姓名＿＿＿＿ 学号＿＿＿＿ 年级＿＿＿＿ 专业、班＿＿＿＿ 学院＿＿＿＿

公平竞争 诚实守信 严肃考纪 拒绝作弊

密 封 线

重庆大学《算法分析与设计》课程试卷

◉ A卷 ○ B卷

2013 — 2014 学年 第 1 学期

开课学院：计算机学院 课程号：＿＿＿＿ 考试日期：2013.12.26

考试方式：○ 开卷 ◉ 闭卷 ○ 其他 考试时间：120 分钟

题 号	一	二	三	四	五	六	七	八	九	十	总 分
得 分											

考试提示

1.严禁随身携带通讯工具等电子设备参加考试；

2.考试作弊，留校察看，毕业当年不授学位；请人代考、替他人考试、两次及以上作弊等，属严重作弊，开除学籍。

一、（10 分）LANDAU 记号

Let $f(n)$ and $g(n)$be asymptotically nonnegative functions. Prove that

(1) $O(f(n))+O(g(n)) = O(f(n)+g(n))$ (5 分)

(2) $\max\{f(n), g(n)\} = \Theta(f(n)+g(n))$ (5 分)

二、 （20 分）快速排序

(1) What is the best time(最好时间) complexity to sort an array with quicksort? Describe such a situation briefly(简短叙述). (3 分)

(2) What is the worst time(最坏时间) complexity to sort the above array with quicksort? Describe such a situation briefly. (3 分)

(3) During each iteration, if we assume that the initial array is split(划分) according to the ration 1:9, then A) write the recurrence(递推函数) of this situation (3 分), B) draw out the corresponding recursion tree (3 分), and C) prove the tight bound (Θ) of your recurrence with substitution(替代法) (8 分).

三、 （20 分）动态规划（0-1 背包问题）

Given N objects and a "knapsack". Assume each object i has weighs $w_i > 0$ kilograms and value $v_i > 0$; Knapsack has capacity of W kilograms. Goal: fill knapsack so as to maximize total value.

(1) Define $OPT(i, w)$ = maximum value for choosing objects $1,\ldots,i$ with weight limit w. Write the recursive formula(递推方程式) to compute $OPT(i, w)$. (8 分)

(2) Suppose W=11, w_1=1, w_2=5, w_3=2, w_4=6 and w_5=7, together with v_1=1, v_2=18, v_3=6, v_4=22 and v_5=28. Draw the **table solution** for $OPT(5, 11)$. (12 分)

四、 （10 分）贪心算法（简化 0-1 背包问题）

Given N objects and a knapsack with capacity W kilograms. Assume all objects have different weights but an identical value(价值一样). Then, the $OPT(N, W)$ can be worked out more easily through greedy choices. Describe the algorithm briefly (5 分) and prove each choice satisfying greedy property (5 分).

五、 （20 分）动态规划（广告牌设置问题）

Suppose you are managing the construction of billboards(广告牌) on a street. The possible n locations for setting billboards(设置广告牌的位置) on the street are given in order as $0\leq x_1<x_2<\ldots<x_n\leq M$ (measured in miles). If you place a billboard at location x_i, you receive a revenue(收益) of $r_i > 0$. Regulations require that no two billboards be within less than or equal to 5 miles of each other(广告牌间距须大于5英里) .The goal is to place billboards at a subset of locations so as to maximize the total revenue. For example, Suppose M = 20, n = 4, $<x_1,x_2,x_3,x_4>$ = $<6,7,12,14>$ and $<r_1,r_2,r_3,r_4>=<5,6,5,1>$. Then the optimal solution would be to place billboards at x_1 and x_3, for a total revenue of 10.

(1) Design a dynamic programming algorithm to solve this problem, write down your brief idea and the recursive formula. (10 分)

(2) Write a program in pseudo code or other popular programming language based on above algorithm. (7 分)

命题人：陈波 组题人：罗辛 审题人：李佳 命题时间：2013.12.20 教务处制

(3) Analyze the computational complexity of your algorithm. (3 分)

六、 （20 分）最大流

Let G=(V, E) be a flow network and $|f|$ be the value of a flow f on G, i.e., $|f| = f(\mathrm{s}, \mathrm{V})$.

(1) Assume $\mathrm{S}, \mathrm{T} \subseteq \mathrm{V}$ be a **cut**. Then prove $|f| = f(\mathrm{S}, \mathrm{T})$. (5 分)

(2) Work out the maximum flow of the following flow network, where the positive integers denote the capacities of each edge respectively. During each iteration, you should draw the residue network(剩余流量图) and find out an augmenting path (增广路径) (if exists). (15 分)