18)

$$\lim_{x\to 0^+} \frac{\ln(\sin x)}{\ln(\tan x)} \quad \left[\text{form } \frac{-\infty}{-\infty}\right]$$

Now, using L' Hospital Rule

$$= \lim_{x\to 0^+} \frac{\frac{\cos x}{\sin x}}{\frac{\sec^2 x}{\tan x}}$$

$$= \lim_{x\to 0^+} \frac{\cos x}{\sin x} \cdot \frac{\tan x}{\sec^2 x}$$

$$= \lim_{x\to 0^+} \frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x} \cdot \cos^2 x$$

$$= \lim_{x\to 0^+} \cos^2 x$$

$$= 1$$

Therefore,

$$\lim_{x\to 0^+} \frac{\ln(\sin x)}{\ln(\tan x)} = 1$$

19)

$$\lim_{x\to 0} \frac{\sin^{-1} 2x}{x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-(2x)^2}} \cdot 2}{1}$$

$$= \lim_{x \to 0} \frac{2}{\sqrt{1-4x^2}}$$

$$= 2$$

Therefore,

$$\lim_{x \to 0} \frac{\sin^{-1} 2x}{x} = 2$$

20)

$$\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x \to 0} \frac{1 - \frac{1}{1+x^2}}{3x^2}$$

$$= \lim_{x \to 0} \frac{1+x^2-1}{(1+x^2) \cdot 3x^2}$$

$$= \lim_{x \to 0} \frac{1}{3(1+x^2)}$$

$$= \frac{1}{3}$$

Therefore,

$$\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3} = \frac{1}{3}$$

21)

$$\lim_{x\to\infty} xe^{-x} = \lim_{x\to+\infty} \frac{x}{e^x} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to+\infty} \frac{1}{e^x}$$

$$= 0$$

Therefore,

$$\lim_{x\to+\infty} x e^{-x} = 0$$

22)

$$\lim_{x\to\pi^-} (x-\pi) \tan \tfrac{1}{2} x = \lim_{x\to\pi^-} \frac{x-\pi}{\cot(\frac{x}{2})} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to\pi^-} \frac{1}{-\frac{1}{2}\cdot \operatorname{cosec}^2 \frac{x}{2}}$$

$$= \frac{1}{-\frac{1}{2}\cdot \operatorname{cosec}^2 \frac{\pi}{2}}$$

$$= -2$$

Therefore,

$$\lim_{x\to\pi^-} (x-\pi) \tan \tfrac{1}{2} x = -2$$

23)

$$\lim_{x\to+\infty} x\sin\frac{\pi}{x} = \lim_{x\to+\infty} \frac{\sin\frac{\pi}{x}}{\frac{1}{x}} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to+\infty} \frac{\frac{-\pi}{x^2}\cdot\cos\frac{\pi}{x}}{\frac{-1}{x^2}}$$

$$= \lim_{x\to+\infty} \frac{-\pi}{x^2}\cdot\cos\frac{\pi}{x}\cdot\frac{-x^2}{1}$$

$$= \lim_{x\to+\infty} \pi\cdot\cos\frac{\pi}{x}$$

$$= \pi$$

Therefore,

$$\lim_{x\to+\infty} x\sin\frac{\pi}{x} = \pi$$

24)

$$\lim_{x\to0^+} \tan x \ln x = \lim_{x\to0^+} \frac{\ln x}{\cot x} \quad \left[\text{form } \frac{-\infty}{\infty}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to0^+} \frac{\frac{1}{x}}{-\operatorname{cosec}^2 x}$$

$$= \lim_{x\to0^+} \frac{1}{x}\cdot\frac{1}{-\operatorname{cosec}^2 x}$$

$$= \lim_{x\to0^+} \frac{1}{x}\cdot(-\sin^2 x)$$

$$= \lim_{x \to 0^+} \frac{-\sin^2 x}{x} \quad \left[\text{form } \frac{-0}{0}\right]$$

$$= \lim_{x \to 0^+} \frac{-2\sin x \cos x}{1}$$

$$= 0$$

Therefore,

$$\lim_{x \to 0^+} \tan x \ln x = 0$$

25)

$$\lim_{x \to \frac{\pi}{2}^-} \sec 3x \cos 5x = \lim_{x \to \frac{\pi}{2}^-} \frac{\cos 5x}{\cos 3x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{-5\sin 5x}{-3\sin 3x}$$

$$= \frac{-5(1)}{-3(-1)}$$

$$= -\frac{5}{3}$$

Therefore,

$$\lim_{x \to \frac{\pi}{2}^-} \sec 3x \cos 5x = -\frac{5}{3}$$

26)

$$\lim_{x\to\pi} (x-\pi)\cot x = \lim_{x\to\pi} \frac{x-\pi}{\tan x} \quad [\text{form } \tfrac{0}{0}]$$

Now, using L' Hospital Rule,

$$= \lim_{x\to\pi} \frac{1}{\sec^2 x}$$

$$= \frac{1}{\sec^2 \pi}$$

$$= 1$$

Therefore,

$$\lim_{x\to\pi} (x-\pi)\cot x = 1$$

27)

$$\lim_{x\to+\infty} (1-3/x)^x \quad [\text{form } 1^\infty]$$

Let,

$$y = \left(1-\frac{3}{x}\right)^x$$

taking ln both side,

$$\ln y = \ln\left(1-\frac{3}{x}\right)^x$$

$$\ln y = x \cdot \ln\left(1-\frac{3}{x}\right)$$

$$\ln y = \frac{\ln\left(1-\frac{3}{x}\right)}{\frac{1}{x}}$$

Taking Limit both side,

$$\lim_{x \to +\infty} \ln y = \lim_{x \to +\infty} \frac{\ln\left(1 - \frac{3}{x}\right)}{\frac{1}{x}} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x \to +\infty} \frac{\frac{1}{1-\frac{3}{x}} \cdot \frac{3}{x^2}}{\frac{-1}{x^2}}$$

$$= \lim_{x \to +\infty} \frac{-3}{1 - \frac{3}{x}}$$

$$= -3$$

Now,

$$\lim_{x \to +\infty} \ln y = -3$$

$$\Rightarrow \ln \lim_{x \to +\infty} y = -3$$

$$\Rightarrow \lim_{x \to +\infty} \left(1 - \frac{3}{x}\right)^x = e^{-3}$$

Therefore,

$$\lim_{x \to +\infty} \left(1 - \frac{3}{x}\right)^x = e^{-3}$$

28)

$$\lim_{x \to 0} (1+2x)^{-\frac{3}{x}}$$

Let,

$$y = (1+2x)^{-\frac{3}{x}}$$

taking ln both side,

$$\ln y = \ln (1+2x)^{-\frac{3}{x}}$$

$$\Rightarrow \ln y = -\frac{3}{x} \ln (1+2x)$$

$$\Rightarrow \ln y = \frac{-3 \ln (1+2x)}{x}$$

taking limit both side,

$$\lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{-3 \ln (1+2x)}{x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x \to 0} \frac{-3 \cdot \frac{1}{1+2x} \cdot 2}{1}$$

$$= \lim_{x \to 0} \frac{-6}{1+2x}$$

$$= -6$$

Now,

$$\lim_{x\to 0} \ln y = -6$$

$$\Rightarrow \ln \lim_{x\to 0} y = -6$$

$$\therefore \lim_{x\to 0} (1+2x)^{-\frac{3}{x}} = e^{-6}$$

Therefore,

$$\lim_{x\to 0} (1+2x)^{-\frac{3}{x}} = e^{-6}$$

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$$\lim_{x\to 0} (e^x+x)^{\frac{1}{x}}$$

Let,

$$y = (e^x + x)^{\frac{1}{x}}$$

taking ln both side,

$$\ln y = \ln(e^x+x)^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x} \cdot \ln(e^x+x)$$

$$\ln y = \frac{\ln(e^x+x)}{x}$$

taking limit both side,

$$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{\ln(e^x+x)}{x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital rule,

$$= \lim_{x \to 0} \frac{e^x + 1}{e^x + x}$$

$$= \frac{1+1}{1+0}$$

$$= 2$$

Now,

$$\lim_{x \to 0} \ln y = 2$$

$$\Rightarrow \ln \lim_{x \to 0} y = 2$$

$$\therefore \lim_{x \to 0} (e^x + x)^{\frac{1}{x}} = e^2$$

Therefore,

$$\lim_{x \to 0} (e^x + x)^{\frac{1}{x}} = e^2$$

30)

$$\lim_{x \to +\infty} \left(1 + \frac{a}{x}\right)^{bx}$$

Let,

$$y = \left(1 + \frac{a}{x}\right)^{bx}$$

taking ln both side,

$$\ln y = \ln \left(1 + \frac{a}{x}\right)^{bx}$$

$$\Rightarrow \ln y = bx \ln \left(1 + \frac{a}{x}\right)$$

$$\Rightarrow \ln y = \frac{b \ln\left(1+\frac{a}{x}\right)}{\frac{1}{x}}$$

taking limit both side,

$$\lim_{x \to +\infty} \ln y = \lim_{x \to +\infty} \frac{b \ln\left(1+\frac{a}{x}\right)}{\frac{1}{x}} \quad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x \to +\infty} \frac{b \cdot \frac{1}{1+\frac{a}{x}} \cdot a \cdot \frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x \to +\infty} \frac{ab}{1+\frac{a}{x}}$$

$$= ab$$

Now,

$$\lim_{x \to +\infty} \ln y = ab$$

$$\Rightarrow \ln\left(\lim_{x \to \infty} y\right) = ab$$

$$\therefore \lim_{x \to +\infty} \left(1+\frac{a}{x}\right)^{bx} = e^{ab}$$

Therefore,

$$\lim_{x \to +\infty} \left(1+\frac{a}{x}\right)^{bx} = e^{ab}$$

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$$\lim_{x\to 1} (2-x)^{\tan[\frac{\pi}{2}.x]}$$

Let,

$$y = (2-x)^{\tan(\frac{\pi x}{2})}$$

taking ln both side,

$$\ln y = \ln (2-x)^{\tan(\frac{\pi x}{2})}$$

$$\Rightarrow \ln y = \tan\frac{\pi x}{2} . \ln(2-x)$$

$$\Rightarrow \ln y = \frac{\ln(2-x)}{\cot\frac{\pi x}{2}}$$

taking limit both side,

$$\lim_{x\to 1} \ln y = \lim_{x\to 1} \frac{\ln(2-x)}{\cot\frac{\pi x}{2}} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to 1} \frac{\frac{1}{2-x}\cdot(-1)}{-\operatorname{cosec}^2\frac{\pi x}{2}\cdot\frac{\pi}{2}}$$

$$= \lim_{x\to 1} \frac{2\sin^2\frac{\pi x}{2}}{\pi(2-x)}$$

$$= \frac{2\sin^2\frac{\pi}{2}}{\pi(2-1)} = \frac{2}{\pi}$$

Now,

$$\lim_{x \to 1} \ln y = \frac{2}{\pi}$$

$$\therefore \lim_{x \to 1} (2-x)^{\tan \frac{\pi}{2}x} = e^{2/\pi}$$

Therefore,

$$\lim_{x \to 1} (2-x)^{\tan \frac{\pi}{2}x} = e^{2/\pi}$$

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$$\lim_{x \to +\infty} \left(\cos \frac{2}{x}\right)^{x^2}$$

Let,

$$y = \left(\cos \frac{2}{x}\right)^{x^2}$$

taking ln both side,

$$\ln y = \ln \left(\cos \frac{2}{x}\right)^{x^2}$$

$$\Rightarrow \ln y = x^2 \ln \left(\cos \frac{2}{x}\right)$$

$$\Rightarrow \ln y = \frac{\ln \left(\cos \frac{2}{x}\right)}{\frac{1}{x^2}}$$

taking limit both side,

$$\lim_{x\to+\infty} \ln y = \lim_{x\to+\infty} \frac{\ln(\cos\frac{2}{x})}{\frac{1}{x^2}} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to+\infty} \frac{\frac{1}{\cos\frac{2}{x}} \cdot -\sin\frac{2}{x} \cdot \frac{-2}{x^2}}{\frac{-2}{x^3}}$$

$$= -\lim_{x\to+\infty} \frac{\tan\frac{2}{x}}{\frac{1}{x}} \quad \left[\text{form } \frac{0}{0}\right]$$

$$= -\lim_{x\to+\infty} \frac{\sec^2\frac{2}{x} \cdot \frac{-2}{x^2}}{\frac{-1}{x^2}}$$

$$= -2 \lim_{x\to+\infty} \sec^2\left(\frac{2}{x}\right)$$

$$= -2$$

Now,

$$\lim_{x\to+\infty} \ln y = -2$$

$$\therefore \lim_{x\to+\infty} \left(\cos\frac{2}{x}\right)^{x^2} = e^{-2}$$

Therefore,

$$\lim_{x\to+\infty} \left(\cos\frac{2}{x}\right)^{x^2} = e^{-2}$$

33|

$$\lim_{x\to 0}\left(\operatorname{cosec} x-\frac{1}{x}\right) \qquad [\infty-\infty]$$

$$=\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$$

$$=\lim_{x\to 0}\frac{x-\sin x}{x\sin x} \qquad \left[\text{form } \frac{0}{0}\right]$$

Now, using L' Hospital Rule,

$$=\lim_{x\to 0}\frac{1-\cos x}{x\cos x+\sin x} \qquad \left[\text{form } \frac{0}{0}\right]$$

$$=\lim_{x\to 0}\frac{\sin x}{\cos x+\cos x-x\sin x}$$

$$=\lim_{x\to 0}\frac{\sin x}{2\cos x-x\sin x}$$

$$=\frac{0}{2\cdot 1-0}$$

$$=0$$

Therefore,

$$\lim_{x\to 0}\left(\operatorname{cosec} x-\frac{1}{x}\right)=0$$

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$$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{\cos 3x}{x^2}\right)$$

$$=\lim_{x\to 0}\frac{1-\cos 3x}{x^2} \quad [\text{form } \tfrac{0}{0}]$$

Now, using L' Hospital Rule,

$$=\lim_{x\to 0}\frac{3\sin 3x}{2x} \quad [\text{form } \tfrac{0}{0}]$$

$$=\lim_{x\to 0}\frac{9\cos 3x}{2}$$

$$=\frac{9}{2}$$

Therefore,

$$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{\cos 3x}{x^2}\right)=\frac{9}{2}$$

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$$\lim_{x\to +\infty}\left(\sqrt{x^2+x}-x\right) \quad [\text{form } \infty-\infty]$$

$$=\lim_{x\to +\infty}\frac{\sqrt{x^2+x}-x}{1}$$

$$=\lim_{x\to +\infty}\frac{\left(\sqrt{x^2+x}-x\right)\left(\sqrt{x^2+x}+x\right)}{\left(\sqrt{x^2+x}+x\right)}$$

$$= \lim_{x \to +\infty} \frac{x^2+x-x^2}{\sqrt{x^2+x}+x}$$

$$= \lim_{x \to +\infty} \frac{x}{\sqrt{x^2+x}+x} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x \to +\infty} \frac{1}{\sqrt{1+\frac{1}{x}}+1}$$

$$= \frac{1}{2}$$

Therefore,

$$\lim_{x \to \infty} \left(\sqrt{x^2+x}-x\right) = \frac{1}{2}$$

36)

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x-1}\right)$$

$$= \lim_{x \to 0} \frac{e^x-1-x}{x(e^x-1)}$$

$$= \lim_{x \to 0} \frac{e^x-1-x}{xe^x-x} \quad \left[\text{form } \frac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x \to 0} \frac{e^x-1}{xe^x+e^x-1} \quad \left[\text{form } \frac{0}{0}\right]$$

$$= \lim_{x\to 0} \frac{e^x}{xe^x + 2e^x}$$

$$= \frac{1}{2}$$

Therefore,

$$\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right) = \frac{1}{2}.$$

37)

$$\lim_{x\to +\infty} [x - \ln(x^2+1)] \quad [\text{form } \infty - \infty]$$

Let,

$$y = x - \ln(x^2+1)$$

$$e^y = e^{x - \ln(x^2+1)}$$

$$= e^x \cdot e^{-\ln(x^2+1)}$$

$$= e^x \cdot e^{\ln\left(\frac{1}{x^2+1}\right)}$$

$$= e^x \cdot \frac{1}{x^2+1}$$

$$\lim_{x\to +\infty} e^y = \lim_{x\to +\infty} \frac{e^x}{x^2+1} \quad [\text{form } \tfrac{\infty}{\infty}]$$

$$= \lim_{x\to +\infty} \frac{e^x}{2x} \quad [\text{form } \tfrac{\infty}{\infty}]$$

$$= \lim_{x\to +\infty} \frac{e^x}{2}$$

$$= +\infty$$

Now,

$$\lim_{x \to +\infty} e^y = +\infty$$

$$\Rightarrow \lim_{x \to +\infty} y = \ln(+\infty)$$

$$\lim_{x \to +\infty} [x - \ln(x^2+1)] = \infty$$

Therefore,

$$\lim_{x \to +\infty} [x - \ln(x^2+1)] = +\infty$$

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$$\lim_{x \to +\infty} [\ln x - \ln(1+x)] \qquad [\text{form } \infty - \infty]$$

$$= \lim_{x \to +\infty} \left(\ln \frac{x}{1+x}\right)$$

$$= \lim_{x \to +\infty} \ln \frac{1}{\frac{1}{x}+1}$$

$$= \ln \frac{\lim_{x \to +\infty} 1}{\lim_{x \to +\infty} \left(\frac{1}{x}+1\right)}$$

$$= \ln(1)$$

$$= 0$$

Therefore,

$$\lim_{x \to +\infty} [\ln x - \ln(1+x)] = 0$$

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$$\lim_{x\to 0^+} x^{\sin x}$$

Let,

$$Y = x^{\sin x}$$

taking ln both side,

$$\ln y = \ln x^{\sin x}$$

$$\ln y = \sin x \ln x$$

$$\ln y = \frac{\ln x}{\operatorname{cosec} x}$$

taking limit both side,

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln x}{\operatorname{cosec} x} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

Now, using L' Hospital Rule,

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\operatorname{cosec} x \cot x}$$

$$= \lim_{x\to 0^+} \frac{\sin x}{x}(-\tan x)$$

$$= 1\cdot(-0)$$

$$\lim_{x\to 0^+} \ln y = 0$$

$$\therefore \lim_{x\to 0^+} x^{\sin x} = e^0 = 1$$

Therefore, $\lim_{x\to 0^+} x^{\sin x} = 1$

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$$\lim_{x\to 0^+} (e^{2x}-1)^x$$

Let,

$$y = (e^{2x}-1)^x$$

taking ln both side,

$$\ln y = \ln (e^{2x}-1)^x$$

$$\ln y = x \cdot \ln (e^{2x}-1)$$

$$\ln y = \frac{\ln (e^{2x}-1)}{\frac{1}{x}}$$

taking limit both side,

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln (e^{2x}-1)}{\frac{1}{x}} \quad \left[\text{form } \tfrac{0}{0}\right]$$

Now using L' Hospital Rule,

$$= \lim_{x\to 0^+} \frac{-2e^{2x} \cdot x^2}{e^{2x}-1} \quad \left[\text{form } \tfrac{0}{0}\right]$$

$$= \lim_{x\to 0^+} \frac{-2 \cdot e^{2x} \cdot 2 \cdot 2x}{e^{2x} \cdot 2}$$

$$= \lim_{x\to 0^+} \frac{-8xe^{2x}}{2e^{2x}}$$

$$= \frac{0}{2}$$

$$= 0$$

Therefore, $\lim_{x\to 0^+} (e^{2x}-1)^x = e^0 = 1$

41)

$$\lim_{x\to 0^+}\left[-\frac{1}{\ln x}\right]^x$$

Let,

$$y=\left[-\frac{1}{\ln x}\right]^x$$

taking ln both side,

$$\ln y = \ln\left(-\frac{1}{\ln x}\right)^x$$

$$= x\ln\left(-\frac{1}{\ln x}\right)$$

taking limit,

$$\lim_{x\to 0^+}\ln y = \lim_{x\to 0^+}\frac{\ln\left(-\frac{1}{\ln x}\right)}{\frac{1}{x}} \quad \left[\text{form } \frac{\infty}{\infty}\right]$$

using L' Hospital Rule,

$$= \lim_{x\to 0^+}\frac{x^2}{x\ln x}$$

$$= \lim_{x\to 0^+}\frac{x}{\ln x}$$

$$\lim_{x\to 0^+}\ln y = 0$$

$$\therefore \lim_{x\to 0^+}\left(-\frac{1}{\ln x}\right)^x = e^0 = 1$$

Therefore

$$\lim_{x\to 0^+}\left(-\frac{1}{\ln x}\right)^x = 1$$

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$$\lim_{x \to +\infty} x^{\frac{1}{x}} \quad [\text{form } \infty^0]$$

Let,

$$y = x^{\frac{1}{x}}$$

taking ln both side,

$$\ln y = \ln x^{1/x}$$

$$\ln y = \frac{1}{x} \ln x$$

taking limit,

$$\lim_{x \to +\infty} \ln y = \lim_{x \to +\infty} \frac{\ln x}{x} \quad [\text{form } \frac{\infty}{\infty}]$$

Using L' Hospital Rule,

$$= \lim_{x \to +\infty} \frac{\frac{1}{x}}{1}$$

$$= \lim_{x \to \infty} \frac{1}{x}$$

$$\lim_{x \to +\infty} \ln y = 0$$

$$\therefore \lim_{x \to +\infty} x^{1/x} = e^0 = 1$$

Therefore,

$$\lim_{x \to +\infty} x^{1/x} = 1$$

43)

$\lim_{x \to +\infty} (\ln x)^{1/x}$ [form ∞^0]

Let

$y = (\ln x)^{1/x}$

taking ln both side

$\ln y = \ln (\ln x)^{1/x}$

$\ln y = \frac{1}{x} \ln \ln x$

taking limit,

$\lim_{x \to +\infty} \ln y = \lim_{x \to +\infty} \frac{\ln \ln x}{x}$ [form $\frac{\infty}{\infty}$]

using L' Hospital Rule,

$= \lim_{x \to +\infty} \frac{\frac{1}{x \ln x}}{1}$

$= \lim_{x \to +\infty} \frac{1}{x \ln x}$

$\lim_{x \to +\infty} \ln y = 0$

$\therefore \lim_{x \to +\infty} y = e^0 = 1$

Therefore,

$\lim_{x \to +\infty} (\ln x)^{1/x} = 1$

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$$\lim_{x \to 0^+} (-\ln x)^x \qquad [\text{form } -\infty^0]$$

Let,

$$y = (-\ln x)^x$$

taking ln both side,

$$\ln y = \ln (-\ln x)^x$$

$$\ln y = x \ln(-\ln x)$$

taking limit,

$$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \frac{\ln(-\ln x)}{\frac{1}{x}} \qquad \left[\text{form } \frac{\infty}{\infty}\right]$$

using L' Hospital Rule,

$$= \lim_{x \to 0^+} \frac{1}{-\ln x} \cdot \frac{1}{-x} \cdot \frac{x^2}{-1}$$

$$= \lim_{x \to 0^+} \frac{-x}{\ln x}$$

$$\lim_{x \to 0^+} \ln y = 0$$

$$\therefore \lim_{x \to 0^+} y = e^0 = 1$$

Therefore,

$$\lim_{x \to 0^+} (-\ln x)^x = 1$$

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$$\lim_{x\to\frac{\pi}{2}^-} (\tan x)^{\frac{\pi}{2}-x}$$

Let, $y = (\tan x)^{\frac{\pi}{2}-x}$

$\ln y = \ln (\tan x)^{\frac{\pi}{2}-x}$ [taking ln both side]

$\ln y = (\frac{\pi}{2}-x) \ln(\tan x)$

taking limit,

$$\lim_{x\to\frac{\pi}{2}^-} \ln y = \lim_{x\to\frac{\pi}{2}^-} \frac{\ln(\tan x)}{\frac{1}{\frac{\pi}{2}-x}} \quad [\text{form } \tfrac{\infty}{\infty}]$$

using L' Hospital Rule,

$$= \lim_{x\to\frac{\pi}{2}^-} \frac{\frac{\sec^2 x}{\tan x}}{\frac{1}{(\frac{\pi}{2}-x)^2}}$$

$$= \lim_{x\to\frac{\pi}{2}^-} \frac{\frac{\pi}{2}-x}{\cos x} \cdot \frac{\frac{\pi}{2}-x}{\sin x}$$

$$= \lim_{x\to\frac{\pi}{2}^-} \frac{\frac{\pi}{2}-x}{\cos x} \cdot \lim_{x\to\frac{\pi}{2}^-} \frac{\frac{\pi}{2}-x}{\sin x}$$

$$\lim_{x\to\frac{\pi}{2}^-} \ln y = 1.0 = 0$$

$$\therefore \lim_{x\to\frac{\pi}{2}^-} y = e^0 = 1$$

Therefore,

$$\lim_{x\to\frac{\pi}{2}^-} (\tan x)^{\frac{\pi}{2}-x} = 1$$