Information Disclosure and Competitive Equilibrium in Capital Market

Rongchuan Tao*[a], Luofeng Zhou*[a]

[a] *Economics and Management School, Wuhan University, China*

Abstract

This paper is intended to analyze the transmitting of information under a competitive price system. Previous researches have claimed that a competitive market with its price observable by a manager can deliver the information to the uninformed investors. Prices are aggregators of information, but not a perfect one in practical. In this case, we are curious about the sensitivity and the stability of a price system. After receiving shocks, the price of a stock, empirically, may react to the shocks, the changing process may also be dependent on the disclosure of information. This paper will establish a model to account for these behaviors. In the part of Monte Carlo simulation, we can see how the state of world accounts for the equilibrium in the asset market. The model will also lay the micro fundamentals for the price system in capital markets existing incomplete information by the formation of general equilibrium.

Keywords:

1. Introduction

This paper provides a model combining the managers' endowment model and pure exchange economy in general equilibrium theory. This model shows that: under complete information, the asset price suffers even more drastic volatility compared to the case where information is not totally disclosed by manager.

In the managers' endowment model (Jung and Kwon, 1988), managers has the ability in determining the disclosed private information. But one of the defects of the model is that it seems rigid to require the manager's disclosing information to agree with the rational expectation made by the uninformed investors. With a observing probability, it would be feasible that manager expose only part of the information in terms of all states of the world. But it's trivial to find a result that when the probability approaches 1, the manager would reveal all information to the investors. One major contribution of the hybrid model is to transform the strategy of manager into an endogenous choice thus maximize the manager's utility. Thus solve the defect appears in the managers' endowment model.

Whether insiders (e.g., managers, sellers) fully disclose their private information has been a research interest in accounting as well as in finance and economics. Grossman and Hart (1980) and Grossman (1981) support full disclosure of private information based on an adverse selection argument. That is, when insiders are known to withhold information, outsiders (e.g., investors, consumers) discount the quality of goods insiders deal with to the lowest possible value consistent with their discretionary disclosure. This in turn drives insiders to make full disclosure.

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While there is also some cost in making full disclosure of information, a full disclosure may result in an over-reacted market thus increasing the instability of the capital market. In the model developed in this thesis, it will be shown that the volatility of relative price of 2 stocks can be very high under full disclosure of information.

In general equilibrium model, it's common to examine a goods market while it's rare to analyze a stock market or a capital market. The pure exchange economy, as a special scenario in general equilibrium model, sets the initial endowments and preferences of consumers, through which a price-equilibrium can be formed. Some observations imply that the stock market may highly resemble the pure exchange market in short period. The main reason lies in thatthe total amount of asset in short-term remains constant, and generally, the generate process of financial asset don't share the same feature with production. So the frame of pure exchange would be most appropriate.

2. The Model

We don't aim to provide a generalized m-players, n-managers and q-properties model immediately. Since it's noticeable that the n-consumers general equilibrium model is derived and developed from a 2-goods and 2-consumers pure exchanging economy, it would also be reasonable to provide and study a toy model first. Next, we begin our construction of the toy model by a series of assumptions.

2.1. Agents' choice

In this part, we finish the construction of the agents' behavior under rational expectation.

First, suppose there are 2 stocks or assets in the capital market. Each asset can be characterized by 2 parameters, denoted as r_{ia} and r_{ib} meaning the property a or b of asset i.

Furthermore, we would like to require that (r_{1a}, r_{1b}) and (r_{2a}, r_{2b}) are independently distributed on a 4-dimensional cube: $(0, a_1) \times (0, b_1) \times (0, a_2) \times (0, b_2)$. Define: $S_1 = (0, a_1) \times (0, b_1)$ and $S_2 = (0, a_2) \times (0, b_2)$. Denote the joint density function of (r_{1a}, r_{1b}) and (r_{2a}, r_{2b}) to be $f_1(r_{1a}, r_{1b})$,$f_2(r_{2a}, r_{2b})$ respectively. We can derive the joint density of $(r_{1a}, r_{1b}, r_{2a}, r_{2b})$ to be $f_1(r_{1a}, r_{1b}) \times f_2(r_{2a}, r_{2b})$, denoting $f(r_{1a}, r_{1b}, r_{2a}, r_{2b})$. The prior belief about the density function is public information. It can be easily seen from the Fig.1 that a total present of each properties and the probability requires a 5-dimensional space. It would also be natural to understand that in the lemma to be present, a quadruple integral on a 4-dimensional cube is needed to obtain the expected utility of investors.

Next, we suppose there are 2 investors in the capital market, each investor has their initial endowment denoting (e_{11}, e_{12}) for investor 1, and (e_{21}, e_{22}) for investor 2, meaning investor1 holds e_{11} amount of asset 1 and e_{12} amount of asset2 and investor2 holds e_{21} amount of asset 1 and e_{22} amount of asset 2.

2 investors have their utility functions towards the assets they hold. The utility depends on 6 variables: x_1, x_2, which are the amount of each kind of asset they hold, and r_{1a}, r_{1b}, r_{2a}, r_{2b}, which are the properties of the assets.

So we denote the utility function of each investors to be:

$$U_1(x_{11}, x_{12}, r_{1a}, r_{1b}, r_{2a}, r_{2b}) \text{ and } U_2(x_{21}, x_{22}, r_{1a}, r_{1b}, r_{2a}, r_{2b})$$

Investors can maximize their utility by pure exchange. But there is some difference, since the investors only know the value of r_{1a}, r_{1b} if and only if they are public information. It's natural to consider the general equilibrium in **an Edgeworth box**.

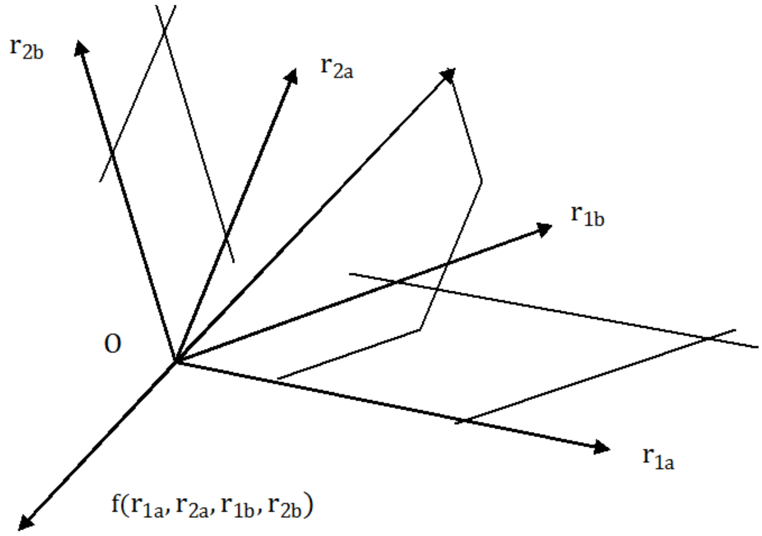


Figure 1: the joint distribution in 4-dimensional linear space

Generally, it's hard to do with the model with the above assumptions only, but we can make progress of analysis by imposing algebraic properties onto the utility functions.

Algebraic Property 1. **Homogeneity** with respect to properties:

$$k \times U(x_1, x_2, r_1 a, r_1 b, r_{2a}, r_{2b}) = U(x_1, x_2, kr_{1a}, kr_{1b}, kr_{2a}, kr_{2b}) \tag{1}$$

Algebraic Property 2. **Muti-linearity** with respect to properties:

$$\begin{aligned} &U(x_1, x_2, r_{1a}+r'_{1a}, r_{1b} + r'_{1b}, r_{2a} + r'_{2a}, r_{2b} + r'_{2b}) \\ &\qquad = U(x_1, x_2, r_{1a}, r_{1b}, r_{2a}, r_{2b}) + U(x_1, x_2, r'_{1a}, r'_{1b}, r'_{2a}, r'_{2b}) \end{aligned} \tag{2}$$

Lemma 1. *Suppose D_1, D_2 are independent events on S_1, S_2 respectively. Then, a utility function satisfying homogeneity and muti-linearity can guarantee:*

$$\begin{aligned} &E(U(x_1, x_2, r_{1a}, r_{1b}, r_{2a}, r_{2b})|D_1, D_2) \\ &\qquad = U(x_1, x_2, E(r_{1a}|D_1), E(r_{1b}|D_1), E(r_{2a}|D_2), E(r_{2b}|D_2)) \end{aligned} \tag{3}$$

Proof. By the homogeneity, we have:

$$\begin{aligned} &E(U(x_1, x_2, r_{1a}, r_{1b}, r_{2a}, r_{2b})|D_1, D_2) \\ &= \frac{1}{|D_1||D_2|} \iint_{D_1 \times D_2} U(x_1, x_2, r_{1a}, r_{1b}, r_{2a}, r_{2b}) f_1(r_{1a}, r_{1b}) f_2(r_{2a}, r_{2b}) \mathrm{d}r_{1a} \mathrm{d}r_{1b} \mathrm{d}r_{2a} \mathrm{d}r_{2b} \\ &= \frac{1}{|D_1||D_2|} \iint_{D_1 \times D_2} U(x_1, x_2, r_{1a} f_1 f_2, r_1 b f_1 f_2, r_{2a} f_1 f_2, r_{2b} f_1 f_2) \mathrm{d}r_{1a} \mathrm{d}r_{1b} \mathrm{d}r_{2a} \mathrm{d}r_{2b} \end{aligned} \tag{4}$$

Since the above integral is Riemann integral, we chop the integrating range into n number of 4-dimensional cubes. Then we substitute the integral by the approximation value on each cube and we have:

$$\begin{aligned}&E(U(x_1,x_2,r_1a,r_1b,r_2a,r_2b)|D_1,D_2)\\=&\frac{1}{|D_1||D_2|}\lim_{n\to\infty}\sum_{i=1}^{n}U(x_{1i},x_{2i},r_{1ai}f_1f_2,r_{1bi}f_1f_2,r_{2ai}f_1f_2,r_{2bi}f_1f_2)|V_i|\end{aligned}\tag{5}$$

V_i is the volume of the $i-th$ cube, it will approach 4-dimensional infinitesimal when n tend to infinity.
By the muti-linearity, we have:

$$\begin{aligned}&E(U(x_1,x_2,r_{1a},r_{1b},r_{2a},r_{2b})|D_1,D_2)\\&=\frac{1}{|D_1||D_2|}\lim_{n\to\infty}U(x_1,x_2,\sum_{i=1}^{n}r_{1ai}f_1f_2|Vi|,\sum_{i=1}^{n}r_{1bi}f_1f_2|V_i|,\\&\qquad\sum_{i=1}^{n}r_{2ai}f_1f_2|V_i|,\sum_{i=1}^{n}r_{2bi}f_1f_2|V_i|)\end{aligned}\tag{6}$$

Notice that $|V_i|$ can be decomposed into the product of 2-dimensional faces and the fact that:

$$\begin{aligned}&\frac{1}{|D_1||D_2|}\lim_{n\to\infty}\sum_{i=1}^{n}r_{1ai}f_1(r_{1ai},r_{1bi})f_2(r_{2ai},r_{2bi})|V_i|\\&=\lim_{n\to\infty}\sum_{i=1}^{n}r_{1ai}f_1(r_{1ai},r_{1bi})s_{1i}f_2(r_{2ai},r_{2bi})s_{2i}\\&=\lim_{n\to\infty}\sum_{i=1}^{n}r_{1ai}f_1(r_{1ai},r_{1bi})s_{1i}=\iint_{D_1}r_{1a}f_1(r_{1a},r_{1b})\mathrm{d}r_{1a}\mathrm{d}r_{1b}\\&=E(r_{1a}|D_1)\end{aligned}\tag{7.1}$$

Following the same process, we obtain

$$\frac{1}{|D_1||D_2|}\lim_{n\to\infty}\sum_{i=1}^{n}r_{1bi}f_1(r_{1ai},r_{1bi})f_2(r_{2ai},r_{2bi})|V_i|=E(r_{1b}|D_1)\tag{7.2}$$

$$\frac{1}{|D_1||D_2|}\lim_{n\to\infty}\sum_{i=1}^{n}r_{2ai}f_1(r_{1ai},r_{1bi})f_2(r_{2ai},r_{2bi})|V_i|=E(r_{2a}|D_1)\tag{7.3}$$

$$\frac{1}{|D_1||D_2|}\lim_{n\to\infty}\sum_{i=1}^{n}r_{2bi}f_1(r_{1ai},r_{1bi})f_2(r_{2ai},r_{2bi})|V_i|=E(r_{2b}|D_1)\tag{7.4}$$

Again, according to the homogeneity and substituting the above equations into (7) , we obtain

$$\begin{aligned}&E(U(x_1,x_2,r_{1a},r_{1b},r_{2a},r_{2b})|D_1,D_2)\\&\qquad=U(x_1,x_2,E(r_{1a}|D_1),E(r_{1b}|D_1),E(r_{2a}|D_2),E(r_{2b}|D_2))\end{aligned}\tag{8}$$

Thus we finish the proof of the lemma. □

This lemma implies a favorable proposition: given managers' strategy, Divide the area D into the Non-disclosure area and disclosure area. If the information is not disclosed, investors would just substitute the expected property value conditional on the event that the information is not disclosed. Under this simplification, we can just treat all the non-disclosed area as a single case in the general equilibrium.

It's easy to verify that the log-form of Cobb-Douglas utility function satisfies the above properties. We adopt the specific form:

$$U_1(x_{11}, x_{12}, r_{1a}, r_{1b}, r_{2a}, r_{2b}) = \alpha_1 r_{1a} ln(x_{11}) + \beta_1 r_{1b} ln(x_{12})$$
$$U_2(x_{21}, x_{22}, r_{1a}, r_{1b}, r_{2a}, r_{2b}) = \alpha_2 r_{2a} ln(x_{21}) + \beta_2 r_{2b} ln(x_{22})$$

It seems necessary to give some interpretation about the above form. According to the above form, it can be inferred that it is favored by investors if the value of property (a) becomes higher or the value of property (b) becomes lower. So, when this theoretical model serves for empirical use, property (a) may represent something like return ratio and property (b) can just be something like the volatility or risks of the capital. Furthermore, it can be interesting to find that for the investor 1, he only consider the information of r_{1a},r_{1b} to be essential in determining utility since r_{2a},r_{2b} is public information, while the other agent only takes the public information into consideration.

Why must we construct a different utility form between the 2 investors? We will soon find that the heterogeneity of the utility function of each investor guarantees the stochastic walk of the general equilibrium.

2.2. Manager's optimal strategy

Suppose there is only one manager who is responsible for asset1 and has the ability to be fully informed of the states of asset1.Since the information about asset2 is public, the manager turns out to be totally informed of each state of the world. The manager can make credible announcements about the states of the world, so that the investors will update the information immediately after the disclosure. Manager's incentives of revealing the information come from the effects he can make on the market's equilibrium. This change in equilibrium may improve his utility.

To become more specific, we may formalize the strategy of manager:

Denote a function $\phi : (0, a_1) \times (0, b_1) \rightarrow \{0, 1\}$.For any state realization (r_{1a}, r_{1b}), the manager disclose the information to the public if and only if $\phi(r_{1a}, r_{1b}) = 1$.

$$D := \{(r_{1a}, r_{1b}) | \phi(r_{1a}, r_{1b}) = 1\}$$
$$ND := \{(r_{1a}, r_{1b}) | \phi(r_{1a}, r_{1b}) = 0\}$$

ND would be the collection of the state which the investors cannot distinguish. D is the area that investors would be fully informed if the world chooses a state in D.

From the above discussion in the investors' optimal reaction, it would be feasible to consider the manager's strategies subject to the investors' reaction. We assume that the manager is an expected utility maximizer, which is equivalent to let the manager to be risk-neutral.

The original idea about the managers' target function is letting it to be the relative price: $P1/P2$. But it appears more general and flexible to first study an abstract, given payoff function, which we shall denote: $v(r_{1a}, r_{1b}, \phi)$. It represents the manager's payoff, given the manager's strategy ϕ and the state of world, r_{1a}, r_{1b}.It's indeed a function since given r_{1a}, r_{1b}, ϕ, the equilibrium can establish uniquely.

According to Lemma 1, we have the following theorem:

Theorem 1. *If investors' utility satisfies muti-linearity and homogeneity and manager is an expected utility maximizer, then manager's optimization is equivalent to :*

$$\max_{\phi} \iint_D [v(r_{1a}, r_{1b}, \phi) \times f(r_{1a}, r_{1b}) \mathrm{d}r_{1a} \mathrm{d}r_{1b} + v(E(r_{1a} \big| ND), E(r_{1b} \big| ND, \phi) \times |ND|] \qquad (*)$$

Proof. The manager is an expected utility maximizer, so the manager's UMP reads:

$$\max_{\phi} E(v(r_{1a}, r_{1b}, \phi))$$

Since the investors can distinguish the world's state if and only if the state (r_{1a}, r_{1b}) is contained by the area D. We can decompose the target function $\max_{\phi} E(v(r_{1a}, r_{1b}, \phi))$ to 2 terms, which differs in whether the information is disclosed.

After decomposing:

$$E(v(r_{1a}, r_{1b}, \phi)) = \iint_D v(r_{1a}, r_{1b}, \phi) \times f(r_{1a}, r_{1b}) \mathrm{d}r_{1a} \mathrm{d}r_{1b} + E(v(r_{1a}, r_{1b}, \phi)|ND) \qquad (8)$$

Lemma 1 indicates that investors' behavior under Non-disclosing information is equivalent to the equilibrium where the information $(E(r_{1a}ND), E(r_{1b}ND))$ are disclosed as the value of r_{1a} and r_{1b} respectively. So:

$$E(v(r_{1a}, r_{1b}, \phi)ND) = \iint_{ND} v(E(r_{1a}ND), E(r_{1b}ND), \phi) \mathrm{d}r_{1a} \mathrm{d}r_{1b} \qquad (9)$$

Notice that: $v(E(r_{1a}ND), E(r_{1b}ND), \phi)$ is constant, so:

$$E(v(r_{1a}, r_{1b}, \phi)ND) = v(E(r_{1a}ND), E(r_{1b}ND), \phi) \times |ND| \qquad (10)$$

Thus the original problem is equivalent to:

$$\max_{\phi} \iint_D [v(r_{1a}, r_{1b}, \phi) \times f(r_{1a}, r_{1b}) \mathrm{d}r_{1a} \mathrm{d}r_{1b} + v(E(r_{1a} \big| ND), E(r_{1b} \big| ND, \phi) \times |ND|] \qquad (*)$$

Q.E.D. □

The further discussion of the manager's optimization problem is shown in Part 6.

3. The solution to the equilibrium

With the above construction, we are able to deal with the competitive equilibrium.

If the information is disclosed by the manager, the utility maximizing problem each investor facing reads:

$$\begin{array}{ll} \text{For investor 1:} & Max\ \alpha_1 r_{1a} ln(x_{11}) + \beta_1 r_{1b} ln(x_{12}) \\ & s.t.\ P_1 x_{11} + P_2 x_{12} = P_1 e_{11} + P_2 e_{12}, \\ \text{For investor 2:} & Max\ \alpha_2 r_{2b} ln(x_{21}) + \beta_2 r_{2a} ln(x_{22}) \\ & s.t.\ P_1 x_{21} + P_2 x_{22} = P_1 e_{21} + P_2 e_{22}. \end{array}$$

The corressponding first order conditions are:

$$(\frac{\alpha_1 r_{1a}}{x_{11}}, \frac{\beta_1 r_{1b}}{x_{12}}) = \lambda_1(\frac{\alpha_2 r_{2b}}{x_{21}}, \frac{\beta_2 r_{2a}}{x_{22}}) = \lambda_2(P_1, P_2).$$

Through simplification, the relationships turn out to be:

$$\frac{x_{12}}{x_{11}} \times \frac{\alpha_1}{\beta_1} \times \frac{r_{1a}}{r_{1b}} = \frac{x_{22}}{x_{21}} \times \frac{\alpha_2}{\beta_2} \times \frac{r_{2b}}{r_{2a}} \tag{11}$$

$$\frac{x_{12} - e_{12}}{x_{11} - e_{11}} = -\frac{\alpha_1}{\beta_1} \times \frac{r_{1b}}{r_{1a}} \times \frac{x_{11}}{x_{12}} \tag{3}$$

$$x_{11} + x_{12} = e_{11} + e_{12} \tag{13}$$

$$x_{21} + x_{22} = e_{21} + e_{22} \tag{14}$$

It's a simple algebraic work to check that (11) and (12) are hyperbolas. The solution of the equilibrium turns out to be the intersection of the 2 hyperbolas.

Notice that $(e_{11}+e_{12}, e_{21}+e_{22})$ and some points in any neighborhood of origin always satisfy (11). The same attempts can deduce that (e_{11}, e_{21}) and some points in any neighborhood of origin always satisfy (12).

If the information of assets is not available, the utility maximizing problem each investor facing reads:

$$\begin{array}{ll} \text{For investor 1:} & Max\ \alpha_1 r_{1a}^E ln(x_{11}) + \beta_1 r_{1b}^E ln(x_{12}) \\ & s.t.\ P_1 x_{11} + P_2 x_{12} = P_1 e_{11} + P_2 e_{12}, \\ \text{For investor 2:} & Max\ \alpha_2 r_{2b}^E ln(x_{21}) + \beta_2 r_{2a}^E ln(x_{22}) \\ & s.t.\ P_1 x_{21} + P_2 x_{22} = P_1 e_{21} + P_2 e_{22}. \end{array}$$

The only difference lies in this way: investors form a expectation of the properties conditional on the *Non-disclosure* event by the prior distribution. According to the lemma, we can just substitute the properties of asset 1 by the expected value, and:

$$r_{1a}^E = \frac{1}{ND} \iint_{ND} r_{1a} f_1(r_{1a}, r_{1b}) dr_{1a} dr_{1b},$$

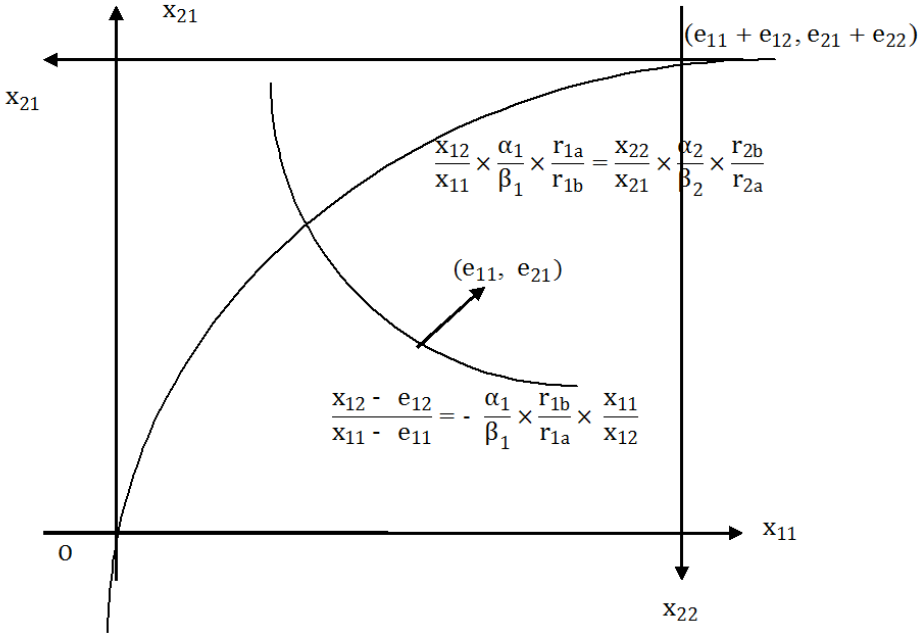
$$r_{1b}^E = \frac{1}{ND} \iint_{ND} r_{1b} f_1(r_{1a}, r_{1b}) dr_{1a} dr_{1b}.$$

Now, we turn our attention to the price system established in the equilibrium. Notice that the first order conditions also indicate that:

$$\frac{P_1}{P_2} = \frac{x_{12} r_{1a}}{x_{11} r_{1b}} \times \frac{\alpha_1}{\beta_1} = \frac{x_{22}}{x_{21}} \times \frac{\alpha_2}{\beta_2} \times \frac{r_{2b}}{r_{2a}}.$$

The above expression is dealing with the relative price between asset 1 and asset 2. So its economic interpretation is well defined, which gives us the motivation in generating the relative price vector in the Monte Carlo experiment to be proceeded.

Given the specific value of each parameter, the above dynamic system will generate a time-series data. We present the Monte Carlo experiment in the case where manager disclose all information.



$$\frac{x_{12}}{x_{11}} \times \frac{\alpha_1}{\beta_1} \times \frac{r_{1a}}{r_{1b}} = \frac{x_{22}}{x_{21}} \times \frac{\alpha_2}{\beta_2} \times \frac{r_{2b}}{r_{2a}}$$

$$\frac{x_{12} - e_{12}}{x_{11} - e_{11}} = -\frac{\alpha_1}{\beta_1} \times \frac{r_{1b}}{r_{1a}} \times \frac{x_{11}}{x_{12}}$$

Figure 2: The Equilibrium

4. Monte Carlo Experiment

4.1. Benchmark simulation

The benchmark simulation set the investors to be full informed of information. Thus we needn't consider the manager's strategy.

To execute the Monte Carlo simulation in Matlab, we first need some calibrations:

- Set $e_{11} + e_{21} = 3, e_{12} + e_{22} = 3$,
- $\frac{\alpha_1}{\beta_1} = 1, \frac{\alpha_2}{\beta_2} = 1, e_{11} = 1, e_{12} = 2$,
- Each of the properties are uniformly distributed in the domain [0,1]

Rename x_{11}, x_{12} as x and y. Let x_t, y_t denote the t-period value of x and y. With the calibration above, the dynamic system becomes:

$$\frac{x_{t+1} - x_t}{y_{t+1} - y_t} = -\frac{r_{1b}}{r_{1a}} \times \frac{x_{t+1}}{y_{t+1}},$$

$$\frac{y_{t+1}}{x_{t+1}} \times \frac{r_{1a}}{r_{1b}} = \frac{3 - y_{t+1}}{3 - x_{t+1}} \times \frac{r_{2b}}{r_{2a}}.$$

In order to evade the interruption of the singular solution i.e. (0,0), simplify the above equations:

$$y_{t+1} = \frac{3r_{1b}r_{2b}x_{t+1}}{3r_{1a}r_{2a} + (r_{1b}r_{2b} - r_{1a}r_{2a})x_{t+1}},$$

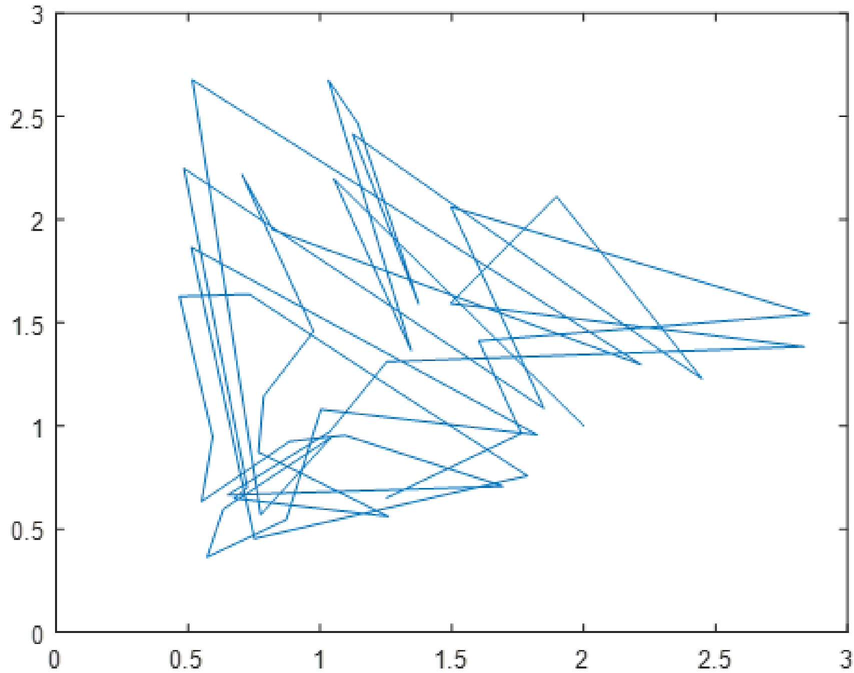


Figure 3: Monte Carlo Simulation

$$y_{t+1} = \frac{r_{1b}x_{t+1}y_t}{(r_{1a}+r_{1b})x_{t+1} - r_{1a}x_t}.$$

Run the code shown in appendix. Slightly different from the code, we analyzed 50 periods in the following experiment by resetting the parameter v to be 50.

The result of Monte Carlo indicates that the assets each investors hold vary from the state of the world in the above stochastic way. In Fig.3, each blue segment begins at the equilibrium of the prior period and ends at the equilibrium of the subsequent period. Also we can observe that the volatility of the holding assets is sometimes very high such that in some period,(i.e. the period 39 in the experiment where $x_{11} = 2.1312, x_{12} = 2.6818$ and $x_{11} + x_{12} = 4.8130$. Notice that the total amount of 2 stocks is 6) nearly all wealth of the economy is concentrated on one investor. The existence of a nearly vertical segment in Fig.3 shows the price may also suffer drastic variation under complete information.

Notice that it is easy to prove that if manager disclose no information from the beginning to end and the information of asset2 is also not available to the investors, the endowment will change only at period1, then stay at that level forever. We can conjecture that the public information contributes to the volatility of a stock market. Next, we wish to verify our conjecture by the Monte Carlo simulation.

It's very convenient to give the relative price according to the price equilibrium (competitive

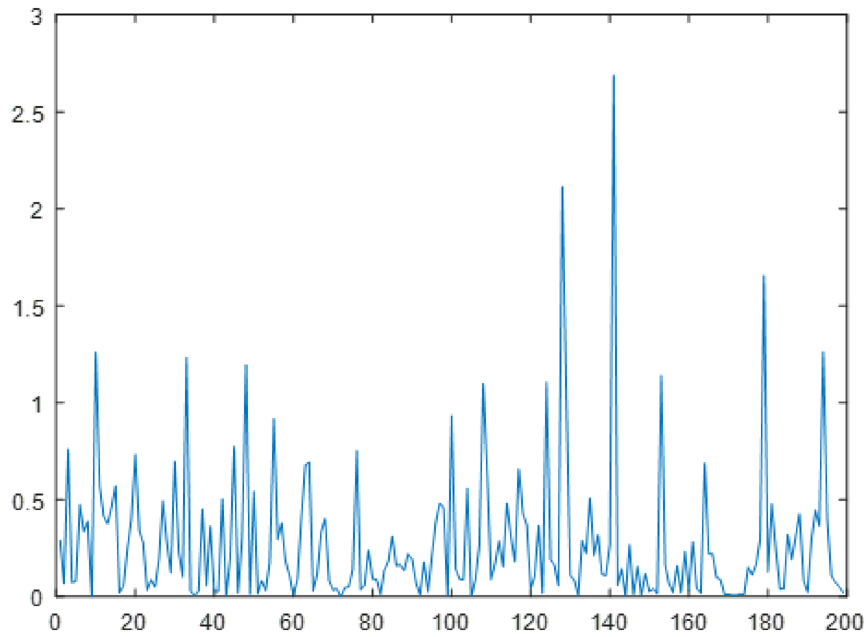


Figure 4

equilibrium), specified by formula:

$$\frac{P_1}{P_2} = \frac{x_{12}}{x_{11}} \times \frac{\alpha_1}{\beta_1} \times \frac{r_{1a}}{r_{1b}} = \frac{x_{22}}{x_{21}} \times \frac{\alpha_2}{\beta_2} \times \frac{r_{2b}}{r_{2a}}.$$

First, we generate the price of each period and graph it in Matlab. In order to reduce the randomness, we set the period to be 200. The result is shown in Fig.4. Next, we would specify a strategy of manager and compare the corresponding price figure with this one.

4.2. Simulation with a given strategy of manager

Take the manager's disclosing map to be:

$$D := \{(r_{1a}, r_{1b}) | r_{1a} - r_{1b} \geq 0.5\}.$$

The intuition of this disclosing choice is: empirically, manager would like to disclose the information if it is favored by investors while they tend to hide the information if the firm runs badly. In the model, r_a represents a favorable property and r_b represents an unfavorable property according to the utility form of each investor.

Run the code that can give the price trend in the appendix, we obtain:

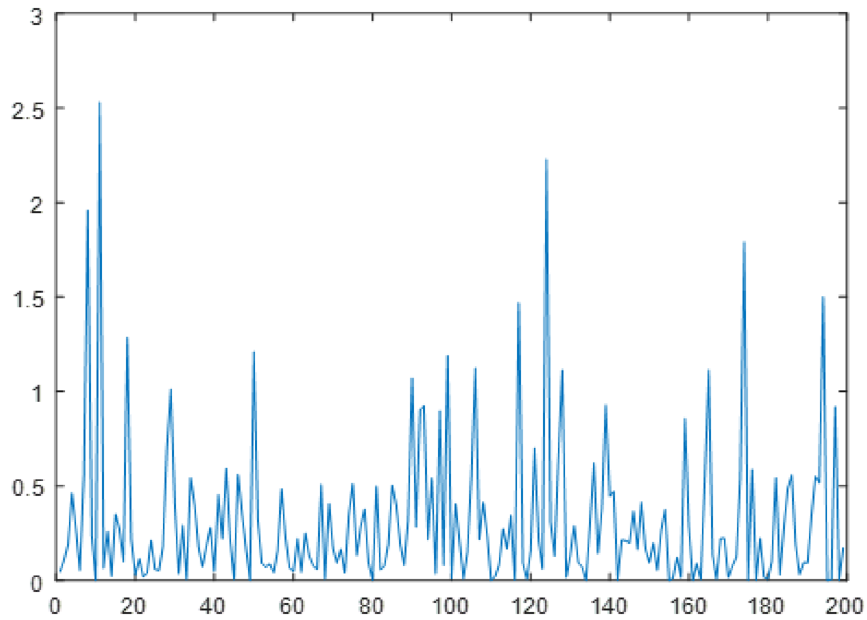


Figure 5

Since the coordinate of Fig.4 and Fig.5 are the same, the volatility of the curve in Fig.4 is obviously higher than that of Fig.5. So we can reasonably conjecture that the volatility of the price would be higher under complete information.

5. Manager's Strategies

We now get back to the manager's strategies. Recall that in the theorem, the manager's optimal problem reads:

$$\max_{\phi}[\iint_{D} v(r_{1a}, r_{1b}, \phi) \times f(r_{1a}, r_{1b}) dr_{1a} dr_{1b} + v(E(r_{1a}|ND), E(r_{ab}|ND, \phi) \times |ND|].$$

The following theorem gives the general necessary conditions serving the constraints of optimal strategy.

Theorem 2. *Suppose the distribution of r_a and r_b is uniform. The optimization area is denoted by D, where $D = (a, b) \times (c, d)$. The manager's utility function is set to be u(x,y), defined on D. Furthermore, we assume that u(x,y) is differentiable on D. Thus u(x,y) determines a differentiable manifold in the 3-dimensional space. We denote the manifold by X. Denote G the barycenter of the non-disclosing area and $\Omega(G)$ the plane through G that is tangent to X, and $\Omega(G)$ has the analytic expression: $z = ax + by + c$. If a non-trivial reporting strategy exists, it must satisfy: $ND = \{(x, y)|u(x, y) \leq ax + by + c\}$.*

Proof. Suppose there is an optimal strategy that maximizes the expected utility of manager that does not satisfy the above condition. We suppose that the non-disclosing area of the optimal strategy to be: K.

Since the strategy does not satisfy $ND = \{(x, y)|u(x, y) \leq ax + by + c\}$, there exists point (x_0, y_0) (denoting by G') such that $u(x_0, y_0) > ax_0 + by_0 + c$ According to some results in real analysis, the point set D
$ND = (x, y)|u(x, y) > ax + by + c$ is open set. So there exists an open measurable neighborhood of G' that is contained by D
ND. Without loss of generality, we take the neighborhood to be an open disc (denoting by U) and (x_0, y_0) is the barycenter of U.

Next, we shall prove that adding this neighborhood into the non-disclosure area will strictly improve the manager's expected utility.

The total effect of adding the neighborhood consists of 3 sub-effects:

The first wave of effect is the changing in the barycenter of the non-disclosing area:

$$S1 = ||G' - G|| \times \frac{|U||ND|}{|U| + |ND|} \times f'_{G \to G'}(x_0, y_0).$$

In the above expression $f'_{G \leftarrow G'}(x)$ is the directional derivative where the directional vector starts at G and end at G'.

This term would approximate:

$$S1 = ||G' - G|| \times |U| \times f'_{G \to G'}(G'),$$

if $|U|$ is very small.

The second wave of effect is the net gain in increasing probability of disclosing no information, which is the measure of the neighborhood multiply by the expected utility:

$$S2 = u(G) \times |U|.$$

The third wave of effect is the loss of the expected return if the information is disclosed:

$$S3 = u(G') \times |U|.$$

So $S1 + S2 + S3 = u(x, y) - ax + by + c$. The sum of the three effects turns out to be positive. A contradiction arises.

Q.E.D. □

6. Conclusion and problems to be solved

This model successfully combines the asymmetric information, rational expectations and general equilibrium, under which we can either develop a more generalized model that contains more managers, more investors and more properties for each kind of stock. The main conclusion for this model is that the spread of information can become the main reason of the changing in the relative prices. Second, given some concrete requirement for the manager's utility function, the manager may have an interior optimal strategy in disclosing information.

We can also easily find many problems to be solved in the future, the most innovative and technical one may be the general solution to the manager's optimal choice. The second problem may be the specific form of the manager's utility function. Shall he take the stability of the market or fairness into consideration? Third, we are curious about what empirical predicting function this model have. At last, is it practicable to release the functional form of the investor's utility without the loss in the feasibility of theoretical research? Since the assumptions about the utility are too strong to fit in the practical usage.

Appendix A. Matlab code for the benchmark simulation

```
% set total time period:50.
v=50;
% generate the corresponding distribution
ra=rand(1,v);
rb=rand(1,v);
ca=rand(1,v);
cb=rand(1,v);

% set initial endowment
x(1)=2;
y(1)=1;

for i= 1:v-1
    g(i)=i;
end
for i=1:v-1
% solve the competitive equilibrium
syms m n
eqn1 = rb(i)*y(i)/((ra(i)+rb(i))*m-ra(i)*x(i))==3*cb(i)*rb(i)
       /(3*ca(i)*ra(i)+(rb(i)*cb(i)-ca(i)*ra(i)*m));
eqn2 = n==3*cb(i)*m/(3*ca(i)+(cb(i)-ca(i))*m);

[solm, soln]=solve([eqn1,eqn2] , [m,n]);
% update the next period endowment according to the solution
x(i+1)=solm;
y(i+1)=soln;

% generate price vector
k(i)=y(i)*ra(i)/x(i)*rb(i);
end

% plot the price trend
plot (g,k)

% plot the trace of the equilibrium
plot (x,y)
axis([0 3 0 3])
```

Appendix B. Matlab code for a non-trivial manager strategy

```
% set total time period:50.
v=50;
% generate the corresponding state of the world in each period
ra=rand(1,v);
```

```
rb=rand(1,v);
cb=rand(1,v);
ca=rand(1,v);

% generate the manager s behavior by the result of the state of world
for j=1: v
if rb(j)-ra(j)≥0.5
      t(j)=1;
else
      t(j)=0;
end
end
% calculate the integral of rb and ra conditional on information is not ...
    disclosed, denoting intb and inta.
for l= 1:20
syms p q
f1=int(p,q,0,p+l/20);
f2=int(p,q,0,1);
inta1=int(f1,p,0,1-l/20);
inta2=int(f2,p,1-l/20,1);
inta(l)=inta1+inta2;

end

for i= 1:199
    g(i)=i;
end

a=inta(10);
b=1-a;

% set initial endowment
x(1)=2;
y(1)=1;


for i=1:v-1
% solve the competitive equilibrium
if t(i)==1
syms m n
eqn1 = rb(i)*y(i)/((ra(i)+rb(i))*m-ra(i)*x(i))
     ==3*cb(i)*rb(i)/(3*ca(i)*ra(i)+(rb(i)*cb(i)-ca(i)*ra(i)*m));
eqn2 = n==3*cb(i)*m/(3*ca(i)+(cb(i)-ca(i))*m);
[solm, soln]=solve([eqn1,eqn2] , [m,n]);
% update the next period endowment according to the solution
x(i+1)=solm;
y(i+1)=soln;

else
% when the information is not disclosed, investor replace ra and rb by a ...
    and b.
syms m n
eqn1 = b*y(i)/((a+b)*m-a*x(i))==3*cb(i)/(3*ca(i)+(cb(i)-ca(i)*m));
eqn2 = n==3*cb(i)*m/(3*ca(i)+(cb(i)-ca(i))*m);

[solm, soln]=solve([eqn1,eqn2] , [m,n]);
% update the next period endowment according to the solution
x(i+1)=solm;
y(i+1)=soln;
```

```
k(i)=y(i)*ra(i)/x(i)*rb(i);
end

end
% plot the price trend
plot (g,k)
% plot the trace of the equilibrium
plot (x,y)
axis([0 3 0 3])
```

Appendix C. Matlab code for numerical solution of optimal ND zone

```
%% Initialize
% define function expression
syms x;
syms y;
%f=(0.3-x)^3+(y-0.5)^3;
f=(y+1)/(x+1)
% decide initial point
x0=0.5;
y0=0.5;
% temp matrices
c0=zeros(100);
% report matrix
reportm=[2];
reportx=[x0];
reporty=[y0];
times=0;
ds=2;
%% Run
% calculate tangent plane, denoted as z
while(ds>0.02)
fx=diff(f,x);
fy=diff(f,y);
fx0=subs(fx,x,x0);
fx00=subs(fx0,y,y0);
fy0=subs(fy,x,x0);
fy00=subs(fy0,y,y0);
f0=subs(f,x,x0);
f00=subs(f0,y,y0);
z =f00 + fx00*(x-x0) + fy00*(y-y0);
%calculate nodeclaration area
nd=c0;
for i=1:100
    for j=1:100
        zk=subs(z,x,i/100);
        zkk=subs(zk,y,j/100);
        fk=subs(f,x,i/100);
        fkk=subs(fk,y,j/100);
        if zkk>fkk
            nd(i,j)=1;
        end
    end
end
s = regionprops(nd,'centroid');
```

```
q1=s.Centroid(1,1)/100;
q2=1-s.Centroid(1,2)/100;
ds=((q1-x0)^2+(q2-y0)^2)^(1/2);
t=reportm;
reportm=[t;ds];
t=reportx;
reportx=[t;q1];
t=reporty;
reporty=[t,q2];
x0=q1;
y0=q2;
times=times+1;
disp(times);
disp(ds);

end
%% show the result
% display the times

% display the contingency

% display the area of nd
[m,n] = size(nd);
[row1,col1] = ind2sub([m,n],find(nd==1));
[row2,col2] = ind2sub([m,n],find(nd==0));
figure, plot(col1,row1,'rs',col2,row2,'bo');
figure,hold on;
scatter(col1,row1,'filled'); scatter(col2,row2);
```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