North South University

Department of Mathematics and Physics

Assignment-3

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Course No : MAT 130

Course Title : Calculus and Analytical Geometry II

Section : 8

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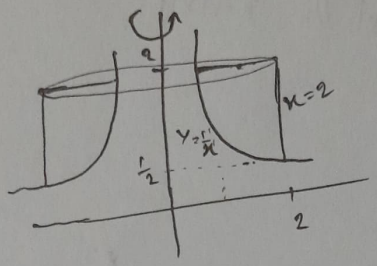
6.2

4]



$y = \frac{1}{x}$

$x = \frac{1}{y}$

$$\therefore \text{Volume}, V = \pi \int_{\frac{1}{2}}^{2} \left[2^2 - \left(\frac{1}{y}\right)^2\right] dy$$

$$= \pi \int_{\frac{1}{2}}^{2} \left(4 - \frac{1}{y^2}\right) dy$$

$$= \pi \left[4y + \frac{1}{y}\right]_{\frac{1}{2}}^{2}$$

$$= \pi \left(8 + \frac{1}{2} - 2 - 2\right)$$

$$= \frac{9\pi}{2}$$

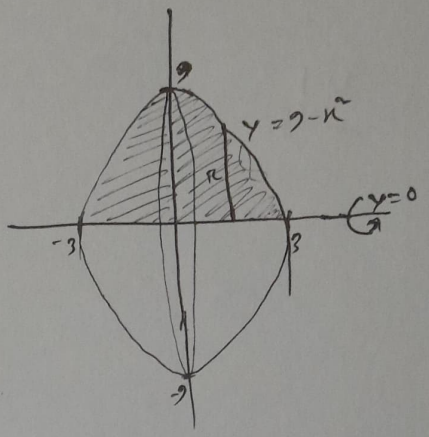
Therefore volume is $\frac{9\pi}{2}$.

12|

$y = 9 - x^2$

$y = 0$

revolved about the x-axis



$$\therefore \text{Volume } V = \pi \int_{-3}^{3} (9 - x^2)^2 \, dx$$

$$= \pi \int_{-3}^{3} (81 - 18x^2 + x^4) \, dx$$

$$= \pi \left[81x - 18\frac{x^3}{3} + \frac{x^5}{5} \right]_{-3}^{3}$$

$$= \pi \left[81x - 6x^3 + \frac{1}{5}x^5 \right]_{-3}^{3}$$

$$= \pi \left(81 \cdot 3 - 6 \cdot 3^3 + \frac{1}{5} \cdot 3^5 - 81(-3) + 6(-3)^3 - \frac{1}{5}(-3)^5 \right)$$

$$= \frac{1296\pi}{5}$$

Therefore volume is $\frac{1296}{5}\pi$

A.

16]

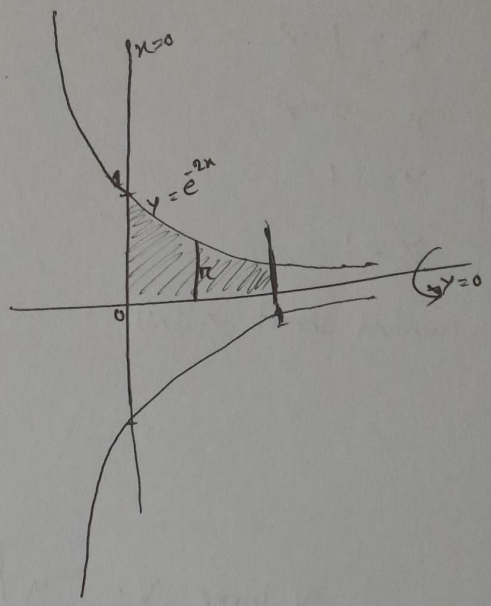
$y = e^{-2x}$

$y = 0$

$x = 0$

$x = 1$

revolved about x-axis.



$$\text{Volume, } V = \pi \int_0^1 (e^{-2x})^2 \, dx$$

$$= \pi \int_0^1 e^{-4x} \, dx$$

$$= \pi \left[\frac{e^{-4x}}{-4} \right]_0^1$$

$$= \pi \left(\frac{e^{-4}}{-4} - \frac{1}{-4} \right)$$

$$= \frac{\pi}{4} (1 - e^{-4})$$

Therefore, volume is $\frac{\pi}{4}(1 - e^{-4})$

24|

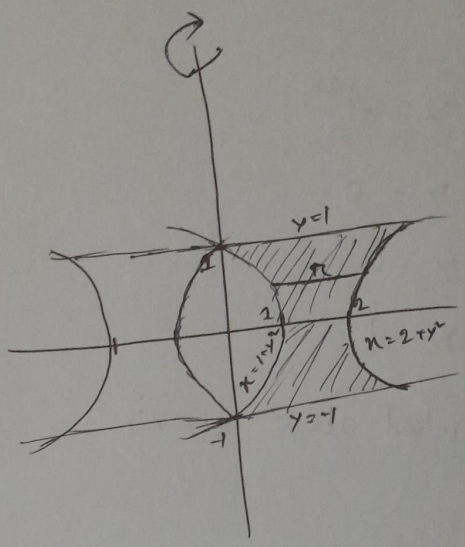
$x = 1-y^2$

$x = 2+y^2$

$y = -1$

$y = 1$

revolved about y-axis



$$\text{Volume, } V = \pi \int_{-1}^{1} \left[(2+y^2)^2 - (1-y^2)^2 \right] dy$$

$$= \pi \int_{-1}^{1} \left[4 + 4y^2 + y^4 - (1 - 2y^2 + y^4) \right] dy$$

$$= \pi \int_{-1}^{1} \left(4 + 4y^2 + y^4 - 1 + 2y^2 - y^4 \right) dy$$

$$= \pi \int_{-1}^{1} (6y^2 + 3)\, dy$$

$$= \pi \left[6 \cdot \frac{y^3}{3} + 3y \right]_{-1}^{1}$$

$$= \pi \left[2y^3 + 3y \right]_{-1}^{1}$$

$$= (2+3+2+3)\pi$$

$$= 10\pi$$

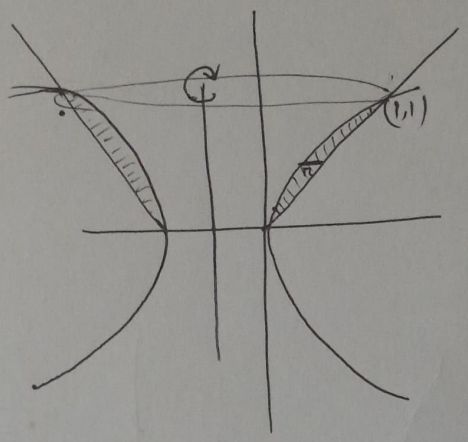
Ans.

42)

$x = y^2$

$x = y$

revolved about $x = -1$



Volume, $V = \pi \int_0^1 \left[(y+1)^2 - (y^2+1)^2 \right] dy$

$= \pi \int_0^1 \left[y^2 + 2y + 1 - (y^4 + 2y^2 + 1) \right] dy$

$= \pi \int_0^1 \left(y^2 + 2y + 1 - y^4 - 2y^2 - 1 \right) dy$

$= \pi \int_0^1 \left(-y^4 - y^2 + 2y \right) dy$

$= \pi \left[-\frac{y^5}{5} - \frac{y^3}{3} + 2\frac{y^2}{2} \right]_0^1$

$= \pi \left(-\frac{1}{5} - \frac{1}{3} + 1 \right)$

$= \frac{7}{15}\pi$

Therefore, Volume is $\frac{7}{15}\pi$

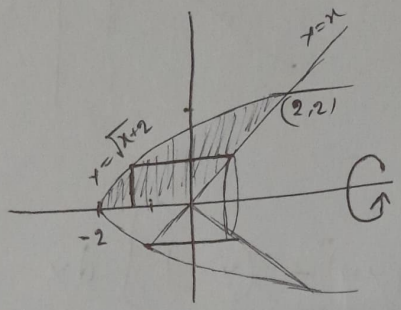
6.3

4)

$y = \sqrt{x+2}$

$y^2 = x+2$

$x = y^2 - 2$



Volume, $V = 2\pi \int_0^2 \left[(y)\left(y-(y^2-2)\right)\right] dy$

$= 2\pi \int_0^2 \left[y\left(y-y^2+2\right)\right] dy$

$= 2\pi \int_0^2 \left(y^2 - y^3 + 2y\right) dy$

$= 2\pi \left[\frac{y^3}{3} - \frac{y^4}{4} + 2\frac{y^2}{2}\right]_0^2$

$= 2\pi \left(\frac{8}{3} - \frac{16}{4} + 4\right)$

$= \frac{16}{3}\pi$

Ans

6)

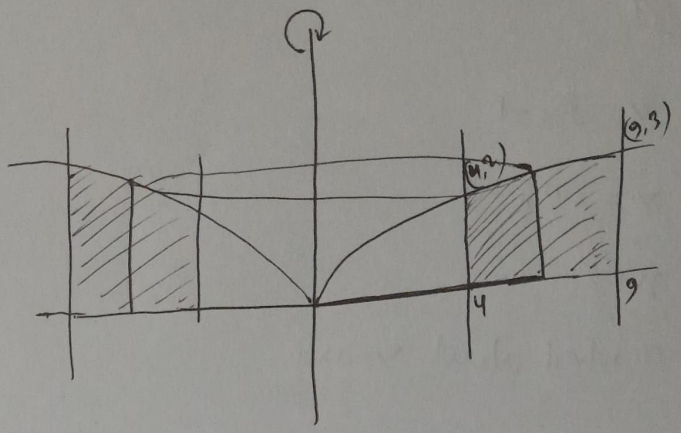
$y = \sqrt{x}$

$x = 4$

$x = 9$

$y = 0$

Revolved about y-axis



$$V = 2\pi \int_4^9 (x)(\sqrt{x})\, dx$$

$$= 2\pi \int_4^9 x^{3/2}\, dx$$

$$= 2\pi \left[\frac{x^{5/2}}{5/2}\right]_4^9$$

$$= 2\pi \left[\frac{2}{5}x^{5/2}\right]_4^9$$

$$= 2\pi \left(\frac{2}{5}\cdot 9^{5/2} - \frac{2}{5}\cdot 4^{5/2}\right)$$

$$= \frac{844}{5}\pi$$

Ans

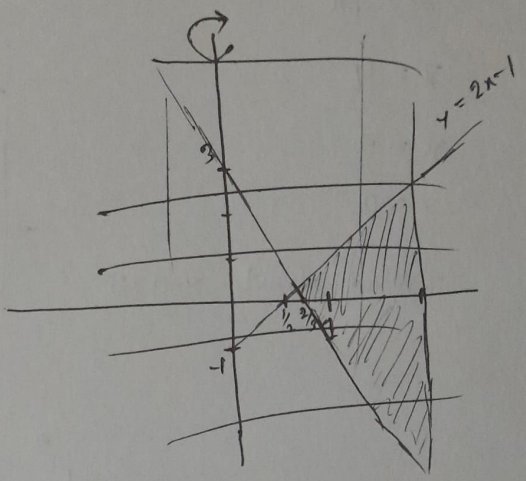
2]

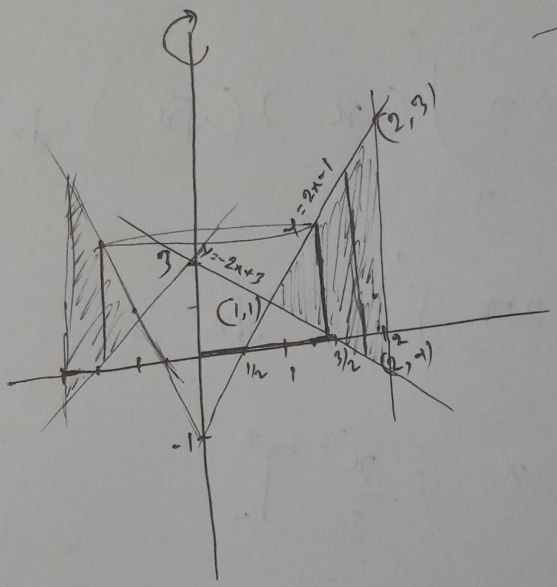
$y = 2x - 1$

$y = -2x + 3$

$x = 2$

revolved about y-axis





$$V = 2\pi \int_1^2 (x)\left((2x-1) - (-2x+3)\right) dx$$

$$= 2\pi \int_1^2 x\left(2x - 1 + 2x - 3\right) dx$$

$$= 2\pi \int_1^2 x\left(4x - 4\right) dx$$

$$= 2\pi \int_1^2 \left(4x^2 - 4x\right) dx$$

$$= 2\pi \left[4 \cdot \frac{x^3}{3} - 4\frac{x^2}{2}\right]_1^2$$

$$= 2\pi\left(4\cdot\frac{8}{3} - 4\cdot 2\cdot 2^2 - 4\frac{1}{3} + 2\cdot 1\right)$$

$$= 2\pi \cdot \frac{10}{3}$$

$$= \frac{20}{3}\pi$$

Ans..

14)

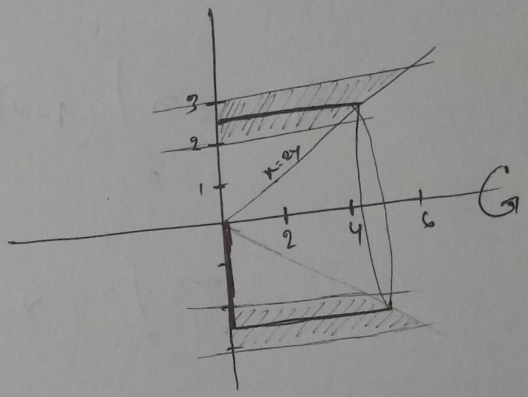
$x = 2y$

$y = 2$

$y = 3$

$x = 0$

revolved about x-axis



Volume $V = 2\pi \int_2^3 x\,(2y)\,dy$

$$= 2\pi \int_2^3 2y^2\,dy$$

$$= 2\pi \left[2\frac{x^3}{3}\right]_2^3$$

$$= 2\pi \left(2\cdot\frac{3^3}{3} - 2\cdot\frac{2^3}{3}\right)$$

$$= \frac{76}{3}\pi$$

Ans

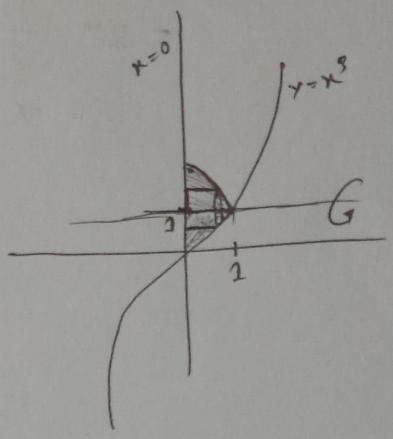
30|

$y = x^3$

$y = 1$

$x = 0$

Revolved about line $y = 1$



$y = x^3$

$x = y^{1/3}$

$$V = 2\pi \int_0^1 (1-y)(y^{1/3}) \, dy$$

$$= 2\pi \int_0^1 (y^{1/3} - y^{4/3}) \, dy$$

$$= 2\pi \left[\frac{y^{4/3}}{4/3} - \frac{y^{7/3}}{7/3} \right]_0^1$$

$$= 2\pi \left[\frac{3}{4} y^{4/3} - \frac{3}{7} y^{7/3} \right]_0^1$$

$$= 2\pi \left(\frac{3}{4} \cdot 1^{4/3} - \frac{3}{7} 1^{7/3} \right)$$

$$= 2\pi \left(\frac{3}{4} - \frac{3}{7} \right)$$

$$= \frac{3}{14}\pi$$

Ans