

North South University
Department of Mathematics and Physics
Assignment - 2

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Course No : MAT-120

Course Title : Calculus and Analytical Geometry I

Section : 13

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3.4101

Given that,

$$x = 1$$

$$y = 2$$

$$\frac{dx}{dt} = -2$$

$$\frac{dy}{dt} = 3$$

and,

$$z = x^3 y^2$$

differentiate w.r.t. t,

$$\frac{dz}{dt} = \frac{d}{dt}(x^3 y^2)$$

$$= x^3 \cdot \frac{d}{dt} y^2 + y^2 \cdot \frac{d}{dt} x^3$$

$$= x^3 \cdot 2y \cdot \frac{dy}{dt} + y^2 \cdot 3x^2 \cdot \frac{dx}{dt}$$

$$= 1^3 \cdot 2 \cdot 2 \cdot 3 - 2^2 \cdot 3 \cdot 1 \cdot 2$$

$$= 12 - 24$$

$$= -12 \text{ unit/second}$$

Therefore, z is decreasing at the rate of 12 units/second

III

Given that,

$$\pi = 4$$

$$\text{and } \frac{d\theta}{dt} = \frac{2\pi}{60} = \frac{\pi}{30} \text{ rad/min}$$

We know that,

$$\text{Area, } A = \frac{1}{2} \pi r^2 \theta$$

$$\Rightarrow A = \frac{1}{2} \cdot 4 \cdot \theta$$

$$\Rightarrow A = 8\theta$$

Now, differentiate w.r.t. t ,

$$\Rightarrow \frac{dA}{dt} = 8 \frac{d\theta}{dt}$$

$$\frac{dA}{dt} = 8 \cdot \frac{\pi}{30} \text{ in}^2/\text{min}$$

$$= \frac{4\pi}{15} \text{ in}^2/\text{min}$$

Therefore,

area swept out rate $\frac{4\pi}{15} \text{ in}^2/\text{min.}$

12)

Let,

$$\text{radius} = r$$

$$\text{area} = A$$

Given that,

$$\frac{dr}{dt} = 3 \text{ ft/sec}$$

$$\begin{aligned}\text{after 10 second radius, } r &= 3 \cdot 10 \text{ ft} \\ &= 30 \text{ ft}\end{aligned}$$

$$\left. \frac{dA}{dt} \right|_{t=10} = ?$$

We know that,

$$A = \pi r^2$$

differentiate both side w.r.t. t

$$\frac{dA}{dt} = \frac{d}{dt} (\pi r^2)$$

$$= \pi \cdot 2r \cdot \frac{dr}{dt} \quad [\pi \text{ is constant}]$$

$$= 2\pi \cdot 30 \cdot 3$$

$$= 180\pi \text{ ft}^2/\text{sec}$$

$$\therefore \frac{dA}{dt}$$

Therefore area is increasing at the rate of $180\pi \text{ ft}^2/\text{sec}$

13]

Given that,

$$\frac{dA}{dt} = 6$$

$$\text{and } A = \pi r^2$$

$$\Rightarrow \pi r^2 = 9$$

$$\Rightarrow r^2 = \frac{9}{\pi}$$

$$\therefore r = \frac{3}{\sqrt{\pi}}$$

We know that,

$$A = \pi r^2$$

$$r^2 = \frac{A}{\pi}$$

differentiate both side w.r.t. t,

$$\frac{d}{dt} r^2 = \frac{1}{\pi} \cdot \frac{dA}{dt}$$

$$\Rightarrow 2r \cdot \frac{dr}{dt} = \frac{1}{\pi} \cdot \frac{dA}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{1}{2\pi r} \cdot \frac{dA}{dt}$$

$$= \frac{1}{2\pi \cdot \frac{3}{\sqrt{\pi}}} \cdot 6$$

$$\therefore \frac{dr}{dt} = \frac{1}{\sqrt{\pi}} \text{ m/h}$$

Therefore, radius increasing at the rate of $\frac{1}{\pi}$ mi/h

141

Given that,

$$\text{radius, } r = 1$$

$$\text{Diameter, } \phi D = 2$$

$$\frac{dv}{dt} = 3$$

We know that,

$$v = \frac{4}{3} \pi r^3$$

$$\Rightarrow v = \frac{4}{3} \pi \left(\frac{D}{2}\right)^3$$

$$\Rightarrow v = \frac{\pi}{6} D^3$$

$$\Rightarrow D^3 = \frac{6}{\pi} v$$

Differentiate both side w.r.t. t,

$$3 \cdot D^2 \cdot \frac{dD}{dt} = \frac{6}{\pi} \frac{dv}{dt}$$

$$\frac{dD}{dt} = \frac{2}{\pi D^2} \frac{dv}{dt}$$

$$= \frac{2}{\pi \cdot 4} \cdot 3$$

$$\therefore \frac{dD}{dt} = \frac{3}{2\pi} \text{ ft/min}$$

Therefore diameter increasing at the rate of $\frac{3}{2\pi}$ ft/min

151

Given that,

$$\text{Radius, } r = 9$$

$$\frac{dr}{dt} = -15 \text{ cm/min}$$

We know that,

$$V = \frac{4}{3} \pi r^3$$

differentiate both side with respect to t ,

$$\begin{aligned}\frac{dV}{dt} &= \frac{4\pi}{3} \cdot 3r^2 \cdot \frac{dr}{dt} \\ &= 4 \cdot 9^2 \cdot (-15) \cdot \pi \\ &= -4860\pi \text{ cm}^3/\text{min}\end{aligned}$$

Therefore,

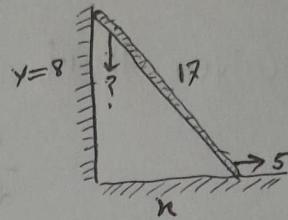
Air must be removed at the rate of $4860\pi \text{ cm}^3/\text{min}$.

161

Given that,

$$\frac{dx}{dt} = 5 \text{ ft/sec}$$

$$y = 8$$



From Pythagorean theorem,

$$x^2 + y^2 = 17^2$$

differentiate both side w.r.t. t ,

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

Hence,
 $x^2 + y^2 = 17^2$
 $x = 15$

$$\Rightarrow 2 \cdot 15 \cdot 5 + 2 \cdot 8 \cdot \frac{dy}{dt} = 0$$

$$\Rightarrow -\frac{dy}{dt} = -\frac{150}{16}$$

$$= -\frac{75}{8} \text{ ft/s}$$

Therefore,

the top of the ladder is moving down the wall

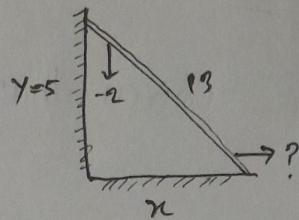
at the rate of $\frac{75}{8}$ ft/sec.

17)

Given that,

$$\frac{dy}{dt} = -2 \text{ ft/sec}$$

$$y = 5 \text{ ft}$$



From Pythagorean theorem,

$$x^2 + y^2 = 13^2$$

differentiate both side w.r.t. t,

$$2x \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

Here,

$$x^2 + 5^2 = 13^2$$

$$x = 12$$

$$\Rightarrow 2 \cdot 12 \cdot \frac{dx}{dt} - 2 \cdot 5 \cdot 2 = 0$$

$$\Rightarrow \frac{dx}{dt} = \frac{20}{24}$$

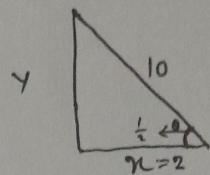
$$= \frac{5}{6} \text{ ft/sec}$$

Therefore, ladder is moving away from the wall at

the rate of $\frac{5}{6}$ ft/sec.

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Given that,



distance of the bottom of the plank from wall, $x = 2 \text{ ft}$

acute angle $= \theta$

$$\frac{dx}{dt} = -6 \text{ in/s} = -\frac{1}{2} \text{ ft/sec}$$

We know,

$$\cos \theta = \frac{x}{10}$$

differentiate both sides w.r.t. t

$$-\sin \theta \cdot \frac{d\theta}{dt} = \frac{1}{10} \frac{dx}{dt}$$

$$\Rightarrow \frac{d\theta}{dt} = -\frac{1}{10 \sin \theta} \cdot \frac{dx}{dt}$$

$$= -\frac{1}{10 \cdot \frac{\sqrt{96}}{10}} \cdot \left(-\frac{1}{2}\right)$$

$$= \frac{1}{2\sqrt{96}} \text{ rad/sec}$$

Hence,

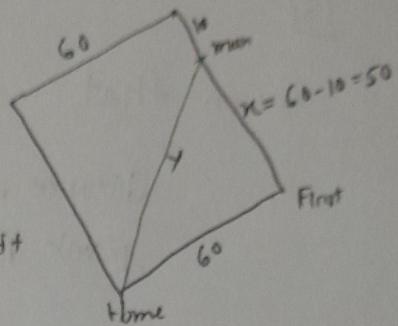
$$\begin{aligned} \sin \theta &= \frac{y}{10} \\ &= \frac{\sqrt{10^2 - x^2}}{10} \\ &= \frac{\sqrt{10^2 - 2^2}}{10} \\ &= \frac{\sqrt{96}}{10} \end{aligned}$$

Therefore, angle is increasing at the rate of $\frac{1}{2\sqrt{96}}$ rad/sec.

12

Let,

$$\text{distance from first base}, x = 60 - 10 \\ = 50 \text{ ft}$$



$$\text{distance from home plate}, y = \sqrt{60^2 + 50^2} \\ = 10\sqrt{61}$$

Given that,

$$\frac{dx}{dt} = 25 \text{ ft/sec}$$

From Pythagorean theorem,

$$x^2 + 60^2 = y^2$$

differentiate both side w.r.t. t,

$$2x \cdot \frac{dx}{dt} = 2y \cdot \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{2x}{2y} \cdot \frac{dx}{dt}$$

$$= \frac{50}{10\sqrt{61}} \cdot 25$$

$$= \frac{125}{\sqrt{61}} \text{ ft/sec}$$

Therefore, distance from home plate is increasing at the rate of $\frac{125}{\sqrt{61}}$ ft/sec.

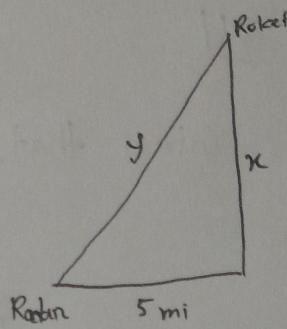
20)

Given that,

$$x = 4 \text{ mi}$$

$$\frac{dy}{dt} = 2000 \text{ mi/h}$$

$$\frac{dx}{dt} = ?$$



From Pythagorean theorem,

$$x^2 + 5^2 = y^2$$

differentiate w.r.t. t,

$$2x \cdot \frac{dx}{dt} = 2y \cdot \frac{dy}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{2y}{2x} \cdot \frac{dy}{dt}$$

$$= \frac{\sqrt{41}}{4} \cdot 2000$$

$$= 500\sqrt{41} \text{ mi/h}$$

Hence,

$$\begin{aligned} x^2 + 25 &= y^2 \\ y &= \sqrt{16+25} \\ &= \sqrt{41} \end{aligned}$$

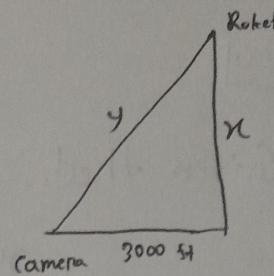
Therefore,

Rocket rising at the rate of $500\sqrt{41}$ mi/h.

21

Given that,

$$x = 4000 \text{ ft}$$



$$\frac{dx}{dt} = 880 \text{ ft/sec}$$

From Pythagorean theorem,

$$x^2 + 3000^2 = y^2$$

differentiate both side w.r.t. t ,

$$2x \cdot \frac{dx}{dt} = 2y \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{2x}{2y} \cdot \frac{dx}{dt}$$

$$= \frac{4000}{5000} \cdot 880$$

$$= 704 \text{ ft/sec}$$

Hence.

$$x^2 + 3000^2 = y^2$$

$$y = \sqrt{4000^2 + 3000^2}$$

$$= 5000$$

Therefore,

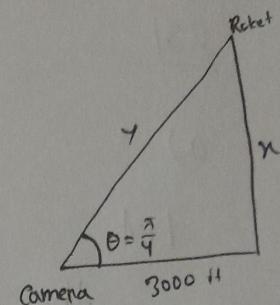
Camera to Rocket distance increasing at the rate of
704 ft/sec.

221

Given that,

$$\theta = \frac{\pi}{4}$$

$$\frac{d\theta}{dt} = 0.2 \text{ rad/sec}$$



We know,

$$\tan \theta = \frac{n}{3000}$$

$$\Rightarrow n = 3000 \tan \theta$$

differentiate both side w.r.t. t,

$$\begin{aligned}\frac{dn}{dt} &= 3000 \cdot (\sec^2 \theta) \cdot \frac{d\theta}{dt} \\ &= 3000 \cdot (\sec^2 \frac{\pi}{4}) \cdot (0.2) \\ &= 1200 \text{ ft/sec}\end{aligned}$$

Therefore,

Rocket rising at the rate of 1200 ft/sec.

23)

a)

Let,

the altitude, n

then,

$$n - n = 3960$$

at perigee, $\theta = 0$

$$\text{So, } n = \frac{4995}{1+0.12 \cos 0} = 4460$$

therefore,

$$\begin{aligned} \text{altitude, } n &= n - 3960 \\ &= 4460 - 3960 \\ &= 500 \text{ miles} \end{aligned}$$

again,

at apogee,

$$\theta = \pi$$

$$\text{So, } n = \frac{4995}{1+0.12 \cos \pi} = 5676$$

therefore,

$$\begin{aligned} \text{altitude, } n &= n - 3960 \\ &= 5676 - 3960 \\ &= 1716 \text{ miles} \end{aligned}$$

Therefore,

altitude at perigee is 500 miles and at apogee
is 1716 miles.

b)

Given that,

$$\theta = 120^\circ$$

$$\therefore r = \frac{4995}{1 + 0.12 \cos 120^\circ} = 5314$$

$$\therefore \text{altitude is, } x = 5314 - 3960 = 1354 \text{ miles}$$

We know,

the rate of change of altitude $\frac{dr}{dt}$ is same as the
rate of change of r , $\frac{dr}{dt}$.

Given that,

$$r = \frac{4995}{1 + 0.12 \cos \theta}$$

differentiate both side w.r.t. t ,

$$\begin{aligned} \frac{dr}{dt} &= \frac{4995 \cdot (0.12) \cdot \sin \theta \cdot \frac{d\theta}{dt}}{(1 + 0.12 \cos \theta)^2} \\ &= \frac{4995 \cdot (0.12) \cdot (\sin 120) \cdot 2.7\pi}{(1 + 0.12 \cos 120) \cdot 180} \end{aligned}$$

$$= 27.68 \text{ mile/min}$$

Therefore,

altitude is increasing at the rate of 27.68 mile/min

Hence, $\frac{d\theta}{dt} = 2.7^\circ/\text{min}$ $= \frac{2.7\pi}{180} \text{ radian}$

241

a)

Let,

horizontal distance is, x

Given that,

$$\frac{dx}{dt} = 300 \text{ mile/h}$$

$$= 440 \text{ ft/sec}$$

We know, $\theta = 30^\circ$

$$\cot \theta = \frac{x}{4000}$$

differentiate both side w.r.t. t ,

$$-\operatorname{cosec}^2 \theta \cdot \frac{d\theta}{dt} = \frac{1}{4000} \cdot \frac{dx}{dt}$$

$$\Rightarrow \frac{d\theta}{dt} = -\frac{1}{4000} \cdot 440 \cdot \sin^2 \theta$$

$$= -0.0275 \text{ rad/sec}$$

$$= -1.6^\circ/\text{sec}$$

therefore,

 θ is decreasing at the rate of $1.6^\circ/\text{sec}$.

b),

Let,

y be the distance between the observation point and the aircraft.

Given that,

$$\theta = 30^\circ$$

$$\frac{d\theta}{dt} = -0.0275 \text{ rad/sec} \quad [\text{from a}]$$

We know,

$$\cosec \theta = \frac{y}{4000}$$

$$\Rightarrow y = 4000 \cosec \theta$$

differentiate both side w.r.t. t ,

$$\frac{dy}{dt} = -4000 (\cosec \theta \cdot \cot \theta) \cdot \frac{d\theta}{dt}$$

$$= -4000 (\cosec 30 \cdot \cot 30) \cdot (-0.0275)$$

$$= 381 \text{ ft/sec}$$

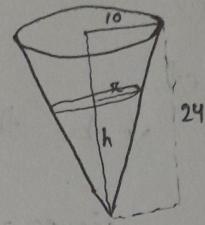
Therefore,

distance increasing at the rate of 381 ft/sec.

251

Given that,

$$\frac{dV}{dt} = 20$$



Using similar triangle,

$$\frac{r}{h} = \frac{10}{24}$$

$$\therefore r = \frac{5}{12} h$$

The volume of water in the tank at a depth h is

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{5}{12} h\right)^2 h$$

$$V = \frac{25}{432} \pi h^3$$

differentiate both side w.r.t. t ,

$$\frac{dV}{dt} = \frac{25}{144} \pi h^2 \cdot \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{144}{25 \cdot \pi \cdot h^2} \cdot \frac{dV}{dt}$$

$$= \frac{144}{25 \cdot \pi \cdot (16)} \cdot 20$$

$$= \frac{9}{20\pi} \text{ ft/min}$$

Therefore, depth increasing at the rate of $\frac{9}{20\pi}$ ft/min

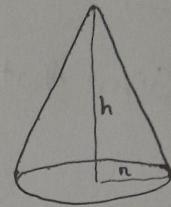
26/

Given that,

$$\frac{dV}{dt} = 8$$

$$h = 6$$

$$\pi = \frac{1}{2} h$$



Now,

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{1}{12} \pi h^3$$

differentiate both side w.r.t. t,

$$\frac{dV}{dt} = \frac{1}{12} \pi \cdot 3h^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{4}{\pi h^2} \cdot \frac{dV}{dt}$$

$$= \frac{4}{\pi \cdot 6^2} \cdot 8$$

$$= \frac{8}{9\pi} \text{ ft/min}$$

Therefore,

height is increasing at the rate of $\frac{8}{9\pi}$ ft/min.

271

Given that,



$$\frac{dh}{dt} = 5$$

$$h = 10$$

$$r = \frac{1}{2}h$$

Now,

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{12} \pi h^3$$

differentiate both side w.r.t. t,

$$\frac{dV}{dt} = \frac{1}{4} \pi h^2 \cdot \frac{dh}{dt}$$

$$= \frac{1}{4} \cdot \pi \cdot 10^2 \cdot 5$$

$$= 125\pi \text{ ft}^3/\text{min.}$$

Therefore,

sand pouring at the rate of $125\pi \text{ ft}^3/\text{min.}$

28/

Let,

circumference is C 

Given that,

$$\frac{dv}{dt} = 10$$

$$h = 8$$

$$r = \frac{1}{2}h$$

Now,

$$C = 2\pi r$$

$$\Rightarrow C = 2\pi \frac{1}{2}h$$

$$C = \pi h$$

differentiate both side w.r.t. t ,

$$\frac{dC}{dt} = \pi \frac{dh}{dt}$$

$$= \pi \cdot \frac{4}{\pi h} \cdot \frac{dh}{dt}$$

$$= \frac{4}{8^2} \cdot 10$$

$$= \frac{5}{8} \text{ ft/min}$$

Hence,

$$v = \frac{1}{3} \pi r^2 h = \frac{1}{12} \pi h^3$$

$$\frac{dv}{dt} = \cancel{\frac{1}{4}} \pi h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{\pi h} \cdot \frac{dv}{dt}$$

Therefore,

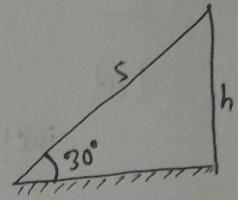
circumference increasing at the rate of

$$\frac{5}{8} \text{ ft/min.}$$

29/

Given that,

$$\frac{ds}{dt} = 500 \text{ mi/h}$$



Now,

$$\sin \theta = \frac{h}{s}$$

$$\Rightarrow h = s \sin \theta$$

$$\Rightarrow h = s \sin 30 = \frac{1}{2} s$$

differentiate both side w.r.t. t ,

$$\frac{dh}{dt} = \frac{1}{2} \frac{ds}{dt}$$

$$= \frac{1}{2} \cdot 500$$

$$= 250 \text{ mi/h.}$$

Therefore,

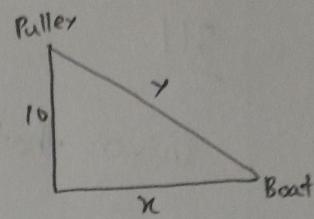
altitude increasing at the rate of 250 mi/h.

301

Given that,

$$\frac{dy}{dt} = -20$$

$$y = 125$$



From Pythagorean theorem,

$$x^2 + 10^2 = y^2$$

differentiate both side w.r.t. t

$$2x \cdot \frac{dx}{dt} = 2y \cdot \frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{2y}{2x} \cdot \frac{dy}{dt}$$

$$= \frac{125}{15\sqrt{69}} \cdot (-20)$$

$$= -\frac{500}{3\sqrt{69}} \text{ ft/min}$$

Hence,
 $x^2 + 10^2 = (125)^2$
 $x = \sqrt{(125)^2 - 10^2}$
 $= \sqrt{15525}$
 $= 15\sqrt{69}$

Therefore,

The boat is approaching the dock at the rate

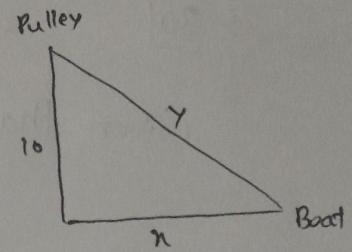
of $\frac{500}{3\sqrt{69}}$ ft/min.

31]

Given that,

$$\frac{dx}{dt} = -12$$

$$y = 125$$



From Pythagorean theorem,

$$x^2 + 10^2 = y^2$$

differentiate both sides w.r.t. t ,

$$2x \cdot \frac{dx}{dt} = 2y \cdot \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{2x}{2y} \cdot \frac{dx}{dt}$$

$$= \frac{15\sqrt{69}}{125} \cdot (-12)$$

$$= -\frac{36\sqrt{69}}{25} \text{ ft/min}$$

Hence,

$$x^2 + 10^2 = y^2$$

$$x = \sqrt{(125)^2 - 10^2}$$

$$= \sqrt{15525}$$

$$= 15\sqrt{69}$$

Therefore,

The rope must be pulled at the rate of

$$\frac{36\sqrt{69}}{25} \text{ ft/min.}$$

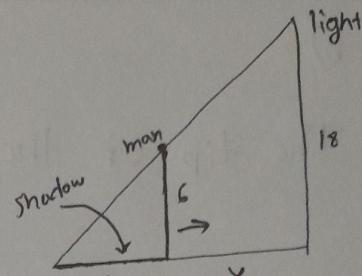
32/

a)

Given that,

$$\frac{dy}{dt} = -3$$

by similar triangle,



$$\frac{x}{6} = \frac{x+y}{18}$$

$$\Rightarrow 18x = 6x + 6y$$

$$\Rightarrow 12x = 6y$$

$$\Rightarrow x = \frac{1}{2}y$$

differentiate both side w.r.t. t ,

$$\frac{dx}{dt} = \frac{1}{2} \frac{dy}{dt}$$

$$= \frac{1}{2} (-3)$$

$$= -\frac{3}{2} \text{ ft/sec}$$

Therefore,

Shadow length is decreasing at the rate of $\frac{3}{2}$ ft/sec.

b)

The tip of the shadow is, z

$$z = x + y$$

Differentiate both side w.r.t. t ,

$$\begin{aligned}\frac{dz}{dt} &= \frac{dx}{dt} + \frac{dy}{dt} \\ &= -\frac{3}{2} + (-3) \\ &= -\frac{9}{2} \text{ ft/sec}\end{aligned}$$

Therefore,

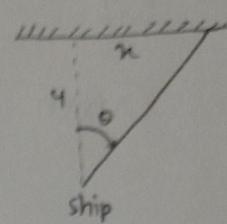
the tip of the shadow is moving at the
rate of $\frac{9}{2}$ ft/sec toward the street light.

33

Given that,

$$\theta = \frac{\pi}{4}$$

$$\frac{d\theta}{dt} = \frac{2\pi}{10} = \frac{\pi}{5} \text{ rad/sec.}$$



Now,

$$\tan \theta = \frac{x}{4}$$

$$\Rightarrow x = 4 \tan \theta$$

differentiate both side w.r.t. t.

$$\begin{aligned}\frac{dx}{dt} &= u \cdot \sec^2 \theta \cdot \frac{d\theta}{dt} \\ &= 4 \cdot (\sec^2 \frac{\pi}{4}) \cdot \frac{\pi}{5} \\ &= \frac{8\pi}{5} \text{ km/sec}\end{aligned}$$

Therefore,
beam moving at the rate of $\frac{8\pi}{5}$ km/sec.

34)

Given that,

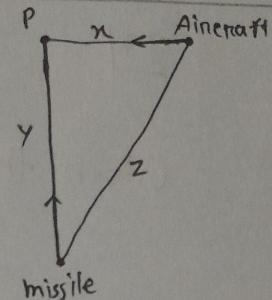
$$x = 2$$

$$y = 4$$

$$\frac{dx}{dt} = -600$$

$$\frac{dy}{dt} = -1200$$

$$\frac{dz}{dt} = ?$$



Now,

$$z^2 = x^2 + y^2$$

differentiate both side w.r.t. t,

$$2z \cdot \frac{dz}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{2 \cdot 2 \cdot (-600) + 2 \cdot 4 \cdot (-1200)}{2 \cdot 2\sqrt{5}}$$

Hence,

$$\begin{aligned} x^2 + y^2 &= z^2 \\ z &= \sqrt{x^2 + y^2} \\ &= 2\sqrt{5} \end{aligned}$$

$$= -600\sqrt{5} \text{ mi/h}$$

Therefore, the distance between missile and aircraft is decreasing at the rate of $600\sqrt{5}$ mi/h.

3.6

[7]

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x} \quad [\text{form } \frac{0}{0}]$$

Now, using L'Hospital Rule,

$$= \lim_{x \rightarrow 0} \frac{e^x}{\cos x}$$

$$= \frac{e^0}{\cos 0}$$

$$= 1$$

Therefore, $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x} = 1$

[8]

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 5x} \quad [\text{form } \frac{0}{0}]$$

Now, using L'Hospital Rule,

$$= \lim_{x \rightarrow 0} \frac{2 \cos 2x}{5 \cos 5x}$$

$$= \frac{2 \cos 0}{5 \cos 0}$$

$$= \frac{2}{5}$$

Therefore, $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 5x} = \frac{2}{5}$.

9]

$$\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} \quad [\text{form } \frac{0}{0}]$$

Now, using L'Hospital Rule,

$$= \lim_{\theta \rightarrow 0} \frac{\sec^2 \theta}{1}$$

$$= \frac{\sec^2 0}{1}$$

$$= 1$$

Therefore,

$$\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = 1$$

10]

$$\lim_{t \rightarrow 0} \frac{te^t}{1-e^t} \quad [\text{form } \frac{0}{0}]$$

Now, using L'Hospital Rule,

$$= \lim_{t \rightarrow 0} \frac{te^t + e^t}{-e^t}$$

$$= \frac{0+1}{-1}$$

$$= -1$$

Therefore,

$$\lim_{t \rightarrow 0} \frac{te^t}{1-e^t} = -1$$

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$$\lim_{x \rightarrow \pi^+} \frac{\sin x}{x - \pi} \quad [\text{form } \frac{0}{0}]$$

Now, using L'Hospital Rule,

$$= \lim_{x \rightarrow \pi^+} \frac{\cos x}{1}$$

$$= \cos \pi$$

$$= -1$$

Therefore,

$$\lim_{x \rightarrow \pi^+} \frac{\sin x}{x - \pi} = -1$$

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$$\lim_{x \rightarrow 0^+} \frac{\sin x}{x^2} \quad [\text{form } \frac{0}{0}]$$

Now, using L'Hospital Rule,

$$= \lim_{x \rightarrow 0^+} \frac{\cos x}{2x}$$

$$= +\infty$$

Therefore,

$$\lim_{x \rightarrow 0^+} \frac{\sin x}{x^2} = +\infty$$

13)

$$\lim_{n \rightarrow +\infty} \frac{\ln n}{n} \quad [\text{form } \frac{\infty}{\infty}]$$

Now, using L'Hospital Rule,

$$= \lim_{n \rightarrow +\infty} \frac{\frac{1}{n}}{1}$$

$$= \lim_{n \rightarrow +\infty} \frac{1}{n}$$

$$= 0$$

Therefore,

$$\lim_{n \rightarrow +\infty} \frac{\ln n}{n} = 0$$

14)

$$\lim_{x \rightarrow +\infty} \frac{e^{3x}}{x^2} \quad [\text{form } \frac{\infty}{\infty}]$$

Now, using L'Hospital Rule,

$$= \lim_{x \rightarrow +\infty} \frac{3e^{3x}}{2x} \quad [\text{form } \frac{\infty}{\infty}]$$

$$= \lim_{x \rightarrow +\infty} \frac{9e^{3x}}{2}$$

$$= +\infty$$

Therefore,

$$\lim_{x \rightarrow +\infty} \frac{e^{3x}}{x^2} = +\infty$$

15)

$$\lim_{x \rightarrow 0^+} \frac{\cot x}{\ln x} \quad [\text{form } \frac{\infty}{-\infty}]$$

Now, using L' Hospital Rule,

$$= \lim_{x \rightarrow 0^+} \frac{-\operatorname{cosec}^2 x}{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0^+} \frac{-x}{\operatorname{cosec}^2 x} \quad [\text{form } \frac{0}{0}]$$

$$= \lim_{x \rightarrow 0^+} \frac{-1}{\sin 2x}$$

$$= \lim_{x \rightarrow 0^+} -\operatorname{cosec} 2x$$

$$= -\infty$$

Therefore,

$$\lim_{x \rightarrow 0^+} \frac{\cot x}{\ln x} = -\infty$$

16)

$$\lim_{x \rightarrow 0^+} \frac{1 - \ln x}{e^{1/x}} \quad [\text{form } \frac{\infty}{\infty}]$$

Now using L' Hospital Rule,

$$= \lim_{x \rightarrow 0^+} \frac{-\frac{1}{x}}{(-\frac{1}{x^2}) \cdot e^{1/x}}$$

$$= \lim_{x \rightarrow 0^+} \left(-\frac{1}{x} \right) \left(-\frac{x}{e^{1/x}} \right)$$

$$= \lim_{x \rightarrow 0^+} \frac{x}{e^{1/x}}$$

$$= 0$$

Therefore,

$$\lim_{x \rightarrow 0^+} \frac{1 - \ln x}{e^{1/x}} = 0$$

[7]

$$\lim_{x \rightarrow +\infty} \frac{x^{100}}{e^x} \quad \left[\text{form } \frac{\infty}{\infty} \right]$$

Now using L'Hospital Rule,

$$= \lim_{x \rightarrow +\infty} \frac{100 \cdot x^99}{e^x} \quad \left[\text{form } \frac{\infty}{\infty} \right]$$

$$= \lim_{x \rightarrow +\infty} \frac{100 \cdot 99 \cdot x^{98}}{e^x} \quad \left[\text{form } \frac{\infty}{\infty} \right]$$

$$= \lim_{x \rightarrow +\infty} \frac{100 \cdot 99 \cdot \dots \cdot 1}{e^x}$$

$$= 0$$

Therefore,

$$\lim_{x \rightarrow +\infty} \frac{x^{100}}{e^x} = 0$$