COMS20011 – Data-Driven Computer Science

Problem Sheet MM04

1 – Calculate the result of the convolution $A*B$ in each of the examples below by hand.

(i) $A = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 2 & 2 & 3 & 3 & 2 \end{pmatrix}$

(ii) $A = \begin{pmatrix} 1 & 1 & 3 & 1 & 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 3 & 3 & -2 & -1 & 0 & 1 & 2 & 3 & 3 \end{pmatrix}$

(iii) $A = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 0 & 5 & 5 & 5 & 0 \\ 0 & 5 & 10 & 5 & 0 \\ 0 & 10 & 10 & 10 & 0 \\ 0 & 5 & 10 & 5 & 0 \\ 0 & 5 & 5 & 5 & 0 \end{pmatrix}$

Now verify your result using Matlab (using *conv* and use *help conv* to determine what convention Matlab uses when convolving at the border points) or using Python.

Answer:

H is the result by hand using the convention seen in the lecture. ***M*** is the result using Matlab and the convention used by *conv* (which does not normalise and leaves it to the user).

(i) *Without normalisation*

$H = \begin{pmatrix} 9 & 11 & 11 \end{pmatrix}$

$M = \begin{pmatrix} 2 & 6 & 9 & 11 & 11 & 7 & 2 \end{pmatrix}$

With normalisation

$H = \frac{1}{4}\begin{pmatrix} 9 & 11 & 11 \end{pmatrix}$

$M = \begin{pmatrix} 2 & 6 & 9 & 11 & 11 & 7 & 2 \end{pmatrix}$

$M = M * 1/4$

(ii) Normalisation factor is $\frac{1}{7}$

$H = \begin{pmatrix} -1 & -1 & 0 & 7 & 13 \end{pmatrix}$ $\quad M = \begin{pmatrix} 3 & 6 & 10 & 9 & -1 & -1 & 0 & 7 & 13 & 14 & 11 & 5 & 2 \end{pmatrix}$

(iii) Normalisation factor is $\frac{1}{8}$

$H = \begin{pmatrix} -35 & 0 & 35 \\ -40 & 0 & 40 \\ -35 & 0 & 35 \end{pmatrix}$ $\quad M = \begin{pmatrix} 0 & -5 & -5 & 0 & 5 & 5 & 0 \\ 0 & -15 & -20 & 0 & 20 & 15 & 0 \\ 0 & -25 & -35 & 0 & 35 & 25 & 0 \\ 0 & -30 & -40 & 0 & 40 & 30 & 0 \\ 0 & -25 & -35 & 0 & 35 & 25 & 0 \\ 0 & -15 & -20 & 0 & 20 & 15 & 0 \\ 0 & -5 & -5 & 0 & 5 & 5 & 0 \end{pmatrix}$

2 – How would low pass filtering be achieved using the Fourier domain? In your answer describe what is meant by Cut-off Frequency.

Answer:

Low pass filtering can be achieved by removing higher frequency information in the Fourier space, i.e. by retaining and letting lower frequencies pass through a filtering operation. Example filters are the ideal low pass filter and the Butterworth low pass filter. Some filter types have an abrupt cut-off point above which no higher frequencies are passed through, while others, like a Butterworth or Gaussian based filters, are more smoothly varying and do not have an abrupt cut-off point.

3 – Consider you are given the Fourier Transform space of an image. Using simple sketches to illustrate your answer, how would you select relevant regions to extract spectral features from

(a) only low frequency regions,
(b) only the very high frequency regions corresponding to prominent variations in intensity in the image that are at around 45 degrees to the horizontal,
(c) all approximately mid-range frequencies.

Answer:

Using conjugate symmetry, we can ignore the bottom half of the Fourier space and extract features from only the top half. We can then extract features from regions defined as

(a) for example a half
disc-shaped region with its centre at the centre of the Fourier space, i.e. at (u=0,v=0)

(b) a bar-shaped region with a small width, say 10, starting at around
(u= - max(ufreq.)/2, v= max(vfreq.)/2) angled at 135 degrees in the Fourier space

(c) a half-ring of a reasonable width, up to a maximum of 2*max(ufreq)/3, and starting from around (u=max(ufreq)/3, v=0).

NOTE: Exact u,v coordinates are not necessary, but approximations plus sketches should give the right indication, e.g. the half-disc must clearly be said to be at the centre of the space.

4 – Imagine you have received a huge shipment of three variety of fruits consisting of *Oranges*, *Satsumas*, and *Red Pears*. The fruit is unfortunately mixed up, but you have access to a vision system you can program to distinguish and separate the fruit as they pass in front of a camera on a converyor belt one at a time. The camera is positioned to give a top-view of the fruit.

(a) State no fewer than three, and no more than five, features you would use in your design to distinguish between the different types. Very briefly explain why your features will pick the correct type each time considering that some measurements maybe somewhat affected by noisy data from the image acquisition process.

Answer:

Shape of fruit, colour of fruit, and size of fruit.
Shape is probably enough to locate pears (roundish for both oranges and satsumas, not round for pear) but due to noise, maybe colour could be used as an additional cue.
Size should be enough to locate satsumas, but best to be combined with colour for more certainty.
All three features should be combined to select oranges to deal with the possibly noisy measurements (i.e. larger than satsumas, rounder than pears, and more orangeness than pears).

(b) Consider you had actually been asked to consider using up to 20 features for this task. Discuss what would you do to find out which features are significant (or which ones are redundant)?

Answer:

Apply some form of step-wise feature selection or feature elimination (e.g. to a training set of the data) and keep only those features whose eigenvalues correspond to say 90% to 95% of the variance in the data. These should hopefully be substantially fewer than 20 dimensions.

5 – Rotate an object, Fourier space rotates too. Translate an object, Fourier space translates too.

(a) Both statements are **True**.
(b) First statement is True and second one is False.
(c) First statement is **False** and the second one is **True**.
(d) Both statements are **False**.

6 – A 3x3 spatial filter with all elements set to **−1,** except the central element set to **16,** has a...

(a) normalisation factor of *1/8*
(b) normalisation factor of *1/16*
(c) normalisation factor of *1/24*
(d) normalisation factor of *1/12*

Sum the absolute value of each element, i.e. abs(-1)*8 + abs(16) = 24

7 – The eigenvalues of a 6D dataset are: **[17, 11, 8, 2, 0.65, 0.35]**. What *variance* in the dataset do the first 3 eigenvalues represent?

(a) 93.2%
(b) 91.3%
(c) 96.3%
(d) 92.3% **(17+11+8) / (17+11+8+2+0.65+0.35) = 0.92307**