North South University

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Assignment-4

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4.7

6) Find the basis for the null space of A.

d)

$$A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$$

$r_2' = r_2 - 3r_1$

$r_3' = r_3 + r_1$

$r_4' = r_4 - 2r_1$

$$\begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 0 & -14 & -14 & -14 & -28 \\ 0 & 4 & 4 & 4 & 8 \\ 0 & -5 & -5 & -5 & -10 \end{bmatrix}$$

$r_2' = -\frac{1}{14} r_2$

$r_3' = \frac{1}{4} r_3$

$r_4' = -\frac{1}{5} r_4$

$$\begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 & 2 \end{bmatrix}$$

$r_3' = r_3 - r_2$

$r_4' = r_4 - r_2$

$$\begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$r_1' = r_1 - 4r_2$ $\begin{bmatrix} 1 & 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Let, the system,

$$x_1 + x_3 + 2x_4 + x_5 = 0$$
$$x_2 + x_3 + x_4 + 2x_5 = 0$$

Solution,

$$x_1 = -x_3 - 2x_4 - x_5$$
$$x_2 = -x_3 - x_4 - 2x_5$$

Let,

$$x_3 = t$$
$$x_4 = s$$
$$x_5 = r$$

$$\therefore \quad x_1 = -t - 2s - r$$
$$x_2 = -t - s - 2r$$

$$\therefore (x_1, x_2, x_3, x_4, x_5) = (-t-2s-r, -t-s-2r, t, s, r)$$
$$= (-t, -t, t, 0, 0) + (-2s, -s, 0, s, 0) + (-r, -2r, 0, 0, r)$$
$$= t(-1, -1, 1, 0, 0) + s(-2, -1, 0, 1, 0) + r(-1, -2, 0, 0, 1)$$

$$= t v_1 + s v_2 + r v_3$$

$\therefore$ $S = \{v_1, v_2, v_3\}$ is a basis for the null space of the matrix A.

11) Find a basis for the subspace of $\mathbb{R}^4$ spanned by the given vectors.

b) $(-1, 1, -2, 0)$, $(3, 3, 6, 0)$, $(9, 0, 0, 3)$

Solutions:

Lets reduce matrix which has rows as given vectors.

$$\begin{bmatrix} -1 & 1 & -2 & 0 \\ 3 & 3 & 6 & 0 \\ 9 & 0 & 0 & 3 \end{bmatrix}$$

$r_1' = -r_1$

$r_2' = \frac{1}{3} r_2$

$r_3' = \frac{1}{3} r_3$

$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 1 & 2 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix}$$

$r_2' = r_2 - r_1$
$r_3' = r_3 - 3r_1$
$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 3 & -6 & 1 \end{bmatrix}$$

$r_2' = \frac{1}{2} r_2$
$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 3 & -6 & 1 \end{bmatrix}$$

$r_3' = r_3 - 3r_2$
$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -6 & 1 \end{bmatrix}$$

$r_3' = -\frac{1}{6} r_3$
$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{6} \end{bmatrix}$$

Therefore,
basis for the subspace $\mathbb{R}^4$ spanned by the given vectors is,

$\{(1, -1, 2, 0), (0, 1, 0, 0), (0, 0, 1, -\frac{1}{6})\}$

c) $(1,1,0,0)$, $(0,0,1,1)$, $(-2,0,2,2)$, $(0,-3,0,3)$

Solutions:

Lets reduces matrix which has rows as given vectors.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ -2 & 0 & 2 & 2 \\ 0 & -3 & 0 & 3 \end{bmatrix}$$

$$\xrightarrow[R_3' = R_3 + 2R_1]{\cancel{R_3' = -\frac{1}{2}R_3}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & -3 & 0 & 3 \end{bmatrix}$$

$$\begin{matrix} R_3' = \frac{1}{2}R_3 \\ R_4' = -\frac{1}{3}R_4 \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

$$\xrightarrow{R_2 \rightleftharpoons R_3} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

$$\xrightarrow{R_4' = R_4 - R_2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

$$\xrightarrow{R_4' = R_4 + R_3} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\xrightarrow{R_4' = -R_4} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore basis for the sub space of $\mathbb{R}^4$ spanned by the given vectors is,

$$\{(1, 1, 0, 0), (0, 1, 1, 1), (0, 0, 1, 1), (0, 0, 0, 1)\}$$

4.8

2) Find the rank and nullity of the matrix; then verify that the values obtained satisfy Formula 4 in the Dimension Theorem.

e)

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

$r_2' = r_2 - 2r_1$

$r_3' = r_3 + r_1$

$$\begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & -7 & -7 & -4 \\ 0 & 7 & 7 & 4 \end{bmatrix}$$

$r_3' = r_2 + r_3$

$$\begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & -7 & -7 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$r_2' = -\frac{1}{7} r_2$

$$\begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 1 & \frac{4}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{r_1' = r_1 - 4r_2} \begin{bmatrix} 1 & 0 & 1 & -2/7 \\ 0 & 1 & 1 & 4/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here,

First and second columns have leading 1.

Therefore, $rank(A) = 2$

Now,

Let the system,

$$x_1 + x_3 - \frac{2}{7}x_4 = 0$$

$$x_2 + x_3 + \frac{4}{7}x_4 = 0$$

Solutions,

$$x_1 = -x_3 + \frac{2}{7}x_4$$

$$x_2 = -x_3 - \frac{4}{7}x_4$$

Let, $x_3 = t$

$x_4 = s$

$$\therefore (x_1, x_2, x_3, x_4) = \left(-t + \frac{2}{7}s,\ -t - \frac{4}{7}s,\ t,\ s\right)$$

$$= (t, -t, t, 0) + \left(\frac{2}{7}s, -\frac{4}{7}s, 0, s\right)$$

$$= t(1, -1, 1, 0) + s\left(\frac{2}{7}, -\frac{4}{7}, 0, 1\right)$$

$$= t\,v_1 + s\,v_2$$

Therefore,

$$\text{nullity }(A) = 2$$

Checking,

$$\text{rank}(A) + \text{nullity }(A) = 2+2$$
$$= 4$$

which is number of collumns of A.

d)

$$A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$$

$$\begin{array}{l} r_2' = r_2 - 3r_1 \\ r_3' = r_3 + r_1 \\ r_4' = r_4 - 4r_1 \end{array} \quad \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 0 & -14 & -14 & -14 & -28 \\ 0 & 4 & 4 & 4 & 8 \\ 0 & -5 & -5 & -5 & -10 \end{bmatrix}$$

$$\begin{array}{l} r_2' = -\frac{1}{14} r_2 \\ r_3' = \frac{1}{4} r_3 \\ r_4' = -\frac{1}{5} r_4 \end{array} \quad \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 & 2 \end{bmatrix}$$

$r_3' = r_3 - r_2$

$r_4' = r_4 - r_2$

$$\begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$r_1' = r_1 - 4r_2$

$$\begin{bmatrix} 1 & 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Here, First and second columns have leading 1.

$\therefore$ rank $(A) = 2$

Let, the system,

$$x_1 + x_3 + 2x_4 + x_5 = 0$$

$$\cancel{x_1}$$

$$x_2 + x_3 + x_4 + 2x_5 = 0$$

Solutions,

$$x_1 = -x_3 - 2x_4 - x_5$$

$$x_2 = -x_3 - x_4 - 2x_5$$

Let,

$x_3 = t$

$x_4 = r$

$x_5 = s$

$$\therefore (x_1, x_2, x_3, x_4, x_5) = (-t - 2r - s,\ -t - r - 2s,\ t,\ r,\ s)$$

$$= (-t, -t, t, 0, 0) + (-2r, -r, 0, r, 0) + (-s, -2s, 0, 0, s)$$

$$= t(-1, -1, 1, 0, 0) + r(-2, -1, 0, 1, 0) + s(-1, -2, 0, 0, 1)$$

Therefore,

$$\text{nullity}(A) = 3$$

Checking,

$$\text{rank}(A) + \text{nullity}(A) = 2 + 3$$
$$= 5$$

which is number of columns of A.

4.9

8/ Find the standard matrix for the transformation defined by the equations.

b)

The matrix transformation $T: R^3 \to \mathbb{R}^3$ defined by the equations

$$w_1 = 7x_1 + 2x_2 - 8x_3$$

$$w_2 = -x_2 + 5x_3$$

$$w_3 = 4x_1 + 7x_2 - x_3$$

can be expressed in matrix form as

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 7 & 2 & -8 \\ 0 & -1 & 5 \\ 4 & 7 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

So the standard matrix for T is,

$$\cancel{A = \begin{bmatrix} 2 & -3 & 1 & 5 \\ 4 & 1 & -2 & 1 \\ 5 & -1 & 4 & 0 \end{bmatrix}}$$

$$A = \begin{bmatrix} 7 & 2 & -8 \\ 0 & -1 & 5 \\ 4 & 7 & -1 \end{bmatrix}$$

c)

The matrix transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by the equations

$$W_1 = -x_1 + x_2$$

$$W_2 = 3x_1 - 2x_2$$

$$W_3 = 5x_1 - 7x_2$$

can be expressed in matrix form as

$$\begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \\ 5 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

So, the standard matrix for T is,

$$A = \begin{bmatrix} -1 & 1 \\ 3 & -2 \\ 5 & -7 \end{bmatrix}$$

10) Find the standard matrix for the operator T defined by the formula.

e) $T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, x_1 + 5x_2, x_3)$

Solutions:

Let, $\{e_1, e_2, e_3\}$ be standard basis for R^3. Then,

$$Te_1 = T(1,0,0) = (1,1,0)$$

$$Te_2 = T(0,1,0) = (2,5,0)$$

$$Te_3 = T(0,0,1) = (1,0,1)$$

standard matrix is constructed from Te_1, Te_2 and Te_3 as columns.

Therefore standard matrix is,

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

d) $T(x_1, x_2, x_3) = (4x_1, 7x_2, -8x_3)$

Solutions:

Let, $\{e_1, e_2, e_3\}$ be standand basis fon $\mathbb{R}^3$.

Then,

$$Te_1 = T(1,0,0) = (4,0,0)$$

$$Te_2 = T(0,1,0) = (0,7,0)$$

$$Te_3 = T(0,0,1) = (0,0,-8)$$

Standand matrix is constructed from Te_1, Te_2, and Te_3

a columns.

Therefore,
standand matrix is,

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 5 & 1 \end{bmatrix}$$

5.1

6] Find the characteristic equations of the following matrix.

c) $$\begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix}$$

Let

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ be a nonzero vector and a scalar λ.

then, $$(A - \lambda I)x = 0$$

$$A - \lambda I = \begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2-\lambda & 0 & 1 \\ -6 & -2-\lambda & 0 \\ 17 & 5 & -4-\lambda \end{bmatrix}$$

Now,

$$\det(A-\lambda I) = \begin{vmatrix} -2-\lambda & 0 & 1 \\ -6 & -2-\lambda & 0 \\ 19 & 5 & -4-\lambda \end{vmatrix}$$

$$= (-2-\lambda)\ (-2-\lambda)(-4-\lambda) - 0 + 1\left(-30 - 19(-2-\lambda)\right)$$

$$= (-2-\lambda)^2(-4-\lambda) - 30 - 19(-2-\lambda)$$

$$= (4 - 2(-2)\cdot\lambda + \lambda^2)\ (-4-\lambda) - 30 + 38 + 19\lambda$$

$$= -16 - 16\lambda - 4\lambda^2 - 4\lambda - 4\lambda^2 - \lambda^3 - 30 + 38 + 19\lambda$$

$$= -\lambda^3 - 8\lambda^2 - \lambda - 8$$

Therefore,

the characteristic equation is,

$$-\lambda^3 - 8\lambda^2 - \lambda - 8 = 0$$

$$\lambda^3 + 8\lambda^2 + \lambda + 8 = 0$$

$$(\lambda^2+1)(\lambda+8) = 0$$

d) $\begin{bmatrix} -1 & 0 & 1 \\ -1 & 3 & 0 \\ -4 & 13 & -1 \end{bmatrix}$

Let,

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be a non zero vector and a scalar λ.

then, $(A - \lambda I)X = 0$

$$A - \lambda I = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 3 & 0 \\ -4 & 13 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-\lambda & 0 & 1 \\ -1 & 3-\lambda & 0 \\ -4 & 13 & -1-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} -1-\lambda & 0 & 1 \\ -1 & 3-\lambda & 0 \\ -4 & 13 & -1-\lambda \end{vmatrix}$$

$$= (-1-\lambda)(3-\lambda)(-1-\lambda) - 0 + 1(-13 + 4(3-\lambda))$$

$$= (-1-\lambda)^2(3-\lambda) - 13 + 4(3-\lambda)$$

$$= (1 - 2(-1)\cdot\lambda + \lambda^2)(3-\lambda) - 13 + 12 - 4\lambda$$

$$= 3 + 6\lambda + 3\lambda^2 - \lambda - 2\lambda^2 - \lambda^3 - 13 + 12 - 4\lambda$$

$$= -\lambda^3 + \lambda^2 + \lambda + 2$$

Therefore,

the characteristic equation is,

$$-\lambda^3 + \lambda^2 + \lambda + 2 = 0$$

$$\lambda^3 - \lambda^2 - \lambda - 2 = 0$$

5.2

6/

Let,

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

a) Find the eigenvalues of A

b) For each eigenvalue λ, find the rank of the matrix $\lambda I - A$.

c) Is A diagonalizeable? Justify your conclusion.

Solution

a)

Let, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be a nonzero vector and a scalar λ

then,

$$(\lambda I - A)x = 0$$

$$\lambda I - A = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix} = \begin{bmatrix} \lambda-4 & 0 & -1 \\ -2 & \lambda-3 & -2 \\ -1 & 0 & \lambda-4 \end{bmatrix}$$

$\det(\lambda I - A) = 0$

$$\Rightarrow \begin{vmatrix} \lambda-4 & 0 & -1 \\ -2 & \lambda-3 & -2 \\ -1 & 0 & \lambda-4 \end{vmatrix} = 0$$

$$\Rightarrow (\lambda-4)\,(\lambda-3)(\lambda-4) - 0 - 1\,(0+1(\lambda-3)) = 0$$

$$\Rightarrow (\lambda-4)^2\,(\lambda-3) - \lambda + 3 = 0$$

$$\Rightarrow (\lambda^2 - 8\lambda + 16)\,(\lambda-3) - \lambda + 3 = 0$$

$$\Rightarrow \lambda^3 - 8\lambda^2 + 16\lambda - 3\lambda^2 + 24\lambda - 48 - \lambda + 3 = 0$$

$$\Rightarrow \lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$\therefore \lambda = 3, 5$$

Therefore, the eigenvalues of the initial matrix are 3 and 5.

b)

For $\lambda = 3$, we have,

$$3I - A = \begin{bmatrix} -1 & 0 & -1 \\ -2 & 0 & -2 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\xrightarrow{r_1' = -r_1} \begin{bmatrix} 1 & 0 & 1 \\ -2 & 0 & -2 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\begin{array}{l} r_2' = r_2 + 2r_1 \\ r_3' = r_3 + r_1 \end{array} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

There is only one nonzero row.

Therefore rank of $3I - A$ is 1.

For $\lambda = 5$, we have,

$$5I - A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ -2 & 2 & -2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow[R_3' = R_3 + R_1]{R_2' = R_2 + 2R_1} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

There are two linearly independent rows.

Therefore, the rank of $5I - A$ is 2.

c)

Note that, since the rank of $3I-A$ is 1, it means that the nullity is 2, meaning that there are two eigenvectors corresponding to the eigenvalue 3.

Similarly, there is 1 eigenvector corresponding to the eigen value 5.

There are 3 eigenvectors, therefore the given 3×3 matrix is diagonalizable.

10) Determine whether the matrix is diagonalizable.

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 3 & 0 \\ -4 & 13 & -1 \end{bmatrix}$$

Let,

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be a nonzero vector and a scalar λ...

then, $(\lambda I - A)X = 0$

$$\lambda I - A = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} -1 & 0 & 1 \\ -1 & 3 & 0 \\ -4 & 13 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda+1 & 0 & -1 \\ 1 & \lambda-3 & 0 \\ 4 & -13 & \lambda+1 \end{bmatrix}$$

Now,

$\det(\lambda I - A) = 0$

$$\begin{vmatrix} \lambda+1 & 0 & -1 \\ 1 & \lambda-3 & 0 \\ 4 & -13 & \lambda+1 \end{vmatrix} = 0$$

$\Rightarrow (\lambda+1)^2(\lambda-3) - (-13-4(\lambda-3)) = 0$

$\Rightarrow (\lambda^2+2\lambda+1)(\lambda-3) + 13 + 4\lambda - 12 = 0$

$\Rightarrow \lambda^3 - 3\lambda^2 + 2\lambda^2 - 6\lambda + \lambda - 3 + 13 + 4\lambda - 12 = 0$

$\Rightarrow \lambda^3 - \lambda^2 - \lambda - 2 = 0$

$\Rightarrow (\lambda^2+\lambda+1)(\lambda-2) = 0$

$\therefore \lambda = 2$ and $\lambda = \frac{-1 \pm i\sqrt{3}}{2}$

Hence, the matrix is not diagonalizable over real numbers.

15/ Find a matrix P that diagonalizes A, and compute $P^{-1}AP$.

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\therefore \cancel{\lambda I - A}$

$$\therefore \; A - \lambda I = \begin{bmatrix} 2-\lambda & 0 & -2 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

Now,

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 0 & -2 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)(3-\lambda)^2 - 0 - 2\left(0 - 0(3-\lambda)\right) = 0$$

$$\Rightarrow (2-\lambda)(9 - 6\lambda + \lambda^2) \cancel{+6 - 2\lambda} = 0$$

$$\Rightarrow 18 - 12\lambda + 2\lambda^2 - 9\lambda + 6\lambda^2 - \lambda^3 \cancel{+6-2\lambda} = 0$$

$$\Rightarrow -\lambda^3 + 8\lambda^2 - 21\lambda \overset{+18}{\cancel{+24}} = 0$$

$$\Rightarrow \lambda^3 - 8\lambda^2 + 21\lambda - \overset{18}{\cancel{24}} = 0$$

$$\therefore \lambda = 2, 3$$

Now,

$$\begin{bmatrix} 2-\lambda & 0 & -2 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \cdots (i)$$

Put, $\lambda = 2$ in (i)

$$\begin{bmatrix} 2-2 & 0 & -2 \\ 0 & 3-2 & 0 \\ 0 & 0 & 3-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2x_3 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore \quad x_2 = 0$

$x_3 = 0$

and $x_1 = r$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} = r \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore P_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is independent eigenvectors of A corresponding to $\lambda = 2$.

Again,

put $\lambda = 3$ in (i)

$$\begin{bmatrix} -1 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_1 - 2x_3 = 0$$

$$\Rightarrow x_1 = -2x_3$$

Let, $x_2 = r$

$x_3 = s$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2s \\ r \\ s \end{bmatrix} = \begin{bmatrix} -2s \\ 0 \\ s \end{bmatrix} + \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix}$$

$$= s \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + r \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore P_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$, $P_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ are independent eigen vectors of A corresponding to $\lambda = 3$

Therefore,

$$P = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Therefore,

$$P^{-1}AP = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$