For $n=3$,

$$Y_4 = Y_3 + h\left(x_3^2 + Y_3^2\right) = 1.3753284 + 0.1\left((0.3)^2 + (1.3753284)^2\right)$$

$$= 1.573481221$$

For $n=4$,

$$Y_5 = Y_4 + h\left(x_4^2 + Y_4^2\right) = 1.573481221 + 0.1\left((0.4)^2 + (1.573481221)^2\right)$$

$$= 1.837065536$$

R-K Method,

we know,

$$Y_{n+1} = Y_n + \frac{1}{2}(k_1 + k_2)$$

$$k_1 = hf(x_n, Y_n)$$

$$k_2 = hf(x_n + h, Y_n + k_1)$$

For $n=0$,

$$k_1 = hf(x_0, Y_0) = 0.1\left((0)^2 + (1)^2\right)$$

$$= 0.1$$

$$k_2 = hf(x_0 + h, Y_0 + k_1) = 0.1\left((0+0.1)^2 + (1+0.1)^2\right)$$

$$= 0.122$$

$$\therefore Y_1 = Y_0 + \frac{1}{2}(k_1 + k_2) = 1 + \frac{1}{2}(0.1 + 0.122)$$

$$= 1.111$$

For $n=1$,

$$k_1 = hf(x_1, y_1) = 0.1\left((0.1)^2 + (1.111)^2\right)$$

$$= 0.1244321$$

$$k_2 = hf(x_1+h, y_1+k_1) = 0.1\left((0.1+0.1)^2 + (1.111+0.1244321)^2\right)$$

$$= 0.1566292474$$

$$\therefore y_2 = y_1 + \frac{1}{2}(k_1+k_2) = 1.111 + \frac{1}{2}(0.1244321 + 0.1566292474)$$

$$= 1.251530674$$

For $n=2$,

$$k_1 = hf(x_2, y_2) = 0.1\left((0.2)^2 + (1.251530674)^2\right) = 0.1606329027$$

$$k_2 = hf(x_2+h, y_2+k_1) = 0.1\left((0.2+0.1)^2 + (1.251530674 + 0.1606329027)^2\right)$$

$$= 0.2084205967$$

$$\therefore y_3 = y_2 + \frac{1}{2}(k_1+k_2) = 1.251530674 + \frac{1}{2}(0.1606329027 + 0.2084205967)$$

$$= 1.436057423$$

For $n=3$,

$$k_1 = hf(x_3, y_3) = 0.1\left((0.3)^2 + (1.436057423)^2\right)$$

$$= 0.2152260923$$

$$k_2 = hf(x_3+h, y_3+k_1) = 0.1\left((0.3+0.1)^2 + (1.436057423 + 0.2152260923)^2\right)$$

$$= 0.2886737249$$

$$y_4 = y_3 + \frac{1}{2}(k_1+k_2) = 1.436057423 + \frac{1}{2}(0.2152260923 + 0.2886737249)$$

$$= 1.688007332$$

For $n=4$,

$$k_1 = hf(x_4, y_4) = 0.1\left((0.4)^2 + (1.688007332)^2\right)$$

$$= 0.3009368753$$

$$k_2 = hf(x_4+h,\ y_4+k_1) = 0.1\left((0.4+0.1)^2 + (1.688007332+0.3009368753)^2\right)$$

$$= 0.420589906$$

$$\therefore y_5 = y_4 + \tfrac{1}{2}(k_1+k_2) = 1.688007332 + \tfrac{1}{2}(0.3009368753 + 0.420589906)$$

$$= 2.048770723$$

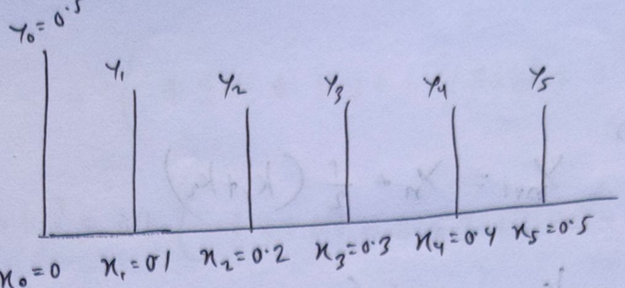
| n | y | Euler | R-k |
|---|---|---|---|
| 0 | $y_1$ | 1.1 | 1.111 |
| 1 | $y_2$ | 1.222 | 1.2515 |
| 2 | $y_3$ | 1.3753 | 1.4361 |
| 3 | $y_4$ | 1.5735 | 1.6880 |
| 4 | $y_5$ | 1.8371 | 2.0488 |

7]

$y' = (x-y)^2$

$y(0) = 0.5$

$y(0.5) = ?$




## Eulen Method

we know,

$$y_{n+1} = y_n + hf(x_n, y_n) \quad \cdots (i)$$

$n = 0, 1, 2, 3, \cdots$

For $n=0$,

$$y_1 = y_0 + hf(x_0, y_0) = 0.5 + 0.1(0 - 0.5)^2 = 0.525$$

Fon $n=1$

$$y_2 = y_1 + hf(x_1, y_1) = 0.525 + 0.1(0.1 - 0.525)^2$$

$$= 0.5430625$$

Fon $n=2$

$$y_3 = y_2 + hf(x_2, y_2) = 0.5430625 + 0.1(0.2 - 0.5430625)^2$$

$$= 0.5548316879$$

Fon $n=3$,

$$y_4 = y_3 + hf(x_3, y_3) = 0.5548316879 + 0.1(0.3 - 0.5548316879)^2$$

$$= 0.5613256068$$

Fon $n=4$,

$$y_5 = y_4 + hf(x_4, y_4) = 0.5613256068 + 0.1(0.4 - 0.5613256068)^2$$

$$= 0.5639282019$$

R-K Method

we knows
$$y_{n+1} = y_n + \frac{1}{2}(k_1+k_2)$$
$$k_1 = hf(x_n, y_n)$$
$$k_2 = hf(x_n+h, y_n+k_1)$$

For $n=0$,
$$k_1 = hf(x_0, y_0) = 0.1(0-0.5)^2 = 0.025$$
$$k_2 = hf(x_0+h, y_0+k_1) = 0.1((0+0.1)-(0.5+0.025))^2$$
$$= 0.0180625$$
$$\therefore y_1 = y_0 + \frac{1}{2}(k_1+k_2) = 0.5 + \frac{1}{2}(0.025+0.0180625)$$
$$= 0.52153125$$

For $n=1$,
$$k_1 = hf(x_1, y_1) = 0.1(0.1-0.52153125)^2 = 0.01776885947$$
$$k_2 = hf(x_1+h, y_1+k_1) = 0.1(0.1+0.1-0.52153125-0.01776885947)^2$$
$$= 0.01151245643$$
$$\therefore y_2 = y_1 + \frac{1}{2}(k_1+k_2) = 0.52153125 + \frac{1}{2}(0.01776885947 + 0.01151245643)$$
$$= 0.5361719\overset{}{0}8$$

For $n=3$

$$k_1 = hf(x_2, y_2) = 0.1(0.2 - 0.536171908) = 0.01130115517$$

$$k_2 = hf(x_2+h, y_2+k_1) = 0.1(0.2+0.1-0.536171908-0.01130115517)$$

$$= 0.006124291$$

$$\therefore y_3 = y_2 + \frac{1}{2}(k_1+k_2) = 0.536171908 + \frac{1}{2}(0.01130115517 + 0.006124291)$$

$$= 0.5448846314$$

For $n=3$,

$$k_1 = hf(x_3, y_3) = 0.1(0.3 - 0.5448846314) = 0.005996848$$

$$k_2 = hf(x_3+h, y_3+k_1) = 0.1(0.3+0.1-0.5448846314-0.005996848)$$

$$= 0.002276522$$

$$y_4 = y_3 + \frac{1}{2}(k_1+k_2) = 0.5448846314 + \frac{1}{2}(0.005996848 + 0.002276522)$$

$$= 0.5490213166$$

For $n=4$,

$$k_1 = hf(x_4, y_4) = 0.1(0.4 - 0.5490213166) = 0.002220735$$

$$k_2 = hf(x_4+h, y_4+k_1) = 0.1(0.4+0.1-0.5490213166-0.002220735)$$

$$= 0.000262574$$

$$\therefore y_5 = y_4 + \frac{1}{2}(k_1+k_2) = 0.5490213166 + \frac{1}{2}(0.002220735 + 0.000262574)$$

$$= 0.5502629716$$

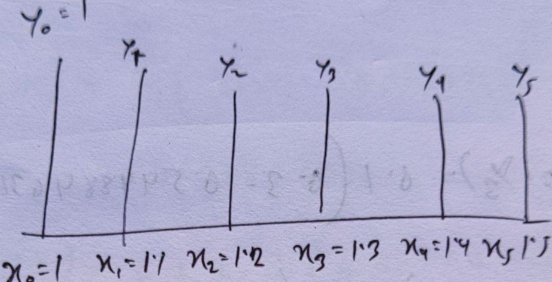
| n | Y | Euler | R-k |
|---|---|---|---|
| 0 | $Y_1$ | 0.525 | 0.5215 |
| 1 | $Y_2$ | 0.5434 | 0.5362 |
| 2 | $Y_3$ | 0.5548 | 0.5449 |
| 3 | $Y_4$ | 0.5613 | 0.5490 |
| 4 | $Y_5$ | 0.5639 | 0.5503 |

9)

$$y' = xy^2 - \frac{y}{x}$$

$$y(1) = 1$$

$$y(1.5) = ?$$




Euler Method,

We know,

$$Y_{n+1} = Y_n + h f(x_n, Y_n) \quad \text{--- (1)}$$

$n = 0, 1, 2, 3, \ldots$

For $n=0$

$$Y_1 = Y_0 + hf(x_0, Y_0) = 1$$

For $n=1$,

$$Y_2 = 1.019090909$$

For $n=2$,

$$Y_3 = 1.05879222$$

For $n=3$,

$$Y_4 = 1.123081991$$

For $n=4$,

$$Y_5 = 1.21944569$$

## R.K Method

We know,

$$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$$

$$k_1 = h f(x_n, y_n)$$

$$k_2 = h f(x_n + h, y_n + k_1)$$

**For $n=0$,**

$$k_1 = h f(x_0, y_0) = 0$$

$$k_2 = h f(x_0 + h, y_0 + k_1) = 0.01909090909$$

$$\therefore y_1 = y_0 + \frac{1}{2}(k_1 + k_2) = 1.009545455$$

**For $n=1$,**

$$k_1 = 0.02033316322$$

$$k_2 = 0.04145477793$$

$$y_2 = 1.040439425$$

**For $n=2$,**

$$k_1 = 0.04319841826$$

$$k_2 = 0.06929846965$$

$$y_3 = 1.096687869$$

**For $n=3$,**

$$k_1 = 0.07199355137$$

$$k_2 = 0.10773770324$$

$$y_4 = 1.186553161$$

**For $n=4$,**

$$k_1 = 0.1123533793$$

$$k_2 = 0.1664799607$$

$$y_5 = 1.325969831$$

| n | y | Euler | R.K |
|---|---|---|---|
| 0 | $y_1$ | 1 | 1·0075 |
| 1 | $y_2$ | 1·0171 | 1·0404 |
| 2 | $y_3$ | 1·0588 | 1·0767 |
| 3 | $y_4$ | 1·1231 | 1·1866 |
| 4 | $y_5$ | 1·2174 | 1·3260 |

H.W. ⇒ From Lecture -

# Assignment-2

$$f(x) = \begin{cases} 0 ; & -\pi \leq x < 0 \\ h ; & 0 \leq x \leq \pi \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi} \left[ \int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx \right]$$

$$= \frac{1}{\pi} \left[ \int_{-\pi}^{0} 0\, dx + \int_{0}^{\pi} h\, dx \right]$$

$$= \frac{1}{\pi} \Big[ hx \Big]_{0}^{\pi}$$

$$= \frac{1}{\pi} . h\pi$$

$$= h$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$= \frac{1}{\pi} \left[ \int_{-\pi}^{0} f(x) \cos nx\, dx + \int_{0}^{\pi} f(x) \cos nx\, dx \right]$$

$$= \frac{1}{\pi} \left[ \int_{-\pi}^{0} 0 \cdot \cos nx\, dx + \int_{0}^{\pi} h \cdot \cos nx\, dx \right]$$

$$= \frac{1}{\pi}\left[ 0 + h \cdot \frac{\sin nx}{n} \right]_0^{\pi}$$

$$= \frac{1}{\pi} \cdot \frac{h}{n}(0-0)$$

$$= 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi}\left[ \int_{-\pi}^{0} f(x) \sin nx \, dx + \int_{0}^{\pi} f(x) \sin nx \, dx \right]$$

$$= \frac{1}{\pi}\left[ \int_{-\pi}^{0} 0 \cdot \sin nx \, dx + \int_{0}^{\pi} h \sin nx \, dx \right]$$

$$= \frac{1}{\pi}\left[ 0 + \frac{h}{n} \cdot \left[ -\cos nx \right]_0^{\pi} \right.$$

$$= -\frac{1}{\pi} \cdot \frac{h}{n}\left[ (-1)^n - 1 \right]$$

$$= -\frac{h}{n\pi}\left\{ (-1)^n - 1 \right\}$$

$$\therefore f(x) = \frac{h}{2} + \sum_{n=1}^{\infty} -\frac{h}{n\pi}\left\{ (-1)^n - 1 \right\} \sin nx$$

$$= h\left[ \frac{1}{2} + \sum_{n=1}^{\infty} -\frac{(-1)^n - 1}{n\pi} \sin nx \right]$$

$$= h\left[ \frac{1}{2} + \frac{2}{\pi} \sin x + 0 + \frac{2}{3\pi} \sin 3x + 0 + \cdots \right]$$

$$f(x) = \begin{cases} 0 & ; -\pi \le x < 0 \\ \sin x & ; 0 \le x \le \pi \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Here

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi} \left[ \int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx \right]$$

$$= \frac{1}{\pi} \left[ \int_{-\pi}^{0} 0 \cdot dx + \int_{0}^{\pi} \sin x\, dx \right]$$

$$= \frac{1}{\pi} \left[ 0 + \left[ -\cos x \right]_0^{\pi} \right]$$

$$= -\frac{1}{\pi} (\cos\pi - 1)$$

$$= -\frac{1}{\pi} (-1-1)$$

$$= \frac{2}{\pi}$$

$$\therefore a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$= \frac{1}{\pi} \left[ \int_{0}^{\pi} \sin x \cdot \cos nx\, dx \right]$$

$$= \frac{1}{\pi}\left[\frac{1}{2}\int_0^{\pi} 2\sin x\cos nx\,dx\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi}\left\{\sin(x+nx)+\sin(x-nx)\right\}dx\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi}\sin(x+nx)\,dx + \int_0^{\pi}\sin(x-nx)\,dx\right]$$

$$= \frac{1}{2\pi}\left\{\left[-\frac{\cos(x+nx)}{(1+n)}\right]_0^{\pi} - \left[\frac{\cos(x-nx)}{(1-n)}\right]_0^{\pi}\right\}$$

$$= \frac{1}{2\pi}\left[\cancel{-\frac{(-1)^{n+1}}{1+n}} + \frac{1-(-1)^{n+1}}{1+n}\ \cancel{\frac{(-1)^{1-n}}{1-n}} + \frac{1-(-1)^{1-n}}{1-n}\right]$$

$$= \frac{1}{2\pi}\left(\cancel{\frac{2}{1-n^2}}\ \frac{1-(-1)^{n+1}}{1+n} + \frac{1-(-1)^{1-n}}{1-n}\right)$$

$$b_n = \frac{1}{\pi}\left[\int_0^{\pi}\sin x\cdot\sin nx\,dx\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} 2\sin x\sin nx\,dx\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi}\left\{\cos(\overset{x-nx}{A-B}) - \cos(\overset{x+nx}{A+B})\right\}dx\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi}\cos(x-nx)\,dx - \int_0^{\pi}\cos(x+nx)\,dx\right]$$

$$= \frac{1}{2\pi} \left\{ \frac{\sin(x-nx)}{(1-n)} \Big]_0^\pi - \frac{\sin(x+nx)}{1+n} \Big]_0^\pi \right\} \quad \Big| \sin(1-n)\pi$$

$$= \frac{1}{2\pi} \{ 0 - 0 )$$

$$= 0$$

$$\therefore f(x) = \frac{2}{\pi \cdot 2} + \sum_{n=1}^{\infty} \frac{1}{2\pi} \left( \frac{1-(-1)^{n+1}}{1+n} + \frac{1-(-1)^{1-n}}{1-n} \right) \cos nx$$

$$= \frac{1}{\pi} + 0 - \frac{2}{3\pi} \cos 2x + 0 - \frac{2}{15\pi} \cdot \cos 4x + 0 - \cdots$$

$$f(x) = x + x^2 \; ; \quad -\pi \le x \le \pi$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} (x + x^2) \, dx$$

$$= \frac{1}{\pi} \left[ \frac{x^2}{2} + \frac{x^3}{3} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left( \frac{\pi^2}{2} + \frac{\pi^3}{3} - \frac{\pi^2}{2} + \frac{\pi^3}{3} \right)$$

$$= \frac{1}{\pi} \cdot \frac{2\pi^3}{3} = \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\cos nx\,dx$$

$$\begin{array}{ccc} x^2 & & \cos nx \\ & + & \\ 2x & & \sin nx/n \\ & - & \\ 2 & & -\cos nx/n^2 \\ & + & \\ 0 & & -\sin nx/n^3 \end{array}$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x\cos nx\,dx + \int_{-\pi}^{\pi} x^2\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left\{\left[\frac{x}{n}\sin nx + \frac{1}{n^2}\cos nx\right]_{-\pi}^{\pi} + \left[\frac{x^2}{n}\sin nx + \frac{2x}{n^2}\cos nx - \frac{2}{n^3}\sin nx\right]_{-\pi}^{\pi}\right\}$$

$$\begin{array}{ccc} x & & \cos nx \\ & + & \\ 1 & & \frac{\sin nx}{n} \\ & - & \\ 0 & & \frac{-\cos nx}{n^2} \end{array}$$

$$= \frac{1}{\pi}\left(\frac{(-1)^n}{n^2} - \frac{(-1)^{-n}}{n^2} + \frac{2\pi(-1)^n}{n^2} - \frac{2\pi(-1)^{-n}}{n^2}\right)$$

$$= \frac{1}{\pi}\left(\frac{(-1)^n - (-1)^{-n} + 2\pi(-1)^n - 2\pi(-1)^{-n}}{n^2}\right)$$

$=$

$$\begin{array}{ccc} x^2+x & & \cos nx \\ & + & \\ 2x+1 & & \frac{\sin nx}{n} \\ & - & \\ 2 & & \frac{-\cos nx}{n^2} \\ & + & \\ 0 & & \frac{-\sin nx}{n^3} \end{array}$$

$$= \frac{1}{\pi}\left[\frac{x^2+x}{n}\sin nx + \frac{2x+1}{n^2}\cos nx - \frac{2}{n^3}\sin nx\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{2\pi+1}{n^2}(-1)^n - \frac{-2\pi+1}{n^2}(-1)^n\right]$$

$$= \frac{1}{\pi}\left[\frac{2\pi+1}{n^2}(-1)^n + \frac{2\pi-1}{n^2}(-1)^n\right]$$

$$= \frac{1}{\pi}(-1)^n\left(\frac{2\pi+1}{n^2} + \frac{2\pi-1}{n^2}\right)$$

$$= \frac{1}{\pi}(-1)^n\left(\frac{4\pi}{n^2}\right)$$

$$= \frac{4(-1)^n}{n^2}$$

| | |
|---|---|
| $x^2+x$ | $\sin nx$ |
| $2x+1$ $\xrightarrow{+}$ | $\frac{-\cos nx}{n}$ |
| $2$ $\xrightarrow{-}$ | $\frac{-\sin nx}{n^2}$ |
| $0$ $\xrightarrow{+}$ | $\frac{\cos nx}{n^3}$ |

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[-\frac{x^2+x}{n}\cos nx + \frac{2x+1}{n^2}\sin nx + \frac{2}{n^3}\cos nx\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{\pi^2+\pi}{n}(-1)^n + \frac{2}{n^3}(-1)^n + \frac{\pi^2-\pi}{n}(-1)^n + \frac{2}{n^3}(-1)^n\right]$$

$$= \frac{(-1)^n}{\pi}\left(-\frac{\pi^2+\pi}{n} + \cancel{\ } - \frac{\pi^2-\pi}{n} + \frac{4}{n^3}\right)$$

$$= \frac{(-1)^n}{\pi}\left(\frac{-\pi^2-\pi-\pi^2+\pi}{n} + \frac{4}{n^3}\right)$$

$$= \frac{(-1)^n}{\pi}\left(\frac{4}{n^3} - \frac{2\pi^2}{n}\right) = \cancel{\frac{(-1)^n}{\pi}\left(\frac{4-2n^2\pi^2}{n^3}\right)}$$

$$b_n = \frac{4(-1)^n}{n^3\pi} - \frac{2(-1)^n\pi}{n}$$

$$= \frac{1}{\pi}\left[\frac{\pi^2 - \pi - \pi^2 - \pi}{n}\right]$$

$$= \frac{1}{\pi} \cdot \frac{-2\pi}{n}$$

$$= \frac{-2}{n}$$

$$\therefore f(x) = \frac{2\pi^2}{3 \cdot 2} + \sum_{n=1}^{\infty}\left(\frac{4(-1)^n}{n^2}\cos nx - \frac{2}{n}\sin nx\right)$$

$$= \frac{\pi^2}{3} \left[-4\cos x + \cos 2x - \frac{4}{9}\cos 3x + \cdots\right.$$
$$\left. - 2\sin x - \sin 2x - \frac{2}{3}\sin 3x - \cdots\right]$$

(*)

$$f(x) = \begin{cases} \frac{1}{4} - x & ; \; 0 \le x < \frac{1}{2} \\ x - \frac{3}{4} & ; \; \frac{1}{2} \le x \le 1 \end{cases}$$

Half range sine series,

$$f(x) = \sum_{n=1}^{\infty}\left(b_n \sin \frac{n\pi x}{1}\right)$$

$$b_n = \frac{2}{1}\int_0^1 f(x)\cdot \sin(n\pi x)\,dx$$

$$= 2\left[\int_0^{\frac12} f(x)\sin(n\pi x)\,dx + \int_{\frac12}^1 f(x)\sin(n\pi x)\,dx\right]$$

$$= 2\left[\int_0^{\frac12}\left(\frac14 - x\right)\sin(n\pi x)\,dx + \int_{1/2}^1\left(x-\frac34\right)\sin(n\pi x)\,dx\right]$$

Tabular Method

| | |
|---|---|
| $-x+\frac14$ | $\sin(n\pi x)$ |
| $-1$ | $\frac{-\cos(n\pi x)}{n\pi}$ |
| $0$ | $\frac{-\sin(n\pi x)}{(n\pi)^2}$ |

| | |
|---|---|
| $x-\frac34$ | $\sin n\pi x$ |
| $1$ | $\frac{-\cos(n\pi x)}{n\pi}$ |
| $0$ | $\frac{-\sin(n\pi x)}{(n\pi)^2}$ |

$$\therefore b_n = 2\left\{\left[\frac{x-\frac14}{n\pi}\cos(n\pi x) - \frac{1}{(n\pi)^2}\sin(n\pi x)\right]_0^{\frac12}\right.$$

$$\left. + \left[-\frac{x-\frac34}{n\pi}\cos(n\pi x) + \frac{1}{(n\pi)^2}\sin(n\pi x)\right]_{\frac12}^1\right.$$

$$b_n = 2\left\{ \left[ \frac{\frac{1}{2}-\frac{1}{4}}{n\pi}\cos\left(\frac{n\pi}{2}\right) - \frac{1}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right) - \frac{0-\frac{1}{4}}{n\pi}\cdot 1 \right] + \right.$$

$$- \frac{1-\frac{3}{4}}{n\pi}(-1)^n + \frac{\frac{1}{2}-\frac{3}{4}}{n\pi}\cos\left(\frac{n\pi}{2}\right) - \frac{1}{(n\pi)^2}\sin\frac{n\pi}{2}$$

$$= 2\left[ \frac{1}{4n\pi}\cos\left(\frac{n\pi}{2}\right) - \frac{2}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{4n\pi} - \frac{1}{4n\pi}(-1)^n - \frac{1}{4n\pi}\cos\left(\frac{n\pi}{2}\right) \right)$$

$$= 2\left[ \frac{1}{4n\pi}\left(1-(-1)^n\right) - \frac{2}{(n\pi)^2}\sin\frac{n\pi}{2} \right]$$

[when n = even, integral is zero]

$$\therefore f(x) = \sum_{n=1}^{\infty} 2\left[ \frac{1-(-1)^n}{4n\pi} - \frac{2}{(n\pi)^2}\sin\frac{n\pi}{2} \right] \sin n\pi x$$

$=$

$$= \left(\frac{1}{\pi} - \frac{4}{\pi^2}\right)\sin \pi x + \left(\frac{1}{3\pi} + \frac{4}{9\pi^2}\right)\sin 3\pi x + \left(\frac{1}{5\pi} - \frac{4}{25\pi^2}\right)\sin 5\pi x + \cdots$$

## H.W. ⇒ From Lecture ⇒ 27

### 1|

$$\int_0^a y^7 \sqrt{a^4 - y^4}\, dy$$

Let,

$y^4 = a^4 x$

$dy = \frac{1}{4y^3} \cdot a^4 \cdot dx$

Limit,

$y = 0\ ;\ x = 0$

$y = a\ ;\ x = 1$

$$= \int_0^1 y^7 \cancel{(a^4 x)}^3 \cdot \sqrt{a^4 - a^4 x} \cdot \frac{1}{4y^3} \cdot a^4\, dx$$

$$= \int_0^1 y^4 \sqrt{a^4 (1-x)}\ \frac{a^4}{4}\, dx$$

$$= \frac{a^6}{4} \int_0^1 a^4 x\ (1-x)^{\frac{1}{2}}\, dx$$

$$= \frac{a^{10}}{4} \int_0^1 x^{2-1}\ (1-x)^{\frac{3}{2}-1}\, dx$$

$$= \frac{a^{10}}{4} \cdot \beta\left(2, \frac{3}{2}\right)$$

$$= \frac{a^{10}}{4} \cdot \frac{\Gamma 2\ \Gamma \frac{3}{2}}{\Gamma\left(2+\frac{3}{2}\right)} = \frac{a^{10}}{4}\ \frac{\lfloor 1\ \lfloor \frac{1}{2}}{\lfloor \frac{5}{2}}$$

$$= \frac{a^{10}}{4}\ \frac{1 \times \frac{1}{2} \times \sqrt{\pi}}{\frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \times \sqrt{\pi}} = \frac{a^{10}}{15}$$ Ans

21

$$\int_0^4 y \sqrt[3]{64-y^3}\, dy$$

Let,

$y^3 = 64x$

$3y^2\, dy = 64\, dx$

$dy = \frac{64}{3y^2}\, dx$

Limit,

$y=0 \; ; x=0$

$y=4 \; ; x=1$

$$= \int_0^1 y \, (64-64x)^{\frac{1}{3}} \frac{64}{3y^2}\, dx$$

$$= \frac{64}{3} \int_0^1 y^{-1} (64)^{\frac{1}{3}} (1-x)^{\frac{1}{3}}\, dx$$

$$= \frac{256}{3} \int_0^1 4^{-1} \cdot x^{-\frac{1}{3}} (1-x)^{\frac{1}{3}}\, dx$$

$$\cancel{= \frac{1024}{3} \int_0^1 x^{4/3-1} (1-x)^{4/3-1}\, dx}$$

$$= \frac{64}{3} \int_0^1 x^{2/3-1} \cdot (1-x)^{4/3-1}\, dx$$

$$= \frac{64}{3} \cdot \beta\left(\frac{2}{3}, \frac{4}{3}\right) = \frac{64}{3} \cdot \frac{\Gamma\frac{2}{3}\,\Gamma\frac{4}{3}}{\Gamma\left(\frac{2}{3}+\frac{4}{3}\right)} = \frac{64}{3} \cdot \frac{\frac{1}{3}\Gamma\frac{2}{3}\,\Gamma\left(1-\frac{2}{3}\right)}{\Gamma 2}$$

$$= \frac{64}{3} \cdot \frac{\frac{1}{3} \cdot \frac{\pi}{\sin\frac{2\pi}{3}}}{1}$$

$$= \frac{64}{3} \cdot \frac{1}{3} \cdot \frac{\pi}{\sin\frac{2\pi}{3}}$$

$$= \frac{64\pi}{9 \sin\frac{2\pi}{3}}$$

$$= \frac{128\pi}{9\sqrt{3}} \text{ Ans}$$

$\Gamma x \, \Gamma 1-x = \frac{\pi}{\sin(\pi x)}, \; x \notin \mathbb{Z}$

Euler's Reflection formula

LANCZOS Approximation

From internet (Quora.com) & Wiki

Calculator = ~~[illegible]~~ 25.7962

$\frac{128\pi}{9\sqrt{3}} = 25.79$

3]

$$\int_0^{\pi/8} \sin^2 4x \cos^5 4x \, dx$$

$$= \frac{1}{4}\int_0^{\pi/2} \sin^2 u \cos^5 u \, du$$

$$= \frac{1}{4} \cdot \frac{\Gamma\frac{2+1}{2}\,\Gamma\frac{5+1}{2}}{2\,\Gamma\frac{2+5+2}{2}}$$

$$= \frac{1}{8} \cdot \frac{\Gamma\frac{3}{2}\,\Gamma 3}{\cancel{2}\,\Gamma\frac{9}{2}}$$

$$= \frac{1}{8} \cdot \frac{\frac{1}{2}\cdot\sqrt{\pi}\cdot 2}{\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}}$$

$$= \frac{2}{105}$$ An

Let,

$4x = u$

$4\,dx = du$

$dx = \frac{1}{4}\,du$

Limit,

$x = 0 \; ; \; u = 0$

$x = \frac{\pi}{8} \; ; \; u = \frac{\pi}{2}$