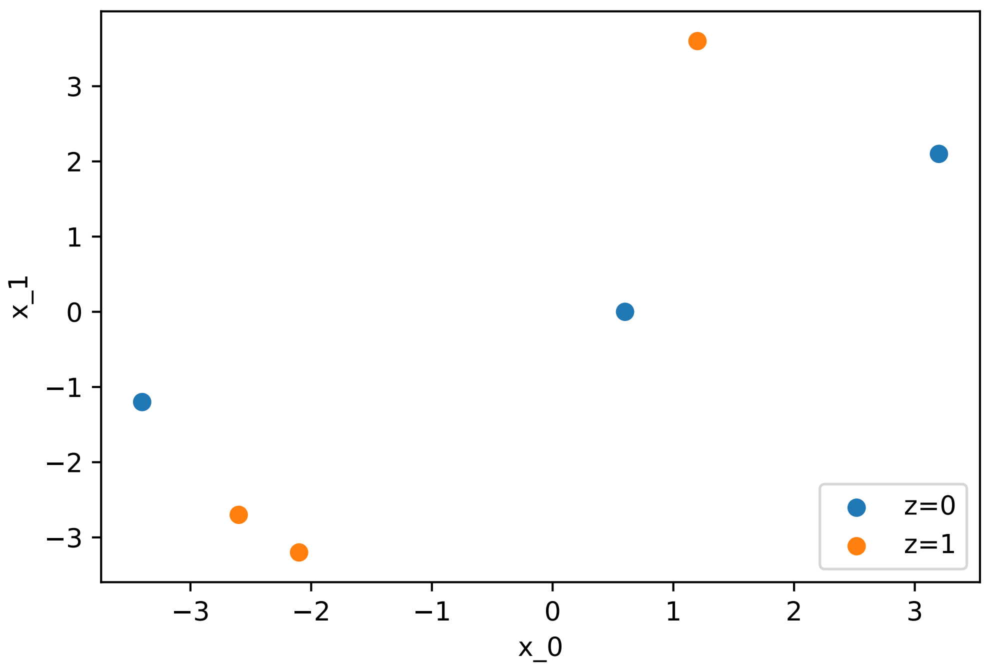
W18 Problem Sheet Explanation: Clustering

K-Means Clustering

So, for this question we have been given the data:

x_0	x_1	z
-2.1	-3.2	1
-3.4	-1.2	0
-2.6	-2.7	1
3.2	2.1	0
1.2	3.6	1
0.6	0	0

We can visualise this data as:



It is important to note that z is not the actual classes of these data points. **The difference between supervised and unsupervised learning is that we do not know what the correct classes are.** Currently, z is just a **prediction** of what we think the classes are - these will be updated as we start to look at the cluster centres.

Clustering is an iterative algorithm, we apply two steps repeatedly until we see no change any more. The aim of it is to estimate the clusters of the data, by determining the cluster centres and updating the clusters. The two steps for the algorithm are:

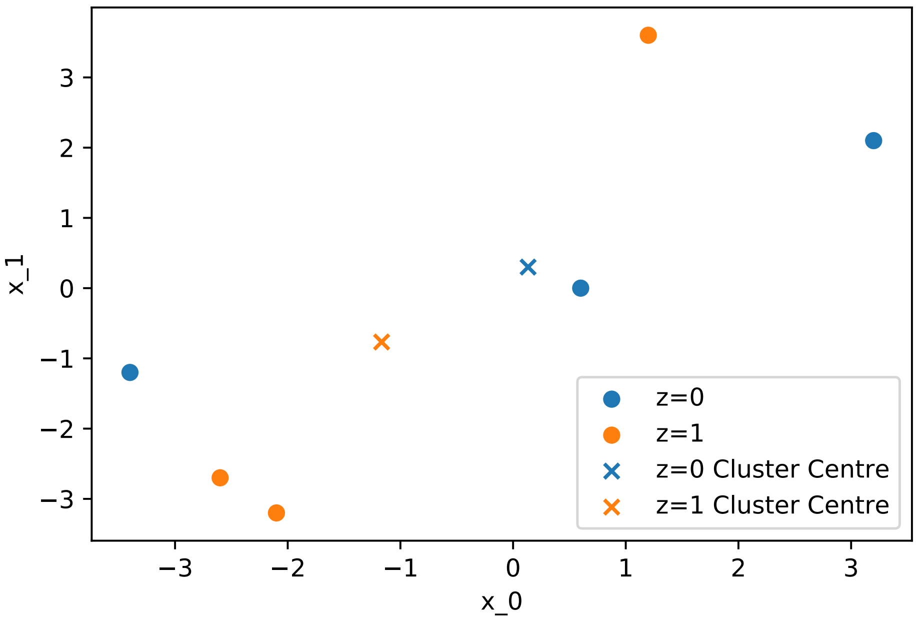
1. Calculate the centre of each cluster - this is the 'E-step'. The clusters are normally initialised as random points or as random data points.

2. Determine the cluster which each data point belong to - this is the 'M-step'. This is often done by calculating the distance between each point and each cluster centre - the cluster which has the closest centre is the one that that data point belongs to.

For this question we can see that the next thing to do is the E-step as the data has already been split into its predicted classes. So, let's find the centre for each class/-cluster:

$$\begin{aligned}
\mu_0 &= \frac{1}{3}\left(\begin{pmatrix}-3.4\\-1.2\end{pmatrix}+\begin{pmatrix}3.2\\2.1\end{pmatrix}+\begin{pmatrix}0.6\\0\end{pmatrix}\right)\\
&= \frac{1}{3}\begin{pmatrix}0.4\\0.9\end{pmatrix}\\
&= \begin{pmatrix}0.133\\0.3\end{pmatrix}\\
\mu_1 &= \frac{1}{3}\left(\begin{pmatrix}-2.1\\-3.2\end{pmatrix}+\begin{pmatrix}-2.6\\-2.7\end{pmatrix}+\begin{pmatrix}1.2\\3.6\end{pmatrix}\right)\\
&= \frac{1}{3}\begin{pmatrix}-3.5\\-2.3\end{pmatrix}\\
&= \begin{pmatrix}-1.167\\-0.767\end{pmatrix}
\end{aligned}$$

Let's plot these centres on top of our data points.



The next thing to do is to update our clusters - the M-step. As you can see currently, our clusters don't seem to make much sense. The top point (orange) is closer to the blue centre than the orange centre, so this should be updated to be a blue point (class 0). This is the process that we will do more formally for each point. I will demonstrate for the first point, then fill the rest of the table in:

$$
\begin{aligned}
D(\boldsymbol{x}, \mu_0) &= \left\| \begin{pmatrix} -2.1 \\ -3.2 \end{pmatrix} - \begin{pmatrix} 0.133 \\ 0.3 \end{pmatrix} \right\| \\
&= \sqrt{(-2.1 - 0.133)^2 + (-3.2 - 0.3)^2} \\
&= \sqrt{17.24} \\
D(\boldsymbol{x}, \mu_1) &= \left\| \begin{pmatrix} -2.1 \\ -3.2 \end{pmatrix} - \begin{pmatrix} -1.167 \\ -0.767 \end{pmatrix} \right\| \\
&= \sqrt{(-2.1 + 1.167)^2 + (-3.2 + 0.767)^2} \\
&= \sqrt{6.79}
\end{aligned}
$$

Therefore, our closest centre is for $z = 1$, and hence it will stay in the same class.

x_0	x_1	z (Previous value)	Distance to μ_0	Distance to μ_1	Closest centre
-2.1	-3.2	1	17.24	6.79	1
-3.4	-1.2	0	14.73	5.18	1
-2.6	-2.7	1	16.47	5.79	1
3.2	2.1	0	12.64	27.29	0
1.2	3.6	1	12.03	24.67	0
0.6	0	0	0.308	3.71	0

Let's plot our updated clusters: