NORTH SOUTH UNIVERSITY

Department of Mathematics & Physics

Assignment – 05

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Course No. : MAT 250

Course Title : Calculus and Analytic Geometry IV

Section : 16

Date : 27 May 2023

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$$F(x,y,z) = x^2\,\hat{i} - 2\,\hat{j} + yz\,\hat{k}$$

Therefore,

$$\text{div } F = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(2) + \frac{\partial}{\partial z}(yz)$$

$$= 2x + 0 + y$$

$$= 2x + y$$

Ans

$$\text{curl } F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 2 & yz \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}(yz) - \frac{\partial}{\partial z}(2)\right) - \hat{j}\left(\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial z}(x^2)\right) + \hat{k}\left(\frac{\partial}{\partial x}(2) - \frac{\partial}{\partial y}(x^2)\right)$$

$$= z\,\hat{i}$$

Ans

19)

Given,

$$F(x,y,z) = 7y^3z^2\,\hat{i} - 8x^2z^5\,\hat{j} - 3xy^4\,\hat{k}$$

Therefore,

$$\operatorname{div} F = \frac{\partial}{\partial x}(7y^3z^2) + \frac{\partial}{\partial y}(8x^2z^5) + \frac{\partial}{\partial z}(3xy^4)$$

$$= 0 + 0 + 0$$

$$= 0$$

Ans

$$\operatorname{curl} F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 7y^3z^2 & 8x^2z^5 & 3xy^4 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}(3xy^4) - \frac{\partial}{\partial z}(8x^2z^5)\right) - \hat{j}\left(\frac{\partial}{\partial x}(3xy^4) - \frac{\partial}{\partial z}(7y^3z^2)\right)$$
$$+ \hat{k}\left(\frac{\partial}{\partial x}(8x^2z^5) - \frac{\partial}{\partial y}(7y^3z^2)\right)$$

$$= (12xy^3 - 40x^2z^4)\,\hat{i} - (3y^4 - 14y^3z)\,\hat{j} + (16xz^5 - 21y^2z^2)\,\hat{k}$$

Ans

21)

Given,

$$F(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}(x\hat{i}+y\hat{j}+z\hat{k})$$

Let,

$$r = \sqrt{x^2+y^2+z^2} \quad ; \quad \frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}$$

Then,

$$F(x,y,z) = \frac{1}{r}(x\hat{i}+y\hat{j}+z\hat{k})$$

$$= \frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}$$

Now,

$$\operatorname{div} F = \frac{\partial}{\partial x}\left(\frac{x}{r}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r}\right) + \frac{\partial}{\partial z}\left(\frac{z}{r}\right)$$

$$= \frac{r - x\cdot\frac{x}{r}}{r^2} + \frac{r - y\cdot\frac{y}{r}}{r^2} + \frac{r - z\cdot\frac{z}{r}}{r^2}$$

$$= \frac{\frac{r^2-x^2}{r}}{r^2} + \frac{\frac{r^2-y^2}{r}}{r^2} + \frac{\frac{r^2-z^2}{r}}{r^2}$$

$$= \frac{r^2-x^2+r^2-y^2+r^2-z^2}{r^3}$$

$$= \frac{3r^2-(x^2+y^2+z^2)}{r^3} = \frac{3r^2-r^2}{r^3} = \frac{2}{r}$$

$$= \frac{2}{\sqrt{x^2+y^2+z^2}}$$

$$\therefore \text{curl } F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{r} & \frac{y}{r} & \frac{z}{r} \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}\left(\frac{z}{r}\right) - \frac{\partial}{\partial z}\left(\frac{y}{r}\right)\right) - \hat{j}\left(\frac{\partial}{\partial x}\left(\frac{z}{r}\right) - \frac{\partial}{\partial z}\left(\frac{x}{r}\right)\right)$$
$$+ \hat{k}\left(\frac{\partial}{\partial x}\left(\frac{y}{r}\right) - \frac{\partial}{\partial y}\left(\frac{x}{r}\right)\right)$$

$$= \left(\frac{-z}{r^2}\cdot\frac{y}{r} + \frac{y}{r^2}\cdot\frac{z}{r}\right)\hat{i} - \left(-\frac{z}{r^2}\cdot\frac{x}{r} + \frac{x}{r^2}\cdot\frac{z}{r}\right)\hat{j}$$
$$+ \left(-\frac{y}{r^2}\cdot\frac{x}{r} + \frac{x}{r^2}\cdot\frac{y}{r}\right)\hat{k}$$

$$= \left(\frac{yz}{r^3} - \frac{yz}{r^3}\right)\hat{i} - 0\hat{j} + 0\cdot\hat{k}$$

$$= 0$$

Ans

22/

Given,

$$F(x,y,z) = \ln x \,\hat{i} + e^{xyz}\,\hat{j} + \tan^{-1}\left(\frac{z}{x}\right)\hat{k}$$

Therefore,

$$\text{div } F = \frac{\partial}{\partial x}(\ln x) + \frac{\partial}{\partial y}\left(e^{xyz}\right) + \frac{\partial}{\partial z}\left(\tan^{-1}\frac{z}{x}\right)$$

$$= \frac{1}{x} + e^{xyz}\cdot xz + \frac{1}{1+\frac{z^2}{x^2}}\cdot\frac{1}{x}$$

$$= \frac{1}{x} + xze^{xyz} + \frac{1}{\frac{x^2+z^2}{x^2}}\cdot\frac{1}{x}$$

$$= \frac{1}{x} + xze^{xyz} + \frac{x}{x^2+z^2} \quad \text{Ans}$$

$$\text{curl } F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \ln x & e^{xyz} & \tan^{-1}\frac{z}{x} \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}\left(\tan^{-1}\frac{z}{x}\right) - \frac{\partial}{\partial z}\left(e^{xyz}\right)\right) - \hat{j}\left(\frac{\partial}{\partial x}\left(\tan^{-1}\frac{z}{x}\right) - \frac{\partial}{\partial z}(\ln x)\right)$$

$$+ \hat{k}\left(\frac{\partial}{\partial x}\left(e^{xyz}\right) - \frac{\partial}{\partial y}(\ln x)\right)$$

$$= \left(0 - e^{xyz}\cdot xy\right)\hat{i} - \left(-\frac{1}{1+\frac{z^2}{x^2}}\cdot\frac{z}{x^2} - 0\right)\hat{j}$$

$$+ \left(e^{xyz}\cdot yz - 0\right)\hat{k}$$

$$= -xye^{xyz}\,\hat{i} + \frac{z}{x^2+z^2}\,\hat{j} + yze^{xyz}\,\hat{k}$$

Ans

23)

Given,

$$F(x,y,z) = 2x\,\hat{i} + \hat{j} + 4y\,\hat{k}$$

$$G(x,y,z) = x\hat{i} + y\hat{j} - z\hat{k}$$

Here,

$$(F \times G) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2x & 1 & 4y \\ x & y & -z \end{vmatrix}$$

$$= \hat{i}\left(-z - 4y^2\right) - \hat{j}\left(-2xz - 4xy\right) + \hat{k}\left(2xy - x\right)$$

$$\therefore \nabla \cdot (F \times G) = \frac{\partial}{\partial x}\left(-z - 4y^2\right) + \frac{\partial}{\partial y}\left(2xz + 4xy\right) + \frac{\partial}{\partial z}\left(2xy - x\right)$$

$$= 0 + 4x + 0$$

$$= 4x$$

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Given,

$$F(x,y,z) = \sin x\,\hat{i} + \cos(x-y)\,\hat{j} + z\hat{k}$$

We know,

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Here,

$$(\nabla \times F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin x & \cos(x-y) & z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}(z) - \frac{\partial}{\partial z}(\cos(x-y))\right) - \hat{j}\left(\frac{\partial}{\partial x}(z) - \frac{\partial}{\partial z}(\sin x)\right) + \hat{k}\left(\frac{\partial}{\partial x}\cos(x-y) - \frac{\partial}{\partial y}\sin x\right)$$

$$= 0 - 0 + \hat{k}\left(-\sin(x-y)\right)$$

$$= -\sin(x-y)\,\hat{k}$$

Therefore,

$$\nabla \cdot (\nabla \times F) = \frac{\partial}{\partial z}\left(-\sin(x-y)\right)$$

$$= 0$$

Q 27)

Given,

$$F(x,y,z) = xy\,\hat{j} + xyz\,\hat{k}$$

We know,

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Here,

$$(\nabla \times F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & xy & xyz \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}(xyz) - \frac{\partial}{\partial z}(xy)\right) - \hat{j}\left(\frac{\partial}{\partial x}(xyz) - 0\right) + \hat{k}\left(\frac{\partial}{\partial x}(xy) - 0\right)$$

$$= xz\,\hat{i} - yz\,\hat{j} + y\,\hat{k}$$

Therefore,

$$\nabla \times (\nabla \times F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & -yz & y \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(yz)\right) - \hat{j}\left(\frac{\partial}{\partial x}(y) - \frac{\partial}{\partial z}(xz)\right) + \hat{k}\left(\frac{\partial}{\partial x}(-yz) - \frac{\partial}{\partial y}(xz)\right)$$

$$= (1+y)\hat{i} + x\hat{j}$$

Ans

41/

Given,

radius vector,

$$r = x\hat{i} + y\hat{j} + z\hat{k}$$

a)

$$\text{curl } r = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}z - \frac{\partial}{\partial z}y\right) - \hat{j}\left(\frac{\partial}{\partial x}z - \frac{\partial}{\partial z}x\right) + \hat{k}\left(\frac{\partial}{\partial x}y - \frac{\partial}{\partial y}x\right)$$

$$= 0 - 0 - 0$$

$$= 0$$

$\therefore$ curl $r = 0$. (verified)

b)

Here,

$$\|r\| = \sqrt{x^2+y^2+z^2}$$

Therefore,

$$\nabla\|r\| = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(\sqrt{x^2+y^2+z^2}\right)$$

$$= \frac{\partial}{\partial x}\left(\sqrt{x^2+y^2+z^2}\right)\hat{i} + \frac{\partial}{\partial y}\left(\sqrt{x^2+y^2+z^2}\right)\hat{j} + \frac{\partial}{\partial z}\left(\sqrt{x^2+y^2+z^2}\right)\hat{k}$$

$$= \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}$$

$$= \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}}$$

$$= \frac{r}{\|r\|}$$ (verified).