48/

Given that,

$$s = s(t)$$

$$v(t) = s'(t)$$

Here,

$$v(t) = \cos t$$

$$\Rightarrow \frac{ds}{dt} = \cos t$$

$$\Rightarrow ds = \cos t \, dt$$

$$\Rightarrow \int ds = \int \cos t \, dt$$

$$\Rightarrow s(t) = \sin t + C$$

Now,

$$s(0) = 2$$

Here, $t = 0$

$s = 2$

$$\therefore 2 = \sin 0 + C$$

$$\Rightarrow c = 2 - 0 = 2$$

Therefore,

$$s(t) = \sin t + 2$$

49)

Given that,

$$s = s(t)$$

$$v(t) = s'(t)$$

Here,

$$v(t) = 3\sqrt{t}$$

$$\Rightarrow \frac{ds}{dt} = 3\sqrt{t}$$

$$\Rightarrow ds = 3\sqrt{t}\, dt$$

$$\Rightarrow \int ds = \int 3 t^{\frac{1}{2}}\, dt$$

$$\Rightarrow s(t) = 3 \cdot \frac{t^{3/2}}{3/2} + c$$

$$= 2 t^{3/2} + c$$

Now,

$$s(4) = 1$$

Here, $t = 4$

$s = 1$

$$\therefore 1 = 2 \cdot 4^{3/2} + c$$

$$\therefore c = 1 - 16 = -15$$

Therefore,

$$s(t) = 2t^{3/2} - 15$$

50/

Given that,

$$s = s(t)$$

$$v(t) = s'(t)$$

Here,

$$v(t) = 3e^t$$

$$\Rightarrow \frac{ds}{dt} = 3e^t$$

$$\Rightarrow ds = 3e^t \, dt$$

$$\Rightarrow \int ds = \int 3e^t \, dt$$

$$\Rightarrow s(t) = 3e^t + c$$

Now,

$$s(1) = 0$$

Here, $t = 1$

$s = 0$

$$\therefore 0 = 3e^1 + c$$

$$\therefore c = -3e$$

Therefore,

$$s(t) = 3e^t - 3e$$

53)

Given that,

$$\frac{dy}{dx} = 2x+1$$

$$\Rightarrow dy = (2x+1)\,dx$$

$$\Rightarrow \int dy = \int (2x+1)\,dx$$

$$\Rightarrow y = 2 \cdot \frac{x^2}{2} + x + c$$

$$\therefore y = x^2 + x + c$$

From point $(-3,0)$ we get that,

$$x = -3$$
$$y = 0$$

$$\therefore 0 = (-3)^2 - 3 + c$$

$$\therefore c = -9+3 = -6$$

Therefore the equation of the curve is,

$$y = x^2 + x - 6$$

54/

Given that,

$$\frac{dy}{dx} = (x+1)^2$$

$$\Rightarrow dy = (x^2+2x+1)\,dx$$

$$\Rightarrow \int dy = \int (x^2+2x+1)\,dx$$

$$\Rightarrow y = \frac{x^3}{3} + 2\cdot\frac{x^2}{2} + x + c$$

$$\therefore y = \frac{1}{3}x^3 + x^2 + x + c$$

From point $(-2, 8)$, we get that,

$$x = -2$$
$$y = 8$$

$$\therefore 8 = \frac{1}{3}(-2)^3 + (-2)^2 + (-2) + c$$

$$\Rightarrow 8 = \frac{-8}{3} + 4 - 2 + c$$

$$\therefore c = \frac{26}{3}$$

Therefore, the equation of the curve is,

$$y = \frac{1}{3}x^3 + x^2 + x + \frac{26}{3}$$

55)

Given that,

$$\frac{dy}{dx} = -\sin x$$

$$\Rightarrow dy = -\sin x \, dx$$

$$\Rightarrow \int dy = \int -\sin x \, dx$$

$$\therefore y = \cos x + c$$

From point (0,2) we get that,

$$x = 0$$
$$y = 2$$

$$\therefore 2 = \cos 0 + c$$

$$\Rightarrow 2 = 1 + c$$

$$\therefore c = 1$$

Therefore,

the equation of the curve is,

$$y = \cos x + 1$$

561

Given that,

$$\frac{dy}{dx} = x^2$$

$$\Rightarrow dy = x^2 dx$$

$$\Rightarrow \int dy = \int x^2 dx$$

$$\Rightarrow y = \frac{x^3}{3} + c$$

$$\therefore y = \frac{1}{3}x^3 + c$$

From the point (-1,2) we get that,

$x = -1$
$y = 2$

$$\therefore 2 = \frac{1}{3}(-1)^3 + c$$

$$\Rightarrow 2 = -\frac{1}{3} + c$$

$$\therefore c = 2 + \frac{1}{3} = \frac{7}{3}$$

Therefore,
the equation of the curve is,

$$y = \frac{1}{3}x^3 + \frac{7}{3}$$

57)

Given that,

$$\frac{d^2y}{dx^2} = 6x$$

$$\Rightarrow \frac{dy}{dx} = \int 6x\,dx$$

$$\therefore \frac{dy}{dx} = 3x^2 + C_1 \quad \cdots (i)$$

Now, the slope of the tangent line, -3

$$\therefore \frac{dy}{dx} = -3 \quad (\text{when } x = 1)$$

From (i),

$$-3 = 3(1)^2 + C_1$$

$$\therefore C_1 = -3 - 3 = -6$$

$$\therefore \frac{dy}{dx} = 3x^2 - 6$$

$$\Rightarrow dy = (3x^2 - 6)\,dx$$

$$\Rightarrow \int dy = \int (3x^2 - 6)\,dx$$

$$\Rightarrow y = 3 \cdot \frac{x^3}{3} - 6x + C_2$$

$$\therefore y = x^3 - 6x + C_2 \quad \cdots (ii)$$

Now,

if, $x = 1$

then

$$y = 5 - 3 \cdot 1 = 2$$

$\therefore$ from (ii),

$$2 = 1^3 - 6 \cdot 1 + c_2$$

$$\therefore c_2 = 2 - 1 + 6 = 7$$

Therefore,

the equation of the curve is,

$$y = x^3 - 6x + 7.$$

5.3

1/

a)

Given integral,

$$\int 2x(x^2+1)^{23}\,dx$$

Let,

$u = x^2+1$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}(x^2+1)$

$\Rightarrow \frac{du}{dx} = 2x$

$\therefore du = 2x\,dx$

Therefore,

$$\int 2x(x^2+1)^{23}\,dx$$

$$= \int u^{23}\,du$$

$$= \frac{u^{24}}{24} + C$$

$$= \frac{(x^2+1)^{24}}{24} + C$$

Therefore,

the value of the integral is,

$$\frac{1}{24}(x^2+1)^{24} + C.$$

b)

Given integral,

$$\int \cos^3 x \sin x \, dx$$

Let,

$u = \cos x$

$\Rightarrow \frac{du}{dx} = -\sin x$

$\therefore -du = \sin x \, dx$

Therefore,

$$\int \cos^3 x \sin x \, dx$$

$$= \int u^3 (-du)$$

$$= -\int u^3 du$$

$$= -\frac{u^4}{4} + c$$

$$= -\frac{\cos^4 x}{4} + c$$

Therefore the value of the given integral is

$$-\frac{\cos^4 x}{4} + c$$

2)

a)

Given integral,

$$\int \frac{1}{\sqrt{x}} \sin\sqrt{x}\, dx$$

Let,

$$u = \sqrt{x}$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$\therefore 2\,du = \frac{1}{\sqrt{x}}\, dx$$

Therefore,

$$\int \frac{1}{\sqrt{x}} \sin\sqrt{x}\, dx$$

$$= \int \sin u \cdot 2\,du$$

$$= 2\int \sin u\, du$$

$$= -2\cos u + C$$

$$= -2\cos\sqrt{x} + C$$

Therefore the value of the given integral is,

$$-2\cos\sqrt{x} + C.$$

b)

Given integral,

$$\int \frac{3x\,dx}{\sqrt{4x^2+5}}$$

Let,

$$u = 4x^2+5$$

$$\Rightarrow \frac{du}{dx} = 8x$$

$$\Rightarrow du = 8x\,dx$$

$$\therefore x\,dx = \frac{1}{8}\,du$$

Therefore,

$$\int \frac{3x\,dx}{\sqrt{4x^2+5}}$$

$$= \int \frac{3 \cdot \frac{1}{8}\,du}{\sqrt{u}}$$

$$= \frac{3}{8} \int \frac{1}{\sqrt{u}}\,du$$

$$= \frac{3}{8} \cdot 2 \cdot \sqrt{u} + c$$

$$= \frac{3}{4}\sqrt{4x^2+5} + c$$

Therefore the value of the given integral is,

$$\frac{3}{4}\sqrt{4x^2+5} + c$$

3/

a)

Given integral,

$$\int \sec^2(4x+1)\,dx$$

Let,
$$u = 4x+1$$

$$\Rightarrow \frac{du}{dx} = 4$$

$$\therefore dx = \frac{1}{4}du$$

Therefore

$$\int \sec^2(4x+1)\,dx$$

$$= \int \sec^2 u \cdot \frac{1}{4}du$$

$$= \frac{1}{4}\int \sec^2 u\,du$$

$$= \frac{1}{4}\tan u + c$$

$$= \frac{1}{4}\tan(4x+1) + c$$

Therefore the value of the given integral is

$$\frac{1}{4}\tan(4x+1) + c$$

b)

Given integral,

$$\int y\sqrt{1+2y^2}\,dy$$

Let,

$u = 1+2y^2$

$\Rightarrow \frac{du}{dy} = 4y$

$\Rightarrow du = 4y\,dy$

$\therefore y\,dy = \frac{1}{4}du$

Therefore,

$$\int y\sqrt{1+2y^2}\,dy$$

$$= \int \sqrt{u}\cdot\frac{1}{4}du$$

$$= \frac{1}{4}\int \sqrt{u}\,du$$

$$= \frac{1}{4}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{4}\cdot\frac{u^{3/2}}{3/2}+c$$

$$= \frac{1}{4}\cdot\frac{2}{3}\cdot u^{3/2}+c$$

$$= \frac{1}{6}\cdot(1+2y^2)^{3/2}+c$$

Therefore the value of the given integral is,

$$\frac{1}{6}(1+2y^2)^{3/2}+c$$

4)

a)

Given integral,

$$\int \sqrt{\sin \pi\theta}\ \cos \pi\theta\ d\theta$$

Let,

$$u = \sin \pi\theta$$

$$\Rightarrow \frac{du}{d\theta} = \cos \pi\theta \cdot \pi$$

$$\therefore \frac{1}{\pi} du = \cos \pi\theta\ d\theta$$

Therefore

$$\int \sqrt{\sin \pi\theta}\ \cos \pi\theta\ d\theta$$

$$= \int \sqrt{u} \cdot \frac{1}{\pi} du$$

$$= \frac{1}{\pi} \int \sqrt{u}\ du$$

$$= \frac{1}{\pi} \cdot \frac{u^{3/2}}{3/2} + c$$

$$= \frac{1}{\pi} \cdot \frac{2}{3} \cdot u^{3/2} + c$$

$$= \frac{2}{3\pi} \cdot (\sin \pi\theta)^{3/2} + c$$

Therefore the value of the given integral is,

$$\frac{2}{3\pi} \sin^{3/2}(\pi\theta) + c$$

b)

Given integral,

$$\int (2x+7)(x^2+7x+3)^{4/5}\,dx$$

Let,

$$u = x^2+7x+3$$

$$\Rightarrow \frac{du}{dx} = 2x+7$$

$$\therefore du = (2x+7)\,dx$$

Therefore,

$$\int (2x+7)(x^2+7x+3)^{4/5}\,dx$$

$$= \int u^{4/5}\cdot du$$

$$= \frac{u^{9/5}}{9/5} + c$$

$$= \frac{5}{9}\cdot u^{9/5} + c$$

$$= \frac{5}{9}(x^2+7x+3)^{9/5} + c$$

Therefore the value of the given integral is,

$$\frac{5}{9}(x^2+7x+3)^{9/5} + c$$

5]

a)

Given integral,

$$\int \cot x \operatorname{cosec}^2 x \, dx$$

Let,

$$u = \cot x$$

$$\Rightarrow \frac{du}{dx} = -\operatorname{cosec}^2 x$$

$$\therefore -du = \operatorname{cosec}^2 x \, dx$$

Therefore,

$$\int \cot x \operatorname{cosec}^2 x \, dx$$

$$= \int u \cdot (-du)$$

$$= -\int u \, du$$

$$= -\frac{u^2}{2} + c$$

$$= -\frac{1}{2} \cot^2 x + c$$

Therefore, the value of the given integral is,

$$-\frac{1}{2} \cot^2 x + c$$

b)

Given integral,

$$\int (1+\sin t)^9 \cos t \, dt$$

Let,

$$u = \cancel{\cot}\ 1+\sin t$$

$$\Rightarrow \frac{du}{dt} = \cos t$$

$$\therefore du = \cos t \, dt$$

Therefore,

$$\int (1+\sin t)^9 \cos t \, dt$$

$$= \int u^9 \, du$$

$$= \frac{u^{10}}{10} + c$$

$$= \frac{1}{10} (1+\sin t)^{10} + c$$

Therefore, the value of the given integral is,

$$\frac{1}{10} (1+\sin t)^{10} + c.$$

6)

a)

Given integral,

$$\int \cos 2x \, dx$$

Let,

$$u = 2x$$

$$\Rightarrow \frac{du}{dx} = 2$$

$$\therefore dx = \frac{1}{2} du$$

Therefore,

$$\int \cos 2x \, dx$$

$$= \int \cos u \; \frac{1}{2} du$$

$$= \frac{1}{2} \int \cos u \, du$$

$$= \frac{1}{2} \sin u + c$$

$$= \frac{1}{2} \sin 2x + c$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \sin 2x + c.$$

b)

Given integral,

$$\int x \sec^2 x^2 \, dx$$

Let,

$$u = x^2$$

$$\Rightarrow \frac{du}{dx} = 2x$$

$$\therefore \frac{1}{2} du = x\,dx$$

Therefore,

$$\int x \sec^2 x^2 \, dx$$

$$= \int \sec^2 u \; \frac{1}{2} du$$

$$= \frac{1}{2} \int \sec^2 u \, du$$

$$= \frac{1}{2} \tan u + c$$

$$= \frac{1}{2} \tan x^2 + c$$

Therefore, the value of the given integral is,

$$\frac{1}{2} \tan x^2 + c.$$

7)

a)

Given integral,

$$\int x^2 \sqrt{1+x}\, dx$$

Let, $u = 1+x$

$\Rightarrow \frac{du}{dx} = 1$

$\therefore dx = du$

again, $u = 1+x$

$x = u-1$

Therefore,

$$\int x^2 \sqrt{1+x}\, dx$$

$$= \int (u-1)^2 \sqrt{u}\, du$$

$$= \int (u^2 - 2u + 1)\, u^{1/2}\, du$$

$$= \int (u^{5/2} - 2u^{3/2} + u^{1/2})\, du$$

$$= \frac{u^{7/2}}{7/2} - 2 \cdot \frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + c$$

$$= \frac{2}{7} u^{7/2} - \frac{4}{5} u^{5/2} + \frac{2}{3} u^{3/2} + c$$

$$= \frac{2}{7} (1+x)^{7/2} - \frac{4}{5} (1+x)^{5/2} + \frac{2}{3} (1+x)^{3/2} + c$$

Ans

b)

Given integral,

$$\int [\operatorname{cosec}(\sin x)]^2 \cos x \, dx$$

Let,

$u = \sin x$

$\Rightarrow \frac{du}{dx} = \cos x$

$\therefore du = \cos x \, dx$

Therefore,

$$\int [\operatorname{cosec}(\sin x)]^2 \cos x \, dx$$

$$= \int [\operatorname{cosec} u]^2 \, du$$

$$= \int \operatorname{cosec}^2 u \, du$$

$$= -\cot u + c$$

$$= -\cot(\sin x) + c$$

Therefore, the value of the given integral is,

$-\cot(\sin x) + c.$

8)

a)

Given integral,

$$\int \sin(x-\pi)\,dx$$

Let, $u = x-\pi$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

Therefore,

$$\int \sin(x-\pi)\,dx$$

$$= \int \sin u\,du$$

$$= -\cos u + c$$

$$= -\cos(x-\pi) + c$$

Therefore the value of the given integral is,

$$-\cos(x-\pi)+c.$$

b)

Given integral,

$$\int \frac{5x^4}{(x^5+1)^2}\,dx$$

Let,

$$u = x^5+1$$

$$\Rightarrow \frac{du}{dx} = 5x^4$$

$$\therefore du = 5x^4\,dx$$

Therefore,

$$\int \frac{5x^4\,dx}{(x^5+1)^2}$$

$$= \int \frac{1}{u^2}\,du$$

$$= \cancel{\ln u + c}$$

$$= \cancel{\ln(x^5+1) + c}$$

$$= \int u^{-2}\,du$$

$$= \frac{u^{-1}}{-1} + c$$

$$= -\frac{1}{u} + c$$

$$= -\frac{1}{x^5+1} + c$$

Ans

2)

a)

Given integral,

$$\int \frac{dx}{x \ln x}$$

Let,

$$u = \ln x$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{x}$$

$$\therefore du = \frac{1}{x} dx$$

Therefore, $\displaystyle\int \frac{dx}{x \ln x}$

$$= \int \frac{1}{u} du$$

$$= \ln u + c$$

$$= \ln(\ln x) + c$$

Therefore the value of the given integral is,

$$\ln(\ln x) + c$$

b)

Given integral,

$$\int e^{-5x}\, dx$$

Let,

$$u = -5x$$

$$\Rightarrow \frac{du}{dx} = -5$$

$$\therefore dx = -\frac{1}{5} du$$

Therefore,

$$\int e^{-5x}\, dx$$

$$= \int e^{u} \cdot \frac{-1}{5} du$$

$$= -\frac{1}{5} \int e^{u} du$$

$$= -\frac{1}{5} \cdot e^{u} + C$$

$$= -\frac{1}{5} e^{-5x} + C$$

Therefore, the value of the given integral is,

$$-\frac{1}{5} e^{-5x} + C$$

10)

a)

Given integral,

$$\int \frac{\sin 3\theta}{1+\cos 3\theta} d\theta$$

Let, $u = 1+\cos 3\theta$

$\Rightarrow \cancel{\frac{du}{d\theta} = -\sin 3\theta \cdot 0}$

$\Rightarrow \frac{du}{d\theta} = -\sin 3\theta \cdot 3$

$\Rightarrow du = -3 \sin 3\theta \, d\theta$

$\therefore \sin 3\theta \, d\theta = -\frac{1}{3} du$

Therefore,

$$\int \frac{\sin 3\theta}{1+\cos 3\theta} d\theta$$

$$= \int \frac{1}{u} \left(-\frac{1}{3}\right) du$$

$$= -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln u + c$$

$$= -\frac{1}{3} \ln(1+\cos 3\theta) + c$$

Therefore, the value of the given integral is,

$$-\frac{1}{3} \ln(1+\cos 3\theta) + c.$$

b)

Given integral,

$$\int \frac{e^x}{1+e^x}\,dx$$

let, $u = \cancel{1+\cos 3\theta}\ 1+e^x$

$$\Rightarrow \frac{du}{dx} = e^x$$

$$\therefore du = e^x\,dx$$

Therefore,

$$\int \frac{e^x}{1+e^x}\,dx$$

$$= \int \frac{1}{u}\,du$$

$$= \ln u + c$$

$$= \ln(1+e^x) + c$$

Therefore the value of the given integral is,

$$\ln(1+e^x) + c$$

15)

Given integral,

$$\int (4x-3)^9\,dx$$

Let,

$$u = 4x-3$$

$$\Rightarrow \frac{du}{dx} = 4$$

$$\therefore dx = \frac{1}{4}du$$

Therefore,

$$\int (4x-3)^9\,dx$$

$$= \int u^9 \tfrac{1}{4} du$$

$$= \frac{1}{4}\int u^9\,du$$

$$= \frac{1}{4}\cdot\frac{u^{10}}{10} + C$$

$$= \frac{1}{40}(4x-3)^{10} + C$$

Therefore, the value of the given integral is,

$$\frac{1}{40}(4x-3)^{10} + C.$$

16)

Given integral,

$$\int x^3\sqrt{5+x^4}\,dx$$

Let,

$$u = 5+x^4$$

$$\Rightarrow \frac{du}{dx} = 4x^3$$

$$\therefore x^3\,dx = \frac{1}{4}\,du$$

Therefore,

$$\int x^3\sqrt{5+x^4}\,dx$$

$$= \int \sqrt{u}\cdot\frac{1}{4}\,du$$

$$= \frac{1}{4}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{4}\cdot\frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{4}\cdot\frac{2}{3}\cdot u^{3/2} + C$$

$$= \frac{1}{6}\cdot(5+x^4)^{3/2} + C$$

Therefore, the value of the given integral is,

$$\frac{1}{6}\cdot(5+x^4)^{3/2} + C.$$

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Given integral,

$$\int \sin 7x \, dx$$

Let,

$$u = 7x$$

$$\Rightarrow \frac{du}{dx} = 7$$

$$\therefore dx = \frac{1}{7} du$$

Therefore, $\int \sin 7x \, dx$

$$= \int \sin u \cdot \frac{1}{7} du$$

$$= \frac{1}{7} \int \sin u \, du$$

$$= \frac{1}{7} \cdot (-\cos u) + c$$

$$= -\frac{1}{7} \cos 7x + c$$

Therefore, the value of the given integral is,

$$-\frac{1}{7} \cos 7x + C$$

18/

Given integral,

$$\int \cos \frac{x}{3} \, dx$$

Let, $u = \frac{x}{3}$

$$\Rightarrow \frac{du}{dx} = \frac{1}{3}$$

$\therefore dx = 3\,du$

Therefore,

$$\int \cos\frac{x}{3}\,dx$$

$$= \int \cos u \; 3\,du$$

$$= 3\int \cos u\,du$$

$$= 3\sin u + c$$

$$= 3\sin\frac{x}{3} + c$$

Therefore, the value of the given integral is,

$$3\sin\frac{x}{3} + c$$

19/

Given integral,

$$\int \sec 4x \tan 4x \, dx$$

Let,

$$u = 4x$$

$$\Rightarrow \frac{du}{dx} = 4$$

$$\therefore dx = \frac{1}{4}du$$

Therefore, $\int \sec 4x \tan 4x \, dx$

$$= \int \sec u \tan u \cdot \frac{1}{4}du$$

$$= \frac{1}{4}\int \sec u \tan u \, du$$

$$= \frac{1}{4}\sec u + c$$

$= \frac{1}{4} \sec 4x + c$

Therefore, the value of the given integral is,

$\frac{1}{4} \sec 4x + C.$

20)

Given integral,

$\int \sec^2 5x \, dx$

Let,

$u = 5x$

$\Rightarrow \frac{du}{dx} = 5$

$\therefore dx = \frac{1}{5} du$

Therefore, $\int \sec^2 5x \, dx$

$= \int \sec^2 u \cdot \frac{1}{5} du$

$= \frac{1}{5} \int \sec^2 u \, du$

$= \frac{1}{5} \tan u + c$

$= \frac{1}{5} \tan 5x + C$

Therefore, the value of the given integral is,

$\frac{1}{5} \tan 5x + C.$

21)

Given integral,

$$\int e^{2x}\,dx$$

Let,

$u = 2x$

$\Rightarrow \frac{du}{dx} = 2$

$\therefore dx = \frac{1}{2}\,du$

Therefore

$$\int e^{2x}\,dx$$

$$= \int e^{u} \cdot \frac{1}{2}\,du$$

$$= \frac{1}{2}\int e^{u}\,du$$

$$= \frac{1}{2} e^{u} \cancel{du} + c$$

$$= \frac{1}{2} e^{2x} + c$$

Therefore the value of the given integral, $\frac{1}{2} e^{2x} + c$

22)

Given integral,

$$\int \frac{dx}{2x}$$

Let, $u = 2x$

$\Rightarrow \frac{du}{dx} = 2$

$\therefore dx = \frac{1}{2}\,du$