CSE 373 / L-28 / 31.05.2024 /

Quiz-4

Chapter-25

All Pair Shortest Path

Recursive Case:

Case-1: Vertex k is not an intermediate

$$d_{ij}^{(k)} = d_{ij}^{(k-1)}$$

Case-2: Vertex k is in intermediate

$$d_{ij}^{(k)} = d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$$

Base Case:

$$d_{ij}^{(0)} = W_{ij}$$

Therefore, the recursive function:

$$d_{ij}^{(k)} = \min\left(d_{ij}^{(k-1)}, \; d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right)$$

Algorithm:

FLOYD-WARSHALL(W) → $O(n^3)$

n = W.rows

$D^{(0)} = W$

for $k = 1$ to n ← Select media

let $D^{(k)} = (d_{ij}^{(k)})$ be a new $n \times n$ matrix

for $i = 1$ to n ← Source

for $j = 1$ to n ← Destination

$d_{ij}^{(k)} = \min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)})$

need to implement here

return $D^{(n)}$

Fun tracking the Shortest Path:

when $k = 0$

$$\pi_{ij}^{(0)} = \begin{cases} NIL & ; i = j \text{ or } w_{ij} = \infty \\ i & ; i \neq j \text{ and } w_{ij} < \infty \end{cases}$$

when $k \geq 1$

$$\pi_{ij}^{(k)} = \begin{cases} \pi_{ij}^{(k-1)} & ; k \text{ is not an intermediate} \\ \pi_{kj}^{(k-1)} & ; k \text{ is in intermediate} \end{cases}$$

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* First for loop of this algorithm will, create $n \times n$ matrix n times. Too much space complexity.

⇒ We can over-write the new shortest path weight. Because for tracking the path we will use another matrix. So only one $n \times n$ matrix is enough for output.

* FLOYD-WARSHALL'(W)

```
n = W.rows
D = W
for k = 1 to n                          O(n^3)
    for i = 1 to n
        for j = 1 to n
            d_ij = min(d_ij, d_ik + d_kj)
return D.
```

Final Syllabus
END Here
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