Chapter : 1

1-1   01.06.2022

Linear Equation:

$$y = mx + c$$

$$x + y = 7$$

⊛

$$\begin{aligned} x + 3y + 5z &= 10 \\ 2x - y + 4z &= -5 \\ 3x + 2y &= 7 \end{aligned}$$

co-efficient

constant part

$$\left[\begin{array}{ccc:c} 1 & 3 & 5 & 10 \\ 2 & -1 & 4 & -5 \\ 3 & 2 & 0 & 7 \end{array}\right] \rightarrow \text{Augmented Matrix}$$

Target Matrix:

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & 5 \end{bmatrix} \rightarrow \text{One Solution}$$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 0 & 5 \end{bmatrix} \rightarrow \text{No solution}$$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \text{Many Solution}$$

L-2 06.06.2022

$$x + y + 2z = 9$$
$$2x + 4y - 3z = 1$$
$$3x + 6y - 5z = 0$$

Augmented Matrix:

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix}$$

$r_2' = r_2 - 2r_1$, $r_3' = r_3 - 3r_1$ $\Rightarrow$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & -11 & -27 \end{bmatrix}$$

$r_2' = \frac{1}{2} r_2$ $\Rightarrow$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & \frac{-7}{2} & \frac{-17}{2} \\ 0 & 3 & -11 & -27 \end{bmatrix}$$

$r_3' = r_3 - 3r$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & \frac{-7}{2} & \frac{-17}{2} \\ 0 & 0 & \frac{-1}{2} & \frac{-3}{2} \end{bmatrix}$$

$r_3' = -2r_3$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & \frac{-7}{2} & \frac{-17}{2} \\ 0 & 0 & 1 & 3 \end{bmatrix}$$ → in row-ichelon form

$\therefore z = 3$

$\therefore y + \frac{-7}{2} z = \frac{-17}{2}$

$\Rightarrow y + \frac{-7 \cdot 3}{2} = \frac{-17}{2}$

$\Rightarrow y = \frac{-17}{2} + \frac{21}{2}$

$\therefore y = \frac{-17+21}{2} = \frac{4}{2} = 2$

$\therefore \quad x + y + 2z = 9$

$\Rightarrow x + 2 + 2\cdot 3 = 9$

$\Rightarrow x = 9 - 8$

$\therefore x = 1$

$\therefore (x, y, z) = (1, 2, 3).$

Ans

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 7 \end{bmatrix} \longrightarrow \text{reduced row-echelon form}$$

|2|

a)

$2x = -1$

$-4x = -6$

$x = -1$

$3x = 0$

b)

$3x_2 - x_3 - x_4 = -1$

$5x_1 + 2x_2 \quad - 3x_4 = -6$

c)

$x + 2y + 3z = 4$

$-4x - 3y - 2z = -1$

$5x - 6y + z = 1$

$-8x \qquad = 3$

d) $3x_1 \qquad + x_3 - 4x_4 = 3$

$-4x_1 \qquad + 4x_3 + x_4 = -3$

$-x_1 + 3x_2 \qquad - 2x_4 = -9$

$-x_4 = -2$

15)

$y_1 = ax_1^2 + bx_1 + c$

$y_2 = ax_2^2 + bx_2 + c$

$y_3 = ax_3^2 + bx_3 + c$

Augmented Matrix:

$$\begin{bmatrix} x_1^2 & x_1 & 1 & y_1 \\ x_2^2 & x_2 & 1 & y_2 \\ x_3^2 & x_3 & 1 & y_3 \end{bmatrix}$$

L-3 / 08.06.2022

1.2

Example - 5:

The augmenet matrix for the system is,

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{bmatrix}$$

$$\xrightarrow[r_3' = r_3 - 2r_1]{r_2' = r_2 - 2r_1} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{bmatrix}$$

$$\xrightarrow{r_2' = -r_2} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{bmatrix}$$

$$\xrightarrow[R_4' = R_4 - 4R_2]{R_3' = R_3 - 5R_2} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 \end{bmatrix}$$

$$\xrightarrow{R_3 \rightleftarrows R_4} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3' = \frac{1}{6} R_3} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2' = R_2 - 3R_3} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$r_1' = r_1 + 2r_2$

$$\xrightarrow{r_1' = r_1 + 2r_2} \begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \text{reduced row-}\cancel{\text{ichelon}}\text{ echelon matrix}$$

$\therefore x_1 + 3x_2 + 4x_4 + 2x_5 = 0$

$x_3 + 2x_4 = 0$

$x_6 = \frac{1}{3}$

$\therefore x_1, x_3, x_6$ are leading variables.

$\therefore x_1 = -3x_2 - 4x_4 - 2x_5$

$\therefore x_3 = -2x_4$

$\therefore x_6 = \frac{1}{3}$

$\therefore x_2, x_4, x_5$ are free variables.

Take, $x_2 = r$

$x_4 = s$

$x_5 = t$

$\therefore \; x_1 = -3r - 4s - 2t$

$x_2 = r$

$x_3 = -2s$

$x_4 = s$

$x_5 = t$

$x_6 = \frac{1}{3}$

where, r, s, t can be any real numbers

(*) $2x + y + 5z = 0$

$\longrightarrow$ linear homogenous equation.

P.T.O.

Example - 6:

The augmented matrix for the system is,

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & 0 \\ 2 & 6 & 0 & 8 & 4 & 18 & 0 \end{bmatrix}$$

$$\xrightarrow[r_4' = r_4 - 2r_1]{r_2' = r_2 - 2r_1} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & 0 \\ 0 & 0 & 4 & 8 & 0 & 18 & 0 \end{bmatrix}$$

$$\xrightarrow{r_2' = -r_2} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & 0 \\ 0 & 0 & 4 & 8 & 0 & 18 & 0 \end{bmatrix}$$

$$\xrightarrow[r_4' = r_4 - 4r_2]{r_3' = r_3 - 5r_2} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 \end{bmatrix}$$

$$\xrightarrow{r_3 \rightleftarrows r_4} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{r_4' = \frac{1}{6} r_4} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$

$$R_2' = R_2 - 3R_3 \longrightarrow \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1' = R_1 + 2R_2 \longrightarrow \begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ $x_1 + 3x_2 + 4x_4 + 2x_5 = 0$

$x_3 + 2x_4 = 0$

$x_6 = 0$

$\therefore$ $x_1 = -3x_2 - 4x_4 - 2x_5$

$x_3 = -2x_4$

$x_6 = 0$

Let,
$x_2 = r$

$x_4 = s$

$x_5 = t$

$\therefore \quad x_1 = -3r - 4s - 2t$

$x_2 = r$

$x_3 = -2s$

$x_4 = s$

$x_5 = t$

$x_6 = 0$

Ans

L-4 / 13.06.2022

1.3

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix}_{\substack{2\times 3 \\ r_1 \times c_1}} \quad \& \quad B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}_{\substack{3\times 4 \\ r_2 \times c_2}}$$

$$c_1 = r_2$$

$$\therefore AB = \begin{bmatrix} 4+0+8 & 1-2+28 & 4+6+20 & 3+2+8 \\ 8+0+0 & 2-6+0 & 8+18+0 & 6+6+0 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 27 & 30 & 13 \\ 8 & -4 & 26 & 12 \end{bmatrix}_{2\times 4}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & -1 \end{bmatrix}_{2\times 3}$$

$$A^T = \begin{bmatrix} 2 & 5 \\ 3 & 6 \\ 4 & -1 \end{bmatrix}_{3\times 2}$$

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a)

$$A = \begin{bmatrix} a & 0 & 0 & 0 & 0 & 0 \\ 0 & b & 0 & 0 & 0 & 0 \\ 0 & 0 & e & 0 & 0 & 0 \\ 0 & 0 & 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & 0 & f \end{bmatrix}$$

b)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & 0 & a_{33} & a_{34} & a_{35} & a_{36} \\ 0 & 0 & 0 & a_{44} & a_{45} & a_{46} \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}$$

d)

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & a_{65} & a_{66} \end{bmatrix}$$

24)

a) $$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

b) $$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix}$$

L-5 / 15.06.2022 /

1.4 /

Theorem : 1.4.1 / Poperties of Matrix Arithmetic

a) $A + B = B + A$

b) $A + (B + C) = (A + B) + C$

c) $A(BC) = (AB)C$

d) $A(B + C) = AB + AC$

e) $(B + C)A = BA + CA$

f) $A(B - C) = AB - AC$

g) $(B - C)A = BA - CA$

h) $a(B + C) = aB + aC$

i) $a(B - C) = aB - aC$

j) $(a + b)C = aC + bC$

k) $(a - b)C = aC - bC$

l) $a(bC) = (ab)C$

m) $a(BC) = (aB)C = B(aC)$

✱ Identity Matrix / Unit Matrix (Product)

✱ In general math: If product of two non-zero number is 1, then thy are inverse of each other.

✱ Square matrix whose determinate is zero is singular matrix

✱ Rules of inverse,

i. Square matrix

ii. Same size

iii. Non- singular

$AB = I = BA$

✱ $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\det(A) = ad - bc$

⊛ $A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow$

$$|A| = \begin{vmatrix} 2 & -5 \\ -1 & 3 \end{vmatrix} = 6 - 5 = 1$$

$$|B| = \begin{vmatrix} 3 & 5 \\ 1 & 2 \end{vmatrix} = 6 - 5 = 1$$

⊛ $A = \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix}$

$$\det(A) = -3 - 8 = -11$$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} -3 & -4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{11} & \frac{4}{11} \\ \frac{2}{11} & -\frac{1}{11} \end{bmatrix}$$

⊛ $B = \begin{bmatrix} 5 & 2 \\ 3 & -1 \end{bmatrix}$

$$\det(B) = -5 - 6 = -11$$

$$\therefore B^{-1} = \frac{1}{\det(B)} \begin{bmatrix} -1 & -2 \\ -3 & 5 \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} -1 & -2 \\ -3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{3}{11} & \frac{-5}{11} \end{bmatrix}$$

(*)

$$A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

$$\det(A) = 6 - 5 = 1$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = B$$

$$B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

$$\det(B) = 6 - 5 = 1$$

$$\therefore B^{-1} = \frac{1}{1}\begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = A$$

(*)

$$2x + 3y = 1$$
$$x - 5y = 7$$

$$\begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$Ax = B$$
$$A^{-1}Ax = A^{-1}B$$
$$x = A^{-1}B$$

$$A^{-1} = \frac{1}{-13}\begin{bmatrix} -5 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{13} & \frac{3}{13} \\ \frac{1}{13} & -\frac{2}{13} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} \frac{5}{13} & \frac{3}{13} \\ \frac{1}{13} & -\frac{2}{13} \end{bmatrix} \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{13} + \frac{21}{13} \\ \frac{1}{13} - \frac{14}{13} \end{bmatrix} = \begin{bmatrix} \frac{26}{13} \\ \frac{-13}{13} \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$(AB)^{-1} = B^{-1}A^{-1}$

$$\Rightarrow (AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1}$$
$$= AIA^{-1}$$
$$= AA^{-1}$$
$$= I$$

$$(B^{-1}A^{-1})AB = B^{-1}(A^{-1}A)B$$
$$= B^{-1}IB$$
$$= B^{-1}B$$
$$= I$$

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1.4 /

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(A \pm B)^{T} = A^{T} \pm B^{T}$$

$$(AB)^{T} = B^{T}A^{T}$$

$$A^{-2} = (A^{2})^{-1} = (A^{-1})^{2}$$

$$A^{2} \cdot A^{3} = A^{5}$$

$$A^{-n} = (A^{-1})^{n} = (A^{n})^{-1}$$

$$(A^{-1})^{-1} = A$$

Matrix Polynomial

$$A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$$

$$P(x) = 2x^{2} - 5x + 4$$

$P(A)$ = matrix polynomial

$$= 2A^{2} - 5A + 4I$$

$\Rightarrow$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4-12 & -6-3 \\ 8+4 & -12+1 \end{bmatrix} = \begin{bmatrix} -8 & -9 \\ 12 & -11 \end{bmatrix}$$

$$P(A) = 2A^2 - 5A + 4I$$

$$= 2\begin{bmatrix} -8 & -9 \\ 12 & -11 \end{bmatrix} - 5\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & -18 \\ 24 & -22 \end{bmatrix} - \begin{bmatrix} 10 & -15 \\ 20 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -16-10+4 & -18+15+0 \\ 24+20+0 & -22-5+4 \end{bmatrix}$$

$$= \begin{bmatrix} -22 & -3 \\ 44 & -23 \end{bmatrix}$$

$$(A^T)^{-1} = (A^{-1})^T$$

$$(A^T)(A^{-1})^T = I$$

$$\Rightarrow (A^{-1}A)^T = I$$

$$\Rightarrow I^T = I$$

$$\therefore I = I$$

Sample Question: $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$

verify that $(A^{-1})^T = (A^T)^{-1}$

A square matrix A is said to be idempotent if

$A^2 = A$.

<u>541</u>

a) $(I-A)^2 = (I-A)(I-A)$

$$= I^2 - IA - AI + A^2$$

$$= I - A - A + A$$

$$= (I-A)$$

$\therefore (I-A)$ is idempotent.

b)

$$(2A-I)(2A-I) = 4A^2 - 2AI - 2I\cdot A + \cancel{2}\, I^2$$

$$= 4A^2 - 2A - 2A + I$$

$$= 4A - 4A + I$$

$$= I$$

So, $(2A-I)^{-1} = (2A-I)$

(Proved)

1·5

Elementary Matrix

Defination: An $n \times n$ matrix is called an elementary matrix if it can be obtained from the $n \times n$ identity matrix I_n by performing a single elementary row operation.

* $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{r_1 \rightleftarrows r_3} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ → an elementary matrix

$\Big\downarrow r_3' = r_3 + 2r_1$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ → an elementary matrix.

Sample Question:

* Find out the operation or an elementary matrix.

* $[A \mid I] \longrightarrow [I \mid A^{-1}]$

$\downarrow$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 5 & 7 & 2 & 0 & 1 & 0 \\ 8 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

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1.5

Find A^{-1}

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[r_3' = r_3 - r_1]{r_2' = r_2 - 2r_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{r_3' = r_3 + 2r_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & +2 & 1 \end{array}\right]$$

$$\xrightarrow{r_3' = -r_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$$\xrightarrow[R_2' = R_2 + 3R_3]{R_1' = R_1 - 3R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 6 & 3 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$$\xrightarrow{R_1' = R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$$= [I \mid A^{-1}]$$

$$\therefore A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

Example-5

$$A = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{bmatrix}$$

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 2 & 4 & -1 & 0 & 1 & 0 \\ -1 & 2 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[r_3' = r_3 + r_1]{r_2' = r_2 - 2r_1} \left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & -8 & -9 & -2 & 1 & 0 \\ 0 & 8 & 9 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{r_2' = -\frac{1}{8} r_2} \left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & 1 & \frac{9}{8} & \frac{1}{4} & -\frac{1}{8} & 0 \\ 0 & 8 & 9 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{r_3' = r_3 - 8r_2} \left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & 1 & \frac{9}{8} & \frac{1}{4} & -\frac{1}{8} & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{array}\right]$$

A is not ~~inter~~ inverseable.

1.6/

(*)

$$x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 + 5x_2 + 3x_3 = 3$$
$$x_1 \quad\quad + 8x_3 = 17$$

Here,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \rightarrow \text{coefficient matrix.}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix}$$

Logic:

$$AX = B$$
$$A^{-1}(AX) = A^{-1}B$$
$$X = A^{-1}B$$

$$\therefore A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix}$$

$$= \begin{bmatrix} -200+48+153 \\ 65-15-51 \\ 25-6-17 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Ans

L-8/ 27.06.2022/

1.7

Diagonal Matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ is a diagonal matrix.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\det(A) = 2\cdot(-3)\cdot(-1) = 6 \neq 0$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$\circledast$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$D^2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

$\circledast$

$$A^4 = \begin{bmatrix} 16 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 81 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

Triangular Matrix

$$A = \begin{bmatrix} 16 & 0 & 0 \\ 5 & 1 & 0 \\ 2 & -3 & 81 \end{bmatrix}$$

$\det(A) = 16 \cdot 1 \cdot 81 = 1296$

Symmetric Matrix

A square matrix A is symmetric if,

$$A = A^T$$

$$\begin{bmatrix} 2 & -3 & 4 \\ -3 & 7 & -2 \\ 4 & -2 & -1 \end{bmatrix}$$

Here,

$a_{12} = a_{21}$

$a_{13} = a_{31}$

$a_{23} = a_{32}$

$a_{ij} = a_{ji}$

$$\begin{bmatrix} -5 & 6 & 3 \\ 6 & 1 & a \\ 3 & a & -3 \end{bmatrix}$$

a = any real number

$$(A^{-1})^T = (A^T)^{-1}$$