,

$$(a^2-16)z = a-4$$

For, $a = 4$,

$$\therefore (4^2 - 16)z = 4 - 4$$

$$\Rightarrow (16 - 16)z = 0$$

$$\Rightarrow 0 \cdot z = 0$$

$$\Rightarrow 0 = 0$$

if $a = 4$, then the system will have infinitely many solutions.

For, $a = -4$,

$$((-4)^2 - 16)z = -4 - 4$$

$$\Rightarrow (16 - 16)z = -8$$

$$\Rightarrow 0 \cdot z = -8$$

$$\Rightarrow 0 = -8$$

if $a = -4$, then the system will be inconsistent. So, there will be no solutions.

If a takes values other than 4 and -4, then the system will be have exactly one solution.

Therefore, the system has no solutions when $a = -4$, infinitely many solutions when $a = 4$, and exactly one solution when $a \neq \pm 4$.

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Solve the following system for x, y, and z.

$$\frac{1}{x} + \frac{2}{y} - \frac{4}{z} = 1$$

$$\frac{2}{x} + \frac{3}{y} + \frac{8}{z} = 0$$

$$-\frac{1}{x} + \frac{9}{y} + \frac{10}{z} = 5$$

Solution:

The given system is

$$\frac{1}{x} + \frac{2}{y} - \frac{4}{z} = 1$$

$$\frac{2}{x} + \frac{3}{y} + \frac{8}{z} = 0$$

$$-\frac{1}{x} + \frac{9}{y} + \frac{10}{z} = 5$$

Let,

$$X = \frac{1}{x}$$

$$Y = \frac{1}{y}$$

$$Z = \frac{1}{z}$$

Now, substitute X, Y, and Z in the given system.

$$X + 2Y - 4Z = 1$$

$$2X + 3Y + 8Z = 0$$

$$-X + 9Y + 10Z = 5$$

The corresponding augmented matrix is,

$$\begin{bmatrix} 1 & 2 & -4 & 1 \\ 2 & 3 & 8 & 0 \\ -1 & 9 & 10 & 5 \end{bmatrix}$$

$$\xrightarrow[r_3' = r_3 + r_1]{r_2' = r_2 - 2r_1} \begin{bmatrix} 1 & 2 & -4 & 1 \\ 0 & -1 & 16 & -2 \\ 0 & 11 & 6 & 6 \end{bmatrix}$$

$$\xrightarrow{r_2' = -r_2} \begin{bmatrix} 1 & 2 & -4 & 1 \\ 0 & 1 & -16 & 2 \\ 0 & 11 & 6 & 6 \end{bmatrix}$$

$$\xrightarrow{r_3' = r_3 - 11r_1} \begin{bmatrix} 1 & 2 & -4 & 1 \\ 0 & 1 & -16 & 2 \\ 0 & 0 & 182 & -16 \end{bmatrix}$$

$$\xrightarrow{r_3' = \frac{1}{182} r_3} \begin{bmatrix} 1 & 2 & -4 & 1 \\ 0 & 1 & -16 & 2 \\ 0 & 0 & 1 & -\frac{8}{91} \end{bmatrix}$$

$$\xrightarrow[r_2' = r_2 + 16r_3]{r_1' = r_1 + 4r_3} \begin{bmatrix} 1 & 2 & 0 & \frac{59}{91} \\ 0 & 1 & 0 & \frac{54}{91} \\ 0 & 0 & 1 & \frac{-8}{91} \end{bmatrix}$$

$$\xrightarrow{r_1' = r_1 - 2r_2} \begin{bmatrix} 1 & 0 & 0 & -\frac{7}{13} \\ 0 & 1 & 0 & \frac{54}{91} \\ 0 & 0 & 1 & -\frac{8}{91} \end{bmatrix}$$

The above matrix is in reduced row echelon form
The corresponding system of equation is

$$X = -\frac{7}{13}$$
$$Y = \frac{54}{91}$$
$$Z = -\frac{8}{91}$$

Now back substitute X, Y, Z and solve for x, y, z.

$$\therefore X = -\frac{7}{13}$$
$$\Rightarrow \frac{1}{x} = -\frac{7}{13}$$
$$\Rightarrow x = -\frac{13}{7}$$

$$\therefore Y = \frac{54}{91}$$
$$\Rightarrow \frac{1}{y} = \frac{54}{91}$$
$$\Rightarrow y = \frac{91}{54}$$

$$\therefore z = -\frac{8}{91}$$

$$\Rightarrow \frac{1}{z} = -\frac{8}{91}$$

$$\Rightarrow z = -\frac{91}{8}$$

Therefore, the solution is

$$x = -\frac{13}{7}$$

$$y = \frac{91}{54}$$

$$z = -\frac{91}{8}$$

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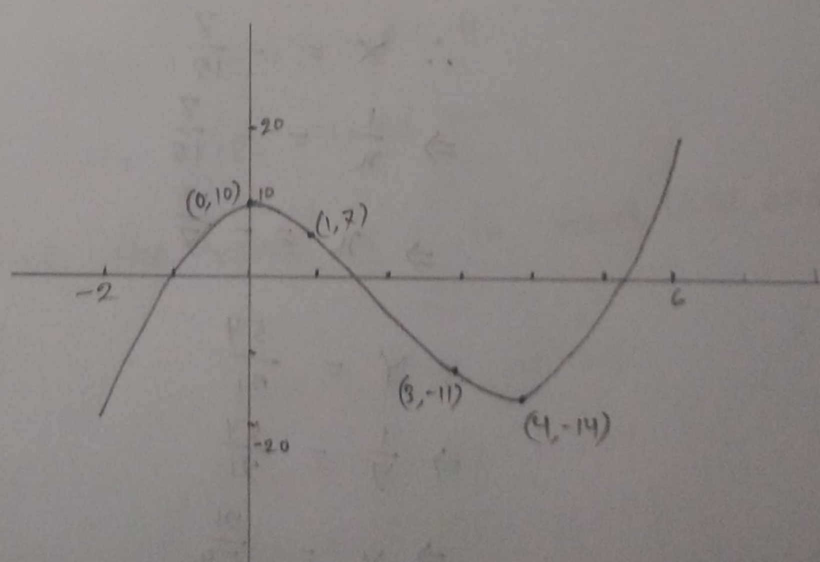
Find the coefficients a, b, c, and d so that the curve shown in the accompanying figure is the graph of the equation

$$y = ax^3 + bx^2 + cx + d$$



Solution:

Given that, the curve $y = ax^3 + bx^2 + cx + d$ passes through the points $(0,10)$, $(1,7)$, $(3,-11)$, $(4,-14)$.

So, these point satisfy the curve equation.

So, we can write that,

$$10 = a\cdot 0^3 + b\cdot 0^2 + c\cdot 0 + d$$

$$7 = a\cdot 1^3 + b\cdot 1^2 + c\cdot 1 + d$$

$$-11 = a\cdot 3^3 + b\cdot 3^2 + c\cdot 3 + d$$

$$-14 = a\cdot 4^3 + b\cdot 4^2 + c\cdot 4 + d.$$

We can simplify and rewrite the above system of equations as follows,

$$d = 10$$

$$a + b + c + d = 7$$

$$27a + 9b + 3c + d = -11$$

$$64a + 16b + 4c + d = -14$$

Lets substitute the value of d in second, third, and fourth equations.

$$a + b + c = -3$$

$$27a + 9b + 3c = -21$$

$$64a + 16b + 4c = -24$$

The corresponding augmented matrix is,

$$\begin{bmatrix} 1 & 1 & 1 & -3 \\ 27 & 9 & 3 & -21 \\ 64 & 16 & 4 & -24 \end{bmatrix}$$

$$\xrightarrow[r_3' = r_3 - 64r_1]{r_2' = r_2 - 27r_1} \begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & -18 & -24 & 60 \\ 0 & -48 & -60 & 168 \end{bmatrix}$$

$$\xrightarrow{r_2' = -\frac{1}{18} r_2} \begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & 1 & \frac{4}{3} & -\frac{10}{3} \\ 0 & -48 & -60 & 168 \end{bmatrix}$$

$$\xrightarrow{r_3' = r_3 + 48 r_2} \begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & 1 & \frac{4}{3} & -\frac{10}{3} \\ 0 & 0 & 4 & 8 \end{bmatrix}$$

$$\xrightarrow{r_3' = \frac{1}{4} r_3} \begin{bmatrix} 1 & 1 & 1 & -3 \\ 0 & 1 & \frac{4}{3} & -\frac{10}{3} \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\xrightarrow[r_2' = r_2 - \frac{4}{3} r_3]{r_1' = r_1 - r_3} \begin{bmatrix} 1 & 1 & 0 & -5 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\xrightarrow{r_1' = r_1 - r_2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

The above matrix is in reduced row echelon form.

The corresponding system of equations is,

$a = 1$

$b = -6$

$c = 2$

Therefore, the solution is,

$a = 1$

$b = -6$

$c = 2$

$d = 10$

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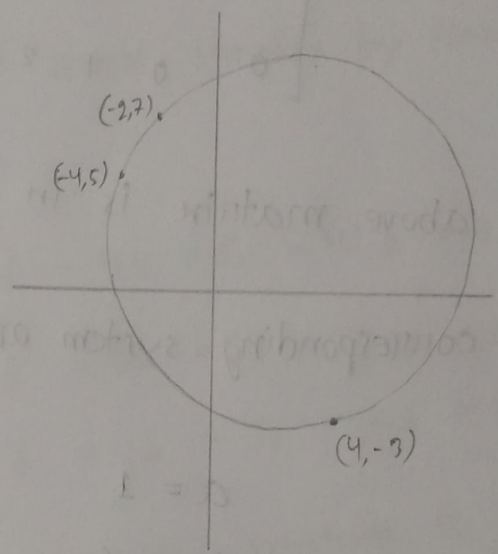
Find the coefficients a, b, c and d so that the curve shown in the accompanying figure is given by the equation

$$ax^2 + ay^2 + bx + cy + d = 0$$



Solution:

Given that, the circle $ax^2 + ay^2 + bx + cy + d = 0$ passes through the points $(-4,5)$, $(-2,7)$ and $(4,-3)$.

So, these points satisfy the circle equation.

So, we can write that,

$$a(-4)^2 + a(5)^2 + b(-4) + c(5) + d = 0$$

$$a(-2)^2 + a(7)^2 + b(-2) + c(7) + d = 0$$

$$a(4)^2 + a(-3)^2 + b(4) + c(-3) + d = 0$$

We can simplify and rewrite the above system of equations as follows,

$$41a - 4b + 5c + d = 0$$

$$53a - 2b + 7c + d = 0$$

$$25a + 4b - 3c + d = 0$$

We can make it more simplify by re-arranging the system equations.

$$d - 4b + 5c + 41a = 0$$

$$d - 2b + 7c + 53a = 0$$

$$d + 4b - 3c + 25a = 0$$

The corresponding augmented matrix is,

$$\begin{bmatrix} 1 & -4 & 5 & 41 & 0 \\ 1 & -2 & 7 & 53 & 0 \\ 1 & 4 & -3 & 25 & 0 \end{bmatrix}$$

$$\xrightarrow[r_3' = r_3 - r_1]{r_2' = r_2 - r_1} \begin{bmatrix} 1 & -4 & 5 & 41 & 0 \\ 0 & 2 & 2 & 12 & 0 \\ 0 & 8 & -8 & -16 & 0 \end{bmatrix}$$

$$\xrightarrow{r_2' = \frac{1}{2} r_2} \begin{bmatrix} 1 & -4 & 5 & 41 & 0 \\ 0 & 1 & 1 & 6 & 0 \\ 0 & 8 & -8 & -16 & 0 \end{bmatrix}$$

$$\xrightarrow{r_3' = r_3 - 8r_2} \begin{bmatrix} 1 & -4 & 5 & 41 & 0 \\ 0 & 1 & 1 & 6 & 0 \\ 0 & 0 & -16 & -64 & 0 \end{bmatrix}$$

$$\xrightarrow{r_3' = -\frac{1}{16} r_3} \begin{bmatrix} 1 & -4 & 5 & 41 & 0 \\ 0 & 1 & 1 & 6 & 0 \\ 0 & 0 & 1 & 4 & 0 \end{bmatrix}$$

$$\xrightarrow[r_2' = r_2 - r_3]{r_1' = r_1 - 5r_3} \begin{bmatrix} 1 & -4 & 0 & 21 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 4 & 0 \end{bmatrix}$$

$$\xrightarrow{r_1' = r_1 + 4r_2} \begin{bmatrix} 1 & 0 & 0 & 29 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 4 & 0 \end{bmatrix}$$

The above matrix is in reduced row echelon form.

The corresponding system of equation is,

$$d + 29a = 0$$

$$b + 2a = 0$$

$$c + 4a = 0$$

Here, b, c, d are leading variables.

Solving for the leading variables we obtain

$$d = -29a$$

$$b = -2a$$

$$c = -4a$$

Here, a is free variables.

Let, $a = r$

So, the solution is,

$$a = r$$

$$b = -2r$$

$$c = -4r$$

$$d = -29r$$

where r can be any real number.