Modèles Linéaires Appliqués / Régression
Régression Logistique: x catégorielle

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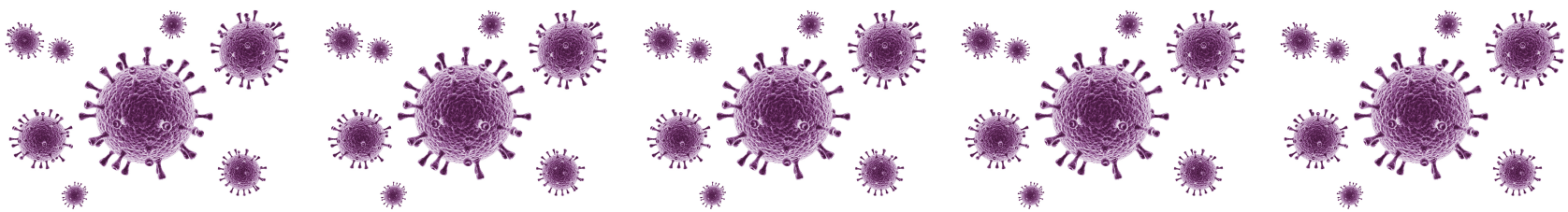
UQAM

Hiver 2020 - COVID-19 # 4



Test d'Indépendance de Pearson

On considère deux variables qualitatives

$$x_1 \in \{a_1, \cdots, a_I\} \text{ et } x_2 \in \{b_1, \cdots, b_J\}$$

et on convertit en un tableau de contingence,

$$n_{i,i} = \sum_k \mathbf{1}(x_{1,k} = a_i)\mathbf{1}(x_{2,k} = b_j$$

```
> loc = "http://freakonometrics.free.fr/titanic.RData"
> download.file(loc, "titanic.RData")
> load("titanic.RData")
> base = base[,1:7]
> table(base$Survived,base$Pclass)

    1   2   3
  0  64  90 270
  1 120  83  85
```

Test d'Indépendance de Pearson

Test : $H_0 : X_1 \perp\!\!\!\perp X_2$, i.e. (cf définition), $\forall i, j$

$$\mathbb{P}[X_1 = a_i, X_2 = b_j] = \mathbb{P}[X_1 = a_i] \cdot \mathbb{P}[X_2 = b_j]$$

i.e. sous H_0, on espère avoir

$$\frac{n_{i,j}}{n} \approx \frac{n_{i,\cdot}}{n} \cdot \frac{n_{\cdot,j}}{n} = \frac{n_{i,j}^{\perp}}{n} \text{ où } n_{i,\cdot} = \sum_{j=1}^{J} n_{i,j}, \ n_{\cdot,J} = \sum_{i=1}^{I} n_{i,j}$$

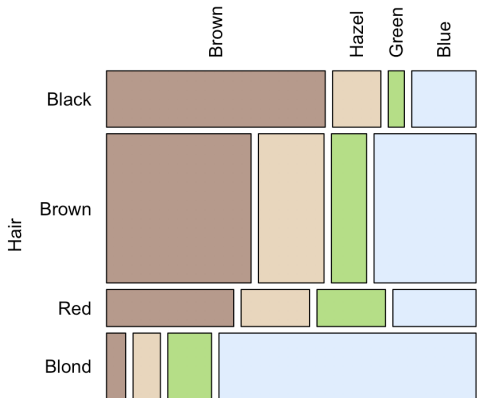
La statistique de test (de Pearson) est

$$Q = \sum_{i,j} \frac{(n_{i,j} - n_{i,j}^{\perp})^2}{n_{i,j}^{\perp}} \sim \chi^2((I-1)(J-1))$$

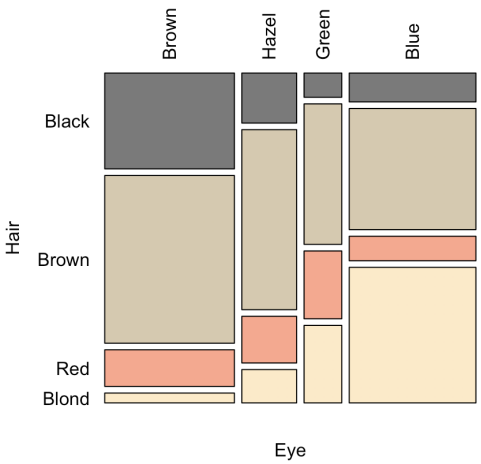
où $u_{i,j} = \dfrac{n_{i,j} - n_{i,j}^{\perp}}{\sqrt{n_{i,j}^{\perp}}}$ est la contribution du couple (i, j).

Test d'Indépendance de Pearson

	brown	hazel	green	blue	
black	63.0%	13.9%	4.6%	18.5%	100.0%
brown	41.6%	18.9%	10.1%	29.4%	100.0%
red	36.6%	19.7%	19.7%	23.9%	100.0%
blond	5.5%	7.9%	12.6%	74.0%	100.0%
	37.2%	15.7%	10.8%	36.3%	

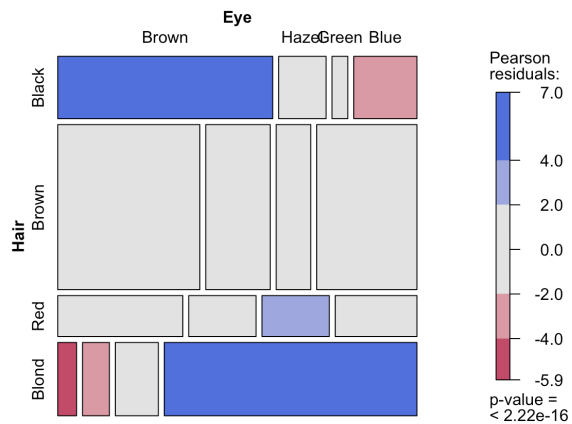


	brown	hazel	green	blue	
black	30.9%	16.1%	7.8%	9.3%	18.2%
brown	54.1%	58.1%	45.3%	39.1%	48.3%
red	11.8%	15.1%	21.9%	7.9%	12.0%
blond	3.2%	10.8%	25.0%	43.7%	21.5%
	100.0%	100.0%	100.0%	100.0%	



Test d'Indépendance de Pearson

	brown	hazel	green	blue	
black	68	15	5	20	108
brown	119	54	29	84	286
red	26	14	14	17	71
blond	7	10	16	94	127
	220	93	64	215	



	brown	hazel	green	blue	
black	40	17	12	39	108
brown	106	45	31	104	286
red	26	11	8	26	71
blond	47	20	14	46	127
	220	93	64	215	

Compare $n_{i,j}$ and $n^{\perp}_{i,j}$

$$n^{\perp}_{i,j} = \frac{n_{i,\cdot} \cdot n_{\cdot,j}}{n}$$

Régression sur x_1

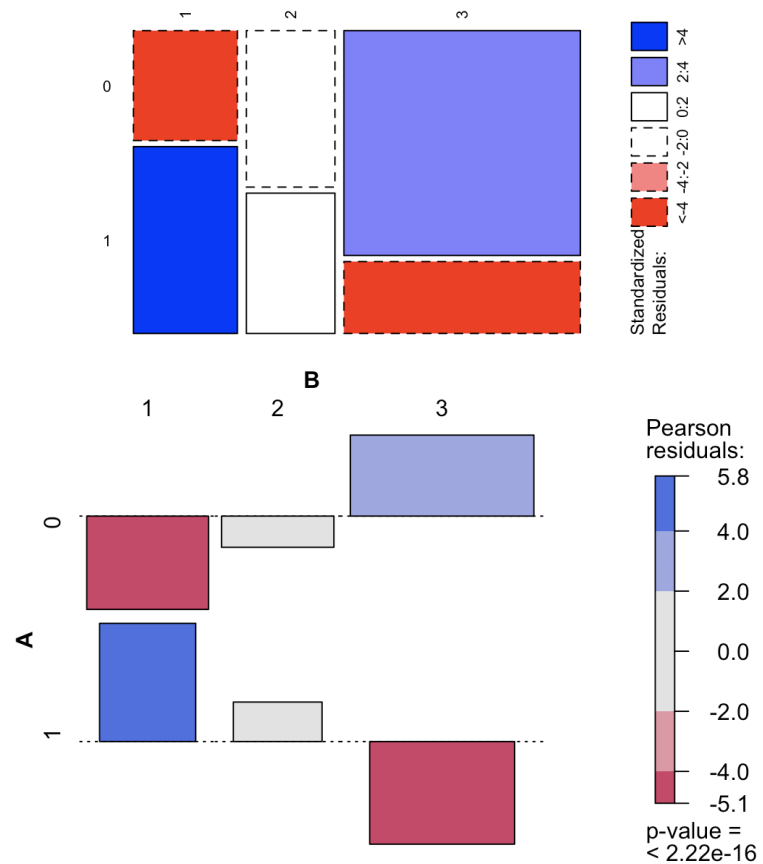
$x_1 \in \{A, B, C\}$, $\boldsymbol{x} = (\mathbf{1}_A, \mathbf{1}_B, \mathbf{1}_C)$

```
> (T=table(base$Survived,
    base$Pclass))
          Pclass
Survived   1   2   3
       0  80  97 372
       1 136  87 119

> chisq.test(T)

 Pearson's Chi-squared test

X-squared = 91.081, df = 2,
    p-value < 2.2e-16
> library("graphics")
> mosaicplot(T)
> library("vcd")
> assoc(T)
```



Régression sur x_1

```
> with(base,aggregate(Survived,by=list(Pclass),mean))
    $x
[1] 0.6296296 0.4728261 0.2423625
```

$$\widehat{p}_A = \frac{1}{n_A} \sum_{i:x_i=A} y_i = 62.96\%, \ \widehat{p}_B = \frac{1}{n_B} \sum_{i:x_i=B} y_i = 47.28\%,$$

```
> reg = glm(Survived ~ Pclass, family = "binomial",
    data = base)
> summary(reg)

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)   0.5306     0.1409   3.766 0.000166 ***
Pclass2      -0.6394     0.2041  -3.133 0.001731 **
Pclass3      -1.6704     0.1759  -9.496  < 2e-16 ***
```

Régression sur x_1

$$\widehat{p}_A = \frac{1}{n_A} \sum_{i:x_i=A} y_i = 62.96\%, \quad \widehat{p}_B = \frac{1}{n_B} \sum_{i:x_i=B} y_i = 47.28\%,$$

```
> beta = coefficients(reg)
> exp(beta[1])/(1+exp(beta[1]))
(Intercept)
  0.6296296
> exp(beta[1]+beta[2])/(1+exp(beta[1]+beta[2]))
(Intercept)
  0.4728261
```

$$\widehat{p}_A = \frac{e^{\widehat{\beta}_0}}{1+e^{\widehat{\beta}_0}}, \quad \widehat{p}_B = \frac{e^{\widehat{\beta}_0+\widehat{\beta}_1}}{1+e^{\widehat{\beta}_0+\widehat{\beta}_1}}, \quad \widehat{p}_C = \frac{e^{\widehat{\beta}_0+\widehat{\beta}_2}}{1+e^{\widehat{\beta}_0+\widehat{\beta}_2}}$$

Régression sur x_1

$$\sum_{i=1}^{n} \widehat{p}_i = \sum_{i=1}^{n} y_i \text{ et } \sum_{i:x_i=j} \widehat{p}_i = \sum_{i:x_i=j} y_i, \ \forall j$$

```
> sum(predict(reg,type="response"))
[1] 342
> sum(base$Survived)
[1] 342
> sum(predict(reg,type="response")[base$Pclass=="1"])
[1] 136
> sum(base$Survived[base$Pclass=="1"])
[1] 136
```

Survie des Passagers du Titanic

y : indicatrice de survie d'un passager du Titanic

```
> reg = glm(Survived ~ Sex, family = "binomial", data
    = base)
> summary(reg)

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)   1.0566     0.1290   8.191 2.58e-16 ***
Sexmale      -2.5137     0.1672 -15.036  < 2e-16 ***
> confint(reg)

                 2.5 %     97.5 %
(Intercept)  0.8085881   1.314934
Sexmale     -2.8465007 -2.190728
```

Survie des Passagers du Titanic

y : indicatrice de survie d'un passager du Titanic

```
> with(base,table(Survived,Sex))
          Sex
Survived female male
       0     81  468
       1    233  109

> with(base,chisq.test(table(Survived,Sex)))

  Pearson's Chi-squared test with Yates' continuity
    correction

data:  table(Survived, Sex)
X-squared = 260.72, df = 1, p-value < 2.2e-16
```

Régression & Données Groupées

La régression binomiale sur données catégorielles peut se faire

- sur les données individuelles, $(y_i, x_{1,i}, x_{2,i}, x_{3,i})$, où $y \in \{0, 1\}$
- sur les données groupes, sur tableau de contingence $N_{i,j,k}$

```
> reg = glm(Survived ~ Sex+Embarked+Pclass, family = "
    binomial", data = base)
> summary(reg)

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)   2.6394     0.2672   9.877  < 2e-16 ***
Sexmale      -2.6081     0.1855 -14.056  < 2e-16 ***
EmbarkedQ    -0.1454     0.3626  -0.401  0.68837
EmbarkedS    -0.5954     0.2278  -2.613  0.00897 **
Pclass2      -0.6691     0.2525  -2.649  0.00806 **
Pclass3      -1.8385     0.2247  -8.182 2.78e-16 ***
```

Régression & Données Groupées

```
> Y = xtabs(Survived ~ Embarked+Pclass+Sex,data=base)
> N = xtabs( (Survived>=0)~ Embarked+Pclass+Sex,data=
    base)
> dfy = as.data.frame(Y)
> dfn = as.data.frame(N)
> df = data.frame(dfy[,1:3],Y=dfy$Freq,N=dfn$Freq)
> tail(df)
   Embarked Pclass  Sex  Y   N
13        C      2 male  2  10
14        Q      2 male  0   1
15        S      2 male 15  97
16        C      3 male 10  43
17        Q      3 male  3  39
18        S      3 male 34 265
```

où $\tilde{Y}_i \sim \mathcal{B}(n_i, p_i),\ p_i = \text{logit}^{-1}(\boldsymbol{x}^\top \gamma)$

Régression & Données Groupées

Les deux modèles sont équivalents, $\widehat{\gamma} = \widehat{\beta}$

```
> regg = glm(cbind(Y,N-Y) ~ Sex+Embarked+Pclass, data
    = df, family=binomial)
> regg

Coefficients:
(Intercept)       Sexmale     EmbarkedQ     EmbarkedS
     2.6394       -2.6081       -0.1454       -0.5954
    Pclass2       Pclass3
    -0.6691       -1.8385
> reg

Coefficients:
(Intercept)       Sexmale     EmbarkedQ     EmbarkedS
     2.6394       -2.6081       -0.1454       -0.5954
    Pclass2       Pclass3
    -0.6691       -1.8385
```