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$$2x^2y'' - xy' + (x^2+1)y = 0 \quad \cdots\cdots \text{(i)}$$

Let,

$$y = \sum_{n=0}^{\infty} c_n x^{n+r}$$ be the solution of (i)

$$y' = \sum_{n=0}^{\infty} (n+r)\, c_n x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1)\, c_n x^{n+r-2}$$

Substituting y'', y', y in (i)

$$2x^2 \sum_{n=0}^{\infty} (n+r)(n+r-1)\, c_n x^{n+r-2} - x\sum_{n=0}^{\infty} (n+r)\, c_n x^{n+r-1} + (x^2+1)\sum_{n=0}^{\infty} c_n x^{n+r} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} 2(n+r)(n+r-1)\, c_n x^{n+r} - \sum_{n=0}^{\infty} (n+r)\, c_n x^{n+r} + \sum_{n=0}^{\infty} c_n x^{n+r+2} + \sum_{n=0}^{\infty} c_n x^{n+r} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} \left[2(n+r)(n+r-1) - (n+r) + 1\right] c_n x^{n+r} + \sum_{n=0}^{\infty} c_n x^{n+r+2} = 0$$

$$\Rightarrow x^r \left[\sum_{n=0}^{\infty} \left\{(2n+2r-2-1)(n+r)+1\right\} c_n x^n + \sum_{n=0}^{\infty} c_n x^{n+2} \right] = 0$$

$$\Rightarrow \left\{r(2r-3)+1\right\} c_0 x^0 + \sum_{n=1}^{\infty} \left\{(2n+2r-3)(n+r)+1\right\} c_n x^n + \sum_{n=0}^{\infty} c_n x^{n+2} = 0$$

Let,

$k = n$	$k = n+2$
$\therefore n = k$	$n = k-2$
if, $n = 1$	if $n = 0$
$k = 1$	$k = 2$

$\cancel{r(2r-3)}+1$

$$(2r^2-3r+1)c_0x^0 + \sum_{k=1}^{\infty}\{(2k+2r-3)(k+r)+1\}c_kx^k + \sum_{k=2}^{\infty}c_{k-2}x^k = 0$$

$$(2r^2-3r+1)c_0x^0 + \{(2+2r-3)(1+r)+1\}c_1x + \sum_{k=2}^{\infty}\{(2k+2r-3)(k+r)+1\}c_kx^k + \sum_{k=2}^{\infty}c_{k-2}x^k = 0$$

$$(2r^2-3r+1)c_0x^0 + ((2r-1)(1+r)+1)c_1x + \sum_{k=2}^{\infty}\left[((2k+2r-3)(k+r)+1)c_k + c_{k-2}\right]x^k = 0$$

Equating coefficient of like terms

$(2r^2-3r+1)c_0 = 0$	$(2r^2-r+2r-1)c_1 = 0$	
$2r^2-3r+1 = 0$	$(2r^2+r-1)c_1 = 0$	$c_1 = 0$
$r = 1, \frac{1}{2}$	$2r^2+r-1 = 0$	
	$r = \frac{1}{2}, -1$	

~~when~~ $r = \frac{1}{2}$

$$((2k+2r-3)(k+r)+1)c_k = -c_{k-2}$$

$$\therefore c_k = \frac{-c_{k-2}}{(2k+2r-3)(k+r)+1}$$

$r = 1$

$$C_k = \frac{-1}{(2k-1)(k+1)+1} C_{k-2}$$

$k=2$,

$$C_2 = -\frac{1}{10} C_0$$

$k=3$,

$$C_3 = -\frac{1}{21} C_1 = 0$$

$k=4$,

$$C_4 = -\frac{1}{36} C_2 = \frac{1}{360} C_0$$

$r = \frac{1}{2}$

$$C_k = \frac{-1}{(2k-2)(k+\frac{1}{2})+1} C_{k-2}$$

$k=2$,

$$C_2 = -\frac{1}{6} C_0$$

$k=3$,

$$C_3 = -\frac{1}{5} C_1 = 0$$

$k=4$,

$$C_4 = -\frac{1}{28} C_2 = \frac{1}{168} C_0$$

When $r = 1$,

$$Y_1 = \sum_{n=0}^{\infty} C_n x^{n+1} = C_0 x + C_1 x^2 + C_2 x^3 + C_3 x^4 + C_4 x^5 + \cdots$$

$$= C_0 x + 0 - \frac{x^3}{10} C_0 + 0 + \frac{x^5}{360} C_0 + \cdots$$

$$= \left(x - \frac{x^3}{10} + \frac{x^5}{360} - \cdots \right) C_0$$

When, $r = \frac{1}{2}$

$$Y_2 = x^{1/2} \left[C_0 x^0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + \cdots \right]$$

$$= x^{1/2} \left[C_0 + 0 + -\frac{x^2}{6} C_0 + 0 + \frac{x^4}{1680} C_0 + \cdots \right]$$

$$= x^{1/2} C_0 \left[1 - \frac{x^2}{6} + \frac{x^4}{1680} - \cdots \right]$$

$\therefore y = C_1 Y_1 + C_2 Y_2$

$$= C_1 \left[\left(x - \frac{x^3}{10} + \frac{x^5}{360} - \cdots \right) C_0 \right] + C_2 \cdot C_0 \cdot x^{1/2} \left[1 - \frac{x^2}{6} + \frac{x^4}{168} - \cdots \right]$$

$$= C_3 \left(x - \frac{x^3}{10} + \frac{x^5}{360} - \cdots \right) + C_4 x^{1/2} \left(1 - \frac{x^2}{6} + \frac{x^4}{168} - \cdots \right)$$

21)

$$2xy'' - (3+2x)y' + y = 0 \quad \text{---} (i)$$

Let,

$y = \sum_{n=0}^{\infty} C_n x^{n+r}$ be the solution of (i)

$$\therefore y' = \sum_{n=0}^{\infty} (n+r) C_n x^{n+r-1}$$

$$\therefore y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1) C_n x^{n+r-2}$$

Substituting y'', y', y in (i),

$$2x \sum_{n=0}^{\infty} (n+r)(n+r-1) C_n x^{n+r-2} - (3+2x) \sum_{n=0}^{\infty} (n+r) C_n x^{n+r-1} + \sum_{n=0}^{\infty} C_n x^{n+r} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} 2(n+r)(n+r-1) C_n x^{n+r-1} - \sum_{n=0}^{\infty} 3(n+r) C_n x^{n+r-1} - \sum_{n=0}^{\infty} 2(n+r) C_n x^{n+r} + \sum_{n=0}^{\infty} C_n x^{n+r} = 0$$

$$\Rightarrow x^r \left[\sum_{n=0}^{\infty} (2n+2r-2-3)(n+r) C_n x^{n-1} - \sum_{n=0}^{\infty} \left(2(n+r)+1 \right) C_n x^{n} \right] = 0$$

$$\sum_{n=0}^{\infty} (2n+2r-5)(n+r) c_n x^{n+r-1} - \sum_{n=0}^{\infty} (2n+2r+1) c_n x^{n+r} = 0$$

$$\Rightarrow r(2r-5)c_0 x^0 + \sum_{n=1}^{\infty} (2n+2r-5)(n+r) c_n x^{n-1} - \sum_{n=0}^{\infty} (2n+2r+1) c_n x^n = 0$$

Let,

$k = n-1$	$k = n$
$n = k+1$	$n = k$
if, $n = 1$	if, $n = 0$
$k = 0$	$k = 0$

$$\Rightarrow r(2r-5)c_0 x^0 + \sum_{k=0}^{\infty} (2k+2+2r-5)(k+r+1) c_{k+1} x^k - \sum_{k=0}^{\infty} (2k+2r+1) c_k x^k = 0$$

$$\Rightarrow r(2r-5)c_0 x^0 + \sum_{k=0}^{\infty} \left[(2k+2r-3)(k+r+1) c_{k+1} - (2k+2r+1) c_k\right] x^k = 0$$

Equating coefficient of like term,

$r(2r-5)c_0 = 0$	$(2k+2r-3)(k+r+1)c_{k+1} - (2k+2r+1)c_k = 0$
$r(2r-5) = 0$	
$\therefore r = 0, \frac{5}{2}$	$\therefore c_{k+1} = \dfrac{2k+2r+1}{(2k+2r-3)(k+r+1)} c_k \;;\; k = 0, 1, 2, \cdots$

$r=0$

$\therefore C_{k+1} = \dfrac{2k+1}{(2k-3)(k+1)} C_k$

$\underline{k=0}$,

$C_1 = -\frac{1}{3} C_0$

$\underline{k=1}$,

$C_2 = -\frac{3}{2} C_1 = \frac{1}{2} C_0$

$\underline{k=2}$,

$C_3 = \frac{5}{3} C_2 = \frac{5}{6} C_0$

$r = \frac{5}{2}$

$C_{k+1} = \dfrac{2k+6}{(2k+2)(k+\frac{7}{2})} C_k$

$\underline{k=0}$,

$C_1 = \frac{6}{7} C_0$

$\underline{k=1}$,

$C_2 = \frac{4}{9} C_1 = \frac{8}{21} C_0$

$\underline{k=2}$,

$C_3 = \frac{10}{33} C_2 = \frac{80}{693} C_0$

When $r=0$,

$$Y_1 = \sum_{n=0}^{\infty} C_n x^n = C_0 x^0 + C_1 x^1 + C_2 x^2 + C_3 x^3 + \cdots$$

$$= C_0 - \frac{x}{3} C_0 + \frac{x^2}{2} C_0 + \frac{5x^3}{6} C_0 + \cdots$$

When $r = \frac{5}{2}$

$$Y_2 = x^{5/2}\left[C_0 x^0 + C_1 x^1 + C_2 x^2 + C_3 x^3 + \cdots\right]$$

$$= x^{5/2}\left[C_0 + \frac{6x}{7} C_0 + \frac{8x^2}{21} C_0 + \frac{80x^3}{693} C_0 + \cdots\right]$$

$$\therefore Y = C_1 Y_1 + C_2 Y_2$$

$$= C_1\left[C_0 - \frac{x}{3} C_0 + \frac{x^2}{2} C_0 + \frac{5x^3}{6} C_0 + \cdots\right] + C_2\left[C_0 + \frac{6x}{7} C_0 + \frac{8x^2}{21} C_0 + \frac{80x^3}{693} C_0 + \cdots\right]$$

$$= C_3\left(1 - \frac{x}{3} + \frac{x^2}{2} + \frac{5x^3}{6} + \cdots\right) + C_4\left(1 + \frac{6x}{7} + \frac{8x^2}{21} + \frac{80x^3}{693} + \cdots\right)$$

$$2xy'' + (1+x)y' + y = 0 \quad \cdots (i)$$

Let,

$$y = \sum_{n=0}^{\infty} C_n x^{n+r} \quad \text{be the solution of (i)}$$

$$\therefore y' = \sum_{n=0}^{\infty} (n+r) C_n x^{n+r-1}$$

$$\therefore y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1) C_n x^{n+r-2}$$

Substituting y'', y', y in (i),

$$2x \sum_{n=0}^{\infty} (n+r)(n+r-1) C_n x^{n+r-2} + (1+x)\sum_{n=0}^{\infty} (n+r) C_n x^{n+r-1} + \sum_{n=0}^{\infty} C_n x^{n+r} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} 2(n+r)(n+r-1) C_n x^{n+r-1} + \sum_{n=0}^{\infty} (n+r) C_n x^{n+r-1} + \sum_{n=0}^{\infty} (n+r) C_n x^{n+r} + \sum_{n=0}^{\infty} C_n x^{n+r} = 0$$

$$\Rightarrow x^r \left[\sum_{n=0}^{\infty} (2n+2r-2+1)(n+r) C_n x^{n-1} + \sum_{n=0}^{\infty} (n+r+1) C_n x^n \right] = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (2n+2r-1)(n+r) C_n x^{n-1} + \sum_{n=0}^{\infty} (n+r+1) C_n x^n = 0$$

$$\Rightarrow r(2r-1) C_0 x^0 + \sum_{n=1}^{\infty} (2n+2r-1)(n+r) C_n x^{n-1} + \sum_{n=0}^{\infty} (n+r+1) C_n x^n = 0$$

Let,

$k = n-1$	$k = n$
$n = k+1$	$n = k$
if, $n = 1$	if, $n = 0$
$k = 0$	$k = 0$

$$\Rightarrow r(2r-1)c_0x^0 + \sum_{k=0}^{\infty}(2k+2+2r-1)(k+r+1)c_{k+1}x^k + \sum_{k=0}^{\infty}(k+r+1)c_kx^k = 0$$

$$\Rightarrow r(2r-1)c_0x^0 + \sum_{k=0}^{\infty}\left[(2k+2r+1)(k+r+1)c_{k+1} + (k+r+1)c_k\right]x^k = 0$$

Equating the coefficient of the like term,

$$r(2r-1)c_0 = 0$$

$$r(2r-1) = 0$$

$$r = 0, \frac{1}{2}$$

$$(2k+2r+1)(k+r+1)c_{k+1} + (k+r+1)c_k = 0$$

$$c_{k+1} = -\frac{(k+r+1)}{(2k+2r+1)(k+r+1)}c_k$$

$$= -\frac{1}{2k+2r+1}c_k \quad ; k = 0, 1, 2, \cdots$$

$r=0$,

$$c_{k+1} = \frac{-1}{2k+1}c_k$$

$k=0$,

$$c_1 = -c_0$$

$k=1$,

$$c_2 = -\frac{1}{3}c_1 = \frac{1}{3}c_0$$

$k=2$,

$$c_3 = -\frac{1}{5}c_2 = -\frac{1}{15}c_0$$

$r = \frac{1}{2}$

$$c_{k+1} = \frac{-1}{2k+2}c_k$$

$k=0$,

$$c_1 = -\frac{1}{2}c_0$$

$k=1$,

$$c_2 = -\frac{1}{4}c_1 = \frac{1}{8}c_0$$

$k=2$,

$$c_3 = -\frac{1}{6}c_2 = -\frac{1}{48}c_0$$

when $r=0$,

$$y_1 = \sum_{n=0}^{\infty}c_nx^n = c_0x^0 + c_1x^1 + c_2x^2 + c_3x^3 + \cdots$$

$$= c_0 + -c_0x + \frac{x^2}{3}c_0 - \frac{x^3}{15}c_0 + \cdots$$

When $r = \frac{1}{2}$,

$$y_2 = x^{1/2} \sum_{n=0}^{\infty} c_n x^n = x^{1/2}\left[c_0 x^0 + c_1 x^1 + c_2 x^2 + c_3 x^3 + \cdots\right]$$

$$= x^{1/2}\left[c_0 - \frac{x}{2}c_0 + \frac{x^2}{8}c_0 - \frac{x^3}{48}c_0 + \cdots\right]$$

$$\therefore y = c_1 y_1 + c_2 y_2$$

$$= c_1\left[c_0 - c_0 x + \frac{x^2}{3}c_0 - \frac{x^3}{15}c_0 + \cdots\right] + c_2 x^{1/2}\left[c_0 - \frac{x}{2}c_0 + \frac{x^2}{8}c_0 - \frac{x^3}{48}c_0 + \cdots\right]$$

$$= c_3\left(1 - x + \frac{x^2}{3} - \frac{x^3}{15} + \cdots\right) + c_4 x^{1/2}\left[1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{48} + \cdots\right]$$

Ans

H.W. ⇒ from Lecture-20

Exercise - 7.2 (Zill's Book)

7|

$$L^{-1}\left\{\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s-2}\right\}$$

$$= L^{-1}\left\{\frac{1!}{s^{1+1}}\right\} - L^{-1}\left\{\frac{0!}{s^{0+1}}\right\} + L^{-1}\left\{\frac{1}{s-2}\right\}$$

$$= t - 1 + e^{2t}$$

Ans.

17| $L^{-1}\left\{\frac{1}{s^2+3s}\right\}$

$= L^{-1}\left\{\frac{1}{s(s+3)}\right\}$

$= L^{-1}\left\{\frac{1}{s(0+3)}+\frac{1}{(s+3)(-3)}\right\}$

$= L^{-1}\left\{\frac{1}{3s}\right\}+L^{-1}\left\{\frac{1}{-3(s+3)}\right\}$

$= \frac{1}{3}-\frac{1}{3}e^{-3t}$

Ans

21| $L^{-1}\left\{\frac{0.9s}{(s-0.1)(s+0.2)}\right\}$

$= L^{-1}\left\{\frac{0.9\times 0.1}{(s-0.1)(0.1+0.2)}+\frac{0.9\times(-0.2)}{(s+0.2)(-0.2-0.1)}\right\}$

$= L^{-1}\left\{\frac{0.09}{0.3(s-0.1)}\right\}+L^{-1}\left\{\frac{-0.18}{-0.3(s+0.2)}\right\}$

$= \frac{3}{10}e^{0.1t}+\frac{3}{5}e^{-0.2t}$

Ans

22|

$$L^{-1}\left\{ \frac{s-3}{(s-\sqrt{3})(s+\sqrt{3})} \right\}$$

$$= L^{-1}\left\{ \frac{\sqrt{3}-3}{(s-\sqrt{3})(\sqrt{3}+\sqrt{3})} + \frac{-\sqrt{3}-3}{(s+\sqrt{3})(-\sqrt{3}-\sqrt{3})} \right\}$$

$$= L^{-1}\left\{ \frac{\sqrt{3}-3}{2\sqrt{3}(s-\sqrt{3})} \right\} + L^{-1}\left\{ \frac{3+\sqrt{3}}{2\sqrt{3}(s+\sqrt{3})} \right\}$$

$$= \frac{\sqrt{3}-3}{2\sqrt{3}} e^{\sqrt{3}x} + \frac{3+\sqrt{3}}{2\sqrt{3}} e^{-\sqrt{3}x}$$

Ans

$$= \frac{1-\sqrt{3}}{2} e^{\sqrt{3}x} + \frac{1+\sqrt{3}}{2} e^{-\sqrt{3}x}$$

Ans

24|

$$L^{-1}\left\{ \frac{s^2+1}{s(s-1)(s+1)(s-2)} \right\}$$

$$= L^{-1}\left\{ \frac{0+1}{s(0-1)(0+1)(0-2)} + \frac{1+1}{(s-1)\cdot 1\cdot(1+1)(1-2)} + \frac{4+1}{(s-2)\,2\cdot(2-1)(2+1)} + \frac{1+1}{(s+1)(-1)(-1-1)(-1-2)} \right\}$$

$$= L^{-1}\left\{ \frac{1}{2s} \right\} + L^{-1}\left\{ \frac{2}{-2(s-1)} \right\} + L^{-1}\left\{ \frac{5}{6(s-2)} \right\} + L^{-1}\left\{ \frac{2}{-6(s+1)} \right\}$$

$$= \frac{1}{2} - e^{x} + \frac{5}{6} e^{2x} - \frac{1}{3} e^{-x}$$

Ans

25)

$$L^{-1}\left\{\frac{1}{s^3+5s}\right\}$$

$$= L^{-1}\left\{\frac{1}{s(s^2+5)}\right\}$$

$$\frac{1}{s(s^2+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+5}$$

$$A(s^2+5)+(Bs+C)s = 1$$

$$As^2+5A+Bs^2+Cs = 1$$

$$(A+B)s^2+Cs+5A = 1$$

$$A = \frac{1}{5} \qquad A+B = 0$$

$$C = 0 \qquad B = -A = -\frac{1}{5}$$

$$= L^{-1}\left\{\frac{1/5}{s} + \frac{-1/5\, s}{s^2+5}\right\}$$

$$= L^{-1}\left\{\frac{1/5}{s}\right\} - L^{-1}\left\{\frac{1}{5}\cdot\frac{s}{s^2+5}\right\}$$

$$= \frac{1}{5} - \frac{1}{5}\cos\sqrt{5}t$$

Ans

26)

$$L^{-1}\left\{\frac{s}{(s+2)(s^2+4)}\right\}$$

$$\frac{s}{(s+2)(s^2+4)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+4}$$

$$\Rightarrow A(s^2+4)+(Bs+C)(s+2) = s \quad \cdots (i)$$

$$As^2+4A+Bs^2+Cs+2Bs+2C = s$$

$$(A+B)s^2+(2B+C)s+(4A+2C) = s$$

$$= L^{-1}\left\{\frac{-1/4}{s+2}\right\} + L^{-1}\left\{\frac{1/4\, s+\frac{1}{2}}{s^2+4}\right\}$$

$$= -\frac{1}{4}e^{-2t} + \frac{1}{4}L^{-1}\left\{\frac{s}{s^2+4}\right\} + \frac{1}{4}L^{-1}\left\{\frac{2}{s^2+2^2}\right\}$$

$$= -\frac{1}{4}e^{-2t} + \frac{1}{4}\cos 2t + \frac{1}{4}\sin 2t$$

Ans

$s=-2$

$$(i) \Rightarrow \quad A(4+4) = -2$$

$$A = \frac{-2}{8} = -\frac{1}{4}$$

$$B = -A = -\frac{1}{4}$$

$$C = 1-2B = \frac{1}{2}$$

H.W. $\Rightarrow$ from Lecture-21

Exercise - 7.2 (Zill's Book)

IVP

36|

$y'' - 4y' = 6e^{3t} - 3e^{-t}$; $y(0) = 1$, $y'(0) = -1$

Applying LT,

$$s^2 Y(s) - sy(0) - y'(0) - 4\left[sY(s) - y(0)\right] = \frac{6}{s-3} - \frac{3}{s+1}$$

$$(s^2 - 4s)Y(s) - s\cdot 1 + 1 + 4\cdot 1 = \frac{6}{s-3} - \frac{3}{s+1}$$

$$(s^2 - 4s)\,Y(s) = \frac{6}{s-3} - \frac{3}{s+1} + s - 5$$

$$\therefore Y(s) = \frac{6}{s(s-3)(s+2)(s-2)} - \frac{3}{s(s+1)(s+2)(s-2)} + \frac{s-5}{s(s+2)(s-2)}$$

$$= \frac{\quad}{s(s-2)(s+1)(s+2)(s-3)}$$

$$= \frac{6s+6-3s+9+(s-5)(s-3)(s+1)}{s(s-3)(s+2)(s-2)(s+1)}$$

$$= \frac{\quad}{s(s-3)(s+1)(s-2)(s+2)}$$

$$\text{.)}\ Y(s) = \frac{6}{s(s-3)(s-4)} + \frac{3}{s(s+1)(s-4)} + \frac{s-5}{s(s-4)}$$

$$= \frac{6s+6 - 3s+9 + (s-5)(s-3)(s+1)}{s(s-3)(s+1)(s-4)}$$

$$= \frac{3s+15}{s(s-3)(s+1)(s-4)}$$

$$= \frac{6}{s(0-3)(0-4)} + \frac{6}{(s-3)(3)(3-4)} + \frac{\quad}{(s-4)}$$

$$= \frac{3s+15}{s(s-3)(s+1)(s-4)} + \frac{s-5}{s(s-4)}$$

$$= \frac{0+15}{s(0-3)(0+1)(0-4)} + \frac{9+15}{(s-3)(3)(3+1)(3-4)} + \frac{-3+15}{(s+1)(-1)(-1-3)(-1-4)}$$

$$+ \frac{12+15}{(s-4)(4)(4-3)(4+1)} + \frac{0-5}{s(0-4)} + \frac{4-5}{(s-4)\cdot 4}$$

Applying inverse LT,

$$y(t) = \frac{15}{12} - \frac{24}{12} e^{3t} - \frac{12}{20} e^{-t} + \frac{27}{20} e^{4t} + \frac{5}{4} - \frac{1}{4} e^{4t}$$

$$= \frac{5}{2} - 2e^{3t} - \frac{3}{5} e^{t} + \frac{11}{10} e^{4t}$$

38/

$$y'' + 9y = e^{t} \quad ; \; y(0) = 0, \; y'(0) = 0$$

Applying LT,

$$s^2 Y(s) - s\,y(0) - y'(0) + 9[s Y(s) - y(0)] = \frac{1}{s-1}$$

$$\Rightarrow (s^2 + 9s) Y(s) - s \cdot 0 - 0 - 9 \cdot 0 = \frac{1}{s-1}$$

$$\Rightarrow Y(s) = \frac{1}{s(s-1)(s+9)}$$

$$Y(s) = \frac{1}{s(0-1)(0+9)} + \frac{1}{(s-1)(1)(1+9)} + \frac{1}{(s+9)(-9)(-9-1)}$$

Applying LT^{-1},

$$y(t) = -\frac{1}{9} + \frac{1}{10} e^{t} + \frac{1}{90} e^{-9t}$$

37)

$y'' + y = \sqrt{2} \sin \sqrt{2} t$; $y(0) = 10$, $y'(0) = 0$

Applying LT,

$$s^2 Y(s) - sy(0) - y'(0) + Y(s) = \sqrt{2} \frac{\sqrt{2}}{s^2+2} = \frac{2}{s^2+2}$$

$$\Rightarrow (s^2+1) Y(s) - s \cdot 10 - 0 = \frac{2}{s^2+2}$$

$$\Rightarrow (s^2+1) Y(s) = \frac{2}{s^2+2} + 10s$$

$$\Rightarrow Y(s) = \frac{2}{(s^2+2)(s^2+1)} + \frac{10s}{(s^2+1)}$$

$$= \frac{2 + 10s^3 + 20s}{(s^2+2)(s^2+1)}$$

$$= -\frac{2}{s^2+2} + \frac{2}{s^2+1} + \frac{10s}{s^2+1}$$

$$\frac{10s^3 + 20s + 2}{(s^2+2)(s^2+1)} = \frac{As+B}{(s^2+2)} + \frac{Cs+D}{(s^2+1)}$$

Applying LT^{-1},

$$y(t) = -\sqrt{2} \sin \sqrt{2} t + 2 \sin t + 10 \cos t$$

38)

$y'' + 9y = e^{t}$; $y(0)=0$, $y'(0)=0$

Applying LT,

$$s^2 Y(s) - s\,y(0) - y'(0) + 9\,Y(s) = \frac{1}{s-1}$$

$$\Rightarrow (s^2+9)Y(s) - s\cdot 0 - 0 = \frac{1}{s-1}$$

$$\therefore Y(s) = \frac{1}{(s-1)(s^2+9)}$$

$$= \frac{1/10}{s-1} + \frac{-1/10\,s - \frac{1}{10}}{s^2+9}$$

$$= \frac{1/10}{s-1} + \frac{-1/10\,s}{s^2+9} - \frac{1/10}{s^2+9}$$

$$\frac{1}{(s-1)(s^2+9)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+9}$$

$$\therefore A(s^2+9) + (Bs+C)(s-1) = 1 \quad \cdots (i)$$

$$As^2 + 9A + Bs^2 + Cs - Bs - C = 1$$

$$(A+B)s^2 + (C-B)s + (9A-C) = 1$$

$s=1$,

$10A = 1$

$\therefore A = \frac{1}{10}$

$B = -A = -\frac{1}{10}$

$C = B = -\frac{1}{10}$

Applying LT^{-1},

$$y(t) = \frac{1}{10}e^{t} - \frac{1}{10}\cos 3t - \frac{1}{10}\cdot\frac{1}{3}\cdot\sin 3t$$

$$= \frac{1}{10}e^{t} - \frac{1}{10}\cos 3t - \frac{1}{30}\sin 3t$$

39) $2y''' + 3y'' - 3y' - 2y = e^{-x}$; $y(0)=0,\ y'(0)=0,\ y''(0)=1$

-2
1
$-\frac{1}{2}$

Applying LT,

$$2[s^3Y(s) - s^2y(0) - sy'(0) - y''(0)] + 3[s^2Y(s) - sy(0) - y'(0)] - 3[sY(s) - y(0)]$$

$$-2Y(s) = \frac{1}{s+1}$$

$$\Rightarrow (2s^3 + 3s^2 - 3s - 2)Y(s) - 2s^2\cdot 0 - 2s\cdot 0 - 2\cdot 1 + 0 = \frac{1}{s+1}$$

$$\Rightarrow (2s^3 + 3s^2 - 3s - 2)Y(s) = \frac{1}{s+1} + 2 = \frac{1+2s+2}{s+1} = \frac{2s+3}{s+1}$$

$$\therefore Y(s) = \frac{(2s+3)}{(s+1)(s-1)(s+2)(s+\frac{1}{2})}$$

$$= \frac{2(-1)+3}{(s+1)(-1-1)(-1+2)(-1+\frac{1}{2})} + \frac{2\cdot 1+3}{(s-1)(1+1)(1+2)(1+\frac{1}{2})}$$

$$+ \frac{2\cdot(-2)+3}{(s+2)(-2+1)(-2-1)(-2+\frac{1}{2})} + \frac{2(-\frac{1}{2})+3}{(s+\frac{1}{2})(-\frac{1}{2}+1)(-\frac{1}{2}-1)(-\frac{1}{2}+2)}$$

$$Y(s) = \frac{1}{(s+1)} + \frac{5/9}{(s-1)} + \frac{2/9}{(s+2)} + \frac{-16/9}{(s+\frac{1}{2})}$$

Applying LT^{-1}

$$\therefore y(x) = e^{-x} + \frac{5}{9}e^{x} + \frac{2}{9}e^{-2x} - \frac{16}{9}e^{-\frac{1}{2}x}$$

40)

$y''' + 2y'' - y' - 2y = \sin 3t \quad ; \; y(0) = 0, \; y'(0) = 0, \; y''(0) = 1$

1
-1
-2

$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) + 2[s^2 Y(s) - s y(0) - y'(0)] - [s Y(s) - y(0)] - 2Y(s)$

$= \frac{3}{s^2+9}$

$\Rightarrow (s^3 + 2s^2 - s - 2) Y(s) - 1 + 0 = \frac{3}{s^2+9}$

$\Rightarrow (s^3 + 2s^2 - s - 2) Y(s) = \frac{3}{s^2+9} + 1 \quad \frac{s^2+12}{s^2+9}$

$\Rightarrow Y(s) = \frac{s^2+12}{(s-1)(s+1)(s+2)(s^2+9)}$

$= \frac{13/60}{s-1} + \frac{-13/20}{s+1} + \frac{16/39}{s+2} + \frac{3/130 \, s}{s^2+9} + \frac{-1/65 \cdot 3}{s^2+9}$

Applying LT^{-1},

$y(t) = \frac{13}{60} e^{t} - \frac{13}{20} e^{-t} + \frac{16}{39} e^{-2t} + \frac{3}{130} \cos 3t - \frac{1}{65} \sin 3t$

Exercise - 7·3 (Zill's Book)

21|

$$y' + 4y = e^{-4t} \quad ; \quad y(0) = 2$$

Applying LT,

$$sY(s) - y(0) + 4Y(s) = \frac{1}{s+4}$$

$$\Rightarrow (s+4)Y(s) - 2 = \frac{1}{s+4}$$

$$\Rightarrow (s+4)Y(s) = \frac{1}{s+4} + 2 = \frac{1+2s+8}{s+4} = \frac{2s+9}{s+4}$$

$$\therefore Y(s) = \frac{2s+9}{(s+4)(s+4)} = \frac{2s+9}{(s+4)^2} = \frac{2s+8+1}{(s+4)^2} = 2\frac{(s+4)}{(s+4)^2} + \frac{1}{(s+4)^2}$$

Applying LT^{-1}

$$y(t) = 2 \cdot e^{-4t} + e^{-4t} t$$

Ans

22|

$$y' - y = 1 + te^{t} \quad ; \quad y(0) = 0$$

Applying LT,

$$sY(s) - y(0) - Y(s) = \frac{1}{s} + \frac{1}{(s-1)^2}$$

$$\Rightarrow (s-1)Y(s) - 0 = \frac{1}{s} + \frac{1}{(s-1)^2}$$

$$\therefore Y(s) = \frac{1}{s(s-1)} + \frac{1}{(s-1)^3} = \frac{1}{s(-1)} + \frac{1}{(s-1)1} + \frac{1}{(s-1)^3}$$

$$Y(s) = -\frac{1}{s} + \frac{1}{s-1} + \frac{1}{(s-1)^3}$$

Applying LT^{-1}

$$y(t) = -1 + e^t + e^t t^2 \cdot \frac{1}{2}$$

$$y(t) = e^t + \frac{1}{2} e^t \cdot t^2 - 1$$

Ans

24|

$$y'' - 4y' + 4y = t^3 e^{2t} \quad ; \quad y(0) = y'(0) = 0$$

Applying, LT,

$$s^2 Y(s) - s y(0) - y'(0) - 4[sY(s) - y(0)] + 4Y(s) = \frac{6}{(s-2)^4}$$

$$\Rightarrow (s^2 - 4s + 4) Y(s) - 0 = \frac{6}{(s-2)^3}$$

$$\Rightarrow Y(s) = \frac{6}{(s-2)^4 (s-2)^2} = \frac{6}{(s-2)^6}$$

Applying LT^{-1},

$$\cancel{y(t) = 6 e^{2t} t^4 \cdot \frac{1}{24} = \frac{1}{4} e^{2t} t^4}$$

$$y(t) = 6 \cdot e^{2t} \cdot t^5 \cdot \frac{1}{120} = \frac{1}{20} e^{2t} t^5$$

Ans

25)

$y'' - 6y' + 9y = t \quad ; \; y(0) = 0, \; y'(0) = 1$

Applying LT,

$$s^2 Y(s) - sy(0) - y'(0) - 6[sY(s) - y(0)] + 9Y(s) = \frac{1}{s^2}$$

$$\Rightarrow (s^2 - 6s + 9) Y(s) - 0 - 1 + 0 = \frac{1}{s^2}$$

$$\Rightarrow (s^2 - 6s + 9) Y(s) = \frac{1}{s^2} + 1 = \frac{1 + s^2}{s^2}$$

$$\therefore Y(s) = \frac{s^2 + 1}{s^2 (s-3)^2}$$

$$= \frac{1}{(s-3)^2} + \frac{1}{s^2 (s-3)^2}$$

$$= \frac{1}{(s-3)^2} + \frac{\frac{2}{27}s + \frac{1}{9}}{s^2} + \frac{-\frac{2}{27}s + \frac{1}{3}}{(s-3)^2}$$

$$\frac{1}{s^2 (s-3)^2} = \frac{As + B}{s^2} + \frac{Cs + D}{(s-3)^2}$$

$$(As + B)(s-3)^2 + s^2 (Cs + D) = 1$$

$$As^3 - 6As^2 + 9As + Bs^2 - 6Bs + 9B + Cs^3 + Ds^2 = 1$$

$$(A + C)s^3 + (B - 6A + D)s^2 + (9A - 6B)s + 9B = 1$$

$$\therefore B = \frac{1}{9}$$

$$\therefore A = \frac{6B}{9} = \frac{2}{27}$$

$$\therefore C = -A = -\frac{2}{27}$$

$$D = 6A - B = \frac{1}{3}$$

LT^{-1},

$$y(t) = e^{3t} \cdot t + \frac{2}{27} + \frac{1}{9} t + L^{-1}\left\{ \frac{-\frac{2}{27}s + \frac{2}{9} + \frac{1}{9}}{(s-3)^2} \right\}$$

$$= e^{3t} \cdot t + \frac{2}{27} + \frac{1}{9} t - \frac{2}{27} e^{3t} + \frac{1}{9} e^{3t} \cdot t$$

$$= \frac{2}{27} + \frac{10}{9} e^{3t} \cdot t + \frac{1}{9} t - \frac{2}{27} e^{3t}$$

A

26|

$$y'' - 4y' + 4y = t^3 \quad ; \quad y(0) = 1, \; y'(0) = 0$$

Applying LT,

$$s^2 Y(s) - sy(0) - y'(0) - 4\left[sY(s) - y(0)\right] + 4Y(s) = \frac{6}{s^4}$$

$$\Rightarrow (s^2 - 4s + 4)Y(s) - s\cdot 1 - 0 + 4\cdot 1 = \frac{6}{s^4}$$

$$\Rightarrow (s-2)^2 Y(s) = \frac{6}{s^4} + s - 4$$

$$\Rightarrow Y(s) = \frac{6}{s^4(s-2)^2} + \frac{s}{(s-2)^2} - \frac{4}{(s-2)^2}$$

$$= \frac{6 + s^5 - 4s^4}{s^4(s-2)^2}$$

$$= \frac{3/4}{s} + \frac{9/8}{s^2} + \frac{3}{4}\cdot\frac{2}{s^3} + \frac{1}{4}\cdot\frac{6}{s^4} + \frac{1}{4}\,\frac{1}{s-2} - \frac{13}{8}\,\frac{1}{(s-2)^2}$$

$$\therefore y(t) = \frac{3}{4} + \frac{9}{8}t + \frac{3}{4}t^2 + \frac{1}{4}t^3 + \frac{1}{4}e^{2t} - \frac{13}{8}te^{2t}$$

Ans

28)

$2y'' + 20y' + 51y = 0 \; ; \quad y(0) = 2, y'(0) = 0$

Applying LT,

$2[s^2Y(s) - sy(0) - y'(0)] + 20[sY(s) - y(0)] + 51Y(s) = 0$

$\Rightarrow (2s^2 + 20s + 51)Y(s) - 2s \cdot 2 - 0 - 20 \cdot 2 = 0$

$\Rightarrow (2s^2 + 20s + 51)Y(s) = 4s + 40$

$\Rightarrow Y(s) = \frac{4s+40}{2s^2+20s+51} = \frac{2s+20}{(s+5)^2 + \frac{1}{2}} = \frac{2(s+5)}{(s+5)^2 + \frac{1}{2}} + \frac{10}{(s+5)^2 + \frac{1}{2}}$

$\therefore y(t) = 2e^{-5t} \cos \frac{1}{\sqrt{2}}t + 10\sqrt{2}\, e^{-5t} \sin \frac{1}{\sqrt{2}}t$

29)

$y'' - y' = e^t \cos t \; ; \quad y(0) = y'(0) = 0$

Applying LT,

$s^2Y(s) - sy(0) - y'(0) - [sY(s) - y(0)] = \frac{s-1}{(s-1)^2 + 1}$

$(s^2 - s)Y(s) - 0 = \frac{s-1}{(s-1)^2 + 1}$

Done in class today (22)

$$y'' + 4y' + 6y = 1 + e^{-t} \quad ; \quad y(0) = y'(0) = 0$$

Applying LT,

$$s^2 Y(s) - s y(0) - y'(0) + 4[sY(s) - y(0)] + 6Y(s) = \frac{1}{s} + \frac{1}{s+1}$$

$$\Rightarrow (s^2 + 4s + 6)Y(s) - 0 = \frac{1}{s} + \frac{1}{s+1} = \frac{2s+1}{s(s+1)}$$

$$\therefore Y(s) = \frac{(2s+1)}{s(s+1)(s^2+4s+6)}$$

Now,

$$\frac{2s+1}{s(s+1)(s^2+4s+6)} = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{(s^2+4s+6)}$$

$$\therefore A(s+1)(s^2+4s+6) + B(s)(s^2+4s+6) + (Cs+D)(s)(s+1) = 2s+1$$

$$\Rightarrow As^3 + 4As^2 + 6As + As^2 + 4As + 6A + Bs^3 + 4Bs^2 + 6Bs + (Cs+D)(s^2+s) = 2s+1$$

$$\Rightarrow (A+B)s^3 + (4A + A + 4B)s^2 + (6A + 4A + 6B)s + 6A + Cs^3 + Ds^2 + Cs^2 + Ds = 2s+1$$

$$\Rightarrow (A+B+C)s^3 + (5A+4B+C+D)s^2 + (11A+6B+D)s + 6A = 2s+1$$

$\therefore A = \frac{1}{6}$ | $C = -\frac{1}{2}$

$\therefore B = \frac{1}{3}$ | $D = -5/3$

$\therefore \dfrac{2s+1}{s(s+1)(s^2+4s+6)} = \dfrac{1/6}{s} + \dfrac{1/3}{s+1} - \dfrac{\frac{s}{2}+5/3}{(s^2+4s+6)}$

s^2+4s+6

$= (s+\frac{1}{2}\cdot 4)^2 + 6 - 4$

$= (s+2)^2 + (\sqrt{2})^2$

$\Rightarrow$ Applying LT^{-1},

$$x(t) = \frac{1}{6} + \frac{1}{3}e^{-t} - \frac{1}{2}L^{-1}\left\{\frac{s}{s^2+4s+6}\right\} + L^{-1}\left\{\frac{5/3}{s^2+4s+6}\right\}$$

$$= \frac{1}{6} + \frac{1}{3}e^{-t} - \frac{1}{2}L^{-1}\left\{\frac{(s+2)-2}{(s+2)^2+(\sqrt{2})^2}\right\} - \frac{5}{3\sqrt{2}}L^{-1}\left\{\frac{\sqrt{2}}{(s+2)^2+(\sqrt{2})^2}\right\}$$

$$= \frac{1}{6} + \frac{1}{3}e^{-t} - \frac{1}{2}L^{-1}\left\{\frac{s+2}{(s+2)^2+(\sqrt{2})^2}\right\} + \frac{1}{2}L^{-1}\left\{\frac{2}{(s+2)^2+(\sqrt{2})^2}\right\} - \frac{5}{3\sqrt{2}}e^{-2t}\sin\sqrt{2}t$$

$$x(t) = \frac{1}{6} + \frac{1}{3}e^{-t} - \frac{1}{2}e^{-2t}\cos\sqrt{2}t + \frac{1}{\sqrt{2}}e^{-2t}\sin\sqrt{2}t - \frac{5}{3\sqrt{2}}e^{-2t}\sin\sqrt{2}t$$

A

HW. ⇒ from lecture - 23

Zill's Book

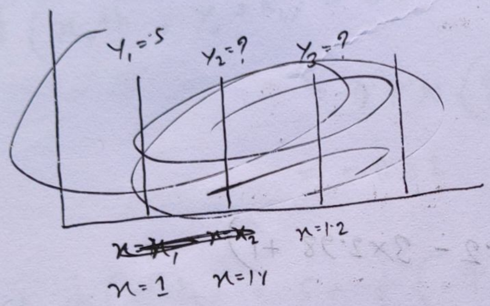
Exercise - 9.1

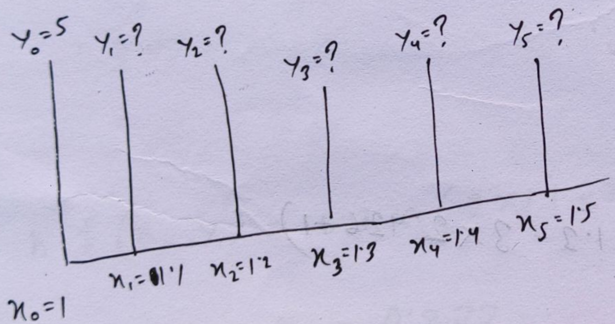
1] $y' = 2x - 3y + 1 \quad \cdots (i)$

$y(1) = 5$

$y(1.5) = ?$

Euler Method.





We know,

$$y_{n+1} = y_n + h f(x_n, y_n) \quad \cdots (i) \; ; \quad n = 0, 1, 2, 3, \ldots$$

For $n=0$,

$$Y_1 = Y_0 + h\,f(x_n, Y_n)$$

$$= 5 + 0.1 \times (2x_0 - 3y_0 + 1)$$

$$= 5 + 0.1 \times (2 \times 1 - 3 \times 5 + 1)$$

$$= 3.8$$

For $n=1$,

$$Y_2 = Y_1 + h\,f(x_1, Y_1)$$

$$= 3.8 + 0.1 \times (2 \times 1.1 - 3 \times 3.8 + 1)$$

$$= 2.98$$

For $n=2$,

$$Y_3 = Y_2 + h\,f(x_2, Y_2)$$

$$= 2.98 + 0.1 \times (2 \times 1.2 - 3 \times 2.98 + 1)$$

$$= 2.426$$

For $n=3$,

$$Y_4 = Y_3 + h\,f(x_3, Y_3)$$

$$= 2.426 + 0.1 \times (2 \times 1.3 - 3 \times 2.426 + 1)$$

$$= 2.0582$$

For $n=4$,

$$Y_5 = Y_4 + h\,f(x_4, Y_4)$$

$$= 2.0582 + 0.1(2 \times 1.4 - 3 \times 2.0582 + 1)$$

$$= 1.82074$$

R-K Method

$$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + h, y_n + k_1)$$

For $n=0$,

$$k_1 = h\, f(x_0, y_0) = 0.1 \times (2x_0 - 3y_0 + 1)$$

$$= 0.1\,(2\times 1 - 3\times 5 + 1)$$

$$= -1.2$$

$$\cancel{k_2 = hf(x_n}$$

$$k_2 = h\, f(x_0 + h,\ y_0 + k_1) = 0.1 \times (2(x_0 + 0.1) - 3(y_0 + k_1) + 1)$$

$$= 0.1 \times (2(1+0.1) - 3(5 - 1.2) + 1)$$

$$= -0.82$$

$$\therefore y_1 = y_0 + \frac{1}{2}(k_1 + k_2) = 5 + \frac{1}{2}(-1.2 - 0.82)$$

$$= 3.99$$

For $n=1$,

$$k_1 = h\, f(x_1, y_1) = 0.1 \times (2\times 1.1 - 3\times 3.99 + 1)$$

$$= -0.877$$

$$k_2 = h\, f(x_1 + h, y_1 + k_1) = 0.1 \times (2(1.1 + 0.1) - 3(3.99 - 0.877) + 1)$$

$$= -0.5939$$

$$\therefore y_2 = y_1 + \frac{1}{2}(k_1 + k_2) = 3.99 + \frac{1}{2}(-0.877 - 0.5939)$$

$$= 3.25455$$

For $n=2$,

$$k_1 = h\, f(x_2, y_2) = 0.1 \times (2\times 1.2 - 3\times 3.25455 + 1)$$

$$= -0.636365$$

$$k_2 = h\, f(x_2+h,\ y_2+k_1) = 0.1\times\left(2\times(1.2+1.1) - 3(3.25455 - 0.636365) + 1\right)$$

$$= -0.4254555$$

$$\therefore y_3 = y_2 + \tfrac{1}{2}(k_1+k_2) = 3.25455 + \tfrac{1}{2}(-0.636365 - 0.4254555)$$

$$= 2.72363975$$

For $n=3$,

$$k_1 = hf(x_3, y_3) = 0.1\times(2\times 1.3 - 3\times 2.72363975 + 1)$$

$$= -0.4570919 25$$

$$k_2 = hf(x_3+h,\ y_3+k_1) = 0.1\times\left(2(1.3+0.1) - 3(2.72363975 - 0.4570919 25) + 1\right)$$

$$= -0.2999643475$$

$$\therefore y_4 = y_3 + \tfrac{1}{2}(k_1+k_2) = 2.72363975 + \tfrac{1}{2}\left(-0.4570919 25 - 0.2999643475\right)$$

$$= 2.345111614$$

For $n=4$,

$$k_1 = hf(x_4, y_4) = 0.1\times(2\times 1.4 - 3\times 2.345111614 + 1)$$

$$= -0.3235334841$$

$$\therefore k_2 = hf(x_4+h,\ y_4+k_1) = 0.1\left(2(1.4+0.1) - 3(2.345111614 - 0.3235334841) + 1\right)$$

$$= -0.2064734389$$

$\therefore Y_5 = Y_4 + \frac{1}{2}(k_1 + k_2) = 2.345111614 + \frac{1}{2}(-0.323533484 - 0.2064734389)$

$= 2.080108152$

n	Y	Euler	R-K	Exact
0	Y_1	3.8	3.77	3.7723
1	Y_2	2.78	3.2546	3.2283
2	Y_3	2.426	2.7236	2.6944
3	Y_4	2.0582	2.3451	2.3162
4	Y_5	1.82074	2.0801	2.0532

$y' + 3y = 2x + 1$

$(D+3)y = 2x+1$

AE

$m + 3 = 0$

$m = -3$

$\therefore Y_c = c_1 e^{-3x}$

$Y_p = \frac{1}{D+3}(2x+1)$

$= \frac{1}{3}\left(1 + \frac{D}{3}\right)^{-1}(2x+1)$

$= \frac{1}{3}\left(1 - \frac{D}{3} + \cdots\right)(2x+1)$

$= \frac{1}{3}\left(2x + 1 - \frac{2}{3}\right)$

$= \frac{1}{3}\left(2x + \frac{1}{3}\right)$

$\therefore Y = c_1 e^{3x} + \frac{1}{3}\left(2x + \frac{1}{3}\right)$

$x = 1,\ y = 5 \quad \therefore c_1 = 84.80560034$

3)

$y' = 1 + y^2$

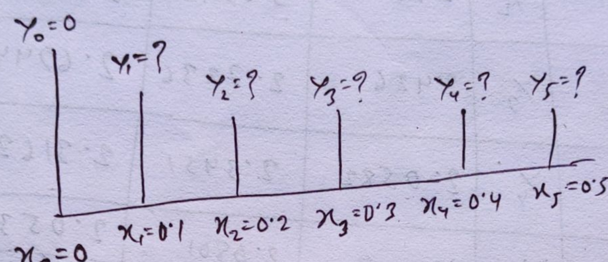
$y(0) = 0$

$y(0.5) = ?$

Euler Method

We know,

$$y_{n+1} = y_n + h f(x_n, y_n) \; ; \; n = 0, 1, 2, 3, \ldots$$



For $n=0$,

$$y_1 = y_0 + h f(x_0, y_0) = 0 + 0.1\left(1 + (y_0)^2\right)$$

$$= 0.1$$

For $n=1$,

$$y_2 = y_1 + h f(x_1, y_1) = 0.1 + 0.1\left(1 + y_1^2\right)$$

$$= 0.1 + 0.1\left(1 + (0.1)^2\right)$$

$$= 0.201$$

For, $n=2$,

$$y_3 = y_2 + h f(x_2, y_2) = 0.201 + 0.1\left(1 + (0.201)^2\right)$$

$$= 0.3050401$$

For $n=3$,

$$y_4 = y_3 + h f(x_3, y_3) = 0.3050401 + 0.1\left(1 + (0.3050401)^2\right)$$

$$= 0.4143450463$$

For $n=4$,

$$Y_5 = Y_4 + hf(x_4, Y_4) = 0.4143450463 + 0.1(1 + (0.4143450463)^2)$$

$$= 0.5315\,13228$$

R-K Method,

We know,

$$Y_{n+1} = Y_n + \frac{1}{2}(k_1 + k_2)$$

$$k_1 = hf(x_n, Y_n)$$

$$k_2 = hf(x_n + h, Y_n + k_1)$$

For, $n=0$,

$$k_1 = hf(x_0, Y_0) = 0.1(1 + Y_0^2) = 0.1(1+0)$$

$$= 0.1$$

$$k_2 = hf(x_0 + h, Y_0 + k_1) = 0.1(1 + (Y_0 + k_1)^2)$$

$$= 0.1(1 + (0 + 0.1)^2)$$

$$= 0.101$$

$$\therefore Y_1 = Y_0 + \frac{1}{2}(k_1 + k_2) = 0 + \frac{1}{2}(0.1 + 0.101)$$

$$= 0.1005$$

For $n=1$,

$$k_1 = hf(x_1, Y_1) = 0.1(1 + (0.1005)^2) =$$

$$= 0.101010025$$

$$k_2 = hf(x_1 + h, Y_1 + k_1) = 0.1(1 + (0.1005 + 0.101010025)^2)$$

$$= 0.104060629$$

$$\therefore Y_2 = Y_1 + \frac{1}{2}(k_1 + k_2) = 0.1005 + \frac{1}{2}(0.101010025 + 0.104060629)$$

$$= 0.203035327$$

For $n=2$,

$$k_1 = hf(x_2, y_2) = 0.1\left(1+(0.203035327)^2\right)$$

$$= 0.1041223344$$

$$k_2 = hf(x_2+h, y_2+k_1) = 0.1\left(1+(0.203035327+0.1041223344)^2\right)$$

$$= 0.1094345829$$

$$\therefore y_3 = y_2 + \frac{1}{2}(k_1+k_2) = 0.203035327 + \frac{1}{2}\left(0.1041223344 \cancel{+0.20303} + 0.1094345829\right)$$

$$= 0.3098137857$$

For $n=3$,

$$k_1 = hf(x_3, y_3) = 0.1\left(1+(0.3098137857)^2\right)$$

$$= 0.1095984582$$

$$k_2 = hf(x_3+h, y_3+k_1) = 0.1\left(1+(0.3098137857+0.1095984582)^2\right)$$

$$= 0.117590663$$

$$\therefore y_4 = y_3 + \frac{1}{2}(k_1+k_2) = 0.3098137857 + \frac{1}{2}\left(0.1095984582 + 0.117590663\right)$$

$$= 0.4234083463$$

For $n=4$,

$$k_1 = hf(x_4, y_4) = 0.1\left(1+(0.4234083463)^2\right)$$

$$= 0.1179274628$$

$$k_2 = hf(x_4+h, y_4+k_1) = 0.1\left(1+(0.4230834463+0.1179274628)^2\right)$$

$$= 0.1293044458$$

$$\therefore y_5 = \cancel{4} y_4 + \frac{1}{2}(k_1+k_2) = 0.4234083463 + \frac{1}{2}\left(0.1179274628 + 0.129304458\right)$$

$$= 0.5470243006$$

Table

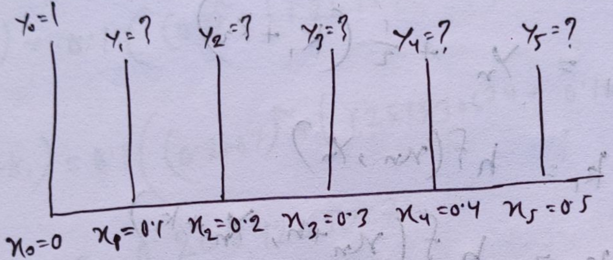
n	Y	Euler	R-K
0	Y_1	0.1	0.1005
1	Y_2	0.201	0.2030
2	Y_3	0.3050	0.3098
3	Y_4	0.4143	0.4234
4	Y_5	0.5315	0.5470

4)

$y' = x^2 + y^2$

$y(0) = 1$

$y(0.5) = ?$



Euler method:

we know,

$$Y_{n+1} = Y_n + hf(x_n, Y_n) \quad \cdots \text{(1)}$$

$$n = 0, 1, 2, 3 \cdots$$

For $n=0$,

$$Y_1 = Y_0 + hf(x_0, Y_0) = 1 + 0.1(0^2 + 1^2)$$

$$= 1.1$$

For $n=1$,

$$Y_2 = Y_1 + 0.1(x_1^2 + Y_1^2) = 1.1 + 0.1((0.1)^2 + (1.1)^2)$$

$$= 1.222$$

For $n=2$,

$$Y_3 = Y_2 + 0.1(x_2^2 + Y_2^2) = 1.222 + 0.1((0.2)^2 + (1.222)^2)$$

$$= 1.3753284$$