Série 2 STT5100

18 septembre 2018

Exerice 1

Observations $(x_i, Y_i), i = 1, ..., n$ following the model $Y_i = \alpha + x_i\beta + \epsilon_i$ where $E\epsilon_i = 0$, $VAR\ \epsilon_i = \sigma^2$, $Cov\ (\epsilon_i, \epsilon_j) = 0$. Find the best linear unbiased estimator of α.

Exerice 2

Observations $(x_i, Y_i), i = 1, ..., n$ following the model $Y_i = \alpha + x_i\beta + \epsilon_i$ where $x_1, ..., x_n$ are fixed constants and $\epsilon_1, ..., \epsilon_n$ iid n(0,σ^2). The model is then reparameterised as

$$Y_i' = \alpha' + (x_i - \bar{x})\beta' + \epsilon_i$$

Let $\hat{\alpha}$ and $\hat{\beta}$ denote the MLEs of α and β, respectively, and $\hat{\alpha}'$ and $\hat{\beta}'$ denote the MLEs of α' and β'.

- Show that $\hat{\beta} = \hat{\beta}'$
- Show that $\hat{\alpha} \neq \hat{\alpha}^{\cdot}$ In fact, show that $\hat{\alpha} = \bar{Y}$. Find the distribution of $\hat{\alpha}'$
- Show that $\hat{\alpha}'$ and $\hat{\beta}'$ are uncorrelated and, hence, independante under normality.

Exerice 3

Observations $(x_i, Y_i), i = 1, ..., n$ following the model $Y_i = x_i^2\theta + \epsilon_i$ where $x_1, ..., x_n$ are fixed constants and $\epsilon_1, ..., \epsilon_n$ iid n(0,σ^2).

- Find the least squares estimator of θ
- Find the MLE of θ
- Find the best unbiased estimator of θ