North South University

Department of Mathematics and Physics

Assignment-6

Name : Joy kumar Ghosh

Student ID : 2211424642

Course No : MAT-130

Course Title : Calculus and Analytical Geometry II

Section : 8

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10·1

46/

$x = \frac{1}{2}t^2 + 1$

$y = \frac{1}{3}t^3 - t$

$t = 2$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{t^2-1}{t}$$

$$= t - \frac{1}{t}$$

$$\therefore \left.\frac{dy}{dx}\right|_{t=2} = 2 - \frac{1}{2}$$

$$= \frac{3}{2}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$= \frac{1+\frac{1}{t^2}}{t}$$

$$= \frac{\frac{t^2+1}{t^2}}{t} = \frac{t^2+1}{t^3}$$

$$\therefore \left.\frac{d^2y}{dx^2}\right|_{t=2} = \frac{2^2+1}{2^3}$$

$$= \frac{4+1}{8}$$

$$= \frac{5}{8}$$

481

$x = \sinh t$

$y = \cosh t$

$t = 0$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{\sinh t}{\cosh t}$$

$$= \tanh t$$

$$\therefore \left.\frac{dy}{dx}\right|_{t=0} = \tanh 0$$

$$= \frac{e^0 - e^0}{e^0 + e^0}$$

$$= \frac{0}{2}$$

$$= 0$$

$$\therefore \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$= \frac{\operatorname{sech}^2 t}{\cosh t}$$

$$= \operatorname{sech}^3 t$$

$$\therefore \left.\frac{d^2y}{dx^2}\right|_{t=0} = \operatorname{sech}^3 0$$

$$= \left(\frac{2}{e^0+e^0}\right)^3$$

$$= 1^3$$

$$= 1$$

66)

$$x = \sqrt{t} - 2 \qquad (1 \leq t \leq 16)$$

$$y = 2t^{3/4}$$

$$\therefore x = \sqrt{t} - 2$$

$$\sqrt{t} = x + 2$$

$$t = (x+2)^2$$

$$\therefore y = 2(x+2)^{2 \cdot \frac{3}{4}}$$

$$= 2(x+2)^{\frac{3}{2}}$$

Now, $t = 1$

then, $x = \sqrt{1} - 2 = -1$

$y = 16$

then, $x = \sqrt{16} - 2 = 2$

$\therefore$ Arc length $= \int_{-1}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$\frac{dy}{dx} = 2 \cdot \frac{3}{2} \cdot (x+2)^{\frac{1}{2}} \cdot 1$

$= \cancel{3\sqrt{3x}}\ 3\sqrt{x+2}$

$\therefore$ Arc length $= \int_{-1}^{2} \sqrt{1+9(x+2)}\ dx$

$= \int_{-1}^{2} \sqrt{1+9x+18}\ dx$

$= \int_{-1}^{2} \sqrt{9x+19}\ dx$

$= \int_{10}^{37} \sqrt{u} \cdot \frac{1}{9}\, du$

Let,

$u = 9x+19$

$du = 9\,dx$

$dx = \frac{1}{9} \cdot du$

x	u
-1	10
2	37

$= \frac{1}{9} \left[\frac{u^{3/2}}{3/2} \right]_{10}^{37}$

$= \frac{1}{9} \cdot \frac{2}{3} \left(37^{3/2} - 10^{3/2} \right)$

$= \frac{2}{27} \left(37\sqrt{37} - 10\sqrt{10} \right)$

Ans.

681

$x = \sin t + \cos t$

$; (0 \le t \le \pi)$

$y = \sin t - \cos t$

$$\text{Arc length} = \int_0^\pi \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^\pi \sqrt{(\cos t - \sin t)^2 + (\cos t + \sin t)^2}\, dt$$

$$= \int_0^\pi \sqrt{2(\cos^2 t + \sin^2 t)}\, dt$$

$$= \int_0^\pi \sqrt{2}\, dt$$

$$= \sqrt{2}\, t \Big]_0^\pi$$

$$= \sqrt{2}\pi$$ Ans.

10.2

3)

e) $(7, 17\pi/6)$

$x = r\cos\theta$
$= 7 \cdot \cos\left(\frac{17\pi}{6}\right)$
$= 7 \cdot \frac{-\sqrt{3}}{2}$
$= -\frac{7\sqrt{3}}{2}$

$y = r\sin\theta$
$= 7 \cdot \sin\left(\frac{17\pi}{6}\right)$
$= 7 \cdot \frac{1}{2}$
$= \frac{7}{2}$

Therefore rectangular coordinates is $\left(-\frac{7\sqrt{3}}{2}, \frac{7}{2}\right)$

4)

e) $\left(-4, -\frac{3\pi}{2}\right)$

$\therefore x = r\cos\theta$
$= -4 \cdot \cos\left(\frac{-3\pi}{2}\right)$
$= 0$

$y = r\sin\theta$
$= -4 \cdot \sin\left(\frac{-3\pi}{2}\right)$
$= -4$

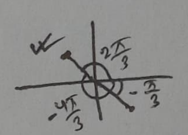
Therefore, rectangular coordinates is $(0, -4)$

5|

e) $(-3, 3\sqrt{3})$

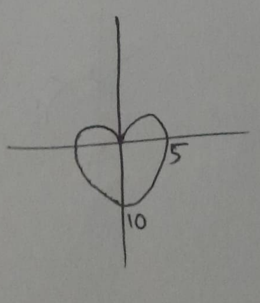
$$r = \sqrt{x^2+y^2}$$
$$= \sqrt{(-3)^2 + (3\sqrt{3})^2}$$
$$= \sqrt{9 + 9.3}$$
$$= \sqrt{36}$$
$$= \pm 6$$

$$\theta = \tan^{-1} \frac{y}{x}$$
$$= \tan^{-1}\left(\frac{3\sqrt{3}}{-3}\right)$$
$$= \tan^{-1}(-\sqrt{3})$$
$$= -\frac{\pi}{3}, \frac{2\pi}{3}$$

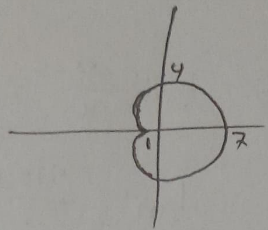


$\therefore$ polar coordinate, $\left(6, \frac{2\pi}{3}\right)$ & $\left(6, \frac{-4\pi}{3}\right)$

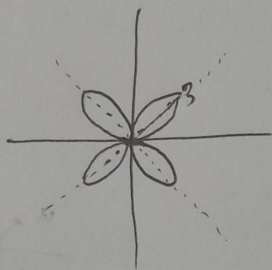
28| $r = 5 - 5\sin\theta$



32) $r = 4 + 3\cos\theta$



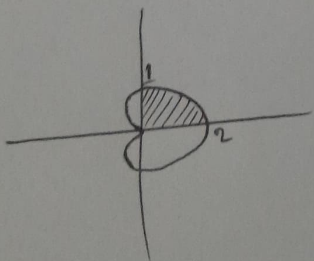
44) $r = 3\sin 2\theta$

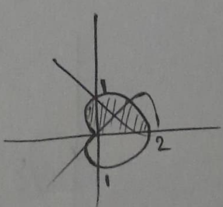


10·3

30)

$r = 1 + \cos\theta$





$$\therefore A = \frac{1}{2}\int_0^{\pi/2} (1+\cos\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} (1 + 2\cos\theta + \cos^2\theta) d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \left(1 + 2\cos\theta + \frac{1}{2}(1+\cos 2\theta)\right) d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \left(1 + 2\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta\right) d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} \left(\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta\right) d\theta$$

$$= \frac{1}{2} \left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{2}\frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left(\frac{3}{2}\cdot\frac{\pi}{2} + 2 + 0\right)$$

$$= \frac{1}{2}\left(\frac{3\pi}{4} + 2\right)$$

$$= \frac{1}{2}\cdot\frac{3\pi+8}{4}$$

$$= \frac{3\pi+8}{8}$$

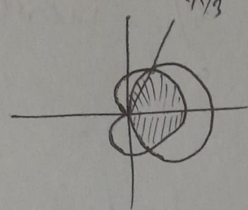
$$= \frac{3\pi}{8} + 1$$

Ans

381

$r = 1 + \cos\theta$

$r = 3\cos\theta$



Intersecting point,

$$1 + \cos\theta = 3\cos\theta$$

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{3}$$

$$\therefore \text{Area} = 2\left[\frac{1}{2}\int_0^{\pi/3} (1+\cos\theta)^2 \, d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2} (3\cos\theta)^2 \, d\theta\right]$$

$$= 2\left[\frac{1}{2}\int_0^{\pi/3} \left(\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta\right) d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2} 9\cos^2\theta \, d\theta\right]$$

$$= 2\left[\frac{1}{2}\left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{2}\frac{\sin 2\theta}{2}\right]_0^{\pi/3} + \frac{9}{2}\cdot\frac{1}{2}\int_{\pi/3}^{\pi/2} (1+\cos 2\theta) \, d\theta\right]$$

$$= 2\left[\frac{1}{2}\left(\frac{3}{2}\cdot\frac{\pi}{3} + 2\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right) + \frac{9}{4}\left[\theta + \frac{\sin 2\theta}{2}\right]_{\pi/3}^{\pi/2}\right]$$

$$= 2\left[\frac{1}{2}\left(\frac{3\pi}{6} + \sqrt{3} + \frac{\sqrt{3}}{8}\right) + \frac{9}{4}\left(\frac{\pi}{2} - \frac{\pi}{3} - \frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right)\right]$$

$$= 2\left[\frac{3\pi}{12} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{16} + \frac{9\pi}{8} - \frac{9\pi}{12} - \frac{9\sqrt{3}}{16}\right]$$

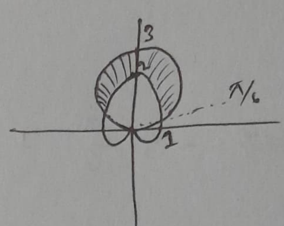
$$= \frac{3\pi}{6} + \frac{9\pi}{4} - \frac{9\pi}{6}$$

$$= \frac{5\pi}{4}$$ Ans

39)

$r = 3\sin\theta$

$r = 1 + \sin\theta$



intersecting point:

$3\sin\theta = 1 + \sin\theta$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{6}$

$$\therefore \text{Area} = 2\cdot\frac{1}{2}\int_{\pi/6}^{\pi/2} (3\sin\theta)^2 - (1+\sin\theta)^2\, d\theta$$

$$= \int_{\pi/6}^{\pi/2} \left(9\sin^2\theta - 1 - 2\sin\theta - \sin^2\theta\right) d\theta$$

$$= \int_{\pi/6}^{\pi/2} \left(8\sin^2\theta - 1 - 2\sin\theta\right) d\theta$$

$$= \int_{\pi/6}^{\pi/2} \left(8\cdot\frac{1}{2}(1-\cos 2\theta) - 1 - 2\sin\theta\right) d\theta$$

$$= \int_{\pi/6}^{\pi/2} (4 - 4\cos 2\theta - 1 - 2\sin\theta)\, d\theta$$

$$= \int_{\frac{\pi}{6}}^{\pi/2} (3 - 4\cos 2\theta - 2\sin\theta)\, d\theta$$

$$= \left[3\theta - 4 \cdot \frac{\sin 2\theta}{2} + 2\cos\theta\right]_{\pi/6}^{\pi/2}$$

$$= 3 \cdot \frac{\pi}{2} - 3 \cdot \frac{\pi}{6} + 2 \cdot \frac{\sqrt{3}}{2} - 2 \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{3\pi}{2} - \frac{\pi}{2}$$

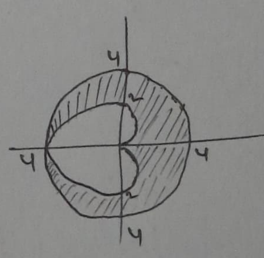
$$= \pi$$

Ans.

40|

$r = 2 - 2\cos\theta$

$r = 4$



$$\text{Area} = 2 \cdot \frac{1}{2} \int_0^{\pi} \left[4^2 - (2 - 2\cos\theta)^2\right] d\theta$$

$$= \int_0^{\pi} (16 - 4 + 8\cos\theta - 4\cos^2\theta)\, d\theta$$

$$= \int_0^{\pi} \left[12 - 8\cos\theta - 4\cdot\tfrac{1}{2}(1+\cos 2\theta)\right] d\theta$$

$$= \int_0^{\pi} (12 - 8\cos\theta - 2 - 2\cos 2\theta)\, d\theta$$

$$= \int_0^{\pi} (10 - 8\cos\theta - 2\cos 2\theta)\, d\theta$$

$$= \left[10\theta - 8\sin\theta - 2\cdot\frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \left[10\theta - 8\sin\theta - \sin 2\theta\right]_0^{\pi}$$

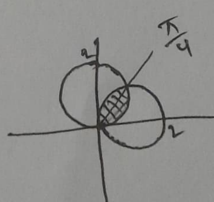
$$= 10\pi$$

Ans

42)

$r = 2\cos\theta$

$r = 2\sin\theta$



$2\cos\theta = 2\sin\theta$

$\cos\theta = \sin\theta$

$1 = \tan\theta$

$\theta = \tan^{-1}(1)$

$= \frac{\pi}{4}$

$$\therefore \text{Area} = 2\cdot\frac{1}{2}\int_0^{\pi/4} (2\sin\theta)^2\, d\theta$$

$$= \int_0^{\pi/4} 4\sin^2\theta\, d\theta$$

$$= 4\int_0^{\pi/4} \frac{1}{2}(1-\cos 2\theta)\,d\theta$$

$$= 2\int_0^{\pi/4} (1-\cos 2\theta)\,d\theta$$

$$= 2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/4}$$

$$= 2\left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$= \frac{\pi}{2} - 1$$

Ans.