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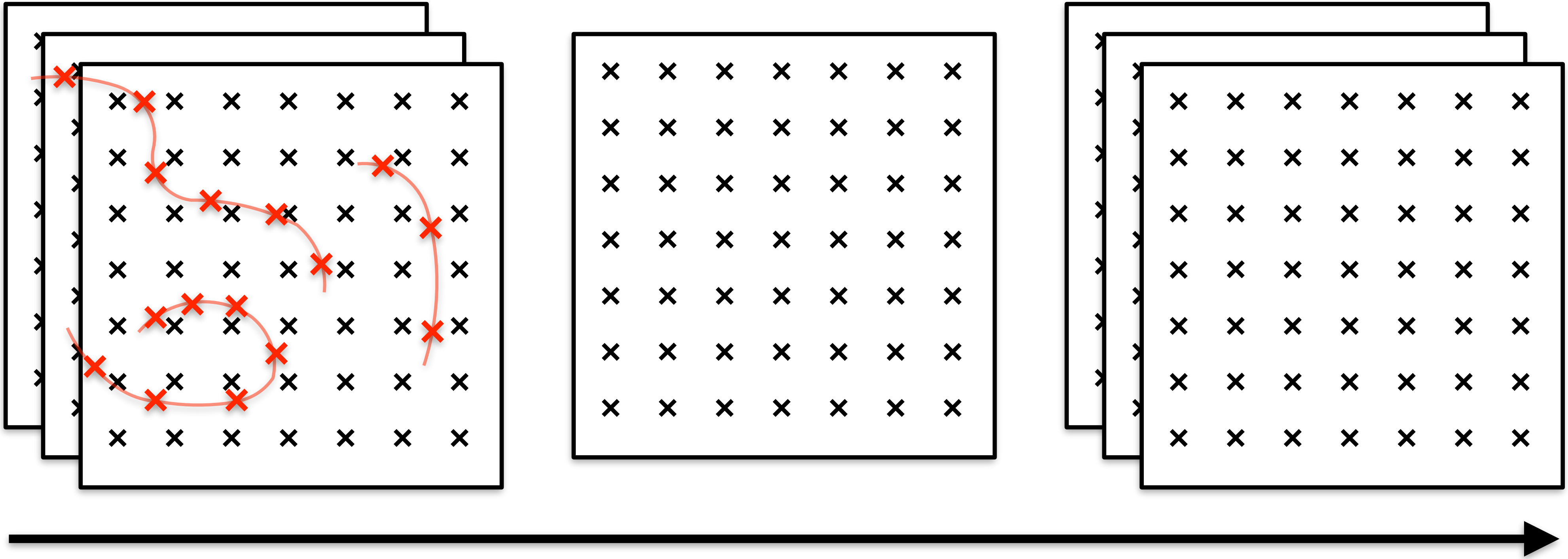
Spatio-Temporal Inference with Disjoint Spatial Locations

Zhang, R.-Y., Moss, H. B., Astfalck, L., Cripps, E. and Leslie, D. S. (2025). BALLAST: Bayesian Active Learning with Look-ahead Amendment for Sea-drifter Trajectories under Spatio-Temporal Vector Fields, *arXiv preprint* arXiv:2509.26005 .



Regress using static GP with <u>extended</u> state space

[Vanilla-SPDE Exchange]

Predict using dynamic GP and <u>extract</u> original states

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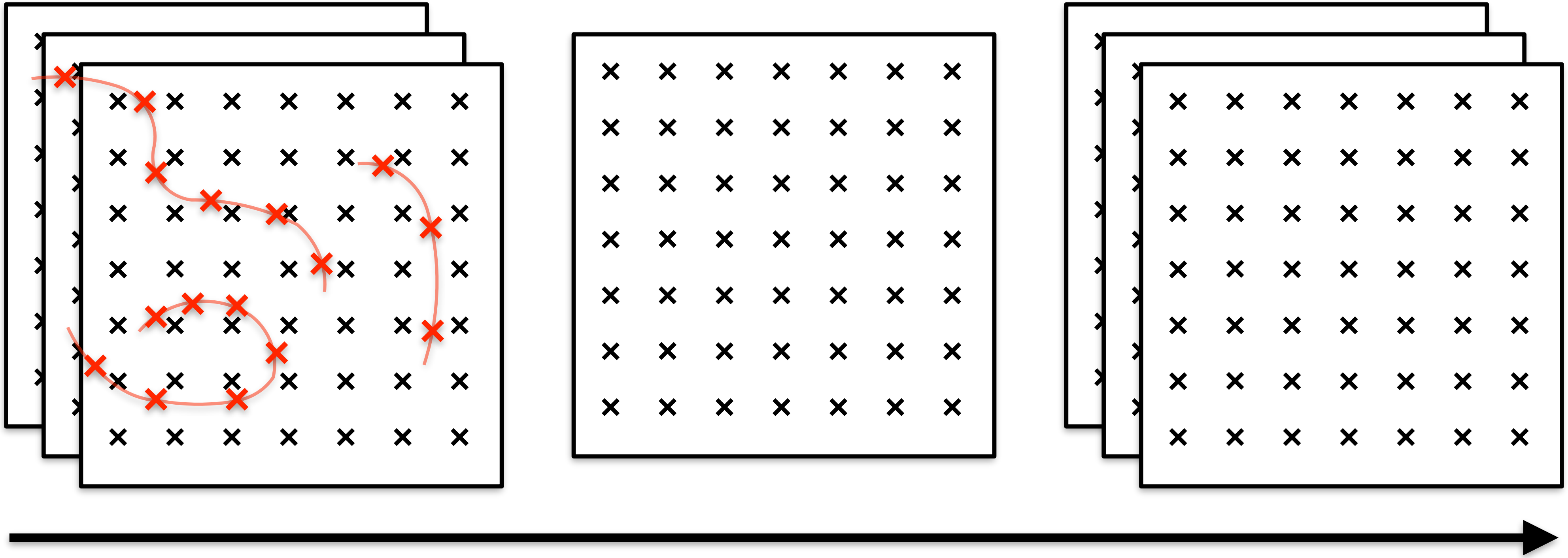

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Denote the number of spatial grid points as N_s for each time slice, and the number of time slices we wish to sample into the future is N_t. So, the total number of test points of posterior prediction is $N_t N_s$. The observation number N_{obs}, which we, for simplicity, assume to be made at distinct locations over $N_{\text{obs,t}}$ time slices. We remark also that the size of both N_t and N_s would often be much larger than N_{obs}, while N_t and $N_{\text{obs,t}}$ are of the same magnitude.

Method	Regression	Sampling	Total
Vanilla	$O(N_{\text{obs}}^3)$	$O(N_s^3 N_t^3)$	$O(N_{\text{obs}}^3 + N_s^3 N_t^3)$
SPDE	$O((N_s + N_{\text{obs}})^3 N_{\text{obs,t}})$	$O(N_s^3 + N_s^2 N_t)$	$O((N_s + N_{\text{obs}})^3 N_{\text{obs,t}} + N_s^3 + N_s^2 N_t)$
VASE	$O(N_{\text{obs}}^3 + N_s^2 N_{\text{obs}} + N_s N_{\text{obs}}^2)$	$O(N_s^3 + N_s^2 N_t)$	$O(N_{\text{obs}}^3 + N_s^2 N_{\text{obs}} + N_s N_{\text{obs}}^2 + N_s^3 + N_s^2 N_t)$

Table 1: Computational cost summary table of different methods. The green indicates the lowest cost in each column, while the red indicates the highest.