

Probability

⊗ Chance or likelihood that a particular outcome will occur.

⊗ Experiment \Rightarrow Die, coin

⊗ Sample Space \Rightarrow Set of all possible outcome.

⊗ Coin tosses three times:

$$S = \{TTT, TTH, THT, THH, HTT, HTH, HHT, HHH\}$$

⊗ Number of sample space elements = n^n \rightarrow Number of trial

⊗ One die tosses two times,

$$\text{Sample space} = 6^2 = 36 \text{ elements}$$

n \rightarrow Number of outcome

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④ Answer from Slide:

$$2 \Rightarrow S = \{2, 3, 5, 7, 11, 13\} \text{ still does not fit}$$

$$3 \Rightarrow S = \{0, 1, 2, 3, \dots, n\}$$

smallest no. fitting in a front board will no. comes

$$4 \Rightarrow S = \{0, 1, 2, 3, 4\}$$

$$\{0, 1, 2, 3, 4\} = 2$$

$$5 \Rightarrow S = \{\text{each day of a year}\}$$

2 Feb if it is
= February

leap year

$$6 \Rightarrow S = \{TS, TU, LS, LU\}$$

: count no. of ways

$$\{HHH, THH, HTH, TTH, HHT, THT, HTT, TTT\} = 8$$

$$\text{Number of sample spaces elements} = 8$$

count out total no. of ways

$$(\text{sample space} - 3) = \text{ways}$$

L-02 / 25.07.2023 /

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Probability Values

$$0 \leq P(A) \leq 1$$

$$\rho = 12.0 - 1 = v + vi$$

Measurement of Probability:

$$\rho = \sqrt{12.0} = v = vi$$

1. Classical approach

$$P(A) = \frac{n(A)}{n(S)}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \text{even numbers} = \{2, 4, 6\}$$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

$$\{2, 4, 6\} = A$$

2. Relative frequency approach

$$P(A) = \lim_{n \rightarrow \infty} \frac{m}{n}$$

$n \rightarrow$ no. of trials
 $m \rightarrow$ no. of trials in favour of

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A in n trials.

3. Subjective Approach

From observation or previous record.

(A) numbered lottery method

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2 | $\text{prob of getting } T_2 \leftarrow \text{sum}$

$$\text{prob of } i + ii + iii = 0.08 + 0.20 + 0.33$$

$$= 0.61$$

$$\text{prob of } i + ii + iii \geq 0$$

$$i_N + v = 1 - 0.61 = 0.39$$

$$i_N = v = 0.39/2 = 0.195$$

$$\textcircled{i} \quad v = 0 \leq P(v) \leq 0.39$$

$$\textcircled{ii} \quad i_N = v = 0.195$$

Practical application of probability events

$$\frac{(A)_N}{(2)_N} = (A)^q$$

4 | $S = \{1, 2, 3, 4, 5, 6\}$ $\{2, 3, 4, 5, 6, 1\} = 2$
 $\{2, 4, 5\} = \text{even numbers} = A$

$$A = \{1, 2\}$$

$$\frac{1}{2} = \frac{2}{6} = (A)^q$$

$$A^c = \{3, 4, 5, 6\}$$

5. Relative frequency approach

~~A~~ point to or Exercise from Slide 8 Book

Class work from

Page - 4 written in A

1.1.4

11.10/

$$P(i) = 6x = \frac{6}{10} = 0.6$$

$$P(ii) = 3x = \frac{3}{10} = 0.3$$

$$P(iii) = x = \frac{1}{10} = 0.1 \quad \begin{matrix} (A/A)^9 \\ (B/B)^9 \end{matrix}$$

$$10x = 1$$

$$x = 0.1$$

Random Variable

& no. of heads A

Random variable $(A)^9 + (B)^9$ gets sum with probability

com - 2 heads total

trials to trisheads

in trisheads distributed binomially how many trials out of

$$x = \text{No. of heads} \times (A)^9 = (A/A)^9$$

$$(A)^9 = \frac{(A)^9 \times (A)^9}{(A)^9} = \frac{(A/A)^9}{(B/B)^9} = (B/B)^9$$

$P(x=0)$ in sample space

~~$P(x=1) = \frac{1}{2}$~~

$83.0 = (B/B)^9$

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Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$\therefore 0 \leq \frac{1}{0} P(B) \geq 0 \quad (\text{iii})$

$\Sigma = 100$
 $L = 10$

already occur
 known
 Given

if A don't depend on B

$$\text{then, } P(A|B) = P(A)$$

Independent of event

⇒ two event will said to be independent if,

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$$

Classworks :

~~I battery lasts longest~~

$$P(Z) = (0.39 + 0.03) \\ = 0.42$$

$$P(Z) \cap P(II) = 0.03$$

$$P(ZZ) = (1 - 0.24 - 0.39) = 0.37$$

$$P(\text{2 lasts long} \mid \text{2 not fail first}) = \frac{0.03}{0.37}$$

L-4 / 01.08.2023 /

Random Variable

⊗ Random variable that relates with probability.

⊗ Coin - 2 times tosses:

$$S = \{HH, HT, TH, TT\}$$

$X = \text{No of heads}$

$$= \{0, 1, 2\}$$

minimum head = 0
maximum head = 2

$$P(X=0) = \frac{1}{4}$$

How many 0 head
main sample space

$$P(X=1) = \frac{2}{4}$$

$$P(X=2) = \frac{1}{4}$$

④ Height is continuous variable
 \Rightarrow Like, minimum height 3' to maximum height 7', it can take any value of height.

i.e.: 4.5, 4.555, 4.9999 ... etc

⑤ PMF \Rightarrow Probability Mass Function is used for discrete random variable.

$x = \{ \text{The score shown on top face} \}$

$$= \{1, 2, 3, 4, 5, 6\}$$

$$P(x) = \left\{ \frac{1}{6}, \frac{1}{6}, \dots, \frac{1}{6} \right\}$$

$$\text{Die has } 6 \text{ faces} \rightarrow \frac{1}{6} = (1-x)^q$$

⑥ Die tosser twitsch:

$$S = 3^2$$

$x = \text{no of 6 comes on } \cancel{\text{the}} \text{ two dies.}$

$$= \{0, 1, 2\}$$

x	0	1	2	Total
$p(x)$	$\frac{25}{30}$	$\frac{10}{30}$	$\frac{1}{30}$	1

$$E(X) = 0 \cdot 0.833 + 1 \cdot 0.333 + 2 \cdot 0.033 = (p+q)q = 0.533$$

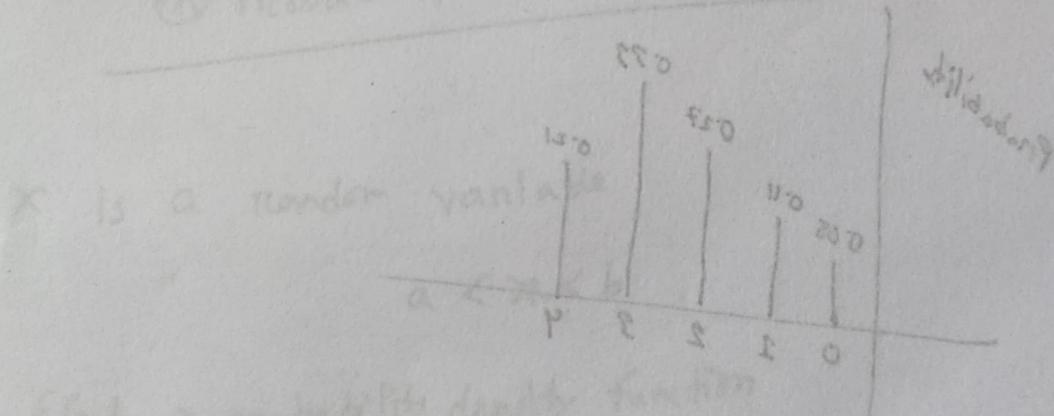
$$1.533 =$$

⊗ CDF = Cumulative Distribution:

x	0	1	2	3	4	5	6	7	8	9	x
$p(x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$								$(x)q$
CDF	$\frac{1}{4}$	$\frac{3}{4}$	1								

$$CDF(x) = p(x) = \#(\rightarrow) \cdot P(X \leq x)$$

⊗ Probability density function



L-5 / 06.08.2023 /

Class - Work

x	0	1	2	3	4
(x)q	0.08	0.11	0.27	0.73	0.21

a) $P(X=4) = 1 - 0.08 - 0.11 - 0.27 - 0.73$
 $= 0.21$

Ans: Cumulative Distribution Function = CDF

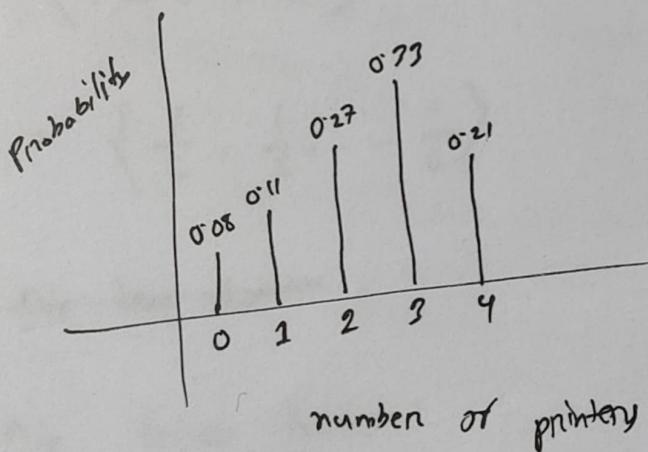
x	0	1	2	3	4
P(x)	0.08	0.11	0.27	0.73	0.21

i. $P(X \geq 0) = 1$

ii. $\sum_{i=0}^4 P(X_i) = 1$

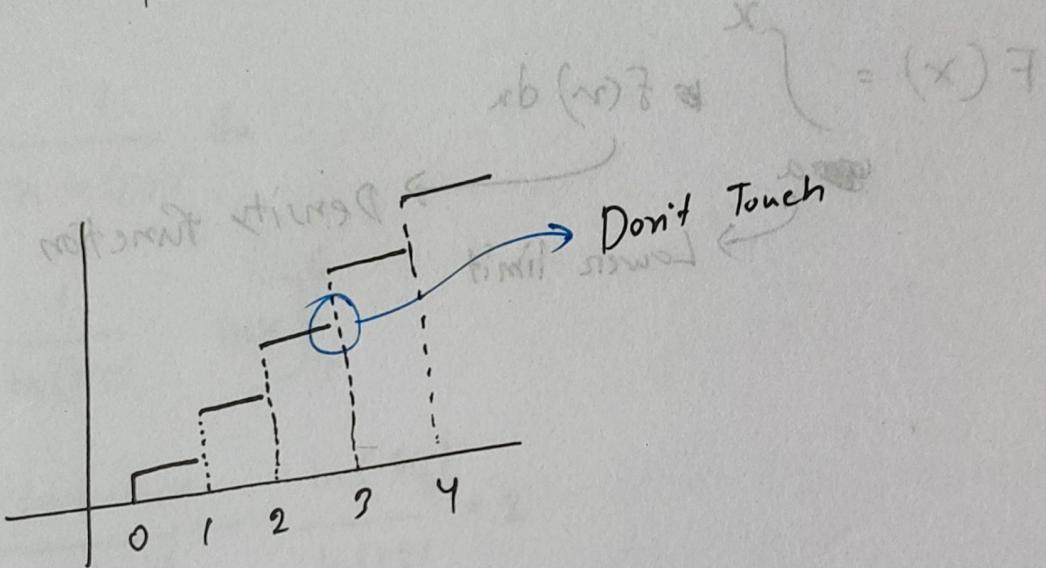
$(X)q = 0.08 + 0.11 + 0.27 + 0.73 = 1$

b)



c)

x	0	1	2	3	4
$P(x)$	0.08	0.11	0.27	0.37	0.21
$F(x)$	0.0	0.19	0.46	0.83	1



~~Probability density function~~

x is a random variable

$$a < x < b$$

$f(n) \rightarrow$ probability density function

i) $f(n) \geq 0$

ii) $\int_a^b f(n) dn = 1$

/ Do enclose from
side

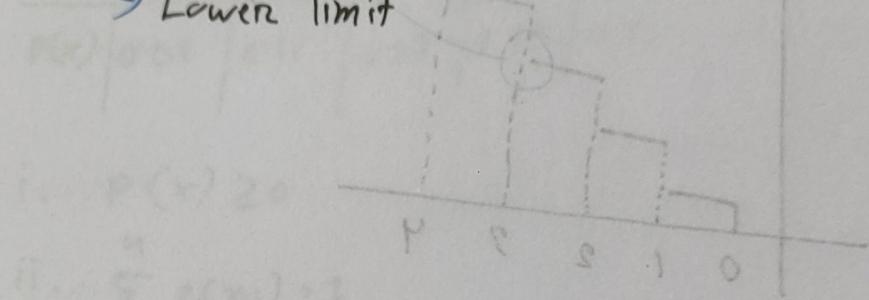
⊗ CDF

$$F(x) = P[x \leq x]$$

Random Variable
Value or a random variable

$$F(x) = \int_a^x f(n) dn$$

Lower limit Density function



differentiable cdf \Leftrightarrow

$$\text{differentiable number } a \text{ in } x \\ d > x > a$$

differentiable pdf \Leftrightarrow (i)

$$\left. \begin{array}{l} \text{differentiable function} \\ \text{at } x \\ \text{at } x \end{array} \right\} \quad \left. \begin{array}{l} 0 \leq x \leq 1 \\ x = n \in \mathbb{N} \end{array} \right\} \quad \left. \begin{array}{l} \text{(i)} \\ \text{(ii)} \end{array} \right.$$

L-6 / 08.08.2023 /

Given density function,

$$f(x) = \frac{1}{x \ln(1.5)}$$

i) $\int_4^6 \frac{1}{x \ln(1.5)} dx$

$$= \frac{1}{\ln(1.5)} \cdot \left[\ln x \right]_4^6$$

$$= \frac{\ln 6 - \ln 4}{\ln(1.5)} = \frac{\ln(5/4)}{\ln(1.5)} = 1$$

ii) Same as i), replace the limit

iii) Same, replace the limit [4, n]

Expectation of a Random variable

x	1	2	3	4
P(x)	$\frac{1}{4}$	$\frac{2}{4}$	0	$\frac{1}{4}$

$$E(x) = \sum x \cdot P(x)$$

$$\begin{aligned} &= (1 \cdot \frac{1}{4}) + (2 \cdot \frac{2}{4}) + (3 \cdot 0) + (4 \cdot \frac{1}{4}) \\ &= \frac{1}{4} + 1 + 0 + 1 \\ &= \frac{7}{4} = 2.25 \end{aligned}$$

if the variable is continuous:

$$E(x) = \int_a^b x \cdot f(x) dx$$

Continuous random variable

IV $E(x) = \int_{49.5}^{50.5} x f(x) dx$

$= \int_{49.5}^{50.5} x [1.5 - 6(x-50.0)^2] dx$

$= \int_{49.5}^{50.5} x [1.5 - 6(x-50.0)^2] dx$

$\therefore E(x) = \frac{(x^2)_{50.5} - (x^2)_{49.5}}{(2.1)_{50.5} - (2.1)_{49.5}}$

Variance:

$$V(x) = E(\tilde{x}) - (E(x))^2$$

$$E(\tilde{x}) = \sum x p(x)$$

$$E(\tilde{x}) = \sum x p(x) = 1.76$$

Standard deviation:

$$SD(x) = +\sqrt{V(x)}$$

$$1 + 0.2 + \frac{1}{p} =$$

$$25.5 = \frac{e}{p} =$$

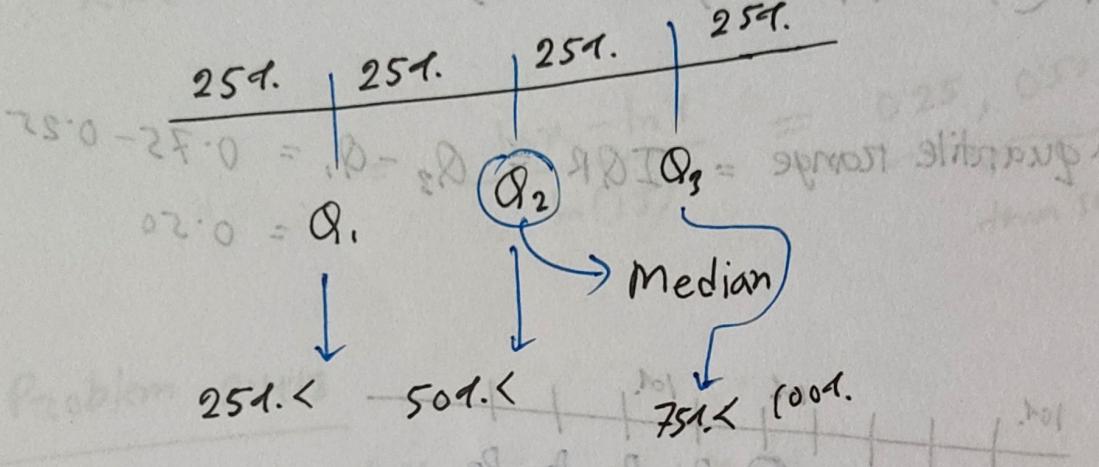
Assignment - 15

Chapter - 2

Date: 20.08.2023

L-7/13.08.2023/

* Quantiles:



(Q₁, Q₃) \Rightarrow Inter quartile range \Rightarrow 501. observation

F(x) = cumulative distribution function

$$= \int_a^x f(x) dx$$

Lower limit

$$= \sum_{\text{low}}^{\infty} f(x)$$

$F(x) = 1$; if $x = b$ (Higher limit)

$b \rightarrow \text{High}$

$$= \int_a^b f(x) dx = 1$$

Low

$$F(x) = 0.25 ; n = Q_1$$

rest of the data $\rightarrow 0.00$ 1st quartile

$$F(x) = 0.5 ; n = Q_2$$

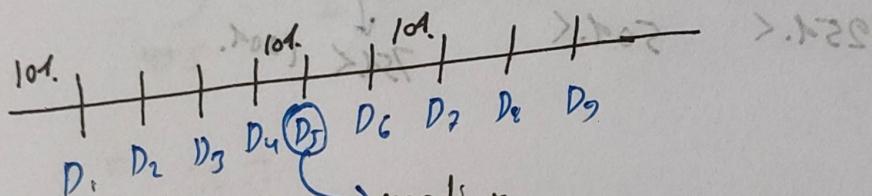
2nd quartile \times

$$F(x) = 0.75 ; n = Q_3$$

2nd quartile | 3rd quartile | 4th quartile | 5th quartile

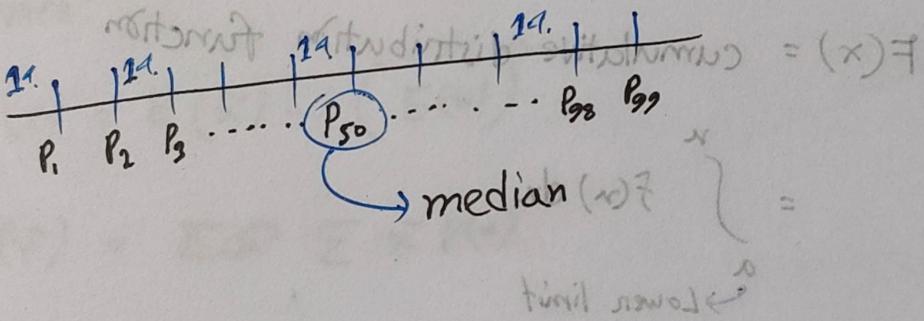
$$\textcircled{2} \text{ Interquartile range} = IQR = Q_3 - Q_1 = 0.75 - 0.25 \\ D = 0.50$$

Decile



rest of the data $\rightarrow 0.2 \Leftarrow$ median subgroup $\rightarrow (Q_1, Q_3)$

Percentile



Example:

$$\textcircled{1} F(x) = 0.25$$

$\Rightarrow \int_{49.5}^n f(n) = 0.25 \Rightarrow$ solve and find value of n .
Then compare with the given range and select the right ans.

Classwork

$$\left(\frac{1}{3} \int_1^n \frac{1}{x \ln(1.5)} dx \right) = \left[\frac{\ln x}{\ln(1.5)} \right]_1^n = nb(n-1) \left(\frac{1}{\ln(1.5)} \right)$$

$$25 \cdot 0 = (2.1 + \frac{2.1 - n}{\ln n - \ln 1}) = 0.25, 0.50, 0.75 \\ 0 = 2.1 - n = \frac{1}{\ln(1.5)} \leftarrow \text{then solve for } n.$$

Problem 2.4.18

$$f(x) = \frac{1-x}{2}; -1 \leq x \leq 1$$

$$E(\tilde{x}) = \int_{-1}^1 \frac{x-\tilde{x}}{2} dx = \int_{-1}^1 \left(\frac{x}{2} - \frac{\tilde{x}}{2} \right) dx$$

$$= \left[\frac{x^2}{4} - \frac{x^3}{6} \right]_{-1}^1 = \frac{1}{4} - \frac{1}{6} - \frac{1}{4} - \frac{1}{6} \\ - - \frac{2}{6} = - \frac{1}{3}$$

$$\sigma(x) = E(\tilde{x}) - [E(\tilde{x})]^2 = \frac{1}{3} - (-\frac{1}{3})^2 = \frac{1}{3} - \frac{1}{9} \\ = \frac{2}{9}$$

$$F(x^2) = \int_{-1}^1 \tilde{x} f(x) dx = \int_{-1}^1 \frac{\tilde{x} - x}{2} dx = \left[\frac{x^3}{6} - \frac{x^4}{8} \right]_{-1}^1$$

$$= \frac{1}{6} - \frac{1}{8} + \frac{1}{6} + \frac{1}{8} = \frac{1}{3}$$

$$SD(x) = \sqrt{\sigma(x)} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$$

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$$\begin{aligned}
 & c) \int_4^n \frac{1-x}{2} dx \\
 & = \frac{1}{2} \int_4^n (1-x) dx = \frac{1}{2} \left[x - \frac{x^2}{2} \right]_4^n = \frac{1}{2} \left(n - \frac{n^2}{2} + 1 - \frac{4^2}{2} \right) \\
 & \Rightarrow \frac{1}{2} \left(n - \frac{n^2}{2} + 1 - 8 \right) = 0.75 \\
 & \Rightarrow n - \frac{n^2}{2} = 1.5 - 1.5 = 0
 \end{aligned}$$

$$\begin{aligned}
 & \Rightarrow n^2 - 2n = 0 \\
 & \Rightarrow n(n-2) = 0 \quad | :n \\
 & \Rightarrow n=0, 2
 \end{aligned}$$

8.11.18 Blätter 8.11.18

$$\begin{aligned}
 & \text{nb} \left(\frac{n}{2} - \frac{n^2}{2} \right) \quad \begin{cases} n=0, 2 \\ \text{in range} \end{cases} \quad \begin{cases} \text{out of range} \\ \frac{n(n-2)}{2} = (x)E \end{cases} \\
 & \therefore n=0, 2
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = \left[\frac{n}{2} - \frac{n^2}{2} \right] = \\
 & \frac{1}{2} - \frac{4}{2} = -\frac{3}{2} =
 \end{aligned}$$

$$\frac{1}{2} - \frac{1}{2} = (x)E - \frac{1}{2} = [(x)E] - (x)E = (x)V$$

$$\begin{aligned}
 & \left[\frac{B(x)3x}{P} \right] = \text{nb} \left[\frac{n^2 - n}{2} \right] \quad \begin{cases} \text{solve for } V \\ = \text{nb}(n)^2 \cdot V \end{cases} = (x)E
 \end{aligned}$$

$$\frac{1}{2} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} - \frac{1}{2} =$$

$$\frac{\pi}{2} = \frac{\pi}{2} = (x)V = (x)q$$

L-8/20.08.2023/

⊗ Joint Probability distribution

$$f(x,y)$$

i) $f(x,y) \geq 0$

ii) $\iint_{-\infty}^{\infty} f(x,y) dx dy = 1$

⊗ Marginal probability distribution of X

$$g(x) = \int_{-\infty}^{\infty} f(x,y) dy$$

$$h(y) = \int_{-\infty}^{\infty} f(x,y) dx$$

⊗ Conditional distribution

$$f(y|x) = \frac{f(x,y)}{g(x)} ; g(x) > 0$$

same as conditional probability
 $P(A|B) = \frac{P(A \cap B)}{P(B)} ; P(B) > 0$

Similarly,

$$f(x|y) = \frac{f(x,y)}{h(y)} ; h(y) > 0$$

⊗ Independence of Random Variable:

if, $f(x, y) = g(x) \cdot h(y)$ joint distribution is independent \otimes

Then, X, Y is independence \times

$$(x, y) \in$$

⊗ Covariance:

$$\text{Cov}(X, Y) = E \left[\{X - E(X)\} \{Y - E(Y)\} \right]$$

or,

$$= E(XY) - E(X)E(Y)$$

⊗ Correlation between X & Y

$$\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \cdot \text{Var}(Y)}}$$

$\rho \Rightarrow -1 \leq \rho \leq 1$
 $\rho \Rightarrow$ for population

$r \Rightarrow$ for other sample

$$0 < r_n : \frac{0 < r_n}{(n)_d} = (n)_d$$

$$\frac{(x, y)^2}{(n)_d} = (x|_n)^2$$

$$0 < (y)_d : \frac{(x, y)^2}{(y)_d} = (y|_n)^2$$

Problem from Lecture-7

$$\begin{aligned}
 a) \iint_A (x-y) dy dx &= A \int_4^6 y \left[\frac{x}{2} - \frac{y^2}{2} \right]_2^5 dy \\
 &= A \int_4^6 -\frac{25}{2} y dy \\
 &= A \cdot -\frac{25}{2} \cdot \left[\frac{y^2}{2} \right]_4^6 \\
 &= -A \frac{25}{2} \cdot 10 \\
 &= -125A
 \end{aligned}$$

Hence,

$$-125A = 1$$

$$\therefore A = -\frac{1}{125}$$

$$\therefore f(x,y) = -\frac{1}{125} (x-y)$$

$$\text{using above eqn. we get } \frac{(3-x)y}{125}$$

$$\begin{aligned}
 b) \iint_A (3-x)y dy dx &= \frac{1}{125} \int_4^5 y \left[3x - \frac{x^2}{2} \right]_0^3 dx \\
 &= \frac{1}{125} \int_4^5 y dx = \frac{1}{125} \cdot \left[\frac{y^2}{2} \right]_4^5 \\
 &= \frac{1}{125} \cdot \frac{9}{2} \\
 &= \frac{9}{100}
 \end{aligned}$$

c)

$$g(x) = \frac{1}{125} \int_{-2}^6 (3-x)y \, dy$$

$$= \frac{3-x}{125} \left[\frac{y^2}{2} \right]_4^6 = xb \cdot b \cdot x(3-x)^3 A$$

$$= \frac{3-x}{125} \cdot 10 = \frac{y}{125} \left[3x - \frac{x^3}{2} \right]_{-2}^6$$

$$= \frac{2(3-x)}{25}; -2 \leq x \leq 3 = \frac{y}{125} \cdot \frac{25}{2}$$

$$A = \frac{y}{10}; 4 \leq y \leq 6$$

d)

$$g(x) \cdot h(y) = \frac{2(3-x)}{25} \cdot \frac{y}{10}$$

$$= \frac{y(3-x)}{125}$$

\rightarrow independence.

Next Sunday 8/12-1

$$xb \cdot \left[\frac{y^2}{2} \right]_{-2}^6 \times \left\{ \frac{1}{125} \right\} = xb \cdot b \cdot x(3-x)^3$$

L 1-5
Assignment = next class

$$y \left\{ \frac{y^2}{2} \cdot \frac{1}{125} \right\} = xb \cdot y \left(\frac{1}{125} \right) =$$

$$\frac{y^3}{6} \cdot \frac{1}{125} =$$

$$=$$

$$e) f(x|y) = \frac{f(x,y)}{h(y)} = \frac{f(x,y)}{h(y)} = \frac{\frac{1}{125} (3-x) \cdot 5}{5/10} = \frac{2}{25} (3-x); -2 \leq x \leq 3$$

$$f) E(x) = \int_{-2}^3 x g(x) dx$$

$$= \int_{-2}^3 x \cdot \frac{2}{25} (3-x) dx$$

$$= \frac{2}{25} \int_{-2}^3 (3x-x^2) dx = (x^2 - \frac{x^3}{3}) \Big|_{-2}^3$$

$$= \frac{2}{25} \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_{-2}^3$$

$$= \frac{2}{25} \cdot \frac{-25}{6} = -\frac{1}{3}$$

$$\sigma^2(x) = E(x^2) - [E(x)]^2$$

$$= \frac{3}{2} - \left(\frac{-1}{3} \right)^2 = \frac{3}{2} - \frac{1}{9} = \frac{25}{18}$$

$$E(\tilde{x}) = \int_{-2}^3 \tilde{x} g(x) dx$$

$$= \frac{2}{25} \int_{-2}^3 \left[\frac{3x^3}{3} - \frac{x^4}{4} \right]_{-2}^3 dx$$

$$= \frac{2}{25} \cdot \frac{75}{4} = \frac{3}{2}$$

$$E(Y) = \int y h(y) dy$$

$$= \int_0^4 \frac{y}{10} dy$$

$$= \left[\frac{1}{30} y^3 \right]_0^4$$

$$= \frac{76}{15}$$

$$E(Y^2) = \int y^2 \cdot \frac{y}{10} dy$$

$$= \frac{1}{40} \cdot y^4 \Big|_0^4$$

$$= 2^4$$

$$\therefore V(Y) = 2^6 - \left(\frac{76}{15} \right)^2 = \frac{74}{225} = 0.329$$

g) $\text{cov}(XY) = E(XY) - E(X)E(Y)$

$$E(XY) = \iint xy f(xy) dx dy$$

$$= \int_0^4 \int_{-x}^3 xy \cdot \frac{1}{125} (3-x)y dx dy$$

$$= \frac{1}{125} \int_0^4 y \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_{-x}^3 dy$$

$$= -\frac{1}{30} \int_{-4}^6 y^2 dy = -\frac{1}{30} \left[\frac{y^3}{3} \right]_{-4}^6 = -\frac{76}{45}$$

$$\therefore \text{Cov}(xy) = E(xy) - E(x)E(y)$$

$$= -\frac{76}{45} + \frac{76}{45} = 0$$

h)

$$\text{Corr}(x,y) = \frac{0}{...} = 0$$



if, x & y are independent then,

$$E(xy) = E(x)E(y)$$

$$\text{Cov}(xy) = 0$$

$$\text{Corr}(xy) = 0$$

	x	y
$E(x)$	1.0	2.0
$E(y)$	0.25	0.35
$E(xy)$	0.1	0.3

\rightarrow x to solve below

$$(x)q \cdot x \bar{z} = (x)\bar{z}$$

$$25.0 \cdot p + 95.0 \cdot q + 15.0 \cdot r + 15.0 =$$

$$82.5 =$$

$$(82.5) - 82.5 \rightarrow [(x)\bar{z}] - (x)\bar{z} = (x)\bar{v}$$

L-10/27.08.2023/

Quiz-1

$$\frac{1}{2P} + \frac{25}{2P} = ?$$

L-11/29.08.2023/

✳ marginal distribution of X $= (r_{xx})_{m \times n}$

x	1	2	3	4	$E(X) = E(r_{xx})$
$P(x)$	0.21	0.24	0.30	0.25	

✳ marginal distribution of Y $= (r_{yy})_{n \times m}$

y	1	2	3	$E(Y) = E(r_{yy})$
$P(y)$	0.32	0.57	0.11	

✳ Expected value of X :

$$E(x) = \sum x \cdot P(x)$$

$$= 0.21 + 2 \cdot 0.24 + 3 \cdot 0.30 + 4 \cdot 0.25$$

$$= 2.59$$

$$\begin{aligned} V(x) &= E(x^2) - [E(x)]^2 = 7.87 - (2.59)^2 \\ &= 1.1619 \end{aligned}$$

$$E(\tilde{n}) = \sum n \tilde{P}(n)$$

$$= 1 \cdot 0.21 + 2 \cdot 0.24 + 3 \cdot 0.30 + 4 \cdot 0.25 \\ = 7.87$$

⊕ $P[x | Y=3] = \frac{P(x, 3)}{P(Y=3)}$

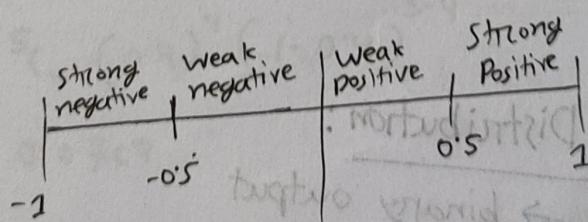
⊕ $P[x=1 | Y=3] = \frac{P(1, 3)}{P(Y=3)} = \frac{0.01}{0.11} = 0.091$

⊕ The conditional distribution of the service time X

X	1	2	3	4
$P(x Y=3)$	0.091	0.091	0.182	0.636

$$E(x, Y) = \sum x \cdot Y \cdot P(x, Y)$$

$$= 4.86$$



L-12/03.09.2023/

Like binary, two outcome, success and failure

Bernoulli Distribution:

$$x = 1, 0$$

$$P(x=1) = P \quad \left. \begin{array}{l} \text{total} = 1 \\ 1-p+P = 1 \end{array} \right\}$$

$$P(x=0) = 1-P \quad \left. \begin{array}{l} \text{total} = 1 \\ 1-p = p \end{array} \right\}$$

$$\frac{(e,x)q}{(e-y)q} = [e=y|x]q$$

$$f(x) = p^x (1-p)^{1-x}; x = 0, 1$$

$$x=0, f(0) = p^0 (1-p)^1$$

$$= 1-p \quad \left. \begin{array}{l} \text{total} = 1-p+p \\ = 1 \end{array} \right\}$$

$$f(1) = p(1-p)^0$$

$$(r,x)q \cdot r \cdot x = (r,x)$$

$$E(x) = p$$

$$\nu(x) = p(1-p)$$

Binomial Distribution:

binary output

Toss a coin in n times

~~successes~~ in n trials.

$$x = 0, 1, 2, \dots, n$$

Discrete random variable.

The probability distribution of x is called Binomial Distribution.

$$f(x; n, p) = \begin{cases} nC_x p^x (1-p)^{n-x} & ; x = 0, 1, 2, \dots, n \\ 0 & ; \text{otherwise} \end{cases}$$

$$E(x) = np$$

$$\sigma^2(x) = np(1-p)$$



$$P(A) = 0.4$$

Baby born, $n=5$

$$P(B) = 0.6$$

Probability of 4th child?

$$P(x=4) = {}^5C_4 (0.4)^4 (1-0.4)^{5-4} = (x=4)q$$

$$= 0.0768$$

at least 2

$$P(x \geq 2) = P(x=2) + P(x=3) + P(x=4) + P(x=5)$$

$$= 0.3456 + 0.2304 + 0.0768 + 0.01024$$

$$= 0.66304$$

$$= 1 - P(x=0) - P(x=1)$$

Voriderm™ IV Injection
Voriconazole 200 mg

L-13 / 05.09.2023

*) Slide Question:

a) $P(X=x) = {}^{20}C_x p^x (1-p)^{20-x}$; $x = 0, 1, 2, \dots, 20$

(simultaneo. ballot 20 x 20 mögliche Möglichkeiten der Anordnung der 20 Bälle)

$$= {}^{20}C_x (0.261)^x (1-0.261)^{20-x}$$

b) $E(X) = np = 20 \cdot 0.261 = 5.22$

$$\sigma^2(X) = np(1-p) = 5 \cdot 2 (1-0.261) = 3.857 \quad \left\{ \begin{array}{l} (q-1)^n \\ (q-1)^n \end{array} \right\} = (q, n, x)^2$$

c) $P(X=7) = {}^{20}C_7 (0.261)^7 (1-0.261)^{20-7}$

$$= 0.125 \quad q^n = (x) E$$

d) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

e) $P(X \geq 2) = 1 - P(X=0) - P(X=1)$

*) Geometric Distribution:

$$P(X=x) = (1-p)^{x-1} p; \quad x=1, 2, 3, \dots$$

$$(E(X)) = \frac{1}{p}$$

$$V(X) = \frac{1-p}{p^2} = 0.324 \quad = (x \leq x) q$$

$$(1-p)q - (0=x)q - 1$$

Bennorali

Binomial Distribution

$$P(X=x) = {}^n C_{x-1} (1-p)^{n-x} p^x$$

$$E(n) = \frac{n}{p}$$

$$\sigma(n) = \sqrt{\frac{n(1-p)}{p^2}}$$

L-14 / 10.09.2023

Normal Distribution

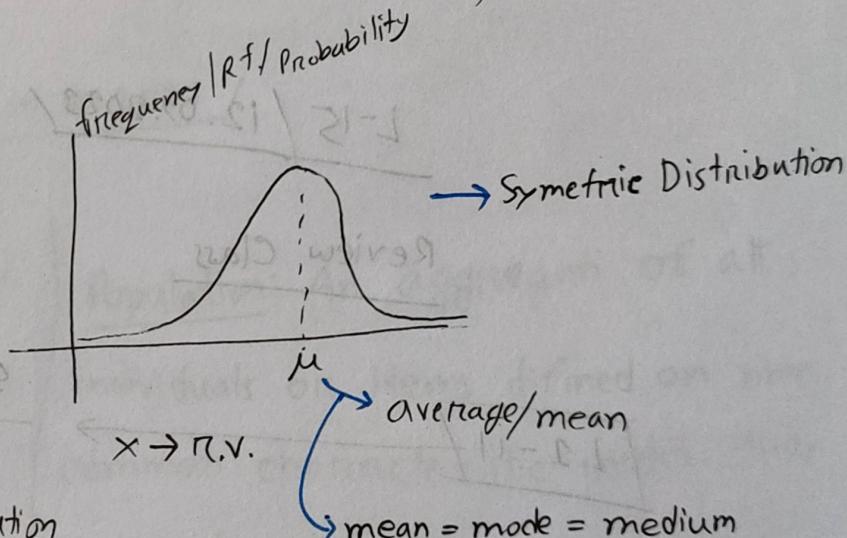
$$f(x; \mu, \sigma^2) = f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\left. \begin{array}{l} -\infty < n < \infty \\ -\infty < \mu < \infty \\ 0 < \sigma^2 < \infty \end{array} \right\}$$

$$E(x) = \mu$$

$$\sigma(x) = \sigma$$

$$SD = \sigma$$



Standard normal distribution

$$\mu = 0$$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$$E(z) = 0$$

$$Z = \frac{x-\mu}{\sigma}$$

$$\sigma(z) = 1$$

Example

$$\text{i) } P(X < 10.5)$$

$$= P\left(\frac{x-\mu}{\sigma} < \frac{10.5-\mu}{\sigma}\right)$$

$$= P\left[Z < \frac{10.5-\mu}{\sigma}\right]$$

$$= P[Z < -1.67]$$

$$= F(-1.67)$$

$$= 0.0475$$

$$\mu > \bar{x} > \mu - \sigma$$

$$\mu - \sigma > \bar{x} > \mu - 2\sigma$$

$$\mu - 2\sigma > \bar{x} > \mu - 3\sigma$$

$$\text{ii) } P[10 < x < 12]$$

$$= P\left[\frac{10-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{12-\mu}{\sigma}\right]$$

$$= P\left[\frac{10-\mu}{\sigma} < Z < \frac{12-\mu}{\sigma}\right]$$

$$= P[-3.33 < Z < 3.33]$$

$$= F(3.33) - F(-3.33)$$

$$= P(Z < 3.33) - P(Z < -3.33)$$

$$= 0.9996 - 0.0004$$

$$= 0.99917 = 0.99917 = (0.4)(x)^2$$

L-15 / 12.09.2023 /

Review Class

L1-11

5 out of 6
70 minutes

All Tent Type

Example

$$\text{i) } P[X > 3.2]$$

$$= P\left[\frac{x-\mu}{\sigma} > \frac{3.2-\mu}{\sigma}\right]$$

$$= P\left[Z > \frac{3.2-\mu}{\sigma}\right]$$

$$= P[Z > 1.67] \Rightarrow (z)^2$$

$$= 1 - P[Z < 1.67]$$

$$= 1 - 0.9525$$

$$= 0.047$$

Question

ii) $P[n < 2.7]$

$$= P[Z < -2.5]$$

$$= P[Z > 2.5]$$

$$= 1 - P[Z < 2.5]$$

$$= 1 - 0.9938$$

$$= 0.062$$

$\textcircled{B} p(A \cup B) = P(A) + P(B) - P(A \cap B)$

L-16 / 17. 09. 2023 /

Midterm Exam

L-17 / 17. 09. 2023 /

⊗ Descriptive Statistic:

⇒ Population

⇒ Sample

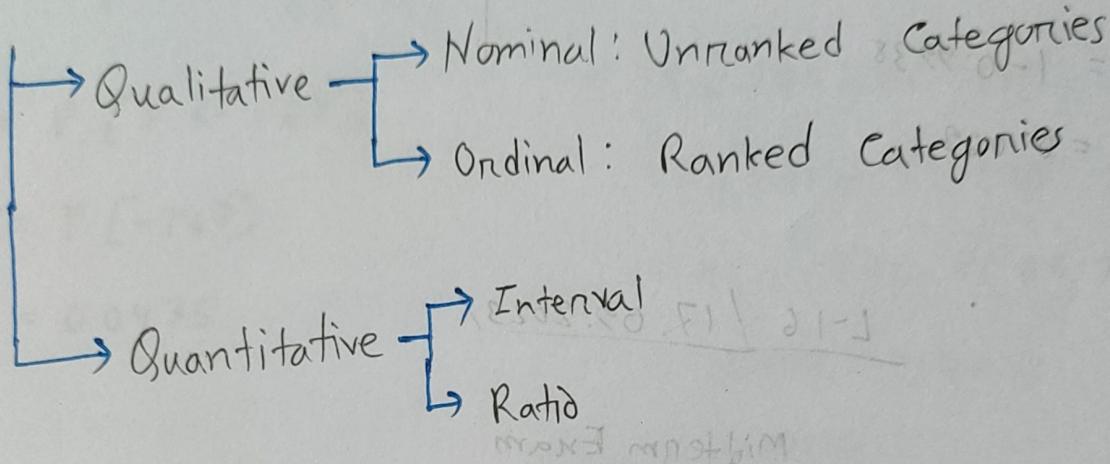
⇒ Data & Variable

⇒ Central Tendency

⇒ Dispersion

Population: An aggregation of all individuals or items defined on some common characteristic under study is called population.

Sample: A subset of population i.e. representative part of population is known as sample.

~~Scales of~~Scales of MeasurementProblem from SlidePractice Exercise \Rightarrow OrdinalCollege Major \Rightarrow NominalReaction time \Rightarrow RatioBlood glucose level \Rightarrow RatioTemperature (Low, Medium, High) \Rightarrow OrdinalTemperature \Rightarrow IntervalMonthly Income \Rightarrow RatioGender \Rightarrow NominalNumber of days absent in class \Rightarrow RatioAge \Rightarrow RatioSex \Rightarrow NominalDate of birth \Rightarrow RatioCountry \Rightarrow NominalDisease \Rightarrow Nominal

Journal Problem from Slide

- ① Continuous, Ratio
- ② Qualitative, Nominal
- ③ Qualitative, Ordinal
- ④ Continuous, Ratio
- ⑤ Qualitative, Nominal

Frequency Distribution

Distribution of data in a table, easy to observe data.

Construction of table:

Step-1: Number of classes,

$$k = 1 + 3.322 \log_{10} N$$

keep, $5 \leq k \leq 10$

Step-2:

Determine interval, $\frac{H - L}{k}$ → Highest

$$i \geq \frac{H - L}{k} \rightarrow \text{Lowest}$$