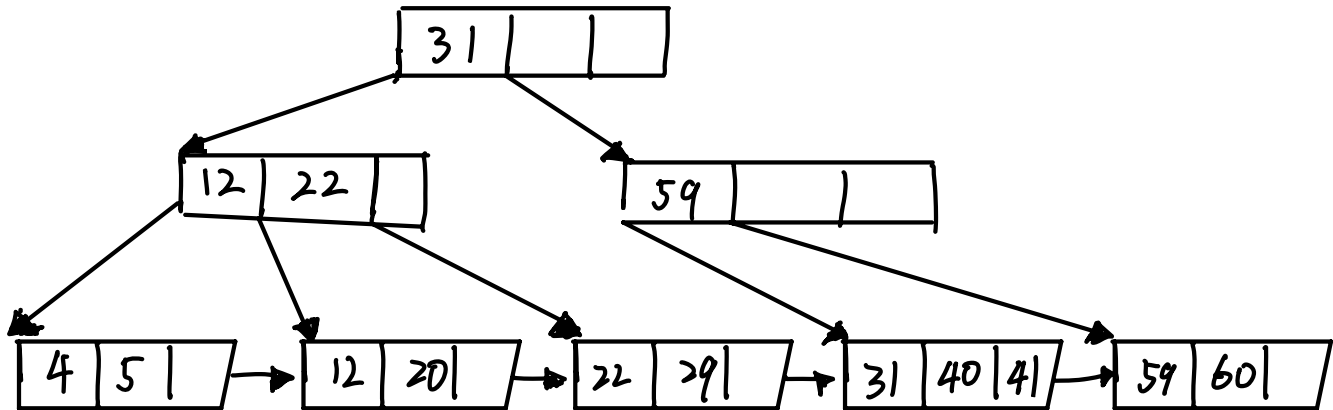


1. $n=4$

For internal node: # of children: $\frac{n}{2} = 2 \leq \# \text{ of children} \leq n = 4$

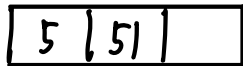
For leaf node: # of value: $\frac{n-1}{2} = 2 \leq \# \text{ of value} \leq n-1 = 3$



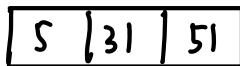
1. ① insert 5



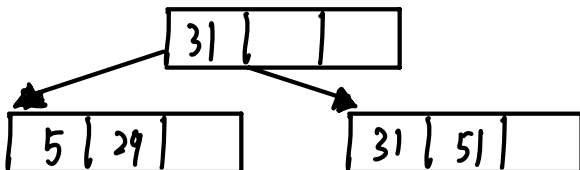
② insert 51



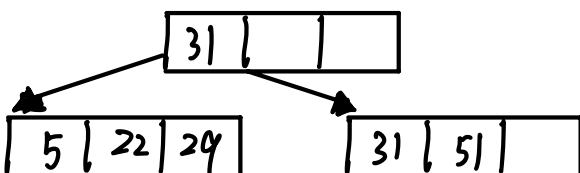
③ insert 31



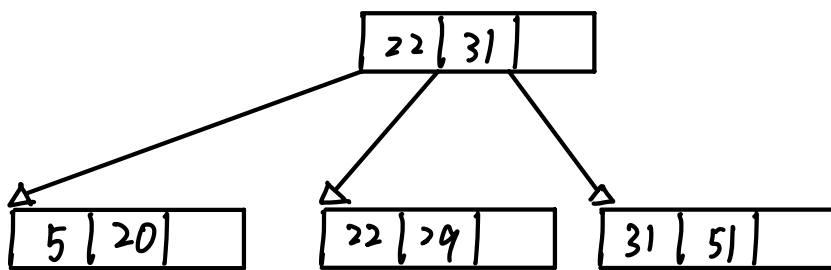
④ insert 29



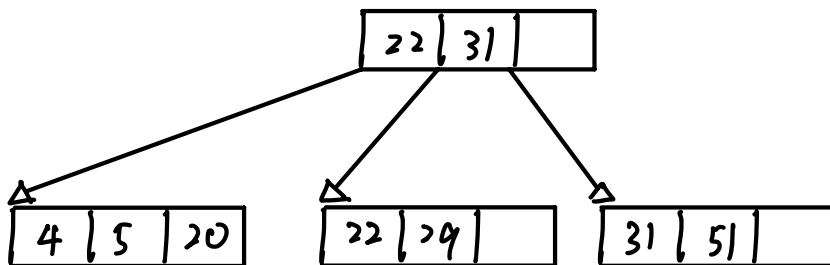
⑤ insert 22



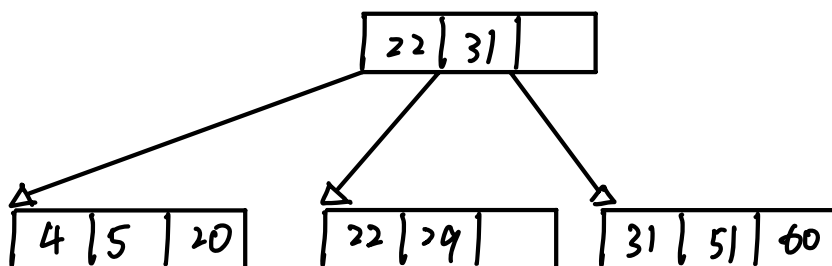
⑥ insert 20



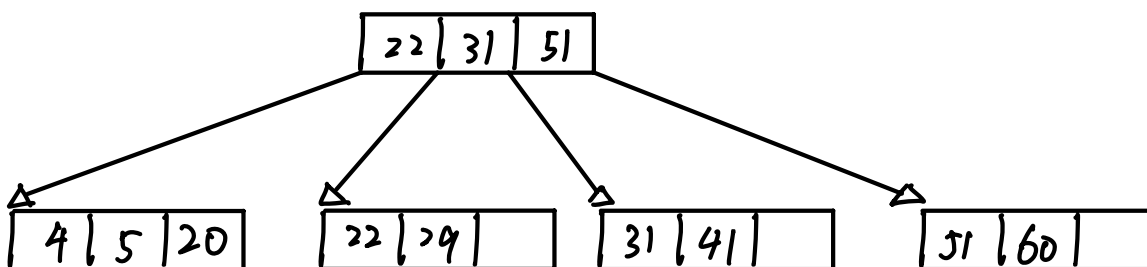
⑦ insert 4



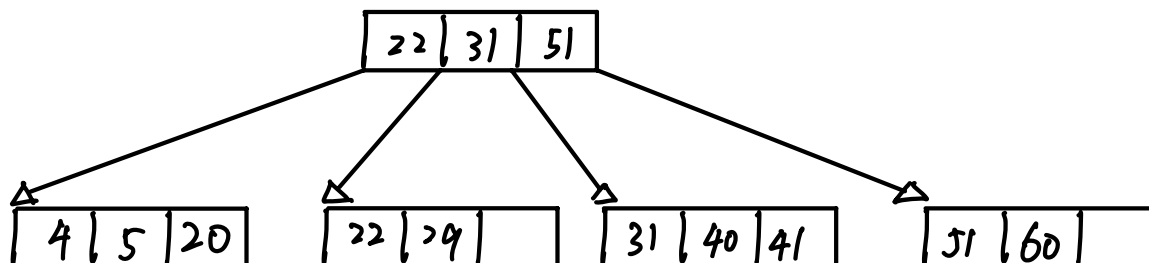
⑧ insert 60



⑨ insert 41

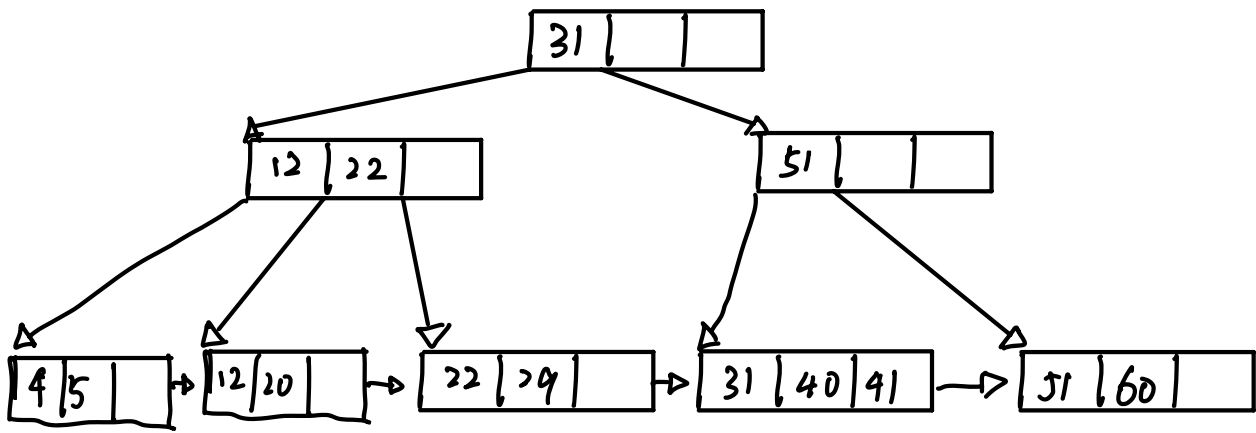


⑩ insert 40

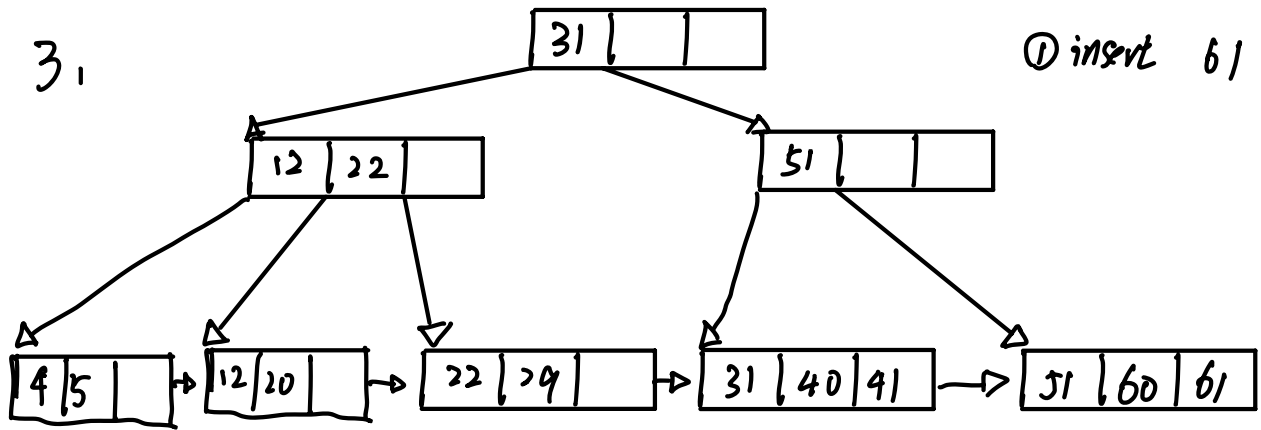


⑪ insert 12

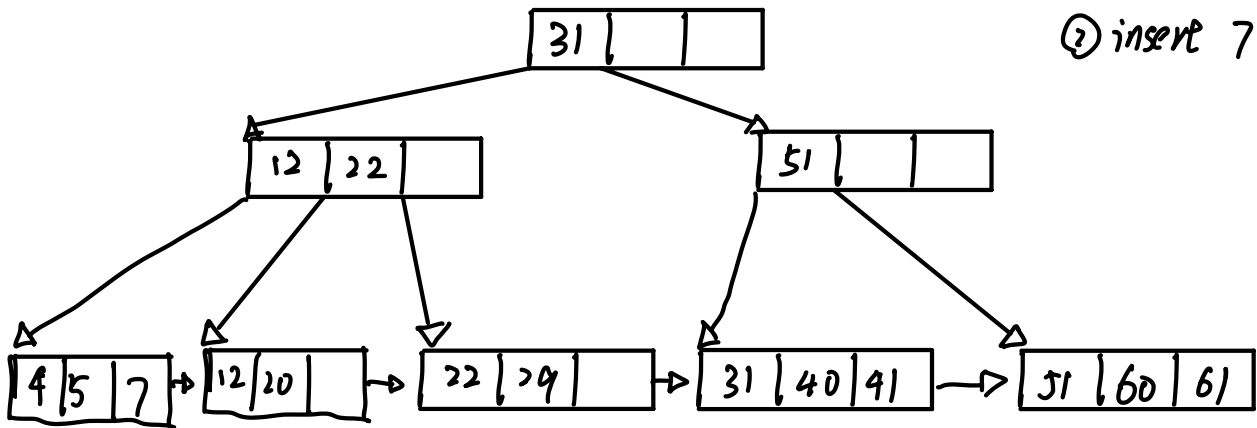
next page



3.

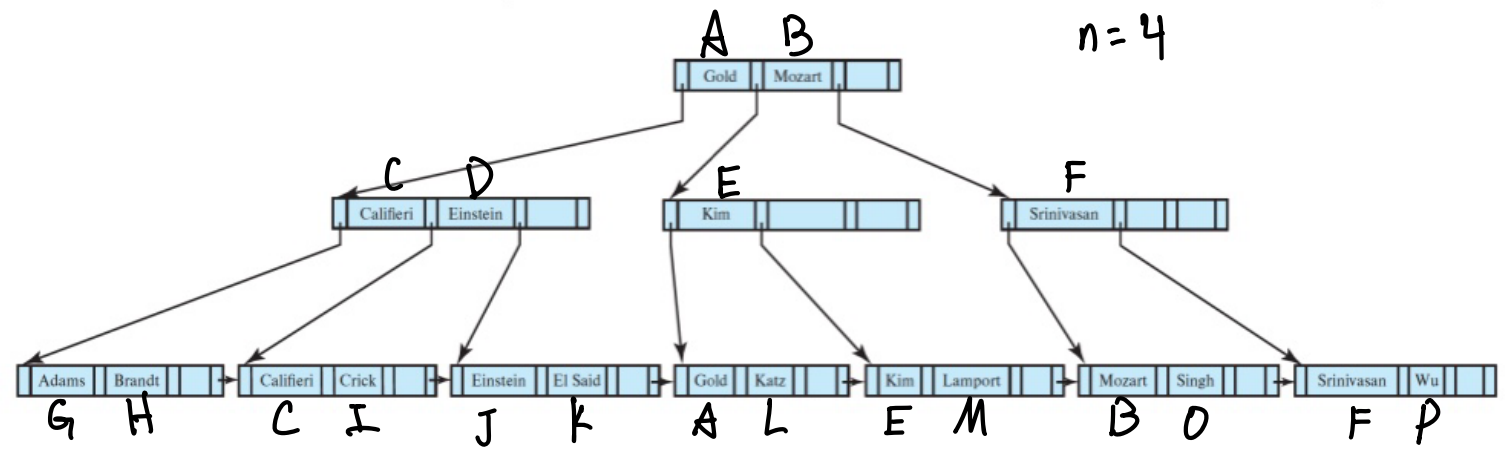


① insert 61



② insert 7

4.



After remove Gold in leaf node, we merge katz to the left sibling \Rightarrow Einstein | El Said | katz

Then we also merge kim in the internal node to its sibling \Rightarrow Califleri | Einstein | kim

Lastly, remove Gold in the root \Rightarrow Mozart | |

use letter to represent the tree, below is the final state

