

ENEL101

Problem set 2

Mathematical operations

Please read the file, “How to complete and upload the solution template” before you begin. Download template_assign2.m and fill it in and then upload the completed file on D2L drop box. Also download test_assign2.m and assign2_solutions.mat to test your answers as you go. Make sure your name and UCID are correctly filled out.

The questions are based on content from chapter 3 of the textbook “MATLAB, An introduction with applications”.

Note the variable Q will not usually be assigned in your function call; just use the return variable ‘answer =’

Notation in this assignment assumes:

$X = [x_1, x_2, \dots, x_N]$, $Y = [y_1, y_2, \dots, y_N]$, $T = [t_1, t_2, \dots, t_N]$, and $Z = [z_1, z_2, \dots, z_N]$ are row vectors of samples.

Use MATLAB to calculate the following problems.

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1. Given a vector of samples $X = [-100, -99, \dots, 99, 100]$, create a vector Y calculated based on element-by-element operations such that $y_i = 0.001 x_i^2 + 1$. Construct matrix Q such that

$$Q = \begin{bmatrix} X \\ Y \\ X + Y \end{bmatrix}$$

2. The electric field intensity, $E(x)$, due to a ring of radius R at any point x along the axis of the ring is given by:

$$E(x) = 9604.5 \left[\frac{Rx}{(x^2 + R^2)^{\frac{3}{2}}} \right]$$

Given $R=6$ cm and a row vector X from elements ranging from 2 cm to 6 cm and spacing of 0.01 cm, use element-by-element operations to calculate a row vector Y whose elements are $y_i = E(x_i)$. Then construct a matrix of

$$Q = \begin{bmatrix} X \\ Y \end{bmatrix}$$

Hint: use 'format longg' to see the significant digits (otherwise you'll just see 0.0000)

3. The voltage $V(t)$ in Volt and the current $i(t)$ in Amp t seconds after closing the switch in an RC circuit are given by

$$V(t) = 24 \left(1 - e^{-\frac{t}{RC}} \right)$$

$$I(t) = \frac{24}{R} e^{-\frac{t}{RC}}$$

given $R = 3800 \Omega$ and $C = 4000 \times 10^{-6}$ F. Create a row vector T , with values of times from 0 to 20 sec with spacing of 2 sec, and use it to create row vectors X and Y such that $x_i = V(t_i)$ and $y_i = I(t_i)$. Then construct a matrix of

$$Q = \begin{bmatrix} T \\ X \\ Y \end{bmatrix}$$

4. Two vectors are given as

$$\mathbf{u} = -1\mathbf{i} + 2.5\mathbf{j} + 0.5\mathbf{k}$$

$$\mathbf{v} = +1\mathbf{i} + 0.2\mathbf{j} + 2\mathbf{k}$$

Compute the dot product as $Q = \mathbf{u} \cdot \mathbf{v}$.

5. Use `cross()` to compute the vector cross product as $Q = \mathbf{u} \times \mathbf{v}$, where Q is a row vector and \mathbf{u} and \mathbf{v} are two row vectors defined in question 4.

6. Use Matlab to show that the reciprocals of square numbers produce a convergent series

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

Do this by computing the following sums

$$X = \sum_{n=1}^{10} \frac{1}{n^2}, Y = \sum_{n=1}^{50} \frac{1}{n^2}, Z = \sum_{n=1}^{100} \frac{1}{n^2}$$

and placing your answers in matrix $Q = [X \ Y \ Z]$

7. Given two matrices

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 & 2 & 0 \\ 3 & 8 & 3 \\ 1 & 3 & 12 \end{bmatrix}$$

Determine $Q = ABB^T$ where the superscript denotes transpose

8. Given two matrices

$$A = \begin{bmatrix} 1 & -3 & 5 \\ 2 & 2 & 4 \\ -2 & 0 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -2 & 1 \\ 5 & 1 & -6 \\ 2 & 7 & -1 \end{bmatrix}$$

Determine $Q = A^{-1}(A + BA^T)$

$$9. \text{ Given matrix } A = \begin{bmatrix} 1 & -3 & 5 \\ 2 & 2 & 4 \\ -2 & 5 & 6 \end{bmatrix}$$

Let B be the matrix formed by having each element of A inverted such that $[B]_{i,j} = 1/[A]_{i,j}$

Then calculate $Q = AB$.

10. The following linear set of equations are determined by applying mesh current analysis to a circuit. Solve the linear set of equations using the \ operator in Matlab

$$-44i_1 + 10i_2 + 16i_3 = -20$$

$$10i_1 - 43i_2 + 6i_3 + 12i_4 = 0$$

$$16i_1 + 6i_2 - 30i_3 + 8i_4 = 12$$

$$12i_2 + 8i_3 - 34i_4 = -40$$

Let $Q = [i_1 \ i_2 \ i_3 \ i_4]$ be a row vector of the results.