NORTH SOUTH UNIVERSITY

Department of Mathematics & Physics

Assignment – 02

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1)

Given, $f(x) = x^2 \; ; \; -\pi \leq x \leq \pi$

Here,

$f(x)$ is an even function and symmetric with respect to y-axis.

$$\therefore b_n = 0$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

Here,

$$\therefore a_0 = \frac{2}{\pi} \int_0^{\pi} x^2 \, dx$$

$$= \frac{2}{\pi} \left[\frac{x^3}{3} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \cdot \frac{\pi^3}{3}$$

$$= \frac{2}{3} \pi^2$$

$$\therefore a_n = \frac{2}{\pi} \int_0^{\pi} x^2 \cos nx \, dx$$

$$= \frac{2}{\pi} \left[\frac{x^2}{n} \sin nx + \frac{2x}{n^2} \cos nx - \frac{2}{n^3} \sin nx \right]_0^{\pi}$$

$$= \frac{2}{\pi} \cdot \frac{2\pi}{n^2} (-1)^n$$

$$= \frac{4}{n^2} (-1)^n$$

x^2	$\cos nx$
$2x$	$\frac{\sin nx}{n}$
2	$\frac{-\cos nx}{n^2}$
0	$\frac{-\sin nx}{n^3}$

$$\therefore f(x) = \frac{2}{3}(\pi)^2 + \sum_{n=1}^{\infty} \frac{4}{n^2}(-1)^n \cos nx$$

$$= \frac{2}{3}\pi^2 + \left[-4\cos x + \cos 2x - \frac{4}{9}\cos 3x + \cdots\right]$$

Ans

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Given,

$$f(x) = x \quad ; \quad 0 \le x \le 2$$

Here,

$f(x) = x$ is an odd function.

$$\therefore a_0 = a_n = 0$$

$$\therefore f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right)$$

x		$\sin\frac{n\pi x}{2}$
	$+$	
1		$\dfrac{-\cos\frac{n\pi x}{2}}{\frac{n\pi}{2}}$
	$-$	
0		$\dfrac{-\sin\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)^2}$

Here,

$$b_n = \frac{2}{2}\int_0^2 x \sin\left(\frac{n\pi x}{2}\right) dx$$

$$= \left[-\frac{2x}{n\pi}\cos\frac{n\pi x}{2} + \left(\frac{2}{n\pi}\right)^2 \sin\frac{n\pi x}{2}\right]_0^2$$

$$= -\frac{4}{n\pi}(-1)^n$$

$$\therefore f(x) = \sum_{n=1}^{\infty} -\frac{4}{n\pi}(-1)^n \sin\left(\frac{n\pi x}{2}\right)$$

$$= \frac{4}{\pi}\sin\frac{\pi x}{2} - \frac{2}{\pi}\sin \pi x + \frac{4}{3\pi}\sin\frac{3\pi x}{2} - \cdots$$

Ans

3)

Given,

$$f(x) = |x| \quad ; \; -2 \leq x \leq 2$$

Here, $f(x)$ is an even function.

$$\therefore b_n = 0$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2}\right)$$

Here,

$$a_0 = \frac{2}{2}\int_0^2 x \, dx = \left[\frac{x^2}{2}\right]_0^2 = 2$$

$$\therefore a_n = \frac{2}{2}\int_0^2 x \cos\left(\frac{n\pi x}{2}\right) dx$$

$$= \left[\frac{2x}{n\pi}\sin\frac{n\pi x}{2} + \left(\frac{2}{n\pi}\right)^2 \cos\left(\frac{n\pi x}{2}\right)\right]_0^2$$

$$= \left(\frac{2}{n\pi}\right)^2 (-1)^n - \left(\frac{2}{n\pi}\right)^2$$

$$= \left(\frac{2}{n\pi}\right)^2 \left\{(-1)^n - 1\right\}$$

x	$\cos\frac{n\pi x}{2}$
$+$	
1	$\dfrac{\sin\frac{n\pi x}{2}}{\frac{n\pi}{2}}$
$-$	
0	$\dfrac{-\cos\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)^2}$

$$\therefore f(x) = \frac{2}{2} + \sum_{n=1}^{\infty} \left(\frac{2}{n\pi}\right)^2 \left\{(-1)^n - 1\right\} \cos\left(\frac{n\pi x}{2}\right)$$

$$= 1 + \left[-\frac{8}{\pi^2}\cos\frac{\pi x}{2} + 0 - \frac{8}{9\pi^2}\cos\frac{3\pi x}{2} + 0 - \cdots\right]$$

Ans

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Given,

$$f(x) = \begin{cases} 0 & ; -\pi \leq x < 0 \\ h & ; 0 \leq x \leq \pi \end{cases}$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

Here,

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx \right]$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 \cdot dx + \int_{0}^{\pi} h\, dx \right]$$

$$= \frac{1}{\pi} \left[hx \right]_0^{\pi}$$

$$= \frac{1}{\pi} \cdot h\pi$$

$$= h$$

$$\therefore a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} f(x) \cos nx\, dx + \int_{0}^{\pi} f(x) \cos nx\, dx \right]$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 \cdot \cos nx\, dx + \int_{0}^{\pi} h \cdot \cos nx\, dx \right]$$

$$= \frac{1}{\pi} \left[0 + \frac{h}{n} \left[\sin nx\right]_0^{\pi} \right] = 0$$

$$\therefore b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx\,dx + \int_{0}^{\pi} f(x)\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\cdot\sin nx\,dx + \int_{0}^{\pi} h\cdot\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[0 + \frac{h}{n}\left[-\cos nx\right]_{0}^{\pi}\right]$$

$$= -\frac{h}{n\pi}\left[(-1)^n - 1\right]$$

Therefore,

$$f(x) = \frac{h}{2} + \sum_{n=1}^{\infty} -\frac{h}{n\pi}\left\{(-1)^n - 1\right\}\sin nx$$

$$= h\left[\frac{1}{2} + \sum_{n=1}^{\infty} -\frac{(-1)^n - 1}{n\pi}\sin nx\right]$$

$$= h\left[\frac{1}{2} + \frac{2}{\pi}\sin x + 0 + \frac{2}{3\pi}\sin 3x + 0 + \cdots\right]$$ Ans

[when, n= even,
function is zero]

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Given,

$$f(x) = \begin{cases} 0 & ; -\pi \le x \le 0 \\ \sin x & ; 0 \le x \le \pi \end{cases}$$

Let,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Here,

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 . dx + \int_{0}^{\pi} \sin x \, dx \right]$$

$$= \frac{1}{\pi} [-\cos x]_0^{\pi}$$

$$= -\frac{1}{\pi} (\cos \pi - 1)$$

$$= -\frac{1}{\pi} (-1-1)$$

$$= \frac{2}{\pi}$$

$$\therefore a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 . \cos nx \, dx + \int_{0}^{\pi} \sin x \cdot \cos nx \, dx \right]$$

$$= \frac{1}{2\pi} \left[\int_{0}^{\pi} 2 \sin x \cos nx \, dx \right]$$

$$= \frac{1}{2\pi} \int_0^{\pi} \{\sin(x+nx) + \sin(x-nx)\} dx$$

$$= \frac{1}{2\pi} \left[\frac{-\cos(x+nx)}{1+n} - \frac{\cos(x-nx)}{1-n} \right]_0^{\pi}$$

$$= \frac{1}{2\pi} \left[\frac{1-(-1)^{n+1}}{1+n} + \frac{1-(-1)^{1-n}}{1-n} \right]$$

$$\therefore b_n = \frac{1}{\pi} \left[\int_{-\pi}^{\pi} f(x) \sin nx \, dx \right]$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 \cdot \sin nx \, dx + \int_0^{\pi} \sin x \cdot \sin nx \, dx \right]$$

$$= \frac{1}{2\pi} \int_0^{\pi} 2 \sin x \cdot \sin nx \, dx$$

$$= \frac{1}{2\pi} \int_0^{\pi} \{\cos(x-nx) - \cos(x+nx)\} dx$$

$$= \frac{1}{2\pi} \left[\frac{\sin(x-nx)}{1-n} - \frac{\sin(x+nx)}{1+n} \right]_0^{\pi}$$

$$= 0$$

Therefore, $f(x) = \frac{2}{\pi \cdot 2} + \sum_{n=1}^{\infty} \frac{1}{2\pi} \left(\frac{1-(-1)^{n+1}}{1+n} + \frac{1-(-1)^{1-n}}{1-n} \right) \cos nx$

$$= \frac{1}{\pi} + 0 - \frac{2}{3\pi} \cos 2x + 0 - \frac{2}{15\pi} \cos 4x + 0 - \ldots$$

Ans

[When, n = odd, function is zero]

6|

Given,

$$f(x) = x + x^2 \quad ; \; -\pi \le x \le \pi$$

Let,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Here,

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} (x + x^2)\, dx$$

$$= \frac{1}{\pi} \left[\frac{x^2}{2} + \frac{x^3}{3} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left(\frac{\pi^2}{2} + \frac{\pi^3}{3} - \frac{\pi^2}{2} + \frac{\pi^3}{3} \right)$$

$$= \frac{1}{\pi} \cdot \frac{2\pi^3}{3}$$

$$= \frac{2\pi^2}{3}$$

$$\therefore a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (x + x^2) \cos nx \, dx$$

Here,

Diff		Int.
$x^2 + x$	$+$	$\cos nx$
$2x + 1$	$-$	$\frac{\sin nx}{n}$
2	$+$	$\frac{-\cos nx}{n^2}$
0		$\frac{-\sin nx}{n^3}$

$$\therefore a_n = \frac{1}{\pi}\left[\frac{x^2+x}{n} \sin nx + \frac{2x+1}{n^2} \cos nx - \frac{2}{n^3} \sin nx \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{2\pi+1}{n^2}(-1)^n - \frac{-2\pi+1}{n^2}(-1)^n \right]$$

$$= \frac{1}{\pi}\left[\frac{2\pi+1}{n^2}(-1)^n + \frac{2\pi-1}{n^2}(-1)^n \right]$$

$$= \frac{1}{\pi} (-1)^n \left(\frac{2\pi+1+2\pi-1}{n^2} \right)$$

$$= \frac{1}{\pi}(-1)^n \left(\frac{4\pi}{n^2} \right)$$

$$= \frac{4}{n^2}(-1)^n$$

$$\therefore b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x+x^2) \sin nx \, dx$$

Here,

	Diff		Int.
			$\sin nx$
	x^2+x	$+$	$\frac{-\cos nx}{n}$
	$2x+1$	$-$	$\frac{-\sin nx}{n^2}$
	2	$+$	$\frac{\cos nx}{n^3}$
	0		

$$\therefore b_n = \frac{1}{\pi}\left[-\frac{x^2+x}{n} \cos nx + \frac{2x+1}{n^2} \sin nx + \frac{2}{n^3} \cos nx \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{\pi^2+\pi}{n}(-1)^n + \frac{2}{n^3}(-1)^n + \frac{\pi^2-\pi}{n}(-1)^n - \frac{2}{n^3}(-1)^n\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi^2-\pi-\pi^2-\pi}{n}\right]$$

$$= \frac{1}{\pi} \cdot \frac{-2\pi}{n} = -\frac{2}{n}$$

Therefore,

$$f(x) = \frac{2\pi^2}{3 \cdot 2} + \sum_{n=1}^{\infty}\left(\frac{4(-1)^n}{n^2}\cos nx - \frac{2}{n}\sin nx\right)$$

$$= \frac{\pi^2}{3} + \left[-4\cos x + \cos 2x - \frac{4}{9}\cos 3x + \cdots \right.$$
$$\left. - 2\sin x - \sin 2x - \frac{2}{3}\sin 3x - \cdots\right]$$

Ans

7)

Given,

$$f(x) = \begin{cases} \frac{1}{4} - x & ; 0 \le x < \frac{1}{2} \\ x - \frac{3}{4} & ; \frac{1}{2} \le x \le 1 \end{cases}$$ find the half range sine series.

We know,

Half range sine series,

$$f(x) = \sum_{n=1}^{\infty}\left(b_n \sin \frac{n\pi x}{1}\right)$$

Here,

$$b_n = \frac{2}{1}\int_0^1 \{f(x)\cdot \sin(n\pi x)\}\,dx$$

$$= 2\left[\int_0^{1/2}\left(\frac{1}{4}-x\right)\sin(n\pi x)\,dx + \int_{1/2}^{1}\left(x-\frac{3}{4}\right)\sin(n\pi x)\,dx\right]$$

Here,

Diff		Int.
$-x+\frac{1}{4}$		$\sin(n\pi x)$
-1	$+$	$\frac{-\cos(n\pi x)}{n\pi}$
0	$-$	$\frac{-\sin(n\pi x)}{(n\pi)^2}$

Diff		Int
$x-\frac{3}{4}$		$\sin(n\pi x)$
1	$+$	$\frac{-\cos(n\pi x)}{n\pi}$
0	$-$	$\frac{-\sin(n\pi x)}{(n\pi)^2}$

$$\therefore b_n = 2\left\{\left[\frac{x-\frac{1}{4}}{n\pi}\cos(n\pi x) - \frac{1}{(n\pi)^2}\sin(n\pi x)\right]_0^{\frac{1}{2}} + \left[-\frac{x-\frac{3}{4}}{n\pi}\cos(n\pi x) + \frac{1}{(n\pi)^2}\sin(n\pi x)\right]_{\frac{1}{2}}^{1}\right\}$$

$$= 2\left\{\frac{\frac{1}{2}-\frac{1}{4}}{n\pi}\cos\left(\frac{n\pi}{2}\right) - \frac{1}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right) - \frac{0-\frac{1}{4}}{n\pi}\cdot 1 - \frac{1-\frac{3}{4}}{n\pi}(-1)^n + \frac{\frac{1}{2}-\frac{3}{4}}{n\pi}\cos\left(\frac{n\pi}{2}\right) - \frac{1}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)\right\}$$

$$= 2\left[\frac{1}{4n\pi}\cos\left(\frac{n\pi}{2}\right) - \frac{2}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{4n\pi} - \frac{1}{4n\pi}(-1)^n - \frac{1}{4n\pi}\cos\left(\frac{n\pi}{2}\right)\right]$$

$$= 2\left[\frac{1}{4n\pi}\left(1-(-1)^n\right) - \frac{2}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)\right]$$

[when, n = even, integral is zero]

Therefore

$$f(x) = \sum_{n=1}^{\infty} 2\left[\frac{1-(-1)^n}{4n\pi} - \frac{2}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)\right]\sin(n\pi x)$$

$$= \left(\frac{1}{\pi} - \frac{4}{\pi^2}\right)\sin \pi x + \left(\frac{1}{3\pi} + \frac{4}{9\pi^2}\right)\sin 3\pi x + \left(\frac{1}{5\pi} - \frac{4}{25\pi^2}\right)\sin 5\pi x + \dots$$

[when, n = even, function is zero]