~~Programación Modular y Orientación a Objetos~~ 1

PARCIAL: 11 de noviembre de 2022

(1) A: "la crema tiene exceso de agua".
B: "crema apta para la venta"

$P(A) = 0{'}05$

$P(\bar{B} \mid A) = 0{'}99$

$P(B \mid \bar{A}) = 0{'}95$

a) $P(B) = P(B \cap A) + P(B \cap \bar{A}) = P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A}) =$

$= P(B|A) \cdot 0{'}05 + 0{'}95 \cdot 0{'}95 = (1 - P(\bar{B}|A)) \cdot 0{'}05 + 0{'}95 \cdot 0{'}95 =$

$= 0{'}01 \cdot 0{'}05 + 0{'}95 \cdot 0{'}95 = \boxed{0{'}903}$

b) $P(\bar{A} \mid \bar{B}) = \dfrac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \dfrac{P(\bar{B}|\bar{A}) \cdot P(\bar{A})}{P(\bar{B}|\bar{A}) \cdot P(\bar{A}) + P(\bar{B}|A) \cdot P(A)} = \dfrac{0{'}05 \cdot 0{'}95}{0{'}05 \cdot 0{'}95 + 0{'}99 \cdot 0{'}0\ldots}$

$= \boxed{0{'}4897}$

c) $750 \cdot P(B) = 750 \cdot 0{'}903 = 677{'}25 \simeq 678$ botes de crema aptos para la venta.

$240€ \cdot 678$ botes de crema $- 650€ \cdot (750 - 678$ botes$) =$

$= \boxed{115.920 €}$.

(2)

X := "Consumo de gas de un hogar de Gerbilla durante los meses de invierno (en m^3)".

$X \sim N(\mu = 90, \sigma)$

a) $P(X > 150) = 0'01 \rightarrow$ ¿$P(X < 100)$?

$P(X > 150) = 1 - P(X \le 150) = \boxed{1 - P\left(Z \le \frac{150-90}{\sigma}\right) = 0'01}$

$qnorm(0'99, 0, 1) = \frac{150-90}{\sigma} \rightarrow \sigma = \frac{150-90}{qnorm(0'99,0,1)}$

$\sigma = \frac{150-90}{2'3263} = 25'7920 \rightarrow \boxed{\sigma = 25'7920}$

$P(X < 100) = P\left(Z \le \frac{100-90}{25'7920}\right) = P\left(Z \le \frac{10}{25'7920}\right) = P(Z \le 0'3877) =$

$= pnorm(0'3877, 0, 1) = \boxed{0'6509}$

b) Y := "Número de hogares cuyo consumo de gas en invierno es superior a 135 m^3 de entre 4000 hogares con calderas de gas".

$Y \sim B(n = 4000, p = P(X > 135))$

$P(X > 135) = 1 - P(X \le 135) = 1 - P\left(Z \le \frac{135-90}{25'7920}\right) = 1 - pnorm(1'7447) =$

$= 1 - 0'9595 = \boxed{0'0405}$

$E(Y) = n \cdot p = 4000 \cdot 0'0405 = \boxed{162 \text{ hogares}}$

c) T := "Consumo de gas ... después de las medidas adoptadas ..."

$T \sim N(\mu = 77, \sigma = 23)$

Se define una nueva variable que será combinación lineal de T:

V := "Consumo de gas de una comunidad"

$V \sim N(\mu = 77 \cdot 16, \sigma = \sqrt{23^2 \cdot 16}) = N(\mu = 1232, \sigma = 92)$

$V \sim N(1232, 92)$

$$P(V > 1200) = 1 - P(V \leq 1200) = 1 - P\left(Z_V \leq \frac{1200-1232}{92}\right) =$$

$$= 1 - P(Z_V \leq -0'3478) = 1 - (1 - P(Z_V \leq 0'3478)) =$$

$$= 1 - 1 + \text{pnorm}(0'3478, 0, 1) = \boxed{0'6360}$$

③ X:= "Potencia en vatios de las bombillas LED de última generación de la empresa AQP".

a) $E(X) \cdot 14$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

$$f(x) = \frac{dF(x)}{dx} = \begin{cases} 0 & \text{en otros casos} \\ \frac{3x^2}{64} & 0 \leq x < 4 \end{cases}$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx = \cancel{\int_{-\infty}^{0} x \cdot 0\, dx} + \int_{0}^{4} \frac{3x^3}{64}\, dx + \cancel{\int_{4}^{\infty} x \cdot 0\, dx} =$$

$$= \frac{3}{64} \cdot \frac{x^4}{4} \Big]_0^4 = \frac{3 \cdot 4^4}{64 \cdot 4} = 3 \text{ vatios}$$

Resultado: $E(X) \cdot 14 = 3 \cdot 14 = \boxed{42 \text{ vatios}}$

b) Y:= "Número de bombillas con potencia inferior a 1'2 W de entre 100 escogidas".

$$Y \sim B(n = 100,\ p = P(X < 1'2)) \begin{cases} np = 2'7 \\ \cancel{n7}\ p \leq 0'1 \end{cases} \cong \mathcal{P}(\lambda = 2'7)$$

$$P(X < 1'2) = P(X \leq 1'2) = F(1'2) = \frac{(1'2)^3}{64} = \boxed{0'027}$$

$$P(Y \leq 5) = P(Y=0) + P(Y=1) + P(Y=2) + P(Y=3) + P(Y=4) + P(Y=5)$$

$$\cong e^{-2'7} \cdot \left(\frac{2'7^0}{0!} + \frac{2'7^1}{1!} + \frac{2'7^2}{2!} + \frac{2'7^3}{3!} + \frac{2'7^4}{4!} + \frac{2'7^5}{5!}\right) = \boxed{0'9433}$$

c) 2000 bombilla $\cdot$ 0'75 = 1500 bombillas cuyo $W > 2'5$

V := "Número de bombillas cuya potencia sea superior a 2'5 W de entre 2000".

$V \sim B(n=2000,\ p = P(X > 2'5))$

$\cdot P(X > 2'5) = P(X \geq 2'5) = 1 - P(X \leq 2'5) = 1 - F(2'5) = 1 - \frac{(2'5)^3}{64} = \boxed{0'755859375}$

$P(V > 1500) = 1 - P(V \leq 1500)$

$\left.\begin{cases} np = 1511'8 \\ nq = 488'2 \end{cases}\right\} \geq 5 \quad \cong N(\mu = 1511'8,\ \sigma = 19'2102)$

$= 1 - P(V \leq 1499'5) = 1 - P\left(Z_V \leq \frac{1499'5 - 1511'8}{19'2102}\right) =$

$= 1 - P(Z_V \leq -0'6403) = 1 - 1 + \text{pnorm}(0'6403, 0, 1) = \boxed{0'7390}$

PARCIAL: 12 de noviembre de 2021

1) $X_A :=$ "Índice de hidroxilos de unos polioles ..."

$X_A \sim N(580, 75)$

$X_B :=$ "Índice de hidroxilos de unos polioles ..."

$X_B \sim N(970, 165)$

A: "Proceso de síntesis A" $P(A) = 0'7$

B: "Proceso de síntesis B" $P(B) = 0'3$

a) $P(B \mid X > 750) = \dfrac{P(B \cap (X > 750))}{P(X > 750)} = \dfrac{P(X_B > 750)}{P(X_B > 750) + P(X_A > 750)} =$

• $P(X_B > 750) = 0'3 \cdot P\left(Z_B > \frac{750 - 970}{165}\right) = 0'3 \cdot P(Z_B > -1'33) =$
$= 0'3 \cdot P(Z_B < 1'33) = 0'3 \cdot \text{pnorm}(1'33, 0, 1) =$
$= 0'3 \cdot 0'9082409 = \boxed{0'27247227}$

• $P(X_A > 750) = 0'7 \cdot P\left(Z_A > \frac{750 - 580}{75}\right) = P(Z_A > 2'26) \cdot 0'7 =$
$= 0'7 \cdot (1 - P(Z_A \leq 2'2666)) = 0'7 \cdot (1 - \text{pnorm}(2'2666, 0, 1)) =$
$= 0'7 \cdot (1 - 0'9882927) = \boxed{0'0081951}$

$P(B \mid X > 750) = \dfrac{0'27247227}{0'0081951 + 0'27247227} = \boxed{0'970801345}$

b) $Y :=$ "Número de unidades que no sirven de entre 400"

$Y \sim B(n = 400, p = P(X \geq 700))$

• $P(X > 700) = P(X_A > 700) + P(X_B > 700) = 0'7 \cdot \left(1 - P\left(Z_A \leq \frac{700 - 580}{75}\right)\right) +$
$+ 0'3 \cdot \left(1 - P\left(Z_B \leq \frac{700 - 970}{165}\right)\right) =$
$= 0'7 \cdot (1 - P(Z_A \leq 1'6)) + 0'3 \cdot (1 - P(Z_B \leq -1'63)) =$
$= 0'7 \cdot (1 - \text{pnorm}(1'6, 0, 1)) + 0'3 \cdot (\text{pnorm}(1'63, 0, 1)) =$
$= 0'7 \cdot 0'9452007 + 0'3 \cdot 0'9491116 = \boxed{0'32309299}$

$Y \sim B(n=400,\ p=0'32309299)\ \begin{cases} np = 129'24 \\ nq = 270'76 \end{cases} \geq 5$

$\cong N(\mu = 129'24,\ \sigma = 9'35329943)$

$P(Y \leq 120) = P(Y \leq 120'5) = P\left(Z_Y \leq \frac{120'5 - 129'24}{9'3533}\right) =$
$= P(Z_Y \leq -0'9376) = 1 - P(Z_Y \leq 0'9376) =$
$= 1 - \text{pnorm}(0'9376, 0, 1) = 1 - 0'8257725 = \boxed{0'174225}$

c) H:= "Altura" $\quad H \sim N(2, \sqrt{0'01})$
W:= "Anchura" $\quad W \sim N(1, \sqrt{0'0036})$

$P = 2H + 2W \qquad P(P < 5'1)$

$P \sim N(\mu_P, \sigma_P) = N(6, 0'2332)$

$\mu_P = 2E(H) + 2E(W) = 2 \cdot 2 + 2 \cdot 1 = 6$
$\sigma_P^2 = 2^2 Var(H) + 2^2 Var(W) = 4 \cdot 0'01 + 4 \cdot 0'0036 = 0'0544$
$\sigma_P = \sqrt{0'0544} = 0'2332380758$

$P(P < 5'1) = P\left(Z_P < \frac{5'1 - 6}{0'2332}\right) = P(Z_P < -3'8593) =$
$= 1 - \text{pnorm}(3'8593) = 1 - 0'9999431 = \boxed{0'0000569}$

2) $X :=$ "Peso en gramos de un microprocesador producido por la empresa MH".

$E(X) = 7/6$ gr.

a) (i) $f(x) \geq 0 \quad \forall x \rightarrow a(x+b) > 0 \begin{cases} a > 0 \\ b > -x \end{cases}$

(ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$\int_{-\infty}^{0} \cancel{0\, dx} + \int_{0}^{2} a(x+b)\, dx + \int_{2}^{\infty} \cancel{0\, dx} = \frac{ax^2}{2} + abx \Big]_0^2 = 2a + 2ab = \boxed{2a(1+b)}$

(iii) $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx = \int_{-\infty}^{0} \cancel{x \cdot 0\, dx} + \int_{0}^{2} a(x^2+bx)\, dx + \int_{2}^{\infty} \cancel{x \cdot 0\, dx} =$

$= \frac{ax^3}{3} + \frac{abx^2}{2} \Big]_0^2 = \boxed{\frac{8}{3}a + 2ab}$

Sistema de 2 ecuaciones y 2 incógnitas:

$\begin{cases} 2a(1+b) = 1 \\ \frac{8}{3}a + 2ab = 7/6 \end{cases} \rightarrow 2a + 2ab = 1 \rightarrow 2ab = 1 - 2a$

$\frac{8}{3}a + 1 - 2a = 7/6 \rightarrow \frac{2}{3}a = 1/6 \rightarrow \boxed{a = 1/4}$

$\boxed{a = 1/4 \quad ; \quad b = 1}$

b) $P(X > 1'5) = P(X \geq 1'5) = \int_{1'5}^{2} \left(\frac{x+1}{4}\right) dx = \frac{1}{4} \cdot \left(\frac{x^2}{2} + x\right) \Big]_{1'5}^{2} =$

$= \frac{1}{4} \cdot \left(\frac{4}{2} + 2\right) - \frac{1}{4} \cdot \left(\frac{2'25}{2} + 1'5\right) = \boxed{0'34375}$

c) T: "Número de micros infructuosos necesarios para que 18 sean válidos".

$$20 \cdot 0'9 = 18 \rightarrow T \sim BN\left(n=18,\ p=\overline{P(0'5 < X < 1'5)}\right)$$

$$P(0'5 < x < 1'5) = P(x < 1'5) - P(x < 0'5) =$$

$$= \int_{0'5}^{1'5} \frac{x+1}{4}\,dx = \frac{1}{4}\cdot\left(\frac{x^2}{2}+x\right)\Bigg]_{0'5}^{1'5} = \frac{1}{4}\cdot\left[\frac{(1'5)^2}{2}+1'5-\frac{(0'5)^2}{2}+0'5\right] =$$

$$= 0'5 \rightarrow \boxed{1-0'5 = 0'5}$$

$$T \sim BN(n=18,\ p=0'5)$$

$$P(T \leq 2) = P(T=0) + P(T=1) + P(T=2) =$$

$$= \binom{17}{0}\cdot 0'5^0 \cdot 0'5^{18} + \binom{18}{1}\cdot 0'5^1 \cdot 0'5^{18} + \binom{19}{2}\cdot 0'5^2\cdot 0'5^{18} =$$

$$= 0'00020122528 = \boxed{2'0122522808 \cdot 10^{-4}}$$

3 A: "Pieza de alta calidad"
B: "Pieza de media calidad"
C: "Pieza de baja calidad".

D: "Máquina está correctamente ajustada"
E: "Máquina no está correctamente ajustada".

$P(D) = 0'9$; $P(E) = 0'1 = P(\bar{D})$

$P(A \mid D) = 0'8$
$P(B \mid D) = 0'15$ $\quad P(B \mid \bar{D}) = 0'6$
$P(C \mid D) = 0'05$ $\quad P(C \mid \bar{D}) = 0'4$

a) $P(A) = P(A \cap D) + P(A \cap \bar{D}) = P(A \mid D) \cdot P(D) + P(A \mid \bar{D}) \cdot P(\bar{D}) =$
$= 0'8 \cdot 0'9 + 0 \cdot 0'1 = \boxed{0'72}$

b) $P(\bar{C}) = P(\bar{C} \cap D) + P(\bar{C} \cap \bar{D}) = P(\bar{C} \mid D) \cdot P(D) + P(\bar{C} \mid \bar{D}) \cdot P(\bar{D}) =$
$= (1 - P(C \mid D)) \cdot P(D) + (1 - P(C \mid \bar{D})) \cdot P(\bar{D}) =$
$= (1 - 0'05) \cdot 0'9 + (1 - 0'4) \cdot 0'1 = 0'95 \cdot 0'9 + 0'6 \cdot 0'1 =$
$= \boxed{0'915}$

c) $X :=$ "Número de piezas de calidad media de entre 10 inspeccionadas"

$X \sim B(n = 10, p = P(B))$

$P(B) = 1 - P(A) - P(C) = 1 - P(A) - (1 - P(\bar{C})) = 1 - 0'72 - (1 - 0'915) =$
$= \boxed{0'195}$

$P(X < 2) = P(X \leq 1) = P(X = 0) + P(X = 1) =$

$= \binom{10}{0} \cdot 0'195^0 \cdot 0'805^{10} + \binom{10}{1} \cdot 0'195^1 \cdot 0'805^9 = \boxed{0'3910970377}$

ORDINARIA 20-21

1

9 bolas (2 blancas y 3 rojas)

Combinaciones posibles que quedan en la urna, y armaremos los casos favorables de forma que haya más bolas blancas que rojas.

Combinaciones restantes en la urna: $CR_{4,2} = \binom{4+2-1}{2} = \binom{5}{2} = \frac{5!}{(5-2)!\,2!} = 10$

Casos favorables dentro de la urna: $\left\{ \begin{array}{l} B\ B\ B\ B\ (2+4B,\ 2R) \\ B\ B\ B\ R\ (2+3B,\ 3R) \end{array} \right\}$

Para que el número de blancas sea mayor que el número de rojas; en la urna tendrá que haber, por lo menos, 3 blancas.

$P(N^o \text{ blancas} > N^o \text{ rojas}) = 2/10 = \boxed{1/5}$

b) $X :=$ "densidad de la bolas blancas" $X \sim N(\mu_1, \sigma_1 = 0'14)$

$Y :=$ "densidad de las bolas rojas" $Y \sim N(\mu_2, \sigma_2 = 0'12)$

muestra x:

$n = 5$

$\bar{x} = \frac{1'38 + 1'34 + 1'41 + 1'36 + 1'48}{5} = 1'394$

muestra y:

$m = 4$

$\bar{y} = \frac{1'42 + 1'38 + 1'45 + 1'39}{4} = 1'41$

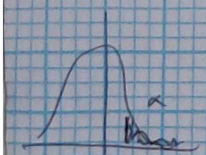
$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2$

$EC = \frac{\bar{X}_1 - \bar{Y}}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} \sim N(0,1)$

$\hookrightarrow = \frac{1'394 - 1'41}{\sqrt{\frac{0'14^2}{5} + \frac{0'12^2}{4}}} = -0'1516$

$\alpha = 0'03$



$S_0 = (-\infty, z_\alpha)$

$S_1 = [z_\alpha, \infty)$

$z_{0'03} = qnorm(0'97, 0, 1) = -qnorm(0'03, 0, 1)$

$= 1'8808$

$S_0 = (-\infty, 1'8808)$; $S_1 = [1'8808, \infty)$

$EC \in S_0 \rightarrow$ Aceptar H_0, con un nivel de significación del 3%, las bolas blanca son menos densas.

c)

$X \sim N(\mu_1 = 1'41 , \sigma_1 = 0'14)$; $Y \sim N(\mu_2 = 1'42 , \sigma_2 = 0'17)$

$$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} \sim N(0,1) \quad ; \quad n = \left(z_{\alpha/2} \cdot \sigma/\varepsilon \right)^2$$

$P(\mu_1 > \mu_2) = 0'9$

$P(\mu_1 - \mu_2 > 0) = 0'9$

$$P\left(\frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} > 0 \right) = 0'9 \rightarrow \text{qnorm}(0'1, 0, 1) = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}}$$

② $X :=$ "Graduación en alcohol del hidrogel"

$X \sim N(\mu = 70,\ \sigma = 2'7)$

<u>muestra:</u>

$n = 120$

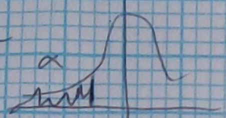
$\bar{x} = 69'3$

$s^2 = 7'2$

a) $H_0:\ \mu = 70$ $\qquad EC = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{69'3 - 70}{2'7/\sqrt{120}} = -2'840042891$

$H_a:\ \mu < 70$

$\alpha = 0'02$



$S_0 = (-z_\alpha, \infty) = (-2'0537, \infty)$

$S_1 = (-\infty, -z_\alpha] = (-\infty, -2'0537)$

$-z_{0'02} = -\text{qnorm}(0'98, 0, 1) = \text{qnorm}(0'02, 0, 1) = -2'0537$

$EC \notin S_0 \rightarrow$ Rechazar H_0, con $\alpha = 0'02$, la empresa auditora tiene razón.

b) $p\text{-valor} = P(EC < -2'8400 \mid EC \sim N(0,1)) =$

$= 1 - \text{pnorm}(2'84, 0, 1) = \text{pnorm}(-2'8400, 0, 1) = \boxed{0'0023}$

c) $P(|\bar{x} - \mu| < 0'25) = 0'99$

$n = \left(z_{\alpha/2} \cdot \sigma/\varepsilon\right)^2 = \left(z_{0'005} \cdot \dfrac{2'7}{0'25}\right)^2 = 773'8767 \simeq \boxed{774}$

$z_{0'005} = \text{qnorm}(0'995, 0, 1) = 2'5758$

d) $P(E_{II}) = P(\text{Aceptar } H_0 \mid H_0 \text{ falsa})$

Destificar, valor límite $= -z_\alpha \cdot \dfrac{\sigma}{\sqrt{n}} + \mu = -z_{0'02} \cdot \dfrac{2'7}{\sqrt{120}} + 70 =$

$= 69'49381398$

$P\left(\bar{x} > 69'4938 \mid \bar{x} \sim N\left(69'3, \dfrac{2'7}{\sqrt{120}}\right)\right) = P\left(Z > \dfrac{69'4938 - 69'3}{2'7/\sqrt{120}}\right) =$

$= 1 - P(Z < 0'7862) = 1 - 0'7842 = \boxed{0'2158}$

(3) $X :=$ "Intensidad de la señal"
$Y :=$ "Marca comercial".

H_0: X e Y son independientes
H_a: X e Y no son independientes.

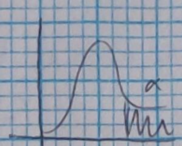
$$EC = \sum_{i=1}^{k} \sum_{j=1}^{m} \frac{(f_i - e_i)^2}{e_i}$$

	Marca A	Marca B	Marca C
Mala	21'12	22	22'88
Aceptable	48'64	$50'6\overline{6}$	$52'69\overline{3}$
Buena	17'92	18'66	$19'41\overline{3}$
Excelente	8'32	$8'6\overline{6}$	$9'01\overline{3}$

$$EC = \frac{(22-21'12)^2}{21'12} + \frac{(24-22)^2}{22} + \frac{(22'88-20)^2}{20} + \dots =$$

$= 6'3533$

$\alpha = 0'05$



$S_0 = [0, \chi^2_i) = [0, 12'5916)$
$S_1 = [\chi^2_i, \infty)$

$i = (k-1)(m-1) = (4-1)(3-1) = 3 \cdot 2 = 6$
$\chi^2_{6,\,0'05} =$ qchisq $(0'95, 6) = 12'5916$

$EC \in S_0 \rightarrow$ Aceptar H_0, no son dependientes.

(4) M: "coger 2 cartas bien franqueadas"

R_i: "Haya i cartas defectuosas".

$P(M \mid R_1) = \frac{99}{100} \cdot \frac{98}{99} =$

...

hasta R_4

$P(R_0 \cup R_1 \mid M)$

$P(R_0) = 5\alpha$

$P(R_1) = 4\alpha$

...

$\boxed{0'6108}$

5) X: "Número de gigabytes descargados diariamente por los clientes de una compañía telefónica".

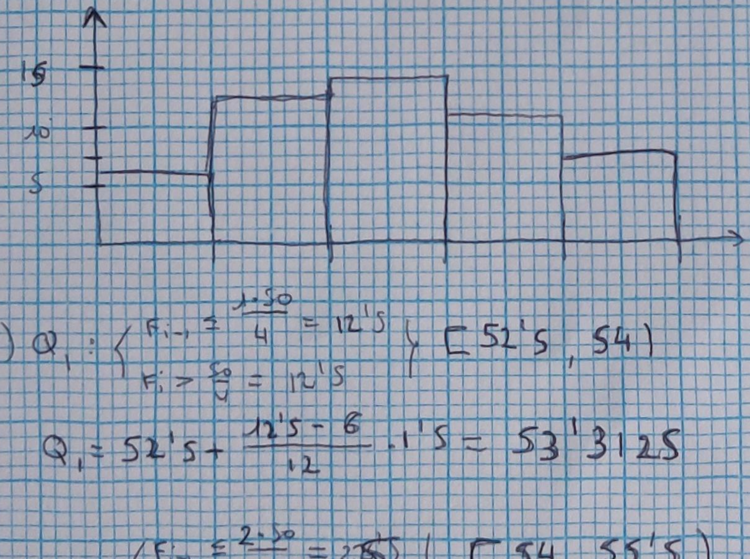
a) $\sqrt{n} = k \to k = \sqrt{50} = 7'0710$

Rango = máx − mín = 58'50 − 51 = 7'5

$\boxed{k = 5}$

$\boxed{\text{amplitud} = \frac{7'5}{5} = 1'5}$

Clases	x_i	f_i	F_i	h_i	H_i
[51, 52'5)	51'75	6	6	6/50	6/50
[52'5, 54)	53'25	12	18	12/50	18/50
[54, 55'5)	54'75	14	32	14/50	32/50
[55'5, 57)	56'25	11	43	11/50	43/50
[57, 58'5]	57'75	7	50	7/50	50/50



b) $Q_1: \left\{ \begin{array}{l} F_{i-1} \leq \frac{1 \cdot 50}{4} = 12'5 \\ F_i > \frac{50}{4} = 12'5 \end{array} \right\}$ [52'5, 54)

$$Q_1 = 52'5 + \frac{12'5 - 6}{12} \cdot 1'5 = 53'3125$$

$Q_2: \left\{ \begin{array}{l} F_{i-1} \leq \frac{2 \cdot 50}{4} = 25 \\ F_i > \frac{2 \cdot 50}{4} = 25 \end{array} \right\}$ [54, 55'5)

$$Q_2 = 54 + \frac{25 - 18}{14} \cdot 1'5 = 54'75$$

$Q_3: \frac{3 \cdot 50}{4} = 37'5 \to$ [55'5, 57)

$$Q_3 = 55'5 + \frac{37'5 - 32}{11} \cdot 1'5 = 56'25$$

$RIC = Q_3 - Q_1 = 2'9375$

$Mo \to [54, 55'5)$

$$Mo = l_i + \frac{f_i - f_{i-1}}{(f_i - f_{i-1}) + (f_i - f_{i+1})} \cdot d_i = 54 + \frac{14-12}{14-12+14-11} \cdot 1'5 = \boxed{54'6}$$

c) ¿$X \sim N(\mu, \sigma)$?

$$\hat{\mu} = \bar{x} = \frac{51'75 \cdot 6 + 53'25 \cdot 12 + 54'75 \cdot 14 + 56'25 \cdot 11 + 57'75 \cdot 7}{50} =$$

$$= \boxed{54'78}$$

$$\hat{\sigma} = \sqrt{S^2} \Rightarrow S^2 = \frac{(51'75 - 54'78)^2 \cdot 6 + \ldots}{49} = 3'442959184$$

$$\hat{\sigma} = \sqrt{3'4430} = 1'8555$$

$$EC = \frac{\sum_{i=1}^{k} (f_i - e_i)^2}{e_i} = 0'6744$$

$p_i = (X = x_i)$	$e_i = n \cdot p_i$	$\frac{(f_i - e_i)^2}{e_i}$

$P(X \leq 52'5) = P(X \leq 52'5) = P(Z \leq -1'2288)$

$= \ldots$

$S_0 = [0, \chi^2_{\alpha,2})$ $\quad \chi^2_{\alpha/2} = \text{qchisq}(0'9, 2) = 4,6052$

$S_1 = [\chi^2_{\alpha,2}, \infty)$

$EC = 0'6744 \in S_0 \to$ Aceptar H_0.

d) $I_{\mu}^{0'95} = \left[\bar{x} \mp z_{\alpha/2} \cdot \sigma/\sqrt{n}\right] = [53'4857, 54'5143]$

$\bar{x} = 54'78$

$z_{\alpha/2} = z_{0'05/2} = z_{0'025} = qnorm(0'975, 0, 1) = 1'9600$

$\sigma/\sqrt{n} = 1'8555/\sqrt{80} = 0'2624$

$I_{\mu}^{0'95} = \left[\bar{x} \mp t_{49; 0'025} \cdot s/\sqrt{n}\right] = [54'2527, 55'3073]$

ORDINARIA 2020

1) A: "llavero A" $P(A) = 1/3$ $P(L|A) = 1/5$

B: "llavero B" $P(B) = 1/3$ $P(L|B) = 1/7$

C: "llavero C" $P(C) = 1/3$ $P(L|C) = 1/8$

L: "llave correcta"

a) $P(L) = P(L\cap A) + P(L\cap B) + P(L\cap C) = P(L|A)\cdot P(A) + P(L|B)\cdot P(B) + P(L|C)\cdot P(C)$

$= 1/5 \cdot 1/3 + 1/7 \cdot 1/3 + 1/8 \cdot 1/3 = \boxed{0'1560}$

b) $P(\bar{L}|C) = 1 - P(L|C) = 1 - 1/8 = \boxed{7/8}$

c) $P(A|L) = \dfrac{P(A\cap L)}{P(L)} = \dfrac{P(L|A)\cdot P(A)}{P(L)} = \dfrac{1/15}{0'1560} = \boxed{0'427350}$

2)

a) X := "Demanda diaria".

$P(X>10) = \int_{10}^{\infty} f(x)\,dx = \int_{10}^{12} \frac{12-x}{64}\,dx + \cancel{\int_{12}^{\infty} 0\,dx} = \frac{1}{64}\left(12x - \frac{x^2}{2}\right)\Big]_{10}^{12} =$

$= \frac{1}{64}\cdot(2) = 1/32 = \boxed{0'03125}$

b)

$P(X<3) = P(X\le 3) = \int_{-\infty}^{3} f(x)\,dx = \cancel{\int_{-\infty}^{0} 0\,dx} + \int_{0}^{3} 1/8\,dx = \frac{1}{8}x\Big]_0^3 =$

$= \boxed{3/8}$

c) $E(X) = \int_{-\infty}^{\infty} x\cdot f(x)\,dx = \cancel{\int_{-\infty}^{0} x\cdot 0\,dx} + \int_{0}^{4} \frac{1}{8}x\,dx + \int_{4}^{12} \frac{12x - x^2}{64}\,dx + \cancel{\int_{12}^{\infty} x\cdot 0\,dx} =$

$= \frac{1}{16}\cdot x^2\Big]_0^4 + \frac{1}{64}\cdot\left(6x^2 - \frac{x^3}{3}\right)\Big]_4^{12} = 1 + \frac{10}{3} = \boxed{\frac{13}{3}}$

d) Beneficio $= -5\cdot\int_0^2 \frac{1}{8}\,dx + 5\cdot\int_2^4 \frac{1}{8}\,dx + 10\int_4^8 \frac{12-x}{64}\,dx + 15\cdot\int_8^{12} \frac{12-x}{64}\,dx =$

$-5\cdot\frac{1}{8}x\Big]_0^2 + 5\cdot\frac{1}{8}x\Big]_2^4 + 10\cdot\frac{1}{64}\cdot\left(12x - \frac{x^2}{2}\right)\Big]_4^8 + 15\cdot\frac{1}{64}\cdot\left(12x - \frac{x^2}{2}\right)\Big]_8^{12} =$

$\frac{-10}{8} + \frac{5}{4} + \frac{15}{4} + \frac{15}{8} = \boxed{45/8}$

3 $X :=$ "Emisión de partículas de diámetro menor a 10 nm".
$X \sim N(\mu, \sigma = 0'1)$

muestra:

$n = 7$

$\bar{x} = 1'98142857$

$$S^2 = \frac{(1'88 - 1'98)^2 + (1'95 - 1'98)^2 + \dots}{7} = 0'00638367428$$

a) $I_{\mu}^{0'95} = [\bar{x} \mp z_{\alpha/2} \cdot \sigma/\sqrt{n}] = [1'9073 \, , \, 2'0555]$

$\bar{x} = 1'9814$

$z_{\frac{0'05}{2}} = z_{0'025} = \text{qnorm}(0'975, 0, 1) = 1'9600$

$$\sigma/\sqrt{n} = \frac{0'1}{\sqrt{7}} = 0'037796447$$

$l = 2 \cdot z_{\alpha/2} \cdot \sigma/\sqrt{n}$

$\downarrow$

$$n = \left(\frac{2 \cdot z_{\alpha/2} \cdot \sigma}{l}\right)^2 = 1536'64 \simeq \boxed{1537}$$

b) $H_0: \mu = 2$ $\quad EC = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \dfrac{1'9814 - 2}{0'1/\sqrt{7}} = -0'4921$
$H_a: \mu > 2$

$\alpha = 0'05$ $\quad S_0 = (-\infty, z_\alpha)$ $\quad z_\alpha = \text{qnorm}(0'95, 0, 1) = 1'6449$
$S_1 = [z_\alpha, \infty)$

$EC \in S_0 \rightarrow$ Aceptar $S_0 \rightarrow$ No es necesario.

c) $p\text{-valor} = P(EC > -0'4921 \mid EC \sim N(0,1)) =$
$= \cancel{pn}\ 1 - \text{pnorm}(0'4921, 0, 1) = \boxed{0'6887}$

d) $P(E_{II}) = P(\text{Aceptar } H_0 \mid H_0 \text{ falsa}) =$

$$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < z_\alpha \rightarrow \bar{x} < 1'6499 \cdot \frac{0'1}{\sqrt{7}} + 2 = 2'062360358$$

$P(\bar{x} > 2'0624 \mid \bar{x} \sim N(2'1, \frac{0'1}{\sqrt{7}})) =$ ~~[illegible]~~ $=$
$= P(Z < -1'0001) = 1 - \text{pnorm}(1'0001, 0, 1) =$
$= 1 - 0'8414 = \boxed{0'1586}$

e) $H_0: \sigma^2 = 0'01$ $\quad EC = \frac{n \cdot s^2}{\sigma^2} \sim \chi^2_{n-1} \rightarrow \frac{7 \cdot 0'00638}{0'01} = 4,466$
$H_a: \sigma^2 > 0'01$

$\alpha = 0'05$ $\quad S_1 = [\chi^2_{6;0'05}, \infty) \Rightarrow \text{qchisq}(0'95, 6) = 12'5916$
$S_0 = [0, \chi^2_{6;0'05})$

$EC \in S_0 \rightarrow$ Aceptar $H_0 \rightarrow$ Sí cumple.

(4) X := "Método"
Y := "Calificaciones"

H_0: X e Y son independientes $\quad EC = \sum_{i=1}^{4} \sum_{j=1}^{m} \frac{(f_i - e_i)^2}{e_i}$
H_a: X e Y no son independientes.

	A	B	C	
D	22	24	20	66
A	50	42	60	152
N	18	22	16	56
S	6	12	8	26
	96	100	104	300

	A	B	C
D	21'12	22	22'88
A	48'64	50'66	52'693
N	17'92	18'66	19'413
S	8'32	8'66	9'013

$EC = 0'58100233 + 2'531662662 + 1'198231697 + 2'205749694 =$
$= \boxed{6'516651383}$ $\quad \chi^2_{6;0'05} = \text{qchisq}(0'95, 6) = 12'5916$

$\alpha = 0'05$ $\quad S_0 = [0, \chi^2_{(4-1)(3-1);0'05})$ $\quad EC \in S_0 \rightarrow X$ e Y son independientes.
$S_1 = [\chi^2_{6;0'05}, \infty)$

5

a) X = "Peso en gramos de las bolsas de patatas".

H_0: X sigue una distribución normal.

H_a: X no sigue una distribución normal.

$$EC = \frac{\sum_{i=1}^{k} (f_i - e_i)^2}{e_i} \qquad \hat{\mu} = \bar{x} = \frac{25 \cdot 20 + 75 \cdot 65 + 125 \cdot 100 + 175 \cdot 95 + 225 \cdot 60 + 275 \cdot 10}{380}$$

$$= 145$$

$$\hat{\sigma} = \sqrt{S^2} = 60'5923$$

Clases	x_i	f_i	p_i	$e_i = n \cdot p_i$	$\frac{(f_i - e_i)^2}{e_i}$
0-50	25	20		17'536	0'3465
50-100	75	65		59'605	0'4883
100-150	125	100		106'435	0'3891
150-200	175	95		99'785	0'2295
200-250	225	60		49'140	2'4001
250-300	275	10		12'705	0'5759
					4'4293

$P(X \leq 50) - P(X \leq 0) = 0'0501$

$P(X \leq 100) - P(X \leq 50) = 0'1703$

$P(X \leq 150) - P(X \leq 100) = 0'3041$

$P(X \leq 200) - P(X \leq 150) = 0'2851$

$P(X \leq 250) - P(X \leq 200) = 0'1404$

$P(X \leq 300) - P(X \leq 250) = 0'0363$

$\chi^2_{0'05,\, k-r-1} = \text{qchisq}(0'95, 3) = 7'8147$

$S_0 = [0, 7'8147)$

$S_1 = [7'8147, \infty)$

$EC \in S_0 \rightarrow$ Aceptar H_0, sí sigue una normal.

b) $P_{85} = \begin{cases} F_{i-1} \leq \frac{85 \cdot 350}{100} \\ F_i > \frac{85 \cdot 350}{100} \end{cases} = 297'5 \rightarrow [200, 250)$

$P_{85} = 200 + \frac{297'5 - 280}{60} \cdot 50 = \boxed{214'58\overline{3}}\ g$

c) $P_{50} = Me = l_i + \frac{n/2 - F_{i-1}}{f_i} \cdot d_i = 100 + \frac{175 - 85}{100} \cdot 50 = \boxed{145\ g}$

$\begin{cases} F_{i-1} \leq n/2 \\ F_i > n/2 \end{cases} = 175 \rightarrow [100 - 150)$

d) $Mo \rightarrow [100 - 150)$

$Mo = 100 + \frac{100 - 65}{100 - 65 + 100 - 95} \cdot 50 = \boxed{143'75\ g}$

e) $[P_5, P_{95}] = [43'75, 243'75]$

EXTRAORDINARIA 20-21 (JUNIO 2021)

1) X := "Contenido de benceno en el agua" $X \sim N(\mu = 30, \sigma = 2)$

a) M := "Número de análisis por hora".

$M \sim \mathcal{P}(\lambda = 21)$ $\{\lambda > 18\}$ $\cong N(21, \sqrt{21})$ }

$P(M > 25) = P(M > 25'5) = P\left(Z > \frac{25'5 - 21}{\sqrt{21}}\right) = P(Z > 0'9820) =$

$= 1 - \text{pnorm}(0'9820, 0, 1) = 1 - 0'8370 = \boxed{0'163}$

b) N := "Número de análisis por día".

$N \sim \mathcal{P}(\lambda = 10 \cdot 21)$ $\lambda > 18 \rightarrow \cong N(210, \sqrt{210})$

$P(N \geq 220) = P(N \geq 219'5) = P\left(Z_n \geq \frac{219'5 - 210}{\sqrt{210}}\right) =$

$= P(Z_n \geq 0'6556) = 1 - P(Z_n \leq 0'6556) = 1 - 0'7440 = \boxed{0'256}$

c) $P(\bar{x} > 32 \mid \bar{x} \sim N(\mu = 30, \sigma/\sqrt{n}))$

$1 - P\left(\bar{x} < \frac{32-30}{2/\sqrt{5}}\right) = 1 - P(Z_{\bar{x}} < 2'2360) = 1 - 0'9873 = \boxed{0'0127}$

2) X := "Anchura de las monedas".

muestra:

$n = 120 > 100$

$\bar{x} = 2'03$

$s = 0'30$

a) $H_0: \mu = 2$

$H_a: \mu > 2$

$EC = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim N(0, 1)$

$EC = \frac{2'03 - 2}{\frac{\sqrt{0'3^2 \cdot \frac{120}{119}}}{\sqrt{120}}} = 1'\cancel{0863}$ 0910

$\alpha = 0'05$

$S_0 = (-\infty, Z_{0'05})$ $Z_{0'05} = \text{qnorm}(0'95, 0, 1) = 1'6449$

$S_1 = [Z_{0'05}, \infty)$

$EC \in S_0 \rightarrow$ Aceptar H_0. Sí pueden considerarse correctos.

b) p-valor $= P(EC > 1'0863 \mid EC \sim N(0, 1)) = 1 - \text{pnorm}(1'\cancel{0863}, 0, 1) =$ 0910

$= 1 - 0'8623 = \boxed{0'1377}$

(3) $X :=$ "Número de empresas familiares en localidades de menos de 1000 habitantes".

H_0: X sigue una distribución binomial

H_a: X no sigue una distribución binomial

$$EC = \frac{\sum_{i=1}^{k} (f_i - e_i)^2}{e_i}$$

$n = 90$

$$\bar{x} = \frac{0 \cdot 5 + 9 + 2 \cdot 20 + 3 \cdot 23 + 4 \cdot 16 + 5 \cdot 11 + 6 \cdot 6}{90} = 3'0\bar{3}$$

$$\hat{p} = \frac{\bar{x}}{k} = \frac{3'0\bar{3}}{\cancel{90}\ 6} = \boxed{\cancel{0'4\bar{3}}}\ 0'0337$$

x_i	f_i	$p_i\ (P(X=x_i))$	$e_i = n \cdot p_i$	$\frac{(f_i - e_i)^2}{e_i}$
0	5		4'1145	0'5388
1	9	0'1434	12'914	
2	20	0'2226	20'042	$9'1 \cdot 10^{-5}$
3	23	0'2278	20'503	0'3038
4	16	0'1728	15'553	0'0128
5	11	0'1036	9'3295	0'6721
6	6	0'0512	4'6094	

$EC = 1'5277$

$$P(X=0) = \binom{90}{0} \cdot 0'0337^{0} \cdot 0'9663^{90-0} = 0'0457$$

. . .

$$\chi^2_{k;0'1} = \text{qchisq}(0'9, 3) = 6'2514$$

$\alpha = 0'1$ $\qquad S_0 = [0, \chi^2_{k;0'1}]$ $\qquad k = l - r - 1 = 5 - 1 - 1 = 3$

$S_1 = [\chi^2_{k;0'1}, \infty)$

$EC \in S_0 \rightarrow$ Aceptar H_0.

JUNIO 2021

3) X := "Número de empresas familiares ubicadas en localidades de menos de 1000 habitantes".

$X \sim B(n = 6,\ p = \frac{\bar{x}}{n})$

$$\bar{x} = \frac{0\cdot 5 + 1\cdot 9 + 2\cdot 20 + 3\cdot 23 + 4\cdot 16 + 5\cdot 11 + 6\cdot 6}{90} = \boxed{3'03}$$

$$p = \frac{3'03}{6} = \boxed{0'50\overline{5}}$$

$$EC = \frac{\sum_{i=1}^{k} (f_i - e_i)^2}{e_i}$$

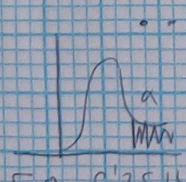
x_i	f_i	$p_i = P(X = x_i)$	$e_i = n \cdot p_i$	$\frac{(f_i - e_i)^2}{e_i}$
0	5	0'0147	1'323	~~15'8344~~ 2'2232
1	9	0'0900	8'1	—
2	20	0'2297	20'673	0'0219692054
3	23	0'3124	28'116	0'9309
4	16	0'2390	21'51	1'4114
5	11	0'0975	8'775	—
6	6	0'0166	1'494	4'4120

$= \boxed{8'9995}$

$$P(X=0) = \binom{6}{0} \cdot 0'505^0 \cdot 0'495^6 = 0'01471$$

...

$\alpha = 0'1$



qchisq (0'9, 3) = $\boxed{6'2514}$

$S_0 = [0,\ 6'2514)$ $S_1 = [6'2514,\ \infty)$

$EC \notin S_0 \rightarrow$ Rechazar H_0.

④ $X \sim N(\mu, 0'5)$

$P(X < 3'5 \mid \mu = \mu_1 = 3) = P\left(Z < \frac{3'5 - 3}{0'5}\right) = P(Z < 1) = \boxed{0'8413}$

$P(X < 3'5 \mid \mu = \mu_2 = 3'2) = P\left(Z < \frac{0'3}{0'5}\right) = P(Z < 0'6) = \boxed{0'7257}$

$P(X < 3'5 \mid \mu = \mu_3 = 3'4) = P\left(Z < \frac{0'1}{0'5}\right) = P(Z < 0'2) = \boxed{0'5793}$

$P(X < 3'5 \mid \mu = \mu_4 = 3'6) = P(Z < -0'2) = \boxed{0'4207}$

$P(X < 3'5 \mid \mu = \mu_5 = 3'8) = P(Z < -0'6) = \boxed{0'2743}$

$$P(\mu_2 \cup \mu_3 \cup \mu_4 \mid X < 3'5) = \frac{P((\mu_2 \cup \mu_3 \cup \mu_4) \cap X < 3'5)}{P(X < 3'5)} =$$

$$= \frac{0'2 \cdot 0'7257 + 0'4 \cdot 0'5793 + 0'2 \cdot 0'4207}{0'1 \cdot 0'8413 + 0'2 \cdot 0'7257 + 0'4 \cdot 0'5793 + 0'2 \cdot 0'4207 + 0'1 \cdot 0'2743} =$$

$$= \frac{0'461}{0'57256} = \boxed{0'8052}$$

ORDINARIA 21-22 (ENERO 2022) $\quad$ 67'9049 $\quad$ 71'2551

(1) $X :=$ "Velocidad de giro del motor" $\quad X \sim N(\mu = 70, \sigma)$

muestra:

$n = 5$

$\bar{x} = 69{,}58$

$s^2 = 3{,}087$ (cuasi)

a) $I_\mu^{0'9} = [\bar{x} \mp t_{\alpha/2, n-1} \cdot s/\sqrt{n}] = [68{,}2875 \,,\, 70{,}8725]$

$\cancel{z_{\alpha/2} = z_{0'1/2} = z_{0'05} = qnorm(0'95, 0, 1) = 1'6449}$

$s = \sqrt{3{,}087} = 1{,}7570$ $\quad t_{n-1;\alpha/2} = qt(0'95, 4) = 2'1318$

muestra 2:

$m = 6$

$\bar{x} = 70{,}083$

b) $\sigma_1 = 1{,}8$

$\sigma_2 = 2{,}1$

$H_0 : \mu_1 = \mu_2$

$H_a : \mu_1 < \mu_2$

$$EC = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} \sim N(0,1)$$

$EC = -0{,}4277$ $\quad \alpha = 0{,}02$ $\quad \begin{cases} S_0 = (-z_\alpha, \infty) \\ S_1 = (-\infty, -z_\alpha] \end{cases}$

$z_\alpha = qnorm(0'98, 0, 1) = 2'0537$

$EC \in S_0 \rightarrow$ Aceptar H_0. No puede afirmarse.

c) $p\text{-valor} = P(EC < -0'4277) = 1 - pnorm(0'4277, 0, 1) = 1 - 0'6657 =$
$= \boxed{0'3343}$

d) $P(E_{II}) = P(\text{Aceptar } H_0 \mid H_0 \text{ falsa})$

$$\bar{x}_1 - \bar{x}_2 > -z_\alpha \cdot \sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}} = -2'4152$$

$$P(T > -2'4152 \mid T \sim N(-1'5, \sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}})) =$$

$$1 - P(Z < -0'7782) = 1 - 1 + pnorm(0'7782, 0, 1) = \boxed{0'7818}$$

enero 22

② a) $X :=$ "Número de epicentros por cada $100 km^2$ de la región".

tamaño de la muestra: 27

H_0: X sigue una distribución de Poisson

H_a: X no sigue una distribución de Poisson.

$$EC = \frac{\sum_{i=1}^{k} (f_i - e_i)^2}{e_i} \qquad \bar{x} = \frac{35}{27} = 1'\overline{296} \qquad \hat{\lambda} = 1'296 \qquad X \sim \mathcal{P}(\lambda = 1'296)$$

x_i	f_i	$p_i = P(X = x_i)$	$e_i = n \cdot p_i$	$\frac{(f_i - e_i)^2}{e_i}$
0	7	0'2736	7'3879	0'0204
1	10	0'3546	9'5747	0'0189
2	6	0'2298	6'2044	0'0067
	10	0'3218	10'0376	
3	3	0'0993	2'6803	0'0381
	4	0'1142	3'8332	0'0651
4	1	0'0427	1'1529	0'0203
	27	1		

$\Sigma = 0'1111$

$$P(X=0) = \frac{e^{-1'296} \cdot 1'296^0}{0!} = 0'2736241034$$

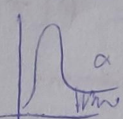
$$P(X=1) = \frac{e^{-1'296} \cdot 1'296^1}{1!} = 0'354616838$$

$$P(X=2) = \frac{e^{-1'296} \cdot 1'296^2}{2!} = 0'229791711$$

$$P(X=3) = \frac{e^{-1'296} \cdot 1'296^3}{3!} = 0'09927001915$$

$$P(X=4) = \frac{e^{-1'296} \cdot 1'296^4}{4!} = 0'03216348621$$

$\alpha = 0'05$



$\chi^2_k \quad k = l - r - 1$

$3 - 1 - 1 = 1$

qchisq $(0'95, 1) = 3'8415$

$S_0 = [0, 3'8415)$

$S_1 = [3'8415, \infty)$

$EC \in S_0 \rightarrow$ Sigue una de $\mathcal{P}$

b) $X \sim \mathcal{P}(\lambda_x = 1'296)$

Y := "Número de epicentros por cada 250 km^2".

$$\frac{250}{100} = 2'5 \rightarrow \lambda_Y = 2'5\,\lambda_X \rightarrow \boxed{\lambda_Y = 3'24}$$

$Y \sim \mathcal{P}(\lambda_Y = 3'24)$

$$P(Y \geq 2) = 1 - P(Y<2) = 1 - [P(X=0) + P(X=1)] =$$

$$= 1 - \left[e^{-3'24} \cdot \left(\frac{3'24^0}{0!} + \frac{3'24^1}{1!}\right)\right] = 1 - 0'166049152 = 0'833945084\,8$$

c)

T := "Tiempo entre dos seísmos de magnitud superior a 7"

$T \sim \mathcal{E}(\beta = 30)$

$\cancel{P(T \leq 9) = 1 - e^{-9/30} = 0'2591817793}$.

$$P(T \leq 21) = 1 - e^{-21/30} = 0'503414696\,2.$$

$$P(T < 21 \mid T > 20) = P(T < 1) = 1 - e^{-1/30} = 0'0327839952.$$

falta de memoria

EXTRAORDINARIA 2022 (JUNIO 2022)

1) X := "Peso del tejido al inicio" $X \sim N(\mu, \sigma)$

Y := "Peso del tejido tras 12h" $Y \sim N(\mu, \sigma)$

muestra x:
$n = 7$
$\bar{x} = 51'1929$
$s^2 = 0'8328$

muestra y:
$m = 7$
$\bar{y} = 49'52714285$
$s^2 = 4'0042$

$S_x^2 = 0'9716$

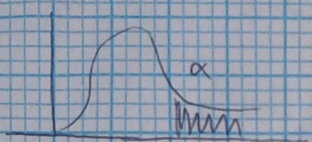
$S_y^2 = 4'6716$

a) $H_0: \sigma_x = 0'8$
$H_a: \sigma_x > 0'8$

$$EC = \frac{n \cdot s^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$= \frac{7 \cdot 0'8328}{0'8^2} = \boxed{9'10875}$$

$\alpha = 0'1$



$S_0 = [0, 10'6446)$
$S_1 = [10'6446, \infty)$

$\chi^2_{6;\,0'1} = qchisq(0'9, 6) = 10'6446$

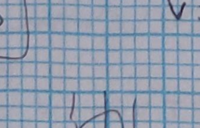
$EC \in S_0 \rightarrow$ Aceptar $H_0 \rightarrow$ El experimento puede considerarse válido.

b) $H_0: \mu_1 = \mu_2$
$H_a: \mu_1 \neq \mu_2$

$$EC = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}} \sim t_v \qquad v = \frac{\left(\frac{S_x^2}{n} + \frac{S_y^2}{m}\right)^2}{\frac{\left(\frac{S_x}{n}\right)^2}{n+1} + \frac{\left(\frac{S_y}{m}\right)^2}{m+1}} - 2 =$$

$= \boxed{1'8553}$ $v = 9'1897 \simeq 9$

$\alpha = 0'15 \rightarrow$



$S_0 = (-t_{9;\,0'15},\ t_{9;\,0'15})$
$S_1 = (-\infty, -t_{9;\,0'15}] \cup [t_{9;\,0'15}, \infty)$

$t_{9;\,0'15} = qt(0'85, 9) =$

c) $p\text{-valor} = P(EC < 1'8553 \mid t_9) =$

② $T :=$ "tiempo de funcionamiento"

a) $P(T \leq 565) = 0'61$

$1 - e^{-565/\beta} = 0'61 \rightarrow 0'39 = e^{-565/\beta}$

$\rightarrow \ln(0'39) = -\frac{565}{\beta} \rightarrow \boxed{\beta = 600'0370388}$

b) $X :=$ "Número de tuberías que funcionan correctamente habiendo transcurrido 300 horas".

$X \sim B(n = 3, \, p = P(T > 300))$

$P(T > 300) = 1 - P(T \leq 300) = 1 - (1 - e^{-300/600}) = \boxed{0'6065}$

$P(X \geq 2) = P(X = 2) + P(X = 3) =$

$= \binom{3}{2} \cdot 0'6065^2 \cdot 0'3935^1 + \binom{3}{3} \cdot 0'6065^3 \cdot 0'3935^0 =$

$= \boxed{0'6573}$

c) $Y :=$ "Número de válvulas que funcionan ... tras 600h y antes de 1000h".

~~Número de válvulas que funcionan ... tras 1000h~~.

$P(T \leq 1000 \mid T \geq 600) = \frac{P(600 \leq T \leq 1000)}{P(T \geq 600)} = \frac{0'1790}{0'3679} = \boxed{0'486}$

$Y \sim B(n = 3, \, p = P(600 \leq \cancel{T} \leq 1000))$

$P(600 \leq \cancel{T} \leq 1000) = P(T \leq 1000) - P(T \leq 600) = \cancel{1} - e^{-1000/600} - \cancel{1} + e^{-600/600} =$

$= \boxed{0'1790}$

$P(T \leq 400) = \boxed{0'4865}$

$P(Y < 2) = P(Y = 0) + P(Y = 1) =$

$\binom{3}{0} \cdot 0'4865^0 \cdot 0'5135^3 + \binom{3}{1} \cdot 0'4865^1 \cdot 0'5135^2 = \boxed{0'52024}$

JUNIO 2022

1)

$X :=$ "Peso del tejido inicial" $\quad X \sim N(\mu_1, \sigma_1)$

$Y :=$ "Peso del tejido tras 12 h" $\quad Y \sim N(\mu_2, \sigma_2)$

Son DEPENDIENTES:

$D :=$ "Diferencia del peso" $\sim N(\mu_D, \sigma_D)$ $\quad \left\{ \begin{array}{l} \mu_D = \mu_1 - \mu_2 \\ \sigma_D = \sqrt{\frac{\sigma_1^2}{n} + \sigma_2^2} \end{array} \right\}$

muestra:							
	50'71	52'72	50'01	52'19	50'63	51'55	50'54
	46'50	52'56	49'76	51'90	48'55	49'63	47'79
	4'21	0'16	0'25	0'29	2'08	1'92	2'75

$\bar{d} = 1'6657142\overline{8}$

$s^2 = 2'0070\overline{53061}$

a) ✓

b) $H_0: \mu_1 = \mu_2$ $\quad$ DEPENDIENTES $\rightarrow$ $\mu_D = 0$

$H_a: \mu_1 > \mu_2$ $\quad$ $\mu_D > 0$

$$EC = \frac{\bar{d} - \mu}{s/\sqrt{n}} = \frac{1'6657}{1'5302/\sqrt{7}} = 2'8800$$

$\alpha = 0'15$

$S_0 = (-\infty, t_{n-1;\alpha})$

$S_1 = (t_{n-1;\alpha}, \infty)$

$t_{n-1;\alpha} = qt(0'85, 6) = 1'1342$

$EC \notin S_0 \rightarrow$ No retiene.

c) $P(T>600) = 1 - P(T \le 600) = 1 - [1 - e^{-1}] = 0'3678794412$

$Y :=$ "Número de válvulas que funcionan correctamente transcurridas 600 h de entre 3".

$Y \sim B(n=3,\ p=0'3679)$

$* \ P(600 < T < 1000) = P(T<1000) - P(T<600) = 1 - e^{-1000/600} - 1 + e^{-1} =$

$= 0'1790$.

$Y \sim B(n=3,\ p=0'1790)$

$P(Y<2) = P(Y \le 1) = P(Y=0) + P(Y=1) =$

$= \binom{3}{0} \cdot 0'1790^0 \cdot 0'821^3 + \binom{3}{1} \cdot 0'1790^1 \cdot 0'821^2 =$

$= 0'821^3 + 3 \cdot 0'120653339 = \boxed{0'915347678}$

3) $X :=$ "Pesos de los corchos".

$X \sim N(\mu=14,\ \sigma=0'75)$

a) $P(12<X<15) = P(X<15) - P(X<12) = P\left(Z \le \frac{15-14}{0'75}\right) - P\left(Z \le \frac{12-14}{0'75}\right) =$

$= P(Z \le 1'33) - P(Z \le -2'66) = P(Z \le 1'33) - [1 - P(Z \le 2'66)] =$

$= 0'9088 - 1 + 0'9962 = \boxed{0'905}$

Probabilidad de que un corcho no cumple: $\boxed{0'095}$

$500 \cdot 0'095 = 47'5 \simeq \boxed{48 \text{ corchos}}$

b) $Y :=$ "Número de corchos que no cumplen de entre 250".

$Y \sim B(n=250,\ p=0'095)$ $\quad n \cdot p = 23'75 \ge 5$ ✓ $\quad n \cdot q = 226'25 \ge 5$ ✓ $\quad \cong N(\mu=23'75,\ \sigma=4'6361)$

$P(Y \le 30) = P(Y \le 30'5) = P\left(Z \le \frac{30'5-23'75}{4'6361}\right) = P(Z \le 1'4559) = \boxed{0'9273}$

c)

$$\cancel{P(13'9 \le X < 14'1) = P\left(Z \le \frac{14'1-14}{0'75}\right) - P\left(Z \le \frac{13'9-14}{0'75}\right) =}$$

$$\cancel{P(Z \le 0'1\overline{3}) - P(Z \le -0'1\overline{3}) = P(Z \le 0'13) - 1 + P(Z \le 0'13) =}$$

?

$$\boxed{P(13'9 \le \bar{X} < 14'1)}$$

$$P(|\bar{x} - \mu| \le 0'1) = P(13'9 \le \bar{x} \le 14'1)$$

$$\bar{x} \sim N(\mu, \sigma/\sqrt{n}) = N\left(14, \frac{0'75}{\sqrt{1250}}\right) \longrightarrow 0'0212$$

$$P(\bar{x} \le 14'1) - P(\bar{x} \le 13'9) = P\left(Z \le \frac{14'1-14}{0'0212}\right) - P\left(Z \le \frac{13'9-14}{0'0212}\right) =$$

$$= P(Z \le 4'7169) - P(Z \le -4'7169) = \cancel{P(Z<4}$$

$$1 - (1-1) = 1$$

④

X: = "Frecuencia de la memoria". $\sigma^2_x = 3844\ MHz^2$

$\sigma_x = 62\ MHz$

<u>muestra:</u>

a) $I^{0'9}_{\mu} = [\bar{x} \mp z_{\alpha/2} \cdot \sigma_x/\sqrt{n}]$

$n = 65 > 30$ ✓

$$\bar{x} = \frac{1625 \cdot 8 + 1675 \cdot 16 + 1725 \cdot 26 + 1775 \cdot 12 + 1825 \cdot 3}{65} = 1714'2\overline{30769}$$

$z_{\alpha/2} = z_{0'1/2} = z_{0'05} = qnorm(0'95, 0, 1) = 1'6449$

$\sigma_x/\sqrt{n} = 62/\sqrt{65} = 7'690153845$

$I^{0'9}_{\mu} = [1701'5812 , 1726'88041]$

b) $H_0: \mu = 1728$ $EC = \dfrac{\bar{X} - \mu}{\sigma_x/\sqrt{n}} = -1'4004$

$H_a: \mu < 1728$

$\alpha = 0'05$ 

$S_0 = (-z_\alpha, \infty)$

$S_1 = (-\infty, -z_\alpha]$

$-z_{0'05} = -qnorm(0'95, 0, 1) = -1'6449$

$EC \in S_0 \rightarrow$ Aceptar H_0 con $\alpha = 0'05 \rightarrow$ Sí, de acuerdo.

c) $P(E_{II}) = P(\text{Aceptar } H_0 \mid H_0 \text{ falso})$

$\dfrac{\bar{X} - \mu}{\sigma_x/\sqrt{n}} > -z_\alpha \rightarrow \bar{X} > -1'6449 \cdot 7'6902 + 1728$

$\bar{X} > 1715'35039$

$P(\bar{X} > 1715'35039 \mid \bar{X} \sim N(1715, 7'6902)) =$

$= P(Z > -0'3445) = P(Z < 0'3445) = pnorm(0'3445, 0, 1)$

$= \boxed{0'6348}$

$\beta = 0'6348 \rightarrow$ Potencia: $1 - \beta = 1 - 0'6348 = \boxed{0'3652}$

d) $P(|\bar{X} - \mu| \leq 25) = 0'9 \rightarrow n = \left(z_{\alpha/2} \cdot \sigma_x/\varepsilon\right)^2$

$n = \left(qnorm(0'05, 0, 1) \cdot 62/25\right)^2 = 16'64111274 \simeq 17$