1. adibidea

$\int 2x\,dx = x^2 + C$, zeren eta $F(x) = x^2 \rightarrow F'(x) = 2x = f(x)$. Konstante gehiago gehituz gero, jatorrizko gehiago lortzen ditugu.

2. adibidea

$$\int \frac{3x^2+1}{x^3+x}dx = \begin{bmatrix} x^3+x=t \\ (3x^2+1)dx = dt \end{bmatrix} = \int \frac{1}{t}\,dt = \ln|t| + C = \ln\left|x^3+x\right| + C$$

3. adibidea

$$\int \arctan x\,dx = \begin{bmatrix} u=\arctan x & du = \dfrac{dx}{1+x^2} \\ dv = dx & v = x \end{bmatrix} = x\cdot\arctan x - \int x\frac{dx}{1+x^2} =$$

$$= x\cdot\arctan x - \frac{1}{2}\int 2x\frac{dx}{1+x^2} = x\cdot\arctan x - \frac{1}{2}\ln\left|1+x^2\right| + C$$

4. adibidea

$$\int \frac{dx}{x^2+x+1} = \int \frac{dx}{\left(x+\dfrac{1}{2}\right)^2+\dfrac{3}{4}} = \begin{bmatrix} x+\dfrac{1}{2}=t \\ dx = dt \end{bmatrix} = \int \frac{dt}{t^2+\dfrac{3}{4}} = \frac{2}{\sqrt{3}}\arctan\frac{2t}{\sqrt{3}} + C =$$

$$= \frac{2}{\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}} + C$$

5. adibidea

$$\int \frac{(x+2)dx}{x^2+2x+2} = \begin{bmatrix} x^2+2x+2=t \\ (2x+2)dx = dt \end{bmatrix} = \frac{1}{2}\int \frac{2x+2+2}{x^2+2x+2}dx = \frac{1}{2}\int \frac{2x+2}{x^2+2x+2} + \int \frac{dx}{(x+1)^2+1} =$$

$$= \frac{1}{2}\ln\left|x^2+2x+2\right| + \arctan(x+1) + C$$

6. adibidea

$$\int \frac{dx}{\sqrt{x^2+2x+3}} = \int \frac{dx}{\sqrt{(x+1)^2+2}} = \begin{bmatrix} x+1=t \\ dx=dt \end{bmatrix} = \int \frac{dt}{\sqrt{t^2+2}} = \ln\left|t+\sqrt{t^2+2}\right| + C =$$

$$= \ln\left|x+1+\sqrt{x^2+2x+3}\right| + C$$

7. adibidea

$$\int \frac{(2x+1)dx}{\sqrt{x^2+2x+3}} = \begin{bmatrix} x^2+2x+3=t^2 \\ (2x+2)dx = 2tdt \end{bmatrix} = \int \frac{(2x+2-1)dx}{\sqrt{x^2+2x+3}} = \int \frac{(2x+2)dx}{\sqrt{x^2+2x+3}} -$$

$$-\int \frac{dx}{\sqrt{(x+1)^2+2}} = 2\sqrt{x^2+2x+3} - \ln\left|x+1+\sqrt{x^2+2x+3}\right| + C$$

8. adibidea

$$\int \frac{x^5}{x^4-1}dx = \int \left[x + \frac{x}{x^4-1}\right]dx = \int x\,dx + \int \frac{x}{x^4-1}dx = \frac{x^2}{2} + \int \frac{x}{x^4-1}dx\,;$$

azken integral hau zatiki sinpleen batura gisa deskonposatu behar da:

$$\frac{x}{x^4-1} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1}$$

$$\frac{x}{x^4-1} = \frac{A(x-1)(x^2+1) + B(x+1)(x^2+1) + (Cx+D)(x+1)(x-1)}{(x+1)(x-1)(x^2+1)}$$

$$x = A(x-1)(x^2+1) + B(x+1)(x^2+1) + (Cx+D)(x+1)(x-1)$$

Baldin $x=1$ $\quad 1=4B \quad \Rightarrow B=1/4$

Baldin $x=-1$ $\quad -1=-4A \quad \Rightarrow A=1/4$

Baldin $x=0$ $\quad 0=-A+B+D \quad \Rightarrow D=0$

Baldin $x=2$ $\quad 2=5A+15B+6C \quad \Rightarrow C=-1/2$

$$\int \frac{x}{x^4-1}dx = \frac{1}{4}\int \frac{dx}{x+1} + \frac{1}{4}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{x\,dx}{x^2+1}$$

$$\int \frac{x}{x^4-1}dx = \frac{1}{4}\ln|x+1| + \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x^2+1| + K$$

Beraz:

$$\int \frac{x^5}{x^4-1}dx = \frac{x^2}{2} + \int \frac{x}{x^4-1}dx = \frac{x^2}{2} + \frac{1}{4}\ln|x+1| + \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x^2+1| + K$$

$$\int \frac{x^5}{x^4-1}dx = \frac{x^2}{2} + \frac{1}{4}\ln|x^2-1| - \frac{1}{4}\ln|x^2+1| + K = \frac{x^2}{2} + \frac{1}{4}\ln\left|\frac{x^2-1}{x^2+1}\right| + K$$

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9. adibidea

$$\int \frac{(3x-1)}{(x-2)^2(x+1)^3}dx = \frac{Ax^2+Bx+C}{(x-2)(x+1)^2} + \int \frac{(Mx+N)}{(x-2)(x+1)}dx$$

Deribatuz:

$$\frac{(3x-1)}{(x-2)^2(x+1)^3} = \frac{(2Ax+B)(x-2)(x+1)^2 - (Ax^2+Bx+C)(3x^2-3)}{(x-2)^2(x+1)^4} + \frac{(Mx+N)}{(x-2)(x+1)}$$

$(x-2)^2(x+1)^3$-z biderkatuz eta gaiak taldekatuz, hauxe lortzen dugu:

$$3x-1 = Mx^4 + (-A+N)x^3 + (A-2B-3M)x^2 + (2B-3C-4A-3N-2M)x + (3C-2B-2N)$$

Gaiak identifikatuz:

$$\begin{cases} 0 = M \\ 0 = -A+N \\ 0 = A-2B-3M \\ 3 = -2B-3C-4A-3N-2M \\ -1 = 3C-2B-2N \end{cases} \Rightarrow \begin{array}{l} M = 0 \\ A = -2/9 \\ N = -2/9 \\ C = -5/9 \\ B = -1/9 \end{array}$$

$$\int \frac{(3x-1)}{(x-2)^2(x+1)^3}dx = \frac{(-2/9)x^2 - (1/9)x - (5/9)}{(x-2)(x+1)^2} + \int \frac{(-2/9)}{(x-2)(x+1)}dx$$

Azken integral hau ebatziz:

$$-\frac{2}{9}\int \frac{dx}{(x-2)(x+1)} = -\frac{2}{27}\ln\left|\frac{x-2}{x+1}\right| + K$$

Azkenean integrala honela geratzen da:

$$\int \frac{(3x-1)}{(x-2)^2(x+1)^3}\,dx = \frac{(-2/9)x^2-(1/9)x-(5/9)}{(x-2)(x+1)^2} - \frac{2}{27}\ln\left|\frac{x-2}{x+1}\right| + K$$

10. adibidea

$$\int \frac{\sin x}{\cos^3 x}\,dx = \begin{bmatrix} \cos x = t \\ -\sin x\; dx = dt \end{bmatrix} = -\int \frac{dt}{t^3} = \frac{t^{-2}}{2} + C = \frac{1}{2\cos^2 x} + C$$

11. adibidea

$$\int \cosh(3x)\cdot\cosh(2x)\,dx = \int \frac{\cosh(5x)+\cosh(x)}{2}\,dx = \frac{1}{2}\int \cosh 5x\,dx + \frac{1}{2}\int \cosh x\,dx = \frac{1}{10}\sinh 5x + \frac{1}{2}\sinh x + C$$

12. adibidea

$$\int \frac{dx}{(1+x)\sqrt{x^2+x+1}} = \begin{bmatrix} \sqrt{x^2+x+1} = x+t \quad \Rightarrow \\ x = \dfrac{t^2-1}{1-2t},\;\; dx = \dfrac{2(-t^2+t-1)}{(1-2t)^2}\,dt \end{bmatrix} = \int \frac{2}{t^2-2t}\,dt = \int -\frac{1}{t}\,dt +$$

$$+\int \frac{1}{t-2}\,dt = -\ln|\,t\,| + \ln|\,t-2\,| + C = -\ln\left|\sqrt{x^2+x+1}-x\right| + \ln\left|\sqrt{x^2+x+1}-x-2\right| + C$$

13. adibidea

$$\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx = \begin{bmatrix} x = \sin t \\ dx = \cos t\; dt \end{bmatrix} = \int \frac{\cos^2 t}{1+\sin^2 t}\,dt = \begin{bmatrix} \tan t = z \\ dt = \dfrac{dz}{1+z^2} \end{bmatrix} =$$

$$= \int \frac{dz}{(1+2z^2)(1+z^2)}$$ zatiki sinpletan deskonposatuz:

$$\frac{1}{(1+2z^2)(1+z^2)} = \frac{A+Bz}{1+2z^2} + \frac{C+Dz}{1+z^2}$$

$$1 = (A+Bz)(1+z^2) + (C+Dz)(1+2z^2)$$

$$\begin{cases} z^3 & \Rightarrow B+2D=0 \\ z^2 & \Rightarrow A+2C=0 \\ z & \Rightarrow B+D=0 \\ z^0 & \Rightarrow A+C=1 \end{cases} \Rightarrow \begin{array}{l} A=2 \\ B=0 \\ C=-1 \\ D=0 \end{array}$$

$$\int \frac{dz}{(1+2z^2)(1+z^2)} = \int \frac{2dz}{1+2z^2} - \int \frac{dz}{1+z^2} =$$

$$= \sqrt{2}\arctan\sqrt{2}z - \arctan z + K = \sqrt{2}\arctan\frac{\sqrt{2}x}{\sqrt{1-x^2}} - \arcsin x + K$$

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14. adibidea

$$\int \frac{1+x^2}{\sqrt{1+x^2}}\,dx = (ax+b)\sqrt{1+x^2} + M\int \frac{dx}{\sqrt{1+x^2}}$$

Deribatuz eta erroaz biderkatuz:

$$1+x^2 = a(1+x^2) + (ax+b)x + M$$

Koefizienteak identifikatuz hurrengo sistema lortzen da:

$$\begin{cases} 1=2a \\ 0=b \\ 1=a+M \end{cases} \Rightarrow \begin{array}{l} a=1/2 \\ b=0 \\ M=1/2 \end{array}$$

$$\int \frac{1+x^2}{\sqrt{1+x^2}}\,dx = \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\int \frac{dx}{\sqrt{1+x^2}} = \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\ln\left| x+\sqrt{1+x^2} \right| + C$$