1. First, find closures of each Attributes sets

For A^+

For $A \to BDE$ $\therefore A^+ = \{B D G\}$

For $BG \to DE$ $\because \{BG\} \subseteq A^+ = \{BDG\}$ $\therefore A^+ = \{BDG\} \cup \{DE\}$

$\therefore A^+ = \{B D E G\}$

For $B \to G$ $\because$ G is already in A^+

For $D \to A$ $\because \{D\} \subseteq A^+ = \{BDEG\}$ $\therefore A^+ = \{BDEG\} \cup \{A\}$

$\therefore A^+ = \{A B D E G\}$

Apply Augmentation role, $CA \longrightarrow ABCDEG = R$

$\therefore$ (CA) is a superkey

For B^+

$\because B \to D$ and $D \to A$ $\therefore B \to A$ by transitivity role

$\because B \to A$ and $A \to BDG$ $\therefore B \to BDG$ by transitivity role

$\because B \to A$ and $B \to BDG$ $\therefore B \to ABDG$ by union role

$\therefore B^+ = \{ABDG\}$

$\because BG \to DE$ and $BG \subseteq B^+ = \{ABDG\}$

$\therefore B^+ = \{ABDG\} \cup \{DE\} = \{ABDEG\}$

Apply Augmentation role, $CB \longrightarrow ABCDEG = R$

$\therefore$ (C, B) is a super key

For D^+

$\because D \to A$ and $A \to BDG$ $\therefore D \to BDG$ by transitivity role

$\because D \to A$ and $D \to BDG$ $\therefore D \to ABDG$ by union role

$\therefore D^+ = \{A\ B\ D\ G\}$

$\because BG \rightarrow DE$ and $\{BG\} \subseteq D^+ = \{ABDG\}$

$\therefore D^+ = \{ABDG\} \cup \{DE\} = \{ABDGE\}$

Apply Augmentation role, $CD \longrightarrow ABCDEG = R$

$\therefore (C, D)$ is a superkey

We can still reach more superkey like (CBG) but none of them are minimal $\therefore$ Candiate keys are (CA), (CB) (CD)

2. a) For AB

$\because A, B \subseteq AB$ $\therefore AB \to AB$

It can't determine C, D, E $\therefore$ not a candidate key

b) For CD

$\because C, D \subseteq CD$ $\therefore CD \to CD$

It can't determine A, B, E $\therefore$ not a candidate key

c) For BD

$\because B, D \subseteq BD$ $\therefore BD \to BD$

$\because BD \to E$ $\therefore BD \to BDE$

It can't determine A, C $\therefore$ not a candidate key

d) For AC

$\because A, C \subseteq AC$ $\therefore AC \to AC$

$\because A \to B$ & $A \subseteq \{AC\}$ $\therefore AC \to ABC$

$\because C \to D$ & $C \subseteq \{ABC\}$ $\therefore AC \to ABCD$

$\because BD \to E$ & $BD \subseteq \{ABCD\}$ $\therefore AC \to ABCDE = R$

$\therefore$ AC is a candidate key

e) For E

E determine nothing $\therefore$ It is not a candidate key

f) For AE

$\because A, E \subseteq AE$ $\therefore AE \to AE$

$\because A \to B$ and $A \subseteq \{AE\}$ $\therefore AE \to ABE$

It can't deter C, D $\therefore$ not a candidate key

$\therefore AC$ is the only candidate key

3. For $A \to BDG$

$\because A \subseteq A \quad \therefore A \to ABDG$

$\because BG \to DE \quad \therefore BDG \to DE$ by augmentation

$\because A \to ABDG$ and $BDG \to DE \quad \therefore A \to ABDEG$

Take $A \to ABDEG$ as example from F^+

It is not trival $\because \{ABDEG\} \not\subseteq \{A\}$

It is not a superkey since it doesn't determe C $\therefore$ doesn't determine R

$\therefore$ Violate BCNF

By Adding $A \to C$, we can make $A^+ = \{ABCDEG\}$

So A is a superkey, then all $\alpha \to \beta$ in F^+ that $\alpha = \{A\}$ satisfy the constraint of BCNF. Same for B^+, D^+.

4. ① Apply union role $\Rightarrow \{J \to KL,\ L \to J,\ MNP \to K,\ KP \to M,\ LJ \to M\}$

- For $J \to KL$
  - remove $K \Rightarrow \{\alpha \to (\beta - K)\} = J \to L$
  - $F' = \{J \to L,\ L \to J,\ MNP \to K,\ KP \to M,\ LJ \to M\}$

    $J^+ = \{JLM\}$ $\because K$ is not in $\{JLM\}$

    $\therefore K$ is not extraneous
  - remove $L \Rightarrow J \to K$
  - $F' = \{J \to K,\ L \to J,\ MNP \to K,\ KP \to M,\ LJ \to M\}$

    $J^+ = \{JK\}$ $\because L$ is not in $\{JK\}$ $\therefore L$ is not extraneous
- $MNP \to K$
  - remove $M \Rightarrow NP \to K$

    $\{K\} \not\subset NP^+$ $\therefore M$ is not extraneous
  - remove $N \Rightarrow MP \to K$

    $\{K\} \not\subset MP^+$ $\therefore N$ is not extraneous
  - remove $P \Rightarrow MN \to K$

    $\{K\} \not\subset MN^+$ $\therefore P$ is not extraneous
- $KP \to M$
  - remove $K \Rightarrow P \to M$

    $\{M\} \not\subset P^+$ $\therefore K$ is not extraneous
  - remove $P \Rightarrow K \to M$

    $\{M\} \not\subset K^+$ $\therefore P$ is not extraneous
- $LJ \to N$

- remove $L \Rightarrow J \to N$

  $\{N\} \not\subseteq J^+$ $\therefore$ L is not extraneous

- remove $J \Rightarrow L \to N$

  $\{N\} \not\subseteq L^+$ $\therefore$ J is not extraneous

$\because$ nothing changed

$\therefore$ $F_C = F = \{J \to KL,\ L \to J,\ MNP \to K,\ KP \to M,\ LJ \to N\}$