NORTH SOUTH UNIVERSITY

Department of Mathematics & Physics

Assignment – 01

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Given,

$$y''' + Py'' + Qy' + Ry = f(x) \quad \cdots (i)$$

$$\therefore y_c = c_1y_1 + c_2y_2 + c_3y_3$$

Let,

$$y_p = u_1y_1 + u_2y_2 + u_3y_3 \quad \cdots (ii)$$

$$\Rightarrow y_p' = u_1y_1' + u_1'y_1 + u_2y_2' + u_2'y_2 + u_3y_3' + u_3'y_3$$

Let,

$$u_1'y_1 + u_2'y_2 + u_3'y_3 = 0 \quad \cdots (iii)$$

$$\therefore y_p' = u_1y_1' + u_2y_2' + u_3y_3' \quad \cdots (iv)$$

$$\Rightarrow y_p'' = u_1y_1'' + u_1'y_1' + u_2y_2'' + u_2'y_2' + u_3y_3'' + u_3'y_3'$$

Let,

$$u_1'y_1' + u_2'y_2' + u_3'y_3' = 0 \quad \cdots (v)$$

$$\therefore y_p'' = u_1y_1'' + u_2y_2'' + u_3y_3'' \quad \cdots (vi)$$

$$\Rightarrow y_p''' = u_1y_1''' + u_1'y_1'' + u_2y_2''' + u_2'y_2'' + u_3y_3''' + u_3'y_3'' \quad \cdots (vii)$$

Now, substituting y_p, y_p', y_p'', y_p''' in (i)

$$\therefore u_1y_1''' + u_1'y_1'' + u_2y_2''' + u_2'y_2'' + u_3y_3''' + u_3'y_3'' + P(u_1y_1'' + u_2y_2'' + u_3y_3'')$$
$$+ Q(u_1y_1' + u_2y_2' + u_3y_3') + R(u_1y_1 + u_2y_2 + u_3y_3) = f(x)$$

$$\Rightarrow u_1(y_1''' + Py_1'' + Qy_1' + Ry_1) + u_2(y_2''' + Py_2'' + Qy_2' + Ry_2)$$
$$+ u_3(y_3''' + Py_3'' + Qy_3' + Ry_3) + u_1'y_1'' + u_2'y_2'' + u_3'y_3'' = f(x)$$

$$\Rightarrow \quad u_1'y_1'' + u_2'y_2'' + u_3'y_3'' = f(x) \quad \text{----(viii)}$$

Now,

$\cancel{(iii) \Rightarrow u_1'y_1}$

$$\left.\begin{aligned} (iii) &\Rightarrow u_1'y_1 + u_2'y_2 + u_3'y_3 = 0 \\ (v) &\Rightarrow u_1'y_1' + u_2'y_2' + u_3'y_3' = 0 \\ (viii) &\Rightarrow u_1'y_1'' + u_2'y_2'' + u_3'y_3'' = f(x) \end{aligned}\right\} \text{System or Linear Equation}$$

$$\therefore D = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} = W$$

$$D_1 = \begin{vmatrix} 0 & y_2 & y_3 \\ 0 & y_2' & y_3' \\ f(x) & y_2'' & y_3'' \end{vmatrix} = -y_2(-y_3' f(x)) + y_3(-y_2' f(x))$$
$$= y_2 y_3' f(x) - y_3 y_2' f(x)$$

$$D_2 = \begin{vmatrix} y_1 & 0 & y_3 \\ y_1' & 0 & y_3' \\ y_1'' & f(x) & y_3'' \end{vmatrix} = y_1(-y_3' f(x)) + y_3(y_1' f(x))$$
$$= y_3 y_1' f(x) - y_1 y_3' f(x)$$

$$D_3 = \begin{vmatrix} Y_1 & Y_2 & 0 \\ Y_1' & Y_2' & 0 \\ Y_1'' & Y_2'' & f(x) \end{vmatrix} = Y_1(Y_2' f(x)) - Y_2(Y_1' f(x))$$

$$= Y_1 Y_2' f(x) - Y_2 Y_1' f(x)$$

$$\therefore U_1' = \frac{D_1}{W} = \frac{Y_2 Y_3' f(x) - Y_3 Y_2' f(x)}{W}$$

$$\Rightarrow U_1 = \int \frac{Y_2 Y_3' f(x) - Y_3 Y_2' f(x)}{W} dx$$

$$\therefore U_2' = \frac{D_2}{W} = \frac{Y_3 Y_1' f(x) - Y_1 Y_3' f(x)}{W}$$

$$\Rightarrow U_2 = \int \frac{Y_3 Y_1' f(x) - Y_1 Y_3' f(x)}{W} dx$$

$$\therefore U_3' = \frac{D_3}{W} = \frac{Y_1 Y_2' f(x) - Y_2 Y_1' f(x)}{W}$$

$$\therefore U_3 = \int \frac{Y_1 Y_2' f(x) - Y_2 Y_1' f(x)}{W} dx$$

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$$y''' + y' = \tan x$$

A.E. $\Rightarrow$ $m^3 + m = 0$

$m(m^2+1) = 0$

$\therefore m = 0, \pm i$

$\therefore y_c = c_1 + c_2 \cos x + c_3 \sin x$

$\therefore y_1 = 1$

$y_2 = \cos x$

$y_3 = \sin x$

$$\therefore W = \begin{vmatrix} 1 & \cos x & \sin x \\ 0 & -\sin x & \cos x \\ 0 & -\cos x & -\sin x \end{vmatrix}$$

$= 1(\sin^2 x + \cos^2 x)$

$= 1 \neq 0$

$$\therefore u_1 = \int (\cos x \cdot \cos x \cdot \tan x - \sin x(-\sin x)\tan x)\, dx$$

$$= \int \left(\cos x \sin x + \sin^2 x \cdot \frac{\sin x}{\cos x}\right) dx$$

$$= \frac{\sin^2 x}{2} + \int \sin x (1 - \cos^2 x) \sec x \, dx$$

$$= \frac{1}{2}\sin^2 x + \int \sin x (\sec x - \cos x)\, dx$$

$$= \frac{1}{2}\sin^2 x + \int (\sin x \sec x - \sin x \cos x)\, dx$$

$$= \frac{1}{2}\sin^2 x - \frac{1}{2}\sin^2 x + \int \sin x \sec x \, dx$$

$$= \int \tan x \, dx = -\ln|\cos x|$$

$$\therefore u_2 = \int -1 \cdot \cos x \cdot \tan x \, dx$$

$$= -\int \sin x \, dx$$

$$= \cos x$$

$$\therefore u_3 = \int 1(-\sin x) \tan x \, dx$$

$$= -\int \sin x \tan x \, dx$$

$$= -\int \frac{\sin^2 x}{\cos x} dx$$

$$= -\int \frac{1-\cos^2 x}{\cos x} dx$$

$$= -\int (\sec x - \cos x) \, dx$$

$$= -\ln|\sec x + \tan x| + \sin x$$

$$\therefore y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$$

$$= -\ln|\cos x| + \cos^2 x + \sin^2 x - \sin x \ln|\sec x + \tan x|$$

$$= -\ln|\cos x| + 1 - \sin x \ln|\sec x + \tan x|$$

$\therefore$ G.S. $\Rightarrow$

$$y = y_c + y_p$$

$$= c_1 + c_2 \cos x + c_3 \sin x - \ln|\cos x| + 1 - \sin x \ln|\sec x + \tan x|$$

$$= c_4 + c_2 \cos x + c_3 \sin x - \ln|\cos x| - \sin x \ln|\sec x + \tan x|$$

An

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$y''' + 4y' = \sec 2x$

A.E. $\Rightarrow$ $m^3 + 4m = 0$

$m(m^2+4) = 0$

$\therefore m = 0, \pm 2i$

$\therefore y_c = c_1 + c_2 \cos 2x + c_3 \sin 2x$

$\therefore y_1 = 1$

$y_2 = \cos 2x$

$y_3 = \sin 2x$

$\therefore W = \begin{vmatrix} 1 & \cos 2x & \sin 2x \\ 0 & -2\sin 2x & 2\cos 2x \\ 0 & -4\cos 2x & -4\sin 2x \end{vmatrix}$

$= 8\sin^2 2x + 8\cos^2 2x$

$= 8 \neq 0$

$\therefore u_1 = \int \frac{1}{8}\left(\cos 2x \cdot 2\cos 2x \cdot \sec 2x - \sin 2x\,(-2\sin 2x)\sec 2x\right) dx$

$= \frac{1}{8}\int \left(2\cos 2x + 2\sin^2 2x \sec 2x\right) dx$

$= \frac{1}{8} \cdot \frac{2}{2}\sin 2x + \frac{1}{8} \cdot 2\int (1-\cos^2 2x)\sec 2x\, dx$

$= \frac{1}{8}\sin 2x + \frac{1}{4}\int (\sec 2x - \cos 2x)\, dx$

$= \frac{1}{8}\sin 2x + \frac{1}{4} \cdot \frac{1}{2} \cdot \ln|\sec 2x + \tan 2x| - \frac{1}{4} \cdot \frac{1}{2} \cdot \sin 2x$

$= \frac{1}{8}\ln|\sec 2x + \tan 2x|$

$\therefore u_2 = \int \frac{1}{8} \cdot (-1)\,(2\cos 2x)\sec 2x \, dx$

$= -\frac{1}{4}\int dx$

$= -\frac{1}{4}x$

$\therefore u_3 = \int \frac{1}{8} \cdot 1 \cdot (-2\sin 2x)\sec 2x \, dx$

$= -\frac{1}{4}\int \sin 2x \sec 2x \, dx$

$= -\frac{1}{4}\int \tan 2x \, dx$

$= +\frac{1}{4} \cdot \frac{1}{2} \cdot \ln|\cos 2x|$

$= \frac{1}{8}\ln|\cos 2x|$

$\therefore y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$

$= \frac{1}{8}\ln|\sec 2x + \tan 2x| - \frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x \ln|\cos 2x|$

$\therefore$ G.S. $\Rightarrow$

$y = y_c + y_p$

$= c_1 + c_2\cos 2x + c_3 \sin 2x + \frac{1}{8}\ln|\sec 2x + \tan 2x| - \frac{1}{4}x\cos 2x$

$+ \frac{1}{8}\sin 2x \ln|\cos 2x|$

Ans

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$$y''' - 2y'' - y' + 2y = e^{4x}$$

A.E. $\Rightarrow$

$$m^3 - 2m^2 - m + 2 = 0$$

$$m^3 + m^2 - 3m^2 - 3m + 2m + 2 = 0$$

$$m^2(m+1) - 3m(m+1) + 2(m+1) = 0$$

$$(m+1)(m^2 - 3m + 2) = 0$$

$$(m+1)(m-1)(m-2) = 0$$

$$\therefore m = -1, 1, 2$$

$$\therefore y_c = c_1 e^{-x} + c_2 e^{x} + c_3 e^{2x}$$

$\therefore y_1 = e^{-x}$

$y_2 = e^{x}$

$y_3 = e^{2x}$

$$\therefore W = \begin{vmatrix} e^{-x} & e^{x} & e^{2x} \\ -e^{-x} & e^{x} & 2e^{2x} \\ e^{-x} & e^{x} & 4e^{2x} \end{vmatrix}$$

$$= e^{-x}(4e^{3x} - 2e^{3x}) - e^{x}(-4e^{x} - 2e^{x}) + e^{2x}(-e^{0} - e^{0})$$

$$= e^{-x}(2e^{3x}) - e^{x}(-6e^{x}) + e^{2x}(-1-1)$$

$$= 2e^{2x} + 6e^{2x} - 2e^{2x}$$

$$= 6e^{2x} \neq 0$$

$$\therefore u_1 = \int \frac{e^x \cdot 2e^{2x} \cdot e^{4x} - e^{2x} \cdot e^x \cdot e^{4x}}{6e^{2x}}\,dx$$

$$= \int \frac{2e^{7x} - e^{7x}}{6e^{2x}}\,dx$$

$$= \int \frac{e^{7x}}{6e^{2x}}\,dx$$

$$= \frac{1}{6}\int e^{5x}\,dx$$

$$= \frac{1}{30}e^{5x}$$

$$\therefore u_2 = \int \frac{e^{2x}(-e^{-x}) \cdot e^{4x} - e^{-x} \cdot 2e^{2x} \cdot e^{4x}}{6e^{2x}}\,dx$$

$$= \int \frac{-e^{5x} - 2e^{5x}}{6e^{2x}}\,dx$$

$$= -\frac{1}{2}\int e^{3x}\,dx$$

$$= -\frac{1}{6}e^{3x}$$

$$\therefore u_3 = \int \frac{e^{-x} \cdot e^x \cdot e^{4x} - e^x \cdot (-e^{-x}) \cdot e^{4x}}{6e^{2x}}\,dx$$

$$= \int \frac{e^{4x} + e^{4x}}{6e^{2x}}\,dx$$

$$= \frac{1}{3}\int e^{2x}\,dx$$

$$= \frac{1}{6}e^{2x}$$

$\therefore Y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$

$= \frac{1}{30} e^{5x} \cdot e^{-x} - \frac{1}{6} e^{3x} \cdot e^{x} + \frac{1}{6} e^{2x} \cdot e^{2x}$

$= \frac{1}{30} e^{4x}$

$\therefore$ G.S. $\Rightarrow$

$y = y_c + y_p$

$= c_1 e^{-x} + c_2 e^{x} + c_3 e^{2x} + \frac{1}{30} e^{4x}$

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$y''' - 3y'' + 2y' = \frac{e^{2x}}{1+e^{x}}$

A.E. $\Rightarrow$ $m^3 - 3m^2 + 2m = 0$

$\Rightarrow m(m^2 - 3m + 2) = 0$

$\therefore m = 0$ | $m^2 - 3m + 2 = 0$

$(m-1)(m-2) = 0$

$\therefore m = 1, 2$

$\therefore y_c = c_1 + c_2 e^{x} + c_3 e^{2x}$

$\therefore y_1 = 1$

$y_2 = e^{x}$

$y_3 = e^{2x}$

$\therefore W = \begin{vmatrix} 1 & e^{x} & e^{2x} \\ 0 & e^{x} & 2e^{2x} \\ 0 & e^{x} & 4e^{2x} \end{vmatrix}$

$= 4e^{3x} - 2e^{3x}$

$= 2e^{3x} \neq 0$

$$\therefore u_1 = \int \frac{e^x \cdot 2e^{2x} \cdot \frac{e^{2x}}{1+e^x} - e^{2x} \cdot e^x \cdot \frac{e^{2x}}{1+e^x}}{2e^{3x}} \, dx$$

$$= \int \frac{\frac{2e^{5x}}{1+e^x} - \frac{e^{5x}}{1+e^x}}{2e^{3x}} \, dx$$

$$= \int \left(\frac{e^{5x}}{1+e^x} \cdot \frac{1}{2e^{3x}} \right) dx$$

$$= \frac{1}{2} \int \frac{e^{2x}}{1+e^x} \, dx$$

$$= \frac{1}{2} \int \left(e^x - \frac{e^x}{1+e^x} \right) dx$$

$$= \frac{1}{2} e^x - \frac{1}{2} \ln(1+e^x)$$

$$\therefore u_2 = \int \frac{-1 \cdot 2e^{2x} \cdot \frac{e^{2x}}{1+e^x}}{2e^{3x}} \, dx$$

$$= -\int \left(\frac{2e^{4x}}{1+e^x} \cdot \frac{1}{2e^{3x}} \right) dx$$

$$= -\int \frac{e^x}{1+e^x} \, dx$$

$$= -\ln|1+e^x|$$

$$\therefore u_3 = \int \frac{1 \cdot e^x \cdot \frac{e^{2x}}{1+e^x}}{2e^{3x}} \, dx$$

$$= \int \left(\frac{e^{3x}}{1+e^x} \cdot \frac{1}{2e^{3x}} \right) dx$$

$$= \frac{1}{2} \int \frac{1}{1+e^x} \, dx$$

$$= -\frac{1}{2} \int \frac{-e^{-x}}{1+e^{-x}} \, dx$$

$$= -\frac{1}{2} \ln \left| 1+e^{-x} \right|$$

$$\therefore y_p = u_1 y_1 + u_2 y_2 + u_3 y_3$$

$$= \frac{1}{2} e^x - \frac{1}{2} \ln \left| 1+e^x \right| - e^x \ln \left| 1+e^x \right| - \frac{1}{2} e^{2x} \ln \left| 1+e^{-x} \right|$$

$\therefore$ G.S. $\exists$

$$y = y_c + y_p$$

$$= c_1 + c_2 e^x + c_3 e^{2x} + \frac{1}{2} e^x - \frac{1}{2} \ln \left| 1+e^x \right| - e^x \ln \left| 1+e^x \right| - \frac{1}{2} e^{2x} \ln \left| 1+e^{-x} \right|$$

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