North South University

Department of Mathematics and Physics

Assignment-5

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7.8

71) $\int_{e}^{\infty} \frac{1}{x \ln^3 x} \, dx$

Let,

$u = \ln x$

$du = \frac{1}{x} dx$

x	u
e	1
b	$\ln b$

$= \lim_{b \to \infty} \int_{e}^{b} \frac{1}{x \ln^3 x} \, dx$

$= \lim_{b \to \infty} \int_{1}^{\ln b} \frac{1}{u^3} \, du$

$= \lim_{b \to \infty} \left. \frac{u^{-2}}{-2} \right]_{1}^{\ln b}$

$= \lim_{b \to \infty} \left[-\frac{1}{2u^2} \right]_{1}^{\ln b}$

$= \lim_{b \to \infty} \left[-\frac{1}{2(\ln b)^2} + \frac{1}{2} \right]$

$= \frac{1}{2}$

Ans.

10) $\int_{-\infty}^{3} \frac{dx}{x^2+9}$

$= \lim_{a \to -\infty} \int_{a}^{3} \frac{1}{x^2+9}\, dx$

$= \lim_{a \to -\infty} \left[\frac{1}{3} \tan^{-1} \frac{x}{3} \right]_{a}^{3}$

$= \lim_{a \to -\infty} \left(\frac{1}{3} \cdot \frac{\pi}{4} - \frac{1}{3} \tan^{-1} \frac{a}{3} \right)$

$= \lim_{a \to -\infty} \left(\frac{\pi}{12} - \frac{1}{3} \tan^{-1} \frac{a}{3} \right)$

$= \frac{\pi}{12} + \frac{\pi}{6} = \frac{\pi}{4}$ Ans.

12) $\int_{-\infty}^{0} \frac{e^x}{3-2e^x}\, dx$

Let,

$u = 3-2e^x$

$du = -2e^x\, dx$

$e^x dx = -\frac{1}{2} du$

$= \lim_{a \to -\infty} \int_{a}^{0} \frac{e^x}{3-2e^x}\, dx$

$\therefore \int \frac{e^x}{3-2e^x}\, dx$

$= -\frac{1}{2} \int \frac{1}{u}\, du$

$= -\frac{1}{2} \ln u + c$

$$= -\frac{1}{2}\ln|3-2e^x| + C$$

$$\therefore \lim_{a\to-\infty} \int_a^0 \frac{e^x}{3-2e^x}\,dx$$

$$= \lim_{a\to-\infty} \left[-\frac{1}{2}\ln|3-2e^x|\right]_a^0$$

$$= \lim_{a\to-\infty} \left(0 + \frac{1}{2}\ln|3-2e^a|\right)$$

$$= \frac{1}{2}\ln|3-0|$$

$$= \frac{1}{2}\ln(3)$$

Ans.

14) $$\int_{-\infty}^{\infty} \frac{x}{\sqrt{x^2+2}}\,dx$$

$$= \int_{-\infty}^{0} \frac{x}{\sqrt{x^2+2}}\,dx + \int_{0}^{\infty} \frac{x}{\sqrt{x^2+2}}\,dx$$

$$= \lim_{a\to-\infty} \int_a^0 \frac{x}{\sqrt{x^2+2}}\,dx + \lim_{b\to\infty} \int_0^{\infty} \frac{x}{\sqrt{x^2+2}}\,dx$$

$$\therefore \lim_{a\to-\infty} \int_a^0 \frac{x}{\sqrt{x^2+2}}\, dx$$

Let,

$u = x^2+2$

$du = 2x\, dx$

$x\,dx = \frac{1}{2} du$

x	u
a	a^2+2
0	2

$$= \lim_{a\to-\infty} \int_{a^2+2}^{2} \frac{1}{\sqrt{u}} \cdot \frac{1}{2} \cdot du$$

$$= \lim_{a\to-\infty} \left[\frac{1}{2} \cdot 2\sqrt{u}\right]_{a^2+2}^{2}$$

$$= \lim_{a\to-\infty} \left[\sqrt{u}\right]_{a^2+2}^{2}$$

$$= \lim_{a\to-\infty} \left[\sqrt{2} - \sqrt{a^2+2}\right]$$

$$= -\infty$$

Therefore, It's divergent. No need to check other part.

16) $$\int_{-\infty}^{\infty} \frac{e^{-t}}{1+e^{-2t}}\, dt$$

$$= \int_{-\infty}^{0} \frac{e^{-t}}{1+e^{-2t}}\, dt + \int_{0}^{\infty} \frac{e^{-t}}{1+e^{-2t}}\, dt$$

$$= \lim_{a\to-\infty} \int_a^0 \frac{e^{-t}}{1+e^{-2t}}\, dt + \lim_{b\to\infty} \int_0^b \frac{e^{-t}}{1+e^{-2t}}\, dt$$

$$\therefore \lim_{a\to\infty} \int_a^0 \frac{e^{-t}}{1+(e^{-t})^2}\,dt$$

$$= \lim_{a\to-\infty} \left[-\tan^{-1}(e^{-t})\right]_a^0$$

$$= \lim_{a\to-\infty} \left(-\frac{\pi}{4} + \tan^{-1}(e^{-a})\right)$$

$$= -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$$

$$\therefore \lim_{b\to\infty} \int_0^{\infty} \frac{e^{-t}}{1+(e^{-t})^2}\,dt$$

$$= \lim_{b\to\infty} \left[-\tan^{-1}(e^{-t})\right]_0^b$$

$$= \lim_{b\to\infty} \left(-\tan^{-1}(e^{-b}) + \frac{\pi}{4}\right)$$

$$= \frac{\pi}{4}$$

$$\therefore \int_{-\infty}^{\infty} \frac{e^{-t}}{1+e^{-2t}}\,dt = \frac{\pi}{4} + \frac{\pi}{4}$$

$$= \frac{\pi}{2} \text{ Ans.}$$

Let,

$$u = e^{-t}$$

$$du = -e^{-t}\,dt$$

$$\therefore \int \frac{e^{-t}}{1+(e^{-t})^2}\,dt$$

$$= -\int \frac{1}{1+u^2}\,du$$

$$= -\tan^{-1}u + c$$

$$= -\tan^{-1}(e^{-t}) + c$$

21/ $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$= \lim_{k \to 1^-} \int_0^k \frac{1}{\sqrt{1-x^2}} dx$

$= \lim_{k \to 1^-} [\sin^{-1} x]_0^k$

$= \lim_{k \to 1^-} (\sin^{-1} k - 0)$

$= \frac{\pi}{2}$ Ans.

24/ $\int_0^{\pi/4} \frac{\sec^2 x}{1-\tan x} dx$

$= \lim_{k \to \frac{\pi}{4}^-} \int_0^k \frac{\sec^2 x}{1-\tan x} dx$

Let,

$u = 1 - \tan x$

$du = -\sec^2 x \, dx$

$\therefore \int \frac{\sec^2 x}{1-\tan x} dx$

$= -\int \frac{1}{u} du$

$= -\ln|u| + c$

$= -\ln|1-\tan x| + c$

$= \lim_{k \to \frac{\pi}{4}^-} [-\ln|1-\tan x|]_0^k$

$= \lim_{k \to \frac{\pi}{4}^-} (-\ln|1-\tan k| + 0)$

$= \infty$

Therefore its divergent.

26) $\int_{-2}^{2} \frac{dx}{x^2}$

$$= \int_{-2}^{0} \frac{1}{x^2}\, dx + \int_{0}^{2} \frac{1}{x^2}\, dx$$

$$= \cancel{\lim_{k \to -2^+} \int_{k}}$$

$$= \lim_{k \to 0^-} \int_{-2}^{k} \frac{1}{x^2}\, dx + \lim_{k \to 0^+} \int_{k}^{2} \frac{1}{x^2}\, dx$$

$$= \lim_{k \to 0^-} \int_{-2}^{k} x^{-2}\, dx + \lim_{k \to 0^+} \int_{k}^{2} x^{-2}\, dx$$

$$= \lim_{k \to 0^-} \left[-\frac{1}{x}\right]_{-2}^{k} + \lim_{k \to 0^+} \left[-\frac{1}{x}\right]_{k}^{2}$$

$$= \lim_{k \to 0^-} \left(-\frac{1}{k} + \frac{1}{-2}\right) + \lim_{k \to 0^+} \left[-\frac{1}{2} + \frac{1}{k}\right]$$

$$= \infty$$

Therefore it's divergent.

30) $\int_{1}^{\infty} \frac{dx}{x\sqrt{x^2-1}}$

$= \int_{1}^{2} \frac{1}{x\sqrt{x^2-1}}\,dx + \int_{2}^{\infty} \frac{1}{x\sqrt{x^2-1}}\,dx$

$= \lim_{k \to 1^+} \int_{k}^{2} \frac{1}{x\sqrt{x^2-1}}\,dx + \lim_{b \to \infty} \int_{2}^{b} \frac{1}{x\sqrt{x^2-1}}\,dx$

$= \lim_{k \to 1^+} \left[\sec^{-1}x\right]_{k}^{2} + \lim_{b \to \infty} \left[\sec^{-1}x\right]_{2}^{b}$

$= \lim_{k \to 1^+} \left(\frac{\pi}{3} - \sec^{-1}k\right) + \lim_{b \to \infty} \left[\sec^{-1}b - \frac{\pi}{3}\right]$

$= \frac{\pi}{3} + \left(\frac{\pi}{2} - \frac{\pi}{3}\right)$

$= \frac{\pi}{2}$ Ans

31) $\int_0^1 \frac{dx}{\sqrt{x}(x+1)}$

$= \lim_{k \to 0^+} \int_k^1 \frac{1}{\sqrt{x}(x+1)} dx$

Let,

$u = \sqrt{x}$

$du = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} dx$

$\cancel{du = \frac{1}{2u} dx}$

$\cancel{dx = 2u\,du}$

$du = \frac{1}{2u} dx$

$dx = 2u\,du$

$\therefore \int \frac{1}{\sqrt{x}(x+1)} dx$

$= \int \frac{1}{u(u^2+1)} 2u\,du$

$= 2 \int \frac{1}{u^2+1} du$

$= 2 \tan^{-1} u + c$

$= 2 \tan^{-1} (\sqrt{x}) + c$

$\therefore \lim_{k \to 0^+} \int_k^1 \frac{1}{\sqrt{x}(x+1)} dx$

$= \lim_{k \to 0^+} \left[2 \tan^{-1}(\sqrt{x}) \right]_k^1$

$= \lim_{k \to 0^+} \left(\frac{\pi}{2} - 2 \tan^{-1}(\sqrt{k}) \right)$

$= \frac{\pi}{2} - 0$

$= \pi/2$ Ans