Modèles Linéaires Appliqués

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Automne 2Q20

OLS #20 (transformations)

Loi de Kleiber

Loi de Kleiber (1947) "si q est le métabolisme et m la masse corporelle de l'animal, $q^4 \sim m^3$"

$$q = \alpha m^{3/4} \text{ ou } q^{4/3} = \beta m$$

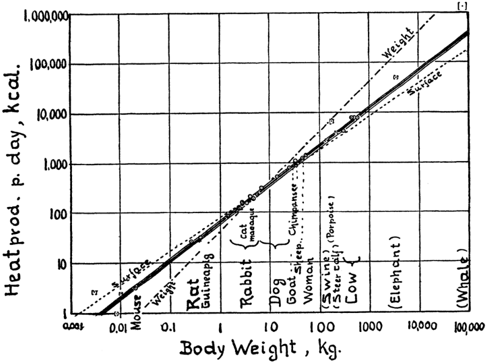


Fig. 1. Log. metabol. rate/log body weight

GROUP	ANIMAL	AUTHOR	BODY WT.	METABOL. RATE PER DAY
a. Data used for calculation of regression line				
			kg.	kcal
1	Mouse	Benedict and Lee, 1936	0.021	3.6
2	Rat 230–300 days old	Kleiber, unpubl.	0.282	28.1
3	Guinea pig	Benedict, 1938	0.410	35.1
4	Rabbit	Tomme and Loria, 1936	2.98	167
5	Rabbit	R. Lee, 1939	1.52	83
6	Rabbit		2.46	119
7	Rabbit		3.57	164
8	Rabbit		4.33	191
9	Rabbit		5.33	233
10	Cat	Benedict, 1938	3.00	152
11	Macaque	Benedict, 1938	4.2	207
12	Dog	Galvão, 1942	6.6	288
13	Dog		14.1	534
14	Dog		24.8	875
15	Dog	de Beer and Hjort, 1938	23.6	872
16	Goat	Benedict, 1938	36.0	800
17	Chimpanzee	Bruhn and Benedict, 1936	38.0	1090
18	Sheep ♀	Lines and Peirce, 1931	46.4	1254
19	Sheep ♂		46.8	1330
20	Woman	McKittrick, 1936	57.2	1368
21	Woman	Lewis, Iliff and Duval, 1943	54.8	1224
22	Woman	McCrery, Wolf and Bavousett, 1940	57.9	1320
23	Cow	Benedict and Ritzman, 1935	300	4221
24	Cow	Kleiber, Regan and Mead, 1945	435	8166
25	Beef heifers	Kleiber, Goss and Guilbert, 1936	482	7754
26	Cow	Benedict and Ritzman, 1935	600	7877
b. Data not used for calculation because conditions not comparable				
	Shrew	Morrison and Pearson, 1946	0.0035	2.9
	Swiss mice	U. S. Navy Res. Unit and Kleiber, 1944	0.0105	3.7
	Dwarf mouse	Benedict, 1938	0.008	1
	Rats (giant)	Benedict, 1938	0.400	33.2
	Rats (growth hormone)	Kleiber and Cole, 1939	0.391	28.6
	Swine	Breirem, 1936	150	2678
	Steer calves	Mitchell et al., 1940	200	3817
	Elephant	Benedict, 1938	3672	49000
	Porpoise	Irving et al., 1941	170	6768
	Whale	Irving, 1941	70000	1.2×10^6

Loi de Kleiber

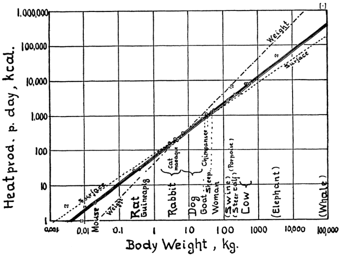
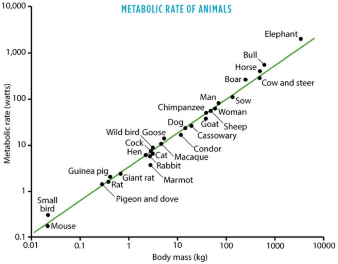
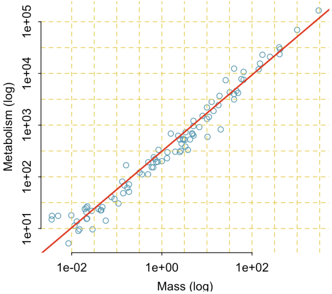
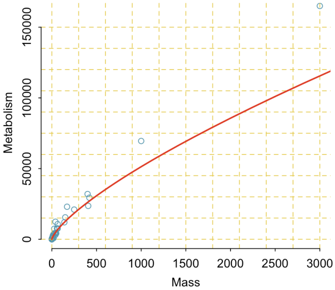


Fig. 1. Log. metabol. rate/log body weight







Loi de Kleiber

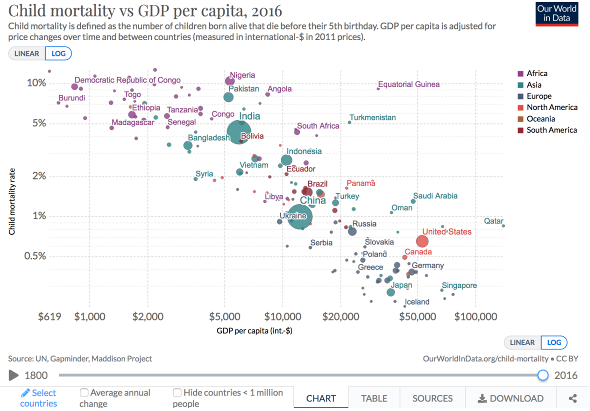
$$q = \alpha m^{3/4} \text{ ou } \underbrace{\log(q)}_{y} = \underbrace{\log\alpha}_{\beta_0} + \underbrace{3/4}_{\beta_1}\underbrace{\log(m)}_{x}$$

```
> library(Sleuth3)
> reg=lm(log(Metab)~log(Mass),data=ex0826)
> summary(reg)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  5.63833    0.04709  119.73   <2e-16 ***
log(Mass)    0.73874    0.01462   50.53   <2e-16 ***

> confint(reg)
                2.5 %    97.5 %
(Intercept) 5.5448128 5.7318485
log(Mass)   0.7097121 0.7677752

> library(car)
> boxTidwell(Metab~Mass,data=ex0826)
 MLE of lambda Score Statistic (z)  Pr(>|z|)
       0.82296             -12.536 < 2.2e-16 ***
```

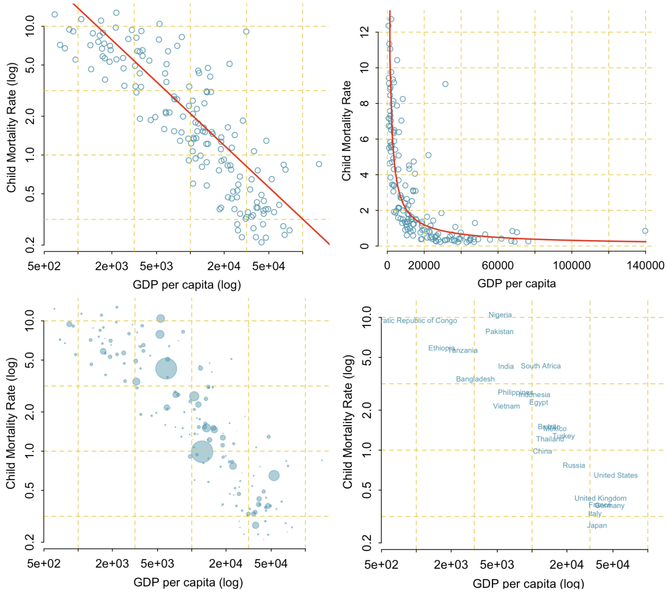
Mortalité Infantile & PIB par Tête

Loi “si g est le PIB par tête et m le taux de mortalité infantile, $m^5 \sim 1/g^4$ ”, cf https://ourworldindata.org/

$$m = \alpha g^{-4/5} \text{ ou } \log(m) = \beta - \frac{4}{5}\log(g)$$



Mortalité Infantile & PIB par Tête



Mortalité Infantile & PIB par Tête

$$m = \alpha g^{-4/5} \text{ ou } \underbrace{\log(m)}_{y} = \underbrace{\log \alpha}_{\beta_0} - \underbrace{4/5}_{\beta_1} \underbrace{\log(g)}_{x}$$

```
> base = read.csv("child-mortality-gdp-per-capita.csv")
> base16 = base[base$Year==2016,]
> base16 = base16[!(is.na(base16[,4])|is.na(base16[,5])),]
> base = base16[,c(5,4)]
> names(base) = c("gpd","mortality")
> reg = lm(log(mortality)~log(gpd),data=base)
> summary(reg)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  8.09330    0.37874   21.37   <2e-16 ***
log(gpd)    -0.81918    0.04068  -20.14   <2e-16 ***

> confint(reg)
                 2.5 %     97.5 %
(Intercept)  7.3453212  8.8412741
log(gpd)    -0.8995162 -0.7388363
```

Sur les Transformations Puissance

```
> reg = lm(log(dist)~log(speed),data=cars)
> summary(reg)

Call:
lm(formula = log(dist) ~ log(speed), data = cars)

Residuals:
     Min       1Q   Median       3Q      Max
-1.00215 -0.24578 -0.02898  0.20717  0.88289

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.7297     0.3758  -1.941   0.0581 .
log(speed)    1.6024     0.1395  11.484 2.26e-15 ***
```

$$\underbrace{\log(d)}_{y} = \underbrace{\log\alpha}_{\beta_0} - \underbrace{8/5}_{\beta_1}\underbrace{\log(s)}_{x} \text{ ou } d = \alpha s^{8/5} \text{ ou } \underbrace{d^{5/8}}_{h(d)} = \gamma s$$