North South University

Department of Mathematics and Physics

Assignment-3

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4.2

2) Use Theorem 4.2.1 to determine which of the following are subspaces of M_{nn}.

b) The set of all $n \times n$ matrices A such that $\det(A) = 0$.

Solution:

Let, W be the set of all $n \times n$ matrices A such that $\det(A) = 0$.

Now, we need to verify that if W is a subspace of M_{nn} or not. W will be a subspace of M_{nn} if and only if the following conditions hold.

i) if $u, v \in W$, then $(u+v) \in W$

ii) if k is any scalar and $u \in W$, then $ku \in W$

Now, checking for first condition,

Let, $u, v \in W$

$u = A_{n \times n}$

$v = B_{n \times n}$

Here,

$\det(A_{n \times n}) = 0$

$\det(B_{n \times n}) = 0$

Now,

$$u + v = A_{n\times n} + B_{n\times n}$$

$$\therefore \det(A_{n\times n} + B_{n\times n}) \neq \det(A_{n\times n}) + \det(B_{n\times n})$$

[determinate is not distributive]

$$\therefore \det(A_{n\times n} + B_{n\times n}) \neq 0$$

Therefore, $(u+v) \notin W$

So, W is not closed under addition.

Therefore,

W is not a subspace of M_{nn}.

c) The set of all $n\times n$ matrices A such that $tr(A) = 0$.

Solution:

Let W be the set of all $n\times n$ matrices A such that

$tr(A) = 0$.

Now, we need to verify that if W is a subspace of M_{nn} or not. W will be a subspace of M_{nn} if and only if the following conditions hold.

i) if $u, v \in W$, then $(u+v) \in W$

ii) if k is any scalar and $u \in W$, then $ku \in W$

Now, checking for first condition,

Let $u, v \in W$

$u = A_{n \times n}$ | Here, $tr(A_{n\times n}) = 0$

$v = B_{n\times n}$ | $tr(B_{n\times n}) = 0$

Now,

$$u+v = A_{n\times n} + B_{n\times n}$$

$$\therefore tr(A_{n\times n} + B_{n\times n}) = tr(A_{n\times n}) + tr(B_{n\times n})$$

$$= 0$$

$$\therefore tr(A_{n\times n} + B_{n\times n}) = 0$$

Therefore, $(u+v) \in W$

So, W is closed under addition.

Now, checking for second condition,

Let, $u \in W$

k is any scalar

$u = A_{n \times n}$

Here, $tr(A_{n \times n}) = 0$

Now,

$$ku = k \cdot A_{n \times n}$$

$$\therefore tr(k \cdot A_{n \times n}) = k \cdot tr(A_{n \times n})$$

$$= k \cdot 0$$

$$= 0$$

$\therefore$ $k \cdot A_{n \times n}$ or $ku \in W$

So, W is closed under scalar multiplication.

Therefore,

W is a subspace of M_{nn}.

11/ In each part, determine whether the given vectors span R^3?

b) $v_1 = (2,-1,3)$, $v_2 = (4,1,2)$. $v_3 = (8,-1,8)$

Solution:

Take any vector $(a,b,c) \in R^3$

Set,

$$(a,b,c) = k_1 v_1 + k_2 v_2 + k_3 v_3$$

$$= k_1 (2,-1,3) + k_2 (4,1,2) + k_3 (8,-1,8)$$

$$= (2k_1, -k_1, 3k_1) + (4k_2, k_2, 2k_2) + (8k_3, -k_3, 8k_3)$$

$$= (2k_1 + 4k_2 + 8k_3 , \quad -k_1 + k_2 - k_3 , \quad 3k_1 + 2k_2 + 8k_3)$$

Therefore,

$$2k_1 + 4k_2 + 8k_3 = a$$

$$-k_1 + k_2 - k_3 = b$$

$$3k_1 + 2k_2 + 8k_3 = c$$

∴ Co-effecient matrix:

$$A = \begin{bmatrix} 2 & 4 & 8 \\ -1 & 1 & -1 \\ 3 & 2 & 8 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 2 & 4 & 8 \\ -1 & 1 & -1 \\ 3 & 2 & 8 \end{vmatrix} = 2(8+2) - 4(-8+3) + 8(-2-3)$$

$$= 20 + 20 - 40$$

$$= 0$$

det (A) is zero. So, system has no solution.

So, k_1, k_2, k_3 do not exist.

Therefore,

v_1, v_2, v_3 do not span R^3.

c) $v_1 = (3,1,4)$, $v_2 = (2,-3,5)$, $v_3 = (5,-2,9)$

Solution:

Take any vector $(a,b,c) \in R^3$

Set,

$$(a,b,c) = k_1 v_1 + k_2 v_2 + k_3 v_3$$

$$= k_1(3,1,4) + k_2(2,-3,5) + k_3(5,-2,9)$$

$$= (3k_1, k_1, 4k_1) + (2k_2, -3k_2, 5k_2) + (5k_3, -2k_3, 9k_3)$$

$$= (3k_1 + 2k_2 + 5k_3,\ k_1 - 3k_2 - 2k_3,\ 4k_1 + 5k_2 + 9k_3)$$

Therefore,

$$3k_1 + 2k_2 + 5k_3 = a$$
$$k_1 - 3k_2 - 2k_3 = b$$
$$4k_1 + 5k_2 + 9k_3 = c$$

$\therefore$ Co-effecient matrix:

$$A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & -3 & -2 \\ 4 & 5 & 9 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 3 & 2 & 5 \\ 1 & -3 & -2 \\ 4 & 5 & 9 \end{vmatrix} = 3(-27+10) - 2(9+8) + 5(5+12)$$

$$= -51 - 34 + 85$$

$$= 0$$

det (A) is zero. So, system has no solution.

So, k_1, k_2, k_3 do not exist.

Therefore, v_1, v_2, v_3 do not span $\mathbb{R}^3$.

4.3

4) Which of the following sets of vectors in P_2 are linearly dependent?

a) $2-x+4x^2,\ 3+6x+2x^2,\ 2+10x-4x^2$

Solution:

Let,

$$P_1 = 2-x+4x^2$$
$$P_2 = 3+6x+2x^2$$
$$P_3 = 2+10x-4x^2$$

Take,

$$k_1P_1 + k_2P_2 + k_3P_3 = 0$$

$$\Rightarrow k_1(2-x+4x^2) + k_2(3+6x+2x^2) + k_3(2+10x-4x^2) = 0$$

$$\Rightarrow \cancel{2k_1 - k_1x}$$

$$\Rightarrow (2k_1+3k_2+2k_3) + (-k_1+6k_2+10k_3)x + (4k_1+2k_2-4k_3)x^2 = 0$$

Therefore, linear system:

$$2k_1 + 3k_2 + 2k_3 = 0$$
$$-k_1 + 6k_2 + 10k_3 = 0$$
$$4k_1 + 2k_2 - 4k_3 = 0$$

Co-effecient matrix:

$$A = \begin{bmatrix} 2 & 3 & 2 \\ -1 & 6 & 10 \\ 4 & 2 & -4 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 2 & 3 & 2 \\ -1 & 6 & 10 \\ 4 & 2 & -4 \end{vmatrix} = 2(-24-20)-3(4-40)+2(-2-24)$$

$$= -88+108-52$$

$$= -32$$

Hence $\det(A) \neq 0$

Therefore,

(P_1, P_2, P_3) is linearly independent. in P_2

d) $1+3x+3x^2$, $x+4x^2$, $5+6x+3x^2$, $7+2x-x^2$

Solution:

Let, $P_1 = 1+3x+3x^2$

$P_2 = x+4x^2$

$P_3 = 5+6x+3x^2$

$P_4 = 7+2x-x^2$

Take,

$$k_1P_1 + k_2P_2 + k_3P_3 + k_4P_4 = 0$$

$$\Rightarrow k_1(1+3x+3x^2) + k_2(x+4x^2) + k_3(5+6+3x^2) + k_4(7+2x-x^2) = 0$$

$$\Rightarrow (k_1 + 5k_3 + 7k_4) + (3k_1 + k_2 + 6k_3 + 2k_4)x + (3k_1 + 4k_2 + 3k_3 - k_4)x^2 = 0$$

Therefore linear system:

$$k_1 \quad + 5k_3 + 7k_4 = 0$$

$$3k_1 + k_2 + 6k_3 + 2k_4 = 0$$

$$3k_1 + 4k_2 + 3k_3 - k_4 = 0$$

This is a homogenus linear system and it have less equation than variables. So, this system have many solutions.

Therefore,

(P_1, P_2, P_3, P_4) is linearly dependent in P_2.

20) By using appropriate identities, where required, determine which of the following sets of vectors in $F(-\infty, \infty)$ are linearly dependent.

c) $1, \sin x, \sin 2x$

Solution:

Consider the set of vectors $1, \sin x, \sin^2 x$, then

$$a(1) + b\sin x + c\sin^2 x = 0$$

for all $x \in [-\inf, \infty]$ not all a, b, c is zero, Let $x = 0$ then

$$a(1) + b\sin 0 + c\sin^2 0 = 0$$

$$a(1) + b(0) + c(0) = 0$$

$$a(1) = 0$$

$$a = 0$$

Let $x = \frac{\pi}{2}$ then

$$a(1) + b\sin\frac{\pi}{2} + c\sin^2\frac{\pi}{2} = 0$$

$$a(1) + b(1) + c(0) = 0$$

$$a + b = 0$$

$$b = 0$$

replace $a=0, b=0$ in

$$a(1) + b\sin x + c\sin^2 x = 0$$

$$0 + 0 \cdot + c\sin^2 x = 0$$

$$c = 0$$

Since the vectors can not be written as a linear combination of the remaining ones, then the set of the vectors are linearly independent.

d) $\cos 2x, \sin^2 x, \cos^2 x$

<u>Solution:</u>

Consider the set of vectors $\cos 2x, \sin^2 x, \cos^2 x$.

We can write $\cos 2x$ as a linear combination $3\sin^2 x$,

$\cos^2 x$ then,

$$a(\sin^2 x) + b(\cos^2 x) = \cos 2x$$

Let $a=1$ and $b=-1$ then,

$$\sin^2 x - \cos^2 x = \cos 2x$$

$$\sin^2 x + (-1)\cos^2 x = \cos 2x$$

The set of the vector is linearly dependent if one vector can be written as a linear combination of the remaining vector, here $\cos 2x$ is written as a linear combination $3\sin^2 x$, $2\cos^2 x$, then the vectors $3\sin^2 x$, $\cos^2 x$ are linearly dependent.