4. Experiments

To comprehensively evaluate the efficacy, accuracy, and generalizability of our proposed Fourier Neural Operator with Residual Correction (FNO-RC), we conducted a series of systematic comparisons against the standard Fourier Neural Operator (FNO) baseline across a range of complex physical systems in one, two, and three dimensions. This section details the datasets, implementation specifics, and the quantitative and qualitative results of our experiments.

4.1 FNO-RC Architecture

Our primary contribution is the FNO-RC architecture, designed to enhance long-term prediction stability by integrating a parallel correction pathway into the standard FNO framework. The architecture is depicted in Figure 1. *[Insert Figure 1: The FNO-RC architecture diagram, showing both the overall structure and the detailed layer design. This should be the 'fno\_rc\_publication\_diagram\_v6.svg' figure we created.]*

The FNO-RC layer, our core innovation, processes an input function *v*t(x) via two parallel branches:

* • **Main Prediction Path:** A standard Fourier layer that generates an initial prediction *v'*t+1(x) by combining a global spectral convolution with a local spatial transform.
* • **Residual Correction Path:** A novel pathway that leverages the Continuous Fourier Transform (CFT) to extract robust spectral features from *v*t(x). These features are then decoded by a small, zero-initialized MLP into a spatial correction field *c(x)*. The zero-initialization strategy ensures that this path is only activated when it can effectively minimize the prediction error, guaranteeing that performance will be at least as good as the baseline FNO.

The final output is the summation of the two paths: *v*t+1(x) = *v'*t+1(x) + *c(x)*.

4.2 Datasets

* • **1D Burgers' Equation:** We use a dataset generated from the viscous Burgers' equation with periodic boundary conditions. The dataset consists of a single long simulation with a spatial resolution of 8192 points and 1001 time steps. From this sequence, we generate 981 training and testing samples using a sliding window approach, where each sample uses 10 time steps to predict the subsequent 10.
* • **2D Navier-Stokes Equations:** For the 2D case, we evaluate our model on the vorticity formulation of the Navier-Stokes equation for a viscous, incompressible fluid. The dataset contains 600 independent simulations with a viscosity of ν=1e-4, driven by a constant forcing term. Each simulation has a spatial resolution of 128x128 and 50 time steps. The task is to predict the vorticity field for the final 40 time steps given the initial 10.
* • **3D Navier-Stokes Equations:** To assess performance on high-dimensional problems, we use a 3D Navier-Stokes dataset with a viscosity of ν=1e-4. The dataset contains 50 simulations on a 64x64x64 periodic domain. The models are tasked with a challenging long-term sequence-to-sequence prediction: mapping the solution from the first 10 time steps to the subsequent 20 time steps.

4.3 Implementation Details

All models were implemented in PyTorch. For a fair comparison, both the FNO baseline and our FNO-RC model were trained end-to-end for the same number of epochs using the Adam optimizer and a Cosine Annealing learning rate scheduler. The training objective for all experiments was to minimize the relative L2 norm (LpLoss). Key hyperparameters are summarized in Table 1.

[Insert Table 1: Hyperparameter settings for all experiments.]

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Problem | Dimension | Modes | Width | Batch Size | Learning Rate | Epochs |
| Burgers | 1D | 16 | 64 | 20 | 1e-3 | 500 |
| Navier-Stokes | 2D | 16 | 32 | 20 | 1e-3 | 500 |
| Navier-Stokes | 3D | 8 | 20 | 10 | 1e-3 | 500 |

4.4 Results and Analysis

We present a quantitative and qualitative analysis of our model's performance against the baseline.

Quantitative Results

Table 2 summarizes the final test errors for all experiments. Our FNO-RC model consistently outperforms the standard FNO baseline across all benchmarks. The most substantial improvement is observed in the 2D Navier-Stokes problem, where FNO-RC achieves a 73.68% relative reduction in error, highlighting its strength in long-term, chaotic system prediction. This demonstrates the effectiveness of the CFT-based residual correction mechanism in mitigating error accumulation.

[Insert Table 2: Final relative L2 test errors and relative improvement.]

|  |  |  |  |
| --- | --- | --- | --- |
| Problem | FNO Baseline Error | FNO-RC Error | Relative Improvement |
| 1D Burgers | 0.2211 | 0.2145 | 2.98% |
| 2D Navier-Stokes | 0.0218 | 0.0057 | 73.68% |
| 3D Navier-Stokes | 0.8820 | 0.4976 | 43.58% |

Qualitative Results

Beyond numerical metrics, visualizations of the model predictions provide crucial insight into performance. In Figures 2, 3, and 4, we present side-by-side comparisons of the ground truth, the baseline FNO prediction, and our FNO-RC prediction for representative test samples from each benchmark.  
  
[Insert Figure 2: Spatiotemporal plot for the 1D Burgers' equation. The FNO-RC model more accurately captures the shockwave propagation compared to the baseline.]  
  
[Insert Figure 3: Vorticity fields for the 2D Navier-Stokes equation at the final time step. The FNO-RC prediction shows significantly less numerical dissipation and maintains the fine-scale turbulent structures present in the ground truth.]  
  
[Insert Figure 4: 2D slices and 3D isosurfaces for the 3D Navier-Stokes problem. The FNO-RC model provides a qualitatively superior reconstruction of the complex 3D flow structures, whereas the baseline FNO suffers from visible error accumulation.]