DISCRETE MATHEMATICS

Computer Engineering in Management and Information Systems

MID-TERM EXAM **November 4, 2022**

EXERCISE 1

Simplify the following statement using properties of logical equivalence:

$$[(q \vee p \to (r \to q)) \to r \wedge q \wedge (q \vee p)] \vee [r \wedge (q \to s) \wedge \neg p \wedge \neg(q \wedge s)]$$

(1.5 points)

EXERCISE 2

Verify that the following argument is valid:

"I study Logic or if I study Set Theory then I learn Congruences. I learn Combinatorics or study Set Theory. I learn Conguences or I do not study Set Theory. Therefore, I learn Combinatorics or Congruences."

(1.5 points)

EXERCISE 3

Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two correspondences defined as follows:

$$f(x) = \begin{cases} x^3 - 1 & x \leq 0 \\ -\cos(x) & 0 < x < \pi \\ \dfrac{x}{\pi} & x \geq \pi \end{cases} \qquad g(x) = \begin{cases} -(x+1)^2 & x \leq -1 \\ x + 1 & -1 < x \leq 0 \\ 1 - x & 0 < x < 1 \\ x - 1 & x \geq 1 \end{cases}$$

a) Plot the correspondences.

b) Determine whether f and g are applications. If so, then analyze their properties.

c) When possible calculate the inverse application.

d) Compute $f \circ g$.

(3 points)

EXERCISE 4

Given set $A = \{1, 2, 3\}$, the following binary relation is defined on $B = A \times A$:

$$(x_1, y_1)R(x_2, y_2) \quad \Leftrightarrow \quad x_1 \leq x_2 \quad \wedge \quad y_1 \geq y_2$$

a) Analyze the properties of the binary relation.

b) Is it an equivalence relation? Is it an order relation? Justify the answers.

c) If it is an equivalence relation, find the quotient set. By contrast, if it is an order relation, draw Hasse's diagram. Justify the answers.

d) Determine what elements are related to (3,1). Is (3,1) related to any element? If so, which ones? Justify the answers.

e) Determine, if possible, the special elements of subset $S = \{(1, 2), (2, 2), (2, 3)\}$.

(2.5 points)

EXERCISE 5

Verify the following expression by using the principle of induction:

$$\frac{1}{2^2 - 1} + \frac{1}{3^2 - 1} + ... + \frac{1}{(n+1)^2 - 1} = \frac{3}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)} \qquad \forall n \in \mathbb{N}$$

(1.5 points)
