NORTH SOUTH UNIVERSITY

Department of Mathematics & Physics

Assignment – 02

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13.3

3)

Given that,

$$z = 9x^2y - 3x^5y$$

$$\therefore \frac{\partial z}{\partial x} = 18xy - 15x^4y$$

$$\therefore \frac{\partial z}{\partial y} = 9x^2 - 3x^5$$

5)

Given that,

$$z = (x^2+5x-2y)^8$$

$$\therefore \frac{\partial z}{\partial x} = 8(x^2+5x-2y)^7(2x+5)$$

$$\therefore \frac{\partial z}{\partial y} = 8(x^2+5x-2y)^7(-2)$$

$$= -16(x^2+5x-2y)^7$$

7)

$$\therefore \frac{\partial}{\partial p}\left(e^{-\frac{7p}{q}}\right) = e^{-\frac{7p}{q}} \cdot \frac{\partial}{\partial p}\left(-\frac{7p}{q}\right)$$

$$= e^{-\frac{7p}{q}} \cdot \left(-7 \cdot \frac{1}{q}\right)$$

$$= -\frac{7}{q} \cdot e^{-\frac{7p}{q}}$$

Ans

$$\therefore \frac{\partial}{\partial q}\left(e^{-\frac{7p}{q}}\right) = e^{-\frac{7p}{q}} \cdot \frac{\partial}{\partial q}\left(-\frac{7p}{q}\right)$$

$$= e^{-\frac{7p}{q}} \cdot \left(-7p \frac{1}{-q^2}\right)$$

$$= \frac{7p}{q^2} \cdot e^{-\frac{7p}{q}}$$

Ans

11)

$$f(x,y) = \sqrt{3x+2y}$$

a)

Given $z = f(x,y)$

$\therefore$ Slope of z in x- direction,

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(f(x,y))$$

$$= \frac{\partial}{\partial x}\left(\sqrt{3x+2y}\right)$$

$$= \frac{1}{2\sqrt{3x+2y}} \cdot 3 = \frac{3}{2\sqrt{3x+2y}}$$

$\therefore$ slope at point $(4,2) = \dfrac{3}{2\sqrt{3\cdot 4+2\cdot 2}}$

$$= \frac{3}{2\sqrt{12+4}}$$

$$= \frac{3}{8}$$

Ans

b)

Slope of z in y-direction,

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(\sqrt{3x+2y}\right)$$

$$= \frac{1}{2\sqrt{3x+2y}} \cdot 2$$

$$= \frac{2}{2\sqrt{3x+2y}}$$

$\therefore$ Slope at point $(4,2) = \dfrac{2}{2\sqrt{3\cdot 4+2\cdot 2}}$

$$= \frac{2}{2\sqrt{12+4}}$$

$$= \frac{1}{4}$$

Ans

13)

a)

Given that,

$$z = \sin(y^2 - 4x)$$

$$\therefore \frac{\partial z}{\partial x} = \cos(y^2-4x)\cdot(-4)$$

$$= -4\cos(y^2-4x)$$

$\therefore$ rate of change at the point $(2,1) = -4\cos(1-4\cdot 2)$

$$= -4\cos(-7)$$

$$= -4\cos 7$$

$$= -3.97$$

Ans

b)

Given that,

$$z = \sin(y^2 - 4x)$$

$$\therefore \frac{\partial z}{\partial y} = \cos(y^2-4x)\cdot(2y)$$

$$= 2y\cos(y^2-4x)$$

$\therefore$ rate of change at the point $(2,1) = (2\cdot 1)\cos(1-4\cdot 2)$

$$= 2\cos 7$$

$$= 1.99$$

Ans

27)

Given that,

$$z = x^3 \ln\left(1+xy^{-3/5}\right)$$

$$\therefore \frac{\partial z}{\partial x} = x^3 \cdot \frac{1}{1+xy^{-3/5}} \cdot y^{-3/5} + 3x^2 \cdot \ln\left(1+xy^{-3/5}\right)$$

$$= \frac{x^3}{y^{-3/5}\left(y^{3/5}+x\right)} \cdot y^{-3/5} + 3x^2 \cdot \ln\left(1+xy^{-3/5}\right)$$

$$= \frac{x^3}{\left(y^{3/5}+x\right)} + 3x^2 \cdot \ln\left(1+xy^{-3/5}\right)$$

Ans

$$\therefore \frac{\partial z}{\partial y} = x^3 \cdot \frac{1}{\left(1+xy^{-3/5}\right)} \cdot x \cdot \frac{-3}{5} \cdot y^{-8/5} + 0 \cdot \ln\left(1+xy^{-3/5}\right)$$

$$= \frac{-3x^4}{5\left(1+xy^{-3/5}\right) \cdot y^{8/5}}$$

$$= \frac{-3x^4}{5\left(y^{8/5}+xy\right)}$$

Ans

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Given that,

$$z = \frac{xy}{x^2+y^2}$$

$$\therefore \frac{\partial z}{\partial x} = \frac{(x^2+y^2).y - xy.2x}{(x^2+y^2)^2}$$

$$= \frac{y(x^2+y^2-2x^2)}{(x^2+y^2)^2}$$

$$= \frac{y(y^2-x^2)}{(x^2+y^2)^2}$$ An

$$\therefore \frac{\partial z}{\partial y} = \frac{(x^2+y^2)x - xy.2y}{(x^2+y^2)^2}$$

$$= \frac{x(x^2+y^2-2y^2)}{(x^2+y^2)^2}$$

$$= \frac{x(x^2-y^2)}{(x^2+y^2)^2}$$ Ans

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Given that,

$$z = \frac{x^2y^3}{\sqrt{x+y}}$$

$$\therefore \frac{\partial z}{\partial x} = \frac{\sqrt{x+y}\cdot 2xy^3 - x^2y^3\cdot\frac{1}{2\sqrt{x+y}}}{\left(\sqrt{x+y}\right)^2}$$

$$= \frac{xy^3\left(2\sqrt{x+y} - \frac{x}{2\sqrt{x+y}}\right)}{(x+y)}$$

$$= \frac{xy^3 \quad \frac{4(x+y)-x}{2\sqrt{x+y}}}{(x+y)}$$

$$= \frac{xy^3(4x+4y-x)}{2(x+y)(x+y)^{\frac{1}{2}}}$$

$$= \frac{xy^3(3x+4y)}{2(x+y)^{3/2}}$$ Ans

$$\therefore \frac{\partial z}{\partial y} = \frac{\sqrt{x+y}\cdot 3x^2y^2 - x^2y^3\cdot\frac{1}{2\sqrt{x+y}}}{\left(\sqrt{x+y}\right)^2}$$

$$= \frac{x^2y^2\left(3\sqrt{x+y} - \frac{y}{2\sqrt{x+y}}\right)}{(x+y)}$$

$$= \frac{x^2y^2\left(\frac{6(x+y)-y}{2\sqrt{x+y}}\right)}{(x+y)}$$

$$= \frac{x^2y^2(6x+6y-y)}{2(x+y)(x+y)^{\frac{1}{2}}}$$

$$= \frac{x^2y^2(6x+5y)}{2(x+y)^{3/2}}$$ Ans

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Given that,

$$f(x,y) = \frac{x+y}{x-y}$$

$$\therefore f_x(x,y) = \frac{(x\cancel{+}y)\cdot 1-(x+y)\cdot 1}{(x-y)^2}$$

$$= \frac{x\cancel{+}y-x-y}{(x-y)^2}$$

$$= \frac{-2y}{(x-y)^2}$$ Ans

$$\therefore f_y(x,y) = \frac{(x-y)\cdot 1 - (x+y)(-1)}{(x-y)^2}$$

$$= \frac{x-y+x+y}{(x-y)^2}$$

$$= \frac{2x}{(x-y)^2} \quad \underline{\text{Ans}}$$

33)

Given that,

$$f(x,y) = y^{-3/2} \tan^{-1}\left(\frac{x}{y}\right)$$

$$\therefore f_x(x,y) = y^{-\frac{3}{2}} \cdot \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{1}{y} + 0$$

$$= \frac{y^{-\frac{5}{2}}}{1+\frac{x^2}{y^2}}$$

$$= \frac{y^{-\frac{5}{2}}}{\frac{x^2+y^2}{y^2}}$$

$$= \frac{y^{-\frac{1}{2}}}{x^2+y^2} \quad \underline{\text{Ans}}$$

$$\therefore f_y(x,y) = y^{-3/2} \cdot \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \left(-x \cdot \frac{1}{y^2}\right) + \left(-\frac{3}{2} \cdot y^{-5/2}\right) \tan^{-1}\left(\frac{x}{y}\right)$$

$$= -\frac{x y^{-7/2}}{1+\frac{x^2}{y^2}} - \frac{3}{2} y^{-5/2} \cdot \tan^{-1}\left(\frac{x}{y}\right)$$

$$= -\frac{x y^{-7/2}}{\frac{x^2+y^2}{y^2}} - \frac{3}{2} y^{-5/2} \cdot \tan^{-1}\left(\frac{x}{y}\right)$$

$$= -\frac{x y^{-3/2}}{x^2+y^2} - \frac{3}{2} y^{-5/2} \tan^{-1} \frac{x}{y}$$

Ans

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Given that,

$$f(x,y) = x^3 e^{-y} + y^3 \sec\sqrt{x}$$

$$\therefore f_x(x,y) = x^3 \cdot 0 + 3x^2 \cdot e^{-y} + y^3 \sec\sqrt{x} \tan\sqrt{x} \frac{1}{2\sqrt{x}} + 0$$

$$= 3x^2 e^{-y} + \frac{1}{2} x^{-1/2} y^3 \sec\sqrt{x} \tan\sqrt{x}$$

Ans

$$\therefore f_y(x,y) = x^3 \cdot e^{-y} \cdot (-1) + 0 + y^3 \cdot 0 + 3y^2 \sec\sqrt{x}$$

$$= 3y^2 \sec\sqrt{x} - x^3 e^{-y}$$

Ans.

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Given that,

$$f(x,y) = (y^2 \tan x)^{-4/3}$$

$$\therefore f_x(x,y) = -\frac{4}{3}(y^2 \tan x)^{-7/3} \cdot (y^2 \sec^2 x + 0)$$

$$= -\frac{4}{3} y^2 \sec^2 x \, (y^2 \tan x)^{-7/3}$$

Ans

$$\therefore f_y(x,y) = -\frac{4}{3}(y^2 \tan x)^{-7/3} \cdot (y^2 \cdot 0 + 2y \cdot \tan x)$$

$$= -\frac{4}{3} \cdot 2y \tan x \cdot (y^2 \tan x)^{-7/3}$$

$$= -\frac{8}{3} y \tan x \, (y^2 \tan x)^{-7/3}$$

Ans

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Given that,

$$f(x,y) = \cosh(\sqrt{x}) \sinh^2(xy^2)$$

$$\therefore f_x(x,y) = \cosh(\sqrt{x}) \cdot 2\sinh(xy^2) \cdot \cosh(xy^2) \cdot y^2 + \sinh(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} \cdot \sinh^2(xy^2)$$

$$= 2y^2 \cosh(\sqrt{x}) \sinh(xy^2) \cosh(xy^2) + \frac{1}{2} x^{-1/2} \cdot \sinh(\sqrt{x}) \sinh^2(xy^2)$$

Ans

$\therefore f_y(x,y) = \cosh(\sqrt{x}) \cdot 2\sinh(xy^2) \cdot \cosh(xy^2) \cdot 2xy + 0$

$= 4xy \cosh(\sqrt{x}) \sinh(xy^2) \cosh(xy^2)$ Ans

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Given that,

$f(x,y,z) = z \ln(x^2 y \cos z)$

$\therefore f_x(x,y,z) = z \cdot \frac{1}{x^2 y \cos z} \cdot 2xy\cos z$

$= \frac{2z}{x}$ Ans

$\therefore f_y(x,y,z) = z \cdot \frac{1}{x^2 y \cos z} \cdot x^2 \cos z$

$= \frac{z}{y}$ Ans

$\therefore f_z(x,y,z) = z \cdot \frac{1}{x^2 y \cos z} \cdot (-x^2 y \sin z) + \ln(x^2 y \cos z)$

$= -\frac{z \sin z}{\cos z} + \ln(x^2 y \cos z)$

$= \ln(x^2 y \cos z) - z\tan z$ Ans

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Given that,

$$f(x,y,z) = y^{-3/2} \cdot \sec\left(\frac{xz}{y}\right)$$

$$\therefore f_x(x,y,z) = y^{-3/2} \cdot \sec\frac{xz}{y} \tan\frac{xz}{y} \cdot \frac{z}{y} + 0$$

$$= y^{-5/2} z \sec\frac{xz}{y} \tan\frac{xz}{y}$$ Ans

$$\therefore f_y(x,y,z) = y^{-3/2} \cdot \sec\frac{xz}{y} \tan\frac{xz}{y} \cdot xz\left(-\frac{1}{y^2}\right) + \left(\frac{-3}{2} y^{-5/2} \cdot \sec\frac{xz}{y}\right)$$

$$= -xy^{-7/2} z \sec\frac{xz}{y} \tan\frac{xz}{y} - \frac{3}{2} y^{-5/2} \sec\frac{xz}{y}$$

Ans

$$\therefore f_z(x,y,z) = y^{-3/2} \cdot \sec\frac{xz}{y} \cdot \tan\frac{xz}{y} \cdot \frac{x}{y} + 0$$

$$= xy^{-5/2} \sec\frac{xz}{y} \cdot \tan\frac{xz}{y}$$

Ans

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Given that,

$$f(x,y,z) = \tan^{-1}\left(\frac{1}{xy^2z^3}\right)$$

$$\therefore f_x(x,y,z) = \frac{1}{1+\left(\frac{1}{xy^2z^3}\right)^2} \cdot \frac{1}{y^2z^3} \cdot \left(-\frac{1}{x^2}\right)$$

$$= -\frac{1}{1+\frac{1}{x^2y^4z^6}} \cdot \frac{1}{x^2y^2z^3}$$

$$= -\frac{1}{\frac{x^2y^4z^6+1}{x^2y^4z^6}} \cdot \frac{1}{x^2y^2z^3}$$

$$= -\frac{x^2y^4z^6}{x^2y^4z^6+1} \cdot \frac{1}{x^2y^2z^3}$$

$$= -\frac{y^2z^3}{x^2y^4z^6+1}$$

Ans

$$\therefore f_y(x,y,z) = \frac{1}{1+\left(\frac{1}{xy^2z^3}\right)^2} \cdot \frac{1}{xz^3} \cdot \left(-2\frac{1}{y^3}\right)$$

$$= -\frac{x^2y^4z^6}{x^2y^4z^6+1} \cdot \frac{2}{xy^3z^3} = -\frac{2xyz^3}{x^2y^4z^6+1}$$ Ans

$$\therefore f_z(x,y,z) = \frac{1}{1+\left(\frac{1}{xy^2z^3}\right)^2} \cdot \frac{1}{xy^2} \cdot \left(-3\frac{1}{z^4}\right)$$

$$= -\frac{x^2y^4z^6}{x^2y^4z^6+1} \cdot \frac{3}{xy^2z^4}$$

$$= -\frac{3xy^2z^2}{x^2y^4z^6+1}$$ Ans

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Given that,

$$f(x,y,z) = \cosh(\sqrt{z})\ \sinh^2(x^2yz)$$

$$\therefore f_x(x,y,z) = \cosh(\sqrt{z}) \cdot 2\sinh(x^2yz) \cdot \cosh(x^2yz) \cdot 2xyz + 0$$

$$= 4xyz \cosh(\sqrt{z}) \sinh(x^2yz) \cosh(x^2yz)$$ Ans

$$\therefore f_y(x,y,z) = \cosh(\sqrt{z}) \cdot 2\sinh(x^2yz) \cdot \cosh(x^2yz) \cdot x^2z + 0$$

$$= 2x^2z \cosh(\sqrt{z}) \sinh(x^2yz) \cosh(x^2yz)$$

$$\therefore f_z(x,y,z) = \cosh(\sqrt{z}) \cdot 2\sinh(x^2yz) \cdot \cosh(x^2yz) \cdot x^2y + \sinh(\sqrt{z}) \cdot \frac{1}{2\sqrt{z}} \cdot \sinh^2(x^2yz)$$

$$= 2x^2y \cosh(\sqrt{z}) \sinh(x^2yz) \cosh(x^2yz) + \frac{1}{2} z^{-1/2} \sinh(\sqrt{z}) \cdot \sinh^2(x^2yz)$$

Ans

48)

Given that,

$$w = \frac{x^2-y^2}{y^2+z^2}$$

$$\cancel{= \frac{x^2}{y^2+z^2} - \frac{y^2}{y^2+z^2}}$$

$$\therefore \frac{\partial w}{\partial x} = \frac{2x}{y^2+z^2} - 0$$

$$= \frac{2x}{y^2+z^2} \quad \text{Ans}$$

$$\frac{\partial w}{\partial y} = \frac{(y^2+z^2)(-2y) - (x^2-y^2)(2y)}{(y^2+z^2)^2}$$

$$= \frac{-2y\,(y^2+z^2+x^2-y^2)}{(y^2+z^2)^2} = -2y\,\frac{(x^2+z^2)}{(y^2+z^2)^2} \quad \text{Ans}$$

$$\therefore \frac{\partial w}{\partial z} = \frac{(y^2+z^2)\cdot 0 - (x^2-y^2)\,2z}{(y^2+z^2)^2}$$

$$= \frac{-2z\,(x^2-y^2)}{(y^2+z^2)^2} \quad \text{Ans}$$

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Given that,

$$w = \sqrt{x^2+y^2+z^2}$$

$$\therefore \frac{\partial w}{\partial x} = \frac{1}{2\sqrt{x^2+y^2+z^2}}\cdot 2x$$

$$= \frac{x}{\sqrt{x^2+y^2+z^2}} \quad \text{Ans}$$

$$\therefore \frac{\partial w}{\partial y} = \frac{1}{2\sqrt{x^2+y^2+z^2}}\cdot 2y$$

$$= \frac{y}{\sqrt{x^2+y^2+z^2}} \quad \text{Ans}$$

$$\therefore \frac{\partial w}{\partial z} = \frac{1}{2\sqrt{x^2+y^2+z^2}}\cdot 2z = \frac{z}{\sqrt{x^2+y^2+z^2}} \quad \text{Ans}$$

55)

Given that,

$$z = x^2 + 3y^2$$

rate of change of z with respect to x,

$$\frac{\partial z}{\partial x} = 2x$$

$\therefore$ rate of change, at the point $(2,1,7)$ is

$$\left.\frac{\partial z}{\partial x}\right|_{(2,1,7)} = 2 \cdot 2 = 4$$

Ans