eman ta zabal zazu

EUSKAL HERRIKO UNIBERTSITATEA
UNIVERSIDAD DEL PAIS VASCO

BILBOKO INGENIARITZA ESKOLA
ESCUELA DE INGENIERÍA DE BILBAO

MATEMATIKA APLIKATUA

INDUSTRIA INGENIARITZA TEKNIKOKO ATALA
SECCIÓN INGENIERÍA TÉCNICA INDUSTRIAL

KALKULUA (EBALUAZIO FINALA)

OHIKO DEIALDIA. 2019ko maiatzak 27

Kudeaketaren eta Informazio Sistemen Informatikaren Ingeniaritzako Gradua

1. Ariketa

Ebatzi honako ekuazio diferentziala:

$$x^2 y'' + 5xy' + 4y = \frac{x^2 - x^{-2}}{2}$$

(2 puntu)

Ebazpena:

Euler-en ekuazio bat da. Beraz, hurrengo aldagai aldaketa planteatzen da:

$$x = e^t \quad \Rightarrow \quad t = \ln x \quad (x > 0) \qquad \frac{dy}{dx} = e^{-t}\frac{dy}{dt}; \quad \frac{d^2y}{dx^2} = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

Beraz:

$$x^2 y'' + 5xy' + 4y = \frac{x^2 - x^{-2}}{2} \xrightarrow{x=e^t} e^{2t}e^{-2t}\left(y''(t) - y'(t)\right) + 5e^t e^{-t} y'(t) + 4y(t) = \frac{e^{2t} - e^{-2t}}{2}$$

$$y''(t) + 4y'(t) + 4y(t) = \frac{e^{2t} - e^{-2t}}{2}$$

Ekuazio karakteristikoa lortzen dugu:

$$r^2 + 4r + 4 = 0 \quad \Rightarrow \quad \frac{-4 \pm \sqrt{16 - 4\cdot 1\cdot 4}}{2} = \frac{-4 \pm 0}{2} = \begin{cases} r = -2 \\ r = -2 \end{cases} \quad \Rightarrow \quad r = -2\,(2)$$

$$y_h = C_1 e^{-2t} + C_2\, t\, e^{-2t} \xrightarrow{x=e^t} y_h = C_1 x^{-2} + C_2 x^{-2} \ln x$$

Orain, ekuazio osoaren soluzio orokorra t parametroaren menpe planteatuko dugu parametroen aldakuntzaren metodoa erabiltzeko:

$$y = L_1(t) e^{-2t} + L_2(t)\, t\, e^{-2t}$$

Baldintzak zehazten ditugu:

$$\left.\begin{array}{l} L_1'(t)e^{-2x}+L_2'(t)t\,e^{-2t}=0 \\ -2L_1'(t)e^{-2t}+L_2'(t)\left(e^{-2t}-2t\,e^{-2t}\right)=\dfrac{e^{2t}-e^{-2t}}{2} \end{array}\right\}$$

Wronskiarra kalkulatzen dugu:

$$W=\begin{vmatrix} e^{-2t} & te^{-2t} \\ -2e^{-2t} & e^{-2t}-2te^{-2t} \end{vmatrix}=e^{-4t}-2te^{-4t}+2te^{-4t}=e^{-4t}$$

$L_1'(t)$ eta $L_2'(t)$ kalkulatuko ditugu Kramerren erregela erabiliz:

$$L_1'(t)=\frac{\begin{vmatrix} 0 & te^{-2t} \\ \dfrac{e^{2t}-e^{-2t}}{2} & e^{-2t}-2te^{-2t} \end{vmatrix}}{W}=\frac{-te^{-2t}\,\dfrac{e^{2t}-e^{-2t}}{2}}{e^{-4t}}=-te^{2t}\,\frac{e^{2t}-e^{-2t}}{2}=\frac{t}{2}-\frac{t}{2}e^{4t}$$

$$L_2'(t)=\frac{\begin{vmatrix} e^{-2t} & 0 \\ -2e^{-2t} & \dfrac{e^{2t}-e^{-2t}}{2} \end{vmatrix}}{W}=\frac{e^{-2t}\,\dfrac{e^{2t}-e^{-2t}}{2}}{e^{-4t}}=e^{2t}\left(\frac{e^{2t}-e^{-2t}}{2}\right)=\frac{e^{4t}-1}{2}$$

$L_1'(t)$ eta $L_2'(t)$ integratuko ditugu $L_1(t)$ eta $L_2(t)$ kalkulatzeko:

$$L_1(t)=\int\left(\frac{t}{2}-\frac{t}{2}e^{4t}\right)dx=\frac{1}{2}\left[\frac{t^2}{2}-\int te^{4t}dt\right]=\frac{1}{2}\left[\frac{t^2}{2}-I_1\right]$$

non $I_1=\int te^{4t}dt=\begin{Vmatrix} u=t & du=dt \\ dv=e^{4t}dt & v=\dfrac{e^{4t}}{4} \end{Vmatrix}=\dfrac{t}{4}e^{4t}-\dfrac{1}{4}\int e^{4t}dt=\dfrac{t}{4}e^{4t}-\dfrac{1}{16}e^{4t}+C$

Beraz, $L_1(t)=\dfrac{1}{2}\left[\dfrac{t^2}{2}-I_1\right]=\dfrac{1}{2}\left[\dfrac{t^2}{2}-\dfrac{t}{4}e^{4t}+\dfrac{1}{16}e^{4t}+C\right]=\dfrac{t^2}{4}-\dfrac{t}{8}e^{4t}+\dfrac{1}{32}e^{4t}+A$

$$L_2(t)=\int\frac{e^{4t}-1}{2}dt=\frac{e^{4t}}{8}-\frac{t}{2}+B$$

$L_1(t)$ eta $L_2(t)$ kalkulatuta daudela, ekuazio osoaren soluzio orokorra hurrengoa izango litzateke:

$$y=L_1(t)e^{-2t}+L_2(t)te^{-2t}=\left[\frac{t^2}{4}-\frac{t}{8}e^{4t}+\frac{1}{32}e^{4t}+A\right]e^{-2t}+\left[\frac{e^{4t}}{8}-\frac{t}{2}+B\right]te^{-2t}=$$

$$= Ae^{-2t} + Bte^{-2t} + \left(\frac{t^2}{4} - \frac{t}{8}e^{4t} + \frac{1}{32}e^{4t} + \frac{t}{8}e^{4t} - \frac{t^2}{2}\right)e^{-2t} = Ae^{-2t} + Bte^{-2t} + \left(\frac{1}{32}e^{4t} - \frac{t^2}{4}\right)e^{-2t} =$$

$$= Ae^{-2t} + Bte^{-2t} + \frac{1}{32}e^{2t} - \frac{t^2}{4}e^{-2t}$$

$$\boxed{y(t) = Ae^{-2t} + Bte^{-2t} + \frac{1}{32}e^{2t} - \frac{t^2}{4}e^{-2t}}$$

$x = e^t \quad \Rightarrow \quad t = \ln x$ aldagai aldaketa desegitea besterik ez da geratzen soluzioa x-ren menpe uzteko:

$$\boxed{y(x) = Ax^{-2} + Bx^{-2}\ln x + \frac{x^2}{32} - \frac{x^{-2}\ln^2 x}{4}}$$

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2. Ariketa

Sailkatu eta ebatzi honako ekuazio diferentziala:

$$\left(xy\cos x + 2x^2e^y\right)dx + \left(x\sin x + x^3e^y\right)dy = 0$$

(2 puntu)

Ebazpena:

Zehatza da?:

$$\left.\begin{aligned} \frac{\partial X}{\partial y} &= x\cos x + 2x^2e^y \\ \frac{\partial Y}{\partial x} &= \sin x + x\cos x + 3x^2e^y \end{aligned}\right\} \rightarrow \text{Desberdinak direnez, ez da zehatza}$$

Faktore integratzaile bat lor daiteke?:

$$\frac{\partial X/\partial y - \partial Y/\partial y}{Y} = \frac{x\cos x + 2x^2e^y - \sin x - x\cos x - 3x^2e^y}{x\sin x + x^3e^y} =$$

$$= -\frac{x^2e^y + \sin x}{x\left(\sin x + x^2e^y\right)} = -\frac{1}{x} = \phi(x) \rightarrow \ z(x) \text{ erako faktore integratzaile bat lor daiteke}$$

Faktore integratzailea hurrengoa da:

$$z(x) = A \cdot e^{\int \phi(x)dx} = A \cdot e^{-\int \frac{1}{x}dx} = A \cdot e^{-\ln x} = \frac{1}{x}$$

Ekuazioa biderkatuz:

$$\left(y\cos x + 2xe^{y}\right)dx + \left(\sin x + x^{2}e^{y}\right)dy = 0 \rightarrow \text{Zehatza da}$$

Soluzio orokorra hurrengoa da:

$$\int_0^x \left(y\cos x + 2xe^{y}\right)dx + \int_0^y 0dy = C$$

$$\left. y\sin x + x^{2}e^{y} \right]_0^x = C$$

$$\boxed{y\sin x + x^{2}e^{y} = C}$$

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3. Ariketa

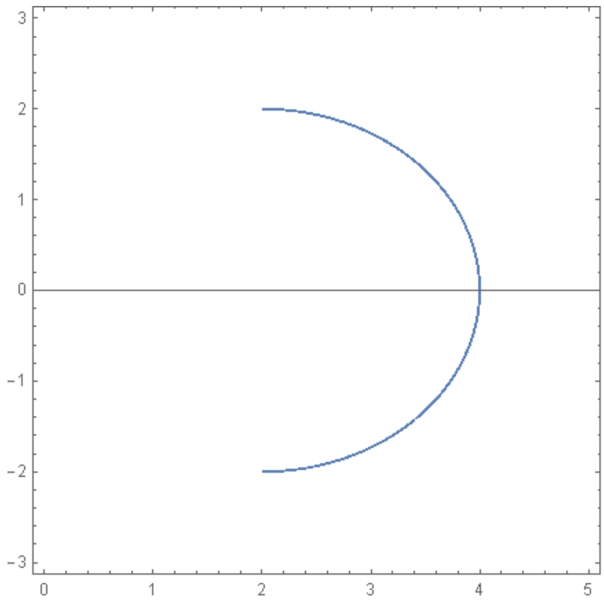
Kalkulatu C kurbaren gaineko honako integral lerromakurra: $\int_C xy^4 \ dS$

C honela definituta egonik: $C = \{x^2 + y^2 = 4x \quad \text{non} \quad x \geq 2$

(2 puntu)

Ebazpena:

C kurba marraztuko dugu:



C kurba parametrizatuz:

$$\begin{cases} x = 2+2\cos\theta \rightarrow \dfrac{dx}{d\theta} = -2\sin\theta \\ y = 2\sin\theta \quad \rightarrow \dfrac{dy}{d\theta} = 2\cos\theta \end{cases} \qquad \theta \in [-\pi/2, \pi/2,]$$

Integralean ordezkatuz:

$$\int_C xy^4\, dS = \int_{-\pi/2}^{\pi/2} (2+2\cos\theta)(2\sin\theta)^4 \sqrt{(-2\sin\theta)^2 + (2\cos\theta)^2}\, d\theta =$$

$$\int_{-\pi/2}^{\pi/2} 2^5 (\sin\theta)^4 + 2^5 \cos\theta (\sin\theta)^4 \sqrt{4(\sin\theta)^2 + 4(\cos\theta)^2}\, d\theta =$$

$$2^6 \underbrace{\int_{-\pi/2}^{\pi/2} (\sin\theta)^4\, d\theta}_{I} + 2^6 \underbrace{\int_{-\pi/2}^{\pi/2} \cos\theta (\sin\theta)^4 d\theta}_{J}$$

I integralaren kalkulua:

$$I = \int_{-\pi/2}^{\pi/2} (\sin\theta)^4\, d\theta = \int_{-\pi/2}^{\pi/2} \left(\frac{1-\cos(2\theta)}{2}\right)^2 d\theta = \int_{-\pi/2}^{\pi/2} \left(\frac{1+\cos^2(2\theta)-2\cos(2\theta)}{4}\right) d\theta =$$

$$= \int_{-\pi/2}^{\pi/2} \left(\frac{1-2\cos(2\theta)}{4}\right) d\theta + \frac{1}{4}\int_{-\pi/2}^{\pi/2} \cos^2(2\theta)\, d\theta = \left[\frac{\theta}{4} - \frac{\sin(2\theta)}{4}\right]_{-\pi/2}^{\pi/2} + \frac{1}{4}\int_{-\pi/2}^{\pi/2} \left(\frac{1+\cos(4\theta)}{2}\right) d\theta =$$

$$= \frac{\pi}{8} + \frac{\pi}{8} + \frac{1}{4}\left[\frac{\theta}{2} - \frac{\sin(4\theta)}{8}\right]_{-\pi/2}^{\pi/2} = \frac{3\pi}{8}$$

J integralaren kalkulua:

$$J = \int_{-\pi/2}^{\pi/2} \cos\theta (\sin\theta)^4 d\theta = \left\| \begin{matrix} t = \sin\theta \\ dt = \cos\theta d\theta \\ d\theta = dt/\cos\theta \end{matrix} \right\| = \int t^4 dt = \left.\frac{\sin^5\theta}{5}\right]_{-\pi/2}^{\pi/2} = \frac{2}{5}$$

Beraz:

$$\int_C xy^4\, dS = 2^6\left[\frac{3\pi}{8}\right] + 2^6\left[\frac{2}{5}\right] = \boxed{64\left(\frac{3\pi}{8} + \frac{2}{5}\right)}$$

4. Ariketa

Izan bedi gainazal hauek mugatzen duten [C] gorputz homogeneoa:

$$x^2 + y^2 - 4z = 0, \quad x^2 + y^2 - z^2 + 16z - 64 = 0 \ \left(z \leq 8\right)$$

Kalkulatu integral hirukoitza erabiliz:

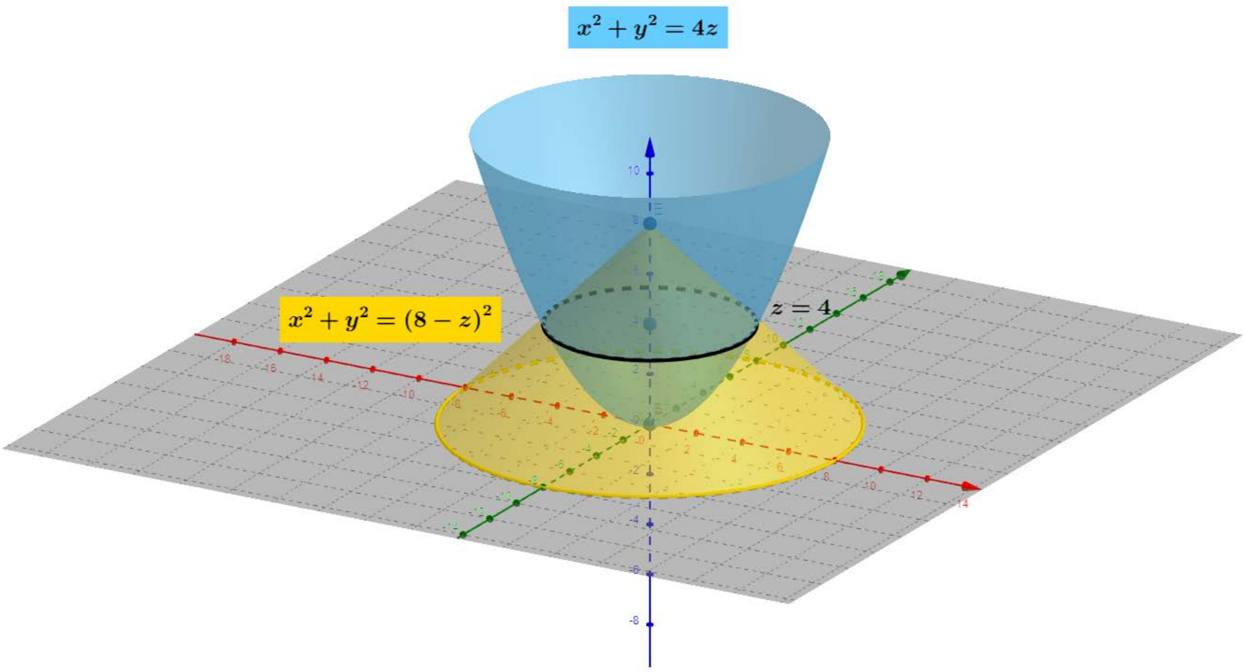
a) C gorputzaren bolumena.

b) C gorputzaren grabitate zentroa.

(2 puntu)

Ebazpena:

a) Irudikapen grafikoan ikus daitekeenez horiz esfera erdi bat ($x^2 + y^2 + \left(z-10\right)^2 = 16$) eta urdinez paraboloide bat ($x^2 + y^2 = z$) ditugu.



Konoak eta paraboloideak mugatutako [C] gorputzaren bolumena, paraboloidearen barrukoa ($x^2 + y^2 = 4z$) eta konoaren barrukoa ($x^2 + y^2 = (8-z)^2$) da. Bolumen hori kalkulatzeko lehendabizi ebakidura planoa kalkulatu behar da.

$$\left\{\begin{array}{l} x^2 + y^2 = 4z \\ x^2 + y^2 = (8-z)^2 \end{array}\right. \left\{ 4z = (8-z)^2 \quad \Rightarrow \quad 4z = z^2 - 16z + 64 \quad \Rightarrow \quad z^2 - 20z + 64 = 0 \right.$$

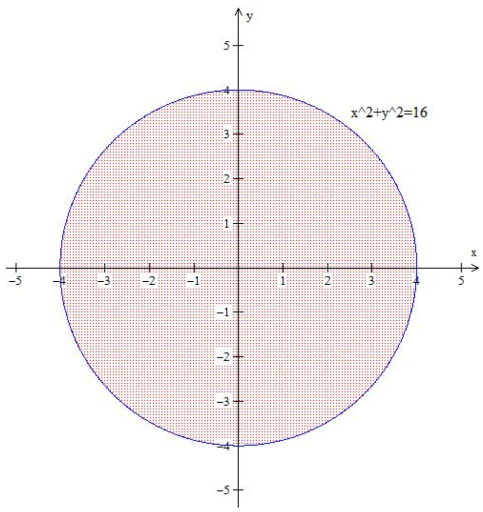
$$\Rightarrow \quad z=\frac{20\pm\sqrt{400-4\cdot 1\cdot 64}}{2} \quad \Rightarrow \quad z=\frac{20\pm 12}{2} \quad \Rightarrow \quad \begin{cases} \boxed{z=4} \\ z=16 \end{cases}$$

Ebakidura planoa $z=4$ da, izan ere konoa $z\leq 8$ -rako definituta baitago.

Koordenatu zilindrikoetan ebatziko da ariketa. Beraz, hurrengo aldagai aldaketa aplikatzen da:

$$\begin{cases} x=\rho\cos\theta \\ y=\rho\sin\theta \\ z=z \\ J(\rho,\theta,z)=\rho \end{cases} \qquad \begin{cases} x^2+y^2=4z \quad \Rightarrow \quad \rho^2=4z \quad \Rightarrow \quad z=\rho^2/4 \\ x^2+y^2=(8-z)^2 \quad \Rightarrow \quad \rho^2=(8-z)^2 \quad \Rightarrow \quad \begin{cases} \boxed{z=8-\rho} \\ z=8+\rho \end{cases} \end{cases}$$

Behin *z*-ren mugak zehaztuta daudela, *XOY* planoaren gaineko proiekzioa egiten dugu eta hurrengoa ikusten da, $x^2+y^2=16$ zirkunferentzia, zentroa *C*(0,0) eta *R*=4.



Ditugun hiru aldagaien mugak orduan hauexek izango dira:

$$\theta=[0,2\pi]; \quad \rho=[0,4]; \quad z=[\rho^2/4, 8-\rho]$$

Orduan, bolumena kalkulatzeko hurrengo integral hirukoitza planteatzen dugu:

$$V=\int_0^{2\pi}d\theta\int_0^4\rho\,d\rho\int_{\rho^2/4}^{8-\rho}dz=\int_0^{2\pi}d\theta\int_0^4\rho(8-\rho-\frac{\rho^2}{4})d\rho=\int_0^{2\pi}d\theta\int_0^4(8\rho-\rho^2-\frac{\rho^3}{4})d\rho=$$

$$=\int_0^{2\pi}\left[4\rho^2-\frac{\rho^3}{3}-\frac{\rho^4}{16}\right]_0^4 d\theta=2\pi\left[64-\frac{64}{3}-16\right]=\frac{160\pi}{3}$$

$$\boxed{V=\frac{160\pi}{3}\quad u^3}$$

b) Behin bolumena kalkulatuta dagoela, grabitate zentroa kalkulatzeko z_c koordenatua soilik lortu behar dugu [C] gorputza simetrikoa baita *OX* eta *OY* ardatzekiko. Beraz, $z_C=\frac{1}{V}\int\int\int_C z\,dx\,dy\,dz$ hurrengo integral kalkulatuko dugu lehenik eta behin:

$$\int_0^{2\pi}d\theta\int_0^4\rho\,d\rho\int_{\rho^2/4}^{8-\rho}z\,dz=\frac{1}{2}\int_0^{2\pi}d\theta\int_0^4\rho\left[(8-\rho)^2-\frac{\rho^4}{16}\right]d\rho=\frac{1}{2}\int_0^{2\pi}d\theta\int_0^4(64\rho-16\rho^2+\rho^3-\frac{\rho^5}{16})d\rho=$$

$$=\pi\left[32\rho^2-\frac{16\rho^3}{3}+\frac{\rho^4}{4}-\frac{\rho^6}{96}\right]_0^4=\pi\left[2^5\cdot2^4-\frac{2^4\cdot2^6}{3}+\frac{2^8}{2^2}-\frac{2^{12}}{3\cdot2^5}\right]=\pi\left[2^9-\frac{2^{10}}{3}+2^6-\frac{2^7}{3}\right]=$$

$$=\pi2^6\left(2^3-\frac{2^4}{3}+1-\frac{2}{3}\right)=64\pi\left(9-\frac{18}{3}\right)=192\pi$$

Beraz, z_c koordenatua hurrengoa da:

$$z_C=\frac{1}{V}\int\int\int_C z\,dx\,dy\,dz=\frac{3\cdot192\pi}{160\pi}=\frac{18}{5}$$

Azkenik, grabitatea zentroa $\left(0,0,\frac{18}{5}\right)$ da.

(2 puntu)

5. Ariketa

Izan bedi hurrengo eran definituriko [*D*] domeinu laua:

$$D=\left\{(x,y)\in\mathbb{R}^2\,/\,x^2+y^2-4x\geq0,\quad(x-2)^2+4y^2-16\leq0,\quad x\geq2\right\}$$

Kalkulatu [*D*] domeinu lauaren azalera **integral bikoitzaren kontzeptua erabiliz**.

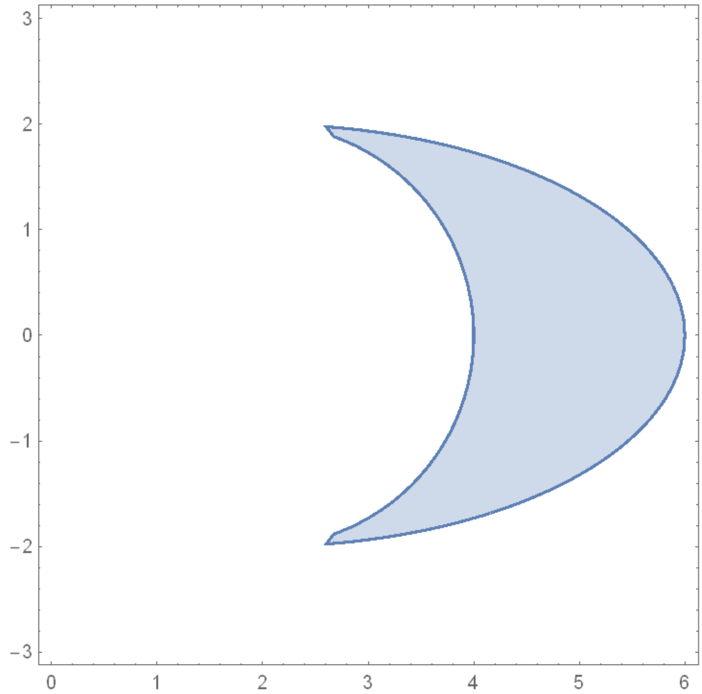
(2 puntu)

Ebazpena:

Lehengo eta behin, *D* domeinua marraztuko dugu:

$(x-2)^2+4y^2-16\leq0\rightarrow$ (2,0) zentroko elipsea

$x^2+y^2-4x\geq0\rightarrow$ (2,0) zentroko eta 2 erradioko zirkunferentzia



Koordenatu polarrak erabiliz:

$$\begin{cases} x = 2 + \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$$

Domeinuan agertzen diren kurben ekuazioak koordenatu polarretan hurrengoak dira:

$$\begin{cases} x^2 + y^2 - 4x = 0 \to \rho = 2 \\ \\ (x-2)^2 + 4y^2 - 16 = 0 \to \rho^2\cos^2\theta + 4\rho^2\sin^2\theta = 16 \to \rho^2\left(1-\sin^2\theta\right) + 4\rho^2\sin^2\theta = 16 \to \\ \to \rho^2\left(1+3\sin^2\theta\right) = 16 \to \rho = \dfrac{4}{\sqrt{1+3\sin^2\theta}} \end{cases}$$

Orain, azalera kalkulatuko dugu, integral bikoitza erabiliz:

$$A=\int_{-\pi/2}^{\pi/2}d\theta\int_{2}^{16/\sqrt{1+3\sin^2\theta}}\rho d\rho=\frac{1}{2}\int_{-\pi/2}^{\pi/2}\left(\frac{16}{\sqrt{1+3\sin^2\theta}}-4\right)d\theta=\frac{1}{2}\cdot 4\int_{-\pi/2}^{\pi/2}\left(\frac{4}{1+3\sin^2\theta}-1\right)d\theta=$$

$$=2\int_{-\pi/2}^{\pi/2}\left(\frac{4-1-3\sin^2\theta}{1+3\sin^2\theta}\right)d\theta=6\int_{-\pi/2}^{\pi/2}\left(\frac{1-\sin^2\theta}{1+3\sin^2\theta}\right)d\theta=6\int_{-\pi/2}^{\pi/2}\left(\frac{\cos^2\theta}{1+3\sin^2\theta}\right)d\theta=$$

$$=\left\|\begin{array}{l}\text{aldagai aldaketa:}\\ t=\tan\theta;d\theta=\dfrac{dt}{1+t^2}\\ \cos^2\theta=\dfrac{1}{1+t^2};\sin^2\theta=\dfrac{t^2}{1+t^2}\end{array}\right\|=6\int_{-\infty}^{\infty}\left(\frac{1}{1+t^2+3t^2}\right)\frac{dt}{1+t^2}=6\int_{-\infty}^{\infty}\frac{dt}{\left(1+t^2\right)\left(1+4t^2\right)}=$$

$$=\left\|\begin{array}{l}\text{zatiki sinpleetan}\\ \text{deskonposatuz}\end{array}\right\|=6\int_{-\infty}^{\infty}\left(\frac{-1/3}{\left(1+t^2\right)}+\frac{4/3}{\left(1+4t^2\right)}\right)dt==6\left[-\frac{1}{3}\arctan t+\frac{2}{3}\arctan 2t\right]_{-\infty}^{\infty}=\boxed{2\pi}$$

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6. Ariketa

Kalkulatu honako integral mugagabeak:

a) $$\int\frac{dx}{(x-1)^2\sqrt{x^2+x-1}}$$

b) $$\int\frac{1}{x^2\sqrt{x^2-4}}\,dx$$

(2 puntu)

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a) $$\int\frac{dx}{(x-1)^2\sqrt{x^2+x-1}}=\left\|\begin{array}{l}x-1=\dfrac{1}{t}\\ dx=-\dfrac{1}{t^2}dt\end{array}\right\|=\int\frac{-\frac{1}{t^2}dt}{\frac{1}{t^2}\sqrt{\left(\frac{t+1}{t}\right)^2+\frac{t+1}{t}-1}}=-\int\frac{dt}{\sqrt{\frac{t^2+2t+1+t^2+t-t^2}{t^2}}}=$$

$$=-\int\frac{t}{\sqrt{t^2+3t+1}}dt=-\int\frac{t}{\sqrt{t^2+3t+1}}dt=-\frac{1}{2}\int\frac{2t+3-3}{\sqrt{t^2+3t+1}}dt=-\frac{1}{2}\left[\int\frac{2t+3}{\sqrt{t^2+3t+1}}dt-\int\frac{3}{\sqrt{t^2+3t+1}}dt\right]=$$

$$=-\frac{1}{2}\left[2\sqrt{t^2+3t+1}-\int\frac{3}{\sqrt{\left(t+\frac{3}{2}\right)^2-\frac{5}{4}}}dt\right]=-\sqrt{t^2+3t+1}+\frac{3}{2}\ln\left|t+\frac{3}{2}+\sqrt{t^2+3t+1}\right|+C=$$

$$=\boxed{-\sqrt{\frac{1}{(x-1)^2}+\frac{3}{x-1}+1}+\frac{3}{2}\ln\left|\frac{3}{x-1}+\frac{3}{2}+\sqrt{\frac{1}{(x-1)^2}+\frac{3}{x-1}+1}\right|+C}$$

b) 1. ebazpen posiblea

$$I=\int\frac{1}{x^2\cdot\sqrt{x^2-4}}\,dx=\int x^{-2}\left(x^2-4\right)^{-1/2}dx=\left\|\begin{matrix} m=-2 & n=2 & p=-\frac{1}{2}\notin\mathbb{Z} \\ \frac{m+1}{n}=-\frac{1}{2}\notin\mathbb{Z} & & \frac{m+1}{n}+p=-1\in\mathbb{Z}\end{matrix}\right\|=\begin{pmatrix}\text{binomia}\\ \text{3. kasua}\end{pmatrix}=$$

$$=\left\|\begin{matrix} x^2=t\to x=t^{1/2} \\ dx=\frac{1}{2}t^{-1/2}dt\end{matrix}\right\|=\int t^{-1}(t-4)^{-1/2}\,\frac{1}{2}t^{-1/2}dt=\frac{1}{2}\int t^{-3/2}(t-4)^{-1/2}\,dt=$$

$$=\frac{1}{2}\int t^{-3/2}\cdot t^{-1/2}\left(\frac{t-4}{t}\right)^{-1/2}dt=\frac{1}{2}\int t^{-2}\cdot\left(\frac{t-4}{t}\right)^{-1/2}dt=\left\|\begin{matrix} \frac{t-4}{t}=z^2\;\Rightarrow\; t=\frac{-4}{z^2-1} \\ dt=\frac{8z}{\left(z^2-1\right)^2}dz\end{matrix}\right\|=$$

$$=\frac{1}{2}\int\left(\frac{-4}{z^2-1}\right)^{-2}\cdot z^{-1}\cdot\frac{8z}{\left(z^2-1\right)^2}dz=\frac{1}{4}\int dz=\frac{1}{4}z+K=\frac{1}{4}\sqrt{\frac{t-4}{t}}+K=\boxed{\frac{1}{4x^2}\sqrt{x^2-4}+K}$$

2. ebazpen posiblea

$$I=\int\frac{1}{x^2\cdot\sqrt{x^2-4}}\,dx=\left\|\begin{matrix} x=\frac{2}{\cos t} & dx=2\cos^{-2}t\cdot\sin t\cdot dt \\ \cos t=\frac{2}{x} & \sin t=\sqrt{1-\left(\frac{2}{x}\right)^2}\end{matrix}\right\|=\int\frac{2\cos^{-2}t\cdot\sin t}{\left(\frac{2}{\cos t}\right)^2\cdot\sqrt{\left(\frac{2}{\cos t}\right)^2-4}}dt=$$

$$=\int\frac{\sin t}{2\sqrt{\frac{4}{\cos^2 t}-4}}dt=\frac{1}{4}\int\frac{\sin t}{\sqrt{\frac{1}{\cos^2 t}-1}}dt=\frac{1}{4}\int\frac{\sin t}{\sqrt{\frac{1-\cos^2 t}{\cos^2 t}}}dt=\frac{1}{4}\int\frac{\sin t}{\sqrt{\frac{\sin^2 t}{\cos^2 t}}}dt=\frac{1}{4}\int\cos t\,dt=$$

$$=\frac{1}{4}\sin t+K=\left\|\cos t=\frac{2}{x}\qquad \sin t=\sqrt{1-\left(\frac{2}{x}\right)^2}\right\|=\frac{1}{4}\sqrt{1-\left(\frac{2}{x}\right)^2}+K=\boxed{\frac{1}{4x^2}\sqrt{x^2-4}+K}$$