7)

a) The plane $3x - 2y + 5z = 0$

$$\Rightarrow 3x = 2y - 5z$$

$$\Rightarrow x = \frac{2}{3}y - \frac{5}{3}z$$

Take,

$$y = r, \quad z = s$$

$$\therefore x = \frac{2}{3}r - \frac{5}{3}s$$

$$(x,y,z) = \left(\frac{2}{3}r - \frac{5}{3}s, r, s\right)$$

$$= \left(\frac{2}{3}r, r, 0\right) + \left(-\frac{5}{3}s, 0, s\right)$$

$$= r\left(\frac{2}{3}, 1, 0\right) + s\left(-\frac{5}{3}, 0, 1\right)$$

$$= rv_1 + sv_2$$

$\{v_1, v_2\}$ is a basis for the solution space of $\mathbb{R}^3$ and the dimension of the solution space $= 2$.

4.7

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & -2 & 5 \end{bmatrix}$$

$r_1 = [2 \quad 3 \quad 5]$

are row vectors

$r_2 = [1 \quad -2 \quad 5]$

members of R^3

$c_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$c_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ members of $\mathbb{R}^2$

$c_3 = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$

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|4.7|

⊛ Basis for the null space

Example - 1:

$$2x_1 + 2x_2 - x_3 + x_5 = 0$$
$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$
$$x_1 + 2x_2 - x_3 - x_5 = 0$$
$$x_3 + x_4 + x_5 = 0$$

Co-effecient matrix,

$$A = \begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 2 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ $x_1 + x_2 + x_5 = 0$

$x_3 + x_5 = 0$

$x_4 = 0$

$\Rightarrow$

$x_1 = -x_2 - x_5$ $\Rightarrow$ $x_1 = -s - t$

$x_3 = -x_5$ $\quad x_2 = s$

$x_4 = 0$ $\quad x_3 = -t$

$x_4 = 0$

$x_5 = t$

$\Rightarrow$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -s-t \\ s \\ -t \\ 0 \\ t \end{bmatrix} = \begin{bmatrix} -s \\ s \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -t \\ 0 \\ -t \\ 0 \\ t \end{bmatrix}$$

$$= s\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} = s v_1 + s v_2$$

$S = \{v_1, v_2\}$ is a basis for the null space of the matrix A

Dimension of the nullspace of A is called the nullity of A and we write $\boxed{\text{nullity}(A) = 2}$

Find ~~the~~ a basis for the row space, and column space of the matrix.

$$A = \begin{array}{c} \begin{array}{cccccc} c_1 & & c_3 & & c_5 & \end{array} \\ \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix} \end{array}$$

$\Rightarrow$

$$A = \begin{array}{c} \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 0 & 0 & 1 & 3 & -2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \rightarrow r_1 \\ \rightarrow r_2 \\ \rightarrow r_3 \\ \\ \end{array}$$

A basis for the row space of A is $S = \{r_1, r_2, r_3\}$

A basis for the column space of A is $S = \{c_1, c_3, c_5\}$

(*) Find the subset of the vectors,

$v_1 = (1,-2,0,3)$, $v_2 = (2,-5,-3,6)$, $v_3 = (0,1,3,0)$, $v_4 = (2,-1,4,-7)$,
$v_5 = (5,-8,1,2)$

$\Rightarrow$

$$\begin{bmatrix} 1 & -2 & 0 & 3 \\ 2 & -5 & -3 & 6 \\ 2 & -1 & 4 & -7 \\ 5 & -8 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & \frac{13}{5} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis for the subspace spanned by the given vectors is

$$\left\{ (1, -2, 0, 3), (0, 1, 3, 0), (0\ 0\ 1\ \tfrac{13}{5}) \right\}$$

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4.8

Theorem 4.8.2

If A is a matrix with n columns, then

$$\text{rank}(A) + \text{nullity}(A) = n$$

(*) A is 5×3 matrix,

rank	nullity
3	0
2	1
1	2

4.9

$$R^n \to R^m$$

$$2x_1 + x_2 - 3x_3 = W_1$$

$$x_1 - x_2 + 5x_3 = W_2$$

$$\begin{bmatrix} 2 & 1 & -3 \\ 1 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}$$

$\therefore \quad A \cdot x = W$

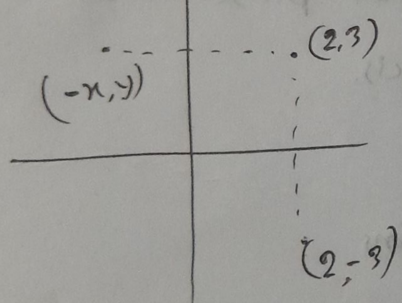
$x \in R^3 \quad , \quad W \in R^2$

$f : V \rightarrow W$

<u>linear transformation</u>

i) $T(A+B) = T(A) + T(B)$

ii) $T(kA) = kT(A)$



$T : R^2 \rightarrow R^2$

$T : (x,y) = (x,-y)$

reflection about x-axis.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

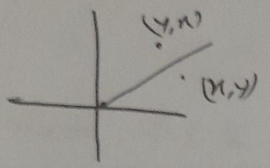
$w_1 = x$

$w_2 = -y$

$T : (x,y) = (-x,y)$

Reflection about y-axis.

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



$T:(x,y) = (y,x)$

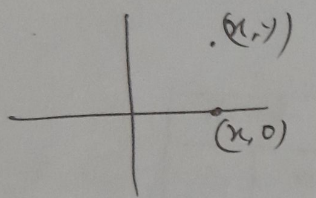
reflection about $y=x$ line

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

⊛

$T: R^3 \to R^3$

⊛



$T: R^2 \to R^2$ defined,

$T: (x,y) = (x,0)$

an orthogonal projection on the x-axis.

⊛ $T(x) = kx$ → $k > 1$ Dilations

→ $0 < k < 1$ contraction

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5.1

Eigenvalues and Eigenvectors

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$Ax = \begin{bmatrix} 3+0 \\ 8-2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= 3x$$

$\therefore Ax = 3x$ → Eigen values, → Eigenvectors

A is $n \times n$, $0 \neq x \in \mathbb{R}^n$, such that

$Ax = \lambda x$; λ is a scalar

Then, x is Eigenvectors

and λ is Eigen values

* Given matrix A, find out Eigenvectors and Eigenvalues

$\Rightarrow$

Take, a nonzero vector x such that,

$$Ax = \lambda x$$

$$\Rightarrow Ax - \lambda x = 0$$

$$\Rightarrow (A - \lambda I)x = 0$$

which is a linear homogenous system.

It passesses nonzero solution only if

$$\det(A - \lambda I) = 0$$

$\det(A - \lambda I) = 0$ is a polynomial equation in λ.

Also it is called the characteristic equation of A.

Example-2

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

Let,

x be a nonzero vector in $\mathbb{R}^2$ and λ is a scalar.

Then, $(A - \lambda I)x = 0 \quad \cdots\cdots (i)$

and $\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\therefore \quad A - \lambda I = \begin{bmatrix} 3-\lambda & 0 \\ 8 & -1-\lambda \end{bmatrix}$$

$$\therefore \begin{vmatrix} 3-\lambda & 0 \\ 8 & -1-\lambda \end{vmatrix} = 0$$

$(3-\lambda)(-1-\lambda) - 0 = 0$

$(3-\lambda)(-1-\lambda) = 0$

$\therefore \lambda = 3, -1.$

Case 1:

Put, $\lambda = 3$ in (i)

$$\begin{bmatrix} 0 & 0 \\ 8 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 8x_1 - 4x_2 = 0$$

$$\Rightarrow x_1 = \frac{1}{2} x_2$$

Let,

$x_2 = 2r$

$\therefore x_1 = r$

So, $\begin{bmatrix} r \\ 2r \end{bmatrix} = r \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\therefore \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an independent Eigen Vectors of A

for $\lambda = 3$.

Case-2

Put, $\lambda = -1$ in (i)

$$\begin{bmatrix} 4 & 0 \\ 8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4x_1 \\ 8x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 = 0$

and $x_2 = r$

$\therefore x = \begin{bmatrix} 0 \\ r \end{bmatrix} = r \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\therefore \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is an independent Eigen vectors of A

for $\lambda = -1$.

5.2

$P = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ is inventable

$P^{-1} AP = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$

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5.1

6)

d) $A = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 3 & 0 \\ -4 & 13 & -1 \end{bmatrix}$

Let,

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be a nonzero vector and a scalar λ ...

then, $(A - \lambda I)x = 0$

$$A - \lambda I = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 3 & 0 \\ -4 & 13 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-\lambda & 0 & 1 \\ -1 & 3-\lambda & 0 \\ -4 & 13 & -1-\lambda \end{bmatrix}$$

$$\therefore \begin{bmatrix} -1-\lambda & 0 & 1 \\ -1 & 3-\lambda & 0 \\ -4 & 13 & -1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \cdots\cdots (i)$$

Now,

$$\begin{vmatrix} -1-\lambda & 0 & 1 \\ -1 & 3-\lambda & 0 \\ -4 & 13 & -1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-1-\lambda)(-3+\lambda)(-1-\lambda) + 1\{-13+4(3-\lambda)\} = 0$$

$$\Rightarrow (-1-\lambda)(-3-3\lambda+\lambda+\lambda^2) - 13 + 12 - \cancel{4}\, 4\lambda = 0$$

$$\Rightarrow (-1-\lambda)(\lambda^2-2\lambda-3) - 1 - 4\lambda = 0$$

$$\Rightarrow -\lambda^2 + 2\lambda + 3 - \lambda^3 + 2\lambda^2 + 3\lambda - 1 - 4\lambda = 0$$

$$\Rightarrow -\lambda^3 + \lambda^2 + \lambda + 2 = 0$$

$$\Rightarrow \lambda^3 - \lambda^2 - \lambda - 2 = 0 \longrightarrow \text{charactenistic equation.}$$

$$\Rightarrow \lambda^2(\lambda-2) + \lambda(\lambda-2) + 1(\lambda-2) = 0$$

$$\Rightarrow (\lambda-2)(\lambda^2+\lambda+1) = 0$$

$$\lambda = 2, \quad \lambda = \frac{-1 \pm \sqrt{-3}}{2}$$

$$\therefore \lambda = 2, \quad \frac{-1 \pm i\sqrt{3}}{2}$$

Put, $\lambda = 2$ in (i) $\Rightarrow$

$$\begin{bmatrix} -3 & 0 & 1 \\ -1 & 1 & 0 \\ -4 & 13 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} -1 & 0 & 0 \\ -7 & 0 & 1 \\ -4 & 13 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} -1 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 9 & -3 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -x_1 + x_2 = 0$

$x_2 - \frac{1}{3} x_3 = 0$

$x_1 = x_2$

$x_3 = 3x_2$

Let,

$x_2 = r$

$x_1 = r \; , x_2 = r$

$x_3 = 2r$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} r \\ r \\ 3r \end{bmatrix} = r \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$= r\, v_1$$

$v_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ is the independent basis eigenvector of A

corresponding to $\lambda = 2$

and $\{v_1\}$ is a basis for the eigenspaces of A.

corresponding to $\lambda = 2$

Dimension of the eigenspace is 1.

$$Ax = \lambda x$$

$$A(Ax) = A(\lambda x)$$

$$\Rightarrow A^2 x = \lambda (Ax)$$

$$= \lambda (\lambda x)$$

$$A^2 x = \lambda^2 x$$

$$A^3 x = A(\lambda^2 x)$$

$$= \lambda^2 (\cancel{A}Ax)$$

$$= \lambda^2 (\lambda x)$$

$$= \lambda^3 x$$

$$\therefore A^k x = \lambda^k x$$

5.2

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -\lambda & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{bmatrix}$$

Now,

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -\lambda & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)\{-\lambda(3-\lambda)+2\} = 0$$

$$\Rightarrow (2-\lambda)(-3\lambda+\lambda^2+2) = 0$$

$$\Rightarrow (2-\lambda)(\lambda^2-3\lambda+2) = 0$$

$$\therefore \lambda = 1, 2, 2$$

$$\therefore \begin{bmatrix} -4 & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Put, $\lambda = 2$, in (i)

$$\begin{bmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_3 = 0$$

$$\Rightarrow x_1 = -x_3$$

Let, $x_2 = r$, $x_3 = s$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -s \\ r \\ s \end{bmatrix} = \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix} + \begin{bmatrix} -s \\ 0 \\ s \end{bmatrix}$$

$$\Rightarrow \quad = r \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$= r P_1 + s P_2$$

$P_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $P_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ are independent eigen vector of A. corresponding to $\lambda = 2$

Again,

put, $\lambda = 1$ in (i)

$$\begin{bmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} x_1 + 2x_3 &= 0 \\ x_2 - x_3 &= 0 \end{aligned}\right\} \quad \begin{aligned} x_1 &= -2x_3 \\ x_2 &= x_3 \end{aligned}$$

Let, $x_3 = k$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2r \\ r \\ r \end{bmatrix} = r \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} = r\, P_3$$

$P_3 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ is the independent eigenvector of A corresponding to $\lambda = 1$.

$$P = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$B = P^{-1} A P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 6 & 2 \end{bmatrix}$$

(*)

$$|B| = |P^{-1} A P| = |P^{-1}|\ |A|\ |P|$$

$$= |P^{-1}| \cdot |P| \cdot |A|$$

$$= |A|$$

$\det(A^{-1}) = \frac{1}{\det(A)}$

$\det(A^{-1}) \cdot \det(A) = 1$

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(*)

Cherry stock = 130 pounds

Mints stock = 170 pounds

x_1 pound with the mixture of half chery and half mints by weight.

x_2 pound with the mixture of $\frac{1}{3}$ cherry and $\frac{2}{3}$ mints by weight.

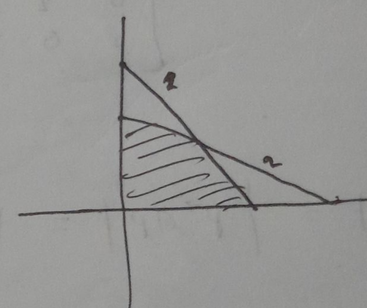
$$Z = 2x_1 + 1.25x_2$$

$$\frac{1}{2}x_1 + \frac{1}{3}x_2 \le 130$$

$$\frac{1}{2}x_1 + \frac{2}{3}x_2 \le 170$$

$$x_1 \ge 0$$

$$x_2 \ge 0$$



(*) $\frac{1}{2}x_1 + \frac{1}{3}x_2 \le 130$

$$\frac{x_1}{260} + \frac{x_2}{390} \le 1 \cdots (i)$$

similarly $\frac{x_1}{340} + \frac{x_2}{255} \le 1 \cdots (ii)$

Entreame Point	Value of Z
(0,0)	00
(0,255)	518.75
(180,120)	510
(260,0)	520

Markov Process

State - 1 : Sunny

State - 2 : Cloudy

State - 3 : Rainy

P_{ij} :

→ Previous state was in j now it will be i, the probality of being i.

$$\begin{array}{c|ccc|c} & 1 & 2 & 3 & \\ \hline & P_{11} & P_{12} & P_{13} & 1 \\ & P_{21} & P_{22} & P_{23} & 2 \\ & P_{31} & P_{32} & P_{33} & 3 \end{array}$$

Preceding State

New state

'p' means Probality

Sum of a column = 1

Rented from location

$$\begin{array}{cccc} 1 & 2 & 3 & \\ \left[\begin{array}{ccc} .8 & .3 & .2 \\ .1 & .2 & .6 \\ .1 & .5 & .2 \end{array}\right] & & & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array}$$

Return to Location

Position $\boxed{23}$ → that means, rented from 3 and return to 2. Probality $.6 = 60\%$

State-1: Alumni donate a year

State-2: Alumni does not donate

$$\begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \left[\begin{array}{cc} 0.8 & 0.3 \\ 0.2 & 0.7 \end{array}\right] & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

→ Probability Matrix

Transition matrix

$\boxed{\text{END}}$