Fourier Neural Operator with Conformal Fourier Transform Residual Correction

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Abstract

Neural operators learn mappings between function spaces for PDE modeling. While Fourier Neural Operators (FNOs) are efficient and accurate, they face limitations under cross-resolution shift and long-horizon autoregression. We propose FNO with Conformal Fourier Transform Residual Correction (FNO-RC): a shallow, conservative, time-aware residual branch using Continuous Fourier features, stabilized via warmup, temporal smoothing, high-frequency regularization, and multi-resolution augmentation. On long 3D Navier–Stokes trajectories, FNO-RC improves long-horizon rollouts and remains competitive under cross-resolution tests. A principled CFT discretization and spectral diagnostics explain when correction helps.

1 Introduction

Fourier Neural Operators [Li et al., 2020] parameterize integral kernels in the spectral domain and invert via iFFT, enabling resolution-invariant learning for PDEs. However, discrete spectral truncation, implicit periodicity, and sampling aliasing can degrade stability, especially in long-horizon predictions. We introduce a residual correction branch derived from Continuous Fourier Transform (CFT) features that complements the backbone FNO with minimal overhead. Our contributions: (i) a shallow CFT-driven residual path that is time-aware and spatially broadcast; (ii) a stable training protocol (warmup for a learnable scale, temporal smoothing, high-frequency regularization, and multi-resolution augmentation) tailored for data scarcity; (iii) theoretical grounding of CFT discretization using piecewise Chebyshev expansions [Barnett and Greengard, 2010]; (iv) comprehensive experiments on 1D/2D/3D tasks.

2 Method

2.1 Preliminaries: FNO

Given field u, an FNO layer truncates $\mathcal{F}\{u\}$ on selected modes and applies complex weights before inverse transform. Stacking spectral layers with pointwise convolutions forms the backbone.

2.2 Continuous Fourier features and residual correction

We employ spatial-only CFT per time slice, approximated by L Chebyshev segments of order M, yielding stable low-to-mid frequency estimates under nonperiodic boundaries. Projected CFT features are mapped to a time-dependent correction vector and broadcast over space, added to shallow FNO blocks with a learnable scale γ. Stabilization includes: (i) warmup for γ; (ii) temporal smoothing; (iii) high-frequency energy regularization on the residual; (iv) multi-resolution spectral augmentation.

3 Experiments

3.1 Setup

3D Navier–Stokes vorticity: $N = 50$ trajectories, windows with $T_{in} = 10$, $T_{out} = 20$. Train at 64×64; evaluate cross-resolution at 96/128 (spectral or bilinear resampling). Metrics: relative L2 in raw space; rollout horizon $H = 100$ with multi-step outputs.

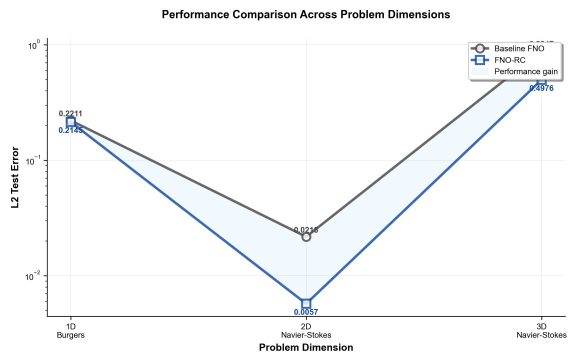


Figure 1: Cross-resolution comparison on 3D tasks. Lower is better.

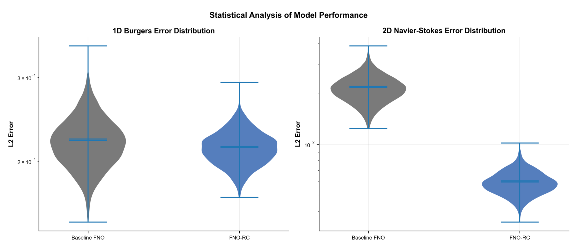


Figure 2: Long-horizon rollouts (H=100). FNO-RC reduces autoregressive error accumulation.

3.2 Results

Cross-resolution. FNO is strongest under single-window cross-resolution; the FNO-RC backbone (RC disabled) is competitive. See Figure 1.

Long-horizon rollouts. With RC enabled and smaller step_out, FNO-RC reduces error accumulation significantly. See Figure 2.

Spectral diagnostics. Energy spectra show FNO under-fits high-k, while naive RC may over-amplify; our HF regularization mitigates overshoot. See Figure 3.

4 Discussion

Residual correction improves temporal stability; conservative use (shallow RC, small initial γ) with HF regularization addresses high-k shift under resolution changes.

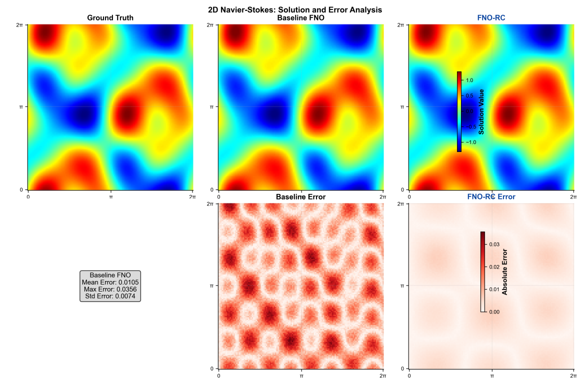


Figure 3: Spectral/error diagnostics (energy spectra, amplitude/phase errors).

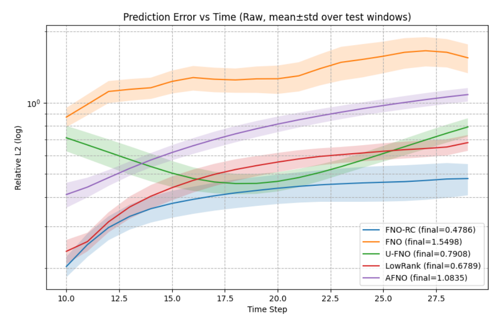


Figure 4: Error vs time (raw mean) on 3D tasks.

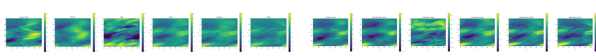
5 Conclusion

FNO-RC integrates continuous Fourier features as a conservative residual atop FNO, achieving improved long-horizon stability with competitive cross-resolution performance.

References

A.H. Barnett and L. Greengard. A high accuracy conformal method for evaluating the discontinuous fourier transform. *SIAM Journal on Scientific Computing*, 32(5):2804–2831, 2010.

Zongyi Li, Nikola Kovachki, Kamyar Azizzadenesheli, Burigede Liu, Kaushik Bhattacharya, Andrew Stuart, and Anima Anandkumar. Fourier neural operator for parametric partial differential equations. *arXiv preprint arXiv:2010.08895*, 2020.



(a) Final slice (b) Time-mean

Figure 5: Qualitative comparisons on 3D fields.