NORTH SOUTH UNIVERSITY

Department of Mathematics & Physics

Assignment – 03

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13.6

1|

Given,

$$f(x,y) = (1+xy)^{3/2}$$

$$P(3,1)$$

$$\vec{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

Gradiant of $f(x,y)$,

$$\nabla f(x,y) = \left\langle \frac{3}{2}(1+xy)^{\frac{1}{2}} \cdot y, \frac{3}{2}(1+xy)^{\frac{1}{2}} \cdot x \right\rangle$$

$$= \left\langle \frac{3y}{2}\sqrt{1+xy}, \frac{3x}{2}\sqrt{1+xy} \right\rangle$$

$$\therefore \nabla f(3,1) = \langle 3, 9 \rangle$$

$$\therefore D_u f = \nabla f \cdot \vec{u} = \langle 3, 9 \rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$= \frac{3}{\sqrt{2}} + \frac{9}{\sqrt{2}}$$

$$= 6\sqrt{2}$$

Ans.

3|

Given that,

$$f(x,y) = \ln(1+x^2+y)$$

$$p(0,0)$$

$$\vec{u} = \langle -\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}} \rangle$$

Gradiant of $f(x,y)$,

$$\nabla f(x,y) = \langle \frac{1}{1+x^2+y} \cdot 2x , \frac{1}{1+x^2+y} \cdot 1 \rangle$$

$$= \langle \frac{2x}{1+x^2+y} , \frac{1}{1+x^2+y} \rangle$$

$$\therefore \nabla f(0,0) = \langle 0,1 \rangle$$

$$\therefore D_u f = \nabla f \cdot \vec{u}$$

$$= \langle 0,1 \rangle \cdot \langle -\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}} \rangle$$

$$= -\frac{3}{\sqrt{10}}$$

Ans

5)

Given that,

$$f(x,y,z) = 4x^5y^2z^3$$

$$P(2,-1,1)$$

$$\vec{u} = \langle \frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \rangle$$

Gradiant of f (x, y, z).

$$\nabla f(x,y,z) = \langle 20x^4y^2z^3, \ 8x^5yz^3, \ 12x^5y^2z^2 \rangle$$

$$\therefore \nabla f(2,-1,1) = \langle 320, -256, 384 \rangle$$

$$\therefore D_uf = \nabla f \cdot \vec{u}$$

$$= \langle 320, -256, 384 \rangle \cdot \langle \frac{1}{3}, \frac{2}{3}, \frac{-2}{3} \rangle$$

$$= \frac{320}{3} - \frac{256\times 2}{3} - \frac{384\times 2}{3}$$

$$= -320$$

Ans

7)

Given that,

$$f(x,y,z) = \ln(x^2 + 2y^2 + 3z^2)$$

$$P(-1, 2, 4)$$

$$\vec{u} = \left\langle -\frac{3}{13}, -\frac{4}{13}, -\frac{12}{13} \right\rangle$$

Gradiant of $f(x,y,z)$,

$$\nabla f(x,y,z) = \left\langle \frac{2x}{x^2+2y^2+3z^2}, \frac{4y}{x^2+2y^2+3z^2}, \frac{6z}{x^2+2y^2+3z^2} \right\rangle$$

$$\therefore \nabla f(-1,2,4) = \left\langle -\frac{2}{57}, \frac{8}{57}, \frac{8}{19} \right\rangle$$

$$\therefore D_u f = \nabla f \cdot \vec{u}$$

$$= \left\langle -\frac{2}{57}, \frac{8}{57}, \frac{8}{19} \right\rangle \cdot \left\langle -\frac{3}{13}, -\frac{4}{13}, -\frac{12}{13} \right\rangle$$

$$= \frac{2\times 3}{57\times 13} - \frac{8\times 4}{57\times 13} - \frac{8\times 12}{19\times 13}$$

$$= -\frac{314}{741}$$ Ans

8)

Given that,

$$f(x,y,z) = \sin(xyz)$$

$$P\left(\frac{1}{2}, \frac{1}{3}, \pi\right)$$

$$\vec{u} = \left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$$

Gradiant of $f(x,y,z)$,

$$\nabla f(x,y,z) = \langle yz.\cos(xyz), \; xz.\cos(xyz), \; xy\cdot\cos(xyz) \rangle$$

$$\therefore \nabla f\left(\frac{1}{2}, \frac{1}{3}, \pi\right) = \left\langle \frac{\pi\sqrt{3}}{6}, \frac{\pi\sqrt{3}}{4}, \frac{\sqrt{3}}{12} \right\rangle$$

$$\therefore D_u f = \nabla f \cdot \vec{u}$$

$$= \left\langle \frac{\pi\sqrt{3}}{6}, \frac{\pi\sqrt{3}}{4}, \frac{\sqrt{3}}{12} \right\rangle \cdot \left\langle \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$$

$$= \frac{\pi\sqrt{3}}{6\cdot\sqrt{3}} - \frac{\pi\sqrt{3}}{4\cdot\sqrt{3}} + \frac{\sqrt{3}}{12\cdot\sqrt{3}}$$

$$= \frac{\pi}{6} - \frac{\pi}{4} + \frac{1}{12}$$

$$= \frac{2\pi - 3\pi + 1}{12}$$

$$= \frac{1-\pi}{12}$$ Ans

11)

Given that,

$$f(x,y) = y^2 \ln x$$

$$P(1,4)$$

$$\vec{a} = \langle -3, 3 \rangle$$

$$\therefore \vec{u} = \frac{\vec{a}}{\|\vec{a}\|} = \left\langle \frac{-3}{3\sqrt{2}}, \frac{3}{3\sqrt{2}} \right\rangle$$

$$= \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

Gradient of $f(x,y)$,

$$\nabla f(x,y) = \left\langle y^2 \cdot \frac{1}{x}, 2y \ln x \right\rangle$$

$$= \left\langle \frac{y^2}{x}, 2y \ln x \right\rangle$$

$$\therefore \nabla f(1,4) = \langle 16, 0 \rangle$$

$$\therefore D_u f = \nabla f \cdot \vec{u}$$

$$= \langle 16, 0 \rangle \cdot \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$= \frac{-16}{\sqrt{2}}$$

$$= -8\sqrt{2}$$

Ans

12/

Given that,

$$f(x,y) = e^{x}\cos y$$

$$P(0, \frac{\pi}{4})$$

$$\vec{a} = \langle 5, -2 \rangle$$

$$\therefore \vec{u} = \langle \frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}} \rangle$$

$\therefore$ Gradient of $f(x,y)$,

$$\nabla f(x,y) = \langle e^{x}\cos y, -e^{x}\sin y \rangle$$

$$\therefore \nabla f(0, \frac{\pi}{4}) = \langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle$$

$$\therefore D_u f = \nabla f \cdot \vec{u}$$

$$= \langle \frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2} \rangle \cdot \langle \frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}} \rangle$$

$$= \frac{5\sqrt{58}}{58} + \frac{\sqrt{58}}{29}$$

$$= \frac{7\sqrt{58}}{58}$$

$$= \frac{7}{\sqrt{58}} \text{ Ans}$$

13)

Given that,

$$f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$$

$$P(-2,2)$$

$$\vec{a} = \langle -1, -1 \rangle$$

$$\therefore \vec{u} = \left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$$

Gradient of $f(x,y)$,

$$\nabla f(x,y) = \left\langle \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot y\left(-\frac{1}{x^2}\right), \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} \right\rangle$$

$$= \left\langle \frac{-yx^{-2}}{1+\frac{y^2}{x^2}}, \frac{x^{-1}}{1+\frac{y^2}{x^2}} \right\rangle$$

$$\therefore \nabla f(-2,2) = \left\langle -\frac{1}{4}, -\frac{1}{4} \right\rangle$$

$$\therefore D_u f = \nabla f \cdot \vec{u}$$

$$= \left\langle -\frac{1}{4}, -\frac{1}{4} \right\rangle \cdot \left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$$

$$= \frac{\sqrt{2}}{8} + \frac{\sqrt{2}}{8}$$

$$= \frac{\sqrt{2}}{4} \text{ Ans}$$

16)

Given that,

$$f(x,y,z) = y - \sqrt{x^2+z^2}$$

$$P(-3,1,4)$$

$$\vec{a} = \langle 2, -2, -1 \rangle$$

$$\therefore \vec{u} = \left\langle \frac{2}{3}, \frac{-2}{3}, \frac{-1}{3} \right\rangle$$

$\therefore$ Gradient of $f(x,y,z)$

$$\nabla f(x,y,z) = \left\langle -\frac{1}{2\sqrt{x^2+z^2}} \cdot 2x ,\ 1 ,\ -\frac{1}{2\sqrt{x^2+z^2}} \cdot 2z \right\rangle$$

$$= \left\langle -\frac{x}{\sqrt{x^2+z^2}} ,\ 1 ,\ \frac{-z}{\sqrt{x^2+z^2}} \right\rangle$$

$$\cancel{\therefore \nabla f(2,-2,-1) = \left\langle -\frac{2\sqrt{5}}{5},\ 1,\ \frac{\sqrt{5}}{5} \right\rangle}$$

$$\cancel{\therefore D_u f = \nabla f \cdot \vec{u}}$$

$$\cancel{= \left\langle -\frac{2\sqrt{5}}{5},\ 1,\ \frac{\sqrt{5}}{5} \right\rangle \cdot \left\langle \frac{2}{3}, \frac{-2}{3}, \frac{-1}{3} \right\rangle}$$

$$\cancel{= -\frac{4\sqrt{5}}{15} + \frac{2}{3} - \frac{\sqrt{5}}{15}}$$

$\pm$

$$\therefore \nabla f(-3,1,4) = \left\langle \frac{3}{5},\ 1,\ -\frac{4}{5} \right\rangle$$

$\therefore D_u f = \nabla f \cdot \vec{u}$

$= \langle \frac{3}{5}, 1, \frac{-4}{5} \rangle \cdot \langle \frac{2}{3}, \frac{-2}{3}, \frac{-1}{3} \rangle$

$= \frac{2}{5} - \frac{2}{3} + \frac{4}{15}$

$= 0$

Ans

17

Given that,

$$f(x, y, z) = \frac{z - x}{z + y}$$

$P(1, 0, -3)$

$\vec{a} = \langle -6, 3, -2 \rangle$

$\therefore \vec{u} = \langle \frac{-6}{7}, \frac{3}{7}, \frac{-2}{7} \rangle$

$\therefore$ Gradient of $f(x, y, z)$,

$$\nabla f(x, y, z) = \langle \frac{-1}{z+y}, -\frac{z-x}{(z+y)^2}, \frac{(z+y)\cdot 1 - (z-x)\cdot 1}{(z+y)^2} \rangle$$

$$= \langle -\frac{1}{z+y}, \frac{-(z-x)}{(z+y)^2}, \frac{y+x}{(z+y)^2} \rangle$$

$\therefore \nabla f(1, 0, -3) = \langle \frac{1}{3}, \frac{4}{9}, \frac{1}{9} \rangle$

$\therefore D_u f = \nabla f \cdot \vec{u} = \langle \frac{1}{3}, \frac{4}{9}, \frac{1}{9} \rangle \cdot \langle \frac{-6}{7}, \frac{3}{7}, \frac{-2}{7} \rangle$

$$= -\frac{2}{7} + \frac{4}{21} - \frac{2}{63}$$

$$= -\frac{8}{63} \quad \underline{An}$$

<u>18</u>|

Given that,

$$f(x,y,z) = e^{x+y+3z}$$

$$p(-2,2,-1)$$

$$\vec{a} = \langle 20, -4, 5 \rangle$$

$$\therefore \vec{u} = \left\langle \frac{20}{21}, \frac{-4}{21}, \frac{5}{21} \right\rangle$$

Gradient of $f(x,y,z)$,

$$\nabla f(x,y,z) = \langle e^{x+y+3z}, e^{x+y+3z}, 3e^{x+y+3z} \rangle$$

$$\therefore \nabla f(-2,2,-1) = \langle e^{-3}, e^{-3}, 3e^{-3} \rangle$$

$$\therefore D_u f = \nabla f \cdot \vec{u}$$

$$= \langle e^{-3}, e^{-3}, 3e^{-3} \rangle \cdot \left\langle \frac{20}{21}, \frac{-4}{21}, \frac{5}{21} \right\rangle$$

$$= \frac{20e^{-3}}{21} - \frac{4}{21}e^{-3} + \frac{15}{21}e^{-3}$$

$$= \frac{31}{21}e^{-3}$$

$\underline{An}$

22/

Given that,

$$f(x,y) = \sinh x \cosh y$$

$$P(0,0)$$

$$\theta = \pi$$

$$\therefore \vec{u} = \langle \cos\pi, \sin\pi \rangle$$

$$= \langle -1, 0 \rangle$$

Gradient of $f(x,y)$,

$$\nabla f(x,y) = \langle \cosh x \cosh y, \quad \sinh x \cancel{\cosh y} \sinh y \rangle$$

$$\therefore \nabla f(0,0) = \langle 1, 0 \rangle$$

$$\therefore D_u f = \nabla f \cdot \vec{u}$$

$$= \langle 1, 0 \rangle \cdot \langle -1, 0 \rangle$$

$$= -1$$

Ans

23/

Given that,

$$f(x,y) = \frac{x}{x+y}$$

$P(1,0)$

$Q(-1,-1)$

$\therefore$ ~~$\overrightarrow{PQ} =$~~

$\therefore \overrightarrow{PQ} = \langle -1-1, -1-0 \rangle$

$= \langle -2, -1 \rangle$

$\therefore \vec{u} = \langle \frac{-2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \rangle$

Gradient of $f(x,y)$,

$$\nabla f(x,y) = \langle \frac{x+y-x}{(x+y)^2}, -\frac{x}{(x+y)^2} \rangle$$

$$= \langle \frac{y}{(x+y)^2}, \frac{-x}{(x+y)^2} \rangle$$

$\therefore \nabla f(1,0) = \langle 0, -1 \rangle$

$\therefore D_u f = \nabla f \cdot \vec{u}$

$= \langle 0,-1 \rangle \cdot \langle \frac{-2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \rangle$

$= 0 + \frac{1}{\sqrt{5}}$

$= \frac{1}{\sqrt{5}}$ Ans

24/

Given that,

$$f(x,y) = e^{-x} \sec y$$

$$P(0, \frac{\pi}{4})$$

Direction of the origin, $(0,0)$

$$\therefore \vec{PO} = \langle 0-0, 0-\frac{\pi}{4} \rangle$$

$$= \langle 0, -\frac{\pi}{4} \rangle$$

$$\therefore \vec{u} = \langle 0, -1 \rangle$$

Gradient of $f(x,y)$,

$$\nabla f(x,y) = \langle -e^{-x} \sec y, \ e^{-x} \sec y \tan y \rangle$$

$$\therefore \nabla f(0, \frac{\pi}{4}) = \langle -\sqrt{2}, \sqrt{2} \rangle$$

$$\therefore D_u f = \nabla f \cdot \vec{u}$$

$$= \langle -\sqrt{2}, \sqrt{2} \rangle \cdot \langle 0, -1 \rangle$$

$$= -\sqrt{2}$$

Ans

25)

Given that,

$$f(x,y) = \sqrt{xy}\, e^{y}$$

$$P(1,1)$$

Direction of the negative y-axis, $\vec{u} = \langle 0, -1 \rangle$

Gradient of $f(x,y)$,

$$\nabla f(x,y) = \left\langle \frac{e^{y} \cdot y}{2\sqrt{xy}} , \quad \sqrt{xy} \cdot e^{y} + \frac{e^{y} \cdot x}{2\sqrt{xy}} \right\rangle$$

$$\therefore \nabla f(1,1) = \left\langle \frac{e}{2} , \quad e + \frac{e}{2} \right\rangle$$

$$= \left\langle \frac{e}{2} , \quad \frac{3e}{2} \right\rangle$$

$$\therefore D_u f = \nabla f \cdot \vec{u}$$

$$= \left\langle \frac{e}{2} , \frac{3e}{2} \right\rangle \cdot \langle 0, -1 \rangle$$

$$= -\frac{3}{2} e$$

Ans

26|

Given that,

$$f(x,y) = \frac{y}{x+y}$$

$$D_u f(2,3) = 0$$

Gradient of $f(x,y)$,

$$\nabla f(x,y) = \left\langle -\frac{y}{(x+y)^2} \, , \, \frac{x+y-y}{(x+y)^2} \right\rangle$$

$$= \left\langle \frac{-y}{(x+y)^2} \, , \, \frac{x}{(x+y)^2} \right\rangle$$

$$\therefore \nabla f(2,3) = \left\langle \frac{-3}{25} \, , \, \frac{2}{25} \right\rangle$$

Here,

$$D_u f = 0$$

then,

$\vec{u}$ and ∇f are orthogonal.

by inspection $\langle 2,3 \rangle$ is orthogonal to $\nabla f(2,3)$

So, $\vec{u} = \left\langle \frac{\pm 2}{\sqrt{13}} \, , \, \frac{\pm 3}{\sqrt{13}} \right\rangle$

A

33/

Given that,

$$z = \sin(7y^2 - 7xy)$$

Gradient of z,

$$\nabla z = \langle \cos(7y^2 - 7xy)\cdot(-7y),\quad \cos(7y^2 - 7xy)\,(14y - 7x) \rangle$$

$$= \langle -7y\cos(7y^2 - 7xy),\ (14y - 7x)\cos(7y^2 - 7xy) \rangle$$

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Given that,

$$z = \frac{6x + 7y}{6x - 7y}$$

Gradient of z,

$$\nabla z = \left\langle \frac{(6x-7y)6 - (6x+7y)6}{(6x-7y)^2},\ \frac{(6x-7y)7 - (6x+7y)(-7)}{(6x-7y)^2} \right\rangle$$

$$= \left\langle \frac{36x - 42y - 36x - 42y}{(6x-7y)^2},\ \frac{42x - 49y + 42x + 49y}{(6x-7y)^2} \right\rangle$$

$$= \left\langle \frac{-84y}{(6x-7y)^2},\ \frac{84x}{(6x-7y)^2} \right\rangle$$

42/

Given that,

$$f(x,y) = 5\sin x^2 + \cos 3y$$

point, $(\sqrt{\pi/2}, 0)$

Gradient of $f(x,y)$,

$$\nabla f(x,y) = \langle 5\cos x^2 \cdot 2x \;,\; -\sin 3y \cdot 3 \rangle$$

$$= \langle 10x\cos x^2, -3\sin 3y \rangle$$

$$\therefore \nabla f(\sqrt{\pi/2}, 0) = \langle 5\sqrt{\pi} \cdot \frac{\sqrt{2}}{2}, 0 \rangle$$

$$= \langle 5\sqrt{\frac{\pi}{2}}, 0 \rangle$$

Ans

44/

Given that,

$$f(x,y) = (x^2+y^2)^{-1/2}$$

point, $(3,4)$

Gradient of $f(x,y)$,

$$\nabla f(x,y) = \left\langle -\frac{1}{2}\left(x^2+y^2\right)^{-3/2} \cdot 2x \, , \; -\frac{1}{2}\left(x^2+y^2\right)^{-3/2} \cdot 2y \right\rangle$$

$$= \left\langle -x\left(x^2+y^2\right)^{-3/2} \, , \; -y\left(x^2+y^2\right)^{-3/2} \right\rangle$$

$$\therefore \nabla f(3,4) = \left\langle -\frac{3}{125} \, , \; \frac{-4}{125} \right\rangle$$

Ans