DISCRETE MATHEMATICS

Computer Engineering in Management and Information Systems

MID-TERM EXAM **November 15, 2019**

EXERCISE 1

Determine by using properties of logical equivalence if the following logical statement is a tautology or a contradiction:

$$[\neg((\neg p \vee \neg q) \vee \neg(\neg r \to s)) \vee ((r \vee s \to (q \to r)) \to q \wedge r \wedge (r \vee s))] \wedge \neg q$$

(1.5 points)

EXERCISE 2

Simplify the following statement using properties of logical equivalence:

$$[(p \to (r \to s)) \wedge r] \vee \neg s$$

(1.25 points)

EXERCISE 3

Verify that the following logical reasoning is valid:

"If Markel drinks beer, then he is not thirsty. Furthermore, it is not true that he wants water and not beer. Today Markel is thirsty. Therefore, today Markel does not want water."

(1.25 points)

EXERCISE 4

Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two correspondences defined as follows:

$$f(x) = \begin{cases} -x^2 - 2 & x \leq 0 \\ 2x - 2 & 0 < x < 1 \\ \ln(x) & x \geq 1 \end{cases} \qquad g(x) = \frac{-1}{x^2 + 1}$$

a) Plot the correspondences.

b) Determine whether f and g are applications. If so, then analyze their properties.

c) Compute $f \circ g$ and $g \circ f$.

d) When possible calculate the inverse application.

(2.5 points)

EXERCISE 5

The binary relation below is defined in $A = \{0, 2, 3, 4, 6, 9, 12, 24, 27, 36, 72, 81\}$:

$$xRy \Leftrightarrow y = 3x$$

a) Analyze the properties of the binary relation.

b) Is it an equivalence relation? Is it an order relation? Justify the answers.

c) What elements are related to element 2?

d) Is element 2 related to any elements? Which ones?

e) Are answers to questions c) and d) the same? Justify the responses.

(2 points)

EXERCISE 6

Verify the following expression by using the principle of induction:

$$1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{(4n^2-1)n}{3} \qquad \forall n \in \mathbb{N}$$

(1.5 points)
