North South University

Department of Mathematics and Physics

Assignment - 1

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Course No.	:	MAT-120
Course Title	:	Calculus and Analytical Geometry I
Section	:	13
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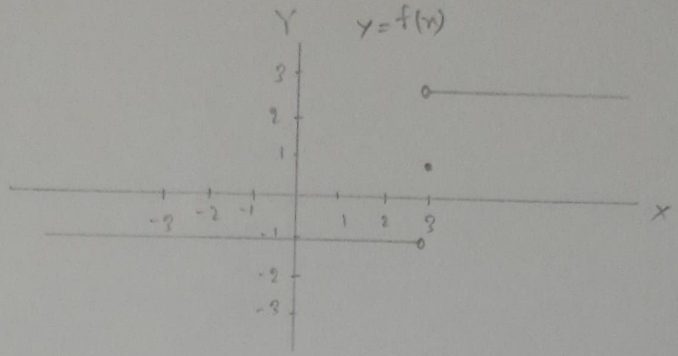
1.1

3)



a)

$$\lim_{x \to 3^-} f(x) = -1$$

b)

$$\lim_{x \to 3^+} f(x) = 3$$

c)

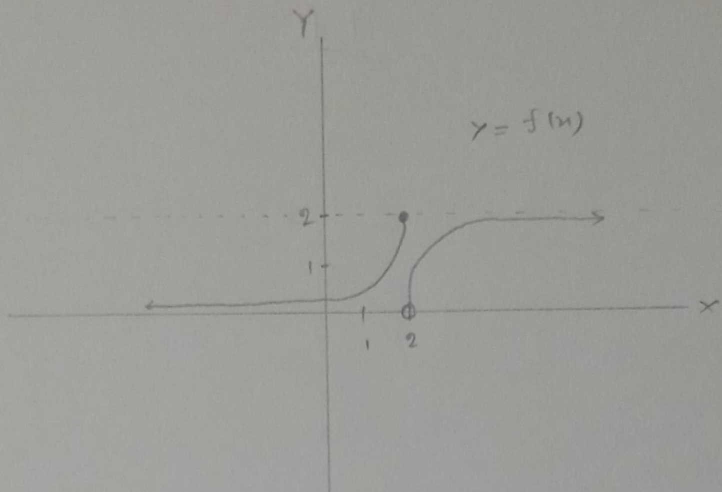
$$\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$$

$\therefore \lim_{x \to 3} f(x)$ does not exist

d)

$$f(3) = 1$$

4)



a)

$$\lim_{x \to 2^-} f(x) = 2$$

b)

$$\lim_{x \to 2^+} f(x) = 0$$

c)

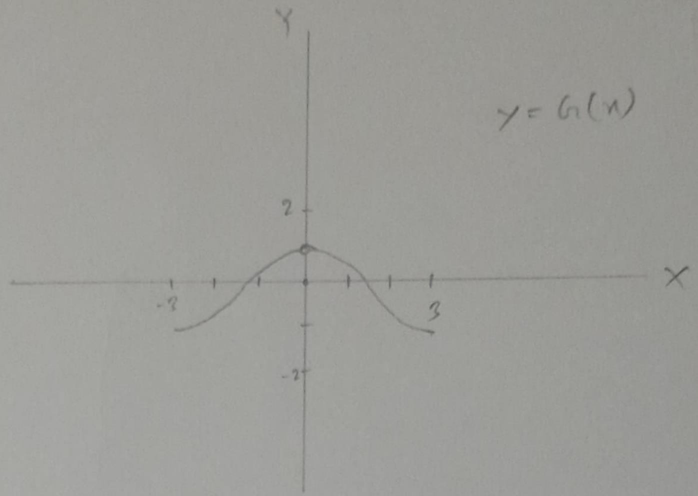
$$\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$$

$\therefore \lim_{x \to 2} f(x)$ does not exist.

d)

$$f(2) = 2$$

6|



a)

$$\lim_{x \to 0^-} G(x) = 1$$

b)

$$\lim_{x \to 0^+} G(x) = 1$$

c)

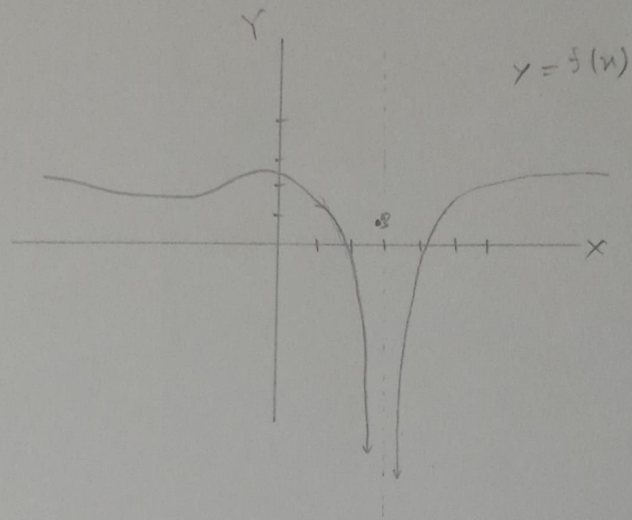
$$\lim_{x \to 0^-} G(x) = \lim_{x \to 0^+} G(x)$$

$$\therefore \lim_{x \to 0} G(x) = 1$$

d)

$$G(0) = 0$$

7]



a)

$$\lim_{x \to 3^-} f(x) = -\infty$$

b)

$$\lim_{x \to 3^+} f(x) = -\infty$$

c)

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$$

$$\therefore \lim_{x \to 3} f(x) = -\infty$$

d)

$$f(3) = 1$$

1.2

3|

Given that,

$$\lim_{x \to 2} x(x-1)(x+1)$$

$$= \lim_{x \to 2} x \cdot \lim_{x \to 2} (x-1) \cdot \lim_{x \to 2} (x+1)$$

$$= 2 \cdot (2-1) \cdot (2+1)$$

$$= 2 \cdot 1 \cdot 3$$

$$= 6$$

Therefore, $\lim_{x \to 2} x(x-1)(x+1) = 6.$

4|

Given that,

$$\lim_{x \to 3} x^3 - 3x^2 + 9x$$

$$= (3)^3 - 3 \cdot (3)^2 + 9 \cdot 3$$

$$= 27 - 27 + 27$$

$$= 27$$

Therefore, $\lim_{x \to 3} x^3 - 3x^2 + 9x = 27.$

5)

Given that,

$$\lim_{x\to 3} \frac{x^2-2x}{x+1}$$

$$= \frac{\lim_{x\to 3}(x^2-2x)}{\lim_{x\to 3}(x+1)}$$

$$= \frac{3^2-2\cdot 3}{3+1}$$

$$= \frac{9-6}{4}$$

$$= \frac{3}{4}$$

Therefore,

$$\lim_{x\to 3} \frac{x^2-2x}{x+1} = \frac{3}{4}.$$

6)

Given that,

$$\lim_{x\to 0} \frac{6x-9}{x^3-12x+3}$$

$$= \frac{\lim_{x\to 0}(6x-9)}{\lim_{x\to 0}(x^3-12x+3)}$$

$$= \frac{6\cdot 0-9}{0^3-12\cdot 0+3}$$

$= \frac{-9}{3}$

$= -3$

Therefore,

$$\lim_{x \to 0} \frac{6x-9}{x^3-12x+3} = -3$$

7]

Given that,

$$\lim_{x \to 1^+} \frac{x^4-1}{x-1}$$

$$= \lim_{x \to 1^+} \frac{(x^2+1)(x^2-1)}{(x-1)}$$

$$= \lim_{x \to 1^+} \frac{(x^2+1)(x+1)(x-1)}{(x-1)}$$

$$= \lim_{x \to 1^+} (x^2+1)(x+1)$$

$$= \lim_{x \to 1^+} (x^2+1) \cdot \lim_{x \to 1^+} (x+1)$$

$= 2 \cdot 2$

$= 4$

Therefore,

$$\lim_{x \to 1^+} \frac{x^4-1}{x-1} = 4.$$

8|

Given that,

$$\lim_{t \to -2} \frac{t^3+8}{t+2}$$

$$= \lim_{t \to -2} \frac{t^3+2^3}{t+2}$$

$$= \lim_{t \to -2} \frac{(t+2)(t^2-2t+4)}{t+2}$$

$$= \lim_{t \to -2} (t^2-2t+4)$$

$$= (-2)^2 - 2\cdot(-2) + 4$$

$$= 4+4+4$$

$$= 12$$

Therefore,

$$\lim_{t \to -2} \frac{t^3+8}{t+2} = 12$$

9|

$$\lim_{x \to -1} \frac{x^2+6x+5}{x^2-3x-4}$$

$$= \lim_{x \to -1} \frac{(x+5)(x+1)}{(x+1)(x-4)}$$

$$= \lim_{x \to -1} \frac{x+5}{x-4}$$

$$= \frac{\lim_{x \to -1} (x+5)}{\lim_{x \to -1} (x-4)}$$

$$= \frac{-1+5}{-1-4}$$

$$= \frac{4}{-5}$$

Therefore,

$$\lim_{x \to -1} \frac{x^2+6x+5}{x^2-3x-4} = -\frac{4}{5}$$

10|

Given that,

$$\lim_{x \to 2} \frac{x^2-4x+4}{x^2+x-6}$$

$$= \lim_{x \to 2} \frac{(x-2)(x-2)}{(x+3)(x-2)}$$

$$= \lim_{x \to 2} \frac{x-2}{x+3}$$

$$= \frac{\lim_{x \to 2} (x-2)}{\lim_{x \to 2} (x+3)}$$

$$= \frac{2-2}{2+3}$$

$$= \frac{0}{5}$$

$$= 0$$

Therefore,

$$\lim_{x\to 2} \frac{x^2-4x+4}{x^2+x-6} = 0$$

11]

Given that,

$$\lim_{x\to -1} \frac{2x^2+x-1}{x+1}$$

$$= \lim_{x\to -1} \frac{(2x-1)(x+1)}{(x+1)}$$

$$= \lim_{x\to -1} (2x-1)$$

$$= 2\cdot(-1)-1$$

$$= -2-1$$

$$= -3$$

Therefore,

$$\lim_{x\to -1} \frac{2x^2+x-1}{x+1} = -3$$

12]

Given that,

$$\lim_{x\to 1} \frac{3x^2-x-2}{2x^2+x-3}$$

$$= \lim_{x\to 1} \frac{(3x+2)(x-1)}{(2x+3)(x-1)}$$

$$= \lim_{x \to 1} \frac{3x+2}{2x+3}$$

$$= \frac{\lim_{x \to 1} (3x+2)}{\lim_{x \to 1} (2x+3)}$$

$$= \frac{3 \cdot 1+2}{2 \cdot 1+3}$$

$$= \frac{5}{5}$$

$$= 1$$

Therefore,

$$\lim_{x \to 1} \frac{3x^2-x-2}{2x^2+x-3} = 1.$$

13)

Given that,

$$\lim_{t \to 2} \frac{t^3+3t^2-12t+4}{t^3-4t}$$

$$= \lim_{t \to 2} \frac{(t-2)(t^2+5t-2)}{t(t^2-4)}$$

$$= \lim_{t-2} \frac{(t-2)(t^2+5t-2)}{t(t+2)(t-2)}$$

$$= \lim_{t-2} \frac{t^2+5t-2}{t^2+2t}$$

$$= \frac{\lim_{x \to 2} (x^2+5x-2)}{\lim_{x \to 2} (x^2+2x)}$$

$$= \frac{2^2+5.2-2}{2^2+2.2}$$

$$= \frac{12}{8}$$

$$= \frac{3}{2}$$

Therefore,

$$\lim_{x \to 2} \frac{x^3+3x^2-12x+4}{x^3-4x} = \frac{3}{2}$$

14)

Given that,

$$\lim_{x \to 1} \frac{x^3+x^2-5x+3}{x^3-3x+2}$$

$$= \lim_{x \to 1} \frac{(x-1)^2 (x+3)}{(x-1)^2 (x+2)}$$

$$= \lim_{x \to 1} \frac{x+3}{x+2}$$

$$= \frac{\lim_{x \to 1} (x+3)}{\lim_{x \to 1} (x+2)}$$

$$= \frac{1+3}{1+2}$$

$= \frac{4}{3}$

Therefore,

$$\lim_{t \to 1} \frac{t^3+t^2-5t+3}{t^2-3t+2} = \frac{4}{3}.$$

15)

Given that,

$$\lim_{x \to 3^+} \frac{x}{x-3}$$

Since the numerator is positive and the denominator $(x-3)$ approaches zero and is greater than zero for x near 3 to the right, the function increases without bound.

Therefore,

$$\lim_{x \to 3^+} \frac{x}{x-3} = +\infty$$

16)

Given that,

$$\lim_{x \to 3^-} \frac{x}{x-3}$$

Since the numerator is positive and the denominator $(x-3)$ approaches zero and is less than zero for x near 3 to the left, the function decreases without bound.

Therefore,

$$\lim_{x \to 3^-} \frac{x}{x-3} = -\infty$$

17)

Given that,

$$\lim_{x \to 3} \frac{x}{x-3}$$

L.H.L :

$$\lim_{x \to 3^-} \frac{x}{x-3} = -\infty$$

R.H.L. :

$$\lim_{x \to 3^+} \frac{x}{x-3} = +\infty$$

L.H.L $\neq$ R.H.L.

Hence, $\lim_{x \to 3} \frac{x}{x-3}$ does not exist.

18)

Given that,

$$\lim_{x \to 2^+} \frac{x}{x^2-4}$$

Since, the numerator is positive and the denominator (x^2-4) approaches zero and is greater than zero for x near 2 to the right, the function increases without bound.

Therefore,

$$\lim_{x \to 2^+} \frac{x}{x^2-4} = +\infty$$

19)

Given that,

$$\lim_{x \to 2^-} \frac{x}{x^2-4}$$

Since the numerator is positive and the denominator (x^2-4) approaches zero and is less than zero for x near 2 to the left, the function decreases without bound.

Therefore,

$$\lim_{x \to 2^-} \frac{x}{x^2-4} = -\infty$$

20)

$$\lim_{x\to 2} \frac{x}{x^2-4}$$

L.H.L:

$$\lim_{x\to 2^-} \frac{x}{x^2-4} = -\infty$$

R.H.L.:

$$\lim_{x\to 2^+} \frac{x}{x^2-4} = +\infty$$

$\therefore$ L.H.L $\neq$ R.H.L.

Therefore,

$\lim_{x\to 2} \frac{x}{x^2-4}$ does not exist.

21)

Given that,

$$\lim_{y\to 6^+} \frac{y+6}{y^2-36}$$

$$= \lim_{y\to 6^+} \frac{y+6}{(y+6)(y-6)}$$

$$= \lim_{y\to 6^+} \frac{1}{y-6}$$

Since, the numerator is positive and the denominator $(y-6)$ approaches zero and is greater than zero for y near 6 to the right, the function increases without bound.

Therefore,

$$\lim_{y \to 6^+} \frac{y+6}{y^2-36} = +\infty$$

22|

$$\lim_{y \to 6^-} \frac{y+6}{y^2-36}$$

$$= \lim_{y \to 6^-} \frac{1}{y-6}$$

Since the numerator is positive and the denominator $(y-6)$ approaches zero and is less than zero for y near ~~to~~ 6 to the left, the function decreases without bound.

Therefore,

$$\lim_{y \to 6^-} \frac{y+6}{y^2-36} = -\infty$$

23|

Given that,

$$\lim_{y \to 6} \frac{y+6}{y^2-36}$$

$$= \lim_{y \to 6} \frac{1}{y-6}$$

L.H.L:

$$\lim_{y - 6^-} \frac{1}{y-6} = -\infty$$

R.H.L.:

$$\lim_{y \to 6^+} \frac{1}{y-6} = +\infty$$

$\therefore$ L.H.L $\neq$ R.H.L

Therefore,

$$\lim_{y \to 6} \frac{y+6}{y^2-36} \text{ does not exist.}$$

24)

Given that,

$$\lim_{x \to 4^+} \frac{3-x}{x^2-2x-8}$$

$$= \lim_{x \to 4^+} \frac{3-x}{(x-4)(x+2)}$$

Since the numerator is negative and the denominator $(x-4)(x-2)$ approaches zero and is greater than zero, for x near 4 to the right, the function decreases without bound.

Therefore,

$$\lim_{x \to 4^+} \frac{3-x}{x^2-2x-8} = -\infty$$

25)

Given that,

$$\lim_{x \to 4^-} \frac{3-x}{x^2-2x-8}$$

$$= \lim_{x \to 4^-} \frac{3-x}{(x-4)(x+2)}$$

Since the numerator is negative and the denominator $(x-4)(x+2)$ approaches zero and is less than zero for x near 4 to the left, the function increases without bound.

Therefore,

$$\lim_{x \to 4^-} \frac{3-x}{x^2-2x-8} = +\infty$$

26)

Given that,

$$\lim_{x \to 4} \frac{3-x}{x^2-2x-8}$$

$$= \lim_{x \to 4} \frac{3-x}{(x-4)(x+2)}$$

L.H.L.:

$$\lim_{x \to 4^-} \frac{3x}{(x-4)(x+2)} = +\infty$$

R.H.L:

$$\lim_{x \to 4^+} \frac{3-x}{(x-4)(x+2)} = -\infty$$

$\therefore$ L.H.L $\neq$ R.H.L.

Therefore,

$$\lim_{x \to 4} \frac{3-x}{x^2-2x-8} \text{ does not exist.}$$

271

Given that,

$$\lim_{x \to 2^+} \frac{1}{|2-x|}$$

Here,

$$f(x) = \frac{1}{|2-x|}$$

$$= \begin{cases} \frac{1}{2-x} & 2-x \geq 0 \\ \frac{1}{x-2} & 2-x < 0 \end{cases}$$

$$= \begin{cases} \frac{1}{2-x} & x \leq 2 \\ \frac{1}{x-2} & x > 2 \end{cases}$$

$$\therefore \lim_{x \to 2^+} \frac{1}{|2-x|} = \lim_{x \to 2^+} \frac{1}{x-2}$$

$$= +\infty$$

Therefore, $\lim_{x \to 2^+} \frac{1}{|2-x|} = +\infty$

28|

Given that,

$$\lim_{x \to 3^-} \frac{1}{|x-3|}$$

Since the numerator is positive and the denominator $|x-3|$ approaches zero and is greater than zero for x near 3 to the left, the function increases without bound.

Therefore,

$$\lim_{x \to 3^-} \frac{1}{|x-3|} = +\infty$$

29|

Given that,

$$\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3}$$

$$= \lim_{x \to 9} \frac{(\sqrt{x})^2 - 3^2}{\sqrt{x}-3}$$

$$= \lim_{x \to 9} \frac{(\sqrt{x}+3)(\sqrt{x}-3)}{(\sqrt{x}-3)}$$

$$= \lim_{x \to 9} (\sqrt{x}+3)$$

$$= \lim_{x \to 9} \sqrt{x} + \lim_{x \to 9} 3$$

$$= \sqrt{\lim_{x \to 9} x} + 3$$

$$= \sqrt{9} + 3$$

$$= 3+3$$

$$= 6$$

Therefore,

$$\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3} = 6$$

30)

Given that,

$$\lim_{y \to 4} \frac{4-y}{2-\sqrt{y}}$$

$$= \lim_{y \to 4} \frac{2^2-(\sqrt{y})^2}{2-\sqrt{y}}$$

$$= \lim_{y \to 4} \frac{(2+\sqrt{y})(2-\sqrt{y})}{(2-\sqrt{y})}$$

$$= \lim_{y \to 4} (2+\sqrt{y})$$

$$= \lim_{y \to 4} 2 + \lim_{y \to 4} \sqrt{y}$$

$$= 2 + \sqrt{\lim_{y \to 4} y}$$

$$= 2 + \sqrt{4}$$

$$= 2+2 = 4$$

Therefore,

$$\lim_{y \to 4} \frac{4-y}{2-\sqrt{y}} = 4$$

31/

Given that,

$$f(x) = \begin{cases} x-1, & x \leq 3 \\ 3x-7, & x > 3 \end{cases}$$

a)

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} x-1$$

$$= 3-1$$

$$= 2$$

Therefore,

$$\lim_{x \to 3^-} f(x) = 2$$

b)

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (3x-7)$$

$$= 3.3-7$$

$$= 9-7$$

$$= 2$$

Therefore,

$$\lim_{x \to 3^+} f(x) = 2$$

c)

L.H.L.:

$$\lim_{x \to 3^-} f(x) = 2$$

R.H.L.:

$$\lim_{x \to 3^+} f(x) = 2$$

$\therefore$ L.H.L. = R.H.L.

Therefore,

$$\lim_{x \to 3} f(x) = 2$$

1.3

9)

Given that,

$$\lim_{x \to +\infty} (1 + 2x - 3x^5)$$

$$= \lim_{x \to +\infty} (-3x^5 + 2x +)$$

$$= \lim_{x \to +\infty} -3x^5$$

$$= -3 \cdot \lim_{x \to +\infty} x^5$$

$$= -\infty$$

Therefore,

$$\lim_{x \to +\infty} (1 + 2x - 3x^5) = -\infty$$

10)

Given that,

$$\lim_{x \to +\infty} (2x^3 - 100x + 5)$$

$$= \lim_{x \to +\infty} 2x^3$$

$$= 2 \cdot \lim_{x \to +\infty} x^3$$

$$= \infty$$

Therefore,

$$\lim_{x \to +\infty} (2x^3 - 100x + 5) = +\infty$$

11]

Given that,

$$\lim_{x \to +\infty} \sqrt{x}$$

$$= \sqrt{\lim_{x \to +\infty} \cdot x}$$

$$= +\infty$$

Therefore,

$$\lim_{x \to +\infty} \sqrt{x} = +\infty.$$

12]

Given that,

$$\lim_{x \to -\infty} \sqrt{5-x}$$

$$= \sqrt{\lim_{x \to -\infty} (5-x)}$$

$$= \sqrt{\lim_{x \to -\infty} 5 - \lim_{x \to -\infty} x}$$

$$= \sqrt{5 + \infty}$$

$$= +\infty$$

Therefore,

$$\lim_{x \to -\infty} \sqrt{5-x} = +\infty$$

13|

Given that,

$$\lim_{x \to +\infty} \frac{3x+1}{2x-5}$$

$$= \lim_{x \to +\infty} \frac{3x}{2x}$$

$$= \lim_{x \to +\infty} \frac{3}{2}$$

$$= \frac{3}{2}$$

Therefore,

$$\lim_{x \to +\infty} \frac{3x+1}{2x-5} = \frac{3}{2}$$

14|

Given that,

$$\lim_{x \to +\infty} \frac{5x^2-4x}{2x^2+3}$$

$$= \lim_{x \to +\infty} \frac{5x^2}{2x^2}$$

$$= \lim_{x \to +\infty} \frac{5}{2} = \frac{5}{2}$$

Therefore,

$$\lim_{x \to +\infty} \frac{5x^2-4x}{2x^2+3} = \frac{5}{2}$$

15)

Given that,

$$\lim_{y \to -\infty} \frac{3}{y+4}$$

$$= 3 \cdot \lim_{y \to -\infty} \frac{1}{y+4}$$

$$= 3 \cdot 0$$

$$= 0$$

Therefore,

$$\lim_{y \to -\infty} \frac{3}{y+4} = 0$$

16)

Given that,

$$\lim_{x \to +\infty} \frac{1}{x-12}$$

$$= \lim_{x \to +\infty} \frac{\frac{1}{x}}{\frac{x-12}{x}}$$

$$= \frac{\lim_{x \to +\infty} \frac{1}{x}}{\lim_{x \to +\infty} 1-\frac{12}{x}}$$

$$= \frac{0}{1-12\cdot 0}$$

$$= 0$$

Therefore,

$$\lim_{x \to +\infty} \frac{1}{x-12} = 0$$

17)

Given that,

$$\lim_{x \to -\infty} \frac{x-2}{x^2+2x+1}$$

$$= \lim_{x \to -\infty} \frac{x}{x^2}$$

$$= \lim_{x \to -\infty} \frac{1}{x}$$

$$= 0$$

Therefore,

$$\lim_{x \to -\infty} \frac{x-2}{x^2+2x+1} = 0$$

18)

Given that,

$$\lim_{x \to +\infty} \frac{5x^2+7}{3x^2-x}$$

$$= \lim_{x \to +\infty} \frac{5x^2}{3x^2}$$

$$= \lim_{x \to +\infty} \frac{5}{3}$$

$$= \frac{5}{3}$$

Therefore,

$$\lim_{x \to +\infty} \frac{5x^2+7}{3x^2-x} = \frac{5}{3}$$

19)

Given that,

$$\lim_{x \to +\infty} \frac{7-6x^5}{x+3}$$

$$= \lim_{x \to +\infty} \frac{-6x^5+7}{x+3}$$

$$= \lim_{x \to +\infty} \frac{-6x^5}{x}$$

$$= \lim_{x \to +\infty} -6x^4$$

$$= -6 \cdot \lim_{x \to +\infty} x^4$$

$$= -\infty$$

Therefore,

$$\lim_{x \to +\infty} \frac{7-6x^5}{x+3} = -\infty$$

20|

Given that,

$$\lim_{x \to -\infty} \frac{5 - 2x^3}{x^2+1}$$

$$= \lim_{x \to -\infty} \frac{-2x^3+5}{x^2+1}$$

$$= \lim_{x \to -\infty} \frac{-2x^3}{x^2}$$

$$= \lim_{x \to -\infty} -2x$$

$$= -2 \cdot \lim_{x \to -\infty} x$$

$$= +\infty$$

Therefore,

$$\lim_{x \to -\infty} \frac{5 - 2x^3}{x^2+1} = +\infty$$

21|

Given that,

$$\lim_{x \to +\infty} \frac{6 - x^3}{7x^3 + 3}$$

$$= \lim_{x \to +\infty} \frac{-x^3+6}{7x^3+3}$$

$$= \lim_{x \to +\infty} \frac{-x^3}{7x^3}$$

$$= \lim_{x \to +\infty} \frac{-1}{7}$$

$$= -\frac{1}{7}$$

Therefore,

$$\lim_{x \to +\infty} \frac{6 - x^3}{7x^3 + 3} = -\frac{1}{7}$$

22|

Given that,

$$\lim_{x \to -\infty} \frac{x + 4x^3}{1 - x^2 + 7x^3}$$

$$= \lim_{x \to -\infty} \frac{4x^3}{7x^3}$$

$$= \lim_{x \to -\infty} \frac{4}{7}$$

$$= \frac{4}{7}$$

Therefore,

$$\lim_{x \to -\infty} \frac{x + 4x^3}{1 - x^2 + 7x^3} = \frac{4}{7}$$

23)

Given that,

$$\lim_{x \to +\infty} \sqrt[3]{\frac{2+3x-5x^2}{1+8x^2}}$$

$$= \sqrt[3]{\lim_{x \to +\infty} \frac{2+3x-5x^2}{1+8x^2}}$$

$$= \sqrt[3]{\lim_{x \to +\infty} \frac{-5x^2}{8x^2}}$$

$$= \sqrt[3]{\lim_{x \to +\infty} \frac{-5}{8}}$$

$$= \sqrt[3]{\frac{-5}{8}}$$

$$= -\frac{\sqrt[3]{5}}{2}$$

Therefore,

$$\lim_{x \to +\infty} \sqrt[3]{\frac{2+3x-5x^2}{1+8x^2}} = -\frac{\sqrt[3]{5}}{2}$$

24)

Given that,

$$\lim_{s \to +\infty} \sqrt[3]{\frac{3s^7-4s^5}{2s^7+1}}$$

$$= \sqrt[3]{\lim_{s \to +\infty} \frac{3s^7-4s^5}{2s^7+1}}$$

$$= \sqrt[3]{\lim_{s \to +\infty} \frac{3s^7}{2s^7}}$$

$$= \sqrt[3]{\lim_{s \to +\infty} \frac{3}{2}}$$

$$= \sqrt[3]{\frac{3}{2}}$$

Therefore,

$$\lim_{s \to +\infty} \sqrt[3]{\frac{3s^7 - 4s^5}{2s^7 + 1}} = \sqrt[3]{\frac{3}{2}}$$

25|

Given that,

$$\lim_{x \to -\infty} \frac{\sqrt{5x^2 - 2}}{x + 3}$$

$$= \lim_{x \to -\infty} \frac{\frac{\sqrt{5x^2 - 2}}{|x|}}{\frac{x+3}{|x|}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{5x^2 - 2}{x^2}}}{\frac{x+3}{-x}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{5 - \frac{2}{x^2}}}{-1 - \frac{3}{x}}$$

$$= \frac{\sqrt{\lim_{x \to -\infty} 5 - 2 \lim_{x \to -\infty} \frac{1}{x^2}}}{\lim_{x \to -\infty} -1 - 3 \cdot \lim_{x \to -\infty} \frac{1}{x}}$$

$$= \frac{\sqrt{5-2\cdot 0}}{-1-3\cdot 0}$$

$$= \frac{\sqrt{5}}{-1}$$

$$= -\sqrt{5}$$

Therefore,

$$\lim_{x\to-\infty} \frac{\sqrt{5x^2-2}}{x+3} = -\sqrt{5}$$

26)

Given that,

$$\lim_{x\to+\infty} \frac{\sqrt{5x^2-2}}{x+3}$$

$$= \lim_{x\to+\infty} \frac{\frac{\sqrt{5x^2-2}}{|x|}}{\frac{x+3}{|x|}}$$

$$= \lim_{x\to+\infty} \frac{\sqrt{\frac{5x^2-2}{x^2}}}{\frac{x+3}{x}}$$

$$= \lim_{x\to+\infty} \frac{\sqrt{5-\frac{2}{x^2}}}{1+\frac{3}{x}}$$

$$= \frac{\sqrt{\lim_{x\to+\infty} 5 - 2\cdot\lim_{x\to+\infty}\frac{1}{x^2}}}{\lim_{x\to+\infty} 1 + 3\cdot\lim_{x\to+\infty}\frac{1}{x}}$$

$$= \frac{\sqrt{5-2\cdot 0}}{1+3\cdot 0}$$

$$= \sqrt{5}$$

Therefore,

$$\lim_{x \to +\infty} \frac{\sqrt{5x^2-2}}{x+3} = \sqrt{5}$$

27)

Given that,

$$\lim_{y \to -\infty} \frac{2-y}{\sqrt{7+6y^2}}$$

$$= \lim_{y \to -\infty} \frac{\frac{2-y}{|y|}}{\frac{\sqrt{7+6y^2}}{|y|}}$$

$$= \lim_{y \to -\infty} \frac{\frac{2-y}{-y}}{\sqrt{\frac{7+6y^2}{y^2}}}$$

$$= \lim_{y \to -\infty} \frac{-\frac{2}{y}+1}{\sqrt{\frac{7}{y^2}+6}}$$

$$= \frac{-2\cdot\lim_{y \to -\infty}\frac{1}{y} + \lim_{y \to -\infty} 1}{\sqrt{7\cdot\lim_{y \to -\infty}\frac{1}{y^2} + \lim_{y \to -\infty} 6}}$$

$$= \frac{-2\cdot 0+1}{\sqrt{7\cdot 0+6}} = \frac{1}{\sqrt{6}}$$

Therefore,

$$\lim_{y \to -\infty} \frac{2-y}{\sqrt{7+6y^2}} = \frac{1}{\sqrt{6}}$$

28)

Given that,

$$\lim_{y \to +\infty} \frac{2-y}{\sqrt{7+6y^2}}$$

$$= \lim_{y \to +\infty} \frac{\frac{2-y}{|y|}}{\frac{\sqrt{7+6y^2}}{|y|}}$$

$$= \lim_{y \to +\infty} \frac{\frac{2-y}{y}}{\sqrt{\frac{7+6y^2}{y^2}}}$$

$$= \lim_{y \to +\infty} \frac{\frac{2}{y} - 1}{\sqrt{\frac{7}{y^2} + 6}}$$

$$= \frac{2 \cdot \lim_{y \to +\infty} \frac{1}{y} - \lim_{y \to +\infty} 1}{\sqrt{7 \cdot \lim_{y \to +\infty} \frac{1}{y^2} + \lim_{y \to +\infty} 6}}$$

$$= \frac{2 \cdot 0 - 1}{\sqrt{7 \cdot 0 + 6}}$$

$$= -\frac{1}{\sqrt{6}}$$

Therefore,

$$\lim_{y \to +\infty} \frac{2-y}{\sqrt{7+6y^2}} = -\frac{1}{\sqrt{6}}$$

29|

Given that,

$$\lim_{x \to -\infty} \frac{\sqrt{3x^4+x}}{x^2-8}$$

$$= \lim_{x \to -\infty} \frac{\dfrac{\sqrt{3x^4+x}}{|x^2|}}{\dfrac{x^2-8}{|x^2|}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\dfrac{3x^4+x}{x^4}}}{\dfrac{x^2-8}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{3+\frac{1}{x^3}}}{1-\frac{8}{x^2}}$$

$$= \frac{\sqrt{\lim\limits_{x \to -\infty} 3 + \lim\limits_{x \to -\infty} \frac{1}{x^3}}}{\lim\limits_{x \to -\infty} 1 - 8 \cdot \lim\limits_{x \to -\infty} \frac{1}{x^2}}$$

$$= \frac{\sqrt{3+0}}{1-8 \cdot 0}$$

$$= \sqrt{3}$$

Therefore, $\lim\limits_{x \to -\infty} \dfrac{\sqrt{3x^4+x}}{x^2-8} = \sqrt{3}$

30|

Given that,

$$\lim_{x \to +\infty} \frac{\sqrt{3x^4+x}}{x^2-8}$$

$$= \lim_{x \to +\infty} \frac{\frac{\sqrt{3x^4+x}}{x^2}}{\frac{x^2-8}{x^2}}$$

$$= \lim_{x \to +\infty} \frac{\sqrt{\frac{3x^4+x}{x^4}}}{1-\frac{8}{x^2}}$$

$$= \lim_{x \to +\infty} \frac{\sqrt{3+\frac{1}{x^3}}}{1-\frac{8}{x^2}}$$

$$= \frac{\sqrt{\lim_{x \to +\infty} 3 + \lim_{x \to +\infty} \frac{1}{x^3}}}{\lim_{x \to +\infty} 1 - \lim_{x \to +\infty} \frac{8}{x^2}}$$

$$= \frac{\sqrt{3+0}}{1-0}$$

$$= \sqrt{3}$$

Therefore,

$$\lim_{x \to +\infty} \frac{\sqrt{3x^4+x}}{x^2-8} = \sqrt{3}$$

31||

Given that,

$$\lim_{x\to+\infty}\left(\sqrt{x^2+3}-x\right)$$

$$=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2+3}-x\right)\left(\sqrt{x^2+3}+x\right)}{\left(\sqrt{x^2+3}+x\right)}$$

$$=\lim_{x\to+\infty}\frac{x^2+3-x^2}{\sqrt{x^2+3}+x}$$

$$=\lim_{x\to+\infty}\frac{3}{\sqrt{x^2+3}+x}$$

$$=\lim_{x\to+\infty}\frac{\frac{3}{|x|}}{\frac{\sqrt{x^2+3}+x}{|x|}}$$

$$=\lim_{x\to+\infty}\frac{\frac{3}{x}}{\frac{\sqrt{x^2+3}}{x}+1}$$

$$=\lim_{x\to+\infty}\frac{\frac{3}{x}}{\sqrt{\frac{x^2+3}{x^2}}+1}$$

$$=\lim_{x\to+\infty}\frac{\frac{3}{x}}{\sqrt{1+\frac{3}{x^2}}+1}$$

$$=\frac{3.\lim_{x\to+\infty}\frac{1}{x}}{\lim_{x\to+\infty}\sqrt{1+\frac{3}{x^2}}+\lim_{x\to+\infty}1}=\frac{3.0}{1+1}=0$$

Therefore,

$$\lim_{x \to +\infty} \left(\sqrt{x^2+3} - x\right) = 0$$

32|

Given that,

$$\lim_{x \to +\infty} \left(\sqrt{x^2-3x} - x\right)$$

$$= \lim_{x \to +\infty} \frac{\left(\sqrt{x^2-3x} - x\right)\left(\sqrt{x^2-3x} + x\right)}{\left(\sqrt{x^2-3x} + x\right)}$$

$$= \lim_{x \to +\infty} \frac{x^2-3x-x^2}{\sqrt{x^2-3x} + x}$$

$$= \lim_{x \to +\infty} \frac{-3x}{\sqrt{x^2-3x} + x}$$

$$= \lim_{x \to +\infty} \frac{\frac{-3x}{x}}{\frac{\sqrt{x^2-3x}}{x} + 1}$$

$$= \lim_{x \to +\infty} \frac{-3}{\sqrt{1-\frac{3}{x}} + 1}$$

$$= \frac{\lim_{x \to +\infty} -3}{\sqrt{\lim_{x \to +\infty} 1 - 3 \cdot \lim_{x \to +\infty} \frac{1}{x}} + \lim_{x \to +\infty} 1}$$

$$= \frac{-3}{\sqrt{1-3 \cdot 0} + 1} = \frac{-3}{1+1} = \frac{-3}{2}$$

Therefore,

$$\lim_{x \to +\infty} \left(\sqrt{x^2-3x} - x\right) = \frac{-3}{2}$$

33/

Given that,

$$\lim_{x \to -\infty} \frac{1-e^x}{1+e^x}$$

$$= \frac{\lim_{x \to -\infty} (1-e^x)}{\lim_{x \to -\infty} (1+e^x)}$$

$$= \frac{\lim_{x \to -\infty} 1 - \lim_{x \to -\infty} e^x}{\lim_{x \to -\infty} 1 + \lim_{e \to -\infty} e^x}$$

$$= \frac{1-0}{1+0}$$

$$= 1$$

Therefore,

$$\lim_{x \to -\infty} \frac{1-e^x}{1+e^x} = 1$$

34|

Given that,

$$\lim_{x \to +\infty} \frac{1-e^{x}}{1+e^{x}}$$

$$= \lim_{x \to +\infty} \frac{e^{-x}-1}{e^{-x}+1}$$

$$= \frac{\lim_{x \to +\infty} e^{-x} - \lim_{x \to +\infty} 1}{\lim_{x \to +\infty} e^{-x} + \lim_{x \to +\infty} 1}$$

$$= \frac{0-1}{0+1}$$

$$= -1$$

Therefore,

$$\lim_{x \to +\infty} \frac{1-e^{x}}{1+e^{x}} = -1$$

35|

Given that,

$$\lim_{x \to +\infty} \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$$

$$= \lim_{x \to +\infty} \frac{1+e^{-2x}}{1-e^{-2x}} \quad \text{[devide by } e^{x}\text{]}$$

$$= \frac{\lim_{x \to +\infty} 1 + \lim_{x \to +\infty} e^{-2x}}{\lim_{x \to +\infty} 1 - \lim_{x \to +\infty} e^{-2x}}$$

$$= \frac{1+0}{1-0}$$

$$= 1$$

Therefore,

$$\lim_{x \to +\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} = 1$$

36|

Given that,

$$\lim_{x \to -\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$= \lim_{x \to -\infty} \frac{\frac{e^x + e^{-x}}{e^{-x}}}{\frac{e^x - e^{-x}}{e^{-x}}}$$

$$= \lim_{x \to -\infty} \frac{e^{2x} + 1}{e^{2x} - 1}$$

$$= \frac{\lim_{x \to -\infty} e^{2x} + \lim_{x \to -\infty} 1}{\lim_{x \to -\infty} e^{2x} - \lim_{x \to -\infty} 1}$$

$$= \frac{0+1}{0-1}$$

$$= -1$$

Therefore, $\lim_{x \to -\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} = -1$

37/

Given that,

$$\lim_{x \to +\infty} \ln\left(\frac{2}{x^2}\right)$$

$$= \ln\left(\lim_{x \to +\infty} \frac{2}{x^2}\right)$$

$$= \ln 0$$

$$= -\infty$$

Therefore,

$$\lim_{x \to +\infty} \ln\left(\frac{2}{x^2}\right) = -\infty$$

38/

Given that,

$$\lim_{x \to 0^+} \ln\left(\frac{2}{x^2}\right)$$

$$= \ln\left(\lim_{x \to 0^+} \frac{2}{x^2}\right)$$

$$= \ln +\infty$$

$$= +\infty$$

Therefore,

$$\lim_{x \to 0^+} \ln\left(\frac{2}{x^2}\right) = +\infty.$$

39)

Given that

$$\lim_{x \to +\infty} \frac{(x+1)^x}{x^x}$$

$$= \lim_{x \to +\infty} \left(\frac{x+1}{x}\right)^x$$

$$= \lim_{x \to +\infty} \left(1+\frac{1}{x}\right)^x$$

$$= e$$

Therefore,

$$\lim_{x \to +\infty} \frac{(x+1)^x}{x^x} = e$$

40)

Given that,

$$\lim_{x \to +\infty} \left(1+\frac{1}{x}\right)^{-x}$$

$$= \lim_{x \to +\infty} \left(1+\frac{1}{x}\right)^{x\cdot(-1)}$$

$$= e^{-1}$$

Therefore,

$$\lim_{x \to +\infty} \left(1+\frac{1}{x}\right)^{-x} = e^{-1}$$

1.5

11/

Given that,

$$f(x) = 5x^4 - 3x + 7$$

Here, $f(x)$ is a polynomial function. Polynomial functions are always continuous.

Therefore,

There is no discontinuous point.

12/

Given that,

$$f(x) = \sqrt[3]{x-8}$$

Here, cubic root is alway defineable. So this function has a continuous curve.

Therefore,

There is no value of x for which the function $f(x)$ is not continuous.

13|

Given that,

$$f(x) = \frac{x+2}{x^2+4}$$

Here, (x^2+4) is ~~aly~~ always positive and greater than zero.

Therefore

there is no discontinuous point.

14|

Given that,

$$f(x) = \frac{x+2}{x^2-4}$$

Here,

$$x^2-4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

Therefore

The function $f(x)$ is not continuous at $x=2$ and $x=-2$.

15)

Given that,

$$f(x) = \frac{x}{2x^2+x}$$

Here,

$$2x^2 + x = 0$$

$$\Rightarrow \cancel{x(2x+1) = 0}$$

$$\Rightarrow x(2x+1) = 0$$

$\therefore x = 0$ and $2x+1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

Therefore,

The function is not continuous at $x = -\frac{1}{2}$ and $x = 0$.

16)

Given that,

$$f(x) = \frac{2x+1}{4x^2+4x+5}$$

$$= \frac{2x+1}{(2x+1)^2+4}$$

Here, $(2x+1)^2+4$ is alway positive and greater than zero

Therefore, the function is continuous everywhere.

17|

Given that,

$$f(x) = \frac{3}{x} + \frac{x-1}{x^2-1}$$

$$= \frac{3x^2-3+x^2-x}{x(x^2-1)}$$

$$= \frac{4x^2-x-3}{x(x^2-1)}$$

Here,

$$x(x^2-1) = 0$$

$$\therefore x = 0 \quad \text{and} \quad x^2-1=0$$

$$x^2 = 1$$

$$x = \pm 1$$

Therefore, the function is not continuous at $x=0$,

$x = 1$ and $x = -1$.

18|

Given that,

$$f(x) = \frac{5}{x} + \frac{2x}{x+4}$$

$$= \frac{5x+20+2x^2}{x(x+4)}$$

$$= \frac{2x^2+5x+20}{x(x+4)}$$

Here,

$$x(x+4) = 0$$

$\therefore x = 0$ and $x+4 = 0$

$x = -4$

Therefore,

the function is not continuous at $x=0$ and $x=-4$.

19|

Given that,

$$f(x) = \frac{x^2+6x+9}{|x|+3}$$

Here, $|x|+3$ is always positive and greater than zero.

Hence, there is no discontinuous point.

20|

Given that,

$$f(x) = \left|4 - \frac{8}{x^4+x}\right|$$

$$= \left|\frac{4x^4+4x-8}{x^4+x}\right|$$

Here,

$$x^4+x=0$$

$$x(x^3+1)=0$$

$\therefore x=0$ and $x^3+1=0$

$$x^3=-1$$

$$x=-1$$

Therefore the function is not continuous at $x=0$ and $x=-1$.

<u>21</u>

Given that,

$$f(x)=\begin{cases} 2x+3, & x\leq 4 \\ 7+\frac{16}{x}, & x>4 \end{cases}$$

For $x<4$, $f(x)=2x+3$ is always continuous at $(-\infty,4)$

For $x>4$, $f(x)=7+\frac{16}{x}$ is always continuous at $(4,\infty)$

For $x=4$,

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} 2x+3$$

$$= 2\cdot 4+3$$

$$= 11$$

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} 7+\frac{16}{x}$$

$$= \lim_{x\to 4^+} 7 + \frac{\lim_{x\to 4^+} 16}{\lim_{x\to 4^+} x}$$

$$= 7+\frac{16}{4}$$

$$= 7+4$$

$$= 11$$

$$f(4) = 2\cdot 4+3$$

$$= 11$$

$$\therefore \lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = f(4) = 11$$

Therefore,

the function is continuous everywhere.

22|

Given that,

$$f(x) = \begin{cases} \frac{3}{x-1} , & x \neq 1 \\ 3 , & x = 1 \end{cases}$$

For $x=1$,

$$f(1) = 3$$

So, function is defined.

Now, checking limit at $x = 1$ for ~~fo~~ the function $f(x)$.

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{3}{x-1}$$

$$= -\infty$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{3}{x-1}$$

$$= +\infty$$

$\therefore$ L.H.L $\neq$ R.H.L.

Hence, ~~function~~ ~~d~~ limit doesn't not exist.

Therefore,

the function is not continuous at $x = 1$.

29/

a)

Given that,

$$f(x) = \begin{cases} 7x-2, & x \leq 1 \\ kx^2, & x > 1 \end{cases}$$

For $x=1$,

$$f(1) = 7.1-2$$
$$= 5$$

So, function is defined.

Now, checking limit,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 7x-2$$
$$= 7.1-2$$
$$= 5$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} kx^2$$
$$= k.1^2$$
$$= k$$

if limit exist, then, L.H.L. = R.H.L.

$$\Rightarrow 5 = k$$

Therefore, value of k is 5.

b)

Given that,

$$f(x) = \begin{cases} kx^2, & x \le 2 \\ 2x + k & x > 2 \end{cases}$$

For $x = 2$,

$$f(2) = k \cdot 2^2 = 4k$$

Now, $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} kx^2$

$$= k \cdot 2^2$$

$$= 4k$$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 2x + k$

$$= 2 \cdot 2 + k$$

$$= 4 + k$$

if limit exist then,

$$L.H.L = R.H.L$$

$$\Rightarrow 4k = 4 + k$$

$$\Rightarrow 3k = 4$$

$$\therefore k = \frac{4}{3}$$

Therefore, value of k is $\frac{4}{3}$.

30)

a)

Given that,

$$f(x) = \begin{cases} 9-x^2, & x \geq -3 \\ \frac{k}{x^2}, & x < -3 \end{cases}$$

For $x = -3$,

$f(-3) = 9-(-3)^2 = 0$

Now,

$$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \frac{k}{x^2}$$

$$= \frac{\lim_{x \to -3^-} k}{\lim_{x \to -3^-} x^2}$$

$$= \frac{k}{9}$$

$$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} 9-x^2$$

$$= 9-(-3)^2$$

$$= 0$$

if limit exist then,

$$\cancel{k}\ L.H.L = R.H.L.$$

$$\Rightarrow \frac{k}{9} = 0$$

$$\therefore k = 0$$

Therefore, value of k is 0.

b)

Given that, $f(x) = \begin{cases} 9-x^2, & x \geq 0 \\ \frac{k}{x^2}, & x < 0 \end{cases}$

For $x=0$,

$$f(0) = 9 - 0^2$$
$$= 9$$

Now,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{k}{x^2}$$
$$= +\infty$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 9 - x^2$$
$$= 9 - 0$$
$$= 9$$

$\therefore$ L.H.L $\neq$ R.H.L

So, limit doesn't exist at $x=0$.

Therefore, the function is not continuous at $x=0$.

So, there is no k value which makes the function continuous every where.