4)

$$x\frac{dy}{dx}+y=(xy)^{3/2}$$

$$\Rightarrow x\frac{dy}{dx}+y = x^{3/2}\cdot y^{3/2}$$

$$\Rightarrow xy^{-3/2}\frac{dy}{dx}+y^{-\frac{1}{2}} = x^{3/2} \quad \text{--- (i)}$$

Let,

$$y^{-\frac{1}{2}} = u$$

$$-\frac{1}{2}y^{-3/2}\frac{dy}{dx} = \frac{du}{dx}$$

$$\text{(i)} \Rightarrow \quad -2x\frac{du}{dx}+u = x^{3/2}$$

$$\Rightarrow \frac{du}{dx} - \frac{1}{2x}\cdot u = \frac{x\cdot x^{1/2}}{-2x}$$

$$\therefore \frac{du}{dx} - \frac{1}{2x}\cdot u = -\frac{1}{2}\sqrt{x} \quad \text{--- (ii)}$$

Here $p(x) = -\frac{1}{2x}$

$$\therefore I.F. = e^{\int -\frac{1}{2x}dx} = e^{-\frac{1}{2}\ln x} = \frac{1}{\sqrt{x}}$$

(ii) $\times I.F.$,

$$\frac{1}{\sqrt{x}}\frac{du}{dx} - \frac{1}{2x\sqrt{x}}\cdot u = -\frac{1}{2}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\cdot u\right) = -\frac{1}{2}$$

$$\Rightarrow \int d\left(\frac{u}{\sqrt{x}}\right) = \int -\frac{1}{2}\,dx$$

$$\Rightarrow \frac{u}{\sqrt{x}} = -\frac{1}{2}x + c$$

$$\therefore \frac{1}{\sqrt{y}\sqrt{x}} = -\frac{1}{2}x + c$$

$$\cancel{\Rightarrow c \Rightarrow}$$

Given,

$$y(1) = 4$$

Then, $c = \dfrac{1}{\sqrt{4}\sqrt{1}} + \dfrac{1}{2}\cdot 1$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$\therefore \frac{1}{\sqrt{x}\sqrt{y}} = -\frac{1}{2}x + 1$$

Ans

# H.W. from Lecture - 7

## Application of ODE (1st)

From Zill's Book

Page- 84

Example- 1:

According to the growth and decay problem, we know,

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \frac{dP}{dt} = kP$$

$$\Rightarrow \int \frac{dP}{P} = \int k\,dt$$

$$\Rightarrow \ln P = kt + C_1$$

$$\Rightarrow P = e^{kt+C_1} = e^{kt} \cdot e^{C_1} = ce^{kt}$$

$$\therefore P(t) = ce^{kt} \quad \cdots \text{(i)}$$

at, $t = 0,\ P = P_0$

$$\therefore P_0 = ce^0 = c$$

$$\text{(i)} \Rightarrow \quad P(t) = P_0 e^{kt} \quad \cdots\cdots \text{(ii)}$$

at, $t = 1$ hour, $P = \frac{3}{2} P_0$

$$\therefore \frac{3}{2} P_0 = P_0 e^{k \cdot 1}$$

$$\Rightarrow e^k = \frac{3}{2}$$

$$\therefore k = \ln\left(\frac{3}{2}\right)$$

$$\text{(ii)} \Rightarrow P(t) = P_0 e^{\ln\left(\frac{3}{2}\right) t} \quad \cdots \text{(iii)}$$

Let, ~~P~~ $t^*$ ~~is~~ be the time for which $P = 3P_0$

$$\text{(iii)} \Rightarrow 3P_0 = P_0 e^{\ln\left(\frac{3}{2}\right) t^*}$$

$$e^{\ln\left(\frac{3}{2}\right) t^*} = 3$$

$$\ln\left(\frac{3}{2}\right) t^* = \ln 3$$

$$\therefore t^* = \frac{\ln 3}{\ln(3/2)} \simeq 2.71 \text{ hour} \simeq 2 \text{ hour } 43 \text{ minutes}$$

Example-2:

Let, $A$ is the ~~number~~ amount of isotope at time, $t$.

From, growth and decay problem, we know,

$$\frac{dA}{dt} \propto A$$

$$\Rightarrow \frac{dA}{dt} = kA$$

$$\Rightarrow \int \frac{dA}{A} = \int k\, dt$$

$$\Rightarrow \ln A = kt + C_1$$

$$\Rightarrow A = e^{kt+C_1} = e^{kt} \cdot e^{C_1} = ce^{kt}$$

$$\therefore A(t) = ce^{kt} \quad \cdots (i)$$

at, $t = 0$, $A = A_0$

$$\therefore A_0 = ce^0 = c$$

$$(i) \Rightarrow A(t) = A_0 e^{kt} \quad \cdots (ii)$$

Here remaining isotope after 15 years $= (100 - 0.043\%)$
$= 99.957\%$

Hence,

when $t = 15$ years, $A = 0.99957 A_0$

$$\therefore 0.99957 A_0 = A_0 e^{k \cdot 15}$$

$$\Rightarrow 0.99957 = e^{15k}$$

$$\Rightarrow 15k = \ln(0.99957)$$

$$\therefore k = \frac{\ln(0.99957)}{15}$$

$$(ii) \Rightarrow A(t) = A_0 e^{\frac{\ln(0.99957)}{15} \cdot t}$$

Let, $t^*$ be the time for which $A = 0.5A_0$

$$\therefore \quad 0.5A_0 = A_0 e^{\frac{\ln(0.99957)}{15} \cdot t^*}$$

$$\Rightarrow \quad \frac{\ln(0.99957)}{15} \cdot t^* = \ln(0.5)$$

$$\Rightarrow \quad t^* = \frac{15 \cdot \ln(0.5)}{\ln(0.99957)}$$

$$\cong 24,174 \text{ years}$$

Ans

Example-3:

Let $x$ be the ~~number of people~~ amount of C-14 at time $t$.

Let, $x_0$ be the initial amount of C-14 at $t=0$.

We know,

half life of C-14 is 5730 years.

~~That means,~~ $\frac{1}{2}\cancel{A_0}$ -

According to the growth and decay problem we know

$$\frac{dx}{dt} \propto x$$

$$\Rightarrow \frac{dx}{dt} = kx$$

$$\Rightarrow \int \frac{dx}{x} = \int k\, dt$$

$$\Rightarrow \ln x = kt + c_1$$

$$\Rightarrow x = e^{kt+c_1} = e^{kt} \cdot e^{c_1} = ce^{kt}$$

$$\therefore x(t) = ce^{kt} \quad \cdots \text{(i)}$$

at, $t = 0$, $x = x_0$

$$\therefore x_0 = ce^0$$

$$\therefore c = x_0$$

$$\text{(i)} \Rightarrow x(t) = x_0 e^{kt} \quad \cdots \text{(ii)}$$

at, $t = 5730$ years, $x = 0.5x_0$

$$\therefore 0.5x_0 = x_0 e^{k \cdot 5730}$$

$$\Rightarrow e^{k \cdot 5730} = 0.5$$

$$\Rightarrow k \cdot 5730 = \ln(0.5)$$

$$\therefore k = \frac{\ln(0.5)}{5730}$$

$$\text{(ii)} \Rightarrow$$

$$\therefore x(t) = x_0 e^{\frac{\ln(0.5)}{5730} \cdot t} \quad \cdots \text{(iii)}$$

Let,

$t^*$ be the time for which $x = 0.001 x_0$

$\cancel{\therefore 0.001 x_0 = x_0 e}$

$$\therefore 0.001 x_0 = x_0 e^{\frac{\ln(0.5)}{5730} \cdot t^*}$$

$$\Rightarrow \frac{\ln(0.5)}{5730} \cdot t^* = \ln(0.001)$$

$$\Rightarrow t^* = \frac{5730 \times \ln(0.001)}{\ln(0.5)}$$

$$\approxeq 57104 \text{ years.}$$

Exercise 3.1

1)

Let, $P$ is the number of people in time $t$.

$P_0$ is the initial number of people at time $t=0$.

According growth and decay problem, we know,

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \frac{dP}{dt} = kP$$

$$\Rightarrow \int \frac{dP}{P} = \int k\,dt$$

$\Rightarrow \quad \ln P = \cancel{kx} \; kt + c_1$

$\Rightarrow \quad P = e^{kt+c_1} = e^{kt} \cdot e^{c_1} = ce^{kt}$

$\therefore P(t) = \cancel{c}\, ce^{kt} \quad \ldots\ldots (i)$

at,

$t = 0, \quad P = P_0$

$\therefore P_0 = ce^0 = c$

$\therefore c = P_0$

$\therefore (i) \Rightarrow \quad P(t) = P_0 e^{kt} \quad \ldots (ii)$

at, $t = 5$ years, $\quad P = 2P_0$

$\therefore 2P_0 = P_0 e^{k \cdot 5}$

$\Rightarrow 2 = e^{5k}$

$\Rightarrow 5k = \ln 2$

$\therefore k = \frac{\ln 2}{5}$

$\therefore (ii) \Rightarrow \quad P(t) = P_0 e^{\frac{\ln 2}{5} \cdot t} \quad \ldots (iii)$

Let,

$t^*$ is the time when, $P = 3P_0$

$\therefore (iii) \Rightarrow$

$3P_0 = P_0 e^{\frac{\ln 2}{5} \cdot t^*}$

$\Rightarrow \frac{\ln 2}{5} t^* = \ln 3$

$\Rightarrow t^* = \frac{5 \ln 3}{\ln 2}$

$\simeq 7.92$ years

2)

From question-1:

$$P(t) = P_0 e^{\frac{\ln 2}{5} t} \quad \cdots \text{(i)}$$

at,

$t = 3$ years, $P = 10,000$ people

$$\therefore \quad 10,000 = P_0 e^{\frac{\ln 2}{5} \cdot 3}$$

$$\Rightarrow \cancel{e^{\frac{3 \cdot \ln 2}{5}} = \frac{10000}{P_0}}$$

$$\therefore \quad P_0 = \frac{10,000}{e^{\frac{\ln 2}{5} \cdot 3}} \approx 6598 \text{ people}$$

Ans

population in 10 yeas,

$$P = 6598 \, e^{\frac{\ln 2}{5} \cdot 10}$$

$$= 26392 \text{ people}$$

Population growth rate

$$\frac{dP}{dt} = \frac{d}{dt}\left(6598 \, e^{\frac{\ln 2}{5} t}\right)$$

$$= 6598 \cdot \frac{\ln 2}{5} \cdot e^{\frac{\ln 2}{5} t}$$

when, $t = 10$,

rate of population = 1829 people per year

3)

Let,

p is the number of population in time t,

According to the growth and decay problem we know,

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \frac{dP}{dt} = kP$$

$$\Rightarrow \int \frac{dP}{P} = \int k\,dt$$

$$\Rightarrow \ln P = kt + c_1$$

$$\Rightarrow P = e^{kt+c_1} = e^{kt} \cdot e^{c_1} = ce^{kt}$$

$$\therefore P(t) = ce^{kt} \cdots (i)$$

when,

$t=0$, $P = 500$

$$\therefore 500 = ce^{0} = c$$

$$\therefore (i) \Rightarrow \quad P(t) = 500\, e^{kt} \cdots (ii)$$

when,

$t = 10$ years, $P = 500 + 500\frac{15}{100} = 575$

$$\therefore 575 = 500\, e^{10k}$$

$$\Rightarrow e^{10k} = \frac{575}{500}$$

$$\Rightarrow 10k = \ln\left(\frac{575}{500}\right)$$

$$\therefore k = \frac{\ln\left(\frac{575}{500}\right)}{10}$$

$\therefore$ (ii) $\Rightarrow$ $p(t) = 500\, e^{\frac{\ln(575/500)}{10} t}$

population at time $t = 30$ years

$$P = 500\, e^{\frac{\ln \frac{575}{500}}{10} \cdot 30}$$

$$\cong 760 \text{ people}$$

rate of population,

$$\frac{dP}{dt} = \frac{d}{dt}\left(500\, e^{\frac{\ln\left(\frac{575}{500}\right)}{10} \cdot t}\right)$$

$$= 500 \cdot \frac{\ln\left(\frac{575}{500}\right)}{10} \cdot e^{\frac{\ln\left(\frac{575}{500}\right)}{10} \cdot t}$$

at $t = 30$ years

$$= 500 \cdot \frac{\ln\left(\frac{575}{500}\right)}{10} \cdot e^{\frac{\ln\left(\frac{575}{500}\right)}{10} \cdot 30}$$

$$= 11 \text{ peoples / year}$$

4|

Let,

$x$ is the number of bacteria at time $t$.

$x_0$ is the initial number of the bacteria at time $t = 0$.

According to growth and decay problem, we know,

$$\frac{dx}{dt} \propto x$$

$$\Rightarrow \frac{dx}{dt} = kx$$

$$\Rightarrow \int \frac{dx}{x} = \int k\,dt$$

$$\Rightarrow \ln x = kt + c_1$$

$$\Rightarrow x = e^{kt+c_1} = ce^{kt}$$

$$\therefore x(t) = ce^{kt} \quad \cdots (i)$$

at,

$t=0, \quad x = x_0$

$$\therefore x_0 = ce^0 = c$$

$$\therefore (i) \Rightarrow x(t) = x_0 e^{kt} \quad \cdots (ii)$$

at,

$t = 3$ hours, $x = 400$

$$\therefore 400 = x_0 e^{3k}$$

$$\Rightarrow e^{3k} = \frac{400}{x_0}$$

$$\Rightarrow 3k = \ln\left(\frac{400}{x_0}\right)$$

$$\therefore k = \frac{\ln\left(\frac{400}{x_0}\right)}{3}$$

$$\therefore (ii) \Rightarrow x(t) = x_0 e^{\frac{\ln\left(\frac{400}{x_0}\right)}{3} t} \quad \cdots (iii)$$

at,

$t = 10$ hours, $x = 2000$

$$\therefore 2000 = x_0 e^{\frac{\ln\left(\frac{400}{x_0}\right) \cdot 10}{3}}$$

$$\Rightarrow \frac{\ln\left(\frac{400}{x_0}\right) \cdot 10}{3} = \ln\left(\frac{2000}{x_0}\right)$$

$$\Rightarrow \ln\left(\frac{400}{x_0}\right)^{10} = \ln\left(\frac{2000}{x_0}\right)^3$$

at,

$t = 3$ hours, $x = 400$

$\therefore 400 = c e^{3k}$

$\therefore c = \frac{400}{e^{3k}}$

$\therefore$ (i) $\Rightarrow$ $x(t) = \frac{400}{e^{3k}} \cdot e^{kt} = 400\, e^{kt-3k}$

$\therefore x(t) = 400\, e^{kt-3k}$ .... (ii)

at,

$t = 10$ hours, $x = 2000$

$\therefore 2000 = 400\, e^{k(10-3)} = 400\, e^{7k}$

$\Rightarrow e^{7k} = \frac{2000}{400}$

$\Rightarrow 7k = \ln\left(\frac{2000}{400}\right)$

$\therefore k = \frac{\ln\left(\frac{2000}{400}\right)}{7}$

$\therefore$ (ii) $\Rightarrow$ $x(t) = 400\, e^{\frac{\ln\left(\frac{2000}{400}\right)}{7} \cdot (t-3)}$ .... (iii)

initial number of bacteria, when $t = 0$,

$x = 400\, e^{0} = 400\, e^{\frac{\ln\left(\frac{2000}{400}\right)}{7} \cdot (-3)}$

$\simeq 201$

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Let,

$x$ is the ~~number of~~ amount of isotop at time, $t$.

according to the growth and decay problem, we know,

$$\frac{dx}{dt} \propto x$$

$$\Rightarrow \frac{dx}{dt} = kx$$

$$\Rightarrow \int \frac{dx}{x} = \int k\,dt$$

$$\Rightarrow \ln x = kt + c_1$$

$$\Rightarrow x = e^{kt+c_1} = ce^{kt}$$

$$\therefore x(t) = ce^{kt} \quad \ldots (i)$$

at, $t = 0$, $x = 1$ gram

$$\therefore 1 = ce^0 = c$$

$$(i) \Rightarrow x(t) = e^{kt} \quad \ldots (ii)$$

at, $t = 3.3$ hours, $x =$ ~~[illegible]~~ $0.5$ gram

$$\therefore 0.5 = e^{3.3k}$$

$$\Rightarrow 3.3k = \ln(0.5)$$

$$\therefore k = \frac{\ln(0.5)}{3.3}$$

$$(ii) \Rightarrow x(t) = e^{\frac{\ln(0.5)}{3.3}t} \quad \ldots (iii)$$

Let,

$t^*$ is the ~~m~~ time when $x = 0.1$ gram.

$$0.1 = e^{\frac{\ln(0.5)}{3.3} \cdot t^*}$$

$$\Rightarrow \frac{\ln(0.5)}{3.3} \cdot t^* = \ln(0.1)$$

$$\Rightarrow t^* = \frac{3.3 \cdot \ln(0.1)}{\ln(0.5)} \cong 10.96 \text{ hours}$$

6)

Let,

$x$ is the mass of the substane at time, $t$.

We know,

$$\frac{dx}{dt} \propto x$$

$$\Rightarrow \frac{dx}{dt} = kx$$

$$\Rightarrow \int \frac{dx}{x} = \int k\,dt$$

$$\Rightarrow \ln(x) = kt + c_1$$

$$\Rightarrow x = e^{kt+c_1} = ce^{kt}$$

$$\therefore x(t) = ce^{kt} \text{ --- (i)}$$

at,

$t = 0, \quad x = 100$ milligram.

$\Rightarrow \quad 100 = ce^{0} = c$

$\therefore (i) \Rightarrow \quad x(t) = 100\, e^{kt} \quad \ldots (ii)$

at,

$t = \cancel{k}\, 6$ hours, $\quad x = (100 - 3) = 97$ miligram.

$\therefore \quad 97 = 100\, e^{6k}$

$\Rightarrow \quad e^{6k} = \frac{97}{100}$

$\Rightarrow \quad 6k = \ln \frac{97}{100}$

$\therefore k = \frac{\ln\left(\frac{97}{100}\right)}{6}$

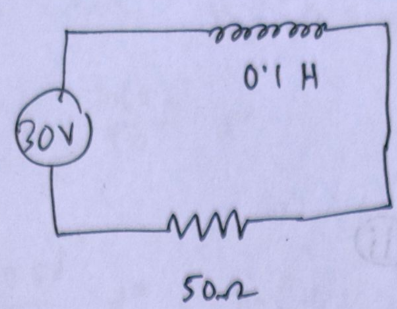
$\therefore (ii) \Rightarrow \quad x(t) = 100\, e^{\frac{\ln\left(\frac{97}{100}\right)}{6} \cdot t} \quad \ldots (iii)$

at, $t = 24$ hours, mass will be,

$x = 100\, e^{\frac{\ln(0.97)}{6} \cdot 24}$

$\cong 88.53$ milligram.

29|




We know,

$$L\frac{di}{dt} + iR = E$$

$$\Rightarrow 0.1\frac{di}{dt} + 50i = 30$$

$$\Rightarrow \frac{di}{dt} + 500i = 300 \quad \cdots (i)$$

Here

$$P(t) = 500$$

$$\therefore I.F. = e^{\int 500 dt} = e^{500t}$$

(i) × I.F,

$$e^{500t}\frac{di}{dt} + 500e^{500t}i = 300e^{500t}$$

$$\Rightarrow \frac{d}{dt}(e^{500t}\cdot i) = 300e^{500t}$$

$$\Rightarrow \int d(ie^{500t}) = \int 300e^{500t}\,dt$$

$$\Rightarrow ie^{500t} = \frac{300}{500}e^{500t} + c$$

$$\Rightarrow i = \frac{3}{5} + ce^{-500t}$$

Given,

$i(0) = 0,$

then, $0 = \frac{3}{5} + ce^{0}$

$\therefore c = -\frac{3}{5}$

$\therefore i(t) = \frac{3}{5}(1 - e^{-500t})$ Ans

if, $t = \infty$,

$i = \frac{3}{5}(1 - e^{-\infty})$

$= \frac{3}{5}$ Amp

Ans

30)

$$L\frac{di}{dt} + Ri = E(t)$$

$$\Rightarrow L\frac{di}{dt} + Ri = E_0 \sin \omega t$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{E_0 \sin \omega t}{L} \quad \text{--- (i)}$$

Here

$$P(t) = \frac{R}{L}$$

$$\therefore I.F. = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L}t}$$

(i) × I.F.,

$$e^{\frac{Rt}{L}}\frac{di}{dt} + \frac{R}{L} \cdot e^{\frac{Rt}{L}} i = \frac{E_0 \sin \omega t \cdot e^{\frac{R}{L} \cdot t}}{L}$$

$$\Rightarrow \frac{d}{dt}\left(i \cdot e^{\frac{R}{L}t}\right) = \frac{E_0 \sin \omega t \cdot e^{\frac{Rt}{L}}}{L}$$

$$\Rightarrow \int d\left(i e^{\frac{R}{L}t}\right) = \int \frac{E_0}{L} \cdot e^{\frac{Rt}{L}} \sin\omega t \, dt$$

$$\Rightarrow \quad i e^{\frac{Rt}{L}} = \frac{E_0 e^{\frac{R}{L}t}\left(R \sin\omega t - \omega L \cos\omega t\right)}{R^2 + \omega^2 L^2} + c$$

$$\therefore \quad i(t) = \frac{E_0 \left(R \sin\omega t - \omega L \cos\omega t\right)}{R^2 + \omega^2 L^2} + c e^{-\frac{R}{L}t}$$

Given

$$i(0) = i_0$$

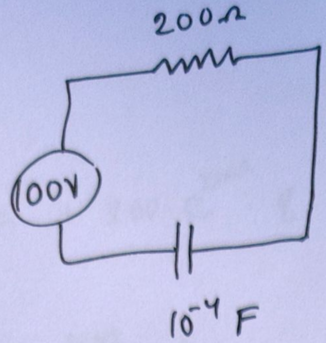
$$\therefore \quad i_0 = \frac{E_0 \left(R \cdot 0 - \omega L \cancel{\cos 0}\right)}{R^2 + \omega^2 L^2} + c e^{0}$$

$$\Rightarrow \quad i_0 = \frac{-E_0 \omega L \cancel{\cos 0}}{R^2 + \omega^2 L^2} + c$$

$$\therefore \quad c = i_0 + \frac{E_0 \omega L \cancel{\cos 0}}{R^2 + \omega^2 L^2}$$

$$\therefore \quad i(t) = \frac{E_0 \left(R \sin\omega t - \omega L \cos\omega t\right)}{R^2 + \omega^2 L^2} + \left(i_0 + \frac{E_0 \omega L}{R^2 + \omega^2 L^2}\right) e^{-\frac{R}{L}t}$$

31|




~~already solved in class.~~

Here,

$$R\frac{dq}{dt} + \frac{q}{c} = E$$

$$\Rightarrow 200\frac{dq}{dt} + 10^4 q = 100$$

$$\Rightarrow \frac{dq}{dt} + \frac{10000}{200} q = \frac{1}{2} \quad \cdots (i)$$

$$\therefore \frac{dq}{dt} + 50q = \frac{1}{2} \quad \cdots (i)$$

$$\therefore I.F. = e^{\int 50\,dt} = e^{50t}$$

(i)×I.F.,

$$e^{50t}\frac{dq}{dt} + 50e^{50t}q = \frac{1}{2}e^{50t}$$

$$\Rightarrow \frac{d}{dt}\left(qe^{50t}\right) = \frac{1}{2}e^{50t}$$

$$\Rightarrow \int d\left(qe^{50t}\right) = \frac{1}{2}\int e^{50t}\,dt$$

$$\Rightarrow qe^{50t} = \frac{1}{2}\cdot\frac{e^{50t}}{50} + C$$

$$\therefore q = \frac{1}{100} + Ce^{-50t}$$

$q(0) = 0,$

$$\therefore \quad 0 = \frac{1}{100} + c \cdot e^{0}$$

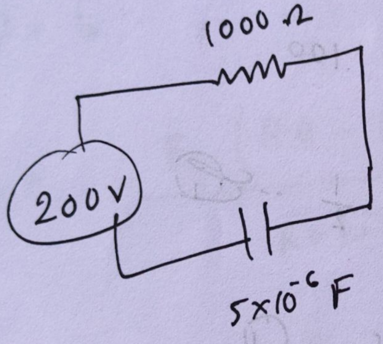
$$\therefore c = -\frac{1}{100}$$

$$\therefore q = \frac{1}{100}\left(1 - e^{-50t}\right)$$ Ans

We know,

$$i = \frac{dq}{dt} = \frac{1}{2} e^{-50t}$$ Ans

32|




Here,

$$R\frac{dq}{dt} + \frac{q}{C} = E$$

$$\Rightarrow 1000\frac{dq}{dt} + 200000\, q = 200$$

$$\Rightarrow \frac{dq}{dt} + 200\, q = \frac{1}{5} \quad \cdots (i)$$

$$\therefore I.F. = e^{\int 200\, dt} = e^{200t}$$

(i) × I.F,

$$e^{200t}\frac{dq}{dt} + 200\cdot e^{200t}\cdot q = \frac{e^{200t}}{5}$$

$$\Rightarrow \frac{d}{dt}\left(e^{200t}\cdot q\right) = \frac{e^{200t}}{5}$$

$$\Rightarrow \int d\left(q e^{200t}\right) = \int 0.2 e^{200t}\cdot dt$$

$$\Rightarrow q e^{200t} = 0.001 e^{200t} + C$$

$$\Rightarrow q(t) = 0.001 + c e^{-200t} \quad \ldots \text{(ii)}$$

Given, we know,

$$i(t) = \frac{dq}{dt} = -200 c e^{-200t}$$

Given, $i(0) = 0.4$

$$\therefore 0.4 = -200 c\cdot e^{0}$$

$$\Rightarrow c = -0.002$$

$$\therefore q(t) = 0.001 - 0.002 e^{-200t} \quad \ldots \text{(iii)}$$

Ans

When,

$t = 0.005$

$$q = 0.001 - 0.002\, e^{-200(0.005)}$$

$$= 2.64 \times 10^{-4} \text{ coul.}$$

$$i = -200 \times 0.002 \times e^{-200(0.005)}$$

$$= 0.15 \text{ Amp}$$

when, $t = \infty$,

$$q = 0.001 - 0.002\, e^{-\infty}$$

$$= 0.001 \text{ coul.}$$

33| confused

$\boxed{\text{H.W}} \Rightarrow$ from Lecture - 8

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Here,

$$\frac{dT}{dt} = k(T-10)$$

$$\int \frac{dT}{T-10} = \int k\, dt$$

$$\ln(T-10) = kt + c_1$$

$$T-10 = e^{kt+c_1} = ce^{kt}$$

$$\therefore T = 10 + ce^{kt} \quad \cdots (i)$$

Let,

$t=0, \quad T = 70°F$

$$\therefore 70 = 10 + ce^{0}$$

$$c = 70 - 10 = 60$$

$$\therefore (i) \Rightarrow T = 10 + 60e^{kt} \quad \cdots (ii)$$

At, $t = 30$ sec, $T = 50°F$

$$\therefore 50 = 10 + 60e^{30k}$$

$$\Rightarrow e^{30k} = \frac{50-10}{60} = \frac{2}{3}$$

$\Rightarrow 30k = \ln\left(\frac{2}{3}\right)$

$\therefore k = \dfrac{\ln\left(\frac{2}{3}\right)}{30}$

(ii) $\Rightarrow$ $T = 10 + 60e^{\frac{\ln(\frac{2}{3})}{30} \cdot t}$ ---- (iii)

When, $t = 60$ sec,

$T = 10 + 60\, e^{\frac{\ln(\frac{2}{3})}{30} \cdot 60}$

$\cong 36.67^\circ F$

Let,

$t^*$ is the time when, $T = 15^\circ F$

$\therefore 15 = 10 + 60\, e^{\frac{\ln(2/3)}{30} \cdot t^*}$

$\Rightarrow e^{\frac{\ln(2/3)}{30} \cdot t^*} = \dfrac{15-10}{60} = \dfrac{1}{12}$

$\Rightarrow \dfrac{\ln\left(\frac{2}{3}\right)}{30} \cdot t^* = \ln\left(\frac{1}{12}\right)$

$\therefore t^* = \dfrac{30 \cdot \ln\left(\frac{1}{12}\right)}{\ln\left(\frac{2}{3}\right)}$

$\cong 183.86$ sec

$\cong$ 3 minutes 4 sec

Ans

14|

We know,

$$\frac{dT}{dt} = k\,(T - T_{avg})$$

$$\Rightarrow \frac{dT}{dt} = k\,(T-5)$$

$$\Rightarrow \int \frac{dT}{T-5} = \int k\,dt$$

$$\Rightarrow \ln(T-5) = kt + c_1$$

$$\Rightarrow T-5 = e^{kt+c_1} = ce^{kt}$$

$$\therefore T = 5 + ce^{kt} \quad \dots (i)$$

at,

$t = 1$ minute $T = 55°F$

$$\therefore 55 = 5 + ce^{k}$$

$$\Rightarrow ce^{k} = 50$$

$$\therefore c = \frac{50}{e^{k}}$$

$$(i) \Rightarrow T = 5 + \frac{50}{e^{k}} \cdot e^{kt} = 5 + 50e^{kt-k} = 5 + 50e^{k(t-1)} \quad \dots (ii)$$

at,

$t = 5$ minute, $T = 30°F$

$$\therefore 30 = 5 + 50e^{4k}$$

$$\Rightarrow e^{4k} = \frac{30-5}{50} = 0.5$$

$\Rightarrow 4k = \ln(0.5)$

$\therefore k = \frac{\ln(0.5)}{4}$

(ii) $\Rightarrow T = 5 + 50\, e^{\frac{\ln 0.5}{4} \cdot (t-1)}$ --- (iii)

when $t = 0$,

$$T = 5 + 50\, e^{\frac{\ln 0.5}{4} \cdot (-1)}$$

$$= 64.46° F$$ Ans.

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We know temperature of boiling water is 100°C.

Here,

$$\frac{dT}{dt} = k(T - T_{avg})$$

$$\Rightarrow \frac{dT}{dt} = k(T - 100)$$

$$\Rightarrow \int \frac{dT}{T-100} = \int k\, dt$$

$$\Rightarrow \ln(T-100) = kt + c_1$$

$$\Rightarrow T - 100 = e^{kt + c_1} = ce^{kt}$$

$$\therefore T = 100 + ce^{kt} \cdots (i)$$

at,

$t = 0, \quad T = 20°C$

$\therefore \quad 20 = 100 + ce^{0}$

$\therefore c = -80$

(i) $\Rightarrow$

$T = 100 - 80e^{kt} \quad \ldots$ (ii)

at,

$t = 1$ sec, $\quad T = 22°C$

$\therefore \quad 22 = 100 - 80e^{k}$

$\Rightarrow \quad e^{k} = \dfrac{22-100}{-80} = 0.975$

$\therefore k = \ln(0.975)$

(ii) $\Rightarrow$

$T = 100 - 80e^{\ln(0.975)t} \quad \ldots$ (iii)

Let,

$t^{*}$ is the time when $T = 90°C$

$\therefore \quad 90 = 100 - 80\, e^{\ln(0.975)t^{*}}$

$\Rightarrow \quad e^{\ln(0.975)t^{*}} = \dfrac{90-100}{-80} = 0.125$

$\Rightarrow \quad \ln(0.975)\, t^{*} = \ln(0.125)$

$\therefore \quad t^{*} = \dfrac{\ln(0.125)}{\ln(0.975)} \approx 82.13$ sec

let,

$t^{**}$ is the time when, $T = 98°C$

$$\therefore \quad 98 = 100 - 80\, e^{\ln(0.975)\, t^{**}}$$

$$\Rightarrow t^{**} = \frac{\ln(0.025)}{\ln(0.975)} \approx 145.70 \text{ sec}$$

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For container A,

initial temperature, 100°C

avg temperature, 0°C

then,

$$\frac{dT}{dt} = k(T - 0)$$

$$\Rightarrow \int \frac{dT}{T} = \int k\, dt$$

$$\Rightarrow \ln T = kt + c_1$$

$$\Rightarrow T = e^{kt + c_1} = ce^{kt}$$

$$\therefore T = ce^{kt} \quad \cdots \text{(i)}$$

at, $t=0$, $T = 100°C$

$\therefore\ 100 = ce^{0} = c$

$(i) \Rightarrow \quad T = 100e^{kt} \quad \cdots (ii)$

at, $t = 1$ minutes, $T = 90°C$

$\therefore\ 90 = 100e^{k}$

$\Rightarrow e^{k} = 0.9$

$\therefore\ k = \ln(0.9)$

$(ii) \Rightarrow \quad T = 100e^{\ln(0.9)t} \quad \cdots (iii)$

at, $t = 2$ minutes,

$$T = 100e^{\ln(0.9).2} \cong 81°C$$

For container B,

initial temperature, $81°C$

average temperature, $100°C$

Here,

$$\frac{dT}{dt} = k(T-100)$$

$$\Rightarrow \int \frac{dT}{T-100} = \int k\,dt$$

$$\Rightarrow \ln(T-100) = kt + c_1$$

$$\Rightarrow T - 100 = e^{kt+c_1} = ce^{kt}$$

$\therefore T = 100 + ce^{kt}$ ---- (iv)

at,

$t = 0, \quad T = 81°C$

$\therefore 81 = 100 + ce^{k \times 0}$

$\therefore c = -19$

(iv) $\Rightarrow$ $T = 100 - 19e^{kt}$ ... (v)

at, $t = 1$ minute, $T = 91°C$

$\therefore 91 = 100 - 19e^{k}$

$\Rightarrow e^{k} = \frac{91-100}{-19} = \frac{9}{19}$

$\therefore k = \ln\left(\frac{9}{19}\right)$

(v) $\Rightarrow$ $T = 100 - 19 . e^{\ln\left(\frac{9}{19}\right) . t}$ --- (vi)

Let, $t^*$ be the time for which $T = 99.9°c$

$\therefore 99.9 = 100 - 19 e^{\ln\left(\frac{9}{19}\right) t^*}$

$\Rightarrow e^{\ln\left(\frac{9}{19}\right) \cdot t^*} = \frac{99.9 - 100}{-19} = \frac{1}{190}$

$\Rightarrow \ln\left(\frac{9}{19}\right) t^* = \ln\left(\frac{1}{190}\right)$

$$\therefore t^* = \frac{\ln\left(\frac{1}{170}\right)}{\ln\left(\frac{2}{17}\right)} \approx 7.02 \text{ minutes}$$

$\therefore$ Total time, $= 2 + 7.02$

$\approx 9.02$ minutes.

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Let,

Average temperature of Oven is $T_0$

Here, $\frac{dT}{dt} = k(T - T_0)$

$$\Rightarrow \int \frac{dT}{(T-T_0)} = \int k \, dt$$

$$\Rightarrow \ln(T - T_0) = kt + c_1$$

$$\Rightarrow T - T_0 = e^{kt + c_1} = ce^{kt}$$

$$\therefore T = T_0 + ce^{kt} \quad \cdots \text{(i)}$$

at, $t = 0$, $T = 70°F$

$$\therefore 70 = T_0 + ce^0$$

$$\therefore c = (70 - T_0)$$

$$\text{(i)} \Rightarrow T = T_0 + (70 - T_0)e^{kt} \quad \cdots \text{(ii)}$$

at,

$t = 0.5$ minuts, $T = 110°$ F

$$\therefore \quad 110 = T_0 + (70 - T_0)\, e^{k \cdot 0.5}$$

$$\Rightarrow \quad e^{0.5k} = \frac{110 - T_0}{70 - T_0}$$

$$\therefore \quad k = \frac{\ln\left(\frac{110 - T_0}{70 - T_0}\right)}{0.5} = 2 \ln\left(\frac{110 - T_0}{70 - T_0}\right)$$

(ii) $\Rightarrow$

$$T = T_0 + (70 - T_0)\, e^{2\ln\left(\frac{110 - T_0}{70 - T_0}\right) \cdot t} \quad \text{---} \quad \text{(iii)}$$

at,

$t = 1$ minutes, $T = 145°$F

$$\therefore \quad 145 = T_0 + (70 - T_0)\, e^{2\ln\left(\frac{110 - T_0}{70 - T_0}\right)}$$

$$\Rightarrow \quad e^{2\ln\left(\frac{110 - T_0}{70 - T_0}\right)} = \frac{145 - T_0}{70 - T_0}$$

$$\Rightarrow \quad 2 \ln\left(\frac{110 - T_0}{70 - T_0}\right) = \ln\left(\frac{145 - T_0}{70 - T_0}\right)$$

$$\Rightarrow \quad \left(\frac{110 - T_0}{70 - T_0}\right)^2 = \frac{145 - T_0}{70 - T_0}$$

$$\Rightarrow \quad \frac{(110 - T_0)^2}{70 - T_0} = 145 - T_0$$

$$\Rightarrow (110 - T_0)^2 = (145 - T_0)(70 - T_0)$$

$$\Rightarrow 12100 - 220T_0 + T_0^2 = 10150 - 145T_0 - 70T_0 + T_0^2$$

$$\Rightarrow 12100 - 220T_0 = 10150 - 215T_0$$

$$\Rightarrow 220T_0 - 215T_0 = 12100 - 10150$$

$$\Rightarrow 5T_0 = 1950$$

$$\therefore T_0 = 390°C$$

18| Can't Understood the Question.

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Here,

$$\frac{dT}{dt} = k(T - 70)$$

$$\Rightarrow \int \frac{dT}{T-70} = \int k\,dt$$

$$\Rightarrow \ln(T-70) = kt + c_1$$

$$\Rightarrow T - 70 = e^{kt+c_1} = \cancel{c} \; ce^{kt}$$

$$\therefore T = 70 + ce^{kt} \quad \cdots (i)$$