L-1 / 28.09.2022 /

Flash-Back of MAT-120

(*) $\frac{d}{dx}(\sin x) = \cos x$; $\int \cos x \, dx = \sin x + c$

$\frac{d}{dx}(\cos x) = -\sin x$; $\int \sin x \, dx = -\cos x + c$

(*) $\frac{d}{dx}(uv) = u \cdot \frac{d}{dx}(v) + v \cdot \frac{d}{dx}(u)$

$\frac{d}{dx}(uvw) = vw \cdot \frac{d}{dx}(u) + uw \cdot \frac{d}{dx}(v) + uv \cdot \frac{d}{dx}(w)$

(*)

$$\frac{d}{dx}(\sin^2 5x) = 2 \sin 5x \cdot \frac{d}{dx}(\sin 5x)$$

$$= 2 \sin 5x \cdot \cos 5x \cdot 5$$

$$= 10 \sin 5x \cdot \cos 5x$$

$\frac{d}{dx}(xy) = x \cdot \frac{dy}{dx} + y$

$\frac{d}{dx}(y^2) = 2y \cdot \frac{dy}{dx}$

(*) $\int \cos x \, e^{\sin x} \, dx$

$u = \sin x$

$du = \cos x \, dx$

$\int e^u \, du$

$= e^u + c$

$= e^{\sin x} + c$

L-2 / 10.10.2022 /

$$\int uv\,dx = u\int v\,dx - \int\left\{\frac{d}{dx}(u)\int v\,dx\right\}dx$$

$$\int u\,dv = uv - \int v\,du$$

LIATE

Left $\Rightarrow u$

Right $\Rightarrow v$

⊛ $\int x e^x\,dx$

$= x\int e^x\,dx - \int\left\{\frac{d}{dx}(x)\int e^x\,dx\right\}dx$

$= xe^x - \int 1\cdot e^x\,dx$

$= xe^x - e^x + c$

⊛ $\int x e^x\,dx$

$u = x$

$du = dx$

$\int dv = \int e^x\,dx$

$v = e^x$

$= x\cdot e^x - \int e^x\,dx$

⊛ $\frac{d}{dx}(e^x) = e^x$

$$\frac{d}{dx}(e^{2x}) = 2e^{2x}$$

$$\int e^{-2x}\,dx = \frac{e^{-2x}}{-2}$$

⊛ $\int x^2 e^{-2x}\,dx$

$$= x^2 \int e^{-2x}\,dx - \int \left\{ \frac{d}{dx}(x^2) \int e^{-2x}\,dx \right\} dx$$

$$= x^2 \cdot \frac{e^{-2x}}{-2} - \int 2x \cdot \frac{e^{-2x}}{-2}\,dx$$

$$= \frac{x^2 e^{-2x}}{-2} + \int x \cdot e^{-2x}\,dx$$

$$= \frac{x^2 e^{-2x}}{-2} + \left[x \cdot \frac{e^{-2x}}{-2} - \int 1 \cdot \frac{e^{-2x}}{-2}\,dx \right]$$

$$= \frac{xe^{-2x}}{-2} - \frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} + c$$

⊛ Tabular Method

Repeated Diff. (u)		Repeated Inte. (v)
x^2		e^{-2x}
	$(+)$	
$2x$		$\dfrac{e^{-2x}}{-2}$
	$(-)$	
2		$\dfrac{e^{-2x}}{4}$
	$(+)$	
0		$\dfrac{e^{-2x}}{8}$

⊛ $$\int x^2 e^{-2x}\,dx = -\frac{x^2 e^{-2x}}{2} - \frac{2x\cdot e^{-2x}}{4} - \frac{2\cdot e^{-2x}}{8}$$

$$= -\frac{x^2 e^{-2x}}{2} - \frac{x e^{-2x}}{2} - \frac{e^{-2x}}{4} + C$$

⊛ $$\int \ln x \cdot 1\,dx$$

$$= \ln x \int 1\,dx - \int \left\{ \frac{d}{dx}(\ln x) \int 1\,dx \right\} dx$$

$$= \ln x \cdot x - \int \frac{1}{x}\cdot x\,dx$$

$$= x\ln x - x + C$$

$$\therefore \int \ln x\,dx = x\ln x - x + C$$

(*) $\int e^x \cos x \, dx$

$= \cos x \int e^x \, dx - \int \left\{ \frac{d}{dx}(\cos x) \int e^x \, dx \right\} dx$

$= \cos x \cdot e^x - \int - \sin x \, e^x \, dx$

$= e^x \cos x + \int \sin x \, e^x \, dx$

$= e^x \cos x + \left[\sin x \int e^x \, dx - \int \left\{ \frac{d}{dx}(\sin x) \int e^x \, dx \right\} dx \right]$

$= e^x \cos x + \left[\sin x \cdot e^x - \int \cos x \cdot e^x \, dx \right]$

$= e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$

$\Rightarrow \int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$

$\Rightarrow 2 \int e^x \cos x \, dx = e^x \cos x + e^x \sin x$

$\therefore \int e^x \cos x \, dx = \frac{e^x \cos x}{2} + \frac{e^x \sin x}{2} + c$

(*) $\int (x^2 - x) \cos x \, dx = (x^2 - x) \sin x + (2x - 1) \cos x - 2 \sin x + c$

$\circledast$ $\int \sqrt{x-1}\, dx$

$u = x-1$

$\frac{du}{dx} = 1$

$du = dx$

$= \int \sqrt{u}\, du$

$= \int u^{\frac{1}{2}}\, du$

$= \frac{u^{\frac{3}{2}}}{3/2} + c$

$= \frac{2}{3}(x-1)^{3/2} + c$

Ans.

$\circledast$ $\int x^2 \sqrt{x-1}\, dx$

$= \int x^2 (x-1)^{\frac{1}{2}}\, dx$

$u = x-1$

$\frac{du}{dx} = 1$

$du = dx$

$u = x-1$

$x = u+1$

$x^2 = (u+1)^2$

$= u^2 + 2u + 1$

$= \int (u^2+2u+1)\, u^{\frac{1}{2}}\, du$

$= \int (u^{5/2} + 2u^{3/2} + u^{\frac{1}{2}})\, du$

$$= \frac{u^{5/2+1}}{5/2+1} + 2\cdot\frac{u^{3/2+1}}{3/2+1} + \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{u^{7/2}}{7/2} + 2\cdot\frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + c$$

$$= \frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + c$$

(*) $\int \tan^{-1}x \, dx = ?$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\int \frac{1}{x}\,dx = \ln|x|$$

(*) $\int \frac{2x}{x^2+5}\,dx$

$u = x^2+5$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$= \int \frac{1}{u}\,du$

$= \ln u + c$

$= \ln(x^2+5) + c$

$$\therefore \boxed{\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c}$$

※ $\int \tan^{-1}x \, dx$

$= \int \tan^{-1}x \cdot 1 \, dx$

$= \tan^{-1}x \int 1 \, dx - \int \left\{ \frac{d}{dx}(\tan^{-1}x) \int 1 \, dx \right\} dx$

$= \tan^{-1}x \cdot x - \int \frac{1}{1+x^2} \cdot x \, dx$

$= x \tan^{-1}x - \int \frac{x}{x^2+1} \, dx$

$= x \tan^{-1}x - \frac{1}{2} \int \frac{2x}{x^2+1} \, dx$

$= x \tan^{-1}x - \frac{1}{2} \cdot \ln(x^2+1) + c$

※ $\int_0^1 \tan^{-1}x \, dx$

$= \left[x \tan^{-1}x - \frac{1}{2} \ln(x^2+1) \right]_0^1$

$= \left(1 \cdot \tan^{-1} 1 - \frac{1}{2} \ln 2\right) - \left(0 \cdot \tan^{-1} 0 - \frac{1}{2} \ln(1)\right)$

$= \frac{\pi}{4} - \frac{1}{2} \ln 2$

$= \frac{\pi}{4} - \ln\sqrt{2}$

$$\boxed{\sin 2x = 2\sin x\cos x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= \cos^2 x - (1 - \cos^2 x)$$

$$= \cos^2 x - 1 + \cos^2 x = 2\cos^2 x - 1$$

$$2\cos^2 x = \cos 2x + 1$$

$$\boxed{\cos^2 x = \frac{1}{2}(\cos 2x + 1)}$$

$$\boxed{\sin^2 x = \frac{1}{2}(1 - \cos 2x)}$$

$$\int \cos x \, dx = ?$$

$$\int \cos^2 x \, dx = \frac{1}{2}\int (1 + \cos 2x)\, dx$$

$$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right)$$

$$= \frac{1}{2}x + \frac{1}{2}\cdot\frac{1}{2}\cdot 2\sin x\cos x$$

$$= \frac{1}{2}\cos x\sin x + \frac{1}{2}x + c$$

⊛ Reduction Formula

⊛ $\boxed{\int \sin^n x\, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x\, dx}$

⊛ $\boxed{\int \cos^n x\, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x\, dx}$

⊛ $\int \cos^3 x\, dx$

$= \frac{1}{3} \cos^2 x \sin x + \frac{2}{3} \int \cos x\, dx$

$= \frac{1}{3} \cos^2 x \sin x + \frac{2}{3} \sin x + C$

MAT-130 / L-3 / 12.10.2022 /

$$\int \ln x \, dx = x \ln x - x + C$$

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + C$$

$$\int \sin(\ln x) \, dx = ?$$

$$= \int \sin(\ln x) \cdot 1 \, dx$$

$$= \sin(\ln x) \int 1 \, dx - \int \left\{ \frac{d}{dx}(\sin(\ln x)) \int 1 \, dx \right\} dx$$

$$= x \sin(\ln x) - \int \cos(\ln x) \cdot \frac{1}{x} \cdot x \, dx$$

$$= x \sin(\ln x) - \int \cos(\ln x) \, dx$$

$$= x \sin(\ln x) - \left[x \cos(\ln x) + \int \sin(\ln x) \cdot \frac{1}{x} \cdot x \, dx \right]$$

$$= x \sin(\ln x) - x \cos(\ln x) - \int \sin(\ln x) \, dx$$

$$\Rightarrow \int \sin(\ln x) \, dx = x \sin(\ln x) - x \cos(\ln x) - \int \sin(\ln x) \, dx$$

$$\Rightarrow \quad 2\int \sin(\ln x)\,dx = x\sin(\ln x) - x\cos(\ln x)$$

$$\therefore \int \sin(\ln x)\,dx = \frac{x\sin(\ln x)}{2} - \frac{x\cos(\ln x)}{2}$$

⊛ $\int \sin\sqrt{x}\,dx = ?$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$du = \frac{dx}{2u}$

$2u\,du = dx$

$$= \int \sin u \cdot 2u\,du$$

$$= 2\int \sin u \cdot u\,du$$

$$= 2\left[u\int \sin u\,du - \int\left\{\frac{d}{du}(u)\int \sin u\,du\right\}du\right]$$

$$= 2\left[-u\cos u + \int \cos u\,du\right]$$

$$= 2\left[\sin u - u\cos u\right]$$

$$= 2\sin\sqrt{x} - 2\sqrt{x}\cos\sqrt{x}$$

⊛ $\int (3x^2 - x + 2)\,e^{-x}\,dx = ?$

By using Tabulan Method:

Repeated Diff.		Repeated Integ.
		e^{-x}
$3x^2-x+2$	$+$	$-e^{-x}$
$6x-1$	$-$	$+e^{-x}$
6	$+$	$-e^{-x}$
0		

$$\therefore \int (3x^2-x+2)e^{-x}\,dx = (3x^2-x+2)(-e^{-x}) - (6x-1)(e^{-x}) + 6(-e^{-x}) + c$$

$$= -e^{-x}(3x^2-x+2+6x-1+6)+c$$

$$= -(3x^2+5x+7)e^{-x}+c$$

Ans

(*) $$\int 4x^4 \sin 2x\,dx = 4x^4\left(-\frac{\cos 2x}{2}\right) + 4x^3 \sin 2x + 6x^2 \cos 2x - 6x \sin 2x - 3\cos 2x + c$$

$$= (4x^3-6x)\sin x - (2x^4-6x^2+3)\cos 2x + c$$

By using Tabular Method.

Reduction Formula

(i) $$\boxed{\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx}$$

(ii) $$\boxed{\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx}$$

(iii) $$\boxed{\int x^n e^x\, dx = x^n e^x - n\int x^{n-1} e^x\, dx}$$

(*) $$\int \tan x = \int \frac{\sin x}{\cos x} = -\int \frac{-\sin x}{\cos x} = -\ln|\cos x| + c$$

$$= \ln|\sec x| + c$$

(*) $$\int \sec x\, dx = \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}\, dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\, dx$$

$$= \ln|\sec x + \tan x| + c$$

(*) $$\int \tan^3 x\, dx = \frac{\tan^2 x}{2} - \int \tan x\, dx$$

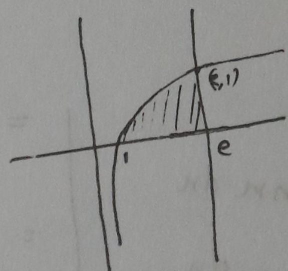
$$= \frac{1}{2}\tan^2 x - \ln|\sec x| + c$$

$\circledast$ $\int x e^x \, dx = xe^x - e^x + c$

$\circledast$ $\int x^2 e^x \, dx = x^2e^x - 2xe^x + 2e^x + c$

$\circledast$ $\int x^3 e^x \, dx = x^3e^x - 3x^2e^x + 6xe^x - 6e^x + c$

$\circledast$ Find the area of the region on closed by $y = \ln x$, the line $x = e$ and the x-axis.



$$A = \int_1^e (\ln x - 0) \, dx$$

$$= \Big[x \ln x - x \Big]_1^e$$

$$= e \cdot 1 - e - 0 + 1$$

$$= e - e + 1$$

$$= \underline{1}$$

Reduction Formula

~~① $\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x$~~

⊛ $\int \sin^5 x \, dx$

$= \int (\sin^2 x)^2 \cdot \sin x \, dx$

$= \int (1-\cos^2 x)^2 \cdot \sin x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$= -\int (1-u^2)^2 \, du$

$= -\int (1-2u^2+u^4) \, du$

$= -u + 2\frac{u^3}{3} - \frac{u^5}{5} + c$

$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + c$

⊛ $\int \sin^4 x \cos^5 x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int u^4 \cdot \cos^4 x \, du$

$= \int u^4 (1-\sin^2 x)^2 \cdot du$

$= \int u^4 (1-u^2)^2 \, du$

$= \int u^4 (1-2u^2+u^4) \, du$

$= \int (u^4 - 2u^6 + u^8)\, du$

$= \frac{u^5}{5} - 2 \cdot \frac{u^7}{7} + \frac{u^9}{9} + c$

$= \frac{1}{5} \sin^5 x - \frac{2}{7} \sin^7 x + \frac{1}{9} \sin^9 x + c$

㊉ $\int \sin^2 x \, dx$

$= -\frac{1}{2} \sin x \cos x + \frac{1}{2} \int 1 \, dx$

$= -\frac{1}{2} \sin x \cos x + \frac{1}{2} x + c$

㊉ $\int \cos^2 x \, dx = \frac{1}{2} x + \frac{1}{2} \sin x \cos x + c$

$= \frac{1}{2} x + \frac{1}{4} \sin 2x + c$

㊉ $\int \cos^3 x \, dx$

$= \frac{1}{3} \cos^2 x \sin x + \frac{2}{3} \int \cos x \, dx$

$= \frac{1}{3} \cos^2 x \sin x + \frac{2}{3} \sin x \cancel{dx} + c$

$= \frac{1}{3} (1 - \sin^2 x) \sin x + \frac{2}{3} \sin x + c$

$= \frac{1}{3} \sin x - \frac{1}{3} \sin^3 x + \frac{2}{3} \sin x + c$

$= \sin x - \frac{1}{3} \sin^3 x + c$

$$\int \sin^3 x\, dx = \int \sin^2 x \sin x\, dx$$

$$= \int (1-\cos^2 x) \sin x\, dx$$

$u = \cos x$

$du = -\sin x\, dx$

$$= -\int (1-u^2)\, du$$

$$= -u + \frac{u^3}{3} + C$$

$$= \frac{1}{3}\cos^3 x - \cos x + C$$

$$\int \cos^4 x\, dx = \frac{1}{4}\cos^3 x + \frac{3}{4}\int \cos^2 x\, dx$$

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\left[\frac{1}{2}\cos x \sin x + \frac{1}{2}\int 1\, dx\right]$$

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{8}\cos x \sin x + \frac{3}{8}x + C$$

Ans.

$$\cancel{= \frac{1}{4}\cos^2 x \cos x \sin x}$$

Similarly:

$$\int \sin^4 x\, dx = -\frac{1}{4}\sin^3 x \cos x - \frac{3}{8}\cos x \sin x + \frac{3}{8}x + C$$

$$= \frac{1}{32}\sin 4x - \frac{1}{4}\sin 2x + \frac{3}{8}x + C$$

⊛ $\int \tan^2 x \sec^4 x \, dx$

$= \int \tan^2 x \sec^2 x \sec^2 x \, dx$

$= \int \tan^2 x (\tan^2 x + 1) \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$= \int u^2 (u^2+1) \, du$

$= \int (u^4 + u^2) \, du$

$= \frac{u^5}{5} + \frac{u^3}{3} + C$

$= \frac{1}{5} \tan^5 x + \frac{1}{3} \tan^3 x + C$

⊛ $\int \tan^3 x \sec^3 x \, dx$

$= \int \tan^2 x \sec^2 x \sec x \tan x \, dx$

$= \int (\sec^2 x - 1) \sec^2 x \sec x \tan x \, dx$

$u = \sec x$

$du = \sec x \tan x \, dx$

$= \int (u^2-1) u^2 \, du$

$= \int (u^4 - u^2) \, du$

⊛ $\int \tan 4x \sec^4 4x \, dx$

$= \int \tan 4x \sec^2 4x \sec^2 4x \, dx$

$u = \tan 4x$

$du = \sec^2 4x \cdot 4 \, dx$

$\frac{1}{4} du = \sec^2 4x \, dx$

$= \int u (\tan^2 4x + 1) \frac{1}{4} du$

$= \frac{1}{4} \int u(u^2+1) \, du$

$= \frac{1}{4} \int (u^3 + u) \, du$

$= \frac{1}{4} \cdot \frac{u^4}{4} + \frac{1}{4} \cdot \frac{u^2}{2} + c$

$= \frac{1}{16} \tan^4 4x + \frac{1}{8} \tan^2 4x + c$

⊛ $\int \tan 4x \sec^4 4x \, dx$

$= \int \sec^3 4x \sec 4x \tan 4x \, dx$

$u = \sec 4x$

$\frac{du}{dx} = \sec 4x \tan 4x \cdot 4$

$\frac{1}{4} du = \sec 4x \tan 4x \, dx$

$= \int u^3 \cdot \frac{1}{4} dx$

$= \frac{1}{4} \int u^3 du$

$= \frac{1}{4} \cdot \frac{u^4}{4}$

$= \frac{1}{16} (\sec^4 4x)$

$$\circledast \int \cos^5 x \, dx$$

$$= \int (\cos^2 x)^2 \cos x \, dx$$

$$= \int (1-\sin^2 x)^2 \cdot \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int (1-u^2)^2 \, du$$

$$= \int (1-2u^2+u^4) \, du$$

$$= u - 2\cdot\frac{u^3}{3} + \frac{u^5}{5} + c$$

$$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$$

$$\circledast \int \sin^5 x \cos^4 x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$$= \int \sin^4 x \cdot \cos^4 x \cdot \sin x \, dx$$

$$= \int (\sin^2 x)^2 \cdot \cos^4 x \cdot \sin x \, dx$$

$$= \int (1-\cos^2 x)^2 \cos^4 x \, \sin x \, dx$$

$$= -\int (1-u^2)^2 u^4 \, du$$

$$= -\int (1-2u^2+u^4) u^4 \, du$$

$$= -\int (u^4 - 2u^6 + u^8) \, du$$

$$= -\frac{u^5}{5} + 2\frac{u^7}{7} - \frac{u^9}{9} + c$$

$$= -\frac{1}{5}\cos^5 x + \frac{2}{7}\cos^7 x - \frac{1}{9}\cos^9 x + c$$

Reduction Formula

⊛ $\int \cot^n x \, dx = -\frac{1}{n-1} \cot^{n-1} x - \int \cot^{n-2} x \, dx$

⊛ $\int \operatorname{cosec}^n x \, dx = -\frac{1}{n-1} \operatorname{cosec}^{n-2} x \cot x + \frac{n-2}{n-1} \int \operatorname{cosec}^{n-2} x \, dx$

L-4 / 17.10.2022 /

7.4

Trigonometric Substitution

⊛ $$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

$\boxed{\text{Form is } \sqrt{a^2-x^2}}$

Let,

$x = 2\sin\theta$

$dx = 2\cos\theta \, d\theta$

Here,

$\sqrt{4-x^2}$

$= \sqrt{4-(2\sin\theta)^2}$

$= \sqrt{4-4\sin^2\theta}$

$= \sqrt{4(1-\sin^2\theta}$

$= 2\cos\theta$

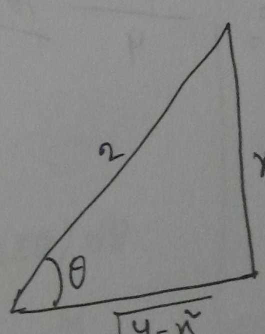
$\therefore \int \frac{2\cos\theta \, d\theta}{4\sin^2\theta \cdot 2\cos\theta}$

$= \frac{1}{4}\int \frac{1}{\sin^2\theta} \, d\theta$

$= \frac{1}{4}\int \operatorname{cosec}^2\theta \, d\theta$

$= -\frac{1}{4}\cot\theta + C$

$= -\frac{\sqrt{4-x^2}}{4x} + C$



$x = 2\sin\theta$

$\sin\theta = \frac{x}{a}$

⊛ $$\int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}}$$

$$= -\frac{\sqrt{4-x^2}}{4x}\Bigg]_1^{\sqrt{2}}$$

$$= -\frac{\sqrt{4-2}}{4\cdot\sqrt{2}} + \frac{\sqrt{4-1}}{4\cdot 1}$$

$$= -\frac{\sqrt{2}}{4\cdot\sqrt{2}} + \frac{\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}\cancel{4}-1}{4}$$ Ans.

⊛ $$\int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}} = -\frac{1}{4}\cot\theta\Bigg]_{\pi/6}^{\pi/4}$$

$$= -\frac{1}{4}\cot\frac{\pi}{4} + \frac{1}{4}\cot\frac{\pi}{6}$$

$$= -\frac{1}{4}\cdot 1 + \frac{1}{4}\cdot\sqrt{3}$$

$$= \frac{\sqrt{3}-1}{4}$$ Ans.

$\sqrt{2} = 2\sin\theta$

$\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}$

again

$1 = 2\sin\theta$

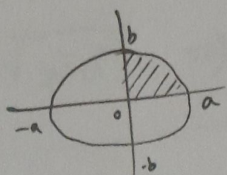
$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

(*) Area of the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$\frac{y^2}{b^2} = \frac{a^2 - x^2}{a^2}$$



$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

$$y = \pm \frac{b}{a}\sqrt{a^2 - x^2}$$

$$\text{Area} = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\, dx$$

Let, $x = a\sin\theta$ | $x = a\sin\theta$

$dx = a\cos\theta\, d\theta$ | $a = a\sin\theta$

$\sin\theta = 1$

$\theta = \frac{\pi}{2}$

$$\therefore \text{Area} = 4\int_0^{\pi/2} \frac{b}{a} \cdot a\cos\theta \cdot a\cos\theta\, d\theta$$

$$= 4ab\int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= 4ab\int_0^{\pi/2} \frac{1}{2}(1 + \cos 2\theta)\, d\theta$$

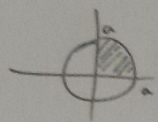
$$= 2ab\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= 2ab\left[\frac{\pi}{2} + 0 - 0\right]$$

$$= \pi ab$$

※ Area of the circle,

$$x^2 + y^2 = a^2$$



$$\Rightarrow y = \pm\sqrt{a^2 - x^2}$$

$$\text{Area} = 4\int_0^a \sqrt{a^2 - x^2}\, dx$$

Let, $x = a\sin\theta$ | if, $x = a$,

$dx = a\cos\theta\, d\theta$ | $a = a\sin\theta$

$\sin\theta = 1$

$\theta = \frac{\pi}{2}$

$$\therefore \text{Area} = 4\int_0^{\pi/2} a\cos\theta \cdot a\cos\theta\, d\theta$$

$$= 4a^2 \int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= 4a^2 \int_0^{\pi/2} \frac{1}{2}(1 + \cos 2\theta)\, d\theta$$

$$= 2a^2 \int_0^{\pi/2} (1 + \cos 2\theta)\, d\theta$$

$$= 2a^2 \left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= 2a^2 \left[\frac{\pi}{2} + 0 - 0\right]$$

$$= \pi a^2$$

Ans

Formula:

$\sqrt{a^2-x^2} \Rightarrow x = a\sin\theta \Rightarrow$ Simplify, $= a\cos\theta$

$\sqrt{a^2+x^2} \Rightarrow x = a\tan\theta \Rightarrow a\sec\theta$

$\sqrt{x^2-a^2} \Rightarrow x = a\sec\theta \Rightarrow a\tan\theta$

⊛ Length of a curve or, arc length:

$$\boxed{L = \int_a^b \sqrt{1+[f'(x)]^2}\, dx}$$

⊛ $\int \frac{\sqrt{x^2-25}}{x}\, dx$

Let,
$x = 5\sec\theta$
$dx = 5\sec\theta\tan\theta\, d\theta$

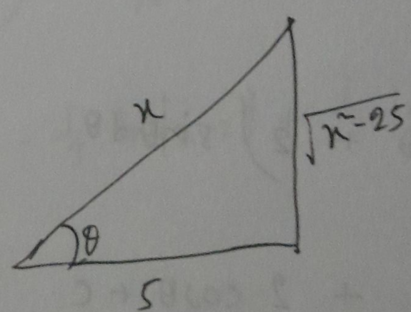
$= \int \frac{5\tan\theta \cdot 5\sec\theta\tan\theta\, d\theta}{5\sec\theta}$

$= 5\int \tan^2\theta\, d\theta$

$= 5\int (\sec^2\theta - 1)\, d\theta$

$= 5[\tan\theta - \theta] + c$

$= 5\tan\theta - 5\theta + c$



$\sec\theta = \frac{x}{5}$

$\cos\theta = \frac{5}{x}$

$$= 5\,\frac{\sqrt{x^2-25}}{5} - 5\cdot \sec^{-1}\frac{x}{5}$$

$$= \sqrt{x^2-25} - 5\sec^{-1}\frac{x}{5}$$

Ans.

⊛ $$\int \frac{3x^3}{\sqrt{1-x^2}}\,dx$$

Let,

$$x = 1\sin\theta$$

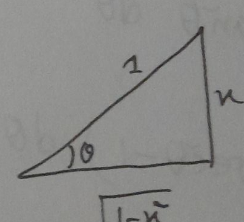
$$dx = \cos\theta\,d\theta$$

$$\therefore \int \frac{3\sin^3\theta\cdot\cos\theta\,d\theta}{\cos\theta}$$

$$= 3\int \sin^3\theta\,d\theta$$

$$= 3\left[-\frac{1}{3}\sin^2\theta\cos\theta + \frac{2}{3}\int\sin\theta\,d\theta\right]$$

$$= -\sin^2\theta\cos\theta + 2\int\sin\theta\,d\theta$$



$$= -\sin^2\theta\cos\theta + 2\cos\theta + C$$

$$= -x^2\cdot\sqrt{1-x^2} + 2\cdot\sqrt{1-x^2} + c = (2-x^2)\sqrt{1-x^2} + c$$

| L-5 | 19.10.2022 |

⊛ $\int \frac{1}{x^2+x-2}\, dx$

Here,

$$\frac{1}{x^2+x-2} = \frac{1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$\frac{1}{(x+2)^2(x-1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1}$$

$$\frac{1}{(x^2+1)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

Now,

$$1 = A(x-1) + B(x+2)$$

$$= Ax - A + Bx + 2B$$

$$= (A+B)x + (2B-A)$$

$\therefore$ $A+B=0$

$2B-A = 1$

$\Rightarrow 2B + B = 1$

$3B = 1$

$\therefore B = \frac{1}{3}$

$\therefore A = -\frac{1}{3}$

$$\therefore \int \frac{1}{x^2+x-2}\, dx$$

$$= \int \frac{-\frac{1}{3}}{x+2}\, dx + \int \frac{1/3}{x-1}\, dx$$

$$= -\frac{1}{3}\ln|x+2| + \frac{1}{3}\ln|x-1| + C$$

Ans

(v) $\int \frac{2x+4}{x^3-2x^2}\,dx$

Here $\frac{2x+4}{x^3-2x^2} = \frac{2x+4}{x^2(x-2)}$

$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$

Now, $(2x+4) = Ax(x-2) + B(x-2) + Cx^2$

$= Ax^2 - 2Ax + Bx - 2B + Cx^2$

$2x+4 = (A+C)x^2 + (B-2A)x - 2B$

$\therefore A+C = 0 \therefore C = 2$

$B-2A = 2 \Rightarrow -2A = 4 \therefore A = -2$

$-2B = 4$

$\therefore B = -2$

$\therefore \int \frac{2x+4}{x^3-2x^2}\,dx$

$= \int \left(\frac{-2}{x} + \frac{-2}{x^2} + \frac{2}{x-2} \right) dx$

$$\circledast \int \frac{x^2+x-2}{3x^3-x^2+3x-1}\,dx$$

$$= \int \left(\frac{-7/5}{3x-1} + \frac{4/5\,x + 3/5}{x^2+1} \right) dx$$

$$= -\frac{7}{5}\cdot\frac{1}{3}\int \frac{3}{3x-1}\,dx + \int \frac{4/5\,x}{x^2+1}\,dx + \int \frac{3/5}{x^2+1}\,dx$$

$$= -\frac{7}{15}\ln|3x-1| + \frac{2}{5}\int \frac{2x}{x^2+1}\,dx + \frac{3}{5}\tan^{-1}x$$

$$= -\frac{7}{15}\ln|3x-1| + \frac{2}{5}\ln|x^2+1| + \frac{2}{5}\tan^{-1}x + c$$

Ans

Here,

$$3x^3 - x^2 + 3x - 1 = x^2(3x-1) + (3x-1)$$

$$= (3x-1)(x^2+1)$$

$$\therefore \frac{x^2+x-2}{3x^3-x^2+3x-1} = \frac{x^2+x-2}{(3x-1)(x^2+1)}$$

$$= \frac{A}{3x-1} + \frac{Bx+C}{x^2+1}$$

$$\therefore x^2+x-2 = A(x^2+1) + (Bx+C)(3x-1)$$

$$= Ax^2 + A + 3Bx^2 - Bx + 3Cx - C$$

$$x^2+x-2 = (A+3B)x^2 + \overset{(3C-B)}{\cancel{(B+3C)}}\,x + (A-C)$$

$$\therefore \begin{aligned} A+3B &= 1 \\ -B+3C &= 1 \\ A-C &= -2 \end{aligned}$$

Do echalon

$$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & -1 & 3 & 1 \\ 1 & 0 & -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & -3 & -1 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & -10 & -6 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & \frac{3}{5} \end{bmatrix}$$

$\rightarrow$ $C = \frac{3}{5}$

$\rightarrow$ $B - 3C = -1$

$B - 3 \cdot \frac{3}{5} = -1$

$B = -1 + \frac{9}{5} = \frac{4}{5}$

$\rightarrow$ $A + 3B = 1$

$A + 3 \cdot \frac{4}{5} = 1$

$A = 1 - \frac{12}{5} = \frac{-7}{5}$

(*)

$$\frac{*}{(x-2)^2(x^2+1)^2} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{Cx+D}{(x^2+1)} + \frac{Ex+F}{(x^2+1)^2}$$

L-6 / 29.10.2022

6.9

$$e^x = \underbrace{\frac{e^x + e^{-x}}{2}}_{\text{even function}} + \underbrace{\frac{e^x - e^{-x}}{2}}_{\text{Odd function}}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

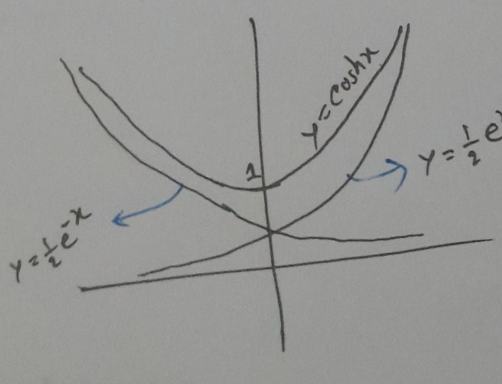
$$\operatorname{cosech} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

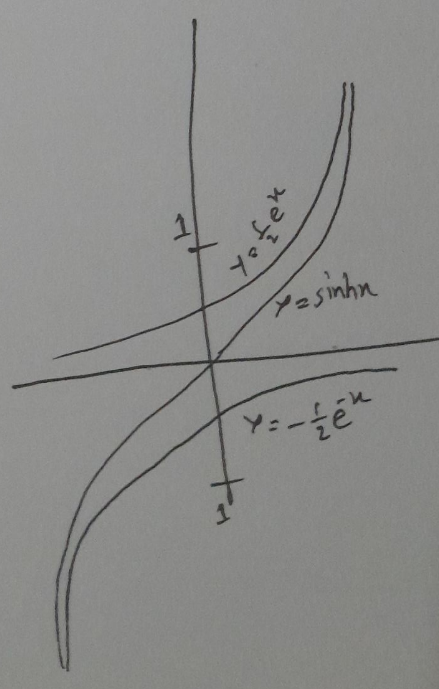
※

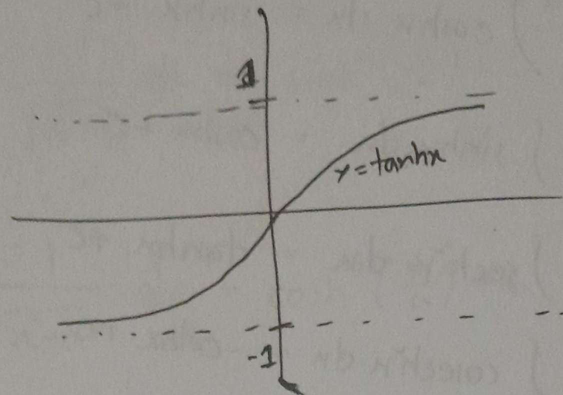
$$\sinh 0 = \frac{e^0 - e^{-0}}{2} = \frac{1-1}{2} = 0$$

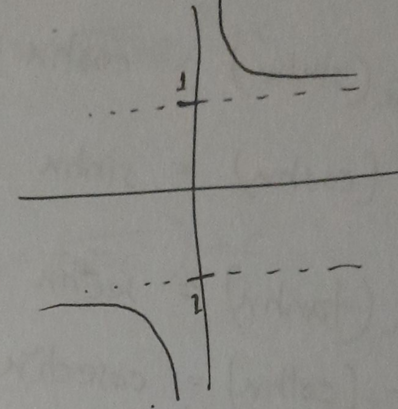
$$\cosh 0 = \frac{e^0 + e^{-0}}{2} = \frac{1+1}{2} = 1$$

※ Graph

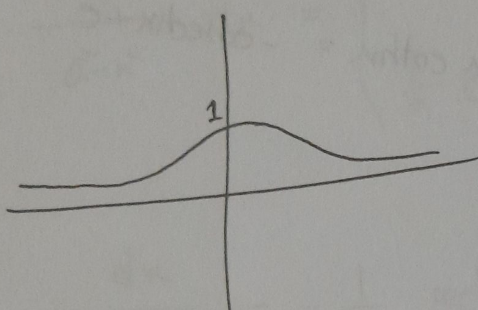




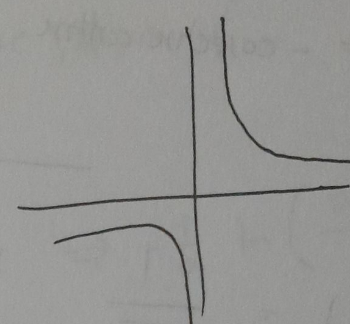




$y = \coth x$



$y = \operatorname{sech} x$



$y = \operatorname{cosech} x$

⊛

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$

$$\cosh x + \sinh x = e^x$$

$$\cosh x - \sinh x = e^{-x}$$

$$\cosh(-x) = \cosh x$$

$$\sinh(-x) = -\sinh x$$