CSE 373 / L-18 / 28.04.2024 /

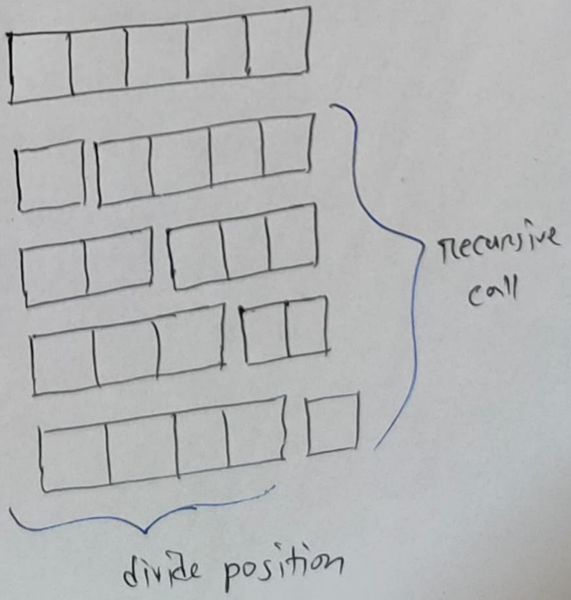
recursive Solution

(*)

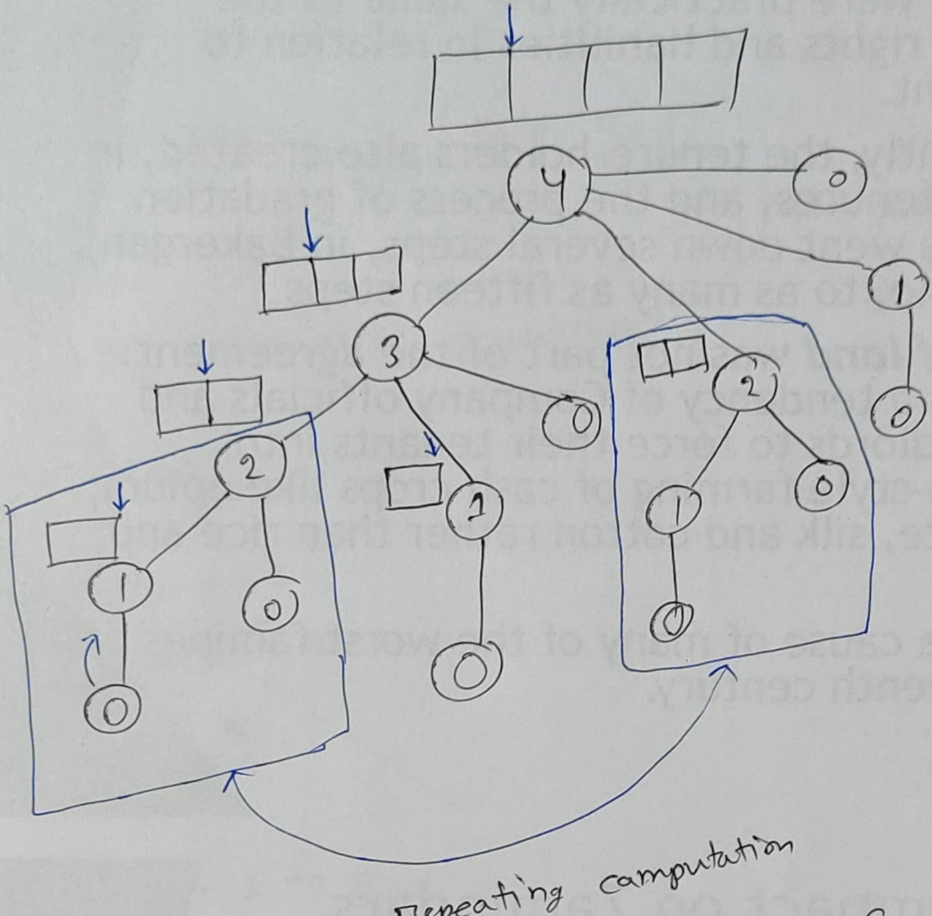
$$r_n = \max_{1 \le i \le n} \left(p_i + \underset{\hookrightarrow \text{ new sub problem}}{r_{n-i}}\right)$$

(*) Top-Down recursive procedure:

```
CUT-ROD (p, n)

    if n == 0
        return 0

    q = -∞

    for i = 1 to n
        q = max(q, p[i] + CUT-ROD(p, n-i))

    return q
```



Cut-Rod (P, 4)



$T(n) = 2^n$, $n > 1$; for repeating comp.

✻ For getting optimal solution, solution of sub-problem need to be optimal.

$$T(n) = 2^n$$

we need to call recursive function 2^n time.

we need to push the call $(n+1)$ time in stack in max

So, if we can memorize this $(n+1)$ solution we can reduce the re-computation time.

⊛ ROD Cutting Problem - Memoization

```
MEMOIZED-CUT-ROD (p,n)

    Let r[0..n] be a new array
    for i= 1 to n
        r[i]= -∞
    return MEMOZZED-CUT-ROD-AUX (p,n,r)

MEMOZZED-CUT-ROD-AUX (p,n,r)

    if r[n] ≥ = 0
        return r[n]
    if n == 0
        q = 0
    else q = -∞
        for i = 1 to n
            q = max (q, p[i] + MEMOZZED-CUT-ROD-AUX (p,n-i,r))
    r[n] = q
    return r[n]
```

⊛ Rod Cutting Problem- Bottom-up

```
BOTTOM-UP-CUT-ROD(p, n)

    Let r[0...n] be a new array

    r[0] = 0

    for j = 1 to n
        q = -∞
        for i = 1 to j
            q = max(q, p[i] + r[j-i])

        r[j] = q

    return r[n]
```

$J = 2$

$r[2] = \max(q, p[1] + r[2-1])$

$(q, p[2] + r[2-2])$

$J = 3$

$r[3] = \max(q, p[1] + r[3-1])$

$(q, p[2] + r[3-2])$

$(q, p[3] + \underbrace{r[3-3]}_{\text{already calculated}})$

⊛ Time Complexity for both top-down and down-top are $\Theta(n^2)$

⊛ Extended version with solutions

```
EXTENDED-BOTTOM-UP-CUT-ROD (p,n)

Let, r[0..n] and s[0..n] be a new array
r[0] = 0
for j = 1 to n
    q = -∝
    for i = 1 to j
        if q < p[i] + r[j-1]          )
            q = p[i] + r[j-1]         ) Difference
            s[j] = i                  )

    r[j] = q
return r and s.
```

```
PRINT-CUT-ROD-SOLUTION (p,n)

(r,s) = ~~EX~~ EXTENDED-BOTTOM-UP-CUT-ROD (p,n)
while n > 0
    print s[n]
    n = n - s[n]
```

⊛ Question Pattern:

Given rod length is 9. Find out the cut position from the table:

Look at 9 ⇒ 3

so, first cut 3 then look for the next 6.

↳ (uncut)

6 (uncut)

L-19/ 30.04.2024 /

Chapter- 15

Matrix Chain Multiplication

$A = m \times n$

$B = n \times p$

$A \cdot B = m \times p$

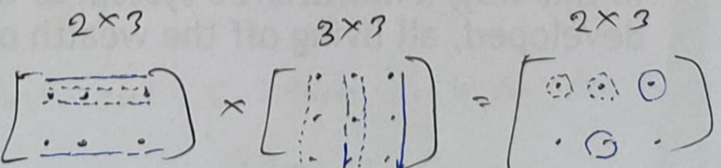
total multiplication

$= m \times p \times n$

$=$ A. rows × A. columns × B. columns

MATRIX-MULTIPLY (A, B)

```
if A. colums ≠ B. rows
    error "incompatible dimensions"
else
    let C be a new A. rows × B. colums matrix
    for i = 1 to A. rows
        for j = 1 to B. columns
            $c_{ij} = 0$
            for k = 1 to A. columns
                $c_{ij} = c_{ij} + a_{ik} \cdot b_{kj}$
    return C
```



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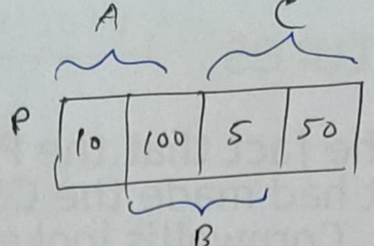
Main Problem

$A = 10 \times 100$

$B = 100 \times 5$

$C = 5 \times 50$



$A_i = P_{i-1} \times P_i$

$A_1 = P_0 \times P_1$

$A_2 = P_1 \times P_2$

$A_3 = P_2 \times P_3$

⊛ $(AB)C$,

$\underbrace{AB}_{}\ \xrightarrow{} 10 \times 5$

$AB = 10 \times 100 \times 5 = 5{,}000$

$(AB)C = 10 \times 5 \times 50 = 2{,}500$

$\Big\}\ 7{,}500$

⊛ $A(BC)$

$BC \rightarrow 100 \times 50$

$BC = 100 \times 5 \times 50 = 25{,}000$

$A(BC) = 10 \times 100 \times 50 = 50{,}000$

$\Big\}\ 75{,}000$

too many multiplication need

⊛ We need to split between the set of matrix, so ~~that~~

new sub-problem will be

A_k and A_{k+1} → $\underbrace{A_{k+1 \cdots n}}$ need to split again

$A_k \rightarrow \underbrace{A_{1 \cdots k}}$ need to split again

where $1 \le k < n$ → we can't split after the last matrix. (lol)

m[i,j] new matrix array

$A_1\ A_2\ A_3$

$n = 3$

$m[n,n]$

	1	2	3
1	A_1A_1 0	A_1A_2	A_1A_3
2	A_2A_1	A_2A_2 0	A_2A_3
3	A_3A_1	A_3A_2	A_3A_3 0

→ we need this one

Repeating. We need to ignore them

→ Store the minimum multiplica count

$$m[i,j] = \underbrace{m[i,k]}_{\text{Subset Solution already known}} + \underbrace{m[k+1,j]}_{\text{Sub-set solution}} + \underbrace{P_{i-1}\, P_k\, P_j}_{\text{number of multiplication of } m[i,k] \text{ and } m[k+1,j]}$$

$1 \leq k < j$

	1	2	3	4	
1	0	✓	✓	$\boxed{✓}$	1
2	✗	0	✓	✓	2
3	✗	✗	0	✓	3
4	✗	✗	✗	0	4

→ we need this value

L-20 / 05. 05. 2024 /

Midterm Exam

L-21/07.05.2024/

(*) Algorithm for compute optimal cost

```
MATRIX-CHAIN-ORDER (P)
    n = p.length - 1
    let m[1...n, 1....n] and s[1....n-1, 2...n] be new tables
    for i = 1 to n
        m[i, i] = 0
    for l = 2 to n
        for i = 1 to n - l + 1
            j = i + l - 1
            m[i, j] = $\alpha$
            for k = i to j - 1
                q = m[i, k] + m[k+1, j] + $P_{i-1} P_k P_j$
                if q < m[i, j]
                    m[i, j] = q
                    s[i, j] = k
    return m and s.
```

$T(n) = O(n^3)$

Space = $O(n^2)$

❋ Algorithm to print Optimal Parenthesis

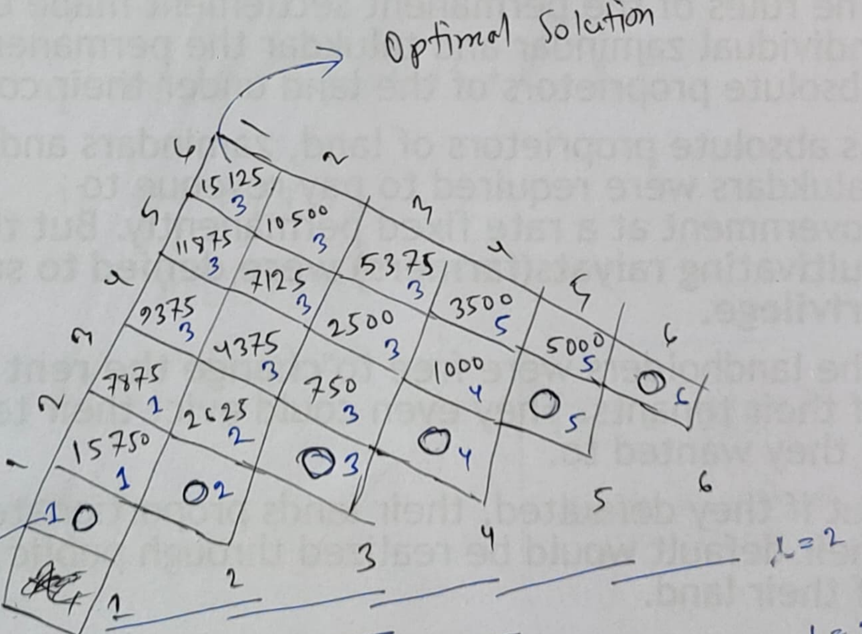
```
PRINT-OPTIMAL-PARENS (s, i, j)
    if  i == j
        print "A";
    else print "("
        PRINT-OPTIMAL-PARENS (s, i, s[i, j])
        PRINT-OPTIMAL-PARENS (s, s[i,j]+1, j)
        print ")"
```

❋ Example:

	0	1	2	3	4	5	6
P	30	35	15	5	10	20	25

Black ⇒ Data of m

BLUE ⇒ Data of S



initial inden or matrin

~~no~~ now best on

$$\underbrace{A_1\ A_2\ A_3}\ \underbrace{A_4\ A_5\ A_6}$$

$$\underbrace{A_1}\quad \underbrace{A_2 A_3}\quad \underbrace{A_4 A_5}\quad \underbrace{A_6}$$

35×15×5 = 2625 [35×5]

5×10×20 = 1000 [5×20]

30×35×5 = 5250 [30×5]

5×20×25 = 2500 [5×25]

30×5×25 = 3750 [30×25]

~~Total time~~ =

Total multiplication

= 2625 + 1000 + 5250 + 2500 + 3750

= 15,125

Optimal solutions

(*) Some breakdown:

$m[i,j] = m[i,k] + m[k+1,j] + P_{i-1} P_k P_j$

$n = 6$

$2 \le l \le 6$

$1 \le i \le n-l+1 = 5$

$j = i+l-1 = 2$

Lets,

$l = 2 \Rightarrow 1 \le k < 2$ $m[1,2]$

$i = 1$

when, $k = 1$

$m[1,1] + m[2,2] + P_0\ P_1\ P_2$

0 0 30 35 15 = 15,750

Let,

$$\left.\begin{array}{l} l=2 \\ i=2 \end{array}\right\} \quad \cancel{2 \le k <} \quad i \le k < i+l-1 = j$$

$$2 \le k < 3 \Rightarrow m[2,3]$$

when, $k=2$

$$m[2,2] + m[3,3] + P_1 \; P_2 \; P_3$$

$$0 \qquad 0 \qquad 35 \; 15 \; 5 = 2625$$

$- - - -$

$- -$

$\cdot \; -$

until, $l=5$

Let,

$$\left.\begin{array}{l} l=3 \\ i=1 \end{array}\right\} \quad 1 \le k < 3 \Rightarrow m[1,3]$$

when, $k = \boxed{1}$ ✓

$$m[1,1] + m[2,3] + P_0 \; P_1 \; P_3$$

$$0 \qquad 2625 \qquad 30 \; 35 \; 5 = \boxed{7875}$$ ✓

when, $k=2$

$$m[1,2] + m[3,3] + P_0 \; P_2 \; P_3$$

$$15750 \qquad 0 \qquad 30 \; 15 \; 5 = 18{,}000$$

Let,

$$\left.\begin{array}{l} l=3 \\ i=2 \end{array}\right\} \quad 2 \le k < 4 \Rightarrow m[2,4]$$

when, $k=2$

$$m[2,2] + m[3,4] + P_1 \; P_2 \; P_4$$

$$0 \qquad 750 \qquad 35 \; 15 \; 10 = 6000$$

when, $\boxed{k=3}$ ✓

$m[2,3] + m[4,4] + P_1 \; P_3 \; P_4$

$2625 \quad 0 \quad 35 \; 5 \; 10 = \boxed{4375}$ ✓

- - - until $i = 4$

Let,

$\left.\begin{array}{l} l = 4 \\ i = 1 \end{array}\right\} \; 1 \leq k < 4 \Rightarrow m[1,4]$

max 3

when, $k = 1$

$m[1,1] + m[2,4] + P_0 \; P_1 \; P_4$

$0 \quad 4375 \quad 30 \; 35 \; 10 = 14875$

$k = 2$

$m[1,2] + m[3,4] + P_0 \; P_2 \; P_4$

$15750 \quad 750 \quad 30 \; 15 \; 10 = 21{,}000$

$\boxed{k=3}$ ✓

$m[1,3] + m[4,4] + P_0 \; P_3 \; P_4$

$7875 \quad 0 \quad 30 \; 5 \; 10 = \boxed{9375}$ ✓

Let,

$\left.\begin{array}{l} l = 5 \\ i = 2 \end{array}\right\} \; 2 \leq k < 6 \Rightarrow m[2,6]$

max 2

when, $k = 2$

$m[2,2] + m[3,6] + P_1 \; P_2 \; P_6$

$0 \quad 5375 \quad 35 \; 15 \; 25 = 18500$

$\boxed{k=3}$

$m[2,3] + m[4,6] + P_1 \; P_3 \; P_6$

2625 3500 35 5 25 = $\boxed{10,500}$

k = 4

$m[2,4] + m[5,6] + P_1 \; P_4 \; P_6$

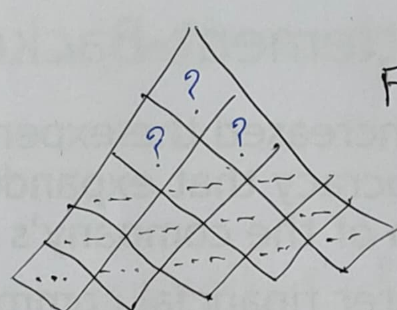
4375 5000 35 10 25 = 18125

k = 5

$m[2,5] + m[6,6] + P_1 \; P_5 \; P_6$

7125 0 35 20 25 = 24625

*** Question Pattern:



Find out the optimal solution on number of multiplication need.

***

Quiz-3
Next Week