DISCRETE MATHEMATICS

Computer Engineering in Management and Information Systems

MID-TERM EXAM **November 5, 2021**

EXERCISE 1

Simplify the following statement using properties of logical equivalence:

$$[(\neg p \vee \neg q) \to \neg(\neg q \wedge r)] \wedge \neg(r \to s)$$

(1.5 points)

EXERCISE 2

Verify that the following argument is valid:

"If a girl does not play with the ball then if she goes to the park she will play on the swing. Today, she neither plays on the swing nor plays with the ball. Therefore, today she is not going to the park."

(1.5 points)

EXERCISE 3

Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two correspondences defined as follows:

$$f(x) = \begin{cases} -x^2 + 1 & x \leq 0 \\ 2^x & 0 < x < 2 \\ \dfrac{x+6}{2} & x \geq 2 \end{cases} \qquad g(x) = \begin{cases} 1 - x & x \leq 1 \\ -1 + x & 1 < x < 4 \\ 15 - 3x & x \geq 4 \end{cases}$$

a) Plot the correspondences.

b) Determine whether f and g are applications. If so, then analyze their properties.

c) Compute $g \circ f$.

d) When possible calculate the inverse application.

(3 points)

EXERCISE 4

Part 1

The following binary relation is defined on set $A = \{x \in \mathbb{Z} \ / \ |x| \leq 5\}$:

$$xRy \Leftrightarrow x - y \quad \text{is even (0 is an even number)}$$

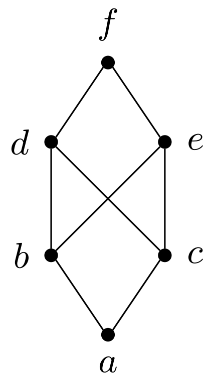
a) Analyze the properties of the binary relation.

b) Is it an equivalence relation? Is it an order relation? Justify the answers.

c) If it is an equivalence relation, find the quotient set. By contrast, if it is an order relation, draw Hasse's diagram. Justify the answers.

Part 2

Let $T = \{a, b, c, d, e, f\}$ be a set on which the binary relation with the following Hasse's diagram has been defined:



a) Find the special elements of subset $S = \{b, c\}$.

b) Is T a chain? Justify the answer.

(2.5 points)

EXERCISE 5

Verify the following expression by using the principle of induction:

$$\frac{1 \cdot 2}{2^3} + \frac{2 \cdot 3}{2^3} + ... + \frac{n \cdot (n+1)}{2^3} = \frac{n(n+1)(n+2)}{24} \qquad \forall n \in \mathbb{N}$$

(1.5 points)