

H.W.

1. $y = 2 + \cos \frac{x}{2}$

2. $y = -2 \cos \pi x$

Solution

$\cos 2x = 4$

1)

$$y = 2 + \cos \frac{x}{2} \quad [\text{period}]$$

$$\Rightarrow y = \cos \frac{x}{2} + 2$$

Now,

$$y = \cos \frac{1}{2} \cdot x$$

Here,

$$\text{Amplitude} = |2| = 2$$

$$\text{Period} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

One circle begin at 0 and end at 4π .

Divided by 4 sub interval, each interval length $\frac{4\pi - 0}{4} = \pi$

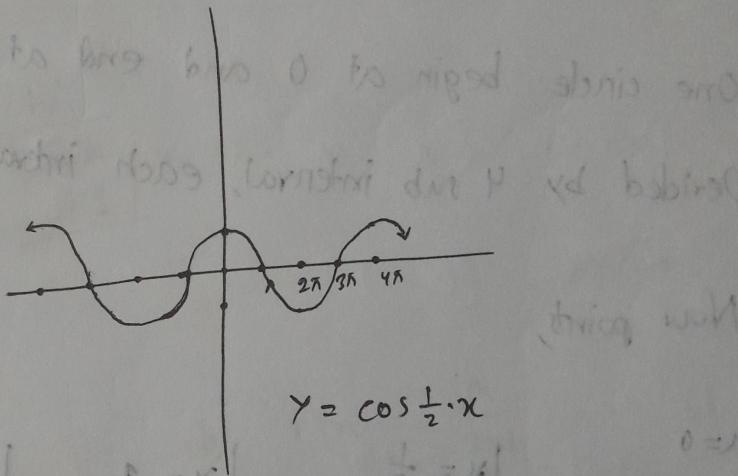
Now points,

$x=0$	$x=\pi$	$x=2\pi$	$x=3\pi$	$x=4\pi$
$y = \cos \frac{1}{2} \cdot 0$	$y = \cos \frac{1}{2} \pi$	$y = \cos \frac{1}{2} \cdot 2\pi$	$y = \cos \frac{1}{2} \cdot 3\pi$	$y = \cos \frac{1}{2} \cdot 4\pi$
$= \cos 0$	$= \cos \frac{\pi}{2}$	$= \cos \pi$	$= \cos \frac{3\pi}{2}$	$= \cos 2\pi$
$= 1$	$= 0$	$= -1$	$= 0$	$= 1$
$(0, 1)$	$(\pi, 0)$	$(2\pi, -1)$	$(3\pi, 0)$	$(4\pi, 1)$

$$\omega = \frac{K\pi}{L} = \text{boiling}$$

Graph:

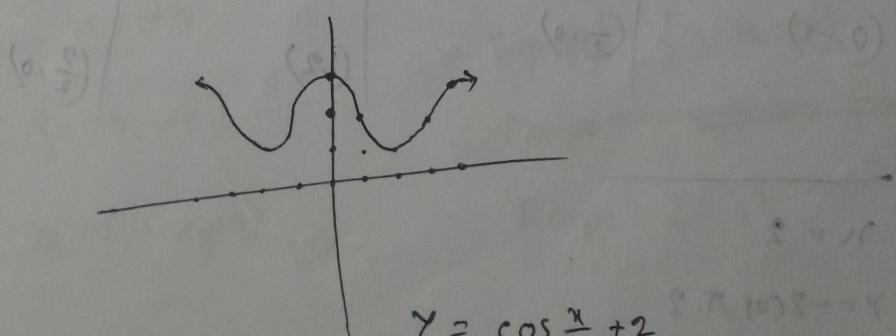
• To draw from 0 to right shift and
 $\frac{1}{2} \rightarrow \frac{0+2}{2}$ after each π (from left to right due to ω is positive)



$$y = \cos \frac{1}{2}x$$

$$\begin{array}{l|l|l|l}
\frac{1}{2}x & 1 = \pi & \frac{1}{2}x & 0 = \pi \\
\frac{1}{2}x \cos \pi = -1 & 1 \cdot \cos \pi = -1 & \frac{1}{2}x \cos 0 = 1 & 0 \cdot \cos 0 = 1 \\
\frac{1}{2}(0) \cos \pi = -1 & 1 \cdot \cos 0 = 1 & \frac{1}{2}(2\pi) \cos 0 = 1 & 0 \cdot \cos 0 = 1
\end{array}$$

\downarrow adding 2



$$y = \cos \frac{x}{2} + 2$$

2]

$$y = -2 \cos \pi x$$

Hence,

$$\text{Amplitude} = |-2| = 2$$

y-co-ordinate line between -2 to 2

$$\text{Period} = \frac{2\pi}{\pi} = 2$$

One circle begin at 0 and end at 2 .

Divided by 4 sub interval, each interval length $\frac{2-0}{4} = \frac{1}{2}$

Now points,

$$\begin{array}{l|l|l|l} n=0 & n=\frac{1}{2} & n=1 & n=\frac{3}{2} \\ y = -2 \cos \pi \cdot 0 & y = -2 \cos \pi \cdot \frac{1}{2} & y = -2 \cos \pi \cdot 1 & y = -2 \cos \pi \cdot \frac{3}{2} \\ = -2 \cos 0 & = -2 \cos \frac{\pi}{2} & = -2 \cos \pi & = -2 \cos \frac{3\pi}{2} \\ = -2 & = 0 & = 2 & = 0 \\ (0, -2) & \left(\frac{1}{2}, 0\right) & (1, 2) & \left(\frac{3}{2}, 0\right) \end{array}$$

$$n=2$$

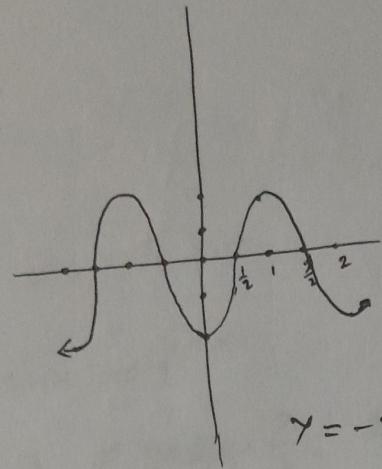
$$y = -2 \cos \pi \cdot 2$$

$$= -2 \cos 2\pi$$

$$= -2$$

$$(2, -2)$$

Graph!



$$y = -2 \cos \pi x$$

$$0 \leq x - \pi$$

$$x \leq \pi$$

domain not

H.W
 $\frac{1}{x^2 + x - 2} : \text{domain}$
 $[0, 1]$

$(\infty, 0]$: sign

9]

a) $f(x) = \frac{1}{x-3}$

b) $f(x) = \frac{x}{x^2 + x - 2} = \frac{x}{(x+2)(x-1)}$

For domain,

$$x-3 \neq 0$$

$$x \neq 3$$

For domain,

$$|x| \neq 0$$

$$0 \leq x^2 + x - 2 \neq 0$$

: Domain: $\{x : x \neq 0\}$

∴ Domain: $\{x : x \neq 3\}$

Range: $\{y : y \neq 0\}$

∴ Range: $\{y : y \neq 0\}$ Range: $\{-1, 1\}$

$(-\infty, 0) \cup (0, \infty)$: domain of

$(-\infty, 1] \cup [1, \infty)$: range of

c)

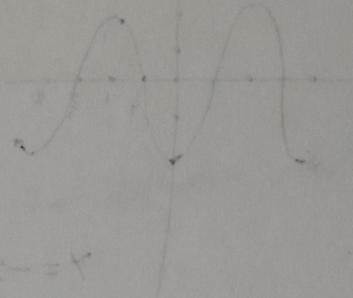
$$g(x) = \sqrt{x-3}$$

For domain,

$$x-3 \geq 0$$

$$x^2 \geq 3$$

$$x \geq \pm\sqrt{3}$$



$$\begin{array}{c} + \\ - \\ \hline -\sqrt{3} & \sqrt{3} \end{array}$$

Domain: $\{x : x \leq -\sqrt{3} \text{ and } x \geq \sqrt{3}\}$

Range: $[0, \infty)$

d)

$$g(x) = \sqrt{x^2 - 2x + 5}$$

For domain,

$$x^2 - 2x + 5 \geq 0$$

$$x^2 - 2x + 1 + 4 \geq 0$$

$$(x-1)^2 + 4 \geq 0$$

This expression is always positive

So, domain: $(-\infty, \infty)$

Range: ~~Ex~~ $[2, \infty)$

e)

(0)

$$h(x) = \frac{1}{1-\sin x}$$

For domain,

$$1-\sin x \neq 0 \quad 0 \leq x < \pi$$

$$0 \leq x < \pi$$

~~$\sin x \neq 1$~~

$$\pi/2 \leq x < \pi$$

$$x \neq \cancel{\pi/2}, \frac{\pi}{2}, \frac{5\pi}{2}$$

$$\pi/2 \leq x < \pi$$

$$0 \leq x < \pi$$

$$(\pi/2 \leq x < \pi) : \text{original}$$

$$\therefore x \neq (2n + \frac{1}{2})\pi ; \quad n = 0, \pm 1, \pm 2, \dots$$

Range:

$$-1 \leq \sin x \leq 1 \quad y \geq \frac{1}{2}$$

$$0 \leq 1 - \sin x \leq 2$$

(b)

$$H(x) = \sqrt{\frac{x^2-4}{x-2}} = \sqrt{x+2}$$

For domain:

$$x-2 \neq 0$$

$$x \neq 2$$

$$\frac{x^2-4}{x-2} \geq 0$$

$$\frac{(x+2)(x-2)}{x-2} \geq 0$$

$$x+2 \geq 0$$

$$x \geq -2$$

$$\therefore \text{domain} : \{x : x \geq -2 \text{ and } x \neq 2\}$$

$$\text{Range: } [0, 2) \cup (2, \infty)$$

10)

a)

$$f(x) = \sqrt{3-x}$$

$$3-x \geq 0$$

$$-x \geq -3$$

$$x \leq 3$$

$$\text{Domain: } \{x : x \leq 3\}$$

b) $f(x) = \sqrt{4-x}$

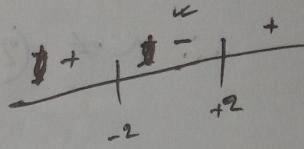
$$4-x \geq 0$$

$$-x \geq -4$$

$$x \leq 4$$

$$x-4 \leq 0$$

$$(x+2)(x-2) \leq 0$$



~~Domain: $(-\infty, -2] \cup [2, \infty)$~~

~~Domain: $[-2, 2]$~~

c)

$$g(x) = 3 + \sqrt{x}$$

For domain,

$$x \geq 0$$

$$D: \{x : x \geq 0\}$$

$$D: \mathbb{R}$$

e) $h(x) = 3 \sin x$

$$D: \mathbb{R}$$

$$f) H(x) = (\sin \sqrt{x})^{-2} = \frac{1}{(\sin \sqrt{x})^2}$$

For domain,

$$x \geq 0$$

And,

$$\sin \sqrt{x} \neq 0$$

$$e^{-(1/x)} e^{-} = 0$$

$$\sqrt{x} \neq 0$$

$$e^{-(1/x)} e^{-} \leftarrow (1/x) e^{-} \leftarrow \sqrt{x} \neq n\pi ; n = 0, \pm 1, \pm 2, \pm 3, \dots$$

Range: $x \geq 1$.

$$e^{-(1/x)} \frac{1}{x} = 0$$

$$e^{-(1/x)} \frac{1}{x} \leftarrow (1/x) \frac{1}{x} \leftarrow (1/x) \leftarrow x = 0$$

$$(1/x) \leftarrow 0 \rightarrow x = 0$$

$$e^{-(1/x)} \cdot 0 + 1/x = 0$$

$$e^{-(1/x)} = 0$$

$$e^{-(1/x)} \leftarrow (1/x) \leftarrow 0 = 0$$

10.2

$$y = x^2 / y = \sqrt{x} / y = \frac{1}{x} / y = |x| / y = \sqrt[3]{x}$$

~~graph~~

$0 \leq x$

5

$$y = -2(x+1)^2 - 3$$

$$y = x^2 \rightarrow (x+1)^2 \rightarrow 2(x+1)^2 \rightarrow -2(x+1)^2 \rightarrow -2(x+1)^2 - 3$$

6

$$y = \frac{1}{2}(x-3)^2 + 2$$

$$y = x^2 \rightarrow (x-3)^2 \rightarrow \frac{1}{2}(x-3)^2 \rightarrow \frac{1}{2}(x-3)^2 + 2$$

7

$$y = x^2 + 6x = x(x+6)$$

$$y = x^2 + 2 \cdot 3x + 3^2 - 9$$

$$y = (x+3)^2 - 9$$

$$y = x^2 \rightarrow (x+3)^2 \rightarrow (x+3)^2 - 9$$

8]

$$y = \frac{1}{2} (n^2 - 2n + 3)$$

$$= \frac{1}{2} (n^2 - 2n + 1 + 2)$$

$$= \frac{1}{2} ((n-1)^2 + 2)$$

$$y = n^2 \rightarrow (n-1)^2 \rightarrow (n-1)^2 + 2 \xrightarrow{\frac{1}{2}((n-1)^2 + 2)}$$

$$\frac{1}{(n-1)^2 + 2} \leftarrow \frac{1}{n^2} \leftarrow \frac{1}{n} = y$$

9]

9]

$$y = 3 - \sqrt{n+1}$$

$$y = \sqrt{n} \rightarrow \sqrt{n+1} \rightarrow -\sqrt{n+1} \rightarrow -\sqrt{n+1} + 3$$

$$\frac{1}{(\sqrt{n})^2 + 2} \leftarrow \frac{1}{n+1} \leftarrow \frac{1}{n} = y$$

10]

10]

$$y = 1 + \sqrt{n-4}$$

$$y = \sqrt{n} \rightarrow \sqrt{n-4} \rightarrow \sqrt{n-4} + 1 = \frac{1}{n} - \frac{n}{n} = \frac{1-n}{n} = y$$

11]

11]

$$y = \frac{1}{2} \sqrt{n} + 1$$

$$1 + \frac{1}{n} \leftarrow \frac{1}{n} \leftarrow \frac{1}{n} = y$$

12]

12]

$$y = -\sqrt{3n}$$

$$1 - \frac{1}{3n} \leftarrow \frac{1}{3n} \leftarrow \frac{1}{n} = y$$

$$y = \sqrt{n} \rightarrow \sqrt{3n} \rightarrow -\sqrt{3n}$$

$$\underline{(3)}$$

$$y = \frac{1}{n} \rightarrow \frac{1}{(n-3)}$$

$$y = \frac{1}{n} \rightarrow \frac{1}{n-1} \rightarrow \frac{1}{-(n-1)}$$

$$\underline{15)} \quad y = 2 - \frac{1}{n+1}$$

$$y = \frac{1}{x} \rightarrow \frac{1}{x+1} \rightarrow -\frac{1}{n+1} \rightarrow -\frac{1}{n+1} + 2$$

$$y = \frac{x-1}{x}$$

$$y = \frac{1}{n} \rightarrow -\frac{1}{n} \rightarrow -\frac{1}{n} + 1$$

$$[7] \quad Y = |n+2| - 2$$

$$y = |x| \rightarrow |x+2| \rightarrow |x+2| - 2$$

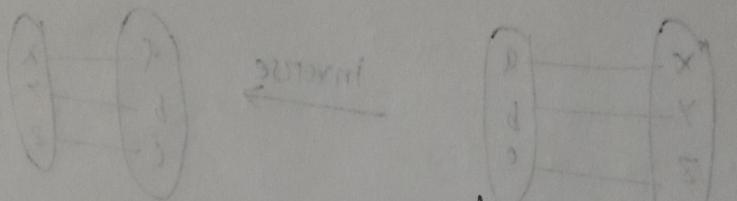
18

$$y = 1 - |x-3|$$

$$y = |x| \rightarrow |x-3| \rightarrow -|x-3| \rightarrow -|x-3| + 1$$

19

$$y = |2x-1| + 1$$



$$y = |x| \rightarrow |2x| \rightarrow |2x-1| \rightarrow |2x-1| + 1$$

mit einem Faktor von 2 ausmultiplizieren

$$\underline{20} \quad y = \sqrt{x^2 - 4x + 4} = \sqrt{(x-2) \cdot x \cdot 2 + 2^2} = \sqrt{(x-2)^2} = |x-2|$$

$$y = |x| \rightarrow |x-2|$$

21

$$y = 1 - 2\sqrt[3]{x}$$

$$y = \sqrt[3]{x} \rightarrow 2\sqrt[3]{x} \rightarrow -2\sqrt[3]{x} \rightarrow -2\sqrt[3]{x} + 1$$

22

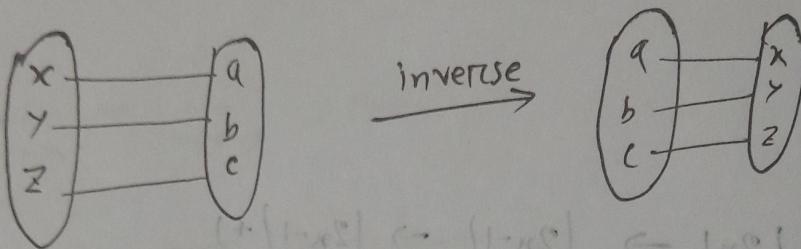
$$y = \sqrt[3]{x-2} - 3$$

$$\Rightarrow y = \sqrt[3]{x} \rightarrow \sqrt[3]{x-2} \rightarrow \sqrt[3]{x-2} - 3$$

(x) \rightarrow zu untersuchen, ob $y = \sqrt[3]{x} - 3$ für $x \in \mathbb{R}$ streng monoton ist

0.4

 Inverse function:



one to one function

If a function one to one, then interchanging range and domain, then we will get the new function.

that is called inverse function.

Domain of f^{-1} = Range of f

Range of f^{-1} = Domain of f

Getting Inverse function from the equation:

Example: If $f(x) = x^3 - 1$, find inverse of $f(x)$.

$$\Rightarrow f(x) = x^3 - 1$$

$$y = x^3 - 1$$

Interchange x and y ,

$$x = y^3 - 1$$

solve for y ,

$$y^3 = x + 1$$

$$\therefore y = \sqrt[3]{x+1}$$

$$\therefore f^{-1}(x) = \sqrt[3]{x+1}$$

Checking:

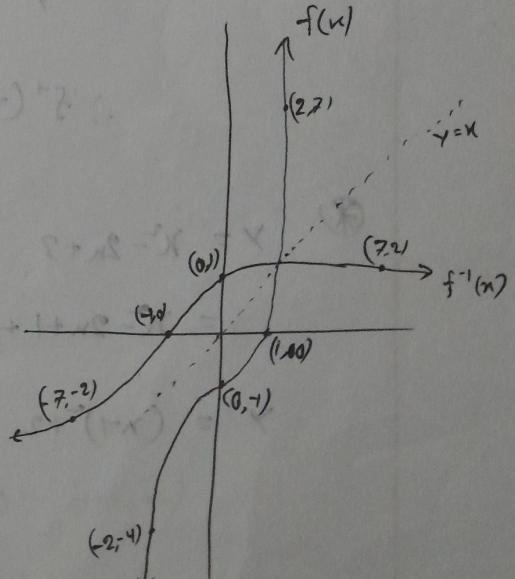
$$\begin{aligned} f(f^{-1}(x)) &= f(\sqrt[3]{x+1}) \\ &= (\sqrt[3]{x+1})^3 - 1 \end{aligned}$$

$$= x+1 - 1$$

$$= x$$

$$\begin{aligned} f^{-1}(f(x)) &= f^{-1}(x^3 - 1) \\ &= \sqrt[3]{x^3 - 1 + 1} \\ &= \sqrt[3]{x^3} \\ &= x \end{aligned}$$

$$\therefore f(f^{-1}(x)) = f^{-1}(f(x)) = x \quad (\text{Proved})$$



④ If $f(x) = (x-1)^2$ are one to one?

restricted domain $x \geq 1$

$$f(x) = (x-1)^2$$

$$y = (x-1)^2$$

interchange, x and y ,

$$x = (y-1)^2$$

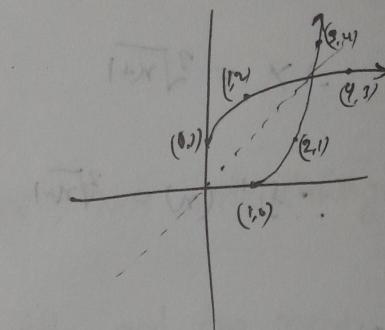
solve for y ,

$$x = (y-1)^2$$

$$y-1 = \sqrt{x}$$

$$y = \sqrt{x} + 1$$

$$\therefore f^{-1}(x) = \sqrt{x} + 1$$

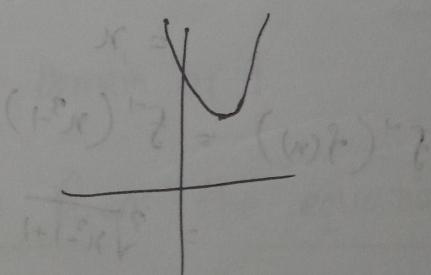


⑤

$$y = x^2 - 2x + 3$$

$$= x^2 - 2x + 1 + 2$$

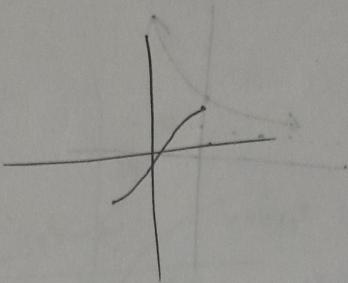
$$y = (x-1)^2 + 2$$



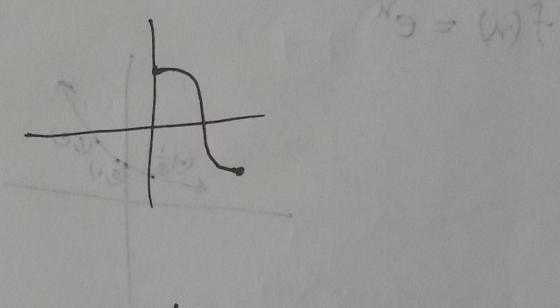
$$D: [1, \infty)$$

$$R: [2, \infty)$$

⊗ $f(x) = \sin x ; -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ $|f''(x) = (x)|$



⊗ $f(x) = \cos x ; 0 \leq x \leq \pi$ $|f''(x) = (x)|$

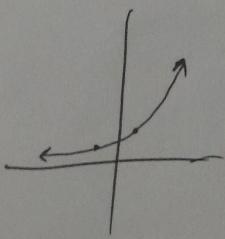


0.5 /

Exponential Function:

$$f(x) = a^x ; a > 0, a \neq 1$$

$$f(x) = 2^x$$



D: $(-\infty, \infty)$

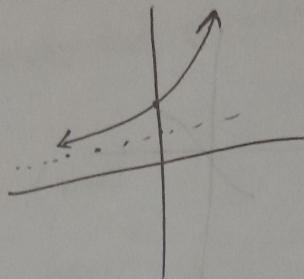
R: $(0, \infty)$

H.A.S $y = 0$

increasing

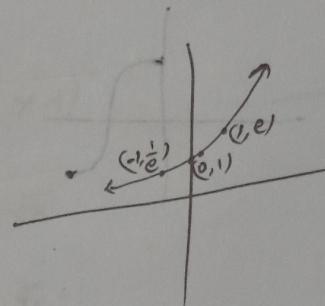
$$(0, 1), (1, a), (-1, \frac{1}{a})$$

$$f(n) = 2^n + 1$$



$$e = 2.7182 \dots \text{ natural no.}$$

$$f(n) = e^n$$



2.0

(*)

$$f(n) = a^n$$

$$y = a^n$$

$$\Rightarrow n = a^y \quad (n > 0, 0 < a < 1) \quad f(n) = (n)^t$$

$$y = \log_a n$$

$$f^{-1}(n) = \log_a n$$



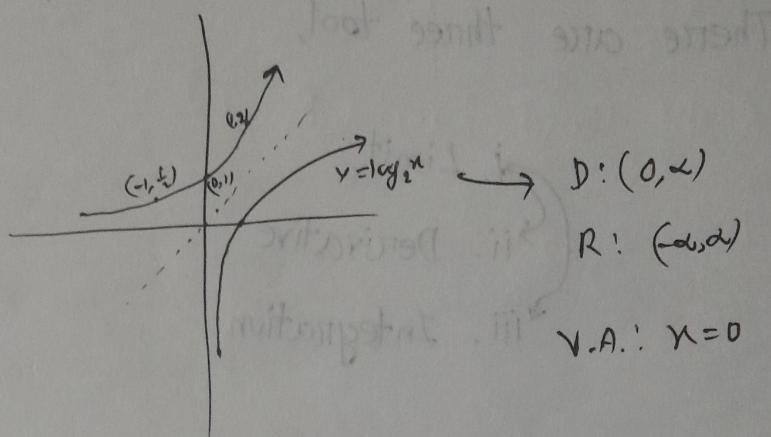
$$n = V - A H$$

minimum

$$(1, 1), (0, 1), (1, 0)$$

⊗

$$y = \log_a x \quad , \text{ if and only if } x = a^y$$



L.E.I
time

movement is to time ⊗

list a begin no to I time is not yet T
so I of words has now step not fast when
now we left is at words have words come x

: stimuli

$$I = (x)^2 \quad \text{with} \quad 0 < x$$

Chapter - 1

There are three tool,

i. Limit

ii. Derivative.

iii. Integration

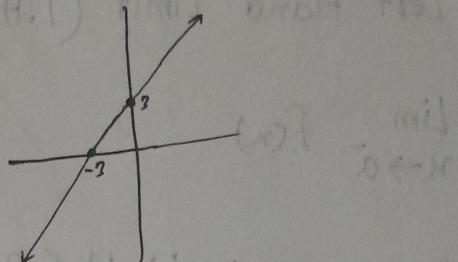
1.1 /
Limit* Limit of a function:

If $f(x)$ has a limit L at an input a , this means that, $f(x)$ gets closer and closer to L as x moves closer and closer to a . Then we can write :

$$\lim_{x \rightarrow a} f(x) = L$$

$$\textcircled{*} \quad f(x) = x+3$$

(J.H.I) limit broof folg (3)



$$\lim_{x \rightarrow 2} x+3 = 5$$

$$\lim_{x \rightarrow 1} f(x) = 4$$

$$\lim_{x \rightarrow -1} f(x) = 2$$

$$\lim_{x \rightarrow -2} f(x) = 1$$

$$f(x) = x+3$$

$$f(1) = 4$$

$$f(-1) = 2$$

$$f(-2) = 1$$

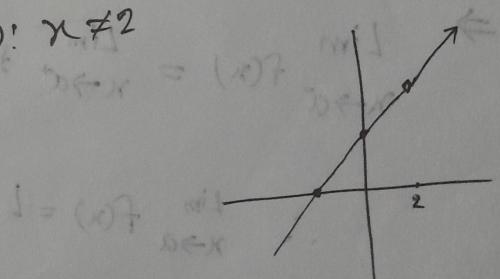
$$f(2) = 5$$

Ko2 limw zw naft

$$\textcircled{*} \quad f(x) = \frac{x^2-4}{x-2}$$

$$f(x) = x+2 \rightarrow D: x \neq 2$$

$$f(2) = \frac{0}{0} = \text{undefined}$$



$$\lim_{x \rightarrow} x+2 = 4$$

$$\lim_{x \rightarrow -1} x+2 = 1$$

$$\lim_{x \rightarrow 4} x+2 = 6$$

⊗ Left Hand Limit (L.H.L.)

$$\lim_{x \rightarrow a^-} f(x)$$

⊗ Right Hand Limit (R.H.L.)

$$\lim_{x \rightarrow a^+} f(x)$$

⊗

$$\text{If, } L.H.L = R.H.L$$

Then we will say

Limit $\lim_{x \rightarrow a} f(x)$ exist.

$$\Rightarrow \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = L$$

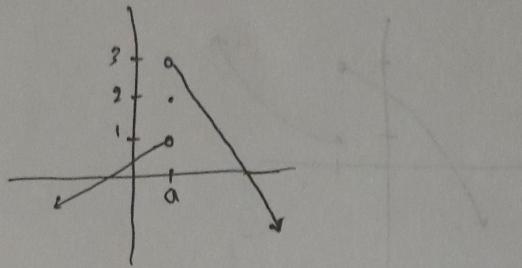
$$\therefore \lim_{x \rightarrow a} f(x) = L$$

⊗ If, $\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x)$

then, limit does not exist. (DNE).

(*)

$$f(x)$$



Hence,

$$\lim_{x \rightarrow a^-} f(x) = 1$$

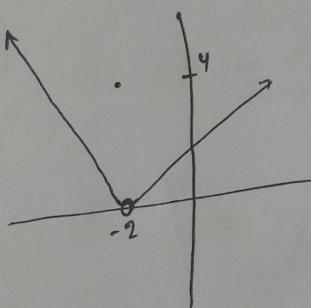
$$\therefore L.H.L \neq R.H.L$$

$$\lim_{x \rightarrow a^+} f(x) = 3$$

\therefore Limit does not exist.

$$f(a) = 2$$

(*)



Hence,

$$\lim_{x \rightarrow -2^+} f(x) = 0$$

$$\therefore L.H.L = R.H.L$$

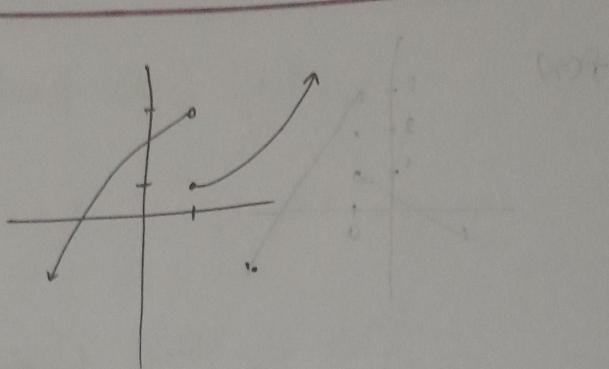
$$\lim_{x \rightarrow -2^-} f(x) = 0$$

\therefore Limit exists.

$$f(-2) = 4$$

$$\therefore \lim_{x \rightarrow -2} f(x) = 0$$

(*)



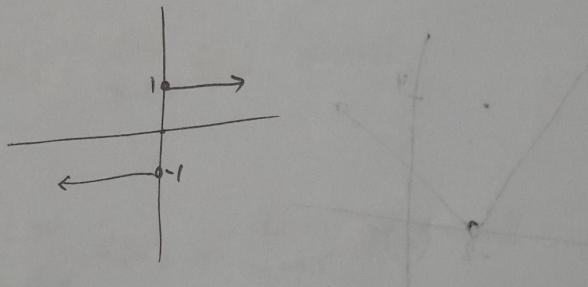
Hence,

$$\lim_{x \rightarrow 3^-} f(x) = 5 \quad \left| \begin{array}{l} \text{from left} \\ \text{f(x) is increasing} \end{array} \right. \quad \therefore L.H.L \neq R.H.L.$$

$$\lim_{x \rightarrow 3^+} f(x) = 2 \quad \left| \begin{array}{l} \text{from right} \\ \text{f(x) is increasing} \end{array} \right. \quad \therefore \text{Limit does not exist.}$$

$$f(x) = 2$$

(*)



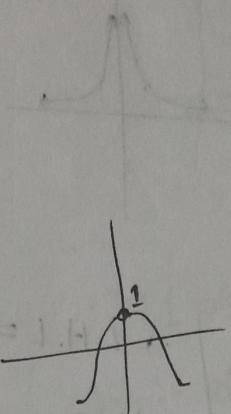
Hence,

$$\lim_{x \rightarrow 0^-} f(x) = -1 \quad \left| \begin{array}{l} \text{from left} \\ \text{f(x) is decreasing} \end{array} \right. \quad \therefore L.H.L \neq R.H.L.$$

$$\lim_{x \rightarrow 0^+} f(x) = 1 \quad \left| \begin{array}{l} \text{from right} \\ \text{f(x) is increasing} \end{array} \right. \quad \therefore \text{Limit does not exist.}$$



$$\lim_{n \rightarrow 0} \frac{\sin x}{n} = ?$$



$$\lim_{n \rightarrow 0} \frac{\sin x}{n} = 1$$

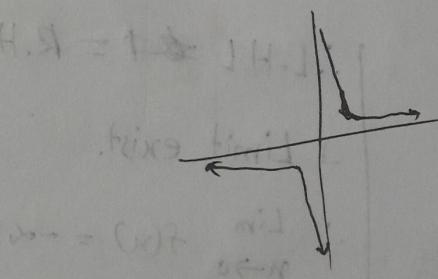
$$\lim_{n \rightarrow 0^+} \frac{\sin x}{n} = 1$$

$f(0) = \text{undefined}$

$$\begin{cases} \therefore L.H.L = R.H.L \\ \therefore \text{Limit exist.} \\ \therefore \lim_{n \rightarrow 0} \frac{\sin x}{n} = 1 \end{cases}$$



$$f(x) = \frac{1}{x}$$



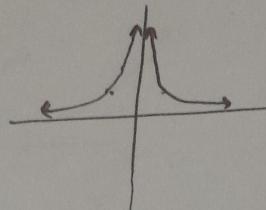
Hence,

$$\lim_{n \rightarrow 0^+} f(n) = \infty$$

$$\lim_{n \rightarrow 0^-} f(n) = -\infty$$

$$\begin{cases} \therefore L.H.L \neq R.H.L \\ \therefore \text{Limit does not exist.} \end{cases}$$

$$f(x) = \frac{1}{x^2}$$



Hence,

$$\lim_{x \rightarrow 0^-} f(x) = \infty$$

$$\lim_{x \rightarrow 0^+} f(x) = \infty$$

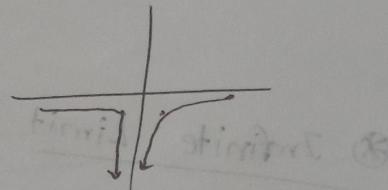
$$\therefore L.H.L = R.H.L$$

\therefore Limit exist.

$$\therefore \lim_{x \rightarrow 0} f(x) = \infty$$

$$\textcircled{*} \quad f(x) = -\frac{1}{x^2}$$

$$L = \frac{\text{R.H.L}}{x} \text{ as } x \rightarrow 0$$



Hence,

$$\lim_{x \rightarrow 0^-} f(x) = -\infty$$

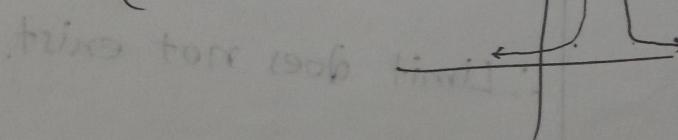
$$\lim_{x \rightarrow 0^+} f(x) = -\infty$$

$$\therefore L.H.L \neq R.H.L$$

\therefore Limit exist.

$$\therefore \lim_{x \rightarrow 0} f(x) = -\infty$$

$$\textcircled{*} \quad f(x) = \frac{1}{(x-2)^2}$$



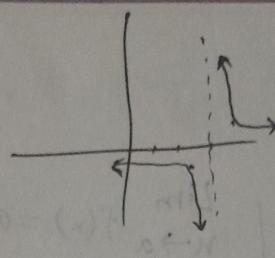
$$\lim_{x \rightarrow 2^-} f(x) = \infty$$

Limit exist.

$$\lim_{x \rightarrow 2^+} f(x) = \infty$$

$$\lim_{x \rightarrow 2} f(x) = \infty$$

$$\textcircled{*} \quad f(x) = \frac{1}{x-3}$$



Hence,

$$\lim_{n \rightarrow 3^-} f(n) = -\infty \quad \left| \begin{array}{l} \therefore L.H.L \neq R.H.L \\ \therefore \text{Limit does not exist.} \end{array} \right.$$

$$\lim_{n \rightarrow 3^+} f(n) = +\infty$$

1.2 / Computing Limit

Two Limit

$$\textcircled{*} \quad f(n) = k \quad \left| \begin{array}{l} \text{L.H.L} = (w)P \underset{n \leftarrow w}{\cancel{k}} \quad R.H.L = (w)T \underset{n \leftarrow w}{\cancel{k}} \end{array} \right. \quad \text{if } w \in N$$

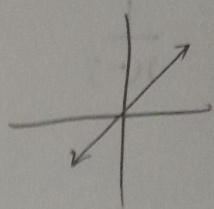
$$\therefore \lim_{n \rightarrow a} f(n) = k \quad \left| \begin{array}{l} \text{L.H.L} = (w)P \underset{n \leftarrow w}{\cancel{k}} \quad R.H.L = (w)T \underset{n \leftarrow w}{\cancel{k}} \\ \pm (w)P \underset{n \leftarrow w}{\cancel{k}} \pm (w)T \underset{n \leftarrow w}{\cancel{k}} = ((w)P \pm (w)T) \underset{n \leftarrow w}{\cancel{k}} \end{array} \right. \quad (i)$$

$$\lim_{n \rightarrow 1} f(n) = k \quad \left| \begin{array}{l} \text{L.H.L} = \\ \text{R.H.L} = \end{array} \right.$$

$$\lim_{n \rightarrow -2} f(n) = k \quad \left| \begin{array}{l} \text{L.H.L} = (w)P \underset{n \leftarrow w}{\cancel{k}} \quad \text{R.H.L} = (w)T \underset{n \leftarrow w}{\cancel{k}} \\ (w)P \underset{n \leftarrow w}{\cancel{k}} \cdot (w)T \underset{n \leftarrow w}{\cancel{k}} = ((w)P \cdot (w)T) \underset{n \leftarrow w}{\cancel{k}} \end{array} \right. \quad (ii)$$

$$\lim_{n \rightarrow -3} g = 3 \quad \left| \begin{array}{l} \text{L.H.L} = \\ \text{R.H.L} = \end{array} \right.$$

$$\textcircled{X} \quad f(n) = n$$



$$f(0) = 0$$

$$\lim_{n \rightarrow a} f(n) = a$$

$$f(1) = 1$$

$$\lim_{n \rightarrow 2} f(n) = 2.$$

$$f(-2) = -2$$

$$\lim_{n \rightarrow 3} f(n) = 3$$



$$\lim_{n \rightarrow 0^+} \frac{1}{n} = \infty$$

$$\lim_{n \rightarrow 0^-} \frac{1}{n} = -\infty$$

Limit Laws:

If, $\lim_{n \rightarrow a} f(n) = L_1$ and $\lim_{n \rightarrow a} g(n) = L_2$

$$\text{i) } \lim_{n \rightarrow a} (f(n) \pm g(n)) = \lim_{n \rightarrow a} f(n) \pm \lim_{n \rightarrow a} g(n) \\ = L_1 \pm L_2$$

$$\text{ii) } \lim_{n \rightarrow a} (f(n) \cdot g(n)) = \lim_{n \rightarrow a} f(n) \cdot \lim_{n \rightarrow a} g(n) \\ = L_1 \cdot L_2$$

$$\textcircled{iii} \quad \lim_{n \rightarrow a} \frac{f(n)}{g(n)} = \frac{\lim_{n \rightarrow a} f(n)}{\lim_{n \rightarrow a} g(n)} = \frac{L_1}{L_2}, \quad L_2 \neq 0$$

$$\textcircled{iv} \quad \lim_{n \rightarrow a} \sqrt[n]{f(n)} = \sqrt[n]{\lim_{n \rightarrow a} f(n)} = \sqrt[n]{L_1}$$

$$\textcircled{v} \quad \lim_{n \rightarrow a} k \cdot f(n) = k \cdot \lim_{n \rightarrow a} f(n) = k \cdot L_1$$

$$\textcircled{vi} \quad \lim_{n \rightarrow a} (f(n))^n = \left(\lim_{n \rightarrow a} f(n) \right)^n = (L_1)^n$$

$$\textcircled{vii} \quad \lim_{n \rightarrow 1} (n^2 + n + 1)$$

$$= \lim_{n \rightarrow 1} n^2 + \lim_{n \rightarrow 1} n + \lim_{n \rightarrow 1} 1 = 1^2 + 1 + 1 = 3$$

equivalent method to find

$$0 < (x)_B \cdot \frac{(x)_A}{(x)_B} = \frac{(x)_A}{(x)_B} \stackrel{n \leftarrow n}{\underset{n \leftarrow n}{\longrightarrow}} \frac{(x)_A}{(x)_B} \stackrel{n \leftarrow n}{\underset{n \leftarrow n}{\longrightarrow}} = \frac{(x)_A}{(x)_B} \stackrel{n \leftarrow n}{\underset{n \leftarrow n}{\longrightarrow}} = (x)_B$$

⊗ Limit of Polynomial Function:

$$f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

$$\lim_{x \rightarrow 1} f(x) = f(1)$$

(*)

$$\begin{aligned} \lim_{x \rightarrow -2} (x^2 + 2x + 3) &= (-2)^2 + 2(-2) + 3 \\ &= 4 - 4 + 3 \\ &= 3 \end{aligned}$$

Ans

(*)

$$\begin{aligned} \lim_{x \rightarrow 3} (3x^2 + 6x - 3) &= 3 \cdot 3^2 + 6 \cdot 3 - 3 \\ &= 3 \cdot 27 + 18 - 3 \\ &= 81 + 18 - 3 \\ &= 96 \end{aligned}$$

Ans

⊗ Limit of Rational Function:

$$f(x) = \frac{P(x)}{Q(x)}$$

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} \frac{P(x)}{Q(x)} = \frac{\lim_{x \rightarrow a} P(x)}{\lim_{x \rightarrow a} Q(x)} = \frac{P(a)}{Q(a)} ; \quad Q(a) \neq 0$$

$$\textcircled{1} \quad \lim_{x \rightarrow 2} \frac{x^2+3}{x^2+6} = \frac{\lim_{x \rightarrow 2} x^2+3}{\lim_{x \rightarrow 2} x^2+6} = \frac{2^2+3}{2^2+6} = \frac{7}{14} = \frac{1}{2}$$

$$\textcircled{2} \quad \lim_{x \rightarrow 2} \frac{x-3}{x-2}$$

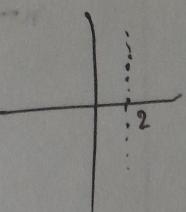
Now,

$$\lim_{x \rightarrow 2^-} \frac{x-3}{x-2} = \infty$$

$$\lim_{x \rightarrow 2^+} \frac{x-3}{x-2} = -\infty$$

L.H.L \neq R.H.L.

\therefore Limit does not exist.



$$0 \leftarrow (0)R : \frac{(0)q}{(0)R} = \frac{(0)q}{(0)R} \text{ and } 0 \leftarrow (0)R$$

$$\frac{2+q}{2+R} \rightarrow (0)P \quad (0)$$

$$2+q \rightarrow \frac{2+q}{2+R} \rightarrow \frac{2+q}{2+R} \approx (0)P \quad \text{and} \quad 2+R$$

⊗ Finding Limit for Polynomial Function

$$f(x) = x^3 + 2x^2 + 3$$

$$\begin{aligned} \lim_{x \rightarrow 1} f(x) &= f(1) \\ &= 1^3 + 2 \cdot 1^2 + 3 \\ &= 1 + 2 + 3 \\ &= 6 \end{aligned}$$

⊗ Finding Limit for Rational Function.

$$f(x) = \frac{P(x)}{Q(x)}$$

$$\lim_{x \rightarrow a} \frac{P(x)}{Q(x)} = \frac{P(a)}{Q(a)} ; Q(a) \neq 0$$

$$⊗ f(x) = \frac{x^2 + 6}{x - 2}$$

$$\lim_{x \rightarrow 1} f(x) = \frac{1^2 + 6}{1 - 2} = \frac{7}{-1} = -7$$

$$\textcircled{*} \quad f(x) = \frac{x+2}{x-3}$$

$$\lim_{x \rightarrow 3} f(x) = ?$$

Hence,

$$\lim_{x \rightarrow 3^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 3^+} f(x) = \infty$$

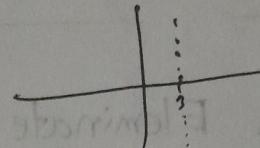
$$\textcircled{*} \quad \lim_{x \rightarrow 3} \frac{1}{|x-3|} = ?$$

Hence,

$$\lim_{x \rightarrow 3^-} \frac{1}{|x-3|} = \infty$$

$$\lim_{x \rightarrow 3^+} \frac{1}{|x-3|} = \infty$$

without graph



$f(2.99) = \frac{4.09}{-0.01} = -499$
$f(2.999) = -4999$
$f(3.001) = \frac{5.001}{0.001} = 50001$
$f(3.00001) = 5000001$

L.H.L. \neq R.H.L.

\Rightarrow Limit does not exist.

$$\frac{(x-2)(x+2)}{(x-2)} = x+2$$

$$f(2.99) = \frac{1}{0.001} = 1000$$

$$f(2.999) = 100000$$

$$f(3.001) = \frac{1}{0.001} = 1000$$

$$f(3.00001) = 1000000$$

\therefore L.H.L. = R.H.L

$$\therefore \lim_{x \rightarrow 3} \frac{1}{|x-3|} = \infty$$

$$\frac{1}{(x-2)(x+2)} = \frac{1}{4x}$$

$$\frac{1}{x-2} = \frac{1}{4}$$

④ Finding Limit involving Radicals:

Rules: Eliminate the zero denominator.

$$\textcircled{1} \lim_{x \rightarrow \sqrt{3}} \frac{x^2 - 3}{x - \sqrt{3}}$$

$$= \lim_{x \rightarrow \sqrt{3}} \frac{x^2 - (\sqrt{3})^2}{(x - \sqrt{3})}$$

$$= \lim_{x \rightarrow \sqrt{3}} \frac{(x + \sqrt{3})(x - \sqrt{3})}{(x - \sqrt{3})}$$

$$= \lim_{x \rightarrow \sqrt{3}} (x + \sqrt{3})$$

$$= \sqrt{3} + \sqrt{3}$$

$$= 2\sqrt{3}$$

Ans

$$\textcircled{2} \lim_{y \rightarrow 4} \frac{y - 4}{2 - \sqrt{y}}$$

$$= \lim_{y \rightarrow 4} \frac{(2 - \sqrt{y})(2 + \sqrt{y})}{2 - \sqrt{y}}$$

$$= \lim_{y \rightarrow 4} (2 + \sqrt{y})$$

$$= 2 + \sqrt{4}$$

$$= 4$$

Ans

⑤

$$\lim_{y \rightarrow c^-} \frac{y+6}{y^2 - 36}$$

$$= \lim_{y \rightarrow c^-} \frac{y+6}{(y+6)(y-6)}$$

$$= \lim_{y \rightarrow c^-} \frac{1}{y-6}$$

$$= -\infty$$

$$\lim_{y \rightarrow c^+} \frac{1}{y-c} = \infty$$

⊗ Limit of Piece-wise defined function.

$$\text{If, } f(x) = \begin{cases} x-1 & x \leq 3 \\ 3x-7 & x > 3 \end{cases}$$

$$\text{i)} \lim_{x \rightarrow 3^-} f(x) \quad \text{ii)} \lim_{x \rightarrow 3^+} f(x) \quad \text{iii)} \lim_{x \rightarrow 3} f(x) \quad \text{iv)} \lim_{x \rightarrow 5} f(x)$$

$$\text{v)} \lim_{x \rightarrow 1} f(x)$$

⇒

$$\text{i)} \lim_{x \rightarrow 3^-} f(x)$$

$$= \lim_{x \rightarrow 3^-} x-1$$

$$= 3-1$$

$$= 2$$

$$\text{ii)} \lim_{x \rightarrow 3^+} f(x)$$

$$= \lim_{x \rightarrow 3^+} 3x-7$$

$$= 3 \cdot 3 - 7$$

$$= 9-7$$

$$= 2$$

$$\therefore \lim_{x \rightarrow 3} f(x) = 2$$

$$\text{iv)} \lim_{x \rightarrow 5} f(x)$$

$$= \lim_{x \rightarrow 5} 3x-7$$

$$= 3 \cdot 5 - 7$$

$$= 15-7$$

$$= 8$$

$$\text{v)} \lim_{x \rightarrow 1} f(x)$$

$$= \lim_{x \rightarrow 1} x-1$$

$$= 0$$

Q) If,

$$f(x) = \begin{cases} x-2 & x < 0 \\ x^2 & 0 \leq x \leq 2 \\ 2x & x > 2 \end{cases}$$

$$\text{i) } \lim_{x \rightarrow 0^-} f(x) \quad \text{ii) } \lim_{x \rightarrow 1} f(x) \quad \text{iii) } \lim_{x \rightarrow 2} f(x)$$

\Rightarrow

$$\text{i) } \lim_{x \rightarrow 0^-} f(x) = ?$$

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} x-2 \\ &= 0-2 \\ &= -2 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} x^2 \\ &= 0^+ \\ &= 0 \end{aligned}$$

$\therefore L.H.L \neq R.H.L$

\therefore Limit does not exist.

$$\text{ii) } \lim_{x \rightarrow 1} f(x)$$

$$= \lim_{x \rightarrow 1} x^2$$

$$= 1^2$$

$$= 1$$

$$\text{iii) } \lim_{x \rightarrow 2} f(x) = ?$$

$$= \lim_{x \rightarrow 2} x^2$$

$$= 2^2$$

$$= 4$$

$$\begin{aligned} \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} 2x \\ &= 2 \cdot 2 \\ &= 4 \end{aligned}$$

$\therefore L.H.L = R.H.L$

$$\therefore \lim_{x \rightarrow 2} f(x) = 4.$$