⊛ $1+2+\cdots+n = \frac{n(n+1)}{2}$

⊛ $1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6} \Rightarrow \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

⊛ $1^3+2^3+3^3+\cdots+n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

⊛

$$\sum_{k=50}^{100} k^2 = ?$$

$$\Rightarrow \frac{100(100+1)(2\cdot 100+1)}{6} - \frac{49(49+1)(2\cdot 49+1)}{6}$$

$1^2+2^2+\cdots+50^2+\cdots+100^2 = ?$ — (i)

$1^2+2^2+\cdots+49^2 = ?$ — (ii)

$$\boxed{\text{(i)} - \text{(ii)}}$$

⊛ $\sum_{i=1}^{4}\sum_{j=1}^{3} ij$

$= \sum_{i=1}^{4}(i+2i+3i) = \sum_{i=1}^{4} 6i = 6\sum_{i=1}^{4} i = 6\cdot 10 = 60$

$$\sum_{i=1}^{4}\sum_{j=1}^{3} ij+2 \quad \Bigg\} \text{ H.W.}$$

$$\sum_{i=1}^{5} 2$$

Recursion

$n! = n(n-1)(n-2) \cdots 3.2.1$

$(n-1)! = (n-1)(n-2) \cdots 3.2.1$

$n! = n(n-1)!$

an object that is defined by the "object itself".

$a_n = 2^n, \quad n = 0, 1, 2, 3$

$a_0 = 1$

$a_1 = 2 = 2 \times a_0$

$a_2 = 4 = 2 \times a_1$

$a_3 = 8 = 2 \times a_2$

$a_4 = 16 = 2 \times a_3$

$a_n = 2 \times a_{n-1} \rightarrow$ Recursive defination

|L-17 / 29.07.2022 /

Inference

Premises

1. $(a \wedge e) \rightarrow (c \vee d)$
2. $b \rightarrow (f \wedge e)$
3. $f \rightarrow a$
4. $\neg d$
5. b
6. $f \wedge e$ modus ponens of 2,5
7. f, e simplify of 6
8. a modus ponens of 3,7
9. $a \wedge e$ conjunction of 7,8
10. $c \vee d$ modus ponens of 1,9
11. c disjunctive sylolism of 4,10

Recursion

Some recursive statement!

(i) $n! = n\,(n-1)!$

(ii) $f(n) = f(n-1) + f(n-2)$

(iii) $a_n = 2^n,\ n = 0, 1, 2, \dots$

$\to a_n = 2a_{n-1}$

$f(0) = 3,\ f(n+1) = 2\,f(n) + 3$

$f(4) = ?$

$\Rightarrow$

$$f(4) = 2 \cdot f(3) + 3 = 2 \cdot 45 + 3 = 93$$

$$f(3) = 2 \cdot f(2) + 3 = 2 \cdot 21 + 3 = 45$$

$$f(2) = 2 \cdot f(1) + 3 = 2 \cdot 9 + 3 = 21$$

$$f(1) = 2 \cdot f(0) + 3 = 2 \cdot 3 + 3 = 9$$

$$f(0) = 3$$

⊛ Given,

$$a_1 = 2, \quad a_k = 5a_{k-1}, \quad k \geq 2$$

It is claimed that terms of the sequence satisfy the equation $2\times 5^{n-1}$, for $n \geq 1$

$\Rightarrow$

Term

L.H.S $= a_1 = 2$ 1

$a_2 = 5a_1 = 5\times 2 = 10$ 2

$a_3 = 5a_2 = 50$ 3

$a_4 = 5a_3 = 250$ 4

$a_5 = 5a_4 = 1250$ 5

. . . . . .

. . . . . .

Basic step:

$n = 1$

R.H.S. $= 2\times 5^{n-1} = 2\times 5^{1-1} = 2 =$ L.H.S.

Inductive steps:

Let's assume that for $n = k$,

the k^{th} term satisfies the given equation,

$$a_k = 2\times 5^{k-1}$$

Now,

we have to show that the given equation is satisfied by the a_{k+1} term that is

$$a_{k+1} = 2 \times 5^{k+1-1} = 2 \times 5^{k}$$

$$a_{k+1} = 5a_k$$
$$= 5 \times 2 \times 5^{k-1}$$
$$= 2 \times 5^{k-1+1}$$
$$= 2 \times 5^{k}$$

(proved)

L-18/ 22.07.2022

Recursive Set

⊛ Set of integers that is divisible by 3.

$\{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, \dots\}$

$\Rightarrow$

Basic Step:

$S = \{3\}$

Recursive step:

if $x \in S, y \in S$, then

$x + y \in S$

iteration 1: $S = 3$

$x \in S, x = 3$

$y \in S, y = 3$

$x + y = 6 \in S$

$S_1 = \{3\} \cup \{6\} = \{3, 6\}$

iteration 2: $x \begin{cases} 3 \\ 6 \end{cases}$

$y \begin{cases} 3 \\ 6 \end{cases}$

	3	6
3	6	9
6	9	12

$S_2 = S_1 \cup \{9, 12\}$

$= \{3, 6, 9, 12\}$

iteration 3:

	3	6	9	12
3	6	9	12	15
6	9	12	15	18
9	12	15	18	21
12	15	18	21	24

$S_3 = S_2 \cup \{15, 18, 21, 24\}$

$= \{3, 6, 9, 12, 15, 18, 21, 24\}$

relation

Relation → $aRb \equiv (a, b) \in R$

Relation

→ Relate two or more sets

→ Basic form of relation is to form ordered pair between the sets.

A, B are sets.

Binary relation form A to B is defined, as the

$\boxed{\text{subset of } A \times B}$ → Cartesian product.

$R \subseteq A \times B$

⊛

$A = \{0, 1, 2\}$

$B = \{p, q\}$

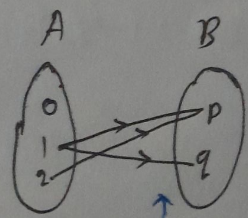
How many relations are possible from A to B and B to A.

⇒ Here

$|A \times B| = 6$

subset number of $A \times B = 2^6 = 64$

⊛ Representation of Relation:



1. Arrow diagram
2. Tabler form
3. Matrix notation

for $\{(1,p), (1,q), (2,p)\}$

	p	q
0		
1	x	x
2	x	

$$\begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

⊛ Matrix notation:

m = row

n = column

$$\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}_{m \times n}$$

$m \rightarrow$ 1st set

$n \rightarrow$ 2nd set

⊛

$A = \{0, 1, 2\}$

$B = \{p, q\}$

⇒ Previous relation:

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} p & q \end{array} \\ \begin{bmatrix} \overset{0}{(0,p)} & \overset{0}{(0,q)} \\ \overset{1}{(1,p)} & \overset{1}{(1,q)} \\ \overset{1}{(2,p)} & \overset{0}{(2,q)} \end{bmatrix} \end{array} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$= M_R$

$\hookrightarrow$ Relation of matrix

✻

Given

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{1, 2, 3, 4, 5\}$$

Draw the M_R , where

$$R = \{(a,b) \mid a \le b,\ a \in A,\ b \in B\}$$

Relation from A to B

$$\left[\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & (1,1) & (1,2) & (1,3) & (1,4) & (1,5) \\ 2 & \cancel{(2,1)} & (2,2) & (2,3) & (2,4) & (2,5) \\ 3 & \cancel{(3,1)} & \cancel{(3,2)} & (3,3) & (3,4) & (3,5) \\ 4 & \cancel{(4,1)} & \cancel{(4,2)} & \cancel{(4,3)} & (4,4) & (4,5) \\ 5 & \cancel{(5,1)} & \cancel{(5,2)} & \cancel{(5,3)} & \cancel{(5,4)} & (5,5) \end{array}\right] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

L-19 / 02. 08.2022

Relation

→ between sets

→ Subset of $A \times B$

→ $2^{|A \times B|} = 2^{|A| \cdot |B|}$

Relation from A to B Is the subset of $A \times B$

Relation $\underbrace{\text{from A to A}}_{\text{on the set A}}$ is the subset of $A \times A$

Types of Relation:

* Reflexive Relation:

a Relation R is Reflexive if ordered pair (a, a) is in R for all $a \in A$

$A = \{1, 2, 3\}$

number of Relation: 2^9

$R_1 = \{(1,2), (2,3), (3,3)\}$

is it reflexive ? ⇒ No

$R_2 = \{(1,1), (2,2), (3,3)\}$

it is Reflexive

$\Delta = \{(1,1), (2,2)\}$

$R \cup \Delta \equiv$ Reflexive closure of R_1

$R_1 = \{(1,2),(2,3),(3,3)\}$

$\Delta = \{(1,1),(2,2)\}$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & \cancel{0}1 & 1 & 0 \\ 2 & 0 & \cancel{0}1 & 1 \\ 3 & 0 & 0 & 1 \end{array}$$

$\rightarrow$ Sum = Trace = 3 ≡ number of rows or collumn

⊛ Is the divides relation on the set of positive integers reflexive? ⇒ Yes.

⊛ Is the divides relation on the set of integers reflexive? ⇒ No

↳ because (0,0) is undefined.

⊛ <u>Symmetric Relation:</u>

↳ if $(a,b) \in R$

then, (b,a) should be in R as well.

$A = \{1,2,3\}$

R : Relation on the set A

↳ $\{(1,2), (1,1), (1,3)\}$ → Not symmetric

Because,

$(2,1) \notin R$

$(3,1) \notin R$

$\Delta = \{(2,1), (3,1)\}$

$R \cup \Delta \equiv$ Symmetric closure of R

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ \cancel{0}1 & 0 & 0 \\ \cancel{0}1 & 0 & 0 \end{bmatrix} \end{array}$$

Foldable/matrix is symmetric

or $A = A^T$

L-20 / 05.08.2022 /

Relation
- → Matrix Notation
- → Reflexive
- → Anti-Symmetric

$A = \{1,2,3\}$

$R_1 = \{(1,2), (2,1), (2,3), (3,2)\}$ ⟶ Symmetric

$R_2 = \{(1,2), (2,3)\}$ ⟶ Anti-Symmetric (No symmetric trace available)

$R_3 = \{(1,2), (2,1), (2,3)\}$ ⟶ Not Symmetric
Not Anti-Symmetric

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

≡ 1 superimpose with 1

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

≡ 1 superimpose with 0

* A relation R is anti-symmetric if for $(a,b) \in R$ there's $(b,a) \in R$ then,

$$a = b$$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Reflexive

Symmetric

Anti-Symmetric

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Symmetric

Anti-Symmetic

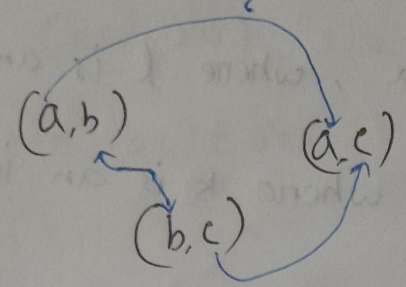
Innocent until proven guilty

* Is the divides relation on the set of positive integers Symmetric / Anti-Symmetric or both?

$\Rightarrow$ Anti-Symmetric.

Transitive

$\forall a \forall b \forall c \left(\left((a,b) \in R \cap (b,c) \in R \right) \rightarrow (a,c) \in R \right)$



$$\begin{tikzcd} a \arrow[r] \arrow[dr] & b \arrow[d] \\ & c \end{tikzcd}$$

A Relation R on the set A is called transitive whenever $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$

$\forall a,b,c \in A$

$A = \{1,2,3,4\}$

$R = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$

→ transitive

For this ⇒ counter example

$R = \{(1,1), (1,2), (2,1)\}$

→ not transitive

* divides relation is transitive.

*

a divides b $\Rightarrow$ $b = la$, where l is an integer

b divides c $\Rightarrow$ $c = kb$, where k is an integer

$\therefore$ $c = kb$ product of two integer

$= kla$

$= ja$ where $j = kl$

$c = ja$

$\Rightarrow$ c is a multiple of a

$\Rightarrow$ "a divides c"

*

Composite Relation:

R is a relation from A to B

S is a relation from B to C

Then a composit relation formed by R and S is

$S \circ R$ where,

$(a,b) \in R$

$(b,c) \in S$

then $(a,c) \in S \circ R$

⊛ $R \circ R = R^2$

$R^3 = R^2 \circ R$

$R^{n+1} = R^n \circ R$

$\{1,2,3\}$ to $\{1,2,3,4\}$

$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$

$\{1,2,3,4\}$ to $\{0,1,2\}$

$S = \{(1,0), (2,0), (3,1), (3,2), (4,0)\}$

$S \circ R = \{(2,1), (3,0), (1,0), (2,2)\}$

Bullian Product

$S \circ R = \begin{bmatrix} M_R \end{bmatrix} \cdot \begin{bmatrix} M_S \end{bmatrix}$

$M_{R^2} = \cancel{R}\, M_R \cdot M_R$

L-21/ 07.08.2022

Counting

Sum Rule & Product Rule

Task 1

|sub1| n_1 ways

|sub2| n_2 ways

$n_1 \times n_2$ ways to finish task 1

List1 $\cap$ List 2

$= |\emptyset| = 0$

Lis 1 $\cap$ List 2

$= \emptyset$

Inclusion - Exclusion

Principle

L-22/ 26.08.2022/

Counting

Sum Rule:

$A \cap B = \emptyset$

$|A \cap B| = |\emptyset| = 0$

Project

List 1

List 2

$B = \{a_1 .. a_{17}\}$

$A = \{1, ... 15\}$

Number of possible ways to chose a project $= |A| + |B|$

Project Rule

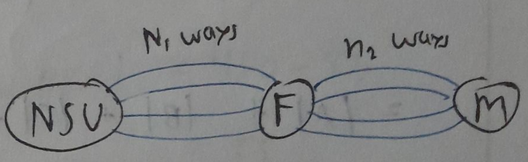
Task

Part A

n_1

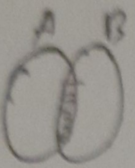
Part B

n_2



$n_1 \times n_2$ ways to travel from NSU to M.

Inclusion. Enclusion. Principle



$\longrightarrow A \cup B$

$$|A \cup B| = \underbrace{|A| + |B|}_{\text{inclusion}} - \underbrace{|A \cap B|}_{\text{enclusion}}$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$= |A \cup (B \cup C)| = |A \cup M|$$

$$= |A| + |M| - |A \cap M|$$

$$= |A| + |B \cup C| - |A \cap (B \cup C)|$$

$$= |A| + |B| + |C| - |B \cap C| - (A \cap (B \cup C))|$$

$$= |A| + |B| + |C| - |B \cap C| - |(A \cap B) \cup (A \cap C)|$$

$$= |A| + |B| + |C| - |B \cap C| - \{ |A \cap B| + |A \cap C| - |A \cap B \cap A \cap C]$$

$$= |A| + |B| + |C| - |B \cap C| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$$

* ~~450 Application~~

* 450 Applicants

$\rightarrow |CSE| = 275$

$\rightarrow |BBA| = 150$

$\rightarrow |CSE \cap BBA| = 30$

How many Job applicants are neither from CSE nor from BBA.

$\Rightarrow$

$$|CSE \cup BBA| = |CSE| + |BBA| - |CSE \cap BBA|$$

$$= 275 + 150 - 30$$

$$= 395$$

$\therefore$ All applicant $- |CSE \cup BBA| = 450 - 395$

$= 55$ Ans

*

Spanish : 1232

French : 879

Russian : 114

$S \cap F = 103$

$S \cap R = 23$

$F \cap R = 14$

2092 students are learning one of the three languages. → $S \cup F \cup R = 2092$

Q. How many students are learning all the three languages. $S \cap F \cap R = ?$

⇒

$$|S \cup F \cup R| = |S| + |F| + |R| - |S \cap F| - |S \cap R| - |F \cap R| + |S \cap F \cap R|$$

$$|S \cap F \cap R| = 2092 - 1232 - 879 - 114 + 103 + 23 + 14$$

$$= 2232 - 2225$$

$$= 7$$ Ans

Permutation & Combination

Permutation

a b c

a c b

b a c

b c a

c a b

c b a

$^3P_3 = 6$

$n! = 3! = 6$

$^3C_3 = 1$

combination $\Rightarrow 1$

$$\boxed{^nC_r \times r! = {^nP_r}}$$

$$\boxed{^nP_r = \frac{n!}{(n-r)!}}$$

$$\boxed{^nC_r = \frac{n!}{(n-r)!\,r!} = \frac{^nP_r}{r!}}$$

ABCDEFG

How many permutations are possible using the given letters that contain

1. String ABC and CEF $\longrightarrow$ ABCEF D G $\quad ^3P_3 = 6$
2. BDE and DAC $\longrightarrow$ Not possible $\Rightarrow 0$.

L-23 / 28.08.2022 /

Counting

$^nP_r = {^nC_r} \cdot r!$

* How many strings of five ASCII characters contains at least one @ ?

$\to 128^5 - 127^5$

with everything

$\underline{128}\ \underline{128}\ \underline{128}\ \underline{128}\ \underline{128}$

without @

$\underline{127}\ \underline{127}\ \underline{127}\ \underline{127}\ \underline{127}$

$\therefore 128^5 - 127^5$

(*) $^nC_r = {}^nC_{n-r}$

$$\text{L.H.S.} = \frac{n!}{(n-r)!\,r!} \qquad \text{R.H.S.} = \frac{n!}{(n-(n-r))!\,(n-r)!}$$

$$= \frac{n!}{(n-n+r)!\,(n-r)!}$$

$$= \frac{n!}{(n-r)!\,r!}$$

$$= \text{L.H.S.}$$

(Proved)

(*) How many poker hands of five can be dealt from standard deck of 52 cards?

$\Rightarrow C(52,5) = {}^{52}C_5$ Ans

(*) How many ways are there to select 47 cards from the same deck of 52

$^{52}P_{47}$

Binomial Expansion

$C(n,r) \equiv {}^nC_r$ act as the coefficient of

Binomial expansion.

$(a+b)^n = ?$ $\quad (x+y)^1 =$

$(x+y)^n = ?$ $\quad (x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4 = x^4 + \dots + y^4$

$\binom{3}{0} x^{n-j} \cdot y^j + \binom{3}{1} x^{3-1} y^1 + \binom{3}{2} x^{3-2} y^2 +$

$\binom{3}{3} x^{3-3} y^3$

$1 \cdot x^3 + 3x^2y + 3xy^2 + 1 \cdot y^3$

$$\sum_{j=0}^{n=3} \binom{n}{j} x^{n-j} y^j = (x+y)^3$$

$$\sum_{j=0}^{n=4} \binom{n}{j} x^{n-j} y^{j} = (x+y)^4 = ?$$

$\Rightarrow$

$$(x+y)^4 = \binom{4}{0} x^{4-0} y^0 + \binom{4}{1} x^{4-1} y^1 + \binom{4}{2} x^{4-2} y^2 + \binom{4}{3} x^{4-3} y^3$$
$$+ \binom{4}{4} x^{4-4} y^4$$

$$= 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 \quad \underline{\text{Ans}}$$

What is the co-efficient of $x^{12}y^{13}$ in the expansion of $(2x-3y)^{25}$?

$${}^{25}C_{13} \cdot (2x)^{25-13} \cdot (-3y)^{13}$$

$$= -\, {}^{25}C_{13} \cdot 2^{12} \cdot 3^{13}$$

Graphs

node

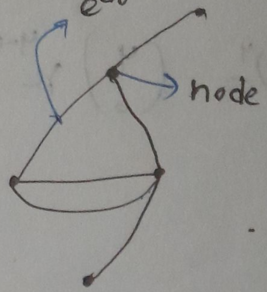
edge

$G \equiv (V, E)$

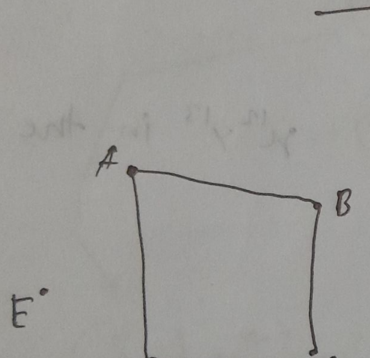
Vertices → nodes

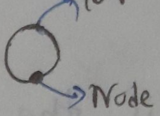
Edges

non empty



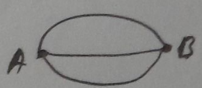
Simple Graph



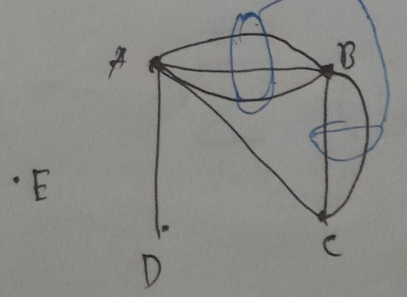


i. No loop exist

ii. No multiple edges.



Multi Graph

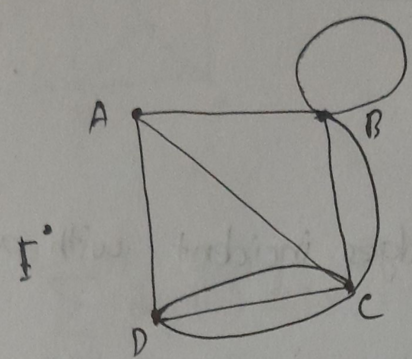


edges of multiplicity of 3 between (A-B)

& 2 between (B-C)

- Allows multiple edges.

⊛ Pseudo - Graph



⊛ Allows loop

⊛ multi edges are allowed

deg (A) = 3

deg (B) = 5

Degree of a node

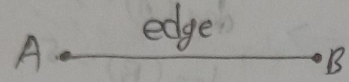
Number of edges attached to a node.

L-24 / 30.08.2022 /

Undirected Graph

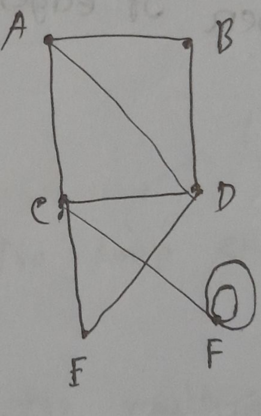
Degree of a node:

"number of edges incident with a node."



• R

$deg(R) = 0$



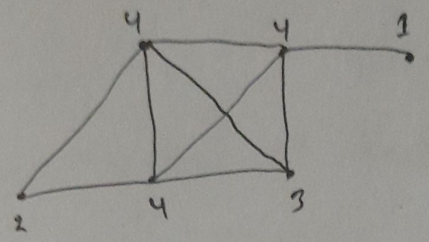
$deg(F) = 5$

$deg(D) = 4$

✻ Handshaking Theorem:

$$\sum_{u \in V} deg(u) = 2 \times e$$

number of edges



9 edges

$$\sum_{u \in V} \deg(u) = 18 = 2 \times e$$

$$\therefore e = 9$$

(*) How many edges are there in a graph with 10 vertices each of degree 6?

$$10 \times 6 = 60 = 2 \times \underbrace{30}_{\text{edges}}$$

$$\sum_{u \in V} \deg(u) = 10 \times 6 = 60 = 2 \times 30$$

$$\therefore e = 30$$

(*) "An undirected graph has an even number of vertices of odd degree."

(Prove it)

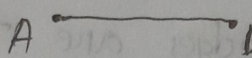
⊛ Simple Graph

K_n

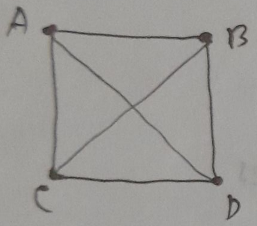
i. Complete Graph:

Any node of the graph should be completed with the rest of the nodes.

K_2 $e = 1$



$^{2}C_{2} = 1$

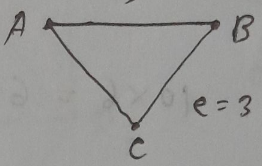


$e = 6$

$^{4}C_{2} = 6$

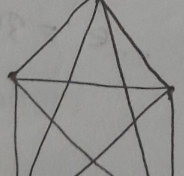
K_4

K_3



$e = 3$

$^{3}C_{2} = 3$



$^{5}C_{2} = 10$

$e = 10$

K_5

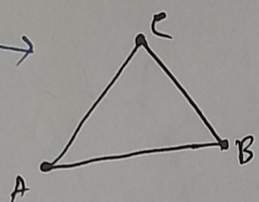
number of nodes

$^{n}C_{2}$ = number of edges we need for a complete graph of size K_n

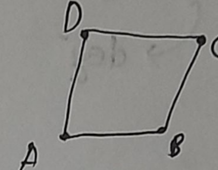
2. Cycle :

required at least 3 nodes.

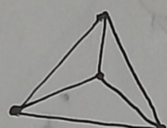
C_3 = Cycle of 3 nodes.



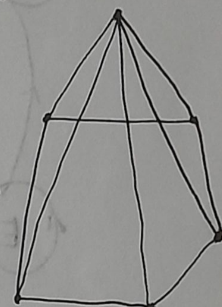
$C_4 \Rightarrow$



3. Wheel :



W_3



W_5

⊛ Directed Graph (Digraph)

$G = (V, E)$

edges are directed

represent as an ordered pair of vertices.

edge associated with (u, v) suggest at that the edge starts u and ends with v.

starts

end

$u \longrightarrow v$

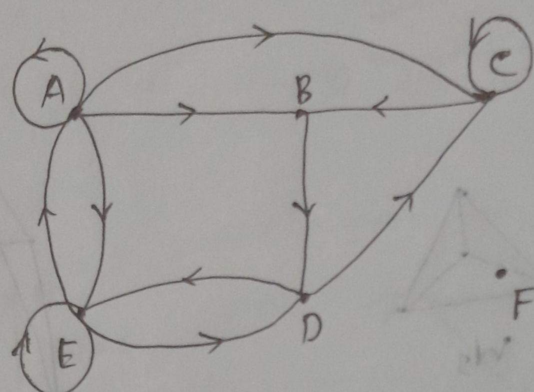
Degree of Directed Graph

Out-degree

$\hookrightarrow deg^+$

In-degree

$\hookrightarrow deg^-$



total edges = 12

$deg^-(A) = 2$

$deg^+(A) = 4$

$deg^-(C) = 3$

$deg^+(C) = 2$

$deg^-(F) = 0$

$deg^+(F) = 0$

$deg^-(B) = 2$

$deg^+(B) = 1$

$deg^-(D) = 2$

$deg^+(D) = 2$

$deg^-(E) = 3$

$deg^+(E) = 3$

$deg^- = 12$

$deg^+ = 12$

END

$e = deg^- = deg^+$