H.W.

1. $y = 2 + \cos \frac{x}{2}$

2. $y = -2 \cos \pi x$

Solution

1)

$$y = 2 + \cos \frac{x}{2}$$

$$\Rightarrow y = \cos \frac{x}{2} + 2$$

Now,

$$y = \cos \cancel{x} \frac{1}{2} \cdot x$$

Here,

$$\text{Amplitute} = |1| = 1$$

$$\text{Period} = \frac{2\pi}{1/2} = 4\pi$$

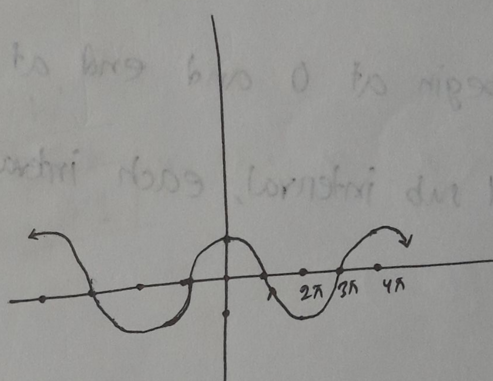
One circle begin at 0 and end at 4π.

Derided by 4 sub interval, each interval length $\frac{4\pi - 0}{4} = \pi$

Now points,

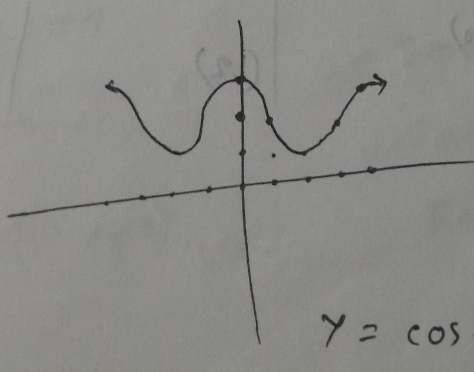
$x=0$	$x=\pi$	$x=2\pi$	$x=3\pi$	$x=4\pi$
$y=\cos\frac{1}{2}\cdot 0$	$y=\cos\frac{1}{2}\pi$	$y=\cos\frac{1}{2}\cdot 2\pi$	$y=\cos\frac{1}{2}\cdot 3\pi$	$y=\cos\frac{1}{2}\cdot 4\pi$
$=\cos 0$	$=\cos\frac{\pi}{2}$	$=\cos\pi$	$=\cos\frac{3\pi}{2}$	$=\cos 2\pi$
$=1$	$=0$	$=-1$	$=0$	$=1$
$(0,1)$	$(\pi,0)$	$(2\pi,-1)$	$(3\pi,0)$	$(4\pi,1)$

Graph:



$y=\cos\frac{1}{2}\cdot x$

$\downarrow$ adding 2



$y=\cos\frac{x}{2}+2$

2)

$Y = -2\cos \pi x$

Here,

Amplitute $= |-2| = 2$

Y- co-ordinat line between -2, to 2.

Period $= \frac{2\pi}{\pi} = 2$

One circle begin at 0 and end at 2.

Devided by 4 sub interval, each interval length $\frac{2-0}{4} = \frac{1}{2}$

Now points,

$x = 0$	$x = \frac{1}{2}$	$x = 1$	$x = \frac{3}{2}$
$y = -2\cos \pi . 0$	$y = -2\cos \pi . \frac{1}{2}$	$y = -2\cos \pi . 1$	$y = -2\cos \pi . \frac{3}{2}$
$= -2\cos 0$	$= -2\cos \frac{\pi}{2}$	$= -2\cos \pi$	$= -2\cos \frac{3\pi}{2}$
$= -2$	$= 0$	$= 2$	$= 0$
$(0,-2)$	$(\frac{1}{2}, 0)$	$(1,2)$	$(\frac{3}{2}, 0)$

$x = 2$

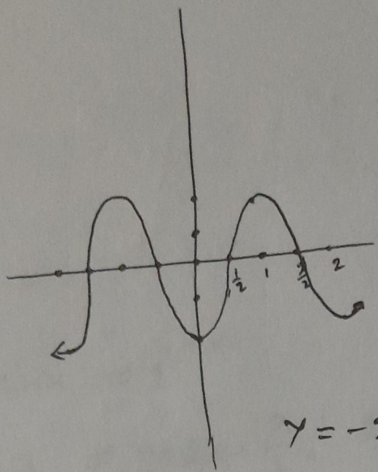
$y = -2\cos \pi . 2$

$= -2\cos 2\pi$

$= -2$

$(2,-2)$

Graph!



$y = -2\cos\pi x$

H.W

$\boxed{0.1}$

9)

a) $f(x) = \frac{1}{x-3}$

For, domain,

$x - 3 \neq 0$

$x \neq 3$

∴ Domain: $\{x : x \neq 3\}$

∴ Range: $\{y : y \neq 0\}$

b) $f(x) = \frac{x}{|x|}$

For domain,

$|x| \neq 0$

$x \neq 0$

∴ Domain: $\{x : x \neq 0\}$

~~Range: $y : y \neq 0$ $\{y : y \neq 0\}$~~

Range: $\{-1, 1\}$

c)

$g(x) = \sqrt{x^2-3}$

For domain,

$x^2 - 3 \geq 0$

$x^2 \geq 3$

~~$x \geq \pm\sqrt{3}$~~

$+ \quad | \quad - \quad | \quad +$

$-\sqrt{3} \qquad \sqrt{3}$

Domain: $\{x : x \leq -\sqrt{3} \text{ and } x \geq \sqrt{3}\}$

Range: $[0, \infty)$

d)

$G(x) = \sqrt{x^2 - 2x + 5}$

For domain,

$x^2 - 2x + 5 \geq 0$

$x^2 - 2x + 1 + 4 \geq 0$

$(x-1)^2 + 4 \geq 0$

This expression is always positive

so, domain: $(-\infty, \infty)$

Range: ~~$[0, \infty)$~~ $[2, \infty)$

e)

$$h(x) = \frac{1}{1-\sin x}$$

For domain,

$1 - \sin x \neq 0$

$\sin x \neq 1$

$x \neq \frac{\pi}{2}, \frac{5\pi}{2}$

$\therefore x \neq (2n + \frac{1}{2})\pi \; ; \quad n = 0, \pm 1, \pm 2 \ldots$

Range: $-1 \leq \sin x \leq 1$ $\quad y \geq \frac{1}{2}$

$0 \leq 1 - \sin x \leq 2$

f) $H(x) = \sqrt{\frac{x^2-4}{x-2}} = \sqrt{x+2}$

For domain:

$x - 2 \neq 0$

$x \neq 2$

$\frac{x^2-4}{x-2} \geq 0$

$\frac{(x+2)(x-2)}{x-2} \geq 0$

$x + 2 \geq 0$

$x \geq -2$

$\therefore$ domain : $\{x : x \geq -2 \text{ and } x \neq 2\}$

Range: $[0, 2) \cup (2, +\infty)$

10)

a)

$f(x) = \sqrt{3-x}$

$3 - x \geq 0$

$-x \geq 0 - 3$

$x \leq 3$

Domain: $\{x : x \leq 3\}$

b) $F(x) = \sqrt{4-x^2}$

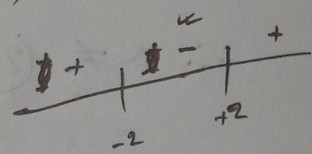
$4 - x^2 \geq 0$

$-x^2 \geq -4$

$x^2 \leq 4$

$x^2 - 4 \leq 0$

$(x+2)(x-2) \leq 0$



~~Domain: $(-\infty, -2] \cup [2, +\infty)$~~

Domain: $[-2, 2]$

c)

$g(x) = 3 + \sqrt{x}$

For domain,

$x \geq 0$

D: $\{x : x \geq 0\}$

d) $G(x) = x^3 + 2$

D: $\mathbb{R}$

e) $h(x) = 3 \sin x$

D: $\mathbb{R}$

f)

f) $H(x) = (\sin\sqrt{x})^{-2} = \dfrac{1}{(\sin\sqrt{x})^2}$

For domain,

$x \geq 0$

And,

$\cancel{\sin\sqrt{x} \neq 0}$

$\sqrt{x} \neq 0$

$\sqrt{x} \neq n\pi \; ; n = 0, \pm 1, \pm 2, \pm 3 \ldots$

Range: $y \geq 1$.

0.2

$y=x^2$ / $y=\sqrt{x}$ / $y=\frac{1}{x}$ / $y=|x|$ / $y=\sqrt[3]{x}$

5)

$y=-2(x+1)^2-3$

$y=x^2 \rightarrow (x+1)^2 \rightarrow 2(x+1)^2 \rightarrow -2(x+1)^2 \rightarrow -2(x+1)^2-3$

6)

$y=\frac{1}{2}(x-3)^2+2$

$y=x^2 \rightarrow (x-3)^2 \rightarrow \frac{1}{2}(x-3)^2 \rightarrow \frac{1}{2}(x-3)^2+2$

7)

$y=x^2+6x$ ~~$= x(x+6)$~~

$y=x^2+2\cdot 3x+3^2-9$

$y=(x+3)^2-9$

$y=x^2 \rightarrow (x+3)^2 \rightarrow (x+3)-9$

8)

$y = \frac{1}{2}(x^2 - 2x + 3)$

$= \frac{1}{2}(x^2 - 2x + 1 + 2)$

$= \frac{1}{2}((x-1)^2 + 2)$

$y = x^2 \rightarrow (x-1)^2 \rightarrow (x-1)^2 + 2 \rightarrow \frac{1}{2}((x-1)^2 + 2)$

9)

$y = 3 - \sqrt{x+1}$

$y = \sqrt{x} \rightarrow \sqrt{x+1} \rightarrow -\sqrt{x+1} \rightarrow -\sqrt{x+1} + 3$

10)

$y = 1 + \sqrt{x-4}$

$y = \sqrt{x} \rightarrow \sqrt{x-4} \rightarrow \sqrt{x-4} + 1$

11)

$y = \frac{1}{2}\sqrt{x} + 1$

$y = \sqrt{x} \rightarrow \frac{1}{2}\sqrt{x} \rightarrow \frac{1}{2}\sqrt{x} + 1$

12)

$y = -\sqrt{3x}$

$y = \sqrt{x} \rightarrow \sqrt{3x} \rightarrow -\sqrt{3x}$

13)

$y = \frac{1}{x-3}$

$y = \frac{1}{x} \rightarrow \frac{1}{(x-3)}$

14)

$y = \frac{1}{1-x} = \frac{1}{-(x-1)}$

$y = \frac{1}{x} \rightarrow \frac{1}{x-1} \rightarrow \frac{1}{-(x-1)}$

15)

$y = 2 - \frac{1}{x+1}$

$y = \frac{1}{x} \rightarrow \frac{1}{x+1} \rightarrow -\frac{1}{x+1} \rightarrow -\frac{1}{x+1} + 2$

16)

$y = \frac{x-1}{x} = \frac{x}{x} - \frac{1}{x} = 1 - \frac{1}{x}$

$y = \frac{1}{x} \rightarrow -\frac{1}{x} \rightarrow -\frac{1}{x} + 1$

17)

$y = |x+2| - 2$

$y = |x| \rightarrow |x+2| \rightarrow |x+2| - 2$

18)

$y = 1 - |x-3|$

$y = |x| \rightarrow |x-3| \rightarrow -|x-3| \rightarrow -|x-3|+1$

19)

$y = |2x-1| + 1$

$y = |x| \rightarrow |2x| \rightarrow |2x-1| \rightarrow |2x-1|+1$

20) $y = \sqrt{x^2-4x+4} = \sqrt{(x - 2\cdot x\cdot 2 + 2^2)} = \sqrt{(x-2)^2} = |x-2|$

$y = |x| \rightarrow |x-2|$

21)

$y = 1 - 2\sqrt[3]{x}$

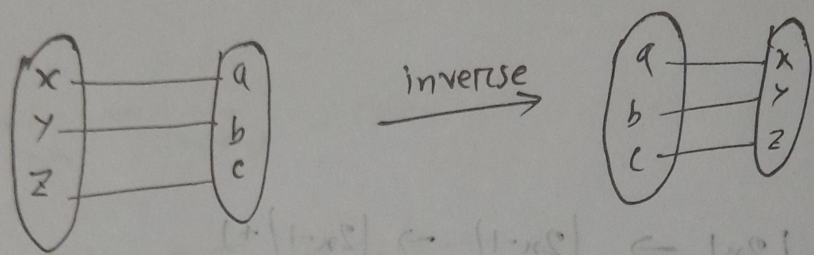
$y = \sqrt[3]{x} \rightarrow 2\sqrt[3]{x} \rightarrow -2\sqrt[3]{x} \rightarrow -2\sqrt[3]{x} + 1$

22) $y = \sqrt[3]{x-2} - 3$

$\Rightarrow y = \sqrt[3]{x} \rightarrow \sqrt[3]{x-2} \rightarrow \sqrt[3]{x-2} - 3$

0.4

Inverse function:



One to one function

If a function one to one, then interchanging range and domain, then we will get the new function, that is called inverse function.

Domain of f^{-1} = Range of f

Range of f^{-1} = Domain of f

Getting Inverse function from the equation:

Example: If $f(x) = x^3 - 1$, find inverse of $f(x)$.

$\Rightarrow \quad f(x) = x^3 - 1$

$y = x^3 - 1$

Interchange x and y,

$x = y^3 - 1$

solve for y,

$y^3 = x + 1$

$\therefore y = \sqrt[3]{x+1}$

$\therefore f^{-1}(x) = \sqrt[3]{x+1}$

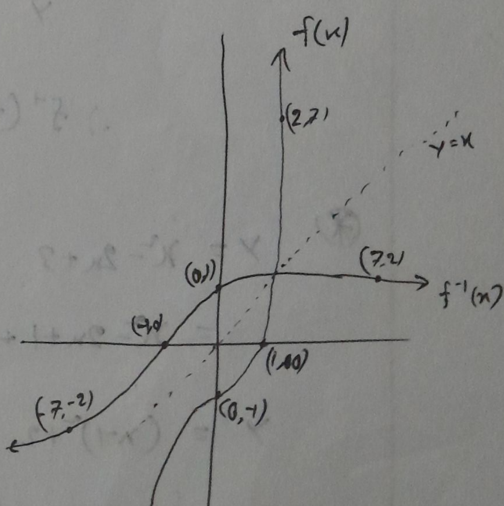
Checking:

$f(f^{-1}(x)) = f(\sqrt[3]{x+1})$

$= (\sqrt[3]{x+1})^3 - 1$

$= x + 1 - 1$

$= x$



$f^{-1}(f(x)) = f^{-1}(x^3 - 1)$

$= \sqrt[3]{x^3 - 1 + 1}$

$= \sqrt[3]{x^3}$

$= x$

$\therefore f(f^{-1}(x)) = f^{-1}(f(x)) = x$ (Proved)

(*) If $f(x) = (x-1)^2$ are one to one?

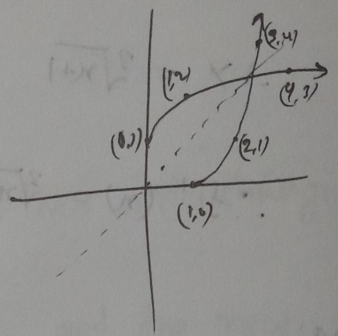
restricted domain $x \geq 1$

$f(x) = (x-1)^2$

$y = (x-1)^2$

interchange, x and y,

$x = (y-1)^2$



solve for y,

$x = (y-1)^2$

$y-1 = \sqrt{x}$

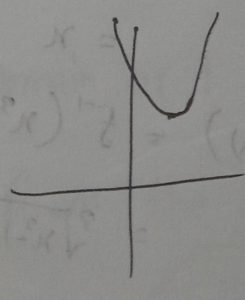
$y = \sqrt{x}+1$

$\therefore f^{-1}(x) = \sqrt{x}+1$

(*) $y = x^2 - 2x + 3$

$= x^2 - 2x + 1 + 2$

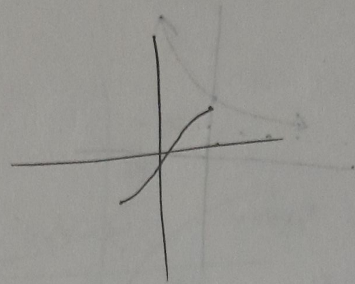
$y = (x-1)^2 + 2$



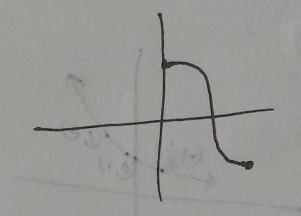
D: $[1, \infty)$

R: $[2, \infty)$

* $f(x) = \sin x \;;\quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$



* $f(x) = \cos x \;;\quad 0 \le x \le \pi$

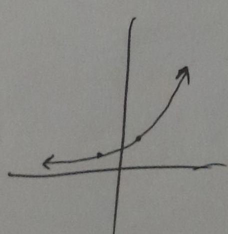


0.5 /

Exponential Function:

$f(x) = a^x \quad ; a > 0,\ a \neq 1$

$f(x) = 2^x$



D: $(-\infty, \infty)$

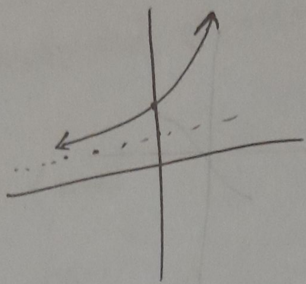
R: $(0, \infty)$

H.A: $y = 0$

increasing

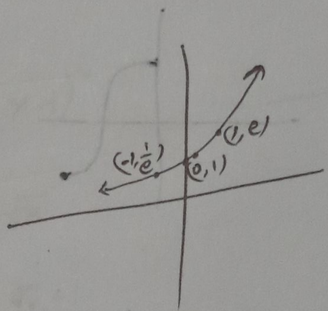
$(0,1)$, $(1,a)$, $(-1, \frac{1}{a})$

$f(x) = 2^x + 1$



$e = 2.7182$ .... natural no.

$f(x) = e^x$



(*)

$f(x) = a^x$

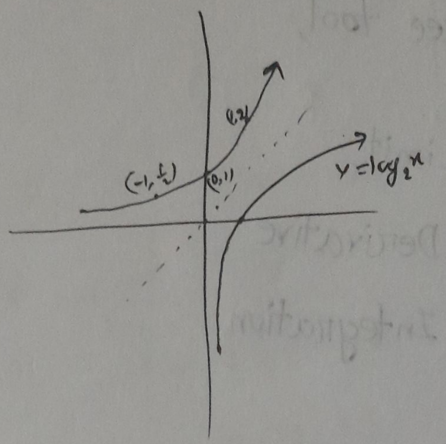
$Y = a^x$

$\Rightarrow x = a^y$

$y = \log_a x$

$f^{-1}(x) = \log_a x$

$y = \log_a x$, if and only if $x = a^y$



$D: (0, \infty)$

$R: (-\infty, \infty)$

V.A.: $x = 0$

Chapter - 1

There are three tool,

i. Limit
ii. Derivative
iii. Integration

1.1 /

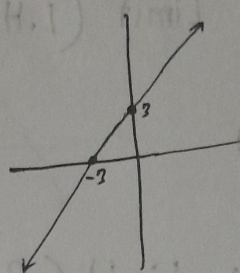
Limit

⊛ Limit of a function:

If $f(x)$ has a limit L at an input a, this means that, $f(x)$ gets closer and closer to L as x moves closer and closer to a, Then we can write :

$$\lim_{x \to a} f(x) = L$$

⊛ $f(x) = x+3$



$\lim_{x \to 2} x+3 = 5$

$\lim_{x \to 1} f(x) = 4$

$\lim_{x \to -1} f(x) = 2$

$\lim_{x \to -2} f(x) = 1$

$f(x) = x+3$

$f(1) = 4$

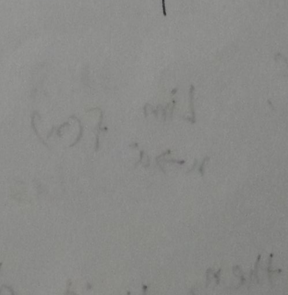
$f(-1) = 2$

$f(-2) = 1$

$f(2) = 5$

⊛ $f(x) = \dfrac{x^2-4}{x-2}$

$f(x) = x+2 \longrightarrow D: x \neq 2$



$f(2) = \frac{0}{0}$ = undefined

$\lim_{x \to} x+2 = 4$

$\lim_{x \to -1} x+2 = 1$

$\lim_{x \to 4} x+2 = 6$

⊛ Left Hand Limit (L.H.L.)

$$\lim_{x \to a^-} f(x)$$

⊛ Right Hand Limit (R.H.L.)

$$\lim_{x \to a^+} f(x)$$

⊛

If, L.H.L = R.H.L

Then we will say

Limit $\lim_{x \to a} f(x)$ exist.

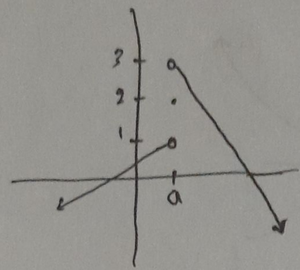
$$\Rightarrow \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$$

$$\therefore \lim_{x \to a} f(x) = L$$

⊛ If, $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$

then, Limit does not exist. (DNE).

⊛ $f(x)$



Here,

$\lim_{x \to a^-} f(x) = 1$

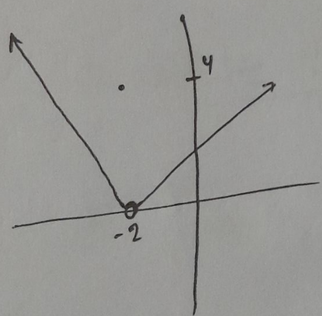
$\lim_{x \to a^+} f(x) = 3$

$f(a) = 2$

$\therefore$ L.H.L $\neq$ R.H.L.

$\therefore$ Limit does not exist.

⊛



Here,

$\lim_{x \to -2^+} f(x) = 0$

$\lim_{x \to -2^-} f(x) = 0$

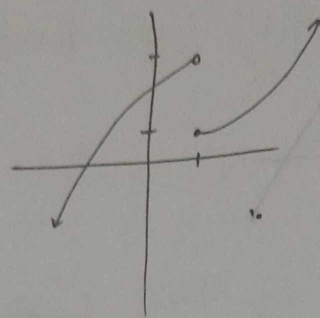
$f(-2) = 4$

$\therefore$ L.H.L = R.H.L

$\therefore$ Limit exist.

$\therefore \lim_{x \to -2} f(x) = 0$

(*)



Here,

$\lim_{x \to 3^-} f(x) = 5$

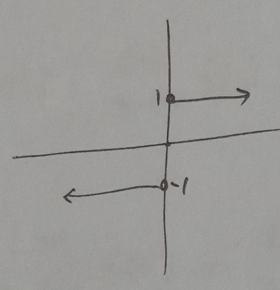
$\lim_{x \to 3^+} f(x) = 2$

$f(x) = 2$

$\therefore$ L.H.L $\neq$ R.H.L.

$\therefore$ Limit does not exist.

(*)



Here,

$\lim_{x \to 0^-} f(x) = -1$

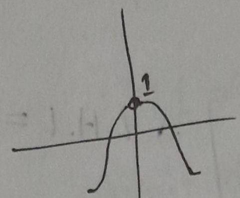
$\lim_{x \to 0^+} f(x) = 1$

$\therefore$ L.H.L $\neq$ R.H.L.

$\therefore$ Limit does not exist.

⊛

$$\lim_{x \to 0} \frac{\sin x}{x} = ?$$



$$\lim_{x \to 0^-} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0^+} \frac{\sin x}{x} = 1$$

$f(0)$ = undefined

$\therefore$ L.H.L = R.H.L

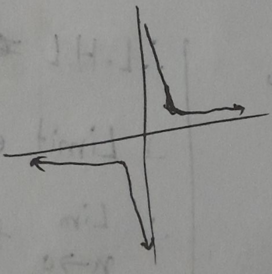
$\therefore$ Limit exist.

$$\therefore \lim_{x \to a} \frac{\sin x}{x} = 1$$

⊛ Infinite Limit

⊛

$$f(x) = \frac{1}{x}$$



Here,

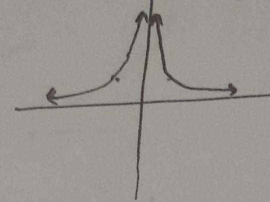
$$\lim_{x \to 0^+} f(x) = \infty$$

$$\lim_{x \to 0^-} f(x) = -\infty$$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ Limit does not exist.

$f(x) = \frac{1}{x^2}$



Here,

$\lim_{x \to 0^-} f(x) = \infty$

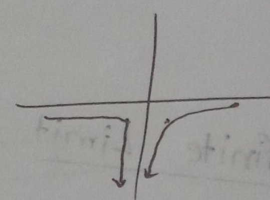
$\lim_{x \to 0^+} f(x) = \infty$

$\therefore$ L.H.L = R.H.L

$\therefore$ Limit exist.

$\therefore \lim_{x \to 0} f(x) = \infty$

⊛ $f(x) = -\frac{1}{x^2}$



Here,

$\lim_{x \to 0^-} f(x) = -\infty$

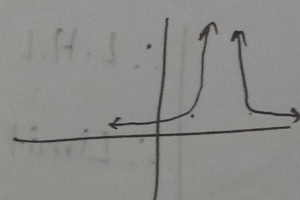
$\lim_{x \to 0^+} f(x) = -\infty$

$\therefore$ L.H.L = R.H.L

$\therefore$ Limit exist.

$\therefore \lim_{x \to 0} f(x) = -\infty$

⊛ $f(x) = \frac{1}{(x-2)^2}$



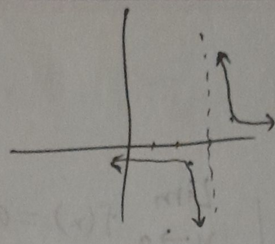
$\lim_{x \to 2^-} f(x) = \infty$

$\lim_{x \to 2^+} f(x) = \infty$

Limit exist.

$\lim_{x \to 2} f(x) = \infty$

(*) $f(x) = \dfrac{1}{x-3}$



Here,

$$\lim_{x \to 3^-} f(x) = -\infty$$

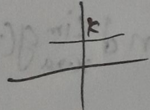
$$\lim_{x \to 3^+} f(x) = +\infty$$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ Limit does not exist.

1.2 / Computing Limit

(*) $f(x) = k$



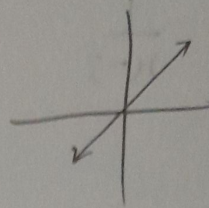
$$\therefore \lim_{x \to a} f(x) = k$$

$$\lim_{x \to -1} f(x) = k$$

$$\lim_{x \to -2} f(x) = k$$

$$\lim_{x \to -3} 3 = 3$$

※ $f(x) = x$



$f(0) = 0$

$f(1) = 1$

$f(-2) = -2$

$\lim_{x \to a} f(x) = a$

$\lim_{x \to 2} f(x) = 2.$

$\lim_{x \to 3} f(x) = 3$

※ $\lim_{x \to 0^+} \frac{1}{x} = \alpha$

$\lim_{x \to 0^-} \frac{1}{x} = -\alpha$

※ <u>Limit Laws:</u>

If, $\lim_{x \to a} f(x) = L_1$ and $\lim_{x \to a} g(x) = L_2$

(i) $\lim_{x \to a} (f(x) \pm g(x)) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

$= L_1 \pm L_2$

(ii) $\lim_{x \to a} (f(x) \cdot g(x)) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

$= L_1 \cdot L_2$

(iii) $$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L_1}{L_2} ; \quad L_2 \neq 0$$

(iv) $$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} = \sqrt[n]{L_1}$$

(v) $$\lim_{x\to a} k \cdot f(x) = k \cdot \lim_{x\to a} f(x) = k \cdot L_1$$

(vi) $$\lim_{x\to a} (f(x))^n = \left(\lim_{x\to a} f(x)\right)^n = (L_1)^n$$

(vii) $$\lim_{x\to 1} (x^2+x+1)$$

$$= \lim_{x\to 1} x^2 + \lim_{x\to 1} x + \lim_{x\to 1} 1$$

$$= 1+1+1$$

$$= 3$$

Ans

Limit of Polynomial Function:

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

$$\lim_{x \to 1} f(x) = f(1)$$

$$\lim_{x \to -2} (x^2 + 2x + 3) = (-2)^2 + 2(-2) + 3$$
$$= 4 - 4 + 3$$
$$= 3$$ Ans

$$\lim_{x \to 3} (3x^2 + 6x - 3) = 3 \cdot 3^3 + 6 \cdot 3 - 3$$
$$= 3 \cdot 27 + 18 - 3$$
$$= 81 + 18 - 3$$
$$= 96$$ Ans.

Limit of Rational Function:

$$f(x) = \frac{P(x)}{Q(x)}$$

$$\lim_{x \to a} f(x) = \lim_{x \to a} \frac{P(x)}{Q(x)} = \frac{\lim_{x \to a} P(x)}{\lim_{x \to a} Q(x)} = \frac{P(a)}{Q(a)} ; \; Q(a) \neq 0$$

$$\lim_{x \to 2} \frac{x^2+3}{x^3+6} = \frac{\lim_{x \to 2} x^2+3}{\lim_{x \to 2} x^3+6} = \frac{2^2+3}{2^3+6} = \frac{7}{14} = \frac{1}{2}$$

Ans

$$\lim_{x \to 2} \frac{x-3}{x-2}$$

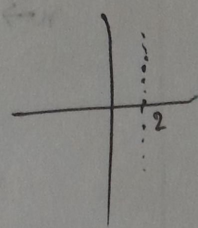
Now,

$$\lim_{x \to 2^-} \frac{x-3}{x-2} = \infty$$

$$\lim_{x \to 2^+} \frac{x-3}{x-2} = -\infty$$

L.H.L $\neq$ R.H.L.

$\therefore$ Limit does not exist.



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⊛ Finding Limit for Polynomial Function

$$f(x) = x^3 + 2x^2 + 3$$

$$\lim_{x \to 1} f(x) = f(1)$$
$$= 1^3 + 2 \cdot 1^2 + 3$$
$$= 1 + 2 + 3$$
$$= 6$$

Ans

⊛ Finding Limit for Rational Function.

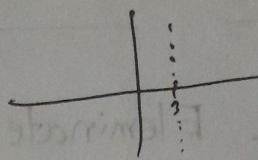
$$f(x) = \frac{P(x)}{Q(x)}$$

$$\lim_{x \to a} \frac{P(x)}{Q(x)} = \frac{P(a)}{Q(a)} \quad ; \; Q(a) \neq 0$$

⊛ $f(x) = \dfrac{x^2+6}{x-2}$

$$\lim_{x \to 1} f(x) = \frac{1^2+6}{1-2} = \frac{7}{-1} = -7$$

⊛ $f(x) = \dfrac{x+2}{x-3}$



$$f(2.99) = \frac{4.99}{-0.01} = -499$$
$$f(2.999) = -4999$$
$$f(3.001) = \frac{5.001}{0.001} = 5001$$
$$f(3.00001) = 500001$$

$\lim\limits_{x \to 3} f(x) = ?$

Here,

$\lim\limits_{x \to 3^-} f(x) = -\infty$

$\lim\limits_{x \to 3^+} f(x) = \infty$

$\therefore$ L.H.L. $\neq$ R.H.L.

$\therefore$ Limit does not exist.

⊛ $\lim\limits_{x \to 3} \dfrac{1}{|x-3|} = ?$

$$f(2.99) = \frac{1}{0.001} = 1000$$
$$f(2.9999) = 100000$$
$$f(3.001) = \frac{1}{0.0001} = 1000$$
$$f(3.00001) = 100000$$

Here,

$\lim\limits_{x \to 3^-} \dfrac{1}{|x-3|} = \infty$

$\lim\limits_{x \to 3^+} \dfrac{1}{|x-3|} = \infty$

$\therefore$ L.H.L = R.H.L

$\therefore \lim\limits_{x \to 3} \dfrac{1}{|x-3|} = \infty$

⊛ Finding Limit involving Radicals:

Rules: Eleminate the zero demominactor.

⊛ $\lim_{x \to \sqrt{3}} \frac{x^2-3}{x-\sqrt{3}}$

$= \lim_{x \to \sqrt{3}} \frac{x^2-(\sqrt{3})^2}{(x-\sqrt{3})}$

$= \lim_{x \to \sqrt{3}} \frac{(x+\sqrt{3})(x-\sqrt{3})}{(x-\sqrt{3})}$

$= \lim_{x \to \sqrt{3}} (x+\sqrt{3})$

$= \sqrt{3}+\sqrt{3}$

$= 2\sqrt{3}$

Ans

⊛ $\lim_{y \to 4} \frac{4-y}{2-\sqrt{y}}$

$= \lim_{y \to 4} \frac{(2-\sqrt{y})(2+\sqrt{y})}{2-\sqrt{y}}$

$= \lim_{y \to 4} (2+\sqrt{y})$

$= 2+\sqrt{4}$

$= 4$

Ans

⊛

$\lim_{y \to 6^-} \frac{y+6}{y^2-36}$

$= \lim_{y \to 6^-} \frac{y+6}{(y+6)(y-6)}$

$= \lim_{y \to 6^-} \frac{1}{y-6}$

$= -\alpha$

$\lim_{y \to 6^+} \frac{1}{y-6} = \alpha$

⊛ Limit of Piece-wise defined function.

⊛ If, $f(x) = \begin{cases} x-1 & x \le 3 \\ 3x-7 & x > 3 \end{cases}$

i) $\lim_{x\to 3^-} f(x)$ ii) $\lim_{x\to 3^+} f(x)$ iii) $\lim_{x\to 3} f(x)$ iv) $\lim_{x\to 5} f(x)$

v) $\lim_{x\to 1} f(x)$

$\Rightarrow$

i) $\lim_{x\to 3^-} f(x)$

$= \lim_{x\to 3} x-1$

$= 3-1$

$= 2$

ii) $\lim_{x\to 3^+} f(x)$

$= \lim_{x\to 3^+} 3x-7$

$= 3\cdot 3-7$

$= 9-7$

$= 2$

iii) L.H.L = R.H.L

$\therefore \lim_{x\to 3} f(x) = 2$

iv) $\lim_{x\to 5} f(x)$

$= \lim_{x\to 5} 3x-7$

$= 3\cdot 5-7$

$= 15-7$

$= 8$

v) $\lim_{x\to 1} f(x)$

$= \lim_{x\to 1} x-1$

$= 1-1$

$= 0$

Q) If,

$$f(x) = \begin{cases} x-2 & x<0 \\ x^2 & 0 \le x \le 2 \\ 2x & x>2 \end{cases}$$

i) $\lim_{x \to 0} f(x)$ ii) $\lim_{x \to 1} f(x)$ iii) $\lim_{x \to 2} f(x)$

$\Rightarrow$

i) $\lim_{x \to 0} f(x) = ?$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x-2$

$= 0-2$

$= -2$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2$

$= 0^2$

$= 0$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ Limit does not exist.

ii) $\lim_{x \to 1} f(x)$

$= \lim_{x \to 1} x^2$

$= 1^2$

$= 1$

iii) $\lim_{x \to 2} f(x) = ?$

$\lim_{x \to 2^-} f(x)$

$= \lim_{x \to 2^-} x^2$

$= 2^2$

$= 4$

$\lim_{x \to 2^+} f(x)$

$= \lim_{x \to 2^+} 2x$

$= 2 \cdot 2$

$= 4$

$\therefore$ L.H.L = R.H.L

$\therefore \lim_{x \to 2} f(x) = 4$.