① ⓐ $\frac{d}{dx}\left(\sin(x^2y^2)\right) = \frac{d}{dx}(xy)$

$\Rightarrow \cos(x^2y^2)\cdot\frac{d}{dx}(x^2y^2) = x\frac{d}{dx}(y) + y\frac{d}{dx}(x)$

$\Rightarrow \cos(x^2y^2)\left[x^2\cdot\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(x^2)\right] = x\cdot\frac{dy}{dx} + y\cdot 1$

$\Rightarrow \cos(x^2y^2)\left[x^2\cdot 2y\cdot\frac{dy}{dx} + y^2\cdot 2x\right] = x\frac{dy}{dx} + y$

$\Rightarrow 2x^2y\cos(x^2y^2)\frac{dy}{dx} + 2xy^2\cos(x^2y^2) - x\frac{dy}{dx} = y$

$\Rightarrow \frac{dy}{dx}\cdot\left[2x^2y\cos(x^2y^2) - x\right] = y - 2xy^2\cos(x^2y^2)$

$$\Rightarrow \frac{dy}{dx} = \frac{y - 2xy^2\cos(x^2y^2)}{2x^2y\cos(x^2y^2) - x}.$$

ⓑ $\frac{dy}{dx} = \frac{d}{dx}\left[(2x^2+7)\sin^2(5x)\ln(5x)\right]$

$= (x^2+7)\sin^2(5x)\cdot\frac{1}{5x}\cdot 5 + (2x^2+7)\cdot\ln(5x)\cdot 2\sin(5x)$

$\cdot\cos(5x)\cdot(5) + \sin^2(5x)\ln(5x)\cdot(4x)$

2(a) $\int e^{\sin\theta} \cos\theta \, d\theta$

put $u = \sin\theta$
$\Rightarrow du = \cos\theta \, d\theta$

$= \int e^u \cdot du$

$= e^u + c$

$= e^{\sin\theta} + C$

(b) $\int_1^3 \frac{x+2}{\sqrt{x^2+4x+7}} dx$

put $u = x^2+4x+7$
$du = (2x+4)\,dx$
$\Rightarrow du = 2(x+2)\,dx$
$\Rightarrow \frac{du}{2(x+2)} = dx$

x	u
1	12
3	28

$= \int_{12}^{28} \frac{\cancel{(x+2)}}{u^{1/2}} \cdot \frac{du}{2\cancel{(x+2)}}$

$= \frac{1}{2}\int_{12}^{28} u^{-1/2} du$

$= \frac{1}{2} \cdot \left[\frac{u^{1/2}}{1/2}\right]_{12}^{28}$

$= \left[\sqrt{u}\right]_{12}^{28}$

$= \sqrt{28} - \sqrt{12}$

≈ 1.8274