Study of Liquid Rope Coiling

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Abstract:

The liquid rope-coil effect of viscous fluid is a common nature phenomenon and needs to be studied quantitatively. Detailed measurement of coiling frequency was done and the frequency-falling height correlation was analyzed in this study. A high speed camera with ability to shoot 5000 pictures per second was used quantitatively to record the motion of this phenomenon. Silicone oil with viscosity from 1000Cst to 100000Cst is used in our experiment. Theoretically, the rheological transition is explained by an asymptotic model which simplifies the coupling between viscous force, gravity and centrifugal force. As a result, model predicts the analytic expression of coiling frequency as a function of falling height, and this relationship is examined by experimental result. Therefore, the model successfully explained the rheological transition in liquid rope coiling. Moreover, experimental study of energy transportation in coiling process is conducted which is novel in this field and the results raises further theoretical problems open to discuss in the future.

1. Introduction:

The origin of liquid-rope coiling effect is based on the un-isotropic nature of the viscous liquid internally, the viscosity plays a significant role to determine whether the phenomenon will be observed or not. The bucking phenomenon is the response of fluid to its impact on substrate and itself is a disclosure of the inhomogeneity inside the viscous liquid (**Fig[1]a**). And the further circling motion is directly related to the viscous nature of the fluid, the viscosity brings elasticity to fluid such that the thin stream of fluid can do oscillation in 2D planes which forms a circle-like trajectory(**Fig[1]b**)*(Maleki et al. 2004)*. When the circling motion is steady, it allows us to analyze it in a rotating frame which yields a balanced dynamical equation for numerical calculation. Experimentally, the dependence of circling frequency on parameters such as viscosity, fluid density, falling height characterizes the dynamical properties of the rope-coiling motion. Interestingly, there exists rheological transition in this motion and it can be seen directly as the configuration of coiling changes with falling height *(Habibi et al. 2006)*. Mentioned above, the theoretical model helps us to understand its physics of such transition. Finally, and on a more personal note, I find that coiling possess its aesthetic value for the beauty visualized by helical and spiral figures.
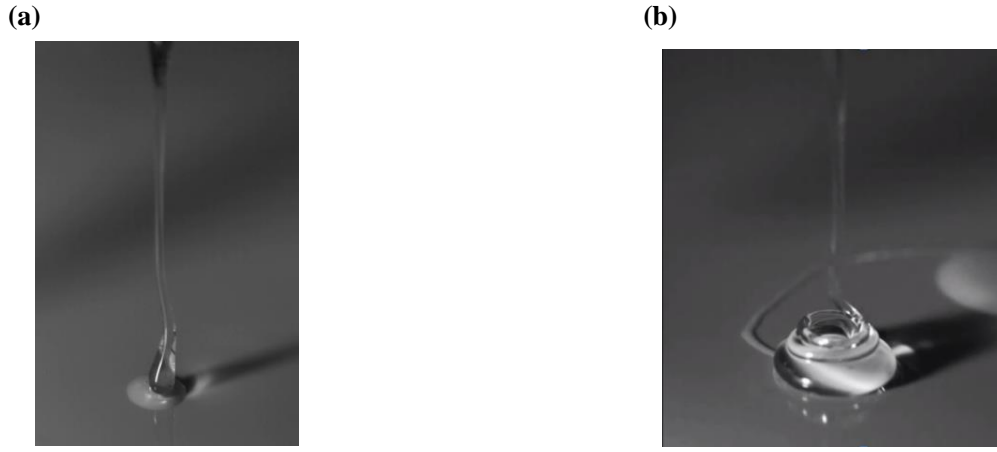
Figure 1

(a) The bucking instability as the starting point of circling-like motion of oil. (b)A specific example of circling configuration at falling height 5cm of silicone oil with $\nu = 10000Cst$

2. Experimental study of steady coiling

Silicone oil with viscosity 1000Cst and 10000Cst with same relative density around 0.976 was chose as our viscous fluid. The first experiment deals with the measurement of circling frequency Ω (HZ) with enhancing vertical falling height (2cm-22cm) (**Fig[2]a**). The initial condition or the fluid flow rate was restricted at 0.00567ml/s by springer connected to a stepper motor (**Fig[2]b**). A high speed camera shooting 5000 frames per second recorded the whole process of motion and further image-processing help us to identify the spatial parameters inner radius of circling stream **d** and the circling radius **D** (**Fig[2]c**). Under these set up, the preciseness was ensured.
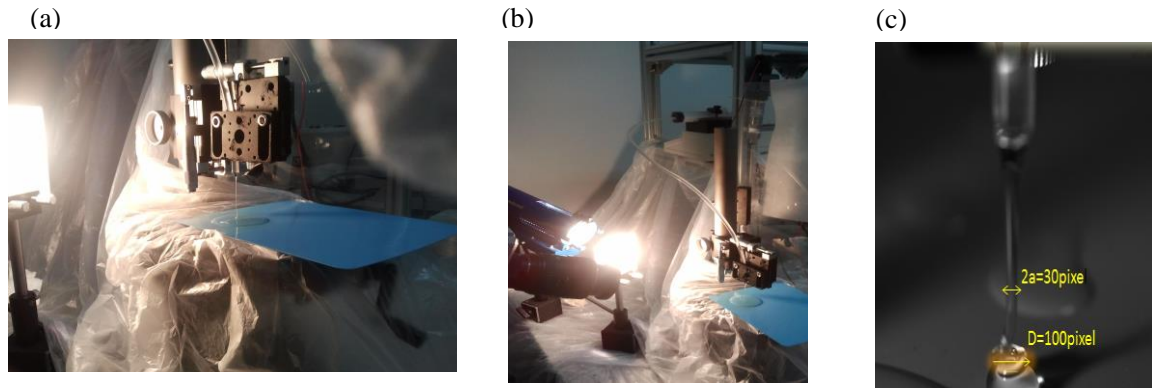

Figure 2

(a) Adjusting falling height of viscous stream. (b) Demonstration of the full experiment apparatus. (c) image processing by the calibration between pixels and real scale(mm).

The fluid falls onto a plate on which it forms a helical coil of radius *R* that rotates with angular frequency _ about a vertical axis. The effective fall height is the distance *H* from the hole down to the first point of contact of the free portion of the rope with the pile of fluid

accumulated on the plate. The radius of the trailing portion of the rope (tail) decreases downward in general under the pull of gravity, and its (nearly **a**), it's clear that there is an equation relating **d, D ,a** and flux **Q** due to the conservation of mass. *(Habibi et al 2006)*(This is simple but useful in our discussion in following experiments)

$$v = R\Omega, Q = \pi a^2 v \qquad (1)$$

With the measurement results of silicone oil of 10000Cst and 1000Cst, the relations of the frequency-height (1000Cst and 10000Cst in comparison) dependence , inner radius (**d**) and coiling radius(**D)** height dependence(only the silicone oil with 10000Cst viscosity measured) are presented in (**Fig[3]**)
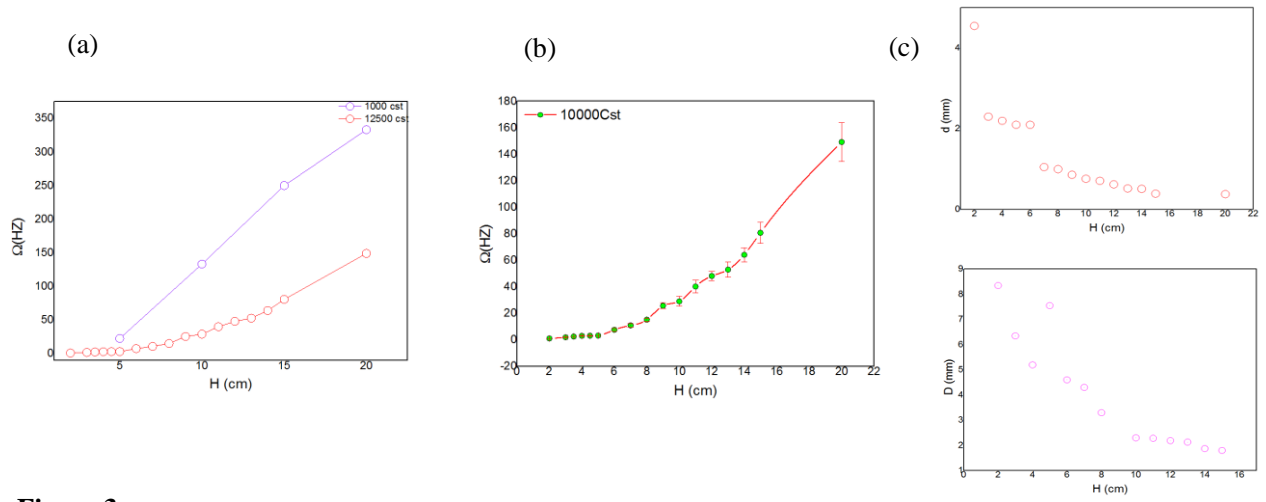

Figure 3

(a)The circling frequency **vs** falling heights, silicone oil with same density but different viscosity (1000Cst and 10000Cst) presented together. (b)The case of 10000Cst individually presented. (c)The upper graph shows the width of liquid coil dependence on height **H**. The lower graph is about the coiling radius dependence on **H.**

In general, the coiling frequency increases with the falling height nonlinearly whiles the coiling radius and the width of coiling radius have negative dependence in height. In comparison, the less viscous silicone coil oscillates slower with height constrained.

Combing the 3 measurements ($\Omega \sim H, d \sim H$ and $D \sim H$), l made an indirect measurement of the energy flux $E = \sum \frac{1}{2} m_s v^2 = \frac{1}{2} \rho S v^2 = \frac{1}{2} \rho \pi r^2 (2\pi f R)^2 \propto D^2 f^2 d^2$ along the coiling stream versus heights (**Fig[4]**).
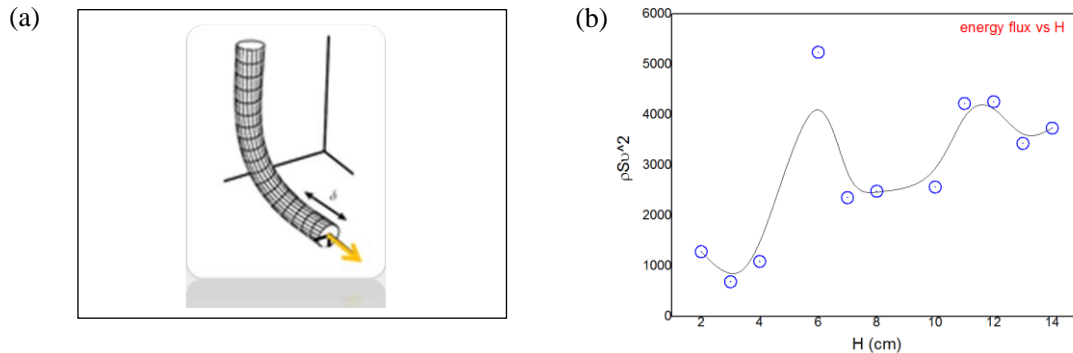
Figure 4

(a) The energy flow along the coiling stream, designated as yellow arrow in the left figure.(b) The energy flux calculated as $\rho S \upsilon^2$ versus **H.**

The result shows that the trend is complicated and a nonlinear curve was fit to the discrete data. It's understandable that the deviations of data points may be due to the coupling between gravity and viscous force. (It will be a straight line if only gravity dominant along the whole transportation process as $\upsilon^2 = 2g\boldsymbol{H}$)

3. **Theoretical Model of Rheological Transition**

This part focuses on explaining the change of coiling configuration in different range of falling heights by raising a theoretical model. In the case of silicone oil with viscosity 10000Cst, as shown in (**Fig[3]**), whatever in the $\Omega \sim \boldsymbol{H}$, $\boldsymbol{d} \sim \boldsymbol{H}$ or $\boldsymbol{D} \sim \boldsymbol{H}$ measurements, there is an obvious discontinuity in increasing(decreasing) rate. Approximately, 3 regimes of flow can be divided with two critical heights 6cm and 12cm. From the graph, the first regime is correspondent to a relatively slow increase of Ω with **H,** and a quite fast increasing in the 3rd regime. The middle regime is somehow strange especially in $\boldsymbol{d} \sim \boldsymbol{H}$ and $\boldsymbol{D} \sim \boldsymbol{H}$ correlations. Hence, we try to understand this transition on the view point of forces balance of steady coiling.

Useful information from the coiling frequency measurement reminds us that before 6cm, the coiling frequency is with the magnitude of 10HZ, while after 12cm, the order of magnitude is already 100HZ. This is a great leap actually. So the asymptotic approach is based on this experimental observation. In a rotating frame with angular velocity ω, steady coiling yields a balance equation between centrifugal force, viscous force and gravity force.

$$F_{inertia} + F_\upsilon + G = 0 \tag{2}$$

Where the magnitudes of each force are $F_{inertia} = \dfrac{\rho S \upsilon^2}{R} \sim \dfrac{\rho a^2 \upsilon^2}{R}$, $F_V \sim \dfrac{\rho v \upsilon a^4}{R^4}$ and $G = \rho S g \sim \rho a^2 g$

(Blount et al. 2010). Our goal is to reach the dependence of Ω on parameters like ρ, v, a ($\boldsymbol{d}$) ,R(**D**) etc. So equation 1 is necessary.

In the first regime, it's confident to state $\dfrac{\upsilon^2}{R} \ll g$, such that $F_{inertia} = o\ (G)$. Omitting gravity makes (Equation 2) becomes

$$G \cong F_V,\ \rho a^2 g \cong \dfrac{\rho v \upsilon^4}{R^4} \tag{3}$$

Combining $\upsilon = R\Omega$, $Q = \pi a^2 \upsilon$, we reach

$$f = \dfrac{\Omega}{2\pi} \cong \left(\dfrac{Q^3 g}{va(H)^8} \right)^{\frac{1}{4}} \tag{4}$$

In the third regime, we can do the same thing except stating $\dfrac{\upsilon^2}{R} \gg g$, under exactly the same procedures we gain

$$f = \frac{\Omega}{2\pi} \cong \left(\frac{Q^4}{\nu a(H)^{10}}\right)^{\frac{1}{3}} \tag{5}$$

Till now, we have reached quantitative expression of coiling frequency as a function of viscosity. Flux and coiling inner radius in the first and third rheological regimes.

4. Discussion and Validation of Model

In the experiment, the coiling frequency as a function of falling heights is given, in order to compare it to theoretical prediction, we make approximation based on experimental fact that inner radius a negatively depends on height H as indicated in **Fig[3b]**. We propose

$$a \propto \frac{1}{H} \tag{6}$$

In this approximation, in the first regime, expression (4) becomes $f = \dfrac{\Omega}{2\pi} \cong \left(\dfrac{Q^3 g}{\nu}\right)^{\frac{1}{4}} \cdot H^2$, the increasing rate is characterized by power 2.

In the third regime $f = \dfrac{\Omega}{2\pi} \cong \left(\dfrac{Q^4}{\nu}\right)^{\frac{1}{3}} \cdot H^{3.3}$, the rate is characterized by power 3.3.

In sum, the model implies that coiling frequency will increase as height increases and it will increase faster in the 3rd regime compared to its increasing speed in first regime. Furthermore, this trend is also observed as experimental results shown in **Fig[3b].**

Overall, theory is consistent with experiments as they show same physical features of rope coiling.it raised the mechanism of rheological transition, let's say from gravity regime to viscosity regime*(Ribe et al. 2008)*. Also, the viscosity term both appears in dominator in (Equation 4,5) which predicts a negative dependence of Ω on it. And this behavior is confirmed by the experimental results shown in **Fig[3a].** The physics picture of change of coiling configurations is clear at this stage.

At last, the configurations are visually presented in **Fig[5].**

(a)	**(b)**	**(c)**

Figure 5

(a)Coiling at 4cm.(b) Coiling at 6cm(c)Coiling at 12cm.

4. Conclusion

In this research, I mainly propound an experimental study of the dynamics of liquid coiling phenomenon of viscous liquid which is considered as Newtonian fluid. In theory, I propose a model based on an asymptotic approach which simplifies the forces coupling. The model agrees with the experiment results for they give same trend of the coiling frequency changes as a function of height. In concrete, frequency is a nonlinear increasing function of falling height, and the increasing rate becomes larger and larger as heights go up. Quantitatively, the

first regime height below 6cm, the increasing rate is characterized by $f \propto H^2$ while in the

third regime height above 12cm, it is characterized by $f \propto H^{3.3}$. In short, the change of

configurations in coiling is quantitatively studied by frequency, radius versus heights plot and identified as a typical rheological transition explained theoretically.

We also studied the energy transportation of fluid in the downward flowing experimentally for the first time in this field. The experiment result shows that the process is complicated and requires further investigation in the future. Further, the non-Newtonian fluid may be a new direction in the study of this field since the current study mainly focus on classical Newtonian fluid.

5. Reference

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