Maximum Sharpe Ratio by Non-Negative Least Square and Bayesian Optimization

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Problem Statement

Choosing the portfolio contains K over N stocks that maximize the Sharpe Ratio:

Sharpe Ratio

$$\text{Sharpe Ratio} = \sqrt{n} * \frac{\mathbb{E}[R - R_f]}{\sqrt{\text{var}[R - R_f]}} = \sqrt{252} * \frac{\mathbb{E}[R]}{\sqrt{\text{var}[R]}}$$

where :

- $R_f = 0$
- $n = 252$
- R is the returns of portfolio
- R is equally distributed by the returns of K stocks: $R = \frac{1}{K}\sum_{i=1}^{K} R_i$

Approach Ideas

- Optimization Methods
- Best subset selection

Project Baseline

- Preprocessing Data
- Using Non-Negative Least Square to remove Short Position
- Using Simple Moving Average to filter downtrend stocks
- Using Bayesian Optimization to searching the best stock combination

Data

- The data has 520432 rows and four attributes of 443 tickers.

	ticker	date	close	volume
0	VN30	2013-01-02	490.82	22641550.0
1	VN30	2013-01-03	491.34	35219262.0
2	VN30	2013-01-04	498.31	21387780.0
3	VN30	2013-01-07	509.18	26031020.0
4	VN30	2013-01-08	525.36	65840432.0

Figure 1: Data

Transforming Data

- The data are transformed into the $T \times N$ data frame with the row's index is the T trading dates and columns index is N ticker ids.

ticker	AAA	AAM	ABT	ACC	ACL	ADS	AGD	AGF	AGM	AGR
date										
2013-01-02	NaN	11.25	22.54	13.44	10.17	NaN	61.0	10.48	13.81	5.21
2013-01-03	NaN	10.98	22.54	13.44	9.84	NaN	64.0	10.96	13.12	5.30
2013-01-04	NaN	10.98	22.44	13.39	9.92	NaN	64.0	10.96	13.18	5.40
2013-01-07	NaN	11.07	22.44	13.34	10.25	NaN	64.0	10.44	13.18	5.49
2013-01-08	NaN	11.44	22.28	13.60	10.57	NaN	61.0	9.96	12.87	5.49

Figure 2: Transform Data

Sample Stock Prices

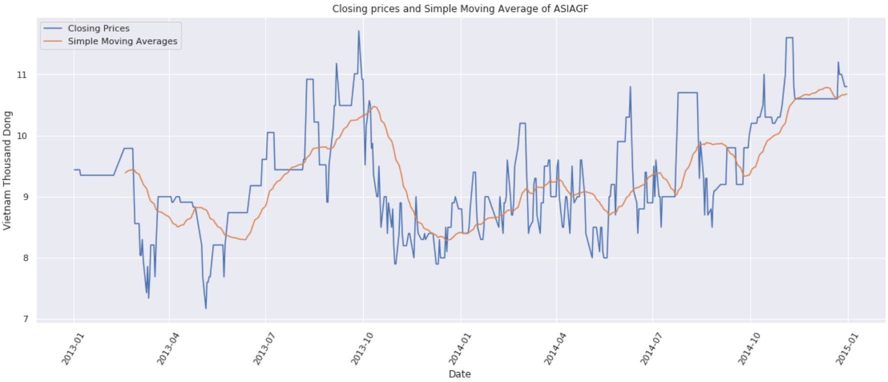

Figure 3: Sample Stocks Price and its Simple Moving Average

Cleaning Data

- Remove all stocks which had missing value in long interval till the end of 31/07/2019.
- Left the stock which had missing from the beginning. This means that the company still not join the stock market until the days that data was available.
- Fill data missing in middle by **Linear Interpolation**.

Selecting Data

- The market could be segment to vary era.
- The further data in past, the lesser affect on current behavior.
- Selecting the Data back to 1 years, which the data using for model is from 31/07/2018 to 31/07/2019.
- The stock which does not join market before 31/07/2018 will be remove from data.

Markowitz's Mean Variances Problems

- Finding the Minimum Variance Portfolio subject to Expected Return.

$$\min_{\mathbf{w}} \quad \frac{1}{2}\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}$$
$$\text{subject to} \quad \mathbf{w}^T\hat{\mu} = p$$

- Solution to Maximize Sharpe Ratio is also the solution of Minimum Variances problems with normalize weight.

$$\mathbf{w}^* = \frac{\sum^{-1}\hat{\mu}}{1_N^T\Sigma^{-1}\hat{\mu}}$$

- The problem could be rewrite as follow:

$$\begin{aligned} \min_{w} \quad & \|p\mathbf{1}_T - R\mathbf{w}\|_2^2 \\ \text{subject to} \quad & \mathbf{w}^T \hat{\mu} = p \\ \text{and} \quad & \mathbf{w}^T \mathbf{1}_N = 1 \end{aligned}$$

- The Entropy problems is the Markowitz' Mean Variances Problems with:
  - Selecting stocks
  - Equally weight
  - No short Position
- Solving by Shrinkage Methods with Non Negative Weight

Non Negative Least Square Methods

$$\min_{\mathbf{w} \succeq 0} \|\tilde{\mu} - R\mathbf{w}\|_2^2$$

where:

- $\tilde{\mu}$ is the expected value of daily return market which each row t^{th} of $\tilde{\mu}$ is calculate by simple average all asset at time $t - 1$

Filtering Downtrend Stock

- The stock that behavior well in training data could drop down in the futures
- Filtering the stock with Simple Moving Average Technical Indicator. The stock with have price path below the SMA is have the uptrend on future.

Transform Problems Objective Functions

$$
\begin{aligned}
& \text{maximize} \quad \sqrt{n}\frac{\mathbb{E}\left[R\right]}{\sqrt{\text{var}\left[R\right]}} \\
\Leftrightarrow \quad & \text{maximize} \quad \frac{\mathbf{w}^T\mu}{\left(\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}\right)^{\frac{1}{2}}} \qquad (n \text{ is annuallization constant factors})
\end{aligned}
$$

Transform Problems Objective Functions

- Because the expect return are compute by the mean of all return of K stock (which $K < N$) we select, then we could rewrite the weight as form:

$$\mathbf{w}^T = \frac{1}{|K|}\mathbf{w'}^T$$

- where
  - $\mathbf{w'}^T = [w_1, w_2, ...w_i] \quad w_i = 0 \text{ or } 1, \quad i \in [0, N]$
  - $|K| = \mathbf{w'}^T \mathbf{1}_N$ ($|K|$ is the number of selected assets)

Transform Problems Objective Functions

$$
\begin{aligned}
\mathbf{w}' &= \arg\max_{\mathbf{w}'} \quad \frac{\frac{1}{|K|}{\mathbf{w}'}^T\mu}{\left(\frac{1}{|K|}{\mathbf{w}'}^T\mathbf{\Sigma}\frac{1}{|K|}\mathbf{w}'\right)^{\frac{1}{2}}} \\
&= \arg\max_{\mathbf{w}'} \quad \frac{{\mathbf{w}'}^T\mu}{\left({\mathbf{w}'}^T\mathbf{\Sigma}\mathbf{w}'\right)^{\frac{1}{2}}}
\end{aligned}
$$

Bayesian Optimization

- Bayesian Optimization is a class of iterative optimization methods using Bayesian Rules to updating knowledge about target function.
- Bayesian Optimization has two feature: Surrogate Model and Acquisition Function
- **Surrogate Model** try to approximate target function by consider it has the Gaussian Process trajectory.
- **Acquisition Function** based on the Surrogate Model to choosing the next point to observe.

Bayesian Optimization

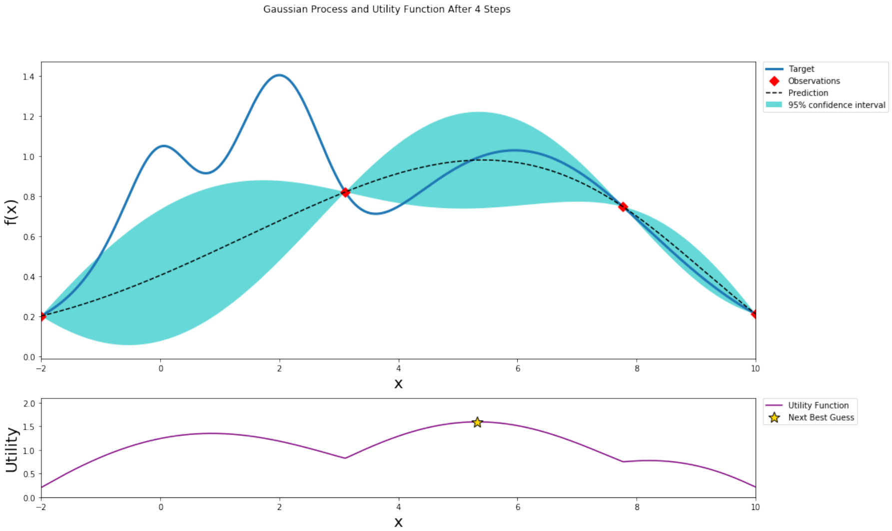

Figure 4: Example of Bayesian Optimization

Surrogate Model

$$f(x) = \frac{\mathbf{g}(x)^T \mu}{(\mathbf{g}(x)^T \mathbf{\Sigma} \mathbf{g}(x))^{\frac{1}{2}}} \quad , \quad x \in [0, 2^{|N|} - 1]$$

where :

- $$g(x)^T = [w_N, w_{N-1}, \dots, w_1]$$
- $$x = 2^{|N|-1} \times w_N + 2^{|N|-2} \times w_{N-1} + \cdots + 2^1 \times w_2 + 2^0 \times w_1$$

Acquisition Function

- Upper confidence bound:

$$\mathbf{x}_{t+1} = \arg\max_{\mathbf{x}} \left(\mu_t(\mathbf{x}) + \kappa \sigma_t(\mathbf{x})\right)$$

- where
  - $\mu_t(\mathbf{x})$ is the mean of Surrogate model at x point
  - $\sigma_t(\mathbf{x})$ is the variance of Surrogate model at x point
  - κ is the hyperparameter
- Setting $\kappa = 5$ for exploration.

Result

- Acquire the combination of **20** stocks : **ABT, AST, BCE, HII, JVC, KMR, L10, LIX, PGD, ROS, SBV, SFC, SJF, TCL, TDC, TMS, TNT, VCF, VIS, VNE**
- The Sharpe Ratio test result in API at 03/09/2019 is **11.132025544144525**.

Thank you for listening !