# Robotics: Science and Systems (R:SS)
# Course Assignment 1

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## 1 Forward and inverse kinematics

### 1.1 Calculate and report the position part of the homogeneous transformation from the base to the gripper

The kinematic map function of this system consists of translation and rotation transformations of every joints:

$$\begin{aligned} T_{base->gripper}(q) = T_{base->L_1}(q)R_{L_1->L_1'}(q)T_{L_1->L_2}(q)R_{L_2->L_2'}(q) \\ T_{L_2->L_3}(q)R_{L_3->L_3'}(q)T_{L_3->L_4}(q)R_{L_4->L_4'}(q)T_{L_4->L_gripper}(q) \end{aligned} \tag{1}$$

We can easily calculate that :

$$T_{base->gripper}(q) = \begin{pmatrix} -0.09311 & -0.97348 & 0.20895 & 0.07395 \\ -0.39624 & 0.22875 & 0.88919 & 0.31468 \\ -0.91341 & 0. & -0.40703 & 0.40686 \\ 0. & 0. & 0. & 1. \end{pmatrix} \tag{2}$$

### 1.2 Calculate the Jacobian martix

$$J_{pos}(q)_{.i} = (a_i \times (p_{eff} - p_i)) \tag{3}$$

And we know

$$a_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

$$a_2 = R_{L_1->L_1'}(q) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{5}$$

$$a_3 = R_{L_1->L_1'}(q)R_{L_2->L_2'}(q) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{6}$$

$$a_4 = R_{L_1->L_1'}(q)R_{L_2->L_2'}(q)R_{L_3->L_3'}(q) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{7}$$

Therefore, we can easily calculate that :

$$J_{pos}(q) = \begin{pmatrix} -0.31468 & -0.00987 & -0.04343 & -0.00931 \\ 0.07395 & -0.04199 & -0.18481 & -0.03962 \\ 0. & -0.32325 & -0.29198 & -0.09134 \end{pmatrix} \tag{8}$$

### 1.3 Compute the optimal joint motion $\Delta q^*$ of the one-shot IK

To solve the inverse kinematic problem, we will use optimality principle to calculate $\Delta q^*$. We have already known W and C, therefore:

$$J^{\sharp} = J^T(JJ^T + C^{-1})^{-1} = \begin{pmatrix} -2.98298 & 0.70095 & 0. \\ 1.25924 & 5.35883 & -3.64466 \\ -1.37767 & -5.86282 & 0.74449 \\ -0.1195 & -0.50854 & -0.267 \end{pmatrix} \tag{9}$$

$$\Delta q* = J^{\sharp}(y^* - y_t) = \begin{pmatrix} -0.89653 \\ -3.89697 \\ 2.69669 \\ 0.07371 \end{pmatrix} \tag{10}$$

## 2 System identification, Signal filtering and state estimation

### 2.1 Explain the following questions:

**a. What are the types of models for classifying different systems?**

- White-box model: a system with all necessary and a priori information available; we have knowledge of its internal structure, or the process (relationship between inputs and outputs).
- Black-box model: a system which can only be observed in terms of its inputs and outputs, without any knowledge of the internal mechanism. A typical black-box is the human brain.
- Grey-box model: a system that can be described by a partial theoretical structure and only needs data to complete the model. The exact parameter values need to be identified from data in order to be fit into the theoretical model.

**b. In which type of model is the system identification needed?**

Grey-box model. As described above, we can not identify black-box model as we know noting about its internal mechanism. And we do not need to identify white-box model as we have already known the relationship between inputs and outputs. But the exact parameter values of grey-box model need to be identified from data.

## 2.2 Explain the following questions:

**a. Why is signal filtering necessary in real world applications?**

- Real world has uncertainties anywhere, which may come from noisy, drifting, limitation of sensors' accuracy and so on.
- A direct let-go of noises will jeopardize state estimation algorithm.
- Direct use of noisy signals cause instability issues in control.
- Necessary pre-filtering can reduce noise to a satisfactory level.

**b. What are the common types of filters and how to choose them?**

- Low-pass filter – low frequencies lower than cut-off frequency are passed
- High-pass filter – high frequencies higher than cut-off frequency are passed
- Band-pass filter – only frequencies within a frequency band are passed
- Band-stop filter – only frequencies within a frequency band are attenuated
- Notch filter – rejects a particular frequency, eg resonance

We choose the filter based on the characteristics of signal and noises. The filter we use should only pass our target signal and reject unexpected noises.z

## 2.3 Assume a scenario of measuring the position of a robot actuator that is following a sinusoidal trajectory of A sin($\omega$), and there are random Gaussian noises mixed in the measurement. (Note: A is the magnitude of the sinusoidal wave; the unit of $\omega$ is rad/s, and 2$\pi$f=$\omega$.) Explain:

**a. Which type of filter should be applied to obtain a clean position signal?**

Low-pass filter with a cut-off frequency f.

**b. What is the justification of selecting this filter?**

We know that the signal we concern has a relative low frequency f. And random Gaussian noises can be considered as high frequence signal, so we obtain a clean position signal by applying low-pass filter to filter the Gaussian noises.

## 2.4 Modelling a car suspension system by a mass-spring-damper model

**a. Discuss if filtering is needed to process some signals prior to system identification, explain the reasons; if yes, which signal should be filtered and which filter should be applied?**

Yes, the signal from on-board transducer shold be processed by applying high-pass filter. Because the on-board measurement drifts slowly during a long measurement. Although the drifting will not affect $\dot{z}$ and $\ddot{z}$, it will create error when we are doing sensor fusing. And the error will make the system unstable and jeopardize state estimation algorithm.

**b. Solve the sensor fusion problem to combine two measurements of the vertical position z.**

The fused estimate is weighting by the inverse of covariance:

$$\widehat{z} = (\frac{L_1}{L} z_1 + \frac{L_2}{L} z_2) \tag{11}$$

where $L_1 = \frac{1}{a_1^2}$, $L_2 = \frac{1}{a_2^2}$ and $L = L_1 + L_2$. Therefore, we can get:

$$\widehat{z} = \frac{a_1 a_2}{a_1^2 + a_2^2} (\frac{1}{a_1^2} z_1 + \frac{1}{a_2^2} z_2) \tag{12}$$

**c. Formulate the dynamical equation of the system using m, k and c.**

The force applied by the spring: $F_x = -kx$

The force by the damper: $F_d = -c\dot{x}$

From Newton's 2nd law we get: $m\ddot{x} = -kx - c\dot{x}$

$$m\ddot{x} = [k, c] \begin{bmatrix} -x \\ -\dot{x} \end{bmatrix} \tag{13}$$

**d. Formulate the identification problem expressed by k and c, explain how to use the data in the formula, and provide a technical solution to solve estimation of k and c.**

For a series of N measurements, 1, 2, ... N:

$$m \begin{bmatrix} \ddot{z_1}, & \ddot{z_2}, & \ldots, & \ddot{z_N} \end{bmatrix} = [k, c] \begin{bmatrix} z_1, & z_2, & \ldots, & z_N \\ \dot{z_1}, & \dot{z_2}, & \ldots, & \dot{z_N} \end{bmatrix} \tag{14}$$

k and c of the car suspension are identified by:

$$[k, c] = m \begin{bmatrix} \ddot{z_1}, & \ddot{z_2}, & \ldots, & \ddot{z_N} \end{bmatrix} \begin{bmatrix} z_1, & z_2, & \ldots, & z_N \\ \dot{z_1}, & \dot{z_2}, & \ldots, & \dot{z_N} \end{bmatrix}^{\dagger} \tag{15}$$

Basically, we can consider the equation 14 as a equation: Y = H X, where H is an observation matrix. We can solve this equation by find out a H that minimizes the sum of squared errors between a model and data (Y-HX). Then $H = Y^* X^+$ where $X^+ = X^T (X^T X)^{-1}$. Therefore, we have $H = Y^* X^T (X^T X)^{-1}$