Analysis of The Performance of Multi-layer Insulated Clothing

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1. Background Information

Some special occupations, such as firefighters, need to operate in high temperatures. In order to meet this requirement, high-temperature work clothes are made of a variety of insulating materials to prevent workers from being burned. In this report, we wish to investigate the thermal insulation performance of a particular type of high-temperature operating clothing. Specifically, We would like to know body temperature of a person, wearing an insulated clothing after a specific time at a certain temperature, given information of the insulating material and the environment temperature.

By the model we make, we can predict the insulation performance of an insulated clothing based on the given parameters. This will effectively reduce the number of experiments in the development of new products, which will help improve efficiency and save budget.

The clothing under this study consists of three layers of different fabric materials, labelled as layers I, II and III, where layer I is in contact with the external environment. There is a gap between layer III and the skin, which is labelled as layer IV. We can set the values of each parameter of each material, including the thickness, density, specific heat, and heat conductivity of each layer, also and we have the coefficients of convective heat transfer between the outermost material and the external environment, the coefficients of convective heat transfer between the innermost gap and the skin.

2. Model Establishment

2.1. Assumption

1.We abstracting the whole heat transfer process as a one-dimensional heat conduction problem.

2.Parameters of each material do not change with increasing temperature.

3.Neglect the heat radiation to the outermost layer of clothing, consider only heat convection as a form of heat transfer.

2.2. Model Analysis

Heat transfer from clothing involves different heat transfer processes: heat diffusion, heat conduction and heat convection. Heat transfer from the surroundings to the clothing, and heat transfer from layer III to the void layer IV, both of these processes involve heat convection. The process of heat transfer within a particular material of clothing is heat diffusion. Heat transfer between different materials is heat conduction.

2.2.1. Heat Convection

For thermal convection, we apply Newton's cooling law. Newton's cooling law states that

$$-\lambda T' = h(T_{higer} - T_{lower}), \tag{2.1}$$

where T denotes temperature, h denotes heat convection coefficient, t denotes time, T_{higher} and T_{lower} denotes higher temperature and lower temperature.

Express the temperature of layer i as T_i. x_i coordinate contact surface sequentially as shown below.

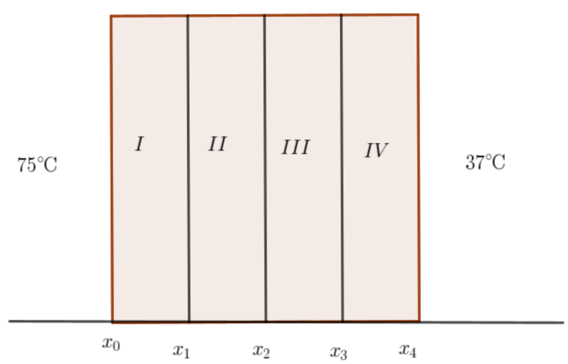

Figure 2.1: Cross-section of clothing

Then the process of heat convection between layer I and the external environment can be expressed as

$$-\lambda_1 \frac{\partial T_1}{\partial x}\bigg|_{x=x_0} = h_e \left[T_{en} - T(x_0, t)\right], \tag{2.2}$$

also, the heat convection equation between layer IV and the skin is

$$-\lambda_4 \frac{\partial T_4}{\partial x}\bigg|_{x=x_4} = h_s \left[-T_{sk} + T(x_4, t)\right], \tag{2.3}$$

λi is the thermal conductivity of the layer i material, h_e and h_s are the thermal convection coefficients between the environment and clothing ,clothing and skin, respectively. T_{en} and T_{sk} denotes temperature of environment and skin. i denotes i-th layer.

2.2.2. Heat Diffusion

For heat diffusion inside the clothing, we use the heat diffusion equation

$$\frac{\partial T_i}{\partial t} = k_i \frac{\partial^2 T_i}{\partial x^2}, k_i = \frac{\lambda_i}{c_i \rho_i}, i = 1, 2, 3, 4 \tag{2.4}$$

k_i is thermal diffusion coefficient, λ_i is thermal convection coefficients. c_i denotes specific heat capacity and ρ_i denotes density.

2.2.3. Heat Conduction

For the heat transfer that occurs between the contact surfaces of two materials,

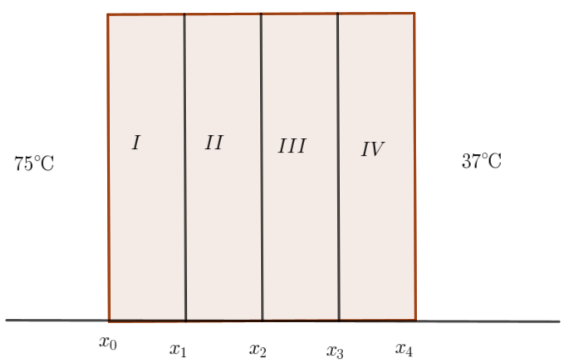

Figure 2.2: Cross-section of clothing

it satisfies

$$T_1\Big|_{x=x_1} = T_2\Big|_{x=x_1}, \tag{2.5}$$

$$T_2\Big|_{x=x_2} = T_3\Big|_{x=x_2}, \tag{2.6}$$

$$T_3\Big|_{x=x_3} = T_4\Big|_{x=x_3}, \tag{2.7}$$

and

$$\lambda_1 \frac{\partial T_1}{\partial x}\Big|_{x=x_1} = \lambda_2 \frac{\partial T_2}{\partial x}\Big|_{x=x_1} \tag{2.8}$$

$$\lambda_2 \frac{\partial T_2}{\partial x}\Big|_{x=x_2} = \lambda_3 \frac{\partial T_3}{\partial x}\Big|_{x=x_2} \tag{2.9}$$

$$\lambda_3 \frac{\partial T_3}{\partial x}\Big|_{x=x_3} = \lambda_4 \frac{\partial T_4}{\partial x}\Big|_{x=x_3} \tag{2.10}$$

we can simplify the above equation as

$$T_i\Big|_{x=x_i} = T_{i+1}\Big|_{x=x_i}, i = 1, 2, 3, \tag{2.11}$$

$$\lambda_i \frac{\partial T_i}{\partial x}\Big|_{x=x_i} = \lambda_{i+1} \frac{\partial T_{i+1}}{\partial x}\Big|_{x=x_i}, i = 1, 2, 3. \tag{2.12}$$

2.2.4. Initial Condition

We also have initial conditions for this problem,

$$T(x, 0) = T_{sk} = T_2 = T_3 = T_4 \tag{2.13}$$

2.3. Model Establishment

From above, we have governing equation

$$\frac{\partial T_i}{\partial t} = k_i \frac{\partial^2 T_i}{\partial x^2}, k_i = \frac{\lambda_i}{c_i \rho_i}, i = 1, 2, 3, 4, \tag{2.14}$$

for boundary conditions, we have

$$-\lambda_1 \frac{\partial T_1}{\partial x}\Big|_{x=x_0} = h_e \left[T_{en} - T(x_0, t)\right], \tag{2.15}$$

$$-\lambda_4 \frac{\partial T_4}{\partial x}\Big|_{x=x_4} = h_s \left[-T_{sk} + T(x_4, t)\right], \tag{2.16}$$

$$T_i\Big|_{x=x_i} = T_{i+1}\Big|_{x=x_i}, i = 1, 2, 3, \tag{2.17}$$

$$\lambda_i \frac{\partial T_i}{\partial x}\Big|_{x=x_i} = \lambda_{i+1} \frac{\partial T_i + 1}{\partial x}\Big|_{x=x_i}, i = 1, 2, 3, \tag{2.18}$$

for initial conditions, we have

$$T(x, 0) = T_{sk} = T_2 = T_3 = T_4 \tag{2.19}$$

3. Problem Solving

To solve this partial differentiable equation, we use numerical method. In this report, we choose finite difference method. The main principle of finite difference method is to perform a discrete difference approximation of the differential terms, thus transforming the differential equation into a system of algebraic equations to be solved.

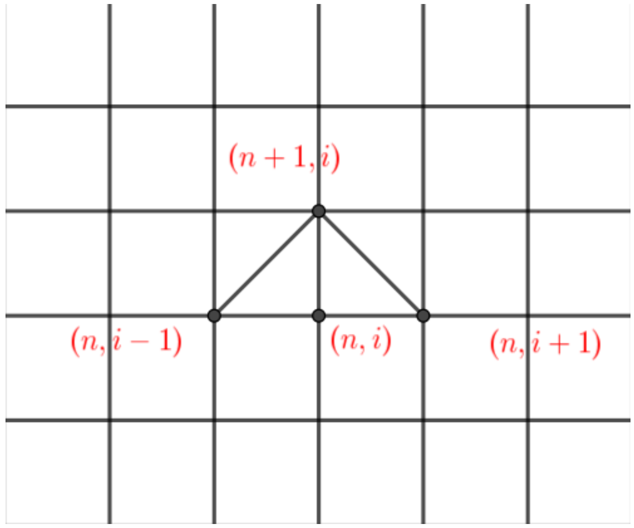

Figure 3.1: Discrete Grid

Firstly, we need to discrete the original equations and boundary conditions. We denote $T(x_i, t_n)$ as T_i^n, then

$$\frac{\partial T}{\partial t} = \frac{T_i^{n+1} - T_i^n}{\Delta t}, \tag{3.1}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{t_{i+1}}^n - 2T_i^n + T_{i-1}^n}{(\Delta x)^2}, \tag{3.2}$$

thus the the discrete control equation is

$$T_i^{n+1} = \frac{k_j \Delta t}{(\Delta x_j)^2}(T_{i+1}^n - 2T_i^n + T_{i-1}^n) + T_i^n, \tag{3.3}$$

denote $\mu_j = \frac{k_j \Delta t}{(\Delta x_j)^2}$, then

$$T_i^{n+1} = \mu_j T_{i+1}^n - (2\mu_j - 1)T_i^n + \mu_j T_{i-1}^n \tag{3.4}$$

For the boundary conditions of two outer sides, they becomes

$$-\lambda_1 \frac{T_1^n - T_0^n}{\Delta x_1} = h_e \left[T_{en} - T(x_0, t)\right], \tag{3.5}$$

$$-\lambda_4 \frac{T_{4M}^n - T_{4M-1}^n}{\Delta x_1} = h_s \left[T_{sk} - T(x_4, t)\right], \tag{3.6}$$

4M denotes the end value of the x-coordinate in IV layer. They can be simplified to

$$(1 + \mu_e)T_1^n - T_0^n = \mu_e T_{en}, \tag{3.7}$$

$$(1+\mu_s)T_{4M}^n - T_{4M-1}^n = \mu_s T_{en}, \tag{3.8}$$

where $\mu_e = \frac{h_e \Delta x}{\lambda_1}$, $\mu_s = \frac{h_s \Delta x}{\lambda_4}$.

For the boundary conditions of three contact surfaces, it becomes

$$\lambda_j \frac{T_{M_j}^n - T_{M_{j-1}}^n}{\Delta x_j} = \lambda_{j+1} \frac{T_{M_{j+1}}^n - T_{M_j}^n}{\Delta x_{j+1}}, \tag{3.9}$$

let $\nu_j = \frac{\lambda_j}{\Delta x_j}$, it can be simplified to

$$\mu_j T_{M_{j-1}}^n + (\mu_j + \mu_{j+1}) T_{M_j}^n = \mu_{j+1} T_{M_{j+1}}^n, \tag{3.10}$$

also we have $T_{M_j}^n = T_{M_{j-1}}^n = T_{M_{j+1}}^n$ in M-th contact surface. The initial condition can also write in this form

$$T_i^0 = T_{sk}. \tag{3.11}$$

Now the original partial differential equation has now been transformed into a system of equations on T_i^{n+1}. We can set values on above parameters and use MATLAB to solve this system of equation. Given these parameters, we would like to know the temperature distribution of this clothing after a person wears it for 60 min in a 75° environment.

Below are the values we set, this data is obtained from CUMCM(China Undergraduate Mathematical Contest in Modeling).

Layer	density(kg/m^3)	specific heat capacity(J/kg*°C)	heat conductivity(W/m*°C)	thicknesses(mm)
I	300	1377	0.082	0.6
II	862	2100	0.37	6
III	74.2	1726	0.045	3.6
IV	1.18	1005	0.028	5

$T_{sk} = 37, T_{en} = 100; h_e = 111, h_s = 8.3$. Here is the code to solve the system of equations.
$Ten = 100; Tsk = 37;$

$h_e = 111; h_s = 8.3;$
$lam = [0.082, 0.37, 0.045, 0.028];$
$c = [1377, 2100, 1726, 1005];$
$den = [300, 862, 74.2, 1.18];$
$dt = 0.002;$
$x = [0.0006, 0.006, 0.0036, 0.005];$
$dx = [0.0001, 0.001, 0.0006, 0.001];$
$I1 = int8(x(1)/dx(1)) + 1;$
$I2 = I1 + x(2)/dx(2);$
$I3 = I2 + x(3)/dx(3);$
$I4 = I3 + x(4)/dx(4);$
$T = zeros(3600/dt, I4);$

$T(1,:) = 37;$

for n=1:3600/dt-1

$u = (h_e * (Ten - T(n,1)) - lam(1) * (T(n,1) - T(n,2))/dx(1)) * dt/(0.5 * dx(1) * den(1) * c(1)) + T(n,1);$

$T(n+1,1) = u;$

for i=2:I1

if i>=2 && i<=I1-1

$= lam(1) * (T(n,i+1) - 2 * T(n,i) + T(n,i-1))/dx(1) * dt/(dx(1) * den(1) * c(1)) + T(n,i);$

$T(n+1,i) = u;$

elseif i==I1

$u = (lam(2) * (T(n,i+1) - T(n,i))/dx(2) + lam(1) * (T(n,i-1) - T(n,i))/dx(1)) * dt/(0.5 * (dx(1) * den(1) * c(1) + dx(2) * den(2) * c(2))) + T(n,i);$

$T(n+1,i) = u;$

end

end

for i=I1+1:I2

if i>=I1+1 && i<=I2-1

$u = lam(2) * (T(n,i+1) - 2 * T(n,i) + T(n,i-1))/dx(2) * dt/(dx(2) * den(2) * c(2)) + T(n,i);$

$T(n+1,i) = u;$

elseif i==I2

$u = (lam(3) * (T(n,i+1) - T(n,i))/dx(3) + lam(2) * (T(n,i-1) - T(n,i))/dx(2)) * dt/(0.5 * (dx(2) * den(2) * c(2) + dx(3) * den(3) * c(3))) + T(n,i);$

$T(n+1,i) = u;$

end

end

for i=I2+1:I3

if i>=I2+1 && i<=I3-1

$u = lam(3) * (T(n,i+1) - 2 * T(n,i) + T(n,i-1))/dx(3) * dt/(dx(3) * den(3) * c(3)) + T(n,i);$

$T(n+1,i) = u;$

elseif i==I3

$u = (lam(4) * (T(n,i+1) - T(n,i))/dx(4) + lam(3) * (T(n,i-1) - T(n,i))/dx(3)) * dt/(0.5 * (dx(3) * den(3) * c(3) + dx(4) * den(4) * c(4))) + T(n,i);$

$T(n+1,i) = u;$

end

end

for i=I3+1:I4

if i>=I3+1 && i<=I4-1

$u = lam(4) * (T(n,i+1) - 2 * T(n,i) + T(n,i-1))/dx(4) * dt/(dx(4) * den(4) * c(4)) + T(n,i); T(n+1,i) = u;$

elseif i==I4

$u = (lam(4) * (T(n,I4-1) - T(n,I4))/dx(4) - h_s * (T(n,I4) - Tsk)) * dt/(0.5 * dx(4) * den(4) * c(4)) + T(n,I4); T(n+1,i) = u;$

end
end
end
$x = [0.0006, 0.006, 0.0036, 0.005];$
$dx = [0.0001, 0.001, 0.0006, 0.001];$
$x1 = dx(1) : dx(1) : x(1);$
$x2 = x(1) + dx(2) : dx(2) : x(1) + x(2);$
$x3 = x(1) + x(2) + dx(3) : dx(3) : x(1) + x(2) + x(3);$ $x4 = x(1) + x(2) + x(3) + dx(4) : dx(4) : x(1) + x(2) + x(3) + x(4);$
$x = [0, x1, x2, x3, x4];$
$T = T(end, :);$
$plot(x, T,' LineWidth', 1.5);$
imagesc(T)

Finally, we obtain the tmeperature distribution of clothing after a person wearing it for 60 minutes in 75° envrionment. We can draw it in a graph. From the graph we can see

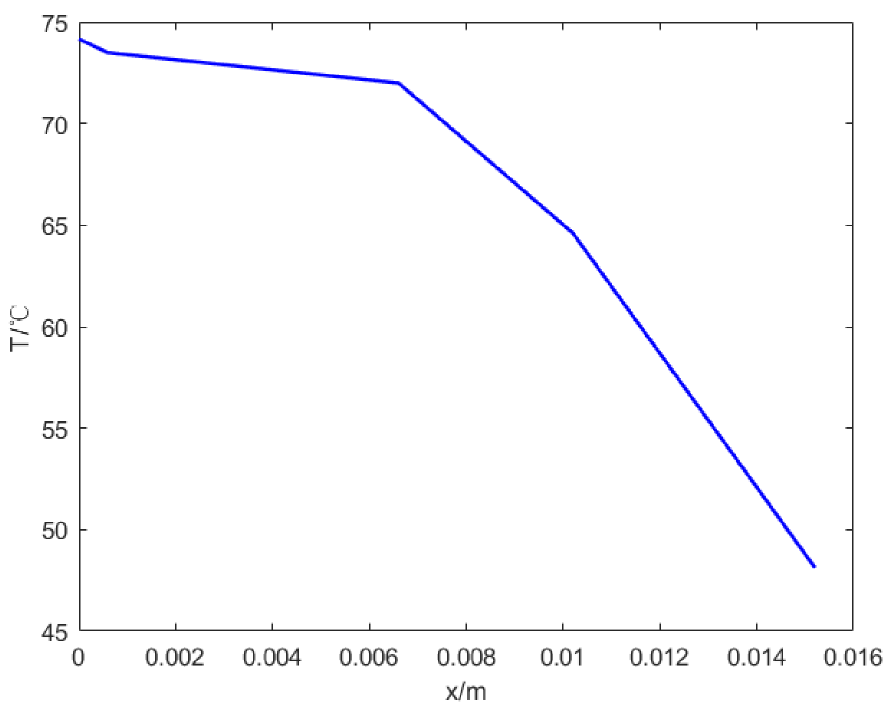

Figure 3.2: This graph shows temperature distribution, while vertical axis shows temperature in degrees Celsius, horizontal axis shows position in centimetres

this clothing is quite well insulated, The temperature gradient on the outside of the clothing changed less, while there was a significant drop in temperature on the inside. Even after an hour in a 75° environment, the temperature inside the clothing is below 50°, which is exactly 48.12°.

Also, we can visualise the x-t grid. This helps us to better observe the temperature change of clothes in the two dimensions of time and location.

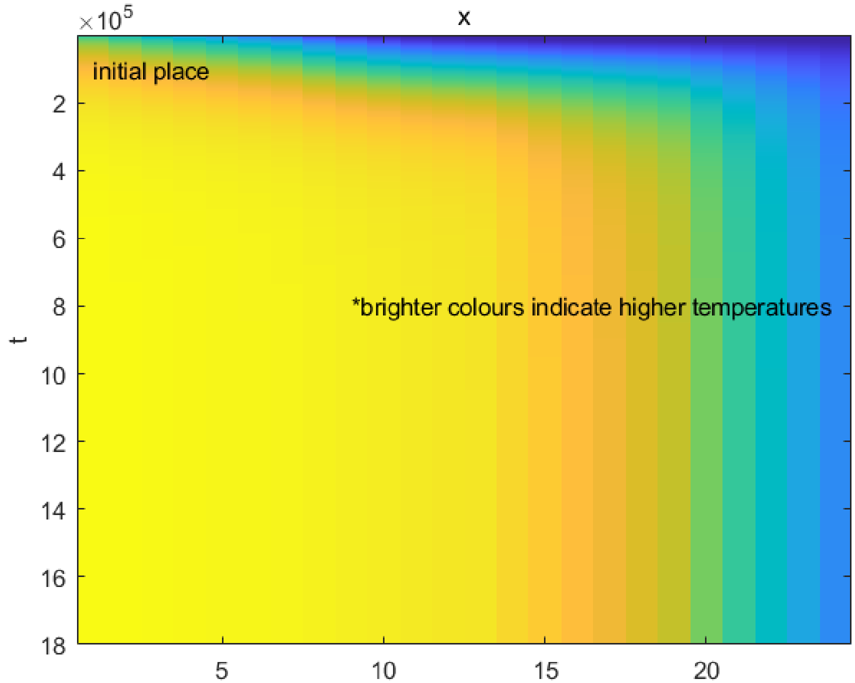

Figure 3.3: x-t Grid, the horizontal coordinate shows the x-value and the vertical coordinate shows the t-value.

Brighter colours indicate higher temperatures. The way to observe this picture is to start from the top right corner, from left to right we can see the temperature distribution of the clothing from the outside to the inside, and from the top to the bottom we can see the temperature distribution advancing with time. Observing from picture we can see as time passing, the overall temperature of the clothing rises from the outside to the inside, then stabilises.

In order to more accurately observe the trend of temperature over time, we plotted the skin temperature, that is, the contact surface of the IV layer with the skin, as it increased over time. From the graph, we can observe that the trend of increasing temperature gradually tends to become more and more moderate, which means that our solution is convergent and realistic.

The code is shown below.

```
T1=zeros(1,3601)
for i=1:500:1800000
T1(1,1+(i-1)*0.002)=T(i,end);
```

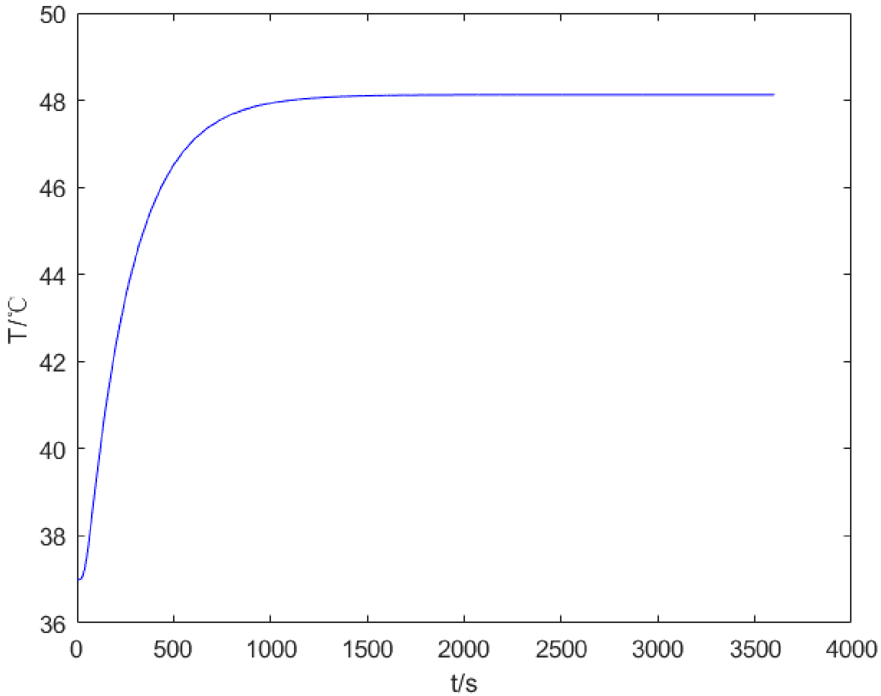

Figure 3.4: Skin temperature icreases over time, while vertical axis shows temperature in degrees Celsius, horizontal axis shows time in seconds

```
end
T1(1,3601)=48.1233
x=1:3601
plot(x,T1,'b');
```

4. Stability Analysis

When using numerical methods, we want to make sure that the error incurred at a given time step does not cause the error to magnify as the calculation continues. If this is satisfied, we call the calculation method stable. Here we use Von Neumann stability analysis because the situation we are analysing satisfies the conditions of the analytical method: the model of the finite difference method is linear, the method uses no more than two time steps, the boundary conditions of the PDE problem are constant coefficients rather than unknown, and there are only two independent variables.

In generally, Von Newmann stability analysis requires

$$r = \frac{k\Delta t}{(\Delta x)^2} < \frac{1}{2}, \tag{4.1}$$

in our model,

$$k_i = \frac{\lambda_i}{c_i \rho_i}, i = 1, 2, 3, 4 \tag{4.2}$$

Here is the code used to calculate the value of r_i

$lam = [0.082, 0.37, 0.045, 0.028];$
$c = [1377, 2100, 1726, 1005];$
$den = [300, 862, 74.2, 1.18];$
$dt = 0.002;$
$x = [0.0006, 0.006, 0.0036, 0.005];$
$dx = [0.0001, 0.001, 0.0006, 0.001];$
for i=1:4
$r = lam(i) * dt/(c(i) * den(i) * dx(i)^2)$
end

We obtain $r_1 = 0.0397, r_2 = 4.0879 \times 10^{-4}, r_3 = 0.0020, r_4 = 0.0472,$ $r_i < \frac{1}{2}$, satisfying Von Newmann stability analysis.

Also, we can draw temperature distribution graph at different moments. From these graphs we can see the temperature distribution gradually converges as time progresses.

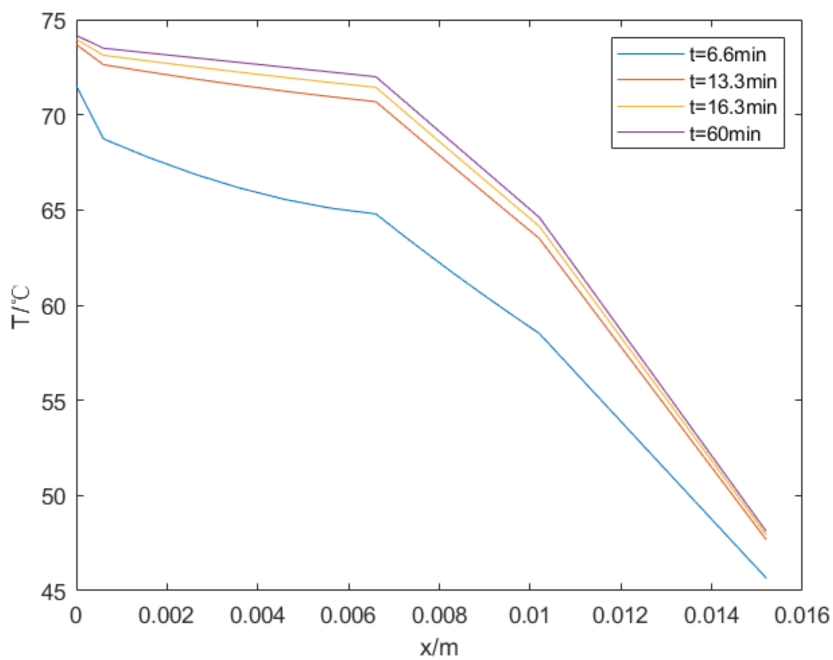

Figure 4.1: Blue line indicates temperature distribution at 6.6 minutes, red line indicates temperature distribution at 13.3 minutes, blue line indicates temperature distribution at 16.3 minutes,blue line indicates temperature distribution at 60 minutes.

Here is the code to display graph.

```
F1 = zeros(180001,I4);
F1(1,:) = T(1,:);
F1(180001,:) = T(1800000,:);
for i = 1:179999
F1(i+1,:) = T(i*10 + 1,:);
end
x=[0,x1,x2,x3,x4];
y1 = F1(20000,:);
y2 = F1(40000,:);
y3 = F1(50000,:);
y4 = F1(180000,:);
plot(x,y1);
hold on
plot(x,y2);
hold on
plot(x,y3);
hold on
plot(x,y4);
```

5. Error Estimate

CUMCM provided measured temperature on the outside of the skin of a dummy wearing an insulated suit. We can use it to compare with our solution.

Denote U_n as the given data in n-th second, we use

$$RSS = \frac{(U_n - T_{4M}^n)^2}{3601} \tag{5.1}$$

to calculate sum of squares of the residuals. By MATLAB, we obtain RSS=0.002. This is a very small error, meaning that the model has a good fit regarding this data. Here is the code to obtain RSS.

```
T1=zeros(1,3601)
for i=1:500:1800000
T1(1,1+(i-1)*0.002)=T(i,end);
end
T1(1,3601)=48.1233
T1t=T1'
T_ex = xlsread('D:\MATH\CUMCMAppendix.xlsx','B3:B3603');
T2 = (T_ex - T1t);
T2t=T2';
RRS=T2t*T2/3601;
```

Also, we could plot both given data and our solution into one picture. We can see the error more visually in the following figure.

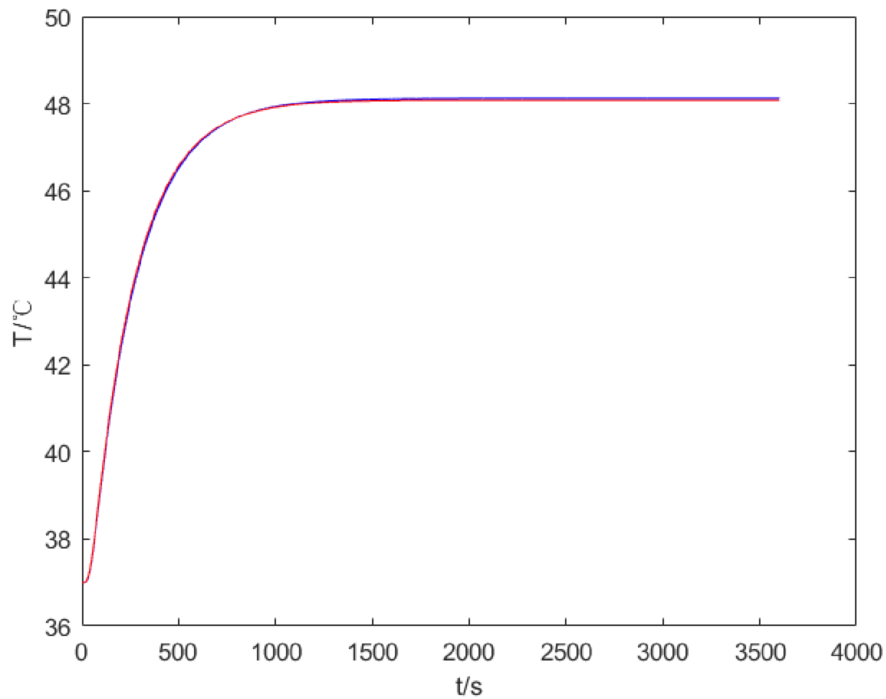

Figure 5.1: Blue line indicates the solution, red line indicates given data.

The code is shown below.
x=1:3601
plot(x,T1,'b');
hold on
$plot(x, T_{ex},' r');$

6. Discussion

6.1. Strength

1.The numerical solution is tested to be stable and convergent;
2.The numerical solution matches well with the given data;
3.The model is realistic, has good predictability of reality, and can obtain temperature predictions for any location at any time.

6.2. Limitation

1.The model abstracts the three-dimensional problem into a one-dimensional problem. For the problem of measuring body surface temperature, this model can only be measured point by point, which is less efficient.

2.Does not take into account the possible perspiration and heat dissipation of the real human body.

3.Only heat convection between the outside of the clothing and the environment was analysed in the model, while the presence of heat radiation is not taken into account.

6.3. Improvement

It is possible to experiment with the heat dissipation of a real human body and combine it with the original heat exchange to make a more accurate prediction of the real situation.

Also, this model uses explicit difference format for numerical solution, in order to upgrade the model, implicit difference format can be used to make the solution more stable and accurate.