L-1 / 28.09.2022 /

Flash-Back of MAT-120

⊛ $\frac{d}{dx}(\sin x) = \cos x$; $\int \cos x \, dx = \sin x + c$

$\frac{d}{dx}(\cos x) = -\sin x$; $\int \sin x \, dx = -\cos x + c$

⊛ $\frac{d}{dx}(uv) = u \cdot \frac{d}{dx}(v) + v \cdot \frac{d}{dx}(u)$

$\frac{d}{dx}(uvw) = vw \cdot \frac{d}{dx}(u) + uw \cdot \frac{d}{dx}(v) + uv \cdot \frac{d}{dx}(w)$

⊛ $\frac{d}{dx}(\sin^2 5x) = 2 \sin 5x \cdot \frac{d}{dx}(\sin 5x)$

$= 2 \sin 5x \cdot \cos 5x \cdot 5$

$= 10 \sin 5x \cdot \cos 5x$

$\frac{d}{dx}(xy) = x \cdot \frac{dy}{dx} + y$

$\frac{d}{dx}(y^2) = 2y \cdot \frac{dy}{dx}$

⊛ $\int \cos x \, e^{\sin x} dx$

$u = \sin x$

$du = \cos x \, dx$

$\int e^u du$

$= e^u + c$

$= e^{\sin x} + c$

L-2 / 10.10.2022 /

$$\int uv\,dx$$

$$= u\int v\,dx - \int\left\{\frac{d}{dx}(u)\int v\,dx\right\}dx$$

$$\int u\,dv$$

$$= uv - \int v\,du$$

LIATE

Left $\Rightarrow u$

Right $\Rightarrow v$

⊛ $\int x e^x\,dx$

$$= x\int e^x\,dx - \int\left\{\frac{d}{dx}(x)\int e^x\,dx\right\}dx$$

$$= xe^x - \int 1\cdot e^x\,dx$$

$$= xe^x - e^x + c$$

⊛ $\int x e^x\,dx$

$u = x$ | $\int dv = \int e^x\,dx$

$du = dx$ | $v = e^x$

$$= x\cdot e^x - \int e^x\,dx$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(e^{2x}) = 2e^{2x}$$

$$\int e^{-2x}\,dx = \frac{e^{-2x}}{-2}$$

$$\int x^2 e^{-2x}\,dx$$

$$= x^2\int e^{-2x}\,dx - \int\left\{\frac{d}{dx}(x^2)\int e^{-2x}\,dx\right\}dx$$

$$= x^2\cdot\frac{e^{-2x}}{-2} - \int 2x\cdot\frac{e^{-2x}}{-2}\,dx$$

$$= \frac{x^2e^{-2x}}{-2} + \int x\cdot e^{-2x}\,dx$$

$$= \frac{x^2e^{-2x}}{-2} + \left[x\cdot\frac{e^{-2x}}{-2} - \int 1\cdot\frac{e^{-2x}}{-2}\,dx\right]$$

$$= \frac{xe^{-2x}}{-2} - \frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} + c$$

Tabular Method

Repeated Diff. (u)		Repeated Inte. (v)
x^2		e^{-2x}
	(+)	
$2x$		$\dfrac{e^{-2x}}{-2}$
	(−)	
		$\dfrac{e^{-2x}}{4}$
2	(+)	
0		$\dfrac{e^{-2x}}{8}$

$$\int x^2 e^{-2x}\,dx = -\frac{x^2 e^{-2x}}{2} - \frac{2x\cdot e^{-2x}}{4} - \frac{2\cdot e^{-2x}}{8}$$

$$= -\frac{x^2 e^{-2x}}{2} - \frac{x e^{-2x}}{2} - \frac{e^{-2x}}{4} + C$$

$$\int \ln x \cdot 1\,dx$$

$$= \ln x \int 1\,dx - \int \left\{ \frac{d}{dx}(\ln x) \int 1\,dx \right\} dx$$

$$= \ln x \cdot x - \int \frac{1}{x}\cdot x\,dx$$

$$= x\ln x - x + C$$

$$\therefore \int \ln x\,dx = x\ln x - x + C$$

⊛ $\int e^x \cos x \, dx$

$= \cos x \int e^x dx - \int \{ \frac{d}{dx}(\cos x) \int e^x dx \} dx$

$= \cos x \cdot e^x - \int -\sin x \, e^x dx$

$= e^x \cos x + \int \sin x \, e^x dx$

$= e^x \cos x + [\sin x \int e^x dx - \int \{ \frac{d}{dx}(\sin x) \int e^x dx \} dx]$

$= e^x \cos x + [\sin x \cdot e^x - \int \cos x \cdot e^x dx]$

$= e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$

$\Rightarrow \int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$

$\Rightarrow 2 \int e^x \cos x \, dx = e^x \cos x + e^x \sin x$

$\therefore \int e^x \cos x \, dx = \frac{e^x \cos x}{2} + \frac{e^x \sin x}{2} + c$

⊛ $\int (x^2 - x) \cos x \, dx = (x^2 - x) \sin x + (2x-1) \cos x - 2 \sin x + c$

(*) $\int \sqrt{x-1}\, dx$

$u = x-1$

$\frac{du}{dx} = 1$

$du = dx$

$= \int \sqrt{u}\, du$

$= \int u^{\frac{1}{2}}\, du$

$= \frac{u^{\frac{3}{2}}}{3/2} + c$

$= \frac{2}{3}(x-1)^{3/2} + c$

Ans.

(*) $\int x^2 \sqrt{x-1}\, dx$

$= \int x^2 (x-1)^{\frac{1}{2}}\, dx$

$u = x-1$

$\frac{du}{dx} = 1$

$du = dx$

$u = x-1$

$x = u+1$

$x^2 = (u+1)^2$

$= u^2+2u+1$

$= \int (u^2+2u+1)\, u^{\frac{1}{2}}\, du$

$= \int (u^{5/2} + 2u^{3/2} + u^{\frac{1}{2}})\, du$

$$= \frac{u^{5/2+1}}{5/2+1} + 2\cdot\frac{u^{3/2+1}}{3/2+1} + \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{u^{7/2}}{7/2} + 2\cdot\frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + c$$

$$= \frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + c$$

(*) $\int \tan^{-1}x \, dx = ?$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\int \frac{1}{x} dx = \ln|x|$$

(*) $\int \frac{2x}{x^2+5} dx$

$u = x^2+5$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$= \int \frac{1}{u} du$

$= \ln u + c$

$= \ln(x^2+5) + c$

$$\therefore \boxed{\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c}$$

※ $\int \tan^{-1}x \, dx$

$= \int \tan^{-1}x \cdot 1 \, dx$

$= \tan^{-1}x \int 1 \, dx - \int \left\{ \frac{d}{dx}(\tan^{-1}x) \int 1 \, dx \right\} dx$

$= \tan^{-1}x \cdot x - \int \frac{1}{1+x^2} \cdot x \, dx$

$= x \tan^{-1}x - \int \frac{x}{x^2+1} \, dx$

$= x \tan^{-1}x - \frac{1}{2} \int \frac{2x}{x^2+1} \, dx$

$= x \tan^{-1}x - \frac{1}{2} \cdot \ln(x^2+1) + c$

※ $\int_0^1 \tan^{-1}x \, dx$

$= \left[x \tan^{-1}x - \frac{1}{2} \ln(x^2+1) \right]_0^1$

$= \left(1 \cdot \tan^{-1} 1 - \frac{1}{2} \ln 2\right) - \left(0 \cdot \tan^{-1} 0 - \frac{1}{2} \ln(1)\right)$

$= \frac{\pi}{4} - \frac{1}{2} \ln 2$

$= \frac{\pi}{4} - \ln\sqrt{2}$

⊛ $\boxed{\sin 2x = 2 \sin x \cos x}$

$\cos 2x = \cos^2 x - \sin^2 x$

$= \cos^2 x - (1 - \cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x = 2\cos^2 x - 1$

$2\cos^2 x = \cos 2x + 1$

$\boxed{\cos^2 x = \frac{1}{2}(\cos 2x + 1)}$

$\boxed{\sin^2 x = \frac{1}{2}(1 - \cos 2x)}$

⊛ $\int \cos x \, dx = ?$

⊛ $\int \cos^2 x \, dx = \frac{1}{2}\int (1 + \cos 2x) \, dx$

$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right)$

$= \frac{1}{2}x + \frac{1}{2} \cdot \frac{1}{2} \cdot 2 \sin x \cos x$

$= \frac{1}{2}\cos x \sin x + \frac{1}{2}x + c$

Reduction Formula

$$\boxed{\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx}$$

$$\boxed{\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx}$$

$$\int \cos^3 x \, dx$$

$$= \frac{1}{3} \cos^2 x \sin x + \frac{2}{3} \int \cos x \, dx$$

$$= \frac{1}{3} \cos^2 x \sin x + \frac{2}{3} \sin x + C$$

MAT-130 / L-3 / 12.10.2022 /

* $\int \ln x \, dx = x \ln x - x + C$

* $\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$

* $\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + C$

* $\int \sin(\ln x) \, dx = ?$

$= \int \sin(\ln x) \cdot 1 \, dx$

$= \sin(\ln x) \int 1 \, dx - \int \left\{ \frac{d}{dx}(\sin(\ln x)) \int 1 \, dx \right\} dx$

$= x \sin(\ln x) - \int \cos(\ln x) \cdot \frac{1}{x} \cdot x \, dx$

$= x \sin(\ln x) - \int \cos(\ln x) \, dx$

$= x \sin(\ln x) - \left[x \cos(\ln x) + \int \sin(\ln x) \cdot \frac{1}{x} \cdot x \, dx \right]$

$= x \sin(\ln x) - x \cos(\ln x) - \int \sin(\ln x) \, dx$

$\Rightarrow \int \sin(\ln x) \, dx = x \sin(\ln x) - x \cos(\ln x) - \int \sin(\ln x) \, dx$

$$\Rightarrow 2\int \sin(\ln x)\,dx = x\sin(\ln x) - x\cos(\ln x)$$

$$\therefore \int \sin(\ln x)\,dx = \frac{x\sin(\ln x)}{2} - \frac{x\cos(\ln x)}{2}$$

(*) $\int \sin\sqrt{x}\,dx = ?$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$du = \frac{dx}{2u}$

$2u\,du = dx$

$$= \int \sin u \cdot 2u\,du$$

$$= 2\int \sin u \cdot u\,du$$

$$= 2\left[u\int \sin u\,du - \int\left\{\frac{d}{du}(u)\int \sin u\,du\right\}du\right]$$

$$= 2\left[-u\cos u + \int \cos u\,du\right]$$

$$= 2\left[\sin u - u\cos u\right]$$

$$= 2\sin\sqrt{x} - 2\sqrt{x}\cos\sqrt{x}$$

(*) $\int (3x^2 - x + 2)\,e^{-x}\,dx = ?$

By using Tabulan Method:

Repeated Diff.		Repeated Integ.
		e^{-x}
$3x^2-x+2$	$+$	$-e^{-x}$
$6x-1$	$-$	$+e^{-x}$
6	$+$	$-e^{-x}$
0		

$$\therefore \int (3x^2-x+2)e^{-x}\,dx = (3x^2-x+2)(-e^{-x}) - (6x-1)(e^{-x}) + 6(-e^{-x}) + c$$

$$= -e^{-x}(3x^2-x+2+6x-1+6) + c$$

$$= -(3x^2+5x+7)e^{-x} + c$$

Ans

$$\int 4x^4 \sin 2x\,dx = 4x^4\left(-\frac{\cos 2x}{2}\right) + 4x^3 \sin 2x + 6x^2 \cos 2x - 6x \sin 2x - 3\cos 2x + c$$

$$= (4x^3-6x)\sin x - (2x^4-6x^2+3)\cos 2x + c$$

By using Tabular Method.

Reduction Formula

(i) $$\boxed{\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx}$$

(ii) $$\boxed{\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx}$$

(iii) $$\boxed{\int x^n e^x\, dx = x^n e^x - n\int x^{n-1} e^x\, dx}$$

* $$\int \tan x = \int \frac{\sin x}{\cos x} = -\int \frac{-\sin x}{\cos x} = -\ln|\cos x| + c$$
$$= \ln|\sec x| + c$$

* $$\int \sec x\, dx = \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}\, dx$$
$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\, dx$$
$$= \ln|\sec x + \tan x| + c$$

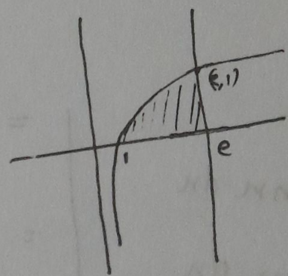
* $$\int \tan^3 x\, dx = \frac{\tan^2 x}{2} - \int \tan x\, dx$$
$$= \frac{1}{2}\tan^2 x - \ln|\sec x| + c$$

⊛ $\int x e^x \, dx = xe^x - e^x + c$

⊛ $\int x^2 e^x \, dx = x^2 e^x - 2x e^x + 2e^x + c$

⊛ $\int x^3 e^x \, dx = x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + c$

⊛ Find the area of the region enclosed by $y = \ln x$, the line $x = e$ and the x-axis.



$$A = \int_1^e (\ln x - 0) \, dx$$

$$= \left[x \ln x - x\right]_1^e$$

$$= e \cdot 1 - e - 0 + 1$$

$$= e - e + 1$$

$$= \underline{1}$$

Reduction Formula

① ~~$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x$~~

⊛ $\int \sin^5 x \, dx$

$= \int (\sin^2 x)^2 \cdot \sin x \, dx$

$= \int (1-\cos^2 x)^2 \cdot \sin x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$= -\int (1-u^2)^2 du$

$= -\int (1-2u^2+u^4) du$

$= -u + 2\frac{u^3}{3} - \frac{u^5}{5} + c$

$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + c$

⊛ $\int \sin^4 x \cos^5 x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int u^4 \cdot \cos^4 x \, du$

$= \int u^4 (1-\sin^2 x)^2 \cdot du$

$= \int u^4 (1-u^2)^2 du$

$= \int u^4 (1-2u^2+u^4) du$

$$= \int (u^4 - 2u^6 + u^8)\, du$$

$$= \frac{u^5}{5} - 2 \cdot \frac{u^7}{7} + \frac{u^9}{9} + c$$

$$= \frac{1}{5}\sin^5 x - \frac{2}{7}\sin^7 x + \frac{1}{9}\sin^9 x + c$$

(*) $\int \sin^2 x \, dx$

$$= -\frac{1}{2}\sin x \cos x + \frac{1}{2}\int 1 \, dx$$

$$= -\frac{1}{2}\sin x \cos x + \frac{1}{2}x + c$$

(*) $\int \cos^2 x \, dx = \frac{1}{2}x + \frac{1}{2}\sin x \cos x + c$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + c$$

(*) $\int \cos^3 x \, dx$

$$= \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\int \cos x \, dx$$

$$= \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x \cancel{dx} + c$$

$$= \frac{1}{3}(1 - \sin^2 x)\sin x + \frac{2}{3}\sin x + c$$

$$= \frac{1}{3}\sin x - \frac{1}{3}\sin^3 x + \frac{2}{3}\sin x + c$$

$$= \sin x - \frac{1}{3}\sin^3 x + c$$

$$\int \sin^3 x \, dx = \int \sin^2 x \sin x \, dx$$

$$= \int (1-\cos^2 x) \sin x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$$= -\int (1-u^2) \, du$$

$$= -u + \frac{u^3}{3} + c$$

$$= \frac{1}{3}\cos^3 x - \cos x + c$$

$$\int \cos^4 x \, dx = \frac{1}{4}\cos^3 x + \frac{3}{4}\int \cos^2 x \, dx$$

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\left[\frac{1}{2}\cos x \sin x + \frac{1}{2}\int 1 \, dx\right]$$

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{8}\cos x \sin x + \frac{3}{8}x + c$$

Ans.

$= \cancel{\frac{1}{4}\cos^2 x \cos x \sin x}$

Similarly:

$$\int \sin^4 x \, dx = -\frac{1}{4}\sin^3 x \cos x - \frac{3}{8}\cos x \sin x + \frac{3}{8}x + c$$

$$= \frac{1}{32}\sin 4x - \frac{1}{4}\sin 2x + \frac{3}{8}x + c$$

$$\circledast \int \tan^2 x \sec^4 x \, dx$$

$$= \int \tan^2 x \sec^2 x \sec^2 x \, dx$$

$$= \int \tan^2 x (\tan^2 x + 1) \sec^2 x \, dx$$

$u = \tan x$

$du = \sec^2 x \, dx$

$$= \int u^2 (u^2+1) \, du$$

$$= \int (u^4 + u^2) \, du$$

$$= \frac{u^5}{5} + \frac{u^3}{3} + C$$

$$= \frac{1}{5} \tan^5 x + \frac{1}{3} \tan^3 x + C$$

$$\circledast \int \tan^3 x \sec^3 x \, dx$$

$$= \int \tan^2 x \sec^2 x \sec x \tan x \, dx$$

$$= \int (\sec^2 x - 1) \sec^2 x \sec x \tan x \, dx$$

$u = \sec x$

$du = \sec x \tan x \, dx$

$$= \int (u^2-1) u^2 \, du$$

$$= \int (u^4 - u^2) \, du$$

⊛ $\int \tan 4x \sec^4 4x \, dx$

$= \int \tan 4x \sec^2 4x \sec^2 4x \, dx$

$u = \tan 4x$

$du = \sec^2 4x \cdot 4 \, dx$

$\frac{1}{4} du = \sec^2 4x \, dx$

$= \int u (\tan^2 4x + 1) \frac{1}{4} du$

$= \frac{1}{4} \int u (u^2+1) \, du$

$= \frac{1}{4} \int (u^3+u) \, du$

$= \frac{1}{4} \cdot \frac{u^4}{4} + \frac{1}{4} \cdot \frac{u^2}{2} + C$

$= \frac{1}{16} \tan^4 4x + \frac{1}{8} \tan^2 4x + C$

⊛ $\int \tan 4x \sec^4 4x \, dx$

$= \int \sec^3 4x \sec 4x \tan 4x \, dx$

$u = \sec 4x$

$\frac{du}{dx} = \sec 4x \tan 4x \cdot 4$

$\frac{1}{4} du = \sec 4x \tan 4x \, dx$

$= \int u^3 \cdot \frac{1}{4} dx$

$= \frac{1}{4} \int u^3 \, du$

$= \frac{1}{4} \cdot \frac{u^4}{4}$

$= \frac{1}{16} (\sec^4 4x)$

⊛ $\int \cos^5 x \, dx$

$= \int (\cos^2 x)^2 \cos x \, dx$

$= \int (1-\sin^2 x)^2 \cdot \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int (1-u^2)^2 \, du$

$= \int (1-2u^2+u^4) \, du$

$= u - 2 \cdot \frac{u^3}{3} + \frac{u^5}{5} + c$

$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$

⊛ $\int \sin^5 x \cos^4 x \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$= \int \sin^4 x \cdot \cos^4 x \cdot \sin x \, dx$

$= \int (\sin^2 x)^2 \cdot \cos^4 x \cdot \sin x \, dx$

$= \int (1-\cos^2 x)^2 \cos^4 x \, \sin x \, dx$

$= -\int (1-u^2)^2 u^4 \, du$

$= -\int (1-2u^2+u^4) u^4 \, du$

$= -\int (u^4 - 2u^6 + u^8) \, du$

$= -\frac{u^5}{5} + 2\frac{u^7}{7} - \frac{u^9}{9} + c$

$= -\frac{1}{5}\cos^5 x + \frac{2}{7}\cos^7 x - \frac{1}{9}\cos^9 x + c$

Reduction Formula

⊛ $\int \cot^n x \, dx = -\frac{1}{n-1} \cot^{n-1} x - \int \cot^{n-2} x \, dx$

⊛ $\int \operatorname{cosec}^n x \, dx = -\frac{1}{n-1} \operatorname{cosec}^{n-2} x \cot x + \frac{n-2}{n-1} \int \operatorname{cosec}^{n-2} x \, dx$

L-4 / 17.10.2022 /

7.4

Trigonometric Substitution

$$\circledast \int \frac{dx}{x^2\sqrt{4-x^2}}$$

$\boxed{\text{Form is } \sqrt{a^2-x^2}}$

Let,

$x = 2\sin\theta$

$dx = 2\cos\theta \, d\theta$

Here,

$\sqrt{4-x^2}$

$= \sqrt{4-(2\sin\theta)^2}$

$= \sqrt{4-4\sin^2\theta}$

$= \sqrt{4(1-\sin^2\theta}$

$= 2\cos\theta$

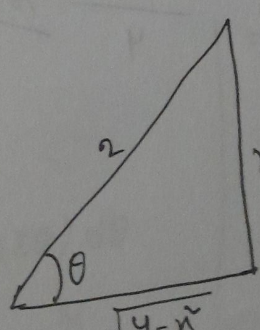
$$\therefore \int \frac{2\cos\theta \, d\theta}{4\sin^2\theta \cdot 2\cos\theta}$$

$$= \cancel{\tfrac{1}{2}}\ \frac{1}{4}\int \frac{1}{\sin^2\theta}\, d\theta$$

$$= \frac{1}{4}\int \operatorname{cosec}^2\theta \, d\theta$$

$$= -\frac{1}{4}\cot\theta + C$$

$$= -\frac{\sqrt{4-x^2}}{4x} + C$$



$x = 2\sin\theta$

$\sin\theta = \frac{x}{a}$

⊛ $\displaystyle\int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}}$

$$= -\frac{\sqrt{4-x^2}}{4x}\Bigg]_1^{\sqrt{2}}$$

$$= -\frac{\sqrt{4-2}}{4\cdot\sqrt{2}} + \frac{\sqrt{4-1}}{4\cdot 1}$$

$$= -\frac{\sqrt{2}}{4\cdot\sqrt{2}} + \frac{\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}-1}{4}$$ Ans.

⊛ $\displaystyle\int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}} = -\frac{1}{4}\cot\theta\Bigg]_{\pi/6}^{\pi/4}$

$$= -\frac{1}{4}\cot\frac{\pi}{4} + \frac{1}{4}\cot\frac{\pi}{6}$$

$$= -\frac{1}{4}\cdot 1 + \frac{1}{4}\cdot\sqrt{3}$$

$$= \frac{\sqrt{3}-1}{4}$$ Ans.

$\sqrt{2} = 2\sin\theta$

$\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}$

again

$1 = 2\sin\theta$

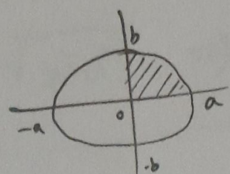
$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

(*) Area of the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$\frac{y^2}{b^2} = \frac{a^2 - x^2}{a^2}$$



$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

$$y = \pm \frac{b}{a}\sqrt{a^2 - x^2}$$

$$\text{Area} = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\, dx$$

Let,

$x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

$x = a\sin\theta$

$a = a\sin\theta$

$\sin\theta = 1$

$\theta = \frac{\pi}{2}$

$$\therefore \text{Area} = 4\int_0^{\pi/2} \frac{b}{a} \cdot a\cos\theta \cdot a\cos\theta\, d\theta$$

$$= 4ab\int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= 4ab\int_0^{\pi/2} \frac{1}{2}(1 + \cos 2\theta)\, d\theta$$

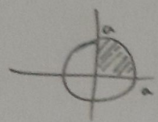
$$= 2ab\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= 2ab\left[\frac{\pi}{2} + 0 - 0\right]$$

$$= \pi ab$$

* Area of the circle,

$$x^2 + y^2 = a^2$$



$$\Rightarrow y = \pm\sqrt{a^2 - x^2}$$

$$\text{Area} = 4\int_0^a \sqrt{a^2 - x^2}\, dx$$

Let, $x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

if, $x = a$,

$a = a\sin\theta$

$\sin\theta = 1$

$\theta = \frac{\pi}{2}$

$$\therefore \text{Area} = 4\int_0^{\pi/2} a\cos\theta \cdot a\cos\theta\, d\theta$$

$$= 4a^2 \int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= 4a^2 \int_0^{\pi/2} \frac{1}{2}(1 + \cos 2\theta)\, d\theta$$

$$= 2a^2 \int_0^{\pi/2} (1 + \cos 2\theta)\, d\theta$$

$$= 2a^2 \left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= 2a^2 \left[\frac{\pi}{2} + 0 - 0\right]$$

$$= \pi a^2$$

Ans

Formula:

$$\boxed{\begin{aligned} &\sqrt{a^2-x^2} \Rightarrow x = a\sin\theta \Rightarrow \text{Simplify}, = a\cos\theta \\ &\sqrt{a^2+x^2} \Rightarrow x = a\tan\theta \Rightarrow a\sec\theta \\ &\sqrt{x^2-a^2} \Rightarrow x = a\sec\theta \Rightarrow a\tan\theta \end{aligned}}$$

* Length of a curve ~~of~~ or, arc length:

$$\boxed{L = \int_a^b \sqrt{1+[f'(x)]^2}\, dx}$$

* $\int \frac{\sqrt{x^2-25}}{x} dx$

Let,
$x = 5\sec\theta$
$dx = 5\sec\theta\tan\theta\, d\theta$

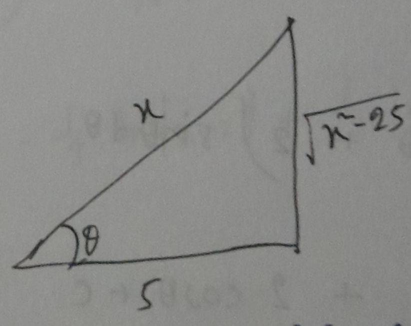
$$= \int \frac{5\tan\theta \cdot 5\sec\theta\tan\theta\, d\theta}{5\sec\theta}$$

$$= 5\int \tan^2\theta\, d\theta$$

$$= 5\int (\sec^2\theta - 1)\, d\theta$$

$$= 5[\tan\theta - \theta] + c$$

$$= 5\tan\theta - 5\theta + c$$



$\sec\theta = \frac{x}{5}$

$\cos\theta = \frac{5}{x}$

$$= 5\,\frac{\sqrt{x^2-25}}{5} - 5\cdot \sec^{-1}\frac{x}{5}$$

$$= \sqrt{x^2-25} - 5\sec^{-1}\cancel{x}\,\frac{x}{5}$$

Ans.

⊛ $$\int \frac{3x^3}{\sqrt{1-x^2}}\,dx$$

Let,

$$x = 1\sin\theta$$

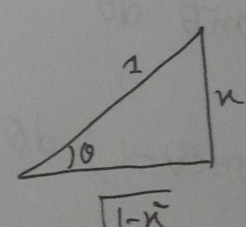
$$dx = \cos\theta\, d\theta$$

$$\therefore \int \frac{3\sin^3\theta\cdot\cos\theta\, d\theta}{\cos\theta}$$

$$= 3\int \sin^3\theta\, d\theta$$

$$= 3\left[-\frac{1}{3}\sin^2\theta\cos\theta + \frac{2}{3}\int \sin\theta\, d\theta\right]$$

$$= -\sin^2\theta\cos\theta + 2\int \sin\theta\, d\theta$$



$$= -\sin^2\theta\cos\theta + 2\cos\theta + C$$

$$= -x^2\cdot\sqrt{1-x^2} + 2\cdot\sqrt{1-x^2} + c = (2-x^2)\sqrt{1-x^2} + c$$

|L-5 | 19.10.2022|

$$\int \frac{1}{x^2+x-2}\,dx$$

Here,

$$\frac{1}{x^2+x-2} = \frac{1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$\frac{1}{(x+2)^2(x-1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1}$$

$$\frac{1}{(x^2+1)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

Now,

$$1 = A(x-1) + B(x+2)$$

$$= Ax - A + Bx + 2B$$

$$= (A+B)x + (2B-A)$$

$\therefore$ $A+B=0$

$2B-A = 1$

$\Rightarrow 2B + B = 1$

$3B = 1$

$\therefore B = \frac{1}{3}$

$\therefore A = -\frac{1}{3}$

$$\therefore \int \frac{1}{x^2+x-2}\,dx$$

$$= \int \frac{-\frac{1}{3}}{x+2}\,dx + \int \frac{1/3}{x-1}\,dx$$

$$= -\frac{1}{3}\ln|x+2| + \frac{1}{3}\ln|x-1| + c$$

Ans

(v) $\int \frac{2x+4}{x^3-2x^2}\,dx$

Here $\frac{2x+4}{x^3-2x^2} = \frac{2x+4}{x^2(x-2)}$

$$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$

Now, $(2x+4) = Ax(x-2) + B(x-2) + Cx^2$

$$= Ax^2 - 2Ax + Bx - 2B + Cx^2$$

$$2x+4 = (A+C)x^2 + \cancel{B}(B-2A)x - 2B$$

$\therefore A+C = 0 \therefore C = 2$

$B-2A = 2 \Rightarrow -2A = 4 \therefore A = -2$

$-2B = 4$

$\therefore B = -2$

$\therefore \int \frac{2x+4}{x^3-2x^2}\,dx$

$$= \int \left(\frac{-2}{x} + \frac{-2}{x^2} + \frac{2}{x-2}\right) dx$$

$$\int \frac{x^2+x-2}{3x^3-x^2+3x-1}\,dx$$

$$= \int \left(\frac{-7/5}{3x-1} + \frac{4/5\,x + 3/5}{x^2+1} \right) dx$$

$$= -\frac{7}{5}\cdot\frac{1}{3}\int \frac{3}{3x-1}\,dx + \int \frac{4/5\,x}{x^2+1}\,dx + \int \frac{3/5}{x^2+1}\,dx$$

$$= -\frac{7}{15}\ln|3x-1| + \frac{2}{5}\int \frac{2x}{x^2+1}\,dx + \frac{3}{5}\tan^{-1}x$$

$$= -\frac{7}{15}\ln|3x-1| + \frac{2}{5}\ln|x^2+1| + \frac{2}{5}\tan^{-1}x + c$$

Ans

Here,

$$3x^3-x^2+3x-1 = x^2(3x-1) + (3x-1)$$
$$= (3x-1)(x^2+1)$$

$$\therefore \frac{x^2+x-2}{3x^3-x^2+3x-1} = \frac{x^2+x-2}{(3x-1)(x^2+1)}$$
$$= \frac{A}{3x-1} + \frac{Bx+C}{x^2+1}$$

$$\therefore x^2+x-2 = A(x^2+1) + (Bx+C)(3x-1)$$
$$= Ax^2 + A + 3Bx^2 - Bx + 3Cx - C$$
$$x^2+x-2 = (A+3B)x^2 + \overset{(3C-B)}{\cancel{(B+3C)}}\,x + (A-C)$$

$$\therefore \quad A+3B = 1$$
$$-B+3C = 1$$
$$A-C = -2$$

Do echalon

$$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & -1 & 3 & 1 \\ 1 & 0 & -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & -3 & -1 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & -10 & -6 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & \frac{3}{5} \end{bmatrix}$$

$\rightarrow C = \dfrac{3}{5}$

$\rightarrow B - 3C = -1$

$B - 3 \cdot \dfrac{3}{5} = -1$

$B = -1 + \dfrac{9}{5} = \dfrac{4}{5}$

$\rightarrow A + 3B = 1$

$A + 3 \cdot \dfrac{4}{5} = 1$

$A = 1 - \dfrac{12}{5} = \dfrac{-7}{5}$

(*) $$\frac{*}{(x-2)^2(x^2+1)^2} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{Cx+D}{(x^2+1)} + \frac{Ex+F}{(x^2+1)^2}$$

L-6 / 29.10.2022

6.9

$$e^x = \underbrace{\frac{e^x + e^{-x}}{2}}_{} + \frac{e^x - e^{-x}}{2}$$

even function

Odd function

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x \ast e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

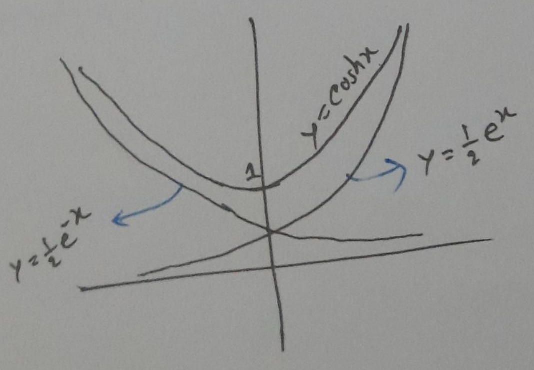
$$\operatorname{cosech} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

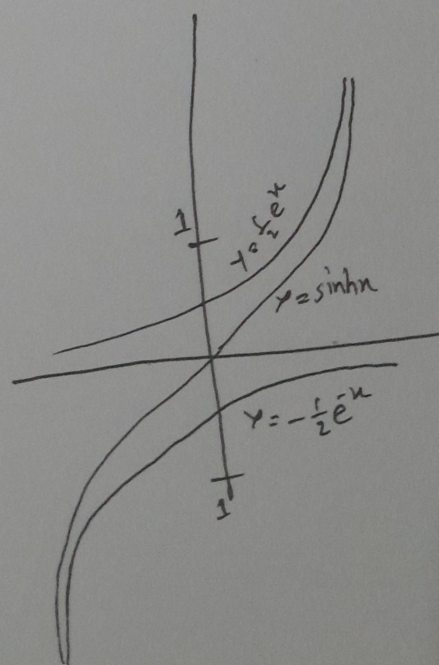
⊛

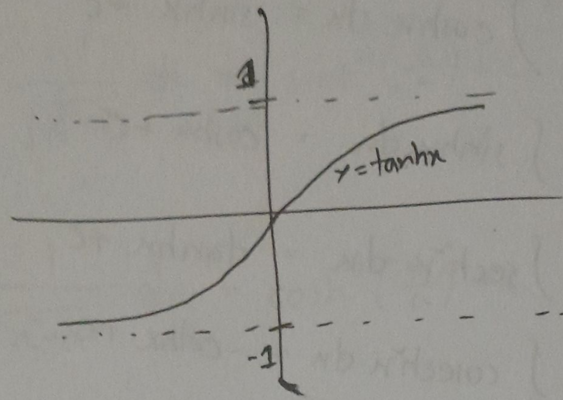
$$\sinh 0 = \frac{e^0 - e^{-0}}{2} = \frac{1-1}{2} = 0$$

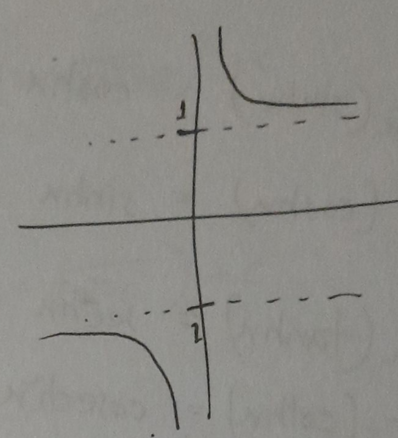
$$\cosh 0 = \frac{e^0 + e^{-0}}{2} = \frac{1+1}{2} = 1$$

⊛ Graph

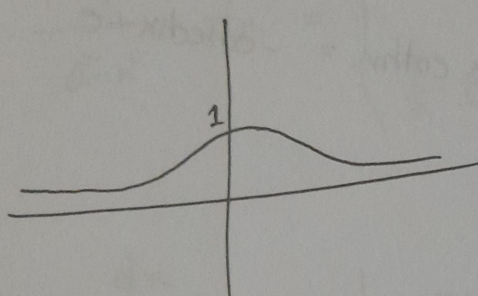




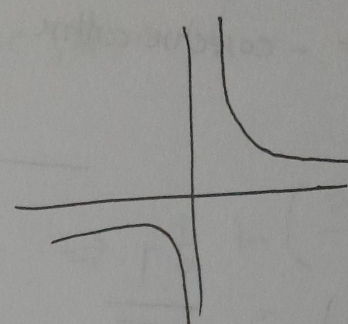




Y = coth x



Y = sech x



Y = cosech x

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$

$$\cosh x + \sinh x = e^{x}$$

$$\cosh x - \sinh x = e^{-x}$$

$$\cosh(-x) = \cosh x$$

$$\sinh(-x) = -\sinh x$$

(*)

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\frac{d}{dx}(\coth x) = \operatorname{cosech}^2 x$

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$

$\int \cosh x \, dx = \sinh x + c$

$\int \sinh x \, dx = \cosh x + c$

$\int \operatorname{sech}^2 x \, dx = \tanh x + c$

$\int \operatorname{cosech}^2 x \, dx = -\coth x + c$

$\int \operatorname{sech} x \tanh x = -\operatorname{sech} x + c$

$\int \operatorname{cosech} x \coth x = -\operatorname{cosech} x + c$

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$

$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$; $x > 1$

$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$ $|x| < 1$

$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$ $|x| > 1$

$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$ $0 < x < 1$

$\frac{d}{dx}(\operatorname{cosech}^{-1} x) = -\frac{1}{|x|\sqrt{1+x^2}}$ $x \neq 0$

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right)+c \Rightarrow \ln\left(x+\sqrt{x^2+a^2}\right)+c$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\left(\frac{x}{a}\right)+c \Rightarrow \ln\left(x+\sqrt{x^2-a^2}\right)+c$$

$$\int \frac{1}{a^2-x^2}\,dx = \left.\begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+c \,;\, |x|<a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right)+c \,;\, |x|>a \end{cases}\right\} \Rightarrow \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+c$$

$; |x| \neq a$

$$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left|\frac{x}{a}\right|+c \Rightarrow -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{|x|}\right)+c$$

$0<|x|<a$

$$\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\operatorname{cosech}^{-1}\left|\frac{x}{a}\right|+c \Rightarrow -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2+x^2}}{|x|}\right)+c$$

$x \neq 0$

L-7/26.10.2022/

$$\int \frac{dx}{\sqrt{4x^2-9}}$$

$$= \int \frac{dx}{2\sqrt{x^2-\frac{9}{4}}}$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{x^2-\frac{9}{4}}}\,dx$$

$$= \frac{1}{2}\cosh^{-1}\left(\frac{x}{3/2}\right)+c$$

$$= \frac{1}{2}\cosh^{-1}\left(\frac{2x}{3}\right)+c$$

Ans.

$y = \cosh x$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\sinh^2 x} = \cosh x$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\cosh^2 x = 1+\sinh^2 x$$

$$\boxed{\sinh x = \frac{e^x - e^{-x}}{2}}$$

㊉ $\int_{0}^{\ln 2} \cosh x \, dx$

$= \sinh x \Big]_0^{\ln 2}$

$= \dfrac{e^x - e^{-x}}{2} \Big]_0^{\ln 2}$

$= \dfrac{e^{\ln 2} - e^{-\ln 2}}{2} - 0$

$= \dfrac{2 - e^{\ln \frac{1}{2}}}{2} = \dfrac{2 - \frac{1}{2}}{2} = \dfrac{\frac{4-1}{2}}{2} = \dfrac{3}{2} \times \dfrac{1}{2} = \dfrac{3}{4}$ Ans

㊉ Show that,

$$\int_{-a}^{a} e^{xt} \, dx = \frac{2}{t} \sinh (at)$$

L.H.S. $= \int_{-a}^{a} e^{xt} \, dx$

$= \dfrac{e^{xt}}{t} \Big]_{-a}^{a}$

$= \dfrac{e^{at}}{t} - \dfrac{e^{-at}}{t} = \dfrac{e^{at} - e^{-at}}{t}$

$$= \frac{2\left(e^{at} - e^{-at}\right)}{2 \cdot t}$$

$$= \frac{2}{t} \cdot \frac{e^{at} - e^{-at}}{2}$$

$$= \frac{2}{t} \cdot \sinh(at)$$

(Showed)

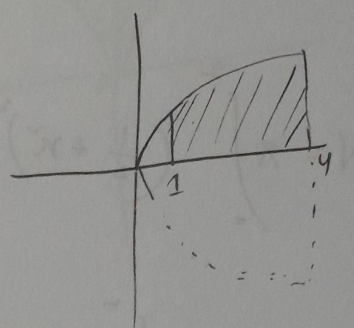
L-8 / 31.10.2022 /

L-9 / 02.11.2022 /

$$V = \int_a^b \pi r^2 \, dx$$

$$= \pi \int_a^b (f(x))^2 \, dx$$

⊛ $y = \sqrt{x}$; $[1, 4]$



$$V = \pi \int_1^4 (\sqrt{x})^2 \, dx$$

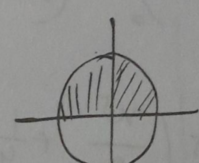
$$= \pi \int_1^4 x \, dx$$

$$= \pi \left[\frac{x^2}{2} \right]_1^4$$

$$= \pi \left(\frac{16}{2} - \frac{1}{2} \right)$$

$$= \frac{15}{2} \pi$$ Ans

⊛



$x^2 + y^2 = r^2$

$y^2 = r^2 - x^2$

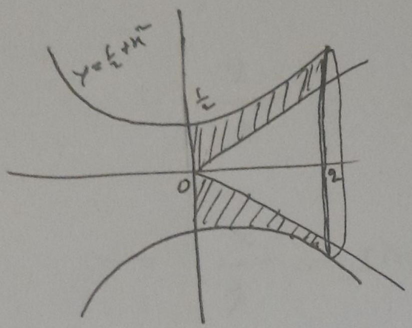
$y = \pm \sqrt{r^2 - x^2}$

$+ \rightarrow$ upper part

$- \rightarrow$ down part

$$\therefore V = 2\pi \int_0^r \left(\sqrt{r^2 - x^2}\right)^2$$

(*)



$$V = \pi \int_0^2 \left(\frac{1}{2}+x^2\right)^2 dx - \pi \int_0^2 x^2\,dx$$

$$= \pi \int_0^2 \left(x^4 + x^2 + \frac{1}{4} - x^2\right) dx$$

$$= \pi \int_0^2 \left(x^4 + \frac{1}{4}\right) dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{1}{4}x\right]_0^2$$

$$= \cdots$$

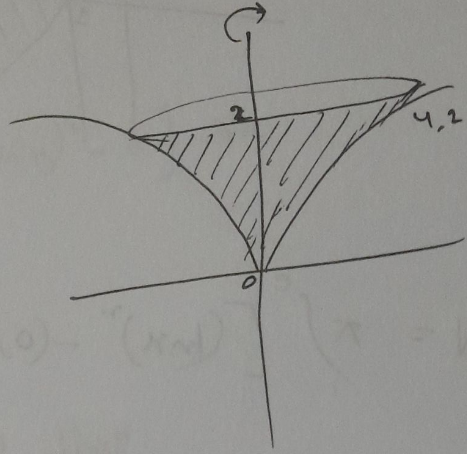
| L-10 / 07.11.2022 |

6.2

$y = \sqrt{x} \Rightarrow y^2 = x$

$y = 2$

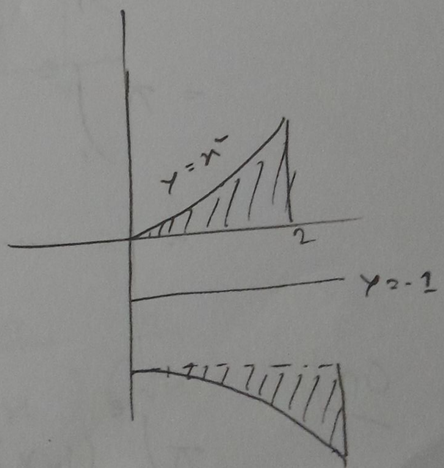
$x = 0$



$$V = \pi \int_0^2 \{(y^2) - (0^2)\} \, dy$$

$y = x^2 \quad [0, 2]$

rotated about $y = -1$



$$V = \pi \int_0^2 \left[(x^2+1)^2 - (1)^2 \right] dx$$

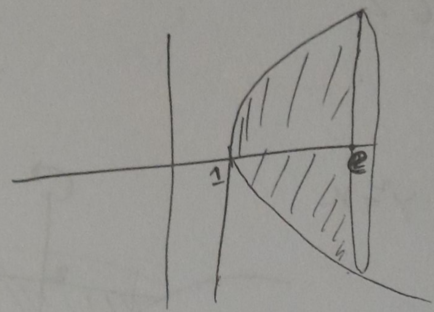
$$= \pi \int_0^2 (x^4 + 2x^2) \, dx$$

<u>6.3</u>

$y = \ln x$

$x = e$

x-axis



$$V = \pi \int_1^e \left[(\ln x)^2 - (0)^2 \right] dx$$

$$= \pi \int_1^e \left[(\ln x)^2 - 0^2 \right] dx$$

$$= \pi \int_1^e (\ln x)^2 \cdot dx$$

$$= \pi \int_1^e u^2 \cdot e^u \, du$$

$$= \ldots —$$

let,

$u = \ln x \Rightarrow e^u = x$

$du = \frac{1}{x} dx$

$x\,du = dx$

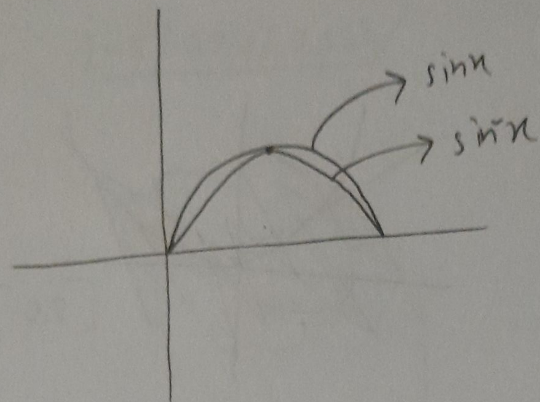
$e^u du = dx$

<u>Or/</u>

$$\pi \int_1^e (\ln x)^2 \cdot 1 \, dx$$

$$= \boxed{\int uv \, dx} \quad \ldots\ldots$$

(*)



$$V = \pi \int_0^{\pi} \left[(\sin^{-1} x)^2 - 0 \right] dx$$

6.3

Cylindical Shell

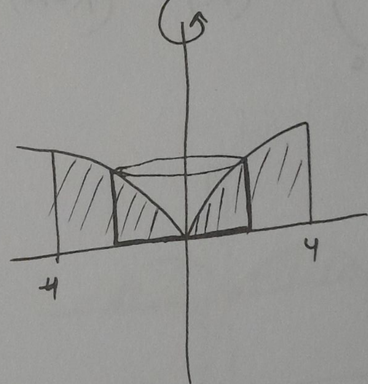
(*) $y = \sqrt{x}$

$x = 1$

$x = 4$

x-axis

rotate about y-axis

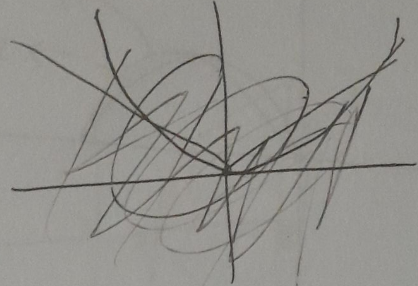


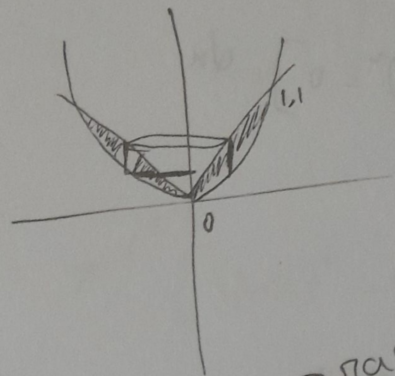
$$V = 2\pi \int_1^4 \overset{r}{(x)} \overset{h}{(\sqrt{x})} \, dx$$

$$= 2\pi \int_1^4 x\sqrt{x} \, dx$$

⊛ $y = x^2$

$y = x$





$$V = 2\pi \int_0^1 \overset{\text{radius}}{(x)} \quad \overset{\text{height}}{(x - x^2)} \, dx$$

L-11 / 02.11.2022

$y = x^2$

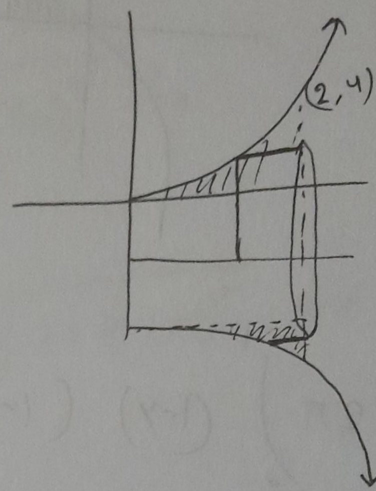
$x = \pm\sqrt{y}$

$x = \sqrt{y}$

⊛ $y = x^2 \quad [0,2]$

$y = -1$

↪ rotate



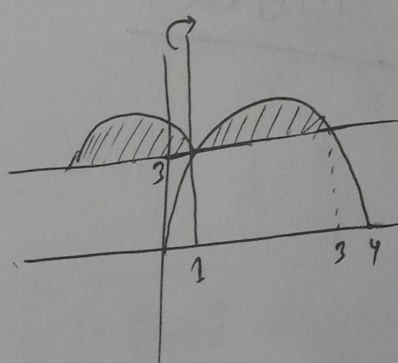
$$V = 2\pi \int_0^4 (y+1)\,(2-\sqrt{y})\,dy$$

⊛ $y = 4x - x^2$

$y = 3$

$x = 1$

↪ rotate



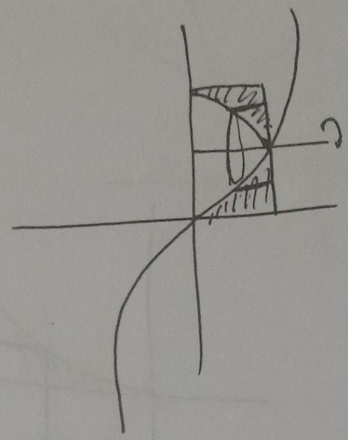
$$V = 2\pi \int_1^3 (x-1)\,(4x-x^2-3)\,dx$$

$y = x^3$

$y = 0$

$y = 1$

↪ rotate



$y = x^3$

$x = \sqrt[3]{y}$

$$V = 2\pi \int_0^1 (1-y)\left(1-\sqrt[3]{y}\right) dy$$

Midterm

L-14/21.11.2022/

6.5

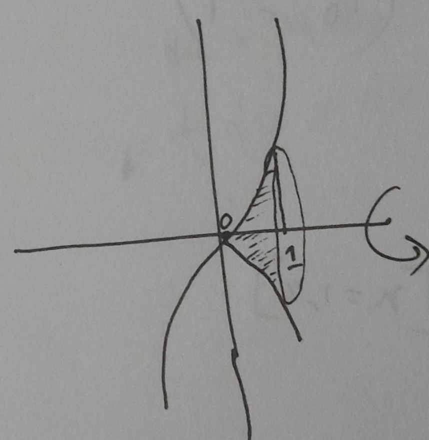
$$S.A. = 2\pi \int_a^b f(x) \cdot \sqrt{1+(f'(x))^2}\, dx$$

or,

$$S.A = 2\pi \int_c^d f(y) \sqrt{1+(f'(y))^2}\, dy$$

$y = x^3$ $[x = 0, 1]$

x- axis



$$SA = 2\pi \int_0^1 x^3 \sqrt{1+(3x^2)^2}\, dx$$

u	x
1	0
10	1

$$= 2\pi \int_0^1 x^3 \sqrt{1+9x^4}\, dx$$

Let,
$u = 1+9x^4$
$du = 36x^3\, dx$
$x^3 dx = \frac{1}{36} du$

$$= 2\pi \int_1^{10} \sqrt{u}\ \frac{1}{36}\, du$$

$$= 2\pi \frac{1}{36} \int_1^{10} \sqrt{u}\, du$$

$$= 2\pi \frac{1}{36} \int_1^{10} u^{\frac{1}{2}}\, du$$

$$= \frac{\pi}{18} \left[\frac{u^{3/2}}{3/2} \right]_1^{10}$$

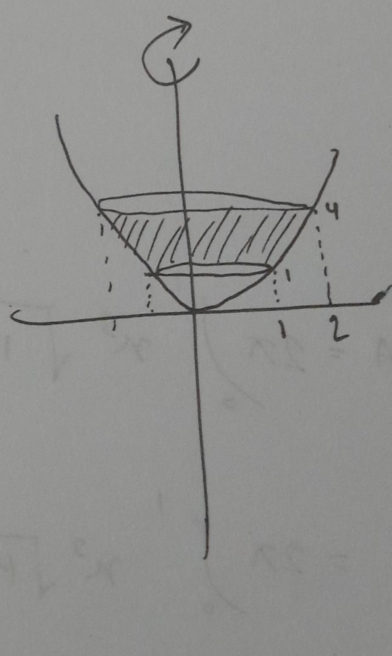
$$= \frac{\pi}{18} \cdot \frac{2}{3} \left[u^{3/2} \right]_1^{10}$$

$$= \frac{\pi}{27} \left(10^{3/2} - 1 \right)$$

(*)

$y = x^2 \; [x = 1, 2]$

y-axis



$$SA = 2\pi \int_1^4 \sqrt{y}\,\sqrt{1+\frac{1}{4y}}\;dy$$

$x = \sqrt{y}$

$\frac{dx}{dy} = \frac{1}{2} y^{-\frac{1}{2}}$

$\left(\frac{dx}{dy}\right)^2 = \frac{1}{4} y^{-1}$

$= \frac{1}{4y}$

$$= 2\pi \int_1^4 \sqrt{y}\,\sqrt{\frac{1}{4y}(4y+1)}\;dy$$

$$= 2\pi \int_1^4 \sqrt{y}\cdot\frac{1}{2\sqrt{y}}\,\sqrt{(4y+1)}\;dy$$

$$= \pi \int_1^4 \sqrt{4y+1}\;dy$$

Let,

$u = 1+4y$

$du = 4\,dy$

$\frac{1}{4}du = dy$

y	u
1	5
4	17

$$= \pi \int_5^{17} \sqrt{u}\;\frac{du}{4}$$

$$= \frac{\pi}{4} \int_5^{17} u^{\frac{1}{2}}\;du$$

$$= \frac{\pi}{4} \left[\frac{u^{3/2}}{3/2} \right]_5^{17}$$

$$= \frac{\pi}{4}\cdot\frac{2}{3}\left[u^{3/2}\right]_5^{17}$$

$$= \frac{\pi}{6}\left[(17)^{3/2} - (5)^{3/2}\right]$$

7.8

Improper Integral

⊛ $\int_{-\infty}^{b} f(x)\,dx$ ⊛ $\int_{a}^{\infty} f(x)\,dx$

⊛ $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{\infty} f(x)\,dx$

⊛ $\int_{1}^{\infty} \frac{1}{x^2}\,dx$

$= \lim_{b \to \infty} \int_{1}^{b} \frac{1}{x^2}\,dx$

$= \lim_{b \to \infty} \left[-\frac{1}{x}\right]_{1}^{b}$

$= \lim_{b \to \infty} \left[-\frac{1}{b} + 1\right]$

$= 1$

⊛ $\int_{1}^{\infty} \frac{1}{x^3}\,dx = \frac{1}{2}$

⊛ $\int_{1}^{\infty} \frac{1}{x}\,dx = \infty$

if, power $> 1 \Rightarrow$ converges
Limited area
finite area

power $\leq 1 \Rightarrow$ diverges, ∞ area

$$\circledast \quad \int_0^{\infty} (1-x)\, e^{-x}\, dx$$

$$= \lim_{b \to \infty} \int_0^{b} (1-x)\, e^{-x}\, dx$$

$$\begin{array}{ccc} 1-x & + & e^{-x} \\ & & -e^{-x} \\ -1 & - & \\ 0 & & e^{-x} \end{array}$$

$$= \lim_{b \to \infty} \left[e^{-x} - (1-x)e^{-x} \right]_0^b$$

$$= \lim_{b \to \infty} \left[e^{-x} - e^{-x} + xe^{-x} \right]_0^b$$

$$= \lim_{b \to \infty} \left[xe^{-x} \right]_0^b$$

$$= \lim_{b \to \infty} \left[be^{-b} \right)$$

$$= \lim_{b \to \infty} \left[\frac{b}{e^b} \right) \quad \left[\text{form } \frac{\infty}{\infty} \right]$$

$$= \lim_{b \to \infty} \left[\frac{1}{e^b} \right) \rightarrow \text{L'Hopital Rules.}$$

$$= 0$$

L-15 / 23.11.2022 /

(x) $\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \int_{-\infty}^{0} \frac{1}{1+x^2} dx + \int_{0}^{\infty} \frac{1}{1+x^2} dx$

$= \lim_{a \to -\infty} \int_{a}^{0} \frac{1}{1+x^2} dx + \lim_{b \to \infty} \int_{0}^{b} \frac{1}{1+x^2} dx$

$\therefore \lim_{a \to -\infty} \int_{a}^{0} \frac{1}{1+x^2} dx = \lim_{a \to -\infty} \tan^{-1} x \Big]_{a}^{0}$

$= \lim_{a \to -\infty} (-\tan^{-1} a)$

$= \frac{\pi}{2}$

$\therefore \lim_{b \to \infty} \int_{0}^{b} \frac{1}{1+x^2} dx = \lim_{b \to \infty} \tan^{-1} x \Big]_{0}^{b}$

$= \lim_{b \to \infty} (\tan^{-1} b)$

$= \frac{\pi}{2}$

$\therefore \int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2} + \frac{\pi}{2} = \pi$ Ans discontinue

$$\circledast \int_a^{b} f(x)\,dx = \lim_{k \to b^-} \int_a^k f(x)\,dx$$

$$\circledast \int_a^b f(x)\,dx = \lim_{k \to a^+} \int_k^b f(x)\,dx$$

Discontinue

discontinue

$$\circledast \int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$= \lim_{k \to c^-} \int_a^k f(x)\,dx + \lim_{k \to c^+} \int_k^b f(x)\,dx$$

$$\circledast \int_0^1 \frac{1}{\sqrt{1-x}}\,dx = \lim_{k \to 1^-} \int_0^k \frac{1}{\sqrt{1-x}}\,dx$$

$$= \lim_{k \to 1^-} \left[-2\sqrt{1-x}\right]_0^k$$

$$= \lim_{k \to 1^-} \left[-2\sqrt{1-k} + 2\right]$$

$$= 2$$

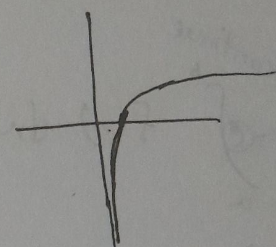
Ans

$$\circledast \int_1^2 \frac{1}{1-x}\,dx = \lim_{k\to 1^+} \int_k^2 \frac{1}{1-x}\,dx$$

$$= \lim_{k\to 1^+} \Big[-\ln|1-x|\Big]_k^2$$

$$= \lim_{k\to 1^+} \left(-\ln(1) + \ln|1-k|\right)$$

$$= \lim_{k\to 1^+} \ln|1-k|$$



$$= -\infty$$

$$\circledast \int_1^4 \frac{dx}{(x-2)^{2/3}} = \int_1^2 \frac{1}{(x-2)^{2/3}}\,dx + \int_2^4 \frac{1}{(x-2)^{2/3}}\,dx$$

$$= \lim_{k\to 2^-} \int_1^k \frac{1}{(x-2)^{2/3}}\,dx + \lim_{k\to 2^+} \int_k^4 \frac{1}{(x-2)^{2/3}}\,dx$$

$$\therefore \lim_{k\to 2^-} \int_1^k \frac{1}{(x-2)^{2/3}}\,dx = \lim_{k\to 2^-} \Big[3(x-2)^{1/3}\Big]_1^k$$

$$= \lim_{k\to 2^-} \left(3(k-2)^{1/3} + 3\right)$$

$$= 3$$

$$\therefore \lim_{k\to 2^+} \int_k^4 \frac{1}{(x-2)^{2/3}}\,dx = \lim_{k\to 2^+} 3(x-2)^{1/3}\Big]_k^4$$

$$= \lim_{k\to 2^+} \left(3\cdot 2^{1/3} - 3(k-2)^{1/3} \right)$$

$$= 3\cdot 2^{1/3}$$

$$\therefore \int_1^4 \frac{dx}{(x-2)^{2/3}} = 3 + 3\cdot\sqrt[3]{2}$$

Ans.

$\circledast$ $$\int_0^2 \frac{dx}{(x-1)^2}$$

$$= -\frac{1}{x-1}\Big]_0^2$$

$= -1-1 = -2$ (Function is positive but area negative) ✗

$\circledast$ $$\int_0^2 \frac{1}{(x-1)^2}\,dx = \int_0^1 \frac{1}{(x-1)^2}\,dx + \int_1^2 \frac{1}{(x-1)^2}\,dx$$

$$= \lim_{k\to 1^-} \int_0^k \frac{1}{(x-1)^2}\,dx + \lim_{k\to 1^+} \int_k^2 \frac{1}{(x-1)^2}\,dx$$

$$\therefore \lim_{k\to 1^-} \left[-\frac{1}{k-1} - 1 \right] = \infty$$ (No need to check other part)

Ans.: diverse.

L-16 / 28.11.2022

7.8

(*) $\int_{-\infty}^{0} e^{4x}\,dx$

$= \lim_{a \to -\infty} \int_{a}^{0} e^{4x}\,dx$

$= \lim_{a \to -\infty} \left[\frac{e^{4x}}{4} \right]_{a}^{0}$

$= \lim_{a \to -\infty} \left[\frac{1}{4} - \frac{e^{4a}}{4} \right]$

$= \frac{1}{4}$ Ans.

(*) $\int_{-\infty}^{\infty} \frac{x}{(x^2+5)^2}\,dx$

$= \int_{-\infty}^{0} \frac{x}{(x^2+5)^2}\,dx + \int_{0}^{\infty} \frac{x}{(x^2+5)^2}\,dx$

$= \lim_{a \to -\infty} \int_{a}^{0} \frac{x}{(x^2+5)^2}\,dx + \lim_{b \to \infty} \int_{0}^{b} \frac{x}{(x^2+5)^2}\,dx$

$$\therefore \lim_{a \to -\infty} \int_{a}^{0} \frac{x}{(x^2+5)^2}\, dx$$

Let,

$u = x^2+5$

$du = 2x \cdot dx$

$x\,dx = \frac{1}{2}\, du$

x	u
a	a^2+5
0	5

$$= \lim_{a \to -\infty} \int_{a^2+5}^{5} \frac{1}{u^2} \cdot \frac{1}{2}\, du$$

$$= \lim_{a \to -\infty} \left[\frac{1}{2} \cdot \frac{1}{-1} \cdot u^{-1} \right]_{a^2+5}^{5}$$

$$= \lim_{a \to -\infty} \left[\frac{1}{-2u} \right]_{a^2+5}^{5}$$

$$= \lim_{a \to -\infty} \left[\frac{1}{-10} + \frac{1}{2(a^2+5)} \right]$$

$$= -\frac{1}{10}$$

$$\therefore \lim_{b \to \infty} \int_{0}^{b} \frac{x}{(x^2+5)^2}\, dx$$

$$= \lim_{b \to \infty} \left[-\frac{1}{2u} \right]_{5}^{b^2+5}$$

$$= \lim_{b \to \infty} \left(-\frac{1}{2(b^2+5)} + \frac{1}{10} \right)$$

$$= \frac{1}{10}$$

$$\therefore \int_{-\infty}^{\infty} \frac{x}{(x^2+5)^2}\, dx = \frac{1}{10} - \frac{1}{10} = 0$$ Ans

$$\circledast \int_{\pi/3}^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}}\,dx$$

$$= \lim_{k\to \pi/3^+} \int_{k}^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}}\,dx$$

$$= \lim_{k\to \pi/3^+} \left[\sqrt{1-2\cos x}\right]_k^{\pi/2}$$

$$= \lim_{k\to \pi/3^+} \left(\sqrt{1-2\cos\frac{\pi}{2}} - \sqrt{1-2\cos k}\right)$$

$$= \lim_{k\to \frac{\pi}{3}^+} \left(1-\sqrt{1-2\cos k}\right)$$

$$= 1-\sqrt{1-2\cdot\frac{1}{2}}$$

$$= 1-0$$

$$= 1$$

Ans,

Let,

$$u = 1-2\cos x$$

$$du = 2\sin x \; dx$$

$$\therefore \int \frac{1}{\sqrt{u}}\cdot\frac{1}{2}\,du$$

$$= \frac{1}{2}\int u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{2}\cdot\frac{u^{\frac{1}{2}}}{1/2}$$

$$= \sqrt{u}$$

$$= \sqrt{1-2\cos x}$$

$\boxed{10\cdot 1}$

We need to eliminate 大 by some alzebra operation.

L-17/30.11.2022/

$x = f(t)$

$y = g(t)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$\frac{dy}{dt} \rightarrow 0, *$

$\frac{dx}{dt} \rightarrow 0, *$

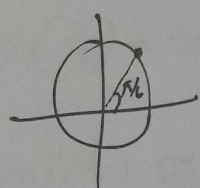
$y = f(x)$

$\frac{*}{*}$ ✓ $\qquad \frac{0}{*}$ $\longleftrightarrow$ $\qquad \frac{*}{0}$ $\updownarrow$ $\qquad \frac{0}{0}$ single point

(*)

$x = \cos t$

$y = \sin t$



$\therefore x^2 + y^2 = 1$

$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{6}} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=\frac{\pi}{6}} = \left.\frac{\cos t}{-\sin t}\right|_{t=\frac{\pi}{6}}$$

$$= -\cot t\,\Big|_{t=\frac{\pi}{6}}$$

$$= -\sqrt{3}$$

$\therefore$ Slope $= -\sqrt{3}$

(*) If need to calculate equation, then use 't' to find the point co-ordinate then use point-slope formula.

point - slope equation

$$\boxed{y - y_1 = m(x - x_1)}$$

⊗ Where the paper plane flying horizontally?

$x = t - 3\sin t$

$y = 4 - 3\cos t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3\sin t}{1 - 3\cos t}$$

$$\frac{dy}{dt} = 0$$

$$3\sin t = 0$$

$$\sin t = 0$$

$$t = \sin^{-1}(0) = 0, \pi, 2\pi, 3\pi$$

when,

$t = 0$, $\frac{dx}{dt} = 1 - 3\cos 0 = -2 \neq 0$

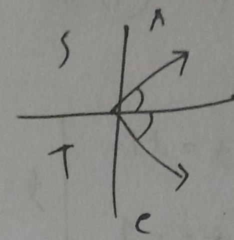
$t = \pi$, $\frac{dx}{dt} = 1 - 3\cos \pi = 1 + 3 = 4 \neq 0$

$t = 2\pi$, $\frac{dx}{dt} = 1 - 3\cos 2\pi = 1 - 3 = -2 \neq 0$

$t = 3\pi$, $\frac{dx}{dt} = 1 - 3\cos 3\pi = 1 + 3 = 4 \neq 0$

$\therefore t = 0, 3.14, 6.28, 9.42$

(*) When flying verticaly:



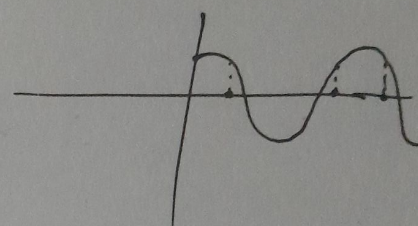
$$\frac{dx}{dt} = 1 - 3\cos t = 0$$

$$-3\cos t = -1$$

$$\cos t = \frac{1}{3}$$

$$t = \cos^{-1}\left(\frac{1}{3}\right)$$

$$t = ?$$



$$t = \cos^{-1}\left(\frac{1}{3}\right) \approx 1.23$$

$$t = 2\pi - \cos^{-1}\left(\frac{1}{3}\right) \approx 5.05$$

$$t = 2\pi + \cos^{-1}\left(\frac{1}{3}\right) \approx 7.51$$

(*) $x = t^2$

$y = t^3$

$$\frac{dy}{dx} = ? \quad (1,1)$$

$\therefore t = \pm\sqrt{x}$

$\therefore y = (\pm\sqrt{x})^3 = \pm x^{3/2}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2}{2t} = \frac{3t}{2}$$

for (1,1)

$1 = t^2 \Rightarrow t = \pm 1$

$1 = t^3 \Rightarrow t = 1$

$$\therefore \left.\frac{dy}{dx}\right|_{t=1} = \frac{3 \cdot 1}{2} = \frac{3}{2}$$

⇒ At point $(1,-1)$

$1 = t^2 \Rightarrow t = \pm 1$
$-1 = t^3 \Rightarrow t = -1$ | $t = -1$

$$\therefore \left.\frac{dy}{dx}\right|_{t=-1} = \frac{3t^2}{2t} = \frac{3}{-2} = -\frac{3}{2}$$

⇒ at point $(0,0)$

$0 = t^2 \Rightarrow t = 0$
$0 = t^3 \Rightarrow t = 0$ | $t = 0$

$$\left.\frac{dy}{dx}\right|_{t=0} = \frac{3t^2}{2t} = \frac{0}{0}$$

✱ $\frac{d^2y}{dx^2} = ?$ $(1,1), (1,-1)$

$(1,1) \to t = 1$, $(1,-1) \to t = -1$

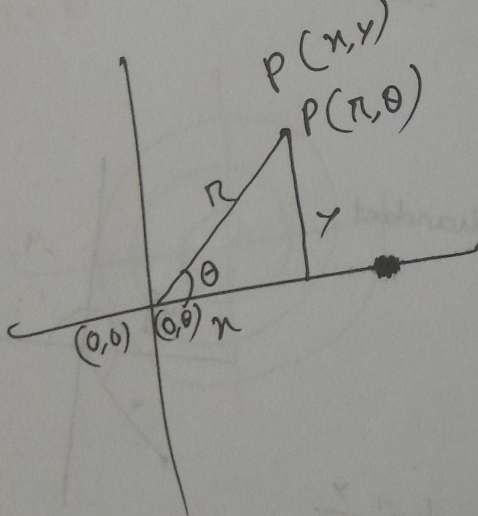
$$y' = \frac{dy}{dx} = \frac{3}{2}t$$

$$\therefore \frac{d^2y}{dx^2} = \frac{dy'}{dx} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}} = \frac{3/2}{2t} = \frac{3}{4t}$$

$$\therefore \left.\frac{d^2y}{dx^2}\right|_{t=1} = \frac{3}{4}$$

$$\therefore \left.\frac{d^2y}{dx^2}\right|_{t=-1} = -\frac{3}{4}$$

Polar Co-ordinate



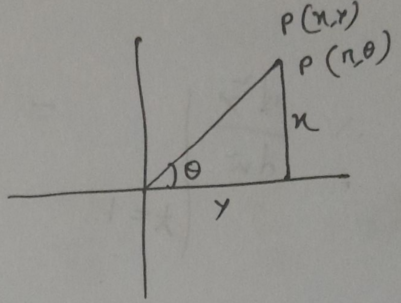
Poll $(0,\theta)$

L-18 / 05.12.2022

10·2

(*) $(r, \theta) = \left(6, \frac{2\pi}{3}\right)$

Find $(x, y) = (-3, 3\sqrt{3})$



$x = r\cos\theta$

$= 6\cos\frac{2\pi}{3}$

$= -3$

$y = r\sin\theta$

$= 6\sin\frac{2\pi}{3}$

$= 3\sqrt{3}$

$x^2 + y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$\boxed{x^2 + y^2 = r^2}$

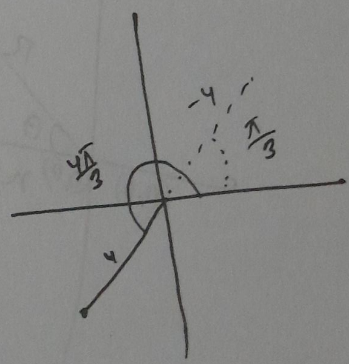
$\frac{y}{x} = \tan\theta$

$\boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$

(*)

$(x, y) = (-2, -2\sqrt{3}) \longrightarrow$ 3rd Quandant

$(r, \theta) = ? = \left(4, \frac{4\pi}{3}\right), \left(-4, \frac{\pi}{3}\right)$



$r = \sqrt{x^2 + y^2} = \sqrt{4 + 4\cdot 3}$

$= \sqrt{4 + 12}$

$= \pm 4$

$\theta = \tan^{-1}\frac{y}{x}$

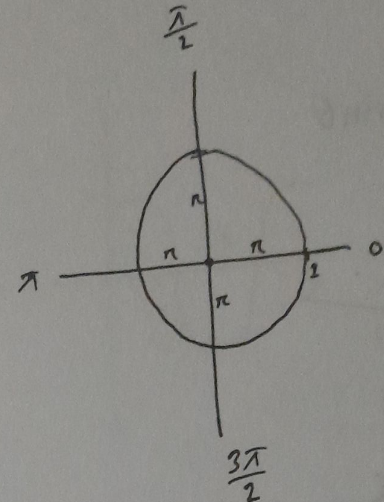
$= \tan^{-1}\frac{-2\sqrt{3}}{-2}$

$= \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}, \frac{4\pi}{3}$

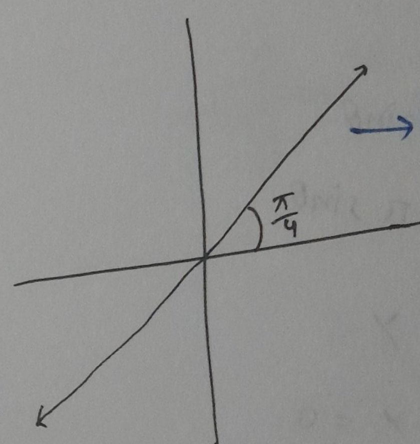
$r = 1$

$\boxed{r = a}$ → families of circle



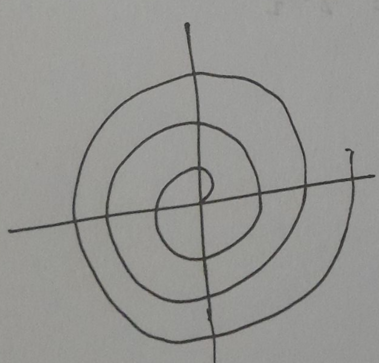
$\theta = \frac{\pi}{4}$

$\boxed{\theta = \theta_0}$ → families of straight line.



→ $\boxed{r \geq 0}$ → including zero (ray)

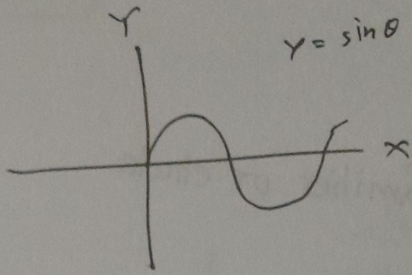
$r = \theta$

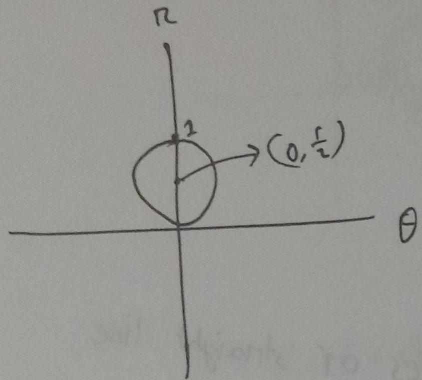


㊍

$r = \sin\theta$

θ	r
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
π	1





$(x-0)^2 + (y-\frac{1}{2})^2 = (\frac{1}{2})^2$

$\Rightarrow \quad r = \sin\theta$

$r^2 = r\sin\theta$

$x^2 + y^2 = y$

$x^2 + y^2 - y = 0$

$x^2 + y^2 - 2\cdot y\cdot\frac{1}{2} + (\frac{1}{2})^2 = (\frac{1}{2})^2$

$(x-0)^2 + (y-\frac{1}{2})^2 = (\frac{1}{2})^2$

families of circle

$\boxed{r = 2\cdot a\cdot \sin\theta}$

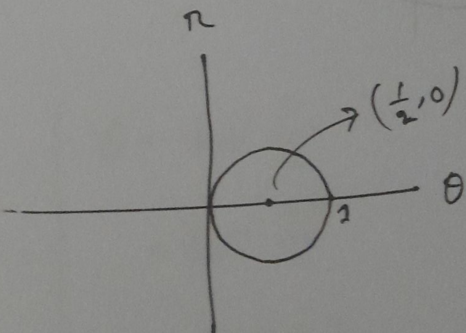
$r = 2\cdot\frac{1}{2}\cdot\sin\theta$

㊍

$r = \cos\theta$

$\boxed{r = 2\cdot a\cdot\cos\theta}$

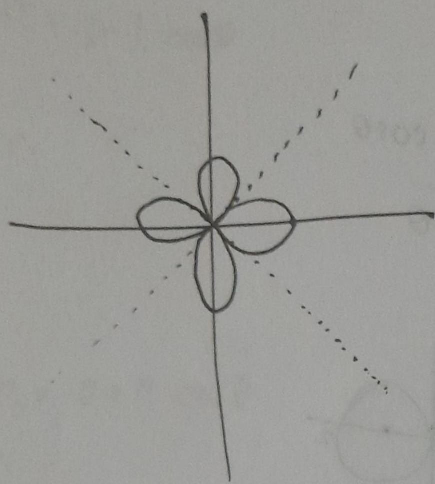
families of circle

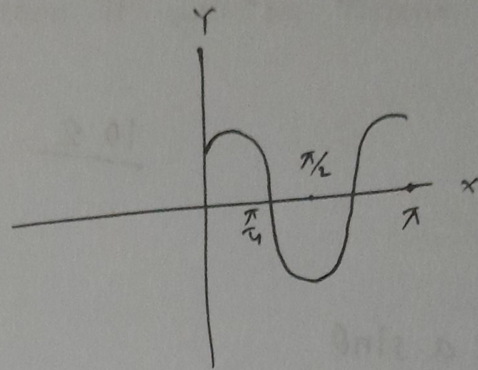


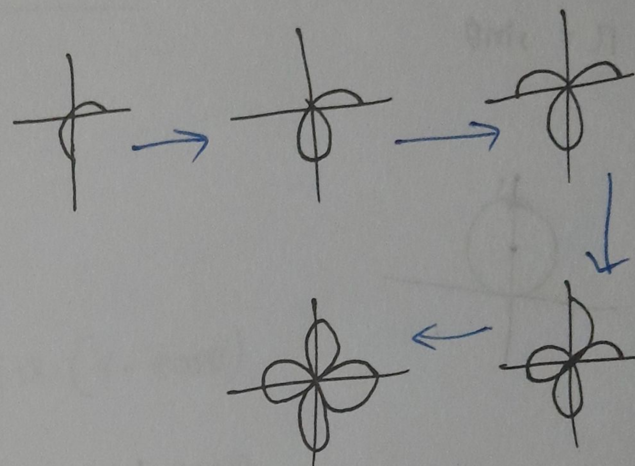
$(x-\frac{1}{2})^2 + (y-0)^2 = (\frac{1}{2})^2$

(*)

$r = \cos 2\theta$







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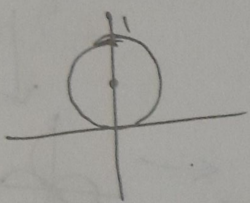
10.2

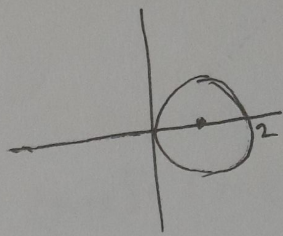
$r = 2a\sin\theta$

$r = \sin\theta$

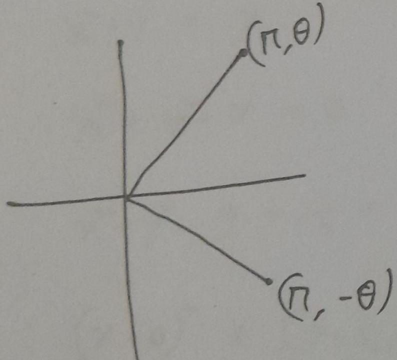


$r = 2a\cos\theta$

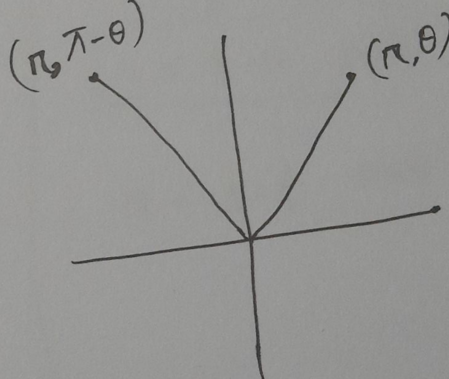
$r = \cos\theta$



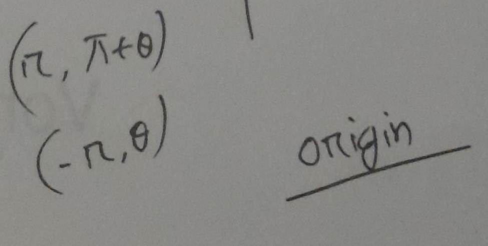
Symetrie



x-axis



y-axis



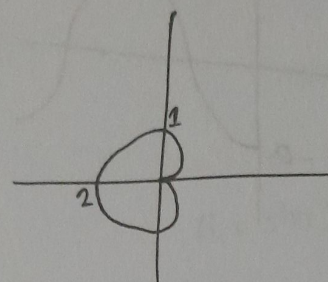
origin

$r = a \pm b \cos\theta$

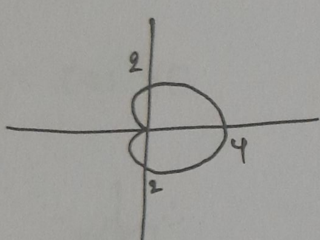
$r = a \pm b \sin\theta$

if $a = b$, then it will be heart shape.

$r = 1 - 1\cos\theta$

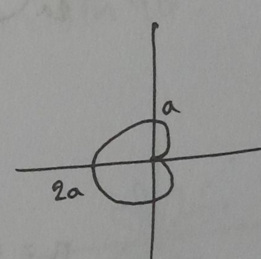


$r = 2 + 2\cos\theta$

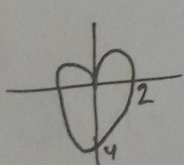


$r = a(1 - \cos\theta)$

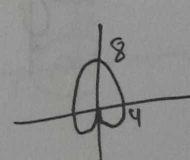
$= a - a\cos\theta$



$r = 2 - 2\sin\theta$

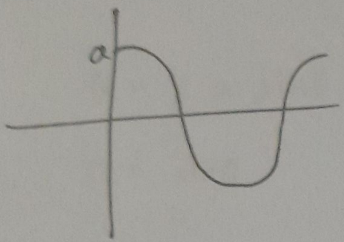


$r = 4 + 4\sin\theta$

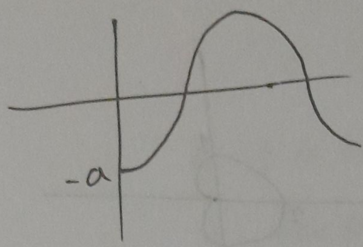


$r = \cos\theta$

$r = a\cos\theta$

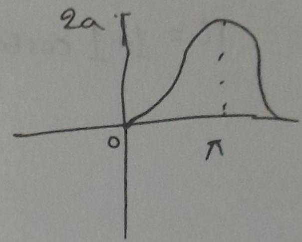


$r = -a\cos\theta$



$r = -a\cos\theta + a$

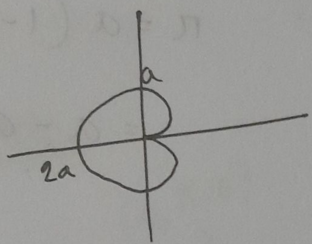
$= a - a\cos\theta$



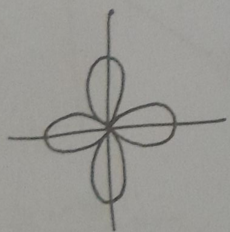
up to π its

increasing

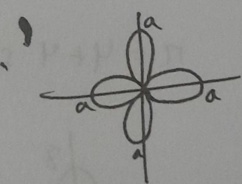
$r = a - a\cos\theta$



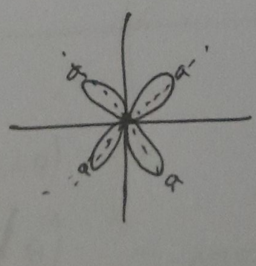
$r = \cos 2\theta$



$r = a\cos 2\theta$



$r = \sin 2\theta$



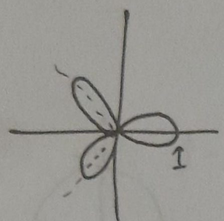
$r = \overset{\text{size}}{a} \sin n\theta$

$r = \overset{\text{size}}{a} \cos n\theta$

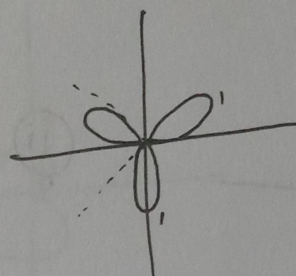
n → odd ⇒ n pattern

n → even ⇒ $2n$ pattern

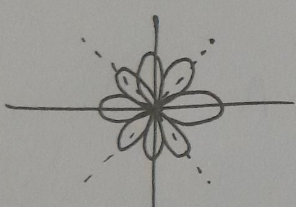
$r = \cos 3\theta$



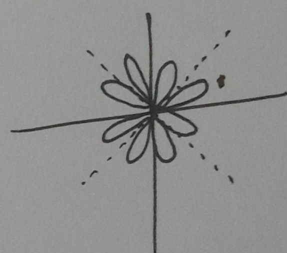
$r = \sin 3\theta$



$r = \cos 4\theta$



$r = \sin 4\theta$



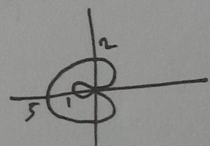
(*)

(i) $r = 2 - 3\cos\theta$

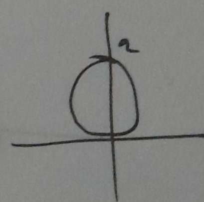
(ii) $r = 2\sin\theta$

(iii) $r = 3\cos 3\theta$

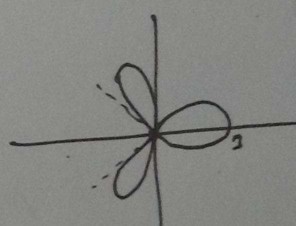
(i)



(ii)



(iii)



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⊛ Draw the graph:

i. $r = 2\sin\theta$

ii. $r = 2\cos 2\theta$

(iii) $r = 2$

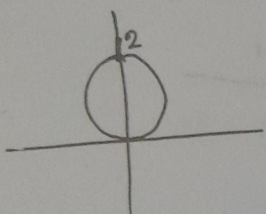
(iv) $r = 2 - 2\sin\theta$

(v) $r = 2\sin 3\theta$

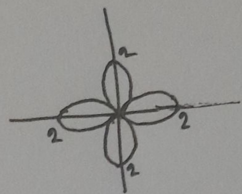
(vi) $r = 5 + 6\cos\theta$

$\Rightarrow$

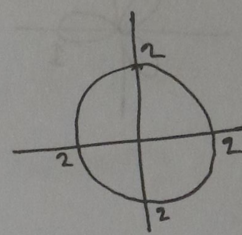
(i)



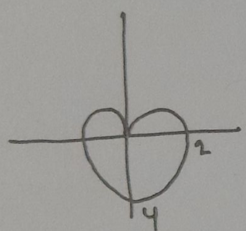
(ii)



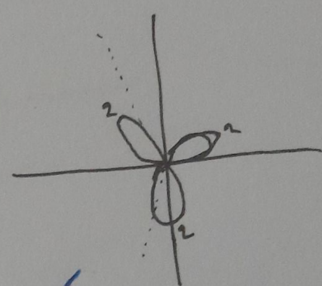
(iii)



(iv)



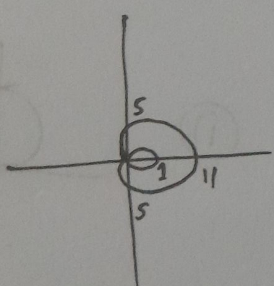
(v)



3 → Down   7 = Down

5 → Up   9 = Up

(vi)

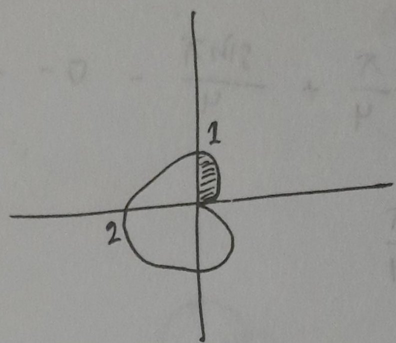


10.3

Area under the polar graph

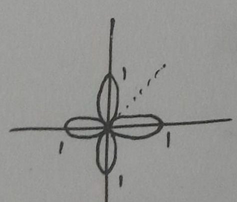
$$\text{area,} = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$$

* $r = 1 - \cos\theta$



$$A = \frac{1}{2} \int_{0}^{\frac{\pi}{2}} (1-\cos\theta)^2 \, d\theta$$

* $r = \cos 2\theta$



$$A = 8 \cdot \frac{1}{2} \int_{0}^{\pi/4} (\cos 2\theta)^2 \, d\theta$$

$$= 4 \int_{0}^{\pi/4} \cos^2 2\theta \, d\theta$$

$$= 4 \int_0^{\pi/4} \frac{1}{2} (1 + \cos 4\theta)\, d\theta$$

$$= 2 \int_0^{\pi/4} (1 + \cos 4\theta)\, d\theta$$

$$= 2 \left[\theta + \frac{\sin 4\theta}{4} \right]_0^{\pi/4}$$

$$= 2 \left[\frac{\pi}{4} + \frac{\sin \pi}{4} - 0 - \frac{\sin 0}{4} \right]$$

$$= 2 \cdot \frac{\pi}{4}$$

$$= \frac{\pi}{2} \quad \underline{\text{Ans}}$$

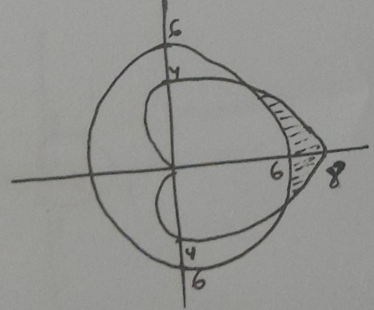
(*) $r = 6$ $\quad$ $r = 4 + 4\cos\theta$

$$4 + 4\cos\theta = 6$$

$$4\cos\theta = 2$$

$$\cos\theta = \frac{1}{2}$$

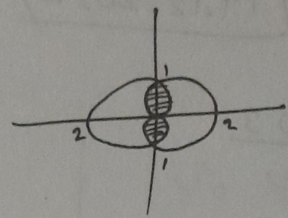
$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$



$$A = 2 \cdot \frac{1}{2} \int_0^{\pi/3} \left((4 + 4\cos\theta)^2 - 6^2 \right) d\theta$$

⊛ $r = 1 - \cos\theta$

$r = 1 + \cos\theta$



$1 - \cos\theta = 1 + \cos\theta$

$2\cos\theta = 0$

$\cos\theta = 0$

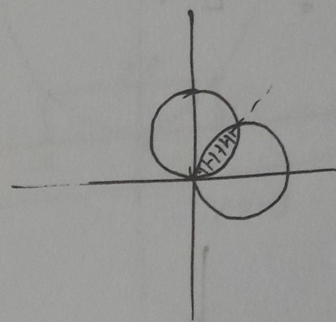
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$$A = 4 \cdot \frac{1}{2} \int_0^{\pi/2} (1 - \cos\theta)^2 \, d\theta$$

⊛

$r = 9\cos\theta$

$r = 9\sin\theta$



$9\cos\theta = 9\sin\theta$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$=$

$$A = 2 \cdot \frac{1}{2} \int_0^{\pi/4} 81 \sin^2\theta \, d\theta$$

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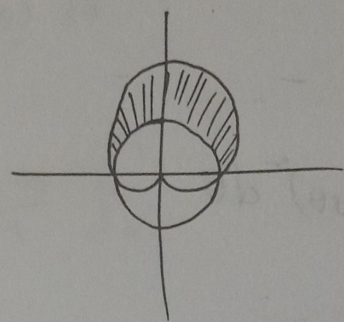
10·3

* $r = 1 + \sin\theta$

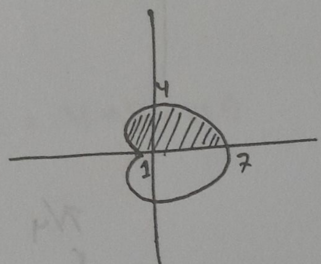
$r = 1$



$\therefore$

$$A = 2 \cdot \frac{1}{2} \int_0^{\pi/2} \left[(1+\sin\theta)^2 - 1^2 \right] d\theta$$

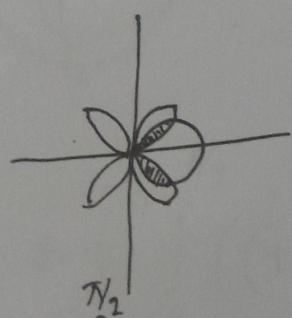
* $r = 4 + 3\cos\theta$



$$A = \frac{1}{2} \int_0^{\pi} (4 + 3\cos\theta)^2 \, d\theta$$

* $r = \sin 2\theta$

$r = \cos\theta$



$$\therefore A = 2 \left[\frac{1}{2} \int_0^{\pi/6} (\sin 2\theta)^2 d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} (\cos\theta)^2 d\theta \right.$$

$\sin 2\theta = \cos\theta$

$2\sin\theta\cos\theta = \cos\theta$

$2\sin\theta = 1$

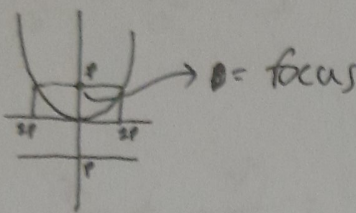
$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

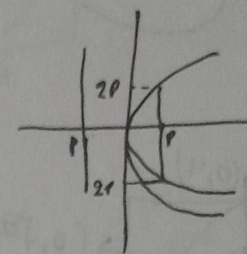
$= \frac{\pi}{6}, \frac{\pi}{2}$

10.4

$x^2 = 4py$

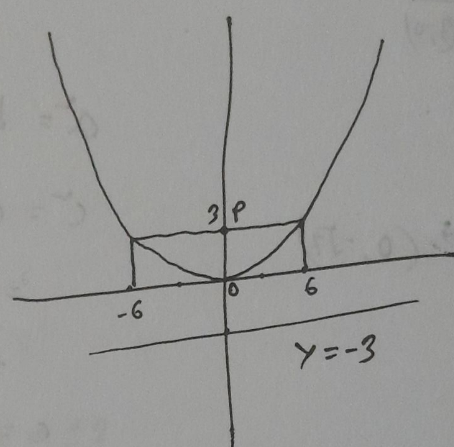


$y^2 = 4px$



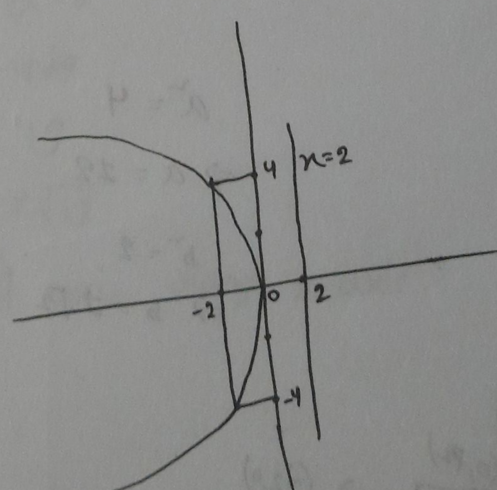
(*) $x^2 = 12y$

$= 4 \cdot 3y$



(*) $y^2 = -8x$

$= 4(-2)x$



(*) Symetric about y-axis
passes through (5,2)
→ Find the equation:

$x^2 = 4py$

$4p = \frac{x^2}{y} = \frac{25}{2}$

$\boxed{\therefore x^2 = \frac{25}{2}y}$

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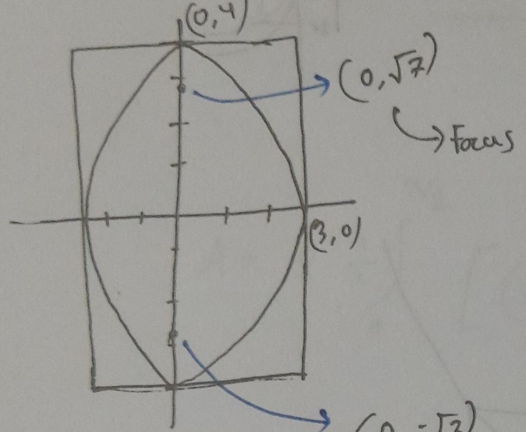
$\circledast$ $\frac{x^2}{9} + \frac{y^2}{16} = 1$ → major axis along the y-axis.

$a^2 = 16$

$\Rightarrow a = \pm 4$

$b^2 = 9$

$\Rightarrow b = \pm 3$



$a^2 = b^2 + c^2$

$c^2 = a^2 - b^2$

$= 16 - 9$

$= 7$

$c = \pm\sqrt{7}$

$\circledast$ $x^2 + 2y^2 = 4$

$x^2 + 2y^2 = 4$

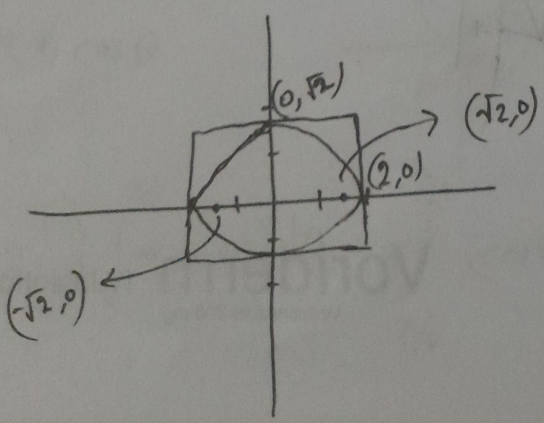
$\frac{x^2}{4} + \frac{y^2}{2} = 1$

$a^2 = 4$

$\Rightarrow a = \pm 2$

$b^2 = 2$

$\Rightarrow b = \pm\sqrt{2}$



$a^2 = b^2 + c^2$

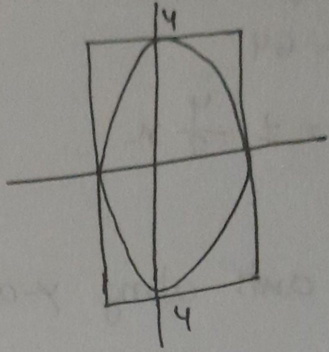
$c^2 = a^2 - b^2$

$= 4 - 2$

$= 2$

$c = \pm\sqrt{2}$

⊛ Focus $(0, \pm 2)$



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{12} + \frac{y^2}{16} = 1$$

$a = \pm 4$

$a^2 = 16$

$c = \pm 2$

$c^2 = 4$

$4 = a^2 - b^2$

$= 16 - b^2$

$b^2 = 16 - 4$

$b^2 = 12$

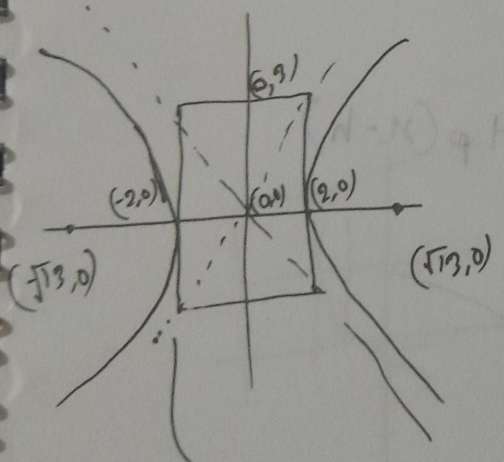
b

⊛

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

$a = \pm 2$

$b = \pm 3$



$c^2 = a^2 + b^2$

$= 4 + 9$

$= 13$

$c = \pm\sqrt{13}$

focal axis along x-axis

asymtote

$y = \pm \frac{3}{2} x$

⊛

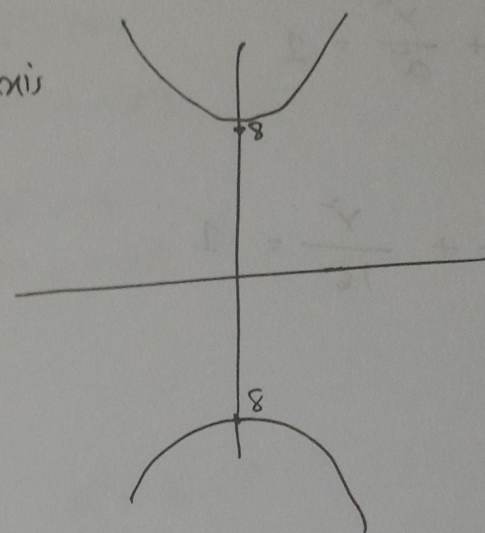
$(0, \pm 8)$ → $a = 8$

$a^2 = 64$

$y = \pm \frac{4}{3} x$

→ focal axis along y-axis

$$\frac{y^2}{64} - \frac{x^2}{36} = 1$$



⊛

$x^2 = 4py$ $(0,0)$ $y^2 = 4px$

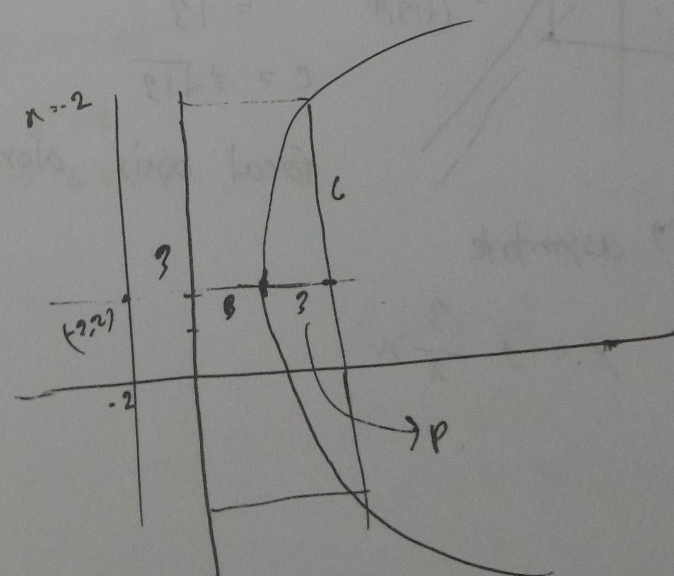
(h,k)

$(x-h)^2 = 4p(y+k)$ $(y-k)^2 = 4p(x-h)$

⊛

$(h,k) = (1,2)$

focus $= (4,2)$



$$(y-k)^2 = 4p(x-h)$$

$$(y-2)^2 = 12(x-1)$$

$$y^2 - 8x - 6y - 23 = 0$$

$$y^2 - 6y = 8x + 23$$

$$y^2 - 2\cdot y\cdot 3 + 3^2 = 8x + 23 + 3^2 = 8x + 32$$

$$(y-3)^2 = 8(x+4)$$

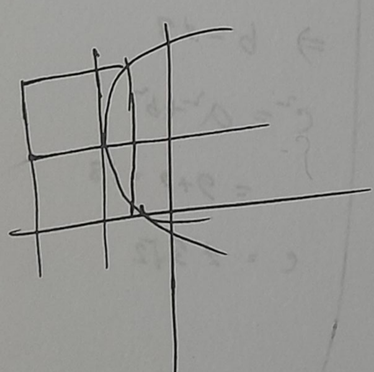
$4p = 8$

$\therefore p = 2$

$$(h,k) = (-4,3)$$

focus $= (-4+2, 3)$

$= (-2, 3)$



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⊛ Describe the Graph

⊛ $x^2-y^2-4x+8y-21=0$

$1\,(x^2-4x) - 1\,(y^2-8y) = 21$

$1\,(x^2-2x\cdot 2+2^2) - 1\,(y^2-2y\cdot 4+4^2) = 21 + 1\cdot 4 - 1\cdot 16$

$1\,(x-2)^2 - 1\,(y-4)^2 = 9$

$$\frac{(x-2)^2}{9} - \frac{(y-4)^2}{9} = 1$$

$\Rightarrow$ 1. Conic: Hyperbola

2. $(h,k) = (2,4)$

3. Focal axis: along- x-axis

4. $a = \pm 3$

$b = \pm 3$

$c = \pm 3\sqrt{2}$

5. focus:

6. Asymptote

$a^2 = 9$

$\Rightarrow a = \pm 3$

$b^2 = 9$

$\Rightarrow b = \pm 3$

$c^2 = a^2 + b^2$

$= 9 + 9 = 18$

$c = \pm 3\sqrt{2}$

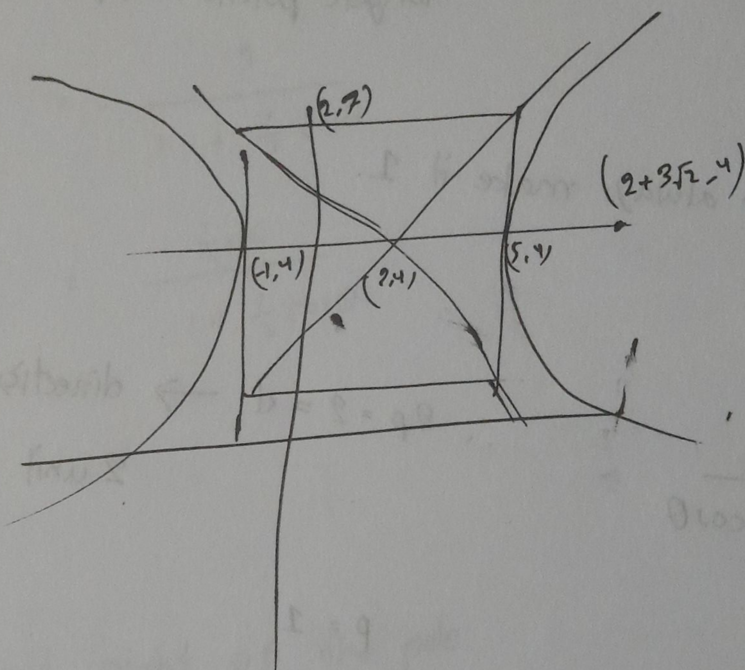
⊛ Assymptote:

$$\frac{(y-4)^2}{9} = \frac{(x-2)^2}{9}$$

$$(y-4) = \pm (x-2)$$

$$y = \pm (x-2) + 4$$

Graph:



10.6

$$r = \frac{e\overset{\to 2p\,(\text{Dinectics})}{d}}{1 \pm e\cos\theta}$$

$$= \frac{ed}{1 \pm e\sin\theta}$$

Equation of polar Condinates

focus in poll

longer portion of condoic = dinectics

always make it 1.

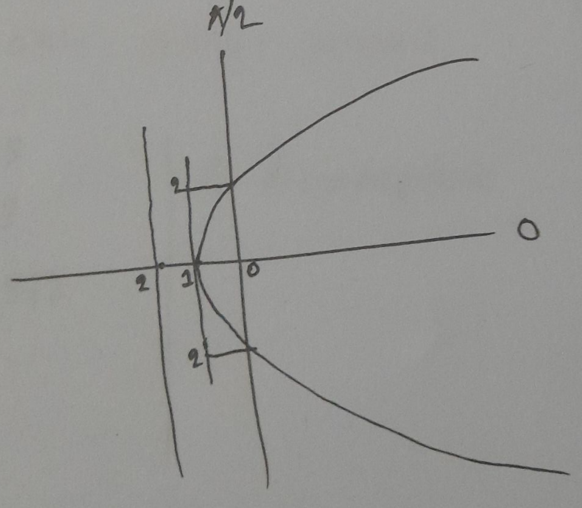
$$r = \frac{2}{1 - \cos\theta}$$

$$= \frac{1 \cdot 2}{1 - 1\cos\theta}$$

$\therefore 2p = 2 = d \rightarrow$ dinectics

2 unit Left of the polar

$p = 1$



⊛ Find a, b, c, d, e ?

Equation : $r = \dfrac{6}{2 + \cos\theta}$

$= \dfrac{6}{2(1 + \frac{1}{2}\cos\theta)}$

$= \dfrac{3}{1 + \frac{1}{2}\cos\theta}$

$= \dfrac{\frac{1}{2} \cdot 6}{1 + \frac{1}{2}\cos\theta}$

$\therefore e = \frac{1}{2}$ = elipse

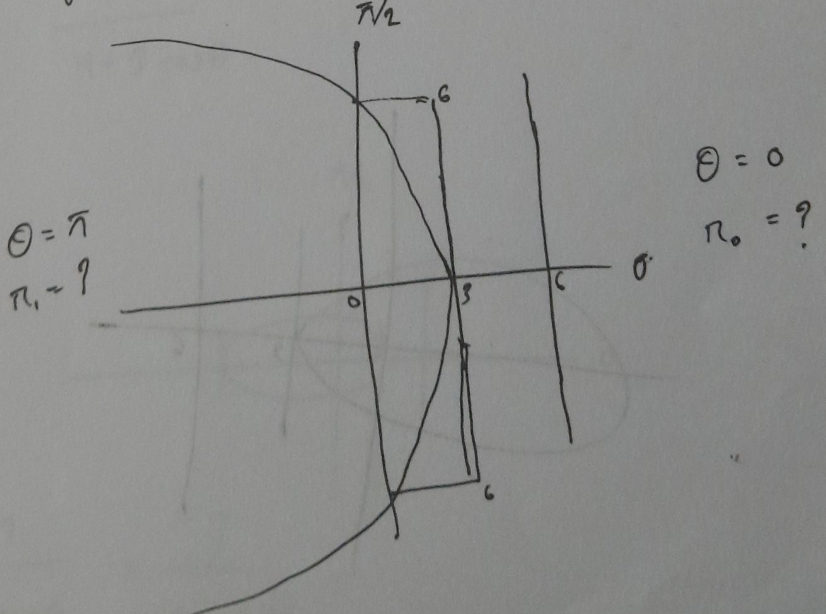
$d = 6$

dinecties : 6 unit right of the pole

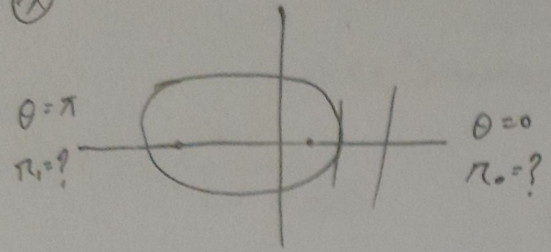
$d = 6$

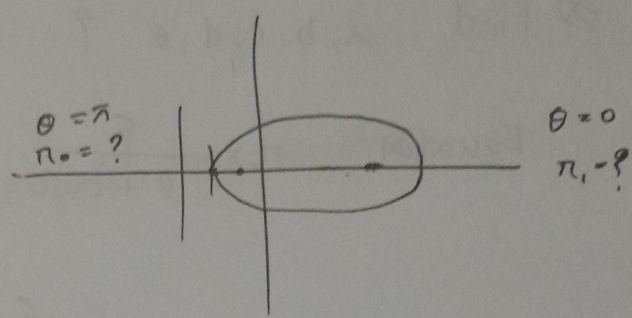
$2p = 6$

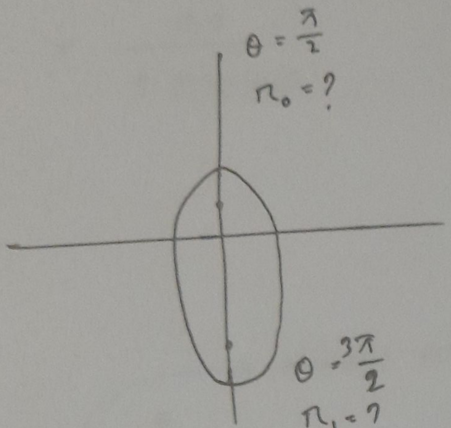
$p = 3$

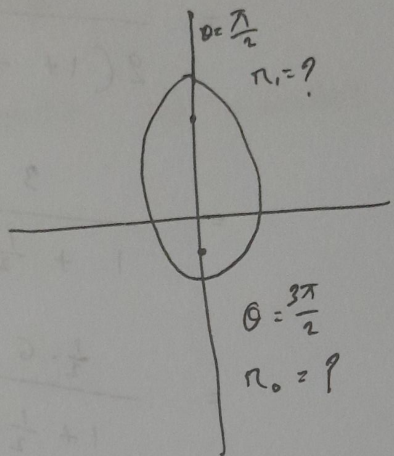


$a = \frac{1}{2}(r_1 + r_0) = \frac{1}{2}(6 + 2) = \frac{1}{2} \cdot 8 = 4$



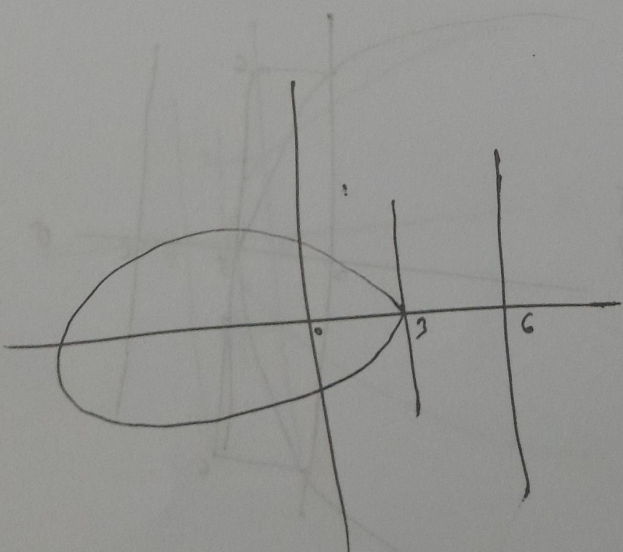






$$b = \sqrt{r_0 r_1} = \sqrt{2 \cdot 6} = \sqrt{12} = 2\sqrt{3}$$

$$c = \frac{1}{2}(r_1 - r_0) = \frac{1}{2}(6-2) = \frac{1}{2}\cdot 4 = 2$$



$r = \dfrac{8}{1-\sin\theta}$

$e = \dfrac{3}{4}$; directix, $x = 4$

$r = \sqrt{x^2+y^2}$

$= \sqrt{16+0}$

$= \pm 4$

$\therefore d = 4$

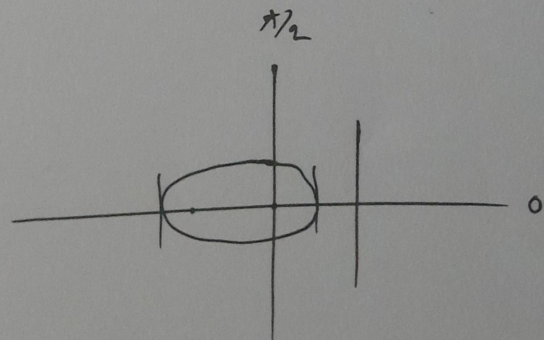
$\therefore \dfrac{\frac{3}{4}\cdot 4}{1+\frac{3}{4}\cos\theta} = \dfrac{3}{1+\frac{3}{4}\cos\theta}$

$= \dfrac{12}{4+3\cos\theta}$

$x = 1$

$d = 1$

$e = 1$

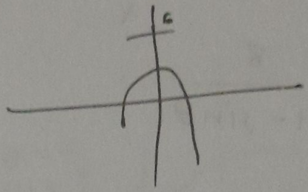


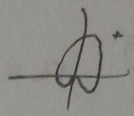
$\dfrac{1\cdot 1}{1+\cos\theta} = \dfrac{1}{1+\cos\theta}$

$e=2$

$y=6$

$d=6$





$$\frac{2\cdot 6}{1+2\sin\theta} = \frac{12}{1+2\sin\theta}$$