CSE 373/L-7/27.02.2024/

Substitution Method

- Guess the form of the solution
- Use mathematical induction to find constants and show that the solution works.

$\Rightarrow T(n) = 2T(\lfloor n/2 \rfloor) + n$

$\Rightarrow$ Guess : $T(n) = O(n \lg n)$

Now, requires to prove that, $T(n) \leq cn \lg n \;;\; c > 0$

Lets,

$m < n \;;\; m = \lfloor n/2 \rfloor$

$\Rightarrow T(\lfloor n/2 \rfloor) \leq c\lfloor n/2 \rfloor \lg \lfloor n/2 \rfloor$

$\Rightarrow T(n) \leq 2(c\lfloor n/2 \rfloor \log \lfloor n/2 \rfloor) + n$

$\leq cn \lg(n/2) + n = cn \lg n - cn \lg 2 + n$

$= cn \lg n - cn + n$

$= cn \lg n \;;\; c \geq 1$

$\therefore T(n) \leq cn \lg n$ (Proved)

⊛ Master Method

Case-1:

If $f(n) = O(n^{\log_b a - \epsilon})$; $\epsilon > 0$

Then, $T(n) = \theta(n^{\log_b a})$

Case-2:

If $f(n) = \theta(n^{\log_b a})$

Then, $T(n) = \theta(n^{\log_b a} \log_2 n)$

Case-3:

If $f(n) = \Omega(n^{\log_b a + \epsilon})$; $\epsilon > 0$

Then, $a \cdot f(n/b) \leq c \cdot f(n)$; $c < 1$

Then, $T(n) = \theta(f(n))$

$\boxed{\text{Slide - 28-30}}$

$\boxed{\text{H.W.} \Rightarrow 4.4-1}$