THEORETICAL ANALYSIS:

The question when analysing this game is not which player should win, since both players play the same independent dice game, that is entirely probability-based, each inning and therefore have equal chances of failing or succeeding. Since this is a zero-sum game, the question becomes how many innings will pass before any of the players wins. To determine this, first the probability distribution of each individual dice game must be determined.

In the dice component the initial die roll really determines the odds of the player succeeding or faulting. There are two important observations here. Firstly, if a number X is rolled, no matter what number that X is there is a $\frac{1}{6}$ chance of rolling that same X on any given roll since it's an independent event. Secondly, the number of opportunities (or rolls) you have to roll X again is entirely dependent on what X is. For example, if I roll one, I only have one chance to roll another one. If I roll a six I have six chances to roll another six. Using these two observations, it is easy to work out the probability of success given the number X rolled:

Case	Probability of rolling X	# of rolls	Probability of success
X=1	P(1) = $\frac{1}{6}$	1	$P(S_1) = P(1) * Rolls = \frac{1}{6} * 1 = \frac{1}{6}$
X=2	P(2) = $\frac{1}{6}$	2	$P(S_2) = P(2) * Rolls = \frac{1}{6} * 2 = \frac{1}{3}$
X=3	P(3) = $\frac{1}{6}$	3	$P(S_3) = P(3) * Rolls = \frac{1}{6} * 3 = \frac{1}{2}$
X=4	P(4) = $\frac{1}{6}$	4	$P(S_4) = P(4) * Rolls = \frac{1}{6} * 4 = \frac{2}{3}$
X=5	P(5) = $\frac{1}{6}$	5	$P(S_5) = P(5) * Rolls = \frac{1}{6} * 5 = \frac{5}{6}$
X=6	P(6) = $\frac{1}{6}$	6	$P(S_6) = P(6) * Rolls = \frac{1}{6} * 6 = 1$

Now given these probabilities we can determine the total probability of success regardless of X. Recall that X is determined by rolling the die initially, therefore the probability of each value of X is $\frac{1}{6}$.

For the probability of success

$P(S) = P(X=1) * P(S_1) + P(X=2) * P(S_2) + \ldots\ldots + P(X=6) * P(S_6)$

$P(S) = \frac{1}{6} * \frac{1}{6} + \frac{1}{6} * \frac{1}{3} + \ldots\ldots. + \frac{1}{6} * 1$

$P(S) = \frac{1}{6} (\frac{1}{6} + \frac{1}{3} + \frac{1}{2} + \frac{2}{3} + \frac{5}{6} + 1)$

$P(S) = \frac{1}{6} * 3 \frac{1}{2}$

$P(S) = 7/12$

Therefore, the probability of success of the dice component is 7/12. Note that this is greater than ½, meaning that most of the time the player should succeed. We can now also calculate the probability of a fault, which is the complement of the probability of success.

For the probability of failure
P(F) = P(‘S)
P(F) = 1 - 7/12
P(F) = 5/12

Now that we know the probability of success or fault in the dice component of the game, we can work out the probability of any given inning ending in a win or a tie. For a win to occur, player 1 must succeed and player 2 must fault or player 1 must fault and player 2 must succeed. For a tie to occur, causing the game to extend to another inning, both players must win or both players must lose.

For the probability of a tie
P(T) = P(S) * P(S) + P(F) * P(F)
P(T) = 7/12 ^ 2 + 5/12 ^2
P(T) = 49/144 + 25/144
P(T) = 74/144
P(T) = 37/72

For the probability of a win
P(W) = P(S) * (F) + P(F) * P(S)
P(W) = 2(P(S) * P(F))
P(W) = 2(7/12 * 5/12)
P(W) = 2 * 35/144
P(W) = 70/144
P(W) = 35/72

To ensure these are correct, P(W) + (T) must be 1 since they are the only two outcomes
P(W) + P(T) = 1
R.S. = 1
L.S. = P(W) + P(T)
L.S. = 35/72 + 37/72
L.S. = 72/72
L.S. = 1
R.S. = L.S Therefore it is correct

With these probabilities established, we can look to determine the game ending on a certain inning. It is important to note that each inning is independent of the others. Advancing to a new inning requires a tie, and the game ending in that inning requires a win. We can show the combination of ties and wins that end the game in each inning.

Inning	1	2	3	4	5	6	7	8	9
Combination	W	TW	TTW	TTTW	TTTTW	TTTTTW	TTTTTTW	TTTTTTTW	TTTTTTTTW
Simplified	W	TW	T^2 W	T^3 W	T^4 W	T^5 W	T^6 W	T^7 W	T^8 W

Based on this table, we can make a rule that for the game to conclude in the n'th inning, there must be n-1 ties and one win. In other words P(n) = P(T)^(n-1) * P(W). We can also observe that the probability of the game ending before the n'th inning is the sum of the probabilities of the game ending in each inning before the n'th (not inclusive). Mathematically:

Probability of the game to end before the y'th inning

$$P(n<y) = \sum_{i=1}^{y-1} P(i)$$

Now we can expand our previous table with numerical values regarding probability that the game ends in that inning and probability that the game ends before that inning respectively.

Inning	1	2	3	4	5
Combination	W	TW	T^2W	T^3W	T^4W
P(n)	35/72 =0.4861	37/72*35/72 = 0.2498	(37/72)^2 *35/72 =0.1284	(37/72)^3 *35/72 =0.0660	(37/72)^4 *35/72 =0.0340
P(n) %	48.61 %	24.98%	12.84%	6.60%	3.40%
P(n<y)	N/A	0.4861	0.7359	0.8643	0.9303
P(n>y) %	N/A	48.61%	73.59%	86.43%	93.03%

Although these numbers are rough because of rounding, this gives us a very good idea what the outcomes of this game should look like. It seems that near 50% of games should be decided in the first inning, around 75% before the third inning and almost 95% before the fifth inning.

This game is a geometric distribution, so finding the expected value is the probability of failing (or in this case tying, since that causes the game to continue) divided by the probability of succeeding (which in this case is the probability of either player winning the game).

Expected value
$E(n) = P(T) / P(W)$
$E(n) = 37/72 / 35/72$
$E(n) = 37/35$

This means the average amount of innings it takes for someone to win is 37/35, but since this game starts on the first inning it really is 37/35 + 1 = 72/35 or approximately 2.0571.

I intend on simulating this game 100 000 times using a Java program, so I'll want to know how many games should end on innings 1, 2 and 3 in 100 000 games. I also would like to know how many games end on inning 10 in 100 000 games, and finally how many games end before the fourth round in 100 000 games. For the purpose of accuracy, I'm going to use the exact fraction probabilities for this and round to the nearest one.

Games that end on inning 1
$E(n=1) = 100000*P(n=1)$
$E(n=1) = 100000*35/72$
$E(n=1) = 48611$

Games that end on inning 2
$E(n=2) = 100000*P(n=2)$
$E(n=2) = 100000*35/72*37/72$
$E(n=2) = 24981$

Games that end on inning 3
$E(n=3) = 100000*P(n=3)$
$E(n=3) = 100000*35/72*(37/72)^2$
$E(n=3) = 12837$

Games that end on inning 10
$E(n=10) = 100000*P(n=10)$
$E(n=10) = 100000*35/72*(37/72)^9$
$E(n=10) = 121$

Games that end before inning 4
$E(n<4) = 100000*P(n<4)$
$E(n<4) = 100000*(P(n=1) + P(n=2) + P(n=3))$
$E(n<4) = 48611 + 24981 + 12837$
$E(n<4) = 86429$

EXPERIMENTAL RESULTS & COMPARISON

	Expected value + Average value	Games ended in 1st inning	Games ended in 2nd inning	Games ended in 3rd inning	Games ended in 10th inning	Games ended before 4th inning
Theoretical number	2.0571	48611	24981	12837	121	86429
Experimental number	2.0117	49787	24909	12690	113	87386
%Error	2.21%	2.42%	0.29%	1.15%	6.61%	1.11%

The table above shows the theoretical distributions and the experimental results after 100 000 tests simulated by the computer (the Java program is attached in a different file for inspection). I also calculated percent error in order to see how accurate the theoretical values were.

Percent error for each of these benchmarks is lower than 3% (with the exception of games ended in the 10th inning) meaning my theoretical calculations must have been fairly accurate. I think if I simulated this game a million times instead of 100 000 then these percent errors would be even smaller.

The games ended after the 10th inning is the only outlier here with 6.61% error. The reason for this is probably just not enough trials. As you can see, the numbers there are considerably lower than the other benchmarks, at 121 and 113. The issue is that with such low numbers, a few improbable games will really mess with the percent error. For this percent error to be reduced this game might have to be run a tens or hundreds of millions of times since the odds of the game ending in the tenth inning are so tiny.