ANALYSIS OF THE PERFORMANCE OF MULTI-LAYER INSULATED CLOTHING

Luo YiWen

email address hc11205@um.edu.mo

Introduction

Some special occupations, such as firefighters, need to operate in high temperatures. In order to meet this requirement, high-temperature work clothes are made of a variety of insulating materials to prevent workers from being burned. In this report, we wish to investigate the thermal insulation performance of a particular type of high-temperature operating clothing. Specifically, We would like to know body temperature of a person, wearing an insulated clothing after a specific time at a certain temperature, given information of the insulating material and the environment temperature.

Model Establishment

Assumption

1.We abstracting the whole heat transfer process as a one-dimensional heat conduction problem.
2.Parameters of each material do not change with increasing temperature.
3.Neglect the heat radiation to the outermost layer of clothing, consider only heat convection as a form of heat transfer.

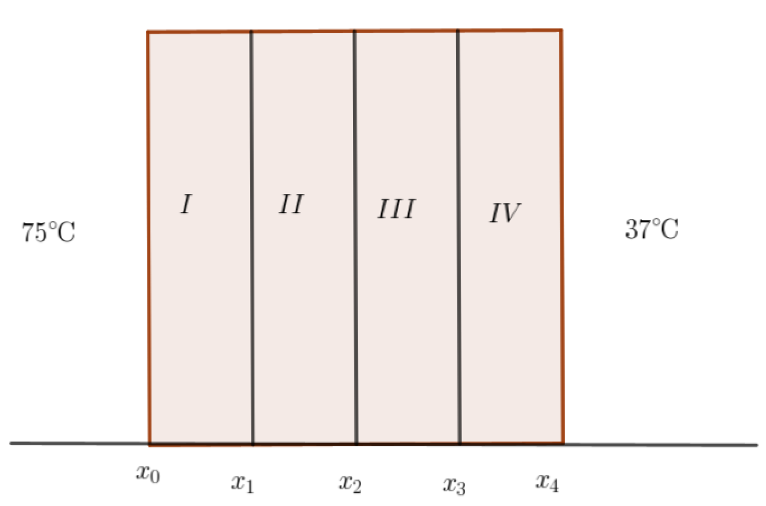

Figura 1: Cross-section of clothing

Heat Convection

$$-\lambda_1 \frac{\partial T_1}{\partial x}\bigg|_{x=x_0} = h_e\left[T_{en} - T(x_0, t)\right], \tag{1}$$

$$-\lambda_4 \frac{\partial T_4}{\partial x}\bigg|_{x=x_4} = h_s\left[-T_{sk} + T(x_4, t)\right], \tag{2}$$

Heat Diffusion

$$\frac{\partial T_i}{\partial t} = k_i \frac{\partial^2 T_i}{\partial x^2}, k_i = \frac{\lambda_i}{c_i \rho_i}, i = 1,2,3,4 \tag{3}$$

Heat Conduction

$$T_i\bigg|_{x=x_i} = T_{i+1}\bigg|_{x=x_i}, i = 1,2,3, \tag{4}$$

$$\lambda_i \frac{\partial T_i}{\partial x}\bigg|_{x=x_i} = \lambda_{i+1} \frac{\partial T_{i+1}}{\partial x}\bigg|_{x=x_i}, i = 1,2,3. \tag{5}$$

Initial Condition

$$T(x,0) = T_{sk} = T_2 = T_3 = T_4 \tag{6}$$

Problem Solving

We choose finite difference method.

$$\frac{\partial T}{\partial t} = \frac{T_i^{n+1} - T_i^n}{\Delta t}, \tag{7}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{(\Delta x)^2}, \tag{8}$$

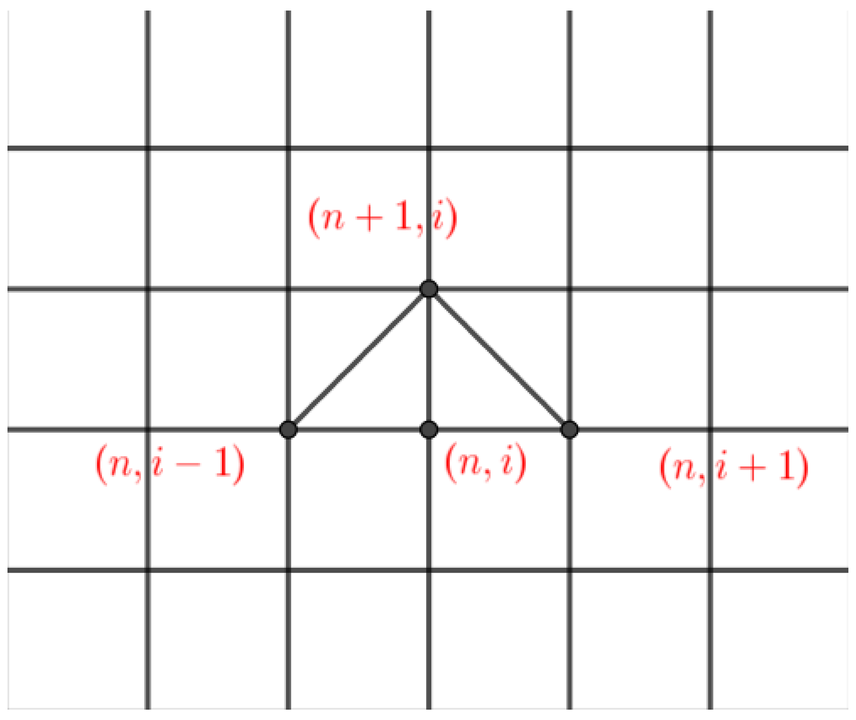

Figura 2: Discrete Grid

Below graph shows temperature distribution, while vertical axis shows temperature in degrees Celsius, horizontal axis shows position in meters.

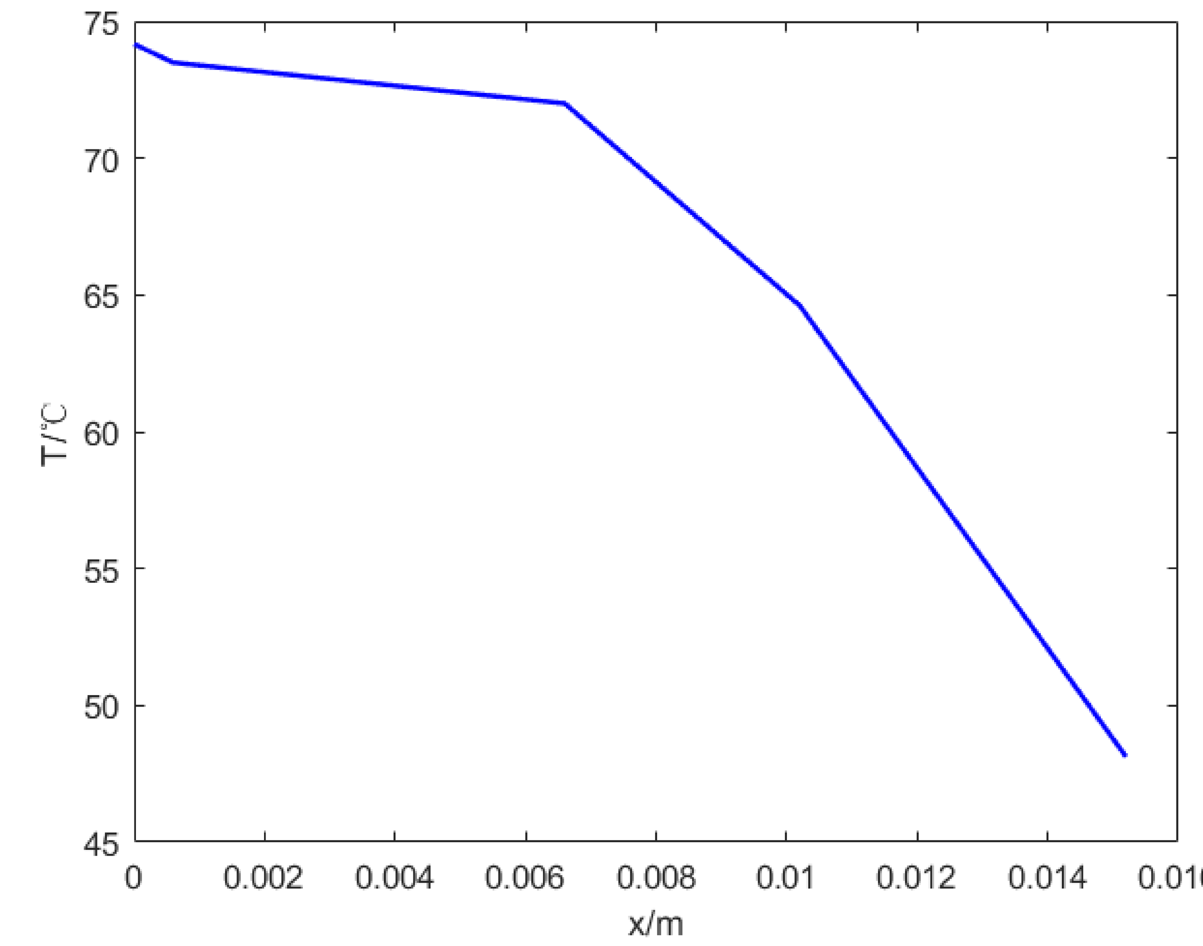

Figura 3

Here is x-t Grid, the horizontal coordinate shows the x-value and the vertical coordinate shows the t-value.

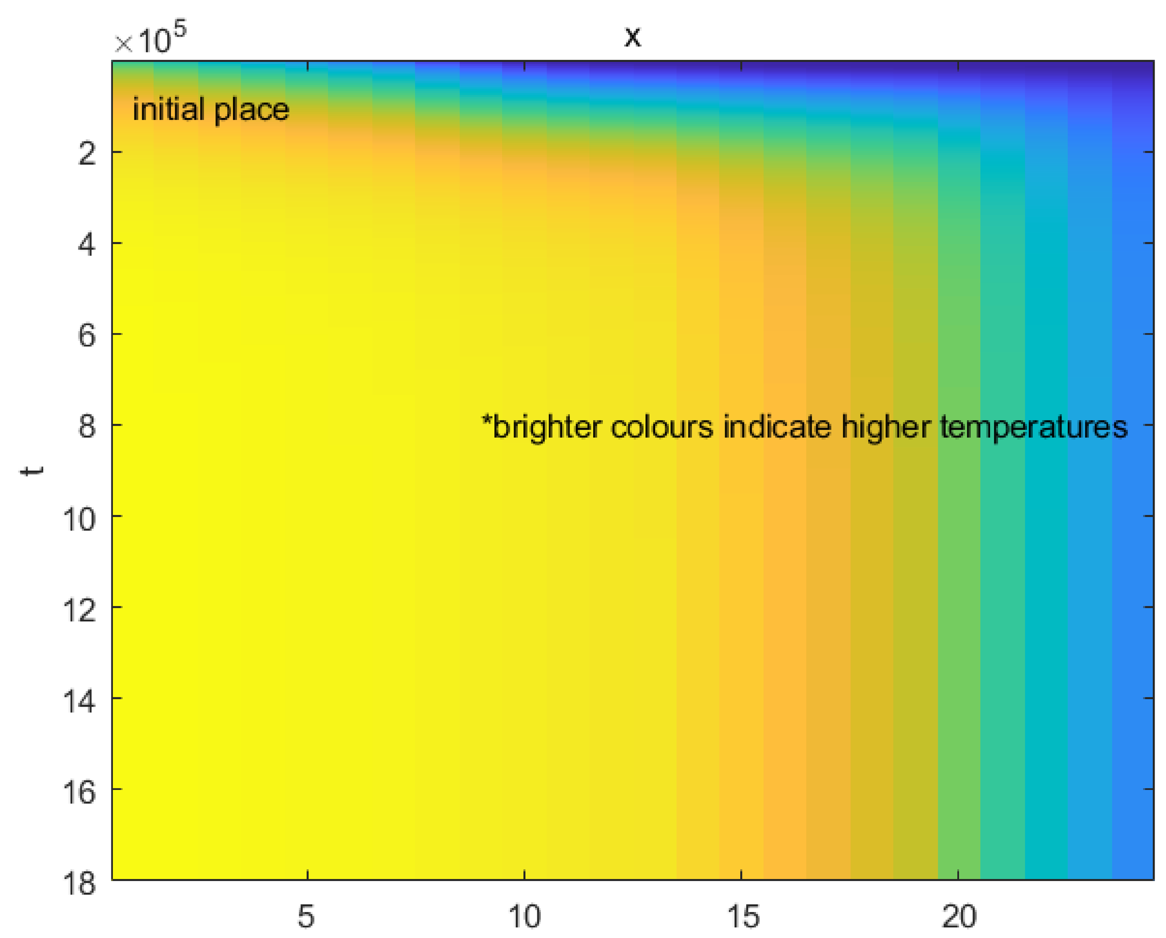

Figura 4

Error and Stability Analysis

Von Newmann stability analysis requires

$$r = \frac{k\Delta t}{(\Delta x)^2} < \frac{1}{2}, \tag{9}$$

in our model,

$$k_i = \frac{\lambda_i}{c_i \rho_i}, i = 1,2,3,4 \tag{10}$$

We obtain $r_1 = 0.0397, r_2 = 4.0879 \times 10^{-4}, r_3 = 0.0020, r_4 = 0.0472, r_i < \frac{1}{2}$, satisfying Von Newmann stability analysis. Also, RSS=0.002 which is quite small.

$$RSS = \frac{(U_n - T_{4M}^n)^2}{3601} = 0.002 \tag{11}$$