NORTH SOUTH UNIVERSITY

Department of Mathematics & Physics

Assignment – 02

Name	: Joy Kumar Ghosh
Student ID	: 2211424 6 42
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a)

Given,

$n = 40$

$\hat{\sigma}_n = 5$

confidence level 95%.

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$Z_{0.025} = 1.96$

$\therefore$ margin of error (ME) $= Z_{\alpha/2} \frac{\hat{\sigma}_n}{\sqrt{n}}$

$= 1.96 \frac{5}{\sqrt{40}}$

$= 1.55$ Ans

b)

for confidence level 99%.

$\alpha = 0.01$

$\frac{\alpha}{2} = 0.005$

$Z_{0.005} = 2.576$

$\therefore$ Margin of error (ME) $= 2.576 \frac{5}{\sqrt{40}}$

$= 2.04$ Ans

4)

Given,

$n = 120$

$\hat{\sigma}_x = 0.28$

$\bar{x} = 3.37$

confidence level 95%.

$\therefore \alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$Z_{0.025} = 1.96$

$\therefore$ mean interval $= 3.37 \pm 1.96 \frac{0.28}{\sqrt{120}}$

$= 3.37 \pm 0.05$

$= 3.32$ to 3.42

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1)

x	f	$\bar{x}-x$	$(\bar{x}-x)^2$
3	3	3.16	9.9856
4	3	2.16	4.6656
5	4	1.16	1.3456
6	3	0.16	0.0256
7	4	-0.84	0.7056
8	6	-1.84	3.3856
9	1	-2.84	8.0656
10	1	-3.84	14.7456

$\therefore$ mean, $\bar{x} = \dfrac{3\times3+4\times3+5\times4+6\times3+7\times4+8\times6+9+10}{25}$

$= 6.16$

$\therefore SD. = \hat{\sigma}_n = \sqrt{\dfrac{9.9856\times3+4.6656\times3+1.3456\times4+0.0256\times3+0.7056\times4+3.3856\times6+8.0656+14.7456}{24}}$

$= \sqrt{3.77}$

$= 1.99$

Here,

$n = 25$

confidence level 95%.

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$\therefore t_{0.025, 24} = 2.064$

$\therefore$ mean rating $= 6.16 \pm 2.064 \frac{1.99}{\sqrt{25}}$

$= 6.16 \pm 0.82$

$= 5.34$ to 6.98

A

3)

Given,

$n = 10$

$\bar{x} = 10.5$

$\hat{\sigma}_n = 0.8$

90% confidence level,

$\alpha = 0.1$

$\frac{\alpha}{2} = 0.05$

$t_{0.05, 9} = 1.833$

$\therefore$ mean weight $= 10.5 \pm 1.833 \frac{0.8}{\sqrt{10}}$

$= 10.5 \pm 0.46$

$= 10.04$ to 10.96

Ans

✱ Matched Pairs t test:

✱ Example-1:

Let, $H_0 : \mu_D = 0$

$H_1 : \mu_D \neq 0$

where, $\mu_D = \mu_Y - \mu_X$

From data table we get,

Id (i)	$D_i = Y_i - X_i$	Id (i)	$D_i = Y_i - X_i$
1	13	6	7
2	3	7	6
3	-1	8	4
4	9	9	-2
5	7	10	2

$$\therefore \bar{D} = \frac{13+3-1+9+7+7+6+4-2+2}{10} = 4.8$$

$$\therefore \tilde{S_D} = \frac{\sum_{i=1}^{10} (Di-\bar{D})^2}{n-1}$$

$$= \frac{(13-4.8)^2+(3-4.8)^2+(-1-4.8)^2+(9-4.8)^2+(7-4.8)^2+(7-4.8)^2}{9} + \frac{(6-4.8)^2+(4-4.8)^2+(-2-4.8)^2+(2-4.8)^2}{9}$$

$$= 20.844$$

$$\therefore \text{Test statistic} = \frac{\bar{D}}{\sqrt{\frac{\tilde{S_D}}{n}}} \sim t_{n-1}$$

$$= \frac{4.8}{\sqrt{\frac{20.844}{10}}}$$

$$= 3.3247$$

Rejection region of 5%. significant

$$= \left(-\alpha, -t_{\frac{0.05}{2},9}\right] \cup \left[t_{\frac{0.05}{2},9}, +\alpha\right)$$

$$= (-\alpha, -2.262] \cup [2.262, +\alpha)$$

Comment: Since the test statistic value falls in the rejection region, so we reject null hypothesis.

Therefore, the fertility treatment with hormones have a significant effect on the systolic blood pressure.

Example-2:

Let,

$H_0: \mu_D = 0$

$H_1: \mu_D > 0$

where, $\mu_D = \mu_Y - \mu_X$

From data table we get,

Id (i)	$D_i = Y_i - X_i$	Id (i)	$D_i = Y_i - X_i$
1	13	6	7
2	3	7	6
3	-1	8	4
4	9	9	-2
5	7	10	2

$$\therefore \bar{D} = \frac{13+3-1+9+7+7+6+4-2+2}{10} = 4.8$$

$$\therefore S_D^2 = \frac{\sum_{i=1}^{10} (D_i - \bar{D})^2}{n-1}$$

$$= \frac{67.24 + 3.24 + 33.64 + 17.64 + 4.84 + 4.84 + 1.44 + 0.64 + 46.24 + 7.84}{9}$$

$$= 20.844$$

$$\therefore \text{Test statistic} = \frac{\bar{D}}{\sqrt{\frac{S_D}{n}}} \sim t_{n-1}$$

$$= \frac{4.8}{\sqrt{\frac{20.844}{10}}}$$

$$= 3.3247$$

$\therefore$ Rejection region of 5% significant,

$$= [t_{0.05,9}, +\infty)$$

$$= [1.833, +\infty)$$

Comment:

Since the test statistics value falls in the rejection region, so we reject null hypothesis.

Therefore, the fertility treatment with hormones increase the systolic blood pressure.

Example- 3:

Let,

$H_0 : \mu_D = 0$

$H_1 : \mu_D < 0$

where, $\mu_D = \mu_Y - \mu_X$

From data table we get,

Id (i)	$D_i = Y_i - X_i$	Id (i)	$D_i = Y_i - X_i$
1	13	6	7
2	3	7	6
3	-1	8	4
4	9	9	-2
5	7	10	2

$$\therefore \bar{D} = \frac{13+3-1+9+7+7+6+4-2+2}{10} = 4.8$$

$$\therefore S_D^2 = \frac{\sum_{i=1}^{10} (D_i - \bar{D})^2}{n-1}$$

$$= \frac{67.24+3.24+33.64+17.64+4.84+4.84+1.44+0.64+46.24+7.84}{9}$$

$$= 20.844$$

∴ Test statistic = $\frac{\bar{D}}{\sqrt{\frac{S_D}{n}}}$

$= \frac{4.8}{\sqrt{\frac{20.844}{10}}}$

$= 3.3247$

∴ Rejection region of 5% significant,

$= (-\infty, -t_{0.05,9}]$

$= (-\infty, -1.833]$

Comment:

Since the test statistic value does not fall in the rejection region, so we can not reject null hypothesis.

Therefore, the ~~for~~ fertility treatment with hormones does not decrease the systolic blood pressure.