Analysis of The Performance of Multi-layer Insulated Clothing

Luo YiWen

May 19, 2024

COLORADO SCHOOL OF MINES

Outline

Introduction

Background Imformation

Firefighters working at fire

- Need
- Application
- Importance

Model Establishment

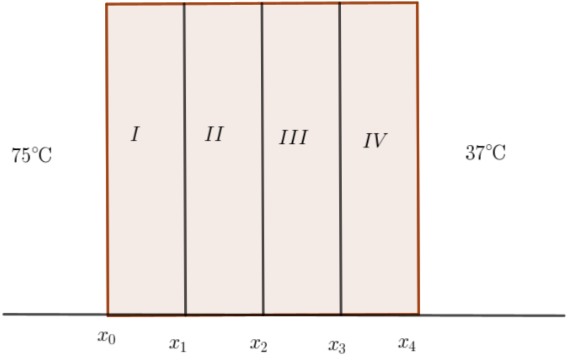
Problem Abstraction

How to simplify this model?

How to simplify this model?

Turning problems into one-dimension.

Model Establishment



Multi-layer insulating materials

- Heat diffusion
- Heat conduction
- Heat convection

Model Establishment

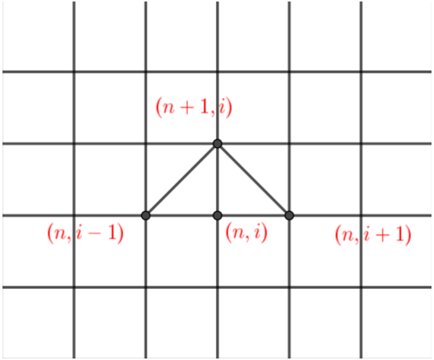
- For control equation,
- $$\frac{\partial T_i}{\partial t} = k_i \frac{\partial^2 T_i}{\partial x^2}, k_i = \frac{\lambda_i}{c_i \rho_i}, i = 1, 2, 3, 4, \tag{1}$$
- For boundary condition
- $$-\lambda_1 \frac{\partial T_1}{\partial x}\bigg|_{x=x_0} = h_e \left[T_{en} - T(x_0, t) \right], \tag{2}$$
- $$-\lambda_4 \frac{\partial T_4}{\partial x}\bigg|_{x=x_4} = h_s \left[-T_{sk} + T(x_4, t) \right], \tag{3}$$
- $$T_i\bigg|_{x=x_i} = T_{i+1}\bigg|_{x=x_i}, i = 1, 2, 3, \tag{4}$$

Model Establishment

- $$\lambda_i \frac{\partial T_i}{\partial x}\bigg|_{x=x_i} = \lambda_{i+1} \frac{\partial T_i + 1}{\partial x}\bigg|_{x=x_i}, i = 1, 2, 3, \tag{5}$$
- For initial condition,
$$T(x, 0) = T_{sk} = T_2 = T_3 = T_4 \tag{6}$$

Numerical Solution

- Numerical Method



Finite Difference Method

Discrete Differential Equation

Discrete Differential Equation

- denote $\mu_j = \frac{k_j \Delta t}{(\Delta x_j)^2}$, then
- $$T_i^{n+1} = \mu_j T_{i+1}^n - (2\mu_j - 1) T_i^n + \mu_j T_{i-1}^n \tag{7}$$
- $$(1 + \mu_e) T_1^n - T_0^n = \mu_e T_{en}, \tag{8}$$
- $$(1 + \mu_s) T_{4M}^n - T_{4M-1}^n = \mu_s T_{en}, \tag{9}$$
- where $\mu_e = \frac{h_e \Delta x}{\lambda_1}$, $\mu_s = \frac{h_s \Delta x}{\lambda_4}$.
- $$T(x, 0) = T_{sk} = T_2 = T_3 = T_4 \tag{10}$$

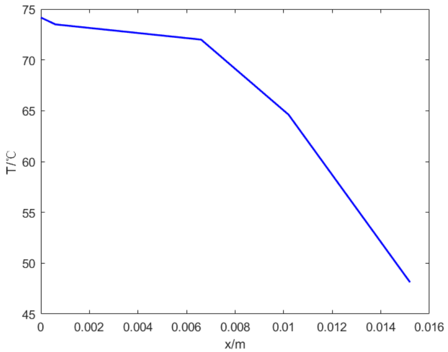
Discrete Differential Equation

- $$\lambda_j \frac{T^n_{M_j} - T^n_{M_{j-1}}}{\Delta x_j} = \lambda_{j+1} \frac{T^n_{M_{j+1}} - T^n_{M_j}}{\Delta x_{j+1}}, \tag{11}$$
- let $\nu_j = \frac{\lambda_j}{\Delta x_j}$, it can be simplified to

$$\mu_j T^n_{M_{j-1}} + (\mu_j + \mu_{j+1}) T^n_{M_j} = \mu_{j+1} T^n_{M_{j+1}}, \tag{12}$$

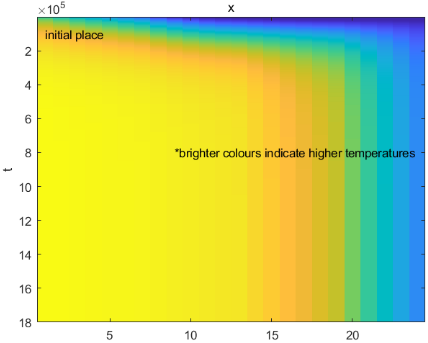
- also we have $T^n_{M_j} = T^n_{M_{j-1}} = T^n_{M_{j+1}}$ in M-th contact surface.
- The initial condition can also write in this form
- $$T^0_i = T_{sk}. \tag{13}$$

Numerical Solution



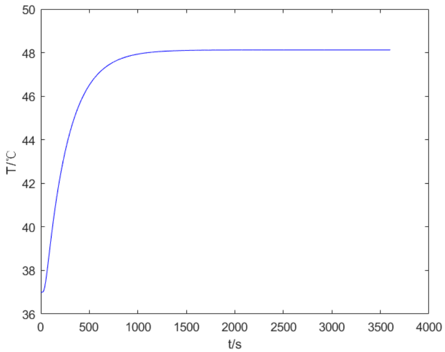
This graph shows temperature distribution, while vertical axis shows temperature in degrees Celsius, horizontal axis shows position in centimetres

Numerical Solution



x-t Grid, the horizontal coordinate shows the x-value and the vertical coordinate shows the t-value.

Numerical Solution

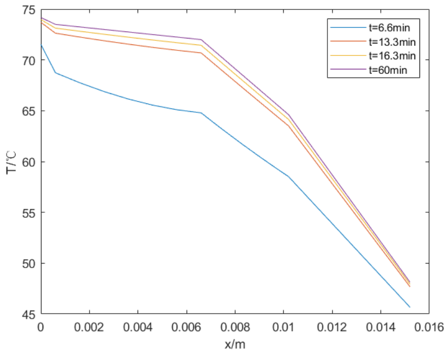


Skin temperature icreases over time, while vertical axis shows temperature in degrees Celsius, horizontal axis shows time in seconds

Stability Analysis

- Von Newmann stability analysis requires
- $$r = \frac{k\Delta t}{(\Delta x)^2} < \frac{1}{2}, \tag{14}$$
- We obtain $r_1 = 0.0397$, $r_2 = 4.0879 \times 10^{-4}$, $r_3 = 0.0020$, $r_4 = 0.0472$, $r_i < \frac{1}{2}$, satisfying Von Newmann stability analysis.

Stability Analysis

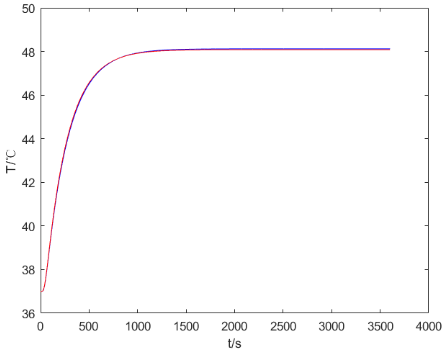


Blue line indicates temperature distribution at 6.6 minutes, red line indicates temperature distribution at 13.3 minutes, blue line indicates temperature distribution at 16.3 minutes,blue line indicates temperature distribution at 60 minutes.

Error Estimate

Error Estimate

- $$RSS = \frac{(U_n - T_{4M}^n)^2}{3601} = 0.002 \tag{15}$$



Discussion

Strength

- Stable and convergent.
- Matches given data.
- realistic

Limitation

- Less efficient.
- Less consider about heat radiation.

Improvement

- More experiences will provide more data.
- Implicit difference format will lead to more stable and accurate solution.

Thank You

Questions?

COLORADO SCHOOL OF MINES