L-1 / 28.02.2022 /

Fundamental Quantity	Derived Quantity
Length	Force
Time	Weight
Mass	Volume
Temp.	
Current	

* $10^{24} \rightarrow$ Yotta

. . . .

. . . . $\longrightarrow$ $\boxed{\text{Mid Question Must}}$

. . . .

. . . .

$10^{-24} \rightarrow$ Yecto

* $E = \frac{1}{2} mv^2$

$= [ML^2T^{-2}]$

$E = mc^2$

$= [ML^2T^{-2}]$

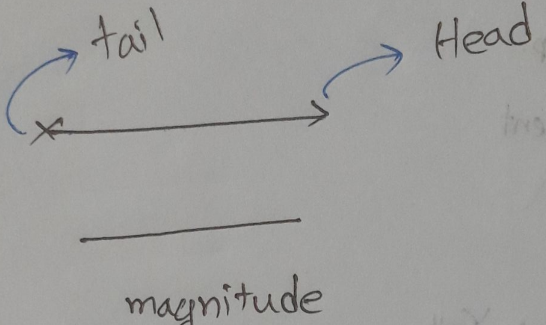
* Why current is not a vector?

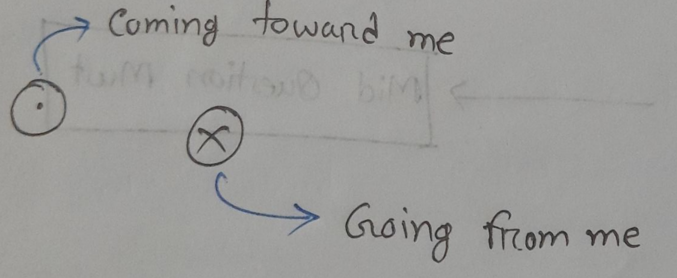
$$x = vt \rightarrow \text{length}$$

$$= [LT^{-1} \cdot T]$$

$$= [L]$$

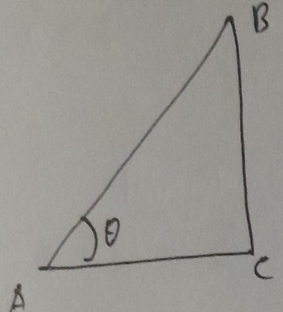
Vector,





|L-2 | 10.10.2022|

⊛ Pythagorian Theorem:



$AB^2 = BC^2 + AC^2$

(i) Calculating unknown length

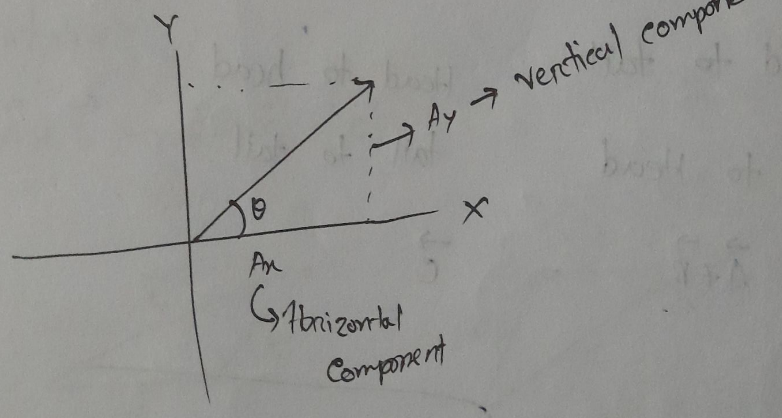
(ii) Checking Right Triangle

$\cos\theta = \frac{AC}{AB}$

$\sin\theta = \frac{BC}{AB}$

$\tan\theta = \frac{BC}{AC}$

⊛



$\cos\theta = \frac{A_x}{A}$

(i) $A_x = A\cos\theta$

(ii) $A_y = A\sin\theta$

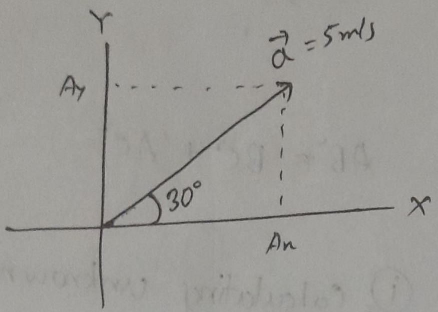
(iii) $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

$A_x^2 + A_y^2 = A^2$

$|A| = \sqrt{A_x^2 + A_y^2}$

$\tan\theta = \frac{A_y}{A_x}$

⊛

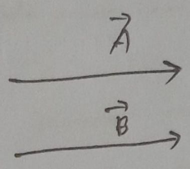


$A_x = 5\cos\theta$

$A_y = 5\sin\theta$

⊛

$\vec{A} = \vec{B}$

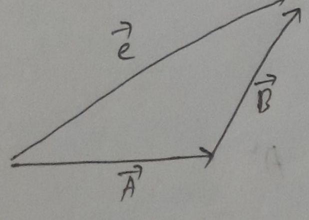


⊛

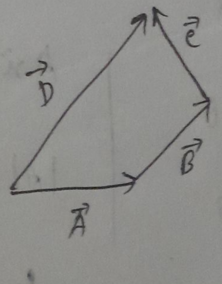
Head to tail   Head to head

tail to Head   tail to tail

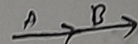
$\vec{A} + \vec{B} = \vec{C}$

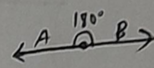


$\vec{A} + \vec{B} + \vec{C} = \vec{D}$



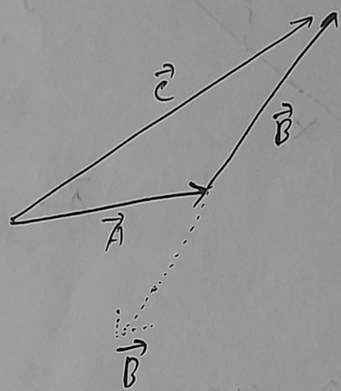
⊛ $\vec{A} = 5$, $\vec{B} = 6$, $\vec{C} = ?$

Maximum $= 11 \rightarrow$ 

Minimum $= 1 \rightarrow$ 

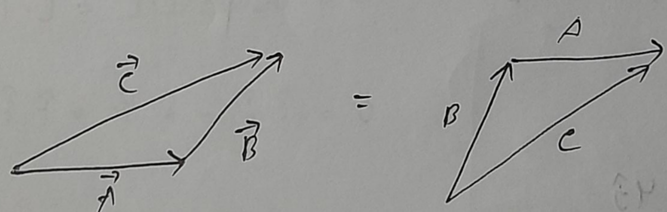
⊛ $\vec{A} - \vec{B} = \vec{C}$

$\vec{A} + (-\vec{B}) = \vec{C}$



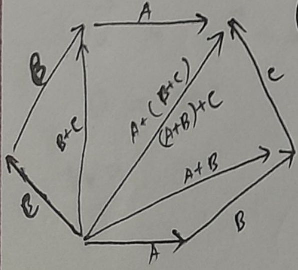
⊛ Commutative:

$\vec{A} + \vec{B} = \vec{B} + \vec{A} = \vec{C}$

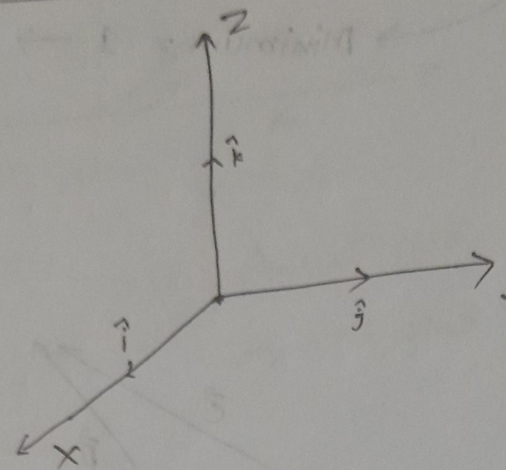


⊛ Associative Rule:

$(\bar{A} + \bar{B}) + C = \bar{A} + (\bar{B} + \bar{C})$

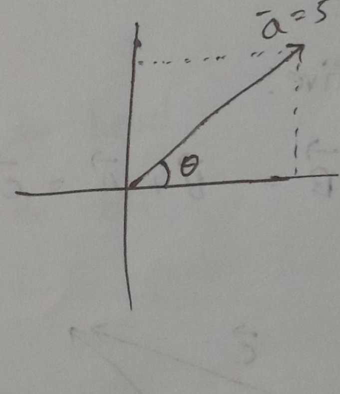


Unit vector



$\vec{A} = A_x \cdot \hat{i} + A_y \cdot \hat{j}$

$\vec{A} = 3\hat{i} + 4\hat{j}$



$|\vec{A}| = \sqrt{3^2 + 4^2} = 5$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1°$

$A = -3\hat{i} - 4\hat{j}$

$|\vec{A}| = \sqrt{3^2 + 4^2} = 5$

$\theta = \tan^{-1}\left(\frac{-4}{-3}\right) = 53.1° + 180°$

L-3/12.10.2022

(*)

No relation with (A,B)

(i) $\bar{A}\cdot\bar{B} = C \longrightarrow \vec{F}\cdot\vec{d} = W \;|\; \vec{F}\cdot\vec{v} = P$

(ii) $\bar{A}\times\bar{B} = \vec{C} \longrightarrow \vec{R}\times\vec{F} = \vec{\tau}$

(*)

(i) $\bar{A}\cdot\bar{B} = |\bar{A}|\cdot|\bar{B}|\cdot\cos\theta$

$= A_x B_x + A_y B_y + A_z B_z$

$\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\bar{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

(i) $\hat{i}\cdot\hat{j} = 1\cdot 1\cos 90^\circ = 0$

$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

(ii) $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

(*) $\vec{F} = 3\hat{i} + 3\hat{j} + \hat{k}$

$\vec{d} = -\hat{j} + 5\hat{k}$

$\theta = 30^\circ$

$W = ?$

$|F| = \sqrt{3^2+3^2+1^2} = \sqrt{19}$

$|d| = \sqrt{1+5^2} = \sqrt{26}$

$W = \sqrt{19}\cdot\sqrt{26}\cos 30^\circ$

$=$

$W = -3+5 = 2$

$\boxed{\theta = \cos^{-1}\frac{|\bar{A}\cdot\bar{B}|}{|A|\cdot|B|}}$

⊛ Cross Product

$$\bar{A} \times \bar{B} = |A|\,|B| \sin\theta$$

(i) $\hat{i} \times \hat{i} = 1 \times 1 \times \sin 0$

$= 0$

$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

(ii) $\hat{i} \times \hat{j} = 1 \times 1 \times \sin 90$

$= 1\,\hat{k}$

$\hat{j} \times \hat{i} = -\hat{k}$

$$\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\bar{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x) + \hat{k}(A_x B_y - A_y B_x)$$

$\vec{r} = \hat{i} + \hat{j} + 3\hat{k}$

$\vec{F} = 2\hat{i} + \hat{k}$

$\vec{\tau} = \vec{r} \times \vec{F}$

$$\boxed{\theta = \sin^{-1} \frac{|\vec{A} \times \vec{B}|}{|A|\,|B|}}$$

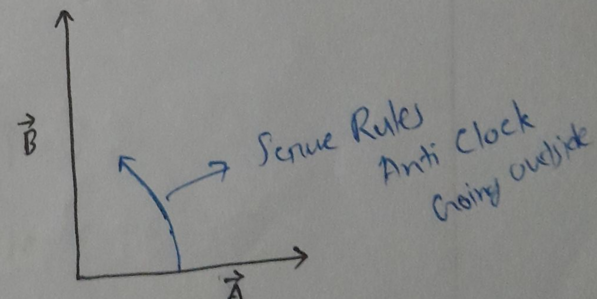
$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ 2 & 0 & 1 \end{vmatrix}$$

$$= \hat{i}(1-0) - \hat{j}(1-6) + \hat{k}(0-2)$$

$$= \hat{i} + 5\hat{j} - 2\hat{k}$$

$$|\tau| = \sqrt{1+5^2+2^2} = \sqrt{30}$$

$$\theta = \sin^{-1} \frac{\sqrt{30}}{\sqrt{11}\cdot\sqrt{5}}$$



$\vec{A} \times \vec{B} = \odot \rightarrow$ Coming Toward me.

$\vec{B} \times \vec{A} = \otimes$

(*) $\vec{A} \times \vec{B} = \vec{C}$

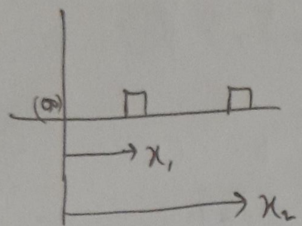
$$\begin{array}{ccc} \vec{A} & \vec{B} & \vec{C} \\ \downarrow & \downarrow & \downarrow \\ -Z & \boxed{-X} & Y \end{array}$$

(*) $\vec{C} = \vec{A} \times \vec{B}$

$$\begin{array}{ccc} \vec{C} & \vec{A} & \vec{B} \\ \downarrow & \downarrow & \downarrow \\ -Z & x & \boxed{-Y} \end{array}$$

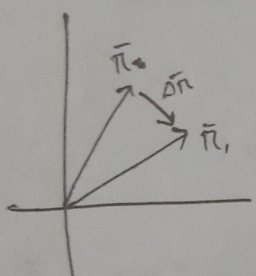
L-4 / 17. 10. 2022 /

kinetics
↳ kinema
↳ motion



$$\Delta \bar{x} = \bar{x}_2 - \bar{x}_1$$

$$\Delta t = t_2 - t_1$$



$$\bar{r} = x\hat{i} + y\hat{j} = 2\hat{i} + 3\hat{j}$$

$$\bar{r}_1 = x_1\hat{i} + y_1\hat{j} = 4\hat{i} + 5\hat{j}$$

$$\bar{r} + \Delta\bar{r} = \bar{r}_1$$

$$\Delta\bar{r} = \bar{r}_1 - \bar{r}$$

$$= (x_1\hat{i} + y_1\hat{j}) - (x\hat{i} + y\hat{j})$$

$$= (x_1 - x)\hat{i} + (y_1 - y)\hat{j}$$

$$= 2\hat{i} + 2\hat{j}$$

⊛ $x(t) = t^2 + 2t + 1$

$y(t) = t + 5$

$t = 5\text{ sec} \Rightarrow x(5) = 36$

$y(5) = 10$

$\vec{r} = x\hat{i} + y\hat{j}$

$= 36\hat{i} + 10\hat{j}$

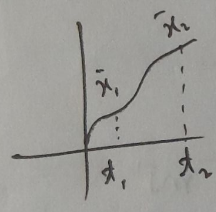
$|\vec{r}| = \sqrt{36^2 + 10^2}$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{10}{36}\right) =$

$V_{avg} = \frac{\Delta \bar{x}}{\Delta t}$ m/s

$\Delta\bar{x} = \bar{x}_2 - \bar{x}_1$

⊛



$\Delta t = t_2 - t_1$

⊛ $x_1 = 2\hat{i} + 3\hat{j}$

$x_2 = 3\hat{i} + 4\hat{j}$

$\Delta\bar{x} = \hat{i} + \hat{j}$

$\Delta t = 10 - 5 = 5$

$t_1 = 5$ sec

$t_2 = 10$ sec

$V_{avg} = \frac{\Delta\bar{x}}{\Delta t} = \left(\frac{1}{5}\hat{i} + \frac{1}{5}\hat{j}\right) ms^{-1}$

$V = \lim_{\Delta t \to 0} \frac{\Delta\bar{x}}{\Delta t} = \frac{dx}{dt} = \dot{x}$ $\frac{d}{dt}$

$x(t) = t^2 + 2t + 1$

$\frac{dx}{dt} = 2t + 2$, $t = 2$ sec

$v = 2 \cdot 2 + 2 = 6 \ ms^{-1}$

$x(t) = t^2 + 2t + 5$

$y(t) = t + 10$

$t = 2$ sec

$v = ?$

$V_x = \frac{dx}{dt} = 2t + 2 = 6 \ ms^{-1}$

$V_y = \frac{dy}{dt} = 1 \ ms^{-1}$

$\vec{v} = V_x \hat{i} + V_y \hat{j}$

$= (6\hat{i} + \hat{j}) \ ms^{-1}$

$|\vec{v}| = \sqrt{6^2 + 1^2} \ ms^{-1}$

$= \sqrt{37} \ ms^{-1}$

$\theta = \tan^{-1}\left(\frac{1}{6}\right) =$

$V_{avg} = \frac{\Delta \bar{x}}{\Delta t} \ ms^{-1}$

$a_{avg} = \frac{\Delta \bar{v}}{\Delta t} \ ms^{-1}$

$a = \lim_{\Delta t \to 0} \frac{\Delta \bar{v}}{\Delta t} = \frac{d\bar{v}}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$

$= \frac{d^2 x}{dt^2} = \ddot{x}$

⊛ $x(t) = t^2 + 2t + 5$

$y(t) = t^3 + 3t + 6$

$t = 2$ sec

$a = ?$

$$v_x = \frac{dx}{dt} = 2t + t$$

$a_x = 2 \ ms^{-2}$ → time independent

$v_y = 3t^2 + 3$

$a_y = 6t$ → time dependent

$= 12 \ ms^{-2}$

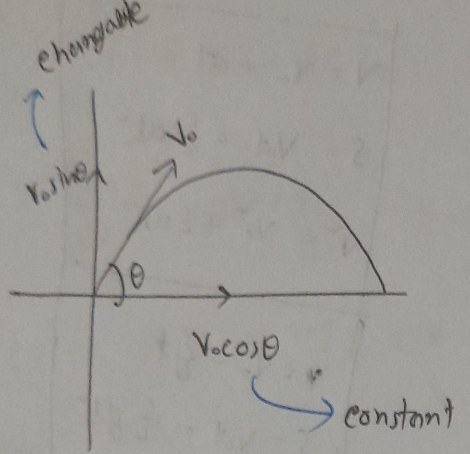
$\vec{a} = a_x \hat{i} + a_y \hat{j}$

$= (2\hat{i} + 12\hat{j}) \ ms^{-2}$

$|\vec{a}| = \sqrt{2^2 + 12^2} = \sqrt{148} \ ms^{-2}$

$$\boxed{\begin{aligned} v &= v_0 + at \\ s &= v_0 t + \tfrac{1}{2} a t^2 \\ v^2 &= v_0^2 + 2as \end{aligned}}$$

$$\boxed{\begin{aligned} v &= v_0 - gt \\ s &= v_0 t - \tfrac{1}{2} g t^2 \\ v^2 &= v_0^2 - 2gs \end{aligned}}$$



$$s = V_0 t - \frac{1}{2} g t^2$$

Vertical

$$y - y_0 = (V_0 \sin\theta) t - \frac{1}{2} g t^2$$

$$y = (V_0 \sin\theta) t - \frac{1}{2} g t^2$$

$$y = V_0 \sin\theta \frac{x}{V_0 \cos\theta} - \frac{1}{2} g \left(\frac{x}{V_0 \cos\theta}\right)^2$$

$$y = x \tan\theta - \frac{1}{2} g \frac{x^2}{V_0^2 \cos^2\theta}$$

a, b

$$y = ax - bx^2$$

$$t = \frac{R}{V_0 \cos\theta}$$

horizontal

$$x - x_0 = (V_0 \cos\theta) t$$

$$x = (V_0 \cos\theta) t$$

$$\boxed{t = \frac{x}{V_0 \cos\theta}}$$

$$y - y_0 = (v_0 \sin\theta)t - \frac{1}{2} g t^2$$

$$0 = v_0 \sin\theta \frac{R}{v_0 \cos\theta} - \frac{1}{2} g \left(\frac{R}{v_0 \cos\theta}\right)^2$$

$$R = \frac{2 v_0^2 \sin\theta \cos\theta}{g}$$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

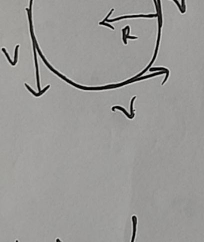
Earth

$v = 460 \ ms^{-1}$

$R = 6371 \times 1000 \ m$

$a = \frac{v^2}{R} = 0.03 \ ms^{-1} < 9.8$

(*) Uniform circular Motion



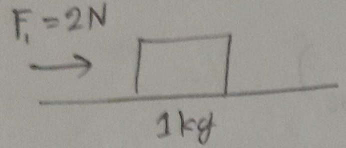
$|\vec{v}| = 10 \ ms^{-1}$

$$a = \frac{v^2}{R}$$

Nuton 2nd Law / $F = ma$

$L-6/24.10.2022$

⊛



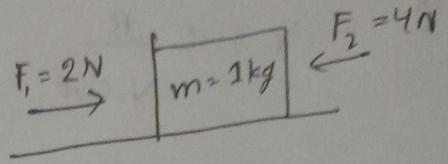
$\bar{F}_{net} = m\bar{a}$

$F_1 = ma$

$a = \frac{F_1}{m} = 2ms^{-2}$

$\longrightarrow$ x axis

⊛



$\bar{F}_{net} = m\bar{a}$

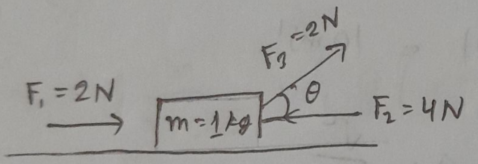
$F_1 - F_2 = m\bar{a}$

$2 - 4 = ma$

$a = \frac{2-4}{1} = -2\ ms^{-2}$

$\longrightarrow$ $-x$ axis

⊛



$\bar{F}_{net} = m\bar{a}$

$F_1 - F_2 + F_3 \cos\theta = ma$

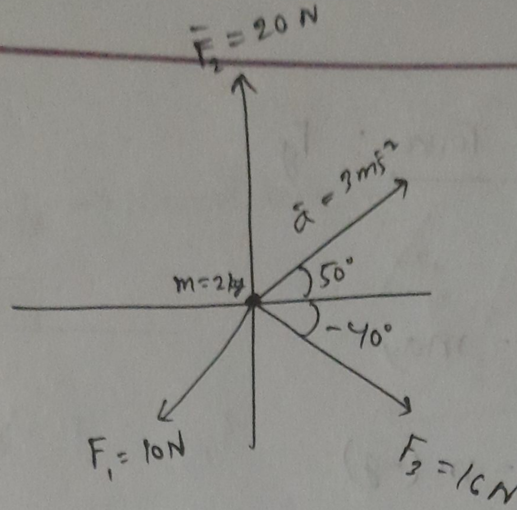
$a = \frac{F_1 - F_2 + F_3 \cos\theta}{m}$

$= \frac{2 - 4 + 2\cos 30^\circ}{1}\ ms^{-2}$

$= -0.267\ ms^{-2}$

$\longrightarrow$ $-x$ axis

(X)



$F_3 = ?$

$$\bar{F}_{net} = m\bar{a}$$

$$\bar{F}_1 + \bar{F}_2 + \bar{F}_3 = m\bar{a}$$

$$\bar{F}_3 = m\bar{a} - \bar{F}_1 - \bar{F}_2$$

x- component:

$$F_{3x} = ma_x - F_{1x} - F_{2x}$$

$$= ma\cos 50° - F_1\cos 210° - F_2\cos 90°$$

$$= 12.5\ N$$

Y- component:

$$F_{3y} = ma_y - F_{1y} - F_{2y}$$

$$= ma\sin 50° - F_1\sin 210° - F_2\sin 90°$$

$$= -10.4\ N$$

$$\therefore F_3 = 12.5\,\hat{i} - 10.4\,\hat{j}$$

$$F_3 = \sqrt{(12.5)^2 + (10.4)^2}$$

$$= 16N$$

$$\theta = \tan^{-1}\left(\frac{F_1}{F_2}\right)$$

$$= -40$$

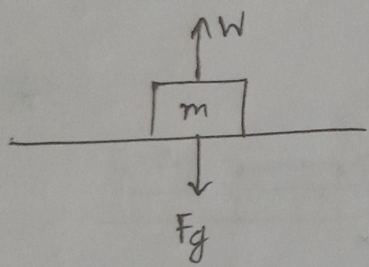
⊗ Gravitational Force : F_g

$$F_{net\,y} = m a_y$$

$$-F_g = m(-g)$$

$$\boxed{F_g = mg}$$

Weight:



$$\vec{F}_{net} = m\vec{a} = 0$$

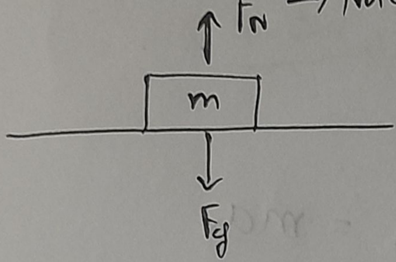
$$\vec{W} - \vec{F}_g = 0$$

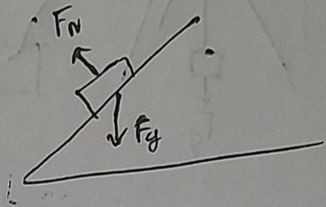
$$\vec{W} = \vec{F}_g = m\vec{g}$$

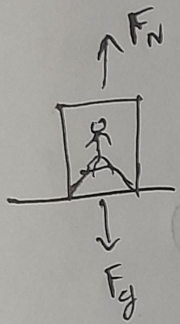
$$\boxed{W = mg}$$

L-7 / 26.10.2022 /

$F_N \to$ Normal Force







$$F_{net} = ma_y$$

$$F_N - F_g = ma_y$$

$$F_N = F_g + ma_y$$

$$\boxed{F_N = m(g + a_y)} \to \text{For moving}$$

For Free Fall

$$F_N = m(g - g) = 0$$

For stationary,

$$a_y = 0$$

$$F_N = F_g = mg$$

a_y = positive

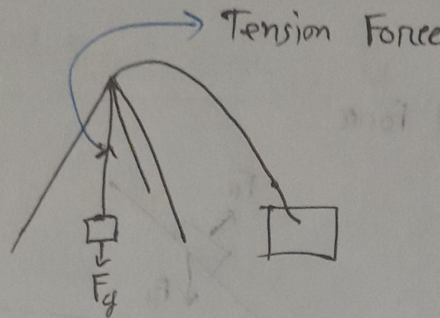
$$\uparrow \Rightarrow F_N = m(9.8 + 2)$$

$$F_N > F_g$$

$$\downarrow \Rightarrow F_N = m(9.8 - a_y);$$

a_y = -ve

$$F_N < F_g$$



$F_{net} = ma_y$

$T - F_g = ma_y$

$T = F_g + ma_y$

$T = m(g + a_y)$

(i) Stationary:

$T = m(g + a_y)$

$T = F_g = mg$

(ii) $\downarrow a_y = 1 ms^{-2}$

$T = m(g-1)$

$T < F_g$

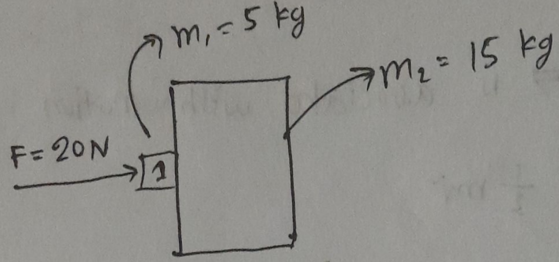
(iii) $\uparrow a_y = 1 ms^{-2}$

$T = m(g+1)$

$T > F_g$

For nuton's third law, we need at least two object.

⊛



$F_{net} = ma$

$20 = (m_1 + m_2) a$

$20 = (5+15) a$

$20 = 20a$

$a = 1\ ms^{-2}$

$\longrightarrow +x$ axis

$\vec{F}_{12} = ?$

$F_{net} = ma$

$F_{12} = 15 \times 1$

$F_{12} = 15\ N$

$\vec{F}_{21} = ma$

$= 5 \times 1$

$= 5\ N$?

$F_{21} = ma$

$F - F_{21} = ma$

$F_{21} = F - ma$

$= 20 - 5 \times 1$

$= 15\ N$

$\boxed{\therefore F_{12} = F_{21}}$

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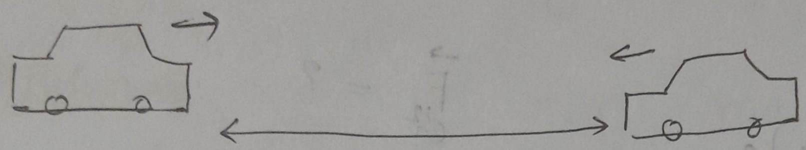
KE $\Rightarrow$ is associated with motion

$KE = \frac{1}{2} mv^2$



$KE = \frac{1}{2} mv^2$

$W = 1.2 \times 10^6 N$

$a = 0.26 \ ms^{-2}$

$v^2 = v_0^2 + 2as$

$m = \frac{W}{g} = 1.22 \times 10^5 \ kg$

$= 0 + 2 \times 0.26 \times 3.2 \times 10^3 m$

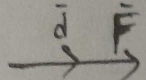
$v = 40.8 \ ms^{-1}$

For two car

$KE = 2 \times \frac{1}{2} mv^2$

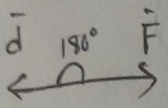
$= mv^2 = 2 \times 10^8 J$

⊛ Energy transfer by a force, then there will be work done.



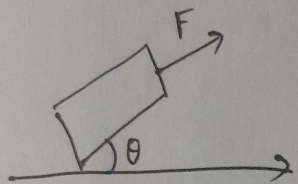
$W = \bar{F} \cdot \bar{d}$

$W \longrightarrow +ve$



$W = Fd \cos 80°$

$= -ve$



$t_1 = 35$

$t_2 = 29$

$\Delta t = t_2 - t_1$

$= 29 - 36$

$= -6\ C$ → Losing energy

$F_x = m a_x$

$v^2 = v_0^2 + 2ad$

$a_x = \frac{v^2 - v_0^2}{2d}$

$F_x = m\left(\frac{v^2 - v_0^2}{2d}\right)$

$F_x d = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 = k_f - k_i = \Delta k$

$Fd \cos\theta = \Delta k$

$W = \bar{F} \cdot \bar{d} = \Delta k$

if, $k_f < k_i$

$W \rightarrow -ve$

$$\boxed{W = \bar{F} \cdot \bar{d} = k_f - k_i = \Delta k}$$

(i) $v_i = 3\ ms^{-1}$

$v_f = -2\ ms^{-1}$

$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = W$

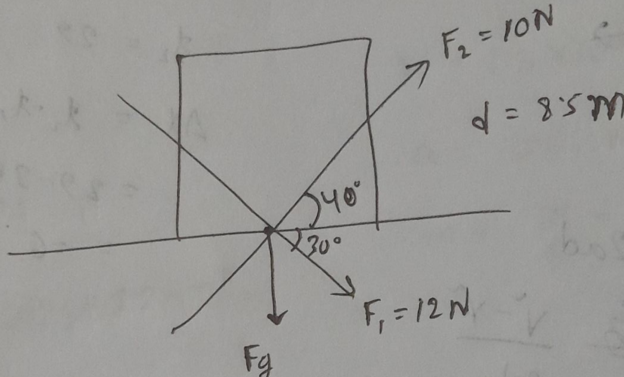
$\frac{1}{2} m (4-9) = -W$

(ii) $v_i = -2\ ms^{-1}$

$v_f = 2\ ms^{-1}$

$\frac{1}{2} m (2^2 - 2^2)$

$= 0$



$W = \bar{F} \cdot \bar{d}$

$W_1 = \bar{F}_1 \cdot \bar{d}$

$= F_1 d \cos 30°$

$= 12 \times 8.5 \times \cos 30°$

$= 88.3\ j$

$W_2 = \bar{F}_2 \cdot \bar{d}$

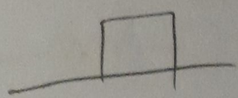
$= F_2 d \cos 40$

$= 65.12\ j$

$W = W_1 + W_2$

$= 153.4\ j$

$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$



$\vec{d} = -3m\hat{i}$

$\vec{F} = 2N\hat{i} - 6N\hat{j}$

$k_i = 10j$

$W = -6j$

$k_f = ?$

$W = k_f - k_i$

$k_f = W + k_i$

$W = \vec{F}\cdot\vec{d}$

$= -6 + 10$

$\therefore k_f = 4j$

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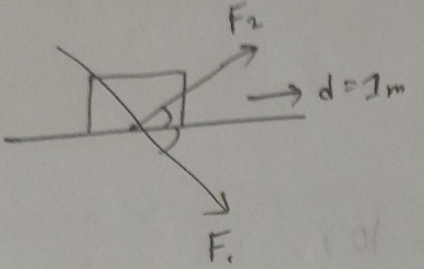
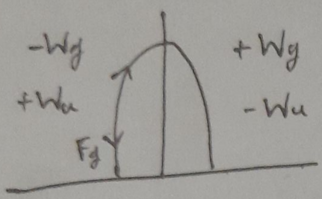
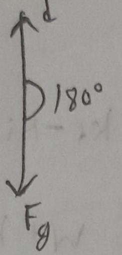






$$\uparrow W_g = \bar{F_g} \cdot \bar{d}$$

$$= F_g \, d \cos 180$$

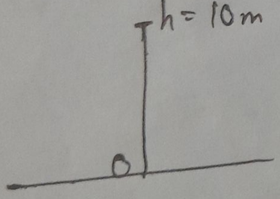
$$= -mgd$$

$$\uparrow W_g = -mgh$$

$$\downarrow W_g = \bar{F_g} \cdot \bar{d}$$

$$= F_g \, d \cos 0^\circ$$

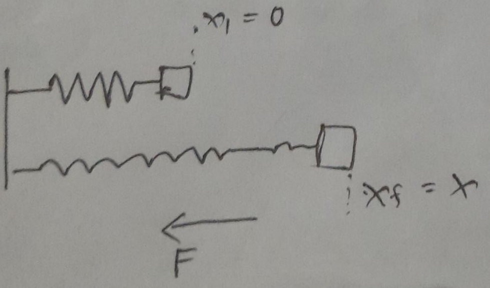
$$\downarrow W_g = mgh$$



$$W_g = -mgh$$

$$= -0.1 \times 9.8 \times 10$$

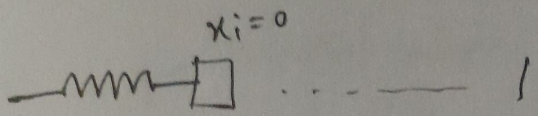
$$= -9.8 \text{ j}$$



$$F \propto -x$$

$$F = -kx$$ Spring Constant

$$k = \frac{F}{x} = \frac{N}{m} = N m^{-1}$$



$$\int dW = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} -F\,dx = -\int_{x_i}^{x_f} kx\,dx$$

$$= -k \int_{x_i}^{x_f} x\,dx$$

$$= -k \left[\frac{x^2}{2} \right]_{x_i}^{x_f}$$

$$= -k \left[\frac{x_f^2}{2} - \frac{x_i^2}{2} \right]$$

$$= -k \frac{x_f^2}{2}$$

$$= -\frac{1}{2} k x_f^2$$

$$\boxed{\begin{aligned} \uparrow W_g &= -mgh \\ W_s &= -\frac{1}{2} k x^2 \end{aligned}}$$

$$\boxed{W = -\frac{1}{2} kx}$$

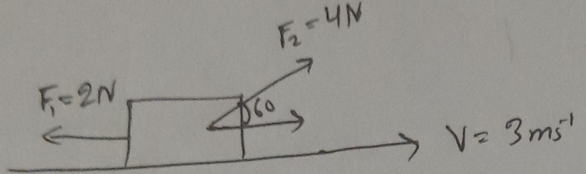
$$P = \frac{W}{t} = \frac{J}{sec} = \text{watt}$$

$$P = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{x}}{dt} = \frac{F\,dx \cos\theta}{dt}$$

$$= F \cos\theta \frac{dx}{dt}$$

$$= F \cos\theta\, v$$

$$P = \vec{F} \cdot \vec{v}$$



$P_1 = \bar{F_1} \cdot \bar{V} = F \cdot V \cos 180$

$= 2 \cdot 3 \cdot (-1) = -6\,W$ → System is loosing energy per unit second

$P_2 = \bar{F_2} \cdot \bar{V} = F \cdot V \cos 60$

$= 4 \cdot 3 \cdot \frac{1}{2}$

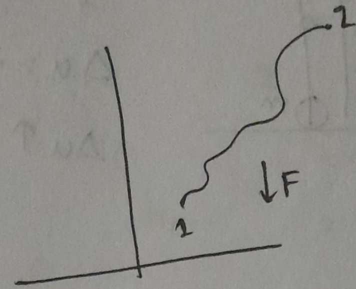
$= 6\,W$

$P = P_1 + P_2 = -6 + 6 = 0\,W$

$\Delta U = -W$ → Potential energy

$W = \Delta k$

$E = k + U$ → Mechanical Energy



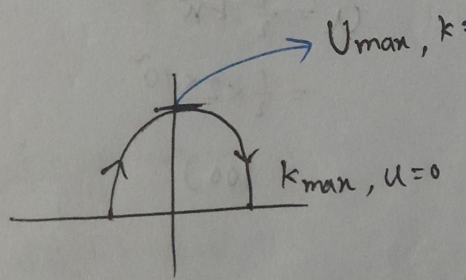
$$W = \int_1^2 dW$$
$$= \int_1^2 \vec{F}\cdot d\vec{y}$$
$$= \int_1^2 F\,dy\cos 180$$
$$= -\int_1^2 F\,dy$$
$$= -mg\int_1^2 dy$$
$$= -mgh$$
$$= -mg[y_2 - y_1]$$

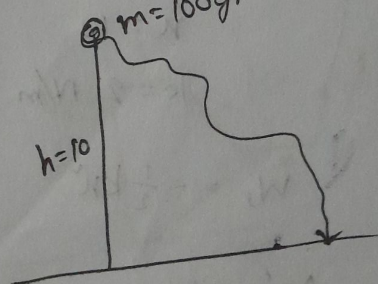
$$W = -mg[y_2 - y_1] = \Delta k = k_2 - k_1$$

$$k_1 + mgy_1 = k_2 + mgy_2$$

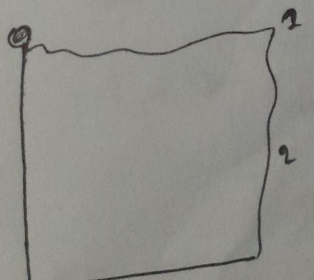
$$\boxed{k_1 + U_1 = k_2 + U_2}$$

$$\boxed{E_1 = E_2}$$





$$W_g = mgh$$
$$= 0.1 \times 9.8 \times 10$$
$$= 9.8\,J$$

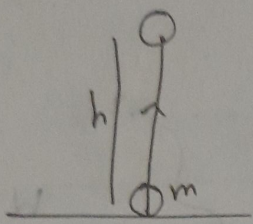


$$W_{g_1} = \vec{F}\cdot\vec{d} = F.d\cos 90^\circ = 0$$

$$W_{g_2} = \vec{F}\cdot\vec{d} = F_g d\cos 0^\circ = 9.8\,J$$

$$\therefore W_g = W_{g_1} + W_{g_2} = 9.8\,J$$

(*)

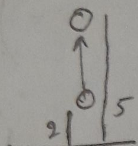


$W_g = mgh$

$\Delta u = -W$

$\Delta u \uparrow = -(-mgh)$

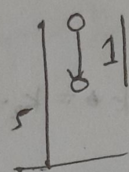
$= mgh$



$u = mgh$

$= 0.5 \times 9.8 \times 3$

$= \ldots$

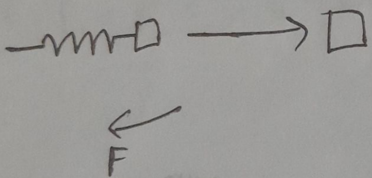


$u = -mgh$

$= -0.5 \times 9.8 \times 1$

$= -4.7 j$

(*)



$x = 10$

$k = 2 \ N/m$

$W_s = -\frac{1}{2}kx^2$

$\Delta u = -W$

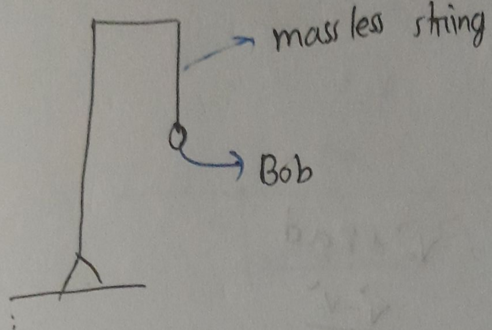
$= -(-\frac{1}{2}kx^2)$

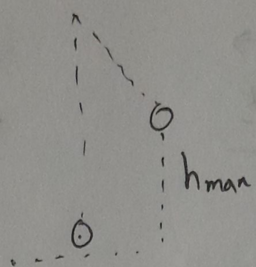
$\Delta u = \frac{1}{2}kx^2$

$u = \frac{1}{2}kx^2$

$= \frac{1}{2} \times 2 \times 10^2$

$= 100 j$

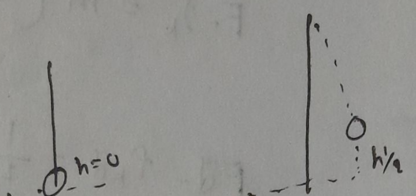




$E = U + K = 5$ jul

$K = 0$

$U = max = 5$



$K = U = 5$ jul

$K = max = 5$

$U = 0$

calculate 'v'

$E = K = \frac{1}{2} m v^2$

⊛ $E = 5J$

$m = 100g$

$E = K = \frac{1}{2} m v^2$

⊛

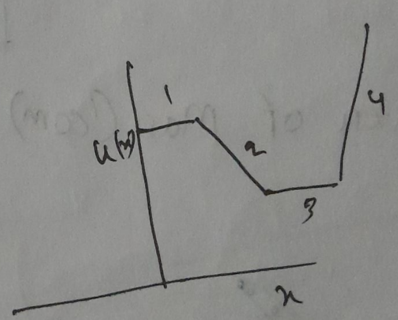
$\Delta U = -W$

$dU = -dW$

$dU = -F\,dx$

$F = \boxed{-\frac{dU}{dx}}$

Slope



$F_4 > F_2 > F_1 = F_3$

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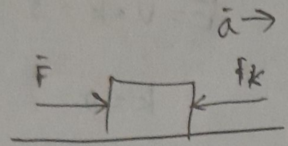
$E_{mec} = U + K$



$v^2 = v_0^2 + 2ad$

$a = \frac{v^2 - v_0^2}{2d}$

$F_{net} = ma$

$F - f_k = ma$

$F - f_k = m\left(\frac{v^2 - v_0^2}{2d}\right)$

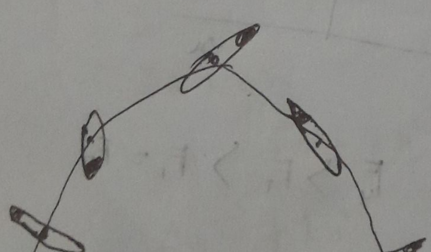
$Fd - f_k d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

$Fd - f_k d = E_{mec}$ $\rightarrow U + K$ max, 0

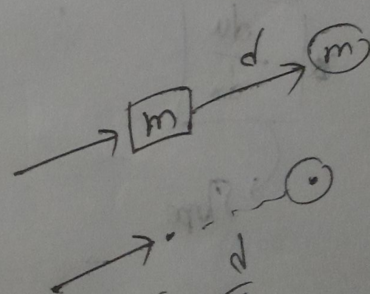
$Fd = f_k d + E_{mec}$

$$\boxed{E = E_{th} + E_{mec} + E_{int}}$$

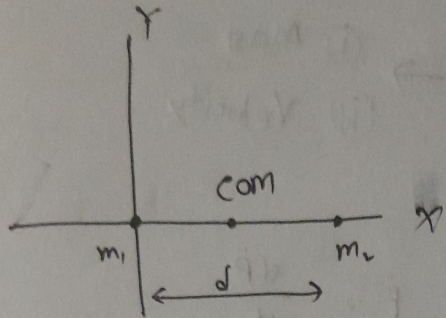
(*) Center of Mass (com)



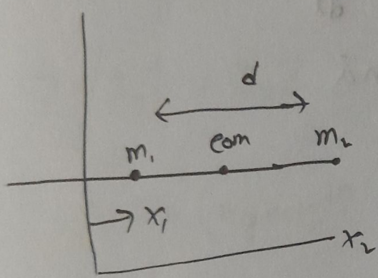
$\vec{v}\vec{a}$



$\vec{v} = \frac{d}{\vec{a}}$



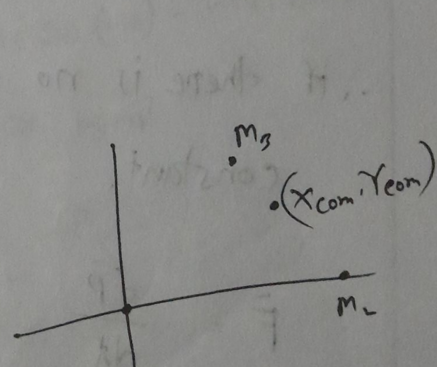
$$X_{com} = \frac{m_2 d}{m_1 + m_2}$$



$$X_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$\sum X_{com} = \frac{1}{m} \sum_{i=1}^{i=n} m_i x_i$$

Particle	Mass	X_{com}	Y_{com}
1	1.2	0	0
2	2.5	140	0
3	3.4	70	12



$$X_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{0 + 2.5 \times 140 + 3.5 \times 70}{1.2 + 2.5 + 3.5} = 83 \text{ cm}$$

$$Y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{0 + 0 + 3.5 \times 120}{1.2 + 2.5 + 3.5} = 58 \text{ cm}$$

$\therefore$ com = (83, 58)

⊛ Linear momentum, $P \rightarrow$ (i) Mass
(ii) Velocity

$$\bar{P} = m\bar{v}$$

Force

$$F = \boxed{\frac{d\bar{p}}{dt}} \rightarrow \text{Slope}$$

$$\bar{F} = \frac{d\bar{P}}{dt}$$

$$= \frac{d(m\bar{v})}{dt}$$

$$= m\frac{d\bar{v}}{dt}$$

$$\bar{F} = m\bar{a}$$

$$F_{net} = 0$$

$$\boxed{\frac{d\bar{P}}{dt} = 0}$$

$$P = \text{constant}$$

$$P_i = P_f$$

∴ if there is no net force, the momentum will be constant.

$$\bar{F} = \frac{d\bar{P}}{dt}$$

$$d\bar{P} = \bar{F}\,dt$$

$$\int_{P_i}^{P_f} d\bar{P} = \int_{P_i}^{P_f} \bar{F}\,dt$$

$$\boxed{\Delta P = \bar{J}}$$

Linear momentum impulse theorem

impulse

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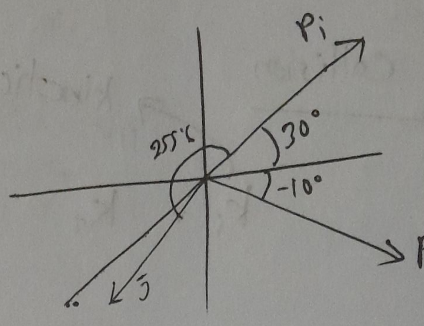
impulse

$$\vec{J} = \int_{t_i}^{t_f} \vec{F} \cdot dt$$



$$\Delta P = \vec{J}$$

$$\bar{P_f} - \bar{P_i} = \bar{J}$$

$m = 80$ kg

$v_i = 70\ ms^{-1}$

$v_f = 50\ ms^{-1}$

$$\bar{J} = J_x \hat{i} + J_y \hat{j}$$

$$= -910\ (kgms^{-1})\hat{i} - 3500 (kgms^{-1})\hat{j}$$

$$J_x = P_{fx} - P_{ix}$$

$$= mv_{fx} - mv_{ix}$$

$$= mv_f \cos(-10) - mv_i \cos 30°$$

$$= -910\ kgms^{-1}$$

$$|\vec{J}| = \sqrt{J_x^2 + J_y^2}$$

$$= 3600\ kgms^{-1}$$

$$\theta = \tan^{-1}\left(\frac{J_y}{J_x}\right)$$

$$= 75.6°$$

$$\theta = 180° + 75.6$$

$$= 255.6°$$

$$J_y = P_{fy} - P_{iy}$$

$$= mv_{fy} - mv_{iy}$$

$$= mv_f \sin(-10) - mv_i \sin 30$$

$$= -3500\ kgms^{-1}$$

$$J = F_{avg}\,\Delta t$$

$$\Delta t = 16\ ms$$

$$\therefore F_{avg} = \frac{J}{\Delta t}$$

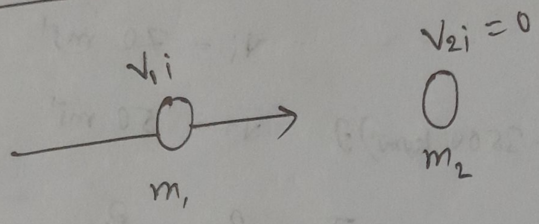
Collision

elastic collision

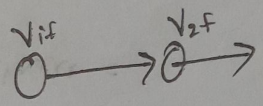
kinetic energy

$$k_i = k_f$$

Before collision



after,



$$m_1 v_1 i = m_1 v_1 f + m_2 v_2 f$$

$$m_1 (v_1 i - v_1 f) = m_2 v_2 f \quad \cdots \text{(i)}$$

Elastic collision:

$$\frac{1}{2} m_1 v_1 i^2 = \frac{1}{2} m_1 v_1 f^2 + \frac{1}{2} m_2 v_2 f^2$$

$$m_1 (v_1 i^2 - v_1 f^2) = m_2 v_2 f^2 \quad \cdots \text{(ii)}$$

(ii) ÷ (i),

(A) $$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

(B) $$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Case-i :

$m_1 = m_2$

$v_{1f} = 0$

$v_{2f} = v_{1i}$

Case-ii :

$m_1 << m_2$

$v_{1f} \approx -v_{1i}$

$v_{2f} \approx 0$

Case-iii

$m_1 >> m_2$

$v_{1f} \approx v_{1i}$

$v_{2f} \approx 2v_{1i}$

Linear Motion

Position, x

Displacement, Δx

Shortage distance between two point

$$\Delta x = x_2 - x_1$$

Average velocity, v_{avg}

$$v_{avg} = \frac{\Delta x}{\Delta t} = ms^{-1}$$

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Average Acceleration,

$a_{avg} =$

Rotational Motion

Angular position, $\theta \rightarrow$ rad

Angular displacement, $\Delta\theta = \theta_2 - \theta_1$

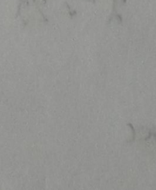
Average Angular velocity, ω_{avg}

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Average angular acceleration,

α_{avg}



$s = r\theta$

$x(t) = t^2 + 2t + 1$

$t = 2$ sec

$v = \frac{dx}{dt} = 2t + 2$

$v = 2 \cdot 2 + 2$

$= 6$

$\theta(t) = t^2 + 2t + 1$

$\omega = \frac{d\theta}{dt} = 2t + 2$

$t = 2$ sec

$\omega = 6$ rad/sec

$a_{avg} = \frac{\Delta v}{\Delta t} = ms^{-2}$

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \ddot{x}$ ms^{-2}

$\Delta\omega = \omega_2 - \omega_1$

$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$

$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \ddot{\theta}$ rad/sec^2

(*) $\theta(t) = t^2 + 2t + 1$

$t = 2$ sec

$\alpha = ?$

$\omega = \frac{d\theta}{dt} = 2t + 2$

$\alpha = \frac{d\omega}{dt} = 2 \rightarrow$ time independent.

$t = 2$ sec ; $\alpha = 2$ rad/sec^2

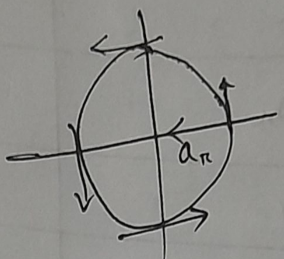
$s = \theta R$ → constant

$$\frac{ds}{dt} = R\frac{d\theta}{dt}$$

$$\boxed{v = R\omega}$$

$$\frac{dv}{dt} = R\frac{d\omega}{dt}$$

$$\boxed{a = R\alpha}$$



$$a_r = \frac{v^2}{R}$$

$$\boxed{a_r = \omega^2 R}$$

$$\boxed{\begin{aligned} s &= R\theta \\ v &= \omega R \\ a_t &= \alpha R \end{aligned}}$$

if, ω = constant

$$a_r = \omega^2 R$$

$$a_t = \alpha R = R \cdot \frac{d\omega}{dt} = 0$$

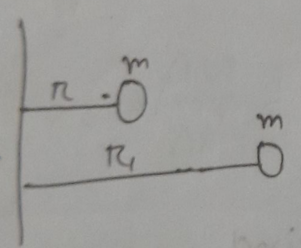
if, ω = increasing or decreasing

$$a_r = \omega^2 R$$

$$a_t = \alpha R$$

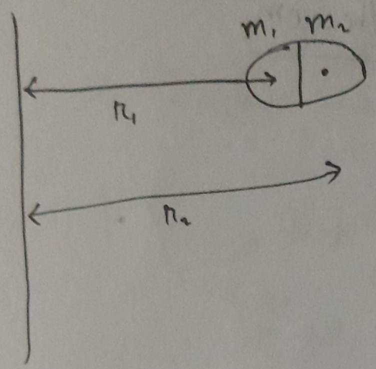
L-13/ 16.11.2022/

Linear	Rotational
x	θ
Δx	$\Delta\theta$
v	ω
a	α
F	τ
p	L
m	$I \rightarrow$ Rotational mass / moment of Inertia



$I = mr^2 \rightarrow kgm^2$

$I_1 > I$



$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$= \sum_{i=1}^{n} \frac{1}{2} m_i v_i^2$$

$$= \sum \frac{1}{2} m_i \omega^2 r_i^2$$

$$= \sum \frac{1}{2} (m_i r_i^2) \omega$$

$$K = \sum \frac{1}{2} I \omega$$

$$\omega = \frac{d\theta}{dt}$$
$$v = \omega r$$

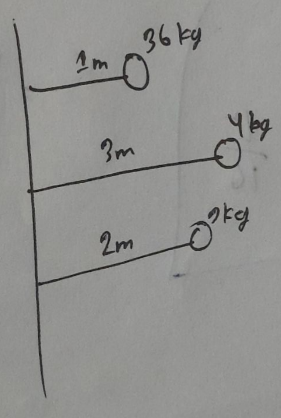
$\omega = 2$ rad/sec

$k = 10$ j

$I = ?$

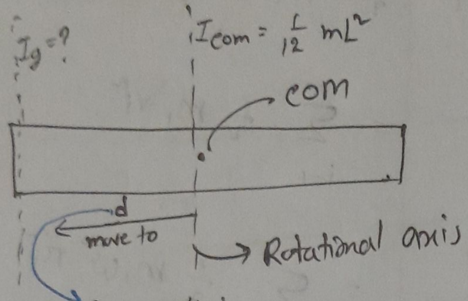
$$I = \frac{2k}{\omega}$$

$\cancel{k = \frac{1}{2} I\omega}$



$$\boxed{I_1 = I_2 = I_3 = 36}$$

Parallal axis theorem



assume that,

$d = \frac{L}{2}$

$\therefore I_g = I_{com} + md^2$

$= \frac{1}{12} mL^2 + m\frac{L^2}{4}$

$= \frac{mL^2 + 3mL^2}{12}$

$= \frac{4mL^2}{12}$

$= \boxed{\frac{mL^2}{3}}$

$m = 1$ kg

$L = 2$ m

$I_g = ?$

if, $d = \frac{L}{4}$

then,

$I_g = \frac{1}{12} mL^2 + m\frac{L^2}{16}$

Midterm

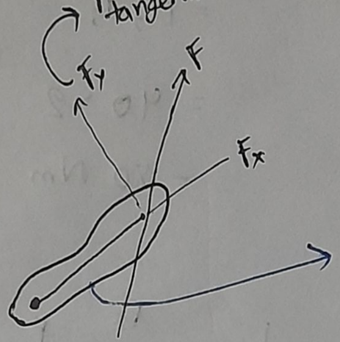
<u>L-15/23.11.2022/</u>

$K = \frac{1}{2} m v^2$

$I \quad \omega$

$\tau \rightarrow$ Torque $\longrightarrow$ rotational force.

$F_{tangential} = F\sin\theta$



For this τ direction $\odot$

$$\vec{\tau} = r F_{tan} = |\bar{r}|\ |\bar{F}|\ \sin\theta$$

$$= \bar{r} \times \bar{F}$$

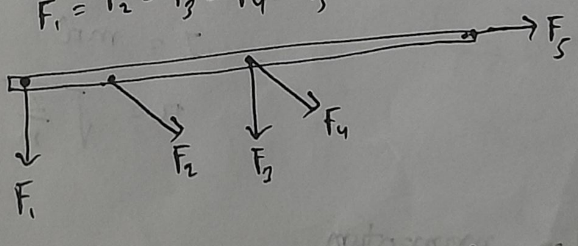
$\circledast$ $F \neq 0$

(i) $\tau = 0,\ r = 0$

(ii) $\tau = 0,\ \theta = 0$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$F_1 = F_2 = F_3 = F_4 = F_5$



$$\tau_3 > \tau_4 > \tau_2 > \tau_1 = \tau_5$$

if,

$F_1 = 3\,N \rightarrow \tau_1 = 0$

$F_2 = 3N$

$F_3 = F_4 = 2N$

$F_5 = 10N \rightarrow \tau_5 = 0$

$\tau_3 = \bar{r}_3 \times \bar{F}_3$

$= r_3 F_3 \sin 90$

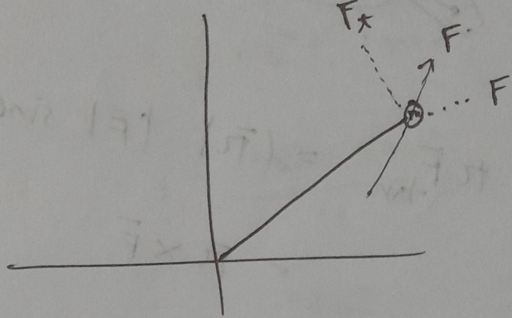
$= r_3 F_3$

$= 40 \times 2$

$= 80$ Nm

$F = ma$

$\tau = I\alpha$



$F_t = ma_t$

$\tau = rF_t$

$= rm\alpha r$

$= mr^2\alpha$

$\tau = I\alpha$

$\tau = 20$ Nm

$\alpha = 2$ rad/sec

$\therefore I = \frac{\tau}{\alpha} = 10\ kgm^2$

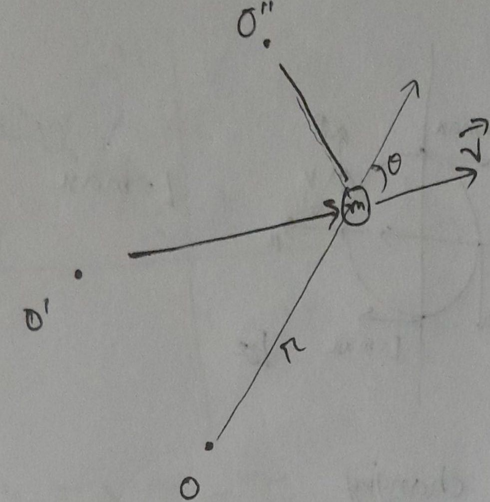
$m = 500$ gm

$r = ?$

$I = mr^2$

$r = \sqrt{\frac{I}{m}}$

$L \rightarrow$ Angular momentum



$\vec{p} = m\vec{v}$

$L = kg\,m^2\,sec^{-1}$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{L} = \vec{r} \times \vec{p}$

$\vec{L}'' = \vec{r} \times \vec{p} = mvr$

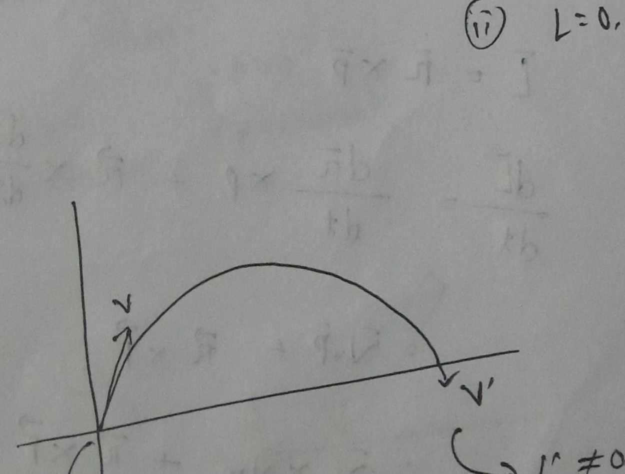
$\vec{L}' = \vec{r} \times \vec{p} = 0$

$\vec{p} \neq 0$

(i) $L = 0, r = 0$

(ii) $L = 0, \theta = 0$

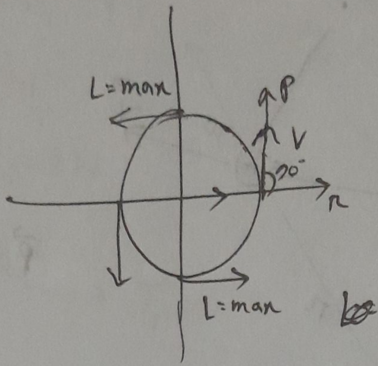
Case-1/



v is changing

L is also changing

case-2/



L=max

V is changing

but, L is constant.

(*)

$F = ma$	$\vec{F} = \frac{d\bar{p}}{dt}$
$\tau = I\alpha$	$\tau = \frac{d\bar{L}}{dt}$

$$\bar{L} = \bar{r} \times \bar{p}$$

$$\frac{d\bar{L}}{dt} = \frac{d\bar{r}}{dt} \times p + \vec{r} \times \frac{d\bar{p}}{dt}$$

$$= \bar{v} \times \bar{p} + \vec{r} \times \vec{F}$$

$$= \underline{\bar{v} \times \bar{v}m} + \vec{r} \times \vec{F}$$

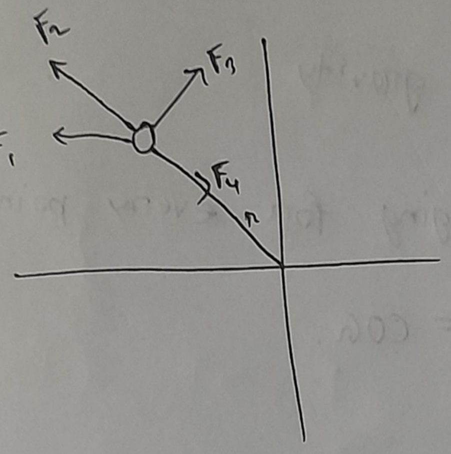
$\hat{i} \times \hat{i} = 0$

$$= \vec{r} \times \vec{F}$$

$$= \tau$$

$$\therefore \vec{\tau} = \frac{d\bar{L}}{dt}$$

⊛



$$\tau = \frac{dL}{dt}$$

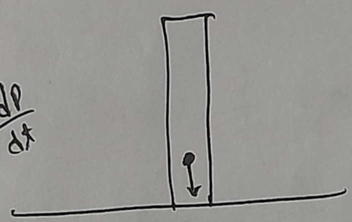
$$\tau_3 > \tau_1 > \tau_2 = \tau_4$$

$$L_3 > L_1 > L_2 = L_4$$

⊛

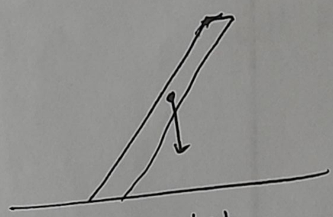
(i)

$$F = \frac{dp}{dt}$$



stable

(ii)



unstable

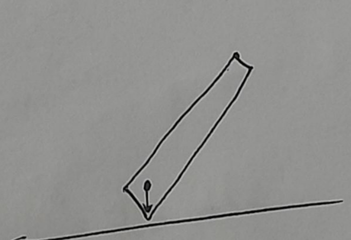
$F = 0$

$\tau = 0$

$p = \text{constant}$

$L = \text{constant}$

(iii)



stable

⊛ if the net torque is zero the angular momentum is constant.

↪ conservation of Angular momentum.

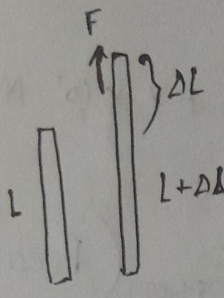
* COG → center of gravity

* If g is not changing for every point of an object then COM = COG.

L-16/ 28.11.2022/

Elasticity

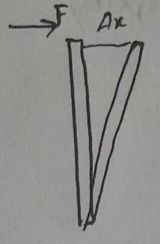
(i) F & L Parallal



$\frac{F}{A} \propto \frac{\Delta L}{L}$

$\frac{F}{A} = E \frac{\Delta L}{L}$ E → Elastic modulus

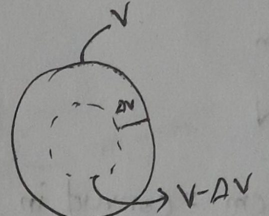
(ii) F & L Perpendicular



$\frac{F}{A} \propto \frac{\Delta x}{L}$

$\frac{F}{A} = G \frac{\Delta x}{L}$ G → Sear Modulus

(iii)



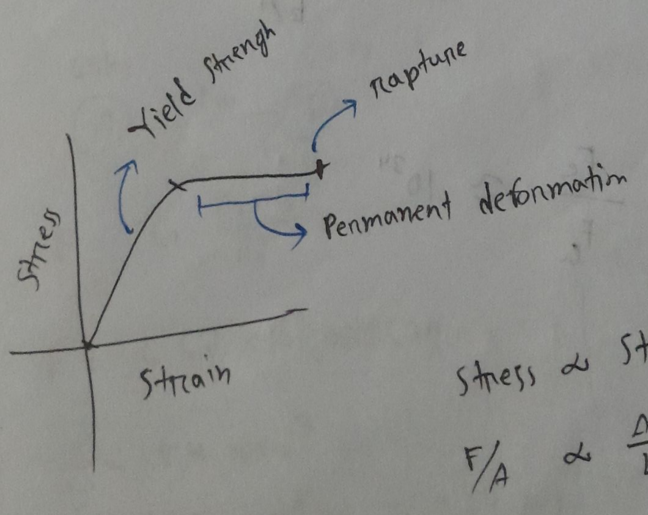
$\frac{F}{A} \propto \frac{\Delta V}{V}$

$\frac{F}{A} = \beta \frac{\Delta V}{V}$ β → Bulk Modulus

Stress : $\frac{F}{A} = N m^{-2}$

Strain : $\frac{\Delta L}{L}$ (Unitless)

(*) Identify which formula will apply.



Stress $\propto$ Strain
$F/A \propto \frac{\Delta L}{L}$ } within Elastic Limit only

$$E = \frac{F/A}{\Delta L/L}$$

Material Property

$E_{steel} = 2.8 \times 10^{8}\ Nm^{-2}$

$E_{silver} = 1.7 \times 10^{8}\ Nm^{-2}$

$E_{steel} > E_{silver}$ (Pressure)

⊛

$L = 10\,m$

$F = 2\,N$

$r = 0.5\,cm$ = convert in m

$E = 2.8 \times 10^{8}\ Nm^{-2}$

Stress = ?

Strain = ?

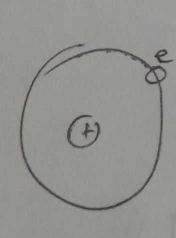
$\Delta L = ?$

$$\text{Stress} = \frac{F}{A} = \frac{2}{\pi r^2}$$

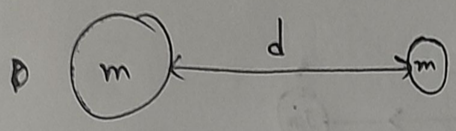
$$\text{strain} = \frac{F}{EA} \quad \left[E = \frac{F/A}{\Delta L/L} \right]$$

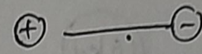
$$\Delta L = \frac{FL}{EA}$$

⊛



$$\frac{F_e}{F_G} \approx 10^{34}$$





$$F_c = k\frac{q_1 q_2}{d^2}$$

$$F \propto m_1 m_2$$

$$F \propto \frac{1}{d^2}$$

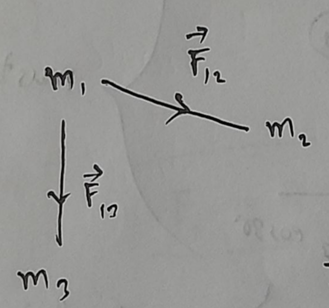
$$F \propto \frac{m_1 m_2}{d^2}$$

$$F = G\frac{m_1 m_2}{d^2}$$ [Limitation: only applieable with two object]

Vector Form:

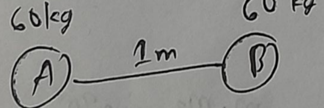
$$\vec{F}_{12} = G\frac{m_1 m_2}{d^2}\hat{r}_{12}$$ $m_1 \to m_2$

(*)



$$\vec{F}_{1net} = \vec{F}_{13} + \vec{F}_{12}$$

(*)



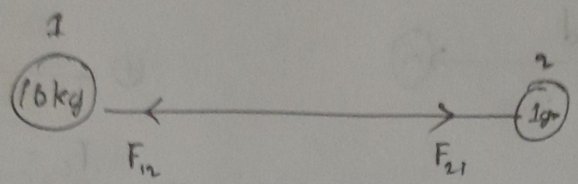
$$F = G\frac{m_1 m_2}{d^2}$$

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$$F = 6.67 \times 10^{-11} \times 3600$$

$$= 2.4 \times 10^{-7}\ N$$

$$= 0.24 \times 10^{-6}\ N = 0.24\ \mu N$$

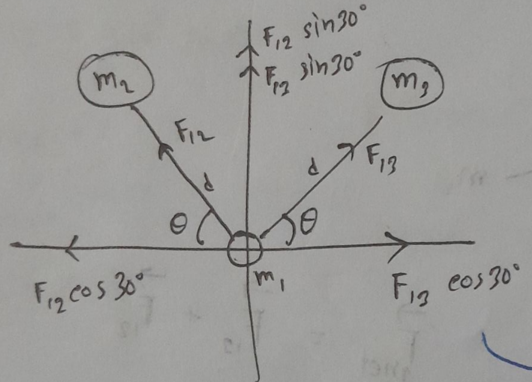


$$F_{12} = G\frac{m_1 m_2}{d^2} \qquad F_{21} = G\frac{m_1 m_2}{d^2}$$

$$\boxed{F_{12} = F_{21}}$$

⊛ World moves towards me $\approx 10^{-24}\ ms^{-2}$

⊛



must 1 Question in Final

Assignment Question

$$F_{1net} = F_{12}\sin 30^\circ + F_{13}\sin 30^\circ$$

$$= G\frac{m_1 m_2}{d^2} \times \sin 30^\circ + G\frac{m_1 m_3}{d^2}\sin 30^\circ$$

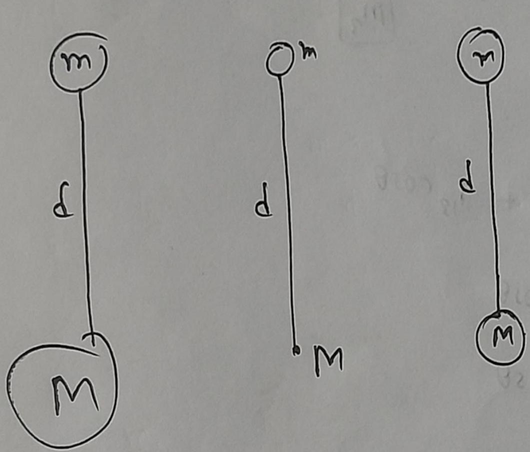
$$F_{1x} = F_{13}\cos 30^\circ - F_{12}\cos 30^\circ = 10N$$

$$F_{1y} = F_{13}\sin 30^\circ + F_{12}\sin 30^\circ = 12N$$

$$\vec{F} = F_{1x}\,\hat{i} + F_{1y}\,\hat{j}$$

$$= 10N\,\hat{i} + 12N\,\hat{j}$$

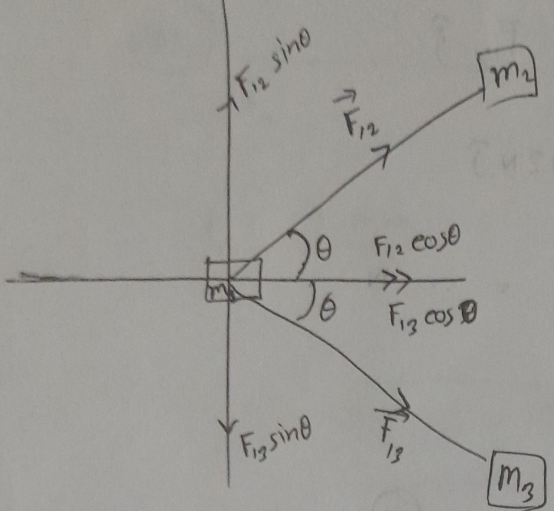
$$|\vec{F}| = \sqrt{10^2+12^2} = \ldots N$$



$$\boxed{F_1 = F_2 = F_3}$$

Not depend on volume

L-17/ 30.11.2022/



mass = same

θ = same

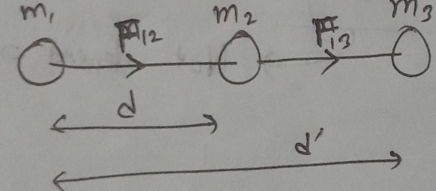
$$F_{1\,net} = F_{12}\cos\theta + F_{13}\cos\theta$$

$$= 2F_{12}\cos\theta$$

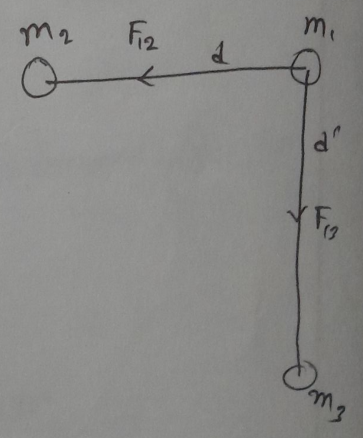
$$= 2F_{13}\cos\theta$$

(*)

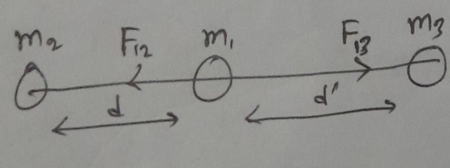
(i)



(ii)



(iii)

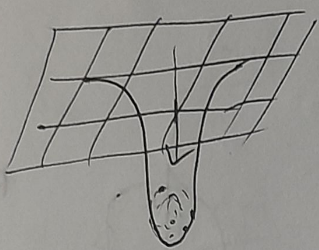


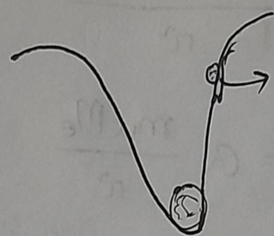
$F_{1\,net}$ 1 > 2 > 3

(*) in which arrangement m_1 will get maximum gravitional force?

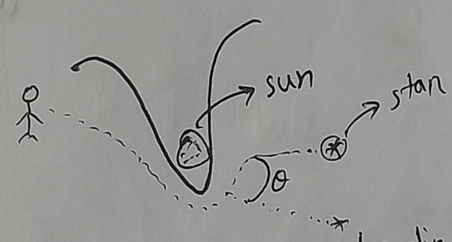
⊛ Space and time is co-related.

⊛ Heavy object in space distorted space more.





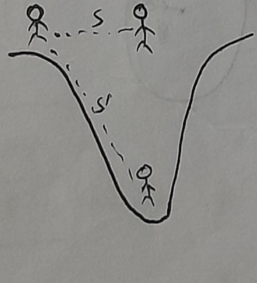
⊛



⊛ experiment only applicable in solar ecilipes

Heavy object makes time and space more distorted.

⊛



$s = vt$ $\qquad v = 3\times10^{8}\ ms^{-1}$

$t = \frac{s}{v}$

$s' = vt'$

$t' = \frac{s'}{v}$

$\therefore$ distortion $\propto$ time

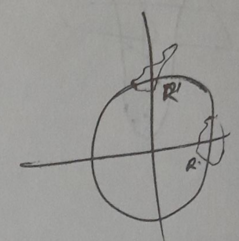
$$F = mg$$
$$= m a_g$$

$$g = a_g$$

$$F_g = \frac{G m_u M_{earth}}{r^2}$$

$$m a_g = G \frac{m_u M_e}{r^2}$$

$$a_g = \frac{GM}{r^2}$$



R > R'

$\therefore$ a_g depends on M and r^2

$$g \approx M$$
$$g \approx r^2$$

When, R is bigger
g is smaller.

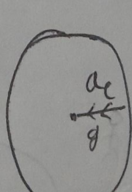
To calculate original weight.

$$mg = \cancel{mg}\ m a_g - \frac{mv^2}{r}$$

$$= \cancel{mg}$$

$$= m a_g - m \omega^2 r$$

$$g = a_g - \omega^2 r$$



$a_c = 0.3\ ms^{-2}$

$g = 9.8\ ms^{-2}$

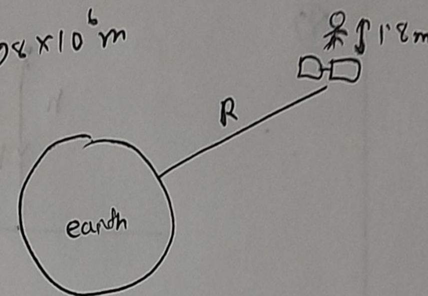
$$a_c = \frac{v^2}{R}$$

$$F = ma$$

$$v = \omega r$$

*

$r = 5.98 \times 10^6 m$



$$g = \frac{GM}{r^2}$$

$$g_{leg} = \frac{GM}{r^2}$$

$$g_{head} = \frac{GM}{(r+1.8)^2}$$

we can't use 1.8m like that, so, we have use another method

$\cancel{da_g = -14.5 \, ms^{-2}}$

$$\frac{dg}{r} = -2\frac{GM}{r^3}$$

$$da_g = -2\frac{GM}{r^3}dr$$

$$da_g = -2\frac{GM}{r^3}dr$$

$$= -4.37 \times 10^{-6} \, ms^{-2}$$

$$da_g = -14.5 \, ms^{-2}$$

$$\boxed{g = \frac{GMR'}{R^3}} \rightarrow$$

R' = diging height

R = earth surface length

m = mass

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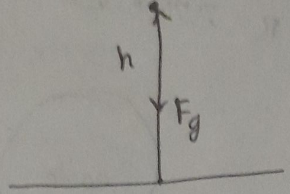


$$W_g = -mgh$$

$$\Delta u = -W$$
$$= -(-mgh)$$
$$= mgh$$

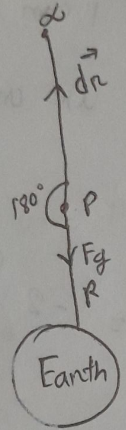
$$U_f - \cancelto{0}{U_i} = mgh$$

$$U_f = mgh$$

i. Reference Point

ii. Work done

iii. $W \rightarrow u$



$$\boxed{F_g = \frac{GM_m}{R^2}}$$

$$dw = \vec{F} \cdot \vec{dr}$$

$$\int dw = \int_R^{\infty} \vec{F} \cdot \vec{dr}$$

$$= -\int_R^{\infty} F\, dr$$

$$= -\int_R^{\infty} \frac{GM_m}{r^2}\, dr$$

$$= -GM_m \int_R^{\infty} r^{-2}\, dr$$

$$= -GM_m \left[-\frac{1}{r}\right]_R^{\infty}$$

$$= -GM_m \left[0 + \frac{1}{R}\right]$$

$$W = -GMm\frac{1}{R}$$

$$= -\frac{GMm}{R}$$

$$\Delta u = -W$$

$$U_\infty - U_i = -\frac{GMm}{R}$$

0, in infinity

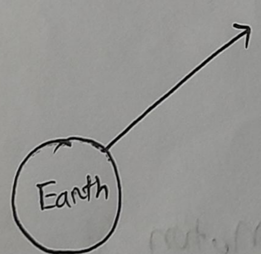
$$U_p = -\frac{GMm}{R}$$

work done positive. So, Potential energy will decrease and negative.

$$\therefore u = -\frac{GMm}{R}$$

Escape Speed!

Assignment Question



$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R}$$

$$V_{es} = \sqrt{\frac{2GM}{R}}$$

Escape Speed

Earth = 11·2 km/s

Moon = 2·38 km/s

Sun = 618 km/s

Neutron Star = 2×10^5 km/s

Simple Harmonic Motion (SHM)

$$F \propto -x$$

$$F = -kx$$

$\hookrightarrow$ Spring Constant $\Rightarrow N m^{-1}$

Hooks Law only works within Elastic Limit.

$$F = -kx$$

$$ma = -kx$$

$$ma + kx = 0$$

$$a + \frac{k}{m}x = 0$$

$$\omega = \sqrt{\frac{k}{m}}$$

$\hookrightarrow$ Angular Frequency

$$\boxed{a + \omega^2 x = 0}$$

$$\boxed{a_{(t)} = -\omega^2 x}$$

$$\boxed{a \propto -x}$$

$\hookrightarrow$ SHM: Simple Harmonic Motion

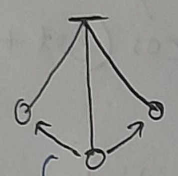
$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\Rightarrow \quad x_{(t)} = \underbrace{x_m}_{\text{Amplitude}} \cos(\overbrace{\omega}^{\text{Angular Frequency}} t + \overbrace{\theta}^{\text{Phase angle}})$$

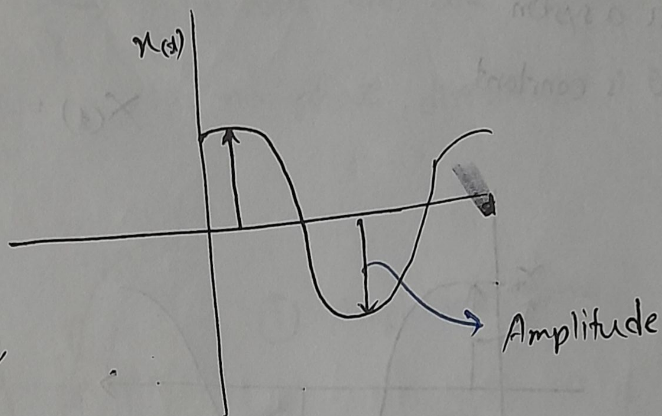
$$v = \frac{dx}{dt}$$

$$a = \frac{d^2x}{dt^2}$$

⊛



maximum distance from equilibinum position, amplitude



T = time Period

T .. sec ... circle ... 1

1 .. " ... " ..... $\boxed{\frac{1}{T}}$ → frequency

$f = \frac{1}{T}$ sec^{-1}/Hz

T 2π

1 ... $\frac{2\pi}{T} = \omega$ → Angular frequency

SHM

$a_{(t)} = -\omega^2 x(t)$ → Angular frequency

$\omega = \sqrt{\frac{k}{m}}$ → Spring Constant, → mass

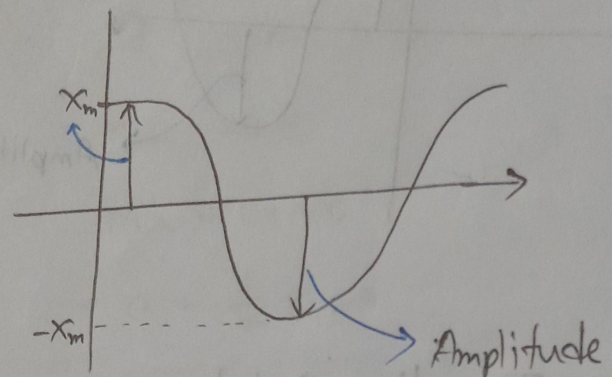
for a system ω is constant

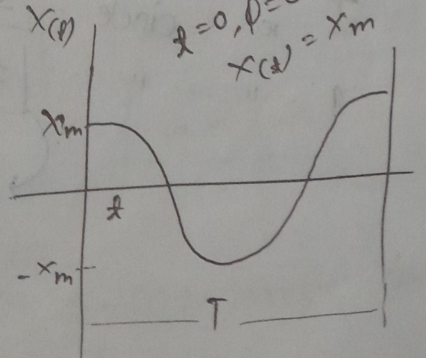
$x_{(t)} = x_m \cos(\omega t + \phi)$

Angular Frequency $\omega = \frac{2\pi}{T}$ = rad/sec

Amplitude

Phase angle



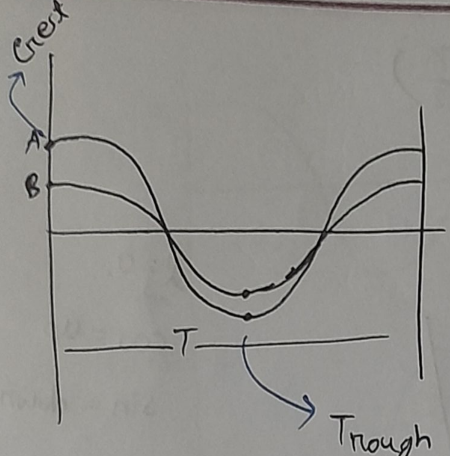


1 circle

Only frequency = $f = \frac{1}{T}$

$= Hz$

⊛



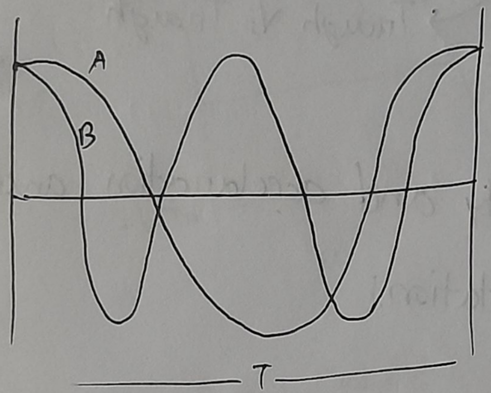
(i) Amplitude (Difference)

(ii) T, f, ω (same)

↳ in x-axis they are at the same point.

So, no phase difference.

⊛



(i) Amplitude same

(ii) $T > T'$

$T = 2T'$

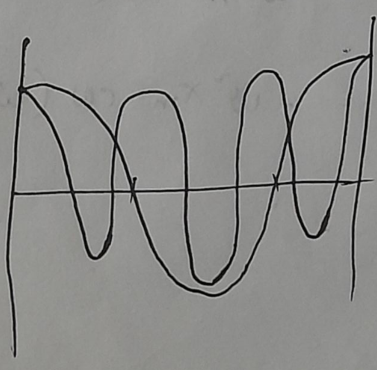
(iii) $f_B > f_A$

$f_B = 2f_A$

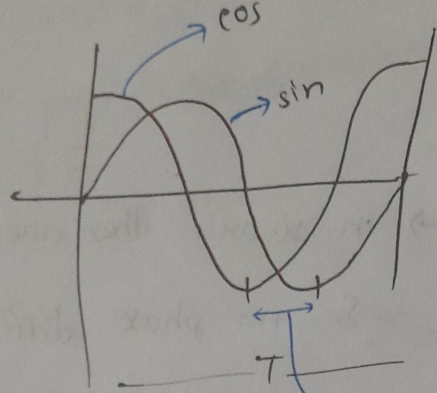
T = A

T' = B

⊛ Can you draw, same amplitude, one has 3 times higher frequency.



$$X_{(t)} = X_m \cos\left(\omega t - \frac{\pi}{2}\right)$$
$$= X_m \sin(\omega t)$$



$t = 0,$

cos = up

sin = down

then, cos leading the graph

Phase Difference

$-\frac{\pi}{2}$

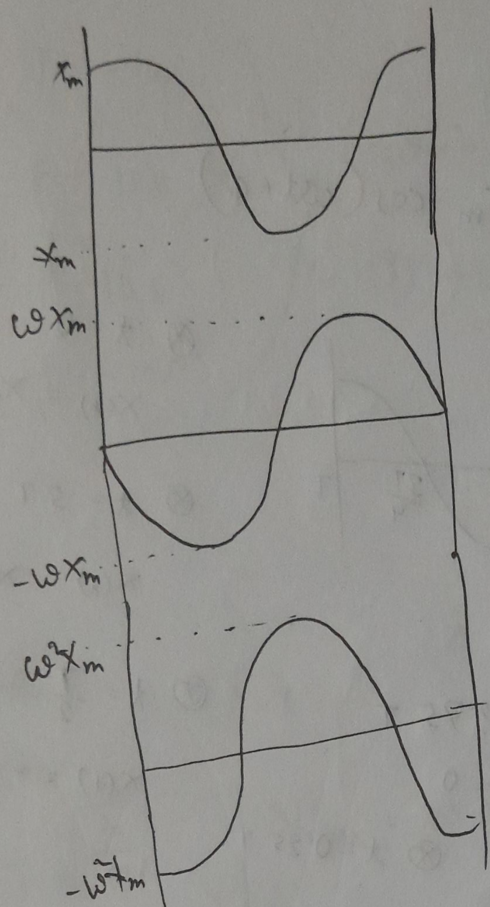
Can you find velocity and accelaration and show graphical representation.

$$X_{(t)} = X_m \cos(\omega t + \phi)$$

derivative,

$$V(t) = -\omega X_m \sin(\omega t + \phi)$$

$$a(t) = -\omega^2 X_m \cos(\omega t + \phi)$$



$t = 0, \ \phi = 0$

$x(t) = x_m$

$t = 0, \ \phi = 0$

$v(t) = 0$

$t = 0, \ \phi = 0$

$a(t) = -\omega^2 x_m$

$$X(t) = 2\cos(2t + \phi)$$

$$v(t) = -4\sin(2t + \phi)$$

$$a(t) = -8\cos(2t + \phi)$$

maximum velocity

$$|v(t)| = |-\omega x_m|$$

$$= 2 \times 2 = 4 \ ms^{-1}$$

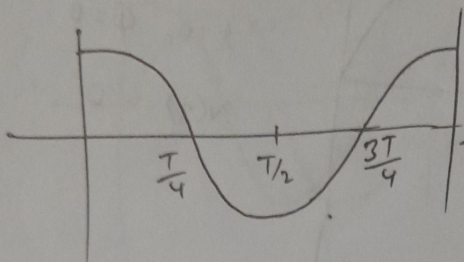
maximum accelaration

$$|a(t)| = |-\omega^2 x_m|$$

$$= 4 \times 2 = 8 \ ms^{-2}$$

Must in Final

$$x(t) = X_m \cos(\omega t + \phi)$$



- $t = 0$

  $x(t) = X_m$

- $t = 5T$

  $x(t) = X_m$

- $t = \frac{T}{2} = 5.5T$

  $x(t) = -X_m$

- $t = 99.75T$

  $x(t) = 0$

- $t = 0.25T$

  $x(t) = 0$

- Given

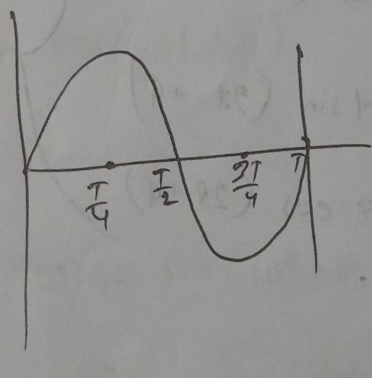
  $t = 0$

  $x(t) = 0$

  $\therefore t = 9.75T$

  $x(t) = ?$

  $= -X_m$



$\boxed{\text{marks} = 3 \text{ point}}$

must in final

(i) $F = -10x$

(ii) $F = 10x$

(iii) $F = 10x^2$

(iv) $F = -10x^2$

which is SHM?

(i) → Because similar to Hook's Law

$F = -kx$

$m =$ defined

$k =$ defined

$\omega = ? = \sqrt{\frac{k}{m}}$

$\omega = 2\pi f$

$T = \frac{1}{f}$

maximum amplitude x

$V_{max} = \omega X_m$

$a_{max} = \omega^2 X_m$

$K = \frac{1}{2} m v^2$

$U = \frac{1}{2} k x^2$

$E = K + U$

$= \frac{1}{2} m v^2 + \frac{1}{2} k x^2$

$= \frac{1}{2} m \omega^2 X_m^2 \sin^2(\omega t + \varphi) + \frac{1}{2} k X_m^2 \cos^2(\omega t + \varphi)$

$$= \frac{1}{2} m \frac{k}{m} X_m^2 \sin^2(\omega t + \phi) + \frac{1}{2} k X_m^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} k X_m^2 \left(\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi) \right)$$

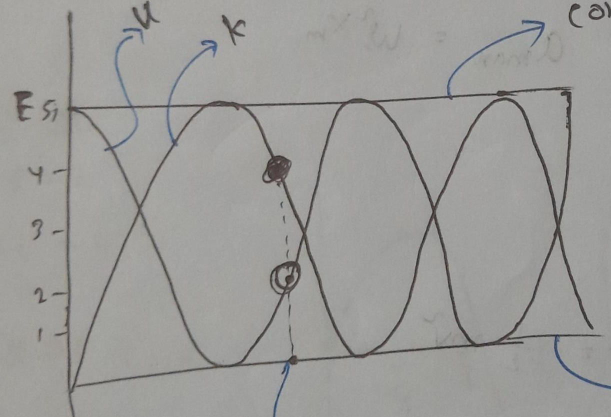
$E = \frac{1}{2} k X_m^2 \longrightarrow$ Total energy time independent.

$k = 2\ N/m$

$X_m = 10\ cm$

$= 0.1\ m$

$E = ?$



constant, Fix

Equilibirum Position

$U = 2j$

$k = 3j$

$E = 5j$

if intenceed point, then,

$U = 2.5\ j$

$k = 2.5\ j$

$E = 5j$

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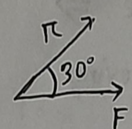
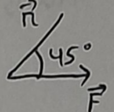
(i) $\uparrow r$ $\uparrow F$

(ii) $\uparrow r$ $\downarrow F$

(iii) 

(iv) 

$$\tau_4 > \tau_3 > \tau_1 = \tau_2$$

Wave is a disturbance that's carries energy.

(i) Mechanieal Wave

⇒ it requires medium

⇒ it follows Newton's Law

⇒ water wave, sound wave

(ii) Electro magnetic Wave

⇒ No medium required

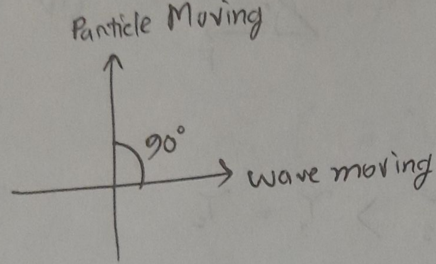
⇒ Doesn't follows Newton's Law

⇒ Light

(iii) Matter Wave

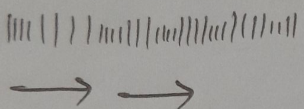
⇒ Electron & Proton

⊛ Transverse Wave ⇒



⇒ water wave

⊛ Longitudinal Wave :



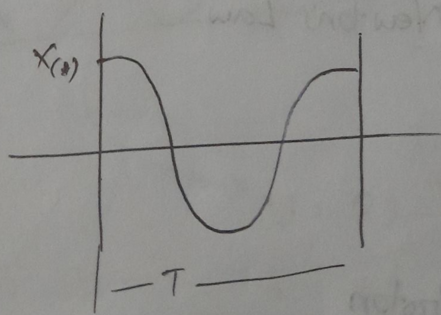
Sound Wave

Amplitude

Angular frequency

Phase Difference

⊛ $$X_{(t)} = X_m \cos(\omega t + \phi)$$

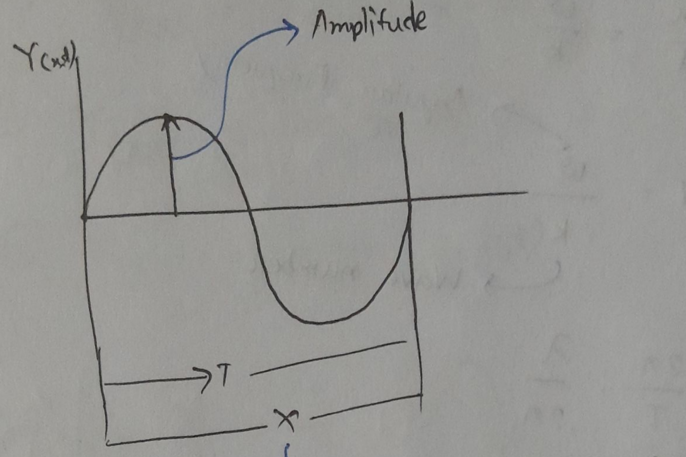


Wave Equation :

Amplitude — Wave Number — Angular Frequency

$$Y_{(x,t)} = Y_m \sin(kx - \omega t + \phi)$$

Not changable phase



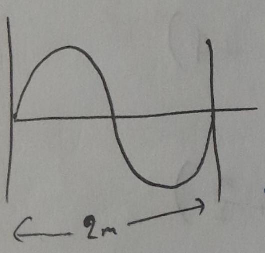
$T \ldots\ldots 2\pi$

$1 \ldots\ldots \frac{2\pi}{T} = \omega$

$\lambda \ldots\ldots 2\pi$

$1 \ldots\ldots \frac{2\pi}{\lambda} = k$

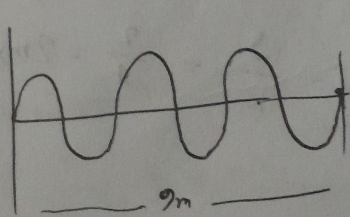
unit length



$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2} = \pi$$

$$k = \pi$$

$$\Rightarrow k = 0.5 = \frac{1}{2} \left[\pi / 2\pi \right]$$



$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3} = \frac{2}{3}\pi = \frac{1}{3}$$

* Phase is not changeable in wave.

$$(kx - \omega t) = \text{constant}$$

$$k \frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k}$$

Angular Frequency

$$v = \frac{\omega}{k}$$

Wave number

$$v = \frac{\omega}{k} = \frac{2\pi}{T} \cdot \frac{\lambda}{2\pi}$$

$$\boxed{v = \frac{\lambda}{T} = f\lambda}$$

$$Y_{(x,t)} = Y_m \sin(kx - \omega t + \phi)$$

$$Y_{(x,t)} = 10 \sin\left(x - 2t + \frac{\pi}{2}\right)$$

Amplitude = 10 m

Wave Number, $k = 1$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1} = 2\pi$$

$$\omega = 2$$

$$f = \frac{\omega}{2\pi}$$

$$v = f\lambda$$

$$f = \frac{v}{\lambda} = \frac{\omega/k}{\frac{2\pi}{k}} = \frac{\omega}{2\pi}$$

$$v = \frac{\omega}{k} = \frac{2}{1} = 2\,ms^{-1}$$

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$$Y_{(x,t)} = \underbrace{Y_m}_{\text{Amplitude}} \sin(\underbrace{k}_{\text{Wave Number}}x - \underbrace{\omega}_{\text{Angular Frequency}} t)$$

$-$ → Moving Left to Right →

(i) $Y_{(x,t)} = 2\sin(4x - 2t)$

(ii) $Y_{(x,t)} = \sin(3x - 4t)$

(iii) $Y_{(x,t)} = 2\sin(3x - 3t)$

$$\left.\begin{aligned} v_1 &= \frac{\omega}{k} = \frac{2}{4} = 0.5 \\ v_2 &= \frac{4}{3} = 1.33 \\ v_3 &= \frac{3}{3} = 1 \end{aligned}\right\}$$

Direction to wave
Wave velocity
$v_2 > v_3 > v_1$

direction y-axis
particle velocity

$$\left\{\begin{aligned} v_1 &= \frac{dY_{(x,t)}}{dt} = -4\cos(4x - 2t) \\ v_2 &= -4\cos(3x - 4t) \\ v_3 &= -6\cos(3x - 3t) \end{aligned}\right.$$

maximum velocity

$v_3 > v_1 = v_2$

⊛

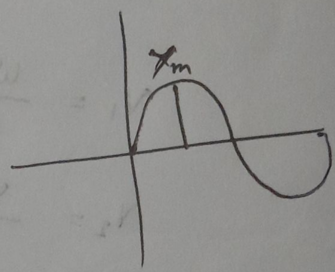
(i) Same amplitude

(ii) Same Wave Length / Frequency

(iii) Same direction → Constructive interference → Can create a big wave

(iv) Different direction opposite → Then they will cancelled each other → Standing Wave

⊛



$$Y_1(x,t) = Y_m \sin(kx - \omega t)$$

$$Y_2(x,t) = Y_m \sin(kx - \omega t + \phi)$$

$$Y = Y_1 + Y_2$$

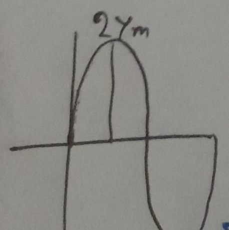
$$= Y_m \sin(kx - \omega t) + Y_m \sin(kx - \omega t + \phi)$$

$$= 2Y_m \sin\left(kx - \omega t + \frac{\phi}{2}\right) \cos\left(\frac{\phi}{2}\right)$$

$$= \underbrace{2Y_m \cos\frac{\phi}{2}}_{\text{Amplitude, Time independent}} \cdot \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

→ Amplitude

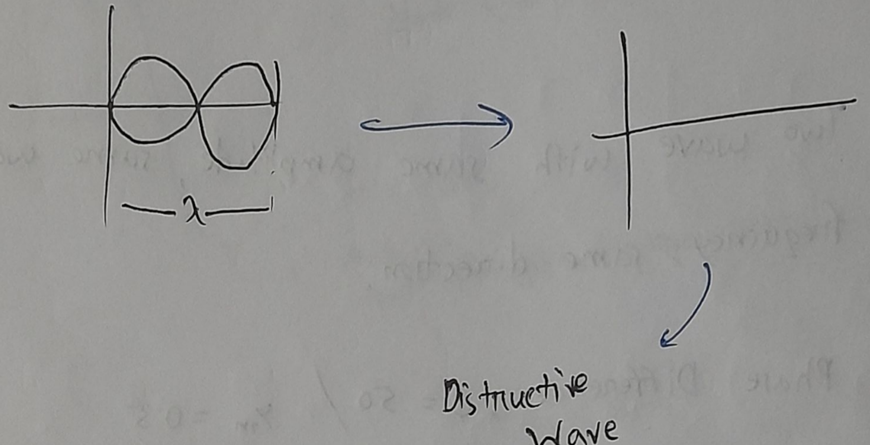
→ Time independent



→ Constructive Interference

⊛ if, $\phi = 0$, then amplitude max
and it two times than before.

⊛ if there is a phase difference of π. opposite direction.
then amplitude will be 0.



⊛

Degree	Radians	Wavelength	Amplitude	Type of Interference
0	0	0	$2y_m$	fully constructive
120	$\frac{2\pi}{3}$	0.33	y_m	intermediate
180	π	0.5	0	fully Distructive
240	$\frac{4\pi}{3}$	0.67	y_m	Intermediate
360	2π	1	$2y_m$	fully constructive
865	15.1	2.4	$0.60 y_m$	Intermediate

$0.2 \rightarrow C$

$0.4 \rightarrow D$

$0.8 \rightarrow C$

Fully Intermediate $\begin{cases} 0.75 \rightarrow I \\ 0.25 \rightarrow I \end{cases}$

Possibilities

$\lambda \approx 0 \rightarrow C$

$\longrightarrow I$

$\lambda \approx 0.5 \rightarrow D$

Two wave with same amplitude, same wave, same frequency, same direction.

Phase Difference, $\phi = 50$ / $y_m = 0.5$

$$Y = ? \longrightarrow \left|2y_m \cos\frac{\phi}{2}\right|$$

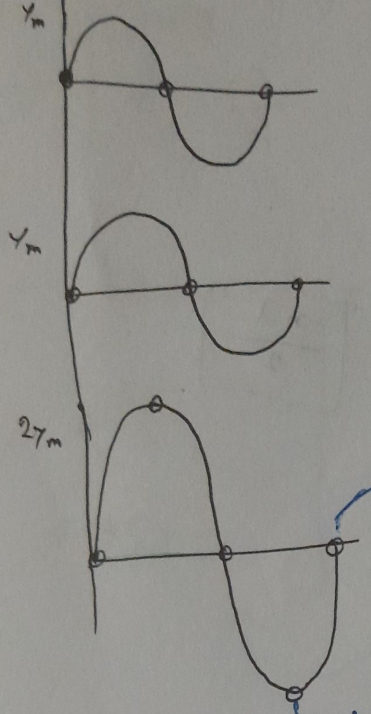
$y_m = 0.5\ m$

$Y = 0.8\ m$

$\phi = ?$

$$Y = \left|2y_m \cos\frac{\phi}{2}\right|$$

Standing Wave



$$Y_1 = y_m \sin(kx - \omega t)$$

$$Y_2 = y_m \sin(kx + \omega t)$$

$$Y = Y_1 + Y_2$$

$$= y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t)$$

$$Y = \underbrace{2y_m \sin(kx)}_{\text{Amplitude : Time independent.}} \cos(\omega t)$$

Node,

$$\sin kx = 0$$

$$\sin kx = \sin n\pi$$

$$kx = n\pi$$

$$x = \frac{n\pi}{k} = \frac{n\pi}{\frac{2\pi}{\lambda}} = \boxed{\frac{n\lambda}{2}}$$

$n=0$	$n=1$	$n=2$
$x=0$	$x=\frac{\lambda}{2}$	$x=\lambda$

Anti node

$$\sin kx = \sin\frac{\pi}{2} = \sin\frac{3\pi}{2} = \sin\frac{5\pi}{2}$$

$$= \sin\frac{(2n+1)\pi}{2}$$

$$kx = (2n+1)\frac{\pi}{2}$$

$$x = \frac{(2n+1)\frac{\pi}{2}}{k}$$

$$= \frac{(2n+1)\frac{\pi}{2}\cdot\lambda}{2\pi}$$

$$x = \left(n+\frac{1}{2}\right)\frac{\lambda}{2}$$

$n = 0$ | $n = 1$
$x = \frac{\lambda}{4}$ | $x = \frac{3\lambda}{4}$

(*) Two red light crossing each other, $\lambda = 650$ nm. Find out node position.

(*) $\lambda = 650$ nm

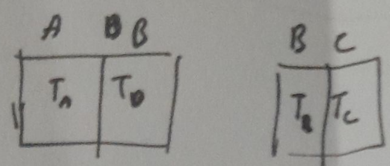
$\therefore x = n \frac{650}{2}$

$= 325 n$

PHY 107 / L-22 / 17.12.2022

Temparature Conversion.:

$T_A = T_B$



Zeroth Law of thermodyn:

$T_A = T_C$

First Law of thermodynamics

→ represent energy conservation

$$Q = W + \Delta u$$

Δu → Internal Energy

20J <20J

System gain energy → $+Q$

System Loss " → $-Q$

Work done by the system → $+W$

Work done on the system → $-W$

$W = -100\,j$

$\Delta u = 74\,j$

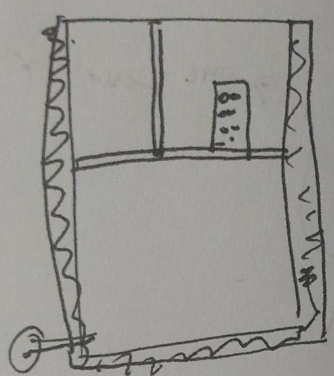
$Q = -100 + 74$

$= -\cancel{3}26\,j$

Comment:

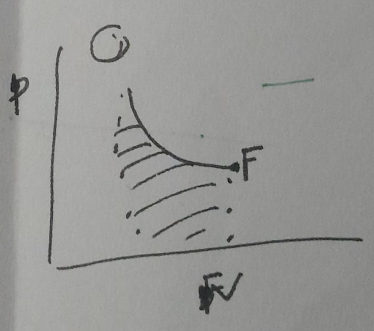
System is losing energy 26 jul as heat.

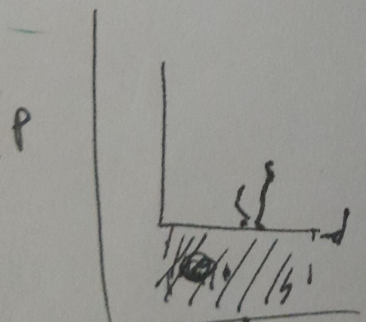
$PV = \frac{F}{A} V$

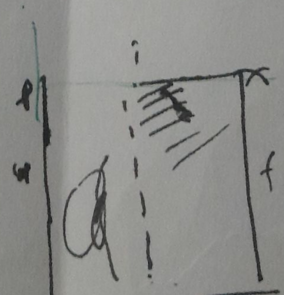


$W = P\Delta V$

$\int dW = \int P dV$







2nd Law

~~N-Tropi~~ N-Tropi

N-tropi enercase in irreversal process

decrease in reversiceel process

irreverse $\Delta s > 0$

reverse = $\Delta s = 0$

$\cancel{\Delta s \geq 0}$

$\boxed{\Delta s \geq 0}$ → 2nd Law of thermodynamics

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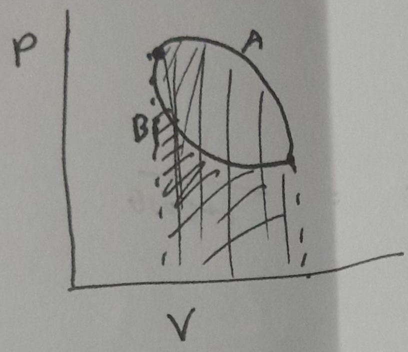
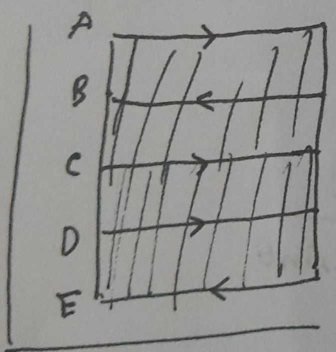
$W = P \Delta V$



Way of A > Way of B

(Work Done)

(under the curve)



$\boxed{A \rightarrow E} \rightarrow$ maximum positive work done.

$\boxed{A \rightarrow E} > \boxed{C \rightarrow E} > \boxed{A \rightarrow B} = \boxed{D \rightarrow E}$

according to positive work done

Rotation

$\theta(t) = t^2 + 2t + 1$

$\omega(t) = \frac{d\theta}{dt} = 2t + 2$

$\alpha(t) = 2$

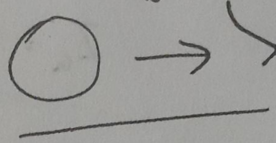
$t = 2$ sec

$a_t = \alpha r$

$a_r = \frac{v^2}{r}$

$= \omega^2 r$

$\alpha = \frac{d\omega}{dt}$



$\omega = 5$ rad/se

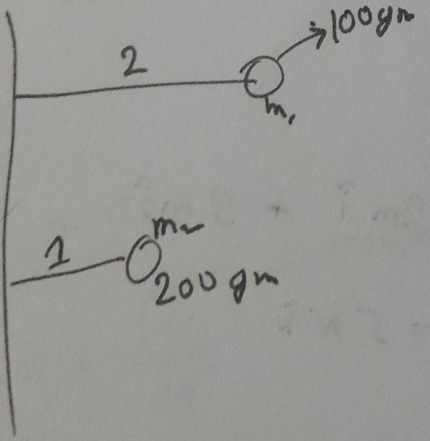
$\omega = 3$ rad/sec

$a_{r} =$ defined

$a_t = \frac{d\omega}{dt} \to 0$

↳ ~~de~~ changing then

$a_r = a_t =$ defined



$I = mr^2$

$I_1 > I_2$

i j k

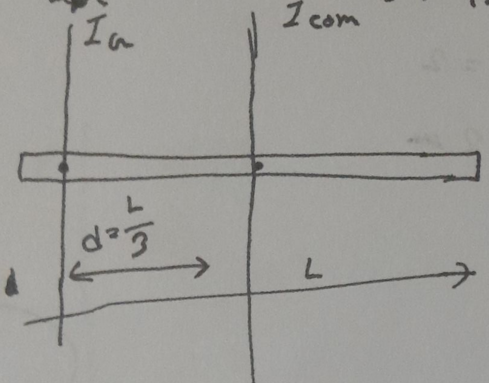
$-J \times k = -i$

Previous Object,

$k = 10\,J$ $\qquad k = \frac{1}{2}mv^2$

$\omega = ?$ $\qquad k = \frac{1}{2}I\omega^2$

$I_{com} = \frac{1}{12}mL^2$



$$\boxed{I_a = I_{com} + md^2}$$

$$= \frac{1}{12}mL^2 + m\left(\frac{L}{3}\right)^2$$

$$= \frac{1}{12}mL^2 + m\frac{L^2}{9}$$

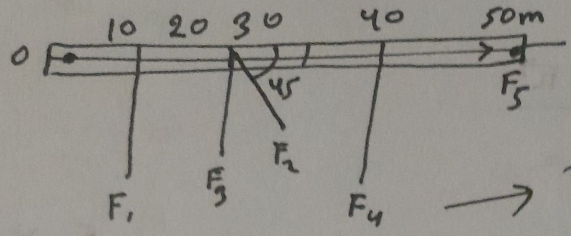
Torque

$\bar{\tau} = \bar{r} \times \bar{F}$ $\qquad r = 2m\,\hat{i} + 3m\,\hat{j}$

$x \quad \boxed{z} \quad -y$ $\qquad \bar{F} = 5N\,\hat{k}$

$$\bar{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 0 & 0 & 5 \end{vmatrix}$$

⊛



$\rightarrow \tau = 400$ Nm

$\to 0$

$F_4 > F_3 > F_1 > F_2 > F_5$

$\tau_4 > \tau_3 > \tau_1 > \tau_2 > \tau_5$

⊛ $\tau = 10$ Nm

$F = ma$

$\alpha = 2$ rad/sec^2

$\tau = I\alpha$

$I = ?$

⊛

$F = 0$

$\xrightarrow{}$ constant if $F = 0$

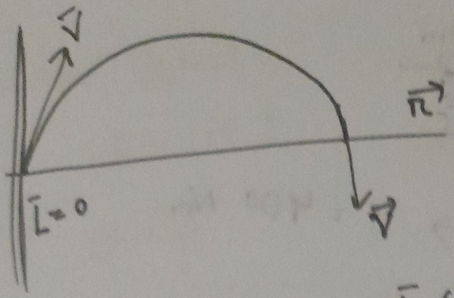
$\vec{P} = ?$ $\boxed{\frac{d\bar{P}}{dt} = F}$

⊛

⊛ $\boxed{\bar{L} = \bar{r} \times \bar{p}}$

$\rightarrow$ constant

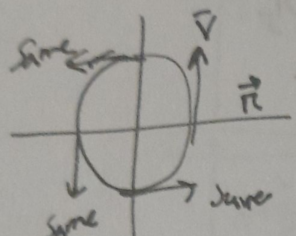
$\bar{\tau} = 0 = \frac{d\bar{L}}{dt}$



$\bar{L} \neq 0$

$L = \bar{r} \times \bar{p}$

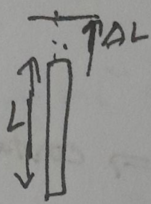
$= \bar{r} \times m\vec{v}$

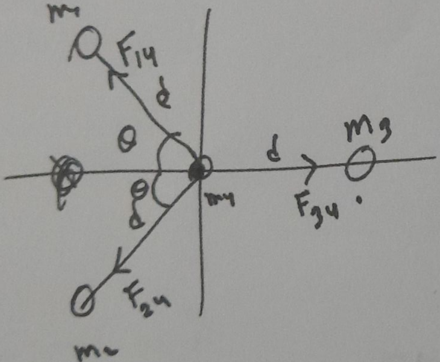


direction but changing and Angular Momentum not changing

$$\frac{F}{A} = E\frac{\Delta L}{L}$$

(Find out formula which is applicable)





$$F_{34} = G\frac{m_3 m_4}{d^2}$$

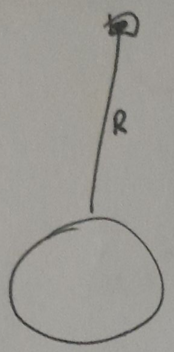
$$F_{net} = F_{34} - F_{14}\cos\theta - F_{24}\cos\theta$$

$= 0000$

$$\frac{Nm^2 \; kg \; kg}{kg^2 \; m^2}$$

$F = N$

⊛



$$u = \frac{-GM_{m}}{R}$$

Earth

m

R → from surface

⊛ $$V_{Escape} = \sqrt{\frac{2GM}{R}}$$

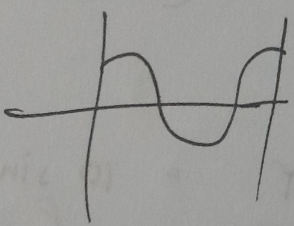
M → kg

R → meter

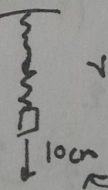
⊛ $X_{(t)} = X_m \cos(\omega t + \phi)$

$t = 0, \phi = 0$

$x_{(t)} = X_m$



$\omega = \sqrt{\frac{k}{m}}$

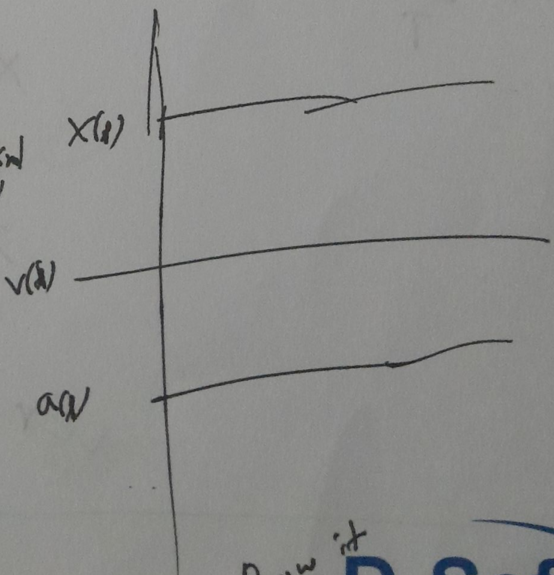


$v = ?$

$= \omega X_m$

$|v_{(t)}| = |\omega X_m|$

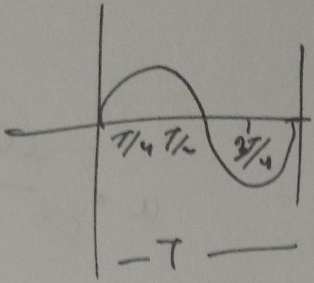
$|a_{(t)}| = |\omega^2 X_m|$



Draw it

$t = 0$

$x(t) = 0$



$t = n.75\,T$

$x(t) = -x_m$

$$\boxed{E = \frac{1}{2} k x_m^2}$$ Total Energy

$\cancel{\phi}\; Y_{(x,t)} = y_m \sin(kx - \omega t + \phi)$

$k = \frac{2\pi}{\lambda}$

$\omega = \frac{2\pi}{T}$

$Y = 10 \sin\left(2x - t + \frac{\pi}{2}\right)$

$x_m = 10$

$k = 2$

$\lambda = ?\quad \frac{2\pi}{k} = \frac{2\pi}{2} = \pi = \frac{\pi}{2\pi} = \frac{1}{2} = 0.5$

$v = \frac{\omega}{k}$

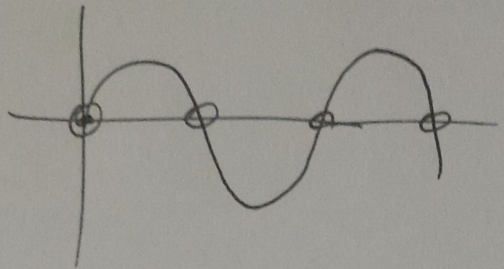
Find V in both axis

$Y' = 2y_m \sin kx \cos\frac{\phi}{2}$

$|Y'| = 2y_m \cos\frac{\phi}{2}$ $\quad y_m = 0.5\,m \quad |Y'| = ?$ amplitude

$\phi = 200^\circ$

⊛



$$x = \frac{n\lambda}{2}$$

$$x = \frac{n\,600}{2} = n\,300 \text{ Nm}$$

$n = 0$	$n = 1$	$n = 2$
$x = 0$	$x = 300$	$x = 600$

⊛ Temparature converte

Q.

⊛ $Q = W + \Delta U$

$\left.\begin{array}{l}\text{by the system positive} \\ \text{on the system negative}\end{array}\right\}$ work done

Calculate Q;

⊛ $\boxed{\text{Today Notes}}$