Therefore,

$$\int \frac{dx}{2x}$$

$$= \int \frac{1}{u} \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln u \; d + c$$

$$= \frac{1}{2} \ln(2x) + c$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \ln(2x) + c$$

23)

Given integral,

$$\int \frac{dx}{\sqrt{1-4x^2}}$$

$$= \int \frac{dx}{\sqrt{1-(2x)^2}}$$

Let,

$u = 2x$

$\Rightarrow \frac{du}{dx} = 2$

$\therefore dx = \frac{1}{2} du$

Therefore,

$$\int \frac{dx}{\sqrt{1-(2x)^2}}$$

$$= \int \frac{1}{\sqrt{1-u^2}} \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \int \frac{1}{\sqrt{1-u^2}} du$$

$$= \frac{1}{2} \sin^{-1} u + c$$

$$= \frac{1}{2} \sin^{-1} 2x + c$$

Therefore the value of the given integral is,

$$\frac{1}{2} \sin^{-1} 2x + c$$

24/

Given integral,

$$\int \frac{dx}{1+16x^2}$$

$$= \int \frac{dx}{1+(4x)^2}$$

Let, $u = 4x$

$$\Rightarrow \frac{du}{dx} = 4$$

$$\therefore dx = \frac{1}{4} du$$

Therefore,

$$\int \frac{dx}{1+(4x)^2}$$

$$= \int \frac{1}{1+u^2} \cdot \frac{1}{4} du$$

$$= \frac{1}{4} \int \frac{1}{1+u^2} du$$

$$= \frac{1}{4} \tan^{-1} u + c$$

$$= \frac{1}{4} \tan^{-1} 4x + c$$

Therefore the value of the given integral is,

$$\frac{1}{4} \tan^{-1} 4x + c.$$

25)

Given integral, $\int t \sqrt{7t^2+12}\, dt$

Let,

$u = 7t^2+12$

$\Rightarrow \frac{du}{dt} = 14t$

$\Rightarrow du = 14t\, dt$

$\therefore t\, dt = \frac{1}{14} du$

Therefore,

$$\int t\sqrt{7t^2+12}\,dt$$

$$= \int \sqrt{u}\cdot\frac{1}{14}\,du$$

$$= \frac{1}{14}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{14}\cdot\frac{u^{3/2}}{3/2}+c$$

$$= \frac{1}{14}\cdot\frac{2}{3}\cdot u^{3/2}+c$$

$$= \frac{1}{21}\cdot(7t^2+12)^{3/2}+c$$

Therefore, the value of the given integral is,

$$\frac{1}{21}\cdot(7t^2+12)^{3/2}+c$$

2(c)

Given integral,

$$\int \frac{x}{\sqrt{4-5x^2}}\,dx$$

Let,

$$u = 4-5x^2$$

$$\Rightarrow \frac{du}{dx} = -10x$$

$$\Rightarrow du = -10x\,dx$$

$$\therefore x\,dx = -\frac{1}{10}\,du$$

Therefore,

$$\int \frac{x}{\sqrt{4-5x^2}}\,dx$$

$$= \int \frac{1}{\sqrt{u}} \cdot \left(-\frac{1}{10}\right) \cdot du$$

$$= -\frac{1}{10} \int \frac{1}{\sqrt{u}}\,du$$

$$\cancel{= -\frac{1}{10} \ln u + c}$$

$$\cancel{= -\frac{1}{10} \ln(4-5x^2) + c}$$

$$= -\frac{1}{10} \int u^{-\frac{1}{2}}\,du$$

$$= -\frac{1}{10} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= -\frac{1}{5}\sqrt{(4-5x^2)} + c$$

Therefore, the value of the given integral is,

$$-\frac{1}{5}\sqrt{(4-5x^2)} + c.$$

27)

Given that,

$$\int \frac{6}{(1-2x)^3}\,dx$$

Let,
$$u = 1-2x$$

$$\Rightarrow \frac{du}{dx} = -2$$

$$\therefore dx = -\frac{1}{2}\,du$$

Therefore,

$$\int \frac{6}{(1-2x)^3}\,dx$$

$$= \int \frac{6}{u^3}\cdot\left(-\frac{1}{2}\right)du$$

$$= -\frac{6}{2}\int u^{-3}\,du$$

$$= -\frac{6}{2}\cdot\frac{u^{-2}}{-2} + c$$

$$= \frac{6}{4}\cdot(1-2x)^{-2} + c$$

$$= \frac{3}{2}\cdot(1-2x)^{-2} + c$$

Therefore, the value of the given integral is,

$$\frac{3}{2}(1-2x)^{-2} + c.$$

28|

Given integral,

$$\int \frac{x^2+1}{\sqrt{x^3+3x}}\, dx$$

Let, $u = x^3+3x$

$$\Rightarrow \frac{du}{dx} = 3x^2+3$$

$$\Rightarrow \frac{du}{dx} = 3(x^2+1)$$

$$\therefore \frac{1}{3}du = (x^2+1)\,dx$$

Therefore,

$$\int \frac{x^2+1}{\sqrt{x^3+3x}}\, dx$$

$$= \int \frac{1}{\sqrt{u}} \cdot \frac{1}{3}\, du$$

$$= \frac{1}{3}\int u^{-\frac{1}{2}}\, du$$

$$= \frac{1}{3} \cdot 2 \cdot \sqrt{u} + c$$

$$= \frac{2}{3}\sqrt{x^3+3x} + c$$

Therefore, the value of the given integral is,

$$\frac{2}{3}\sqrt{x^3+3x} + c$$

29)

Given integral,

$$\int \frac{x^3}{(5x^4+2)^3}\, dx$$

Let,

$$u = 5x^4 + 2$$

$$\Rightarrow \frac{du}{dx} = 20x^3$$

$$\therefore x^3 dx = \frac{1}{20} du$$

Therefore,

$$\int \frac{x^3}{(5x^4+2)^3}\, dx$$

$$= \int \frac{1}{u^3} \cdot \frac{1}{20} du$$

$$= \frac{1}{20} \int u^{-3} du$$

$$= \frac{1}{20} \cdot \frac{u^{-2}}{-2} + c$$

$$= -\frac{1}{40} \cdot (5x^4+2)^{-2} + c$$

$$= -\frac{1}{40} \cdot (5x^4+2)^{-2} + c$$

Therefore, the value of the given integral is,

$$-\frac{1}{40} (5x^4+2)^{-2} + c.$$

30)

Given integral,

$$\int \frac{\sin(1/x)}{3x^2}\,dx$$

Let,

$$u = \frac{1}{x}$$

$$\Rightarrow \frac{du}{dx} = -\frac{1}{x^2}$$

$$\therefore \frac{1}{x^2}\,dx = -du$$

Therefore,

$$\int \frac{\sin(1/x)}{3x^2}\,dx$$

$$= \int \frac{\sin u}{3}(-du)$$

$$= -\frac{1}{3}\int \sin u\,du$$

$$= -\frac{1}{3}(-\cos u) + c$$

$$= \frac{1}{3}\cos\frac{1}{x} + c$$

Therefore, the value of the given integral is

$$\frac{1}{3}\cos\frac{1}{x} + c$$

31)

Given integral,

$$\int e^{\sin x} \cos x \, dx$$

Let,

$u = \sin x$

$\Rightarrow \frac{du}{dx} = \cos x$

$\therefore du = \cos x \, dx$

Therefore,

$$\int e^{\sin x} \cos x \, dx$$

$$= \int e^{u} \, du$$

$$= e^{u} + c$$

$$= e^{\sin x} + c$$

Therefore, the value of the given integral is,

$$e^{\sin x} + c.$$

32)

Given integral,

$$\int x^3 e^{x^4} \, dx$$

Let, $u = x^4$

$\Rightarrow \frac{du}{dx} = 4x^3$

$\therefore \frac{1}{4} du = x^3 \, dx$

Therefore,

$$\int x^3 e^{x^4} dx$$

$$= \int e^u \cdot \frac{1}{4} du$$

$$= \frac{1}{4} \int e^u du$$

$$= \frac{1}{4} e^u + c$$

$$= \frac{1}{4} e^{x^4} + c$$

Therefore the value of the given integral is,

$$\frac{1}{4} e^{x^4} + c.$$

33)

Given integral,

$$\int x^2 e^{-2x^3} dx$$

Let, $u = -2x^3$

$$\Rightarrow \frac{du}{dx} = -6x^2$$

$$\Rightarrow du = -6x^2 dx$$

$$\Rightarrow -\frac{1}{6} du = x^2 dx$$

Therefore, $\int x^2 e^{-2x^3} dx$

$$= \int e^u \cdot \left(-\frac{1}{6}\right) \cdot du$$

$$= -\frac{1}{6}\int e^u \, du$$

$$= -\frac{1}{6} e^u + C$$

$$= -\frac{1}{6} e^{-2x^3} + C$$

Therefore, the value of the given integral is,

$$-\frac{1}{6} e^{-2x^3} + C$$

34|

Given integral,

$$\int \frac{e^x + e^{-x}}{e^x - e^{-x}} \, dx$$

Let, $u = e^x - e^{-x}$

$$\Rightarrow \frac{du}{dx} = e^x - e^{-x} \cdot (-1)$$

$$\Rightarrow \frac{du}{dx} = e^x + e^{-x}$$

$$\therefore du = (e^x + e^{-x}) \, dx$$

Therefore, $\displaystyle\int \frac{e^x + e^{-x}}{e^x - e^{-x}} \, dx$

$$= \int \frac{1}{u} \, du$$

$$= \ln u + C$$

$$= \ln(e^x - e^{-x}) + C \quad \text{Ans.}$$

35|

Given integral,

$$\int \frac{e^x}{1+e^{2x}}\,dx$$

Let,

$$u = e^x$$

$$\Rightarrow \frac{du}{dx} = e^x$$

$$\therefore du = e^x\,dx$$

Therefore,

$$\int \frac{e^x}{1+e^{2x}}\,dx$$

$$= \int \frac{1}{1+u^2}\,du$$

$$= \tan^{-1}u + c$$

$$= \tan^{-1}(e^x) + c$$

Therefore, the value of the given integral is,

$$\tan^{-1}(e^x) + c$$

36|

Given integral,

$$\int \frac{t}{t^4+1}\,dt$$

$$= \int \frac{t}{(t^2)^2+1}\,dt$$

Let,

$u = t^2$

$\Rightarrow \frac{du}{dt} = 2t$

$\Rightarrow t\,dt = \frac{1}{2} du$

Therefore,

$$\int \frac{t}{(t^2)^2+1} dt$$

$$= \int \frac{1}{u^2+1} \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \int \frac{1}{u^2+1} du$$

$$= \frac{1}{2} \tan^{-1} u + c$$

$$= \frac{1}{2} \tan^{-1}(t^2) + c$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \tan^{-1}(t^2) + c$$

37/

Given integral,

$$\int \frac{\sin(5/x)}{x^2} dx$$

Let, $u = \frac{5}{x}$

$\Rightarrow \frac{du}{dx} = -\frac{5}{x^2}$

$\Rightarrow -\frac{1}{5} du = \frac{1}{x^2} dx$

Therefore,

$$\int \frac{\sin(5/x)}{x^2}\,dx$$

$$= \int \sin u \left(-\frac{1}{5}\right) du$$

$$= -\frac{1}{5}\int \sin u\, du$$

$$= -\frac{1}{5}(-\cos u) + C$$

$$= \frac{1}{5}\cos\frac{5}{x} + C$$

Therefore the value of the given integral is,

$$\frac{1}{5}\cos\frac{5}{x} + C$$

38)

Given integral,

$$\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}\,dx$$

Let,

$$u = \sqrt{x}$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$\therefore 2\,du = \frac{1}{\sqrt{x}}\,dx$$

Therefore,

$$\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx$$

$$= \int \sec^2 u \cdot 2\,du$$

$$= 2\int \sec^2 u \, du$$

$$= 2 \tan u + c$$

$$= 2 \tan \sqrt{x} + c$$

Therefore the value of the given integral is,

$$2 \tan \sqrt{x} + c$$

39/

Given integral,

$$\int \cos^4 3t \sin 3t \, dt$$

Let,

$$u = \cos 3t$$

$$\Rightarrow \frac{du}{dx} = -\sin 3t \cdot 3$$

$$\Rightarrow \frac{du}{dx} = -3 \sin 3t$$

$$\therefore -\frac{1}{3} du = \sin 3t \, dx$$

Therefore,

$$\int \cos^4 3t \sin 3t \, dt$$

$$= \int u^4 \cdot (-\tfrac{1}{3}) du$$

$$= -\frac{1}{3} \int u^4 \, du$$

$$= -\frac{1}{3} \cdot \frac{u^5}{5} + c$$

$$= -\frac{1}{3} \cdot \frac{\cos^5(3t)}{5} + c$$

Ans

40|

Given integral,

$$\int \cos 2x \sin^5 2x \, dx$$

Let,

$u = \sin 2x$

$\Rightarrow \frac{du}{dx} = \cos 2x \cdot 2$

$\Rightarrow \frac{1}{2} du = \cos 2x \, dx$

Therefore,

$$\int \cos 2x \sin^5 2x \, dx$$

$$= \int u^5 \cdot \frac{1}{2} \cdot du$$

$$= \frac{1}{2} \int u^5 \, du$$

$$= \frac{1}{2} \cdot \frac{u^6}{6} + c$$

$$= \frac{1}{12} \cdot \sin^6 (2x) + c$$

Therefore, the value of the given integral is,

$$\frac{1}{12} \cdot \sin^6 (2x) + c.$$

41|

Given integral,

$$\int x \sec^2(x^2)\, dx$$

Let,
$$u = x^2$$

$$\Rightarrow \frac{du}{dx} = 2x$$

$$\therefore \frac{1}{2} du = x\, dx$$

Therefore

$$\int x \sec^2(x^2)\, dx$$

$$= \int \sec^2 u \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \int \sec^2 u\, du$$

$$= \frac{1}{2} \tan u + c$$

$$= \frac{1}{2} \tan x^2 + c$$

Therefore the value of the given integral is,

$$\frac{1}{2} \tan x^2 + c$$

42|

Given integral,

$$\int \frac{\cos 4\theta}{(1+2\sin 4\theta)^4}\, d\theta$$

Let,
$$u = 1 + 2\sin 4\theta$$

$$\Rightarrow \frac{du}{d\theta} = 2\cos 4\theta \cdot 4 = 8\cos 4\theta$$

$\therefore \frac{1}{8} du = \cos 4\theta \, d\theta$

Therefore,

$$\int \frac{\cos 4\theta}{(1+2\sin 4\theta)^4} d\theta$$

$$= \int \frac{1}{u^4} \cdot \frac{1}{8} \cdot du$$

$$= \frac{1}{8} \int u^{-4} du$$

$$= \frac{1}{8} \cdot \frac{u^{-3}}{-3} + c$$

$$= -\frac{1}{24} \cdot (1+2\sin 4\theta)^{-3} + c$$

Therefore, the value of the given integral is,

$$-\frac{1}{24} \cdot (1+2\sin 4\theta)^{-3} + c$$

43|

Given integral,

$$\int \cos 4\theta \sqrt{2-\sin 4\theta} \, d\theta$$

Let, $u = 2 - \sin 4\theta$

$\Rightarrow \frac{du}{d\theta} = -\cos 4\theta \cdot 4$

$\Rightarrow \frac{du}{d\theta} = -4\cos 4\theta$

$\therefore -\frac{1}{4} du = \cos 4\theta \, d\theta$

Therefore,

$$\int \cos 4\theta \sqrt{2-\sin 4\theta}\, d\theta$$

$$= \int \sqrt{u} \cdot \left(-\frac{1}{4}\right) du$$

$$= -\frac{1}{4} \int u^{\frac{1}{2}}\, du$$

$$= -\frac{1}{4} \cdot \frac{u^{3/2}}{3/2} + c$$

$$= -\frac{1}{4} \cdot \frac{2}{3} \cdot u^{3/2} + c$$

$$= -\frac{1}{6} (2-\sin 4\theta)^{3/2} + c$$

Therefore the value of the given integral is,

$$-\frac{1}{6} (2-\sin 4\theta)^{3/2} + c$$

44)

Given integral,

$$\int \tan^3 5x \sec^2 5x \, dx$$

Let,

$$u = \tan 5x$$

$$\Rightarrow \frac{du}{dx} = \sec^2 5x \cdot 5$$

$$\therefore \frac{1}{5} du = \sec^2 5x \, dx$$

Therefore,

$$\int \tan^3 5x \sec^2 5x \, dx$$

$$= \int u^3 \cdot \frac{1}{5} du$$

$$= \frac{1}{5} \int u^3 du$$

$$= \frac{1}{5} \cdot \frac{u^4}{4} + c$$

$$= \frac{1}{20} \tan^4 5x + C$$

Therefore the value of the given integeral,

$$\frac{1}{20} \tan^4 5x + C$$

451

Given integral,

$$\int \frac{\sec^2 x \, dx}{\sqrt{1-\tan^2 x}}$$

Let, $u = \tan x$

$$\Rightarrow \frac{du}{dx} = \sec^2 x$$

$$\therefore du = \sec^2 x \, dx$$

Therefore, $\int \frac{\sec^2 x \, dx}{\sqrt{1-\tan^2 x}}$

$$= \int \frac{1}{\sqrt{1-u^2}} du$$

$= \sin^{-1} u + c$

$= \sin^{-1}(\tan x) + c$

Therefore the value of the given integral is,

$\sin^{-1}(\tan x) + c.$

46)

Given integral,

$$\int \frac{\sin\theta}{\cos^2\theta + 1} d\theta$$

Let, $u = \cos\theta$

$\Rightarrow \frac{du}{d\theta} = -\sin\theta$

$\therefore -du = \sin\theta \, d\theta$

Therefore, $\int \frac{\sin\theta}{\cos^2\theta + 1} d\theta$

$= \int \frac{1}{u^2+1} (-du)$

$= -\int \frac{1}{u^2+1} du$

$= -\tan^{-1} u + c$

$= -\tan^{-1}(\cos\theta) + c$

Ans

47)

Given integral,

$$\int \sec^3 2x \tan 2x \, dx$$

Let, $u = \sec 2x$

$\Rightarrow \frac{du}{dx} = \sec 2x \tan 2x \cdot 2$

$\therefore \frac{1}{2} du = \sec 2x \tan 2x \, dx$

Therefore,

$$\int \sec^3 2x \tan 2x \, dx$$

$$= \int u^2 \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \int u^2 du$$

$$= \frac{1}{2} \cdot \frac{u^3}{3} + c$$

$$= \frac{1}{6} \cdot \sec^3 2x + c$$

Therefore the value of the given integral,

$$\frac{1}{6} \cdot \sec^3 2x + c.$$

48/

Given Integral,

$$\int [\sin(\sin\theta)] \cos\theta \, d\theta$$

Let,
$$u = \sin\theta$$

$$\Rightarrow \frac{du}{d\theta} = \cos\theta$$

$$\therefore du = \cos\theta \, d\theta$$

Therefore,
$$\int [\sin(\sin\theta)] \cos\theta \, d\theta$$

$$= \int \sin u \, du$$

$$= -\cos u + c$$

$$= -\cos(\sin\theta) + c$$

Therefore the value of the given integral is, $-\cos(\sin\theta) + c$

49/

Given integral,
$$\int \frac{dx}{e^x}$$

$$= \int e^{-x} dx$$

Let, $u = -x$

$$\Rightarrow \frac{du}{dx} = -1$$

$\therefore dx = -du$

Therefore,
$$\int e^{-x}\, dx$$

$$= \int e^{u} (-du)$$

$$= -\int e^{u}\, du$$

$$= -e^{u} + c$$

$$= -e^{-x} + c$$

Therefore the value of the given integral is,

$$-e^{-x} + c.$$

50|

Given integral,
$$\int \sqrt{e^{x}}\, dx$$

$$= \int e^{x/2}\, dx$$

Let,
$$u = \frac{x}{2}$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2}$$

$$\therefore 2du = dx$$

Therefore,
$$\int e^{x/2}\,dx$$
$$= \int e^u \cdot 2\,du$$
$$= 2\int e^u\,du$$
$$= 2e^u + c$$
$$= 2e^{x/2} + c$$

Therefore the value of the given integral is,
$$2e^{x/2} + c.$$

51)

Given integral,
$$\int \frac{dx}{\sqrt{x}\,e^{(2\sqrt{x})}}$$

Let, $u = 2\sqrt{x}$

$$\Rightarrow \frac{du}{dx} = \frac{1}{\sqrt{x}}$$
$$\therefore du = \frac{1}{\sqrt{x}}\,dx$$

Therefore, $$\int \frac{dx}{\sqrt{x}\,e^{2\sqrt{x}}}$$
$$= \int \frac{1}{e^u}\cdot du$$

$$= -e^{-u} + c$$

$$= -e^{-2\sqrt{x}} + c$$

Therefore, the value of the given integral,

$$-e^{-2\sqrt{x}} + c.$$

52)

Given integral,

$$\int \frac{e^{\sqrt{2y+1}}}{\sqrt{2y+1}} dy$$

Let, $u = \sqrt{2y+1}$

$$\Rightarrow \frac{du}{d\cancel{x}} = \frac{1}{2\sqrt{2y+1}} \cdot 2$$

$$\therefore du = \frac{1}{\sqrt{2y+1}} dy$$

Therefore, $\int \frac{e^{\sqrt{2y+1}}}{\sqrt{2y+1}} dy$

$$= \int e^{u} du$$

$$= e^{u} + c$$

$$= e^{\sqrt{2y+1}} + c$$

Ans.

53)

Given integral,

$$\int \frac{y}{\sqrt{2y+1}}\, dy$$

Let,

$u = 2y+1$ again,

$u = 2y+1$

$\Rightarrow \frac{du}{dy} = 2$

$y = \frac{1}{2}(u-1)$

$\therefore \frac{1}{2} du = dy$

Therefore,

$$\int \frac{y}{\sqrt{2y+1}}\, dy$$

$$= \int \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\, dy$$

$$= \frac{1}{2}\int (u-1)\, u^{-\frac{1}{2}}\, \cancel{dy}\, \frac{1}{2} du$$

$$= \frac{1}{4}\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du$$

$$= \frac{1}{4}\left(\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right) + c$$

$$= \frac{1}{6} u^{3/2} - \frac{1}{2} u^{1/2} + c$$

$$= \frac{1}{6}(2y+1)^{3/2} - \frac{1}{2}(2y+1)^{1/2} + c$$

Ans.

54)

Given integral,

$$\int x\sqrt{4-x}\,dx$$

Let,

$u = 4-x$ again, $u = 4-x$

$\Rightarrow \frac{du}{dx} = -1$ $x = 4-u$

$\therefore dx = -du$

Therefore,

$$\int x\sqrt{4-x}\,dx$$

$$= \int (4-u)\sqrt{u}\,(-du)$$

$$= -\int (4 \cdot u^{1/2} - u^{3/2})\,du$$

$$= -4 \cdot \frac{u^{3/2}}{3/2} + \frac{u^{5/2}}{5/2} + c$$

$$= -\frac{8}{3}u^{3/2} + \frac{2}{5}u^{5/2} + c$$

$$= -\frac{8}{3}(4-x)^{3/2} + \frac{2}{5}(4-x)^{5/2} + c$$

Therefore, the value of the given integral,

$$-\frac{8}{3}(4-x)^{3/2} + \frac{2}{5}(4-x)^{5/2} + c$$

55|

Given integral,

$$\int \sin^3 2\theta \, d\theta$$

$$= \int \sin^2 2\theta \sin 2\theta \, d\theta$$

$$= \int (1-\cos^2 2\theta) \sin 2\theta \, d\theta$$

Let,

$$u = \cos 2\theta$$

$$\Rightarrow \frac{du}{d\theta} = -\sin 2\theta \cdot 2$$

$$\Rightarrow \frac{du}{d\theta} = -2\sin 2\theta$$

$$\therefore -\frac{1}{2} du = \sin 2\theta \, d\theta$$

Therefore,

$$\int (1-\cos^2 2\theta) \sin 2\theta \, d\theta$$

$$= \int (1-u^2) \cdot (-\tfrac{1}{2}) du$$

$$= -\frac{1}{2} \int (1-u^2) \, du$$

$$= -\frac{1}{2}\left(u - \frac{u^3}{3}\right) + C$$

$$= -\frac{1}{2}\cos 2\theta + \frac{1}{6}\cos^3 2\theta + C$$

Ans

57/

Given integral,

$$\int \frac{t+1}{t} \, dt$$

$$= \int \left(\frac{t}{t} + \frac{1}{t} \right) dt$$

$$= \int \left(1 + \frac{1}{t} \right) dt$$

$$= \int 1 \, dt + \int \frac{1}{t} \, dt$$

$$= t + \ln t + c$$

Therefore the value of the given integral is,

$$t + \ln t + c.$$

58/

Given integral,

$$\int e^{2\ln x} \, dx$$

$$= \int e^{\ln x^2} \, dx$$

$$= \int x^2 \, dx$$

$$= \frac{x^3}{3} \cancel{dx} + c$$

$$= \frac{1}{3} x^3 + c$$

Therefore the value of the given integral,

$$\frac{1}{3} x^3 + c.$$

59)

Given integral,

$$\int [\ln (e^x) + \ln (e^{-x})] dx$$

Here,

$$\ln e^x + \ln e^{-x} = \ln (e^x e^{-x})$$
$$= \ln 1$$
$$= 0$$

$$\therefore \int (\ln e^x + \ln e^{-x}) dx = c$$

Therefore, the value of the given integral, is c.

60)

Given integral,

$$\int \cot x \, dx$$
$$= \int \frac{\cos x}{\sin x} dx$$

Let, $u = \sin x$

$$\Rightarrow \frac{du}{dx} = \cos x$$

$$\therefore du = \cos x \, dx$$

Therefore,

$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{u} du$$
$$= \ln u + c$$
$$= \ln (\sin x) + c$$

Therefore, the value of the given integral,

$\ln (\sin x) + c$.

6)

a) Given integral,

$$\int \frac{dx}{\sqrt{9-x^2}}$$

$$= \int \frac{dx}{\sqrt{3^2-x^2}}$$

$$= \sin^{-1}\frac{x}{3} + c$$

Therefore the value of the given integral is, $\sin^{-1}\frac{x}{3} + c$

b)
Given integral,

$$\int \frac{dx}{5+x^2}$$

$$= \int \frac{dx}{(\sqrt{5})^2 + x^2}$$

$$= \frac{1}{\sqrt{5}} \tan^{-1}\frac{x}{\sqrt{5}} + c$$

Therefore the value of the given integral is,

$$\frac{1}{\sqrt{5}} \tan^{-1}\frac{x}{\sqrt{5}} + c.$$

e)

Given integral,

$$\int \frac{dx}{x\sqrt{x^2-\pi}}$$

$$= \int \frac{dx}{x\sqrt{x^2-(\sqrt{\pi})^2}}$$

$$= \frac{1}{\sqrt{\pi}} \sec^{-1} \frac{x}{\sqrt{\pi}} + c$$

Therefore the value of the given integral is,

$$\frac{1}{\sqrt{\pi}} \sec^{-1} \frac{x}{\sqrt{\pi}} + c$$

62)

a) Given integral,

$$\int \frac{e^x}{4+e^{2x}} dx$$

Let,

$$u = e^x$$

$$\frac{du}{dx} = e^x$$

$$\therefore du = e^x dx$$

Therefore $\int \frac{e^x}{4+e^{2x}} dx$

$$= \int \frac{du}{2^2+u^2}$$

$$= \frac{1}{2} \tan^{-1} \frac{u}{2} + c$$

$$= \frac{1}{2} \tan^{-1} \frac{e^x}{2} + c$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \tan^{-1} \frac{e^x}{2} + c$$

b)

Given integral;

$$\int \frac{dx}{\sqrt{9-4x^2}}$$

$$= \int \frac{dx}{2\sqrt{\frac{9}{4}-x^2}}$$

$$= \frac{1}{2} \int \frac{dx}{\sqrt{\left(\frac{3}{2}\right)^2 - x^2}}$$

$$= \frac{1}{2} \sin^{-1} \frac{x}{3/2} + c$$

$$= \frac{1}{2} \sin^{-1} \frac{2x}{3} + c$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \sin^{-1} \frac{2x}{3} + c.$$