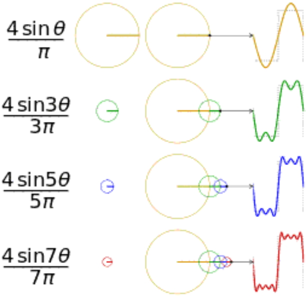
COMS20011 – Data-Driven Computer Science

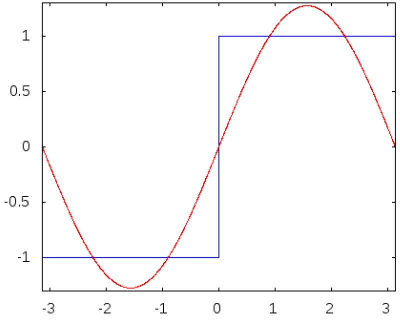


Lecture Video MM07 – Fourier Series

March 2021

Majid Mirmehdi

Next in DDCS

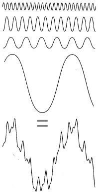


Feature Selection and Extraction

- Signal basics and **Fourier Series**
- 1D and 2D Fourier Transform
- Another look at features
- Convolutions

Frequency Analysis

Trigonometric Fourier Series: Any *periodic* function can be expressed as the sum of sines and/or cosines of different frequencies, each multiplied by a different coefficient. → *Jean Baptiste Joseph Fourier* (1822).



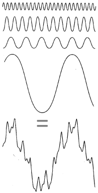
$$f(x) = \sum_{n=0}^{\infty} a_n \cos\left(\frac{2\pi n x}{T}\right) + b_n \sin\left(\frac{2\pi n x}{T}\right)$$

A function with period T is represented by two infinite sequences of coefficients. n is the no. of cycles/period.

- The sines and cosines are the Basis Functions of this representation. a_n and b_n are the Fourier Coefficients.
- The sinusoids are harmonically related: each one's frequency is an integer multiple of the fundamental frequency of the input signal.

Frequency Analysis

Trigonometric Fourier Series: Any periodic function can be expressed as the sum of sines and/or cosines of different frequencies, each multiplied by a different coefficient. → *Jean Baptiste Joseph Fourier* (1822).



$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi nx}{T}\right) \;+\; b_n \sin\left(\frac{2\pi nx}{T}\right)$$

A function with period T is represented by two infinite sequences of coefficients. n is the no. of cycles/period.

- The sines and cosines are the Basis Functions of this representation. a_n and b_n are the Fourier Coefficients.
- The sinusoids are harmonically related: each one's frequency is an integer multiple of the fundamental frequency of the input signal.
- **a_0 is often referred to as the DC term or the average of the signal**

Fourier Series Solution

A *Fourier series* provides an equivalent representation of the function:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi nx}{T}\right) \; + \; b_n \sin\left(\frac{2\pi nx}{T}\right)$$

The coefficients are:

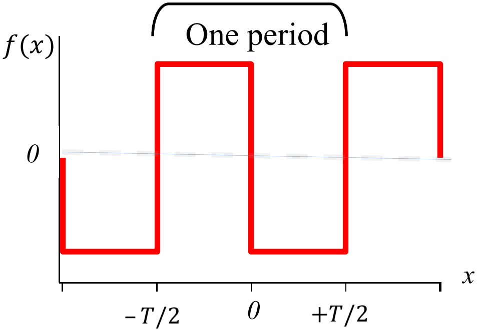
$$a_n = \frac{2}{T} \int_{-T/2}^{+T/2} f(x) \cos(\frac{2\pi nx}{T}) dx$$

$$b_n = \frac{2}{T} \int_{-T/2}^{+T/2} f(x) \sin(\frac{2\pi nx}{T}) dx$$

Fourier Series Example: Square Wave

$f(x)$ → a square wave

$$f(x) = \begin{cases} +1 & \frac{-T}{2} \leq x < 0 \\ -1 & 0 \leq x < \frac{T}{2} \end{cases}$$



Example periodic function on $-T/2, +T/2$

Fourier Series Example: Square Wave

f(x) → a square wave

$$f(x) = \begin{cases} +1 & \dfrac{-T}{2} \le x < 0 \\ -1 & 0 \le x < \dfrac{T}{2} \end{cases}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{+T/2} f(x)\cos(2\pi nx/T)dx$$

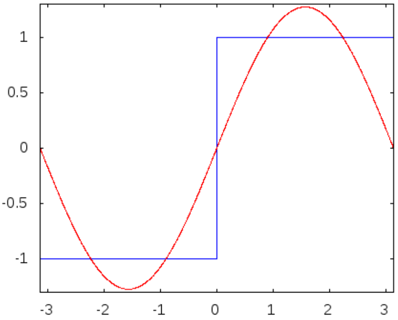
$$= \frac{2}{T}\int_{-T/2}^{0} \cos(2\pi nx/T)dx - \frac{2}{T}\int_{0}^{+T/2} \cos(2\pi nx/T)dx = \mathbf{0}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{+T/2} f(x)\sin(2\pi nx/T)dx$$

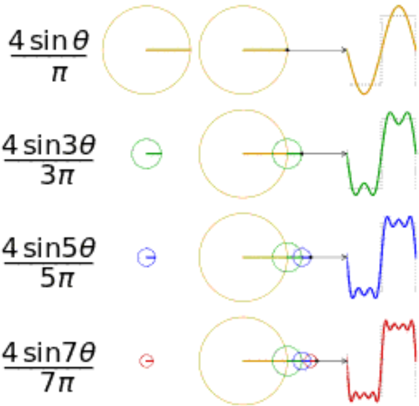
$$= \begin{cases} \dfrac{4}{n\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

$$f(x) = \frac{4}{\pi}\cdot\sin\frac{2\pi x}{T} + \frac{4}{3\pi}\cdot\sin 3\cdot\frac{2\pi x}{T} + \frac{4}{5\pi}\cdot\sin 5\cdot\frac{2\pi x}{T} + \dots$$

n = 1,3,5,7,…



Approximating the Square Wave



$$\theta = \frac{2\pi x}{T}$$

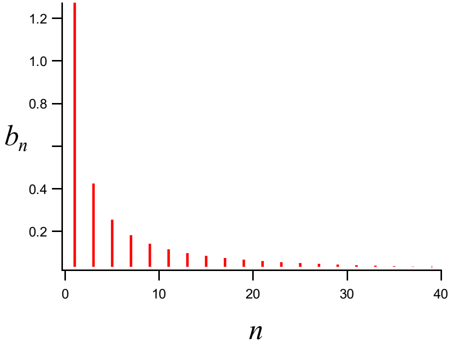
$$f(x) = \frac{4}{\pi} \cdot \sin\ \frac{2\pi x}{T} + \frac{4}{3\pi} \cdot \sin\ 3 \cdot \frac{2\pi x}{T} + \frac{4}{5\pi} \cdot \sin\ 5 \cdot \frac{2\pi x}{T} + \frac{4}{7\pi} \cdot \sin\ 7 \cdot \frac{2\pi x}{T} + \ldots$$

Fourier Space/Domain for the Square Wave

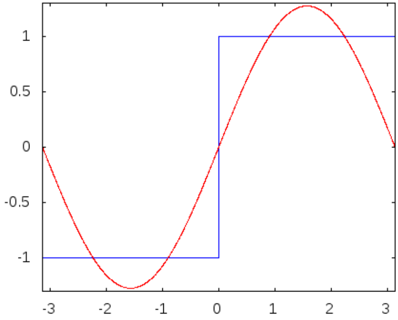
- The set of *Fourier Space* coefficients b_n contain complete information about the function

- Although *f(x)* is periodic to infinity, b_n is negligible beyond a finite range

- Sometimes the Fourier representation is more convenient to use, or just view



Next in DDCS



Feature Selection and Extraction

- Signal basics and Fourier Series
- **1D** and 2D **Fourier Transform**
- Another look at features
- Convolutions